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TRADE IN INTERMEDIATE INPUTS AND BUSINESS CYCLE COMOVEMENT

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ABSTRACT

Does input trade synchronize business cycles across countries? I incorporate input trade into a dynamic multi-sector model with many countries, calibrate the model to match bilateral input-output data, and estimate trade-comovement regressions in simulated data. With correlated productivity shocks, the model yields high trade- comovement correlations for goods, but near-zero correlations for services and thus low aggregate correlations. With uncorrelated shocks, input trade generates more comovement in gross output than real value added. Goods comovement is higher when (a) the aggregate trade elasticity is low, (b) inputs are more substitutable than final goods, and (c) inputs are substitutable for primary factors.

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An online appendix is available at:
http://www.nber.org/data-appendix/w18240

1 Introduction

A large empirical literature suggests that international trade transmits shocks and synchronizes economic activity across borders. For example, bilateral trade is strongly (and robustly) correlated with bilateral GDP comovement.[1] The theoretical underpinnings of this empirical relationship remain poorly understood. For example, the workhorse international real business cycle (IRBC) model struggles to replicate the quantitative magnitude of the empirical correlation between bilateral trade and GDP comovement. Kose and Yi (2006) have dubbed this the "trade-comovement puzzle."

In addressing this puzzle, recent empirical work has turned attention to the role of cross-border intermediate input linkages as a conduit for shocks. For example, Ng (2010) documents that proxies for bilateral production fragmentation predict bilateral GDP correlations, while Di Giovanni and Levchenko (2010) document that bilateral trade is more important in explaining output comovement for home and foreign sectors that use each other as intermediates. Further, Burstein, Kurz, and Tesar (2008) show that countries that intensively engage in intra-firm trade with United States multinational parents display higher manufacturing output correlations with the U.S. In a related vein, Bergin, Feenstra, and Hanson (2009, 2011) document that Mexican export assembly (maquiladora) industries are twice as volatile as their US counterparts, suggesting strong transmission of US shocks to Mexico through input linkages.

This focus on input trade is potentially important, since intermediate inputs account for roughly 60% of international trade. Yet, the standard IRBC model does not distinguish trade in final goods versus intermediate inputs, and thus is ill-suited to study propagation of shocks through input chains. To remedy this problem, I develop a many country, multi-sector extension of the IRBC model that includes sector-to-sector input-output linkages both within and across countries. This model is an open economy analog to closed economy models of sectoral linkages, pioneered by Long and Plosser (1983). I calibrate the model to data on bilateral final and intermediate goods trade flows for 22 countries and a composite rest-of-the-world region, and simulate model responses to sector-specific productivity shocks. Using simulated data, I assess the ability of the model to account for observed trade-comovement correlations, highlighting the role of input trade in transmitting shocks.

In the model, input trade transmits shocks across borders independent of, and in addition to, standard IRBC transmission mechanisms. In the IRBC model, idiosyncratic shocks generate output comovement by inducing comovement in factor supplies. Specifically, a

[1] Among others, see Frankel and Rose (1998), Imbs (2004), Baxter and Kouparitsas (2005), Kose and Yi (2006), Calderón, Chong, and Stein (2007), Inklaar, Jong-A-Pin, and Haan (2008), Di Giovanni and Levchenko (2010), and Ng (2010).

positive shock in the home country raises home output and depreciates home's terms of trade, which induces increased factor supply and hence value added abroad. This mechanism continues to operate in the augmented model with intermediate inputs. However, with traded intermediates, productivity shocks are passed downstream through the production chain directly. For example, an increase in productivity in country A lowers the marginal cost of producing gross output for downstream countries that import inputs from country A, and is therefore associated with increased gross output in downstream countries in equilibrium. Further, gross output in downstream countries can increase even if factor supplies are held constant. Therefore, comovement in gross output may be delinked from comovement in real value added. Thus, the production chain puts significant new structure to how shocks are transmitted, above and beyond standard IRBC mechanics.

To evaluate these channels quantitatively, I calibrate the model using data from the World Input Output Database (WIOD). This database provides a sequence of 'global bilateral input-output tables' that record final and intermediate goods shipments across countries and sectors. This type of data has several advantageous features for calibration of international macro models. First, the framework respects national accounts definitions of final and intermediate goods, and therefore is consistent with standard macro aggregates. Second, the data includes consistent gross output, value added, and bilateral final and intermediate shipments. This data enables a more realistic calibration of openness and bilateral linkages than has been previously possible in the literature.[2]

Proceeding to the quantitative analysis, I first simulate the model using an estimated productivity process in which shocks are allowed to be correlated across countries.[3] Despite the introduction of intermediate inputs in the model, the aggregate trade-comovement puzzle is alive and well: the aggregate trade-comovement correlations for real value added and gross output are at most 10-20% the size of the observed correlations. Thus, introducing intermediate inputs into the IRBC model does not resolve the trade-comovement puzzle. Nonetheless, this disappointing *aggregate* result hides a number of interesting *disaggregate* features of the model and data.

To shed light on the origins of the aggregate puzzle, I examine trade-comovement correlations for goods and services sectors separately. In the data, output comovement for both goods and services sectors is strongly and positively related to bilateral trade intensity. The

[2] Exports are commonly treated as comparable to GDP in the prior literature, despite the fact that exports are recorded on a gross (not value added) basis. This makes the economy look 'too open' and distorts the strength of bilateral trade linkages. I will discuss this issue further below, but see also Johnson (2013).

[3] I estimate this productivity process using sector-level productivity data from Groningen's EU KLEMS and 10-Sector databases. In the main text, I simulate the model with complete financial markets. I present supplementary results with incomplete financial markets (constant nominal trade balances) in the appendix. All the main results are robust to changing the financial market structure.

model matches the trade-comovement correlation for the goods sector well (with correlations over 3/4 as large as in the data), but generates a near-zero trade-comovement correlation for the services sector. This implies that the low aggregate trade-comovement correlation in the model is largely due to the model's inability to match services comovement.

These differences in sector-level comovement in the model could be explained in two ways. First, sector-level differences in output comovement could arise because trade propagates shocks for goods, but not services, sectors. Second, sector-level differences in output comovement could arise because the cross-country correlation of productivity shocks differs across sectors. To separate the role of correlated shocks from the transmission of idiosyncratic shocks via trade, I simulate the model again with uncorrelated shocks across countries, and re-estimate trade-comovement regressions in the new simulated data.

With uncorrelated shocks, the trade-comovement correlation in the goods sector falls substantially. For gross output, the trade-comovement correlation with uncorrelated shocks is about 1/4 as large as with correlated shocks. For real value added, it is less than 1/10 as large. This implies that the cross-country correlation of shocks itself is primarily responsible for the high degree of goods sector comovement in the model. To further illustrate this point, I compare output comovement and productivity correlations directly. In the model, cross-country output correlations for both sectors are tightly related to measured productivity correlations. By contrast, cross-country output correlations in the data are related to productivity correlations for the goods sector, but not the services sector. Therefore, the poor model-fit for the services sector is largely a byproduct of a weak empirical link between productivity and output in the services sector.

As for the role of traded inputs, there is an important asymmetry in how idiosyncratic shocks influence comovement for gross output versus value added in the model. In all simulations, gross output comoves more strongly than real value added. This difference is attributable to comovement in input use in the model. Because gross output is a composite of real value added and intermediate inputs, comovement in input use can synchronize gross output even if real value added is held fixed. In practice, intermediate input trade serves to synchronize input use in the model, and therefore leads gross output to be more synchronized than real value added across countries.

As a final step in the analysis, I explore how changes in elasticities influence output comovement in the model. I focus on changes in three key elasticities in the model: (a) the within-sector elasticity of substitution across final goods from alternative source countries, (b) the elasticity of substitution across inputs from alternative countries or sectors, and (c) the elasticity of substitution between primary factors and intermediate inputs in production.

The experiments I implement address three main issues. First, as is well known, the elasticity of substitution between home and foreign output (the 'trade elasticity') is an important parameter in shaping comovement in IRBC-type models. In my model, the trade elasticity depends directly on both (a) and (b), so I quantify the role of the trade elasticity by varying these two parameters. Second, Burstein et al. (2008) and Di Giovanni and Levchenko (2010) have emphasized the potential role of input complementarity in explaining comovement. To isolate the role of complemetarity separate from the trade elasticity, I simulate the model for alternative values of (a) and (b) that leave the aggregate trade elasticity approximately constant. Third, changes in the complementarity between primary factors and inputs may alter the degree of value added comovement in the model. The third set of simulations therefore varies (c), again holding the trade elasticity constant.

This analysis confirms one standard result (with a twist) and yields two new findings. The standard result is that lower aggregate trade elasticities yield more output comovement in the model, particularly for goods output. The twist is that this heightened comovement for the goods sector has only a minimal impact on aggregate comovement, since services output remains weakly correlated across countries. The two new results are that – holding the aggregate trade elasticity constant – the goods trade-comovement correlation is higher when inputs are substitutable relative to final goods, or primary factors are substitutable for intermediate inputs. With the 'most favorable' configuration of parameters, the model with uncorrelated shocks generates trade-comovement correlations equal to about 3/4 of the data for goods gross output, and 1/4 for value added.

In addition to the empirical work cited above, this paper is related to a number of recent attempts to incorporate input trade into business cycle models. The closest antecedent is a two-country, two-sector IRBC model with intermediates by Ambler, Cardia, and Zimmerman (2002).[4] The framework also shares many features with Bems and Johnson (2012), who study how international relative prices influence demand for domestic value added when inputs are traded. It is also related to Bems (2013), who studies how input trade influences relative price adjustment during external rebalancing episodes.

This paper is also related in spirit to recent models by Burstein, Kurz, and Tesar (2008), Arkolakis and Ramanarayan (2009), and Bergin, Feenstra, and Hanson (2011). Among these papers, the contrast with Burstein et al. is most relevant.[5] They study a two sector IRBC

[4]This paper is distinguished from Amber et al. in both scope and focus. Whereas Amber et al. focus on a stylized two country case, I calibrate and simulate a many country model to match newly available global input-output data. Further, I focus on the trade-comovement puzzle, where Ambler et al. emphasize general business cycle moments. Lastly, Ambler et al. devote attention to analyzing the role of investment frictions and capital depreciation in their framework, where I focus on the role of elasticities.

[5]Arkolakis and Ramanarayan (2009) adopt a multi-stage production function, an approach that deviates more strongly from the IRBC tradition. Bergin, Feenstra, and Hanson (2011) work with a two sector model, in which the 'offshoring sector' involves Ricardian trade in a continuum of goods. They emphasize the role

model, in which the 'production sharing' sector combines foreign and domestic value added to produce final goods. This sector features a low elasticity of substitution between domestic and foreign value added, so increasing the size of the production sharing sector lowers the aggregate trade elasticity and raises comovement. In contrast to their model, my framework features production functions that combine domestic value added with traded gross inputs, rather than domestic and foreign value-added directly. This allows my model to match global input-output tables. Further, I analyze the role of trade elasticities versus complementarity of inputs separately, where their analysis does not separate these parameters.

More broadly, the model in this paper shares important characteristics with closed-economy models of sectoral linkages, as developed in Long and Plosser (1983).[6] This literature provides many insights that are applicable to cross-border input trade. However, there is an important difference to keep in mind. Within the domestic economy, factors may be reallocated across sectors following a shock, whereas factors are comparatively immobile across countries. This weakens the link between gross output and value added comovement in my international model relative to domestic models.

Finally, in simulating an international macro model with more than two heterogeneous countries, the paper is also related to work by Zimmerman (1997), Kose and Yi (2006), Juvenal and Monteiro (2010), and Ishise (2012a,b). These papers emphasize that third-country effects may be important in driving bilateral correlations, effects that are picked up in my many country framework. None feature trade in inputs, however.

The remainder of the paper proceeds as follows. In Section 2, I describe the model and discuss several features of the framework. In Section 3, I describe how I calibrate the model and estimate the stochastic processes for productivity. I discuss the simulation results in Section 4, starting with data facts in Section 4.1, the baseline model results in Section 4.2, and results for alternative elasticities in Section 4.3. Section 5 concludes.

2 A Many Country, Multi-Sector Sector Model with Cross-Border Input Linkages

I begin by articulating a multi-sector, many country international real business cycle model that allows trade in both final and intermediate goods. The key difference between this model and the standard IRBC framework is that I specify production functions for gross

of the extensive margin of offshoring, which is not included in my framework.

[6]The subsequent literature includes Hornstein and Praschnik (1997), Horvath (1998, 2000), Dupor (1999), Shea (2002), Carvalho (2008), and Foerster, Sarte, and Watson (2011), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012).

output and define preferences over purchases of final goods. This has two implications.

First, I can calibrate the production structure in the model to match cross-border input shipments, while calibrating preferences to match shipments of final goods. As I discuss further below, this eliminates the inconsistent treatment of gross versus value added objects in standard calibrations of the IRBC framework. Second, there is a new channel for transmission of shocks through the production chain that is not operative in the standard IRBC framework. After introducing the model, I discuss both these features in greater detail.

2.1 Production

Consider a multi-period world economy with many countries ($i, j \in \{1, \ldots, N\}$). Country i produces a tradable differentiated good in sector s using capital $K_{it}(s)$, labor $L_{it}(s)$, and composite intermediate good $X_{it}(s)$, which is an aggregate of intermediate goods produced by different source countries. I assume that the sector-level production function takes a nested CES form:

$$Q_{it}(s) = Z_{it}(s) \left(\theta_i(s)^{1-\sigma} V_{it}(s)^\sigma + (1-\theta_i(s))^{1-\sigma} X_{it}(s)^\sigma\right)^{1/\sigma} \tag{1}$$

$$\text{with} \quad X_{it}(s) = \left(\sum_{j=1}^{N} \sum_{s'=1}^{S} \omega_{ji}^x(s', s)^{1-\eta} X_{jit}(s', s)^\eta\right)^{1/\eta} \tag{2}$$

$$\text{and} \quad V_{it}(s) = K_{it}(s)^\alpha L_{it}(s)^{1-\alpha}, \tag{3}$$

where $X_{jit}(s', s)$ is the quantity of intermediate goods from sector s' in country j used by sector s in country i, $V_{it}(s)$ is a composite domestic factor input composed of capital and labor, $Z_{it}(s)$ is sector-specific productivity, and $\{\theta_i(s), \omega_i^x(s', s), \alpha\}$ are parameters that govern shares of inputs in gross output, individual inputs in total input use, and individual factors in value added respectively.

Output is produced under conditions of perfect competition. A representative firm in country i, sector s takes the prices for its output and inputs as given, and the firm rents capital and hires labor to solve:

$$\begin{aligned} \max \quad & p_{it}(s)Q_{it}(s) - w_{it}L_{it}(s) - r_{it}K_{it}(s) - \sum_{j=1}^{N}\sum_{s'=1}^{S} p_{jt}(s')X_{jit}(s', s) \\ \text{s.t.} \quad & L_{it}(s) \geq 0, K_{it}(s) \geq 0, X_{jit}(s', s) \geq 0, \end{aligned} \tag{4}$$

where $p_{it}(s)$ denotes the price of output, w_{it} is the wage, r_{it} is the rental rate for capital, and the production function for $Q_{it}(s)$ is given above. This problem can be broken into two steps. In the first step, the firm chooses the amount of the composite factor $V_{it}(s)$ and intermediate $X_{it}(s)$ to use, given the prices of the composite factor $p^v_{it}(s) = \left(\frac{r_{it}}{\alpha}\right)^{\alpha}\left(\frac{w_{it}}{1-\alpha}\right)^{1-\alpha}$ and intermediate $p^x_{it}(s) = \left(\sum_{j=1}^{N}\sum_{s'=1}^{S}\omega^x_{ji}(s',s)p_{jt}(s')^{\eta/(\eta-1)}\right)^{(\eta-1)/\eta}$. In the second step, the firm then chooses capital, labor, and the use of individual intermediates.

Output is used both as an intermediate input in production and to produce a composite final good. Denoting final goods shipments from country i to country j in sector s as $F_{ijt}(s)$, then gross output from sector s in country i equals shipments used as intermediates plus shipments used to produce final composite goods:

$$Q_{it}(s) = \sum_{j}\left(F_{ijt}(s) + \sum_{s'=1}^{S} X_{ijt}(s,s')\right). \tag{5}$$

Sector-level shipments of final goods are aggregated by competitive firms to form a composite final goods as follows. Within each sector, final goods from all sources are combined via a CES aggregator to form a sector-level composite: $F_{it}(s) = \left(\sum_{j=1}^{N}\omega^f_{ji}(s)^{1-\rho}F_{jit}(s)^{\rho}\right)^{1/\rho}$. And these sector-level composites are combined via a Cobb-Douglas aggregator to form a composite final good $F_{it} = \prod_{s=1}^{S} F_{it}(s)^{\gamma_i(s)}$, where $\gamma_i(s)$ is the expenditure share on final goods of type s in country i.[7] A representative final goods firm maximizes:

$$\begin{aligned} \max \quad & p^f_{it}F_{it} - \sum_{j=1}^{N}\sum_{s=1}^{S} p_{jt}(s)F_{jit}(s), \\ \text{s.t.} \quad & F_{jit}(s) \geq 0, \end{aligned} \tag{6}$$

where p^f_{it} is the price of the composite final good and the production function for F_{it} is given above. As above, this can be thought of as a two step process, where first firms choose the amount of each composite final good $F_{it}(s)$ to use given prices for those composites $p^f_{it}(s) = \left(\sum_{j=1}^{N}\omega^f_{ji}(s)p_{jt}(s)^{\rho/(\rho-1)}\right)^{(\rho-1)/\rho}$ and then choose final goods from individual sources to form the composites.

The composite final good in each country is used for consumption and investment: $F_{it} = C_{it} + I_{it}$. The aggregate capital stock evolves according to: $K_{it+1} = I_{it} + (1-\delta)K_{it}$, where $K_{it} = \sum_{s=1}^{S} K_{it}(s)$.

