NBER WORKING PAPER SERIES

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Working Paper 18148
http://www.nber.org/papers/w18148

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
June 2012

We thank William Shore and Wei Dou for research assistance and several helpful comments. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 18148
June 2012
JEL No. G31,G32

ABSTRACT

We show how the value of a real option depends on corporate income taxes and the option's "debt capacity," defined as the amount of debt supported or displaced by the option. The value of the underlying asset must be an adjusted present value (APV). The risk-free rate of interest must be after-tax. Debt capacity depends on the APV and target debt ratio for the underlying asset, on the option delta and on the amount of risk-free borrowing or lending that would be needed for replication. The target debt ratio for a real call option is almost always negative. Observed debt ratios for growth firms that follow the tradeoff theory of capital structure will be lower than target ratios for assets in place. Our results can rationalize some empirical financing patterns that seem inconsistent with the tradeoff theory, but rigorous tests of the theory for growth firms seem nearly impossible.

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1. INTRODUCTION

Consider a tax-paying corporation with an option to invest in a real asset. The option is to be valued as a derivative by the risk-neutral method, that is, by calculating the payoffs to the option *as if* the expected rate of return on the underlying asset is equal to the risk-free rate and then discounting. What is the discount rate? It is the risk-free rate, of course, but pre-tax or after-tax?

Accurate valuation of real options depends on the correct answer to this question. The answer also has important implications for debt policy and tests of the tradeoff theory of capital structure. If the tradeoff theory is correct and risk and debt capacity of assets in place are constant, then a firm with no growth options will have a constant target debt ratio, and will adjust back towards that ratio when its actual ratio is off-target. But a firm with valuable growth options will *not* operate at a constant debt ratio, and will usually move to a *lower* debt ratio when the values of its assets and options increase. As we will show, some empirical results that seem inconsistent with the tradeoff theory may actually be explained by the dynamics of target debt ratios for growth options.

Back to the beginning question on taxes and discounting: Taxes are important, not just because corporations pay them, but also because interest tax shields are valuable. Project NPVs are typically calculated by discounting at a tax-adjusted rate, for example at an after-tax weighted average cost of capital (WACC). Tax-adjusted discount rates depend on the debt supported by a project's cash flows as well as on the cash flows' business risk. To value a real option, we must therefore determine the amount of debt supported or displaced by the option.

Black-Scholes (1973) and Merton (1973) demonstrated that a call option can be replicated by delta (δ) units of the underlying risky asset less a specific amount of riskless borrowing. We can always express the value of a real option as the net value of these two components of a replicating position. The first component (δ units of the underlying) is perfectly correlated with the underlying asset and should have the same proportional debt capacity. The second component (borrowing) should displace ordinary debt on a dollar-for-dollar basis. Thus the debt capacity of a real option will depend on the value and debt capacity of the underlying asset and the amount of implicit borrowing. The interest on the debt in a replicating portfolio is not tax-deductible, however. Thus there is a tax cost when implicit option debt displaces ordinary borrowing.

We show how to calculate real option values using a risk-and-debt-neutral method. The method is most easily implemented by the following two steps.

1. Calculate the APV of the underlying real asset, including the present value of interest tax shields on debt supported by the asset. Use this APV as the value of the underlying asset. (The APV is often calculated in one step by discounting cash flows at an after-tax WACC).)

2. Calculate the all-equity after-tax payoffs to the real option as if the expected rate of return on the underlying asset were equal to the *after-tax* risk-free rate. Then discount the payoffs to the option at the after-tax rate.

The second step operates in an *after-tax* risk-neutral setting, where the risk-neutral rate is set so that the expected rate of return on the underlying asset equals the after-tax risk-free rate.

Our method assumes that capital structure is continuously rebalanced to maintain a debt ratio equal to the target for corporate assets, *including* the corporation's real options. This assumption can be only approximately correct, but it follows common practice. For example, discounting at an after-tax weighted average cost of capital (WACC) requires that future debt ratios stay constant. The assumption also follows from the tradeoff theory of capital structure. If the theory is correct, the firm will always be on or moving back towards the target.

Preview of Valuation Results

Consider a European option to purchase a real asset at date 1. The exercise price is a fixed amount X. The real asset is worth PV_0 at date 0. Assume for simplicity that it generates no cash flows between dates 0 and 1. Its cash flows after date 1 are net of corporate taxes that would be paid by an all-equity firm.

Standard practice would value this option by the risk-neutral method, discounting at the pre-tax risk-free rate r. Suppose this standard valuation has been done properly. We can write the call value as:

$$\text{PV}(call) = \delta\,\text{PV}_0 - D_C,$$

where δ is the option delta and D_C is the implicit option leverage, that is, the amount of debt that would be needed to replicate the option.

This valuation procedure does not consider how ownership of the real option affects a corporation's debt capacity and interest tax shields. The debt capacity of the option depends on the two terms given above. The first term δPV_0 is positive and proportional to PV_0. This term supports debt. The second term D_C *displaces* regular borrowing, and the implicit interest on D_C is *not* tax-deductible. When option leverage substitutes for explicit leverage, interest tax shields are lost.

Therefore we must convert both terms to APVs that account for taxes and leverage:

$$\text{APV}(call) = \delta\,\text{APV}_0 - \text{APV}(D_C(1+r)), \qquad (1)$$

where APV_0 is the APV of the underlying real asset and $\text{APV}(D_C(1+r))$ is the APV of the option leverage.

We write the second APV term as a function of $D_C(1+r)$, because this APV calculation discounts both the principal and implicit interest for D_C. The discount rate is the *after-tax* risk-free rate. Thus this APV calculation grosses up D_C by the factor $\frac{1+r}{1+r(1-T)}$, where T is the marginal corporate tax rate. We derive APV formulas in the next section.

Now suppose we redo the option valuation in a risk-neutral setting where the risk-free rate is defined after-tax. That is, the risk-neutral drift term is $r(1-T)$. The asset value PV_0 is not affected, because the drift rate and the discount rate "cancel" when specified consistently. But now the correct APV(*call*) is calculated automatically, with no need for a tax gross-up.

This valuation logic can be applied generally. Consider a European call option with a fixed exercise price *X*, written on an asset worth PV_0 now. The asset will generate no cash flows or dividends prior to option expiration τ periods in the future. The Black-Scholes-Merton value would normally be calculated as:

$$\text{PV}(call) = N(d_1)\text{PV}_0 - N(d_2)X(1+r)^{-\tau}$$

We adapt the Black-Scholes-Merton formula for leverage and taxes by substituting APV_0 for PV_0 and the after-tax rate $r(1-T)$ for the pre-tax rate *r*. The APV of the call is then:

$$\text{APV}(call) = N(d_1)\text{APV}_0 - N(d_2)X(1+r(1-T))^{-\tau} \qquad (2)$$

where d_1 and d_2 are now the arguments of the normal distribution function calculated using APV_0 for the underlying asset and the after-tax risk-free rate.

Preview of Implications for Capital Structure

The tradeoff theory assumes that the firm has a target debt ratio, which is determined by the risk and other attributes of the firm's assets, and that the actual debt ratio equals the target on average and reverts to the target over time. Many tests of the tradeoff theory take its predictions as qualitative. We can be more precise.

Suppose that the static-tradeoff theory holds exactly. Then the implicit debt in real (call) options must displace explicit borrowing. Of course only explicit borrowing shows up in observed debt ratios. Several empirical implications follow:

1. The target debt ratio for a real call option is always less than the target debt ratio for the underlying asset. Therefore firms with valuable real options will have lower market debt ratios.

2. The net debt capacity of real call options is in most cases negative. Therefore valuable call options will in most cases lead to lower borrowing relative to assets in place. If book values are adequate proxies for assets in place, then firms with more valuable growth options will usually operate at lower book debt ratios.
3. The net debt capacity of real options depends on the value of the underlying asset, the exercise price of the options, and other option-pricing inputs, including volatility and interest rates. Therefore the firm's target book and market debt ratios cannot be stable over time, even if the firm's portfolio of assets in place and real options is held constant.
4. Growth options should not increase the equity standard deviation or beta. Growth options are of course more volatile than assets in place, but debt policy cancels out any increase.

Of course these predictions assume that growth options account for a material part of the market value of the firm. This is not true for all firms, but is true for many.

Prior Research

The interplay of taxes, financing and the cost of capital has been well-covered in the finance literature on the valuation of real assets. Key papers include Myers (1974), which introduces the APV method, Miles and Ezzell (1980), Taggart (1991) and Ruback (1986).[1] Taxes and financing are also important in practice. For example, they affect valuations when cash flows are discounted at an after-tax weighted average cost of capital (WACC) and in APV calculations. The PV of interest tax shields motivates corporate borrowing in the tradeoff theory of capital structure. But as far as we know, the effects of corporate leverage and taxes on the valuation of real options have never been addressed.

The pre-tax risk-free rate is used to value real options in practice. Discount rates are not adjusted for taxes and leverage. For example, Dixit and Pindyck "largely ignore taxes" (1994, p. 55). They and Copeland and Antikarov (2001) do not adjust the discount

[1] For a summary, see Brealey, Myers and Allen (2010, Chapter 19). See also Ruback (2002).

rate for taxes. Brennan and Schwartz's (1985) real-options analysis of natural resource investments values cash flows after taxes but uses a pre-tax risk-free rate. All of the examples in McDonald's (2006) practice-oriented survey discount at the pre-tax rate. Brealey, Myers and Allen (2010) cover taxes and leverage in detail when explaining WACC and APV, but then use the pre-tax risk-free rate in their chapter on real options.

Ruback (1986) explained why the after-tax rate $r(1-T)$ should be used to value risk-free cash flows. See also Brealey, Myers and Allen (2011, pp. 498-501). Intuition suggests that the after-tax rate should also be used to discount certainty-equivalent option payoffs. We show that the intuition is correct when valuing implicit option leverage.

Modigliani and Miller (MM) (1963) derive the tax- and leverage-adjusted discount rate $r_i^* = r_i(1 - \lambda T)$, where r_i is the opportunity cost of capital for assets in risk class i and λ is the target debt ratio for the firm.[2] Nowadays r_i would be defined as an unlevered cost of capital, likely derived from the capital asset pricing model (CAPM). Myers, Dill and Bautista (1976) derive the discount rate for financial leases as $r = r_D(1 - \lambda T)$, where r_D is the pre-tax cost of debt and λ is the fraction of debt supported or displaced by the lease. These discount rates are useful for calculating APVs in some circumstances, but are not adapted to value real options. But we will derive a similar discount-rate formula for valuing certainty equivalent cash flows from real assets.

