NBER WORKING PAPER SERIES

THE INVARIANCE OF R&D TO THE NUMBER OF FIRMS IN THE INDUSTRY

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Working Paper No. 1798

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
January 1986

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NBER Working Paper #1798
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ABSTRACT

This paper presents certain remarkably simple results concerning market's allocation to R+D and its comparison to socially efficient allocations. We posit that a firm can undertake more than one project aimed at the same innovation, and consider a product market characterized by Bertrand competition. Among the results we obtain is that the market R+D (that is, the number of projects undertaken, and the effort spent on different projects) is invariant to the number of firms. We also examine the effects of the number of firms on the gains from innovation to consumers, firms, and society, and show, in particular, that the market undertakes less R+D than is socially desirable.

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THE INVARIANCE OF R+D TO THE NUMBER OF FIRMS IN THE INDUSTRY

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1. Introduction

A major concern of the recent research in the theory of innovation has been the effect of market structure on private marginal returns from innovation, and, thus, on the equilibrium level of market R+D. Recent work has also emphasized the relationship between marginal private returns and social returns which, in general, may not be the same.[1] For instance, in some patent races, the private return is either zero, when the firm is not first to invent, or the total (appropriable) return when it is; while the social return is the increase in the present value of social gain from having the invention earlier than it otherwise would have been available.

The present analysis is based on a model in which a firm can undertake more than one project aimed at the same innovation, and the product market is characterized by Bertrand competition. The main results of this paper are:

(i) <u>The number of firms in the industry has no effect on the pace of innovation</u>. That is, the marginal decisions of a firm to undertake an additional research project, or to spend additional efforts on a project, are unaffected by the number of firms. The resulting invariance of the market equilibrium is in marked contrast with some previous studies[2] which have found the number of firms in the industry to be a critical determinant of the market R+D.

(ii) Any 'interior' market equilibrium is 'quasi-efficient.'[3] That is, the set of projects undertaken in the market as well as the intensities with which they are undertaken maximize the economy-wide probability of a successful innovation, given the level of expenditure on R+D, but the market expenditure on R+D is smaller than what is socially optimal.

These results are fairly general; they hold, for instance, whether R+D projects have independent outcomes or not, whether there is symmetric equilibrium or not. In a more restricted model we also establish that:

(iii) The intensity at which a research project is pursued in the market is invariant to the magnitude of (appropriable) rent from successful innovation. If the rent is larger, then the number of projects undertaken is larger.

(iv) The intensity at which a project is undertaken in the market is socially optimal but, in general, the market undertakes fewer projects than is socially desirable.

(v) The number of firms in the industry affects the gains from innovation to firms and consumers and, thus, it affects aggregate social gains. A larger number of firms lowers industry profit as well as the profit of an individual firm. Also, for a class of innovations, a larger number of firms raises consumers' gains as well as the aggregate social gains from innovation.

A key feature of our model is that a firm may undertake more than one research project aimed at the same innovation, if it is profitable to do so. This assumption, we believe, is more plausible from an economic viewpoint than the one underlying some previous models in which a firm can undertake only one research project. It is easy to understand why this difference in assumption has a significant effect on the analysis of R+D.

Under our assumption, a firm has a larger set of instruments (it can select a portfolio consisting of projects at different levels of intensity) and thus, in general, its behavior is quite different from that when it is arbitrarily constrained to undertake a single project. The properties of the resulting market equilibrium in research are also, therefore, different. This insight has critical implications for the analysis of R+D, regardless of the particular model one uses (for example, the particular assumptions one makes concerning the nature of competition in the product market); though the specific consequences of our assumption would, of course, depend on the characteristics of the model. The present analysis is conducted in a context where there is Bertrand competition in the product market. We begin, in the next section, with a simple model; a more general model is investigated in Section 3.

