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Péter Hudomiet
Robert J. Willis

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ABSTRACT

Based on subjective survival probability questions in the Health and Retirement Study (HRS), we use an econometric model to estimate the determinants of individual-level uncertainty about personal longevity. This model is built around the Modal Response Hypothesis (MRH), a mathematical expression of the idea that survey responses of 0, 50 or 100 percent to probability questions indicate a high level of uncertainty about the relevant probability. We show that subjective survival expectations in 2002 line up very well with realized mortality of the HRS respondents between 2002 and 2010. We show that the MRH model performs better than typically used models in the literature of subjective probabilities. Our model gives more accurate estimates of low probability events and it is able to predict the unusually high fraction of focal 0, 50 and 100 answers observed in many datasets on subjective probabilities. We show that subjects place too much weight on parents' age at death when forming expectations about their own longevity, while other covariates such as demographics, cognition, personality, subjective health and health behavior are underweighted. We also find that less educated people, smokers and women have less certain beliefs; and recent health shocks increase uncertainty about survival, too.

Péter Hudomiet
University of Michigan
Department of Economics
503 Thompson Street
Ann Arbor, MI 48109
hudomiet@umich.edu

Robert J. Willis
3254 ISR
University of Michigan
P. O. Box 1248
426 Thompson Street
Ann Arbor, MI 48106
and NBER
rjwillis@isr.umich.edu

Introduction

Textbook models of uncertainty usually treat probabilities as known or knowable objects. This is the case for outcomes of symmetric devices with known properties that are subject to random forces such as the toss of a coin or die, the shuffling of a deck of cards or the shaking of an urn containing balls of different colors. In contrast, the probabilities that confront economic decision makers are usually less precisely known. [1,2]

In this paper, we investigate the probability beliefs held by a given individual about personal mortality risks elicited from survey questions on the Health and Retirement Study (HRS) that ask respondents about the numerical probability that he or she will survive to a given age that is 10-20 years in the future. In forming his belief about mortality risk, a person might consult a life table based on the experience of millions of individuals. The life table provides an estimate of the mean probability of survival for persons of given age and sex with essentially no sampling error. However the individual may be ambiguous about whether this probability represents the risk he himself faces. He may have personal information that makes him think that he has a higher or lower risk than the average person of his age and sex. Moreover, he may be unsure about the influence that personal information such as the ages at death of parents and relatives, health history and current symptoms, or exercise and dietary habits will have on his likely longevity. Finally, in answering the survey question, a person must construct a probability judgment "on the fly," accessing whatever frequentist data or epistemic beliefs he may have stored in his brain and

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[1] Frank Knight, as early as in 1921, introduced the distinction between, as he called, "risk" for known and "uncertainty" for unknown probabilities.

[2] Paté-Cornell (1996, pp. 96-97) observes that *"... uncertainties in decision and risk analyses can be divided into two categories: those that stem from variability in known (or observable) populations and, therefore, represent randomness in samples (aleatory uncertainties), and those that come from basic lack of knowledge about fundamental phenomena (epistemic uncertainties also known in the literature as ambiguity)*."

manipulating this information through reasoning or gut reaction to produce an answer to the specific question about, say, the probability he will survive to age 80, all within less than a minute.[3]

In this paper, we assume that probability beliefs about survival are ambiguous in the sense that an individual has in mind a range of possible values of the probability that can be described by a density function, $g(p)$, that can take on a variety of shapes depending on both its mean and the degree of ambiguity. The theory literature calls this density function second order probability beliefs. (See, for example, Gilboa and Maracini, 2011).

We compare the predicted survival rates of our sample members to the actual survival of sample members eight years later, as reported in the 2010 wave of HRS. The predictions from our models track actual mortality fairly closely for sample members who are below age 80, but begin to diverge substantially at the oldest ages, with older respondents being overly optimistic. The predicted survival rates co-vary with demographic characteristics, health status, parental mortality, smoking behavior and cognitive status in largely the same way that they do in regressions that explain actual mortality. Thus, it appears that the subjective survival probability answers are good candidates for modeling individual level heterogeneity in survival chances.[4]

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[3] The attempt to determine the probabilistic beliefs of lay people using direct questions on surveys has only become commonplace in the past two decades (Manski, 2004). There is a related but somewhat separate tradition of eliciting probability beliefs of experts as part of risk assessments in engineering and operations research applications. See Paté-Cornell (1996) for a summary of this tradition.

[4] Our findings about the external validity of the HRS subjective probability questions are consistent with those in earlier papers that have studied these questions in the HRS (Hurd and McGarry, 2002; Smith et. al., 2001). The overly optimistic expectations of people over 75 in HRS has also been noted by Hurd et al. (2008) who find the same pattern across eleven European countries in the SHARE (Survey of Health, Ageing and Retirement in Europe). There are also a few papers that use these questions in models of behavior under uncertainty such as, for example, Picone, et al. (2004) who find that people who expect to live longer are more likely to choose medical screening tests.

The major contribution of this paper is to provide an estimate of ambiguity about survival probabilities that is embodied in the spread of $g(p)$. We find that survival expectations are very uncertain in the Health and Retirement Study (HRS) and that uncertainty varies in the population: more educated people have more certain beliefs, women have less certain beliefs, and deterioration of health, especially from previously excellent levels, leads to more uncertainty about survival chances.

Identifying second order probability beliefs is possible under the assumptions of our survey response model, the Modal Response Hypothesis (MRH) which is a mapping from probability beliefs $g(p)$ to survey responses. It assumes that people report the mode of $g(p)$ whenever it exists and they report 50%, whenever $g(p)$ is so ambiguous that it does not have a unique mode. The motivation for the MRH is twofold. First, imagine that people estimate the probabilities of certain events after observing some successes and failures of these events. Under some conditions a naïve estimator, which is the ratio of "good cases", is exactly equal to the mode of the Bayesian updated posterior probability distribution. Moreover, this estimator is biased in finite samples. The MRH assumes that people report this simple, naïve estimator in surveys. The second motivation comes from the literature using numerical subjective probabilities on surveys. When people are asked to report a numerical probability using any digit between 0 and 100, an unusually large fraction of answers are heaped on 50. The excessive use of 50 has been interpreted as occurring because many people treat "50-50" as a synonym for "I don't know" or even for "God only knows," a sentiment that suggests that the true probability is unknowable (Fischoff and Bruine de Bruin, 1999; Bruine de Bruin and Carman, 2012; Lillard and Willis, 2001).[5,6] Researchers have realized that these focal answers might introduce bias into

[5] In recent waves of the Health and Retirement Study, respondents have been asked a follow-up question if they answered 50 to survival probability question: Do you think that it is about equally likely that you will

estimates of subjective probabilities. As a remedy, Manski and Molinari (2010) think of focal responses as extreme versions of rounding, where some people always round to 0, 50 or 100 percent. Lumsdaine and Potter van Loon (2012) model the probability of providing a focal answer as a separate equation in their econometric model. These papers found that accounting for focal responses is important for valid inference. However, they are agnostic about the reasons so many people provide such answers. Our approach, instead, is to model focal responses with an economic model and relate it to the precision of beliefs of individuals. In our model people answer with an epistemic 50 percent as a way of saying "I am very unsure", when their beliefs are too ambiguous. As far as we know the MRH model is the first in the literature that makes use of the focal responses to learn something about the beliefs of individuals.

For comparison, we present an alternative model of survey response which assumes that the person reports the mean of $g(p)$. We show that the MRH model can account quite well for heaping at 0, 50 and 100 while the mean response model cannot. The overuse of focal responses can be especially problematic when very low or very high probabilities are modeled since the high ratio of 50 percent responses can arbitrarily push the estimated mean probabilities away from their true, extreme values. We show in this paper that our modal response model which explicitly models focal answers works better than the simple mean model typically used in research on subjective probabilities. When the modal response model is used for extreme probabilities both the estimated unconditional means and the average partial effects are closer to ones estimated from realized mortality data. Alternative models of focal responses, such as

die before 75 as it is that you will live to 75 or beyond, or are you just unsure about the chances? About two-thirds say that they are "just unsure." In this paper, we use data from the 2002 wave of HRS which did not have a follow-up question.

[6] In addition, the answers to the survival questions also exhibit some heaping at 0 and 100. Taken literally, of course, these answers cannot represent rational probability beliefs except, perhaps, in the case of zero for persons who know themselves to be terminally ill.

Manski and Molinari (2010) and Lumsdaine and Potter van Loon (2012) might work equally well to reduce this bias and to predict enough focal answers. The main advantage of the MRH is that it is based on an economic model of limited information and uncertain beliefs.[7]

Learning about the degree of uncertainty in individuals' beliefs can be important for several reasons. First, the value of information about mortality risk, for example, should be a function of this uncertainty. Uncertain people might value information more, and certain people might rationally ignore any new information since they have already established a good understanding of the risks they face. Even in a fully Bayesian Subjective Expected Utility (SEU) framework the degree of uncertainty might play an important role if the utility function is not linear in probabilities. This is the case, for example, if people can invest in learning about their own mortality risks.[8] Second, learning about the degree of uncertainty in individuals' beliefs is useful from a survey methodological point of view, too. People whose probability beliefs are more certain may answer probabilistic survey questions with greater precision. However, if beliefs are uncertain, as is the case with mortality expectations, we expect large measurement error in survey responses, that might even account for the discrepancy between subjective and objective survival probabilities at later ages. Future research should investigate the potential role of measurement error in subjective expectation data and its link to uncertainty in individuals' beliefs. Third,

[7] Manski and Molinari (2010) discuss an alternative, more direct approach to learn about the imprecision of belief. They make use of a follow-up question in a different survey (the ALP) asking about the range of probabilities responders had in mind when they provided their answers. Roughly half of the sample reported that their answers were "exact", and the rest reported relatively wide ranges of probabilities with the average width being 18 percent. We believe that this approach is very informative about general uncertainty in the population, but it is less obvious how to make use of the provided ranges. First, the meaning of an "exact" answer is ambiguous. Second, one would expect that 50 percent responders provide wider ranges of possible probabilities than people whose answers ended with 0 or 5 (like 10 or 15 percent). However, they did not find a strong pattern like that in their data.

[8] Another example is Bommier and Villeneuve (2012), who discuss a model of mortality risk aversion, where the utility function is not additively separable over time, and thus, mortality probabilities enter the model in a non-linear fashion as well.

learning about the degree of uncertainty in individuals' beliefs might be very important in Non-Bayesian/Non-Expected Utility models. While we don't work in these frameworks, empirical evidence on ambiguous beliefs may be useful to those who do.

The paper is organized as follows. In Section 1 we provide a quick overview of the literature on ambiguity and its role in economic decisions. Section 2 describes the subjective survival data in HRS that we use in this paper. Section 3 introduces the MRH model that can be used to model any subjective expectation data that uses the HRS framework. Section 4 describes a simple and tractable model of individual survival curves. Section 5 discusses the estimation method and identification and Section 6 shows the results.

1. Ambiguity in economics

Knightian uncertainty, epistemic uncertainty and ambiguity are roughly synonymous terms that figure prominently in a longstanding and ongoing debate about the link between rationality and probability beliefs, on the one hand, and the relationship between beliefs and decisions, on the other hand. In an excellent and authoritative review of this debate since Pascal's famous bet on the existence of God in 1670, Gilboa and Marinacci (2011) discuss the different models of expectation formation, including Bayesian and Non-Bayesian models. They call the model we use in this paper "the smooth model of ambiguity" or "second order probability beliefs." In typical models of ambiguity, agents have a set of possible probability distributions in mind but they are not able to compound this information into a single probability distribution. The smooth model, however, makes compounding possible. The real question in the smooth model is whether agents have a preference for known probabilities (in other words they are ambiguity-averse) or not (in which case they are simple Bayesians). As Gilboa and Marinacci (2011) write "... *beyond the above mentioned separation [between beliefs and utilities], the smooth preferences model enjoys an additional advantage of tractability. Especially if one specifies a simple functional form for* *[preferences for known probabilities], one gets a simple model in which uncertainty/ambiguity attitudes can be analyzed in a way that parallels the treatment of risk attitudes in the classical literature*.", pp. 45.

The major contribution of this paper is to provide an estimate of second order probability beliefs about survival probabilities that are embodied in the spread of $g(p)$. It is important to clarify the role of this object in alternative views of uncertainty. Conventional economic theory of behavior under uncertainty, rooted in subjective expected utility (SEU) theory (Savage, 1954), is often interpreted as having erased the distinction between known and unknown probabilities because expected utility is a linear function of probabilities. That is, assume that the person's ambiguous probability beliefs are described by $g(p)$. His expected utility is

$$EU = \int_0^1 \left[pU_{live} + (1-p)U_{die} \right] g(p)dp = \overline{p}U_{live} + (1-\overline{p})U_{die} ,$$

where $\overline{p}$ is the mean of this ambiguous distribution. Clearly, expected utility is invariant to a mean preserving spread of $g(p)$; hence, decisions based on expected utility are unaffected by ambiguity.

In the famous Ellsberg experiments (Ellsberg, 1961) subjects were presented with choices of drawing balls from different urns whose compositions were either known or ambiguous. Most people revealed distaste for ambiguity which was at odds with the SEU theory. The survey by Gilboa and Marinacci (2011) provides a comprehensive and insightful discussion of this.

We separate the issues concerning ambiguity of probability beliefs from those concerning the effects of epistemic uncertainty and ambiguity aversion on decisions. We do so by utilizing survey data that asks directly about people's probability beliefs. As Manski (2004) emphasizes, this approach differs from the practice in much of applied economics of assuming that individuals have exogenously given probabilities. It also differs from Savage's theory which infers probability beliefs from choice situations. This means that we can explore empirically how beliefs about risk and uncertainty vary in the population without being required to take a stand on how decisions are affected by probability beliefs.

