

NBER WORKING PAPER SERIES

VOLATILITY, THE MACROECONOMY AND ASSET PRICES

Ravi Bansal  
Dana Kiku  
Ivan Shaliastovich  
Amir Yaron

Working Paper 18104  
<http://www.nber.org/papers/w18104>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
May 2012

We thank seminar participants at NBER Spring 2012 Asset-Pricing Meeting, AFA 2012, SED 2011, Arizona State University, Duke University, London School of Economics, NYU-Five Star conference, The Wharton School, Vanderbilt University, University of British Columbia, University of New South Wales, University of Sydney, and University of Technology Sydney for their comments. Shaliastovich and Yaron thank the Rodney White Center for financial support. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

NBER working papers are circulated for discussion and comment purposes. They have not been peer-reviewed or been subject to the review by the NBER Board of Directors that accompanies official NBER publications.

© 2012 by Ravi Bansal, Dana Kiku, Ivan Shaliastovich, and Amir Yaron. All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission provided that full credit, including © notice, is given to the source.

Volatility, the Macroeconomy and Asset Prices  
Ravi Bansal, Dana Kiku, Ivan Shaliastovich, and Amir Yaron  
NBER Working Paper No. 18104  
May 2012  
JEL No. E0,G0,G01,G12

**ABSTRACT**

We show that volatility movements have first-order implications for consumption dynamics and asset prices. Volatility news affects the stochastic discount factor and carries a separate risk premium. In the data, volatility risks are persistent and are strongly correlated with discount-rate news. This evidence has important implications for the return on aggregate wealth and the cross-sectional differences in risk premia. Estimation of our volatility risks based model yields an economically plausible positive correlation between the return to human capital and equity, while this correlation is implausibly negative when volatility risk is ignored. Our model setup implies a dynamics capital asset pricing model (DCAPM) which underscores the importance of volatility risk in addition to cash-flow and discount-rate risks. We show that our DCAPM accounts for the level and dispersion of risk premia across book-to-market and size sorted portfolios, and that equity portfolios carry positive volatility-risk premia.

Ravi Bansal  
Fuqua School of Business  
Duke University  
1 Towerview Drive  
Durham, NC 27708  
and NBER  
ravi.bansal@duke.edu

Ivan Shaliastovich  
The Wharton School  
University of Pennsylvania  
2423 Steinberg Hall-Dietrich Hall  
3620 Locust Walk  
Philadelphia, PA 19104  
ishal@wharton.upenn.edu

Dana Kiku  
Finance Department  
Wharton School  
University of Pennsylvania  
3620 Locust Walk  
Philadelphia, PA 19104-6367  
kiku@wharton.upenn.edu

Amir Yaron  
The Wharton School  
University of Pennsylvania  
2256 Steinberg-Dietrich Hall  
Philadelphia, PA 19104-6367  
and NBER  
yaron@wharton.upenn.edu

# 1 Introduction

Financial economists are interested in understanding risk and return and the underlying economic sources of movements in asset markets. In this paper we show that macroeconomic volatility is an important and separate source of risk which critically affects the aggregate economy (i.e., consumption) and asset prices. Our analysis yields a dynamic asset-pricing framework with three sources of risks: cash-flow, discount rate, and volatility risks. We show that volatility risk affects consumption and ignoring volatility news results in a sizable mis-specification of the stochastic discount factor (SDF). Our empirical analysis yields two central results: (i) the impact of volatility on consumption is important for understanding joint dynamics of the return to human capital and the return to equity; (ii) volatility risks carry a sizeable risk premium and help explain the level and dispersion of expected returns in the cross-section of assets. In sum, our evidence suggests that in addition to cash-flow and discount-rate fluctuations, volatility risk is an important channel for understanding the macroeconomy and financial markets.

Bansal and Yaron (2004) provide a structural framework to analyze volatility risk. In their model risk premia is increasing in volatility of aggregate wealth, and importantly, shocks to volatility carry a separate risk premium. In this article we show that consumption news is directly affected by news about aggregate volatility. This equilibrium result reveals the importance of volatility risks for a proper measurement of consumption news and the stochastic discount factor (SDF), and for inference about sources of risks in asset markets. Clearly the effect of volatility on consumption, SDF, and consequently asset prices is overlooked in the literature that assumes a constant volatility process (e.g., Campbell (1996)). We quantify the magnitude of the mis-specification caused by ignoring volatility risks and document that it is significant for both the SDF and risk premia.

We present empirical evidence that both macroeconomic- and return-based volatility measures feature persistent predictable variation, which suggests that volatility is potentially an important source of economic risks. Our empirical findings of long-run predictable variation in volatility are consistent with earlier work by Bollerslev and Mikkelsen (1996) in the context of market-return volatility, and by Stock and Watson (2002) and McConnell and Perez-Quiros (2000) in the context of macroeconomic volatility. We incorporate the evidence of time-variation in volatility in our dynamic asset pricing model, and use it to evaluate the implications of volatility risk for consumption, returns to human capital and equity, and the cross-sectional dispersion in equity returns.

In a model with constant volatility, Lustig and Van Nieuwerburgh (2008) show that returns to human capital and the market are puzzlingly negatively correlated. Standard economic models would imply a positive correlation between the two re-

turns, as both of these claims are long aggregate economic outcomes. In this paper we provide a potential resolution to their puzzling finding by highlighting the importance of time-varying volatility of consumption. We document that in the data, high macro-volatility states are high-risk states associated with significant consumption declines and high risk premia and discount rates. In contrast, model specifications used in earlier empirical work ignore volatility risk and therefore counterfactually imply that expected consumption should rise in these states. We show that when volatility risks are incorporated, an increase in the risk premium is indeed a bad state for consumption; this allows the model to capture consumption and expected return dynamics in a manner consistent with the data and generates a positive correlation between human capital and equity returns.

To explore the importance of volatility risks for a broad cross-section of asset returns and their ability to account for the cross-sectional differences in risk premia, we assume that the return to aggregate wealth is perfectly correlated with the return to market equity (e.g., Epstein and Zin (1991), Campbell (1996)). Under this assumption, volatility of aggregate wealth is observed and can be used in empirical analysis. Our model yields a dynamic CAPM (DCAPM) which has three sources of risks: cash-flow, discount rate, and volatility risks. The market price of cash-flow risk equals the risk aversion coefficient; the market prices of discount rate risks and volatility risks are both equal to -1. As in Bansal and Yaron (2004), the market price of volatility risk is negative, and assets with large payoffs in high volatility states (such as long straddle positions) should have positive volatility betas, and therefore negative risk premia. Empirical evidence in Bansal, Khatchatrian, and Yaron (2005b) shows that equity has negative exposure to aggregate volatility since high volatility lower equity prices; hence, equity should have a positive volatility risk premium. As discussed below, we find considerable empirical support for these implications. Note that when volatility risks are absent, as in Campbell (1996), all risk premia ought to be constant and discount-rate news simply reflects risk-free rate news. If the risk-free rate is also assumed constant, there is no discount-rate variation and the entire risk premium in the economy is due to cash-flow risk.

To estimate the unobservable return to human capital, we assume that the expected return on human component of wealth is linear in economic states. This allows us to extract the underlying news in consumption, wealth return and the stochastic discount factor using a standard VAR-based methodology.<sup>1</sup> We find that in the model without volatility risks, as in Lustig and Van Nieuwerburgh (2008), the correlation between human capital and market returns is strongly negative. For the benchmark preferences (risk aversion of 5 and intertemporal elasticity of substitution of 2), the correlations in realized and expected returns range between -0.50 and -0.70. In contrast, when volatility risks are incorporated, the two assets are positively correlated:

---

<sup>1</sup>The importance of human capital component of wealth for explaining equity prices has been illustrated in earlier work by Jagannathan and Wang (1996) and Campbell (1996).

the correlation in return innovations is 0.20, and the correlations in discount rates and five-year expected returns are 0.25 and 0.40, respectively. The model-implied risk premia of the wealth portfolio, human capital and equity are 2.6%, 1.4% and 7.2%, respectively. Volatility risks account for about one-third of the total risk premium of human capital, and about one-half of the risk premium of the wealth portfolio. The inclusion of volatility risks has important implications for time-series dynamics of the underlying economic shocks. For example, in our volatility risk-based model, discount rates are high and positive in recent recessions of 2001 and 2008, which is consistent with a sharp increase in economic volatility and risk premia during those times. The constant-volatility specification, on the other hand, generates negative discount rate news in the two recessions.

To test the pricing implications of our volatility-based DCAPM, we exploit vector-autoregression dynamics of state variables and estimate the model under the null. That is, we impose theoretical restrictions on the market prices of risks as well as the riskless rate. In estimation, we utilize both time-series moments and pricing restrictions for a cross section of book-to-market and size sorted portfolios.<sup>2</sup> We find that all equity portfolios have negative volatility betas, i.e., equity prices fall on positive news about volatility. Given that investors attach a negative price to volatility shocks, volatility risks carry positive premia. Our volatility-based DCAPM accounts for more than 95% of the cross-sectional variation in risk premia and is not rejected by the overidentifying restrictions. We find that volatility risk accounts for up to 2% of the overall risk premium of the market portfolio, and quantitatively matter more for growth rather than value portfolios.

We document a strong positive correlation between the risk premium and ex-ante market volatility, which reflects a positive correlation between discount rates and volatility. We find that in periods of recessions and those with significant economic stress, such as the Great Recession, both discount-rate news and volatility news are large and positive. Our evidence based on the expected return to aggregate wealth and its volatility and that of expected market return and its volatility are consistent in that volatility and discount rates in both cases are strongly positively correlated.

The rest of the paper is organized as follows. In Section 2 we present a theoretical framework for the analysis of volatility risks. We set up the long-run risks model to gain further understanding of how volatility affects quantitative inference about consumption dynamics and the stochastic discount factor. In Section 3 we develop an empirical framework to quantify the role of the volatility channel in the data and discuss the model implications for the market, human capital and wealth portfolio. Section 4 discusses the implications of the volatility-based DCAPM and the role

---

<sup>2</sup>To measure economic news, we use a standard list of predictive variables (comprising the realized market variance, price-dividend ratio, dividend growth, term and default spreads, and a long-term interest rate), and unlike Campbell, Giglio, Polk, and Turley (2011), we do not use any portfolio-specific characteristics such as the small-stock value spread.

of volatility risks for explaining a broader cross-section of assets. We confirm the robustness of our results in Section 6. Conclusion follows.

## 2 Theoretical Framework

In this section we consider a general economic framework with recursive utility and time-varying economic uncertainty and derive the implications for the innovations into the current and future consumption growth, returns, and the stochastic discount factor. We show that accounting for the fluctuations in economic uncertainty is important for a correct inference about economic news, and ignoring volatility risks can alter the implications for the financial markets.

### 2.1 Consumption and Volatility

We adopt a discrete-time specification of the endowment economy where the agent's preferences are described by a Kreps and Porteus (1978) recursive utility function of Epstein and Zin (1989) and Weil (1989). The life-time utility of the agent  $U_t$  satisfies

$$U_t = \left[ (1 - \delta)C_t^{1-\frac{1}{\psi}} + \delta (E_t U_{t+1}^{1-\gamma})^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right]^{\frac{1}{1-\frac{1}{\psi}}}, \quad (2.1)$$

where  $C_t$  is the aggregate consumption level,  $\delta$  is a subjective discount factor,  $\gamma$  is a risk aversion coefficient,  $\psi$  is the intertemporal elasticity of substitution (IES), and for notational ease we denote  $\theta = (1 - \gamma)/(1 - \frac{1}{\psi})$ . When  $\gamma = 1/\psi$ , the preferences collapse to a standard expected power utility.

As shown in Epstein and Zin (1989), the stochastic discount factor  $M_{t+1}$  can be written in terms of the log consumption growth rate,  $\Delta c_{t+1} \equiv \log C_{t+1} - \log C_t$ , and the log return to the consumption asset (wealth portfolio),  $r_{c,t+1}$ . In logs,

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} \Delta c_{t+1} + (\theta - 1)r_{c,t+1}. \quad (2.2)$$

A standard Euler condition

$$E_t [M_{t+1} R_{t+1}] = 1 \quad (2.3)$$

allows us to price any asset in the economy. Assuming that the stochastic discount factor and the consumption asset return are jointly log-normal, the Euler equation for the consumption asset leads to:

$$E_t \Delta c_{t+1} = \psi \log \delta + \psi E_t r_{c,t+1} - \frac{\psi - 1}{\gamma - 1} V_t, \quad (2.4)$$

where we define  $V_t$  to be the conditional variance of the stochastic discount factor plus the consumption asset return:

$$\begin{aligned} V_t &= \frac{1}{2} \text{Var}_t(m_{t+1} + r_{c,t+1}) \\ &= \frac{1}{2} \text{Var}_t m_{t+1} + \text{Cov}_t(m_{t+1}, r_{c,t+1}) + \frac{1}{2} \text{Var}_t r_{c,t+1}. \end{aligned} \quad (2.5)$$

The volatility component  $V_t$  is equal to the sum of the conditional variances of the discount factor and the consumption return and the conditional covariance between the two, which are directly related to the movements in aggregate volatility and the risk premia in the economy. In this sense, we interpret  $V_t$  as a measure of the aggregate economic volatility. In our subsequent discussion we show that, under further model restrictions, the economic volatility  $V_t$  is proportional to the conditional variance of the future aggregate consumption, and the proportionality coefficient is always positive and depends only on the risk aversion coefficient. As can be seen from equation (2.4) economic volatility shocks do not impact expected consumption when there is no stochastic volatility in the economy (so  $V_t$  is a constant), or when the IES parameter is one,  $\psi = 1$ . These cases have been entertained in Campbell (1983), Campbell (1996), Campbell and Vuolteenaho (2004), and Lustig and Van Nieuwerburgh (2008). In the paper we argue for economic importance of the variation in aggregate uncertainty and  $IES > 1$  to interpret movements in consumption and in asset markets.

