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ABSTRACT

International consumption risk sharing studies have largely ignored their models' counterfactual implications for asset returns although these returns incorporate direct market measures of risk. In this paper, we modify a canonical risk-sharing model to generate more plausible asset return behavior and then consider the effects on welfare gains. Matching the mean and variance of equity returns and the risk-free rate requires persistent consumption risk, leading to three main findings: (1) risk-sharing gains decrease as the ability to diversify persistent consumption risk decreases; (2) the international correlation of equity returns is high relative to the correlation of consumption and dividends, implying low diversification potential for persistent consumption risk; and (3) increasing persistent consumption risk reduces the gains. Taken together, our findings suggest that asset returns imply more international risk sharing than previously thought.

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1 Introduction

How much welfare improvement can be generated by optimal international consumption risk-sharing? The obvious importance of this question has motivated a significant body of research.[1] As this literature shows, international risk-sharing gains depend directly upon the value of consumption risk and the ability to diversify across countries. Clearly, asset prices in international financial markets provide a direct measure of this consumption risk. Nevertheless, consumption risk-sharing studies typically ignore implications for asset return behavior.[2] Indeed, assumptions about risk and intertemporal substitution in consumption often generate counterfactual implications for the magnitude of asset returns.

This gap between models of international risk-sharing and asset return behavior appears significant given advances in consumption-based asset pricing. Specifically, several lines of research demonstrate that introducing low frequency variation into the intertemporal marginal rate of substitution in consumption helps to fit asset return behavior.[3] The persistence in this consumption risk implies that investors do not view all uncertainty as temporary, in contrast to typical international risk-sharing models.[4]

In this paper, we begin to bridge this gap using a canonical international consumption risk-sharing framework in the tradition of Obstfeld (1994a,b). Within this tradition, the current degree of implied risk sharing is measured with observed consumption data. These data are used to construct parameters that form the benchmark for evaluating potential diversification gains. The consumption data parameters are then input into a fully diversified international risk sharing equilibrium. Comparing the lifetime utility of the consumption path from the current economy to that of the optimal risk sharing economy provides the welfare gains to international risk-sharing. To evaluate the implications from disciplining the model with asset return behavior, we take this

[1] Surveys that discuss the literature on international risk-sharing welfare gains include Tesar (1995), Lewis (1999,2011), and Coeurdacier and Rey (2011).

[2] Obstfeld (1994b) serves as a notable counter-example. He chooses risk parameters to match means of equity returns more closely, though not other asset return behavior such as the risk-free rate and the volatility of equity returns. We describe this paper in more detail below.

[3] Campbell and Cochrane (1999) employ habit persistence with time-varying risk aversion in preferences. Bansal and Yaron (2006) assume that consumption growth has a persistent component. Barro (2006) follows Reitz (1984) in assuming that investors price disaster risk.

[4] Colacito and Croce (2010) and Stathopoulos (2011a) are important exceptions. Below we describe how our analyses differ.

canonical model typically based on transitory risk and introduce low frequency variation in risk. For this purpose, we introduce persistent risk as a small autoregressive component in consumption following Bansal and Yaron (2006). We choose this approach because it incorporates the same recursive preferences as our canonical framework. As such, our analysis of persistent risk naturally nests the more typical transitory-only risk case as in Obstfeld (1994b).[5]

Our analysis generates at least three important findings. First, we show that the risk-sharing gains depend strongly on how much of the persistent risk can be diversified. If persistent consumption risk correlations are low and, hence, can be diversified under optimal international risk-sharing, the welfare gains are very large.

Second, we use equity returns and consumption growth correlations to identify the magnitude of persistent risk correlations across countries. For this purpose, we assume that equity returns pay consumption dividends following Obstfeld (1994b) and Mehra and Prescott (1985). The model implies that the persistent risk correlations are very high and near one across our sample of advanced economies.[6] This finding follows from the higher correlation of equity returns relative to consumption growth in the data. Within the model, equity return correlations depend more strongly on persistent risk than do consumption correlations. Therefore, the model points to high correlations in this consumption risk.

Even though this model with persistent risk generates implied asset return moments that are closer to the data, the implied equity premium and asset return variances remain too low. Therefore, in our final scenario, we consider a version of the Bansal and Yaron (2006) model that assumes equity pays out dividends as measured by the data. Evaluating this model leads to our third main finding: the gains from risk-sharing are dampened when the variability of persistent risk best fits asset returns. This result may seem surprising since the higher dividend volatility implies greater persistent consumption risk. When we analyze the source of these lower gains, we find an intuitive explanation. Matching asset returns increases the implied variability in persistent risk, but also reduces the counterpart variability in transitory risk. At the same time, persistent consumption

[5] In order to measure welfare gains when economies grow, Obstfeld (1994a) demonstrates the importance of decoupling risk aversion and intertemporal elasticity as in Epstein and Zin (1989) and Weil (1990) preferences. Bansal and Yaron (2006) also use Epstein-Zin preferences. By contrast, Barro (2006) assumes constant-relative-risk-aversion preferences, implicitly constraining the parameters. Campbell and Cochrane (1999) use habit-preferences that do not naturally nest the Obstfeld (1994a,b) framework.

[6] The analysis below only covers advanced economies. Our results below are likely to be mitigated for developing countries if their returns are less correlated with the world.

risk is already highly diversified in the current economy since implied correlations are close to one. Thus, fully opening the country's asset markets to international trade only allows diversification on a smaller transitory risk, thereby reducing the implied risk-sharing gains.

Taken together, these findings show that modifying consumption-based models to match asset return behavior do not substantially increase international risk-sharing gains. The high correlations between equity returns across countries in combination with low correlations between consumption or dividends mean that persistent risk is already highly diversified. Furthermore, consumption-based models that match asset returns require higher persistent consumption risk relative to transitory consumption risk. The interaction of these two effects suppress the gains from international risk-sharing.

Importantly, this result stands in contrast to a conventional view that disciplining consumption-based models to match asset returns necessarily generates high welfare gains. The conventional view follows by presuming that implausibly high risk aversion parameters are required to match the equity premium. By contrast, including persistent consumption risk can produce basic features of asset returns with risk aversion coefficients of 10 or below. Moreover, our findings suggest that international risk-sharing cannot be determined simply by looking at the data co-movements in consumption. In the presence of persistent consumption risk, additional information from asset returns is required to identify the degree of integration.

In our analysis, we use a Simulated Method of Moments (SMM) approach to fit consumption and preference parameters to data moments for seven industrialized countries Specifically, we target the means and standard deviations of equity returns and the risk-free rate, along with moments from consumption or dividends for three versions of the model. Although the asset return implications improve with each successive version, no single model completely matches all the moments we consider for all the countries. As such, we view our results as methodological rather than prescriptive. Consumption-based risk sharing models that incorporate persistent risk to match asset return moments will tend to find the risk sharing relationships highlighted above.

We analyze the canonical risk sharing model as in Obstfeld (1994b). In this framework, all countries share the risk of a common consumption good with identical preferences. To our knowledge, this paper is the first to consider the impact of persistent consumption risk in a common consumption multi-country model. In related papers, Colacito and Croce (2010) and Stathopoulos (2011a) consider the gains from international risk-sharing in a two-country model aimed at explaining exchange rate behavior In these models, consumers in each country are assumed to prefer their home goods in order to deliver variation in the relative price of goods. By contrast, we study a common good framework that ignores the direct role of the real exchange rate but indirectly incorporates that risk into the common good. As such, we can consider multiple countries without the need to restrict our sample to two countries. As another difference, Colacito and Croce (2010,2011) examine a two country model with otherwise symmetric consumption. By contrast, we use SMM to fit asset returns to multiple countries generating asymmetric consumption parameters. Overall, we view our analyses as complementary to each other as we focus on different aspects of risk-sharing.

The structure of the paper is as follows. In Section 2, we describes the basic risk-sharing framework. In Section 3, we evaluate that framework under the assumption that all consumption risk is transitory. In Section 4, we modify this framework by incorporating a small persistent component in consumption. Section 5 considers the Bansal and Yaron (2006) model based upon dividend data. Section 6 extends the analysis in several ways including differing means, population sizes and a wider set of countries. Section 7 gives concluding remarks

2 Risk-Sharing and Returns: The Framework

How well are countries diversified? And given their degree of integration, what are the benefits to complete international risk-sharing? The obvious importance of these questions has motivated a large literature that studies consumption risk-sharing.[7] These studies typically evaluate the benefits of risk sharing by comparing implications for welfare from observed consumption to that of an alternative fully integrated world economy. As in Lucas (1987), the consumption increase that makes individuals in the observed equilibrium indifferent between moving to the fully integrated equilibrium provides a measure of the risk-sharing gains.

In this paper, we ask how international risk-sharing gains are affected when consumption based models match asset return behavior. While asset return behavior is clearly only one way to discipline the model, it is arguably the most important for the question at hand. Trade in international capital markets is often viewed as the primary mechanism for sharing risks globally. As such,

[7] Backus, Kehoe, and Kydland (1991) observed that consumption correlations are lower than output correlations, thus violating the implications of perfect risk-sharing. Explanations range from incomplete markets (e.g., Baxter and Crucini (1995)), hedging labor risk (e.g., Baxter and Jermann (1997), Heathcote and Perri (2008)), hedging non-tradeables (e.g., Stockman and Tesar (1995)), and transactions costs (e.g., Tesar and Werner (1995), Warnock (2002)). For a more detailed survey, see Lewis (1999).

the prices of assets in these markets reflect equilibrium views toward risk. For this reason, we take these asset returns as the basis for disciplining our models of international consumption and implied risk parameters.

To develop the framework, we first review a canonical consumption model and then evaluate the effects of different consumption risk specifications in the following sections.

2.1 The Basic Framework

We begin with a canonical consumption risk-sharing model in the tradition of Obstfeld (1994a,b). The world consists of J different countries. A continuum of identical investors live in each country j, for $j = 1, ...J$. All of these investors have preferences over a common aggregate non-durable good, C. Investors in each country j differ from investors in other countries because they originally own the rights to j country output, denoted Y^j. This output is an exogenous endowment process that in turn depends upon a state vector that spans all J country production processes. For expositional simplicity, we subsume the dependence of output on the state variables below and write the variables as conditional realizations of these variables at time t. Below, we also refer to the investor in country j interchangeably as "country j" as well as "consumer" and "agent" from country j.

The log output growth rate processes of each of these agents is defined by $g_{y,t}^j \equiv \ln(Y_t^j / Y_{t-1}^j)$ When capital markets become integrated, the mean growth rates of investor wealth may be affected. For this reason, we consider the effects on welfare from changing growth paths as well as reducing consumption variability.

Since both growth rates and volatility may be altered under optimal risk sharing, we require preferences that allow risk aversion and the intertemporal elasticity of substitution to differ. As demonstrated by Obstfeld (1994a), standard constant relative risk aversion preferences can lead to misleading results about risk-sharing since these preferences restrict the risk aversion parameter to equal the inverse of the intertemporal elasticity of substitition (IES) in consumption. The risk aversion parameter measures how much consumers value reduction in the variability of current period consumption while the IES measures the value of consumption changes over time. As such, analyzing gains from risk-sharing requires preferences that do not impose this restriction. Therefore, we assume consumers in each country have recursive preferences following Epstein and Zin (1989) and Weil (1989). Further, to ensure that our risk-sharing results do not arise from country-specific views toward risk, we also assume all countries have the same preference parameters over a common non-durable good, C. Specifically, utility for each country j at time t can be written:

$$U^j(C_t^j, U_{t+1}^j) = \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \tag{1}$$

where U_{t+1}^j is the utility function at $t+1$; $0 < \beta < 1$ is the time discount rate; $\gamma \geq 0$ is the risk-aversion parameter; $\theta \equiv \frac{1-\gamma}{1-\frac{1}{\psi}}$ for $\psi \geq 0$, the intertemporal elasticity of substitution; and where $\mathrm{E}_t(\cdot)$ is the expectation operator conditional on the information set at time t.[8] As described by Epstein and Zin (1989), this utility function specializes to standard time-additive constant-relative risk aversion preferences when $\gamma = \frac{1}{\psi}$.

For an equilibrium to exist we require that lifetime utility be bounded and rational along the equilibrium path for each country. For this reason, we also require the condition that on an equilibrium path:

$$U^j(C_t^j, E_t[U_{t+1}^j]) < \infty \tag{2}$$

with $U^j(C_t^j, E_t[U_{t+1}^j]) \in \mathbb{R}$.

2.2 International Risk Sharing Economy

We first consider the optimal international risk sharing equilibrium. In this equilibrium, each country initially owns its home country output claims. Here we describe this equilibrium as a decentralized asset market although we show in Appendix A that this equilibrium also solves the social planner problem. Also, in this section, we assume that each country has a single representative agent. Later, we consider the effects of differing population weights.

We consider an economy with fully open financial markets at some initial time defined as $t = 0$. Consumers in each country can sell off the rights to their own output streams. Since output is non-durable, total consumption equals total output in each period.[9] We define the price of claims to country j's output in world markets at time t as P_t^{j*}. The agent's optimization problem can be written:

$$\underset{\substack{\left\{C_t, \varpi_t^j\right\} \\ \forall t}}{Max} \; U^j(C_t^j, U_{t+1}^j)$$

[8] In equation (1), we follow the form in Epstein and Zin (1989), equation (5.3). While we use this equation for parsimonious price and the value functions, our gains and return calculations are also consistent with the Epstein and Zin (1991) form.

[9] While many of the international risk-sharing models consider endowment economies, some studies analyze business cycle models (see for example, Tesar (1995)). However, business cycle studies generally consider a common steady-state growth rate across countries and constant-relative risk aversion utility.

$$\text{s.t.} \quad C_t^j + \underline{P}_t^{*\prime}\underline{\varpi}_t^j \leq W_t^{j*}$$
$$\left(\underline{Y}_{t+1} + \underline{P}_{t+1}^*\right)'\underline{\varpi}_t^j = W_{t+1}^{j*}$$

where $U^j(C_t^j, U_{t+1}^j)$ is given by equation (1), $\underline{\varpi}_t^j = \{\varpi_t^{j1}, \varpi_t^{j2}, ..., , \varpi_t^{jJ}\}$ is the vector of claims held by country j investors on each of the country outputs, $\underline{Y}_t$ is the Jx1 vector of the output realizations, $\underline{P}_t^*$ is the price vector of these claims, and W_t^{j*} is the wealth of country j at world prices.

Assuming the budget constraint holds with equality, i.e., $C_t^j + \underline{P}_t^{*\prime}\underline{\varpi}_t^j = W_t^{j*}$, country j agent's problem can be written as the Bellman equation:

$$V_t(C_t^j, W_t^j) = \underset{\substack{\{C_t^j\} \\ \forall t}}{Max} \left[C_t^{j\left(\frac{1-\gamma}{\theta}\right)} + \beta E_t \left[V_{t+1}^j(C_{t+1}^j, W_{t+1}^j)^{1-\gamma} \right]^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}} \tag{3}$$

$$\text{s.t.} \quad W_{t+1}^{j*} = (W_t^{j*} - C_t^j) * R_{t+1}^j \tag{4}$$

where $R_{t+1}^j = \frac{P_{c,t+1}^{j*} + Y_{t+1}^j}{P_{c,t}^{j*}}$ is the gross return on the wealth portfolio paying out consumption of country j at world prices and $P_{c,t}^{j*}$ is the value of that country's consumption path generated by portfolio holdings.

The problem is simplified by noting that the utility function is homogeneous in consumption and wealth so that all agents hold the same portfolio shares in each country's output.[10] Therefore, all countries buy shares in a world mutual fund. They each sell off claims to their own output and seek to buy the most claims on the world mutual fund as possible. Defining the world mutual fund payout as $Y_t^w \equiv \Sigma_{j=1}^J Y_t^j$ and ϖ^{jw} as the claim of country j on world output in period $t = 0$, the investor in country j faces the constraint in the first period[11]:

$$\left(Y_0^w + P_0^{w*}\right)\varpi^{jw} \leq \left(Y_0^j + P_0^{j*}\right) \tag{5}$$

where P_0^{w*} is the price of the mutual fund in world markets. Since the agent chooses to maximize utility, the portfolio constraint holds with equality so that: $\varpi^{jw} = (Y_0^j + P_0^{j*})/(Y_0^w + P_0^{w*})$. Subsequent to this initial sell-off, all countries own the same mutual fund albeit with differing shares. As a result, there is no incentive for future rebalancing so that ϖ^{jw} is constant over time.

Using the homogeneity of Epstein-Zin preferences, the solution to the Bellman equation (3) has

[10] See Ingersoll (1987) or the discussion in Obstfeld (1994b).