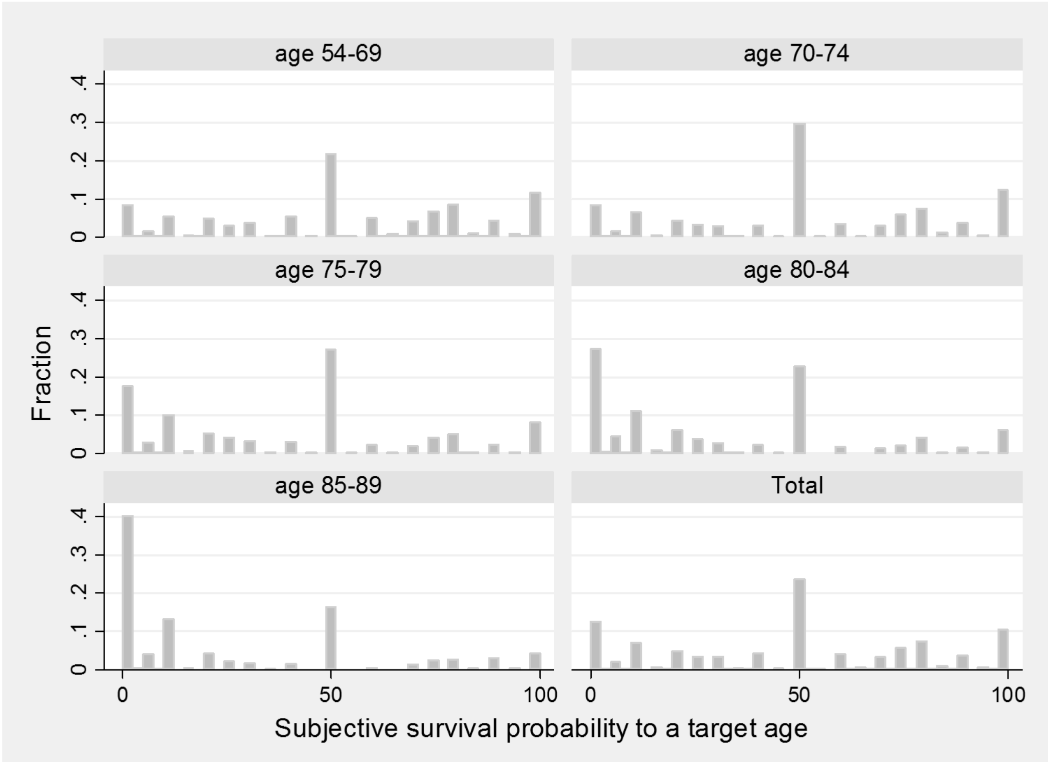
2. Subjective Survival Probability Questions on the HRS

The Health and Retirement Study has asked probabilistic expectation questions on various topics since its beginning in 1992. The survival question that we use in this paper comes from the 2002 wave and it reads as follows: "What is the percent chance that you will live to be [TARGET AGE] or more?" The target age exceeds the individual's age by at least 10 years: it is 80 years for people below 70, and 85, 90, 95 and 100 for individuals in successive five year age intervals.

Although the subjective probability responses in HRS seem reasonable when averaged across respondents, individual responses appear to contain considerable noise and are often heaped on values of "0", "50" and "100" (See for example Manski, 2004, for a discussion). Considering the whole group of probability questions in HRS-1998, for example, while only 5% of respondents refused to answer the probability questions, 52% of questions were heaped on either "0" or "100" and an additional 15% were heaped on "50".

These patterns are illustrated in Figure 1 by histograms of responses to the HRS-2002 survival probability question. We have included separate histograms by the target age used in the survival question and a total histogram in the six panels of Figure 1. Each histogram shows a high frequency of focal answers, especially at 50. The ratio of 50 responses is somewhat smaller at old ages where the actual survival probabilities are low, but it still accounts for 16 percent of the responses for people above 84.

Figure 1: Distribution of Survival Probabilities to Target Age, by Age of Respondent, HRS-2002



Note: Target age is 80 years for people below 70, and it is 85, 90, 95 and 100 for individuals in successive five year age intervals

Some psychologists, especially Fischhoff, Bruine de Bruin and their colleagues (Fischhoff and Bruine de Bruin, 1999; Bruine de Bruin et al., 2000; Bruine de Bruin and Carman, 2012) have argued that answers of "50" may reflect "epistemic uncertainty;" that is, a failure to have any probability belief at all about the event in question or, at least, to have no clear idea of what the probability could be. Alternatively, of course, an answer of "50" might reflect a very precise belief about the probability that a fair coin will come up heads or perhaps a somewhat less precise belief that a given event is about equally likely to occur or not occur. Indeed, while HRS probability questions offer participants a scale of integers from 0 to 100, the large majority of "non-focal" answers are integers ending in "5" or "0", suggesting that responses from most people involve rounding or approximation. See Manski and Molinari (2010) for a discussion of the different rounding practices of survey respondents in the HRS and of potential remedies.

There has been much less emphasis in the psychological literature on focal answers at "0" or "100". When a probability question concerns an event such as the chance of being alive ten or fifteen years from now, it does not seem credible to assume that a respondent who gives such an answer of "100" is completely certain he will be alive then and, apart from a person diagnosed with a terminal illness, an answer of "0" should not be taken at face value, either[9].

Previous researchers have found that the tendency to give focal answers is associated with low education, lower cognitive ability and it varies with other demographic variables, too. (Lillard and Willis, 2001; Hurd and McGarry, 1995; Lumsdaine and Potter van Loon, 2012). These covariates are known to correlate with mortality. Focal responses, thus, might bias estimated average survival probabilities if we take them at face value and the bias might be stronger in situations where the underlying probability is far from 50 percent. In order to test whether this is the case, in the empirical part of the paper we will compare estimated individual survival curves to actual survival of the respondents eight years later in 2010, which is the last available wave of the HRS.

3. Probability Beliefs and the Modal Response Hypothesis

In this section, we describe a theoretical model which attempts to relate answers that an individual gives to a survey question about the subjective probability of a given event and his underlying probability beliefs. In our model we distinguish between *ambiguity* and *epistemic uncertainty*, with the latter corresponding to cases when information is limited.

[9] It is possible to regard answers of "0" or "100" as approximations which are no different in kind than rounded answers of "5", "40" or "95". However, in a discussion of Gan, Hurd and McFadden (2005), Willis (2005) provides evidence against this interpretation.

Let us assume that person i is faced with the problem of estimating the probability p_i of an event A. Initially he has no information about the probability of this event, he has an uninformed prior $p_i^{prior} \sim U(0,1)$, where U denotes the uniform distribution. The person observes event A happening $\alpha_i - 1$ times and not happening $\beta_i - 1$ times. In the survival context, for example, this means that a person is aware of $\alpha_i - 1$ people similar to himself who survived to the given target age and $\beta_i - 1$ similar people who died before reaching that age. It is well known that if this new information is used to Bayesian update one's beliefs about p_i, the posterior distribution has a Beta distribution with parameters α_i and β_i, $p_i \sim Beta(\alpha_i, \beta_i)$.

When faced with a survey question about the probability of event A, the person might respond with the mean or the mode of this distribution. When α_i and β_i are larger than one[10], the mean and the mode of the Beta distribution are

$$p_i^{mean} = \frac{\alpha_i}{\alpha_i + \beta_i}, \tag{1}$$

$$p_i^{\mathrm{mode}} = \frac{\alpha_i - 1}{\alpha_i + \beta_i - 2}. \tag{2}$$

A Bayesian agent would report p_i^{mean} which is the expected value of the Bayesian updated posterior distribution. Note that p_i^{mode} is exactly equal to the naïve estimator of the probability that can be computed by the number of "good cases" which is $\alpha_i - 1$ over the number of all cases which is $\alpha_i + \beta_i - 2$. A frequentist agent, thus, would not report the mean but, rather, the mode

[10] Other cases will be discussed below.

of the distribution $g_i\left(p\right)$. The modal response hypothesis assumes that people report $p_i^{\text{mod}e}$ rather than p_i^{mean} to probabilistic survey questions for at least two reasons. First, as Lillard and Willis (2001) argue, it is cognitively less burdensome for a respondent to answer a survey probability question quickly by reporting the most likely value of p, given by the mode of $g(p)$, than it is to report the expected value given by $p_i^{mean} = \int pg_i(p)dp$. Second, $p_i^{\text{mod}e}$ is equal to a very simple rule-of-thumb estimator for the probability in question: the frequentist response. In a survey situation where people have to answer many questions in a very short timeframe, it seems a reasonable assumption that they give frequentist approximations to probability questions instead of Bayesian updating their priors. Moreover, in this model the mode is often a good approximation of the mean.

The formula in (2) does not give the mode of the distribution when either α_i or β_i is smaller than 1. Whenever $\alpha_i < 1, \beta \geq 1$ the distribution is always decreasing and has a unique mode at zero. Whenever $\alpha_i \geq 1, \beta < 1$ the distribution is always increasing and its unique mode is at one. Finally, if $\alpha_i < 1, \beta < 1$ the distribution has a U-shape and it has two maxima at zero and one. In this case one finds it more probable that the probability of event A is zero or one than that it is 50 percent.

We have motivated the use of the Beta distribution with a Bayesian framework where agents observe certain numbers of successes and failures. As we shall show, however, the distributions that occur when either α_i or β_i is smaller than 1 cannot be derived from Bayesian updating based on evidence. Indeed, these distributions correspond to situations in which, in effect, the agent has very little objective evidence on which to base his beliefs and, lacking evidence, tends to give conventional “epistemic “ responses to survey questions about his probabilistic beliefs. In particular, we hypothesize that the person will respond with an extreme value of either zero or one when $g(p)$ is monotonically decreasing or increasing. When the distribution is U-shaped, we hypothesize that the person will answer "50" as a synonym for "God only knows" rather than as necessarily a belief that the outcome in question is equally likely to occur or not.[11]

To show how the shape of $g(p)$ is related to the amount of evidence on which an individual bases his beliefs, we introduce two more parameters that are functions of α and β:

$$\mu_i = \frac{\alpha_i}{\alpha_i + \beta_i}, \tag{3}$$

$$n_i = \alpha_i + \beta_i. \tag{4}$$

μ_i is the expected value of the distribution of the probability in question, and n_i is a measure of the precision of beliefs. Higher n_i means more precise beliefs: that is, a tighter $g_i(p)$ density function. Earlier, we argued that an uninformed agent with a uniform prior over the unit interval would update his prior after observing $\alpha - 1$ successes and $\beta - 1$ failures in a sample of $N = \alpha + \beta - 2$. Note that $B(1,1)$ is a uniform distribution so that $n_i = \alpha + \beta = 2$ for an uninformed agent. Equivalently, such an agent observes no data since $N = 0$. Thus, a necessary condition for Bayesian updating is that the agent observes a positive amount of data, which implies that $\alpha > 1$, $\beta > 1$ and $N > 2$. As we have seen, any Beta function satisfying this condition is unimodal where the mode falls in the interval, $0 < p < 1$. Conversely, when $\alpha < 1$ or $\beta < 1$ $g(p)$ may be monotonically increasing, decreasing or U-shaped depending on the

[11] In the survival context, a U-shaped distribution could represent the beliefs of someone who is unsure whether he had inherited a genetically transmitted disease. In case he did, he might face a low survival probability to the target age, but if he did not he has a high probability of surviving. The posterior distribution of the survival probability in this case can have a U shape, where the extreme probabilities are more likely than any middle values. However, it is not plausible that such situations are common enough to account for the large number of "50" responses that we see in survey responses.

value of μ and if $\alpha+\beta<1$, $g(p)$ is always U-shaped. Obviously, Bayesian updating cannot be the source of such beliefs since one cannot observe a negative number of signals! That is why we label such beliefs as "epistemic" and distinguish them from "ambiguous" ones.

In our development of the Beta model, the precision parameter, $n_i=\alpha_i+\beta_i$, is assumed to be an integer equal to the size of the sample less two that is observed by agent *i*. A broader and more useful interpretation of precision is that it measures the confidence that an individual has in his judgment of the risk of a given event. For instance, educated individuals can utilize, in addition to their personal experience, a broader knowledge of evidence about mortality and its causes from past coursework, wider reading and better informed family and social networks. Thus, we may interpret precision as a measure of a person's capacity to assess his survival risks based on his knowledge of mortality risks and his ability to translate personal information about his own health, health behavior and family history into its implications for survival chances.

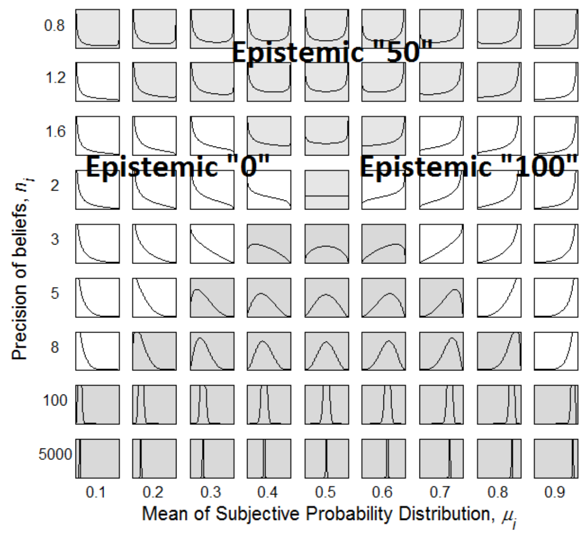
The relationship between n_i, μ_i and $g(p)$ is depicted in Figure 2. The figure presents a matrix of 81 probability density functions—$g(p\,|\,\mu_i,n_i)$—corresponding to nine different values of the mean of $g(p)$, given by μ_i on the horizontal axis, and nine different degrees of precision, measured by n_i, on the vertical axis. The figure also illustrates the boundary between ambiguous beliefs that can be represented by a second order probability distribution based on Bayesian principles and epistemic uncertainty in which the individual has too little knowledge about the risk in question to be able to form an evidence-based probability judgment. Possible ambiguous densities appear in the darkly shaded, inverted U-shaped region in Figure 2 for which $n_i>2$. Note that throughout this region, reports of p^{mean} and p^{mode} tend to be very close to one another. The lightly shaded area at the top of the figure corresponds to epistemic 50 percent responses. As we can see, when uncertainty is high (n_i is low) for any values of μ_i the model predicts a 50 percent response, including cases when the mean probability is very low or very high. The MRH, thus, predicts that the large fraction of 50 percent answers, typical in subjective probabilistic expectation data, can in fact correspond to mean probabilities that are far from 50 percent. The two unshaded triangular regions in Figure 2 correspond to epistemic 0 and 100 responses. Such responses are typical in cases when the mean probability, μ_i is also close to 0 or 100. However, when n_i is close to 2, that is beliefs are almost uniform, 0 and 100 percent answers can occur when μ_i is close to 50 percent. Thus, the MRH can predict a large fraction of focal 0, 50 and 100 answers and the corresponding bias can be either positive or negative.

