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DEFYING GRAVITY:
HOW LONG WILL JAPANESE GOVERNMENT BOND PRICES REMAIN HIGH?

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Working Paper 18287
<http://www.nber.org/papers/w18287>

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
August 2012

The research started when both authors were visiting researchers at the Research Department of the IMF in the summer of 2011. We thank Olivier Blanchard and the research department for financial support and comments on earlier drafts as well as for hospitality during our stay. We also benefited from the seminar participants at the IMF, University of California at San Diego (IR/PS), Columbia University (Center for Japanese Economy and Business), Indiana University (Economics), the University of Chicago Booth School of Business, Japan Economic Association annual meeting, and NBER Japan Project meeting. We are especially grateful to comments by Olivier Blanchard, John Cochrane, Takero Doi, Mitsuhiro Fukao, Robert Hodrick, Kenneth Kang, Anil Kashyap, Eric Leeper, Phillip Lipsy, Hugh Patrick, Eric Rasmussen, Kiichi Tokuoka, Masaya Sakuragawa and David Weinstein. We thank Cathleen Cimino for editorial assistance. Remaining errors are our own.

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Hoshi is an author of the reports "Why Did Japan Stop Growing?" and "Policy Options for Japan's Revival" that are commissioned by National Institute of Research Advancement (NIRA). Hoshi is also an independent director of UnionBanCal Corporation, a wholly owned subsidiary of Bank of Tokyo Mitsubishi UFJ. Ito has at various times in the last five years made compensated presentations at meetings hosted by various financial institutions. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 18287

August 2012, Revised August 2012

JEL No. E62,H63,H68,J11,O47,O53

ABSTRACT

Recent academic papers have shown that the Japanese sovereign debt situation is not sustainable. The puzzle is that the bond rate has remained low and stable. Some suggest that the low yield can be explained by domestic residents' willingness to hold Japanese government bonds (JGBs) despite its low return, and that as long as domestic residents remain home-biased, the JGBs are sustainable. About 95% of JGBs are currently owned by domestic residents. This paper argues that even with such dominance of domestic investors, if the amount of government debt breaches the ceiling imposed by the domestic private sector financial assets, the JGB rates can rapidly rise and the Japanese government can face difficulty rolling over the existing debt. A simulation is conducted on future paths of household saving and fiscal situations to show that the ceiling would be breached in the next 10 years or so without a drastic fiscal consolidation. This paper also shows that the government debt can be kept under the ceiling with sufficiently large tax increases. The JGB yields can rise even before the ceiling is hit, if the expectation of such drastic fiscal consolidation disappears. This paper points out several possible triggers for such a change in expectation. However, downgrading of JGBs by credit rating agencies is not likely to be a trigger, since past downgrades have not produced any change in the JGB yield. If and when the JGB rates rapidly rise, the Japanese financial institutions that hold a large amount of JGBs will sustain losses and the economy will suffer from fiscal austerity, financial instability, and inflation.

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1. Introduction

It is well-known that Japan has the highest debt to GDP ratio among OECD countries. Many academic papers written in the last decade have concluded that the current Japanese government deficits and debts are not sustainable. But anyone who had thought the sovereign crisis would come to Japan has been proven wrong so far.¹ In some occasions, the yield on the 10-year JGB jumped by 100 basis points within a few months (1998 and 2003), but it soon came back down to a low level. Downgrades by credit rating companies in the last thirteen years did not bring down Japanese bond prices. Even the ongoing sovereign debt crisis in Europe has not rattled the JGB yields. If anything, the turmoil outside Japan has lowered the JGB yields even further. Japanese government bond prices appear to defy the law of gravity.

Almost all recent papers on Japanese government debt reach the same conclusion: the current course of fiscal debt dynamics is not sustainable. For example, Doi (2009), Doi, Hoshi, and Okimoto (2011), Doi and Ihuri (2009), Sakuragawa and Hosono (2011), Ito (2011), Ito, Watanabe, and Yabu (2011), and Ostry et al. (2010) all find that without a drastic change in fiscal policy, the Japanese government debt to GDP ratio cannot be stabilized. Imrohoroğlu and Sudo (2011) find that an unlikely high jump in productivity growth would be necessary to stabilize the debt to GDP ratio without changing fiscal policy. These results are a stark contrast to those of Broda and Weinstein (2005), which conclude that a reasonable increase in the tax rate can stabilize the debt to GDP ratio, using the data available in 2003. This suggests the situation has deteriorated substantially in less than 10 years.

The fiscal problem of Japan has been highlighted by the IMF as well. For example, IMF (2011) reports “stabilizing the net debt ratio by 2016 and reducing it to around 135% of GDP by 2020 would require a reduction of the primary fiscal deficit by 10 percent of GDP over a 10 year horizon” (IMF 2011, p.11). Without such substantial adjustment, the net debt to GDP ratio is predicted to grow without bound and to exceed 200% by 2023.

In many papers, Japan’s fiscal policy is regarded as sustainable if the debt to GDP ratio is expected to come back down to where it is now in some distant future. No attention is paid however, to how high the debt to GDP ratio goes before it starts to fall. This paper differs from many previous analyses, in considering an explicit ceiling that the government debt should not exceed at any point of time. Such a ceiling is given by the amount of financial assets held by the domestic private sector. We show that even under an extreme assumption that all the private sector savings from now on are invested in government debt, the debt eventually will exceed the amount of private sector financial assets.² When the market believes the point when the debt

¹ We do not claim a credit for the analogy of Japanese debts to defying gravity. From serious press to financial newsletters, descriptions like “Mt. Fuji” of Japanese government bonds and government bonds “defy gravity” have been very popular.

² IMF (2009), Tokuoka (2010), and Oguro and Kobayashi (2011) all discuss the ceiling of private sector savings and provide simple calculations. This paper provides refinements over these earlier attempts by considering the dynamics carefully under several alternative scenarios.

exceeds the domestic private sector financial assets is close, the yield will have to rise. This paper also points out such a change can be sudden and unpredictable, and describes possible scenarios of the economic impacts of such a debt crisis.

The rest of the paper is organized as follows. Section 2 starts by reviewing the various definitions of Japanese government debt and clarifying our preferred definition used in this paper. Section 3 describes and explains the reasons for the persisting low and stable JGB rates. Section 4 simulates the expected paths for government debt and private sector financial assets under the assumption that the fiscal policy stance of the Japanese government does not change in the future. We find that the amount of government debt is expected to exceed the private sector financial assets in the next 10 years. The simulations depend on several important assumptions. Section 5 discusses how the results are sensitive to those assumptions. Section 6 starts out by identifying an alternative future tax path that would keep the future government debt below the private sector financial assets. Thus, if the market believes that Japan can still embark on such fiscal consolidation in the not so distant future, the low JGB yields are understandable. If and when the expectation changes, however, a fiscal crisis can be triggered even before the government debt hits the ceiling of the private sector financial assets. Section 6 discusses what would trigger such a change in expectation. Section 7 examines the impacts of financing reconstruction costs of the earthquake/tsunami damages on the possibility of a debt crisis. Section 8 considers the likely consequences of such a debt crisis. Section 9 concludes by summarizing the paper's findings.

2. Defining Japanese Government Debt

When one discusses the amount of Japanese government debt, it is useful to clarify the assumed definition because there are several different definitions of Japanese government debt, and depending on which definition is used, the amount can be very different. Table 1 shows the three definitions of Japanese government debt that are often used and how they differ.

<Table 1> about here

The first column shows the definition of Japanese government debt as reported to the IMF. This definition captures the liabilities incurred by the central government. It includes the total outstanding of JGBs including FILP (Fiscal Investment and Loan Program) bonds, other long-term borrowings, Financial Bills, and (explicit) government guarantees. As of the end of fiscal year 2010, the amount of JGBs including FILP bonds amounted to about 750 trillion yen (about 150% of GDP).

FILP bonds are the JGBs issued to finance some government agencies covered by the FILP, such as the Japan Finance Corporation, the Urban Renaissance Agency and others. Local governments are also major recipients of the FILP funds. Before the FILP reform of 2001, the

FILP was financed by the Trust Funds Bureau of the Ministry of Finance that collected funds from the postal savings and the national pension funds in the form of deposits. After the reform of 2001, the postal savings and national pension funds were no longer required to deposit their funds to the Ministry of Finance. The agencies in the FILP are encouraged to raise the funds in the market by issuing FILP Agency Bonds, but those that have trouble raising sufficient funds in the market can rely on the Fiscal Loan Funds (created by reorganizing the Trust Fund Bureau), which are funded by FILP bonds (Zaito bonds). The distinction between the regular JGBs and the FILP-JGBs is purely accounting.³

In addition to the JGBs and FILP bonds, the central government has long-term borrowings of about 60 trillion yen and short-term Financing Bills (FBs) of about 110 trillion yen. Financing Bills (FBs) are issued primarily to fund foreign reserves that are held in a special account of the government. They are rolled over every 3 months and have offsetting entries on the asset side of the balance sheet (foreign reserves).

The central government also guarantees bonds issued by some government agencies such as the Deposit Insurance Corporation, the Development Bank of Japan, and Japan Highway Corporation. The total guaranteed liabilities amount to about 45 trillion yen. In total, the government debt according to this definition was about 970 trillion yen as of the end of fiscal 2010 (March 2011).

The second column of Table 1 shows another definition of Japanese government debt, which covers both central and local governments. Compared with the definition in the first column, this definition adds bonds issued by local governments, but excludes the FILP bonds, Financial Bills (FBs), and government guarantees. According to this definition, Japanese government debt was forecasted to be about 890 trillion yen as of the end of fiscal 2011.

The third column of the table shows the definition used in national income accounting. Compared with the second column, this definition adds FBs as well as the government guarantees back into the debt calculation, but excludes FILP bonds (Zaito bonds). Moreover, this definition adds the liabilities in the social security funds, which are included in the general government sector in the national income accounting. Altogether, the Japanese government debt at the end of fiscal 2009 was 1,023 trillion yen (214% of GDP) according to this definition.

In this paper, we adopt the definition in the third column of the table – the national income accounting definition. Thus, our definition excludes the FILP bonds. To the extent that the government has contingent liabilities to support those government agencies financed by FILP bonds, we are underestimating the degree of government indebtedness.

³ For more details on the FILP, see MOF (2010). The FILP bonds are physically indistinguishable from JGBs, but they are excluded from some definitions of government liabilities because they are supposed to be redeemed by future revenues of the FILP agencies rather than tax revenues. The idea that the taxpayers would not be called on to pay for the losses in FILP agencies is questionable at best as Doi and Hoshi (2003) points out.

The national income accounting definition is also the one used by the OECD. Figure 1 shows the debt to GDP ratios for G7 countries reported by the OECD. The figure shows that Japan has moved from around the middle to the highest in debt to GDP ratio among the G7 countries during the last two decades. OECD also reports the net debt number, which subtracts the financial assets that the government owns from the gross debt number. Figure 2 shows the net debt to GDP ratios for the G7 countries and also tells the same story. Japan had one of the lowest net debt to GDP ratios in the beginning of the 1990s, but Japan's net debt ratio climbed to the highest by the end of the 2000s.

<Figure 1 > about here

<Figure 2> about here

In comparing the government debt to the amount of private sector financial assets, it is sensible to exclude government debt that is held by the government itself from the definition of debt that we use. Thus, we use the concept of adjusted net debt as advocated by Doi (2008). The adjusted net debt is defined as the gross debt of the government sector in the national income accounting minus the government sector financial assets that are considered to be readily disposable. Thus, different from the standard definition of the net debt, some financial assets such as the fiscal adjustment funds of local governments are not subtracted. These financial assets are held as a buffer for unexpected losses and not expected to be used to redeem the government debt.

3. Why is the JGB yield so low?

Economic research has been accumulating overwhelming evidence against the fiscal sustainability of Japan. Many international financial institutions, credit rating agencies, and private-sector analysts agree over this assessment. Yet, the JGB interest rate has been low and stable. The 10-year JGB rate has been below 2% since 1999, and between 0.8% and 1.5% in the last few years. The rate is much lower than the bond rate of other advanced countries. This is despite the fact that Japan has a higher debt to GDP ratio than the European countries that have suffered from sovereign debt crises in the last two years—Greece, Ireland, Portugal, Spain, and Italy. JGB yield actually fell as the Japanese debt to GDP ratio increased in the 1990s and 2000s as Figure 3 shows. Why has the JGB yield not risen?

<Figure 3 > about here

Several factors are identified to have contributed to the low and stable JGB yield (see Ito (2011), for example). This paper focuses on the oft-stated explanation that the JGB yields are low because there are high private savings in Japan that continue to be invested into government

bonds. Thus, one does not have to worry about increasing bond yields as long as the massive amount of private savings is there.

Indeed most of Japanese government debt is held by Japanese residents. Table 2 shows the distribution of JGB ownership by type of investor from 2005 to 2010. Looking at the column for 2010, the largest share of 39% is owned by Japanese commercial banks, including the Japan Post Bank. The insurance companies hold about 20% of the total and other private-sector financial institutions own about 10%. The government social security fund (Government Pension Insurance Fund or GPIF) also owns about 10%. The Bank of Japan holds about 8%. An additional 5% is held by the households directly, and about 3% is held by other domestic investors. Thus, in total more than 95% of JGBs are owned by domestic residents.

<Table 2 > about here

The large amount of private savings does not necessarily imply that it has to be used to purchase low yielding domestic government bonds. The savings could chase higher returns and move abroad on a massive scale. Indeed this happened to some extent as Japan accumulated foreign assets over time. However, a large proportion of the savings stayed domestic, despite the persistent difference of returns in recent years. The proportion of yen-denominated domestic assets for Japan is extremely high as compared to investors in other advanced countries.

What is important here is the home bias of the institutional investors. As Table 2 shows, the direct holdings of government bonds by households and non-financial corporations are not that large. Private savings are mostly deposited into the banks, and the banks buy Japanese government bonds.

Banks find the JGBs attractive because the investment does not involve currency risk, which has been historically high for foreign bonds. The capital adequacy requirements (Basle I, II, and III) also make JGBs desirable for banks: JGBs (and sovereign debts of advanced countries) are assigned zero weights in calculating the risk-weighted assets, either by regulation or by internal models, that determine the minimum amount of capital banks must hold. Pension funds and insurance companies seem to also be content with holding a large amount of long-term JGBs because their liabilities are also in the yen.

The stagnation of the Japanese economy also makes JGBs attractive to banks. The returns from alternative investments such as corporate loans have been quite low. The sustained near-zero interest rate policy of the Bank of Japan was another reason for low rates of return in general. Finally, continued deflation means that the real yields of JGBs for Japanese consumers have been higher than the nominal yields.

The regression analysis by Tokuoka (2010) finds that the low yields of JGBs may be indeed related to its ownership predominantly by Japanese residents. Table 3 shows a representative regression result. The results show that high household and corporate net savings

are associated with low JGB yield. The regression results also show that high foreign ownership of the JGB is associated with high bond yield. One percentage point increase of foreign ownership of JGBs pushes up the yield by 11 basis points. When these two factors are controlled for, a standard negative relation between the debt to GDP ratio and the government bond yield re-emerges for Japan. The point estimate suggests that the bond yield rises by 2 basis points for each one percentage point increase in the debt to GDP ratio.

<Table 3 > about here

4. Sustainability calculation

If the growth of private savings decelerates and government debt continues to increase, the amount of government debt will eventually exceed the amount of private savings. At that point, even if all the private sector financial assets are invested in the JGBs, leaving nothing for private sector credit, at least some JGBs must be held by foreign investors. As soon as the market sees that the current course definitely leads to such a situation, the government will have trouble selling new JGBs at low interest rates. In this paper, we call such a situation a “crisis.” In a crisis, new JGBs cannot be sold at low interest rates and the interest rate would rise.

There is good reason to believe that household saving will decline, which will slow down the growth of the private sector financial assets. The baby boomer generation will retire in the next ten years and they will start consuming out of their financial assets. Further, the working-age population is expected to decline by 8% over the next ten years.

To get an idea about when the government debt is expected to catch up with the private savings in the absence of fiscal reform, we carry out the following calculation. The government debt is assumed to follow the following dynamics:

$$b_{t+1} = \frac{1+r_t}{1+\eta_t} b_t + g_t - \tau_t \quad (1)$$

where b_t is the government debt to GDP ratio at the beginning of period t , r_t is the real interest rate, η_t is the real GDP growth rate, g_t is the government expenditures including transfers divided by GDP in period t , and τ_t is the tax rate (relative to GDP). For the initial value of the debt to GDP ratio, we use 153%, which is the amount of adjusted net debt calculated by Doi, Hoshi, and Okimoto (2011). The future government expenditure series also comes from Doi, Hoshi, and Okimoto (2011). The series is based on the 2008 estimates of healthcare and long-term care expenditures by the National Congress on Social Security and the 2009 estimates of social security related expenditures by the Ministry of Health, Labor, and Welfare and assumes no drastic future reform. The sum of total tax revenues and social security contribution is assumed to stay at 30% of GDP, the approximate level for fiscal 2010.

We consider three alternative assumptions for the interest rate.

R1: Interest rate is equal to the larger of the growth rate (η_t) and the level in 2010 (1.3%).

R2: Interest rate rises by 2 basis points for every one percentage point that the debt to GDP ratio at the beginning of the period exceeds the 2010 level (153%) ($r_t = 1.3\% + 0.02*(b_t - 1.53)$).

R3: Interest rate rises by 3.5 basis points for every one percentage point that the debt to GDP ratio at the beginning of the period exceeds the 2010 level (153%) ($r_t = 1.3\% + 0.035*(b_t - 1.53)$).

R1 is motivated by the fact that the average yield on 10 year JGBs over the last several years has been about the same as the GDP growth rate, but constrains the interest rate not to be lower than the current rate even when the GDP growth rate declines further. R2 and R3 assume that the interest rate rises as the government accumulates more debt. Many empirical studies have demonstrated such a relation. R2 (2.0 basis points increase) uses the finding of Tokuoaka (2010) for Japan. R3 (3.5 basis points increase) assumes the coefficient estimate used by Gagnon (2010) and is the median estimate from studies of various advanced economies. In both cases, the interest rate is assumed to respond linearly to increases in the debt to GDP ratio. This is a conservative assumption in the light of some evidence that suggests the interest rate increases at a higher rate once the debt to GDP ratio exceeds certain a threshold, as Ardagna, Caselli, and Lane (2004) find.

The debt calculated by (1) is compared to the amount of domestic private financial assets that can be potentially used to finance government debt. As the measure of such domestic private savings, we consider:

Net financial assets of the household sector – Value of shares and other equities held by the household sector + Cash, deposits, government bonds, and public corporation bonds held by the private nonfinancial sector

The private savings thus defined was 261.3% of GDP at the end of fiscal year 2010.⁴

Starting from this initial value of the private financial assets, we assume the future private financial assets will evolve according to the following equation.

$$a_{t+1} = \frac{1+r_t}{1+\eta_t} b_t + \max(a_t - b_t, 0) + s_t \quad (2)$$

where a_t is the private financial assets to GDP ratio at the beginning of time t , s_t is the (flow) saving divided by GDP in year t . The dynamics assume that all the government debt is held by the private sector and that the portion of the private financial assets grows at the rate of the

⁴ The data on the financial assets are taken from the Bank of Japan Flow of Funds Data.

interest on the government debt. The rest of the private financial assets are assumed to grow at the rate of GDP growth.

The aggregate saving rate is a function of the demography. Appendix 1 describes how we estimate the aggregate saving rate from 2010 to 2050. The result is shown in Figure 4. The saving rate starts out above 3% in 2010, but quickly goes below 2% by 2017. It then holds steady and start to decline again in the 2030s, falling almost to -3% by the end of the 2040s.