[7]Note that I assume that there is no value added at this stage to be consistent with the accounting conventions in the input-output data, which records the value of retail and distribution services as output of the services sector.

2.2 Consumption and Labor Supply

Each country is populated by a representative consumer, who consumes final goods and supplies labor L_{it} for production, with $L_{it} = \sum_{s=1}^{S} L_{it}(s)$. The consumer's utility function is given by:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \left[\log(C_{it}) - \frac{\chi\epsilon}{1+\epsilon} L_{it}^{(1+\epsilon)/\epsilon} \right]. \tag{7}$$

where ϵ is the Frisch elasticity of labor supply and β is the rate of time preference.

2.3 Asset Markets

In specifying the equilibrium in the model, I need to take a stand on financial market structure. In the main text and simulations, I assume that financial markets are complete. To write out the budget constraint in this case, I introduce explicit state notation here, which is suppressed elsewhere.

Let the state of the world at time t by indexed by ϖ_t, with transition probability density $f(\varpi_{t+1}, \varpi_t)$. Then let $B_i(\varpi_{t+1})$ denote country i's holdings of a one-period state-contingent bonds, paying off one unit of the numeraire good in state ϖ_{t+1}, and let $b(\varpi_{t+1}, \varpi_t)$ be the price of that security in state ϖ_t at date t. Further, these state-contingent bonds are in zero net supply in all states: $\sum_i B_i(\varpi_{t+1}) = 0$. Assuming the consumer owns the domestic capital stock, her budget constraint is then:

$$p_{it}^f (C_{it} + I_{it}) + \int b(\varpi_{t+1}, \varpi_t) B_i(\varpi_{t+1}) d\varpi_{t+1} = r_{it} K_{it} + w_{it} L_{it} + B_i(\varpi_t). \tag{8}$$

The consumer's problem is then to choose $\{C_{it}, L_{it}, K_{it+1}\}$ and asset holdings $\{B_i(\varpi_{t+1})\}$ given prices and initial asset endowments $\{B_i(\varpi_0)\}$ to maximize Equation (7) subject to Equation (8).

In the appendix, I report results for a second version of the model with restricted financial markets. Specifically, I solve and simulate the model holding nominal trade balances constant over time at their initial steady-state level.[8] In terms of explaining output comovement, this alternative model produces results that are both qualitatively and quantitatively similar to the complete markets markets model.[9] See the Online Appendix for details.

[8]The polar opposite of complete markets is obviously financial autarky, equivalently balanced trade. Complete financial autarky is inconsistent with steady state trade balances, which arise in the data. Therefore, I calibrate this alternative version of the model to match steady-state trade imbalances, and then hold those nominal imbalances constant. The dynamics in this case are similar to those from a model with true financial autarky, where trade balances are held constant at zero.

[9]Obviously, the models yield different results for the degree of consumption comovement in the model. The fact that output comovement is similar in both versions of the model suggest that "resource shifting"

2.4 Equilibrium

Given a stochastic process for productivity and initial asset holdings $\{B_i(\varpi_0)\}$, an equilibrium in the model is a collection of quantities $\{C_{it}, F_{it}, B_i(\varpi_t)\}_i$ for each country, quantities $\{Q_{it}(s), K_{it}(s), L_{it}(s), \{F_{jit}(s)\}_j, \{X_{jit}(s', s)\}_{j,s'}\}_{i,s}$ for each country-sector, and prices $\{r_{it}, w_{it}, p^f_{it}, \{p_{it}(s)\}_s\}_i$ and $b(\varpi_{t+1}, \varpi_t)$. These must solve the consumer's and producers' problems, and clear goods, factor, and asset markets. The equilibrium conditions are collected explicitly in Appendix A.

2.5 Discussion

The model articulated above differs from the standard IRBC framework in that I specify a production function for gross output (Equations (1)-(3)), and therefore account directly for intermediates that are 'used up' in the production process. As mentioned above, this means that the transmission mechanisms and calibration procedure are different than the standard IRBC model. I pause here to discuss both these issues in greater detail.

2.5.1 Mechanics of Comovement

In examining comovement on the production side, it is important to distinguish between real gross output and real value added. With the general CES formulation of the production function, one cannot write real value added as a closed form function of capital, labor, and productivity alone. So I will take an indirect approach and define real value added as a subfunction of gross output, and characterize how real value added changes over time. This approach is consistent with the national accounts practice of defining real GDP via double deflation [Sims (1969)].

Suppose that gross output can be written as: $Q_{it}(s) = g(RVA_{it}(s), X_{it}(s); t, s)$, where $RVA_{it}(s) = h(K_{it}(s), L_{it}(s); t)$ is a function defining how real value added is produced from primary factors and $g(\cdot)$ is homogeneous of degree one. Given constant returns to scale and perfect competition, then write proportional changes in gross output as:

$$\hat{Q}_{it}(s) = s^v_i(s)\widehat{RVA}_{it}(s) + s^x_i(s)\hat{X}_{it}(s), \tag{9}$$

where $s^v_i(s) \equiv \frac{p^v_i V_i(s)}{p_i(s)Q_i(s)}$ and $s^x_i(s) \equiv \frac{p^x_i(s)X_i(s)}{p_i(s)Q_i(s)}$ are the steady-state shares of value added and

effects are not important in explaining my results. This is in contrast to Kose and Yi (2006), who suggest that financial autarky improves the ability of IRBC models to replicate the trade-comovement relationship.

intermediate inputs in gross output. Then manipulation of this expression yields:

$$\begin{aligned}\widehat{RVA}_{it}(s) &= \frac{1}{s_i^v(s)}\left[\hat{Q}_{it}(s) - s_i^x(s)\hat{X}_{it}(s)\right] \\ &= \frac{1}{s_i^v(s)}\hat{Z}_{it}(s) + \hat{V}_{it}(s),\end{aligned} \tag{10}$$

where the transition from the first to the second line uses the fact that $\hat{Q}_{it}(s) = \hat{Z}_{it}(s) + s_i^v(s)\hat{V}_{it}(s) + s_i^x(s)\hat{X}_{it}(s)$ in the model above.[10]

The need to distinguish comovement in gross output from comovement in real value added is evident on examination of these equations. Gross output is a composite of real value added and intermediate inputs, while real value added depends on productivity and factor inputs alone. Real output growth may be correlated across countries either because real value added growth is correlated, or because growth in input use is correlated across countries. Thus, traded intermediates loosen the link between real output and value added in the model.

In an extreme case, gross output could be correlated across countries even if real value added is constant in all countries. I pause to discuss this special case to provide intuition regarding the role of input linkages in the model. I make two simplifying assumptions to move from the general model to this special case. First, I assume that each country and sector is endowed with a fixed amount of the composite factor. This shuts down both model dynamics and endogenous comovement in real value added. Second, I assume that the production function, input aggregators, and final goods aggregators are all Cobb-Douglas.

As described in Appendix A, the proportional change output following productivity innovations in this special case is given by:

$$\hat{Q} = [I - \Omega']^{-1}\hat{Z}, \tag{11}$$

where Q and Z are vectors that stack gross output and productivity in all countries and sectors. The Ω matrix is a global bilateral input-output matrix that summarizes flows of intermediates across countries and sectors. The matrix $[I - \Omega']^{-1}$ provides a set of weights that indicate how production of sector s in country i responds to productivity shocks to sector s' in country j. The weights can be interpreted as the total cost share of intermediates from sector s' in country j in production of sector s in country i, which include both direct purchases of inputs from j and indirect purchases of inputs from j embodied in purchases of

[10]Note that if we instead assume the production function is Cobb-Douglas in $V_{it}(s)$ and $X_{it}(s)$, we skip these steps and write gross output explicitly as a function of real value added: $Q_{it}(s) = RVA_{it}(s)^{\theta_i(s)}X_i(s)^{1-\theta_i(s)}$, where $RVA_{it}(s) = Z_i(s)^{1/\theta_i(s)}V_{it}(s)$ is real value added.

inputs from third countries.

These total cost shares summarize how shocks are transmitted through the structure of cross-border input linkages. Put simply, a positive productivity shock in country k raises output in countries that use country k goods as inputs. This is true whether they use k goods directly or whether they rely on country k goods indirectly, in the sense that they source intermediates from some third country that itself relies heavily on inputs from country k. This has the implication that output will be correlated for country i and country j when they have similar overall sourcing patterns.

This logic underlying how input linkages transmit shocks across borders is intimately related to how input linkages transmit shocks across sectors. Not surprisingly, therefore, variants on Equation (11) are embedded in closed economy models by Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Carvalho (2008), and Foerster, Sarte, and Watson (2011), and Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi (2012). These papers all study the role of sector-level shocks in generating aggregate fluctuations. Hornstein and Praschnik (1997), Shea (2002) and Conley and Dupor (2003) focus on the role of domestic input-output linkages in explaining output comovement across sectors in the United States. Shea (2002), for example, uses a closed economy version of Equation (11) to measure the strength with which cost shocks in upstream sectors propagate downstream.

Broadening our focus beyond the special case, the general model features these input linkages alongside the standard IRBC transmission of shocks via relative prices and factor supply. If intermediates are removed from the model (setting $\theta_i(s) = 1$), then the production function is linear in the composite factor: $Q_{it}(s) = Z_{it}(s)V_{it}(s)$. When productivity shocks are uncorrelated across countries, output in country i will then be correlated with output in country j only if factor supplies V_i and V_j co-move.

Comovement in factor supplies, in turn, reflects two well-known forces. On the one hand, terms of trade movements following productivity shocks tend to generate positive comovement, particularly in labor inputs. As in the standard IRBC model, a productivity increase in country i causes the relative price of output from country i to fall. From the foreign perspective, the resulting terms of trade appreciation raises factor returns and hence induces increased factor supply and output. The strength of this channel depends on how responsive prices are to the underlying shocks, with lower elasticities of substitution between home and foreign output yielding larger price movements. Offset against this force for positive comovement, the model with complete markets also features 'resource shifting effects', whereby a positive productivity shock at home raises the return to investment at home and hence draws capital into the country. This dampens the positive comovement in total factor inputs (capital plus labor). In practice, we will see that the first channel tends to dominate the second, yielding positive comovement in real value added in response to idiosyncratic productivity shocks.[11]

2.5.2 Taking the Model to Data

Before turning to calibration details, there are several broad points about matching this model to data that deserve comment. The production function and resource constraints above represent a multi-stage production process with an effectively infinite number of production stages, where value is added at each stage in a decreasing geometric sequence. Because production requires both domestic and imported intermediates, gross trade in the model will be a multiple of the actual value added exchanged between countries, as goods cross borders many times throughout the production process. In this sense, the model allows for 'double counting' in trade statistics associated with input trade.

The standard IRBC framework is not compatible with 'double counting' in trade data, or the use of imports to produce exports.[12] In the IRBC literature, the convention has been to write down production functions for value added, where value added is produced output of domestic factors (e.g., capital and labor). This production structure introduces several complications for calibration using conventional data.

Consistent with the value added production structure, IRBC models are typically calibrated treating gross exports and imports *as if* they are measured in value added terms. Put differently, they are calibrated under the implicit assumption that the domestic value added content of exports is equal to one. This procedure creates a model economy that is 'too open' relative to reality. Johnson and Noguera (2012) report that the ratio of value added to gross trade is about 0.7 for the median country. Therefore, treating gross exports as if they are value added implies that the economy is roughly 40% too open in the standard calibration. By calibrating a model with a production structure for gross output, I am able to circumvent this problem.

On top of this problem, there are also complications in calibrating preferences in the standard IRBC framework. To be consistent with production that is measured in value added terms, the standard model must implicitly specify preferences over value added. This is problematic in the sense that substitution elasticities are always estimated using data on gross expenditure or gross trade flows. Therefore, they may not be appropriate for models

[11] Another way to see that 'resource shifting' plays a small role in explaining the results below is that I find similar comovement results in versions of the model with or without complete markets.

[12] Some semantic confusion may arise in comparing these frameworks. Starting at least with Backus, Kehoe, and Kydland (1994), IRBC models typically talk about trade in "intermediate goods," which are aggregated to produce a "composite final good." Despite this nomenclature, trade in these models should be thought of as trade in value added or quasi-final goods, wherein output crosses an international border only once.

with production/preferences in value added models.[13] Because I specify preferences over final goods directly, conventional expenditure-based elasticity estimates are appropriate in the context of my framework.

3 Calibration

I solve for model dynamics in a two-sector version (goods versus services) of the model using standard linearization techniques. For reference, I include the linearized equilibrium conditions in Appendix A. In this section, I briefly describe how I parametrize the linearized model and estimate the stochastic process for productivity, with details in the appendices.

3.1 Parameters

To simulate the linearized model, I need values for several structural parameters, along with information on some steady-state value shares. Starting with the parameters, I need to assign values to $\{\beta, \epsilon\}$ for preferences and $\{\sigma, \eta, \rho, \alpha, \delta\}$ for the technology. Some of these parameters are identical across simulations, while others change. In all simulations, I set $\alpha = .33$, $\delta = .1$, $\beta = .96$, and $\epsilon = 4$ based on standard values in the literature.[14]

The elasticity parameters $\{\sigma, \eta, \rho\}$ vary across simulations to allow different degrees of complementary versus substitutability in production and preferences. In the baseline simulation below, I set $\rho = .5$, so the elasticity of substitution between final goods from different sources is 2. On the production side, I set $\sigma = \eta = 0$ in the baseline simulation. This implies that the production function is Cobb-Douglas in real value added and the composite intermediate, and that the composite intermediate is itself Cobb-Douglas in inputs from different source countries. In Section 4.3, I consider alternative elasticities, and defer discussion of those cases till then.

3.2 Steady-State Shares

The remaining data needed to parametrize the linearized model are steady-state value shares (e.g., the share of inputs in production, the share of foreign goods in final demand and input

[13]See Herrendorf, Rogerson, and Valentinyi (forthcoming) for discussion of this issue in the context of models of structural transformation.

[14]On the Frisch elasticity, see King and Rebelo (1999) or Chetty, Guren, Manoli, and Weber (2011). While a Frisch elasticity of 4 is required to generate fluctuations in hours worked similar to data in the standard RBC model, it has been criticized as too high relative to micro estimates. In unreported results, I have simulated the model with a Frisch elasticity of labor supply set to 1, and the performance of the model is both qualitatively and quantitatively very similar.

use, etc.). Data on value added and gross output by sector $\{p_i(s)Q_i(s), p_i^v V_i(s)\}$ plus bilateral final and intermediate goods shipments $\{p_i(s)F_{ij}(s), p_i(s)X_{ij}(s, s')\}$ are sufficient to compute the shares.

I obtain these data from the World Input-Output Database (WIOD) for the year 1995.[15] Due to limitations on the availability of time series data on output and productivity (see below), I include 22 countries from the WIOD database separately in the model, covering approximately 80% of world GDP, and aggregate the remaining countries to form a composite "rest-of-the-world" region. Further, I aggregate the WIOD data to form two composite sectors, defined as "goods" (including agriculture, natural resources, and manufacturing) and "services."

By taking this all information directly from the data, the steady state matches country/sector sizes and bilateral trade flows exactly. Further, to match both value added and expenditure data, I allow trade to be unbalanced in the steady state. Therefore, steady state trade balances match those observed in data, and then fluctuate around those values in the simulations.

3.3 Productivity Process

In the model, $Z_{it}(s)$ is TFP for the production of gross output. Since data on gross output TFP is unavailable for many countries and years, I estimate the stochastic process for productivity in a two step procedure. I discuss these steps briefly here, and provide additional discussion of the procedure in Appendix B.

In the first step, I estimate a stochastic process for value-added TFP. Since value-added TFP data is also not widely available, I follow the literature and use data on value-added labor productivity (LP) in place of value-added TFP data in estimation. In practice, this means that I estimate the following productivity process:

$$\log LP_{it}^{VA}(s) = \lambda_i(s) \log LP_{it-1}^{VA}(s) + \epsilon_{it}(s), \tag{12}$$

where $LP_{it}^{VA}(s)$ is value-added labor productivity.[16]

[15] The WIOD database directly measures cross-country shipments of final and intermediate goods using disaggregate commodity trade data classified according to the BEC system, which links Harmonized System codes to national accounts end uses. While the WIOD covers 1995-2009, I choose the earliest year, since it is the closest year to the midpoint of my output and productivity time series.

[16] Note that there are no cross-country or cross-sector spillovers in this productivity process. With N countries and 2 sectors, I cannot estimate unrestricted cross-country spillovers given the relatively short length of the time series available. I have experimented with allowing cross-sector spillovers within countries. Point estimates for cross-sector spillovers are generally unstable across countries and imprecisely estimated (often indistinguishable from zero). Therefore, I omit them for simplicity.

In the second step, I convert the estimated productivity process in Equation (12) into an equivalent stochastic process for gross output TFP. This entails converting the shocks $\epsilon_{it}(s)$, which apply to value-added TFP, into equivalent shocks for gross output TFP. To do this, I multiply each residual by the steady-state ratio of value added to gross output: $\tilde{\epsilon}_{it}(s) \equiv s_i^v(s)\epsilon_{it}(s)$. I then use $\tilde{\epsilon}_{it}(s)$ to form the covariance matrix of shocks to $\log(Z_{it}(s))$, denoted $\tilde{\Sigma}$.

I use data on annual sectoral labor productivity growth over the 1970-2007 period from the Groningen Growth and Development Centre's EU KLEMS and 10-Sector databases.[17] To extract the cyclical component of productivity, corresponding to $\log LP_{it}^{VA}(s)$ above, I use the Hodrick-Prescott (HP) filter. Since the data frequency is annual, I set the value of the HP smoothing parameter to 6.25 in my baseline estimates.[18] With this degree of smoothing, the estimated autocorrelation in cyclical productivity is low, and not statistically distinguishable from zero, in nearly all countries and sectors. Therefore, for simulations in the main text, I set $\lambda_i(s)$ equal to zero, and treat the cyclical component of labor productivity as a measure of $\epsilon_{it}(s)$. In the Online Appendix, I document that all the key results below are robust to setting the HP smoothing parameter to a larger value, which generates more persistence in the productivity process.[19]

In the simulations below, I will use the covariance matrix $\tilde{\Sigma}$ in two ways. One set of simulations will allow shocks to be correlated across countries, with correlations determined by the estimated covariance matrix. This is the standard approach in the literature. The shortcoming of this approach is that comovement in this set of simulations is driven both by transmission of shocks across countries via trade linkages and the direct correlation of the underlying shocks themselves.