The after-tax weighted average cost of capital (WACC) is widely used in corporate finance. It is useful for valuing real assets but not for valuing real options. Application of a constant WACC to discount multi-period cash flows assumes not only that the systematic risk of the cash flows is constant, but also that debt capacity is constant fraction of value. This assumption is wrong for real options.

We are not the first to consider how real options could affect capital structure choices. Some empirical results have been attributed in a qualitative way to the presence of growth options. For example, growth firms with high market-to-book ratios tend to use less debt, which makes sense if growth options increase market values but have little

[2] Myers (1974) shows that the MM formula works only for level perpetuities and fixed borrowing. The correct formula for finite-lived projects with expected cash flows that vary over time is in Miles and Ezzell (1980).

or no debt capacity. See Barclay, Smith and Watts (1995) for a summary of this view of the evidence.

Barclay, Morellec and Smith (2006) present an agency model in which growth options have *negative* debt capacity and present empirical results indicating that firms with growth options tend to operate at low *book* debt ratios. We agree with these authors that growth options usually have negative debt capacity, but our modeling approach is different from theirs. They present an agency model in which debt constrains managers' impulse to overinvest. We start with standard option valuation methods – no agency issues – and simply derive the implications of the tradeoff theory when real options are important.

We also note a related literature on how growth options affect the risk of levered common equity. Gomez and Smidt (2010) and Jacquier, Titman and Yalcin (2010) are recent examples. The latter article shows that equity betas are more closely linked to the implicit leverage in growth options than to explicit borrowing.[3] Again we agree, but our approach is different, because we use the tools of option valuation and corporate finance in order to derive specific predictions about capital structure if the tradeoff theory is correct.

The logic of tax- and leverage-adjusted valuation of real options is introduced by binomial examples in Section 2. Section 3 generalizes the binomial analysis, introduces the modified Black-Scholes-Merton formula and calculates examples of errors when real options are valued without adjusting for taxes and leverage. Section 4 summarizes the implications of real options for optimal capital structure and tests of the static tradeoff theory.

2. EXAMPLES

The logic of our APV approach to option valuation is best introduced in one-period binomial examples. We will concentrate on calls, although the same logic applies to puts. We assume a pre-tax risk-free rate of 6%. For simplicity we also assume that the

[3] Jacquier, Titman and Yalcin (2010) refer to option leverage as "operating leverage."

corporation can borrow or lend at 6% pre-tax. The opportunity cost of capital is 10%. The marginal corporate tax rate is T = 35%.

APV of the Underlying Asset

Start with the value of the underlying real asset; the growth option will enter in a moment. Assume that the asset generates a single expected cash payoff of V_1 = 110 after corporate taxes at t = 1. There are no intermediate cash flows between t = 0 and t = 1. Discounting at the opportunity cost of capital gives a present value V_0 = 110/1.10 = 100. The asset's payoff can also be expressed as a certainty-equivalent payoff of CEQ(V_1) = 106. Discounting at the 6% risk-free rate gives the same V_0 = 100.

This valuation assumes all-equity financing and does not include the value of interest tax shields on the debt supported by the asset. Assume that the firm's target debt ratio for assets in place is λ = 50%, expressed as a fraction of APV.

$$\begin{aligned}\text{APV}(V_1) &= \frac{\text{CEQ}(V_1)}{1+r} + \frac{\lambda r T \text{APV}(V_1)}{1+r} = \frac{\text{CEQ}(V_1)}{1+r(1-\lambda T)} \\ &= \frac{106}{1+0.06\,(1-0.5\times 0.35)} = \frac{106}{1.0495} = 101\end{aligned} \tag{3}$$

APV(V_1) includes the value of one period's interest tax shields on debt of 50.5. The tax shields are worth 1.0. Note that the certainty-equivalent discount rate is $r\left(1-\lambda T\right)$. The term $\left(1-\lambda T\right)$ adjusts for leverage and taxes, as in the MM cost of capital formula. Note too that the relationship between the PV and the tax-adjusted APV is

$$\text{APV}(V_1) = \text{PV}(V_1)\left(\frac{1+r}{1+r\left(1-\lambda T\right)}\right).$$

We will work with certainty-equivalent cash flows in this paper to give a clearer comparison of discount rates for real assets and real options. But the APV of 101 could have been calculated by discounting the expected payoff of 110 at an after-tax weighted average cost of capital (WACC). WACC can be calculated by the Miles-Ezzell (1980) formula. Let k be the opportunity cost of capital. Then[4]

$$\text{WACC} = k - \lambda r T\left(\frac{1+k}{1+r}\right) = 0.10 - 0.5\times0.06\times0.35\times\left(\frac{1.10}{1.06}\right) = 0.0891$$

Discounting the expected cash flow of 110 at 8.91% gives the APV of 101.

We will interpret standard practice as starting with the all-equity PV of the underlying real asset, so that a conversion to APV is necessary. But some applications may start with an APV-equivalent valuation, for example a PV calculated at an after-tax WACC. In these cases no explicit APV calculation is required.

We will refer to the "debt capacity" of real assets and of options on those assets, with the understanding that "debt capacity" is not the maximum amount that could be borrowed, but the optimal target amount that the firm chooses to borrow, taking into account taxes, costs of financial distress, asset risk characteristics, etc.

APV of a Forward Contract

The simplest derivative is a one-period forward contract to purchase the asset, say for $X = 100$. The long position on the forward receives the cash flow V_1. The APV of the forward contract is

$$\text{APV}(\textit{forward}) = \text{APV}(V_1) - \text{APV}(X).$$

We have already calculated APV(V_1), but now have to calculate APV(X). If we forget about debt and taxes, the PV (not APV) of the forward purchase price is $D_F = X/(1 + r) = 100/1.06 = 94.34$. D_F is the implicit forward leverage and the amount of debt needed to replicate the forward contract. But X is a fixed, debt-equivalent obligation, so D_F displaces debt dollar for dollar. Therefore we discount at the after-tax risk-free rate

[4] The WACC can also be calculated from the common formula WACC = $r_D(1 - T)\lambda + r_E(1 - \lambda)$. Here the cost of debt r_D is the risk-free rate of $r = 6\%$. The cost of equity r_E can be calculated by different routes. One route starts with the expected rate of return to an investor who buys a 50-50 portfolio of debt and equity. The portfolio is a claim on the unlevered asset, worth 100, and the safe interest tax shield, worth 1. The expected return on this portfolio is 9.96%. The cost of equity can be backed out as 13.92%. Then WACC = .06×(1 − .35)×.5 + .1392×.5 = .0891.

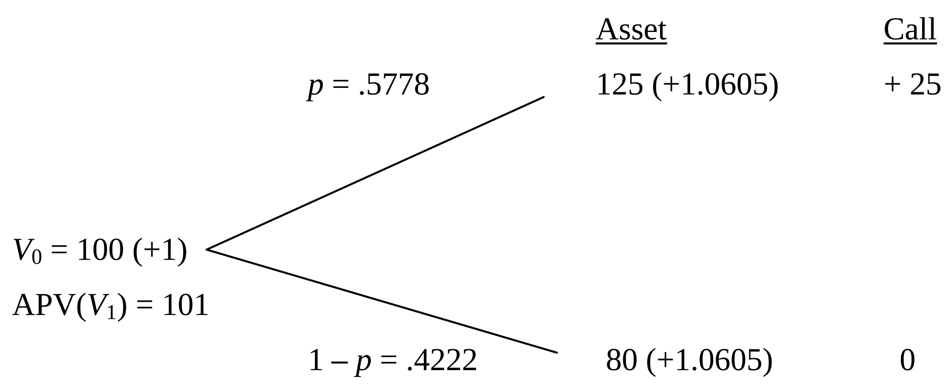
.06(1 − .35) = .039, which gives APV(X) = 100/1.039 = 96.25. (This APV calculation follows from Eq. (3) when λ = 100%.)

The APV of the forward contract is 101 – 96.25 = +4.75. Notice that calculating the contract's APV requires two discount rates, $r(1-\lambda T)$) for the underlying asset and $r(1-T)$ for the forward purchase price. Notice also that using the after-tax rate to discount the forward price grosses up D_F by the factor $\frac{1+r}{1+r(1-T)}$.

The debt capacity of the forward contract is negative. APV(V_1) supports debt of 50.50. APV(X) displaces explicit borrowing of 96.25. The net debt capacity is – 45.75. A firm holding the forward purchase contract would borrow 45.75 less against its other assets and appear to operate at an excessively conservative debt ratio.[5]

APV of a Real Call Option

Now consider a one-period call option for the same real asset with an exercise price of X = 100. We start with standard valuation practice, using a one-period, pre-tax binomial event tree with rate of return outcomes of +25% (the "up" branch) and –20% (the "down" branch). The risk-neutral probabilities of the up and down branches are p = .5778 and 1 – p = .4222, so that the expected rate of return is equal to the pre-tax risk-free rate.



[5] Marking the forward contract to its market value of 4.75 and putting this net value on the asset side of the balance sheet does not solve this problem. The firm would still appear to have unused debt capacity from its assets in place. Fully revealing accounting would split the forward contract, showing APV(V_1) on the left side of the balance sheet and APV(X) as an explicit debt-equivalent liability on the right.

Interest tax shields on debt supported by the asset are split out and shown in parentheses. The tax shields are determined by borrowing at t = 0 and do not depend on the up or down outcome at t = 1. The payoffs to the call do not yet take account of interest tax shields.

The PV (not yet APV) of the call is (.5778×25)/1.06 = 13.63. We can break out this value as the difference between two values, just as for the forward contract. The two values are δ = .5556 units of the asset and implicit debt of D_C = 41.93.

$$\text{PV}(\text{call}) = \delta V_0 - D_C = 55.56 - 41.93 = 13.63.$$

These two values show the replicating portfolio for the call.[6] The first term is δ units of the real asset. The second term D_C is the PV of a future fixed payment of 41.93 plus interest of rD_C = .06×41.93 = 2.52. We now have to calculate the APV of each term.