<Figure 4> about here

The upper-bound for the debt to GDP ratio is defined as the level when the new issue of government bonds exceeds the total (flow) saving of that year and the amount of the private sector financial assets that are not in the form of the government debt yet. Thus, in order to avoid the upper bound, the debt must satisfy the following constraint.

$$B_t - B_{t-1} \leq S_{t-1} + (A_{t-1} - B_{t-1}) \quad (3)$$

Or rewriting this in terms of the ratios to GDP,

$$b_t \leq \frac{s_{t-1} + a_{t-1}}{1 + \eta_{t-1}} \quad (4)$$

We consider several different future growth rates. We start by a simple, but unrealistically optimistic assumption that Japan's GDP will grow at 2% annually for the next 40 years. This is an assumption often used by the Japanese government for future economic projections. Figure 5 shows the path of the debt to GDP ratio under the alternative scenarios on the interest rate. The series DebtX (X=1, 2, 3) is the path of the debt to GDP ratio under the interest rate scenario RX. The figure also shows the right hand side of the constraint (4) as MaxDebtX (X=1, 2, 3). For every scenario on the interest rate, the figure shows that the government debt exceeds MaxDebt in 2023 at the latest.

<Figure 5> about here

The assumption of 2% real GDP growth indefinitely is probably too optimistic. The assumption ignores the tremendous shift in demography, namely the shrinking working-age (age 20-65) population. A more reasonable assumption may be that GDP per working-age population rather than GDP itself will grow at a constant rate.

Note that:

$$rGDP = POP * \frac{wPOP}{POP} * \frac{rGDP}{wPOP}, \quad (5)$$

where $rGDP$ is the (level) of real Gross Domestic Product, POP is total population, and $wPOP$ is the working-age population. In terms of the growth rate, we have:

$$\Delta rGDP = \Delta POP + \Delta \frac{wPOP}{POP} + \Delta \frac{rGDP}{wPOP}, \quad (6)$$

where Δ is the growth rate operator, $\Delta x = \{x(t)-x(t-1)\}/x(t-1)$. Thus, the real GDP growth rate, $\Delta rGDP$, is the sum of the population growth rate, ΔPOP , the growth rate of the ratio of working-age population to population, $\Delta(wPOP/POP)$, and the growth rate of GDP per working-age person, $\Delta(rGDP/wPOP)$. The last term, the growth rate of GDP per working-age person, can be roughly regarded as the rate of labor productivity growth. Thus, we refer to this term as the labor productivity growth for short.⁵ Table 4 demonstrates the demographic growth decomposition for the years 1955 to 2010.

<Table 4 > about here

During the rapid growth period from 1955 to 1970, the average annual growth rate was 9.7%. This is the left-hand side of equation (6). The right hand side of equation (6) breaks down the growth rate of GDP into several contributing factors; population growth (1%), increase in the proportion of working-age population (1.0%), and growth rate of GDP per working-age person (7.7%). The boost in the overall growth rate by the population factor is called demographic dividend.⁶ The overall growth rate fell over time as all of these sources of growth declined. The growth rate of GDP per working-age person, however, seems to have stabilized after the 1990s.

Future projection of demographic growth decomposition requires the expected future demographic changes as inputs. We take the (mid-point) future projection of the total population and working-age population from the National Institute of Population and Social Security Research (IPSS).⁷ This produces the first two terms of the right-hand side of the equation (6).

Given these expected demographic changes, how much growth of GDP per working-age person is necessary to maintain the GDP growth of 2%? Table 5 gives the answer. The 2% real economic growth implies that the growth rate of GDP per working-age person must be at around 3% in the next twenty years (2011-2030) and at 3.5% for the following twenty years (2031-2050). The productivity growth rate of 3% and above has not been observed for Japan since the 1980s. It seems implausible that the Japanese economy can repeat the miraculous growth of the 1970s and 1980s in the next 40 years.

⁵ The number of actual workers who engage in production is the working-age population times the labor participation rate, plus anyone who may be working in the retired age group (age 65 and over). The decline in working-age population can be alleviated if the participation rate increases, or more elderly participate in the labor market. This paper ignores these possibilities.

⁶ For demographic dividend in general, see Bloom, Canning, and Sevilla (2003), and for application of demographic dividend to Japan, see Komine and Kabe (2009).

⁷ For the IPSS population forecasts, see <http://www.ipss.go.jp/pp-newest/e/ppfj02/top.html>.

<Table 5 > about here

Thus, a more reasonable approach is to assume the growth rate of GDP per working-age person to be similar to that of the 1990s and the 2000s. We consider two alternative growth rates of GDP per working-age population. The low growth scenario assumes 1.05% (average of 1994-2010) and the high growth scenario assumes 2.09% (average of 2001-2007).⁸ Table 6 shows the growth decomposition under the assumption of 1.05% growth rate of GDP per-working-age person.

<Table 6> about here

The result for the debt sustainability calculation under the low growth scenario is reported in Figure 6 and is very similar to the one in Figure 5. The upper bound for the debt accumulation is reached by 2024 at the latest.

<Figure 6> about here

Table 7 shows the growth decomposition under the high growth scenario. The growth rate will be close to 1% for the next twenty years until 2030, then falls to 0.6% in the 2030s and the 2040s. The result of the dynamics for government debt and private sector saving shown in Figure 7, again is pretty much the same as the one in the previous figure. The Japanese government is expected to run out of room to sell more bonds domestically by 2024 at the latest.

<Table 7> about here

< Figure 7> about here

In the calculations for Figures 6 and 7, we use future GDP growth rates that are different from the government projection of 2% per year. The future government expenditure to GDP ratios, however are still based on the government projection of 2% GDP growth. This procedure is correct if the government expenditure falls at the same rate as GDP when the GDP growth rate falls below 2%. If government expenditure does not fall as much, our procedure results in underestimation of the future government expenditure to GDP ratio. To see the impact of this potential misspecification, we calculated the dynamics for government debt under an alternative extreme assumption: the level of government expenditure does not depend on the GDP growth rate, so that the government expenditure to GDP ratio at year t when the GDP growth rate is x ($x < 2\%$) is given by (the government estimates under 2% growth assumption) $\times (1.02) / (1+x)$. The result (not reported here) shows that the government debt can exceed the MaxDebt sooner, but not by more than one year.

⁸ The years 2001-2007 roughly correspond to the years when Junichiro Koizumi was the Prime Minister. Junichiro Koizumi became Prime Minister in April 2001. He resigned as Prime Minister in September 2006, succeeded by Mr. Shizo Abe, who was widely regarded as a protégé of Koizumi. Prime Minister Abe lasted only one year in office.

The results in Figures 5 through 7 suggest that the year when the Japanese debt would exceed the private financial assets does not depend very much on the growth rate, which is adversely affected by the aging of the Japanese population. This does not mean that the population dynamics do not matter for the sustainability of the fiscal situation. On the contrary, expected increases in social security related spending in the future as a result of aging, which is also embedded in all of the simulations above, is at the heart of the sustainability problem.

Appendix 2 provides a simple example to show the power of population dynamics that could make a previously solvent pay-as-you-go pension system unsustainable. It is a simple stylized example of the current Japanese situation, where the government finances, especially the social security system, have not adjusted to the reality of declining population. Appendix 3 extends the analysis by using a continuous time version of the overlapping generation model developed by Blanchard (1985).

5. Sensitivity to different assumptions

The above simulations are based on several assumptions. First, we assume that the outstanding balance of corporate savings will remain constant for the forecast horizon. Given that corporate saving has been positive (balance has been increasing) in recent years, this assumption may appear too conservative. If Japanese corporations continue to save and increase their deposits, then absorption of the JGB by domestic financial institutions may continue forever. However, high corporate saving may have been motivated by potential large investment in the near future. Direct investment and M&A activities abroad that draw down corporate savings may occur in the future. Indeed some corporations seem to have stepped up their efforts on overseas investment taking advantage of the appreciated yen. When any sign of JGB vulnerability appears, corporate savings are more likely to find alternative investment outlets other than household savings. Thus, assuming corporate savings to remain constant seems to be reasonable.

Second, our simulations assume that the private financial assets already outside Japan would not be called back to be invested in the JGBs. In other words, the Japanese investors that already invested substantially in foreign assets are assumed to be free from the home bias. In this sense, these Japanese investors would behave similarly to foreign investors. They would repatriate foreign assets to hold the JGBs only when the JGB interest rate rises significantly to match the yields of foreign assets. Thus, for our calculation of the ceiling of domestic financial assets, ignoring foreign assets owned by Japanese residents seems to be justifiable.

Third, for the new flow of household and corporate saving, we assume that all of these savings can be invested into the JGBs with no outflow to hold foreign assets. If this assumption does not hold, the government debt would catch up with the domestic private financial assets sooner.

What is important is the amount of private sector savings that exhibit substantial home bias. This reasoning casts a doubt on an oft-heard argument that a fiscal crisis cannot happen as

long as the current account is in surplus. The current account deficit is neither a necessary nor sufficient condition for a fiscal crisis. Government debt may become unsustainable even when the current account is still in surplus if domestic savers refuse to purchase JGBs at a low rate and shift their portfolio to hold other domestic or foreign assets. On the contrary, a fiscal crisis may not happen even when the current account turns to a deficit if the current account deficit is a result of large capital inflow to Japan.

Finally, our analysis does not deal with the exchange rate explicitly. Exchange rate fluctuations that anticipate fiscal problems in the future can influence the fiscal crisis in several ways. On the one hand, a threat of fiscal crisis may lead to yen depreciation before a fully developed crisis. This may help export-oriented manufacturing firms in Japan and lead to an increase in government revenues. This may also postpone the fiscal crisis to a later date than our analysis suggests. On the other hand, the currency depreciation may be quicker and more violent, which was the case in many currency crises in emerging economies. Domestic investors may shift their assets abroad, intensifying the fiscal crisis.

6. Triggers of the Crisis

The simulations above suggest that Japanese government debt will soon exceed private sector financial assets. As soon as the market expects this, we would expect the interest rate on government debt to start increasing, but the JGB yields do not seem to behave as if a crisis is expected in the next 10 years or less. One explanation is that the market participants believe the government will implement a drastic fiscal reform to restore fiscal sustainability well before government debt exceeds the private sector financial assets. A source of such expectation may be the fact that the total tax burden to GDP ratio for Japan is still low at around 30% (including payment into the social security system).⁹ Japan may be able to eliminate fiscal deficits if it increases the tax burden to a level that is comparable to many European countries. For example, the deficit of the central government was 44 trillion yen in fiscal year 2011. Using the widely used estimate that a marginal 1% increase in the consumption tax rate brings in additional revenue of 0.5% of GDP (about 2.5 trillion yen), an 18% increase in the consumption tax rate would increase the government revenue by 45 trillion yen.¹⁰ Thus, if the consumption tax rate is raised from the current 5% to 23%, the entire deficit can be eliminated. Most European countries have a VAT rate more than 15% and some as high as 25% (e.g. Sweden), so the 23% consumption tax may not be outrageous. Thus, at least in theory, Japan has huge room for increasing the consumption tax rate.

⁹ The tax burden ratio for fiscal 2011 is estimated to be 29.2% (http://www.mof.go.jp/budget/fiscal_condition/basic_data/201104/sy2302n.pdf).

¹⁰ To be precise, one percent out of five percent increase of consumption tax will be given to local governments, so that only 80% of the consumption tax increase is reserved for the central government tax revenue. However, if the central government reduces the transfers to the local government by the same amount of consumption tax revenue increase for local governments, the central government ends up receiving 100% of the consumption tax increase.

We can use the simple simulation model described above to see what kind of future tax rates would make the fiscal situation sustainable. We start from the expected tax rates for the next several years that are implied by the latest plan for the integrated reform of social security and tax systems, which is being debated in the Diet as of June 2012. One percentage point increase of the consumption tax rate is assumed to increase the tax revenue by 0.5% of GDP. Thus, the tax rate increase of 3% planned for April 2014 would raise the tax and social security contribution to GDP ratio (hereafter, tax burden ratio) in fiscal 2014 from the current 30% to 31.5%. For fiscal 2015, the planned mid-year increase of the consumption tax rate of 2% would increase the tax revenue by another 0.5% to 32% of GDP. The tax burden ratio for fiscal 2016 would be 32.5%.

For each assumption about the interest rate R1, R2, and R3, we find a path for the future tax burden ratio that starts with these and eventually brings down the debt to GDP ratio to 153% (the initial value in 2010). Such a path is not unique, but Figure 8 shows one possible path for each interest rate assumption that makes the debt process sustainable. Here the growth rate of GDP divided by working-age person is assumed to be 1.05% (the average for 1994-2010).

<Figure 8> about here

Under R1, the tax burden ratio increases from 32% in 2015 to 33% in 2016, and increases by 1% a year until it reaches 46% in 2029. The ratio stays at 46% until 2070 and then gradually falls to 43%. This tax burden ratio path brings the debt to GDP ratio down to 153.9% by 2100 as Figure 9 shows.

<Figure 9 > about here

Under R2 or R3, the 1% tax rate increase each year would take the tax rates to an unrealistically high level before the debt to GDP ratio starts falling. Looking at the tax and social security collection to GDP ratio across OECD countries, no country has a ratio above 50%. The highest rate observed ratio between 2003 and 2010 is 50.8% for Denmark in 2005.¹¹ Thus, we search for a tax burden ratio path that does not go over 50%. For R2, this implies that the tax burden ratio needs to increase by 3% a year from 2016 to 2020. The tax burden ratio reaches 50% in 2023 and stays there for 3 years. Then, the ratio can gradually fall to 47% by 2100. Figure 9 shows that the tax policy brings the debt to GDP ratio down to 153.5% by 2100.

The sustainable tax burden ratio under R3 must jump by 9% in 2016 and another 8% in 2017. The tax burden ratio reaches 50% as early as 2018 and stays there for 24 years until 2041. Eventually the tax rate falls gradually to 47% by 2100. The tax policy brings the debt to GDP ratio down to 153.4% by 2100 as Figure 14 shows.

Figure 10 provides another way to look at the debt dynamics under the sustainable policies. The figure shows the path of the ratio of government debt to the maximum amount of

¹¹ http://www.oecd-ilibrary.org/taxation/total-tax-revenue_20758510-table2

government debt that the private sector can absorb by using the current financial assets and the new saving (MaxDebt) for each interest rate scenario. For all interest rate scenarios, the ratio never exceeds 100%, suggesting that the amount of government debt continues to be below the private financial assets.

<Figure 10> about here

If the market participants expect future tax increases such as the ones in Figure 8, it is quite understandable why we have not yet observed high yields on JGBs. We know, however, that the market expectation can change quickly. If and when market participants are convinced that tax increases are unlikely to come, it can trigger a crisis.

One can consider several potential triggers for such a change in the expectation. First, a change in the expectation may be triggered by some events in the financial market. An example is a downgrade by credit rating agencies. Investors may revise their assessment of bonds and re-balance portfolios based on (unexpected) changes in credit rating. The past experience of downgrades in Japan, however, suggests that a downgrade is not likely to be an important trigger, as we discuss further below.

Second, a change in the expectation may be triggered by a political event. For example, if the bill for integrated reform of social security and tax systems fails to pass in the Diet, the public may conclude that the government is less likely to achieve fiscal consolidation in the future. Another example is a change in the policy of government financial institutions to buy JGBs. In 1998, some confusion over how many government bonds the Fiscal Investment and Loan Program (FILP) would buy led to a sudden rise of the JGB yield by more 100 basis points in less than three months.

Third, contagion from a foreign country that is experiencing a debt crisis may trigger a change in the expectation for Japan. For example, the debt crisis in Greece has spread to other highly indebted countries in the euro area, including Ireland, Portugal, Spain and Italy. One cannot rule out the possibility that such a contagion may influence countries outside the euro area, such as Japan.

Fourth, a change in the expected future interest rates may trigger a crisis. Such a change in the interest expectation may be a consequence of an unexpected monetary policy action. For example, high energy price inflation can prompt the Bank of Japan to tighten monetary policy by raising the interest rate. With a higher future interest rate path, the expected fiscal policy may not be sustainable anymore. On the contrary, if the Bank of Japan does not tighten when the energy prices rise, this may also raise future interest rates by raising the future inflation expectation. Thus, on both accounts a sustained rise in energy prices can put the Bank of Japan in a tough place.

An event in the summer of 2003 illustrates how a small rise in the JGB yield can be amplified quickly. The yield of 10-year JGB went up from 0.5% in June to 1.6% in September in 2003. A sudden price decline raised the VaR (value at risk) of the JGB, and many financial institutions sold JGBs to reduce the risk. This behavior, which was individually prudent, resulted in depressing the JGB price further.

As we argued previously, a JGB downgrade by a credit rating agency is a potential trigger of a crisis. Japan's past experience suggests, however, that the credit rating agencies are not likely to be a trigger. The JGB has already undergone a series of downgrades by Moody's, S&P and Fitch in the late 1990s to the beginning of 2000s, and again recently. Looking back at these experiences, we can judge the likelihood that credit rating agencies would pull the trigger for the Japanese sovereign debt crisis. The history of the above-mentioned changes in credit rating is summarized in Figure 11 and Table 8.

< Figure 11 and Table 8 > about here

If a downgrading of JGB influences the expectations of investors and makes them more reluctant to hold JGBs, we should expect the bond yield to rise, the yen to depreciate, and the stock prices to decline. In order to see whether these reactions have been observed at the time of downgrading in the past, market reactions in various event windows (from 1 to 150 days) are examined. Figure 12 shows such results. Panels 1 to 3 show the reaction of the JGB bond rate to S&P downgrades and negative watch announcements (Panel 1); to Moody's downgrade and negative watch announcements (Panel 2); and to Moody's down- and up-grades with opposite signs (Panel 3). The vertical axis is the interest rate change (percentage point changes), and the horizontal axis is the window of changes from day $t-k$ to t , or from t to $t+k$, where $k=1, 5, 25, 75, 150$. Since other news and events influence the bond yield, immediate changes ($k=+1$ and $+5$) are more appropriate in evaluating the impact of credit rating changes. The changes before the credit rating change are shown in order to check whether there were significant trends before the credit rating downgrade, either for rationally anticipating credit rating changes or for other reasons, which may carry over to the post-event days. In any panel, the changes in the bond yields around the event date are hardly positive. If anything, downgrades tend to lower the yields. Similarly, Panels 4 to 6 show the reactions of the yen/dollar rate to various categories (S&P, Moody's, and Moody's including upgrades) of credit rating changes. The (+) means yen depreciation and the unit is in percentage changes. These panels suggest that the yen/dollar rate has been unpredictable before or after the credit rate changes, but downgrades do tend to be followed by slight yen appreciation, if any. With respect to a stock price index, Panels 7 (S&P downgrades), 8 (Moody's downgrades) and 9 (Moody's including upgrades) suggest that the stock price index tended to fall both before and after the downgrades, as the changes are mostly below 0 in the vertical axis, which measures the percentage change in the stock price index. Downgrades tended to occur in the phase of negative market trend, but in sum, we do not find evidence that any of the expected reactions occurred when the JGB was downgraded.

The most dramatic downgrade event so far was the 2-notch downgrading by Moody's on May 31, 2002. The Ministry of Finance was quite upset and wrote an open letter to Moody's questioning the judgment. The market remained calm, however, sending the bond yield actually lower. Thus from such evidence, it seems safe to conclude that credit rating agencies are not likely actors capable of pulling a trigger for a crisis, at least for Japan.¹² This assessment has been confirmed by what followed after the downgrading by Moody's on August 24, 2011 and the most recent one by Fitch on May 22, 2012. The downgrades were more or less expected and the JGB yield did not change at all following the announcements.