3. A Simple Model

A research project has a binary outcome: it is either successful or not.[4] If e is the variable effort (expenditure) on a research project, then the probability of its success is $p(e)$, where $e \geq 0$, $1 \geq p \geq 0$, and $p_e > 0$.[5] The outcomes of different projects are independent of one another, regardless of firm affiliation. A firm can undertake as many projects as it desires, all of which are aimed at the same innovation. Thus, if e_{ij} denotes the effort by the i-th firm on its project j, and if this firm undertakes $j = 1, \ldots, k_i$ projects, then the probability that at least one of the projects undertaken by this firm is successful is given by $q_i = 1 - \prod_{j=1}^{k_i}(1 - p(e_{ij}))$.

The product market is characterized by Bertrand competition.

Specifically, the (positive) rent gained by a firm is R if it innovates and if no other firm innovates. If two or more firms innovate, then none of them get any rent and the benefits of innovation accrue solely to consumers. h_i denotes the probability that all firms, other than the i-th firm are unsuccessful. That is, $h_i = \prod_{f\neq i} \prod_{j=1}^{k_f} (1 - p(e_{fj}))$, where $f = 1, \ldots, N$ denotes the firms. $N \geq 1$, and it is finite. Then, the (expected) profit of firm i is $\pi_i = Rh_i q_i - \sum_{j=1}^{k_i} (e_{ij} + a)$, where a is the fixed cost of undertaking a project.

We focus at present on the symmetric interior Nash equilibrium in which all projects have the same $p(e)$ function, each firm undertakes the same number of projects and, further, if a firm undertakes more than one project, then all projects are undertaken at the same level of effort.[6] At an interior equilibrium, $e > 0$, $k \geq 1$, and both e and k are finite. Therefore

(1) $$q = 1 - (1 - p(e))^k, \text{ and}$$

(2) $$h = (1 - p(e))^{Nk-k}.$$

The first order conditions with respect to e and k, for a firm's optimum, are: $Rhq_e - k = 0$, and $Rhq_k - (e + a) = 0$, respectively. These equilibrium conditions can be restated, using (1) and (2), as[7]

(3) $$R(1 - p)^{n-1} p_e - 1 = 0, \text{ and}$$

(4) $$-R(1 - p)^n \ln(1 - p) - (e + a) = 0,$$

where $n = Nk$ is the total number of projects undertaken in the market.

Note that the above expressions determine the effort per project, e, and the total number, n, of projects undertaken in the market. A change in N simply changes k, keeping n and e unchanged. Thus, the only effect of N is on the number of projects a firm undertakes, which is $k = n/N$. In a duopoly, for instance, each of two firms undertakes half as many projects as a monopoly would have undertaken. It follows then that the number of firms in the market has no impact on (i) the total number of research projects undertaken; (ii) the intensity of each of the projects; and, therefore (iii) the probability of a successful innovation.

The intuitive idea behind this result is as follows. Consider the marginal decision of a firm to undertake the last project (or to invest the last dollar on a project). This project (or dollar) yields a benefit only if the other projects undertaken by this firm fail, <u>as well as</u> if all of the projects undertaken by other firms fail. The marginal decisions are thus influenced by the <u>total</u> number of projects undertaken in the market; and not by how these projects are partitioned between the firm making the decision and other firms. Thus, whether the marginal project yields a return, as well as the return from the marginal effort invested in a project are independent of the number of firms. Furthermore, it is easily verified that this independence holds even when a firm has a vector of control variables, and when the expected cost of a project is a general function of the control variables.

A still stronger result is obtained by solving (4) for $(1 - p)^n$ and substituting the resulting expression into (3). This yields

$$(5) \qquad -(e + a)p_e/(1 - p)\ln(1 - p) - 1 = 0 .$$

The above expression characterizes the optimal e, and it does not contain R or N. Thus, the optimal effort per project is independent not only of the number of firms in the industry, but also of the magnitude of rent from successful innovation. Further, by perturbing (3) with respect to R, and noting that e is invariant to this perturbation, we obtain

(6) $$dn/dR = -1/R\ln(1 - p) > 0 .$$

Thus, a larger number of projects is undertaken in the market if the rent from innovation is larger.