The first term δV_0 has the exactly same risk as the asset in place and therefore the same fractional debt capacity of λ = 50%. Thus the APV of the first term is δAPV(V_1) = .5556×101 = 56.11.[7] The second term D_C displaces debt dollar for dollar. Its APV must take account of the tax shields lost on the debt that it displaces. We therefore discount the future payment of $D_C(1+r)$ = 41.93 + 2.52 = 44.45 at the after-tax risk-free rate of 3.9%. So $\text{APV}(D_C(1+r))$ = 44.45/1.039 = 42.78. The APV of the call is:

$$\text{APV}(call) = \delta\,\text{APV}(V_1) - \text{APV}(D_C(1+r)) = 56.11 - 42.78 = 13.33$$

As for the forward contract, there are two tax-adjusted discount rates. APV(V_1) can be valued by discounting the certainty equivalent of V_1 at $r(1-\lambda T)$. $\text{APV}(D_C(1+r))$ is valued by discounting D_C, plus interest on D_C, at the after-tax risk-free rate $r(1-T)$.

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[6] Most real options cannot be explicitly replicated and hedged, but that does not change valuation principles. See Brealey, Myers and Allen (2011, pp. 569-570).

[7] In this example the option delta does not depend on whether V_1 or APV(V_1) is used for the underlying real asset.

The calculation for $\text{APV}(D_C(1+r))$ grosses up the option leverage D_C by a factor of $\frac{1+r}{1+r(1-T)}$.

The debt capacity of the call is negative. The first term supports debt of $\lambda\delta\text{APV}(V_1)$ = .5×.5556×101 = 28.06. The option's implicit leverage displaces debt of 42.78. Net debt capacity is 28.06 – 42.78 = − 14.72. Real call options usually displace explicit debt, although they can have positive debt capacity if they are far enough in the money.[8]

Suppose the firm has one asset in place worth APV = 101 and a real call option to buy one more. If the firm's debt is on target, its market-value balance sheet (showing explicit debt only) is:

APV	=	101.00	*D*	=	35.78
APV(*call*)	=	13.33	*E*	=	78.55
		114.33			114.33

The asset in place has debt capacity of 50.50, but the firm only borrows 35.78, because the call's debt capacity is –14.72. The market debt ratio would be recorded as only 35.78/114.33 =.31, far below the target debt ratio of 50% against the APV of assets in place. The firm would appear to operate at a conservatively low debt ratio. In fact it is at its debt target. It is borrowing the same 50% fraction of the APV of the asset in place and of $\delta\text{APV}(V_1)$, the option's position in an identical asset by way of the call. But it is also reducing borrowing dollar for dollar to compensate for the APV of the leverage in the call.

We can also write the balance sheet making showing the option leverage explicitly:

[8] Change our example so that X = 25 rather than 100. Now δ = 1 and $\delta\lambda\text{APV}(V_1)$ = 50.50. The APV of the option debt is 25.51. The net contribution to debt capacity is 50.50 – 25.51 = +24.99.

APV	=	101.00	D	=	35.78
			D_C	=	42.78
δAPV(V_1)	=	56.11	E	=	78.55
		157.11			114.33

Notice that total debt (save for one penny added in rounding) is exactly 50% of firm value. Therefore the standard deviation or beta of the equity is not changed by the growth option.[9] The risk on the left of the balance sheet is the same with or without the growth option, because δAPV(V_1) is strictly proportional to the APV of the asset in place. The financial risk on the right is the same, because the debt ratio remains at λ =50% when option leverage is included. Of course the growth option is riskier than the asset in place, but the firm offsets the additional risk by borrowing less.

The APV of the real call option is only 30 cents less (13.63 vs. 13.33) than its PV calculated by standard methods, which ignore debt and taxes. This is not a material difference. The example looks at only one period in a simple binomial tree, however. We will show later that the valuation errors from ignoring debt and taxes can be large in more realistic cases.

APV in an After-Tax Binomial Tree

We have calculated APVs in a conventional pre-tax binomial event tree—a tree in which the risk-neutral drift rate equals the pre-tax interest rate. Suppose we switch to an *after-tax* binomial tree. That tree is identical to the conventional tree except for the probabilities, which are reset so that the expected return is the after-tax return $r(1-T)$ = .06(1 – .35) = .039. The new probabilities are p = .5311 and $1-p$ = .4689. The APV of the underlying asset is not changed. But option leverage increases, because the discount rate is lower. In this tree, option leverage of 42.78 is calculated in one step, but the result is exactly equal to the $\text{APV}(D_C(1+r))$ calculated in the pre-tax tree. Thus:

[9] We thank William Shore for pointing out this result.

$$\text{APV}(call) = \delta\,\text{APV}(V_1) - \text{APV}(D_C(1+r)) = 56.11 - 42.78 = 13.33$$

Switching from the pre-tax to the after-tax tree increases option leverage by the factor $\frac{1+r}{1+r(1-T)}$, the same as the gross-up required in the pre-tax tree. Therefore we can bypass the conversion of D_C to $\text{APV}(D_C(1+r))$ and just value real-option leverage in an after-tax risk-free setup. Details are in the next section.

3. BINOMIAL AND BLACK-SCHOLES-MERTON FORMULAS FOR REAL OPTIONS

Now we generalize to a real call option on a longer-lived asset. We present valuation formulas for valuing a European call on the asset for each time interval in a binomial event tree. The formulas converge to Black-Scholes-Merton as calendar time is divided into smaller intervals and the number of intervals becomes very large.

Assume that the asset generates a known constant cash-flow yield y that is proportional to end-of-period APV.[10] The payoff at date t + 1 from purchasing the asset at date t is $\text{APV}_{t+1}(1 + y) + \lambda r T \text{APV}_t$. APV_{t+1} captures the present value at t + 1 of all subsequent cash flows and associated interest tax shields.

The asset can be valued recursively, discounting expected payoffs at a tax- and leverage-adjusted discount rate. If the expected payoffs are certainty equivalents, as in Eq. (3), APV_0 is:

$$\text{APV}_0 = \frac{\text{CEQ}\left[\text{APV}_1(1+y) + \lambda r T\,\text{APV}_0\right]}{1+r}$$

$$= \frac{\text{CEQ}\left[\text{APV}_1(1+y)\right]}{1+r(1-\lambda T)}$$

[10] A constant cash-flow yield implies that the asset is a growing, declining or constant perpetuity. The cash flow yield for a finite-lived asset would have to increase as the asset approached its final date.

Once APV_0 is calculated we can proceed to value a call on the asset. We start as before with a conventional, pre-tax binomial tree with up and down steps u and $d = 1/u$. We use APV as the value of the underlying asset.

		Asset	Call
		$p\,[u\text{APV}_0\,(1+y) + \lambda rT\text{APV}_0]$	$C^u - rTD_C$
APV_0			
		$(1-p)\,[d\text{APV}_0\,(1+y) + \lambda rT\text{APV}_0]$	$C^d - rTD_C$

The option payoffs in this tree include two terms. The first terms C^u and C^d are standard. For example, if the option matures in the next period, $C^u = \text{Max}(u\text{APV}_0 - X, 0)$ and $C^d = \text{Max}(d\text{APV}_0 - X, 0)$. (The option does not get the cash flow yield $y\text{APV}_0$ or the interest tax shield $\lambda rT\text{APV}_0$ on debt supported by the underlying real asset.) The second term rTD_C captures the interest tax shields lost because of debt displaced by the option's implicit leverage.

The equations for option replication in the pre-tax tree are:

$$C^u - rTD_C = \delta u\text{APV}_0 - D_C(1 + r) \text{ and } C^d - rTD_C = \delta d\text{APV}_0 - D_C(1 + r),$$

$$C^u = \delta u\text{APV}_0 - D_C(1 + r(1 - T)) \text{ and } C^d = \delta d\text{APV}_0 - D_C(1 + r(1 - T)). \tag{4}$$

Eq. (4) implies that the fractional debt capacity of a real call option is always less than for the underlying asset. The debt capacity would be the same (λ) but for option leverage D_C, which enters as a negative term in Eq. (4).

The lost tax shields rTD_C are the same in the up and down states, so the formula for the option delta is $\delta = \dfrac{C^u - C^d}{(u-d)APV_0}$. Given δ, the amount of debt displaced is:

$$D_C = \frac{\delta u\text{APV}_0 - C^u}{1 + r(1-T)} = \frac{\delta d\text{APV}_0 - C^d}{1 + r(1-T)} \tag{5}$$

Eq. (5) for D_C discounts at the after-tax rate $r(1 - T)$. The standard formula, which ignores interest tax shields on displaced debt, would discount at the pre-tax rate r. Thus Eq. (5) grosses up option leverage by the factor $(1+r)/(1+r(1 - T))$.

Of course the gross-up is not needed in an after-tax binomial tree, where the expected risk-neutral rate of return on the asset is $r\left(1-T\right)$. The APV of option leverage is generated automatically in the after-tax tree, with no required explicit adjustment for interest tax shields lost because of displaced debt. The after-tax tree is identical to the conventional tree, except for the pseudo probabilities, which depend on the after-tax risk-free rate. The probability of the "up" branch is:

$$p = \frac{\dfrac{1+r\left(1-T\right)}{1+y} - d}{u-d}$$

Moving from the pre-tax to after-tax tree does not change the state variable $APV_0(1 + y)$. The amount of debt in the replicating portfolio changes, because of the gross-up for lost interest tax shields on ordinary debt displaced by option leverage.

Volatility of APV in discrete time

We used APV_0 as the state variable in the binomial event tree. APV_0 includes the interest tax shield $\lambda rT APV_0$, which is fixed by borrowing at t = 0 and does not depend on the outcome at t = 1. This tax shield does not affect the payoffs to the call or the dollar spread between up and down payoffs to the real asset, but it does reduce the risk of the next payoff to the real asset. Therefore the shield could affect volatility and the choice of u and d, the rate of return outcomes in the tree. Thus we pause to consider what determines volatility when APV is used as the state variable in a discrete-time binomial setup.