[11] The equal portfolio weights derive from equal population weights. Below, we relax this assumption.

the general form:[12]

$$V_t(C_t, W_t) = C_t \left[\frac{W_t}{C_t}\right]^{\left(\frac{1}{1-(1/\psi)}\right)} \tag{6}$$

where $W_t = C_t + P_{c,t}$, where $P_{c,t}$ is the present value of all future consumption. In the risk-sharing equilibrium, consumption levels for all countries are proportional to world output according to: $C_t^{j*} = \varpi^{jw} Y_t^w$. Therefore, country j wealth can be written: $W_t^{j*} = C_t^{j*} + P_{c,t}^{j*} = \varpi^{jw}(Y_t^w + P_t^{w*})$ and the value function becomes:

$$V_t(C_t^{j*}, W_t^{j*}) = C_t^{j*} \left[\frac{W_t^{j*}}{C_t^{j*}}\right]^{\left(\frac{1}{1-(1/\psi)}\right)} = \varpi^j Y_t^w \left[1 + \frac{P_t^{w*}}{Y_t^w}\right]^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \tag{7}$$

Note from this equation that since $W_t^{j*}/C_t^{j*} = 1 + \left(\varpi^j P_t^{w*}/\varpi^j Y_t^w\right) = 1 + (P_t^{w*}/Y_t^w)$, the wealth-to-consumption ratios are equalized across countries; that is, $\left(W_t^{j*}/C_t^{j*}\right) = \left(W_t^{i*}/C_t^{i*}\right), \forall i, j$. Thus, under perfect risk-sharing, all countries share the same welfare from the wealth-to-consumption ratio, but their consumption levels differ according to their shares in world output, ϖ^j.

Solving for the value function then requires solving for the equity prices of world output P_t^{w*} as well as country j output in world markets. Epstein and Zin (1991) derive the first-order condition for any asset ℓ using these recursive preferences as:

$$E_t \left\{\beta^\theta (C_{t+1}/C_t)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^P)^{(\theta-1)} R_{t+1}^\ell\right\} = 1 \tag{8}$$

where R_{t+1}^P is the gross return on the market portfolio paying out consumption and R_{t+1}^ℓ is the gross return on asset ℓ.

In the open economy, the market portfolio that pays out consumption for all countries is the world mutual fund so that: $R_{t+1}^P = R_{t+1}^{w*} = \left(P_{t+1}^{w*} + Y_{t+1}^w\right)/P_t^{w*}$. Since equation (8) must be satisfied for all returns, the price of the world output can be solved by setting: $R_{t+1}^\ell = R_{t+1}^{w*}$ and solving the Euler equation:

$$E_t \left\{\beta^\theta (C_{t+1}^j/C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^\theta\right\} = 1 \tag{9}$$

Similarly, the return on this common world consumption growth rate can be used to derive the prices of country output in world markets. That is, the price of country j's output at world prices is determined by the Euler equation:

$$E_t \left\{\beta^\theta (C_{t+1}^w/C_t^w)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{w*})^{(\theta-1)} R_{t+1}^{j*}\right\} = 1 \tag{10}$$

[12] See for example Campbell (1993) and Obstfeld (1994a).

where $R^{j*}_{t+1} = \left(P^{j*}_{t+1} + Y^{j}_{t+1}\right)/P^{j*}_{t}$ for P^{j*}_{t+1} is the price of country j output at world prices. Solving these two Euler equations for the price of the world output P^{w*}_{t+1} and the country price P^{j*}_{t+1} and substituting the result into the value function determines the welfare of each country under open markets.

2.3 Benchmark Economy

So far, we have described a perfect risk-sharing equilibrium. To measure the potential gains to attaining this equilibrium, we also require a benchmark measure for the current degree of risk sharing. For this purpose, we treat assets as though they were priced from a closed economy. We adopt this benchmark for four important reasons. First, this assumption is standard in the asset pricing literature that uses domestic economy Euler equations to value assets.[13] Second, this assumption is also the standard in the international risk sharing literature spanning at least two decades.[14] Third, although the benchmark model treats the asset markets as closed, the consumption and asset returns used to discipline the model incorporate the current level of integration. Since consumption and asset returns are clearly influenced by foreign factors, these foreign effects appear in the data, reflecting the current level of integration. Finally, this benchmark is likely to bias the analysis against our findings. A closed economy benchmark typically generates larger gains than a partially open economy benchmark. At the same time, our analysis below shows that the gains from international risk sharing implied by asset returns are smaller than previously thought. Therefore, using a closed economy pricing benchmark tends to make our conclusions conservative.

We now describe this benchmark economy. Each agent from country j is endowed with realizations of output from his own country given by Y^j_t. In each period, he consumes this output and then buys claims on the endowment process for the following period at price P^j_t. Thus, the agent's optimization problem is given by:

$$\underset{\substack{\{C^j_t,\varpi^{jj}_t\}\\ \forall t}}{Max}\; U^j(C^j_t, U^j_{t+1})$$

$$\begin{aligned} \text{s.t.}\quad C^j_t + P^j_t\varpi^{jj}_t &\leq W^j_t \\ \left(Y^j_{t+1} + P^j_{t+1}\right)\varpi^{jj}_t &= W^j_{t+1} \end{aligned} \tag{11}$$

[13]Moreover, recent empirical studies of assets such as equities continue to find that country effects are important. See Bekaert, Hodrick and Zhang (2009).

[14]See for example, Cole and Obstfeld (1991), Obstfeld (1994a,b), van Wincoop (1994), Tesar (1995), Lewis (2000), and Callen, Imbs, and Mauro (2011).

where $U^j(C_t^j, U_{t+1}^j)$ is given by equation (1). The agent in country j consumes his own output because the consumption good is non-durable and claims on this process cannot be sold internationally; that is, $C_t^j = Y_t^j$. Note that in this case, the price of output is also equal to the price of consumption so that $P_{c,t}^j = P_t^j$. Since the number of shares is time invariant, we normalize the number of outstanding shares to one. Clearly the agent in country j holds all of his own country's shares so that $\varpi_t^{jj} = 1$ and $\varpi_t^{ji} = 0$, $\forall i \neq j$. Therefore, assuming the budget constraint in equation (11) holds with equality, $C_t^j + P_t^j = W_t^j$. Substituting wealth and consumption into the value function (6) above, the solution to the country j agent's problem can be written:

$$V_t(C_t^{jB}, W_t^{jB}) = C_t^{jB} \left[\frac{W_t^{jB}}{C_t^{jB}}\right]^{\left(\frac{1}{1-(\frac{1}{\psi})}\right)} = Y_t^j \left[1 + \frac{P_t^j}{Y_t^j}\right]^{\left(\frac{1}{1-(\frac{1}{\psi})}\right)} \tag{12}$$

where we have used the superscript B to denote the benchmark economy equilibrium.

Solving for the value function then requires solving for P_t^j, the value of consumption from the country's output process. As above, this price can be determined from the Euler equation (8). In the benchmark case, the return on the consumption asset is $R_{t+1}^P = R_{t+1}^j \equiv \left(P_{t+1}^j + Y_{t+1}^j\right)/P_t^j$ so that:

$$E_t\left\{\beta^\theta (Y_{t+1}^j / Y_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^j)^\theta\right\} = 1 \tag{13}$$

Once the price P_t^j is determined, the value function in equation (12) gives welfare in the benchmark economy in terms of output.

2.4 Evaluating International Risk-Sharing Gains

We can now evaluate the international risk sharing gains. These gains are the permanent consumption increase in lifetime utility that would make a consumer indifferent between remaining in a benchmark economy or moving to completely open markets. Thus, the welfare gains for country j are given by Δ^j in the following:

$$V_0((1+\Delta^j)C_0^{jB}, (1+\Delta^j)W_0^{jB}) = V_0(C_0^{j*}, W_0^{j*})$$

Using the solution for the value function for the benchmark economy in equation (12)) and the value function for the perfect risk-sharing economy in equation (7), this welfare gain has the form:

$$(1+\Delta^j) = \left\{\frac{W_0^*/C_0^*}{W_0^{jB}/C_0^{jB}}\right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \left(\frac{C_0^{j*}}{C_0^{jB}}\right) = \left\{\frac{1+\frac{P_0^{w*}}{Y_0^w}}{1+\frac{P_0^j}{Y_0^j}}\right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \left(\frac{\varpi^j Y_0^w}{Y_0^j}\right)$$

In the following sections, we use consumption processes to evaluate these gains.

3 Risk-Sharing and Returns With Transitory Consumption Risk

In this section, we start to use consumption processes to determine risk sharing gains and asset prices. We begin with a standard assumption in the consumption risk sharing literature that all risk is temporary around a trend. Thus, the log output growth rate processes of each of these countries is determined by a mean growth rate μ^j, and an independent and identically distributed (I.I.D.) transitory innovation, η^j, given by:

$$g_{y,t+1}^j = \mu^j + \eta_{t+1}^j \tag{14}$$

where $\eta_{t+1}^j \backsim N(0, \sigma^j)$.

We now use this simple formulation to evaluate the welfare in the benchmark and international risk sharing equilibria. In the benchmark case, consumption equals output so that the benchmark consumption process is $g_{c,t+1}^j$ is given by equation (14).

Substituting this consumption process into the Euler equation and solving for the price of output implies:[15]

$$P_t^j = \frac{Y_t^j \beta M_j^{\left(1-\frac{1}{\psi}\right)}}{1-\beta M_j^{\left(1-\frac{1}{\psi}\right)}} \tag{15}$$

where $M_j \equiv \exp\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^2\right]$ for $\widetilde{\mu}^j = \mu^j + \frac{1}{2}\sigma^2$ or the mean of log $\left(\frac{C_{t+1}^j}{C_t^j}\right)$. Note that $-\frac{1}{2}\gamma\sigma^2$ is the effect on the logarithm of marginal utility due to variations in future utility through the recursive next period utility, $E_t\left[U_t^{(1-\gamma)}\right]$. So $\ln(M_j)$ is the logarithm of the certainty equivalent consumption path for the period utility function in the recursive preferences, $\left(U_{t+1}^j\right)^{1-\gamma}$, while $\left(1-\frac{1}{\psi}\right)$ measures how consumers value this consumption path over time. In other words, the price is the discounted present value of the country output process.

[15]Appendix B details this solution as well as the other prices used to determine the value function.

Substituting the price in equation (15) into (12) gives the value function in the benchmark economy as:

$$V_t(C_t^{jB}, W_t^{jB}) = Y_t^j \left[\frac{1}{1 - \beta M_j^{\left(1-\frac{1}{\psi}\right)}} \right]^{\left(\frac{1}{1-(\frac{1}{\psi})}\right)} \tag{16}$$

As we noted above, utility must be bounded and lie on the real line along the equilibrium path (equation (2)). Therefore, the benchmark economy equilibrium requires that $1 - \beta M_j^{\left(1-\frac{1}{\psi}\right)} > 0$, a condition that also ensures positive prices in equation (15).

Similarly, the mutual fund price in the world economy is given by:

$$P_t^{w*} = \frac{Y_t^w \beta M_*^{\left(1-\frac{1}{\psi}\right)}}{1 - \beta M_*^{\left(1-\frac{1}{\psi}\right)}} \tag{17}$$

where $M_* \equiv \exp\left[\mu^* + \frac{1}{2}(1-\gamma)\sigma^{*2}\right]$ for $\mu^* \equiv \frac{1}{J}\Sigma_{j=1}^J \mu^j$ and $\sigma^{*2} = \left(\frac{1}{J}\right)^2 \iota' \ \Sigma \ \iota$ for Σ, the variance-covariance matrix of consumption growth rates, and ι, a J-dimensional unit vector. In other words, μ^* is the mean output growth rates for an equally-weighted portfolio of J countries while σ^* is its standard deviation.[16] Note that M_* is the certainty equivalent consumption path as before, but now this path is determined by the world per capita consumption growth process.

The value function also depends upon the country's share of world output, ϖ^j, determined by the price of output from country j in world markets, P_t^{j*}. As we show in the appendix, this price depends upon the value of the growth in country j as well as the hedging properties of the country's output in world markets.

Armed with the solutions to the benchmark economy price of consumption, P_t^j, the price of world consumption, P_t^{w*}, and the price of output from country j in the world economy, P_t^{j*}, we can determine the solutions for the value functions in the two economies. Using these solutions, the welfare gain has the form:

$$(1+\Delta^j) = \left\{ \frac{W_0^*/C_0^*}{W_0^{jB}/C_0^{jB}} \right\}^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \left(\frac{C_0^{j*}}{C_0^{jB}}\right) = \left[\frac{1 - \beta M_*^{\left(1-\frac{1}{\psi}\right)}}{1 - \beta M_j^{\left(1-\frac{1}{\psi}\right)}} \right]^{-\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \frac{\varpi^j Y_0^w}{Y_0^j} \tag{18}$$

These gains therefore depend upon consumption means and variances given by the data.

[16] As above, the countries have equal weights by the assumption of a measure one of representative agents in each country. We allowing for differing population weights below. Also, forming the means and variances as a sum of the countries is not strictly correct since the sum of logs does not equal the log of sums. In the appendix, we describe Monte Carlo analysis that considers the approximation error to this assumption.

3.1 Gains: An Ilustrative Example

To develop intuition about the gains, we take the logarithm of equation (18) to yield:

$$\Delta^j \approx \ln(1+\Delta^j) = \ln\left(W_0^*/C_0^*\right)^{-\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} - \ln\left(W_0^{jB}/C_0^{jB}\right)^{-\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} + \ln\left(\frac{C_0^{j*}}{C_0^{jB}}\right)$$

This decomposition shows that the gains depend upon two components: (a) the percentage difference between the optimal risk sharing and benchmark economy wealth-to-consumption ratios in utility terms, $\ln\left(W_0/C_0\right)^{-\left(\frac{1}{1-(1/\psi)}\right)}$; and (b) the percentage difference between the risk sharing and benchmark economy initial consumption, $\ln\left(\frac{C_0^{j*}}{C_0^{jB}}\right)$.[17]

Consider an example to illustrate these two components. Assume there are two symmetric countries that have identical mean growth rates (μ) of 2%, standard deviations (σ) of 1.8% and initial outputs, Y_0.[18] Furthermore, to make the best case for diversification, assume that the correlation in consumption growth is -1. Since the countries are identical, the equilibrium prices imply equal shares in the world or $\varpi^1 = \varpi^2 = 0.5$. Thus, the second component is absent since $\ln\left(\frac{C_0^{j*}}{C_0^{jB}}\right) = 0$.

Figure 1 shows the effects on the utility and welfare gains over time assuming risk aversion of 10 and IES of 1.5.[19] The figure depicts the benchmark economy welfare in the blue line as $\ln\left(W_0^*/C_0^*\right)^{-\left(\frac{1}{1-(1/\psi)}\right)}$ while the optimal risk sharing economy welfare is given in the red line. The gains per year are depicted in the green line as the difference between the two lines. As this figure shows, the gains from risk-sharing derive from the higher wealth-to-consumption ratio in the optimal risk sharing economy. Figure 1b provides a picture of how the certainty-equivalent portion of consumption, $\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^2$, differs between the benchmark and international risk sharing economy. Clearly, the gains in this symmetric country case derive completely from the lower volatility in the open economy.

By contrast, Figures 2 consider a case when one country has a 50% higher volatility. As Figure 2a demonstrates, the gains are higher for the "High σ" country in the purple line than the "Low σ" country in the green line. The breakdown between the two components can be seen in the other panels. where we denote the high volatility country with "H" and the low volatility country with "L". First, Figure 2b shows that the gain in welfare for the low volatility country is higher

[17] In Obstfeld (1994b), the initial reallocation of consumption occurs because countries differ in the returns on their risky capital and the technology in some parts of the world are shut down.

[18] Below we show that these numbers correspond to the United States over our sample.

[19] Below we show how gains are affected for a wider range of parameters.

since the expected utility is lower in the benchmark economy. By contrast, Figure 2c shows the effect from initial shifts in consumption allocations. The benchmark economy certainty equivalent consumption paths appear in the solid lines showing that the low σ country has a steeper path than the high σ counterpart. The open economy paths are given in the dashed lines. Since the low volatility country has a higher price, it can buy more shares in the world mutual fund so that $\varpi^{Low} > \varpi^{High}$. Accordingly, the high volatility country initial consumption declines while the low volatility country consumption increases. Thus, $\ln\left(C_0^{L*}/C_0^{LB}\right) > 0$ while $\ln\left(C_0^{H*}/C_0^{HB}\right) < 0$. As the Figure 2c shows, both countries gain when this initial reallocation is combined with the gain in wealth-to-consumption.