To more cleanly identify the trade transmission mechanism, I will also simulate the model under the (counterfactual) assumption that shocks are uncorrelated across countries. To pa-

[17]See http://www.rug.nl/research/ggdc/databases. The EU KLEMS database includes 19 OECD countries: Australia, Austria, Belgium, Canada, Denmark, Spain, Finland, France, Germany, Greece, Ireland, Italy, Japan, Korea, Netherlands, Portugal, Sweden, United Kingdom, and the United States. Where possible, I use the 2009 version (revised 2011) of the EU KLEMS data, and fill in using the 2008 version of the data where data is missing in the 2009 version. I obtain data for Brazil, India, and Mexico from the 10-sector database. In the EU KLEMS data, labor productivity growth is measured as real value added growth less growth in hours worked. In the 10-Sector database, productivity is measured as real value added growth less growth in the number of workers employed.

[18]Ravn and Uhlig (2002) demonstrate that a smoothing parameter of 6.25 generates the same degree of smoothing as a value of 1600 in quarterly data, which is the common default value for quarterly data. See also the textbook discussion in Canova (2011). Baxter and King (1999) also argue for a low value (10) for smoothing parameter in annual data.

[19]In the appendix, I examine results with a smoothing parameter equal to 100, as used by Backus, Kehoe, and Kydland (1994). In a previous working paper, I also presented results based on linearly detrending the data.

rameterize this counterfactual scenario, I zero out the "off-diagonal" elements of the covariance matrix, loosely following Horvath (1998). Specifically, I impose $\text{cov}(Z_{it}(s), Z_{jt}(s')) = 0$ for all $i \neq j$. This allows shocks to be correlated across sectors within countries, but uncorrelated for any cross-country sector pairs. While this eliminates cross-country correlations in shocks, it should be noted that $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is an upper bound to the size of the truly independent productivity shocks.[20] This implies that simulated shocks using this method will be somewhat too large relative to the truly idiosyncratic shocks that countries face. Thus, one should interpret simulation results using these idiosyncratic shocks as an upper bound on the ability of the model to generate comovement from true idiosyncratic country shocks.

One last detail regarding the simulation is that I include a composite rest-of-the-world region in the simulations, but do not have directly measured productivity data for this composite region. Therefore, I assume that productivity shocks in the rest-of-the-world are uncorrelated with productivity shocks to countries in my sample. I parameterize the variance and cross-sector correlations of the shocks to this region based on median values in the data.

4 Results

To frame the analysis, I open this section by briefly presenting two sets of stylized facts concerning the relationship between trade and comovement at the aggregate and sector levels. I then examine the model's ability to match these facts. I begin by describing trade-comovement correlations allowing productivity shocks to be correlated across countries, as in the data. The analysis focuses on three questions. First, how do sector-level correlations aggregate up to generate the aggregate trade-comovement correlation? Second, what role do trade linkages versus the correlation of shocks play in explaining these patterns? Third, what role does input trade play in explaining how idiosyncratic shocks are transmitted across countries in the model? Finally, I examine how trade-comovement correlations change as I vary the degree of complementarity/substitutability in production and demand.

4.1 Trade and Output Comovement: Data

There are two sets of stylized facts in the data that deserve to be highlighted at the outset, since these serve as benchmarks against which I will evaluate model fit.

[20]For example, suppose that there are global shocks and i.i.d. country shocks. Then $\text{cov}(Z_{it}(s), Z_{it}(s'))$ is equal to the sum of the variance of the global shock plus the variance of the idiosyncratic country shock, and hence an upper bound on the variance of the idiosyncratic shock.

First, bilateral trade intensity is positively correlated with bilateral comovement in both aggregate value added and gross output.[21] I plot these these relationships in Figure 1, and report the corresponding regression point estimates in Table 1 [Panel A, columns (1) and (5)]. In the figures and table, output comovement is measured by the pairwise correlation of year-on-year growth rates. Bilateral trade intensity is defined as $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$, and is computed for the benchmark calibration year (1995).[22] The regression point estimates indicate that a one point increase in log bilateral trade intensity translates into a bilateral output correlation that is 0.1 larger. To fix ideas, this means that moving from say the US-Spain (-6.57) to US-Canada (-3.45) levels of log bilateral trade is associated in an average increase of about 0.3 in output correlations. This estimated aggreagte trade-comovement relationship is large, but in line with the literature.

Second, bilateral trade is positively correlated with comovement in sector-level value added and gross output. And this relationship is strong for goods-goods, services-services, and goods-services (cross) sector pairs. These sector-level correlations are depicted in Figure 2, with point estimates in Table 1 [Panel A, columns (2)-(4) and (6)-(8)]. Similar to the previous figures, correlations here are computed for year-on-year growth rates of real sector-level output. Further, to facilitate comparison to the aggregate results, the x-variable is aggregate bilateral trade intensity.[23]

The uniformity of these results for value added vs. gross output, and for aggregate vs. sector-level output, is striking. Underlying these results are two meta-results.[24] The first is that cross-country correlations in value-added and gross output are very similar. In the aggregate, the correlation between the cross-country correlation in value added and the cross-country correlation in gross output is 0.82. Similar results hold at the sector level as well. The second is that country pairs with high bilateral comovement in goods production also tend to have high comovement in services production.

[21] Real value added and gross output data is taken from the EU KLEMS data for all countries, with three exceptions. Real value added data for Brazil, India, and Mexico is from Groningen's 10-sector Database. Gross output data is not available for these three countries, so gross output correlations presented below are computed among the remaining 19 countries (171 bilateral pairs). For most countries, data covers the 1970-2007 period. However, several countries have truncated time series: Brazil (1970-2005), Canada (1970-2004), India (1970-2005), Japan (1973-2006), and Portugal (1970-2006). Correlations with these countries are computed over these slightly shorter time periods.

[22] In the Online Appendix, I estimate trade-comovement regressions in both the model and data using the level of bilateral trade intensity. All results emphasized below go through with this alternative specification. I prefer the log specification, due to the apparent linearity of the relationship depicted in the figures.

[23] Supplemental estimates using sector-level measures of bilateral trade intensity are in the Online Appendix. The point estimates are similar, since sector-level measures of trade intensity are highly correlated with aggregate trade intensity.

[24] See the Online Appendix for illustration of these results.

4.2 Trade and Comovement: Model

To compare model with data, I compute pairwise correlations of year-on-year output growth in the model with correlated shocks as averages over 500 replications of 35 years each. In Table 1 [Panel B], I project these model correlations on bilateral trade intensity. Looking at columns (1) and (4), the model with correlated shocks produces a small, positive aggregate trade-comovement coefficient. The coefficient is roughly 13-24% the size of the coefficient estimated in the data, and only statistically different than zero for gross output.

These results indicate that the aggregate trade-comovement puzzle is alive and well. Neither the correlation of productivity shocks across countries, nor the transmission of idiosyncratic shocks through trade, is strong enough to fully replicate the strength of the observed aggregate relationship. However, these aggregate results obscure important differences in model fit across sectors.

Starting with the positive, the model does generate a strong trade-comovement relationship for goods output. The regression coefficients for goods sector comovement are 3/4 as large as in data for value added [column (2)], and even higher for gross output [column (6)].[25] This high trade-comovement coefficient for goods contrasts sharply with a low (slightly negative) trade-comovement coefficient for services sectors [columns (3) and (7)], which is grossly at odds with the data. The model does better for cross-sector correlations, yielding a positive trade-comovement coefficient equal to about 30-35% of that in the data [columns (4) and (8)].

Bringing the aggregate and sector-level results together, the aggregate trade-comovement coefficient can be interpreted as a weighted average of the sector-level coefficients. Therefore, these sector-level results imply that the low aggregate trade comovement correlation in the model is largely driven by the model's failure to match services comovement across countries.

To illustrate the differential performance of the model for goods and services directly, I plot cross-country correlations of output for the goods sector and the services sector in Figure 3. The correlations between model-predicted and actual output correlations for the goods sector are about 0.6 for real value added and 0.4 for gross output. For services, the correlation is about 0.2 for value added and zero for gross output.[26] This dichotomy – the model fits relatively well for goods and poorly for services – is at the heart of the aggregate

[25] Restricting the sample to include only the 171 pairs for which we have both gross output and value added data, the trade-comovement coefficient for real value added is roughly 65% as large as in data. See the Online Appendix.

[26] The model underpredicts the median bilateral correlation for both sectors. The median correlation for goods is 0.19 for gross output and 0.14 for real value added in the model, versus 0.40 and .28 respectively in data. The median correlation for services is 0.05 for gross output and 0.02 for real value added in the model, versus 0.25 and 0.21 respectively in data.

puzzle.

4.2.1 Propagation of Idiosyncratic Shocks vs. Correlated Shocks

The strong comovement in goods production in the model with correlated shocks could be explained in two ways. Goods sectors might comove because they are more tightly linked across countries through trade. Or, they might comove because goods productivity shocks are more highly correlated across countries. That is, the trade-comovement correlation could genuinely capture the role of trade in propagating shocks, or bilateral trade intensity could simply be a proxy for the underlying correlation of productivity shocks.

In this section, I focus on distinguishing between these two alternatives. To do so, I first present results from model simulations with uncorrelated shocks, which quantify how strongly output comoves in response to idiosyncratic shocks. I then examine the correlation of productivity shocks directly.

Idiosyncratic Shocks In Table 1 [Panel C], I report the results of trade-comovement regressions in simulated data from the model with uncorrelated productivity shocks. The aggregate trade-comovement correlation declines substantially in these simulations [columns (1) and (5)], falling to less than 10% of the true coefficient for gross output, and even lower for value added.

At the sector level, the coefficients are also attenuated toward zero. Only the trade-comovement coefficient for gross output of goods is appreciably different than zero, equal to about 1/4 the size of the coefficient with correlated shocks [column (6), Panel C vs. Panel B]. Yet, despite the fact that trade is correlated with gross output comovement for goods, it is not strongly correlated with value added comovement [column (2)]. This result deserves separate attention, as it highlights the role that intermediates play in this framework, so I will return to it below.

Stepping back, the weak propagation of idiosyncratic shocks in the model is related to the findings in Kose and Yi (2006). Since our methods differ, I pause to compare my approach to theirs. In a three-country IRBC model, Kose and Yi vary bilateral trade-intensity exogenously (by manipulating trade costs), holding the correlation of shocks across countries constant. They then compare value added correlations across equilibria with different trade patterns and compute a trade-comovement quasi-regression coefficient off these comparisons. They find that this quasi-regression coefficient is at most 1/10th the size of the coefficient in the data, a similar order of magnitude to the coefficients here. The exercise I perform is somewhat different. Rather then exogenously changing trade patterns, I instead change the correlation of shocks across countries, holding trade patterns constant.[27] It turns out that both approaches point to similar conclusions.

Productivity and Output Correlations Given that idiosyncratic shocks do not generate strong comovement in the model, we turn our attention to productivity shocks themselves. In the model with correlated shocks, sector-level output comovement is tightly linked to the correlation of shocks. I illustrate this relationship for value added in the top two panels of Figure 4, where the x-axis records the sector-level correlation of $\tilde{\epsilon}_{it}(s)$ across countries and the y-axis records the correlation of real value added.[28] Recall, however, that the model replicates data on goods comovement well, and services comovement poorly. This implies that productivity comovement should explain output comovement in the data for the goods sector, but not the services sector. I verify both these implications in the bottom panels Figure 4. The model fails to replicate output comovement in the services sector primarily because productivity shocks are not tightly linked to output comovement in services.

Summing Up Three conclusions follow from this discussion. First, trade does not transmit idiosyncratic shocks strongly enough in the model to account for the observed trade-comovement relationship, even in the goods sector. Second, the large trade-comovement coefficient obtained in the model with correlated shocks for the goods sector is primarily attributable to the correlation of shocks itself. Third, on the flip side, the poor fit of the model for services lies in the low explanatory power for productivity in explaining fluctuations in services output.

4.2.2 Mechanics of Comovement in Goods Production

In the discussion above, I noted that the one place that idiosyncratic shocks do play a role in synchronizing output is for gross output in the goods sector. I now examine this result more closely, since it sheds light on how input trade transmits shocks in the model.

To recap, propagation of independent shocks explains roughly one-quarter of the observed comovement of goods gross output in the data. For reference, I plot these correlations in data versus the model with uncorrelated shocks in Figure 5. There is a clear positive relationship, particularly among EU country pairs. The U.S.-Canada outlier is instructive. The predicted correlation is near 0.25, while the actual correlation in the data is near 0.75, roughly a ratio of three to one. More generally, this magnitude is consistent with the overall spread in the

[27] One could implement the Kose-Yi approach in my framework, by calibrating and simulating the model for two different steady-states. Instead, I borrow from the domestic sectoral linkages literature [e.g., Horvath (1998)], and manipulate the shock structure.

[28] Results for gross output are similar, and therefore omitted for brevity.

data. Focusing on EU-pairs, predicted correlations vary in the range $(0, 0.2)$ while actual correlations lie in the range $(0.2, 0.8)$, so the ratio of the ranges is roughly .6/.2, or three to one.

In contrast to these results, the model does not generate significant comovement in value added in the goods sector following idiosyncratic shocks. This is reflected the low trade-comovement coefficient for value-added [Table 1, Panel C, column (2)]. This points to role of inputs in driving a wedge between comovement in gross output and value added in the model, as discussed in Section 2.5.1.

To illustrate comovement in value added versus gross output in data and model, I plot the correlation of gross output against the correlation for real value added for the goods sector in Figure 6. The top figure depicts the relationship in the data, the middle figure depicts the relationship in the model with correlated shocks, and the bottom figure presents this relationship in the model with uncorrelated shocks. Clearly, the model with correlated shocks generates similar correlations for both gross output and value added, which is consistent with the data. In contrast, there are large differences between the gross output and value added correlations in the model with uncorrelated shocks. Two points are worth nothing. First, dispersion in correlations of real value added across country pairs is much smaller than the variance of correlations in gross output. Second, the correlation of gross output is typically larger (sometimes much larger) than the correlation of real value added for individual country pairs.

These discrepancies shed light on the role of intermediate goods in the model. Recall from Section 2.5 that gross output is a composite of real value added and intermediate inputs, as in Equation (9). The correlation of gross output growth can then be decomposed into a weighted sum of the correlation of real value added growth across countries, the correlation of input use growth across countries, and the cross-correlation of real value added and input use growth:

$$\rho_{ij}(\hat{Q}) = w_{ij}^{vv}\rho_{ij}(\widehat{RVA}) + w_{ij}^{xx}\rho_{ij}(\hat{X}) + w_{ij}^{vx}\rho_{ij}(\widehat{RVA}, \hat{X}) + w_{ij}^{xv}\rho_{ij}(\widehat{RVA}, \hat{X}), \tag{13}$$

where $w_{ij}^{vv}, w_{ij}^{xx}, w_{ij}^{vx}, w_{ij}^{xv}$ are the appropriate weighting terms for each correlation, themselves functions of the shares and standard deviations of gross output, real value added, and input use.

I plot the correlations $\rho_{ij}(\widehat{RVA})$ and $\rho_{ij}(\hat{X})$ for selected country pairs in Figure 7. These correlations in the data and model with correlated shocks are presented in the top and middle figures. As in the data, the model with correlated shocks features correlations for gross output, value added, and input use of roughly similar magnitudes. The correlations in the model with uncorrelated are presented in the bottom figure. In this case, the correlation in input use across countries dwarfs the correlation in real value added for countries with high gross output comovement. Further, the correlation of gross output is nearly equal to the simple average of these two correlations.[29] Thus, the correlation of gross output is high in the model with uncorrelated shocks because intermediate use is highly correlated, not because value added is highly correlated. Put differently, the transmission of shocks through input linkages synchronizes input use across countries, and this translates into significant gross output comovement. However, the model struggles to translate this gross output comovement into comovement in value added.

4.3 Complementarity and Comovement

In this section, I examine how changes in elasticities in the model alter output comovement and trade-comovement correlations. To frame the discussion, I evaluate two mechanisms suggested by the existing literature for generating additional comovement.

First, prior work (using models without an input-output structure) has established that low elasticities of substitution between home and foreign output (low trade elasticities) tend to increase output comovement across countries [Heathcote and Perri (2002), Kose and Yi (2006), Corsetti, Dedola, and Leduc (2008), Burstein, Kurz, and Tesar (2008), and Drozad and Nosal (2008)]. One objective of this section is to quantify how the trade elasticity influences output comovement in my model with input-output linkages.

Second, a recent strain of thought holds that disruptions in input-sourcing produce large output losses because inputs are complements in production [Burstein, Kurz, and Tesar (2008), Di Giovanni and Levchenko (2010), Jones (2011)].[30] This is intuitively plausible. Negative input supply shocks should be particularly painful to downstream input users who have limited ability to substitute toward alternative suppliers, or have limited ability to substitute away from using inputs toward using their own factor inputs (i.e., capital and labor) more intensively. A second objective of this section is to quantify how varying the degree of complementarity in input sourcing and/or between primary factors and intermediate inputs influences output comovement.

[29] In the simulated data, the weights on each term are approximately equal (roughly 1/4) and the typical cross-correlation ($\rho_{ij}(\widehat{RVA}, \hat{X})$ or $\rho_{ij}(\widehat{RVA}, \hat{X})$) is relatively close to $\rho_{ij}(\hat{Q})$, lying between the extremes of $\rho_{ij}(\widehat{RVA})$ and $\rho_{ij}(\hat{X})$. Hence, the simple average of $\rho_{ij}(\widehat{RVA})$ and $\rho_{ij}(\hat{X})$ approximates $\rho_{ij}(\hat{Q})$ quite well.