The above analysis also brings out clearly the difference between the consequences of our assumption that k is determined endogenously, and the more restrictive assumption under which k is exogenously fixed at unity. In the latter case, it is apparent from (3) that the optimal effort per project (and hence the probability of a successful innovation in the market) depends, in general, on the number of firms.

Welfare Analysis: The invariance results we have derived might give an impression that public policy (affecting the number of firms in the industry) has no role to play in the context of research and innovation. This is not correct because, though the number of firms does not affect the aggregate probability of innovation, it does affect the division of this probability between the two cases: (i) when only one firm innovates, and (ii) when more than one firm innovates. Since the post-innovation gains to consumers (or firms) are different under these two cases, their expected gains are affected by the number of firms in the industry.

To see this, let z denote the probability of innovation, and let g denote the probability that two or more firms innovate. That is

$$(7) \qquad z = 1 - (1 - p)^n$$

$$(8) \qquad g = z - Nhq$$

where, recalling our earlier notation, Nhq is the probability that only one firm innovates. Clearly, z is independent of N but its division between g and Nhq is not. Specifically[8]

$$(9) \qquad d(Nhq)/dN = h[k\ln(1 - p) + q] < 0 .$$

This is what we would expect, because if the same number of total projects is divided among a larger number of firms then the probability that two or more firms innovate is higher and, correspondingly, the probability that only one firm innovates is lower.

The above reasoning also suggests that a larger number of firms would lower the aggregate profit of firms. This can be ascertained as follows. The aggregate corporate profit is given by

$$(10) \qquad N\pi = RNhq - Nk(e + a) .$$

Now, note that the last term in the above right hand side does not depend on N, whereas, from (9), the first term is decreasing in N. Thus, $d(N\pi)/dN < 0$. Further, $d\pi/dN = [d(N\pi)/dN - \pi]/N < 0$, if a firm's profit is nonnegative (which we assume). Therefore, a larger number of firms lowers the profit for a single firm, as well as for the industry as a whole.

Next, consider consumers. They face a monopoly on the fruits of innovation if only one firm innovates, but get the entire benefit from innovation if two or more firms innovate. If their gains in these two cases are represented by S_1 and S_2 respectively, then $S_2 - S_1$ represents the loss due to monopoly, relative to the case when consumers receive the full benefit of innovation. Normally, $S_2 - S_1$ will be positive.[9] Now, the expected gain to consumers is $S = S_1Nhq + S_2g$, which can be restated as

$$S = S_2z - (S_2 - S_1)Nhq \tag{11}$$

where the first term represents the full gain from innovation, and the second term represents the loss due to monopoly. Using (9), it is obvious that the consumers gain is larger if the number of firms is larger.

Since the number of firms has opposite effects on consumers and firms, we combine these two effects to study the societal implications. Our analysis here assigns equal weights to the gains of consumers and firms, but the results can be easily rephrased if the weights are different. The social gain is $B = S + N\pi$, which, from (10) and (11), can be expressed as

$$B = S_2z - (S_2 - S_1 - R)Nhq - Nk(e + a) \ . \tag{12}$$

It is apparent from (9) and (12) that whether the social gain is increasing or decreasing in the number of firms depends on whether the consumers loss due to monopoly, $(S_2 - S_1)$, is larger or smaller than the firms' rent from monopoly, R . In typical cases in which the innovation is meant to reduce a product's production cost, consumers suffer deadweight losses when a monopoly captures any rents; that is, $S_2 - S_1 > R$.[10] In these cases, clearly, a larger number of firms yields a larger social gain.