To summarize, the modal response hypothesis claims that survey respondents report a potentially rounded version of the mode of $g_i\left(p\right)$ whenever it exists and they report 50 percent whenever $g_i\left(p\right)$ has a U shape,

$$p_i^{mrh} = \begin{cases} round\left(\dfrac{\alpha_i - 1}{\alpha_i + \beta_i - 2}\right) & \text{if} \quad \alpha_i > 1, \beta_i > 1 \\ 1 & \text{if} \quad \alpha_i > 1, \beta_i \leq 1 \\ 0 & \text{if} \quad \alpha_i \leq 1, \beta_i > 1 \\ 0.5 & \text{if} \quad \alpha_i \leq 1, \beta_i \leq 1. \end{cases} \tag{5}$$

The *round* function can be rounding to the closest 1 percent, 5 percent, 10 percent or anything that seems appropriate in the context of the survey. Manski and Molinari (2010), for example, use a framework where there are individual differences in rounding practices. Their approach can also be modeled in our framework by letting the *round* function vary across individuals.

Figure 2: Density of probability beliefs ($g_i(p)$) for different mean (μ_i) and precision (n_i) values



The hypothesis of this paper is that people answer subjective probabilistic expectation questions according to the MRH. It would be desirable, however, to test the MRH against other survey response models. A natural candidate for comparison is the mean model where people respond a potentially rounded version of the mean of $g_i(p)$

$$p_i^{mm} = round(\mu_i). \quad (6)$$

In the mean model the precision of beliefs (n_i) is not identified, only the mean (μ_i) is. The mean and the mode models, however, converge to each other as n_i goes to infinity. That is, the mean model is embedded in the MRH, and thus, a relatively small estimated belief precision (*n)* would be evidence in favor of the MRH.

4. Individual subjective survival curves

In the previous section, we introduced two survey response models that transform second order probability distributions $g\left(p \mid \mu_i, n_i\right)$ into the survey response p_i^{mrh} or p_i^{mm}. These models can be applied to any subjective probabilities in the HRS format and not just to survival data. To close the model, however, we need to specify the mean (μ_i) and the precision (n_i) of beliefs. There is no unique way of modeling these two variables; it is the task of the researcher to find the appropriate model in the context of the particular project. In this section, we show how μ_i and n_i can be modeled in the context of survival probabilities.

The so-called Gompertz model of longevity has been widely used in both demography and biology because its increasing mortality hazard assumption lines up with mortality data of humans and other species very well (Vaupel, 1997). The Gompertz model assumes that the hazard of death is exponentially increasing with age:

$$h\left(a\right) = \gamma_0 \gamma_1 \exp\left(\gamma_1 a\right), \tag{7}$$

where γ_0 is a positive scale and γ_1 is a positive shape parameter. By simple calculation, (7) leads to the following survival probability from age *a* to age *t*:

$$S\left(a,t\right) = \exp\left(-\gamma_0 \left(\exp\left(\gamma_1 t\right) - \exp\left(\gamma_1 a\right)\right)\right). \tag{8}$$

The main advantage of subjective survival data is that we can estimate individual heterogeneity in survival chances. With objective survival data we can only identify group-specific survival probabilities by computing the ratio of survivors in a particular group. Unobserved heterogeneity within groups, however, is not identified. In contrast, subjective survival data enables us to estimate individual heterogeneity in survival chances as we collect probability data on the individual level. We follow Vaupel (1979) by allowing the scale parameter (γ_0) to have a gamma distribution with shape parameter k and scale parameter θ and we assume that the shape parameter (γ_1) is fixed in the population:

$$\gamma_{0i} \sim \Upsilon(k,\theta). \tag{9}$$

The expected value of the gamma distribution is $k\theta$ and thus both parameters increase mortality chances and decrease the probability of survival (see equation (8)). The main advantage of using the gamma distribution, as we shall argue, is analytic tractability. Note, however, that this is a flexible 2 parameter distribution with both parameters being estimated, and hence this assumption is not very restrictive. The first advantage of the gamma distribution is that the average survival probabilities can be derived analytically. As we show in Online Appendix C2, in the gamma-gompertz model the average survival probabilities from age *a* to age *t* is

$$E\left(S_i\left(a,t \mid k,\theta,\gamma_1\right)\right)=\left(1+\theta\left(\exp\left(\gamma_1 t\right)-\exp\left(\gamma_1 a\right)\right)\right)^{-k}. \tag{10}$$

The second advantage of the gamma-gompertz framework is that we can analytically derive the effect of individual heterogeneity on sample selection. Different survival chances are modeled by letting γ_0 have a distribution in the population. In this paper we refer to γ_0 as "frailty" (Vaupel, 1979), which includes genetic, environmental and behavioral factors that affect the underlying mortality of individuals other than age. As long as survival chances are heterogeneous in a population, fit individuals will be overrepresented in the sample over time as frail individuals are more likely to die and not participate in the HRS. By applying the formula from Vaupel (1979) we can analytically characterize this sample selection. See the online appendix C1 for details.

To sum up, we use the following structural equations for individual survival chances.

$$\mu_i \equiv S_i\left(a,t\right)=\exp\left(-\gamma_{0i}\left(\exp\left(\gamma_1 t\right)-\exp\left(\gamma_1 a\right)\right)\right), \tag{11}$$

$$\gamma_{0i} \sim \Upsilon\left(k, \theta_i\right), \tag{12}$$

$$\theta_i = \frac{1}{\frac{1}{\theta^r} + \left(\exp\left(\gamma_1 a\right) - \exp\left(\gamma_1 r\right)\right)} \text{ and} \tag{13}$$

$$\theta^r = x_i^{'} \beta_\theta. \tag{13}$$

θ^r represents the scale parameter of the gamma distribution in the reference cohort which is the 50 year old ($r = 0.5$). Equation (13) shows the effect of sample selection. The scale parameter $\left(\theta_i\right)$ is decreasing with age, as fit individuals are increasingly overrepresented in the sample. Equation (13) shows that we add covariates to the scale parameter of the 50 year old cohort. Finally Online appendix C3 shows how average partial effects of the different covariates can be derived after fitting this model.

So far we have only talked about how to model the mean survival probability, μ_i. For modeling the precision of beliefs (n_i), we use a very simple log-normal framework.

$$\ln\left(n_i\right) = z_i^{'} \beta_n + u_{ni}, \tag{14}$$

$$u_{ni} \sim N\left(0, \sigma_n^2\right). \tag{15}$$

Equations (11)-(15) together with the survey response models of the previous section fully specify our model.

5. Estimation and identification

Our structural model has two unobservables: λ_{0i} which is a function of the mean survival probability and u_{ni} which is the unobserved heterogeneity in the precision of beliefs. Based on the distributional assumptions from the previous section, the model is fully specified and it can be estimated with maximum likelihood.

We only observe one survival probability answer in HRS. In Section 3, we proposed two survey response models. The mean model assumed that people report a rounded version of their true survival chances, while the MRH model assumed that people report a rounded version of the mode of the distribution of probability beliefs $g\left(p \mid \mu_i, n_i\right)$ or 50 percent when the mode does not exist.

The joint distribution of the two random variables λ_{0i} and u_{ni} is complicated because one is gamma, the other is normal and they enter the model in a non-linear fashion. The estimation of the MRH, thus, can be carried out by maximum simulated likelihood. The estimation of the mean model is more straightforward as the precision of beliefs plays no role in the model. Online Appendix C4 and C5 show how the likelihood function of these two models can be constructed.

It is worth discussing how our main parameters are identified. We seek to estimate the following set of parameters: $\lambda_1, k, \beta_\theta, \beta_n, \sigma_n$. In the case of the mean model, parameters of the belief precision, β_n and σ_n are not identified. Parameter λ_1 is the shape parameter of the individual survival function and it is identified from how fast the probability responses (p_i^{hrs}) change with age $\left(\partial E\left(p_i^{hrs} \mid a\right) / \partial a\right)$. Parameters k and β_θ determine the scale parameter of the individual subjective survival curves (λ_{0i}) and they are identified from the location and dispersion of the individual responses ($E\left(p_i^{hrs} \mid x_i\right)$ and $V\left(p_i^{hrs} \mid x_i\right)$).

The identification of the belief precision parameters β_n and σ_n primarily comes from the fraction of different focal answers in different demographic groups. If we have many focal answers, we expect n_i to be small. If we have many different types of focal answers (0, 50 and 100), we expect a high σ_n, indicating a high dispersion of belief precision in the population.

6. Empirical Analysis

Beliefs about subjective survival probabilities presumably depend on an individual's knowledge of different risk factors and demographic differences in the society; his personal information about his own health, habits and family members' longevity; his ability to translate this information into a probability and on his level of optimism or pessimism. In the empirical model estimated in this section, we use the information available in the HRS to try to capture several of the major determinants of beliefs in a parsimonious fashion.

6.1. Sample and Measures Employed

The sample used in the empirical analysis consists of 13,038 age eligible respondents to the 2002 Health and Retirement Study, over age 54 in 2002,[12] who provided responses to the subjective survival probability question. Excluded from the sample are proxy respondents and non-respondents in the 2000 wave of data collection. Also excluded are persons over 90 who were not asked the survival probability questions and people for whom we did not have realized survival information in 2010.[13] Table A1 in Appendix A presents descriptive statistics for the variables used in our analyses. The average age of our sample members is just over 68 years (ranging from 54 to 89 years) and on average the target age was 16 years from their current age. The modal sample member is a white female with a high school education although there is substantial variance in each of these dimensions.

Our theory suggests that people may utilize personal information in forming their subjective survival beliefs. Parents' age at death, one's own current health status and health behavior are such variables. As Table A1 shows, 16 percent of our sample still has a living mother but only 5

[12] The 2002 HRS is a representative panel sample of the 54+ population and their spouses.

[13] Actual mortality of HRS respondent is very precisely measured from administrative data (the National Health Index) and it is even available for people who dropped out of the survey in a later wave.

percent has a living father. The average age at death of the mothers is 76 years while the corresponding number for the fathers is 72 years. We construct six variables about parental mortality. Three of them correspond to the mortality of the same sex parent (father-son, mother-daughter pairs) and three correspond to the opposite sex parent. Within each pair we first take the age at death of the parent if he/she is dead. If (s)he is alive we impute the expected age at death of the parent based on his/her gender and age. In all models we include dummy variables about whether the parents are alive to control for potential imputation bias. Finally we create a linear spline from the imputed parental mortality with a single cut-off point at the age of the interviewee. This approach is motivated by the idea that individuals might consider parental mortality less informative about their own survival chances if they have already lived longer than their parents.[14] These four splines (two for each parent) together with the two dummies constitute our six parental mortality variables.

We also include in our analysis three sets of variables on health related behavior. As Table A1 shows, 43 percent of our sample reports regular exercise at least three times a week. While only 14 percent of the sample smoked in 2002, almost 60 percent reported having smoked in the past. There is a big variation in the sample in drinking behavior. Roughly half of our sample (48 percent) reports that they drink alcohol sometimes, but the majority are not regular alcohol consumers. Among those who are, the average number of days when they drink is 3.4 days a week, and the average number of alcoholic beverages consumed is 1.9.

Self-rated health in the HRS is measured on a five-point scale--1) Excellent, 2) Very Good, 3) Good, 4) Fair and 5) Poor. We translate these into three categories: 1) excellent/very good; 2)

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[14] Simple exploratory work suggested that parental mortality has a stronger effect on expectations when the parent died at an old age. The specification we use in this paper is not the only way to allow for such non-linearity and future work should explore alternative models.

good; 3) fair/poor[15]. We then construct dummy variables representing the combination of self-rated health in 2000 and 2002 for each respondent with excellent/very good in both years as the baseline case. In Table A1 we only show the marginals of this joint distribution. As we can see the fraction of people in "excellent/very good" health decreased from 47 percent in 2000 to 43 percent in 2002. The fraction of people in "good" health did not change much (31 and 32 percent) and the fraction in "fair/poor" health increased from 22 to 25 percent.

We also include two cognitive measures ("Vocabulary" and the "27-point cognitive capacity scale"[16]), and the CESD depression score which measures depressive symptoms.[17] Table A1 shows that the average member of our sample has higher cognitive and lower depression scores than the average HRS respondent.

6.2 Maximum Likelihood Model Estimates

The objective of this section is to test the modal response hypothesis (MRH) through a series of performance tests. All the results we present in this section are based on the twelve estimated models shown in Table A2 in Appendix A and B1, B5 and B9 in Online appendix B. Table A2 reports models without covariates, Table B1 adds basic demographic information, Table B5 expands the model with cognition, personality and parental mortality and finally Table B9 contains all variables including subjective health. All tables present three models. The first columns show actual eight year survival of the HRS respondents between 2002 and 2010 estimated with nonlinear least squares. The second columns shows results using subjective

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[15] These three categories represent relatively good, average and relatively bad health. The reason for the aggregation is that we interact subjective health in 2000 and 2002 and adding 5 x 5 = 25 interactions would make the interpretation of these variables difficult.