[30] This argument also appears in the popular press. For example, see press coverage of the economic repercussions of the 2011 earthquake and tsunami in Japan for countries/sectors connected to Japan via global supply chains (e.g., in the U.S. auto industry).

4.3.1 Intermediate Inputs and Final Goods

The parameters η and ρ control the degree of complementarity/substitutability in input and final goods aggregators, respectively. These parameters also control the 'aggregate trade elasticity' in the model. Therefore, care is needed in choosing parameters to isolate the role of the trade elasticity separately from the role of the relative elasticities in input and final goods demand.

For concreteness, let us define the aggregate trade elasticity in the model to be the elasticity of substitution between gross output from alternative source countries. Focusing on domestic versus foreign output, the aggregate trade elasticity governs how the ratio of real imports to real expenditure on domestic output (i.e., spending on domestically produced inputs plus final goods) responds to changes in relative prices. Since total imports and expenditure consist of both final and intermediate goods, then the trade elasticity depends on both η and ρ. As an approximation, it is helpful to think of the trade elasticity as a weighted average of $1/(1-\eta)$ and $1/(1-\rho)$, with weights that depend on the shares of final and intermediate goods in trade.[31] Since the share of inputs in trade is about 0.6, then the trade elasticity in the baseline version of the model is roughly 1.4.[32]

To isolate the role of changes in the relative elasticities in final and intermediate demand, separate from changes in the aggregate trade elasticity, I first consider changes in η and ρ that hold the trade elasticity (approximately) constant. That is, as I raise/lower η, I simultaneously lower/raise ρ to keep the weighted average of $1/(1-\eta)$ and $1/(1-\rho)$ constant (with a weight of 0.6 attached to $1/(1-\eta)$ and 0.4 attached to $1/(1-\rho)$, as above). Since there is scant evidence on the relative size of η and ρ, I examine several extreme scenarios: (a) near-Leontief complementarity for inputs and substitutability for final goods ($\eta = -19, \rho = .71$), (b) substitutability for inputs and near-Leontief complementarity for final goods ($\eta = .57, \rho = -19$), and (c) equal elasticities for both final and intermediate goods ($\eta = \rho = .29$). Further, in these simulations, I hold the value of σ constant at its baseline value.

For each configuration of elasticities, I simulate the model and estimate trade-comovement regressions as in previous sections. The results for goods-goods sector pairs and aggregate output are presented in Table 2 [columns (2)-(4)]. In column (2) of Panel A, the trade-comovement coefficients for both value added and gross output falls relative to the baseline model when inputs are complements and final goods are substitutable. In contrast, in column (3), we see that the trade-comovement coefficients for both rise relative to the baseline model

[31] See Appendix A for further discussion. This approximation holds the composition of expenditure constant and assumes that home vs. foreign price changes are uniform across countries and sectors.

[32] Because the share of inputs in trade is not identical across countries, the trade elasticity implicitly varies across countries. Nonetheless, it is close to 1.4 for most countries.

when inputs are substitutable and final goods are complements. Further, substitutable inputs and complementary final goods generate a stronger trade comovement relationship than does having equal elasticities [column (4)].

In terms of quantitative magnitudes, the substitutable inputs and complementary final goods configuration of parameters raises the trade-comovement coefficient for gross output by 50% and more than doubles the coefficient for real value added, bringing the coefficients up to 1/3 and 1/10 of the coefficients in data.[33] Nonetheless, despite these increases in goods comovement, the coefficients for aggregate output are virtually unchanged.

Thus far, I have held the aggregate trade elasticity roughly constant while changing η and ρ. Columns (5)-(7) of Table 2 examine simulations in which the aggregate elasticity is higher/lower than the baseline value. When I raise the aggregate trade elasticity (with $\eta = \rho$) in column (5), the trade-comovement correlation virtually disappears, even for goods gross output. When I lower the trade elasticity instead in columns (6) and (7), the trade-comovement relationship strengthens relative to both the baseline and the case with substitutable inputs and complementary final goods. Further, the trade-comovement relationship is slightly stronger with unequal elasticities, where inputs being more substitutable than final goods (consistent with the comparison between columns (3) and (4)), but the difference is not large. In terms of magnitudes, the trade-comovement relationship for goods output strengthens a lot as the trade elasticity falls. For goods, the trade-comovement coefficient is about 2/3 the size of the coefficient in data for goods gross output, and about 1/5 as large for real value added. This increased comovement for goods is large enough to raise aggregate comovement relative to the baseline, but aggregate comovement is still small relative to the data.

4.3.2 Primary Factors and Intermediate Inputs

In evaluating the role of η and ρ, I held the elasticity of substitution between intermediate inputs and primary factors constant at its baseline value. To examine the role of this elasticity itself, I simulate the model with higher/lower values of σ, holding the trade elasticity approximately constant by fixing η and ρ at the their baseline values. Similar to the analysis above, I consider two alternate cases for the elasticity: (a) near-Leontief complementarity between inputs and factors ($\sigma = -19$), and (b) substitutability between inputs and factors ($\sigma = .5$). The results are reported in Table 3 [columns (2)-(3)].

In column (2), we see that making primary factors and inputs complements actually reduces the trade-comovement regression coefficient, for both gross output and real value

[33] The trade-comovement coefficients also rise relative to the baseline model in simulations with correlated shocks.

added. In column (3), the opposite result obtains: the trade-comovement coefficient rises when inputs are more substitutable for primary factors. Further, one subtle result gives insight into the role of this substitution elasticity. While the trade-comovement coefficients for gross output and real value added rise/fall in tandem, the spread between them changes across the columns. Specifically, the spread narrows when inputs and factors are complements, which reflects the fact that output and value added are locked more tightly together when producers cannot substitute between factors and inputs in production. The primary effect of this lack of substitutability is to lower gross output comovement, rather than to raise comovement in real value added.

Finally, in column (4), I simulate the model with the configuration of parameters that these simulations, combined with the previous simulations varying η and ρ, suggest should yield the highest trade-comovement correlation: a low trade elasticity, with $\rho < \eta$, combined with substitutable inputs and primary factors. In fact, this configuration of parameters does yield the 'best' performance across all the simulations, with the trade-comovement coefficient rising to 0.073 for gross output (3/4 of the data) and 0.027 for real value added (1/4 of the data). This is sizable goods comovement obtained from idiosyncratic shocks.

4.3.3 Bringing the Results Together

Four observations help bring together the results of the simulations above to form a coherent view of the combined role of the three elasticities in the model.

The first observation is that bilateral comovement in value added and gross output are linked together across simulations. While the correlation of value added comovement with bilateral trade intensity is always lower that for gross output comovement, these correlations rise and fall together across simulations. These changes in trade-comovement coefficients reflect changes in bilateral comovement patterns, since bilateral trade intensity is identical across simulations. Given that demand for factors is linked to the level of gross output, the fact that value added and gross output comovement are linked together is intuitive.

The second observation is that lower aggregate trade elasticities generate more output comovement and strengthen the trade-comovement relationship in the model. This result echoes the previous literature, but differs in an important respect. In those papers, gross trade is treated as comparable to GDP, so the trade elasticity in conventional IRBC models controls substitution between imports (implicitly foreign value added) and absorption of domestic value added. A low elasticity in the conventional model therefore amounts to an explicit assumption that foreign value added is complementary to domestic value added. In my model with intermediate inputs, I define the trade elasticity over domestic versus foreign gross output, respecting the fact that imports are measured on a gross basis. The mapping from this elasticity to the elasticity between home versus foreign value added is not direct, as it depends on other model parameters. Nonetheless, similar to the prior literature, lower trade elasticities generate larger fluctuations in the gross output terms of trade, and these serve to generate more comovement in equilibrium labor inputs.

The third and fourth observations concern the role of individual elasticities, holding the aggregate trade elasticity constant. The third observation is that the model generates higher trade-comovement correlations when inputs are more substitutable than final goods ($\eta > \rho$). The fourth is that larger substitution elasticities ($\sigma > 0$) between inputs and primary factors generate more comovement. Both results are initially surprising, as they run counter to the standard intuition that complementarity in the production function should increase the strength with which shocks are transmitted across borders. Therefore, these results merit additional discussion.[34]

In the model, a higher substitution elasticity for inputs and primary factors (σ) is associated with more output comovement, with the largest increases among countries with the strongest trade linkages. Both aggregate and goods comovement rise in the model as σ increases. The increased comovement in value added is associated both with increased aggregate labor comovement and increased comovement in labor employed in goods production. To understand this labor comovement, note that in the model the first-order condition for labor supply is $\chi L_{it}^{1/\epsilon} = \left(\frac{1}{C_{it}}\right) \frac{w_{it}}{p_{it}^f}$, so $\hat{L}_{it} = -\epsilon[\hat{p}_{it}^f + \hat{C}_{it}] + \epsilon \hat{w}_{it}$. With log preferences over consumption, $\hat{p}_{it}^f + \hat{C}_{it}$ is equal for all countries, so $\hat{L}_{it} = \zeta_t + \epsilon \hat{w}_{it}$, where ζ_t denotes the component common to all countries. Then, labor comovement rises because nominal wage changes $\hat{w}_{it}$ are more strongly correlated across countries when inputs are substitutable for factors.

The story for the role of η vs. ρ is somewhat different. Here, $\eta > \rho$ is associated with slightly more aggregate gross output comovement (due to higher comovement in aggregate input use), but slightly less aggregate real value added comovement (due to lower comovement in aggregate labor). For the goods sector, comovement in both gross output and value-added increases with $\eta > \rho$, driven mainly by increased comovement in both labor and capital inputs in the goods sector. Given the decline in aggregate labor comovement, this implies that factors are being drawn out of services and reallocated to manufacturing following shocks in countries with strong trade linkages, amplifying goods comovement.

[34] I summarize changes in bilateral correlations under alternative elasticities in the Online Appendix. These figures underlie the discussion the following paragraphs.

5 Conclusion

This paper uses a two-sector, many country extension of the IRBC model with trade in both final and intermediate goods to refine our understanding of the trade-comovement puzzle along several dimensions.

First, with correlated shocks, the model generates high output comovement for goods, but not services. Thus, the low aggregate trade-comovement correlation in the model is mostly attributable to the services sector. Differences in the correlation of productivity shocks across sectors account for the bulk of this differential comovement across sectors. Trade linkages play a secondary role, as evidenced by the higher comovement of goods than services in the model with uncorrelated shocks. One approach to closing the gap between model and data would be to expand the set of shocks beyond productivity to include shocks that synchronize services more forcefully.

Second, input trade does not resolve the trade-comovement puzzle in a straightforward manner. Following a home productivity shock, input use in downstream countries rises in response, which leads to gross output comovement between the home country and downstream input users. Nonetheless, while input linkages promote gross output comovement, value added comovement is typically much weaker. Further, increased complementarity between inputs and primary factors or complementarity among intermediate goods, holding the aggregate trade elasticity constant, generates less output comovement in the model.

These results push back against the idea that input trade is the missing link in understanding the strong predictive power of bilateral trade in explaining business cycle synchronization. Set against prior empirical results linking input trade to comovement [Ng (2010), Burstein, Kurz, and Tesar (2008), Di Giovanni and Levchenko (2010)], they point to a new "input trade and comovement puzzle."

Despite these results, a broad takeaway is that the introduction of intermediates into macro models alters the role of trade as a conduit of shocks. Several topics require attention in future work. First, the relationship between the model outlined here and the conventional IRBC framework should be explored in greater detail. The core issue is that the model in this paper is a gross model, while the conventional IRBC framework is a value-added model. More work is needed to map out how the gross parameters estimated in the literature (e.g., trade elasticities) map into key parameters value-added models (e.g., the elasticity of substitution between home and foreign value added). Second, while input trade does not resolve the trade-comovement puzzle in the IRBC-style framework, this may speak to the shortcomings of the IRBC framework. For example, the model ignores the fact that the bulk of intermediates are traded within multinational firms, and this concentration of input trade among the largest firms in the economy may mean shocks to intermediate suppliers are passed to aggregates more forcefully. More careful consideration of microeconomic features of input trade would be useful in future work.

References

Acemoglu, Daron, Vasco M. Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi. 2012. "The Network Origins of Aggregate Fluctuations." *Econometrica*, 80(5): 1977–2016.

Ambler, Steve, Emanuela Cardia, and Christian Zimmerman. 2002. "International transmission of the business cycle in a multi-sector model." *European Economic Review*, 46: 273–300.

Arkolakis, Costas, and Anath Ramanarayanan. 2009. "Vertical Specialization and International Business Cycle Synchronization." *Scandinavian Journal of Economics*, 111(4): 655–680.

Backus, David, Patrick Kehoe, and Finn Kydland. 1994. "Dynamics of the Trade Balance and the Terms of Trade: The J-Curve?" *The American Economic Review*, 84(1): 84–103.

Baxter, Marianne. 1995. "International Trade and Business Cycles." In *Handbook of International Economics*. Vol. 3, , ed. Gene Grossman and Kenneth Rogoff, Chapter 35, 1801–1864. Elsevier.

Baxter, Marianne, and Michael Kouparitsas. 2005. "Determinants of business cycle comovement: a robust analysis." *Journal of Monetary Economics*, 52: 113–157.

Baxter, Marianne, and Robert G. King. 1999. "Measuring Business Cycles: Approximate Band-Pass Filters for Economic Time Series." *The Review of Economics and Statistics*, 81(4): 575–593.

Bems, Rudolfs. 2013. "Intermediate Inputs, External Rebalancing and Relative Price Adjustment." Unpublished Manuscript, International Monetary Fund.

Bems, Rudolfs, Robert Johnson, and Kei-Mu Yi. 2010. "Demand Spillovers and the Collapse of Trade in the Global Recession." *IMF Economic Review*, 58(2): 295–326.

Bergin, Paul, Robert Feenstra, and Gordon Hanson. 2009. "Offshoring and Volatility: Evidence from Mexico's Maquiladora Industry." *The American Economic Review*, 99(4): 4.

Burstein, Ariel, Christopher Kurz, and Linda Tesar. 2008. "Trade, production sharing, and the international transmission of business cycles." *Journal of Monetary Economics*, 55: 775–795.

Calderon, Cesar, Alberto Chong, and Ernesto Stein. 2007. "Trade intensity and business cycle sychronization: Are developing countries any different?" *Journal of International Economics*, 71: 2–21.

Canova, Fabio. 2007. *Methods for Applied Macroeconomic Time Series.* Princeton, NJ:Princeton University Press.

Carvalho, Vasco M. 2008. "Aggregate Fluctuations and the Network Structure of Intersectoral Trade." Unpublished Manuscript, CREi and Universitat Pompeu Fabra.

Chetty, Raj, Adam Guren, Day Manoli, and Andrea Weber. 2011. "Does Indivisible Labor Explain the Difference between Mico and Macro Elasticities? A Meta-Analysis of Extensive Margin Elasticities." *American Economic Review Papers and Proccedings*, 101(2).

Di Giovanni, Julian, and Andrei Levchenko. 2010. "Putting the Parts Together: Trade, Vertical Linkages and Business Cycle Comovement." *American Economic Journal: Macroeconomics*, 2(2): 95–124.

Drozad, Lukasz, and Jaromir Nosal. 2008. "Long-Run Price Elasticity of Trade and the Trade-Comovement Puzzle." Unpublished Manuscript, University of Pennsylvania.

Dupor, Bill. 1999. "Aggregation and irrelevance in multi-sector models." *Journal of Monetary Economics*, 43: 391–409.

Foerster, Andrew, Pierre-Danieal Sarte, and Mark Watson. 2011. "Sectoral vs. Aggregate Shocks: A Structural Factor Analysis of Industrial Production." *The Journal of Political Economy*, 119(1): 1–38.

Frankel, Jeffrey, and Andrew Rose. 1998. "The Endogeneity of the Optimum Currency Area Criteria." *The Economic Journal*, 108: 1009–1025.

Heathcote, Jonathan, and Fabrizio Perri. 2002. "Financial autarky and international business cycles." *Journal of Monetary Economics*, 49: 601–627.

Herrendorf, Berthold, Richard Rogerson, and Akos Valentinyi. forthcoming. "Two Perspectives on Preferences and Structural Transformation." *The American Economic Review.*

Hornstein, Andreas, and Jack Praschnik. 1997. "Intermediate Inputs and Sectoral Comovement in the Business Cycle." *Journal of Monetary Economics*, 40: 573–595.

Horvath, Michael. 1998. "Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks." *Review of Economic Dynamics*, 1: 781–808.

Horvath, Michael. 2000. "Sectoral shocks and aggregate fluctuations." *Journal of Monetary Economics*, 45: 69–106.

Imbs, Jean. 2004. "Trade, Finance, Specialization, and Synchronization." *The Review of Economics and Statistics*, 86: 723–734.

Inklaar, Robert, Richard Jong-A-Pin, and Jakob de Haan. 2008. "Trade and business cycle synchronization in OECD countries – A re-examination." *European Economic Review*, 52(4): 646–666.

Ishise, Hirokazu. 2012*a*. "Trade Costs and Business Cycle Transmission in a Multi-Country, Multi-Sector Model." Unpublished Manuscript, University of Iowa.

Ishise, Hirokazu. 2012*b*. "The World Has More Than Two Countries: Implications of Multi-Country International Real Business Cycle Models." Unpublished Manuscript, University of Iowa.

Johnson, Robert C. 2013. "A Guide to Value-Added Export Data for Macroeconomic and Trade Research." Unpublished Manuscript, Dartmouth College.

Johnson, Robert C., and Guillermo Noguera. 2012. "Accounting for Intermediates: Production Sharing and Trade in Value Added." *Journal of International Economics*, 86(2): 224–236.

Johnson, Robert C., and Rudolfs Bems. 2012. "Value-Added Exchange Rates." NBER Working Paper No. 18498.