The call does not receive next period's interest tax shield or the cash flow yield $yAPV_1$. The call's value depends only on APV_1, which has the same volatility as the unlevered asset value V_1. Thus the up and down moves u and $d = 1/u$ should be based on the volatility of APV_1.[11]

The only safe interest tax shield is the first one, $\lambda rT APV_0$. Later tax shields share all the risks of future cash flows and APVs. For example, the value at date t = 1 of the tax shield at date t = 2 is proportional to the realized V_1 and APV_1. It is proportional because the firm is assumed to rebalance its capital structure at date t = 1 and borrow the fraction λ of the realized outcome APV_1. Viewed from date t = 0, the interest tax shield for t = 2 has exactly the same risk as APV_1 or the unlevered payoff V_1. So do the interest tax shields for t = 3, 4, … . When the firm rebalances its capital structure every period, the tax shields have the same risk as future cash flows and unlevered asset values.

The volatility of APV_0 is different from the volatility of the unlevered value V_0 because we are working in discrete time. The difference disappears in continuous time. Continuous time would, strictly speaking, require continuous rebalancing to the firm's target debt ratio.

Our APV analysis assumes that firms rebalance their capital structures exactly to target, either continuously or at the start of each discrete period. This assumption is of course a stretch. Standard DCF valuation practice assumes for simplicity that firms do rebalance, however. For example, discounting at the after-tax WACC assumes that the firm rebalances to keep its market-value debt ratio constant.[12] In this paper we accept this standard assumption and work out its implications for valuing real options.

Valuing puts

Corporations also hold real put options, that is, options to abandon existing assets. Puts are valued in the same way as calls, except that puts support debt, not displace it. Thus we distinguish D_P, the debt supported by a put's negative leverage, from D_C, the

[11] The cash flow yield is proportional to APV_1 and does not affect percentage volatility.
[12] The APV method does not require rebalancing every period. It can incorporate fixed debt repayment schedules or debt policies that adjust with lags to target debt capacity. The assumption of prompt rebalancing could also be relaxed for APVs of real options. We do not consider such extensions here, however.

debt displaced by the option leverage in a call. The replication equations for a put in the pre-tax binomial tree are:

$$C^u + rTD_P = (\delta - 1)u\text{APV}_0 + D_P(1 + r) \text{ and } C^d + rTD_P = (\delta - 1)d\text{APV}_0 + D_P(1 + r)$$

Option leverage D_P now has a positive sign, because a put is replicated by a short position in the underlying asset plus lending. The put-option delta is $\delta - 1$, where δ is the call-option delta. Thus

$$C^u = (\delta - 1)u\text{APV}_0 + D_P(1 + r(1 - T)) \text{ and } C^d = (\delta - 1)d\text{APV}_0 + D_P(1 + \text{r}(1 - T)). \qquad (6)$$

The amount of lending required to replicate the put is:

$$D_C = \frac{(\delta - 1)u\text{APV}_0 - C^u}{1 + r(1 - T)} = \frac{(\delta - 1)d\text{APV}_0 - C^d}{1 + r(1 - T)} \qquad (7)$$

Notice that D_P is calculated by discounting at the after-tax risk-free rate. The discounting is automatic in the after-tax binomial tree.

The fractional debt capacity of a real put option is never negative and always greater than for the underlying asset. This follows from put-option replication, which requires APV(*put*) = $(\delta - 1)\text{APV}(V_1) + D_P$, where D_P is given by Eq. (7). APV(*put*) is non-negative, so $D_P \geq (\delta - 1)\text{APV}(V_1)$. The debt displaced by the short position in the underlying asset is less than the short position, assuming that the underlying asset is risky and $\lambda < 1$. The put's debt capacity $\lambda(\delta - 1)\text{APV}(V_1) + D_P$ must therefore be positive. This debt capacity can approach zero when the put is far out of the money, however.

A Modified Black-Scholes-Merton Formula

Cox, Ross and Rubinstein (1979) showed that in the continuous-time limit, as the length of the time interval approaches zero and the return parameters *r*, *u* and *d* are rescaled accordingly, their binomial option pricing formula converges to the Black-

Scholes-Merton option pricing formula. The convergence proof does not depend on the nature of the underlying asset (here an APV) or on the definition of the risk-free rate (here the after-tax rate $r(1-T)$).[13] Therefore, we can modify the Black-Scholes-Merton formula to take account of taxes and leverage by (1) substituting the APV of the underlying asset for the PV and (2) substituting the after-tax risk-free rate for the pre-tax rate.[14] Suppose the underlying asset generates all-equity cash flows at a known rate y in proportion to its market value. If τ denotes the time remaining to expiration, the APV of a call is:

$$\begin{aligned} \text{APV}(call) &= N(d_1)\text{APV}(V_\tau) - N(d_2)\text{APV}(X) \\ &= N(d_1) APV_0 (1+y)^{-\tau} - N(d_2) X (1 + r(1-T))^{-\tau} \end{aligned}$$

$$d_1 = \frac{\ln\left\{\frac{APV_0}{X}\right\} + \left(\ln\{1 + r(1-T)\} - \ln\{1+y\} + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}$$

$$d_2 = d_1 - \sigma\sqrt{\tau}$$

APV_0 is the spot value of the stream of cash flows and associated interest tax shields generated by the underlying real asset. If the asset generates income before date τ ($y > 0$), the APV of the underlying delivered immediately will be greater than APV(V_τ), which is a deferred claim. This is accounted for by “discounting” at the cash-flow yield rate y.

The Black-Scholes-Merton formula for the value of a real put option is:

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[13] In a multi-period problem, the composition of the replicating portfolio changes as time elapses and the price of the underlying asset evolves. The multi-period binomial option pricing formula was obtained by imagining a dynamic trading program under which the replicating portfolio is revised at the start of each time interval, so that its payoffs at the end of the time interval again replicate the option payoffs. (In the continuous-time market setting of Black-Scholes-Merton, the dynamic trading program requires continuous portfolio revisions.) Our modified option valuation formula thus reflects an additional implicit transaction at the end of each time interval: corporate borrowing is revised to maintain a target debt ratio in market value terms.

[14] Merton (1973) presents the formula for the value of an option on the common stock of a firm that follows a proportional dividend policy.

$$\begin{aligned}\text{APV}(put) &= N(-d_2)\text{APV}(X) - N(-d_1)\text{APV}(V_\tau)\\ &= N(-d_2)X\,(1+r(1-T))^{-\tau} - N(-d_1)APV_0(1+y)^{-\tau}\end{aligned}$$

The drift term in the modified valuation formulas is the after-tax interest rate adjusted for payouts. Thus, real options are valued *as if* the APV of the underlying real asset is expected to appreciate at the after-tax risk-free rate. One might therefore call the modified formula "risk-and-debt-neutral valuation". Thus whether one applies an analytical formula (such as the modified Black-Scholes-Merton formula), simulation methods, or numerical approximation procedures, one can work with the APV of the underlying asset and use an after-tax risk-neutral drift rate (equal to the after-tax risk-free rate adjusted for payouts).

Notice that the target debt ratio λ does not appear in the valuation formulas. Thus, the value of a real option does not depend directly on the debt capacity of the underlying asset, just as the value of an option does not depend directly on the expected rate of return on the underlying asset. Debt capacity and expected rate of return influence option values only insofar as they affect the adjusted present value of the underlying.

Option Debt

European calls and puts are equivalent to simple combinations of digital options—a call (put) option is equivalent to the combination of a short (long) cash-or-nothing option and a long (short) asset-or-nothing option, both with the same strike price and time to expiration as the call (put). The debt capacity of a real option is equivalent to a weighted combination of the same digital components, where the weight on the asset-or-nothing option is equal to the debt capacity of the underlying asset and the weight on the cash-or-nothing option is equal to one. Therefore, the debt capacities of real European-type calls and puts can be calculated using the modified Black-Scholes-Merton formulas with one further change: the APV of the underlying asset is weighted by its debt capacity λ. If we let B denote debt capacity, the debt capacities of calls and puts are given by the following formulas:

$$\mathrm{B}(call) = \lambda N(d_1)APV_0(1+y)^{-\tau} - N(d_2)X(1+r(1-T))^{-\tau}$$

$$\mathrm{B}(put) = N(-d_2)X(1+r(1-T))^{-\tau} - \lambda N(-d_1)APV_0(1+y)^{-\tau}$$

Figure 1 is a pair of graphs of option debt in relation to the adjusted present value of the underlying asset. The options have a strike price of 100 and a time to expiration of one year. There are three cases: the debt capacity of the underlying asset is 0%, 25%, or 50%. Panel A gives the results for calls, Panel B for puts. Option debt is always less than or equal to zero for calls *if* the underlying asset has no debt capacity. Option debt for calls can be positive if the underlying asset has positive debt capacity and the option is very deep in the money. Option debt is always greater than or equal to zero for puts.

The relationship between option debt and the value of the underlying asset is further illuminated by Figures 2 and 3, which are graphs of the option debt deltas and gammas. (The deltas and gammas are the first and second derivatives, respectively, of option debt with respect to the value of the underlying asset.) Notice that the option debt deltas can be well in excess of one in absolute value. Also, the deltas for calls on assets with debt capacity change sign when they are deep in the money. Notice too that the debt gammas change sign as the options shift from out-of-the-money states to in-the-money states. Thus, while the relationship between the value of an option and the value of the underlying asset is rich, the relationship between option debt capacity and the value of the underlying is even richer.

Examples

The modified Black-Scholes-Merton formula will tell us whether the valuation errors from ignoring taxes and the debt capacity of real options are important. Table 1 reports the values of real call options with exercise prices of 100. The underlying asset values (APV) range from 40 to 200. The options are European, with maturities of 1, 3 or 5 years. The tax rate is 35%. Panel A assumes a pre-tax risk-free interest rate of 6%, a volatility of 20% and a cash flow yield of 10%. Panels B, C and D, respectively, show results for a 30% volatility, a 9% interest rate and a 0% cash flow yield.

Each entry in Table 1 gives the correct option value, taking account of the option's debt capacity and the interest tax shields lost because of debt displaced by the option, and below it the absolute (dollar) and percentage errors from *not* adjusting for taxes and leverage. Take for example a 5-year at-the-money option in panel A. The option is worth 5.01 using the modified Black-Scholes-Merton formula. Valuing the option by the conventional method – that is, Black-Scholes-Merton with a pre-tax interest rate – would value the option at 6.79, an overvaluation of 1.78 or 36%.