<Figure 12> about here

Another possible trigger in the financial market may be a hike in the Credit Default Swap (CDS) spread. Figure 13 shows that the CDS spread for the JGB has been trending upward since late 2007. It peaked at around 100 basis points (bp) after the Lehman failure. Although it went down to about 40 bp in mid-2009, it started to climb again to 90 bp in 2011, and spiked up again immediately after the March 11 earthquake/tsunami. The CDS spread reflects the default probability assessment by the market. Figure 13 suggests that the market assessment of the default probability for the JGB increased over the last three years. Although the level of spread for Japan is still low compared to some European countries that have experienced fiscal crises in 2010-11, the upward trend is obvious. Just for comparison, CDS spreads for Spain and Italy were at around 100 bp in November/December 2009, before rising higher than 100 in 2010, and then above 300 in the summer of 2011.

<Fig 13 > about here

7. Implications of the earthquake/tsunami

The strong earthquake and tsunami that hit Japan on March 11, 2011 may turn out to be important because it can affect the expectation on future fiscal consolidation. The government decided to issue Reconstruction Bonds to finance the reconstruction costs. The Third Supplementary Budget for Fiscal 2011 included the issue of 11.6 trillion yen (about 2.4% of GDP) of Reconstruction Bonds. According to the government plan, an additional 12.7 trillion (about 2.6% of GDP) yen of Reconstruction Bonds are slated to be issued in fiscal year 2012. Although the Reconstruction Bonds are supposed to be redeemed using the revenues from income tax surcharge for 25 years and categorized in the special account, they add to the real burden of government debt until it is fully redeemed. In addition, decontamination of nuclear

¹² Because Japan has not experienced any crisis despite repeated downgrading by credit rating agencies, some compare the credit rating agencies to "a boy who cried wolf" in Aesop's fable. A relevant lesson from the fable, however, is that a wolf eventually showed up and the people in the village were not prepared. Thus, whether the evidence that credit downgrading so far has not triggered a crisis does not imply the crisis itself would not happen, with or without warnings.

substances and compensations for nuclear-accident-related businesses and evacuation are estimated to be as much as 20-30 trillion yen (about 4-6% of GDP).¹³

Although these are large expenditures, the amount seems to be relatively small compared to more than 1,000 trillion yen of outstanding debt. The simple simulation model in the last section can be used to examine the impacts of the reconstruction financing on the debt dynamics. For the interest rate scenario, we consider R2. Let us start from the sustainable debt path in Figure 10. The path assumes the tax rates shown in Figure 8. Figure 14 shows the debt to GDP ratio divided by the MaxDebt under this scenario as defined by the series “sustainable”. Note that the ratio is well below 100%, which suggests the private savings ceiling is never breached under the “sustainable” policy. The series “With Reconstruction Expenses” shows the path of the debt-to-MaxDebt ratio when the government expenditure is increasing by 2.4% of GDP in 2011 and 2.6% of GDP in 2012. The ratio still stays well under 100%. Thus, we confirm that additional debt of a few percentage of GDP does not change the sustainability condition very much.

<Figure 14> about here

An important assumption here is that the tax increase proceeds as it has been expected. If the tax increase is delayed during the reconstruction period, the debt dynamics can change substantially. The debt path “With Reconstruction and Delayed Tax Increase” in Figure 18 shows what happens if the start of the tax increase was delayed by five years from 2014 to 2019 (in addition to increased bond issues in 2011 and 2012). The debt to MaxDebt ratio comes dangerously close to 100% in the early 2020s.

Figure 15 repeats the same exercise by adding 1% of GDP as additional expenses each year from 2012 to 2015 to clean up the aftermath of the nuclear accident and to compensate for victims. With these additional costs, the delay becomes critical. In 2023, the debt exceeds the MaxDebt even with the tax policy that used to be sufficient to achieve the sustainability.

<Figure 15> about here

Thus, how to finance the public costs of reconstruction can be important in the sustainability question. It could become either the last straw to large debts that break the back of the Japanese economy or the first step toward the right direction.

The year 2012 is expected to be a boom year supported by the reconstruction demand. The people will reason that if the government is unable to raise taxes in a boom year, it will never raise taxes. The loss of this confidence can trigger a crisis. Alternatively, if the

¹³ Estimates widely vary depending on how much coverage one may consider. The 30 trillion estimate includes compensation for those households affected in the 20-30km radius, businesses that were affected by the accident, and costs to decommission the reactors at the Fukushima Daiichi. See a Japanese article in Diamond: <http://diamond.jp/articles/-/12436>

government starts the necessary fiscal consolidation process by financing a substantial portion of the reconstruction expenditure in the form of taxes, this will boost the confidence that the government is serious about the long-term fiscal health and equitable intergenerational burden sharing. Then Japan may even have a chance of avoiding a fiscal crisis.

8. Consequences of a crisis

If and when the fiscal crisis occurs, how will the crisis evolve? First, the rise in JGB yields will raise other interest rates, such as the mortgage rate and corporate bond rate, and increase the cost of funds for households, firms and banks. This will reduce the consumption of non-durables and durables at least temporarily, and hurt corporate investment. The magnitude of the impacts on consumption and investment, however, may not be large. Muellbauer and Murata (2011), for example, find that consumption in Japan did not increase at all when the interest rate fell during the 1990s and 2000s. They even find that consumption responded positively to a rise in the interest rate in some specifications. Similarly, many large corporations have substantial corporate saving and do not feel constrained by the cost of funds. Small and medium firms that have been helped by the low interest rate environment and forgiving lending attitude by banks under the Act to Facilitate Financing for SMEs may suffer from higher cost of funds. This would exacerbate the crisis.

Second, the crisis would have a large adverse impact on financial institutions, because the majority of long-term government bonds are held by Japanese banks and insurance companies. When the interest rate rises, they suffer valuation losses. For example, Japanese banks collectively hold about 142 trillion yen of central and local government bonds as of the end of March 2010. This is about 32% of total bank loans. The interest rate risk is large, too. According to Bank of Japan (2010), 100 basis points increase in JGB yields is estimated to cause about 4.7 trillion yen of losses for Japanese banks collectively (BOJ 2010, Chart 3-2-3, p.39). This is about 11.7% of the Tier I capital at the end of March 2010 and about twice as much as the income before tax for the accounting year ending on March 31, 2010. The interest rate risk as of March 2008 was estimated to be around 3.5 trillion yen. This may not reduce regulatory capital immediately because the banks are not required to mark all the securities to the market, but many will none the less tighten their credit provision.

Third, if the short-term interest rate rises sharply, along with the long-term bond rate, this would cause another serious problem. The foreign exchange special account is where most of foreign reserves are held. On the asset side of the special account, U.S. Treasuries are held as international foreign reserves; and on the liability side of the special account, short-term government securities (FBs) are issued to the market. In the last 20 years, the interest rate of the U.S. treasuries exceeded the short-term yen interest rate. However, if the yen interest rate becomes higher than the U.S. interest rate, the difference incurs the cash flow from the regular

budget to the special account. Ito (2003) showed how much profit the Japanese authority made from the interest rate differential between the U.S. and Japan. The size of FBs outstanding is about 110 trillion yen. If the yen interest rate becomes higher than the dollar interest rate by 2 percentage points, it would require 2 trillion yen of interest income subsidy from the general account to the special account.

Finally, the crisis will force the government to respond. Here we list some likely responses of the government and the central bank.

First, to the extent that the government still has room for tax increases when the crisis happens, the government can implement emergency tax increases. That may impress the market and the yield may come down. Since it takes time for the debt to GDP ratio to come down, however, the reduction in the yield may be very limited. Thus, a drastic tax hike alone may not be sufficient to end the crisis by this time. Even worse, the higher tax rate may further depress the aggregate demand, resulting in actually lower tax revenue. Whether the tax increases save the economy or destroy the economy depends on some factors that this paper does not consider, such as the elasticity of the aggregate demand to a tax increase.

Second, the government is likely to be forced to cut expenditures as well. Because a substantial part of the increase of government expenditure in recent years and in the near future comes from the increased liability of the national pension system and the healthcare system, cuts in these areas would have maximum impacts financially. Cutting the benefits of retirees is in the direction toward correcting the intergenerational imbalance that currently exists in the public transfers. For example, Keen, Pradhan, Kang, and de Mooij (2011) find “those over age 60 as of 2003 are expected to receive about 100 million yen more in *net* social benefits over their lifetime than are those not yet born” (p.16).

Third, the government may try to force domestic financial institutions to roll over the debt or to accept restructuring of the debt. The government may be especially effective in arranging such a deal with the Japan Post Bank, which is still owned by the government and is a large holder of the government bonds. The forced restructuring, however, will impose losses on the financial institutions and may cause a financial crisis.

Fourth, instead of negotiating with the private sector financial institutions, the government may pressure the Bank of Japan to buy government bonds, including newly issued bonds. The Bank of Japan has been clear that they do not endorse such a monetization policy because it undermines the fiscal discipline. However, at the time of crisis, the central bank may find it as the option that is least destructive to the financial system. If such money financing is used to respond to the liquidity crisis, it will lead to very high inflation, which will depreciate the yen. This will partially stimulate the economy via an export boom, provided that Japan does not suffer a major banking crisis at the same time.

Unexpected inflation will result in the redistribution of wealth from lenders to borrowers. This is also redistribution from the old generations to young generations, since the older generation has much higher financial assets whose value might decline, or would not rise at the same pace with the inflation rate. However, this may not have such detrimental impacts on the economy, since many who participate in production and innovation (corporations and entrepreneurs) are borrowers rather than lenders.

9. Concluding remarks

The Japanese government debt is clearly unsustainable without a drastic change in fiscal policy. The interest rates of Japanese government bonds, however, have been low. Market participants do not seem to worry about the problem of high and rising debts. The continuing low JGB yields may reflect the market's view that the ample amount of private sector financial assets in Japan will always be there to absorb additional JGBs, but the current calm situation is not likely to continue. The rapid aging of the Japanese population means that the growth of private sector savings is slowing down and eventually will turn negative. The Japanese government cannot rely on the private sector to continue buying JGBs beyond a certain point. Thus, the hope for fiscal consolidation can be abruptly dashed with some trigger of a crisis.

In order to avoid its debt hitting an absolute ceiling of private saving, the government may raise taxes to close the deficit gap in time. Since the Value Added Tax (consumption tax) rate is still 5%, a gradual increase in the tax rate can bring the debt dynamics to a sustainable path. If the government fails to act, the market is likely to force some reforms on the government during a crisis.

This paper formalizes the above ideas into a simulation model. If the government successfully increases taxes in time, the debt becomes sustainable in the sense that it does not breach the ceiling of the private sector financial assets. If the government fails to implement fiscal reform, however, a crisis of high bond yield will unfold. At that point, it may be too late to avert serious consequences of a drastic cut in government services (like European countries in the current ongoing crisis), a drastic cut in living standards caused by depreciation, and a high rate of inflation that results from the central bank financing of government debt.

The tsunami and nuclear disaster in March 2011 made it necessary for the government to issue more debt to finance reconstruction. The disaster may become the last straw on the back of the Japanese government finance, or it may prompt the government to take a first step towards the right direction. It is up to the Japanese government and the public to decide to implement fiscal reform before it is too late.

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Appendix 1. Estimation of aggregate saving rate: 2010-2050

Let s_{it} be the saving per capita in year t for the generation who were born at year i . The aggregate saving in year t is given by:

$$S_t = \sum_{i=0}^t N_{it} s_{it},$$

where N_{it} is the number of people who were born at year i . Thus, the aggregate saving to GDP ratio is:

$$\frac{S_t}{Y_t} = \frac{\sum_{i=0}^t \frac{N_{it}}{N_t} s_{it}}{Y_t / N_t} = \sum_{i=0}^t \frac{N_{it}}{N_t} \theta_{it}, \text{ where } \theta_{it} \equiv \frac{s_{it}}{Y_t / N_t}$$

Thus, the aggregate saving rate is the weighted average of the generational saving rate measured as the saving per capita divided by GDP per capita, which we denote as θ_{it} . If we have θ_{it} and N_{it}/N_t , we can calculate the aggregate saving rate for year t .

We use the data from *Family Income and Expenditure Survey* to calculate the saving rate for each age bracket. The survey reports the income and expenditure for 11 age brackets according to the age of the head of the household: 24 or younger, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69, 70 or older. The survey covers a sample of households with two or more members. Thus the survey does not cover single households. The income and expenditure items are collected for the sample of households whose heads are employees, but only expenditure items are collected for all other households, which include not only retirees but also self-employed. Using the tabulation for all the households and another tabulation for the employee households only, we construct the saving rate in the following way.

First, we estimate the number of households headed by retirees and the number of households headed by non-employees (self-employed, farmers, etc.). We have the following information from the survey for each age bracket.

- N_T : Total number of all types of households in the sample
- H_T : Average number of household members for all types of households
- W_T : Average number of household members who earn income for all types of households
- N_E : Total number of employee households in the sample
- H_E : Average number of household members for employee households
- W_E : Average number of household members who earn income for employee households

Let N_R be the number of retiree households and N_S be the number of the other households (self-employed and others). Assuming the proportion of the household members who earn income is

the same for both employee households and other households, but zero for retiree households, we know:

$$(N_E + N_S) \frac{W_E}{H_E} = N_T \frac{W_T}{H_T}, \text{ and}$$

$$N_E + N_S + N_R = N_T$$

By solving these, we can calculate N_S and N_R .

Next, we calculate the per capita income for each generation by multiplying the average household income for employee households by $N_E + N_S$. The assumption here is that the average income is the same for both employee households and other non-retiree households. The consumption for each generation is calculated by multiplying the average consumption for all households by the number of all sample households.

Finally, the saving is calculated by subtracting consumption from income for each age bracket. We estimate the number of people covered by the survey by multiplying the number of all households and the average number of household members. The saving per capita is calculated by dividing the saving by this estimated number of people in the survey. The saving rate relative to GDP per capita is calculated by dividing the saving for each age bracket by the average income per capita. Figure A1 shows the saving rate for each age bracket calculated in this way for each year from 2000 to 2010. The number for each age bracket did not change very much over the decade. We take the average saving rate for each age bracket over a 2000-2010 interval and use that as θ_{it} for $t \in [2010, 2050]$.

<Figure A1>

The population weight for each generation is calculated from the mid-point projection by the National Institute of Population and Social Security Research (IPSS). Figure A2 shows the population distribution for 2010, 2020, 2030, 2040, and 2050. We can see that the Japanese population is expected to age rapidly.

<Figure A2>

Appendix 2. A simple overlapping generation model with an unexpected shift in the population growth rate

We consider a pure endowment economy. A consumer lives for two periods. The consumer receives endowment W when young, and nothing when old. The amount of endowment does not change over time (no economic growth). The consumer has access to the storage technology that transforms one unit of endowment this period to one unit of consumption goods in the next period.

Let $C(t,1)$ and $C(t,2)$ denote the consumption when young and when old respectively of a consumer who was born at the beginning of the period t . The lifetime utility function is assumed to be $U = u(C(t,1)) + u(C(t,2))$. The function is concave and there is no time preference: $u' > 0$, $u'' < 0$. Population of generation j , denoted by $P(j)$, is assumed to grow at rate g : $P(j+1) = (1+g)P(j)$.

(1) No intergenerational transfer

Without any government pension program, one can show that the following is the equilibrium consumption pattern.

$$C(t,1) = C(t,2) = W/2 \text{ for all } t.$$

The consumer consumes one-half in the first period, saves the rest and consumes it in the second period. The equilibrium lifetime utility is:

$$U^* = u(W/2) + u(W/2)$$

(2) Government social security program

The government imposes the tax τ on the endowment to the young with a promise to pay the social security benefit $(1+r)\tau$ when they retire. The τ can be considered as either social security tax or the offer of government bonds. The system is Pay-as-you-go (PAYGO): all the

tax revenues from (or bond sales to) generation t will be distributed to the retired generation $t-1$. This government budget constraint implies $(1+g)\tau = (1+r)\tau$, or $r=g$.

The budget constraint for an individual consumer who was born at the beginning of period t is:

$$\begin{aligned} C(t, 1) + C(t, 2) &= (1 - \tau)W + (1+g)\tau W \\ &= (1 + \tau g)W \end{aligned}$$

$$\text{Saving is } S(t, 1) = (1 - \tau)W - C(t,1)$$

The optimal consumption pattern for the individual consumer is calculated by maximizing the lifetime utility with respect to the consumption pattern subject to the following two constraints.

$$C(t,1)+C(t,2) = (1 + \tau g)W \quad (\text{Individual budget constraint})$$

$C(t, 1) \leq (1 - \tau)W$ (Financial constraint, an individual cannot borrow against the future government transfer)

Given τ and g , the optimal individual consumption pattern is:

$$(A) C(t, 1) = C(t, 2) = ((1 + \tau g)W)/2, \quad \text{if } (1 + \tau g)/2 \leq (1 - \tau);$$

$$(B) C(t, 1) = (1 - \tau)W \text{ and } C(t,2)=(1+r)\tau W, \quad \text{if } (1 + \tau g)/2 > (1 - \tau)$$

So, given g , the complete consumption smoothing (A) is possible if and only if $\tau \leq 1/(2 + g)$. If τ is too large, an individual ends up spending more when old than when young, but the consumer cannot borrow against the future transfer. There is a resource constraint for the economy as a whole: the government cannot collect more than the endowment. Thus, $\tau \leq 1$.

Note that the lifetime wealth for an individual consumer is increasing in τ . This implies that higher τ increases the lifetime utility as long as it is not too large to prevent complete consumption smoothing. Thus, if the government wants to maximize the lifetime utility of every generation, it is optimal to set $\tau = 1/(2+g)$.

Numerical Example:

Suppose $\tau = 0.3$, $g=r=0.5$. Disposable income $(1 - \tau)W = 0.7W$. The equilibrium consumption is $C(t,1) = C(t,2) = 0.575W$, and private saving is $(1 - \tau)W - C(t,1) = 0.125W$. This can be sustained forever, and every generation is better off than an original situation of no social security ($C(t,1) = C(t,2) = 0.5W$).

(3) Social security going wrong when population stops growing

To get an idea about what happens when the population growth rate declines as in Japan and the government does not adjust the social security system that was introduced by assuming high population growth, suppose that at generation $t=T$, the population growth suddenly stops:

$$g_0 > 0 \text{ for } t = 0, 1, 2, \dots, T, \text{ and } P(t) = (1+g)P(t-1),$$

$$g_1 = 0, \text{ for } t = T+1, T+2, \dots; \text{ that is, the } P(t) = P(t-1) = P(T).$$

At time T , the government keeps the promise to generation $T-1$, to pay $r=g_0$ by adjusting τ for generation T : τ_0 for $t = 0, 1, 2, \dots, T$. Then, the government has to adjust the tax rate on generation T to:

$$\tau_1 = (1+g_0) \tau_0,$$

with a promise that they will also receive $\tau_1(1+g_0)$. To do that, the government has to adjust the tax rate on generation $T+1$ to:

$$\tau_2 = (1+g_0) \tau_1 = (1 + g_0)^2 \tau_0$$

The tax rate has to continue increasing to keep the promise, until $\tau > 1$, hitting the wall of the resource constraint.

Numerical Example:

We start from the sustainable example that we calculated above. $\tau = 0.3$, $g=0.5$. Disposable income $(1 - \tau)W = 0.7W$. The equilibrium consumption is $C(t,1) = C(t,2) = 0.575W$ and private saving is $(1 - \tau)W - C(t,1) = 0.125W$.

Now at T+1, population growth goes to zero, but the government will keep $r=0.5$. The tax rate for generation T+1 has to be:

$$\tau_1 = 0.3 \times (1 + 0.5) = 0.45.$$

The rate continues to increase to keep the untenable promise to the previous generation.

$$\tau_2 = 0.45 \times (1 + 0.5) = 0.675$$

$$\tau_3 = 0.675 \times (1 + 0.5) > 1$$

And the social security system will become absolutely unsustainable.