Jones, Charles. 2011. "Intermediate Goods and Weak Links in the Theory of Economic Development." *American Economic Journal: Macroeconomics*, 3(2): 1–28.

Juvenal, Luciana, and Paulo Santos Monteiro. 2010. "Trade and Synchronization in a Multi-Country Economy." Federal Reserve Bank of St. Louis Working Paper 2010-038C.

King, Robert, and Sergio Rebelo. 1999. "Resuscitating Real Business Cycles." In *Handbook of Macroeconomics*. Vol. 1, , ed. J. B. Taylor and M. Woodford, Chapter 14, 927–1007. Elsevier.

Kose, M Ayhan, and Kei Mu Yi. 2006. "Can the standard international business cycle model explain the relation between trade and comovement?" *Journal of International Economics*, 68: 267–295.

Long, John, and Charles Plosser. 1983. "Real Business Cycles." *Journal of Political Economy*, 91(1): 39–69.

Ng, Eric C.Y. 2010. "Production fragmentation and business-cycle comovement." *Journal of International Economics*, 82(1): 1–14.

O'Mahoney, Mary, and Marcel Timmer. 2009. "Output, Input and Productivity Measures at the Industry Level: The EU KLEMS Database." *The Economic Journal*, 119: F374–F403.

Ravn, Morten O., and Harald Uhlig. 2002. "On Adjusting the Hodrick-Prescott FIlter for the Frequency of Observations." *The Review of Economics and Statistics*, 84(2): 371–385.

Shea, John. 2002. "Complementarities and Comovements." *Journal of Money, Credit, and Banking*, 34: 412–433.

Sims, Christopher. 1969. "Theoretical Basis for A Double Deflated Index of Real Value Added." *The Review of Economics and Statistics*, 51(4): 470–471.

Uhlig, Harald. 1999. "A toolkit for analysing nonlinear dynamic stochastic models easily." In *Computational Methods for the Study of Dynamic Economies.* , ed. Ramon Marimon and Andrew Scott, Chapter 3, 30–61. Oxford University Press.

Zimmerman, Christian. 1997. "International Real Business Cycles among Heterogeneous Countries." *European Economic Review*, 41: 319–355.

Table 1: Trade-Comovement Regressions in the Data and Baseline Model

	Real Value Added				Gross Output			
	Aggregate (1)	Goods (2)	Services (3)	Cross (4)	Aggregate (5)	Goods (6)	Services (7)	Cross (8)
Panel A: Data								
Log Bilateral Trade	0.112***	0.112***	0.074***	0.072***	0.093***	0.097***	0.072***	0.076***
	(0.013)	(0.012)	(0.012)	(0.008)	(0.011)	(0.012)	(0.014)	(0.009)
Observations	231	231	231	462	171	171	171	342
R-squared	0.26	0.31	0.14	0.15	0.27	0.27	0.15	0.18
Panel B: Model with Correlated Shocks								
Log Bilateral Trade	0.015	0.081***	-0.013	0.022**	0.022*	0.087***	-0.009	0.027***
	(0.013)	(0.012)	(0.012)	(0.009)	(0.013)	(0.013)	(0.012)	(0.009)
Observations	231	231	231	462	231	231	231	462
R-squared	0.01	0.18	0.00	0.01	0.01	0.19	0.00	0.02
Panel C: Model with Uncorrelated Shocks								
Log Bilateral Trade	0.003***	0.006***	-0.003***	0.007***	0.008***	0.022***	-0.002***	0.012***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.001)	(0.001)
Observations	231	231	231	462	231	231	231	462
R-squared	0.13	0.28	0.09	0.24	0.35	0.54	0.04	0.29

Goods, Services, and Cross columns contain sector-level regressions for goods paired with goods, services paired with services, and goods paired with services. Log bilateral trade in all regressions is $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$, and all regressions include a constant. Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 2: Trade-Comovement Regressions with Alternative Elasticities for Intermediate Inputs and Final Goods in Model with Uncorrelated Shocks

	Trade Elasticity $\approx$ 1.4				Trade Elasticity $\approx$ 4	Trade Elasticity $\approx$ .5	
	Baseline Model $\eta = 0$, $\rho = .5$	Comp. Inputs Subs. Final Goods $\eta = -19$, $\rho = .71$	Subs. Inputs Comp. Final Goods $\eta = .57$, $\rho = -19$	Equal Elast. $\eta = \rho = .29$	Equal Elast. $\eta = \rho = .75$	Equal Elast. $\eta = \rho = -1$	Unequal Elast. $\eta = -.25$, $\rho = -19$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Panel A: Goods Output							
Panel A1: Real Value Added							
Log Bilateral Trade	0.006***	0.002***	0.013***	0.008***	-0.001	0.019***	0.021***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.002)	(0.002)
Observations	231	231	231	231	231	231	231
R-squared	0.28	0.05	0.44	0.35	0.01	0.48	0.49
Panel A2: Gross Output							
Log Bilateral Trade	0.022***	0.017***	0.033***	0.025***	0.003***	0.064***	0.068***
	(0.002)	(0.002)	(0.003)	(0.002)	(0.001)	(0.006)	(0.006)
Observations	231	231	231	231	231	231	231
R-squared	0.54	0.48	0.54	0.55	0.08	0.52	0.51
Panel B: Aggregate Output							
Panel B1: Real Value Added							
Log Bilateral Trade	0.003***	0.003***	0.003***	0.003***	0.001	0.005***	0.005***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	231	231	231	231	231	231	231
R-squared	0.13	0.14	0.11	0.12	0.01	0.22	0.21
Panel B2: Gross Output							
Log Bilateral Trade	0.008***	0.008***	0.009***	0.009***	0.004***	0.012***	0.012***
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Observations	231	231	231	231	231	231	231
R-squared	0.35	0.34	0.36	0.35	0.18	0.42	0.42

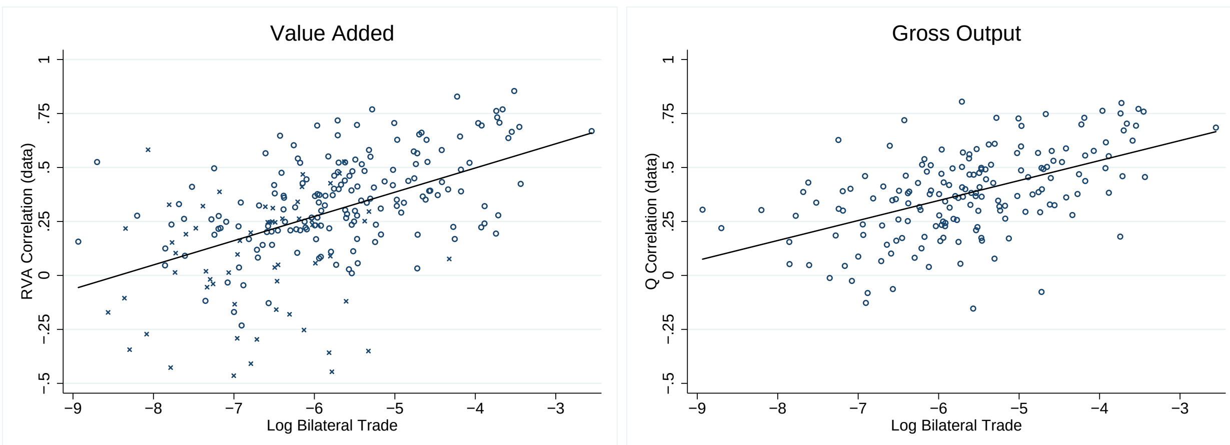
The elasticity of substitution between primary factors and intermediate inputs is equal to one ($\sigma = 0$) in all simulations. Log bilateral trade in all regressions is $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$, and all regressions include a constant. Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$.

Table 3: Trade-Comovement Regressions with Alternative Elasticities between Intermediate Inputs and Primary Factors in Model with Uncorrelated Shocks

	Trade Elasticity ≈ 1.4			Trade Elasticity $\approx .5$
	Baseline Model	Comp. Inputs & Factors	Subs. Inputs & Factors	Subs. Inputs & Factors
	$\sigma = 0$	$\sigma = -19$	$\sigma = .5$	$\sigma = .5$, $\eta = -.25$, $\rho = -19$
	(1)	(2)	(3)	(4)
Panel A: Goods Output				
Panel A1: Real Value Added				
Log Bilateral Trade	0.006***	0.005***	0.008***	0.027***
	(0.001)	(0.001)	(0.001)	(0.003)
Observations	231	231	231	231
R-squared	0.28	0.21	0.30	0.48
Panel A2: Gross Output				
Log Bilateral Trade	0.022***	0.011***	0.032***	0.073***
	(0.002)	(0.001)	(0.003)	(0.007)
Observations	231	231	231	231
R-squared	0.54	0.44	0.56	0.51
Panel B: Aggregate Output				
Panel B1: Real Value Added				
Log Bilateral Trade	0.003***	0.002***	0.004***	0.006***
	(0.001)	(0.001)	(0.001)	(0.001)
Observations	231	231	231	231
R-squared	0.13	0.08	0.18	0.27
Panel B2: Gross Output				
Log Bilateral Trade	0.008***	0.006***	0.010***	0.013***
	(0.001)	(0.001)	(0.001)	(0.002)
Observations	231	231	231	231
R-squared	0.35	0.25	0.39	0.44

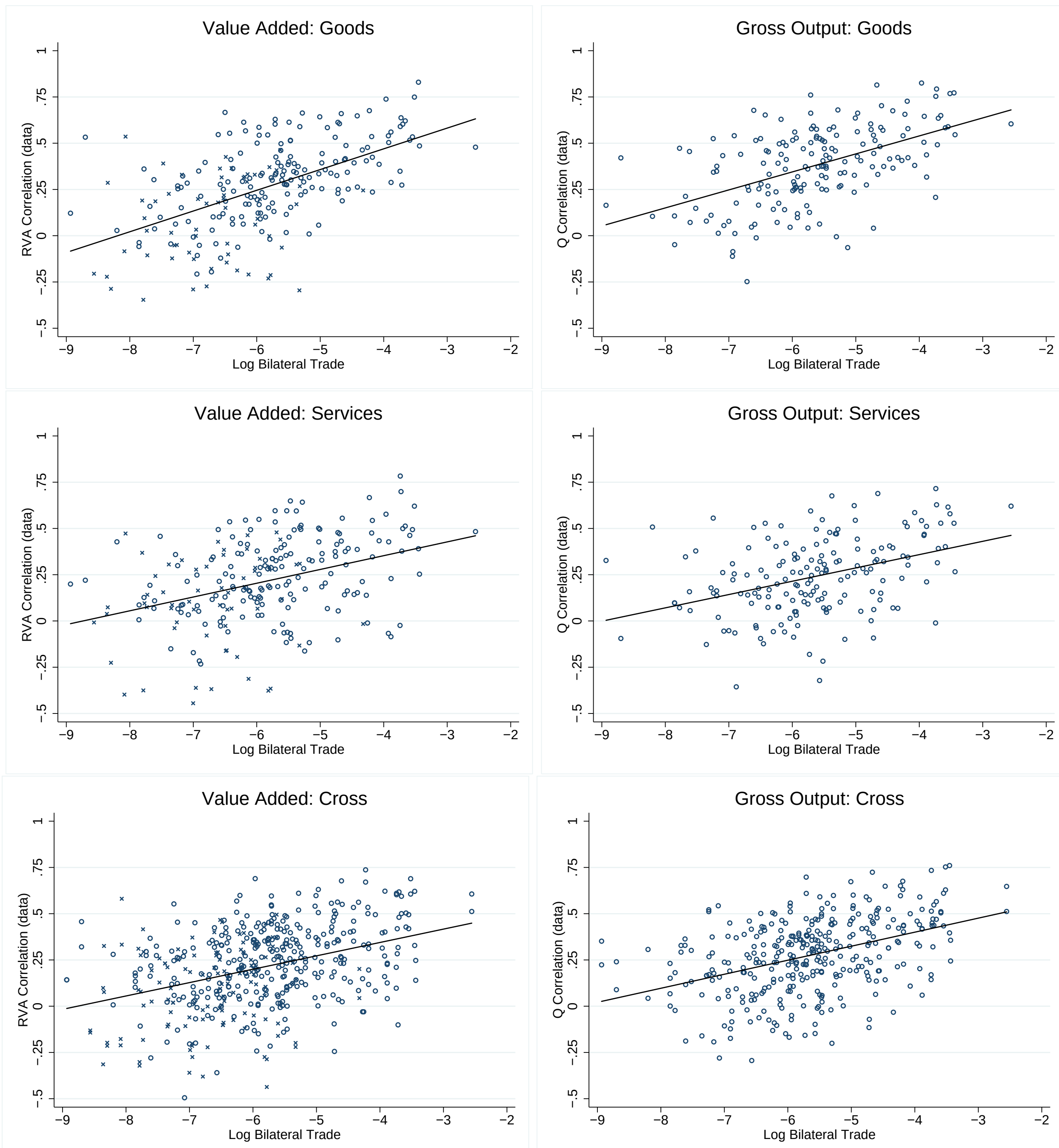
The elasticities of substitution for input and final goods aggregators are set to baseline values ($\eta = 0$, $\rho = .5$) in columns (1)-(3). Log bilateral trade in all regressions is $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$, and all regressions include a constant. Robust standard errors in parentheses. Significance levels: * $p < .1$, ** $p < .05$, *** $p < .01$.

Figure 1: Value Added and Gross Output Correlations vs. Trade Intensity



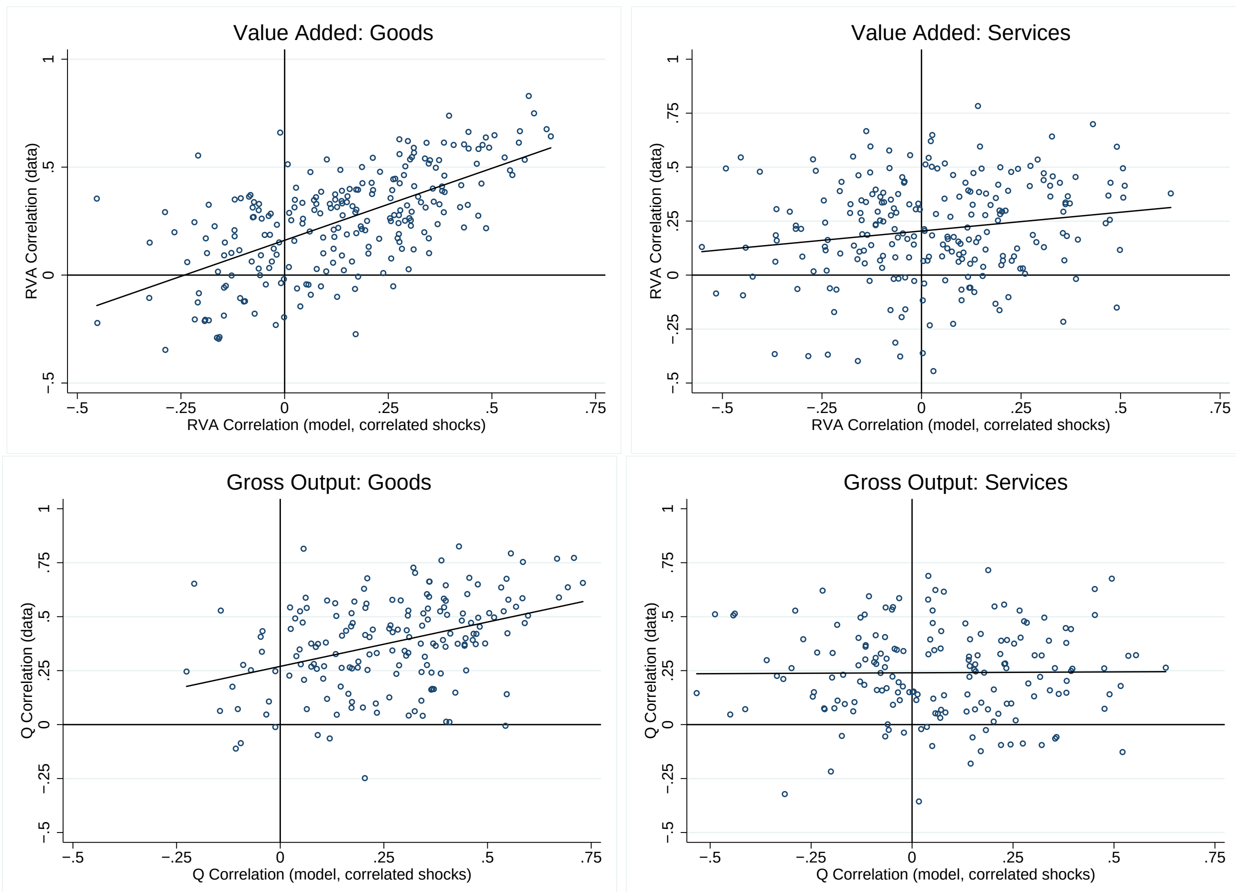
Real Value Added (RVA) and Real Gross Output (Q) correlations are the sample correlation of year-on-year growth rates for each country pair. Log Bilateral Trade is defined as: $\log\left(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j}\right)$. Marker symbol x in the Value Added figure denotes observations where no corresponding gross output correlation is available due to missing data. The solid black line in each figure is a regression line, with coefficients presented in Table 1.