Table 1 shows that real call options are always *overvalued* by the conventional method. Dollar and percentage errors are material. Percentage errors are larger for longer maturities and larger for out-of-the-money calls than for in-the-money calls. Absolute errors are large for in-the-money calls. Dollar and percentage errors are also greater when the pre-tax interest rate is high (Panel C). This is as expected, since the chief difference between the standard and modified Black-Scholes-Merton model is use of an after-tax interest rate.[15] The tax adjustment to the interest rate matters more when interest rates are high.

Table 2 shows values and valuation errors for real put options. Puts are systematically *undervalued* by conventional methods. Dollar and percentage errors are again material. For example, the 5-year at-the-money put option in panel A is worth 25.50 using the modified Black-Scholes-Merton formula. The conventional method would value the option at 25.50, a valuation error of −6.08 or −24%.

These examples show that ignoring taxes and debt capacity can introduce serious errors in valuing real options. Of course there will also be cases where the errors are not material and where conventional practice is an acceptable approximation. We do not see the point of the approximation, however, since the correct tax- and leverage-adjusted valuation method is no more complicated or burdensome than the conventional method.

Tables 1 and 2 are restricted to simple European calls and puts with fixed exercise prices. The real options held by corporations are more complex. Our methods should generalize from simple to more complex options, however. If we assume that the

[15] The other potential difference is use of APV for the underlying asset rather than the asset's unlevered PV. Whether conventional practice would use APV or PV is not so clear. An asset value calculated by discounting expected cash flows at an after-tax WACC is equivalent to an APV. The same cash flows discounted at an unlevered cost of capital give an estimate of PV.

tradeoff theory holds, so that firms have meaningful target debt ratios against assets in place, then taxes and debt capacity have to be accounted for when valuing real options.

4. EMPIRICAL IMPLICATIONS

Growth options have implicit leverage. Other things equal, the tradeoff theory must therefore predict lower debt ratios for firms with valuable growth options. Abandonment (put) options have the opposite effect. Puts increase debt capacity and should allow the corporation to "lever up."

We do not endorse the tradeoff theory as generally correct and complete, but it's nevertheless useful to work out the theory's empirical implications when the firm holds valuable real options. We will not attempt a review of research on capital structure, which is enormous,[16] but will give examples of prior research below.

The tradeoff theory predicts that firms will trade off the tax advantages of borrowing (interest tax shields) against costs of financial distress. Costs of financial distress include direct costs of bankruptcy or reorganization and also costs from moral hazard and agency problems caused by default risk. The tradeoff determines a target capital structure that maximizes overall firm value.

The tradeoff theory makes two broad predictions. The first is cross-sectional: observed debt ratios should equal target debt ratios on average. Target debt ratios should vary depending on the firm's tax status, the risk or other attributes of its assets (tangible vs. intangible assets, for example) and on how much value would be lost in financial distress. The second, time-series prediction is that observed debt ratios will fluctuate, but adjust back towards target debt ratios over time.

The implicit leverage in real options has immediate implications for cross-sectional tests. The debt capacity of real call options is usually negative and always less than the debt capacity of assets in place. Therefore a firm with valuable growth options will appear to operate at a too-conservative debt ratio, compared to the debt capacity of its assets in place. A mature firm with valuable abandonment (put) options will appear to

[16] Review articles include Harris and Raviv (1991), Myers (2003) and Frank and Goyal (2008).

operate at an aggressively high debt ratio, because puts contribute debt capacity rather than displacing it.

Cross-sectional tests of the tradeoff theory

Most cross-sectional tests start with a standard list of variables to "explain" differences in market or book debt ratios.[17] See Rajan and Zingales (1995) and Harris and Raviv (1991). The most common variables include the following (plus or minus signs in parentheses show typical effects on debt ratios in the cross section[18]): profitability (−); market-to-book ratio (−); tangibility of assets, for example the ratio of property, plant and equipment to total assets (+), and size, usually the log of total assets (+). Many other variables have also been tested, but it is useful to start here with these old standards.

The positive signs for size and tangibility seem reasonable if the tradeoff theory is correct. Large firms ought to borrow more; they are presumably safer and more likely to pay taxes. Firms with more tangible assets are less likely to be damaged in financial distress and should therefore have higher target debt ratios.

The negative signs for profitability and market-to-book are harder to rationalize. The market-to-book ratio might measure intangible assets in place, which are more liable to damage in financial distress. But intangibles ought to contribute some positive debt capacity and at least increase *book* debt ratios. In fact the market-to-book ratio usually correlates with lower book as well as market debt ratios. The negative relationship between profitability and book debt ratios is even harder to explain by the tradeoff theory. Higher profitability increases debt servicing capacity and also increases taxable income and the potential value of interest tax shields. Therefore profitable firms should operate at higher book and market debt ratios, but by and large they don't.

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[17] The tradeoff theory is most often interpreted as applying to market-value debt ratios. Book debt ratios are brought in as a check or backup.

[18] There are of course exceptions to these "typical effects." See Appendix C in Antoniou et al. (2008), which tabulates results from dozens of prior research papers that have used these and other variables.

These typical results are easier to square with the tradeoff theory when we recognize real options. Firms with higher market-to-book ratios are more likely to be firms with more valuable growth options. Our analysis says that such firms will operate at *lower* market debt ratios, because growth options have lower debt capacities than underlying assets in place, and in most cases at lower book debt ratios, because debt capacity for growth options is usually negative. The negative relationship between profitability and financial leverage also makes sense if more profitable firms have more valuable growth options.

Large firms are usually mature firms, which should borrow more if assets in place account for a greater fraction of their market value than for younger growth firms. Mature firms are also likely to have valuable abandonment options, which add debt capacity. Recognizing real put options therefore reinforces the tradeoff theory's prediction that large firms should borrow more.

Examples

Table 3 reports debt capacities and debt ratios for firms that have assets in place worth APV = 100 and real call options to invest 100 in the same assets in year 3. That is, the firms have options to double in size in year 3. Option values are from Panel A of Table 1. Thus the tax rate is 35%, the pre-tax risk-free interest rate is 6%, the volatility is 20% and the cash-flow yield rate is 10%. (Versions of Table 3 based on 1- or 5-year maturities or on other panels of Table 1 tell similar stories.) The top panel of Table 3 assumes a target debt ratio of 25% for assets in place. The bottom panel assumes the target debt ratio is 50%. The firm is assumed on-target with respect to the debt capacity of its option as well as its asset in place. The value of the asset in place ranges from 40 to 200.

Table 3 reports negative debt capacity for the call option in all cases. The negative debt capacity can be double or triple the option's APV. For example, the at-the-money option in Panel A is worth 5.62. Its debt capacity is – 15.47.

Also, with one exception, option debt capacity *decreases* as the option moves farther in the money. The exception occurs in Panel B when the underlying asset value increases from 150 to 200. In this case option debt capacity increases (becomes less negative) from −16.54 to −9.39. Debt capacity would continue to increase for still-higher asset values and eventually turn positive. In the limit, where the ratio of asset value to exercise price becomes extremely high, the fractional debt capacity of a real call option approaches λ, the target ratio for the underlying asset.

Table 3 also reports the target debt ratio for the option—the ratio of the option's debt capacity to its value—which always increases (becomes less negative) as the option moves more in the money.

Negative debt capacity means that the option's implicit leverage displaces reported leverage. Thus the firm's market debt ratio in Table 3 is always less than the 25% and 50% target debt ratios for assets in place. In Panel A, the firm operates at a *negative* debt ratio when underlying asset value is 150 or higher. The firm is a net lender. The "book" ratio of debt to assets in place is also less than the target ratios in Table 3. The book ratio would eventually move above the targets, however, because option debt capacity turns positive when the option is far enough in the money.

Table 4 reports debt ratios for an aging firm that has assets in place worth APV = 100 and options to abandon (put) its assets for 100 in year 3. Put option values come from Panel A of Table 2. Format and inputs are the same as in Table 3.

The firm's market and book debt ratios in Table 4 are always greater than the 25% and 50% target debt ratios for assets in place. The fractional debt capacity of a real put option is never negative and always greater than for the underlying asset. The put's contribution to debt capacity can be double or triple the value of the put itself. For example, the at-the-money put option in Panel A of Table 4 is worth 19.64. Its debt capacity is 54.91.

Table 4 probably overstates the debt capacity of real put options, because it assumes that the exercise price is fixed and risk-free. In practice the proceeds from abandonment will be uncertain, and will not support debt dollar for dollar. Nevertheless, debt capacity from puts can help explain why takeovers of mature, cash-cow firms are often highly levered. For example, LBOs are often diet deals motivated by options to sell assets and shrink operations. The options increase debt capacity relative to target debt ratios for assets in place.

The results summarized in Tables 3 and 4 were obtained based on the assumption that the firm holds a single call or put. Essentially the same conclusions follow from more realistic examples, in which the firm holds a portfolio of calls and/or puts with a range of expiration dates and exercise prices. Table 5 reports debt ratios for firms holding a portfolio of fifty call options: ten calls expiring each year from one to five years and exercise prices ranging in five steps from 50 to 200. Results are reported for firms with the potential to grow 50%, 100% and 150% over five years. Panels A and B report debt ratios when the assets in place and underlying assets have a target debt ratio of 25% and 50%, respectively.

Like the hypothetical firms with a single call options in Table 3, the debt-to-value ratios for the firms with a portfolio of options in Table 5 are equal or very close to the target debt ratio when the options are out of the money (underlying value equal to 40), but decline as the value of the underlying real asset increases. (The cases with 100% growth potential in Table 5 correspond most closely to the single-option results in Table 3.) The decline in debt-to-value ratios is substantial in all cases—even in the case of 50% growth potential and a very high target ratio of 50% (Panel B). Debt-to-value ratios are also not stable—they change when the value of the underlying asset changes.

The ratio of debt to assets in place—the "book" debt ratio—for firms with a portfolio of options likewise behaves much as the book debt ratio for firms with a single option.[19] The book ratio is close to the target debt ratio when the options are well out of the money, declines as the value of the underlying assets increases (moving from left to right in Tables 3 and 5), then becomes flat or even rises as the underlying assets become very valuable and the options are deep in the money. Also, the book debt ratio is close to the market debt ratio—the ratio of debt to all assets, including growth options—in most cases. The exception arises when the target debt ratio for the underlying assets is high (50%) and the options are deep in the money.