Appendix3. Blanchard (1986) model of perpetual use with an unexpected decline in the population growth rate

This appendix extends the analysis of Appendix 2 using a continuous time version of overlapping generation model developed by Blanchard (1985). In the model, a consumer faces a constant probability of death each period. Let p denote this probability of death. By buying an actuarially fair annuity contract that makes the consumer pay the entire amount of wealth, w , upon his death, and to receive pw when she survives, the consumer can avoid leaving wealth behind unintentionally. Thus, Blanchard (1985) derives a well-specified aggregate consumption function that can be analyzed.

We add the population growth (or decline) to this model to motivate introduction of a pay-as-you-go social security system. Let β be the growth rate of population and normalize the size of population at time $t=0$ to be one. This can be achieved by setting the size of the new born at time t to be $n_t = (p + \beta)e^{\beta t}$. The case considered in Blanchard (1985) is a special case where $\beta=0$. It is straightforward to confirm:

$$\int_{-\infty}^t n_s e^{-p(t-s)} ds = \left[e^{\beta s - p(t-s)} \right]_{-\infty}^t = e^{\beta t}$$

The individual utility function is assumed to be logarithmic. Since the consumer dies with probability p each period, she maximizes the following lifetime expected utility function:

$$E_t \left[\int_t^{\infty} \log c(s, v) e^{(\theta+p)(t-v)} dv \right], \quad (\text{A3.1})$$

where $c(s, v)$ stands for the consumption at time v , of a consumer born at time s . Assuming the annuity contract that pays pw as long as the consumer lives and collects w when she dies, the wealth for the individual evolves as:

$$\frac{dw(s, t)}{dt} = [r(t) + p]w(s, t) + y(s, t) - c(s, t),$$

where $y(s, t)$ is the consumer who was born at time s receives at time t . Thus, the lifetime budget constraint is given by:

$$\int_t^{\infty} c(s, v) e^{-\int_t^v [r(\mu)+p]d\mu} dv = w(s, t) + \int_t^{\infty} y(s, v) e^{-\int_t^v [r(\mu)+p]d\mu} dv \equiv w(s, t) + h(s, t) \quad (\text{A3.2})$$

The human capital, $h(s, t)$ is defined to be the present discount value of the future labor income earned by an individual who was born at time s . Maximization of (A3.1) subject to (2) yields a familiar consumption function:

$$c(s, t) = (p + \theta) [w(s, t) + h(s, t)]$$

We can calculate a per capita quantity, which we denote in the uppercase text as:

$$X(t) = \frac{1}{e^{\beta t}} \int_{-\infty}^t x(s, t) n_s e^{-p(t-s)} ds \quad (\text{A3.3})$$

Using this notation:

$$C(t) = (p + \theta)[H(t) + W(t)] \quad (\text{A3.4})$$

Note that the per capita human wealth, $H(t)$, is given by:

$$\begin{aligned} H(t) &= \frac{1}{e^{\beta t}} \int_{-\infty}^t h(s, t) n_s e^{-p(t-s)} ds = \frac{1}{e^{\beta t}} \int_{-\infty}^t \left[\int_t^{\infty} y(s, v) e^{-\int_t^v [r(\mu)+p] d\mu} dv \right] n_s e^{-p(t-s)} ds \\ &= \frac{1}{e^{\beta t}} \int_t^{\infty} \left[\int_{-\infty}^t y(s, v) n_s e^{-p(t-s)} ds \right] e^{-\int_t^v [r(\mu)+p] d\mu} dv \end{aligned}$$

Assuming the labor income is distributed equally ($y(s, v) = Y(v)$) and noting $\int_{-\infty}^t n_s e^{-p(t-s)} ds = e^{\beta t}$

$$H(t) = \int_t^{\infty} Y(v) e^{-\int_t^v [r(\mu)+p] d\mu} dv$$

Taking the derivative with respect to time, the dynamics of $H(t)$ is given by:

$$\dot{H}(t) \equiv \frac{dH(t)}{dt} = (r(t) + p)H(t) - Y(t) \quad (\text{A3.5})$$

We introduce a pay-as-you-go social security system to this economy. Let us assume the government introduces the system that collects z from every consumer younger than age T . Then, the total social security collection at time t is given by:

$$\int_{t-T}^t z n_s e^{-p(t-s)} ds = \int_{t-T}^t z(p + \beta) e^{\beta s - p(t-s)} ds = z e^{-pT + \beta(t-T)} \left[e^{(\beta+p)T} - 1 \right] \quad (\text{A3.6})$$

If the per capita benefit paid to the consumers who are older than the age T is q , the total benefit payment at time t is given by:

$$\int_{-\infty}^{t-T} q n_s e^{-p(t-s)} ds = \int_{-\infty}^{t-T} q(p + \beta) e^{\beta s - p(t-s)} ds = q e^{-pT + \beta(t-T)} \quad (\text{A3.7})$$

To make this pay-as-you-go social security system sustainable, we can set q and z to satisfy:

$$q = z \left[e^{(\beta+p)T} - 1 \right] \quad (\text{A3.8})$$

Under this pay-as-you-go system, the present discount value of the net lifetime benefit at time t for an individual who was born at time s , $b(s, t)$, is given by:

$$b(s,t) = \begin{cases} -\int_t^{s+T} z e^{-\int_t^v (r(\mu)+p)d\mu} dv + \int_{s+T}^{\infty} q e^{-\int_{s+T}^v (r(\mu)+p)d\mu} dv & t-T < s \leq t \\ \int_t^{\infty} q e^{-\int_t^v (r(\mu)+p)d\mu} dv & s \leq t-T \end{cases} \quad (\text{A3.9})$$

The net benefit is smallest for the generation that is just born ($s=t$). Even for this generation, one can show the benefit is positive if the growth rate of population exceeds the interest rate ($\beta > r(t)$). Let us see that for the case where the interest rate is constant. If the interest rate is constant at r , the equation (A3.9) is simplified to:

$$b(t,t) = \frac{z}{r+p} e^{-(r+p)(t+T)} - \frac{z}{r+p} e^{-(r+p)t} + \frac{q}{r+p} e^{-(r+p)(t+T)} \quad (\text{A3.10})$$

Substituting (A3.8) into (A3.10), we get:

$$\begin{aligned} b(t,t) &= \frac{z}{r+p} e^{-(r+p)(t+T)} - \frac{z}{r+p} e^{-(r+p)t} + \frac{z}{r+p} e^{-(r+p)(t+T)+(\beta+p)T} - \frac{z}{r+p} e^{-(r+p)(t+T)} \\ &= \frac{z}{r+p} e^{-(r+p)t} \left[e^{(\beta-r)T} - 1 \right] \end{aligned} \quad (\text{A3.11})$$

The expression is greater than zero if $\beta > r$.

The value of benefits per capita, $B(t)$, is defined using (A3.3). For the simple case of the constant interest rate, one can show, through a tedious calculation using (A3.8), the per capita benefits do not change over time:

$$B(t) = \frac{z}{r+p} \frac{p+\beta}{\beta-r} \left[e^{(\beta-r)T} - 1 \right] \quad (\text{A3.12})$$

With the introduction of the social security system, the lifetime budget constraint for the individual who was born at time s changes to:

$$\int_t^{\infty} c(s,v) e^{-\int_t^v [r(\mu)+p]d\mu} dv = w(s,t) + h(s,t) + b(s,t) \quad (\text{A3.13})$$

Now an individual maximizes (A3.1) subject to (A3.13). The maximization implies:

$$c(s,t) = (\theta + p) \left[w(s,t) + h(s,t) + b(s,t) \right]$$

Or in per capita terms,

$$C(t) = (\theta + p) \left[W(t) + H(t) + B(t) \right] \quad (\text{A3.14})$$

We consider a closed economy version of the model. First, let us look at the case where the pay-as-you-go social security system is working as it was planned. The government collects from the young exactly the same amount as they distribute to the old, so there is no revenue shortfall to finance by issuing government bonds. In this case, the only financial asset that the individuals hold is the claim against the productive capital. Thus, $W=K$. Following Blanchard (1985), let us define the net per capita production function:

$$f(K) = F(K,1) - \delta K, \quad (\text{A3.15})$$

where δ is the depreciation rate. The dynamics of K is given by:

$$\dot{K} = f(K) - C \quad (\text{A3.16})$$

The dynamics of the per capita consumption is given by:

$$\dot{C} = (p + \theta)(\dot{H} + \dot{K})$$

because B does not change over time. Using (A3.4), (A3.5), and (A3.16), this is simplified to:

$$\dot{C} = (r - \theta)C - p(p + \theta)K \quad (\text{A3.17})$$

As Blanchard (1985) showed, the equilibrium can be characterized using a phase diagram on K - C plane. Noting that the interest rate is determined as the marginal product of capital ($r = r(K)$ and $r'(K) < 0$), Figure A3 shows the $\dot{C} = 0$ curve and $\dot{K} = 0$ curve implied by (A3.17) and (A3.16).

The $\dot{K} = 0$ curve traces the production function and the $\dot{C} = 0$ curve asymptotes toward $K=K^*$ such that $r'(K^*) = \theta$. The model has a unique steady state. The equilibrium path is the saddle path that goes through the steady state denoted by A in the figure.

Now suppose the population growth drops at time x and the number of new borns at time t and after becomes $n_s = pe^{\beta x}$. When this happens, the population eventually converges to $e^{\beta x}$, the population at time x . The government starts experiencing a revenue shortfall for the social security program if the same amount z is collected from all individuals younger than T . For time $t (>x)$, the revenue shortfall is given by:

$$\int_x^t z \left[(p + \beta)e^{\beta s} - pe^{\beta x} \right] e^{-p(t-s)} ds = ze^{\beta x} \left[e^{\beta(t-x)} - 1 \right]$$

After time $x+T$, the revenue shortfall will be partially offset by reduced payments since the smaller cohorts now start receiving benefits. The net revenue shortfall for $t (>x+T)$ is:

$$\begin{aligned} & \int_{t-T}^t z \left[(p + \beta)e^{\beta s} - pe^{\beta x} \right] e^{-p(t-s)} ds - \int_x^{t-T} q \left[(p + \beta)e^{\beta s} - pe^{\beta x} \right] e^{-p(t-s)} ds \\ & = ze^{\beta x} [e^{\beta T} - 1] \end{aligned}$$

using (A3.8) again. After time $x+T$, the net revenue shortfall becomes constant. This suggests that the smaller collection from the young is exactly offset by the lower payments for the old cohorts as soon as the smaller cohorts start receiving the benefits.

Suppose the government finance the revenue short fall by issuing bonds. The government debt starts to accumulate after time x according to the following dynamics.

$$\dot{D} = rD(t) + M(t) \quad (\text{A3.18})$$

$M(t)$ is the revenue shortfall of the social security system and given by:

$$M(t) = \begin{cases} ze^{\beta x} [e^{\beta(t-x)} - 1] & x \leq t < x+T \\ ze^{\beta x} [e^{\beta T} - 1] & t \geq x+T \end{cases} \quad (\text{A3.19})$$

Now there are two financial assets: the productive capital (K) and the government debt (D), so $\dot{W} = \dot{K} + \dot{D}$. The dynamic equation for the consumption changes to:

$$\dot{C} = (r - \theta)C - p(p + \theta)(K + D) - (p + \theta)M \quad (\text{A3.20})$$

Thus, the $\dot{C} = 0$ curve in the phase diagram continues to shift upward as long as the revenue shortfall exists and the debt accumulates. The dynamic equation for the capital also changes slightly to:

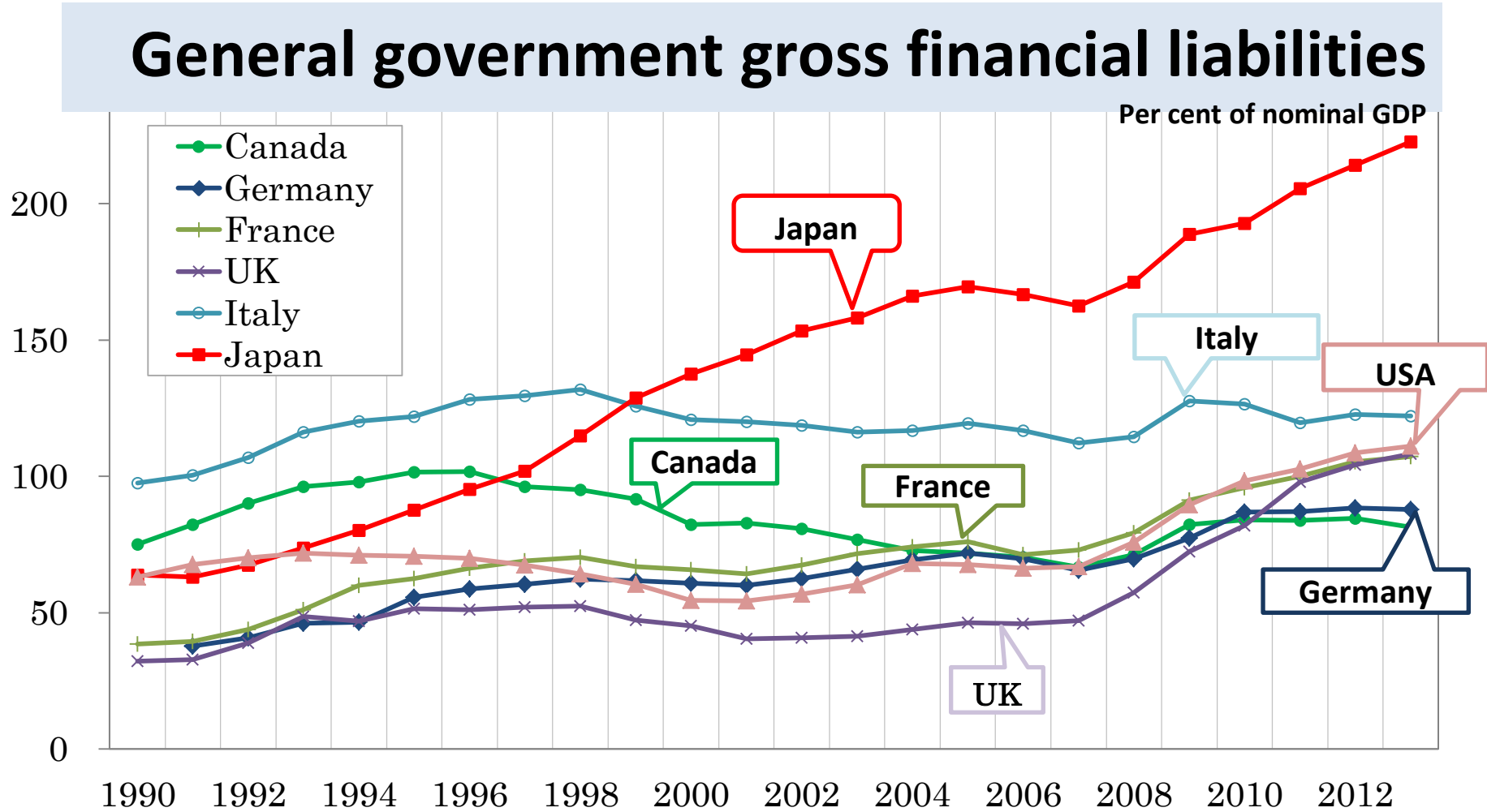
$$\dot{K} = F(K) - C - \dot{D} \quad (\text{A3.21})$$

Now the government debt starts to crowd out the capital. The $\dot{K} = 0$ curve in the phase diagram shifts downward as the debt increases.

Figure A4 shows a possible change in the steady state and the equilibrium caused by a sudden decline of fertility and the revenue shortfall. The new steady state is characterized by lower capital and lower consumption. It is not clear if the new equilibrium path lies above or below the original equilibrium path. If the new equilibrium path is above the original one, the consumption momentarily increases following the accumulation of the government debt but eventually the consumption starts to decline as the capital level falls.

Although the model explored in this appendix is very different from a mechanical simulation model that we use in the text, it shows the process where the accumulation of government debt continues to crowd out the productive capital.

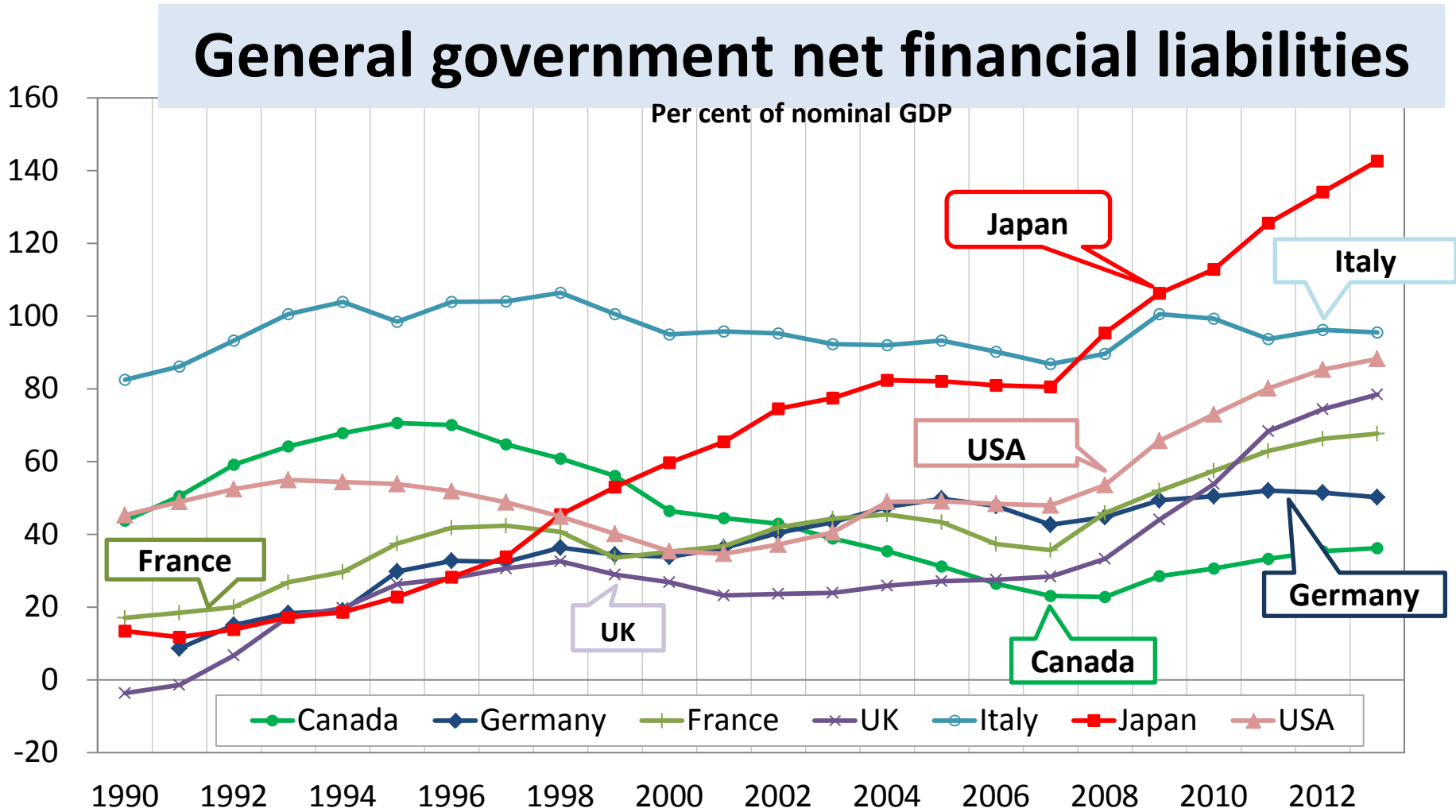
Figure 1. Gross Government Debt to GDP ratio



Source: OECD Economic Outlook 91 database.