Figure 2: Value Added and Gross Output Correlations vs. Trade Intensity, by Sector



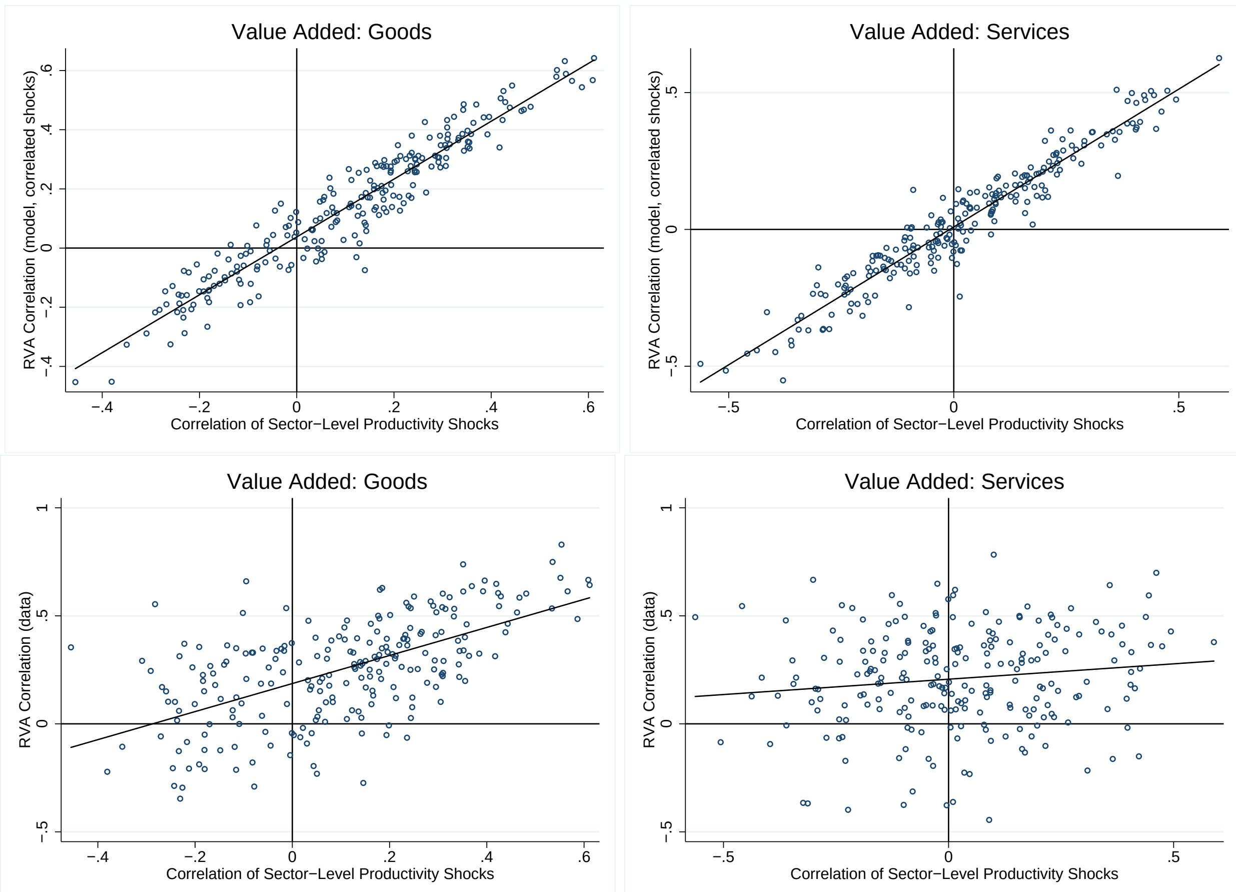
Figures labeled 'Goods' present correlations for the goods sector in i with the goods sector in j. Figures labeled 'Services' present correlations for the services sector in i with the services sector in j. Figures labeled 'Cross' present correlations for goods in i with services in j. Log bilateral trade in these figures is $\log(\frac{EX_{ij}+EX_{ji}}{GDP_i+GDP_j})$ for sector pair $\{s, s'\}$. Marker symbol x in the Value Added figures denotes observations where no corresponding gross output correlation is available due to missing data. The solid black line in each figure is a regression line, with coefficients presented in Table 1.

Figure 3: Value Added and Gross Output Correlations in Data vs. Model with Correlated Shocks



Figures labeled 'Goods' present correlations for the goods sector in i with the goods sector in j. Figures labeled 'Services' present correlations for the services sector in i with the services sector in j. The solid black line in each figure is a regression line.

Figure 4: Value Added and Productivity Shock Correlations, by Sector



Figures labeled 'Goods' present correlations for the goods sector in i with the goods sector in j. Figures labeled 'Services' present correlations for the services sector in i with the services sector in j. The solid black line in each figure is a regression line.

Figure 5: Goods Gross Output Correlations in Data vs. Model with Uncorrelated Shocks

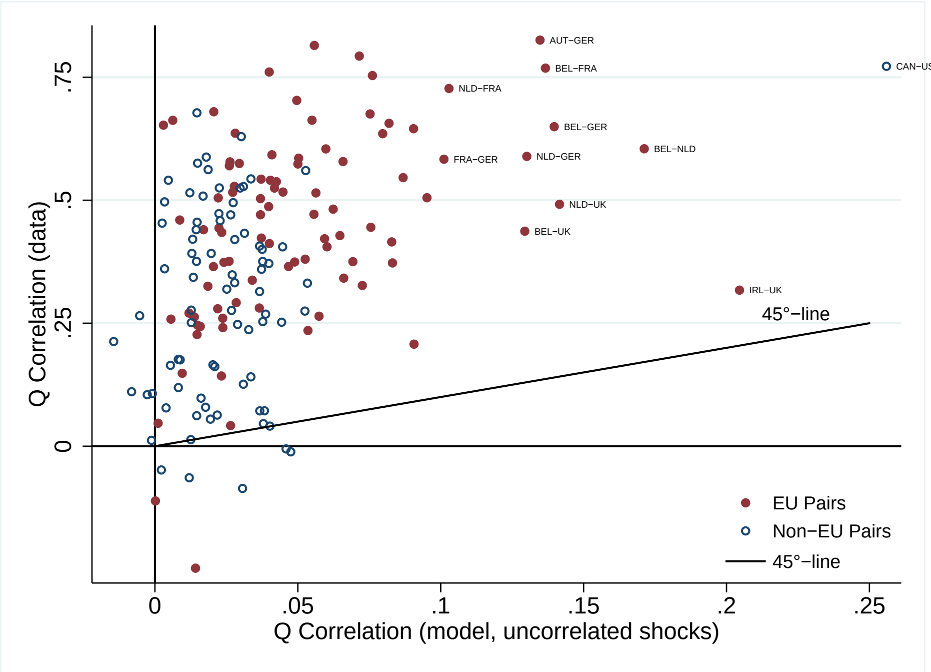


Figure 6: Goods Gross Output vs. Value Added Correlations in Data vs. Model

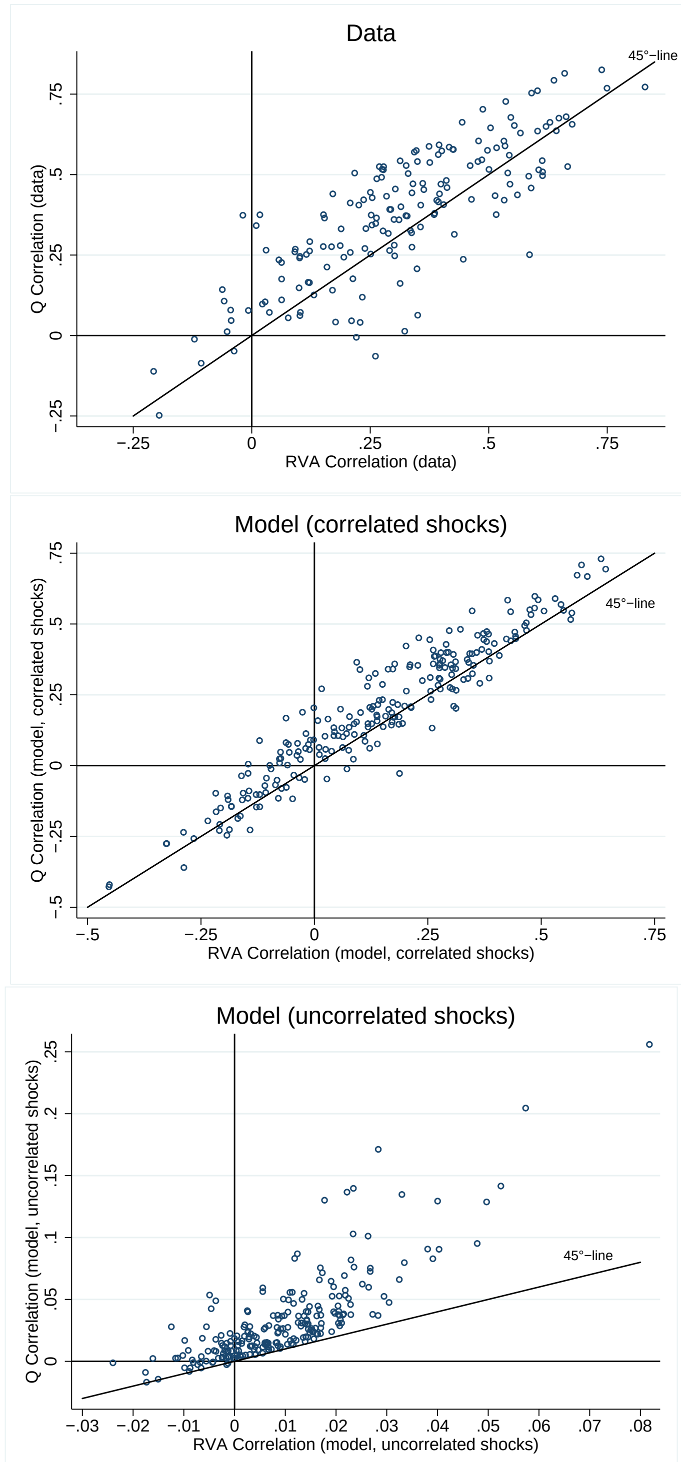
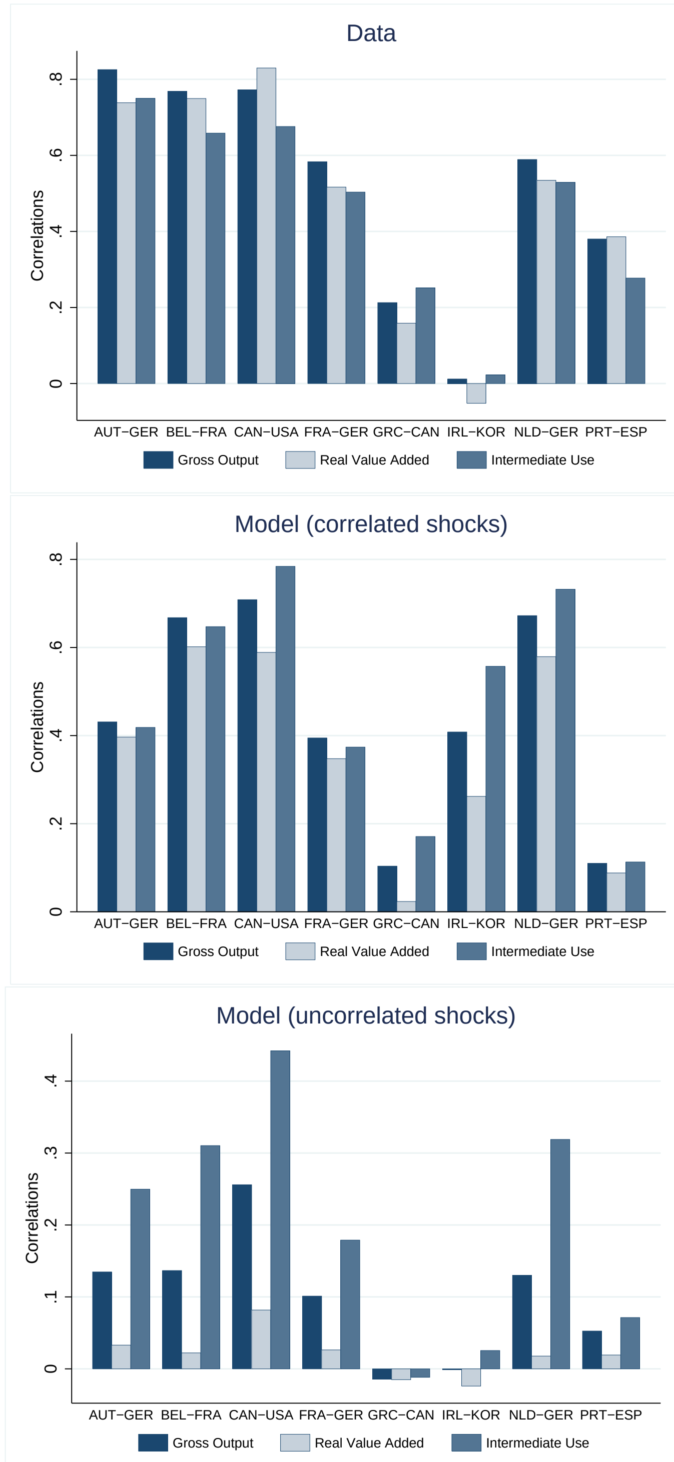


Figure 7: Correlations of Goods Gross Output, Real Value Added, and Intermediate Input Use in Data vs. Model



Appendix A

This appendix first describes the equilibrium and linearization of the model with complete asset markets, introduced in Section 2. I then discuss the example underlying Equation (11) in Section 2.5.1. Finally, I describe how I approximate the aggregate trade elasticity in the model, which I use to organize the simulations in Tables 2 and 3.

A.1 Equilibrium and Linearization

To simplify the exposition, I manipulate the equilibrium conditions of the model to eliminate the need to track state-contingent asset holdings over time. The collapsed equilibrium conditions here correspond to what one obtains from solving the social planner's problem. The equilibrium conditions are as follows:

$$1 = \beta E_t \left[\left(\frac{C_{it}}{C_{it+1}} \right) \left(\frac{r_{it+1}}{p^F_{it+1}} + (1-\delta) \right) \right] \tag{A1}$$

$$\chi L_{it}^{1/\epsilon} = \left(\frac{1}{C_{it}} \right) \frac{w_{it}}{p^F_{it}} \tag{A2}$$

$$\frac{p^f_{it} C_{it}}{p^f_{it+1} C_{it+1}} = \frac{p^f_{1t} C_{1t}}{p^f_{1t+1} C_{1t+1}} \tag{A3}$$

$$p^f_{it}(s) F_{it}(s) = \gamma_i(s) p^f_{it} F_{it} \tag{A4}$$

$$F_{jit}(s) = \omega^f_{ji}(s) \left(\frac{p_{jt}(s)}{p^f_{it}(s)} \right)^{1/(\rho-1)} F_{it}(s) \tag{A5}$$

$$V_{it}(s) = Z_{it}(s)^{\sigma/(1-\sigma)} \theta_i(s) \left(\frac{p^v_{it}(s)}{p_{it}(s)} \right)^{1/(\sigma-1)} Q_{it}(s) \tag{A6}$$

$$X_{it}(s) = Z_{it}(s)^{\sigma/(1-\sigma)} (1-\theta_i(s)) \left(\frac{p^x_{it}(s)}{p_{it}(s)} \right)^{1/(\sigma-1)} Q_{it}(s) \tag{A7}$$

$$r_{it} K_{it}(s) = \alpha p^v_{it}(s) V_{it}(s) \tag{A8}$$

$$w_{it} L_{it}(s) = (1-\alpha) p^v_{it}(s) V_{it}(s) \tag{A9}$$

$$X_{jit}(s', s) = \omega^x_{ji}(s', s) \left(\frac{p_{jt}(s')}{p^x_{it}(s)} \right)^{1/(\eta-1)} X_{it}(s) \tag{A10}$$

The market clearing conditions are given by:

$$Q_{it}(s) = \sum_{j=1}^{N}\sum_{s'=1}^{S} F_{ijt}(s) + X_{ijt}(s,s') \tag{A11}$$

$$F_{it} = C_{it} + K_{it+1} - (1-\delta)K_{it} \tag{A12}$$

$$K_{it} = \sum_{s=1}^{S} K_{it}(s) \tag{A13}$$

$$L_{it} = \sum_{s=1}^{S} L_{it}(s). \tag{A14}$$

And remaining production functions and composite aggregators are given by:

$$Q_{it}(s) = Z_{it}(s)\left(\theta_i(s)^{1-\sigma}V_{it}(s)^{\sigma} + (1-\theta_i(s))^{1-\sigma}X_{it}(s)^{\sigma}\right)^{1/\sigma} \tag{A15}$$

$$X_{it}(s) = \left(\sum_{j}\sum_{s'} \omega^{x}_{ji}(s',s)^{1-\eta}X_{jit}(s',s)^{\eta}\right)^{1/\eta} \tag{A16}$$

$$V_{it}(s) = K_{it}(s)^{\alpha}L_{it}(s)^{1-\alpha} \tag{A17}$$

$$F_{it}(s) = \left(\sum_{j} \omega^{f}_{ji}(s)^{1-\rho}F_{jit}(s)^{\rho}\right)^{1/\rho} \tag{A18}$$

$$F_{it} = \prod_{s} F_{it}(s)^{\gamma_i(s)}. \tag{A19}$$

These equations represent $7N + 10(S \times N) + 6N^2$ equations (minus one after choosing a numeraire) in the same number of unknowns. The unknowns include $\{C_{it}, F_{it}, K_{it}, L_{it}\}$ for each country, $\{Q_{it}(s), V_{it}(s), X_{it}(s), K_{it}(s), L_{it}(s), F_{it}(s), \{F_{jit}(s)\}_j, \{X_{jit}(s',s)\}_{j,s'}\}_{i,s}$ for each country-sector, and prices $\{r_{it}, w_{it}, p^{f}_{it}, \{p^{f}_{it}(s), p^{v}_{it}(s), p^{x}_{it}(s), p_{it}(s)\}_s\}_i$.