We use the equilibrium restriction in the Equation (2.4) to derive the immediate consumption news. The return to the consumption asset  $r_{c,t+1}$  which enters the equilibrium condition in Equation (2.4) satisfies the usual budget constraint:

$$W_{t+1} = (W_t - C_t)R_{c,t+1}. \quad (2.6)$$

A standard log-linearization of the budget constraint yields:

$$r_{c,t+1} = \kappa_0 + w_{c,t+1} - \frac{1}{\kappa_1} w_{c,t} + \Delta c_{t+1}, \quad (2.7)$$

where  $w_{c,t} \equiv \log(W_t/C_t)$  is the log wealth-to-consumption ratio (inverse of the savings ratio), and  $\kappa_0$  and  $\kappa_1$  are the linearization parameters. Solving the recursive equation forward, we obtain that the immediate consumption innovation can be written as the revision in expectation of future returns on consumption asset minus the revision in expectation of future cash flows:

$$c_{t+1} - E_t c_{t+1} = (E_{t+1} - E_t) \sum_{j=0}^{\infty} \kappa_1^j r_{c,t+1+j} - (E_{t+1} - E_t) \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1}. \quad (2.8)$$

Using the expected consumption relation in (2.4), we can further express the consumption shock in terms of the immediate news in consumption return,  $N_{R,t+1}$ , revisions

of expectation of future returns (discount rate news),  $N_{DR,t+1}$ , as well as the news about future volatility  $N_{V,t+1}$  :

$$N_{C,t+1} = N_{R,t+1} + (1 - \psi)N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1}N_{V,t+1}, \quad (2.9)$$

where for convenience we denote

$$\begin{aligned} N_{C,t+1} &\equiv c_{t+1} - E_t c_{t+1} & N_{R,t+1} &\equiv r_{c,t+1} - E_t r_{c,t+1}, \\ N_{DR,t+1} &\equiv (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j r_{c,t+j+1} \right), & N_{V,t+1} &\equiv (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j V_{t+j} \right), \\ N_{CF,t+1} &\equiv (E_{t+1} - E_t) \left( \sum_{j=0}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right) = N_{DR,t+1} + N_{R,t+1} \end{aligned} \quad (2.10)$$

To highlight the intuition for the relationship between consumption, asset prices and volatility, let us define the news in future expected consumption  $N_{ECF,t+1}$  :

$$N_{ECF,t+1} = (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right). \quad (2.11)$$

Note that the consumption innovation equation in (2.4) implies that the news in future expected consumption is driven by the discount rate news to the wealth portfolio and the news in economic volatility:

$$N_{ECF,t+1} = \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}. \quad (2.12)$$

In a similar way, we can decompose the shock in wealth-to-consumption ratio into the expected consumption and volatility news:

$$\begin{aligned} (E_{t+1} - E_t)w_{c,t+1} &= N_{ECF,t+1} - N_{DR,t+1} \\ &= \left( 1 - \frac{1}{\psi} \right) \left( N_{ECF,t+1} - \frac{1}{\gamma - 1} N_{V,t+1} \right). \end{aligned} \quad (2.13)$$

When the IES is equal to one, the substitution effect is equal to the income effect, so the future expected consumption moves one-to-one with the discount rate news. As the two news exactly offset each other, the wealth-to-consumption ratio is constant so that the agent consumes a constant fraction of total wealth. On the other hand, when the IES is not equal to one, the movements in expected consumption no longer correspond to the movements in discount rates when aggregate volatility is time-varying. Indeed, fluctuations in economic volatility lead to the time variation



in the risk premia which directly affects the discount rates.<sup>3</sup> In Sections 3 and 4 we empirically document that "bad" economic times are associated with low future expected growth, high risk premia and high uncertainty, so that the volatility news co-move significantly positively with the discount rate news and negatively with the cash-flow news. This evidence is consistent with the economic restriction in (2.12) when volatility risks are accounted for: when the IES is above one, volatility shocks directly lower future expected consumption and can offset a simultaneous increase in discount rates. However, ignoring the volatility news, the structural equation (2.12) would imply that news to future consumption and discount rates news are perfectly positively correlated, so that the bad times of high volatility and high discount rates would correspond to the good times of positive news to future consumption. This stands in a stark contrast to the empirical observations and economic intuition, and highlights the importance of volatility risks to correctly interpret the movements in consumption and asset prices.

## 2.2 Asset Prices and Volatility

The innovation into the stochastic discount factor implied by the representation in Equation (2.2) is given by,

$$m_{t+1} - E_t m_{t+1} = -\frac{\theta}{\psi}(\Delta c_{t+1} - E_t \Delta c_{t+1}) + (\theta - 1)(r_{c,t+1} - E_t r_{c,t+1}). \quad (2.14)$$

Substituting the consumption shock in Equation (2.9), we obtain that the stochastic discount factor is driven by future cash flow news,  $N_{CF,t+1}$ , future discount rate news,  $N_{DR,t+1}$ , and volatility news,  $N_{V,t+1}$  :

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1}. \quad (2.15)$$

As shown in the above equation, the market price of the cash-flow risk is  $\gamma$ , and the market prices of volatility and discount rate news are equal to negative 1. Notably, the volatility risks are present at any values of the IES. Thus, even though with IES equal to one ignoring volatility does not lead to the mis-specification of the consumption shock, the inference on the stochastic discount factor is still incorrect and can significantly affect the interpretation of the asset markets.

---

<sup>3</sup>Time-varying risk aversion would also induce time-varying risk premia. However, the volatility dynamics we use is directly estimated from the observable macro quantities. In contrast, the process for variation in risk aversion is more difficult to directly measure in the data.

Given this decomposition for the stochastic discount factor, we can rewrite the expression for the ex-ante economic volatility  $V_t$  in (2.5) in the following way:

$$\begin{aligned}
V_t &= \frac{1}{2} \text{Var}_t(m_{t+1} + r_{c,t+1}) \\
&= \frac{1}{2} \text{Var}_t(-\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1} + N_{R,t+1}) \\
&= \frac{1}{2} \text{Var}_t((1 - \gamma)N_{CF,t+1} + N_{V,t+1}),
\end{aligned} \tag{2.16}$$

where in the last equation we used the identity that the sum of the immediate and future discount rate news on the wealth portfolio is equal to the current and future consumption news. Consider the case when the variance of volatility news  $N_{V,t+1}$  and its covariance with cash-flow news are constant (i.e., volatility shocks are homoscedastic). In this case,  $V_t$  is driven by the variance of current and future consumption news, where the proportionality coefficient is determined only by the coefficient of risk-aversion:

$$V_t = \text{const} + \frac{1}{2}(1 - \gamma)^2 \text{Var}_t(N_{CF,t+1}). \tag{2.17}$$

Hence, the news in  $V_t$  correspond to the news in the future variance of long-run consumption shocks; in this sense,  $V_t$  is the measure of the ex-ante economic volatility. Further, note that when there is a single consumption volatility factor, we can identify  $V_t$  from the rescaled volatility of immediate consumption news,  $V_t = \text{const} + \frac{1}{2}(1 - \gamma)^2 \chi \text{Var}_t(\Delta c_{t+1})$ , where  $\chi$  is the scaling factor which is equal to ratio of the variance of long-run consumption growth to the variance of current consumption growth,

$$\chi = \text{Var}(N_{CF}) / \text{Var}(N_C). \tag{2.18}$$

We impose this structural restriction to identify economic volatility shocks in our empirical work.

Using Euler equation, we obtain that the risk premium on any asset is equal to the negative covariance of asset return  $r_{i,t+1}$  with the stochastic discount factor:

$$E_t r_{i,t+1} - r_{ft} + \frac{1}{2} \text{Var}_t r_{i,t+1} = \text{Cov}_t(-m_{t+1}, r_{i,t+1}). \tag{2.19}$$

Hence, knowing the exposures (betas) of a return to the fundamental sources of risk, we can calculate the risk premium on the asset, and decompose it into the risk compensations for the future cash-flow, discount rate, and volatility news:

$$\begin{aligned}
&E_t r_{i,t+1} - r_{ft} + \frac{1}{2} \text{Var}_t r_{i,t+1} \\
&= \gamma \text{Cov}_t(r_{i,t+1}, N_{CF,t+1}) - \text{Cov}_t(r_{i,t+1}, N_{DR,t+1}) - \text{Cov}_t(r_{i,t+1}, N_{V,t+1}).
\end{aligned} \tag{2.20}$$

Consider a case when the volatility is constant and all the economic shocks are homoscedastic. First, it immediately implies that the revision in expected future volatility news is zero,  $N_{V,t+1} = 0$ . Further, when all the economic shocks are homoscedastic, all the variances and covariances are constant, which implies that the risk premium on the consumption asset is constant as well. Thus, the discount rate shocks just capture the innovations into the future expected risk-free rates. Hence, under homoscedasticity, the economic sources of risks include the revisions in future expected cash flow, and the revisions in future expected risk-free rates:

$$m_{t+1}^{NoVol} - E_t m_{t+1}^{NoVol} = -\gamma N_{CF,t+1} + N_{RF,t+1}, \quad (2.21)$$

for  $N_{RF,t+1} = (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j r_{f,t+j} \right)$ .

When volatility is constant, the risk premia are constant and determined by the unconditional covariances of asset returns with future risk-free rate news and future cash-flow news. Further, the beta of returns with respect to discount rate shocks,  $N_{DR,t+1}$ , should just be equal to the return beta to the future expected risk-free shocks,  $N_{RF,t+1}$ . In several empirical studies in the literature (see e.g., Campbell and Vuolteenaho (2004)), the risk-free rates are assumed to be constant. Following the above analysis, it implies, then, that the news about future discount rates is exactly zero, and so is the discount-rate beta, and all the risk premium in the economy is captured just by risks in future cash-flows. Thus, ignoring volatility risks can significantly alter the interpretation of the risk and return in financial markets.

### 2.3 Mis-Specification of Consumption and SDF

To gain further understanding of how volatility affects quantitative inference about consumption, the stochastic discount factor, asset prices and risk premia, we utilize a standard long-run risks model of Bansal and Yaron (2004). This model captures many salient features of the macroeconomic and asset market data and importantly ascribes a prominent role for the volatility risk.<sup>4</sup>

In a standard long-run risks model, consumption dynamics satisfies

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \quad (2.22)$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t \epsilon_{t+1}, \quad (2.23)$$

$$\sigma_{t+1}^2 = \sigma_c^2 + \nu(\sigma_t^2 - \sigma_c^2) + \sigma_w w_{t+1}, \quad (2.24)$$

---

<sup>4</sup>See Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2007a) for a discussion of the long-run risks channels for the asset markets and specifically the role of volatility risks, Bansal *et al.* (2005b) for early extensive empirical evidence on the role of volatility risks, and Eraker and Shaliastovich (2008), Bansal and Shaliastovich (2010), and Drechsler and Yaron (2011), for the importance of volatility risks for derivative markets.

where  $x_t$  drives the persistent variation in expected consumption and  $\sigma_t^2$  is the conditional variance of the consumption shocks. Innovation  $\eta_t$  is a short-run consumption shock,  $\epsilon_t$  is the shock to the expected consumption growth, and  $w_t$  is the volatility shock; for parsimony, these three shocks are assumed to be *i.i.d* Normal.

As shown in Bansal and Yaron (2004), the innovation in the equilibrium stochastic discount factor satisfies,

$$(E_{t+1} - E_t)m_{t+1} = -\lambda_c\sigma_t\eta_{t+1} - \lambda_x\varphi_e\sigma_t\epsilon_{t+1} - \lambda_\sigma\sigma_w w_{t+1}, \quad (2.25)$$

where  $\lambda_c, \lambda_x$  and  $\lambda_\sigma$  denote the market prices of short-run, long-run and volatility risk, respectively; their expressions are provided in the Appendix. In particular, with preference for early resolution of uncertainty ( $\psi > 1/\gamma$ ), the market price of volatility risks is negative:  $\lambda_\sigma < 0$ , so that high volatility represents bad states for investors in which their marginal utility goes up. Further, when IES parameter is above one, the equilibrium equity valuations fall at times of high volatility, consistent with the empirical evidence documented in Bansal *et al.* (2005b). Negative asset beta coupled with negative market price of volatility risks leads to a positive risk premium for volatility shocks in the model. Further, note that the risk premium in the economy is time-varying and driven by the conditional volatility of consumption  $\sigma_t^2$ . When the conditional volatility of economic shocks is constant, the case considered in Campbell (1996), the risk premia on all the assets should be constant over time.

We use a standard calibration of the model to evaluate the extent of the mis-measurement of the innovations into consumption and stochastic discount factor if one ignores the presence of volatility. The parameter configuration used in the model simulation is similar to Bansal, Kiku, and Yaron (2009b) and is presented in Table A.1 in the Appendix. The model is calibrated to match a wide range of asset-market and consumption moments in the data and thus provides a realistic laboratory for our analysis. We document the key moments of the consumption and asset-market data in Table 1, and provide the model-implied output in the Appendix Table A.2. Notably, the model produces a significant positive correlation between the discount rate news and the volatility news: it is 60% for the consumption asset, and 90% for the market. Further, for both consumption and market return, most of the risk compensation comes from the cash-flow and volatility news, while the contribution of the discount rate news is quite small.

Table 2 reports the implied consumption innovations when volatility is ignored, that is when the term  $N_V$  is not accounted for in constructing the consumption innovations. In constructing the implied consumption innovations via equation (2.9) we use the equilibrium solutions for  $N_{R,t+1}$ ,  $N_{DR,t+1}$ , and  $N_{V,t+1}$  implied by the long-run risks model. In particular, we assume that the consumption return news  $N_{R,t+1}$  and  $N_{DR,t+1}$  can be identified correctly in the simulated data even if the econometrician ignores volatility component in the analysis.

The top panel of Table 2 shows that when IES is not equal to one, the implied consumption innovations are distorted. In particular, when IES is equal to two, the volatility of consumption innovations is about twice that of the true consumption innovations, and the correlation between the true consumption shock and the implied consumption shock is only 50%. Similar distortions are present when the IES is less than one. In the bottom panel of Table 2 we report the implications of ignoring volatility for the stochastic discount factor. When volatility is ignored, for all values of the IES the SDF's volatility is downward biased by about one-third. The market risk premium is almost half that of the true one, and the correlations of the SDF with the return, discount rate, and cash-flow news are distorted. Finally, it is important to note that even when the IES is equal to one, the SDF is still misspecified. In all, the evidence clearly demonstrates the potential pitfalls that might arise in interpreting asset pricing models and the asset markets sources of risks if the volatility channel is ignored.