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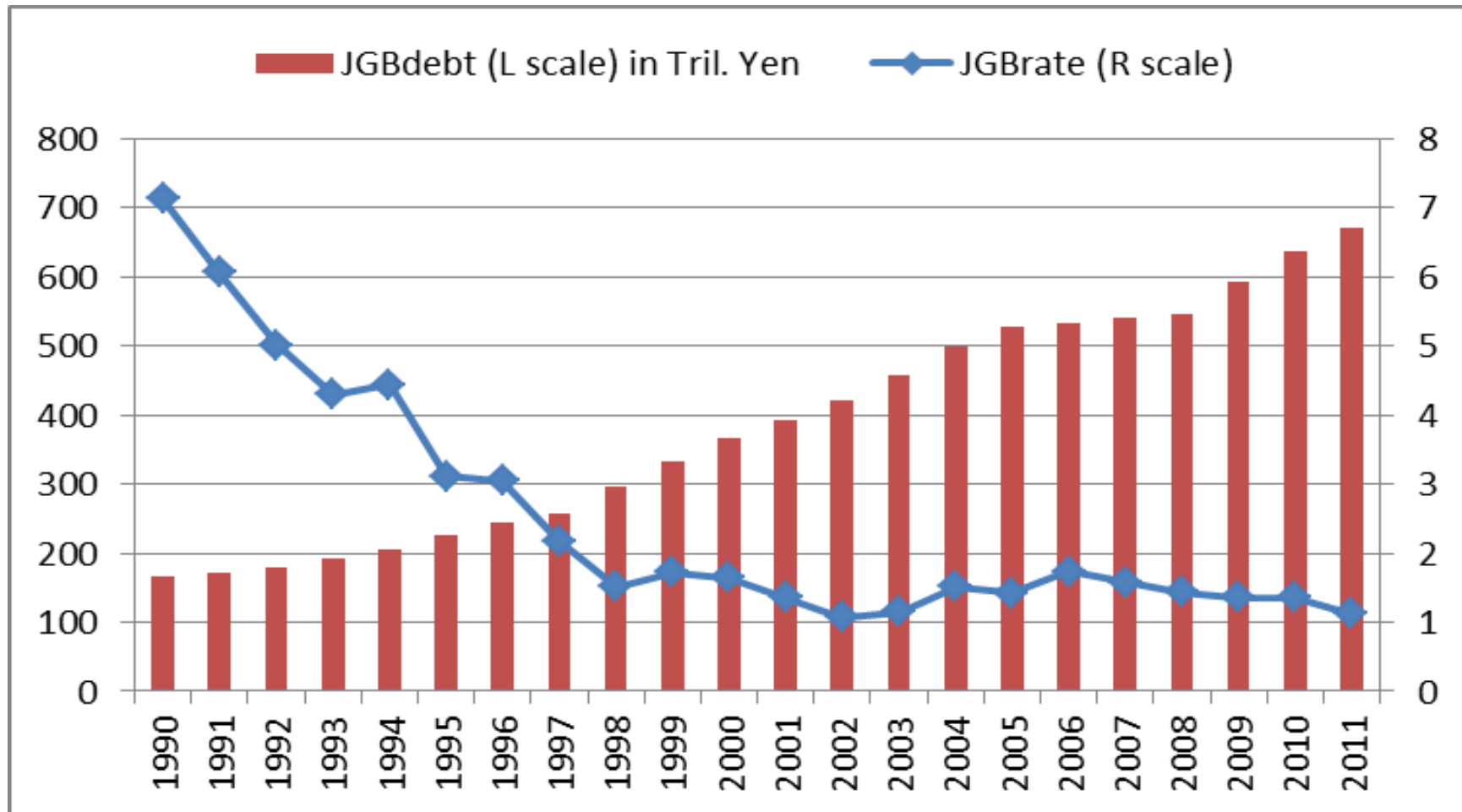
Figure 2. Net Government Debt to GDP ratio



Source: OECD Economic Outlook 91 database.

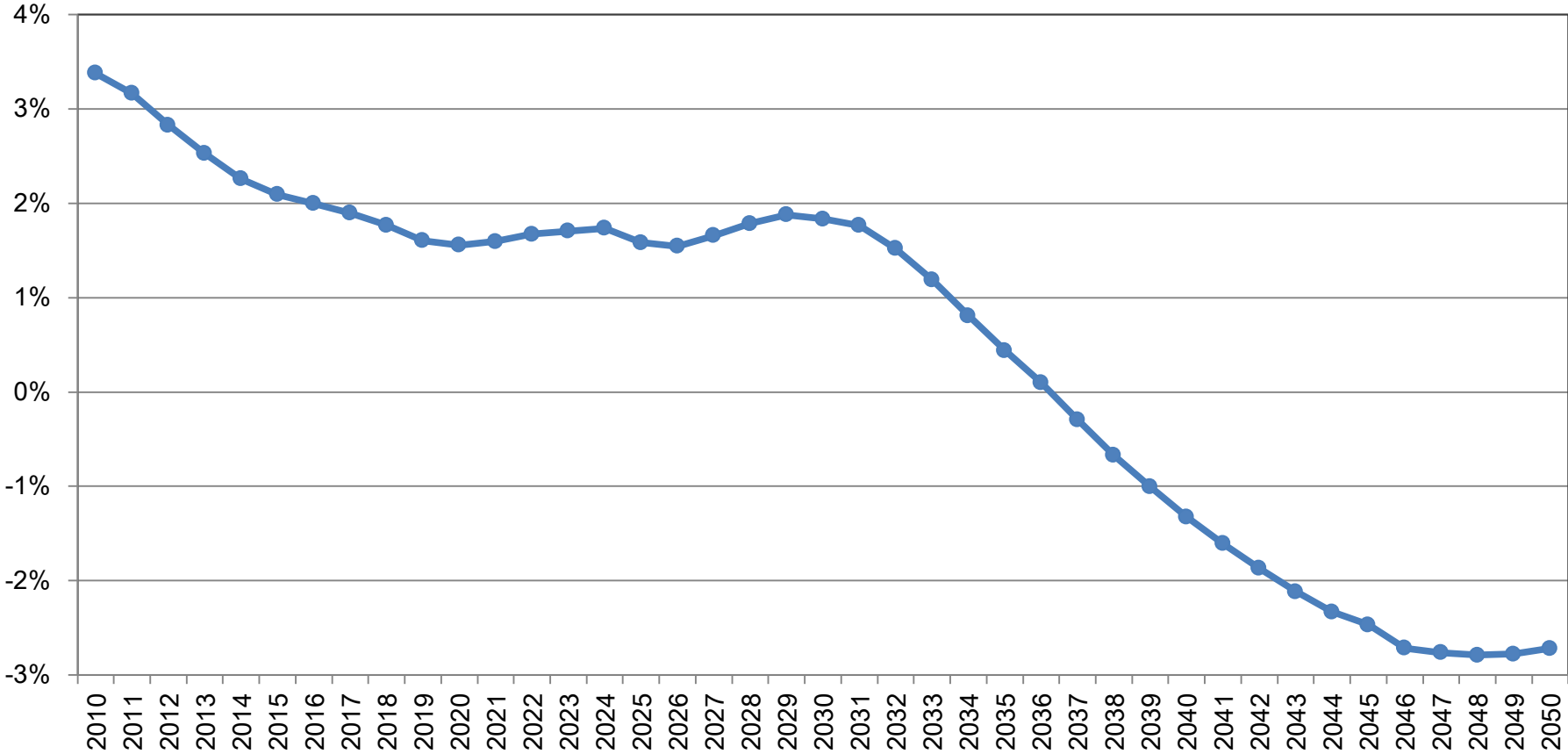
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Figure 3: JGB outstanding amount and JGB interest rate



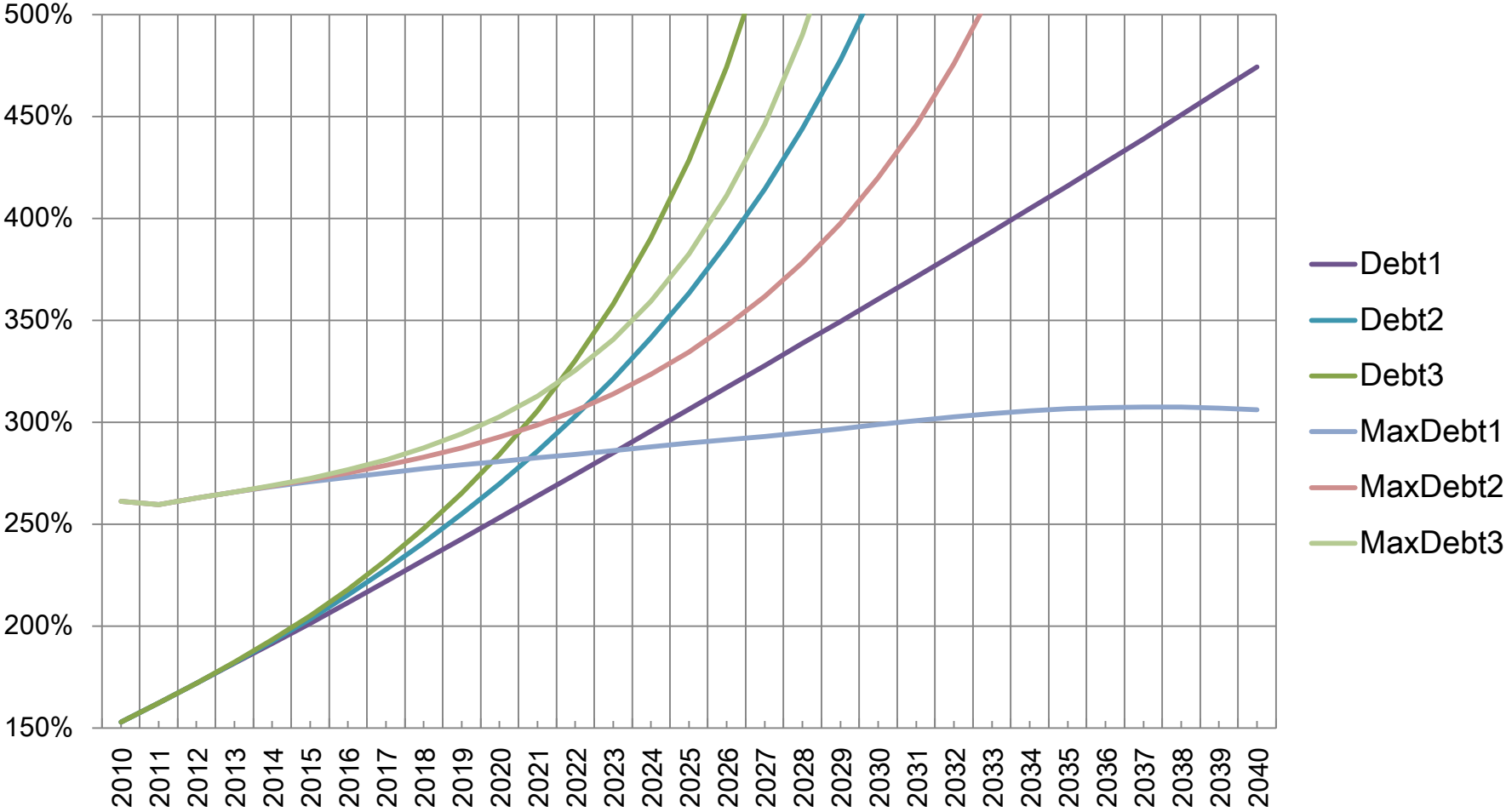
Source: Ministry of Finance.

Figure 4. Aggregate Saving to GDP Ratio: 2010-2050



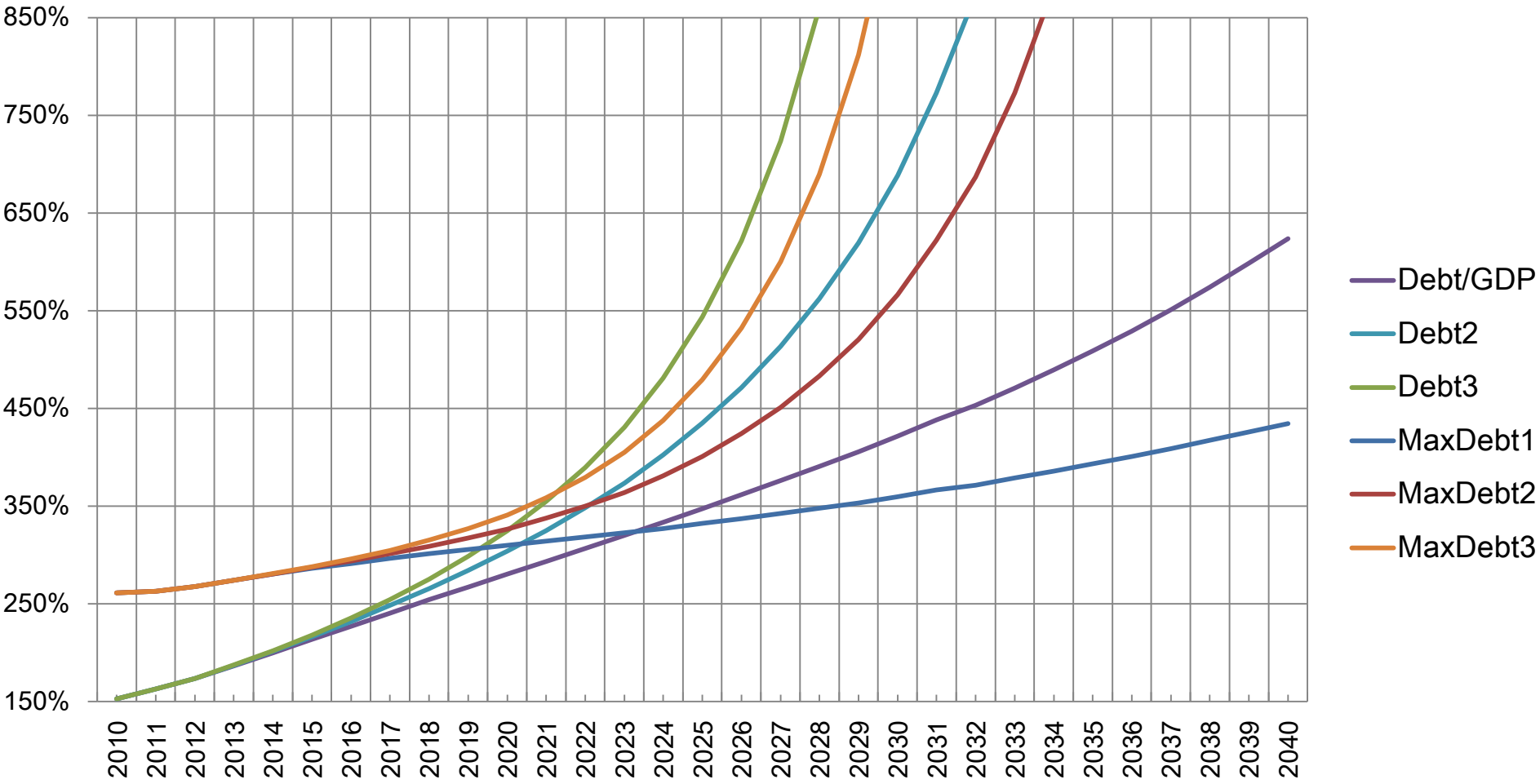
Note: Authors' calculation.

Figure 5. Government Debt and Private Sector Financial Assets: 2010-2040 (2% GDP Growth)



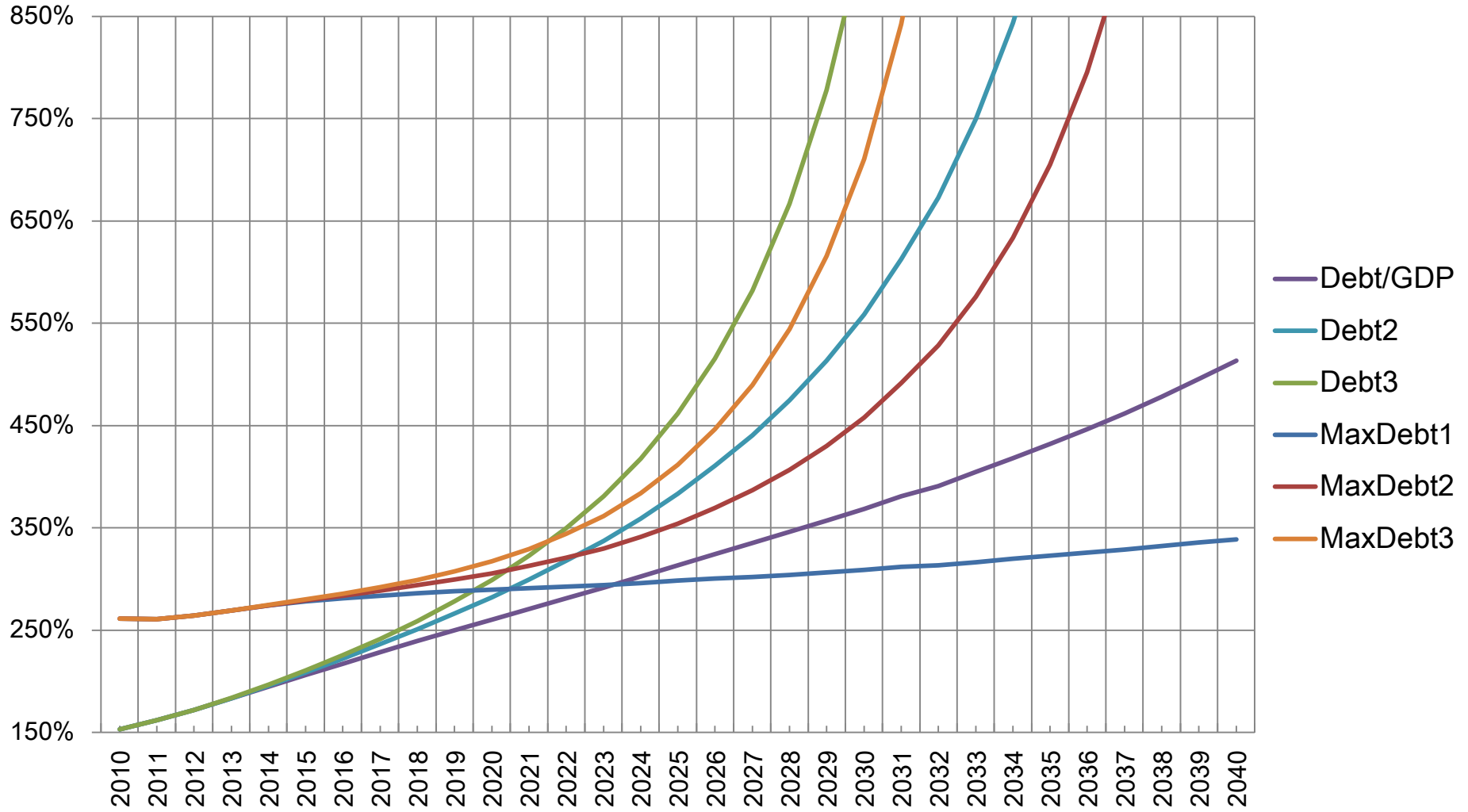
Note: Authors' calculation.