These equilibrium conditions can be linearized around the steady state as follows. The linearized and stacked first order conditions are given by:

$$0 = E_t\left[\hat{C}_t - \hat{C}_{t+1} + (1-\beta(1-\delta))\left(\hat{r}_{t+1} - \hat{p}^f_t\right)\right] \tag{A20}$$

$$0 = \frac{1}{\epsilon}\hat{L}_t + \hat{C}_t - \hat{w}_t + \hat{p}^f_t \tag{A21}$$

$$0 = D\hat{p}^f_t + D\hat{C}_t \tag{A22}$$

$$0 = \hat{p}^f_t + \hat{F}_t - \hat{p}^f_t(s) - \hat{F}_t(s) \tag{A23}$$

$$0 = \hat{\mathbb{F}}_t(s) + \frac{1}{1-\rho}M_i\hat{p}_t(s) - \frac{1}{1-\rho}M_j\hat{p}^f_t(s) - M_j\hat{F}_t(s) \tag{A24}$$

$$0 = \hat{V}_t(s) - \frac{\sigma}{1-\sigma}\hat{Z}_t(s) + \frac{1}{1-\sigma}\hat{p}^v_t(s) - \frac{1}{1-\sigma}\hat{p}_t(s) - \hat{Q}_t(s) \tag{A25}$$

$$0 = \hat{X}_t(s) - \frac{\sigma}{1-\sigma}\hat{Z}_t(s) + \frac{1}{1-\sigma}\hat{p}^x_t(s) - \frac{1}{1-\sigma}\hat{p}_t(s) - \hat{Q}_t(s) \tag{A26}$$

$$0 = \hat{\mathbb{X}}_t(s', s) + \frac{1}{1-\eta}M_i\hat{p}_t(s') - \frac{1}{1-\eta}M_j\hat{p}^x_t(s) - M_j\hat{X}_t(s) \tag{A27}$$

$$0 = \hat{p}^v_t(s) + \hat{V}_t(s) - \hat{r}_t - \hat{K}_t(s) \tag{A28}$$

$$0 = \hat{p}^v_t(s) + \hat{V}_t(s) - \hat{w}_t - \hat{L}_t(s), \tag{A29}$$

where $M_i \equiv I_{N\times N} \otimes \mathbf{1}_{N\times 1}$, $M_j \equiv \mathbf{1}_{N\times 1} \otimes I_{N\times N}$, and $D = \begin{bmatrix} -\mathbf{1}_{N\times 1} & I_{(N-1)\times(N-1)} \end{bmatrix}$. The objects $\{r_t, w_t, p^f_t, p^f_t(s), p^x_t(s), p_t(s)\}$ and $\{C_t, L_t, F_t, Q_t(s), X_t(s), K_t(s), L_t(s), F_t(s)\}$ are vectors of prices and quantities, with elements i equal to the relevant variable for country i. The vector $\mathbb{F}_t(s)$ is a N^2 dimensional vector that records final goods shipments for sector s, while $\mathbb{X}_t(s', s)$ is a N^2 dimensional vector that records intermediates goods flows from sector s' to sector s:

$$\hat{\mathbb{F}}_t(s) = [\hat{F}_{11t}(s), \hat{F}_{12t}(s), \ldots, \hat{F}_{1Nt}(s), \hat{F}_{21t}(s), \hat{F}_{22t}(s), \ldots]'$$
$$\hat{\mathbb{X}}_t(s', s) = [\hat{X}_{11t}(s', s), \hat{X}_{12t}(s', s), \ldots, \hat{X}_{1Nt}(s', s), \hat{X}_{21t}(s', s), \hat{X}_{22t}(s', s), \ldots]'$$

The stacked and linearized market clearing conditions are given by:

$$0 = \hat{Q}_t(s) - S_F(s)\hat{\mathbb{F}}_t(s) - \sum_{s'} S_X(s, s')\hat{\mathbb{X}}(s, s') \tag{A30}$$

$$0 = \hat{F}_t - diag\left(\frac{\bar{C}_i}{\bar{F}_i}\right)\hat{C}_t - diag\left(\frac{\bar{K}_i}{\bar{F}_i}\right)\hat{K}_{t+1} + diag\left(\frac{\bar{K}_i(1-\delta)}{\bar{F}_i}\right)\hat{K}_t \tag{A31}$$

$$0 = \hat{K}_t - \sum_{s=1}^{S} diag\left(\frac{\bar{K}_i(s)}{\bar{K}_i}\right)\hat{K}_t(s) \tag{A32}$$

$$0 = \hat{L}_t - \sum_{s=1}^{S} diag\left(\frac{\bar{L}_i(s)}{\bar{L}_i}\right)\hat{L}_t(s). \tag{A33}$$

The bar notation denotes steady state values.[35]

[35] To be clear about the calibration, I make the assumption in the model that the capital share of income

The matrices $S_F(s)$ and $S_X(s,s')$ collect the share of output allocated to final and intermediate use in destinations as follows:

$$S_F(s) \equiv \begin{pmatrix} s_1^f(s) & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^f(s) & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix} \quad \text{and} \quad S_X(s,s') \equiv \begin{pmatrix} s_1^x(s,s') & \mathbf{0} & \cdots \\ \mathbf{0} & s_2^x(s,s') & \cdots \\ \vdots & \cdots & \ddots \end{pmatrix}$$

$$\text{with} \quad s_i^f(s) = [s_{i1}^f(s), \cdots, s_{iN}^f(s)], \quad s_{ij}^f(s) = \frac{p_i(s)F_{ij}(s)}{p_i(s)Q_i(s)},$$

$$s_i^x(s,s') = [s_{i1}^x(s,s'), \cdots, s_{iN}^x(s,s')], \quad s_{ij}^x(s,s') = \frac{p_i(s)X_{ij}(s,s')}{p_i(s)Q_i(s)}.$$

Finally, the stacked and linearized production functions and aggregators are given by:

$$0 = \hat{Q}_t(s) - \hat{Z}_t(s) - diag\left(\frac{p_i^v V_i(s)}{p_i(s)Q_i(s)}\right)\hat{V}_t(s) - diag\left(\frac{p_i^x(s)X_i(s)}{p_i(s)Q_i(s)}\right)\hat{X}_t(s) \tag{A34}$$

$$0 = \hat{V}_t(s) - \alpha\hat{K}_t(s) - (1-\alpha)\hat{L}_t(s) \tag{A35}$$

$$0 = \hat{X}_t(s) - \sum_{s'} W_X(s',s)\hat{\mathbb{X}}_t(s',s) \tag{A36}$$

$$0 = \hat{F}_t(s) - W_F(s)\hat{\mathbb{F}}_t(s) \tag{A37}$$

$$0 = \hat{F}_t - \sum_s diag\left(\frac{p_i^f(s)F_i(s)}{p_i^f F_i}\right)\hat{F}_t(s). \tag{A38}$$

The matrices $W_F(s)$ and $W_X(s',s)$ are sourcing shares for final and intermediate goods:

$$W_F(s) \equiv [diag(w_1^f(s)), diag(w_2^f(s)), \ldots]$$

$$\text{with} \quad w_i^f(s) = [w_{i1}^f(s), \cdots, w_{iN}^f(s)], \quad w_{ij}^f(s) \equiv \frac{p_i(s)F_{ij}(s)}{p_j^f(s)F_j(s)},$$

$$\text{and} \quad W_X(s',s) \equiv [diag(w_1^x(s',s)), diag(w_2^x(s',s)), \ldots]$$

$$\text{and} \quad w_i^x(s',s) = [w_{i1}^x(s',s)), \cdots, w_{iN}^x(s',s)], \quad w_{ij}^x(s',s) \equiv \frac{p_i(s')X_{ij}(s',s)}{p_j^x(s)X_j(s)}.$$

To compute the dynamics, one needs to modify these conditions to reflect the choice of numeraire. Further, to reduce the computational burden, I manually substitute out for final and intermediate goods shipments $\{\{F_{jit}(s)\}_j, \{X_{jit}(s',s)\}_{j,s'}\}_{i,s}\}$, thereby reducing the size of the system by $6N^2$ (3174 with 23 countries) elements. Obviously other manual substitutions further reduce the dimensionality, but eliminating unknowns that increase in the square of the number of countries is most helpful. I use Harald Uhlig's "Toolkit for

α is common across sectors. This implies that $\frac{\bar{K}_i(s)}{\bar{K}_i}$ and $\frac{\bar{L}_i(s)}{\bar{L}_i}$ equal the share of each sector in total value added, which I observe. I also use the model to allocate final expenditure across consumption and investment in the steady state. Since I observe capital income $r_iK_i = \alpha\sum_s p_i^v V_i(s)$, and hence the ratio of capital income to final expenditure $\frac{r_iK_i}{p_i^f F_i}$, I can solve for $\frac{K_i}{F_i}$ using the fact that $\frac{r_i}{p_i^f} = 1/\beta - (1-\delta)$ in the steady state. From this, the shares of investment and consumption in final expenditure follow.

Analyzing Nonlinear Dynamic Stochastic Models" in MATLAB to compute solutions to this system.[36]

A.2 Example: Cobb-Douglas Model with Fixed Factor Inputs

I now turn to the details underlying the derivation of Equation (11) in Section 2.5.1. As stated in the text, I make two assumptions in moving from the general model to the special case. First, I assume that each country and sector is endowed with a fixed amount of the composite factor, denoted $\bar{V}_i(s)$. Second, I assume that the production functions and final goods aggregators are Cobb-Douglas.

With these assumptions, preferences and production functions can be written as:

$$U_{it} = \log\left(\prod_s \prod_j F_{jit}(s)^{\omega^f_{ji}(s)}\right) \tag{A39}$$

$$Q_{it}(s) = Z_{it}(s)\bar{V}_i(s)^{\theta_i(s)} \left(\prod_{s'} \prod_j X_{jit}(s', s)^{\omega^x_{ji}(s',s)}\right)^{1-\theta_i}, \tag{A40}$$

where $\omega^f_{ji}(s)$ and $\omega^x_{ji}(s', s)$ are shares of goods from j in preference and technologies for country i. Further, note that $C_{it} = F_{it}$, since there is no investment or capital accumulation in this example.

With this set-up, we can characterize the equilibrium response to productivity shocks by solving a social planner's problem. Suppose the social planner maximizes $\sum_i N_i U_{it}$ by choosing $\{F_{jit}, X_{jit}\}_{\forall j,i}$, given the production function and resource constraint $Q_{it}(s) = \sum_j F_{ijt}(s) + \sum_j \sum_{s'} X_{ijt}(s, s')$. This yields four sets of linearized equilibrium conditions:

$$\hat{\mathbb{F}}(s) = -M_i \hat{\lambda}(s) \tag{A41}$$

$$\hat{\mathbb{X}}(s', s) = -M_i \hat{\lambda}(s') + M_j \hat{\lambda}(s) + M_j \hat{Q}(s) \tag{A42}$$

$$\hat{Q}(s) = S_F(s)\hat{\mathbb{F}}(s) + \sum_{s'} S_X(s, s')\hat{\mathbb{X}}(s, s') \tag{A43}$$

$$\hat{Q}(s) = \hat{Z}(s) + diag(1 - \theta_i(s)) \sum_{s'} W_X(s', s)\hat{\mathbb{X}}(s', s), \tag{A44}$$

where $\lambda_j(s)$ denotes the shadow price of output from country j and the remaining notation follows the previous section.

Then derivation of Equation (11) proceeds in two steps. The first step is to combine Equations (A41), (A42), and (A43) to solve for prices as a function of quantities. After some algebra, one obtains $\hat{Q}(s) = -\hat{\lambda}(s)$, which says that relative prices are proportional to relative quantities. This is a familiar result from Cobb-Douglas models. The second step is to solve for quantities produced as a function of the shocks. To do so, substitute Equation (A42) into Equation (A44), and then eliminate prices using the result from the first step.

[36] See Uhlig (1999) or http://www2.wiwi.hu-berlin.de/institute/wpol/html/toolkit.htm

Rearranging the result yields:

$$\underbrace{\begin{pmatrix} \hat{Q}(1) \\ \hat{Q}(2) \end{pmatrix}}_{\hat{Q}} = \underbrace{\begin{pmatrix} \Omega(1,1) & \Omega(2,1) \\ \Omega(1,2) & \Omega(2,2) \end{pmatrix}}_{\Omega'} \underbrace{\begin{pmatrix} \hat{Q}(1) \\ \hat{Q}(2) \end{pmatrix}}_{\hat{Q}} + \underbrace{\begin{pmatrix} \hat{Z}(1) \\ \hat{Z}(2) \end{pmatrix}}_{\hat{Z}}, \tag{A45}$$

where Ω' is an input-output matrix, with block elements $\Omega(s, s')$, which are composed of entries equal to the expenditure share on inputs from sector s in country i used by sector s' in country j. Manipulation of this equation then completes the derivation of equation (11).

A.3 Approximate Trade Elasticity

In Section 4.3, I present simulations with alternative values for $\{\eta, \rho, \sigma\}$. I made the argument there that unrestricted changes in η and ρ change the aggregate elasticity of substitution between home and foreign gross output (the 'trade elasticity') in the model. Further, I asserted there that one can approximate the aggregate trade elasticity using a trade-weighted average of $1/(1-\eta)$ and $1/(1-\rho)$. I present to details underlying that approximation here.

Starting with definitions, aggregate imports for country i are defined as: $IM_{it} = \sum_s IM_{it}(s)$, where $IM_{it}(s) = \sum_{j \neq i} p_{jt}(s) F_{jit}(s) + \sum_{j \neq i} \sum_{s'} p_{jt}(s) X_{jit}(s, s')$. Let us also define country i's expenditure on its own gross output as: $E_{it}(s) = p_{it}(s) F_{iit}(s) + \sum_{s'} p_{it}(s) X_{iit}(s, s')$, with $E_{it} = \sum_s E_{it}(s)$.

To characterize the trade elasticity, I study how the ratio of expenditure on imports to domestic goods IM_{it}/E_{it} depends on the relative price of foreign goods. As is standard, the elasticity of the expenditure ratio IM_{it}/E_{it} with respect to relative prices will be equal to one minus the elasticity of substitution between home and foreign goods.

Because there are multiple sectors and heterogeneous elasticities for final and intermediate goods, changes in IM_{it}/E_{it} depend on substitution between home/foreign suppliers within each sector and end use category, as well changes in the composition of expenditure across sectors and end use categories. I focus on how direct substitution effects influence IM_{it}/E_{it}, holding composition constant. Specifically, I approximate the change in IM_{it}/E_{it} holding the sector-level allocation of domestic expenditure on domestic output ($E_{it}(s)/E_{it}$) and the allocation of domestic expenditure on domestic final vs. intermediate goods ($p_{it}(s) F_{iit}(s)/E_{it}(s)$ and $p_{it}(s) X_{iit}(s, s')/E_{it}(s)$) constant. Further, to obtain a simple aggregate elasticity, I derive the response of IM_{it}/E_{it} to a uniform change in foreign relative to domestic prices (i.e., equal across countries and sectors). There are four steps in this approximation.

First, by construction, IM_{it}/E_{it} can be written as:

$$\frac{IM_{it}}{E_{it}} = \sum_s \left(\frac{E_{it}(s)}{E_{it}} \right) \left(\frac{IM_{it}(s)}{E_{it}(s)} \right).$$

Then holding $E_{it}(s)/E_{it}$ constant, changes in IM_{it}/E_{it} are approximated as:

$$\widehat{IM}_{it} - \hat{E}_{it} \approx \sum_s \left(\frac{IM_{it}(s)}{IM_{it}} \right) \left(\widehat{IM}_{it}(s) - \hat{E}_{it}(s) \right), \tag{A46}$$

where hat's denote log changes from date t to $t+1$.

Second, by construction, $IM_{it}(s)/E_{it}(s)$ can be written as:

$$\frac{IM_{it}(s)}{E_{it}(s)} = \sum_{j \neq i} \left(\frac{p_{jt}(s)F_{jit}(s)}{p_{it}(s)F_{iit}(s)}\right)\left(\frac{p_{it}(s)F_{iit}(s)}{E_{it}(s)}\right) + \sum_{j \neq i}\sum_{s'} \left(\frac{p_{jt}(s)X_{jit}(s,s')}{p_{it}(s)X_{iit}(s,s')}\right)\left(\frac{p_{it}(s)X_{iit}(s,s')}{E_{it}(s)}\right).$$

Then holding $p_{it}(s)F_{iit}(s)/E_{it}(s)$ and $p_{it}(s)X_{iit}(s,s')/E_{it}(s)$ constant, changes in $\frac{IM_{it}(s)}{E_{it}(s)}$ can be approximated as:

$$\widehat{IM}_{it}(s) - \hat{E}_{it}(s) \approx \frac{\rho}{\rho - 1} \sum_{j \neq i} \left(\frac{p_{jt}(s)F_{jit}(s)}{IM_{it}(s)}\right)(\hat{p}_{jt}(s) - \hat{p}_{it}(s)) + \frac{\eta}{\eta - 1} \sum_{j \neq i}\sum_{s'} \left(\frac{p_{jt}(s)X_{jit}(s,s')}{IM_{it}(s)}\right)(\hat{p}_{jt}(s) - \hat{p}_{it}(s)), \quad \text{(A47)}$$

where I used the first order conditions to write changes in $p_{jt}(s)F_{jit}(s)/p_{it}(s)F_{iit}(s)$ and $p_{jt}(s)X_{jit}(s,s')/p_{it}(s)X_{iit}(s,s')$ as functions of relative prices and the elasticity parameters.

Third, if we further assume that price changes are uniform across foreign countries and sectors – so $\hat{p}_{jt}(s) - \hat{p}_{it}(s)$ is equal for all j and s, denoted $\hat{p}_{-i,t} - \hat{p}_{it}$ – then Equation (A47) simplifies to:

$$\widehat{IM}_{it}(s) - \hat{E}_{it}(s) \approx \left[\frac{\rho}{\rho - 1}\left(\frac{IM_{it}^F(s)}{IM_{it}(s)}\right) + \frac{\eta}{\eta - 1}\left(\frac{IM_{it}^X(s)}{IM_{it}(s)}\right)\right](\hat{p}_{-i,t} - \hat{p}_{it}), \quad \text{(A48)}$$

where $IM_{it}^F(s)$ and $IM_{it}^X(s)$ are total imports of sector s final and intermediate goods.