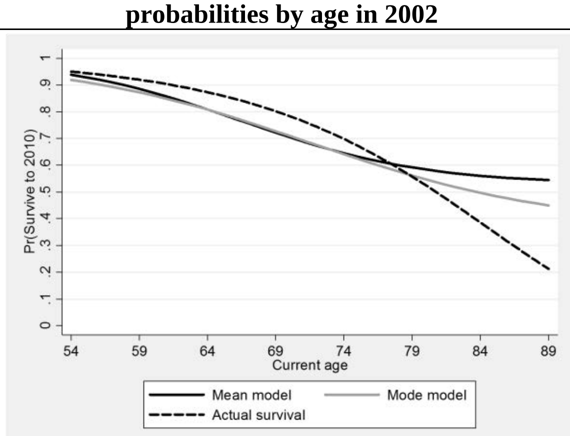
[16] The 27-point scale Langa-Weir method is discussed in Crimmins, et al. (2011). HRS cognitive measures are described in Fisher, et al. (2012).

[17] See Ofstedal et. al. (2002).

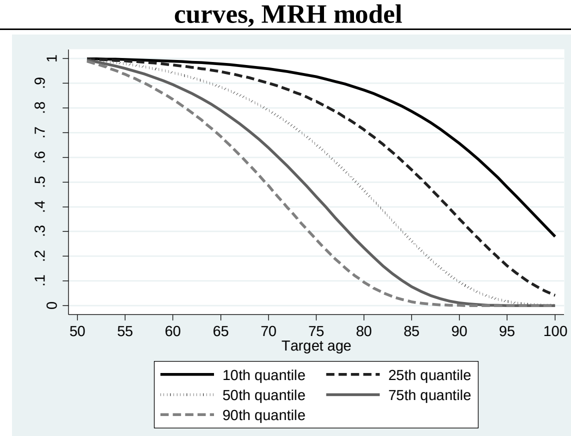
expectations based on the mean model and the last two columns show results from the MRH. All the parameters of our model ($\gamma_0, \theta^{50}, n, \gamma_1, k, sd(n)$) are assumed to be positive and thus their logarithms enter the likelihood function. Covariates potentially enter two equations. The first is the equation of θ^{50} which is the scale parameter of the gamma distribution of the mortality hazard at the age of 50. In the actual survival models we model γ_0 directly. Covariates with positive coefficients are estimated to increase the mortality hazard and decrease the survival chances. The magnitudes of these coefficients will be analyzed later when we derive average partial effects of them on various survival probabilities. The second equation where covariates appear is the equation of the precision of beliefs (n). Positive coefficients mean tighter, more precise probability beliefs.

Figure 3: Average actual and expected survival probabilities and dispersion in beliefs

Panel A: 8 year actual and expected survival probabilities by age in 2002



Panel B: Heterogeneity in the subjective survival curves, MRH model

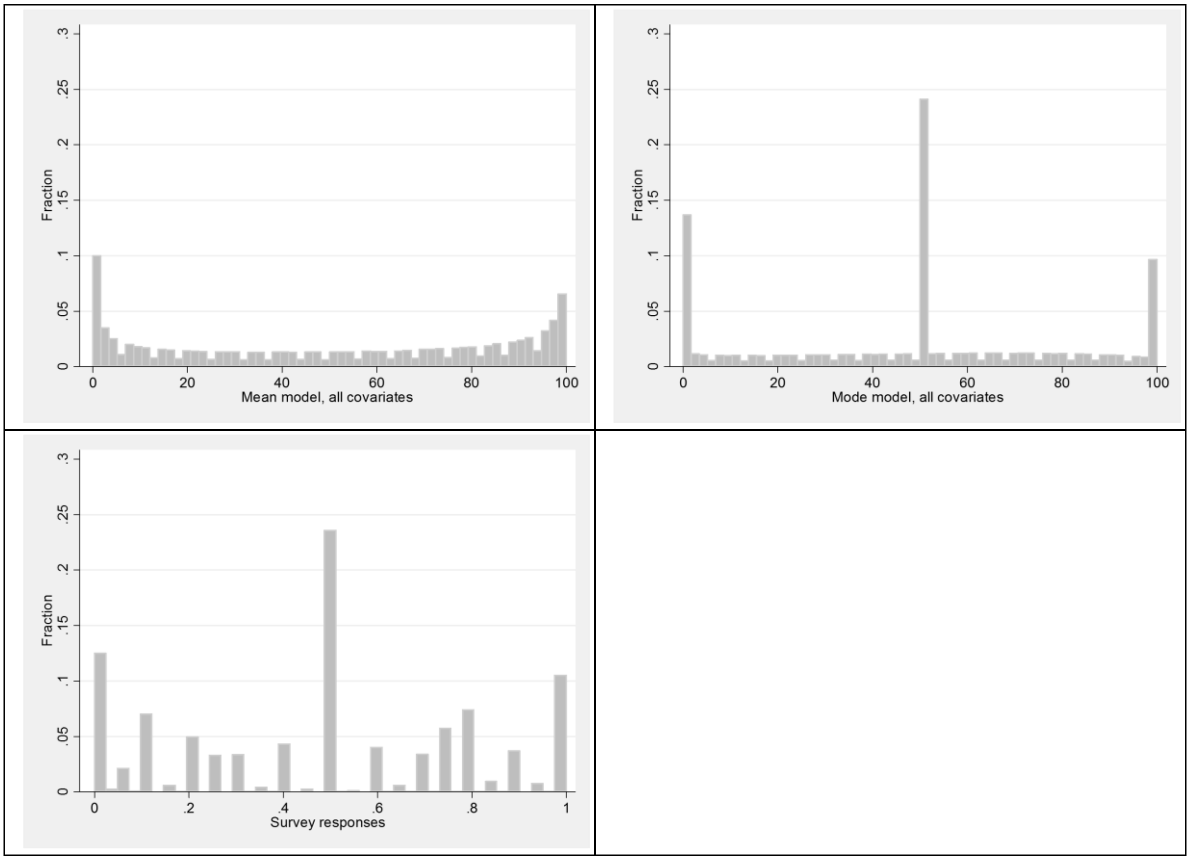


Panel A of Figure 3 compares estimated actual 8 year survival probabilities of HRS respondents to subjective survival beliefs in 2002 computed from the models in Table A2 with no covariates. The horizontal axis shows the current age of respondents in 2002 and the vertical axis shows the fitted average 8 year survival chances from the three models. As we discussed in Section 3, heterogeneity in survival chances leads to sample selection as people with better fitness are more likely to survive and become respondents of the HRS survey at older ages. In the case of realized survival, only the interpretation of the estimates changes, but we do not need to make any further adjustments of the parameters. The demography literature calls survival tables of this sort "Survival probabilities of the survivors." In the case of subjective survival chances, however, we do have to properly adjust for the unmeasured genetic and environmental differences of cohorts as discussed in Section 4. As we can see, both the mean and the MRH model track the actual survival chances very well up until about age 84, when the subjective probabilities become too optimistic. While the 8 year actual survival chance of a 90 year old is roughly 20 percent, the corresponding numbers in the MRH and mean models are ~45 and 55 percent, respectively. Thus, although the MRH model provides numbers that are closer to the true survival chances at old ages, these numbers are still too large on average. It is not obvious, however, whether these overly optimistic numbers are biases in people's heads or biases due to measurement error in the survey. In this paper we do not try to separate these two types of bias and we simply compare the mean, the mode and the actual survival models using the raw data.

Panel B of Figure 3 shows the estimated heterogeneity of survival chances in our sample. The different curves correspond to different values of γ_0 with lower values meaning better fitness. As we can see, there is notable variability in survival chances. For example, the difference in median survival (i.e., half-life) between those in the 10th and 90th percentiles of the estimated frailty distribution is about 25 years. That is, comparing two groups of 50 year olds, half of those in the 90th percentile are expected to survive to age 70 while among those in the 10th percentile half are expected to survive to age 95. Using only mortality data, one cannot identify the unobserved heterogeneity in survival chances.[18]

[18] There is a long history of discussion about the difficulty in separately identifying duration effects and unobserved heterogeneity. See, for example, Vaupel (1979) and Heckman and Singer (1984). Using subjective survival data, however, identifying unobserved heterogeneity in frailty is easier, because we

Figure 4: Simulated survey responses based on the mean and the mode models with all covariates and the empirical distribution of survey responses



The reason the MRH model is somewhat better than the mean model in predicting low probability events is that the high fraction of 50 percent answers are allowed to be focal answers that do not arbitrarily push the mean survival chances up. To visualize this effect we simulated survey responses based on the estimated models of subjective survival chances in Table B9. In order to get precise numbers, we used 651.900 observations for simulation which is 50 times the size of our dataset ($651,900 = 50 \times 13,038$). As we can see in Figure 4, the MRH model is able to predict histograms of responses that are very similar to the histogram of actual responses in the

observe probabilities of survival on the individual level. This contrasts with the use of mortality data, where it is hard to know which survivor is fit and which is simply luckier than other non-survivors. Even though we use a particular functional form for how unobserved heterogeneity enters the model (the gamma-gompertz framework) it is important to note that these functional form assumptions are not needed for identifying unobserved heterogeneity in subjective frailty.

bottom panel. The ratio of 50 percent answers is around 25 percent, while the ratio of 0 and 100 percent answers are both around 10 percent. What is more important, the MRH model recognizes that the high fraction of focal answers should not be taken at face value as a large fraction of them only reflect imprecise knowledge. The mean model, however, takes all the focal answers at face value. Consequently, the mean model is not able to predict a histogram similar to actual responses, and it seriously biases the estimation of low or high probability events.

Much of a person's personal information about health is likely embodied in his assessment of the level and trajectory of self-rated health. To explore the effects of other covariates, we present estimates of models with and without subjective health in Table 2 and 1 respectively. Both tables show estimated average partial effects of surviving from age 55 to75 and from 75 to 95. Online appendix B contains more detailed versions of these tables and further specifications, including models with only demographic variables and partial effects of surviving from 2002 to 2010.

The tables show that the majority of the coefficients are smaller in absolute value when subjective as opposed to objective information is used, but the coefficients based on the MRH are closer to objective values. This pattern is more obvious for low probability events (surviving from age 75 to 95), as the coefficients are roughly two times as big in the MRH. Thus, not only the average survival probabilities (Figure 3), but the average partial effects are also closer to objective values compared to the mean model. We take it as evidence that the MRH model is more successful for modeling subjective probabilistic expectations, particularly for low probability events, because the bias from focal answers are explicitly modeled.

Table 1: Average partial effects of surviving from age 55 to 75 and from 75 to 95 in three models with demographic, personality and personal information variables: actual survival and the mean and MRH models of subjective survival expectations

	55-75			75-95		
	Actual survival	Mean	MRH	Actual survival	Mean	MRH
Demographics						
Education	-0.003**	0.009***	0.009***	-0.004**	0.003***	0.005***
Female	0.073***	0.035***	0.051***	0.097***	0.012***	0.029***
Black	0.043***	0.031***	0.07***	0.058***	0.011***	0.04***
Hispanic	0.055***	-0.052***	-0.04**	0.073***	-0.019***	-0.023**
Cognition and personality, standardized scores						
Cognition	0.055***	0.02***	0.019***	0.073***	0.007***	0.011***
Vocabulary	0.003	0.003	-0.003	0.004	0.001	-0.002
Depression	-0.031***	-0.053***	-0.06***	-0.041***	-0.019***	-0.034***
Parents' age at death, linear splines with cut-off at own age						
Same sex, sp1	0.026	0.068	-0.048	0.035	0.024	-0.028
Same sex, sp2	0.159***	0.314***	0.415***	0.21**	0.112***	0.239***
Same sex lives	0.011	0.001	0.006	0.014	0	0.003
Opp. sex, sp1	0.038	0.186***	0.156***	0.051	0.066***	0.09***
Opp. sex, sp2	0.144**	0.097**	0.116**	0.191**	0.035**	0.067**
Opp. sex lives	0.028	0.007	0.007	0.038	0.002	0.004
Health related behavior						
Exercises	0.096***	0.054***	0.061***	0.128***	0.019***	0.035***
Ever smoked	-0.076***	-0.02***	-0.015**	-0.101***	-0.007***	-0.008**
Smokes now	-0.097***	-0.059***	-0.06***	-0.128***	-0.021***	-0.035***
Ever drinks	0.056***	0.044***	0.033***	0.074***	0.016***	0.019***
# of days drinks	0.007***	0.004*	0.005**	0.01***	0.001*	0.003**
# of drinks	-0.012***	-0.009**	-0.008**	-0.016***	-0.003**	-0.004**

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table 2: Average partial effects of surviving from age 55 to 75 and from 75 to 95 in three models with demographic, personality, personal information and subjective health variables: actual survival and the mean and MRH models of subjective survival expectations

	55-75			75-95		
	Surv	Mean	MRH	Surv	Mean	MRH
Demographics						
Education	-0.006***	0.006***	0.004***	-0.007***	0.003***	0.003***
Female	0.067***	0.026***	0.039***	0.086***	0.01***	0.024***
Black	0.051***	0.046***	0.086***	0.067***	0.018***	0.053***
Hispanic	0.071***	-0.04***	-0.019	0.092***	-0.016**	-0.012
Cognition and personality, standardized scores						
Cognition	0.05***	0.009**	0.008**	0.064***	0.004**	0.005*
Vocabulary	0.002	0.003	-0.003	0.003	0.001	-0.002
Depression	-0.009***	-0.026***	-0.031***	-0.012***	-0.01***	-0.019***
Parents' age at death, linear splines with cut-off at own age						
Same sex, sp1	0.019	0.104**	-0.022	0.025	0.041**	-0.014
Same sex, sp2	0.122**	0.268***	0.375***	0.157**	0.105***	0.231***
Same sex lives	0.011	-0.001	0.002	0.014	0	0.001
Opp. sex, sp1	0.043	0.206**	0.155***	0.056	0.081***	0.095***
Opp. sex, sp2	0.093	0.061	0.1**	0.12	0.024	0.061**
Opp. sex lives	0.041	0.006	0.003	0.053	0.002	0.002
Health related behavior						
Exercises	0.066***	0.025***	0.03***	0.086***	0.01***	0.019***
Ever smoked	-0.067***	-0.005	-0.002	-0.086***	-0.002	-0.001
Smokes now	-0.093***	-0.053***	-0.056***	-0.121***	-0.021***	-0.034***
Ever drinks	0.045***	0.021***	0.013*	0.059***	0.008**	0.008*
# of days drinks	0.005**	0.002	0.003	0.007**	0.001	0.002
# of drinks	-0.01**	-0.009**	-0.007**	-0.013**	-0.003**	-0.004*
Subjective health in 2000/2002						
Excel./excel.	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*	*ref.*
Excel./good	-0.095***	-0.063***	-0.071***	-0.123***	-0.025***	-0.044***
Excel./poor	-0.177***	-0.118***	-0.137***	-0.23***	-0.046***	-0.084***
Good/excel.	-0.05***	-0.05***	-0.055***	-0.064***	-0.02***	-0.034***
Good/good	-0.095***	-0.097***	-0.113***	-0.124***	-0.038***	-0.069***
Good/poor	-0.192***	-0.17***	-0.173***	-0.248***	-0.067***	-0.106***
Poor/excel.	-0.124***	-0.1***	-0.108***	-0.161***	-0.04***	-0.066***
Poor/good	-0.142***	-0.139***	-0.146***	-0.183***	-0.055***	-0.09***
Poor/poor	-0.209***	-0.237***	-0.263***	-0.27***	-0.093***	-0.162***