The last result also suggests that if the government can alter the number of firms in a non-distortive manner (for instance through an entry subsidy) and if there are no fixed costs associated with establishing a firm, then the optimal number of firms is such that each firm undertakes a single project. Obviously, if there are fixed costs, we can use (12) to calculate the corresponding optimal number of firms.[11]

Social Optimum: Our objective here is to contrast the socially optimal resource allocation to R+D with the market allocation described above. Let n denote the number of projects undertaken by the planner. Then z, given in (7), is the probability that at least one project is successful; in which case consumers receive the full benefits of innovation. The expected social gain is: $S_2 z - n(e + a)$.[12] The corresponding first order conditions, with respect to e and n, characterizing the internal optimum, can be expressed as

(13) $$S_2(1 - p)^{n-1}p_e - 1 = 0$$

(14) $$-S_2(1 - p)^n \ln(1 - p) - (e + a) = 0 .$$

Note the similarity between the social allocation described above, and the market equilibrium described by (3) and (4). The two sets of expressions are identical except that the gain from successful innovation is R for a firm, whereas it is S_2 for the planner. This similarity should not be surprising because, once again, the marginal decision of the planner (to undertake the last project, or to invest the last dollar on a project) depends on the total number of projects that have already been undertaken; just the way it did for a firm in the market. Now, recall that $de/dR = 0$. It follows that the market effort per project is at the socially efficient level.

Further, recalling (6), the similarity between the market equilibrium and the social optimum also implies that whether the number of projects undertaken in the market is smaller (larger) than the socially optimal number depends on whether S_2 is larger (smaller) than R. Once again, in a wide variety of circumstances (for instance, for innovations dealing with cost reduction), the full consumers' gain from innovation is larger than the rents to a firm from monopolizing the innovation; that is, $S_2 > R$. In these cases (on which we focus in the rest of this paper), the market undertakes fewer projects than is socially desirable.

In fact, the economic content of the above result is a consequence of Bertrand competition, and it does not depend on some of the details of the model (for example, whether a firm can undertake many or only one project). The reason is simple. Under Bertrand competition, a firm captures rents only when it turns out to have monopoly over innovation. It follows that, so long as the rents to a firm when it is a monopoly are smaller than the full consumers' gain from innovation, the market investment in R+D is smaller than what is socially desirable.

3. General Invariance Results

The model in the preceding section assumed that there is a single technology for innovation (though the effort level could vary) and that the outcomes of different projects are statistically independent; also, we focussed on a symmetric equilibrium. In fact, our central result that the market's allocation to R+D is invariant to the number of firms is more general. The main reason behind this invariance is that, under Bertrand competition, there is return from undertaking the marginal project only if all other projects are unsuccessful; regardless of (i) how these projects are partitioned among firms, (ii) whether these projects are based on the same or different technologies and effort levels, or (iii) whether the outcomes of these projects are correlated or independent. Moreover, the return from the marginal project (when it is the only successful project), R, is also independent of what the portfolio of unsuccessful projects is. In the following paragraphs, we make this intuition more precise.

Let $t = 1, \ldots, T$ denote different types of projects, where e^t is the effort corresponding to a project of type t, and where different types of projects represent different technologies as well as different levels of effort spent on any particular technology. The vector $M = [M^1, \ldots, M^T]$ denotes a portfolio of projects where M^t is the number of projects of type t. $M^t \geq 0$. Define $r(M)$ to be the probability that at least one project in the set M is successful.

Now, consider the portfolio which maximizes $Rr(M) - M\hat{e}$, where $\hat{e} = [e^1, \ldots, e^T]$. We refer to this portfolio as the 'quasi-efficient portfolio.' Let $M_{-t} = [M^1, \ldots, M^t - 1, \ldots, M^T]$, and $r_t(M_{-t}) = r(M) - r(M_{-t})$. $r_t(M_{-t})$ is thus the probability that the marginal project of type t is successful and all other projects in the portfolio M_{-t} are unsuccessful. (Note that the last deduction does not depend on whether the outcomes of projects in the portfolio M are statistically correlated or independent.) Analogously, define $M_{+j} = [M^1, \ldots, M^j + 1, \ldots, M^T]$, $M_{-t+j} = [M^1, \ldots, M^t - 1, \ldots, M^j + 1, \ldots, M^T]$, $r_j(M) = r(M_{+j}) - r(M)$, and $r_j(M_{-t}) = r(M_{-t+j}) - r(M_{-t})$.