Figure 6. Government Debt and Private Sector Financial Assets: 2010-2040 (1.05% GDP per worker growth)



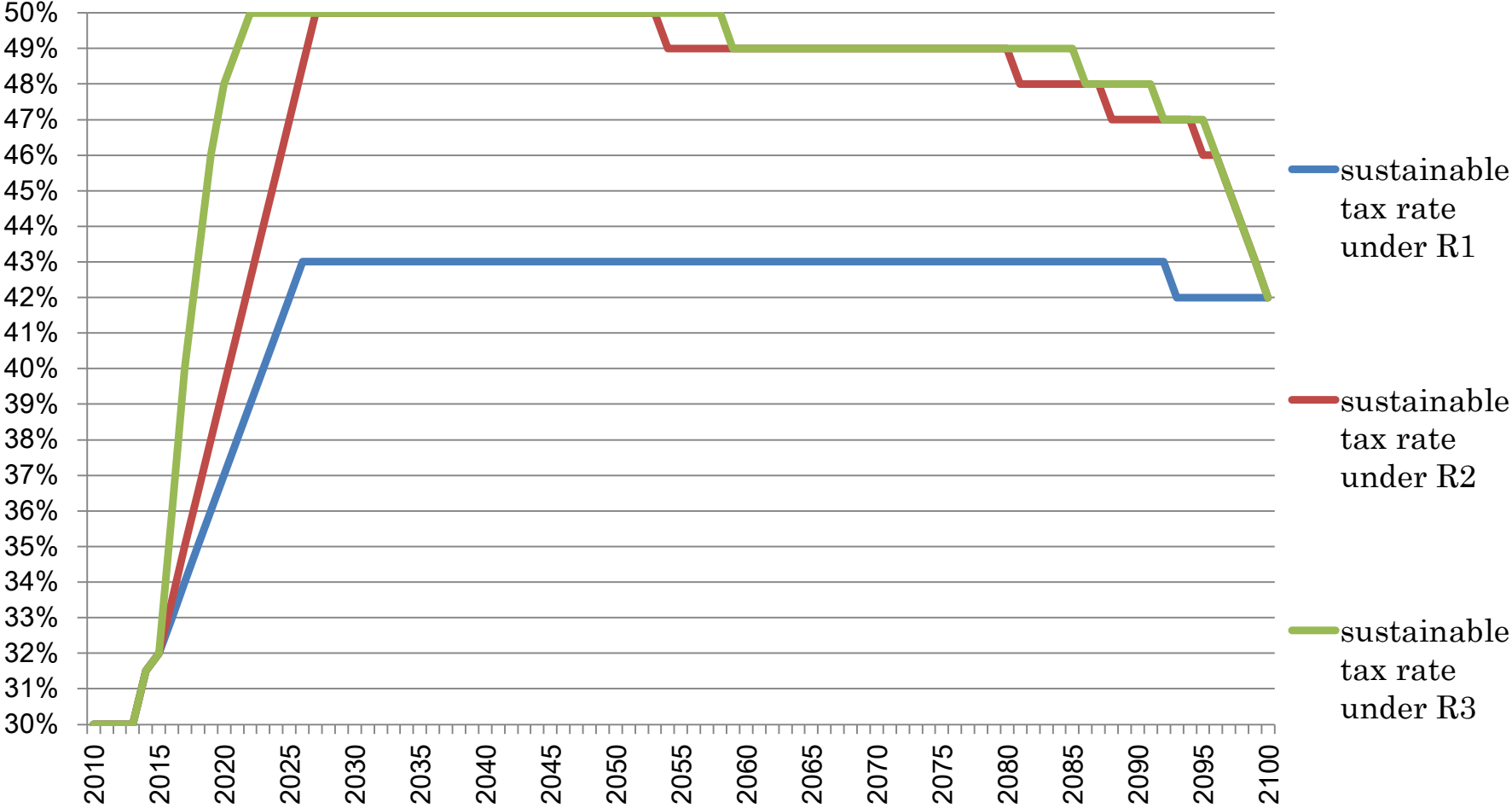
Note: Authors' calculation.

Figure 7. Government Debt and Private Sector Financial Assets: 2010-2040 (2.09% GDP per worker growth)



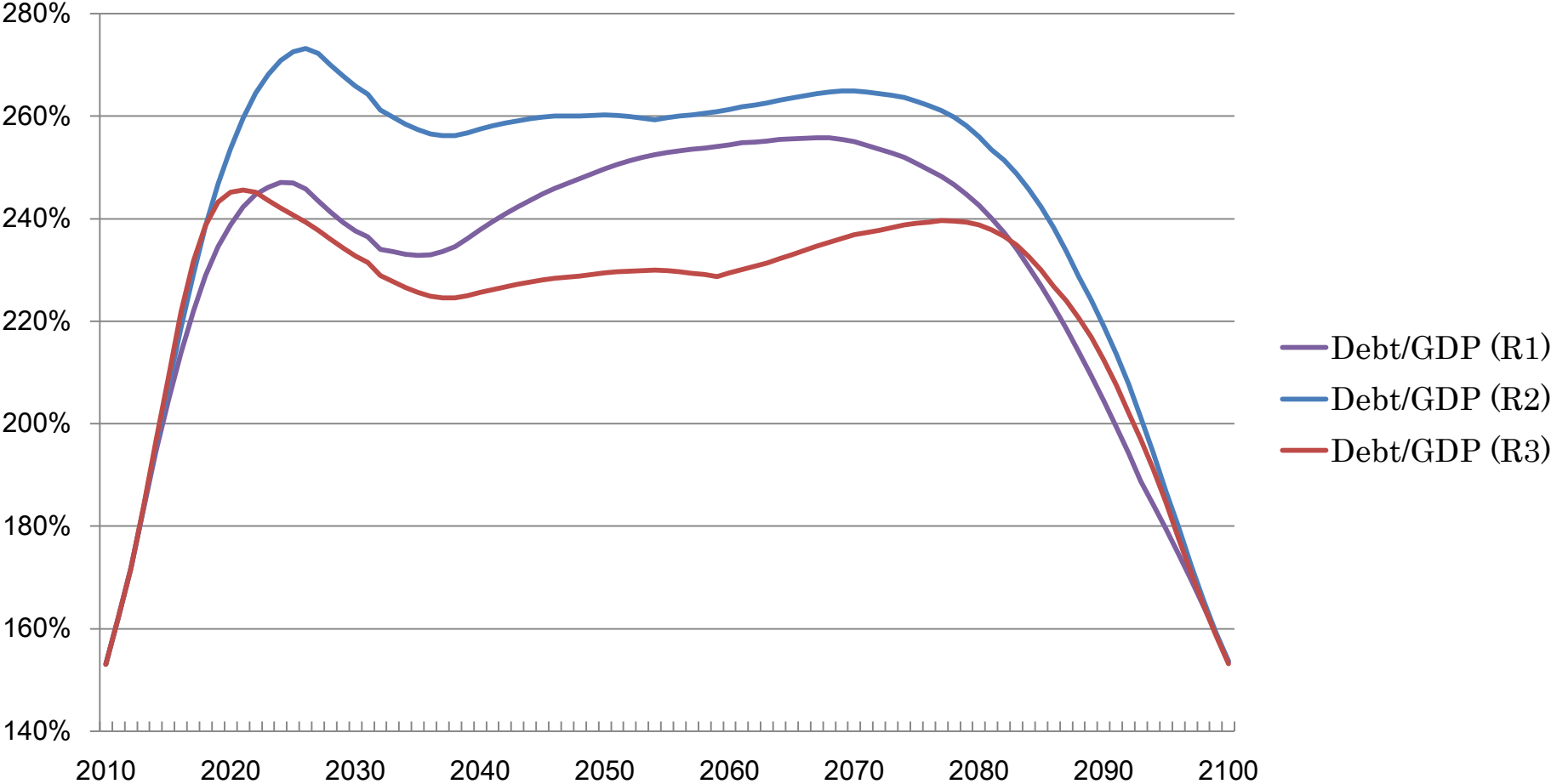
Note: Authors' calculation.

Figure 8. Sustainable Tax Policy under Each Interest Rate Assumption



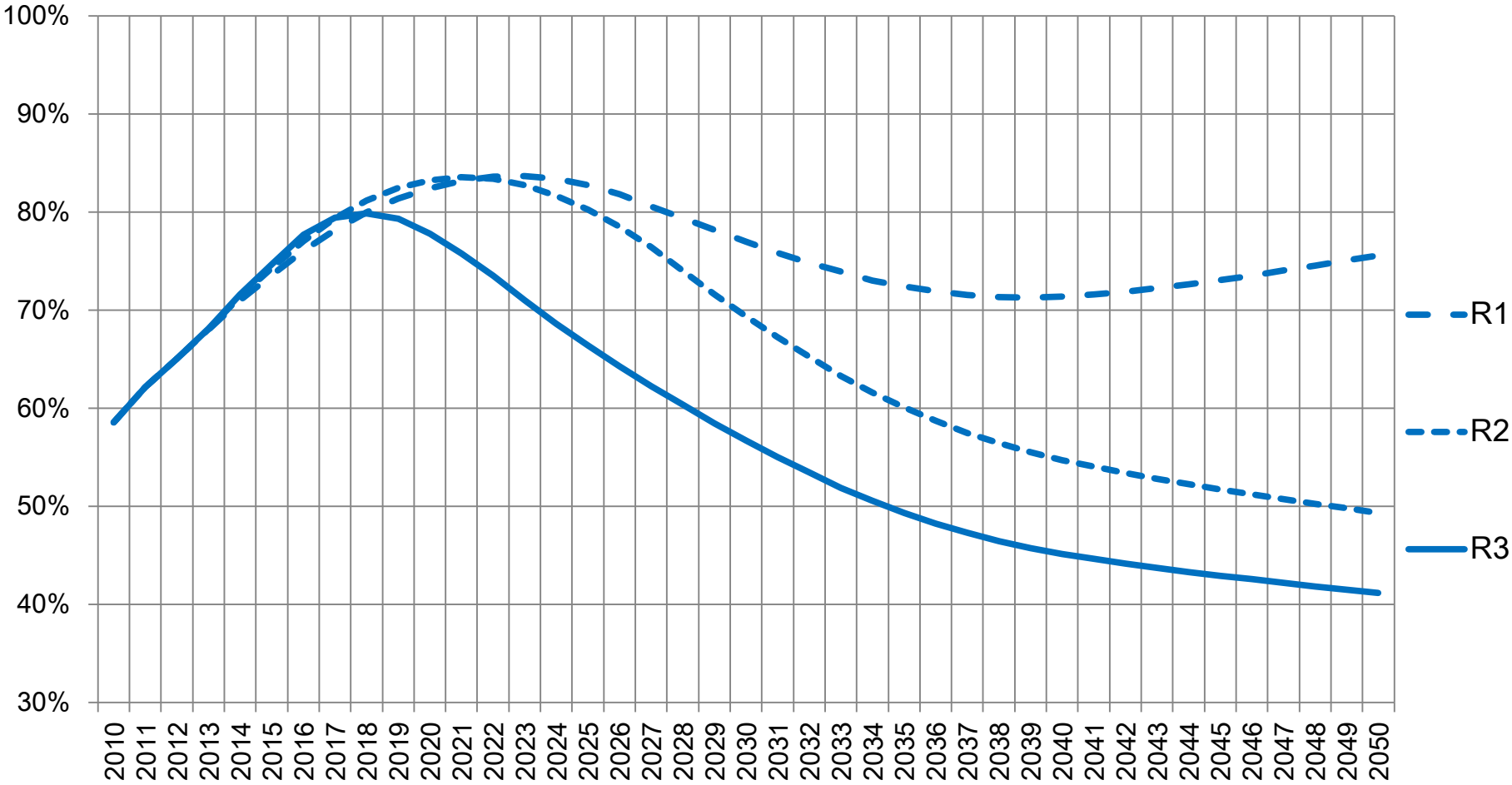
Note: Authors' calculation.

Figure 9. Debt/GDP Ratio with Sustainable Tax Policy



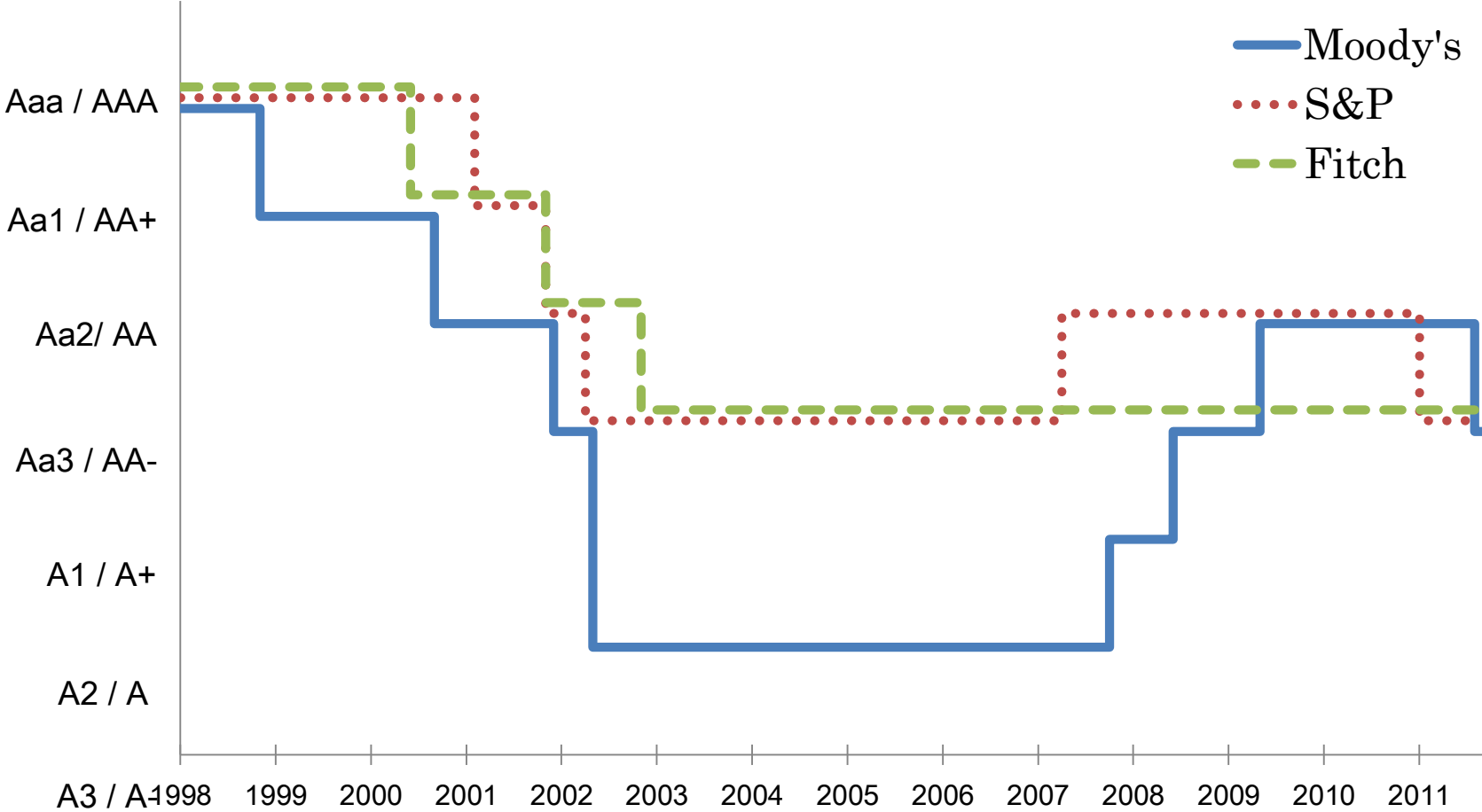
Note: Authors' calculation.

Figure 10. Debt to MaxDebt Ratio with Sustainable Tax Policy



Note: Authors' calculation.

Figure 11. Credit Rating



Source: Bloomberg.