These numerical experiments confirm our cross-sectional predictions for market and book debt ratios when firms strictly follow the tradeoff theory.

[19] In this paper both "book" and "market" debt ratios are calculated based on market values. "Book" debt ratio here means the ratio of debt to the market value of assets in place. We use "book" to distinguish a debt ratio that (like corporate balance sheets) does not reflect the value of growth and abandonment options.

1. The target debt ratio for the APV of a real call option is always less than the target debt ratio for the underlying asset. Therefore growth firms with valuable real call options will have lower market debt ratios. Tables 3 and 5 indicate that growth firms should operate at debt ratios that are much lower than their targets for assets in place. In some cases optimal debt ratios will be negative.
2. The net debt capacity of real call options is in most cases negative. Therefore growth firms will in most cases reduce borrowing relative to assets in place. If book values are adequate proxies for assets in place, then firms with more valuable growth options will usually operate at lower book debt ratios. The differences between observed debt ratios and targets for assets in place can be dramatic.
3. The predictions for mature firms with valuable abandonment (put) options and few growth options are reversed. These firms will operate at higher market and book debt ratios. Table 4 suggests that the differences between observed debt ratios and targets for assets in place can be dramatic.

One lesson of this paper is that it's useful to consider the tradeoff theory's predictions for book as well as market debt ratios. The impact of real options on observed book and market debt ratios can be different. Of course book ratios will less informative when book values are poor measures of the values of assets in place.

Changes in debt ratios over time

Our analysis does not make life easier for time-series tests of the tradeoff theory. Observed debt ratios will clearly evolve in complex ways, even when target debt ratios for real assets are constant. If the tradeoff theory is correct and real options are important, any time-series test that assumes a constant target book or market debt ratio is mis-specified.

Our numerical experiments only hint at the complexity likely to be encountered in real firms. It is not just a matter of changes in underlying asset values and option values and debt capacities. Firms will invest to create new options, but also discard options that are too far out of the money. Changes in technology and competition will extinguish options but also create new ones.

But we can say something more definite about how target debt ratios should change over time in response to random shocks to the firm's profitability and the value of its assets in place. We will assume for simplicity that the firm's portfolio of real options is held constant.

A positive shock to the value of a firm's assets in place moves the firm from left to right in Tables 3, 4 and 5. Notice that *the target market debt ratio falls when market value increases*. In other words, the derivative of the target ratio with respect to change is value is *negative*, both for growth firms (Tables 3 and 5) and for mature firms with abandonment options (Table 4).[20] For growth firms, the target debt ratio falls even though the debt capacity of real options increases (or becomes less negative). The reason is that a positive shock to profitability and asset values increases the proportion of market value accounted for by the growth options, which always have less debt capacity than assets in place.

Our numerical experiments are simplified, but it is nevertheless plausible to predict that firms with valuable real options – abandonment options as well as growth options – will target a *lower* market debt ratio when market value goes up. They may also target a lower book debt ratio – notice how the ratio of debt to assets in place declines from left to right in Tables 3, 4 and 5.

This negative relationship could explain otherwise difficult puzzles in the empirical literature on capital structure. Take Welch (2004) as an example. This paper finds that year-to-year changes in market debt ratios are mostly explained by changes in stock price, and that net issues of debt and equity do not counteract the effects of stock-price changes, at least in the short run. If the corporations in Welch's sample are really trying to get back to a stable target debt ratio, they are not working very hard at it. In addition, Welch finds that net issuing activity appears to amplify the effect of stock price

[20] Our theory does not require this negative effect for real call options—the derivative turns positive when growth options are far in the money. The negative effect is required for puts, because puts have higher fractional debt capacity than the underlying assets in place. This additional debt capacity dissipates when asset value increases and the put moves out of the money.

changes on market debt ratios, at least in the short run. “Over one year, firms respond to poor [stock price] performance with more debt issuing activity and to good performance with more equity issuing activity.” (Welch 2004, p. 111.) This behavior is a deep puzzle if one ignores real options and assumes that the firms are following the tradeoff theory with a stable target debt ratio. The behavior makes sense if real options are important and target ratios incorporate the options’ debt capacity. If poor stock price performance means less valuable growth options and more valuable abandonment options, then target debt ratios should go up, leading to more debt issuing activity. Good stock price performance should have the opposite effect.

Predictions about equity risk

Assume the firm has a definite target debt ratio λ for its assets in place, based on the risks of those assets. If the firm follows the tradeoff theory, then it should rebalance its capital structure to keep its total leverage – the sum of ordinary borrowing and the implicit debt in its real options – equal to λ times the sum of the APVs of assets in place and the sum of the underlying APVs for its real options, each multiplied by its current option delta. Assume that the assets underlying the real options have the same business risks as assets in place. Then equity risk (standard deviation or beta) will not depend on the value of its real options. The real options are of course riskier than the assets in place, but debt policy compensates.

Equity risk will be strictly constant if asset risk is constant and the firm can adjust capital structure every period, with no lags or frictions. If capital structure is rebalanced with lags, as in a target adjustment model, then a positive (negative) shock to profitability should at first increase (decrease) equity risk. But equity risk will converge gradually to a target level. This is a testable prediction of the tradeoff theory.

5. CONCLUSIONS

Standard valuation methods take account of project debt capacity and the interest tax shields on debt supported by the project. “Debt capacity” is not the maximum amount that could be borrowed, but a target amount. The tradeoff theory of capital structure says that the target should strike a balance (at the margin) between the tax advantages of borrowing and possible future costs of financial distress.

This paper analyzes growth (call) and abandonment (put) options in this standard valuation framework. We work with certainty-equivalent cash flows to facilitate comparison with standard option valuation methods. These standard methods ignore debt capacity and interest tax shields, however. Certainty-equivalent option payoffs are just discounted at a pre-tax risk-free rate. This discounting procedure is not correct.

Our results include the following:

1. Valuing assets in place: The correct discount rate for unlevered certainty-equivalent cash flows is $r(1-\lambda T)$, where r is the pre-tax interest rate, T is the marginal corporate tax rate and λ is the target debt ratio, expressed as a fraction of the asset's adjusted present value (APV).
2. Valuing real options: Use a tax- and leverage-adjusted APV to value of the underlying real asset. Value real-option payoffs (in a risk-neutral setting) at the *after-tax* interest rate $r(1-T)$. For example, the modified Black-Scholes-Merton formula uses APV as the asset value and $r(1-T)$ as the interest rate.
3. Real call options have implicit leverage. The fractional debt capacity of a call is usually negative, and always less than for the underlying asset.
4. The fractional debt capacity of a real put option is always positive and greater than for the underlying asset.

The following results assume that the firm manages its capital structure according to the tradeoff theory and that its debt ratio is always on target.

5. The implicit debt in real call options displaces ordinary borrowing. Observed debt ratios for firms with valuable growth options will be less than target debt ratios for assets in place. This result may help explain why profitability and market-to-book ratios are negatively correlated with debt ratios.
6. The implicit lending in real put options supports additional ordinary borrowing. Debt ratios for firms with valuable abandonment options will be greater than target debt ratios for assets in place. This result may help explain why large, mature firms tend to operate at high debt ratios, especially in LBOs.

7. When a growth firm's profitability and value improve, its target market debt ratio falls. When profitability and value decline, the target ratio increases. (This result assumes that growth options are valuable, but not too far in the money.)
8. Capital structures should adjust to offset the additional risks of real options, leaving the standard deviation or beta of equity constant.

Results 5 to 8 may assist our empirical understanding of corporate debt policies. These empirical predictions clearly require further investigation, however. A detailed empirical study of options and capital structure is beyond the scope of this paper.

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Figure 1: Option debt as a function of the APV of the underlying asset

Debt capacity for options with an exercise price of 100 and a time to expiration of one year. The top panel describes real calls, the bottom real puts. Underlying assets have a debt capacity of 0%, 25%, or 50%. In all cases the pre-tax risk-free rate is 6% and the marginal tax rate is 35%. The underlying cash-flow yield and volatility are 10% and 20%, respectively.