3.2 Gains: A Three Country Version

We now analyze these gains using data for three countries: the United States, the United Kingdom, and Canada. Colacito and Croce (2010) and Stathopoulos (2011a) consider two country models focusing upon asset returns for the U.S. and the U.K. alone. However, our framework allows for multiple countries and does not require symmetry.[20] To demonstrate the effects of asymmetry and multiple countries, we begin parsimoniously by evaluating these two countries together with Canada. Below, we extend this analysis to seven OECD countries.

Since agents in our model have the same preferences over a common consumption good basket, we require data that incorporate changes in the value of this consumption across countries. Therefore, we follow Obstfeld (1994b) in analyzing annualized consumption growth adjusted for purchasing power parity deviations in the Penn World Tables from 1950 to 2009. Details of the data construction are in Appendix C.

Table 1, Panel A shows the means and standard deviations for the three country set, along with the first order autocorrelation, and cross-country correlation. The mean growth rates range from 1.96% for Canada to 2.08% for the U.S. However, the standard deviations in all three countries are large and are close to the mean growth rates. For this reason, we assume in the preliminary analysis that the mean growth rates are equal across countries. The table also shows that the first order autocorrelations are lowest for the U.S. at 0.27 and highest for the U.K. at 0.40. The table also reports the correlation matrix for consumption.

[20]Martin (2010) considers gains in a multi-country model of disaster risk, but only calibrates to one moment such as the mean of equity returns. Stathopoulos (2011b) analyzes a multiple country model of home bias but does not evaluate risk sharing.

Table 1 Panel B gives the estimates of welfare gains for risk aversion of 4 and 10 as well as intertemporal elasticity of substitition (IES) equal to 0.5 and 1.5. In order to be consistent with our analysis with persistent consumption below, we evaluate the consumption growth rates as a percent of monthly consumption.[21] For a given ψ, the welfare gains increase in risk aversion γ. For example, the welfare gains for the US increase from 0.3% to 0.9% when $\psi = 0.5$ and from 1% to 3.4% when $\psi = 1.5$.

Table 1 Panel B also reports the share of the world fund owned by each country in the open economy. As risk aversion increases, the hedging properties of each country's endowment becomes more valuable so that price shares in mutual funds begin to diverge. For example, when $\gamma = 4$ and $\psi = 0.5$, the price shares are virtually identical, ranging from 33.3% for the US and Canada to 33.4% for the U.K. But when risk aversion increases to $\gamma = 10$ and IES is $\psi = 1.5$ the U.K. receives a higher share at 33.6% compared to the U.S. at 33.1%.

Panel C of Table 1 shows the implied asset return moments from this model in addition to the data counterparts from 1971 to 2009. For brevity, we give the results for the U.S and U.K. alone, though the implied results for Canada are quite similar. First, we note the well-known equity premium puzzle (Mehra and Prescott (1985)). The equity premium over our sample in the data is 4.3% and 4.5% for the U.S. and the U.K, respectively. However, the implied equity premium is only a fraction of these numbers ranging from 0.1% to 0.3% for higher risk aversion. Second, the numbers demonstrate the inability of the standard model to generate sufficient volatility in equity returns. The standard deviation for equity returns are about 18% for the US and 23.5% for the U.K, but the model can only generate numbers less than 2%. Finally, since the implied risk-free rate is higher than the data, the numbers highlight the "risk-free rate puzzle" (Weil (1989)). Higher IES reduces the risk-free rate to below 3%, insufficient to match the rate in the U.S. data. Moreover, the model counterfactually predicts a constant risk free rate.

4 Risk Sharing and Returns with Persistent Consumption Risk

The disconnect between asset return means and the data shown above confirms the standard findings. Not only does the size of the equity premium demand a high risk aversion coefficient, but the historically low risk-free rates require a high intertemporal elasticity of substitution in

[21] Since the gains are specified as a percentage of the initial consumption levels, the gains are measured as a percent of permanent consumption as in Lucas (1987).

consumption. To match mean returns, these parameters must often be implausibly high, generating exceedingly high welfare gains.[22] Moreover, even relatively high risk aversion of 10 and IES of 1.5 cannot generate the volatility of equity returns observed in the data shown above. And the model implies a constant risk-free rate, clearly inconsistent with the data.

By contrast, persistent consumption risk provides higher volatility in asset returns and also delivers a higher equity premium and lower risk-free rate with less extreme preference parameters. Several approaches have been suggested in the literature to incorporate this risk. The leading candidates are habit persistence (Campbell and Cochrane (1999)), long-run risk (Bansal and Yaron (2004), and disaster risk (Reitz (1988), Barro (2006)). Among these, Bansal and Yaron (2006) alone use recursive preferences, permitting a straightforward comparison to the canonical model above. Therefore, we add a small persistent component to consumption growth as in the long-run risk approach.

The model in the previous section assumed that equity is an asset that pays out consumption. This assumption follows most asset pricing studies beginning with Mehra and Prescott (1985) as well as the international risk sharing counterpart. On the other hand, Bansal and Yaron (2004) assume that equity pays out dividends as measured by the data. In order to understand the implications of persistent consumption risk on risk-sharing, we continue to assume equity pays out consumption in this section. Later, in Section 4 we re-evaluate the model when equity is assumed to pay out dividends, a version closer to Bansal and Yaron (2006).

To consider the impact of persistent consumption risk on risk-sharing, we now include a persistent stochastic component x_t^j in the output growth.[23]

$$\begin{aligned} g_{y,t+1}^j &= \mu^j + x_t^j + \eta_{t+1}^j \\ x_{t+1}^j &= \rho^j x_t^j + e_{t+1}^j \end{aligned} \quad (19)$$

where $\eta_{t+1}^j \backsim N(0, \sigma^j)$ and $e_{t+1}^j \backsim N(0, \sigma_e^j)$. Since the volatility of the persistent shock, e_t^j, is very small compared to the transitory shock, η_t, we sometimes write the relationship in proportional form: $\sigma_e^j \equiv \varphi_e^j \sigma^j$. One reason risk-sharing models often assume the purely transitory process in equation (14) is that the autocorrelation in consumption growth tends to be insignificantly different

[22] See, for example, the discussion in Coeurdacier and Rey (2011). Obsteld (1994b) finds gains in excess of 100% when risk aversion matches the equity premium and growth rates differ. Lewis (2000) also finds large gains with high risk aversion even when means are the same.

[23] van Wincoop (1999) also examines risk-sharing assuming an auto-regressive consumption growth process but with no transitory component.

from zero. Indeed, when we fit the output process with persistent risk below, we find the estimates of the variance to be quite small.

4.1 Evaluating Risk-Sharing Gains with Persistent Consumption Risk

To evaluate the gains when consumption risk is persistent, we now reconsider the value function for the consumers in the benchmark economy and in the integrated world economy. We use the new output processes to determine the prices of consumption in the benchmark economy, P_t^j, and the price of consumption in world markets, P_t^{w*}, as well as the share of each country's endowment at world market prices, ϖ^j. The presence of persistent risk means that these prices can no longer be solved explicitly. In Appendix B, we detail our solution for these prices using the Campbell and Shiller (1988) approximation. Our results below use the full numerical approximation of this solution.

To help build intuition for these results, we compare the approximation to the transitory consumption risk case. As noted in equation (15), the benchmark economy price depends upon the certainty equivalent consumption path given by $M_j \equiv \exp\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^2\right]$ where $\widetilde{\mu}^j = \mu^j + \frac{1}{2}\sigma^2$. With the introduction of persistent consumption risk, a similar discount factor applies to this price where now:

$$M_j \equiv \exp\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^2\left(1 + \frac{\varphi_e^2\ k^2}{(1-k\rho)^2}\right)\right]$$

Here k is an approximating constant close to one and now $\widetilde{\mu}^j = \mu^j + \frac{1}{2}\sigma^2\left(1 + \frac{\varphi_e^2\ k^2}{(1-k\rho)^2}\right)$. In other words, certainty equivalent consumption has an additional source of risk due to the persistent consumption, captured by its variance measured by $\frac{\varphi_e^2\ k^2}{(1-k\rho)^2}$. Due to Jensen's inequality, this variance impacts the growth rate directly through $\widetilde{\mu}^j$ as well as the riskiness of the consumption path through marginal utility measured by $-\frac{1}{2}\gamma$.

Similarly, the discount factor under open economies is given by:

$$M^* \equiv \exp\left[\widetilde{\mu}^* - \frac{1}{2}\gamma\sigma^{*2}\left(1 + \frac{\varphi_e^{*2}\ k^2}{(1-k\rho)^2}\right)\right]$$

where $*$ refers to the parameters in the world portfolio.

The relationship between the certainty equivalent consumption paths in the benchmark and open economy determine the gains from integration. As in equation (18), the gains for country j

in time 0 are:

$$(1+\Delta^j) = \left(\frac{C_0^{j*}}{C_0^{jB}}\right)\left\{\frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}} = \left(\frac{C_0^{j*}}{C_0^{jB}}\right)\left[\frac{1+\frac{P_0^{w*}}{C_0^*}}{1+\frac{P_0^j}{C_0^j}}\right]^{\left(\frac{1}{1-\frac{1}{\psi}}\right)} = \frac{\varpi^j Y_0^w}{Y_0^j}\left[\frac{1+\frac{P_0^{w*}}{Y_0^w}}{1+\frac{P_0^j}{Y_0^j}}\right]^{\left(\frac{1}{1-\frac{1}{\psi}}\right)} \tag{20}$$

The price of the world mutual fund, P_t^{w*}, the benchmark economy price of country j output, P_t^j, and the open economy share of country j in world markets, ϖ^j, now include the effects of persistent consumption risk.

4.2 Identifying Country-Specific Persistent Consumption Risk

The persistent component in consumption must be small since deviations from annual consumption growth look close to transitory. Therefore, we follow Bansal and Yaron (2006) in assuming that consumption decisions are made at the monthly frequency and persistence is difficulty to detect at the annual level. To discipline our model, we choose the consumption parameter values that come closest to generating the consumption and asset return moments we observe in the data. Here we briefly summarize this identification, relegating the details to Appendix C.

To generate the parameters values, we first calibrate the monthly rates μ to the annual means of consumption growth. We then implement Simulated Method of Moments (SMM) to provide the best fit to the parameters for each country. That is, for every set of parameter values, we first solve the model using the analytical solutions for returns in the benchmark economy. We then compute the difference between a targeted set of model generated moments and the data return and consumption moments. We weight these moments equally to give the same importance to consumption and returns. The set of parameter values that minimizes this difference is the SMM estimate.

We target six data moments for each country: the standard deviation and first order autocorrelation of annual consumption growth, the mean equity premium, the mean risk free rate, the standard deviation of the market return, and of the risk free rate. Using these six moments per country, we use SMM to obtain three parameters for each country: (a) the standard deviation of the transitory component of consumption, σ^j; (b) the standard deviation of the persistent component, σ_e^j, and (c) the autocorrelation of the persistent risk component, ρ^j. In all our estimates, we find that the autocorrelation parameters ρ^j are quite similar to each other. Therefore, in the results below we set $\rho^j = \rho$ for all j across countries for parsimony.

Our SMM analysis requires a set of preference parameters. Above we showed that higher IES and risk aversion parameters help deliver the higher equity premia and lower risk-free rates shown in the data. For this reason, we continue to assume $\beta = .998$ at the monthly horizon and restrict our attention to the higher end of our parameter range with $\psi = 1.5$ and $\gamma = 10$. These parameters are also consistent with the preferred values chosen by Bansal and Yaron (2006).

Table 2, Panel A shows the resulting SMM-generated parameters of (σ^j, σ_e^j) conditional on the monthly calibrated means of consumption. The monthly growth rates, μ^j, are near 0.17% for all three countries. The transitory risk standard deviation ranges from 0.6% for the U.K. to 0.9% for the U.S. However, as measured by σ_e, persistent consumption is only a tiny fraction of transitory volatility.

Table 2, Panel B gives the targeted moments for asset returns used to fit these parameters. In addition to these moments, we target the annual mean and standard deviation of consumption growth in Table 1. Panel C reports the implied moments from our simulation. Compared to the purely transitory consumption risk in Table 1, the addition of persistent consumption risk increases the mean equity premium and lowers the risk-free rate. Also, the standard deviation of equity returns increases to 3.6% for the U.S. and the risk-free rate is no longer constant. Clearly, the data returns are much more volatile than the model implies. Nevertheless, the addition of persistent consumption risk moves the model predictions in the direction of higher equity premium, lower risk-free rate, and more volatile asset returns.

Panel C also shows the fit for consumption moments. The implied consumption volatility is higher than the data for all three countries. In the data, the standard deviation is about 1.7, but the model generates higher volatility ranging from 2.9 for the U.S. to 2.2 for Canada. On the other hand, the implied consumption autocorrelation fits the data quite well for all three countries.

Finally, Panel D shows the implications of the model for the world variability. Clearly, the volatility is reduced for both the transitory and persistent components of risk, thereby generating potential risk-sharing gains.

4.3 International Risk-Sharing Gains and Persistent Risk Correlation

Evaluating risk-sharing gains requires measures of the correlation of consumption across countries. While Table 1 provides the correlation of consumption across countries in the data, the consumption variance and covariance depends upon both the transitory shocks, η_t, and the persistent shocks, e_t. We begin by analyzing the welfare gains when the correlation between transitory shocks are assumed to be given by the data correlations as in the transitory-only case. For persistent risk correlations, we consider a wide range to understand the effects or this risk. Below, we describe an identification that pins down this correlation.

Table 3 illustrates the effects of the correlation in persistent consumption risk on the welfare gains. The top numbers for each country report the gains as a percent of permanent consumption while the numbers in parentheses below give the percent of the country's share in world output, ϖ^j. For reference, the first column gives the results using the same parameter estimates when there is no persistent risk so that $\sigma_e = 0$.[24]

The following five columns provide welfare gains numbers assuming correlations between persistent consumption ranging from 0 to 1, implying a decreasing ability to diversify this risk. When the correlation is equal to zero, the gains increase dramatically for all countries relative to the case with no persistent risk. For example, the gains for the U.S are 10.2% when $\sigma_e = 0$ but increase to 70% if persistent consumption shocks are uncorrelated. As the estimates show for increasing correlations of $Corr(e_t^i, e_t^w)$, the U.S. gains decline steadily to about 8%. Similar patterns hold for the other countries.[25]

As illustrated in Figure 2, we can decompose the gains defined in Section 2.4 into two components. The first component is the gain from the change in the wealth-to-consumption ratio: $\left\{\frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$. We report these percentage gains in Table 3 in the rows labeled "Gain from W^j/C^j" for each country. When the persistent risk is uncorrelated, the gains from wealth-to-consumption are substantially higher, ranging from about 90% for the U.S. and the U.K. to 57% for Canada. The lower measures of transitory and persistent risk volatilities for Canada means that the gains are lower than the counterpart for the other countries. As the correlation of the persistent component increases, the gains from wealth-to-consumption decline. When $Corr(e^j, e^w) = 1$, no further diversification on persistent risk is possible and the wealth-to-consumption ratio for Canada declines so that the "gain" is actually a loss of 4%.

The second component, C^{j*}/C^{jB}, captures the compensation to countries such as Canada with better diversification potential. The change in the initial consumption allocation reflects the value of each country's endowment at world prices, $\varpi^j = \left(Y_0^j + P_0^{j*}\right) / (Y_0^w + P_0^{w*})$. Thus, this

[24] These gains are higher than the I.I.D. case in Table 1 because here we target asset returns, thereby generating greater consumption volatility. We return to a comparison of gains with the same consumption volatility below.

[25] Similarly, Martin (2010) finds that the gains from risk sharing with disaster risk are much higher when disasters are not synchronized.

component is greater for countries with higher endowments and prices. The percentage gains from the initial consumption allocations are reported in Table 3 in the rows labeled "Gain from C^{j*}/C^{jB}". When persistent risk is perfectly correlated, the U.S. output is least valuable since it has the highest volatility in persistent and transitory risk as well as the greatest correlation with the world. As such, the intial value of consumption is less than the benchmark economy by -11%. By contrast, Canada's output is the most valuable and therefore the percent gain is positive at 12%. As the persistent risk correlation $Corr(e^j, e^w)$ increases to one, the declining value of the wealth-to-consumption gains to Canada are offset by the price effect.

Table 3 demonstrates that the international correlation in persistent risk is important for determining risk-sharing gains. However, these measures treat the persistent correlation as a free parameter. We next identify the cross-country correlations in both components of consumption using asset return correlations.

4.4 Identifying Persistent Risk Correlation when Equity Pays Consumption

We have shown that international risk sharing gains depend crucially upon the correlation in persistent consumption risk. We now use the model together with asset return and consumption data to identify this correlation.