Fourth, plugging (A48) into (A46), yields the approximation:

$$\widehat{IM}_{it} - \hat{E}_{it} \approx \left[\frac{\rho}{\rho - 1}\left(\frac{IM_{it}^F}{IM_{it}}\right) + \frac{\eta}{\eta - 1}\left(\frac{IM_{it}^X}{IM_{it}}\right)\right](\hat{p}_{-i,t} - \hat{p}_{it}), \quad \text{(A49)}$$

where IM_{it}^F and IM_{it}^X are total imports of final and intermediate goods.

This says that the ratio of imports to domestic expenditure on home gross output depends on relative prices with an elasticity that is a trade-weighted average of the expenditure elasticities for final and intermediate goods, where the trade weights depend on import shares of final and intermediate goods. To convert this to an elasticity over home and foreign quantities, we take one minus this expenditure elasticity, which yields:

$$\text{Aggregate Trade Elasticity} \approx \frac{1}{1-\rho}\left(\frac{IM_{it}^F}{IM_{it}}\right) + \frac{1}{1-\eta}\left(\frac{IM_{it}^X}{IM_{it}}\right). \quad \text{(A50)}$$

This is the approximate formula I use to frame discussion in the main text and tables. Note that this formula implies that the aggregate elasticity will differ across countries due to differences in import shares by end use. For simplicity, I suppress this dependence in my discussion in the main text. I instead focus on the average trade elasticity in the model, as if these shares are the same across countries (and equal to average shares of final and intermediate goods in total world trade).

Appendix B

In this appendix I present details and discussion concerning the procedure I use to parametrize the stochastic process for gross output TFP ($Z_{it}(s)$). As noted in the text, the procedure has two steps. The first step is to estimate a stochastic process for value-added TFP, using value-added labor productivity as a proxy for value-added TFP. The second step is to convert the productivity process for value-added TFP into an equivalent process for gross output TFP, which I execute by multiplying shocks to value-added TFP by the steady state value-added to output ratio.

Value-Added Labor Productivity as Proxy for Value-Added TFP In the first step, I use value-added labor productivity in place of value-added TFP due to data availability. Data on value-added labor productivity is available for all my sample countries between 1970 and 2007.[37] In contrast, value-added TFP is only available for a subset of 15 countries. Further, the TFP time-series is much shorter for most countries; where available, the data typically begin around 1980 (though some countries, like Germany, have an even shorter time series). The longer labor productivity time series helps increase the precision of the estimates for the productivity process, both due to the increased number of observations and because the longer time series includes recession episodes in the 1970's and early 1980's. Further, in practice, labor productivity is likely measured more accurately than TFP, since it does not require estimates of the capital stock or labor quality.

Using value-added labor productivity in place of value-added TFP implicitly assumes that capital dynamics do not drive variation in labor productivity at business cycle frequencies. This assumption is commonly employed in the aggregate IRBC literature, where for example Backus, Kehoe, and Kydland (1992), Heathcote and Perri (2002), Kose and Yi (2006) all use prior labor productivity data to parametrize the productivity process. Because the EU KLEMS database contains both value-added labor productivity and TFP for a subset of countries and years, I can examine how well this assumption performs.

The most direct approach to examining this assumption is to compare value-added TFP growth to value-added labor productivity growth.[38] For each country and sector, I compute the correlation between growth rates of value-added labor productivity and TFP – defined as $\rho_i(s) \equiv \text{corr}\left(\Delta \log(TFP_i^{VA}(s)), \Delta \log(LP_i^{VA}(s))\right)$. The histogram of $\rho_i(s)$ is plotted in Figure 8. As is evident, the correlations are uniformly high. The median correlation is 0.93, and only three (out of 30 country-sector pairs) are less than 0.85.

This high correlation of sector-level growth rates of value-added TFP and labor productivity means that cross-country correlations in sector-level productivity are also similar for both measures. To show this, I compute cross-country correlations $\rho_{ij}^{TFP}(s,s') \equiv \text{corr}\left(\Delta \log TFP_i^{VA}(s), \Delta \log TFP_i^{VA}(s')\right)$ and $\rho_{ij}^{LP}(s,s') \equiv \text{corr}\left(\Delta \log LP_i^{VA}(s), \Delta \log LP_i^{VA}(s')\right)$

[37] While almost all countries cover this entire period, there are a few countries truncated time series. In the EU KLEMS data, Canada has data for 1970-2004, Japan has data for 1973-2006, and Portugal has data from 1970-2006. In the 10-sector database, Brazil and Mexico have data for 1970-2005, while India has data for 1970-2004.

[38] For these comparisons, I use the 2009 version (2011 revision) of the EU KLEMS data, but the same patterns hold in the 2008 version of the data.

and plot $\rho_{ij}^{TFP}(s,s')$ versus $\rho_{ij}^{LP}(s,s')$ in Figure 9.[39] As is evident, $\rho_{ij}^{LP}(s,s')$ is a good proxy for $\rho_{ij}^{TFP}(s,s')$ overall. Breaking down results by sector pair, the correlation of $\rho_{ij}^{TFP}(s,s')$ with $\rho_{ij}^{LP}(s,s')$ is 0.9 for goods sectors paired with goods sectors, and about 0.8 for both services paired with services and goods paired with services. As such, using labor productivity in place of TFP to estimate the cross-country correlations is very likely not important in understanding the results in the main text.

Converting Value-Added TFP to Gross Output TFP In the second step, I adjust the estimated stochastic process to make it applicable to gross output. To understand the nature of the adjustment, recall the discussion in Section 2.5.1 about distinguishing gross output from real value added. TFP measured using gross output is $\widehat{TFP}_{it}^{Q}(s) = \hat{Z}_{it}(s)$, while TFP measured using real value added is $\widehat{TFP}_{it}^{V}(s) = \frac{1}{s_i^v(s)}\hat{Z}_{it}(s)$, as in Equation (10). The two TFP measures are related by $\widehat{TFP}_{it}^{Q}(s) = s_i^v(s)\widehat{TFP}_{it}^{V}(s)$, so shocks to productivity measured using value added will be larger than the corresponding shocks measured using gross output. Drawing on Equation (12), if $\Sigma = \frac{1}{T}\sum_t \hat{\epsilon}_t\hat{\epsilon}_t'$ is the covariance matrix for shocks to value-added TFP, then the covariance matrix for shocks to gross output TFP is $\widetilde{\Sigma} = \frac{1}{T}\sum_t \hat{\tilde{\epsilon}}_t\hat{\tilde{\epsilon}}_t'$, where $\hat{\tilde{\epsilon}}_{it}(s) \equiv (1-\theta_i(s))\hat{\epsilon}_{it}(s)$ as in the main text. The persistence parameter $\lambda_i(s)$ obtained in estimation of (12) can be directly used to describe persistence in gross output TFP, as it is common to both gross output and value-added TFP.

In implementing this procedure, I use the steady-state value of $s_i^v(s)$ for each country and sector. This approach is consistent with the assumptions in the baseline model simulations, where I assume that the production function is Cobb-Douglas, so the ratio of value added to gross output is constant in the model. In the data, the ratio of value-added to gross output is time-varying. This raises a question as to how well this simplifying assumption performs in practice.

To address that question, note that the maintained assumption $\widehat{TFP}_{it}^{Q}(s) = s_i^v(s)\widehat{TFP}_{it}^{V}(s)$ says that growth rate of gross output TFP is a linear transformation of the growth rates for value-added TFP, with a zero intercept. I can examine whether this holds in a subset of the EU KLEMS data (2008 version) in which chain-weighted value-added and gross output TFP are both reported. To be explicit, the fact that the data are constructed using chain weighting means that the ratio of value-added to gross output is allowed to be time-varying in constructing the data.[40] Nonetheless, the punchline is that gross output TFP growth is in fact nearly a constant rescaling of value-added TFP growth.[41]

I illustrate this using data for the United States from 1978 to 2005 in Figure 10, where I plot annual growth rates for gross output TFP on the y-axis and annual growth rates for value-added TFP on the x-axis. As is evident, the growth rates line up nearly on a straight

[39] In this figure and the subsequent statistics, I focus on the 12 countries that have more than 25 years of data for both value-added labor productivity and TFP.

[40] This data sub-sample is different than the data used above. Whereas I used the 2009 version (2011 revision) of the data above, that data does not include both value-added and gross output TFP measures. Therefore, I use the 2008 version here, where both are reported.

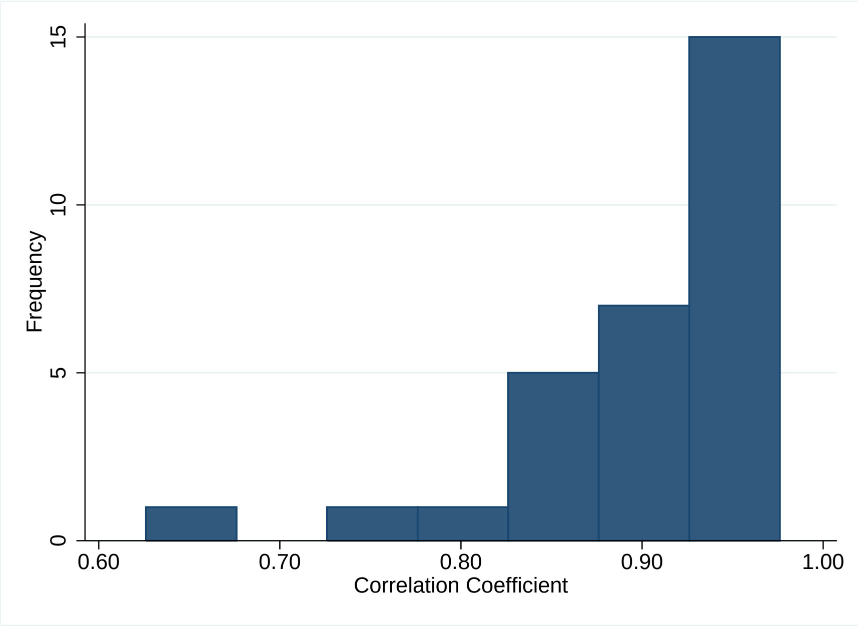
[41] This holds in the EU KLEMS data that I use. Whether it also holds in alternative data is an open question.

line. The slope of the relationship in each figure is shallower than the 45 degree line. The reason is that the value added to output ratio is less than one in both sectors. Further, the slope is shallower for goods than services, since the value added to output ratio is smaller for goods than services. This basic result that growth rates for gross output TFP are a linear transformation of growth rates for value-added TFP holds generally in the EUKLEMS data. For example, if I pool productivity growth rates in the goods sector for all countries and regress the growth rate for gross output TFP on the growth rate for value-added TFP (with no constant), then I get a coefficient of 0.34 (roughly the share of value added in gross output for goods) and an R-squared of 0.97.

Detrending the Data As discussed in the text, I use the HP filter to detrend the value-added labor productivity data prior to estimating Equation (12). In the main simulations, I filter the data with the HP smoothing parameter set to 6.25, as suggested by Ravn and Uhlig (2002). When I then estimate Equation (12) on the cyclical component of productivity, the autocorrelation estimates are low and not statistically significant in nearly all countries and sectors. Literally, only 3 of the 44 country-sector coefficients are significantly different than zero at the 5% level, and number of point estimates are actually negative. I present the distribution of coefficient estimates in the left panel of Figure 11. Based on these estimates, I set the persistence parameter to zero for all countries and sectors in my baseline simulations. And I construct the covariance matrix for productivity shocks across sectors and countries ($\tilde{\Sigma}$) using the cyclical component of labor productivity directly.

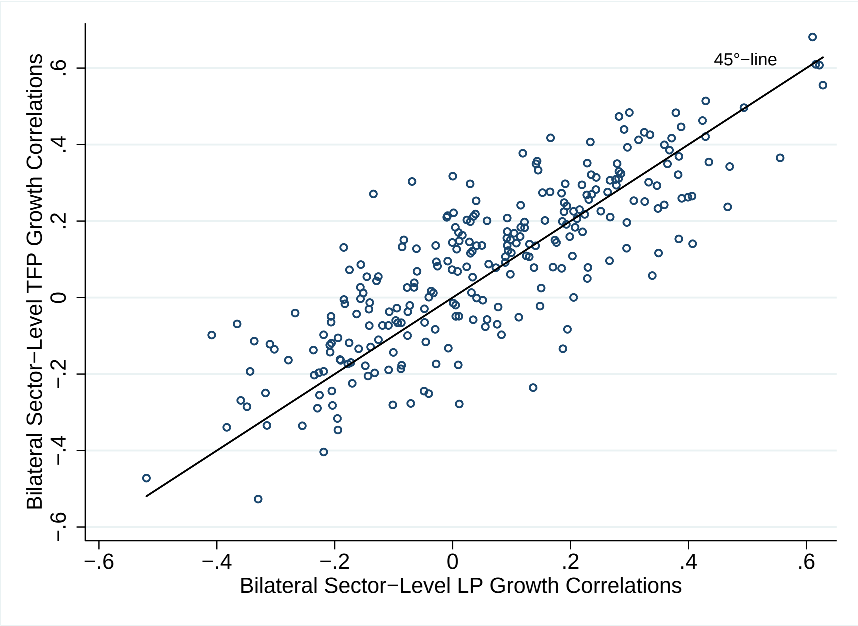
As a robustness check, I re-estimate the productivity process using an alternative value for the HP smoothing parameters. Following Backus, Kehoe, and Kydland (1994), I filter the data using a persistence parameter equal to 100. For comparison to the previous results, I present the distribution of AR(1) coefficient estimates from estimating Equation (12) with this detrended productivity measure in the right panel of Figure 11. As is obvious, the cyclical component of productivity displays a markedly higher degree of persistence in this case. Therefore, to check the baseline results, I re-run the simulations using these persistence estimates and the corresponding residuals from Equation (12) to parameterize the productivity process. These results are included in the Online Appendix.

Figure 8: Histogram of Estimated Within-Country Correlations Between Sector-Level Value-Added TFP and Value-Added Labor Productivity Growth



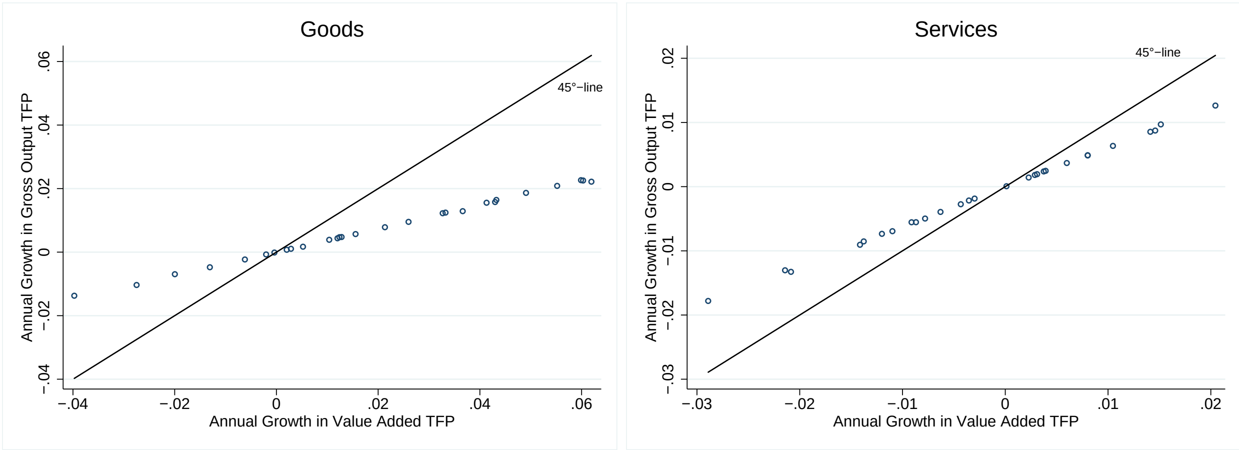
Source: EU KLEMS Database (2009 version, 2011 revision).

Figure 9: Estimated Cross-Country Correlation of Sector-Level Value-Added TFP vs. Value-Added Labor Productivity Growth



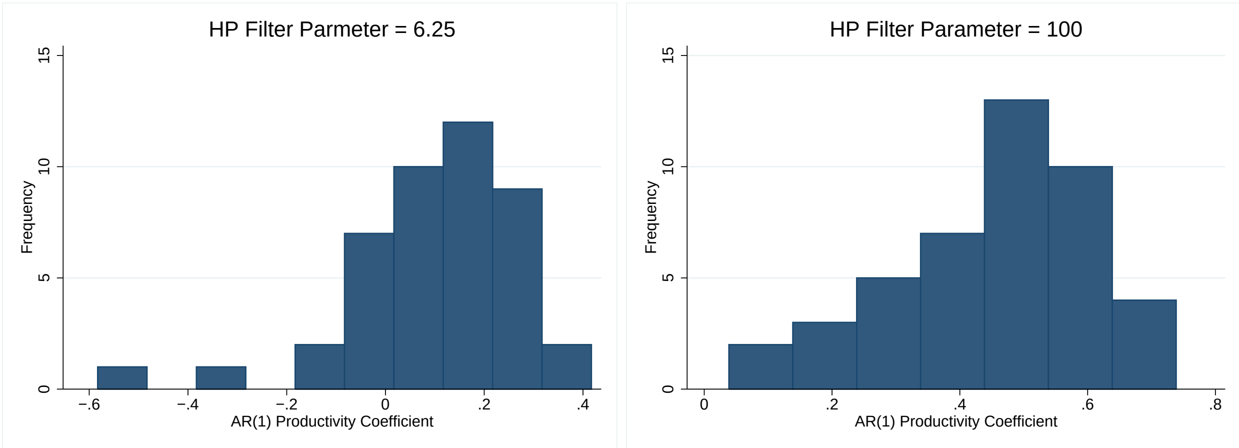
Source: EU KLEMS Database (2009 version, 2011 revision).

Figure 10: United States Gross Output TFP Growth vs. Value-Added TFP Growth



Source: EU KLEMS Database (2008 version).

Figure 11: AR(1) Coefficients for Cyclical Component of Value-Added Labor Productivity



Source: EU KLEMS Database and author's calculations.