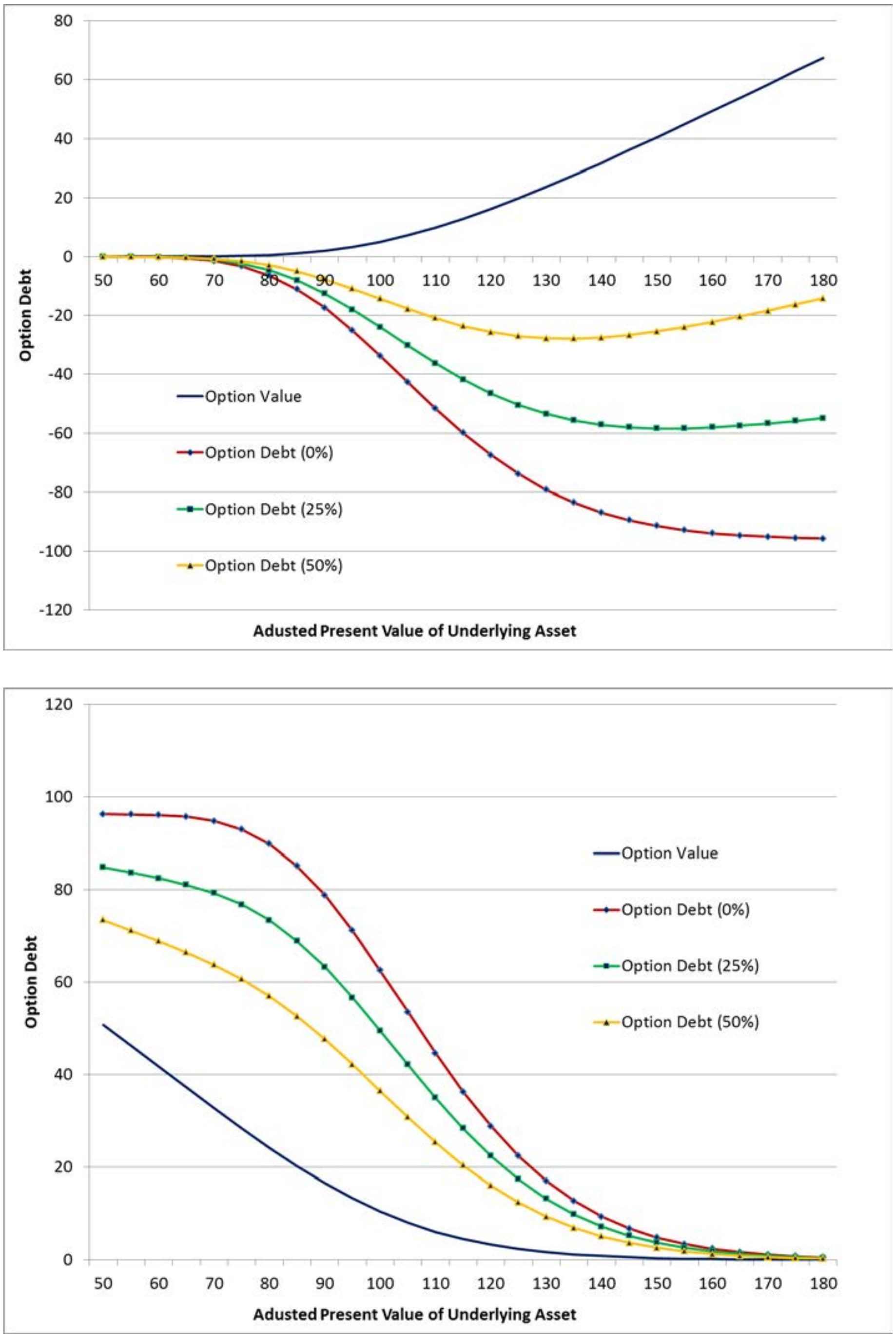
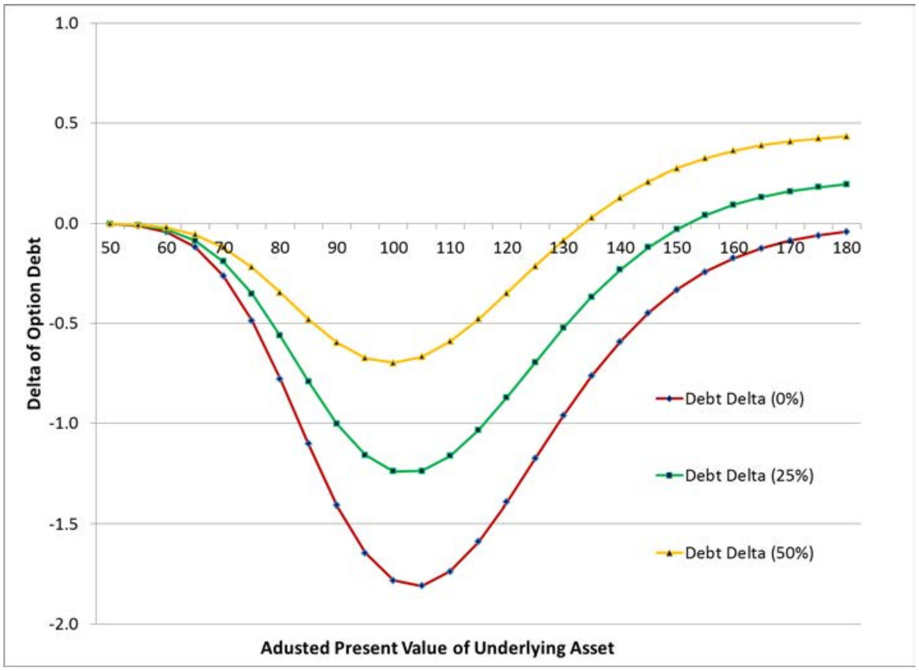


Figure 2: Delta of option debt as a function of the APV of the underlying asset

The delta of option debt capacity is the derivative of option debt with respect to the APV of the underlying. Options have an exercise price of 100 and a time to expiration of one year. The top panel describes real calls, the bottom real puts. Underlying assets have a debt capacity of 0%, 25%, or 50%. In all cases the pre-tax risk-free rate is 6% and the marginal tax rate is 35%. The underlying cash-flow yield and volatility are 10% and 20%, respectively.



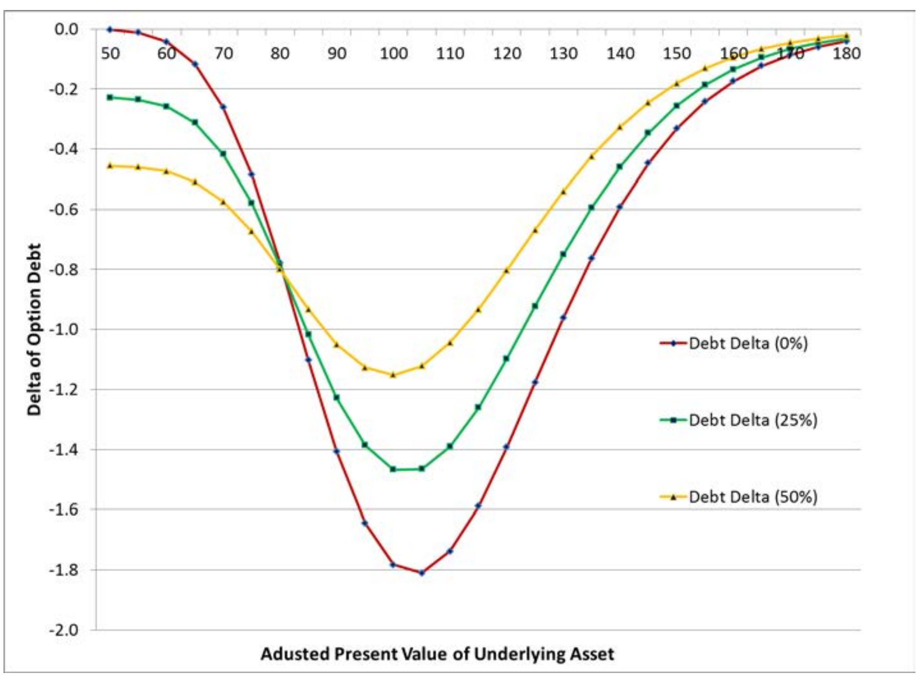


Figure 3: Option debt gamma as a function of the APV of the underlying asset

The gamma of option debt capacity is the second derivative of option debt with respect to the APV of the underlying. Options have an exercise price of 100 and a time to expiration of one year. Gammas are identical for calls and puts with the same underlying, exercise price and time to expiration. Underlying assets have a debt capacity of 0%, 25%, or 50%. In all cases the pre-tax risk-free rate is 6% and the marginal tax rate is 35%. The underlying cash-flow yield and volatility are 10% and 20%, respectively.

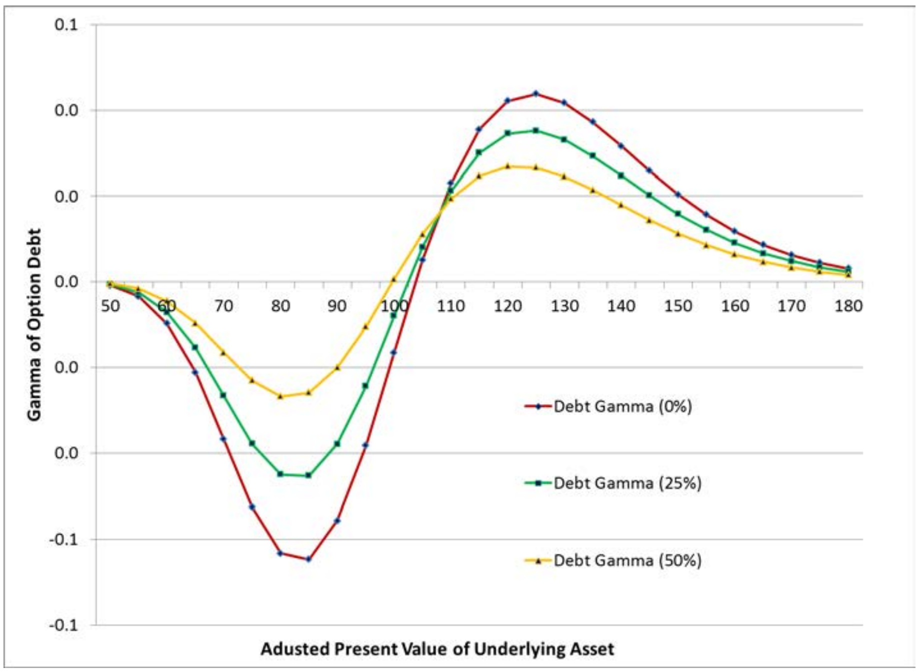


Table 1: Tax and leverage-adjusted value of real call (growth) options

Exercise price is 100. Present values are calculated using the modified Black-Scholes-Merton (BSM) formula, which accounts for option leverage and taxes. Absolute and percentage errors from using the standard BSM model are reported below the correct present values. Present values and errors are reported for different combinations of the pre-tax risk-free rate (*r*), the cash-flow yield on the underlying asset (*y*) and the volatility of the return on the underlying asset (σ). The marginal tax rate is 35%.