Figure 12. Event Analysis, downgrade on JGB rate

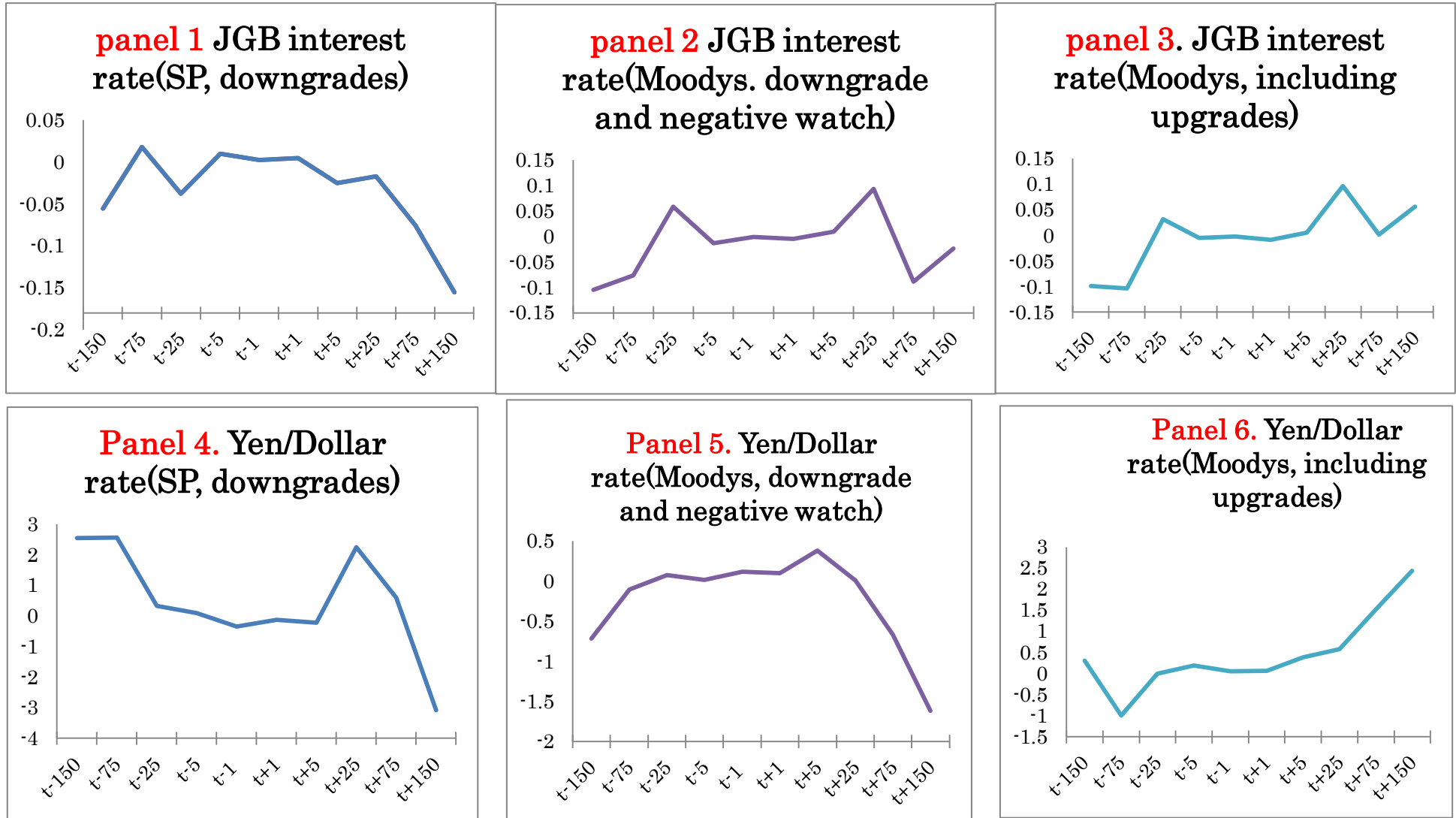
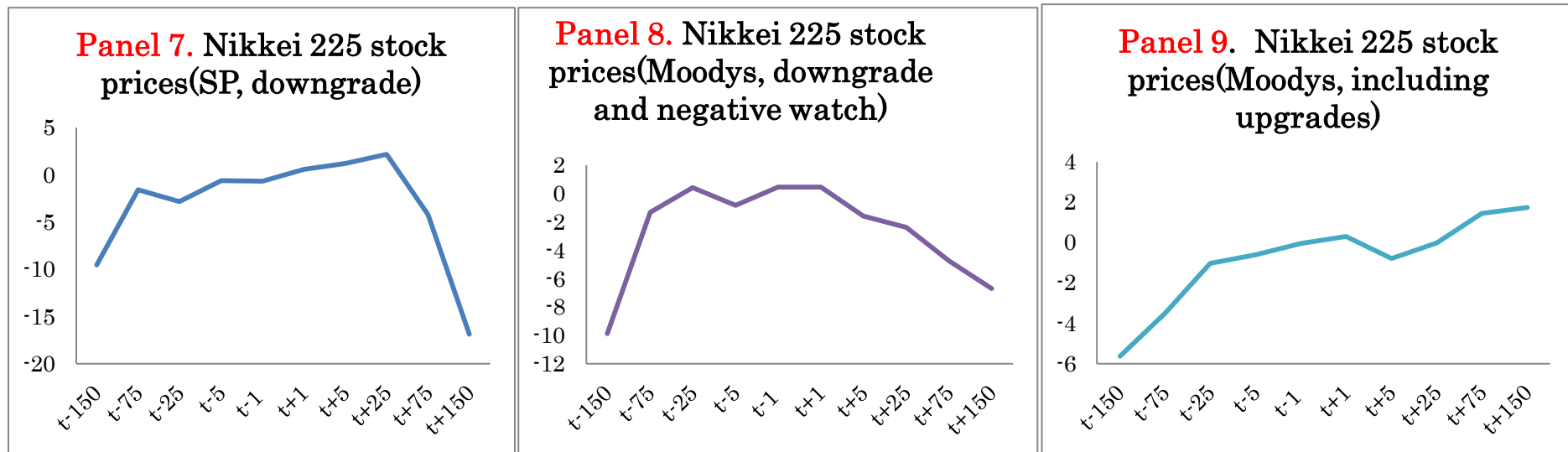
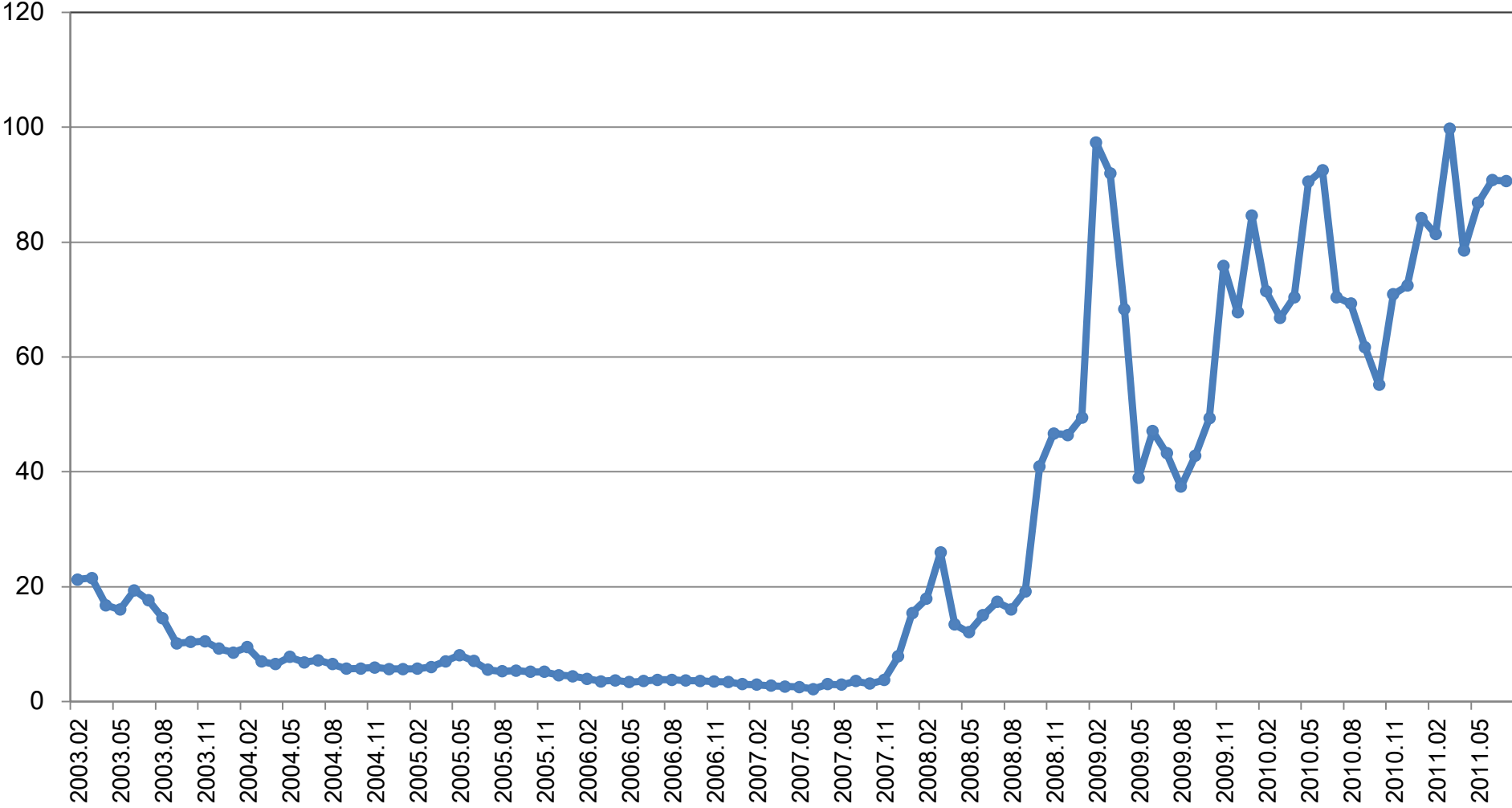


Figure 12 (continued). Event Analysis, downgrade on JGB rate (continued)



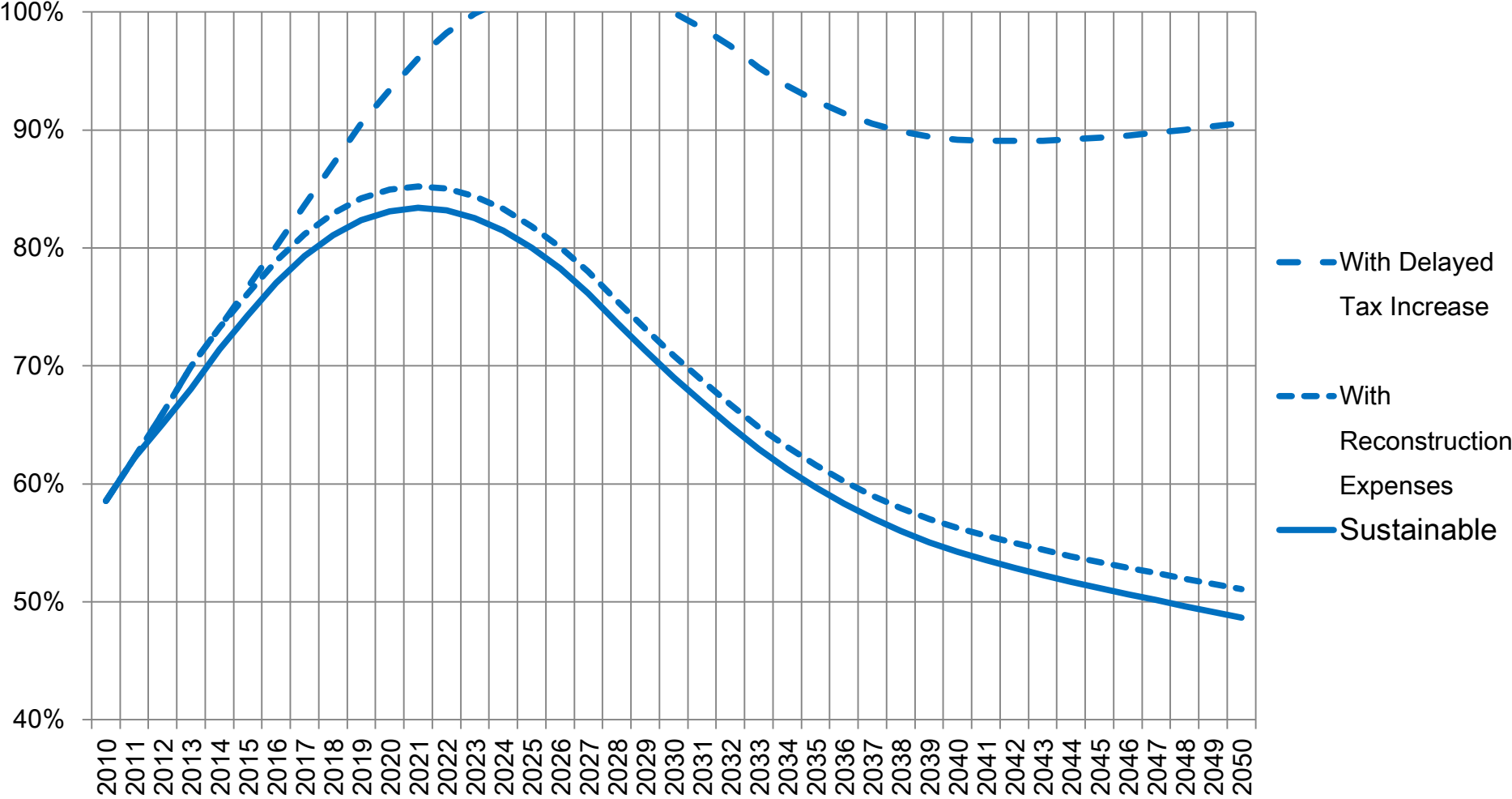
Note: Author's calculation.

Figure 13. JGB CDS Spreads: 2003-2011 (Weekly)



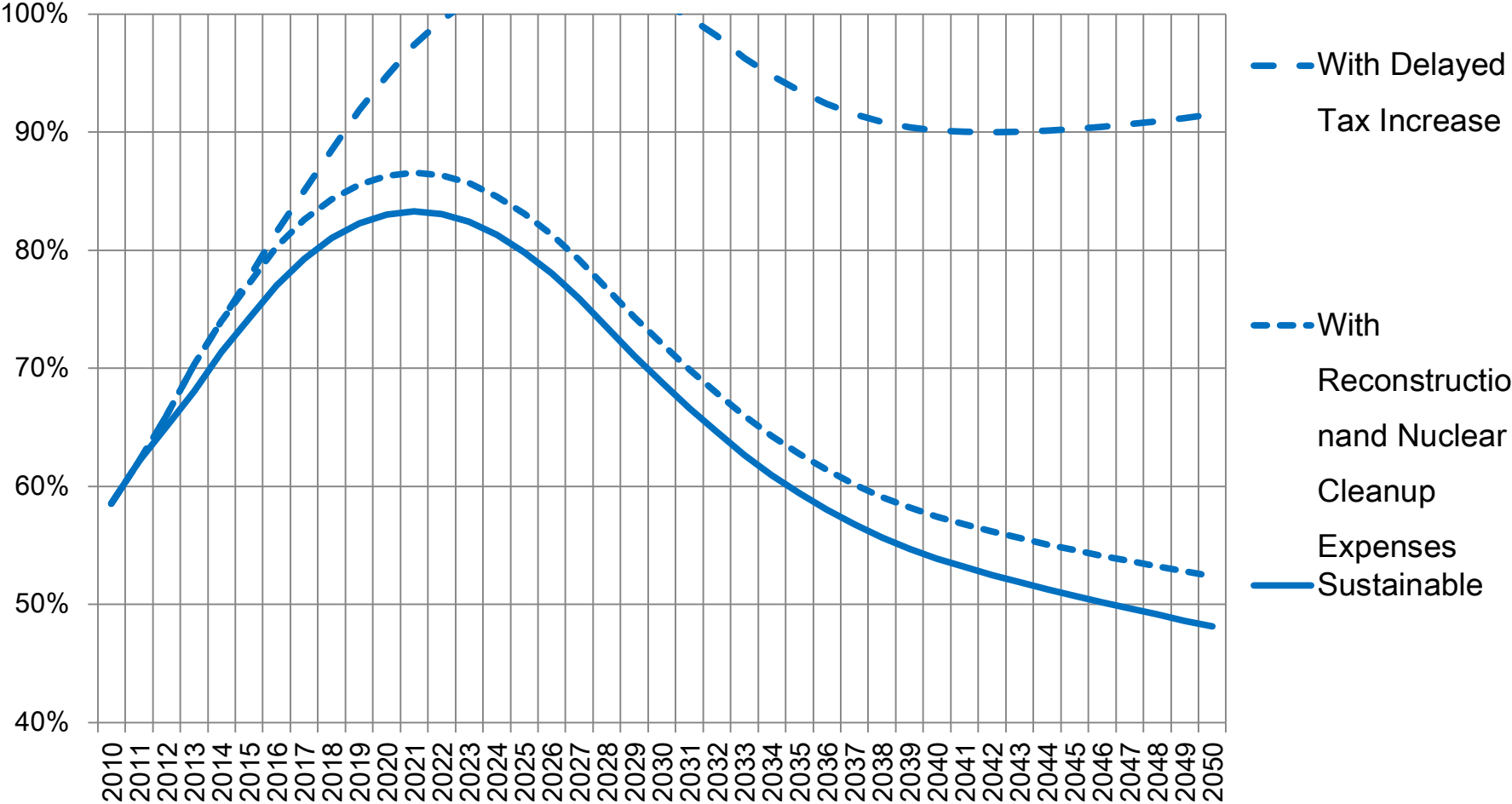
Source: Authors' calculation

Figure 14. Debt to MaxDebt ratio with reconstruction expenditures



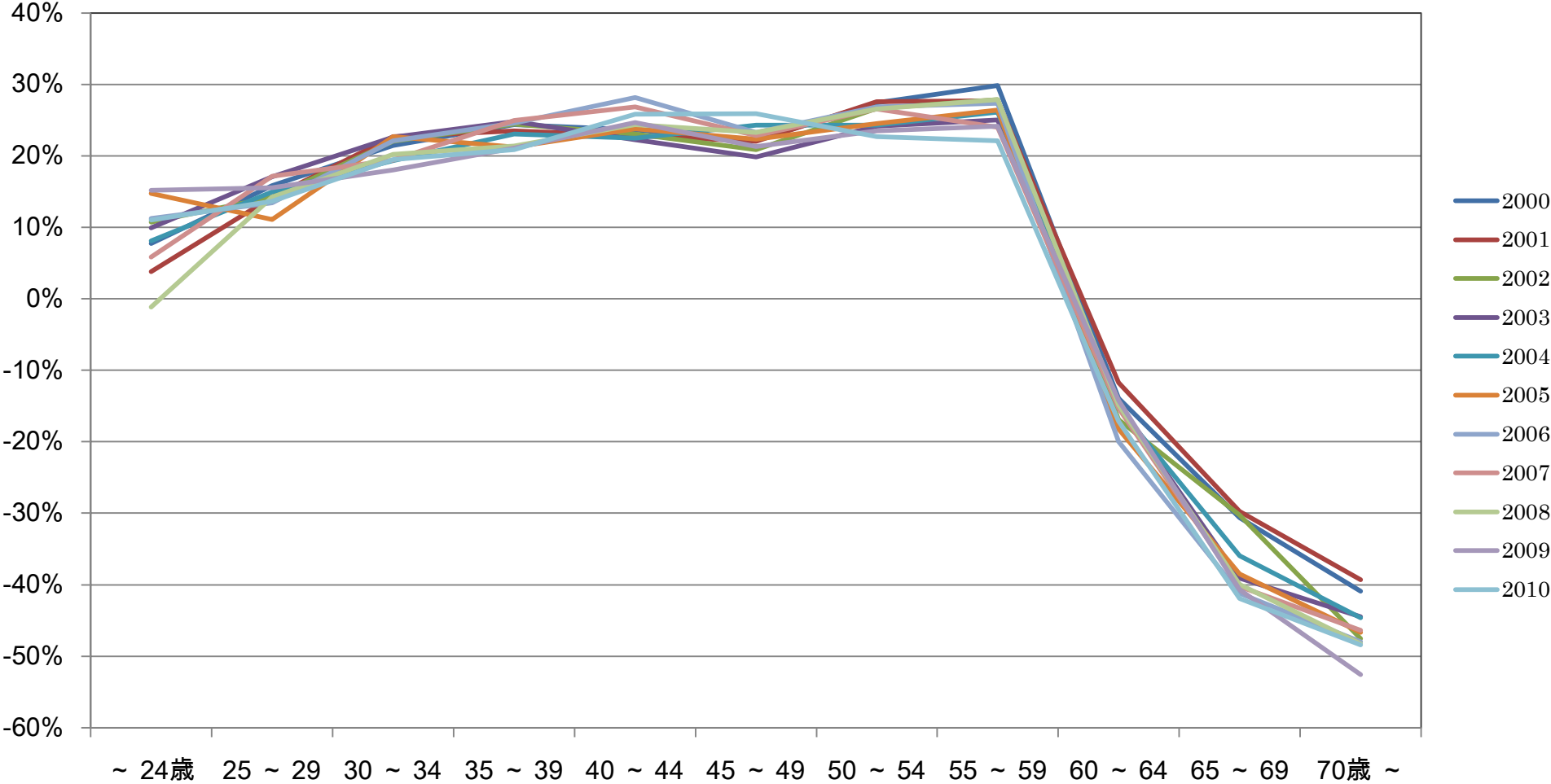
Source: Authors' calculation

Figure 15. Debt to MaxDebt ratio with reconstruction and nuclear cleanup expenses



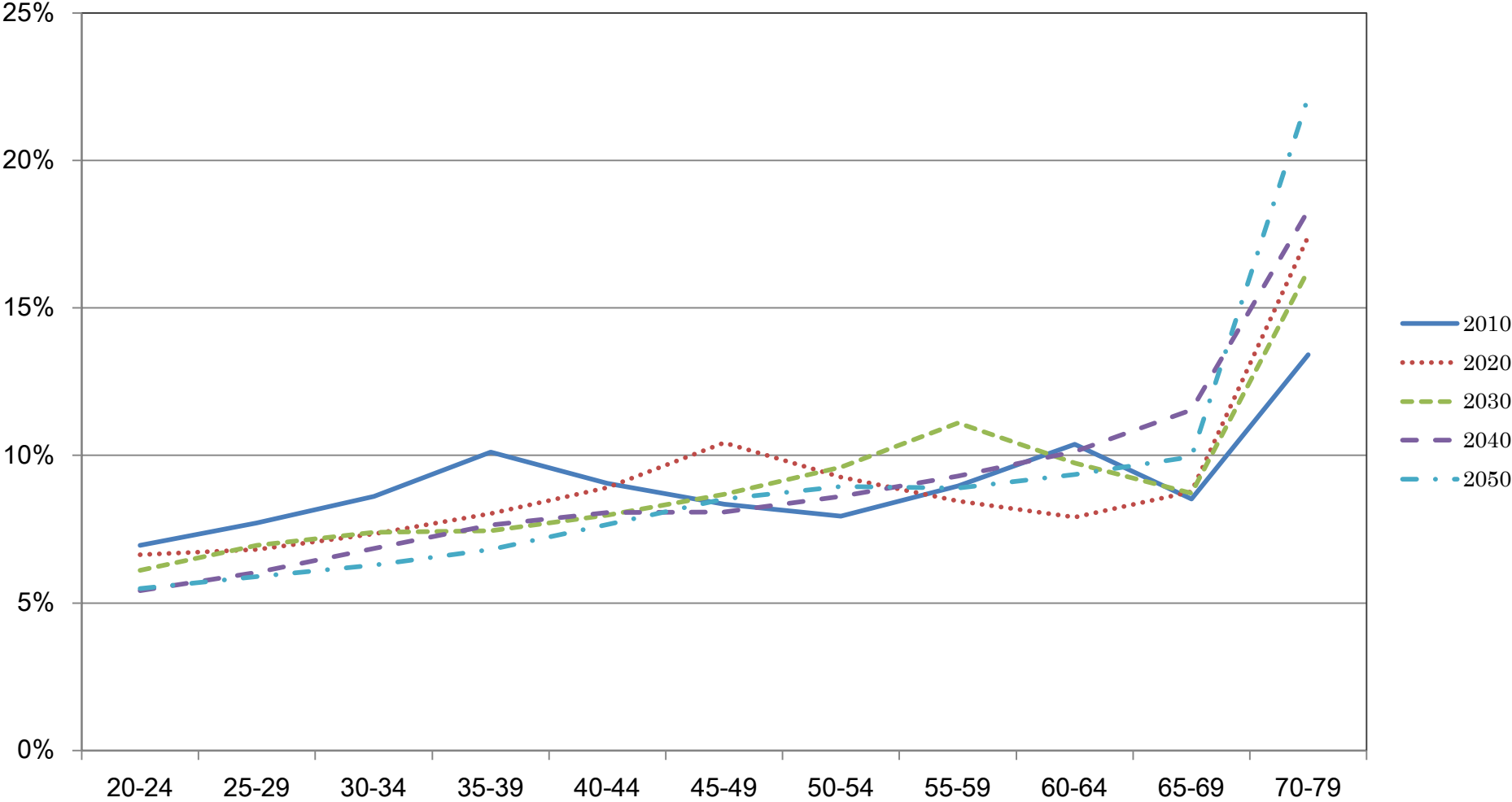
Note: Authors' calculation.