In the benchmark economy, the covariance in consumption growth across countries using equation (19) can be written:

$$Cov(g_c^i, g_c^j) = \sigma^i \sigma^j Corr(\eta^i, \eta^j) + \frac{\sigma_e^i \sigma_e^j}{1-\rho^2} Corr(e^i, e^j) \tag{21}$$

Thus, the observed covariance is comprised of two sources of correlation: the component due to the temporary shock, η, and the persistent shock, e, where $1-\rho^2$ adjusts for the autocorrelation

We now turn to the correlation of equity returns generated by the model. As we show in the appendix, the Campbell-Shiller approximation of returns imply that equity returns for country i can be written in the form:

$$R_{t+1}^i = a_0^i + a_1^i x_t^i + a_2^i e_{t+1}^i + \eta_{t+1}^i \tag{22}$$

where a_0^i, a_1^i, a_2^i are constants. Calculating the covariance of equity returns across countries provides a second observable depending on consumption correlations:

$$Cov(R^i, R^j) = \sigma^i \sigma^j Corr(\eta^i, \eta^j) + \left[\frac{a_1^i a_1^j}{1-\rho^2} + a_2^i a_2^j \right] \sigma_e^i \sigma_e^j Corr(e^i, e^j) \tag{23}$$

Note that equity covariances and consumption covariance depend identically upon the transitory correlation, $Corr(\eta^i, \eta^j)$. However, the variability in returns also depends upon the current level of persistence risk through the two terms in square brackets. First, it depends upon the current level of persistent risk, x_t, measured by the autoregressive effect $a_1^i a_1^j/(1-\rho^2)$. Second , it depends upon the current innovation in persistent risk through $a_2^i a_2^j$.

Given the two observable covariances in consumption growth in equation (21) and equity returns in equation (23), we can identify the two sets of correlations, $Corr(\eta^i, \eta^j)$ and $Corr(e^i, e^j)$, for each pair of covariances across countries.[26] Combining the consumption covariances in equation (21) with the equity covariance in equation (23), we solve for the correlation in the persistent shock as:

$$Corr(e^i, e^j) = D_o \frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} \left[Corr(R^i, R^j) - \frac{\sigma_c^i \sigma_c^j}{\sigma_R^i \sigma_R^j} Corr(g_c^i, g_c^j) \right] \tag{24}$$

where $D_o \equiv \left[\frac{a_1^i a_1^j - 1}{1-\rho^2} + a_2^i a_2^j \right]^{-1}$. In the appendix, we show that $D_o > 0$.

Equation (24) highlights the implications of consumption and equity covariances for the correlation on persistent risk. As the correlation in equity returns, $Corr(R^i, R^j)$, increases relative to the correlation in consumption, $Corr(g_c^i, g_c^j)$, the implied correlation of persistent shocks rises. Furthermore, this effect is exacerbated if the variability in equity returns, σ_R^i exceeds the variability in consumption, σ_c^i .

Table 4 Panel A reports the equity return correlations in the data. The correlations between equity returns are generally higher than the correlations between consumption growth rates in Table 1.[27] In particular, the equity return correlations are higher than 0.5. By contrast, the correlations between consumption growth rates are lower than 0.5 with the exception of the correlation between Canada and the U.S. The pattern for equity return correlations to be higher than consumption correlations is even more pronounced when we expand the set of countries below. At the same time, the variability of stock returns is much higher than the variability of consumption growth rates. As a result, the relationship between equity and consumption correlations generates high correlations for the persistent consumption risk, $Corr(e^i, e^j)$. This tendency is reinforced by the much lower variability of the persistent risk relative to equity returns, implying that $\frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} >> 1$. Therefore, the combination of equity and consumption covariances imply high correlation of e_t^i

[26] The appendix provides details about this construction.

[27] Dumas et al (2003) also find that equity correlations across countries are higher than output correlations, and use this observation to analyze the degree of integration. Bansal and Lundblad (2002) use the high international correlation in equity returns to argue that cash flow growth rates contain a small predictable component.

across countries.[28] Indeed, across all seven of our countries studied and all versions of our model, the implied correlations for persistent risk are never below 0.8.

Table 4 Panel B then shows that the implied correlations for persistent and transitory risk. As expected, the combination of consumption and equity covariances imply a very high degree of correlation in persistent risk near one. In the interest of parsimony, we report only the correlation of each country against the world. Panel B also shows the implied correlation on the transitory risk. Comparing these correlations to the consumption correlations in Table 1 shows that the high correlations on persistent risk generate lower correlations on the transitory risk.

Panel C of Table 4 gives the welfare gains based upon the implied consumption correlations. Since the identified correlations on the persistent component are essentially equal to one, persistent risk is already fully diversified thereby attenuating the welfare gains. The gains range from 7.8% for Canada to 9.4% for the U.K. Once again, the slightly lower volatility and better hedging properties of Canadian consumption mean that Canada loses on the change in wealth to consumption ratio at −4%, but gains on the value of its endowment implying a higher gain from initial consumption at 12%. By contrast, both the U.S. and the U.K. gain from the world wealth to consumption ratio, but lose from initial risk-sharing consumption relative to the benchmark economy consumption at −7% and −5%, respectively.

4.5 Summary: Persistent Risk with Consumption-Paying Equity

In this section, we examined the gains from risk sharing when asset returns are used to discipline consumption parameters. We assumed that equity pays out consumption as measured by the data. To determine the correlations of persistent versus transitory consumption across countries, we used data on correlations in equity and consumption. Since cross-country equity correlations are higher than consumption correlations and since the volatility of equity is higher than consumption, the model implied high correlations in persistent consumption risk. As a result, this risk is almost completely diversified, even without fully open markets.[29]

Although this model generated better asset pricing implications than the transitory-only case, the fitted asset return and consumption moments remain far from the data. In the next section, we address a version of this model to improve the fit.

[28] Since the standard deviations of e and η are fitted to the data, the implied correlations of the components can in principle exceed 1. In such instance, we restrict the correlations to equal 1.

[29] Similarly, Brandt, Cochrane and Santa-Clara (2006) argue that the high correlations in exchange rates imply consumption risk is well-diversified.

5 Risk Sharing and Dividend-Paying Asset Returns

So far, we have assumed that equity returns pay out consumption, following in the tradition of Mehra and Prescott (1985) and Obstfeld (1994). However, as the analysis above shows, even persistent consumption risk does not generate a sufficiently high equity premium or volatility in returns. Moreover, the better fit for asset returns comes at the cost of higher variability in consumption than observed in the data. Bansal and Yaron (2004) (hereafter BY) have argued that equity returns are better explained when persistence consumption risk depends upon dividend payments. In this section, we employ this framework to identify consumption risk and then re-examine the implications for international consumption risk-sharing.

5.1 Persistent Consumption Risk and Dividends

We now reconsider the consumption growth process with persistent consumption risk in equation (19) repeated here for convenience.

$$\begin{aligned} g^j_{c,t+1} &= \mu^j + x^j_t + \eta^j_{t+1} \\ x^j_{t+1} &= \rho^j x^j_t + e^j_{t+1} \end{aligned}$$

where $\eta^j_{t+1} \sim N(0, \sigma^j)$, $e^j_{t+1} \sim N(0, \sigma^j_e)$, $\eta^j_{t+1} \perp e^j_{t+1}$. In order to match asset return behavior, BY fit the behavior of dividends and consumption growth rates to the implied estimates of asset return moments. For this purpose, they assume that the growth rate of dividends, $g_{d,t}$, depends upon the persistent component of consumption. Using a superscript to identify the country j, we rewrite their assumed dividend process as:

$$g^j_{d,t+1} = \mu^j_d + \phi^j x^j_t + u^j_{t+1} \tag{25}$$

where $u^j_{t+1} \sim N(0, \sigma^j_u)$, $u^j_{t+1} \perp \eta^j_{t+1} \perp e^j_{t+1}$ and μ^j_d is the growth rate of dividends. Note that in equation (25), dividends depend upon persistent consumption risk according to the coefficient ϕ^j. BY argue that this coefficient acts as a leverage ratio that relates dividends to consumption.

5.2 Identifying Country-Specific Consumption Risk with Dividends

We now amend our asset return model to assume equity pays the dividend process (equation (25)). As BY have observed, the variability in dividends along with the leverage ratio helps generate greater variability in persistent consumption growth, thereby generating better fit to asset returns.

We therefore use the model to provide fitted values for the dividend parameters along with new estimates of the original consumption parameters. For this purpose, we add to the set of target moments the standard deviation and growth rate of dividends. As with consumption growth, we first calibrate the monthly growth rate of dividends. We then use SMM to fit the four parameters $\left[\sigma^j, \sigma_e^j, \rho^j, \sigma_d^j,\right]$ to eight moments: the prior six consumption and asset return moments augmented by the dividend growth standard deviation and autocorrelation. As before, the fitted values for ρ^j are quite close to each other across countries so we restrict them to be equal in our reported results.

Table 5 reports the analysis for the three countries. Panel A gives the parameter estimates. Compared to the consumption asset model in Table 2, the variability due to persistent risk is higher for all three countries at around 0.04%. As equation (25) shows, this higher volatility is in part generated by the greater volatility of dividends as well as the leverage ratio, ϕ^j. In our analysis, we follow Bansal and Yaron (2004) in setting this parameter to 3.

Table 5, Panel B shows the asset return and dividend target moments for the SMM analysis. The asset return moments are the same as before: the equity premium, equity standard deviation, risk-free rate means, and standard deviations. The analysis also continues to target the standard deviations and first order autocorrelation of consumption. However, we now add two moments from dividend growth: the standard deviation and the first order autocorrelation.

Panel C provides the best fit estimates from SMM. The fitted equity premium is now close to the data at 5% for the U.S. and 6.5% for Canada, though the number for the U.K. is somewhat larger than the data. Similarly, the estimates of the risk-free rate are now closer to the data. Importantly, the standard deviation of equity is close to the data with implied estimates between 15% to 18.5%. The standard deviation of the risk free rate is also higher, though still considerably lower than the data suggest.[30] The model also tends to predict a more volatile dividend process for the U.S. and Canada as well as somewhat greater persistence. As in the consumption asset case, the implied consumption volatility and autocorrelation is higher than in the data. Nevertheless, compared to the prior model, the dividend-based model gets closer to matching target moments.

[30] Bansal and Yaron (2004) address this issue by assuming stochastic volatility. For parsimony, we do not include this risk in the present paper. Nevertheless, the high degree of correlation across countries in volatility measures suggests that this risk is also highly diversified.

5.3 Identifying Persistent Risk Correlation when Equity Pays Dividends

We now use the newly fitted parameters to re-evaluate international risk-sharing gains. As before, we require additional restrictions from equity returns to identify the correlation in persistent consumption risk. When equity pays out dividends, we show in the appendix that the Campbell-Shiller approximation of returns imply that equity returns for country i can be written in the form:

$$R_{t+1}^i = b_0^i + b_1^i x_t^i + b_2^i e_{t+1}^i + u_{t+1}^i \tag{26}$$

where b_0^i, b_1^i, b_2^i are constants. Note that by contrast to the case when equity pays out consumption, returns now depend upon the innovation to dividend growth, u_{t+1}^i, instead of the innovation to transitory consumption, η. Calculating the covariance of equity returns across countries as before implies:

$$Cov(R_t^i, R_t^j) = \sigma_u^i \sigma_u^j Corr(u^i, u^j) + \left[\frac{b_1^i b_1^j}{1-\rho^2} + b_2^i b_2^j \right] \sigma_e^i \sigma_e^j Corr(e^i, e^j) \tag{27}$$

Comparing the covariance in equation (27) with the implied covariances when equity pays out consumption in equation (23) shows that the relationships are similar except that the correlation and volatility in transitory dividend shocks (u) replace their counterparts for transitory consumption shocks (η).

Therefore, to identify the effects of dividend shocks, we require the covariance of dividends across countries. Using the dividend growth process (equation (25)), this covariance is:

$$Cov(g_{d,t}^i, g_{d,t}^j) = \sigma_u^i \sigma_u^j Corr(u^i, u^j) + \phi^i \phi^j \frac{\sigma_e^i \sigma_e^j}{1-\rho^2} Corr(e^i, e^j) \tag{28}$$

Note that the dividend covariance in equation (28) has the same form as the covariance of consumption growth in equation (21) with two important changes. First, the covariance in transitory consumption shocks is replaced by the covariance in transitory dividend shocks. Second, "leverage" parameters $\phi^i \phi^j$ now appear in the second term, reflecting the covariance of persistent consumption risk.

Given the covariances in equity returns and in dividends, we can now solve for the correlation in persistent consumption risk, $Corr(e^i, e^j)$ as before:

$$Corr(e^i, e^j) = B_o \frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} \left[Corr(R^i, R^j) - \frac{\sigma_d^i \sigma_d^j}{\sigma_R^i \sigma_R^j} Corr(g_d^i, g_d^j) \right] \tag{29}$$

where $B_o \equiv \left[\frac{b_1^i b_1^j - \phi^i \phi^j}{1-\rho^2} + b_2^i b_2^j \right]^{-1}$. In the appendix, we show that $B_o > 1$ when $\phi^i \phi^j > 1$, a condition that is satisfied by the BY assumptions that $\phi = 3$. Thus, equation (29) illustrates a similar relationship as in the consumption asset case. As the correlation in equity returns, $Corr(R^i, R^j)$, increases relative to the correlation in dividends, $Corr(g_d^i, g_d^j)$, the implied correlation of persistent shocks rises, an effect reinforced when the variability in dividends, σ_d^i, is less than that of equity returns, σ_R^i.

Table 6, Panel A reports the dividend correlation while Panel B gives the equity return correlation. Consistent with the pattern observed between equity returns and consumption, the correlations between equity returns are higher than dividends. Furthermore, as previously reported, the standard deviations of dividends are smaller than the standard deviations of equity returns. As a result, Panel C shows that the implied correlations on persistent consumption risk are all equal or close to one. The high correlations on persistent risk also identify a lower correlation on transitory risk, since the $Corr(\eta^i, \eta^j)$ estimates in Panel C are all lower than their data counterparts (in Table 1).

Panel D of Table 6 gives the gains implied by this decomposition, ranging from 2.7% for the U.S. and Canada to 4.2% for the U.K. When equity is assumed to pay dividends instead of consumption, the implied standard deviation on persistent consumption, σ_e, is lowest for the U.K.. Thus, many of the features previously observed for the lowest persistent risk country, Canada, hold here for the U.K. In particular, the U.K. has the highest share of world output at 38.3%. Furthermore, the percentage certainty equivalent change from the wealth-to-consumption ratio worsens for the U.K. at −9% while both the U.S. and Canada gain at 13% and 9%, respectively. At the same time, the U.K. gains from an improvement of initial consumption of 15% relative to the benchmark economy, while this ratio is lower for both the U.S. and Canada.

5.4 Persistent Risk and Risk-Sharing Gains: Uncovering the Channels

Comparing the gains from risk-sharing when equity pays consumption and when equity pays dividends highlights a surprising pattern. Although the dividend case implies greater persistent risk, the risk-sharing gains are uniformly lower. That is, Table 4C shows that the gains from risk-sharing range from around 8% to 9.5% when equity pays consumption. But Table 6D reports the counterpart gains when equity pays dividends at around 3% to 4%. This result might seem counterintuitive since greater persistent risk should make international diversification more valuable.

A problem with this comparison is that the two scenarios differ in other respects as well. Importantly, the model implied consumption variability is lower in the dividend asset case than the consumption asset case. The standard deviation of monthly consumption ranges from 0.9% to 0.7%

when equity pays consumption (Table 2 Panel A) but is lower at 0.5% to 0.6% when equity pays dividends (Table 4 Panel A). Thus, the lower gains may simply reflect lower overall consumption risk.

To disentangle these two effects, we conduct a thought experiment. We constrain the data consumption volatility σ_{gc} to the US estimates in the two asset cases. We then increase the volatility of persistent risk and recalculate the gains. Figure 3a illustrates the results. Strikingly, the gains for both cases decline as the persistent risk increases. Moreover, due to the lower volatility, the dividend case remains everywhere below the consumption asset. The triangles mark the fitted numbers from the table. Clearly, whether the data volatility were higher, as in the consumption case or lower as as in the dividend case, higher persistent risk would reduce welfare gains.

Therefore, the lower gains in the dividend case arise from greater persistent risk and not lower consumption volatility. The intuition is clear. When persistent risk is almost perfectly diversified, an increase in the volatility of persistent risk dampens the gains to diversification. Greater persistent volatility implies lower transitory volatility. Therefore, the transitory diversifiable risk is reduced.