The analysis above assumed the researcher has access to the return on wealth,  $r_{c,t+1}$ . In many instances, however, that is not the case (e.g., Campbell and Vuolteenaho (2004), Campbell (1996)) and the return on the market  $r_{d,t+1}$  is utilized instead. The fact the market return is a levered asset relative to the consumption/wealth return exacerbate the inference problems shown earlier. In particular, Table A.3 in the Appendix shows that when the IES is equal to two, the volatility of the implied consumption shocks is about 14.3%, relative to the true volatility of only 2.5%. Campbell (1996) (Table 9) reports the implied consumption innovations based on equation (2.9) when volatility is ignored and the return and discount rate shocks are read off a VAR using observed financial data. The volatility of the consumption innovations when the IES is assumed to be 2 is about 22%, not far from the quantity displayed in our simulated model in Table A.3.<sup>5</sup> As in our case, lower IES values lead to somewhat smoother implied consumption innovations. While Campbell (1996) concludes that this evidence is more consistent with a low IES, the analysis here suggests that in fact this evidence is consistent with an environment in which the IES is greater than one and the innovation structure contains a volatility component.

### 3 Volatility, Aggregate Wealth and Consumption

In this section we develop and implement a volatility-based permanent income hypothesis framework to quantify the role of the volatility channel for the asset markets. As the aggregate consumption return (i.e., aggregate wealth return) is not directly observed in the data we assume that it is a weighted combination of the return to the stock market and human capital. This allows us to adopt a standard VAR-based

---

<sup>5</sup>The data used in Campbell (1996) is from 1890-1990 which leads to slightly higher volatility numbers than the calibrated model produces.

methodology to extract the innovations to consumption return, volatility, SDF, and assess the importance of the volatility channel for returns to human capital and equity.

### 3.1 Econometric Specification

Denote  $X_t$  a vector of state variables which include annual real consumption growth  $\Delta c_t$ , real labor income growth  $\Delta y_t$ , real market return  $r_{d,t}$ , market price-dividend ratio  $pd_t$ , and the realized variance measure  $RV_t$  :

$$X_t = [\Delta c_t \quad \Delta y_t \quad r_{d,t} \quad pd_t \quad RV_t]'. \quad (3.1)$$

For parsimony, we focus on a minimal set of economic variables in our benchmark empirical analysis, and in Section 6 we confirm that our main results are robust to the choice of predictive variables, volatility measurements and estimation strategy.

The vector of state variables  $X_t$  follows a VAR(1) specification, which we refer to as Macro VAR:

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \quad (3.2)$$

where  $\Phi$  is a persistence matrix and  $\mu_X$  is an intercept. Shocks  $u_{t+1}$  are assumed to be conditionally Normal with a time-varying variance-covariance matrix  $\Omega_t$ .

To identify the fluctuations in the aggregate economic volatility, we include as one of the state variables a realized variance measure based on the sum of squares of monthly industrial production growth over the year:

$$RV_{t+1} = \sum_{j=1}^{12} \Delta ip_{t+j/12}^2. \quad (3.3)$$

Constructing the realized variance from the monthly data helps us capture more accurately the fluctuations in the aggregate macroeconomic volatility in the data, and we use industrial production because high-frequency real consumption data is not available for a long sample. For robustness, we checked that our results do not materially change if we instead construct the measure based on the realized variance of annual consumption growth. To ensure consistency, we re-scale industrial production based realized variance to match an average level of consumption variance.

The expectations of  $RV_{t+1}$  implied by the dynamics of the state vector capture the ex-ante macroeconomic volatility in the economy; this way of extracting conditional aggregate volatility is similar to Bansal *et al.* (2005b), Bansal, Kiku, and Yaron (2007b), among others. Following the derivations in Section 2, the economic volatility

$V_t$  then becomes be proportional to the ex-ante expectation of the realized variance  $RV_{t+1}$  based on the VAR(1):

$$\begin{aligned} V_t &= V_0 + \frac{1}{2}\chi(1-\gamma)^2 E_t RV_{t+1} \\ &= V_0 + \frac{1}{2}\chi(1-\gamma)^2 i'_v \Phi X_t, \end{aligned} \tag{3.4}$$

where  $V_0$  is an unimportant constant which disappears in the expressions for shocks,  $i_v$  is a column vector which picks out the realized variance measure from  $X_t$ , and  $\chi$  is a parameter which captures the link between the observed aggregate consumption volatility and  $V_t$ . In the model with volatility risks, we fix the value of  $\chi$  to the ratio of the variances of the cash-flow to immediate consumption news, consistent with the theoretical restriction in Section 2. In the specification where volatility risks are absent, the parameter  $\chi$  is set to zero.

Following the above derivations, the revisions in future expectations of the economic volatility can be calculated in the following way:

$$N_{V,t+1} = \frac{1}{2}\chi(1-\gamma)^2 i'_v (I + Q) u_{t+1}, \tag{3.5}$$

where  $Q$  is the matrix of the long-run responses,  $Q = \kappa_1 \Phi (I - \kappa_1 \Phi)^{-1}$ .

The VAR specification implies that shocks into immediate market return,  $N_{R,t+1}^d$ , and future market discount rate news,  $N_{DR,t+1}^d$ , are given by<sup>6</sup>

$$N_{R,t+1}^d = i'_r u_{t+1}, \quad N_{DR,t+1}^d = i'_r Q u_{t+1}, \tag{3.6}$$

where  $i_r$  is a column vector which picks out market return component from the set of state variables  $X_t$ .

While the market return is directly observed and the market return news can be extracted directly from the VAR(1), in the data we can only observe the labor income but not the total return to human capital. We make the following identifying assumption, identical to Lustig and Van Nieuwerburgh (2008), that expected labor income return is linear in the state variables:

$$E_t r_{y,t+1} = \alpha + b' X_t, \tag{3.7}$$

where  $b$  captures the loadings of expected human capital return to the economic state variables. Given this restriction, the news into future discounted human capital returns,  $N_{DR,t+1}^y$ , is given by,

$$N_{DR,t+1}^y = b' \Phi^{-1} Q u_{t+1}, \tag{3.8}$$

---

<sup>6</sup>In what follows, we use superscript "d" to denote shocks to the market return, and superscript "y" to identify shocks to the human capital return. Shocks without the superscript refer to the consumption asset, consistent with the notations in Section 2.

and the immediate shock to labor income return,  $N_{R,t+1}^y$ , can be computed as follows:

$$\begin{aligned} N_{R,t+1}^y &= (E_{t+1} - E_t) \left( \sum_{j=0}^{\infty} \kappa_1^j \Delta y_{t+j+1} \right) - N_{DR,t+1}^y \\ &= i'_y (I + Q) u_{t+1} - b' \Phi^{-1} Q u_{t+1}, \end{aligned} \quad (3.9)$$

where the column vector  $i_y$  picks out labor income growth from the state vector  $X_t$ .

To construct the aggregate consumption return (i.e., aggregate wealth return), we follow Jagannathan and Wang (1996), Campbell (1996), Lettau and Ludvigson (2001) and Lustig and Van Nieuwerburgh (2008) and assume that it is a portfolio of the returns to the stock market and returns to human capital:

$$r_{c,t} = (1 - \omega)r_{d,t} + \omega r_{y,t}. \quad (3.10)$$

The share of human wealth in total wealth  $\omega$  is assumed to be constant. It immediately follows that the immediate and future discount rate news on the consumption asset are equal to the weighted average of the corresponding news to the human capital and market return, with a weight parameter  $\omega$ :

$$\begin{aligned} N_{R,t+1} &= (1 - \omega)N_{R,t+1}^d + \omega N_{R,t+1}^y, \\ N_{DR,t+1} &= (1 - \omega)N_{DR,t+1}^d + \omega N_{DR,t+1}^y. \end{aligned} \quad (3.11)$$

These consumption return innovations can be expressed in terms of the VAR(1) parameters and shocks and the vector of the expected labor return loadings  $b$  following Equations (3.6)-(3.9).

Finally, we can combine the expressions for the volatility news, immediate and discount rate news on the consumption asset to back out the implied immediate consumption shock following the Equation (2.9):

$$\begin{aligned} c_{t+1} - E_t c_{t+1} &= N_{R,t+1} + (1 - \psi)N_{DR,t+1} + \frac{\psi - 1}{\gamma - 1} N_{V,t+1} \\ &= \underbrace{[(1 - \omega)i'_r Q + \omega(i'_y(I + Q) - b' \Phi^{-1} Q)]}_{N_{R,t+1}} u_{t+1} \\ &\quad + (1 - \psi) \underbrace{[(1 - \omega)i'_r Q + \omega b' \Phi^{-1} Q]}_{N_{DR,t+1}} u_{t+1} + \left( \frac{\psi - 1}{\gamma - 1} \right) \underbrace{\frac{1}{2} \chi (1 - \gamma)^2 i'_v Q}_{N_{V,t+1}} u_{t+1} \\ &\equiv q(b)' u_{t+1}. \end{aligned} \quad (3.12)$$

The vector  $q(b)$  defined above depends on the model parameters, and in particular, it depends linearly on the expected labor return loadings  $b$ . On the other hand, as



consumption growth itself is one of the state variables in  $X_t$ , it follows that the consumption innovation satisfies,

$$c_{t+1} - E_t c_{t+1} = i'_c u_{t+1}, \quad (3.13)$$

where  $i_c$  is a column vector which picks out consumption growth out of the state vector  $X_t$ . We impose this important consistency requirement that the model-implied consumption shock in Equation (3.12) matches the VAR consumption shock in (3.13), so that

$$q(b) \equiv i_c, \quad (3.14)$$

and solve the above equation, which is linear in  $b$ , to back out the unique expected human capital return loadings  $b$ . That is, in our approach the specification for the expected labor return ensures that the consumption innovation implied by the model is identical to the consumption innovation in the data.

## 3.2 Data and Estimation

In our empirical analysis, we use an annual sample from 1930 to 2010. Real consumption corresponds to real per capita expenditures on non-durable goods and services, and real income is the real per capita disposable personal income; both series are taken from the Bureau of Economic Analysis. Market return data is for a broad portfolio from CRSP. The summary statistics for these variables are presented in Table 1. The average labor income and consumption growth rates are about 2%. The labor income is more volatile than consumption growth, but the two series co-move quite closely in the data with the correlation coefficient of 0.80. The average log market return is 5.7%, and its volatility is almost 20%. The realized consumption variance is quite volatile in the data, and spikes up considerably in the recessions. Notably, the realized variance is negatively correlated with the price-dividend ratio: the correlation coefficient is about -0.25, so that asset prices fall at times of high macroeconomic volatility, consistent with findings in Bansal *et al.* (2005b).

We estimate the Macro VAR specification in (3.2) using equation-by-equation OLS. For robustness, we also consider an MLE approach where we incorporate the information in the conditional variance of the residuals; the results are very similar, and are discussed in the robustness section. To derive the implications for the market, human capital, and wealth portfolio returns, we set the risk aversion coefficient  $\gamma$  to 5 and the IES parameter  $\psi$  to 2. The share of human wealth in the overall wealth  $\omega$  is set to 0.8, the average value used in Lustig and Van Nieuwerburgh (2008). In the full model specification featuring volatility risks we fix the volatility parameter  $\chi$  according to the restriction in Equation (2.18). To discuss the model implications in the absence of volatility risks, we set  $\chi$  equal to zero.

The Macro VAR estimation results are reported in Table 3. The magnitudes of  $R^2$  in the regressions vary from 10% for the market return to 80% for the price-dividend ratio. Notably, the consumption and labor income growth rates are quite predictable with this rich setting, with the  $R^2$  of 60% and nearly 40%, respectively. Because of the correlation between the variables, it is hard to interpret individual slope coefficients in the regression. Note, however, that the ex-ante consumption volatility is quite persistent with an autocorrelation coefficient of 0.63 on annual frequency, and it loads significantly and negatively on the market price-dividend ratio.<sup>7</sup>

We plot the ex-ante consumption volatility and the expected consumption growth rate on Figure 2. The evidence on persistent fluctuations in the ex-ante macroeconomic volatility and the gradual decline in the volatility over time is similar to the findings in Stock and Watson (2002) and McConnell and Perez-Quiros (2000). Notably, the volatility process is strongly counter-cyclical: its correlation with the NBER recession indicator is -40%, and it is -30% with the expected real consumption growth. Consistent with this evidence, the news in future expected consumption implied by the Macro VAR,  $N_{ECF}$ , is sharply negative at times of high volatility. Indeed, as shown in Table 4, future expected consumption news are on average -1.70% at times of high (top 25%) versus 2.23% in low (bottom 25%) volatility times. Further, in Figure 2 we plot a Macro VAR impulse response of consumption growth to one standard deviation shock in ex-ante consumption volatility,  $Var_t \Delta c_{t+1}$ ; see Appendix for the details of the computations. One standard deviation volatility shock corresponds to an increase in ex-ante consumption variance by  $(1.95\%)^2$ . As shown in the Figure, consumption growth significantly declines by almost 1% on the impact of volatility news and remains negative up to five years in the future. The response of the labor income growth is similar.

In the full model with volatility risks, the volatility news is strongly correlated with the discount rate news in the data. As documented in Table 4, the correlation between the volatility news and the discount rate news on the market reaches nearly 90%, and the correlations of the volatility news with the discount rate news to labor return and the wealth portfolio are 30% and 80%, respectively. A high correlation between the volatility news and the discount rate news to the wealth return is evident on Figure 4. These findings are consistent with the intuition of the economic long-run risks model where a significant component of the discount rate news is driven by shocks to consumption volatility (see Section 2.3). On the other hand, when the volatility risks are absent, the discount rate news no longer reflect the fluctuations in the volatility, but rather mirror the revisions in future expectations of consumption. As a result, the correlation of the implied discount rate news with volatility news becomes -0.85 for the labor return, and -0.3 for the wealth portfolio. The implied discount rate news ignoring volatility risks is very different from the discount rate news when volatility risks are taken into account. For example, when volatility risks

---

<sup>7</sup>The process for realized volatility is obviously more volatile and less persistent.

are accounted for, the measured discount rate news is 5.14% in the latest recession of 2008 and 0.91% in 2001. Without the volatility channel, however, it would appear that the discount rate news is negative at those times: the measured discount rate shock is -2.86% in 2008 and -0.73% in 2001. Thus, ignoring the volatility channel, the discount rate on the wealth portfolio can be significantly mis-specified due to the omission of the volatility risk component, which would alter the dynamics of the aggregate returns as we discuss in subsequent section.