Figure A1. Saving Rate by Household Age Bracket



Source: Family Income and Expenditure Survey.

Figure A2. Population Distribution



Source: National Institute of Population and Social Security Research (IPSS), mid-point projection.

Figure A3. Equilibrium in a Blanchard (1985) type model

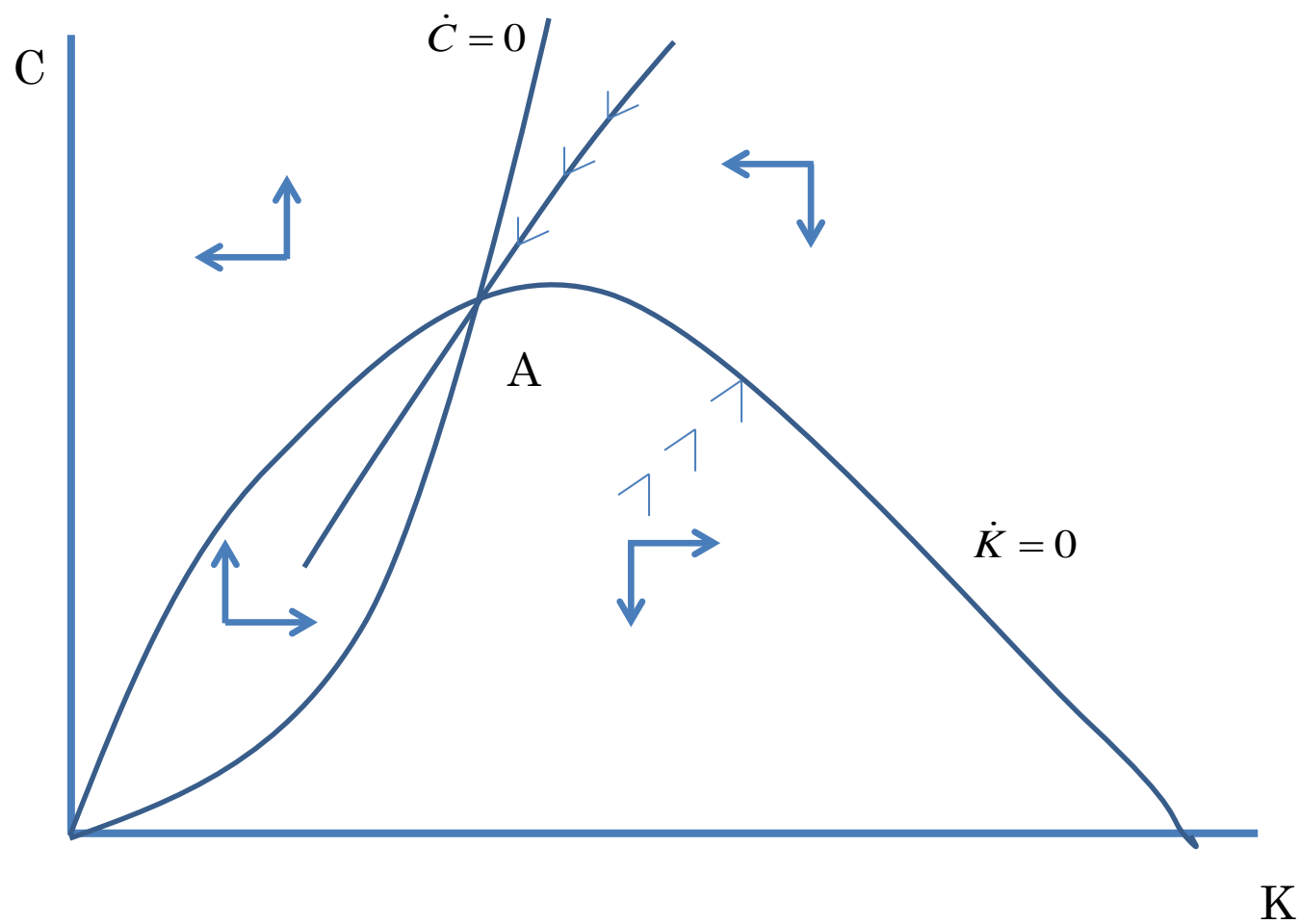


Figure A4. When the government starts running deficits

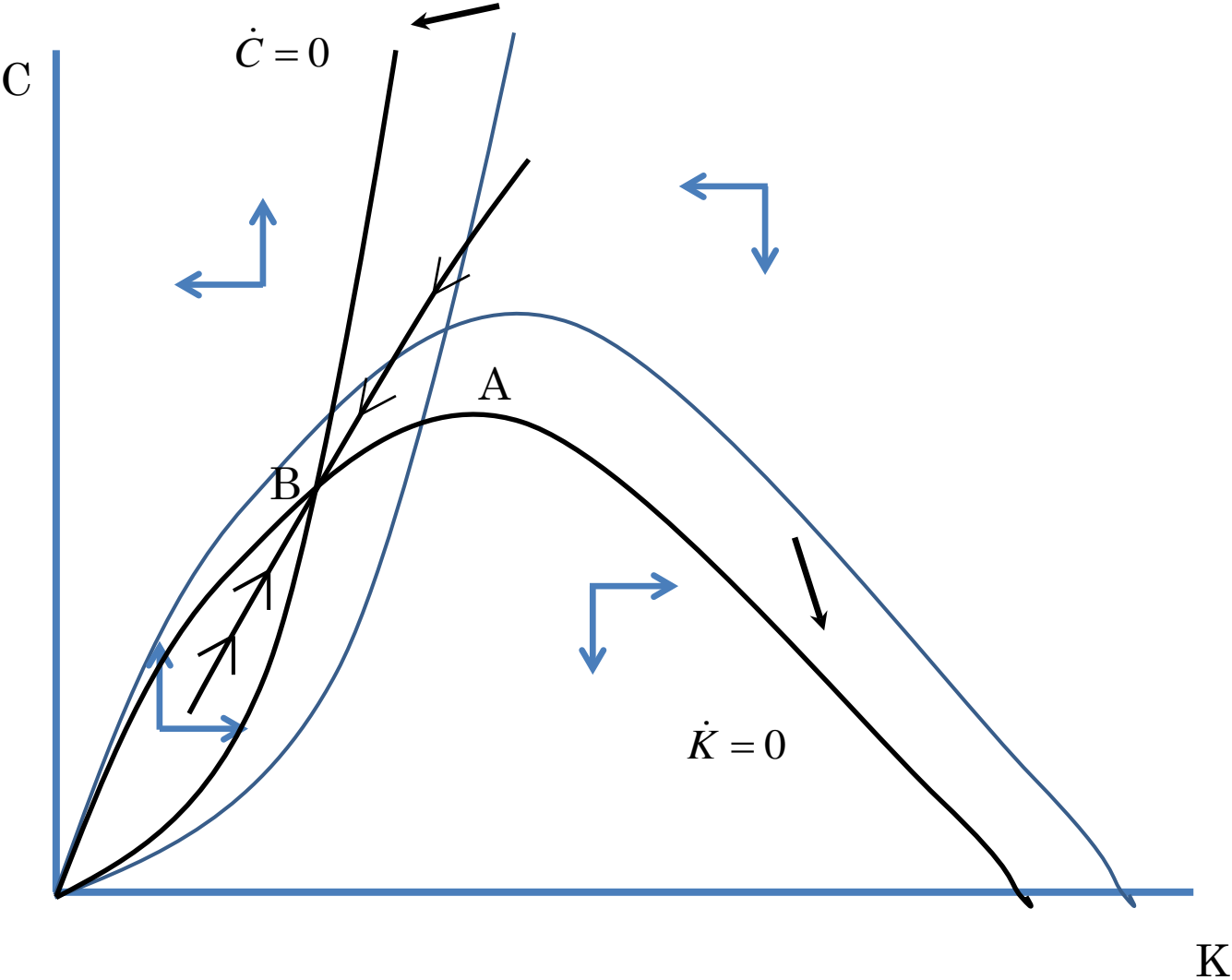


Table 1. Different concepts of Government liabilities

National Liabilities: JGB, Borrowings and Guarantees as reported to IMF			National and Local Government Longterm liability (to be redeemed by mainly future)			General Government Gross Liability (National Account concept)		
At the end of March 2011	(Trillion yen)	ratio to GDP	Forecast for March 2012	(Trillion yen)	ratio to GDP	End of March 2000	(Trillion yen)	ratio to GDP
Longterm Liability of National Government			Longterm Liability of National Government			National Government Liabilities		
JGB	636.3		JGB	668.0		JGB (+)	559	
Zaito Bonds	118.2		Zaito Bonds (**)	Not included		Zaito Bonds	Not included	
Others(*)	59.1		Others (*)	24.0		Others (*)	63	
Financing Bills (**)	110.8		Financing Bills (**)	Not included		Financing Bills and Discount Bonds(+)	149	
Government guarantee	44.7		Government Guarantee	Not included		Government Guarantee (**)	54	
Total	969.1	202%	Local, Long-term liability	200.0		Local liabilities	183	
GDP(2010)	479.0		Total	892.0	186%	Liabilities in Social Security Fund	15	
						Total	1,023	214%
(Source) The Ministry of Finance, homepage http://www.mof.go.jp/jgbs/reference/gbb/data.htm			(Source) The Ministry of Finance, homepage http://www.mof.go.jp/jgbs/reference/gbb/data.htm			(Source) Cabinet Office, Economic and Social Research Institute, http://www.esri.cao.go.jp/jp/sna/kakuhou/kekka/h21_kaku/h21_kaku_top.html#c1		
(*) Others includes government bonds that are issues as capital of public entities (Kofu Kokusai) and borrowings			(*) Borrowings for accounts to be distributed to local governments (Kofu tax), about 34 trillion yen, is categorized in the local liabilities instead of National government liabilities			(+ Short-term discount bonds are excluded from JGB and included in Financing bills and discount bonds		
(**) Financing Bills are issued primarily to fund the foreign reserves that are held in the special account of the government. They are rolled over every 3 months, and considered to be short-term liabilities that have assets, that is foreign reserves, to match the liabilities. FBs should be excluded from long-term liabilities, and from "net" government liabilities.			(**) Zaito bonds are not included because they are in theory repaid from income from government investment; Financing bills are not included because they are short-term, and also they are backed by assets (foreign reserves); and gouarantee is not included because they are only contingent liabilities.			(**) Government guarantee includes liabilities at the public agencies (Dokuritsu Gyosei Hojin)		

Table 2. Owners of JGBs

	2005 March tril. Yen (%)	2006 March tril. Yen (%)	2007 March tril. Yen (%)	2008 March tril. Yen (%)	2009 March tril. Yen (%)	2010 March tril. Yen (%)	2010 Dec tril. Yen (%)
General Government	2.0 0.3%	7.4 1.1%	3.6 0.5%	2.5 0.4%	2.5 0.4%	1.9 0.3%	2.3 0.3%
Public Pension	57.6 9.0%	61.5 9.2%	68.3 10.1%	78.1 11.2%	80.1 11.8%	76.3 11.2%	74.8 10.3%
FILP	48.8 7.6%	39.4 5.9%	23.9 3.6%	10.9 1.6%	1.2 0.2%	0.8 0.1%	0.8 0.1%
Postal Saving	109.7 17.1%	126.2 18.9%	140 20.8%	— —	— —	— —	— —
Postal Insurance	55.1 8.6%	57.0 8.5%	61.0 9.1%	— —	— —	— —	— —
Bank of Japan	92.1 14.3%	86.7 13.0%	71.0 10.6%	63.7 9.2%	55.9 8.2%	51.2 7.5%	58.1 8.0%
Private Financial Institutions	218.6 34.1%	218.5 32.7%	216.1 32.1%	439.7 63.3%	441.6 64.9%	464.5 68.1%	502.6 69.1%
banks	111.6 17.4%	114.5 17.2%	101.6 15.1%	246.4 35.5%	246.2 36.2%	258.7 37.9%	282.7 38.9%
insurance	54.8 8.5%	58.4 8.7%	61.8 9.2%	129.2 18.6%	135.1 19.8%	139.9 20.5%	147.0 20.2%
private pension funds	21.3 3.3%	24.0 3.6%	26.2 3.9%	26.8 3.9%	25.6 3.8%	28.0 4.1%	28.2 3.9%
others	31.0 4.8%	21.6 3.2%	26.5 3.9%	37.3 5.4%	34.7 5.1%	37.9 5.6%	44.7 6.1%
Overseas	26.4 4.1%	30.2 4.5%	40.2 6.0%	47.4 6.8%	43.9 6.5%	31.6 4.6%	35.1 4.8%
Household	21.8 3.4%	28.0 4.2%	33.4 5.0%	36.3 5.2%	36.0 5.3%	34.4 5.0%	33.0 4.5%
Others	9.6 1.5%	12.4 1.9%	15.2 2.3%	16.5 2.4%	19.6 2.9%	21.4 3.1%	20.5 2.8%
Total	641.8 100.0%	667.3 100.0%	672.7 100.0%	695.0 100.0%	680.9 100.0%	682.1 100.0%	727.1 100.0%

Source: Ministry of Finance.

Table 3. Explaining the JGB yield

Variable	Gross debt including FILP	JGB held by Bank of Japan	Net financial wealth held by household and corporate sectors	Share of foreign holdings of JGBs	R square
Estimate	0.02	0.01	-0.02	0.11	0.38
t-stat	(3.52) ^{***}	(0.36)	(-3.37) ^{***}	(2.06) ^{**}	
<p>Tokuoka (2010) Table II.6</p> <p>Notes: FILP is the government investment program, which used to be in the special account that were funded by Postal Bank surplus funds, and later became a part of government bond issues.</p>					

Table 4. History of Demographic Dividend

	Δ rGDP =	Δ POP +	Δ (wPOP / POP) +	Δ (rGDP / wPOP)
1955-1970	9.70%	1.00%	1.03%	7.77%
1971-1980	4.46%	1.22%	0.01%	3.46%
1981-1990	4.68%	0.55%	0.18%	3.92%
1991-2000	1.06%	0.27%	0.10%	0.69%
2001-2010	0.72%	0.09%	-0.49%	1.12%

Notes: Authors' calculation. Each row does not exactly add up as the equation suggests, due to approximation in ten-year average growth rates

Data Source: GDP from Cabinet Office, Japan for GDP; and population from National Institute of Population and Social Security Research

Table 5. Implication of 2% growth

	$\Delta rGDP =$	$\Delta POP +$	$\Delta(wPOP / POP) +$	$\Delta(rGDP / wPOP)$
2011-20	2.00%	-0.31%	-0.77%	3.09%
2021-30	2.00%	-0.62%	-0.15%	2.77%
2031-40	2.00%	-0.83%	-0.68%	3.51%
2041-50	2.00%	-0.99%	-0.50%	3.49%

Notes: Authors' calculation. Each row does not exactly add up as the equation suggests, due to approximation in ten-year average growth rates. ΔPOP and $\Delta(wPOP/POP)$ are calculated from forecasts of IPSS, then $\Delta(rGDP/wPOP)$ is assumed to be 2.09%, which was the average of 2001-2007. $\Delta rGDP$ was derived from the identity;

Data Source: GDP from Cabinet Office, Japan for GDP; and population from National Institute of Population and Social Security Research (IPSS).

Source: Authors' calculation.

Table 6. Growth per worker productivity at 1.05%

	Δ rGDP =	Δ POP +	Δ (wPOP / POP) +	Δ (rGDP / wPOP)
2011-20	-0.04%	-0.31%	-0.77%	1.05%
2021-30	0.28%	-0.62%	-0.15%	1.05%
2031-40	-0.46%	-0.83%	-0.68%	1.05%
2041-50	-0.44%	-0.99%	-0.50%	1.05%

Notes: Authors' calculation. Each row does not exactly add up as the equation suggests, due to approximation in ten-year average growth rates .

Δ POP and Δ (wPOP/POP) are calculated from forecasts of IPSS, then Δ (rGDP/wPOP) is assumed to be 1.05%, which was the average of 1994-2010. Δ rGDP was derived from the identity;

Data Source: GDP from Cabinet Office, Japan for GDP ; and population from National Institute of Population and Social Security Research (IPSS).

Table 7. Per-worker labor productivity increase of 2.09%

	$\Delta rGDP =$	$\Delta POP +$	$\Delta(wPOP / POP) +$	$\Delta(rGDP / wPOP)$
2011-20	0.98%	-0.31%	-0.77%	2.09%
2021-30	1.30%	-0.62%	-0.15%	2.09%
2031-40	0.55%	-0.83%	-0.68%	2.09%
2041-50	0.57%	-0.99%	-0.50%	2.09%

Notes: Authors' calculation. Each row does not exactly add up as the equation suggests, due to approximation in ten-year average growth rates .

ΔPOP and $\Delta(wPOP/POP)$ are calculated from forecasts of IPSS, then $\Delta(rGDP/wPOP)$ is assumed to be 2.09%, which was the average of 2001-2007. $\Delta rGDP$ was derived from the identity;

Data Source: GDP from Cabinet Office, Japan for GDP ; and population from National Institute of Population and Social Security Research (IPSS)

Table 8: Changes in Credit Rating Changes

	Moody's		S&P		Fitch	
	Date	Change to	Date	Change to	Date	Change to
1993	5/3	Aaa				
1998	7/23 11/17	Watch (-) Aa1				
2000	2/17 9/8	Watch (-) Aa2			6/29	AA+
2001	9/6 12/4	Watch (-) Aa3	2/22 11/27	AA+ AA	11/26	AA
2002	2/13 5/31	Watch (-) A2	4/15	AA-	11/21	AA-
2007	7/4 10/11	Watchi (+) A1	4/22	AA		
2008	6/30	Aa3				
2009	5/18	Aa2				
2011			1/27	AA-		
2012					5/22	A+

(*) 2-notch downgrade

Source: Bloomberg