				Maturity	APV of Underlying Asset						
				(years)	40	60	80	100	125	150	200
Panel A	Interest rate = 6%	Yield = 10%	Volatility = 20%		0.00	0.01	0.61	5.09	19.76	40.49	85.58
				1	0.00	0.00	0.14	0.70	1.49	1.82	1.91
					66%	38%	23%	14%	8%	4%	2%
					0.00	0.20	1.60	5.62	15.11	28.66	62.25
				3	0.00	0.10	0.57	1.46	2.73	3.75	4.79
					80%	52%	36%	26%	18%	13%	8%
					0.02	0.37	1.80	5.01	11.82	21.37	45.98
				5	0.02	0.23	0.83	1.78	3.15	4.42	6.19
					93%	63%	46%	36%	27%	21%	13%
Panel B	Interest rate = 6%	Yield = 10%	Volatility = 30%		0.00	0.24	2.35	8.69	23.06	42.19	85.81
				1	0.00	0.04	0.28	0.72	1.28	1.62	1.86
					27%	17%	12%	8%	6%	4%	2%
					0.17	1.43	4.82	10.71	21.23	34.47	65.97
				3	0.06	0.34	0.87	1.55	2.39	3.11	4.10
					33%	24%	18%	14%	11%	9%	6%
					0.42	2.07	5.38	10.37	18.64	28.71	52.57
				5	0.16	0.58	1.21	1.94	2.84	3.67	4.99
					37%	28%	23%	19%	15%	13%	10%
Panel C	Interest rate = 9%	Yield = 10%	Volatility = 20%		0.00	0.01	0.74	5.74	21.15	42.19	87.35
				1	0.00	0.01	0.25	1.13	2.22	2.63	2.73
					108%	59%	34%	20%	11%	6%	3%
					0.01	0.30	2.13	6.96	17.63	32.14	66.71
				3	0.01	0.23	1.09	2.50	4.29	5.57	6.72
					127%	77%	51%	36%	24%	17%	10%
					0.04	0.58	2.56	6.65	14.74	25.46	51.73
				5	0.06	0.53	1.66	3.20	5.15	6.77	8.76
					144%	92%	65%	48%	35%	27%	17%
Panel D	Interest rate = 6%	Yield = 0%	Volatility = 20%		0.00	0.05	1.68	9.83	29.73	53.86	103.75
				1	0.00	0.02	0.31	1.06	1.71	1.88	1.91
					59%	33%	19%	11%	6%	3%	2%
					0.07	1.58	7.61	19.13	39.01	62.00	111.00
				3	0.04	0.57	1.79	3.10	4.22	4.78	5.13
					60%	36%	24%	16%	11%	8%	5%
					0.56	4.33	13.13	26.28	46.69	69.48	117.92
				5	0.33	1.61	3.36	4.90	6.22	6.99	7.62
					59%	37%	26%	19%	13%	10%	6%

Table 2: Tax and leverage-adjusted value of real put (abandonment) options

Exercise Price is 100. Present values are calculated using the modified Black-Scholes-Merton (BSM) formula, which accounts for option leverage and taxes. Absolute and percentage errors from using the standard BSM model are reported below the correct present values. Present values and errors are reported for different combinations of the pre-tax risk-free rate (*r*), the cash-flow yield on the underlying asset (*y*) and the volatility of the return on the underlying asset (σ). The marginal tax rate is 35%.

				Maturity	APV of Underlying Asset						
				(years)	40	60	80	100	125	150	200
Panel A	Interest rate = 6%	Yield = 10%	Volatility = 20%		59.88	41.71	24.13	10.42	2.37	0.38	0.01
				1	-1.91	-1.90	-1.77	-1.20	-0.42	-0.09	0.00
					-3%	-5%	-7%	-12%	-17%	-23%	-32%
					59.11	44.28	30.65	19.64	10.35	5.12	1.14
				3	-5.19	-5.09	-4.62	-3.74	-2.47	-1.45	-0.41
					-9%	-11%	-15%	-19%	-24%	-28%	-35%
					57.77	45.70	34.71	25.50	16.79	10.82	4.38
				5	-7.84	-7.63	-7.03	-6.08	-4.71	-3.45	-1.67
					-14%	-17%	-20%	-24%	-28%	-32%	-38%
Panel B	Interest rate = 6%	Yield = 10%	Volatility = 30%		59.89	41.94	25.87	14.03	5.67	2.07	0.24
				1	-1.91	-1.86	-1.63	-1.18	-0.63	-0.28	-0.04
					-3%	-4%	-6%	-8%	-11%	-14%	-18%
					59.28	45.50	33.87	24.73	16.48	10.93	4.86
				3	-5.14	-4.86	-4.32	-3.65	-2.81	-2.09	-1.10
					-9%	-11%	-13%	-15%	-17%	-19%	-23%
					58.18	47.40	38.30	30.87	23.62	18.16	10.97
				5	-7.71	-7.28	-6.65	-5.93	-5.02	-4.19	-2.87
					-13%	-15%	-17%	-19%	-21%	-23%	-26%
Panel C	Interest rate = 9%	Yield = 10%	Volatility = 20%		58.11	39.94	22.49	9.30	1.99	0.30	0.00
				1	-2.73	-2.72	-2.48	-1.60	-0.51	-0.10	0.00
					-5%	-7%	-11%	-17%	-25%	-33%	-44%
					54.27	39.54	26.34	16.15	8.04	3.76	0.76
				3	-7.09	-6.87	-6.01	-4.60	-2.82	-1.53	-0.38
					-13%	-17%	-23%	-28%	-35%	-41%	-49%
					50.46	38.58	28.15	19.82	12.38	7.58	2.80
				5	-10.21	-9.73	-8.61	-7.07	-5.11	-3.49	-1.50
					-20%	-25%	-31%	-36%	-41%	-46%	-54%
Panel D	Interest rate = 6%	Yield = 0%	Volatility = 20%		56.25	36.29	17.93	6.08	0.98	0.11	0.00
				1	-1.91	-1.89	-1.59	-0.85	-0.20	-0.03	0.00
					-3%	-5%	-9%	-14%	-20%	-26%	-35%
					49.23	30.74	16.77	8.28	3.16	1.16	0.15
				3	-5.15	-4.63	-3.40	-2.10	-0.97	-0.41	-0.06
					-10%	-15%	-20%	-25%	-31%	-35%	-43%
					43.15	26.92	15.71	8.87	4.28	2.07	0.51
				5	-7.54	-6.26	-4.51	-2.96	-1.64	-0.88	-0.25
					-17%	-23%	-29%	-33%	-38%	-42%	-49%

Table 3: Debt capacity and target debt ratios for firms with real call options

Option values from Table 1, Panel A, for 3-year maturity. The firm has the option to double its scale at year 3. Target debt ratios for assets in place are 25% (Panel A) and 50% (Panel B). "Debt ratio for option" equals the option's debt capacity to its market value. The pre-tax risk-free rate is 6% and the marginal tax rate is 35%. The underlying cash-flow yield and volatility are 10% and 20%, respectively.

		APV of Underlying Asset						
		40	60	80	100	125	150	200
Panel A: Debt Ratio for Assets in Place = 25%	Option value	0.00	0.20	1.60	5.62	15.11	28.66	62.25
	Option debt capacity	-0.03	-1.03	-5.94	-15.47	-29.04	-39.15	-45.21
	Debt ratio for option	-729%	-512%	-372%	-275%	-192%	-137%	-73%
	Assets in place	40	60	80	100	125	150	200
	Debt capacity of assets in place	10	15	20	25	31	38	50
	Debt to market value for firm	25%	23%	17%	9%	2%	-1%	2%
	Debt to assets in place for firm	25%	23%	18%	10%	2%	-1%	2%
Panel B: Debt Ratio for Assets in Place = 50%	Option value	0.00	0.20	1.60	5.62	15.11	28.66	62.25
	Option debt capacity	-0.02	-0.62	-3.43	-8.44	-14.32	-16.54	-9.39
	Debt ratio for option	-453%	-308%	-215%	-150%	-95%	-58%	-15%
	Assets in place	40	60	80	100	125	150	200
	Debt capacity of assets in place	20	30	40	50	63	75	100
	Debt to market value for firm	50%	49%	45%	39%	34%	33%	35%
	Debt to assets in place for firm	50%	49%	46%	42%	39%	39%	45%

Table 4: Debt capacity and target debt ratios for firms with real put options

Option values from Table 2, Panel A, for 3-year maturity. The firm has the option to liquidate at year 3. Target debt ratios for assets in place are 25% (Panel A) and 50% (Panel B). "Debt ratio for option" equals the option's debt capacity to its market value. The pre-tax risk-free rate is 6% and the marginal tax rate is 35%. The underlying cash-flow yield and volatility are 10% and 20%, respectively.

				APV of Underlying Asset						
				40	60	80	100	125	150	200
Panel A	Debt Ratio for	Assets in Place = 25%	Option value	59.11	44.28	30.65	19.64	10.35	5.12	1.14
			Option debt capacity	81.61	76.86	68.19	54.91	36.64	21.84	6.38
			Debt ratio for option	138%	174%	222%	280%	354%	427%	558%
			Assets in place	40	60	80	100	125	150	200
			Debt capacity of assets in place	10	15	20	25	31	38	50
			Debt to market value for firm	92%	88%	80%	67%	50%	38%	28%
			Debt to assets in place for firm	229%	153%	110%	80%	54%	40%	28%
Panel B	Debt Ratio for	Assets in Place = 50%	Option value	59.11	44.28	30.65	19.64	10.35	5.12	1.14
			Option debt capacity	74.11	66.00	55.67	43.15	27.87	16.26	4.64
			Debt ratio for option	125%	149%	182%	220%	269%	318%	405%
			Assets in place	40	60	80	100	125	150	200
			Debt capacity of assets in place	20	30	40	50	63	75	100
			Debt to market value for firm	95%	92%	86%	78%	67%	59%	52%
			Debt to assets in place for firm	235%	160%	120%	93%	72%	61%	52%

Table 5: Debt ratios for firm with a portfolio of options

Debt ratios for hypothetical firms holding a portfolio of real call options. Each portfolio consists of an equal number of calls with exercise prices ranging from 50 to 200 and expiration dates ranging from one to five years. The target debt ratio for the assets in place and underlying assets is 25% in Panel A and 50% in Panel B. The pre-tax risk-free rate is 6% and the marginal tax rate is 35%. The underlying cash flow yield and volatility are 10% and 20%, respectively.

Ratio of Debt to Value

		Growth	APV of Underlying Asset						
		Potential	40	60	80	100	125	150	200
Panel A	Debt ratio 25%	50%	24%	22%	19%	17%	15%	14%	14%
		100%	24%	19%	13%	10%	7%	6%	6%
		150%	23%	15%	8%	3%	0%	-1%	-1%
Panel B	Debt ratio 50%	50%	50%	48%	46%	45%	44%	43%	43%
		100%	49%	46%	42%	40%	38%	37%	37%
		150%	49%	44%	38%	35%	33%	32%	33%

Ratio of Debt to Assets in Place

		Growth	APV of Underlying Asset						
		Potential	40	60	80	100	125	150	200
Panel A	Debt ratio 25%	50%	24%	22%	19%	18%	17%	16%	16%
		100%	24%	19%	14%	10%	8%	7%	7%
		150%	23%	16%	8%	3%	0%	-2%	-1%
Panel B	Debt ratio 50%	50%	50%	48%	47%	47%	47%	48%	49%
		100%	49%	47%	45%	44%	44%	45%	49%
		150%	49%	45%	42%	41%	41%	43%	48%