### 3.3 Labor, Market and Wealth Return Dynamics

Table 5 reports the evidence on correlations between immediate and future returns on the market, human capital and wealth portfolio. Without the volatility risk channel, shocks to the market and human capital returns are significantly negatively correlated, which is consistent with the evidence in Lustig and Van Nieuwerburgh (2008). Indeed, as shown in the top panel of the Table, the correlations between immediate stock market and labor income return news,  $N_{R,t+1}^d$  and  $N_{R,t+1}^y$ , the discount rate news,  $N_{DR,t+1}^d$  and  $N_{DR,t+1}^y$ , and the future long-term (5-year) expected returns,  $E_t r_{t \rightarrow t+5}^d$  and  $E_t r_{t \rightarrow t+5}^y$ , range between -0.50 and -0.70. All these correlations turn positive when the volatility channel is present: the correlation of immediate return news increases to 0.20; and for discount rates and the expected 5-year returns it goes up to 0.25 and 0.40, respectively. Figure 3 plots the implied time-series of long-term expected returns on the market and human capital. A negative correlation between the two series is evident in the model specification which ignores volatility risks. The evidence for the co-movements of returns is similar for the wealth and human capital, and the market and wealth portfolios, as shown in the middle and lower panels of Table 5. Because the wealth return is a weighted average of the market and human capital returns, these correlations are in fact positive without the volatility channel. These correlations increase considerably and become closer to one once the volatility risks are introduced. For example, the correlations between immediate and future expected news on the market and wealth returns rise to 80% with the volatility risk channel, while without volatility risks the correlation is 0.07 for the discount rates, 0.26 for the 5-year expected returns and 0.46 for the immediate return shocks.

To understand the role of the volatility risks for the properties of the wealth portfolio, consider again the consumption equation in (2.12), which for convenience we reproduce below:

$$\begin{aligned}
 (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j \Delta c_{t+j+1} \right) &= \psi N_{DR,t+1} - \frac{\psi - 1}{\gamma - 1} N_{V,t+1} \\
 &= \psi (\omega N_{DR,t+1}^y + (1 - \omega) N_{DR,t+1}^d) - \frac{\psi - 1}{\gamma - 1} N_{V,t+1}
 \end{aligned} \tag{3.15}$$

When the volatility risks are not accounted for,  $N_v = 0$  and all the variation in the future cash flows is driven by news to discount rates on the market and the human capital. However, as shown in Table 4, in the data consumption growth is much smoother than asset returns: the volatility of cash-flow news is about 5% relative to 14% for the discount rate news on the market. Hence, to explain relatively smooth variation in cash flows in the absence of volatility news, the discount rate news to human capital must offset a large portion of the discount rate news on the market, which manifests as a large negative correlation between the two returns documented in Table 5. On the other hand, when volatility news is accounted for, they remove the risk premia fluctuations from the discount rates and isolate the news in expected cash flows. Indeed, a strong positive correlation between volatility news and discount rate news in the data is evident in Table 4. This allows the model to explain the link between consumption and asset markets without forcing a negative correlation between labor and market returns.

We use the extracted news components to identify the innovation into the stochastic discount factor, according to Equation (2.15), and document the implications for the risk premia in Table 6. At our calibrated preference parameters, in the model with volatility risks the risk premium on the market is 7.2%; it is 2.6% for the wealth portfolio, and 1.4% for the labor return. Most of the risk premium comes from the cash-flow and volatility risks, and the volatility risks contribute about one-half to the overall risk premia. The discount rate shocks contribute virtually nothing to the risk premia. These findings are consistent with the economic long-run risks model (see Table A.2). Without the volatility channel, the risk premia are 2.3% for the market, 0.9% for the wealth return and 0.5% for the labor return.

While the main results in the paper are obtained with preference parameters  $\gamma = 5$  and  $\psi = 2$ , in Table 7 we document the model implications for a range of the IES parameter from 0.5 to 3.0. Without the volatility channel, the correlations between labor and market returns are negative and large at all considered values for the preference parameters, which is consistent with the evidence in Lustig and Van Nieuwerburgh (2008). In the model with volatility risks, one requires IES sufficiently above one to generate a positive link between labor and market returns – with IES below one, the volatility component no longer offsets risk premia variation in the consumption equation, which makes the labor-market return correlations even lower than in the case without volatility risks. The evidence is similar for other values of risk aversion parameter. Higher values for risk aversion lead to higher risk premium, that is why we chose moderate values of  $\gamma$  in our analysis.

## 4 Volatility-Based Dynamic CAPM

To further highlight the importance of volatility risks for understanding the dynamics of asset prices, we use a market-based VAR approach to news decomposition. As frequently done in the literature, here, we assume that the wealth portfolio corresponds to the aggregate stock market and, therefore, is observable. This assumption allows us to measure cash-flow, discount-rate and volatility news directly from the available stock market data. As shown in Section 3, in a more general setting that explicitly makes a distinction between aggregate and financial wealth and accounts for time-variation in volatility, realized and expected returns on wealth and stock market are highly correlated. This evidence suggests that we should be able to learn about time-series dynamics of fundamental risks and their prices from the observed equity data. Furthermore, to sharpen identification of underlying risks, we will extract them by exploiting both time-series and cross-sectional moment restrictions.

The theoretical framework here is same as the one in Section 2 with the return on the consumption asset equal to the return on the market portfolio. Hence, the equilibrium risk premium on any asset is determined by its exposure to the innovation in the market return and news about future discount rates and future volatility. The multi-beta implication of our model is similar to the multi-beta pricing of the intertemporal CAPM of Merton (1973) where the risk premium depends on the market beta and the asset exposure to state variables that capture changes in future investment opportunities. What distinguishes our volatility-based dynamic capital asset pricing model (DCAPM) from the Merton's framework is that, in our model, both relevant risk factors and their prices are identified and pinned down by the underlying model primitives and preferences. This is important from an empirical perspective as it provides us with testable implications that can be taken to the data. Note also that in our volatility-based DCAPM, derived from recursive preferences, the relevant economic risks comprise not only short-run fluctuations (as in the equilibrium C-CAPM of Breeden (1979)) but also risks that matter in the long run.

### 4.1 Market-Based Setup

As derived above, the stochastic discount rate of the economy is given by:

$$m_{t+1} - E_t m_{t+1} = -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1}. \quad (4.1)$$

In order to measure news components from equity data, we assume that the state of the economy is described by vector:

$$X_t \equiv (RV_{r,t}, z_t, \Delta d_t, ts_t, ds_t, i_t)',$$

where  $RV_{r,t}$  is the realized variance of the aggregate market portfolio;  $z_t$  is the log of the market price-dividend ratio;  $\Delta d_t$  is the continuously compounded dividend

growth of the aggregate market;  $ts_t$  is the term spread defined as a difference in yields on the 10-year Treasury bond and three-month T-bill;  $ds_t$  is the yield differential between Moody's BAA- and AAA-rated corporate bonds; and  $i_t$  is the log of the real long-term interest rate. The data are real, sampled on an annual frequency and span the period from 1930 till 2010. The realized variance is constructed by summing up squared monthly rates of market return within a year. The real long-term rate is measured by the yield on the 10-year Treasury bond adjusted by inflation expectations. Excess returns on the market and a cross section are constructed by subtracting the annualized rate on the three-month Treasury bill from annual, nominal equity returns. Our state vector comprises variables that are often used in the return-forecasting literature. We discuss the robustness of our evidence to the state specification below.

We model the dynamics of  $X_t$  via a first-order vector-autoregression:

$$X_{t+1} = \mu_X + \Phi X_t + u_{t+1}, \quad (4.2)$$

where  $\Phi$  is a  $(6 \times 6)$ -matrix of VAR coefficients,  $\mu_X$  is a  $(6 \times 1)$ -vector of intercepts, and  $u_{t+1}$  is a  $(6 \times 1)$ -vector of zero-mean, conditionally normal VAR innovations. Note that the dynamics of the log return on the aggregate market portfolio ( $r$ ) are implied by the dynamics of its price-dividend ratio and dividend growth:

$$r_{t+1} = \kappa_0 + \Delta d_{t+1} + \kappa_1 z_{t+1} - z_t, \quad (4.3)$$

where  $\kappa_0$  and  $\kappa_1$  are constants of log-linearization. To construct cash-flow, discount-rate and volatility news we iterate on the VAR, using the same algebra as in Section 3.1 with a simplification that all news components are now directly read from the VAR since the return on the market is assumed to represent the return on the overall wealth. For example, discount rate news is computed as:

$$N_{DR,t+1} = \left( i_{\Delta d} + \kappa_1 i_z \right)' Q u_{t+1}, \quad (4.4)$$

where  $i_z$  and  $i_{\Delta d}$  are  $(6 \times 1)$  indicator vectors with an entry of one in the second and third positions, respectively, and  $Q = \kappa_1 \Phi (I - \kappa_1 \Phi)^{-1}$ . Other news components are computed in a similar way.

We use the extracted news to construct the innovation in the stochastic discount factor and price a cross section of equity returns by exploiting the Euler equation, i.e.,

$$\begin{aligned} E_t [R_{i,t+1} - R_{ft}] &= -Cov_t(m_{t+1} - E_t m_{t+1}, r_{i,t+1} - E_t r_{i,t+1}) \\ &= \gamma Cov_t(N_{CF,t+1}, \epsilon_{i,t+1}) - Cov_t(N_{DR,t+1}, \epsilon_{i,t+1}) - Cov_t(N_{V,t+1}, \epsilon_{i,t+1}) \end{aligned} \quad (4.5)$$

where  $E_t [R_{i,t+1} - R_{ft}]$  is the arithmetic risk premium of asset  $i$ , and  $\epsilon_{i,t+1} \equiv r_{i,t+1} - E_t r_{i,t+1}$  is the innovation into asset- $i$  return. It is important to emphasize that we

carry out estimation under the null of the model. In particular, we restrict the premium of a zero-beta asset and impose the model's restrictions on the market prices of discount-rate and volatility risks. The price of cash-flow risks (i.e., risk aversion) is estimated along with time-series parameters of the model.

To extract return innovations for the cross section, we use an econometric approach similar to Bansal, Dittmar, and Lundblad (2005a) and Bansal, Dittmar, and Kiku (2009a) that allows for a sharper identification of long-run cash-flow risks in asset returns. In particular, for each equity portfolio, we estimate its long-run cash-flow exposure ( $\phi_i$ ) by regressing portfolio's dividend growth rate on the three-year moving average of the market dividend growth:

$$\Delta d_{i,t} = \mu_i + \phi_i \overline{\Delta d}_{t-2 \rightarrow t} + \epsilon_{i,t}^d, \quad (4.6)$$

where  $\Delta d_{i,t}$  is portfolio- $i$  dividend growth,  $\overline{\Delta d}_{t-2 \rightarrow t}$  is the average growth in market dividends from time  $t - 2$  to  $t$ , and  $\epsilon_{i,t}^d$  denotes idiosyncratic portfolio news. Using the log-linearization of return:

$$r_{i,t+1} = \kappa_{i,0} + \Delta d_{i,t+1} + \kappa_{i,1} z_{i,t+1} - z_{i,t}, \quad (4.7)$$

the innovation into asset- $i$  return is then given by:

$$\epsilon_{i,t+1} = \phi_i (\Delta d_{t+1} - E_t \Delta d_{t+1}) + \epsilon_{i,t+1}^d + \kappa_i \epsilon_{i,t+1}^z, \quad (4.8)$$

where  $z_{i,t}$  is the price-dividend ratio of portfolio  $i$ ,  $\kappa_{i,0}$  and  $\kappa_{i,1}$  are portfolio-specific constants of log-linearization,  $(\Delta d_{t+1} - E_t \Delta d_{t+1})$  is the VAR-based innovation in the market dividend growth rate, and  $\epsilon_{i,t+1}^z$  is the innovation in the portfolio price-dividend ratio obtained by regressing  $z_{i,t+1}$  on the VAR state variables. We use the extracted innovation in the portfolio return to construct the risk-premium restriction given in Equation (4.5).<sup>8</sup>

To extract news and construct the innovation in the stochastic discount factor, we estimate time-series parameters and the coefficient of risk aversion using GMM by exploiting two sets of moment restrictions. The first set of moments comprises the VAR orthogonality moments; the second set contains the Euler equation restrictions for the market portfolio and a cross-section of five book-to-market and five size sorted portfolios. To ensure that the moment conditions are scaled appropriately, we weight each moment by the inverse of its variance and allow the weights to be continuously up-dated throughout estimation. Further details of the GMM estimation are provided in Appendix C.

---

<sup>8</sup>Our empirical results remain similar if instead we rely on the cointegration-based specification of Bansal *et al.* (2009a).



## 4.2 Ex-Ante Volatility and Discount-Rate Dynamics

The GMM estimates of the market-based VAR dynamics are presented in Table 8. As shown in the first row of the table, the realized variance of the market return is highly predictable with an  $R^2$  of more than 60%. Time-variation in the one-year ahead expected variance is coming mostly from variation in realized variance, term and default spreads, and the risk-free rate, all of which are quite persistent in the data. The conditional variance, therefore, features persistent time-series dynamics with a first-order autocorrelation coefficient of about 0.69. These persistent dynamics are consistent with empirical evidence of low-frequency fluctuations in market volatility documented in the literature (see, for example, Bollerslev and Mikkelsen (1996) among others).