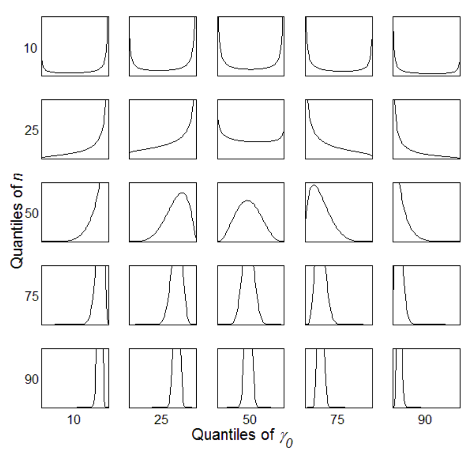
***, ** and *** denote significance at 10, 5 and 1 percent level**

However, the majority of the coefficients are still smaller in absolute value in the MRH than in the actual survival models. There are two potential explanations for this. Either there is measurement error in the data or people are underestimating the effect of some variables when forming expectations about their longevity. Without further assumptions, measurement error, other than focal responses, is not identified in our model. Note, however, that not all coefficients are attenuated equally. In fact two coefficients are systematically higher in absolute value in the subjective models: same sex parents' age at death and depression. This is consistent with a model in which agents base their expectations on easily observable determinants of mortality (parent's survival) because they have limited information on demographic differences in the society and on the role of different behavioral factors, such as smoking and exercising, on survival. In this case the partial effects of personal information is expected to be higher for subjective than for objective survival; and the pattern is expected to be the reverse for other variables. This is exactly what we see in the data.

Table 1 and 2 also show that people are aware that regular exercising is beneficial, and smoking is harmful for them, although they underestimate the role of these factors. We can also see that regular but limited alcohol consumption is not damaging, while teetotalling and heavy drinking is unhealthy. Even more interesting is the comparison of the models with and without controlling for subjective health. Not surprisingly both the objective and subjective survival probabilities are less affected by behavior when subjective health is controlled. However, the subjective values shrink more strongly. For example, as Table 1 shows those who quit smoking expect a 2 percentage point lower chance of surviving from age 55 to 75 compared to those who never smoked. However, this effect disappears when subjective health is taken into account (Table 2). It means that healthy quitters falsely believe that their survival chances are the same as those who never smoked; whereas quitters who had already acquired a disease understand its consequences.

It is also worth noting the demographic differences in subjective and objective survival. As expected, females are more likely to live longer and this is reflected in their subjective expectations. Racial differences, however, are more complex. Conditional on education, personal information, health and behavior, blacks and Hispanics have a higher chance of survival than whites in this age range. African Americans' expectations reflect this difference, but Hispanics are more pessimistic than Non-Hispanics. Table B2-B4 in the online appendix show roughly similar patterns when only demographic variables enter the model. Finally even though the educated are more likely to live longer (Table B2-B4), when personality, personal information and behavior are controlled, the effect of education becomes negative (!). Further investigation, not shown in the paper, shows that this result is driven by the cognitive capacity variable. Education and cognition has a strong positive correlation but cognition is a better predictor of survival than education. Moreover, the educated also have better subjective health, and thus the effect of education becomes even more negative when health is controlled (Table 2). In the meantime the educated believe to have a better chance of surviving independent of the control variables used.

Figure 5: Estimated distribution of probability beliefs $g_i(p)$ of surviving from age 50 to age 80



Finally let us take a look at the estimated distribution of probability beliefs (second order probability distribution, $g_i(p)$) of HRS respondents. Based on the MRH model without covariates (Table A2) we computed the 10th, 25th, 50th, 75th and 90th quantiles of belief precision (n) and the scale parameter of the survival function (γ_0) for the cohort of age 50. These numbers can be found in Table A3. Figure 5 shows the corresponding probability belief distributions of the probability of surviving from age 50 to age 80. As we can see, there is an enormous heterogeneity in probability beliefs. The median responder in HRS (3rd row and 3rd column of Figure 5) has a belief distribution that is single peaked but wide, having significant probability mass for any possible probability values between zero and one. It means that although the median responder's best guess for the probability of surviving from age 50 to age 80 is roughly 50 percent, she is quite unsure about this probability. People with even less precise beliefs are very unsure. For example, already at the 25th percentile of belief precision (where $n = 1.58$) everyone provides a focal response of either 0, 50 or 100. At the 10th percentile (where $n = 0.4$) everyone has U-shaped beliefs and, thus, responds with an epistemic 50%.[19] As we increase belief precision to the 75th percentile, the second order probability belief distribution becomes quite tight, having most of its mass in the neighborhood of the mean probability. It means that at least 25 percent of the respondents have very precise beliefs about their own survival chances.

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[19] Note, however, that the particular shape of the distribution of beliefs is only identified from the lognormal functional form in the region where *n* is below 1. Because there are many focal responses in the HRS data, the model estimates many uncertain responses where $n < 1$. It is hard to know, however, what the distribution of n looks like, conditional on being below 1. The log-normality assumption might or might not describe this conditional distribution well. It is possible, for example, that no-one has U-shaped beliefs, but all epistemic 50 responses come from a uniform distribution. If that is the case, then the log-normality assumption of n is inappropriate.

Table 3: Predictors of uncertainty in the MRH models of subjective survival expectations

	ln(n)		
	[1]	[2]	[3]
Demographics			
Education	0.056***	0.051***	0.053***
Female	-0.139***	-0.156***	-0.157***
Black	0.053	0.041	0.02
Hispanic	0.188*	0.155	0.132
Cognition and personality, standardized scores			
Cognition		0.021	0.027
Vocabulary		0.008	0.009
Depression		0.067***	0.043*
Parents' age at death, linear splines with cut-off at own age			
Same sex, sp1		-0.493*	-0.493*
Same sex, sp2		0.467	0.5
Same sex lives		-0.064	-0.071
Opp. sex, sp1		-0.122	-0.117
Opp. sex, sp2		0.528	0.522
Opp. sex lives		0.026	0.032
Health related behavior			
Exercises		0.015	0.02
Ever smoked		-0.091*	-0.091*
Smokes now		0.007	0.006
Ever drinks		0.174***	0.177***
# of days drinks		-0.014	-0.015
# of drinks		-0.035	-0.033
Subjective health in 2000/2002			
Excel./excel.			*ref.*
Excel./good			-0.172**
Excel./poor			0.122
Good/excel.			-0.092
Good/good			-0.05
Good/poor			-0.059
Poor/excel.			-0.106
Poor/good			-0.042
Poor/poor			0.177**
Age and time horizon of the HRS question; variables divided by 100			
Age	-2.513*	-2.26	-2.333
Horizon	-7.503	-9.823	-10.037*
Age X Horizon	25.438**	27.698***	27.82***
Constant	1.572*	1.845*	1.882**

***, ** and *** denote significance at 10, 5 and 1 percent level**

Determinants of belief precision appear in Table 3 and the last columns of Table B1, B5 and B9 in Online Appendix B. Positive coefficients mean tighter, more precise beliefs. As we can see more educated people have more certain beliefs. This is consistent with our hypothesis, discussed in Section 3, that more educated people may have a broader knowledge of evidence about mortality and its causes. We can also see that the deterioration of health, especially from previously excellent levels, leads to more uncertainty about survival chances, perhaps because of uncertainty about the future course of a new disease. Those who were in poor health both in 2000 and in 2002, however, hold the most certain and pessimistic beliefs about their survival chances.

The effect of age and the time horizon of the survival question in HRS have complicated relations to uncertainty. For a fixed time horizon, the net effect of age on uncertainty is negative, because the interaction term dominates for any time horizon values used in HRS (from 11 years to 26 years). Thus, older people seem to have more precise beliefs about their survival chances which might reflect learning. For a fixed age, the net effect of the time horizon on uncertainty is also negative, because the interaction term dominates again. This means that people hold more precise views about their long run than their short run survival chances.

The two splines measuring same sex parental mortality in Table B5 and B9 shows a "V" shape. It means that people are the most unsure about their survival chances when they are around the age when their same sex parent died. We can also see that active smokers and those who have already quit have less precise survival expectations compared to those who never smoked; infrequent alcohol consumption leads to more precise beliefs; women are less sure than men and depressed people are relatively more certain about their otherwise poor survival expectations.

Conclusion

The modal response hypothesis is used in this paper as the foundation for an econometric model that is intended to provide a mapping between survey responses to probability questions and the underlying subjective probability beliefs of individuals about their chances of surviving to a target age. In this paper, we have presented the MRH as a hypothesis designed to capture the kinds of "gut response" to such questions that would be made after about 15 seconds of consideration by persons who vary in the amount of information they have about actuarial risks to health, about their own health-related circumstances and in their capacity to process such information into subjective beliefs. We argued in Section 3 that reporting the mode is relatively easier from a cognitive point of view than the mean or the median; the mode is equal to a very simple rule-of-thumb estimator for the probability in question; and that the mode often provides a good approximation to the expected probability that is called for in *SEU* theory.

Our empirical findings suggest that there is considerable heterogeneity in subjective survival risks, some of it associated with age, sex, race, education, health related behavioral factors, parental mortality and cognitive capacity. We have shown that subjective survival expectations line up with actual mortality very well when the objective probabilities are moderate. The subjective survival probabilities, however, become overly optimistic at old ages when the true survival probabilities are relatively low. We have shown that the MRH model does a better job compared to a standard mean model in reducing this bias as the MRH models focal answers in an explicit way. It remains for future research to learn whether the overly optimistic subjective expectations are biases in individuals' head, potentially having behavioral consequences, or they are a result of survey measurement error, potentially being related to uncertain beliefs.

In the empirical section of this paper, we have also found substantial uncertainty about mortality risks which is manifested by considerable spread in the estimated distribution of subjective survival probabilities for a typical respondent. In addition, we found significant variation in uncertainty, holding expected survival risk constant. It remains for future work to explore the explanation of these findings more deeply and to see whether survival risk and uncertainty about this risk play a role in decisions made by HRS respondents.

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Appendix A: Tables and figures

Table A1: Descriptive statistics, HRS-2002

	mean	sd
Alive in 2010	0.77	0.42
Subjective survival probability to target age	48.40	32.13
Age	68.15	8.69
Target age less actual age	15.97	4.17
Female	0.59	0.49
Black	0.12	0.33
Hispanic	0.06	0.24
Years of education	12.54	3.00
Mother is alive	0.16	0.37
Mother's age of death/100 or current age	0.76	0.15
Father is alive	0.05	0.23
Father's age of death/100 or current age	0.72	0.14
Exercises at least 3 times a week	0.43	0.49
Ever smoked	0.59	0.49
Smokes now	0.14	0.34
Ever drinks alcohol	0.48	0.50
# of days a week when drinks alcohol	1.10	2.08
# of days a week when drinks alcohol if positive	3.42	2.34
# of drinks when drinks alcohol	0.61	1.18
# of drinks when drinks alcohol if positive	1.92	1.36
Health excellent / very good, 2002	0.43	0.49
Health good, 2002	0.32	0.47
Health fair / poor, 2002	0.25	0.43
Health excellent / very good, 2000	0.47	0.50
Health good, 2000	0.31	0.46
Health fair / poor, 2000	0.22	0.42
Cognition score, std.	0.08	1.01
Vocabulary score, std.	0.11	0.97
CESD depression score, std.	-0.04	1.04
N	13038	

Table A2: Actual survival until 2010 and the mean and MRH models of subjective survival expectations, models without covariates

	Actual survival	Mean model	MRH
$\ln(\gamma_0)$	-8.404		
	[0.21]***		
$\ln(\theta^{50})$		-11.174	-8.827
		[0.150]***	[0.169]***
ln(n)			1.978
			[0.040]***
$\ln(\gamma_1)$	2.277	2.73	2.397
	[0.024]***	[0.012]***	[0.020]***
ln(k)		-0.656	0.121
		[0.011]***	[0.025]***
ln(sd(n))			0.814
			[0.025]***
N	13038	13038	13038
Log-likelihood		-57961.459	-47058.606

*, ** and *** denote significance at 10, 5 and 1 percent

Table A3: Quantiles of belief precision (n) and probabilities of surviving from age 50 to age 80

quantiles	n	γ_0	S(50,80)
10	0.40	0.000022	0.87
25	1.58	0.000054	0.71
50	7.23	0.000120	0.47
75	33.12	0.000229	0.23
90	130.31	0.000370	0.10

Online Appendix (not to shown in the paper)