The optimality conditions for the quasi-efficient portfolio are

$$Rr_t(M_{-t}) \geq e^t \tag{15}$$

(16) $\quad Rr_j(M) \leq e^j$

(17) $\quad Rr_t(M_{-t}) - e^t \geq Rr_j(M_{-t}) - e^j$

for all t for which $M^t \geq 1$, and for all j. The above expressions have obvious meanings. Expression (15) implies that all marginal projects that are undertaken at least breakeven. Expression (16) implies that it does not pay to undertake a project not already undertaken. Expression (17) ensures that each marginal project undertaken maximizes the incremental profit. In the analysis below we assume for brevity that there is a unique quasi-efficient portfolio but, as we shall see, our qualitative conclusions are not affected by this assumption.

It is straightforward to establish that: The quasi-efficient portfolio is identical to the portfolio of a social planner who is constrained to spend no more than what is spent on the quasi-efficient portfolio. To see this, let M^* denote the quasi-efficient portfolio; the corresponding total effort is $M^*\hat{e}$. Clearly, for this level of effort, $r(M^*)$ is the maximum probability of at least one successful project. Therefore, a social planner, attempting to maximize $S_2r(M)$, but constrained to spend no more than $M^*\hat{e}$, can do no better than to choose the portfolio M^*. (This result explains why we have referred to M^* as the quasi-efficient allocation.) Further, the optimal expenditure of a social planner would exceed $M^*\hat{e}$ if he did not face any constraint on spending; this is because the social gain from a successful project, S_2, exceeds R. Thus, the expenditure on the quasi-efficient portfolio is smaller than what is socially optimal.

Next, consider market allocations. $m^f = [m^{f1}, \ldots, m^{fT}]$ denotes a portfolio of firm f, and $\bar{m}^f = [\bar{m}^{f1}, \ldots, \bar{m}^{fT}]$ denotes the constraints on the number of projects of different types that a firm can undertake.[13] That is, $m^{ft} \leq \bar{m}^{ft}$ or, equivalently, $m^f \leq \bar{m}^f$. The corresponding <u>market portfolio</u>, and the constraint on the market portfolio, respectively, are $m = \sum_f m^f$, and $\bar{m} = \sum_f \bar{m}^f$. $s(m^i, \sum_{f \neq i} m^f)$ represents the probability that at least one of the projects within the set m^i is successful, and all projects in the set $\sum_{f \neq i} m^f$ are unsuccessful. Therefore, the profit of firm i can be expressed as: $Rs(m^i, \sum_{f \neq i} m^f) - m^i \hat{e}$. Our interest here is to examine the market portfolios resulting from firms' choices in the context of Nash equilibria.

Suppose that, in a Nash market equilibrium, the firm i is undertaking at least one project of type t. Then, the increment in its probability of 'success' (that is, in its probability of capturing the rent R) from undertaking the marginal project of type t is: $s(m^i, \sum_{f \neq i} m^f) - s(m^i_{-t}, \sum_{f \neq i} m^f)$. This expression is the same as the probability that the marginal project is successful, and all other projects (that is, those in the set $\sum_{f \neq i} m^f$, as well as in the set m^i_{-t}) are unsuccessful. An earlier definition, therefore, allows us to restate the above incremental probability as[14]

$$(18) \qquad r_t(m_{-t}) = s(m^i, \sum_{f \neq i} m^f) - s(m^i_{-t}, \sum_{f \neq i} m^f)$$