We find that the extracted discount-rate and volatility risks are strongly counter-cyclical and positively correlated. Both news tend to increase during recessions and decline during economic expansions. At the one-year horizon, the correlation between discount-rate and volatility news is 0.47. This evidence aligns well with economic intuition. As the contribution of risk-free rate news is generally small, discount-rate risks are mostly driven by news about future risk premia, and the latter is tied to expectations about future economic uncertainty. Consequently, discount rates and conditional volatility of the market portfolio share common time-series dynamics, especially at low frequencies. We illustrate their co-movement in Figure 5 by plotting the 5-year expected market return and the 5-year conditional variance implied by the estimated VAR. As the figure shows, both discount rates and the conditional variance feature counter-cyclical fluctuations, and almost mirror the dynamics of one another. The correlation between the two time series is 0.75. Motivated by the documented high correlation between discount rates and volatility, in Section 4.4 we consider a restricted version of our volatility-based DCAPM that constraints variations in the discount rate to variations in the conditional variance.

## 4.3 Pricing Implications of the Volatility-based DCAPM

The cross-sectional implications of the volatility-based DCAPM are given in Table 9. In this specification, no restrictions are imposed on the dynamics of discount rates. The table presents sample average excess returns of the market portfolio and the cross section, risk premia implied by the market-based model, and asset exposure to cash-flow, discount-rate and volatility risks. The bottom panel of the table shows the estimate of the market price of cash-flow risks. The evidence reported in the table yields several important insights. First, we find that cash-flow risks play a dominant role in explaining both the level and the cross-sectional variation in risk premia. At the aggregate market level, cash-flow risks account for 4.8% or, in relative terms, for about 60% of the total risk premium. Cash-flow betas are monotonically increasing



in book-to-market characteristics and monotonically declining with size. Value and small stocks in the data are more sensitive to persistent cash-flow risks than are growth and large firms, which is consistent with the evidence in Bansal *et al.* (2005a), Hansen, Heaton, and Li (2008) and Bansal *et al.* (2009a).

Second, we find that all assets have negative exposure to discount-rate and volatility risks. That is, in the data, prices of all equity portfolios tend to fall when discount rates or volatility are expected to be high. Because the prices of discount-rate and volatility risks, according to the model, are equal to negative one, both risks carry positive premia. The documented positive compensation for volatility risks is consistent with the evidence of a positive variance premium reported in Drechsler and Yaron (2011), and Bollerslev, Tauchen, and Zhou (2009). These papers show that the estimate of the variance risk premium defined as a difference between expected variances under the risk-neutral and physical measure is not only positive on average, it is almost always positive in time series. Our findings are also confirmed by the option-based evidence in Coval and Shumway (2001) who show that, in the data, average returns on zero-market-beta straddles are significantly negative. Recently, Campbell *et al.* (2011) also consider an ICAPM framework with time-varying volatility. They, however, report a negative compensation for volatility risks which conflicts with the discussed evidence on the variance risk premium and expected straddle returns. From an economic standpoint, high volatility states are states of low prices and low aggregate wealth. Hence, volatility risks should carry a positive risk premium.

The evidence in Table 9 also shows that discount-rate and volatility risks, each, account for about 20% of the overall market risk premium, and seem to affect the cross section of book-to-market sorted portfolios in a similar way. Both discount-rate and volatility risks matter more for the valuation of growth firms than that of value firms. This is consistent with economic intuition that growth firms, whose cash flows are shifted to the future, are more exposed to risk-premia and discount-rate variation.

Overall, our volatility-based DCAPM accounts for about 96% of the cross-sectional variation in risk premia, and implies a value premium of 6.1% and a size premium of about 6.8%. The cross-sectional fit of the model is illustrated in Figure 6(a). The estimate of the market price of cash-flow risks is statistically significant:  $\hat{\gamma} = 2.64$  (SE=0.41), and the model is not rejected by the overidentifying restrictions: the  $\chi^2$  test statistic is equal to 7.74 with a p-value of 0.65.

Our empirical evidence is fairly robust to economically reasonable changes in the VAR specification, sample period or frequency of the data. For example, omitting term and default spreads from the VAR yields a p-value of 0.25. The estimation of the model using post-1964 quarterly-sampled data results in a  $\chi^2$  statistic of 8.1 with a corresponding p-value of 0.33. Across these alternative specifications, the estimates of the market price of cash-flow risks continue to be significant, and the extracted discount-rate and volatility risks remain strongly positively correlated.

## 4.4 Asset Pricing Implications of the Restricted DCAPM

In addition to the unrestricted model discussed above, we consider a specification that ties the dynamics of the expected return of the market portfolio to the dynamics of its conditional variance. This restricted specification has the advantage of allowing for a better identification of the role of volatility risks and is motivated by a documented tight link between discount-rate and volatility news. The estimation details of this set-up are presented in Appendix D.

Table 10 presents the asset pricing implications of the restricted volatility-based DCAPM that incorporates the constraint on the dynamics of the risk premium. It reports the model-implied premia of the aggregate market and the cross section, and portfolio betas with respect to each risk source. Consistent with the evidence from the unrestricted model, cash-flow risks remain the key determinant of the level of the risk premium and its dispersion in the cross section. Still, volatility risks contribute significantly. At the aggregate level, about 2% of the premium is due to volatility risks. Thus, volatility risks account for about 25% of the overall market premium. At the cross-sectional level, the contribution of volatility risks is fairly uniform across size-sorted portfolios, but displays some tangible heterogeneity in the book-to-market sort. Value firms in the data seem to be quite immune to volatility risks, and therefore carry an almost zero volatility risk premium. Growth firms, on the other hand, are relatively sensitive to news about future economic uncertainty. The restricted specification is not rejected by the test of overidentifying restrictions and, as shown in Figure 6(b), accounts for a large portion of the cross-sectional variation in risk premia. Once again notice that all equity portfolios have negative exposure to volatility risks, and therefore, provide investors with positive volatility premia.

The variance decomposition of the stochastic discount factor reveals that 52% of the overall variation in the SDF is due to cash-flow risks and about 12% is due to volatility risks. While the direct contribution of volatility risks may seem modest, they account for another 32% of the variation in the SDF through their covariation with cash-flow news. Similar to the consumption-based evidence presented in Section 3, cash-flow news rises during expansions and falls in recessions, while news about future uncertainty exhibits strongly counter-cyclical dynamics. Volatility risks have a sizable effect on the dynamics of asset prices. A one-standard deviation increase in volatility news leads to a negative 11% fall in the return of the aggregate market portfolio.

To summarize, our empirical evidence in Section 4 highlights the importance of volatility risks for understanding the cross-sectional risk-return tradeoff. We show that revisions in expectations about future volatility contribute significantly to the overall variation in the stochastic discount factor. In the data, equity prices fall when volatility news and marginal utility are high. Therefore, volatility risks carry a sizable and positive risk premium.

## 5 Robustness

We conduct a number of robustness checks to ensure that our main results are not sensitive to volatility measurements, choice of the predictive variables and estimation strategy. In our benchmark Macro VAR specification, we measure the realized variance using squared monthly industrial production growth rates, scaled to match the overall consumption volatility. First, we check that our results remain broadly similar if we instead compute the realized variance based on the square of the annual consumption growth; in this case, the implied correlations between immediate news to the stock market and labor return is 0.20; it is 0.03 for the discount rate news and 0.26 for the 5-year expected returns. Second, we confirm that adding other predictive variables into the VAR(1) does not materially change our results, either. For example, if we include the data on interest rate, slope of the term structure and default spread into our benchmark Macro VAR specification, the implied correlations between human capital and market return are all in 0.2-0.4 range. Finally, we consider an alternative MLE estimation strategy where we allow the variance of the VAR(1) residuals of consumption and income growth to be time-varying and driven by the ex-ante volatility variable. The results are very similar to our benchmark Macro VAR: the correlations between human capital and market returns are positive and range between 5% for the immediate news to 45% for the 5-year expected returns.

In Section 4 we assume that the volatility of volatility shocks is constant. To confirm robustness of our DCAPM evidence, we estimate a more general set-up that allows for time-variation in the variance of volatility shocks. In particular, we assume that state vector  $X_t$  follows the first-order dynamics as in Equation (4.2), with  $u_{t+1} \sim N(0, \sigma_{r,t}\Omega)$ , and  $\sigma_{r,t} = Var_t(N_{R,t+1})$ . Here, we assume that all time-variation in conditional second moments of the VAR innovations (including the innovation to the variance component) is driven by a single state variable.<sup>9</sup> We maintain the assumption that the conditional risk premia are proportional to the conditional variance of the return of the market portfolio.<sup>10</sup> One can show that in this set-up, the dynamics of the aggregate volatility component  $V_t$  are given by:

$$V_t \approx V_0 + \xi \sigma_{r,t}^2, \quad (5.1)$$

where  $\xi$  is a non-linear function of the underlying preference and time-series parameters.

We find that the asset pricing implications of this generalized DCAPM set-up are comparable to the ones in our benchmark specification. Consistent with the evidence discussed above, cash-flow risks play a dominant role in explaining the cross-sectional risk-return tradeoff. The contribution of volatility risks remains significant and, in

---

<sup>9</sup>The estimation is carried out by imposing positivity restriction on  $\hat{\sigma}_{r,t}$ .

<sup>10</sup>Empirical evidence is robust if the assumption on the dynamics of risk premia is relaxed.

fact, is slightly larger relative to the case when volatility shocks are homoscedastic. On average, volatility risks account for about 25% of the overall risk premia in the cross-section, and about 40% of the total premium of the market portfolio. The contribution of volatility risks varies significantly across stocks sorted on book-to-market characteristic. Similar to the implications of our benchmark specification, growth stocks are more sensitive to volatility (and discount rate) variation than value stocks are.

## Conclusions

In this paper we show that volatility news is an important source of risk that affects the measurement and interpretation of underlying risks in the economy and financial markets. Our theoretical analysis yields a dynamics asset pricing model with three sources of risks: cash-flow, discount-rate and volatility risks, each carrying a separate risk premium. We show that ignoring volatility risks may lead to sizable mis-specifications of the dynamics of the stochastic discount factor and equilibrium consumption, and distorted inferences about risk and return. Calibrating an off-the-shelf, long-run risks model, we find that potential distortions caused by neglecting time-variation in economic volatility are indeed significant and manifest in large upward biases in the volatility of consumption news and large downward biases in the volatility of the implied stochastic discount factor and, consequently, risk premia.

Consistent with the existing empirical evidence, we document that both macroeconomic and return-based measures of volatility are highly persistent. Importantly, we also find that, in the data, a rise in volatility is typically accompanied by a significant decline in realized and expected consumption, a fall in equity prices and an increase in risk premia. That is, high volatility states are states of high risk reinforced by low economic growth and high discount rates. This evidence is consistent with the equilibrium relationship among volatility, consumption and asset prices implied by our model. A specification that ignores time-variation in volatility, in contrast to the data, would imply an upward revision in expected lifetime consumption following an increase in discount rates and would clearly fail to account for a strong positive correlation between volatility and discount-rate risks.

The empirical evidence we present highlights the importance of volatility risks for the joint dynamics of human capital and equity returns, and the cross-sectional risk-risk tradeoff. Our dynamic volatility-based asset pricing model is able to reverse the puzzling negative correlation between equity and human-capital returns documented previously in the literature in the context of a homoscedastic economy. By incorporating empirically robust positive relationship between ex-ante volatility and discount rates (a missing link in the homoscedastic case), our model implies a positive correlation between returns to human capital and equity while, simultaneously, matching

time-series dynamics of aggregate consumption. We also show that quantitatively, volatility risks help explain both the level and variation in risk premia across portfolios sorted on size and book-to-market characteristics. We find that during times of high volatility in financial markets (hence, high marginal utility), equity portfolios tend to realize low returns. Therefore, equity markets carry a positive premium for volatility risk exposure.

## A Long-Run Risks Model Setup

In a standard long-run risks model of Bansal and Yaron (2004), consumption dynamics satisfies

$$\Delta c_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}, \quad (\text{A.1})$$

$$\Delta d_{t+1} = \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{d,t+1}, \quad (\text{A.2})$$

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t \epsilon_{t+1}, \quad (\text{A.3})$$

$$\sigma_{t+1}^2 = \sigma_c^2 + \nu(\sigma_t^2 - \sigma_c^2) + \sigma_w w_{t+1}, \quad (\text{A.4})$$

where  $\rho$  governs the persistence of expected consumption growth  $x_t$ , and  $\nu$  determines the persistence of the conditional aggregate volatility  $\sigma_t^2$ .  $\eta_t$  is a short-run consumption shock,  $\epsilon_t$  is the shock to the expected consumption growth, and  $w_{t+1}$  is the shock to the conditional volatility of consumption growth; for parsimony, these three shocks are assumed to be *i.i.d* Normal.

The equilibrium solution to the price-consumption ratio,  $pc_t$ , is linear in the expected growth and consumption volatility:

$$pc_t = A_0 + A_x x_t + A_\sigma \sigma_t^2, \quad (\text{A.5})$$

where the equilibrium price-to-consumption ratio parameters satisfy

$$A_x = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho}, \quad A_\sigma = (1 - \gamma) \left(1 - \frac{1}{\psi}\right) \left[ \frac{1 + \left(\frac{\kappa_1 \varphi_x}{1 - \kappa_1 \rho}\right)^2}{2(1 - \kappa_1 \nu)} \right], \quad (\text{A.6})$$

and  $\kappa_1$  is the log-linearization parameter.