To verify this conjecture, we consider a counterfactual experiment. We conduct the same experiment as depicted in Figure 3a, but assume instead that the correlation on persistent risk is 0.8 instead of 1. Thus, some persistent risk can be diversified. The pattern is shown in Figure 3b. As the variance of the persistent shock σ_e increases, the welfare gains now increase since this risk is diversifiable. When persistent risk can be diversified, the dividend asset case has the greatest gains, highlighting once again the role of persistent risk.

5.5 Summary: Persistent Risk with Dividend-Paying Equity

In this section, we re-evaluated the model assuming that equity pays out dividends as measured by the data. This version of the model provided better fits for asset return and consumption moments. It also required a new identification of the correlations of persistent consumption across countries based on dividends. As with the consumption case, however, we found that persistent consumption risk is almost completely diversified, even without fully open markets.

Comparing the two versions of the model generated surprising results. Despite greater persistent risk when equity pays dividends, the gains were lower than the consumption asset case. Further analysis yielded a straightforward explanation, however. Higher volatility in persistent risk implies lower volatility in transitory risk as measured by the data. Since the model implies the persistent risk is essentially diversified, the lower risk on the diversifiable component means lower gains.

6 Risk-Sharing and Other Considerations

In order to understand key features of risk-sharing across countries, we have focused upon a number of simplifying assumptions. First, we have assumed that all countries have the same mean growth rates. Second, we have treated all countries as though they are the same size. Third, we have considered a small set of three countries. In this section, we relax all three of these assumptions.

6.1 Differing Means

In the quantitative analysis, we have so far assumed common growth rates across countries. Here we consider the effects of relaxing this assumption.

The effects on welfare gains are straightforward. The price of a country's output in the open economy is increasing in the mean growth rate. We described this relationship for the transitory only case in Section 2 as well as the persistent case in Section 3. At the same time, a higher growth rate economy will not benefit as much from the common growth rate in the open economy since it must share the lower growth rates of the others.

Table 7 shows that this intuition holds in our quantitative analysis as well. Panel A repeats the mean annualized growth rates in Table 1 showing that the U.S. has the higher growth rate in the sample at about 2.1%. Panels B and C repeat the analysis with differing μ^i for Table 4 when equity pays consumption and for Table 6 when equity pays dividends, respectively.

Compared to the common means analysis, the U.S. receives a greater share of world output but also has a lower welfare gain than the other countries. For the dividend asset case, for example, when mean growth rates are common as in Table 6, the share of world output is 30.2% but this share increases to 31.3% with the higher U.S. mean in Table 7. At the same time, the gains to the U.S. decline from 2.7% with common means in Table 6 to 2.3% with the differing means in Table 7. Overall, allowing for differing means imply a higher world share for high growth countries, but also lower welfare gains as they share in a lower growth world economy.

6.2 Differing Sizes

Above we treated the three countries as though they were all the same size, though this assumption is clearly counterfactual. Since our consumption data are measured in per capita units, we can easily recover aggregate consumption by multiplying population size. Accounting for differing sizes requires a modification of the framework described in Section 2. Here we describe the new equilibrium as a decentralized economy. Appendix A shows that this equilibrium is also the solution to a social planner's problem that puts equal weight on each person in the population.

6.2.1 The modified equilibrium

Each person in each country is endowed with the claim to one unit of *per capita* output in his home country, Y_t^j, $\forall t$. Defining the number of people in country j as N^j, total output in country j is $N^j Y^j$. In the perfect risk-sharing economy, each person owns the per capita output of his own country. Thus, there are now N^j claims to output of country j available. At time 0, each person in country j sells his share and purchases shares in the world output process. Thus, the budget constraint for country j as a whole implies

$$N^j \left(Y_0^j + P_0^{j*} \right) = N^w \left(Y_0^w + P_0^{w*} \right) \varpi_0^{jw}$$

where total population is $N^w = \Sigma_{j=1}^J N^j$. Solving for the share of country j agents in world markets then implies: $\varpi^{jw} = n^j \left(Y_0^j + P_0^{j*} \right) / \left(Y_0^w + P_0^{w*} \right)$ where $n^j \equiv N^j / N^w$. That is, the share of country j in the world market is equal to its share in the world wealth, $\frac{Y_0^j + P_0^{j*}}{Y_0^w + P_0^{w*}}$, multiplied by its share in world population, n^j.

Using these population-weighted shares, the world parameters become $\mu^* \equiv \Sigma_{j=1}^J n^j \ \mu^j$ for the mean growth rate and $\sigma^{*2} = \left(\frac{1}{J}\right)^2 n' \ \Sigma \ n$ and $\sigma_e^{*2} = \left(\frac{1}{J}\right)^2 n' \ \Sigma_e \ n$ for the transitory and persistent variance, respectively, where n is the j dimensioned vector of population shares and Σ and Σ_e are the variance-covariance matrices of transitory and persistent shocks, respectively.

6.2.2 The implied gains

Table 8 reports the results assuming differing country sizes. Panel A gives the new parameters based upon fitting the model with population differences. Panels B and C repeat the analysis based on these population weights when equity pays consumption and when equity pays dividends, respectively. For both of these cases, the price-weighted gains do not provide a steady-state equilibrium because one or more countries have unbounded utility, violating the equilibrium condition (equation (2)).

Since the pricing equilibrium does not have a steady state, we consider a more general the set of Pareto efficient allocations.[31] In particular, we report the gains if each country receives all the gains while leaving every other country indifferent. Thus, the upper bound in gains for each efficient equilibrium is given by calculating for country j the gains from receiving all of the initial surplus consumption allocation while setting $\Delta^i = 0, \forall i \neq j$. Thus, we first solve for $\widehat{C}_0^{i*}$, the initial consumption allocations for residents in country i that implies no gains for residents of all countries except j.

$$1 = \left(\frac{\widehat{C}_0^{i*}}{Y_0^i} \right) \left\{ \frac{W_0^*/C_0^*}{W_0^{iB}/C_0^{iB}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{30}$$

where W_0^{iB}/C_0^{iB} are calculated from the benchmark economy as above. Similarly, the open economy wealth-to-consumption ratios W_0^*/C_0^* must be the same among the set of efficient allocations since state prices are equalized in the competitive equilibrium. The upper bounds on country j welfare among this set of allocations is then given by:

$$(1 + \Delta_{Max}^j) = \left(\frac{\widehat{C}_{0,Max}^{j*}}{Y_0^j} \right) \left\{ \frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} = \left(\frac{Y_0^w - \sum_{i \neq j} n^j \widehat{C}_0^{i*}}{n^j Y_0^j} \right) \left\{ \frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \tag{31}$$

The set of efficient allocations for residents in each country i are then bracketed by the minimum consumption, $\widehat{C}_0^{i*}$, that will give zero welfare gains and the maximum consumption, $\widehat{C}_{0,Max}^{i*}$, that will give all the world welfare gains to country i.

Panel A shows the efficient set range of gains under the consumption asset case for each country. The first row shows the gains from the improvement in the wealth-to-consumption ratio. As before these are positive for the U.S. and U.K., but negative for Canada. The remaining rows show the range in gains depending upon which country receives all the surplus. For the U.S, the gains are as high as 8.8% when Americans receive all the surplus at 71% of world output. On the other hand, when the US is indifferent, thereby receiving no gains, that share of world output is 65%. By contrast, Canada loses on the wealth-to-consumption ratio but if compensated to the maximum share of 13% of world output, receives a large 78.6% gain. Similar patterns hold for the dividend asset case in Panel B. In this case, the U.K. has better hedge properties so its role switches with Canada. Overall, the results in Table 8 highlight the scope for gains to trade across countries.

[31] Details are in Appendix A.

6.3 More Countries

In the analysis so far, we have focused upon a small group of countries comprised of three countries. This analysis demonstrates how the framework can expand the number of countries over the two-country models of Colacito and Croce (2010) and Stathopoulos (2011a). In principle, however, the multi-country framework described in Section 2 applies to an arbitrary number of countries. To show the analysis with more countries, we now apply our framework over seven countries. For our analysis, we consider the three countries above and include Australia, France, Germany, and Japan.[32]

As above, we consider the effects of persistent consumption risk under the two alternative assumptions that link equity returns to the data: (1) equity pays out consumption; and (2) equity pays out dividend. We first use the target moments to try to fit consumption parameters for the new countries. We then use these parameters together with the parameters for the U.S., U.K., and Canada above to re-evaluate the risk-sharing gains. In the interest of parsimony, we only report the results for the dividend asset case since it fits returns better.

6.3.1 Identifying Persistent Consumption Risk for Additional Countries

We first implement our Simulated Method of Moments approach on consumption and asset return data for the four new countries. Table 9 Panel A reports the set of consumption and dividend parameters $[\mu, \sigma, \sigma_e, \sigma_{gc}, \mu_d, \sigma_d]$. Panels B and C give the set of target data moments and implied moments, respectively. The variability in persistent consumption risk σ_e, is similar across countries. Although Japan has the lowest variability of persistent consumption, it also has the highest variability in transitory consumption risk. Note that to fit asset returns, implied consumption variability is higher than the data as found for the other three countries. Moreover, while the autocorrelation in consumption is close to the data for most countries, it is clearly too high to match the tiny autocorrelation for Australia.

6.3.2 Risk-Sharing Gains with More Countries

We next consider the implications for risk-sharing using the fitted parameters for the four new countries together with the corresponding parameters for the U.S., U.K., and Canada previously reported in Table 5. Panel D of Table 9 shows these results. The first column reports the data

[32] This set of countries is close to the Group of Seven countries analyzed in Lewis (2000), except that Australia replaces Italy to provide a wider geographic coverage of industrialized countries.

correlations between dividends. To preserve space, we simply report the correlation of dividends for each country and the world, although the full matrix is used in estimation. The correlations demonstrate the low correlations in dividends relative to those between equity returns, noted earlier. All correlations are less than 0.55 and that of the UK is as low as 0.33. The next two columns report the implied correlations between the world and country persistent shock, e^i as well as the transitory shock, η^i. Once again, the correlations in persistent risk are very high and close to one.

The final columns show the welfare gains. As in the population-weighted case, the decentralized economy does not have a steady state equilibrium. The U.K. has a low persistent and transitory risk volatility and its share is correspondingly high. Thus, with the expanded set of countries, its price becomes undefined.

We therefore report the range of Pareto efficient allocations. Under the column labeled "Gains", we report the maximum gain for the row country while setting the gains for all other countries equal to zero. The following three columns report the maximum share of output for that country when setting all other country gains to zero along with the gain due to increases in wealth-to-consumption "W/C" and the change in initial consumption "C^*/C^B". For example, the gain for the U.S. is 127% when the gains are zero for all other countries so that residents receive 28% of world per capita income. The gains from wealth-to-consumption are only 16% while the gains from receiving initial consumption is 96%. On the other hand, the last column reports the lowest world consumption share so that the U.S. is not made worse off in the world economy. At 12%, this share is significantly lower than the maximum.

The welfare gains may appear high relative to earlier tables, but the reasons are clear. First, the reported gains are the maximum if all surplus were given to one country. For the U.S. gains, for example, dividing by seven would imply an average gain per country of only about 17%. Second, the gains are larger because there are more countries, increasing the potential gains from trade.

7 Conclusion

International asset returns incorporate market valuations of risk and these valuations are central to understanding potential gains from global consumption risk sharing. Nevertheless, most international risk sharing studies ignore the implications of these markets. In this paper, we have begun to bridge this gap by noting how features that bring the model closer to data impact views about risk sharing.

Low frequency variations in consumption risk are key to generating the size of equity premia and volatility of asset returns. In this paper, we consider these variations as a small but persistent component of consumption shocks. For this purpose, we use data on consumption, asset returns, and in the final version, dividends to determine the best fit for seven industrialized economies. Our analysis produces three main findings.

First, we find that the magnitude of risk-sharing gains depend inversely on the degree of correlation in persistent consumption risk across countries. In other words, the consumption risk-sharing gains increase with the ability to diversify persistent risk.

Second, we provide an identification for the persistent risk correlation using consumption and equity return correlations across countries. This identification implies high correlations on persistent risk and, hence, a low diversification potential. In the data, equity return correlations are higher than consumption correlations across countries. In the model, equity returns and, hence, their correlations depend more strongly on the persistent risk component than the transitory risk component. Taken together, the model implies a high correlation in persistent risk.

Third, we show that higher volatility in persistent risk reduces the implied gains from risk sharing. Once we disentangle the diversification benefits of transitory versus persistent risk, the intuition is clear. Greater volatility in persistent risk implies lower volatility in transitory risk. Since persistent risk is already diversified, only transitory risk can be shared. Higher persistent risk therefore implies lower diversifiable transitory risk, thereby reducing risk sharing gains. Thus, significant international consumption risk is already shared.

Overall, our results overturn conventional views about the gains from international consumption risk sharing when disciplined by asset returns. Calibrating models to asset return moments such as the equity premium do not translate into significantly higher risk sharing gains. Indeed, incorporating persistent risk implies lower risk sharing gains even though this risk helps fit a number of asset return moments. Thus, this paper provides a new departure in understanding international consumption risk sharing.

References

[1] Backus, David K; Kehoe, Patrick J; Kydland, Finn E., *International Real Business Cycles*, Journal of Political Economy, vol. 100, no. 4, August 1992, pp. 745-75

[2] Barro, Robert J., *Rare Disasters, Asset Prices, and Welfare Costs*, American Economic Review, vol. 99, no. 1, March 2009, pp. 243-264.

[3] Bansal, Ravi; Lundblad, Christian, *Market Efficiency, Asset Returns, and the Size of the Risk Premium in Global Equity Markets,* Journal of Econometrics, vol. 109, 2002, pp. 195-237.

[4] Bansal, Ravi; Yaron, Amir, *Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles*, Journal of Finance, vol. 59, no. 4, August 2004, pp. 1481-1509

[5] Baxter, Marianne; Crucini, Mario J., *Business Cycles and the Asset Structure of Foreign Trade*, International Economic Review, vol. 36, no. 4, November 1995, pp. 821-54

[6] Baxter, Marianne; Jermann, Urban J., *The International Diversification Puzzle Is Worse Than You Think*, American Economic Review, vol. 87, no. 1, March 1997, pp. 170-80

[7] Bekaert, Geert; Hodrick, Robert; Zhang, Xiaoyan, *International Stock Return Comovement*, Journal of Finance, vol. 64, no. 6, December 2009, pp. 2591-2626

[8] Brandt, Michael W; Cochrane, John H; Santa-Clara, Pedro.*International Risk Sharing Is Better Than You Think, or Exchange Rates Are Too Smooth,* Journal of Monetary Economics, vol. 53. no. 4 May 2006, pp.671-698.

[9] Callen, Michael; Imbs, Jean; Mauro, Paulo, *Pooling Risk Among Countries*, Paris School of Economics Working Paper, July 2011.

[10] Campbell, John Y., *Consumption-Based Asset Pricing*, "Handbook of the Economics of Finance" Vol. IB, George Constantinides, Milton Harris, and Rene Stulz eds., North-Holland, Amsterdam, 2003.

[11] Campbell, John Y., *Intertemporal Asset Pricing without Consumption Data*, American Economic Review, vol. 83, no. 3, June 1993, pp. 487-512

[12] Campbell, John Y; Cochrane, John H., *By Force of Habit: A Consumption-Based Explanation of Aggregate Stock Market Behavior*, Journal of Political Economy, vol. 107, no. 2, April 1999, pp. 205-51

[13] Campbell, John Y; Shiller, Robert J., *The dividend-price ratio and expectations of future dividends and discount factors*, Review of Financial Studies, vol. 1,1988, pp. 195-227.

[14] Coeurdacier, Nicolas; Rey, Helene; *Home Bias in Open Economy Financial Macroeconomics*, Working Paper, London Business School, September 2011.

[15] Colacito, Riccardo; Croce, Mariono M. *The Short and Long Run Benefits of Financial Integration,*American Economic Review, vol. 100, no. 2, May 2010, 527-531.

[16] Colacito, Riccardo; Croce, Mariono M. *Risks for the Long Run and the Real Exchange Rate*, Journal of Political Economy, vol. 119, no. 1, February 2011, 153-181.

[17] Cole, Harold L; Obstfeld, Maurice, *Commodity Trade and International Risk Sharing: How Much Do Financial Markets Matter?*, Journal of Monetary Economics, vol. 28, no. 1, August 1991, pp. 3-24

[18] Dumas, Bernard; Harvey, Campbell R; Ruiz, Pierre. *Are Correlations of Stock Returns Justified by Subsequent Changes in National Outputs?* Journal of International Money and Finance, vol. 22. no. 6, Nov 2003, pp. 777-811.

[19] Epstein, Larry G; Zin, Stanley E., *Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework*, Econometrica, vol. 57, no. 4, July 1989, pp. 937-69

[20] Gallant, A Ronald; Tauchen, George.*The Relative Efficiency of Method of Moments Estimators* Journal of Econometrics vol. 92, no.1, Sep 1999, pp. 149-172.