Consequently, the increment in the profit of firm i from undertaking the marginal project of type t is: $Rr_t(m_{-t}) - e^t$. It follows then that a market portfolio m is sustainable only if the breakeven condition

$$(19) \qquad Rr_t(m_{-t}) - e^t \geq 0$$

is satisfied for all firms which undertake one or more projects of type t . The main point to note here is that the breakeven condition for the sustainability of a market portfolio, (19), is the same as the breakeven condition for the quasi-efficient portfolio, (15). Analogous derivations show that the other two optimality conditions for the quasi-efficient portfolio, (16) and (17), also characterize the sustainability of a market portfolio. This is intuitive because, under Bertrand competition, a firm's decision to undertake or not to undertake a marginal project (of any type) turns out to be based on the same considerations which are relevant in determining the quasi-efficient portfolio.

The above characterization of the market portfolio leads to the following result: If the quasi-efficient portfolio is feasible in the market, then it is sustainable as a market portfolio. (What we mean here by feasibility is that there is at least one way to spread the quasi-efficient portfolio among the firms in the market, without violating the firm's constraints; that is $\bar{m} \geq M^*$.) This is because if firms' portfolios are such that the market portfolio is the same as the quasi-efficient portfolio, then no firm has an incentive to change its portfolio.

Consider now two economies which differ in the number of firms as well as in the constraints faced by different firms, but the quasi-efficient portfolio is feasible in both of them. A corollary of the above result is that the quasi-efficient portfolio is sustainable in both economies. Thus: Among the equilibria in the two different economies are at least two (one in each economy) which entail the same market portfolio of research projects, provided the quasi-efficient portfolio is feasible in both economies.

We would, of course, have liked to prove a stronger invariance result: that the set of equilibria in two different economies are identical. But this does not appear to be the case. There may be market equilibria in which the constraints faced by one or more firms are strictly binding on the portfolios they have chosen. As a consequence, if one firm undertakes a project at an inefficient effort level, then it may lead some other firms to undertake projects which are also at inefficient effort levels. This is because, as pointed out earlier, the marginal gains to a firm are influenced by what is undertaken in the market.

The stronger result does, however, hold if the relevant difference among firms is only due to the access they have to different technologies. To see this, suppose different types of firms have access to different subsets of the economy-wide set of technologies, but the choices of firms (in an equilibrium) are not constrained due to any other reason. That is, no firm undertakes all of the projects (of a technology to which it has access) that it could. We refer to such equilibria as 'interior equilibria.' Once again, the sustainability conditions for a market portfolio (corresponding to an interior equilibrium) are the same as the optimality conditions for the quasi-efficient portfolio (where the latter portfolio maximizes $Rr(M) - M\hat{e}$ subject to the economy-wide set of technologies). Thus: All interior equilibria in a market economy entail a market portfolio which is the same as the quasi-efficient portfolio. Next, consider two different market economies (A and A') which have the same types of firms (though the number of firms of different types are different). It follows from the last result that: The market portfolio is identical for all interior equilibria in the two economies.[15]

4. Conclusions

The relationship between the market structure and the nature of market R+D, and that between the private and social marginal returns from innovative activity are, in general, complicated. This paper establishes an important invariance result in the central case of Bertrand competition: the market R+D (that is, the number of research projects undertaken as well as the intensities of individual projects) is invariant to the number of firms in the industry. We also show that though the market expenditure on R+D is efficiently spent (in the sense that the market portfolio of research projects is socially optimal, given the market expenditure), the market expenditure is smaller than what is socially desirable.

In a simplified version of our model, we have established additional results. We show, for instance, that the intensity of an R+D project in the market is socially optimal (in particular, the intensity does not depend on the magnitude of rent that a firm gains from successful innovation), but the market undertakes a smaller number of projects than is socially optimal. We have also hinted at some policy implications: for example, the desirability of increasing the number of firms (which yields larger gains to consumers, smaller gains to firms, and larger social welfare gains).