The innovation into the stochastic discount factor is determined by the short-run, expected consumption and volatility shocks:

$$m_{t+1} - E_t m_{t+1} = -\lambda_c \sigma_t \eta_{t+1} - \lambda_x \varphi_e \sigma_t \epsilon_{t+1} - \lambda_\sigma \sigma_w w_{t+1}, \quad (\text{A.7})$$

where the discount rate parameters and market prices of risks satisfy

$$\begin{aligned} m_x &= -\frac{1}{\psi}, \quad m_\sigma = (1 - \theta)(1 - \kappa_1 \nu) A_\sigma, \quad m_0 = \theta \log \delta - \gamma \mu - (\theta - 1) \log \kappa_1 - m_\sigma \sigma_c^2, \\ \lambda_c &= \gamma, \quad \lambda_x = (1 - \theta) \kappa_1 A_x, \quad \lambda_\sigma = (1 - \theta) \kappa_1 A_\sigma. \end{aligned} \quad (\text{A.8})$$

Given the equilibrium model solution, we can provide explicit expressions for the immediate consumption returns news,  $N_{R,t+1}$ , the discount-rate news,  $N_{DR,t+1}$ , and the volatility

news,  $N_{V,t+1}$ , in terms of the underlying shocks and model parameters. The consumption return shock,  $N_{R,t+1}$  is driven by all three shocks in the economy,

$$N_{R,t+1} = A_x \kappa_1 \varphi_x \sigma_t \epsilon_{t+1} + A_\sigma \kappa_1 \sigma_w w_{t+1} + \sigma_t \eta_{t+1}, \quad (\text{A.9})$$

while the discount rate news,  $N_{DR,t+1}$  is driven only by the expected growth and volatility innovations:

$$N_{DR,t+1} = \frac{1}{\psi} \frac{\kappa_1}{1 - \kappa_1 \rho} \varphi_e \sigma_t \epsilon_{t+1} - \kappa_1 A_\sigma \sigma_w w_{t+1}. \quad (\text{A.10})$$

The economic volatility component,  $V_t$ , is directly related to the conditional variance of consumption growth:

$$V_t = \frac{1}{2} \text{Var}_t(r_{c,t+1} + m_{t+1}) = \text{const} + \frac{1}{2} \chi (1 - \gamma)^2 \sigma_t^2, \quad (\text{A.11})$$

where the proportionality coefficient  $\chi$  satisfies,

$$\chi = \left( \frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 + 1. \quad (\text{A.12})$$

Consistent with our discussion in Section 2, as volatility shocks are homoscedastic and there is a single consumption volatility factor, the volatility parameter  $\chi$  is unambiguously positive and is equal to the ratio of variances of the long-run cash flows news,  $N_{CF,t+1}$ , to the immediate consumption news,  $N_{C,t+1}$ :

$$\chi = \frac{\text{Var}(N_{CF,t+1})}{\text{Var}(N_{C,t+1})}. \quad (\text{A.13})$$

The innovation into the future expected volatility  $N_{V,t+1}$  satisfies

$$N_{V,t+1} = \frac{1}{2} \chi (1 - \gamma)^2 \frac{\kappa_1}{1 - \kappa_1 \rho} \sigma_w w_{t+1}. \quad (\text{A.14})$$

Notice that all the three shocks,  $N_{R,t+1}$ ,  $N_{DR,t+1}$  and  $N_{V,t+1}$ , are correlated with each other as they depend on the underlying shocks in the economy. In particular, if IES is above one, the discount rate shocks and the volatility shocks are positively correlated, because the volatility is driving the risk premium which is an important component of discount rate innovations.

The expression for consumption innovations,  $N_{C,t+1}$ , and the stochastic discount factor can now be written in terms of the innovations to the consumption return, discount rate and volatility shocks, as shown in Equations (2.9) and (2.15). Under the null of the model, the consumption shock is equal to  $\sigma_t \eta_{t+1}$ , and the innovation into the stochastic discount factor matches the expression in Equation (A.7).

The price-dividend ratio satisfies

$$pd_t = H_0 + H_x x_t + H_\sigma \sigma_t^2, \quad (\text{A.15})$$

where

$$H_x = \frac{\phi - \frac{1}{\psi}}{1 - \kappa_{1d}\rho}, \quad H_\sigma = \frac{m_s + 0.5((\pi - \gamma)^2 + (\lambda_x - \kappa_{1d}H_x)^2\varphi_e^2 + \varphi_d^2)}{1 - \kappa_{1d}\nu}, \quad (\text{A.16})$$

for a log-linearization parameter  $\kappa_{1d}$

$$\log \kappa_{1d} = m_0 + \mu_d + H_\sigma \sigma_0^2 (1 - \kappa_{1d}\nu) + 0.5(\lambda_\sigma - \kappa_{1d}H_\sigma)^2 \sigma_w^2. \quad (\text{A.17})$$



Table A.1: Configuration of Long-Run Risks Model Parameters

Preferences	$\delta$	$\gamma$	$\psi$	
	0.9984	10	2	
Consumption	$\mu$	$\rho$	$\varphi_e$	
	0.0015	0.975	0.037	
Volatility	$\sigma_g$	$\nu$	$\sigma_w$	
	0.0072	0.999	2.8e-06	
Dividend	$\mu_d$	$\phi$	$\varphi_d$	$\pi$
	0.0015	2.5	3.5	2.0

Baseline calibration of the long-run risks model.

Table A.2: Consumption and Asset Market Calibration

	Mean	Std. Dev.	AR(1)
Consumption:	1.82	2.90	0.43
Dividend:	1.82	10.54	0.34
Risk-free Rate:	1.52	1.14	0.98
	Wealth	Market	
Corr. of discount rate with vol shock	0.59	0.96	
Total Risk Premium	2.28	6.01	
Cash-flow Risk Premium	1.22	3.44	
Discount Rate Risk Premium	0.03	0.03	
Vol. Risk Premium	1.06	2.54	

Long-run risks model implications for consumption growth and asset market. Based on a long model sample of monthly data. Consumption is time-aggregated to annual frequency.

Table A.3: Consumption Innovation Ignoring Volatility and Consumption Return

	IES = 2			IES= 1			IES = 0.75		
	Ignore Vol	Mkt Vol	True	Ignore Vol	Mkt Vol	True	Ignore Vol	Mkt Vol	True
<b>Panel A: Model with Time-varying Volatility</b>									
Vol of cons. shock	14.32	12.73	2.49	11.53	10.87	2.49	8.67	8.82	2.49
Corr. with True cons. shock	0.35	0.39	1.00	0.43	0.46	1.00	0.58	0.57	1.00
<b>Panel B: Model with Constant Volatility</b>									
Volatility of cons. shock	8.48	8.48	2.49	8.47	8.47	2.49	8.41	8.41	2.49
Corr. with True cons shock	0.59	0.59	1.00	0.59	0.59	1.00	0.59	0.59	1.00

Consumption shock, the stochastic discount factor shock and the market risk premium under the calibrated long-run risks model (column "True"), and implied when dividend return is substituted for consumption return (column "Mkt Vol"), and when the volatility risks are ignored (column "Ignore Vol"). Population values based on the long-run risks model with stochastic volatility. Volatility is annualized, in percent.

## B Impulse Response Computations

The VAR(1) dynamics for the state variables follows,

$$X_{t+1} = \mu + \Phi X_t + u_{t+1}, \quad (\text{B.1})$$

where the unconditional variance-covariance matrix of shocks is  $\Omega = \Sigma \Sigma'$ .

The ex-ante consumption variance is  $Var \Delta c_t = v_0 + v_1' X_t$ , for  $v_1 = i_{rv}' \Phi$ . Hence, ex-ante volatility shocks are  $v_1' u_{t+1}$ . To generate a one-standard deviation ex-ante volatility shock, we choose a combination of primitive shocks  $\tilde{u}_{t+1}$  proportional to their impact on the volatility:

$$\tilde{u}_{t+1} = \frac{(v_1' \Sigma)'}{\sqrt{v_1' \Sigma \Sigma' v_1}}. \quad (\text{B.2})$$

Based on the VAR, we can compute impulse responses for consumption growth, labor income growth, price-dividend ratio and expected market return in the data. Using the structure of the model and the solution to the labor return sensitivity  $b$ , we can also compute the impulse response of model-implied consumption return and price-consumption ratio to the volatility shocks.

## Appendix C: GMM Estimation

The dynamics of the state vector are described by a first-order VAR:

$$X_t = \Phi_0 + \Phi X_{t-1} + u_t$$

where  $X_t$  is a  $(6 \times 1)$ -vector of the state variables,  $\Phi_0$  is a  $(6 \times 1)$ -vector of intercepts,  $\Phi$  is a  $(6 \times 6)$ -matrix, and  $u_t$  is a  $(6 \times 1)$ -vector of gaussian shocks. The VAR orthogonality moments compose the first set of moment restrictions in our GMM estimation:

$$E[h_t^{VAR}] = E \begin{bmatrix} u_t \\ u_t \otimes X_{t-1} \end{bmatrix} = 0.$$

The second set of moments comprises the Euler conditions for 11 portfolios (the aggregate market and the cross section of five size and five book-to-market sorted portfolios):

$$E[h_t^{CS}] = E \left[ R_{i,t}^e - \text{RiskPrem}_i \right]_{i=1}^{11} = 0,$$

where  $R_{i,t}^e$  is the excess return of assets  $i$ , and  $\text{RiskPrem}_i \equiv -Cov(m_{t+1} - E_t m_{t+1}, r_{i,t+1} - E_t r_{i,t+1})$  is the model-implied risk premium of asset  $i$ .

Let  $\widehat{h}$  denote the sample counterpart of the combined set of moment restrictions, i.e.,

$$\widehat{h} = \frac{1}{T} \sum_{t=1}^T \begin{bmatrix} h_t^{VAR} \\ h_t^{CS} \end{bmatrix}.$$

The parameters of the VAR dynamics and the coefficient of risk aversion are estimated by minimizing a quadratic form of the sample moments:

$$\{\widehat{\Phi}_0, \widehat{\Phi}, \widehat{\gamma}\} = \underset{\Phi_0, \Phi, \gamma}{\operatorname{argmin}} \widehat{h}' W \widehat{h},$$

where  $W$  is a weight matrix. The moments are weighted by the inverse of their corresponding variances; the off-diagonal elements of matrix  $W$  are set at zero. We allow the weights to be updated throughout estimation (as in a continuously up-dated GMM). The reported standard errors are based on the New-West variance-covariance estimator.

When we incorporate restrictions on the variation in risk premia, the set of moment conditions is augmented by the two orthogonality moments implied by equation (B.3). In particular, let  $\varepsilon_t \equiv r_t^e - (\alpha_0 + \alpha \sigma_{r,t-1}^2)$ , where  $r_t^e$  is the log excess return of the market portfolio. Then,

$$E[h_t^{RP}] = E \begin{bmatrix} \varepsilon_t \\ \varepsilon_t \sigma_{r,t-1}^2 \end{bmatrix} = 0.$$

The estimation of the parameter vector, which in addition includes  $\alpha_0$  and  $\alpha$ , is set-up in the same way as described above.

## Appendix D: Incorporating Restrictions on Risk-Premia Variation

To facilitate the interpretation of risks and identify the contribution of volatility risks, we make the following assumption:

$$E_t[r_{t+1} - r_{f,t}] = \alpha_0 + \alpha \sigma_{r,t}^2. \quad (\text{B.3})$$

That is, we assume that risk premia in the economy are driven by the conditional variance of the market return,  $\sigma_{r,t}^2 \equiv \operatorname{Var}_t(N_{R,t+1})$ . We can now re-write the innovation into the stochastic discount factor in terms of cash-flow news, risk-free rate news and long-run news in  $\sigma_{r,t}^2$ . In particular, using the definition of  $V_t$  and the dynamics of the SDF (see equations (2.5) and (2.15)):

$$\begin{aligned} V_t &= \frac{1}{2} \operatorname{Var}_t(m_{t+1} + r_{t+1}) = \frac{1}{2} \operatorname{Var}_t(-\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1} + N_{R,t+1}) \quad (\text{B.4}) \\ &= \frac{1}{2} \operatorname{Var}_t(-\gamma(N_{R,t+1} + N_{RP,t+1} + N_{RF,t+1}) + N_{RP,t+1} + N_{RF,t+1} + N_{V,t+1} + N_{R,t+1}) \\ &\approx 0.5(1 - \gamma)^2 \sigma_{r,t}^2. \end{aligned}$$

Note that the second line in equation (B.4) makes use of the decomposition of discount-rate news into risk-premia ( $N_{RP}$ ) and risk-free rate ( $N_{RF}$ ) news, and the last line exploits assumption (B.3) and homoscedasticity of volatility shocks. Since variation in the risk-free rate in the data is quite small, we ignore its contribution to the conditional variance and use equation (B.4) as an approximation. We can now express the innovation in the SDF as:

$$\begin{aligned} m_{t+1} - E_t[m_{t+1}] &= -\gamma N_{CF,t+1} + N_{DR,t+1} + N_{V,t+1} \\ &\approx -\gamma N_{CF,t+1} + N_{RF,t+1} + (\alpha + 0.5(1 - \gamma)^2) N_{\sigma^2,t+1} , \end{aligned} \tag{B.5}$$

where  $N_{\sigma^2,t+1} \equiv (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j \sigma_{r,t+j}^2 \right)$ , and  $N_{RF,t+1} \equiv (E_{t+1} - E_t) \left( \sum_{j=1}^{\infty} \kappa_1^j r_{f,t+j} \right)$ . Note that if the volatility channel is shut down (i.e., risk premia are constant), the last term of the innovation in the stochastic discount factor disappears.

We exploit the same market-based VAR set-up as earlier. Note that the first equation in the VAR allows us to estimate the dynamics of the conditional variance,  $\sigma_{r,t}^2$ , which we then use to obtain the estimate of the market risk premium. We continue to rely on GMM in estimation of the VAR parameters, the parameters of the risk-premium dynamics ( $\alpha_0$  and  $\alpha$ ), and risk aversion. The set of moment restrictions is augmented by the two moments of the risk-premium regression implied by equation (B.3).

## References

- Bansal, R., R. Dittmar, and C. Lundblad, (2005a), Consumption, dividends, and the cross-section of equity returns, *Journal of Finance* 60, 1639–1672.
- Bansal, R., R. F. Dittmar, and D. Kiku, (2009a), Cointegration and consumption risks in asset returns, *Review of Financial Studies* 22, 1343–1375.
- Bansal, R., V. Khatchatrian, and A. Yaron, (2005b), Interpretable asset markets, *European Economic Review* 49, 531–560.
- Bansal, R., D. Kiku, and A. Yaron, (2007a), A note on the economics and statistics of predictability: A long run risks perspective, Working paper.
- Bansal, R., D. Kiku, and A. Yaron, (2007b), Risks for the long run: Estimation and inference, Working paper.
- Bansal, R., D. Kiku, and A. Yaron, (2009b), An empirical evaluation of the long-run risks model for asset prices, Working paper.
- Bansal, R. and I. Shaliastovich, (2010), Confidence risks and asset prices, *American Economic Review, papers and proceedings* 100, 537–41.
- Bansal, R. and A. Yaron, (2004), Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.
- Bollerslev, T. and H. O. Mikkelsen, (1996), Modeling and pricing long memory in stock market volatility, *Journal of Econometrics* 73.
- Bollerslev, T., G. Tauchen, and H. Zhou, (2009), Expected stock returns and variance risk premia, *Review of Financial Studies* 22.
- Breeden, D. T., (1979), An intertemporal asset pricing model with stochastic consumption and investment opportunities, *Journal of Financial Economics* 7, 265–296.
- Campbell, J., (1983), Intertemporal asset pricing without consumption data., *American Economic Review* 83, 487–512.
- Campbell, J., (1996), Understanding risk and return, *Journal of Political Economy* 104, 298–345.
- Campbell, J., S. Giglio, C. Polk, and R. Turley, (2011), An intertemporal capm with stochastic volatility, Working paper, Harvard University.
- Campbell, J. and T. Vuolteenaho, (2004), Bad beta, good beta, *American Economic Review* 94, 1249–1275.