Appendix B: Supplemental Tables

Table B1: Outputs of the ML estimation models with demographic variables; actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH	
	$\ln(\gamma_0)$	$\ln(\theta^{50})$	$\ln(\theta^{50})$	$\ln(n)$
Demographics				
Education	-0.048	-0.13	-0.089	0.056
	[0.006]***	[0.007]***	[0.005]***	[0.009]***
Female	-0.343	-0.193	-0.209	-0.139
	[0.033]***	[0.037]***	[0.027]***	[0.051]***
Black	0.037	-0.007	-0.151	0.053
	[0.055]	[0.053]	[0.038]***	[0.076]
Hispanic	-0.157	0.329	0.198	0.188
	[0.079]**	[0.083]***	[0.062]***	[0.110]*
Age and time horizon of the HRS question; variables divided by 100				
Age				-2.513
				[1.478]*
Horizon				-7.503
				[6.314]
Age X Horizon				25.438
				[10.693]**
Constant	-7.538	-9.358	-7.234	1.572
	[0.223]***	[0.190]***	[0.181]***	[0.946]*
Other parameters				
$\ln(\gamma_1)$	2.268	2.719	2.358	
	[0.024]***	[0.012]***	[0.020]***	
ln(k)		-0.629	0.158	
		[0.011]***	[0.025]***	
ln(sd(n))			0.808	
			[0.025]***	
N	13038	13038	13038	
Log-likelihood		-57714.3	-46739.4	

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B2: Average partial effects of surviving 8 more years in three models with demographic variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	0.008	0.011	0.012
	[0.001]***	[0.002]***	[0.002]***
Female	0.061	0.017	0.027
	[0.006]***	[0.004]***	[0.006]***
Black	-0.007	0.001	0.02
	[0.010]	[0.005]	[0.006]***
Hispanic	0.028	-0.028	-0.026
	[0.014]**	[0.008]***	[0.009]***

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B3: Average partial effects of surviving from age 55 to 75 in three models with demographic variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	0.01	0.023	0.021
	[0.001]***	[0.004]***	[0.004]***
Female	0.074	0.035	0.048
	[0.008]***	[0.008]***	[0.010]***
Black	-0.008	0.001	0.035
	[0.012]	[0.009]	[0.011]***
Hispanic	0.034	-0.059	-0.046
	[0.017]**	[0.017]***	[0.016]***

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B4: Average partial effects of surviving from age 75 to 95 in three models with demographic variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	0.013	0.007	0.011
	[0.002]***	[0.001]***	[0.002]***
Female	0.093	0.011	0.025
	[0.010]***	[0.003]***	[0.007]***
Black	-0.01	0	0.018
	[0.015]	[0.003]	[0.006]***
Hispanic	0.042	-0.019	-0.024
	[0.022]**	[0.006]***	[0.009]***

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B5: Outputs of the ML estimation models with demographic, personality and personal information variables; actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH	
	$\ln(\gamma_0)$	$\ln(\theta^{50})$	$\ln(\theta^{50})$	$\ln(n)$
Demographics				
Education	0.015	-0.053	-0.037	0.051
	[0.006]**	[0.008]***	[0.005]***	[0.010]***
Female	-0.34	-0.198	-0.215	-0.156
	[0.037]***	[0.039]***	[0.029]***	[0.055]***
Black	-0.201	-0.177	-0.297	0.041
	[0.055]***	[0.053]***	[0.038]***	[0.076]
Hispanic	-0.254	0.297	0.171	0.155
	[0.078]***	[0.081]***	[0.060]***	[0.106]
Cognition and personality, standardized scores				
Cognition	-0.254	-0.115	-0.079	0.021
	[0.020]***	[0.021]***	[0.015]***	[0.029]
Vocabulary	-0.013	-0.017	0.013	0.008
	[0.019]	[0.021]	[0.015]	[0.028]
Depression	0.142	0.303	0.253	0.067
	[0.015]***	[0.020]***	[0.015]***	[0.025]***
Parents' age at death, linear splines with cut-off at own age				
Same sex, sp1	-0.121	-0.386	0.205	-0.493
	[0.153]	[0.292]	[0.184]	[0.291]*
Same sex, sp2	-0.733	-1.784	-1.766	0.467
	[0.279]***	[0.214]***	[0.161]***	[0.342]

Same sex lives	-0.049	-0.003	-0.025	-0.064
	[0.119]	[0.059]	[0.044]	[0.100]
Opp. sex, sp1	-0.177	-1.058	-0.664	-0.122
	[0.156]	[0.297]***	[0.190]***	[0.292]
Opp. sex, sp2	-0.667	-0.553	-0.494	0.528
	[0.282]**	[0.210]***	[0.155]***	[0.340]
Opp. sex lives	-0.131	-0.037	-0.031	0.026
	[0.120]	[0.061]	[0.045]	[0.107]
Health related behavior				
Exercises	-0.445	-0.309	-0.258	0.015
	[0.038]***	[0.035]***	[0.025]***	[0.051]
Ever smoked	0.352	0.113	0.063	-0.091
	[0.038]***	[0.038]***	[0.027]**	[0.054]*
Smokes now	0.447	0.337	0.257	0.007
	[0.050]***	[0.053]***	[0.039]***	[0.076]
Ever drinks	-0.259	-0.25	-0.141	0.174
	[0.044]***	[0.043]***	[0.031]***	[0.062]***
# of days drinks	-0.033	-0.021	-0.019	-0.014
	[0.011]***	[0.011]*	[0.008]**	[0.016]
# of drinks	0.056	0.049	0.033	-0.035
	[0.019]***	[0.019]**	[0.014]**	[0.028]
Age and time horizon of the HRS question; variables divided by 100				
Age				-2.26
				[1.471]
Horizon				-9.823
				[6.149]
Age X Horizon				27.698
				[10.367]***
Constant	-6.712	-8.595	-5.796	1.845
	[0.288]***	[0.239]***	[0.202]***	[0.946]*
Other parameters				
ln(γ_1)	2.104	2.684	2.179	
	[0.041]***	[0.014]***	[0.029]***	
ln(k)		-0.573	0.296	
		[0.011]***	[0.026]***	
ln(sd(n))			0.773	
			[0.026]***	
N	13038	13038	13038	
Log-likelihood		-57265.8	-46196.4	

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B6: Average partial effects of surviving 8 more years in three models with demographic, personality and personal information variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	-0.003	0.005	0.005
	[0.001]**	[0.001]***	[0.001]***
Female	0.058	0.017	0.029
	[0.006]***	[0.004]***	[0.007]***
Black	0.034	0.015	0.039
	[0.009]***	[0.005]***	[0.010]***
Hispanic	0.043	-0.025	-0.023
	[0.013]***	[0.008]***	[0.009]**
Cognition and personality, standardized scores			
Cognition	0.043	0.01	0.011
	[0.003]***	[0.002]***	[0.003]***
Vocabulary	0.002	0.001	-0.002
	[0.003]	[0.002]	[0.002]
Depression	-0.024	-0.026	-0.034
	[0.003]***	[0.004]***	[0.007]***
Parents' age at death, linear splines with cut-off at own age			
Same sex, sp1	0.02	0.033	-0.027
	[0.026]	[0.024]	[0.027]
Same sex, sp2	0.124	0.152	0.234
	[0.047]***	[0.032]***	[0.059]***
Same sex lives	0.008	0	0.003
	[0.020]	[0.005]	[0.006]
Opp. sex, sp1	0.03	0.09	0.088
	[0.026]	[0.026]***	[0.026]***
Opp. sex, sp2	0.113	0.047	0.065
	[0.048]**	[0.020]**	[0.028]**
Opp. sex lives	0.022	0.003	0.004
	[0.020]	[0.005]	[0.006]
Health related behavior			
Exercises	0.075	0.026	0.034
	[0.007]***	[0.005]***	[0.008]***
Ever smoked	-0.06	-0.01	-0.008
	[0.007]***	[0.004]***	[0.004]**
Smokes now	-0.076	-0.029	-0.034
	[0.009]***	[0.006]***	[0.009]***
Ever drinks	0.044	0.021	0.019

	[0.008]***	[0.005]***	[0.006]***
# of days drinks	0.006	0.002	0.003
	[0.002]***	[0.001]*	[0.001]**
# of drinks	-0.009	-0.004	-0.004
	[0.003]***	[0.002]**	[0.002]**

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B7: Average partial effects of surviving from age 55 to 75 in three models with demographic, personality and personal information variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	-0.003	0.009	0.009
	[0.001]**	[0.002]***	[0.002]***
Female	0.073	0.035	0.051
	[0.009]***	[0.009]***	[0.012]***
Black	0.043	0.031	0.07
	[0.012]***	[0.011]***	[0.017]***
Hispanic	0.055	-0.052	-0.04
	[0.017]***	[0.016]***	[0.016]**
Cognition and personality, standardized scores			
Cognition	0.055	0.02	0.019
	[0.005]***	[0.005]***	[0.005]***
Vocabulary	0.003	0.003	-0.003
	[0.004]	[0.004]	[0.004]
Depression	-0.031	-0.053	-0.06
	[0.004]***	[0.009]***	[0.013]***
Parents' age at death, linear splines with cut-off at own age			
Same sex, sp1	0.026	0.068	-0.048
	[0.033]	[0.050]	[0.047]
Same sex, sp2	0.159	0.314	0.415
	[0.060]***	[0.067]***	[0.100]***
Same sex lives	0.011	0.001	0.006
	[0.026]	[0.010]	[0.010]
Opp. sex, sp1	0.038	0.186	0.156
	[0.034]	[0.054]***	[0.046]***
Opp. sex, sp2	0.144	0.097	0.116
	[0.060]**	[0.042]**	[0.048]**
Opp. sex lives	0.028	0.007	0.007
	[0.026]	[0.011]	[0.011]

Health related behavior			
Exercises	0.096	0.054	0.061
	[0.009]***	[0.011]***	[0.014]***
Ever smoked	-0.076	-0.02	-0.015
	[0.009]***	[0.007]***	[0.007]**
Smokes now	-0.097	-0.059	-0.06
	[0.012]***	[0.013]***	[0.015]***
Ever drinks	0.056	0.044	0.033
	[0.010]***	[0.010]***	[0.010]***
# of days drinks	0.007	0.004	0.005
	[0.002]***	[0.002]*	[0.002]**
# of drinks	-0.012	-0.009	-0.008
	[0.004]***	[0.004]**	[0.004]**

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B8: Average partial effects of surviving from age 75 to 95 in three models with demographic, personality and personal information variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	-0.004	0.003	0.005
	[0.002]**	[0.001]***	[0.002]***
Female	0.097	0.012	0.029
	[0.012]***	[0.003]***	[0.009]***
Black	0.058	0.011	0.04
	[0.016]***	[0.004]***	[0.012]***
Hispanic	0.073	-0.019	-0.023
	[0.023]***	[0.006]***	[0.010]**
Cognition and personality, standardized scores			
Cognition	0.073	0.007	0.011
	[0.008]***	[0.002]***	[0.004]***
Vocabulary	0.004	0.001	-0.002
	[0.005]	[0.001]	[0.002]
Depression	-0.041	-0.019	-0.034
	[0.005]***	[0.004]***	[0.009]***
Parents' age at death, linear splines with cut-off at own age			
Same sex, sp1	0.035	0.024	-0.028
	[0.043]	[0.018]	[0.028]
Same sex, sp2	0.21	0.112	0.239
	[0.086]**	[0.027]***	[0.071]***

Same sex lives	0.014	0	0.003
	[0.034]	[0.004]	[0.006]
Opp. sex, sp1	0.051	0.066	0.09
	[0.044]	[0.020]***	[0.029]***
Opp. sex, sp2	0.191	0.035	0.067
	[0.086]**	[0.016]**	[0.030]**
Opp. sex lives	0.038	0.002	0.004
	[0.035]	[0.004]	[0.006]
Health related behavior			
Exercises	0.128	0.019	0.035
	[0.013]***	[0.004]***	[0.010]***
Ever smoked	-0.101	-0.007	-0.008
	[0.012]***	[0.003]***	[0.004]**
Smokes now	-0.128	-0.021	-0.035
	[0.015]***	[0.005]***	[0.010]***
Ever drinks	0.074	0.016	0.019
	[0.013]***	[0.004]***	[0.006]***
# of days drinks	0.01	0.001	0.003
	[0.003]***	[0.001]*	[0.001]**
# of drinks	-0.016	-0.003	-0.004
	[0.006]***	[0.001]**	[0.002]**

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B9: Outputs of the ML estimation models with demographic, personality, personal information and subjective health variables; actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH	
	$\ln(\gamma_0)$	$\ln(\theta^{50})$	$\ln(\theta^{50})$	$\ln(n)$
Demographics				
Education	0.026	-0.037	-0.019	0.053
	[0.006]***	[0.007]***	[0.005]***	[0.010]***
Female	-0.31	-0.152	-0.169	-0.157
	[0.036]***	[0.038]***	[0.027]***	[0.054]***
Black	-0.24	-0.264	-0.369	0.02
	[0.053]***	[0.052]***	[0.036]***	[0.075]
Hispanic	-0.332	0.229	0.083	0.132
	[0.077]***	[0.078]***	[0.056]	[0.105]
Cognition and personality, standardized scores				
Cognition	-0.231	-0.052	-0.035	0.027
	[0.019]***	[0.020]**	[0.015]**	[0.028]
Vocabulary	-0.011	-0.018	0.011	0.009
	[0.018]	[0.020]	[0.014]	[0.028]
Depression	0.043	0.148	0.131	0.043
	[0.016]***	[0.020]***	[0.014]***	[0.026]*
Parents' age of death, linear splines with cut-off at own age				
Same sex, sp1	-0.09	-0.6	0.094	-0.493
	[0.154]	[0.281]**	[0.172]	[0.287]*
Same sex, sp2	-0.567	-1.552	-1.608	0.5
	[0.272]**	[0.207]***	[0.153]***	[0.337]
Same sex lives	-0.051	0.006	-0.007	-0.071
	[0.114]	[0.057]	[0.042]	[0.099]
Opp. sex, sp1	-0.203	-1.193	-0.663	-0.117
	[0.156]	[0.285]***	[0.177]***	[0.287]
Opp. sex, sp2	-0.434	-0.353	-0.427	0.522
	[0.274]	[0.205]*	[0.148]***	[0.336]
Opp. sex lives	-0.191	-0.032	-0.014	0.032
	[0.116]*	[0.059]	[0.043]	[0.105]
Health related behavior				
Exercises	-0.309	-0.146	-0.13	0.02
	[0.038]***	[0.034]***	[0.024]***	[0.051]
Ever smoked	0.31	0.031	0.009	-0.091
	[0.038]***	[0.036]	[0.026]	[0.053]*
Smokes now	0.436	0.308	0.239	0.006
	[0.049]***	[0.051]***	[0.037]***	[0.075]
Ever drinks	-0.211	-0.119	-0.054	0.177