An important ingredient behind a theory of the firm which our analysis has left out is the economies and diseconomies of scope; that is, there may be important spillover effects among the research projects undertaken by a firm. Also, the relationships we have established here between social and private returns will not in general obtain under other forms of competition (for example, Cournot). Our analysis suggests, further, the need to compare the outcomes of policies aimed at encouraging price competition versus other forms of competition (for example, quantity competition). The determinants of and means by which the government may affect a choice in the modes of competition is, however, a question beyond the scope of this paper.

FOOTNOTES

*We thank two annonymous referees for their valuable comments.

1. See Barzel (1968), Dasgupta and Stiglitz (1980), Kamian and Swartz (1982), Loury (1979), and Stiglitz (forthcoming), among others.

2. See footnote 1.

3. See below for precise definitions of quasi-efficiency and of an interior equilibrium.

4. Here we abstract from issues concerning the timing and the scale of innovations; that is, by spending more resources, one can alter the date of innovation or the magnitude of rent. But the analysis can be readily modified to incorporate these aspects.

5. Subscripts e and k denote partial derivatives with respect to these variables.

6. As is well known, there may not always exist a symmetric interior Nash equilibrium, because of the non-concavity of the relevant functions. Also, we are assuming at present that there are no binding constraints (such as on credit) which might prevent a firm from undertaking the desired set of projects. A more general framework is considered later.

7. For simplicity, we are treating k as a continuous variable. If k is treated as an integer, then the expression analogous to (4) is: $R(1 - p)^{n-1}p \geq (e + a) \geq R(1 - p)^{n}p$, with at least one strict inequality. This does not affect the invariance result derived below.

8. The sign of the right hand side of (9) is obtained as follows. $q(k)$ is easily seen to be strictly concave in k. Thus $q(k) - q(k = 0) < q_k(k = 0)k$. Using (1), then, $k\ln(1 - p) + q(k) < 0$, Thus, (9) is negative.

9. The simplest case is that of a cost reducing innovation for a product. Suppose the innovation reduces the (fixed) unit cost of the product from c_0 to c_2, where c_0 is the current (competitive) price. If only one firm innovates then it sets a monopoly price c_1, where $c_0 \geq c_1 > c_2$. The rent to this firm is $R = (c_1 - c_2)D(c_1)$ where D is the aggregate demand function. If more than one firm innovates then, due to Bertrand competition, the new competitive price is c_2. Obviously, then, $S_2 - S_1 > 0$. Also, unless the demand is entirely insensitive to the price, the standard consumer surplus arguments show that $S_2 - S_1 > R$.

10. See footnote 9.

11. These conclusions, naturally, do not extend to distortive instruments such as investment tax credits. It should also be pointed out that certain instruments of policy may not be feasible due to informational problems. For example, it may be difficult to monitor the number of projects undertaken by a firm.

12. As in some earlier literature [Dasgupta and Stiglitz (1980), for example], the present treatment of social optimum assumes that the revenue required to finance the R+D can be raised in a non-distortive manner. If only distortive instruments (such as commodity taxes) are available for raising revenue then, under some circumstances, the wel-

fare consequence of the market allocation may not be significantly different from that of the social optimum. See Stiglitz (forthcoming).

13. A firm might be facing some other type of constraints; for instance, on the total effort that it can spend (credit constraint) or on the types of technologies available to it. These different formulations, however, do not affect our results.

14. Note, once again, that (18) does not depend on whether the outcomes of projects within or across portfolios m^i and $\sum_{f \neq i} m^f$ are correlated or independent.

15. If there is a multiplicity of quasi-efficient portfolios then the last two results are modified as follows: (i) Every interior equilibrium in a market economy entails a portfolio which is the same as <u>a</u> quasi-efficient portfolio, and (ii) Consider an interior equilibrium in economy A. If the corresponding market portfolio is feasible in economy A', then it is sustainable in economy A'.

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