- Coval, J. D. and T. Shumway, (2001), Expected option returns, *Journal of Finance* 56(3).
- Drechsler, I. and A. Yaron, (2011), What's vol got to do with it, *Review of Financial Studies* 24, 1–45.
- Epstein, L. G. and S. Zin, (1989), Substitution, risk aversion and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.
- Epstein, L. G. and S. Zin, (1991), Substitution, risk aversion and the temporal behavior of consumption and asset returns: An empirical analysis, *Journal of Political Economy* 99, 263–286.
- Eraker, B. and I. Shaliastovich, (2008), An equilibrium guide to designing affine pricing models, *Mathematical Finance* 18, 519–543.
- Hansen, L. P., J. C. Heaton, and N. Li, (2008), Consumption strikes back? Measuring long-run risk, *Journal of Political Economy*, forthcoming.
- Jagannathan, R. and Z. Wang, (1996), The conditional capm and the cross-section of expected returns, *Journal of Finance* 51, 3–53.
- Kreps, D. M. and E. L. Porteus, (1978), Temporal resolution of uncertainty and dynamic choice theory, *Econometrica* 46, 185–200.
- Lettau, M. and S. Ludvigson, (2001), Consumption, aggregate wealth, and expected stock returns, *Journal of Finance* 56(3).
- Lustig, H. and S. Van Nieuwerburgh, (2008), The returns on human capital: Good news on wall street is bad news on main street, *Review of Financial Studies* 21, 2097–2137.
- McConnell, M. and G. Perez-Quiros, (2000), Output fluctuations in the united states: What has changed since the early 1980s?, *American Economic Review* 90, 1464–76.
- Merton, R. C., (1973), An intertemporal capital asset pricing model, *Econometrica* 41, 867–887.
- Stock, J. and M. Watson, (2002), Has the business cycle change and why?, in Gertler, M. and K. Rogoff, eds., *NBER Macroeconomics Annual: 2002*, Cambridge, MA: MIT Press.
- Weil, P., (1989), The equity premium puzzle and the risk free rate puzzle, *Journal of Monetary Economics* 24, 401–421.

## Tables and Figures

Table 1: **Data Summary Statistics**

	Mean	Std. Dev.	AR(1)
Consumption growth	1.86	2.18	0.48
Labor income growth	2.01	3.91	0.39
Market return	5.70	19.64	-0.01
Price-dividend ratio	3.38	0.45	0.88
Realized variance	4.76	11.13	0.44

Summary statistics for real consumption growth, real labor income growth, real market return, stock market price-dividend ratio and the realized variance. Realized variance is based on the sum of squared monthly industrial production growth rates over the year, and is re-scaled to match the unconditional variance of consumption growth. Annual observations from 1930 to 2010. Consumption growth, labor income growth and market return statistics are in percent; realized variance is multiplied by 10000.



Table 2: Mis-Specification of Consumption and SDF Dynamics

	IES = 2		IES= 1		IES = 0.75	
	Ignore Vol	True	Ignore Vol	True	Ignore Vol	True
<b>Implied Consumption Shock</b>						
Vol of cons. shock	5.46	2.49	2.49	2.49	2.79	2.49
Corr. with true cons. shock	0.46	1.00	1.00	1.00	0.89	1.00
<b>Implied SDF Shock</b>						
Vol of SDF shock	41.37	61.89	39.96	59.65	39.14	58.46
Market Risk Premium	3.48	6.02	2.75	4.96	2.28	3.84
Corr. with true SDF shock	0.71	1.00	0.67	1.00	0.64	1.00

Consumption shock, the stochastic discount factor shock and the market risk premium under the calibrated long-run risks model (column "True"), and implied when the volatility risks are ignored (column "Ignore Vol"). Population values based on the long-run risks model with stochastic volatility. Volatility is annualized, in percent.

Table 3: Macro VAR Estimates

	$\Delta c_t$	$\Delta y_t$	$r_{dt}$	$pd_t$	$RV_t$	$R^2$
$\Delta c_{t+1}$	0.221 [0.090]	0.149 [0.037]	0.058 [0.012]	0.002 [0.003]	2.628 [1.091]	0.61
$\Delta y_{t+1}$	-0.271 [0.271]	0.507 [0.146]	0.079 [0.026]	0.005 [0.007]	2.991 [3.417]	0.36
$r_{d,t+1}$	-3.081 [0.881]	1.109 [0.369]	0.065 [0.085]	-0.073 [0.037]	-10.653 [10.265]	0.08
$pd_{t+1}$	-3.586 [0.939]	0.974 [0.647]	-0.234 [0.147]	0.921 [0.041]	-8.872 [10.856]	0.80
$RV_{t+1}$	-0.007 [0.005]	-0.006 [0.004]	0.001 [0.0004]	-0.001 [0.0003]	0.310 [0.091]	0.35

Parameter estimates of the Macro VAR which includes real consumption growth  $\Delta c$ , real labor income growth  $\Delta y$ , real market return  $r_d$ , market price-dividend ratio  $pd$  and the realized variance  $RV$ . Realized variance is based on the sum of squared monthly industrial production growth rates over the year, and is re-scaled to match the unconditional variance of consumption growth. Annual observations from 1930 to 2010. Robust standard errors are in the brackets.

Table 4: Role of Volatility for Economic News

	$N_{ECF}$	$N_V$	$N_{DR}^d$	With Vol Risk			No Vol Risk		
				$N_{DR}^y$	$N_{DR}$	$N_M$	$N_{DR}^y$	$N_{DR}$	$N_M$
Std. Dev.	5.19	29.72	14.22	2.58	3.91	47.27	4.65	2.60	26.41
Corr. with $N_V$	-0.27	1.00	0.86	0.28	0.77	0.84	-0.85	-0.27	0.24
Volatility News Periods:									
Lowest 25%	2.23	-32.57	-12.37	-0.61	-2.96	-44.77	4.48	1.11	-8.13
Highest 25%	-1.70	36.94	16.02	0.68	3.75	49.90	-5.09	-0.86	8.35

Standard deviation of economic news, correlation with volatility news and the magnitude of the news in lowest 25% and highest 25% volatility periods. Economic news include news in expected future consumption  $N_{ECF}$ , future volatility  $N_V$ , stochastic discount factor  $N_M$ , and discount rates on the market  $N_{DR}^d$ , labor  $N_{DR}^y$  and wealth return  $N_{DR}$ . "With Vol Risk" columns show the economic news when the volatility risks are present in the construction of the aggregate wealth portfolio, while "No Vol Risk" columns document the implications when volatility risks are ignored. Risk aversion is set at  $\gamma = 5$ , and IES  $\psi = 2$ .

Table 5: **Labor, Market and Aggregate Wealth Return Correlations**

		No Vol Risk	With Vol Risk
<b>Market and Labor Return:</b>			
Immediate Shocks	$Corr(N_R^d, N_R^y)$	-0.60	0.20
Discount Shocks	$Corr(N_{DR}^d, N_{DR}^y)$	-0.72	0.25
5-year Expectations	$Corr(E_t r_{t \rightarrow t+5}^d, E_t r_{t \rightarrow t+5}^y)$	-0.50	0.38
<b>Market and Wealth Return:</b>			
Immediate Shocks	$Corr(N_R^d, N_R)$	0.46	0.80
Discount Shocks	$Corr(N_{DR}^d, N_{DR})$	0.07	0.86
5-year Expectations	$Corr(E_t r_{t \rightarrow t+5}^d, E_t r_{t \rightarrow t+5})$	0.26	0.78
<b>Wealth and Labor Return:</b>			
Immediate Shocks	$Corr(N_R, N_R^y)$	0.43	0.74
Discount Shocks	$Corr(N_{DR}, N_{DR}^y)$	0.65	0.71
5-year Expectations	$Corr(E_t r_{t \rightarrow t+5}, E_t r_{t \rightarrow t+5}^y)$	0.71	0.87

Correlations between immediate shocks, discount rate shocks and 5-year expected returns to the market, labor, and aggregate wealth.  $N_R, N_{DR}$  and  $E_t r_{t \rightarrow t+5}$  stand for the immediate news, discount rate news, and 5 year expected return to the wealth portfolio, respectively. Subscripts "y" and "d" denote the corresponding news for the labor return and stock market return. "With Vol Risk" columns show the implications the volatility risks are present in the construction of the aggregate wealth portfolio, while "No Vol Risk" columns document the implications when volatility risks are ignored. Risk aversion is set at  $\gamma = 5$ , and IES  $\psi = 2$ .

Table 6: Labor, Market and Aggregate Wealth Return Risk Premium

		No Vol Risk	With Vol Risk
<b>Market Return:</b>			
Risk Premium	$Cov(-N_M, N_R^d)$	2.34	7.16
Cash-Flow Risk Premium	$Cov(-\gamma N_{CF}, N_R^d)$	2.57	2.57
Vol Risk Premium	$Cov(-N_V, N_R^d)$	0	4.28
Discount Rate Risk Premium	$Cov(-N_{DR}, N_R^d)$	-0.23	0.30
<b>Wealth Return:</b>			
Risk Premium	$Cov(-N_M, N_R)$	0.86	2.57
Cash-Flow Risk Premium	$Cov(-\gamma N_{CF}, N_R)$	0.94	1.20
Vol Risk Premium	$Cov(-N_V, N_R)$	0	1.31
Discount Rate Risk Premium	$Cov(-N_{DR}, N_R)$	-0.08	0.06
<b>Labor Return:</b>			
Risk Premium	$Cov(-N_M, N_R^y)$	0.53	1.42
Cash-Flow Risk Premium	$Cov(-\gamma N_{CF}, N_R^y)$	1.27	0.86
Vol Risk Premium	$Cov(-N_V, N_R^y)$	0	0.57
Discount Rate Risk Premium	$Cov(-N_{DR}, N_R^y)$	-0.04	0.00

Total risk premium on labor, market and wealth return, and its decomposition into cash-flow, discount rate and volatility risk premium components.  $N_M$  denotes the stochastic discount factor shock, and  $N_{CF}, N_{DR}$  and  $N_V$  represent the current and future cash-flow news, discount rate news on the wealth portfolio and volatility news.  $N_R, N_R^y$  and  $N_R^m$  denote the immediate return news to wealth, labor and market, respectively. "With Vol Risk" columns show the implications the volatility risks are present in the construction of the aggregate wealth portfolio, while "No Vol Risk" columns document the implications when volatility risks are ignored. Risk aversion is set at  $\gamma = 5$ , and IES  $\psi = 2$ .

Table 7: **Role of Preferences for Aggregate Returns**

$\psi$	Lbr-Mkt Corr			Risk Premia			Lbr-Mkt Corr			Risk Premia		
	$N_R$	$N_{DR}$	$Er$	Mkt	Lbr	Wealth	$N_R$	$N_{DR}$	$Er$	Mkt	Lbr	Wealth
	<b>With Vol Risk</b>						<b>No Vol Risk</b>					
0.5	-0.89	-0.54	-0.35	4.86	-5.03	-3.06	-0.81	-0.20	0.05	1.65	-1.38	-0.77
1.0	-0.94	-0.43	-0.16	6.39	-1.38	0.18	-0.94	-0.43	-0.16	2.11	-0.30	0.18
1.5	-0.29	-0.17	0.13	6.90	0.41	1.71	-0.76	-0.60	-0.35	2.27	0.21	0.62
2.0	0.20	0.25	0.38	7.16	1.42	2.57	-0.60	-0.72	-0.50	2.34	0.49	0.86
2.5	0.36	0.52	0.50	7.31	2.05	3.11	-0.51	-0.79	-0.62	2.39	0.67	1.01
3.0	0.44	0.62	0.56	7.42	2.49	3.48	-0.44	-0.84	-0.71	2.42	0.79	1.12

Correlations between labor and market returns (immediate news  $N_R$ , discount rate news  $N_{DR}$  and 5-year expected returns  $Er$ ), and the risk premia on the market, labor and aggregate wealth returns. "With Vol Risk" columns show the implications the volatility risks are present in the construction of the aggregate wealth portfolio, while "No Vol Risk" columns document the implications when volatility risks are ignored. Risk aversion is set at  $\gamma = 5$ , and the IES varies between 0.5 and 3.

Table 8: Market-based VAR Estimates

	$RV_{r,t}$	$z_t$	$\Delta d_t$	$ts_t$	$ds_t$	$i_t$	$R^2$
$RV_{r,t+1}$	0.274 [0.167]	-0.017 [0.012]	0.023 [0.035]	-0.696 [0.336]	2.579 [1.031]	0.353 [0.202]	0.63
$z_{t+1}$	-0.961 [0.863]	0.904 [0.074]	-0.584 [0.261]	0.856 [1.276]	4.981 [5.213]	0.593 [0.556]	0.80
$\Delta d_{t+1}$	0.715 [0.408]	0.050 [0.031]	0.161 [0.131]	2.347 [0.952]	-7.887 [3.121]	-0.407 [0.370]	0.27
$ts_{t+1}$	-0.071 [0.023]	0.004 [0.003]	-0.016 [0.011]	0.410 [0.103]	1.036 [0.147]	-0.029 [0.019]	0.50
$ds_{t+1}$	0.020 [0.017]	-0.001 [0.002]	0.009 [0.006]	-0.127 [0.034]	0.682 [0.146]	0.021 [0.024]	0.56
$i_{t+1}$	-0.352 [0.105]	0.001 [0.007]	-0.050 [0.026]	-0.197 [0.182]	2.075 [0.792]	0.613 [0.109]	0.48

Table 8 presents GMM estimates of the market-based VAR.  $RV_{r,t}$  denotes the realized variance of the aggregate market portfolio;  $z_t$  is the log of the price-dividend ratio;  $\Delta d_t$  is the log dividend growth;  $ts_t$  and  $ds_t$  are term- and default spreads, respectively;  $i_t$  is the log of the interest rate. Robust standard errors are presented in brackets. The data used in estimation are real and cover the period from 1930 to 2010.