	[0.043]***	[0.042]***	[0.030]*	[0.061]***
# of days drinks	-0.024	-0.012	-0.011	-0.015
	[0.011]**	[0.011]	[0.008]	[0.016]
# of drinks	0.045	0.05	0.029	-0.033
	[0.019]**	[0.019]***	[0.014]**	[0.028]
Subjective health in 2000/2002				
Excel./excel.	*ref.*	*ref.*	*ref.*	*ref.*
Excel./good	0.442	0.363	0.306	-0.172
	[0.065]***	[0.055]***	[0.040]***	[0.084]**
Excel./poor	0.827	0.682	0.585	0.122
	[0.091]***	[0.121]***	[0.085]***	[0.157]
Good/excel.	0.231	0.29	0.237	-0.092
	[0.079]***	[0.061]***	[0.044]***	[0.095]
Good/good	0.445	0.562	0.483	-0.05
	[0.059]***	[0.050]***	[0.035]***	[0.074]
Good/poor	0.894	0.983	0.74	-0.059
	[0.065]***	[0.087]***	[0.061]***	[0.104]
Poor/excel.	0.579	0.582	0.461	-0.106
	[0.119]***	[0.156]***	[0.106]***	[0.196]
Poor/good	0.66	0.805	0.624	-0.042
	[0.075]***	[0.088]***	[0.063]***	[0.116]
Poor/poor	0.973	1.372	1.126	0.177
	[0.057]***	[0.070]***	[0.052]***	[0.085]**
Age and time horizon of the HRS question; variables divided by 100				
Age				-2.333
				[1.449]
Horizon				-10.037
				[6.053]*
Age X Horizon				27.82
				[10.204]***
Constant	-7.288	-9.366	-6.379	1.882
	[0.293]***	[0.233]***	[0.197]***	[0.933]**
Other parameters				
$\ln(\gamma_1)$	2.088	2.686	2.149	
	[0.041]***	[0.013]***	[0.029]***	
ln(k)		-0.54	0.377	
		[0.011]***	[0.026]***	
ln(sd(n))			0.758	
			[0.026]***	
N	13038	13038	13038	
Log-likelihood		-56994.3	-45850.0	

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B10: Average partial effects of surviving 8 more years in three models with demographic, personality, personal information and subjective health variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	-0.004	0.003	0.003
	[0.001]***	[0.001]***	[0.001]***
Female	0.052	0.013	0.023
	[0.006]***	[0.004]***	[0.006]***
Black	0.04	0.023	0.05
	[0.009]***	[0.006]***	[0.012]***
Hispanic	0.056	-0.02	-0.011
	[0.013]***	[0.007]***	[0.008]
Cognition and personality, standardized scores			
Cognition	0.039	0.004	0.005
	[0.003]***	[0.002]**	[0.002]**
Vocabulary	0.002	0.002	-0.002
	[0.003]	[0.002]	[0.002]
Depression	-0.007	-0.013	-0.018
	[0.003]***	[0.003]***	[0.004]***
Parents' age at death, linear splines with cut-off at own age			
Same sex, sp1	0.015	0.052	-0.013
	[0.026]	[0.024]**	[0.024]
Same sex, sp2	0.095	0.134	0.217
	[0.046]**	[0.029]***	[0.055]***
Same sex lives	0.009	-0.001	0.001
	[0.019]	[0.005]	[0.006]
Opp. sex, sp1	0.034	0.103	0.09
	[0.026]	[0.027]***	[0.025]***
Opp. sex, sp2	0.073	0.03	0.058
	[0.046]	[0.019]	[0.026]**
Opp. sex lives	0.032	0.003	0.002
	[0.020]*	[0.005]	[0.006]
Health related behavior			
Exercises	0.052	0.013	0.018
	[0.006]***	[0.004]***	[0.005]***
Ever smoked	-0.052	-0.003	-0.001
	[0.006]***	[0.003]	[0.004]
Smokes now	-0.073	-0.027	-0.032
	[0.008]***	[0.006]***	[0.008]***
Ever drinks	0.035	0.01	0.007

	[0.007]***	[0.004]***	[0.004]*
# of days drinks	0.004	0.001	0.001
	[0.002]**	[0.001]	[0.001]
# of drinks	-0.008	-0.004	-0.004
	[0.003]**	[0.002]**	[0.002]*
Subjective health in 2000/2002			
Excel./excel.	*ref.*	*ref.*	*ref.*
Excel./good	-0.074	-0.031	-0.041
	[0.011]***	[0.007]***	[0.010]***
Excel./poor	-0.139	-0.059	-0.079
	[0.016]***	[0.014]***	[0.020]***
Good/excel.	-0.039	-0.025	-0.032
	[0.013]***	[0.007]***	[0.009]***
Good/good	-0.075	-0.048	-0.065
	[0.010]***	[0.009]***	[0.015]***
Good/poor	-0.15	-0.085	-0.1
	[0.012]***	[0.015]***	[0.023]***
Poor/excel.	-0.097	-0.05	-0.062
	[0.020]***	[0.015]***	[0.020]***
Poor/good	-0.111	-0.069	-0.084
	[0.013]***	[0.013]***	[0.020]***
Poor/poor	-0.163	-0.118	-0.152
	[0.010]***	[0.020]***	[0.033]***

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B11: Average partial effects of surviving from age 55 to 75 in three models with demographic, personality, personal information and subjective health variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	-0.006	0.006	0.004
	[0.001]***	[0.002]***	[0.001]***
Female	0.067	0.026	0.039
	[0.008]***	[0.008]***	[0.010]***
Black	0.051	0.046	0.086
	[0.012]***	[0.012]***	[0.020]***
Hispanic	0.071	-0.04	-0.019
	[0.017]***	[0.015]***	[0.013]
Cognition and personality, standardized scores			
Cognition	0.05	0.009	0.008
	[0.004]***	[0.004]**	[0.004]**
Vocabulary	0.002	0.003	-0.003
	[0.004]	[0.003]	[0.003]
Depression	-0.009	-0.026	-0.031
	[0.003]***	[0.005]***	[0.007]***
Parents' age at death, linear splines with cut-off at own age			
Same sex, sp1	0.019	0.104	-0.022
	[0.033]	[0.048]**	[0.042]
Same sex, sp2	0.122	0.268	0.375
	[0.058]**	[0.059]***	[0.090]***
Same sex lives	0.011	-0.001	0.002
	[0.024]	[0.010]	[0.010]
Opp. sex, sp1	0.043	0.206	0.155
	[0.034]	[0.053]***	[0.043]***
Opp. sex, sp2	0.093	0.061	0.1
	[0.058]	[0.038]	[0.044]**
Opp. sex lives	0.041	0.006	0.003
	[0.025]	[0.010]	[0.010]
Health related behavior			
Exercises	0.066	0.025	0.03
	[0.008]***	[0.007]***	[0.008]***
Ever smoked	-0.067	-0.005	-0.002
	[0.009]***	[0.006]	[0.006]
Smokes now	-0.093	-0.053	-0.056
	[0.012]***	[0.012]***	[0.014]***
Ever drinks	0.045	0.021	0.013

	[0.010]***	[0.008]***	[0.007]*
# of days drinks	0.005	0.002	0.003
	[0.002]**	[0.002]	[0.002]
# of drinks	-0.01	-0.009	-0.007
	[0.004]**	[0.004]**	[0.003]**
Subjective health in 2000/2002			
Excel./excel.	*ref.*	*ref.*	*ref.*
Excel./good	-0.095	-0.063	-0.071
	[0.015]***	[0.014]***	[0.017]***
Excel./poor	-0.177	-0.118	-0.137
	[0.021]***	[0.028]***	[0.034]***
Good/excel.	-0.05	-0.05	-0.055
	[0.017]***	[0.013]***	[0.015]***
Good/good	-0.095	-0.097	-0.113
	[0.013]***	[0.017]***	[0.024]***
Good/poor	-0.192	-0.17	-0.173
	[0.016]***	[0.031]***	[0.038]***
Poor/excel.	-0.124	-0.1	-0.108
	[0.026]***	[0.031]***	[0.033]***
Poor/good	-0.142	-0.139	-0.146
	[0.017]***	[0.027]***	[0.033]***
Poor/poor	-0.209	-0.237	-0.263
	[0.015]***	[0.039]***	[0.054]***

***, ** and *** denote significance at 10, 5 and 1 percent level**

Table B12: Average partial effects of surviving from age 75 to 95 in three models with demographic, personality, personal information and subjective health variables: actual survival and the mean and MRH models of subjective survival expectations

	Actual survival	Mean	MRH
Demographics			
Education	-0.007	0.003	0.003
	[0.002]***	[0.001]***	[0.001]***
Female	0.086	0.01	0.024
	[0.011]***	[0.003]***	[0.007]***
Black	0.067	0.018	0.053
	[0.015]***	[0.005]***	[0.015]***
Hispanic	0.092	-0.016	-0.012
	[0.022]***	[0.006]**	[0.009]
Cognition and personality, standardized scores			
Cognition	0.064	0.004	0.005
	[0.007]***	[0.002]**	[0.003]*
Vocabulary	0.003	0.001	-0.002
	[0.005]	[0.001]	[0.002]
Depression	-0.012	-0.01	-0.019
	[0.004]***	[0.002]***	[0.005]***
Parents' age at death, linear splines with cut-off at own age			
Same sex, sp1	0.025	0.041	-0.014
	[0.042]	[0.019]**	[0.026]
Same sex, sp2	0.157	0.105	0.231
	[0.080]**	[0.026]***	[0.068]***
Same sex lives	0.014	0	0.001
	[0.032]	[0.004]	[0.006]
Opp. sex, sp1	0.056	0.081	0.095
	[0.042]	[0.022]***	[0.029]***
Opp. sex, sp2	0.12	0.024	0.061
	[0.079]	[0.015]	[0.029]**
Opp. sex lives	0.053	0.002	0.002
	[0.032]	[0.004]	[0.006]
Health related behavior			
Exercises	0.086	0.01	0.019
	[0.012]***	[0.003]***	[0.006]***
Ever smoked	-0.086	-0.002	-0.001
	[0.012]***	[0.003]	[0.004]
Smokes now	-0.121	-0.021	-0.034
	[0.015]***	[0.005]***	[0.010]***
Ever drinks	0.059	0.008	0.008

	[0.012]***	[0.003]**	[0.005]*
# of days drinks	0.007	0.001	0.002
	[0.003]**	[0.001]	[0.001]
# of drinks	-0.013	-0.003	-0.004
	[0.005]**	[0.001]**	[0.002]*
Subjective health in 2000/2002			
Excel./excel.	*ref.*	*ref.*	*ref.*
Excel./good	-0.123	-0.025	-0.044
	[0.020]***	[0.006]***	[0.013]***
Excel./poor	-0.23	-0.046	-0.084
	[0.029]***	[0.012]***	[0.025]***
Good/excel.	-0.064	-0.02	-0.034
	[0.022]***	[0.006]***	[0.011]***
Good/good	-0.124	-0.038	-0.069
	[0.018]***	[0.008]***	[0.019]***
Good/poor	-0.248	-0.067	-0.106
	[0.024]***	[0.014]***	[0.029]***
Poor/excel.	-0.161	-0.04	-0.066
	[0.035]***	[0.013]***	[0.023]***
Poor/good	-0.183	-0.055	-0.09
	[0.024]***	[0.012]***	[0.025]***
Poor/poor	-0.27	-0.093	-0.162
	[0.023]***	[0.019]***	[0.042]***

***, ** and *** denote significance at 10, 5 and 1 percent level**

Appendix C: Derivations

C.1. Effect of individual survival heterogeneity on sample selection

In our model there is individual heterogeneity in survival chances. One consequence of this is that fit individuals will be overrepresented in the sample over time as frail individuals are more likely to die. This sample selection can be conveniently modeled in our framework. Let k^a and θ^a denote the shape and scale parameters in cohort a. As Vaupel (1979) shows the following is true for any cohorts

$$k^a = k^r \text{ and} \tag{15}$$

$$\theta^a = \frac{1}{\frac{1}{\theta^r} + \left(\exp\left(\gamma_1 a\right) - \exp\left(\gamma_1 r\right)\right)}, \tag{15}$$

where r is a reference cohort. As we can see, older and younger cohorts share the same shape parameter k^a, but older cohorts have lower scale parameter θ^a (lower frailty) than younger cohorts.