[21] Hansen, Lars P. Dynamic *Valuation Decomposition within Stochastic Economies* Econometrica, forthcoming.

[22] Jonathan Heathcote; Perri, Fabrizio, *The international diversification puzzle is not as bad as you think*, University of Minnesota Working Paper, April 2008.

[23] Ingersoll, Johnathan E., *Theory of Financial Decision Making*, Rowman and Littlefield, 1987.

[24] Lewis, Karen K., *Why Do Stocks and Consumption Imply Such Different Gains from International Risk Sharing?*, Journal of International Economics, vol. 52, no. 1, October 2000, pp. 1-35

[25] Lewis, Karen K.,*Trying to Explain Home Bias in Equities and Consumption*, Journal of Economic Literature, vol. 37, no. 2, June 1999, pp. 571-608.

[26] Lucas, Robert E., Jr., *Models of Business Cycles*, Oxford: Basil Blackwell, 1987.

[27] Lucas, Robert E., Jr.; Stokey, Nancy L. *Recursive Methods in Economic Dynamics*, Harvard University Press, 1989.

[28] Martin, Ian, *How Much Do Financial Market Matter? Cole-Obstfeld Revisited,* Stanford University, Working Paper, November 2010.

[29] Meese, Richard A; Rogoff, Kenneth, *Empirical Exchange Rate Models of the Seventies: Do They Fit Out of Sample?*, Journal of International Economics, vol. 14, no. 1-2, February 1983, pp. 3-24

[30] Mehra, Rajnish; Prescott, Edward C., *The Equity Premium: A Puzzle*, Journal of Monetary Economics, vol. 15, no. 2,March 1985, pp. 145-61

[31] Obstfeld, Maurice, *Risk-Taking, Global Diversification, and Growth*, American Economic Review, vol. 84, no. 5, December 1994b, pp. 1310-29

[32] Obstfeld, Maurice, *Evaluating Risky Consumption Paths: The Role of Intertemporal Substitutability*, European Economic Review, vol. 38, no. 7, August 1994a, pp. 1471-86

[33] Rietz, Thomas A., *The Equity Risk Premium: A Solution*, Journal of Monetary Economics, vol. 22, no. 1, July 1988, pp. 117-131

[34] Stathopoulos, Andreas, *Asset Prices and Risk Sharing in Open Economies*, Working Paper, University of Southern California, January 2011a

[35] Stathopoulos, Andreas, *Portfolio Home Bias and External Habit Formation*, Working Paper, University of Southern California, November 2011b

[36] Stockman, Alan C; Tesar, Linda L., *Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements*, American Economic Review, vol. 85, no. 1, March 1995, pp. 168-85

[37] Tesar, Linda L., *Evaluating the Gains from International Risksharing*, Carnegie-Rochester Conference Series on Public Policy, vol. 42, June 1995, pp. 95-143

[38] Tesar, Linda L; Werner, Ingrid M., *Home Bias and High Turnover*, Journal of International Money and Finance, vol. 14, no. 4, August 1995, pp. 467-92

[39] van Wincoop, Eric, *Welfare Gains from International Risksharing*, Journal of Monetary Economics, vol. 34, no. 2, October 1994, pp. 175-200

[40] van Wincoop, Eric, *How Big are Potential Welfare Gains from International Risksharing?* Journal of International Economics, Vol. 47, 1999, pp. 109-135.

[41] Varian, Hal R., *Microeconomic Analysis*, W.W. Norton and Company, 1987.

[42] Warnock, Francis E., *Home Bias and High Turnover Reconsidered*, Journal of International Money and Finance, vol. 21, no. 6, November 2002, pp. 795-805

[43] Weil, Philippe, *Nonexpected Utility in Macroeconomics*, Quarterly Journal of Economics, vol. 105, no. 1, February 1990, pp. 29-42

[44] Weil, Philippe, *The Equity Premium Puzzle and the Risk-Free Rate Puzzle*, Journal of Monetary Economics, vol. 124, no. 3, 1989, pp. 401-21

Table 1: Consumption, Welfare Gains and Asset Returns

Panel A: Consumption Statistics

	Mean	S.D.	AC	Correlation U.S.	Correlation U.K.	Correlation Can
United States	2.08	1.76	0.27	1.00	0.49	0.63
United Kingdom	1.99	1.72	0.40	0.49	1.00	0.32
Canada	1.96	1.73	0.38	0.63	0.32	1.00

Panel B: Welfare Gains

Intertemporal Elasticity of Substitution	$\psi = 0.5$		$\psi = 1.5$	
Risk Aversion	$\gamma = 4$	$\gamma = 10$	$\gamma = 4$	$\gamma = 10$
US	0.3	0.9	1.0	3.4
	(33.3)	(33.2)	(33.1)	(33.1)
UK	0.6	1.7	2.4	6.7
	(33.4)	(33.5)	(33.6)	(33.6)
Canada	0.4	1.3	1.8	5.2
	(33.3)	(33.4)	(33.4)	(33.4)

Panel C: Return Moments

	Model Implied:				Data
US:					
Equity Premium-Mean	0.1	0.3	0.1	0.3	4.3
Equity Return-Std Dev	1.8	1.8	1.8	1.8	17.6
Risk-free Rate - Mean	5.6	5.3	2.9	2.7	1.5
Risk-free Rate - Std Dev	0.0	0.0	0.0	0.0	2.2
UK:					
Equity Premium-Mean	0.1	0.3	0.1	0.2	4.5
Equity Return-Std Dev	1.7	1.7	1.7	1.7	23.5
Risk-free Rate - Mean	5.4	5.1	2.8	2.7	3.9
Risk-free Rate - Std Dev	0.0	0.0	0.0	0.0	2.8

Table 2: Parameters and Targeted Moments			
Country	US	UK	Can
Panel A: Monthly Parameters			
Mean (μ)	.173	.166	.164
Transitory Std Dev (σ)	.920	.630	.660
Persistence Std Dev (σ_e)	.027	.030	.026
Cons Std Dev (σ_{gc})	.929	.648	.673
Panel B: Targeted Moments			
Equity Premium-Mean	4.3	4.5	6.5
Equity Return-Std Dev	17.6	23.5	17.6
Risk-free Rate - Mean	1.5	3.9	2.5
Risk-free Rate - Std Dev	2.2	2.8	6.0
Consn Growth - Std Dev	1.8	1.7	1.7
Consn Growth - Autocorrelation	0.3	0.4	0.4
Panel C: Simulated Moments			
Equity Premium-Mean	1.6	1.2	1.1
Equity Return-Std Dev	3.6	2.8	2.7
Risk-free Rate - Mean	1.8	2.2	2.2
Risk-free Rate - Std Dev	0.5	0.5	0.4
Consn Growth - Std Dev	2.9	2.3	2.2
Consn Growth - Autocorrelation	0.3	0.5	0.4
Panel D: Implied World	σ^*	σ_e^*	σ_{gc}^*
	.599	.028	.614
Notes: All variables in percent. Model assumes common mean $\mu^* = .168$. All reported simulations based upon $\rho = 0.979$, $\gamma = 10, \psi = 1.5$, and annual $\beta = 0.985$			

Table 3: Welfare Gains and the Persistent Risk Correlation						
Cross-Country Correlation		**Corr**$(\mathbf{e}^j, \mathbf{e}^w)$ =				
	σ_e= **0**	**0.0**	**0.2**	**0.5**	**0.8**	**1.0**
US	10.2	70.0	54.2	34.0	17.4	7.8
Portfolio Share	(28.4)	(29.7)	(30.0)	(37.4)	(30.7)	(30.9)
Gain from W^j/C^j	29	91	71	47	27	16
Gain from C^{j*}/C^{jB}	-11	-11	-10	-9	-8	-7
UK	12.6	86.0	65.5	40.3	20.2	9.0
Portfolio Share	(36.9	(33.0)	(32.6)	(32.2)	(31.9)	(31.7)
Gain from W^j/C^j	2	88	69	45	26	15
Gain from C^{j*}/C^{jB}	11	-1	-2	-3	-4	-5
Canada	8.4	75.7	58.1	35.9	17.8	7.6
Portfolio Share	(34.7)	(37.3)	(37.4)	(30.4)	(37.4)	(37.4)
Gain from W^j/C^j	4	57	41	21	5	-4
Gain from C^{j*}/C^{jB}	4	12	12	12	12	12

Notes: All variables in percent. For each country, first line gives total % gains in consumption implied by Table 2 parameters. Second line in parenthesis reports percentage shares in world output,ϖ^j. Third line is % $\frac{W^{j*}/C^{j*}}{W^{jB}/C^{jB}}^{\frac{1}{1-\frac{1}{\psi}}}$. The fourth line is % change in initial consumption allocation,.$\left(C_0^{j*}/C_0^{jB}\right)$.

Table 4: Equity Correlations and Gains

Equity as Consumption Asset

	United States	United Kingdom	Canada
A. Equity Return Correlation:			
United States	1.00	0.75	0.72
United Kingdom	0.75	1.00	0.59
Canada	0.72	0.59	1.00
B. Implied Correlations[a]			
Corr$(\mathbf{e}^i, \mathbf{e}^w)$:	1.00	1.00	1.00
Corr(η^i, η^j):			
United States	1.00	0.48	0.62
United Kingdom	0.48	1.00	0.29
Canada	0.62	0.29	1.00
Corr(η^i, η^w):	0.70	0.59	0.64
C: Welfare Gains			
Gain	7.9	9.4	7.8
Portfolio Share	(30.9)	(31.7)	(37.4)
Gain from W^j/C^j	17	15	-4
Gain from C^{j*}/C^{jB}	-7	-5	12

[a]Implied Correlations determined from cross-country equity and consumption correlations (Table 1A)

Table 5: Parameters - Dividend Model			
Country	US	UK	Can
Panel A: Monthly Parameters			
Mean (μ)	.173	.166	.164
Transitory Std Dev (σ)	.604	.469	.454
Persistence Std Dev (σ_e)	.044	.040	.044
Cons Std Dev (σ_{gc})	.641	.509	.499
Dividend Mean (μ_d)	.186	.339	.201
Dividend SD (σ_d)	3.03	3.74	3.63
Panel B: Targeted Moments			
Equity Premium-Mean	4.3	4.5	6.5
Equity Return-Std Dev	17.6	23.5	17.6
Risk-free Rate - Mean	1.5	3.9	2.5
Risk-free Rate - Std Dev	2.2	2.8	6.0
Consn Growth - Std Dev	1.8	1.7	1.7
Consn Growth - Autocorrelation	0.3	0.4	0.4
Dividend - Std Dev	7.1	6.8	13.0
Dividend - Autocorrelation	0.1	0.3	0.3
Panel C: Simulated Moments			
Equity Premium-Mean	5.0	5.7	6.5
Equity Return-Std Dev	15.2	18.5	18.3
Risk-free Rate - Mean	2.0	2.0	1.9
Risk-free Rate - Std Dev	0.7	0.7	0.8
Consn Growth - Std Dev	2.6	2.4	2.6
Consn Growth - Autocorrelation	0.6	0.6	0.7
Dividend - Std Dev	9.6	12.1	12.2
Dividend - Autocorrelation	0.4	0.4	0.4
Panel D: Implied World	σ^*	σ_e^*	σ_{gc}^*
	.397	.043	.449

Notes: All variables in percent. Model assumes common $\mu^* = .168$, $\mu_d^* = .201$. All reported simulations set $\rho = 0.979$, $\gamma = 10$, $\psi = 1.5$, and annual $\beta = 0.985$

Table 6: Dividend Correlations and Gains *Equity as Dividend Asset*			
	United States	United Kingdom	Canada
A. Dividend Correlation:			
United States	1.00	0.35	0.37
United Kingdom	0.35	1.00	0.12
Canada	0.37	0.12	1.00
B. Equity Return Correlation:			
United States	1.00	0.75	0.72
United Kingdom	0.75	1.00	0.59
Canada	0.72	0.59	1.00
C. Implied Correlations[a]			
$\mathbf{Corr}(\mathbf{e}^i, \mathbf{e}^w)$:	0.996[b]	0.996[b]	1.000
$\mathbf{Corr}(\eta^i, \eta^j)$:			
United States	1.00	0.42	0.57
United Kingdom	0.42	1.00	0.19
Canada	0.57	0.19	1.00
D: Welfare Gains	2.7	4.2	2.7
Portfolio Share	(30.2)	(38.3)	(31.4)
Gain from W^j/C^j	13	-9	9
Gain from C^{j*}/C^{jB}	-9	15	-6

[a] Implied Correlations based on dividend correlations in A, on equity in B, and consumption in Table 1A

[b] Corr(e^{US},e^{UK})=0.987

Table 7: Differing Means and Gains			
Country	United States	United Kingdom	Canada
Annual Means	2.08	1.99	1.96
A.Equity paying Consumption			
Welfare Gains	8.3	9.5	7.5
Portfolio Share	(32.6)	(31.4)	(36.0)
Gain from W^j/C^j	10.8	16.3	-0.6
Gain from C^{j*}/C^{jA}	-2.2	-5.9	8.1
B. Equity paying Dividends			
Welfare Gains	2.3	4.5	2.4
Portfolio Share	(31.3)	(38.1)	(30.6)
Gain from W^j/C^j	8.9	-8.5	11.4
Gain from C^{j*}/C^{jA}	-6.1	14.2	-8.1

Notes: All variables in percent. Panel A reports gains for the consumption asset case (Table 4) with differing means. Panel B gives the gains for dividend asset case (Table 6).

Table 8: Differing Sizes and Efficient Range

Country	United States	United Kingdom	Canada
Population Weights	.70	.23	.06
A.Equity paying Consumption			
Gain from W^j/C^j	8	6	-11
Maximum Gains[a]	8.8	26.3	78.6
Portfolio Share	(71)	(27)	(13)
Gain from C^{j*}/C^{jB}	1	19	101
Minimum Gains	0	0	0
Portfolio Share[b]	(65)	(22)	(7)
B. Equity paying Dividends			
Gain from W^j/C^j	8	-14	4
Maximum Gains[a]	3.0	7.2	31.0
Portfolio Share	(67)	(29)	(8)
Gain from C^{j*}/C^{jB}	-5	24	26
Minimum Gains	0	0	0
Portfolio Share[b]	(65)	(27)	(6)

Notes: All variables in percent. Panel A reports gains for the consumption asset case (Table 4) with population sizes Panel B gives the gains for dividend asset case (Table 6).

[a]Results give bounds for efficient allocations where $\Delta^j = 0$ for all countries but column country.

[b]Shares that imply $\Delta^\ell = 0$ for column country ℓ.

Table 9: Many Countries and Gains

A. Parameters	Consumption Mean and SD				Dividend			
	Mean (μ)	Trans (σ)	Persist (σ_e)	Total (σ_{gc})	Mean (μ_d)	SD (σ_d)		
Australia	0.170	0.620	0.051	0.668	0.280	4.32		
France	0.212	0.672	0.049	0.714	0.267	5.25		
Germany	0.157	0.562	0.044	0.602	0.398	4.50		
Japan	0.322	1.092	0.036	1.111	0.233	5.15		
Implied World	0.195	0.403	0.042	0.454	NA	NA		
B: Target Moments	Equity Prem	Equity S.D.	Rfree Mean	Rfree. S.D	Con S.D	Con A.C.	Div S.D	Div A.C.
Australia	7.1	22.1	1.6	6.3	2.2	0.03	11.8	0.48
France	7.6	25.6	1.8	5.9	1.8	0.52	14.0	0.19
Germany	6.4	23.1	4.2	4.5	1.6	0.61	12.6	0.43
Japan	2.2	25.0	2.6	5.2	3.2	0.68	10.2	0.61
C: Implied Moments								
Australia	7.6	20.4	1.6	0.8	3.0	0.62	13.7	0.4
France	7.9	23.4	1.9	0.9	3.1	0.60	16.4	0.4
Germany	6.6	21.2	1.8	0.8	2.7	0.62	14.3	0.4
Japan	3.6	20.9	2.4	0.6	3.6	0.39	15.0	0.3
D: Welfare Gains and Correlations	Div Corr with World	Implied Corr		Efficient Set Range				
				Max[a]				Min[b]
		(e^i, e^w)	(η^i, η^w)	Gains	Share	Δ W/C	C^*/C^B	Share
United States	0.44	0.96	0.42	127	28	16	96	12
United Kingdom	0.33	1.0	0.28	100	31	-9	119	16
Canada	0.54	0.91	0.37	122	29	11	100	13
Australia	0.50	0.88	0.06	192	24	75	67	08
France	0.47	1.0	0.35	180	24	64	70	09
Germany	0.51	0.92	0.32	125	28	14	97	13
Japan	0.48	0.83	0.38	111	30	1	108	14

Notes: All reported simulations based upon $\gamma = 10$, $\psi = 1.5$,and annual $\beta = 0.985$

[a]Results give bounds for efficient allocations where $\Delta^j = 0$ for all countries but row country.