Table 9: Asset Pricing Implications of the Volatility-Based DCAPM

	Risk Premia (%)		Betas $\times$ 100		
	Data	Model	CF	DR	Vol
Market	7.9	7.7	1.80	-1.67	-1.23
BM1	7.2	7.2	1.72	-1.53	-1.09
BM2	7.6	8.3	2.11	-1.59	-1.13
BM3	9.4	10.0	2.66	-1.58	-1.37
BM4	10.8	11.3	3.29	-1.58	-1.04
BM5	13.1	13.2	4.64	-0.91	-0.09
Size1	14.8	14.2	4.57	-1.55	-0.62
Size2	13.0	12.4	3.47	-1.98	-1.29
Size3	11.6	11.2	2.97	-1.94	-1.38
Size4	10.4	9.7	2.43	-1.88	-1.44
Size5	7.4	7.4	1.91	-1.44	-0.98
			Price(CF)	Price(DR)	Price(Vol)
			2.64	-1	-1
			[0.38]	[na]	[na]

Table 9 shows risk premia implied by the volatility-based Dynamic CAPM and risk exposures (betas) of the aggregate market and a cross section of five book-to-market and five size sorted portfolios. The bottom panel presents the estimates of the market prices of risks and the corresponding robust standard errors (in brackets). According to the model, prices of discount-rate and volatility risks are fixed at -1. “Data” column reports average returns in excess of the three-month Treasury bill rate in the 1930-2010 sample.

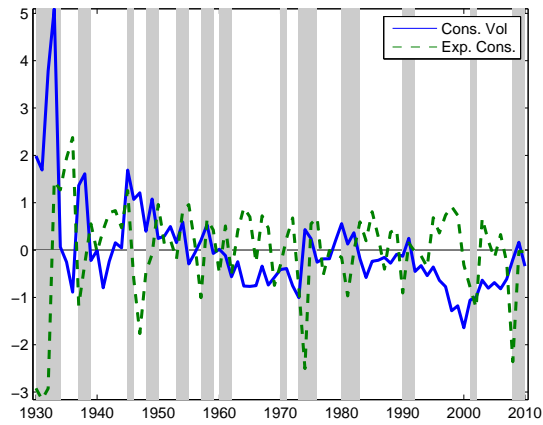


Table 10: **Asset Pricing Implications of the Restricted Volatility-Based DCAPM**

	Risk Premia (%)		Betas $\times$ 100		
	Data	Model	CF	RF	Vol
Market	7.9	7.6	1.98	-0.02	-1.93
BM1	7.2	7.0	1.92	0.19	-1.76
BM2	7.6	8.3	2.30	0.10	-1.81
BM3	9.4	10.2	2.80	-0.09	-2.09
BM4	10.8	11.4	3.39	-0.17	-1.58
BM5	13.1	13.0	4.54	-0.04	-0.05
Size1	14.8	14.1	4.63	0.07	-0.96
Size2	13.0	12.5	3.67	0.07	-2.06
Size3	11.6	11.3	3.19	0.05	-2.20
Size4	10.4	9.8	2.63	-0.05	-2.25
Size5	7.4	7.4	2.04	-0.04	-1.52
			Price(CF)	Price(RF)	Price(Vol)
			2.85	-1	-1
			[0.45]	[na]	[na]

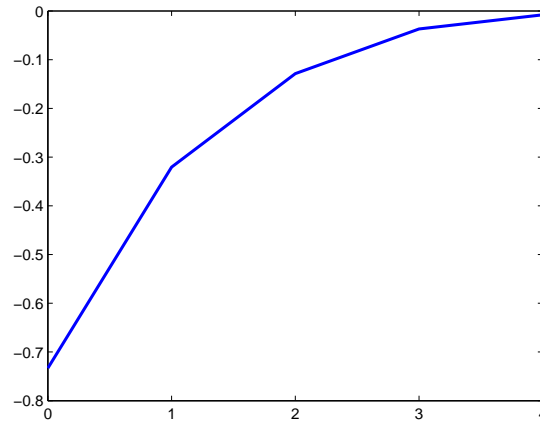
Table 10 shows risk premia implied by the restricted volatility-based Dynamic CAPM and risk exposures (betas) of the aggregate market and a cross section of five book-to-market and five size sorted portfolios. In this specification, discount-rate variation is decomposed into variation in risk premia (which are proportional to the conditional market variance) and variation in the risk-free rate. The bottom panel presents the estimates of the market prices of risks and the corresponding robust standard errors (in brackets). According to the model, prices of risk-free and volatility news are fixed at -1. “Data” column reports average returns in excess of the three-month Treasury bill rate in the 1930-2010 sample.

Figure 1: Volatility and Expected Consumption Growth from Macro VAR



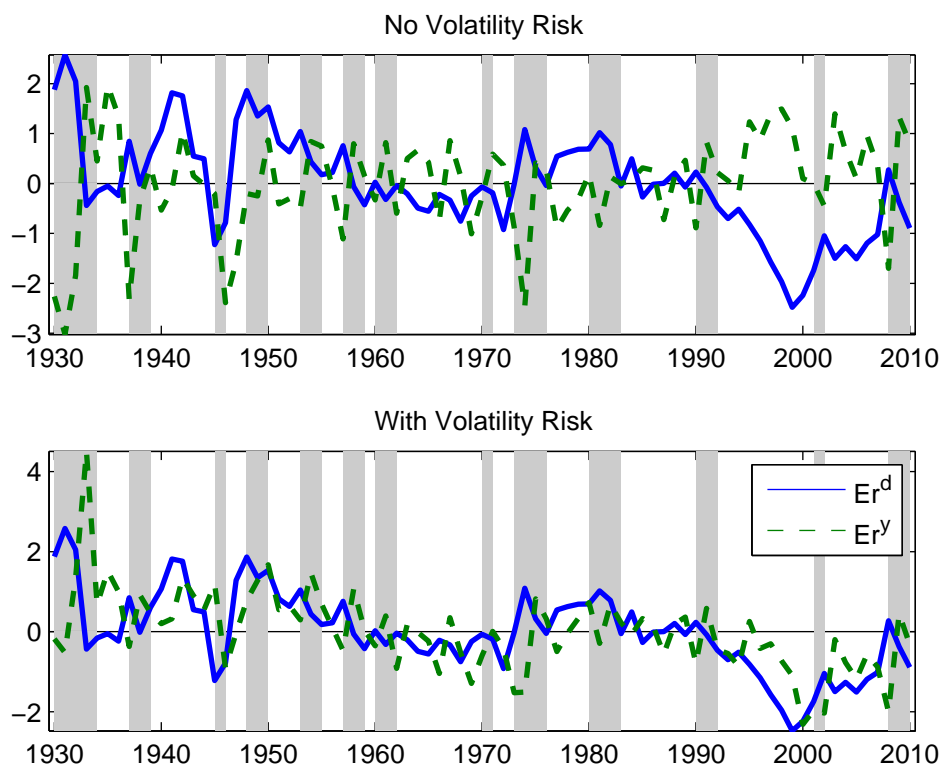
Ex-ante consumption volatility (solid line) and expected consumption growth (dashed line) implied from Macro VAR. The two series are standardized. Shaded areas represent the NBER recession dates.

Figure 2: Consumption Response to Volatility Shock from Macro VAR



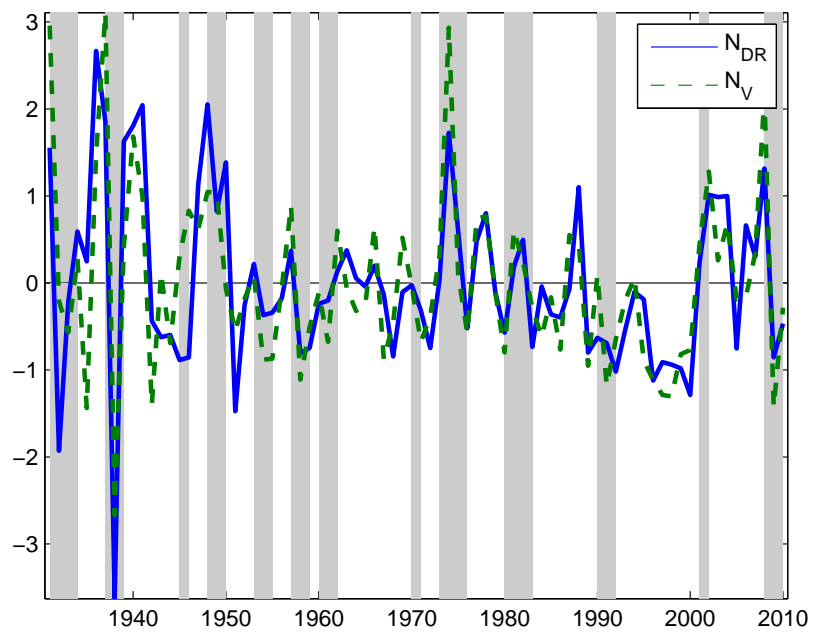
Impulse response of consumption growth to one standard deviation shock in ex-ante volatility of consumption, implied by the Macro VAR. One standard deviation volatility shock corresponds to an increase in ex-ante consumption variance by  $(1.95\%)^2$ . Consumption growth is annual, in per cent.

Figure 3: 5-year Expected Market and Labor Returns



Five-year expected returns on the market (solid line) and human capital (dashed line). "With Volatility Risk" panel shows the implied expected returns when the volatility risks are present in the construction of the aggregate wealth portfolio, while "No Volatility Risk" panel documents the return implications when volatility risks are ignored. The return series are standardized. Shaded areas represent the NBER recession dates.

Figure 4: Discount Rate and Volatility News



Discount rate news on the wealth portfolio  $N_{DR}$  and the volatility news  $N_V$ . The news series are standardized. Shaded areas represent the NBER recession dates.

Figure 5: Discount Rate and Conditional Variance of the Market Portfolio

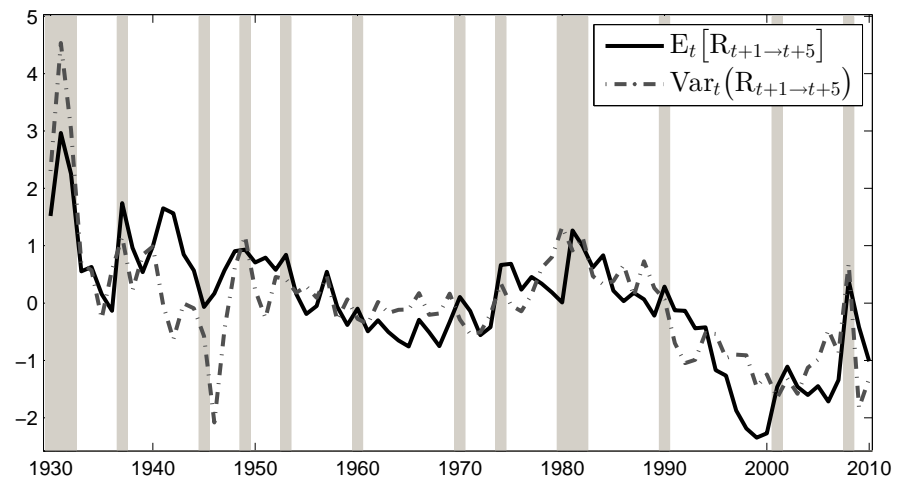
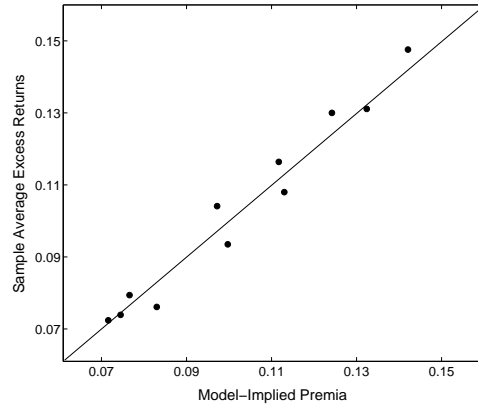
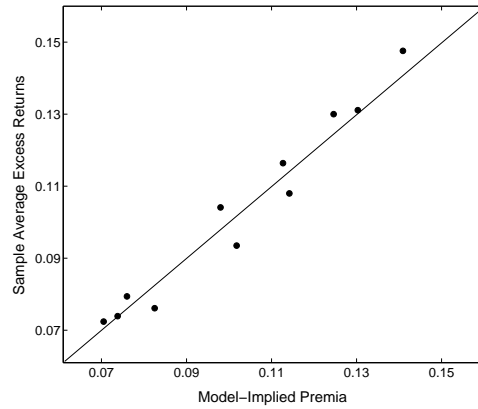


Figure 5 plots time-series of the 5-year expected market return and 5-year conditional variance of the market portfolio implied by the unrestricted market-based VAR estimates. The two lines are normalized to have a zero mean and a unit variance; shaded areas represent the NBER recession dates.

Figure 6: Data and Model-Implied Risk Premia



(a) Volatility-Based DCAPM



(b) Restricted Volatility-Based DCAPM

Figure 6 presents a scatterplot of sample average excess returns versus model-implied risk premia of the aggregate market and a cross section of five book-to-market and five size sorted portfolios. Panel (a) shows cross-sectional fit of the unrestricted volatility-based DCAPM. Panel (b) corresponds to the restricted version of the volatility-based DCAPM. In the restricted specification, variation in the market risk premium is proportional to variation in the conditional variance of the market portfolio. Sample averages correspond to average returns in excess of the three-month Treasury bill rate in the 1930-2010 sample.