C.2. Proof of equation (10)

The density function of the gamma distribution is

$$f\left(x\right) = \frac{1}{\Upsilon\left(k\right)\theta^k} x^{k-1} exp\left(-\frac{x}{\theta}\right) = c\left(k,\theta\right) x^{k-1} exp\left(-\frac{x}{\theta}\right).$$

It is well known that expected value of the gamma function is

$$E\left(x\right) = \int_0^\infty xc\left(k,\theta\right) x^{k-1} exp\left(-\frac{x}{\theta}\right) dx = \int_0^\infty c\left(k,\theta\right) x^k exp\left(-\frac{x}{\theta}\right) dx = k\theta.$$

The expected value of a scaled gamma function is also gamma and its expected value is

$$E\left(cx\right) = ck\theta.$$

The expected value of the negative exponentiated gamma function is

$$E\left(exp(-x)\right)=\int_0^\infty exp(-x)c(k,\theta)x^{k-1}exp\left(-\frac{x}{\theta}\right)dx$$

$$=\int_0^\infty c(k,\theta)x^{k-1}exp\left(-\frac{x}{\theta}-x\right)dx=\int_0^\infty c(k,\theta)x^{k-1}exp\left(-\frac{x}{\frac{\theta}{\theta+1}}\right)dx=*$$

Note that this is very similar to the expected value formula of the gamma function, thus

$$*=\frac{(k-1)\theta}{\theta+1}\frac{c(k,\theta)}{c\left(k+1,\frac{\theta}{\theta+1}\right)}=\frac{(k-1)\theta}{\theta+1}\frac{\Upsilon(k-1)\left(\frac{\theta}{\theta+1}\right)^k}{\Upsilon(k)\theta^k}=*$$

Note that $\Upsilon(k)=(k-1)\Upsilon(k-1)$ and thus

$$*=\frac{k-1}{k-1}\frac{\theta^k}{\theta^k}(1+\theta)^{-k}=(1+\theta)^{-k}.$$

Thus if $\lambda_{0i}\sim\Upsilon(k,\theta)$ then

$$E\left[S_i(a,T)\right]=E\left[exp\left(-\lambda_{0i}\left[exp(\lambda_1 T)-exp(\lambda_1 a)\right]\right)\right]=\left(1+\theta\left[exp(\lambda_1 T)-exp(\lambda_1 a)\right]\right)^{-k}.$$

C.3. Details about the use of the Delta method to derive average partial effects

The goal is to derive point estimates and standard errors of the partial effects of any covariate x_j

on the survival probability from age a to age T

$$APE_j(a_1,a_2)=E_x\left[\frac{\partial S_i(a_1,a_2)}{\partial x_j}\right]. \qquad \textbf{(15)}$$

The point estimates are.

$$APE_j(a,T)=E_x\left[\frac{\partial\left(1+\theta^a\left[exp(\lambda_1 T)-exp(\lambda_1 a)\right]\right)^{-k}}{\partial x_j}\right]=*$$

Let us denote $e(a',a)=exp(\lambda_1 a')-exp(\lambda_1 a)$. Then

$$* = E_x \left[\frac{\partial \left(1 + \frac{e(T,a)}{exp(-\beta' x_i) + e(a,r)} \right)^{-k}}{\partial x_j} \right]$$

$$= E_x \left[-k \left(1 + \frac{e(T,a)}{exp(-\beta' x_i) + e(a,r)} \right)^{-k-1} \frac{e(T,a) exp(-\beta' x_i) \beta_j}{\left(exp(-\beta' x_i) + e(a,r) \right)^2} \right]$$

$$= E_x \left[-k \left(exp(-\beta' x_i) + e(T,r) \right)^{-k-1} \left(exp(-\beta' x_i) + e(a,r) \right)^{k-1} e(T,a) exp(-\beta' x_i) \beta_j \right].$$

By substituting the estimated coefficients into this formula we have a point estimate for the average partial effect of x_j on the survival probability from age a to age T. The standard errors can be computed with the delta method. For any differentiable transformation $g(\beta)$ and variance-covariance matrix Σ, the variance covariance matrix of $g(\beta)$ is $(\nabla g)^T \Sigma (\nabla g)$.

Thus, we only need to compute the first derivatives of g. Let us see them one-by-one.

$$\frac{\partial APE_j(a,T)}{\partial \beta_l} = -I(j = l) \times$$

$$\times E_x \left[k \left(exp(-\beta' x_i) + e(T,r) \right)^{-k-1} \left(exp(-\beta' x_i) + e(a,r) \right)^{k-1} e(T,a) exp(-\beta' x_i) \right]$$

$$- E_x \left[k(k+1) \left(exp(-\beta' x_i) + e(T,r) \right)^{-k-2} \left(exp(-\beta' x_i) + e(a,r) \right)^{k-1} e(T,a) exp(-2\beta' x_i) \beta_j x_{il} \right]$$

$$+ E_x \left[k(k-1) \left(exp(-\beta' x_i) + e(T,r) \right)^{-k-1} \left(exp(-\beta' x_i) + e(a,r) \right)^{k-2} e(T,a) exp(-2\beta' x_i) \beta_j x_{il} \right]$$

$$+ E_x \left[k \left(exp(-\beta' x_i) + e(T,r) \right)^{-k-1} \left(exp(-\beta' x_i) + e(a,r) \right)^{k-1} e(T,a) exp(-\beta' x_i) \beta_j x_{il} \right].$$

$$\frac{\partial APE_j(a,T)}{\partial k} = -E_x \left[\left(exp(-\beta' x_i) + e(T,50) \right)^{-k-1} \left(exp(-\beta' x_i) + e(a,50) \right)^{k-1} e(T,a) exp(-\beta' x_i) \beta_j \right]$$

$$- E_x \left[k \left(exp(-\beta' x_i) + e(T,r) \right)^{-k-1} \left(exp(-\beta' x_i) + e(a,r) \right)^{k-1} e(T,a) exp(-\beta' x_i) \times \right.$$

$$\left. \times \beta_j \left(-ln\left(exp(-\beta' x_i) + e(T,50) \right) \right) \right] + E_x \left[ln\left(exp(-\beta' x_i) + e(a,50) \right) \right].$$

$$\frac{\partial APE_j(a,T)}{\partial \lambda_1} = \left[Texp(\lambda_1 T) - 50exp(\lambda_1 50) \right] \times$$

$$\times \Big\{ E_x \Big[k(k+1) \big(exp(-\beta' x_i) + e(T,50) \big)^{-k-2} \big(exp(-\beta' x_i) + e(a,50) \big)^{k-1} e(T,a) exp(-\beta' x_i) \beta_j \Big]$$

$$- E_x \Big[k(k-1) \big(exp(-\beta' x_i) + e(T,50) \big)^{-k-1} \big(exp(-\beta' x_i) + e(a,50) \big)^{k-2} e(T,a) exp(-\beta' x_i) \beta_j \Big] \Big\}.$$

By substituting the estimated coefficients into these formulas we have an estimator for the variance covariance matrix of the average partial effects.

C.4. The likelihood function of the mean model

The likelihood function can be written as

$$l_i = \Pr\left(\underline{p}_i \leq S_i(a,t) \leq \overline{p}_i \right), \tag{15}$$

where $\underline{p}_i$ and $\overline{p}_i$ denote the lower and upper bound probabilities that would be rounded to the survey response. For example, if the rounding function rounds to the closest 1 percent and the survey response is 27 percent, then $\underline{p}_i = 0.265$ and $\overline{p}_i = 0.275$. If the rounding function rounds to the closest 5 percent, then the corresponding probabilities would be $\underline{p}_i = 0.225$ and $\overline{p}_i = 0.275$.[20] The likelihood function, thus, is

$$l_i = \Pr\left(\frac{-\ln(\underline{p}_i)}{\left(\exp(\gamma_1 t) - \exp(\gamma_1 a)\right)} \geq \lambda_{0i} \geq \frac{-\ln(\overline{p}_i)}{\left(\exp(\gamma_1 t) - \exp(\gamma_1 a)\right)} \right), \tag{15}$$

[20] As we can see, a response that is not a multiple of 5 percent cannot be a rounded version of a true latent probability when we round to the closest 5 percent. The rounding model we have in mind is one where agents only identify bins (e.g. 0%-2.5%, 2.5%-7.5%, …, 97.5%-100%) and any response in a particular bin only tells the econometrician that the true latent probability is also in the same bin. An alternative model would be that of Manski and Molinari (2010) where they identify individuals' rounding practices across many probability questions and use different rounding functions for each individual.

which can be easily computed from the c.d.f. of the gamma distribution in with parameters k and θ_i.

C.5. The likelihood function of the modal response model

The estimation of the MRH can be carried out by maximum simulated likelihood (MSL). MSL computes the likelihood function by drawing many values from the distribution of one (or several) random variables and computing the average conditional likelihood, conditioning on those values. In our case, it is worth simulating values from the distribution of u_{ni}. A standard version of the simulated likelihood would look like the following.[21]

$$l_i = \frac{1}{S}\sum_{s=1}^{S} l_i\left(u_{ni}^s\right), \tag{15}$$

where S is the number of simulation draws. The problem with this approach is that our model is full of discontinuities that make this approach infeasible. Let us take a look at the MRH formula by rewriting (5) with our new variables, μ_i and n_i. Let us denote $\underline{n}_i \equiv \frac{1}{n_i}$ and $\overline{n}_i \equiv 1 - \frac{1}{n_i}$. The MRH model assumes that the survey response is

[21] Note that in case the simulated values are drawn from the distribution of u_{ni}, we do not need to weight the terms in **Error! Reference source not found.** by the density of the draw as the simulation itself already weights the data.

$$p_i^{mrh} = \begin{cases} round\left(\frac{\mu_i n_i - 1}{n_i - 2}\right) & \text{if} \quad \overline{n}_i > \mu_i > \underline{n}_i \\ 1 & \text{if} \quad \mu_i > \underline{n}_i, \overline{n}_i \\ 0 & \text{if} \quad \mu_i < \underline{n}_i, \overline{n}_i \\ 0.5 & \text{if} \quad \overline{n}_i < \mu_i < \underline{n}_i \end{cases} . \qquad (15)$$

As we can see, whenever $n_i \leq 1$ an answer can only be 50 percent. Whenever $n_i \leq 2$, an answer can only be 0, 50 or 100 percent. These discontinuities make the simulation model in **Error! Reference source not found.** hard for the following reason. Imagine that during the maximization of the likelihood function we get into a region where the precision of beliefs n_i is always below 2 for each simulation draw. This could happen if $z_i^{'}\beta^k{}_n \ll 2$ and $\sigma_n^{2,k}$ is small, where k indexes the actual guesses for the parameters. In this case, the likelihood function would be undefined and the numerical maximization would fail. As a remedy, we recommend drawing separate simulation values from each region of n_i; this assures that the likelihood is well-defined in each iteration of the maximization.[22] The likelihood function, thus, is written

$$\begin{aligned} l_i = & \frac{1}{S_1}\sum_{s=1}^{S_1} l_i\left(u_{ni}^s, n_i \leq 1\right)\Pr\left(n_i \leq 1\right) + \frac{1}{S_2}\sum_{s=1}^{S_2} l_i\left(u_{ni}^s, 1 < n_i \leq 2\right)\Pr\left(1 < n_i \leq 2\right) \\ & + \frac{1}{S_3}\sum_{s=1}^{S_3} l_i\left(u_{ni}^s, 2 < n_i\right)\Pr\left(2 < n_i\right). \end{aligned} \qquad (15)$$

Now, whenever $z_i^{'}\beta^k{}_n \ll 2$ and $\sigma_n^{2,k}$ is small in a particular iteration for a non-focal answer, the likelihood is a well-defined, extremely small number.

[22] Hill, Perry and Willis (2004) did not use this trick when they estimated a very similar model. The consequence was that they had to make restrictions on their model to be able to carry out the numerical estimation. It turned out that our model is identified and estimable in practice under mild conditions once these discontinuities are properly taken care of.

The probabilities of the different regions of n_i are trivial since n_i is assumed to have a log-normal distribution. Whenever $n_i \leq 1$ an answer can only be fifty and thus the conditional likelihood is

$$l_i\left(u_{ni}^s, n_i \leq 1\right) = \begin{cases} 1 & \text{if } p_i^{hrs} = 50 \\ 0 & \text{otherwise} \end{cases}, \tag{15}$$

where p_i^{hrs} represents the survey response to the HRS mortality question. When $1 < n_i \leq 2$, it is assured that $\underline{n}_i \geq \overline{n}_i$ and thus

$$l_i\left(u_{ni}^s, 1 < n_i \leq 2\right) = \begin{cases} \Pr\left(\overline{n}_i < S_i\left(a,t\right) < \underline{n}_i\right) & \text{if } p_i^{hrs} = 50 \\ \Pr\left(S_i\left(a,t\right) \leq \overline{n}_i\right) & \text{if } p_i^{hrs} = 0 \\ \Pr\left(S_i\left(a,t\right) \geq \underline{n}_i\right) & \text{if } p_i^{hrs} = 100 \\ 0 & \text{otherwise} \end{cases}. \tag{15}$$

These probabilities can be computed analogously to the mean model derived in Section 5.1.

The most complicated, although still very straightforward, case is the conditional likelihood in the region where $n_i > 2$. In this region $\underline{n}_i < \overline{n}_i$. The only complication is that a 0 and a 100 answer can now be either a focal answer or an exact rounded answer. 50 answers in this region cannot be focal answers as $n_i > 2$ and thus either α_i or β_i is larger than one. The conditional likelihood is

$$l_i\left(u_{ni}^s, n_i > 2\right) = \begin{cases} \Pr\left(S_i\left(a,t\right) \leq \underline{n}_i \text{ OR } \dfrac{S_i\left(a,t\right)n_i - 1}{n_i - 2} \leq \overline{p}_i\right) & \text{if } p_i^{hrs} = 0 \\ \Pr\left(S_i\left(a,t\right) \geq \overline{n}_i \text{ OR } \dfrac{S_i\left(a,t\right)n_i - 1}{n_i - 2} \geq \underline{p}_i\right) & \text{if } p_i^{hrs} = 100. \\ \Pr\left(\underline{p}_i \leq \dfrac{S_i\left(a,t\right)n_i - 1}{n_i - 2} \leq \overline{p}_i\right) & \text{otherwise} \end{cases} \tag{15}$$

After straightforward algebra the conditional likelihood becomes

$$l_i\left(u_{ni}^s, n_i > 2\right) = \begin{cases} \Pr\left(S_i\left(a,t\right) \le \max\left\{ \frac{\overline{p}_i\left(n_i - 2\right) + 1}{n_i}, \underline{n}_i \right\} \right) & \text{if } p_i^{hrs} = 0 \\ \Pr\left(\min\left\{ \frac{\underline{p}_i\left(n_i - 2\right) + 1}{n_i}, \overline{n}_i \right\} \le S_i\left(a,t\right) \right) & \text{if } p_i^{hrs} = 100, \\ \Pr\left(\frac{\underline{p}_i\left(n_i - 2\right) + 1}{n_i} \le S_i\left(a,t\right) \le \frac{\overline{p}_i\left(n_i - 2\right) + 1}{n_i} \right) & \text{otherwise} \end{cases} \quad \textbf{(15)}$$

and again these probabilities can be computed analogously to the mean model derived in Section C4.