[b]Shares that imply $\Delta^\ell = 0$ for row country ℓ.

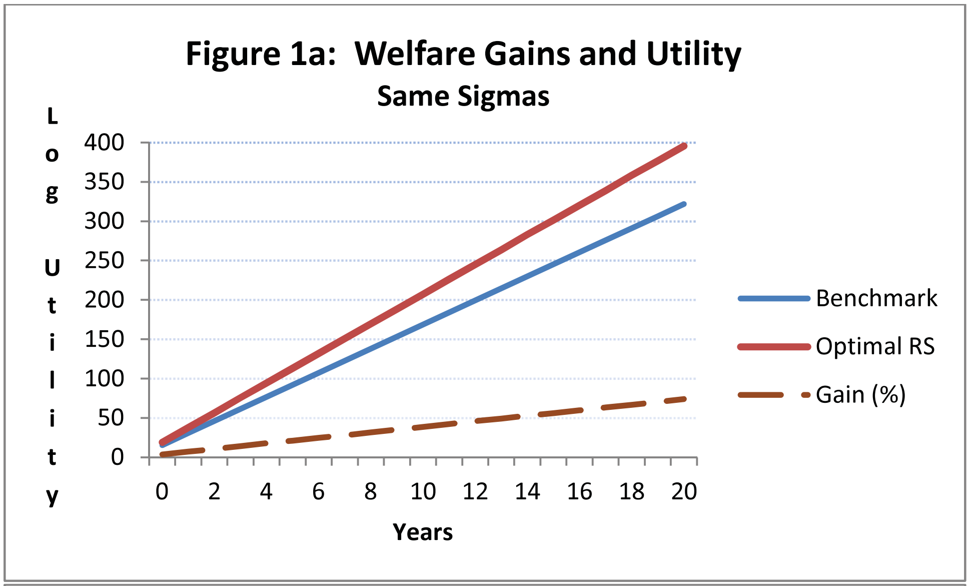
Figure 1a: Welfare Gains and Utility
Same Sigmas
Log Utility
400
350
300
250
200
150
100
50
0
0 2 4 6 8 10 12 14 16 18 20
Years
Benchmark
Optimal RS
Gain (%)

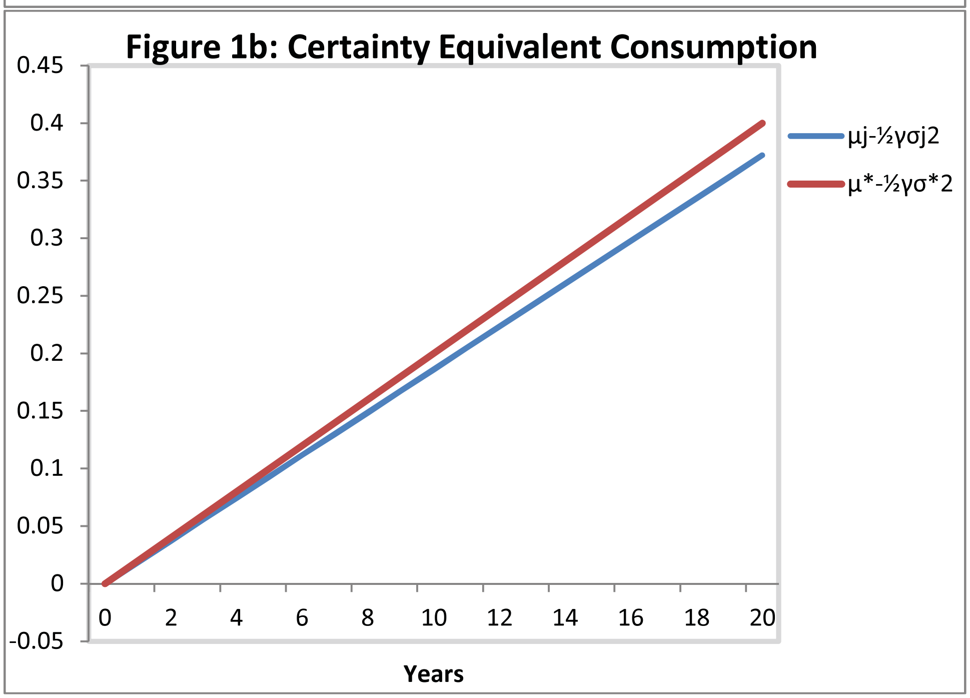
Figure 1b: Certainty Equivalent Consumption
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
-0.05
0 2 4 6 8 10 12 14 16 18 20
Years
μj-½γσj2
μ*-½γσ*2

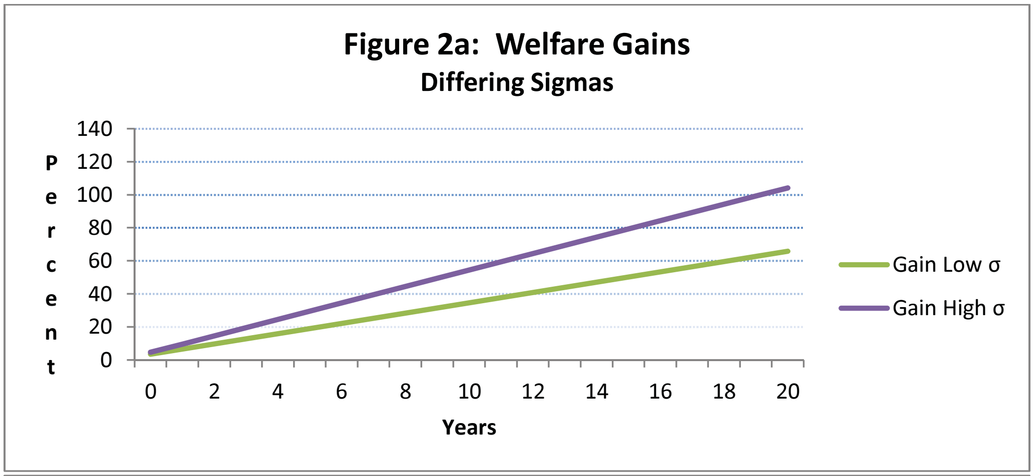
Figure 2a: Welfare Gains
Differing Sigmas
Percent
140
120
100
80
60
40
20
0
0 2 4 6 8 10 12 14 16 18 20
Years
Gain Low σ
Gain High σ

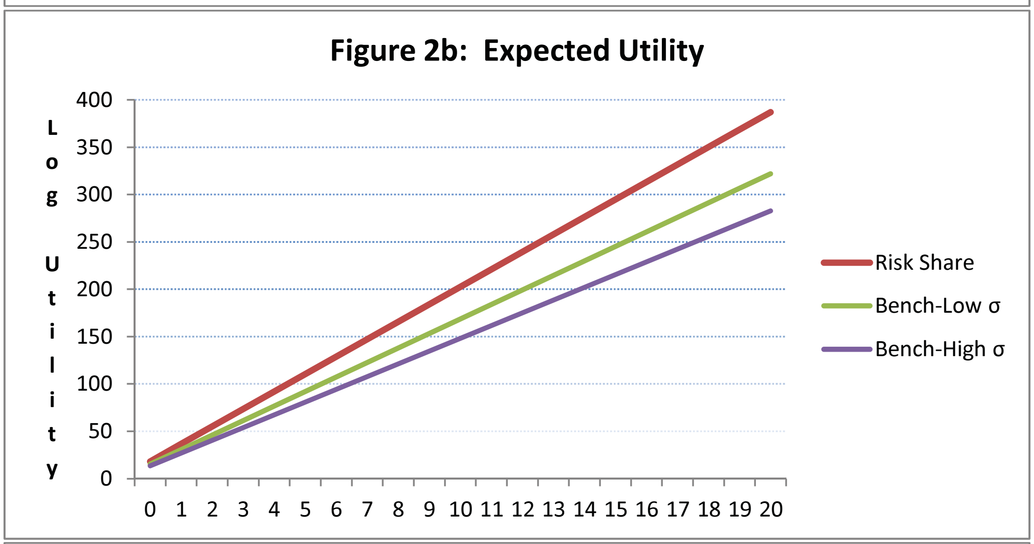
Figure 2b: Expected Utility
Log Utility
400
350
300
250
200
150
100
50
0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
Risk Share
Bench-Low σ
Bench-High σ

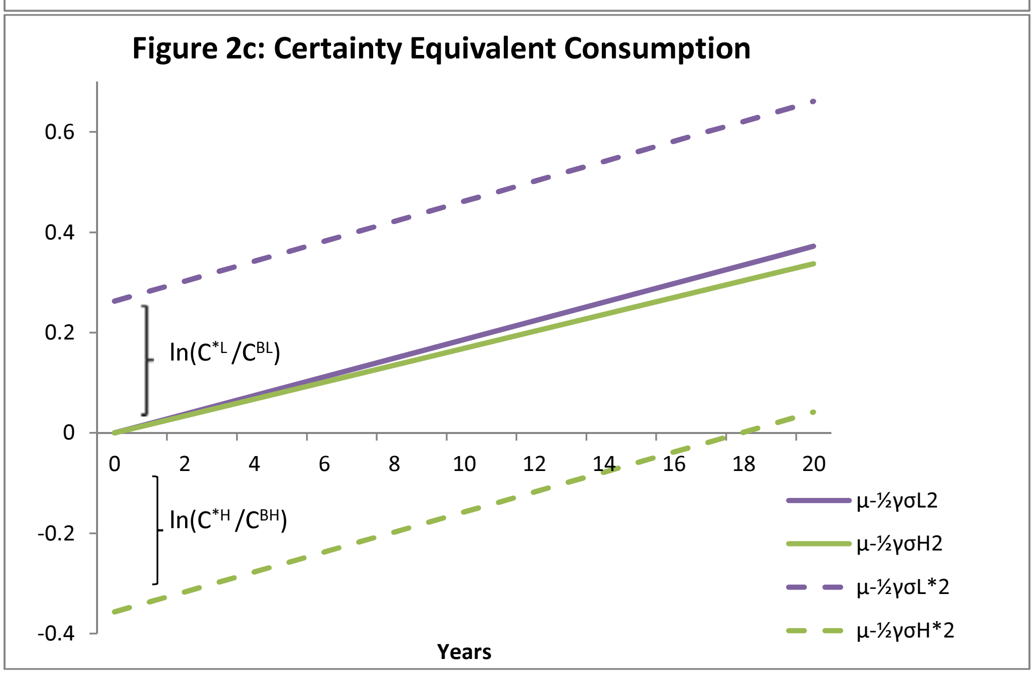
Figure 2c: Certainty Equivalent Consumption
0.6
0.4
0.2
0
-0.2
-0.4
0 2 4 6 8 10 12 14 16 18 20
ln(C*L /CBL)
ln(C*H /CBH)
Years
μ-½γσL2
μ-½γσH2
μ-½γσL*2
μ-½γσH*2

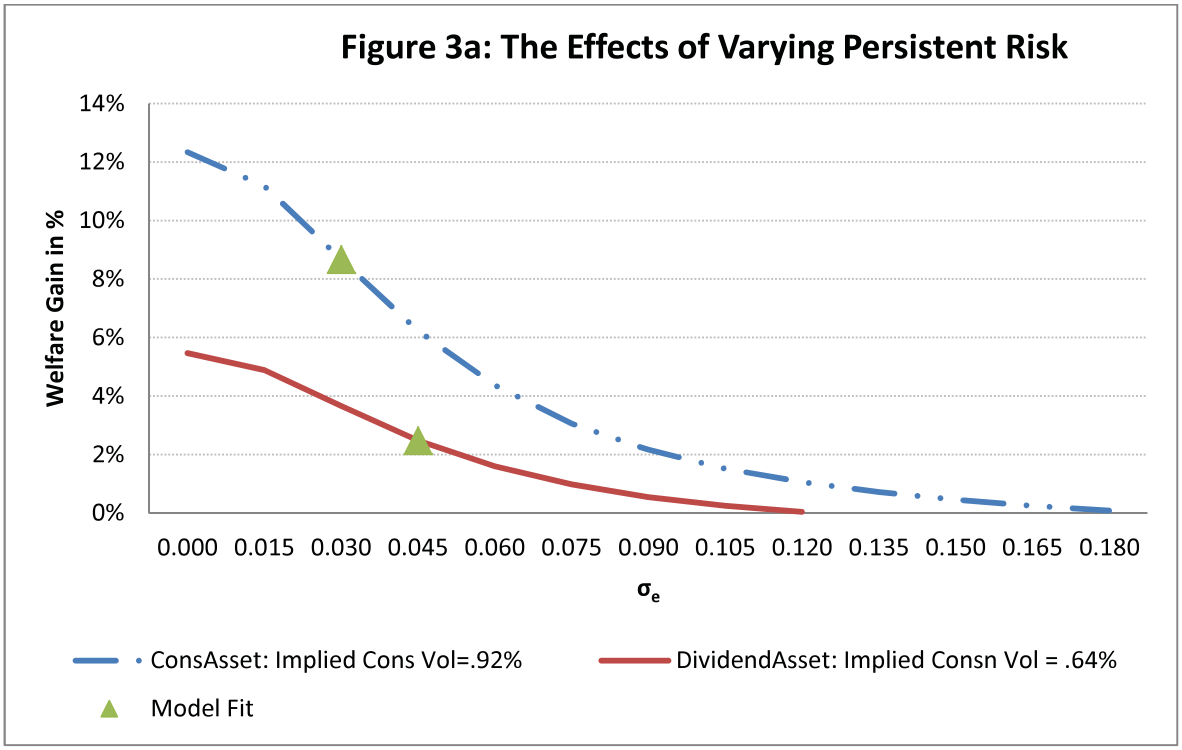
Figure 3a: The Effects of Varying Persistent Risk
14%
12%
10%
8%
6%
4%
2%
0%
Welfare Gain in %
0.000 0.015 0.030 0.045 0.060 0.075 0.090 0.105 0.120 0.135 0.150 0.165 0.180
σe
ConsAsset: Implied Cons Vol=.92%
DividendAsset: Implied Consn Vol = .64%
Model Fit

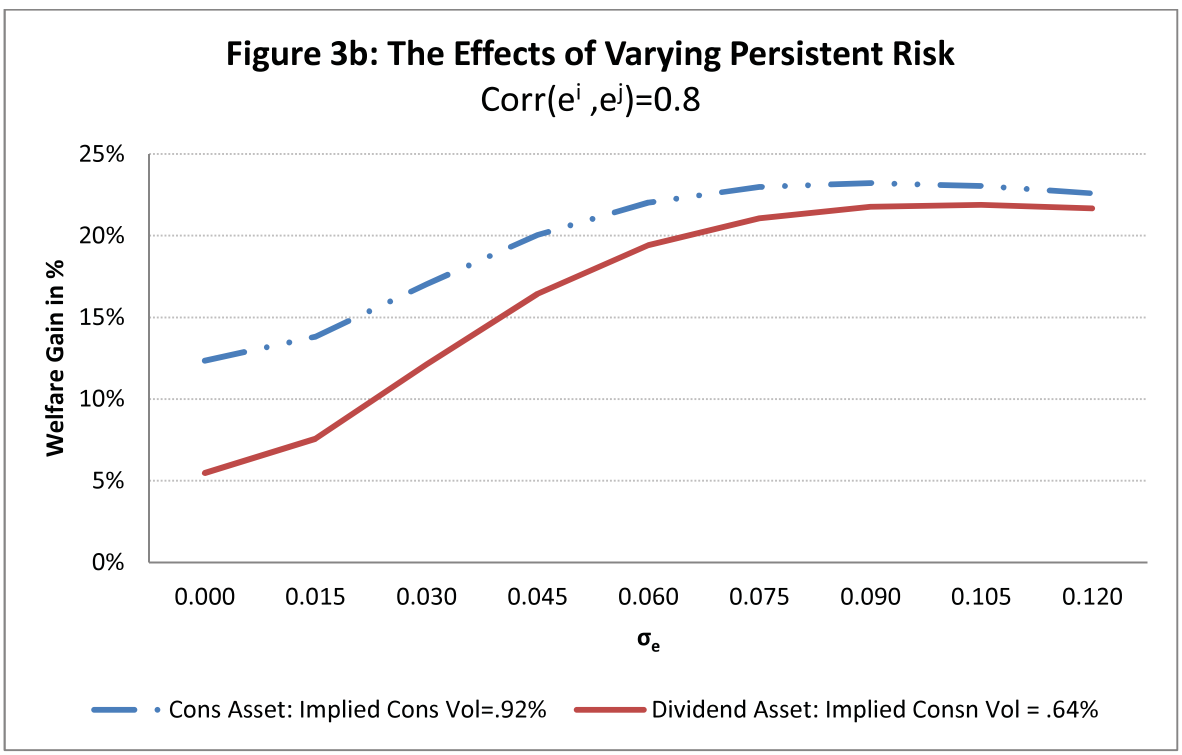
Figure 3b: The Effects of Varying Persistent Risk
Corr(ei ,ej)=0.8
25%
20%
15%
10%
5%
0%
Welfare Gain in %
0.000 0.015 0.030 0.045 0.060 0.075 0.090 0.105 0.120
σe
Cons Asset: Implied Cons Vol=.92%
Dividend Asset: Implied Consn Vol = .64%

A Appendix: Country Consumption Weights

In this appendix we show that the country weights in aggregate consumption are determined by the solution to a planner's problem. We first assume identically sized economies and then extend these results to differing population weights. Finally, we characterize the set of Pareto efficient allocations.

A.1 The Consumption Allocation with Identically Sized Countries

Proposition 1: *Let a^j be the planner weights on utility of country j, Q^j_τ be the state-price for country j at time τ. and $U^j(C^j_t, U^j_{t+1})$ be given by:*

$$U^j(C^j_t, U^j_{t+1}) = \left\{ C^j_t{}^{\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U^j_{t+1} \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}. \tag{32}$$

Then the solution to the planner's problem:

$$\underset{\substack{\{C^j_t\} \\ \forall t}}{MaxS} = \sum_{j=1}^{J} a^j U^j(C^j_0, U^j_1) \tag{33}$$

$$\text{s.t.} \sum_{j=1}^{J} C^j_t = \sum_{j=1}^{J} Y^j_t, \forall t \tag{34}$$

$$E_0 \sum_{\tau=0}^{\infty} Q^j_\tau C^j_\tau = E_0 \sum_{\tau=0}^{\infty} Q^j_\tau Y^j_\tau, \forall j \tag{35}$$

$$U^j(C^j_0, U^j_1) \in R, < \infty, \forall j \tag{36}$$

is given by:

$$C^j_t = \varpi^j Y^w_t, \forall t,$$

where

$$\varpi^j = \frac{Y^j_0 + P^{j*}_0}{Y^w_0 + P^{w*}_0} \tag{37}$$

for $Y^w_t \equiv \sum_{j=1}^{J} Y^j_t$, the world output in each period and for $P^{j*}_0 = E_0 \sum_{\tau=1}^{\infty} Q^*_\tau Y^j_\tau$ and $P^{w*}_0 = E_0 \sum_{\tau=1}^{\infty} Q^*_\tau Y^w_\tau$, the present value of country j's output and the world output, respectively, at the world stochastic discount factor, Q^*_τ.

Discussion: Note that the planner maximizes utility across agents in each country given three constraints. The first constraint given in equation (34) is the resource constraint that total output equals total consumption in each period. The second constraint given in equation (35) is the lifetime budget constraint for each country. This constraint says that the expected lifetime value of consumption for each country equals the expected lifetime value of its output. The budget constraint holds in expectations both because of uncertainty (Lucas and Stokey (1989), pp. 487-490) and because current utility depends upon expected future utility. Finally, the third constraint in equation (36) requires utility to be bounded and rational along the equilibrium path.

Proof: The planner's problem can be simplified by solving for the value function of each country given the budget constraint (35). For this purpose, first note that since the state-price at time 0 is one (i.e., $Q_0^j = 1$), the lifetime budget constraint can be rewritten as:

$$C_0^j + E_0 \sum_{\tau=1}^{\infty} Q_\tau^j C_\tau^j = Y_0^j + E_0 \sum_{\tau=1}^{\infty} Q_\tau^j Y_\tau^j, \forall j$$

Or as:

$$C_0^j + P_0^{jc} = W_0^j \equiv Y_0^j + P_0^{jy}, \forall j \tag{38}$$

where P_t^{jy} is the price of country j output at state prices Q_τ^j and P_t^{jc} is the price of country j consumption at these same prices. Thus, the budget constraint simply states that current consumption plus the future expected value of consumption equals current country output plus the expected value of future output. Alternatively, the value of current and future output is identically equal to country wealth.

To solve for the value function of each country, we then solve for the Bellman equation:

$$\begin{aligned} V(C_0^j, W_0^j) &= \underset{\substack{\{C_t^j\} \\ \forall t}}{Max} \left\{ C_t^{j\,\frac{1-\gamma}{\theta}} + \beta E_t \left[\left(U_{t+1}^j \right)^{1-\gamma} \right]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\ & \quad s.t.(38) \text{ holds} \end{aligned} \tag{39}$$

This problem has the solution (Campbell (1993), Obstfeld (1994a)):

$$V(C_t^j, W_t^j) = \left(C_t^j \right)^{-\frac{1/\psi}{1-(1/\psi)}} \left(W_t^j \right)^{\frac{1}{1-(1/\psi)}} = C_t^j \left(\frac{W_t^j}{C_t^j} \right)^{\frac{1}{1-(1/\psi)}} \tag{40}$$

Then the value function can be determined given the solution to equilibrium wealth. This solution in turn depends upon the equilibrium price of output, P_t^{jy}. But this price can be determined using the Euler equation for the return on the asset paying output, P_t^{jy} (Epstein and Zin (1989)):

$$E_t \left\{ \beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^{jc})^{(\theta-1)} R_{t+1}^{jy} \right\} = 1 \tag{41}$$

where $R^{jc}_{t+1} \equiv (C^j_{t+1} + P^{jc}_{t+1})/P^{jc}_t$ and $R^{jy}_{t+1} \equiv (Y^j_{t+1} + P^{jy}_{t+1})/P^{jy}_t$. Similarly, the equilibrium price of the consumption asset can be determined using the Euler equation for its return

$$E_t \left\{ \beta^\theta (C^j_{t+1}/C^j_t)^{\left(-\frac{\theta}{\psi}\right)} (R^{jc}_{t+1})^\theta \right\} = 1 \tag{42}$$

So these solutions to the value functions give us J value functions in terms of J sets of country state prices, Q^j_τ. However, in a Pareto competitive equilibrium with heterogeneous agents but identical preferences, these state prices must be equal across agents (See for example, Varian (1978), p. 152.) Using our notation above, these equilibrium state prices correspond to the common Q^*_τ. Therefore, all agents share the same in the Euler equations (41) and (42). As a consequence, consumption growth rates are equated:

$$(C^j_{t+1}/C^j_t) = (C^i_{t+1}/C^i_t), \forall i, j, t$$

Thus, in the equilibrium, per capita consumption levels are proportional to aggregate output. Defining this proportion for country j as ϖ^{jw},

$$C^j_t = \varpi^{jw} Y^w_t, \forall t. \tag{43}$$

In this case, the lifetime expected consumption in the budget constraint (38) becomes:

$$E_0 \sum_{\tau=0}^{\infty} Q^*_\tau C^j_\tau = \varpi^{jw} Y^w_0 + E_0 \sum_{\tau=1}^{\infty} Q^*_\tau \varpi^{jw} Y^w_\tau = \varpi^{jw} \left(Y^j_0 + P^{w*}_0 \right)$$

We now substitute the value function for each country (40) into the planner problem (33) to rewrite the problem as:

$$\underset{\substack{\{C^j_t\} \\ \forall t}}{Max} S = \sum_{j=1}^{J} a^j V(C^j_0, W^j_0)$$

$$\begin{aligned} \text{s.t.} \sum_{j=1}^{J} C^j_t &= \sum_{j=1}^{J} Y^j_t, \forall t \\ \varpi^{jw} \left(Y^j_0 + P^{w*}_0 \right) &= W^j_0 \equiv Y^j_0 + P^{*j}_0, \forall j \end{aligned}$$

Clearly then $\varpi^{jw} = \frac{Y^j_0 + P^{*j}_0}{Y^j_0 + P^{w*}_0}$ as in equation (37), verifying the proposition above.

Using the definition of wealth, note also that:

$$\frac{W^j_t}{C^j_t} = \frac{\varpi^{jw} \left(Y^j_t + P^{w*}_t \right)}{\varpi^{jw} Y^w_t} = \frac{Y^j_t + P^{w*}_t}{Y^w_t}$$

Therefore, the wealth-consumption ratio is equal for all countries in equilibrium.

Moreover, the planner weights are equalized across countries. To see why, note that the first-order condition for period 0 is:

$$\frac{\partial S}{\partial C_0^j} = \frac{a^j}{1-\psi}\left(\frac{W_t^j}{C_t^j}\right)^{\frac{1}{1-(1/\psi)}} - \lambda = 0 \tag{44}$$

Rearranging equation (44) and using the fact that $\frac{W_t^j}{C_t^j} = \frac{W_t^i}{C_t^i}, \forall i, j$ implies that $a^i = a^j$ as required for a utilitarian planner (Varian (1978), pp. 152-154.), thus verifying the proposition.

A.2 The Consumption Allocation with Differing Population Sizes

The consumption allocations above are derived assuming all countries have the same number of agents or, alternatively, the planner cares about countries equally regardless of size. Here we recalculate the planner allocations assuming that in each country j there are N^j people and the planner cares about maximizing over all individual utilities. Individuals in each country are endowed with the claims to one per capita unit of country output Y_t^j.

Proposition 2: *Let a^{ji} be the planner weights on utility of resident i in country j, Q_τ^j be the state-price for country j at time τ, and consumption and utility of agent i in country j at time t be C_t^{ji} and U_t^{ji}, respectively, where U_t^{ji} is the Epstein-Zin utility given in equation (32). Then the solution to the planner's problem:*

$$\underset{\{C_t^j\}_{\forall t}}{Max} S = \sum_{j=1}^{J}\sum_{i=1}^{N^j} a^{ji} U^j(C_0^{ji}, U_1^{ji}) \tag{45}$$

$$\text{s.t.} \sum_{j=1}^{J}\sum_{i=1}^{N^j} C_t^{ji} = \sum_{j=1}^{J}\sum_{i=1}^{N^j} Y_t^j, \forall t \tag{46}$$

$$E_0\sum_{\tau=0}^{\infty} Q_\tau^j C_\tau^{ji} = E_0\sum_{\tau=0}^{\infty} Q_\tau^j Y_\tau^{ji}, \forall i, j \tag{47}$$

is given by:

$$C_t^j = \varpi^{jw} N^w \widetilde{Y}_t^w, \forall t,$$

where

$$\varpi^{jw} = n^j \frac{Y_0^j + P_0^{j*}}{\widetilde{Y}_0^w + \widetilde{P}_0^{w*}} \tag{48}$$

for $C_t^j \equiv N^j C_t^{ji}$, the consumption in each country j; $N^w \equiv \Sigma_{j=1}^J N^j$, the world population; $\widetilde{Y}_t^w = Y_t^w / N^w$, the world per capital output; $n^j \equiv \left(N^j/N^w\right)$, the country j population share;

$\widetilde{P}_0^{w*} = E_0 \sum_{\tau=1}^{\infty} Q_\tau^* Y_\tau^w$, *the price of world per capita output. and as before,* $P_0^{j*} = E_0 \sum_{\tau=1}^{\infty} Q_\tau^* Y_\tau^j$, *the price of country j per capital output at world prices.*

Proof: The population-weighted planner problem can be solved as a straightforward extension to the identical sized country version above. First note that as above the identical preferences implies that consumption growth rates are equalized or that:

$$(C_{t+1}^{j\ell}/C_t^{j\ell}) = (C_{t+1}^{iq}/C_t^{iq}), \forall i, j, \ell, q, t$$

Therefore, consumption across individuals differ only by a proportional initial condition. Moreover, since agents in each country are identical, in equilibrium $C_t^{ji} = C_t^{j\ell} \forall i, \ell$ so each agent holds identical shares in world output of ϖ_0^{jw}/N^j. As a result, individual consumption can be rewritten:

$$C_t^{ji} = \left(\varpi_0^{jw}/n^j\right) \widetilde{Y}_t^w, \forall t,$$

where we have used the fact that aggregate world output can be written as world per capita output times world population or $Y_t^w = \widetilde{Y}_t^w \Sigma_{\ell=1}^J N^\ell$. Using this solution in the individual lifetime budget constraint in equation (47) and solving for ϖ^{jw} verifies the consumption allocations in equation (48). Moreover, by the competitive equilibrium, $Q_\tau^j = Q_\tau^*$ as before. Thus,substituting the solutions for the prices P_0^{j*} and $\widetilde{P}_0^{w*}$ into the individual value function in (40) and then solving for the initial period first order condition to the planner problem (45), verifies that $a^{ji} = a^{\ell q}, \forall j, i, \ell, q$ corresponding to utilitarian planner weights, thereby proving the proposition.

Discussion: Thus, equation (48) implies the shares in world output are the same as the equal population case in Proposition 1 except for two differences. First, the shares are weighted by population shares, n^j. As such larger countries have higher shares in world output. Second, the price of world output is now a population weighted average of country output.

A.3 The Set of Pareto Efficient Allocations

The solution to the planner's problem does not always correspond to a steady state equilibrium. This tendency becomes more pronounced when the consumption parameters differ significantly. For these cases, we characterize the set of efficient allocations so that risk-sharing generates gains for some countries without making others worse off. These allocations provide the boundaries for the efficient set.

Proposition 3: *Let* $V(C_t^i, W_t^i)$ *be the value functions given by the individual Bellman equation (39) and let* $V(Y_0^i, W_0^{iB}) = Y_t^j \left[1 + \frac{P_t^j}{Y_t^j}\right]^{\left(\frac{1}{1-(\frac{1}{\psi})}\right)}$ *be the value function in the benchmark economy,* *equation (12). Then the initial consumption allocations that maximize country j utility without making all other countries worse off solves the problem:*

$$\underset{C_\tau^\ell,\forall\tau,\ell}{Max}\, V(C_0^j, W_0^j) \tag{49}$$

$$\begin{aligned} \text{s.t. } V(C_0^i, W_0^i) &\geq V(Y_0^i, W_0^{iB}), \forall i \neq j \\ \text{s.t. } \sum_{j=1}^{J} C_t^j &= Y_t^w \equiv \sum_{j=1}^{J} Y_t^j, \forall t \end{aligned}$$

and is given by the set of $\left\{\widehat{C}_0^{j*}, \widehat{C}_0^{i*} \forall i \neq j\right\}$ determined by giving the reservation initial consumption levels $\widehat{C}_0^{i*}$ to all $i \neq j$ countries (equation (30)):

$$1 = \left(\frac{\widehat{C}_0^{i*}}{Y_0^i}\right) \left\{\frac{W_0^*/C_0^*}{W_0^{iB}/Y_0^i}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

and giving the residual consumption from the resource constraint to country j (equation (31)):

$$(1+\Delta^j) = \left(\frac{Y_0^w - \sum_{i\neq j} \widehat{C}_0^{i*}}{Y_0^j}\right) \left\{\frac{W_0^{j*}/C_0^{j*}}{W_0^{jB}/C_0^{jB}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

Proof: Since the efficient Pareto allocation implies common state prices, the consumption growth rates across countries are shared as above and consumption levels are a constant share of world output. Thus, as before, the wealth-to-consumption ratios are equalized across countries and the problem to determine the boundary of the efficient set in equation (49) can be rewritten:

$$\begin{aligned} \underset{C_\tau^\ell,\forall\tau,\ell}{Max}\, C_0^j V\left(\frac{W_0^j}{C_0^j}\right) \text{ s.t. } C_0^i V\left(\frac{W_0^*}{C_0^*}\right) &= Y_0^i V\left(\frac{W_0^{iB}}{Y_0^i}\right), \forall i \neq j \\ \text{s.t. } \sum_{j=1}^{J} C_0^j &= Y_0^w \end{aligned}$$

Using the fact that the gains can be written as:

$$1+\Delta^i = \left(\frac{C_0^{i*}}{Y_0^i}\right) \left\{\frac{W_0^*/C_0^*}{W_0^{iB}/Y_0^i}\right\}^{\frac{1}{1-\frac{1}{\psi}}}$$

the constraints clearly imply that for all but country j, the allocations are determined by setting $\Delta^i = 0$ so that (equation (30) holds. Maximizing the utility to country j then means allocating all remaining consumption to country j so that $\widehat{C}_0^{j*}$ is determined by equation (31) verifying the proposition.

B Appendix: Model Solutions and Analysis

In this appendix, we describe the solutions to the risk sharing gains as well as asset returns for the model. To calculate the gains from risk sharing, we require solutions to the value function under the benchmark economy and the international risk-sharing economy. As noted above, the general solution to this value function is[33]:

$$V(C_t^j, W_t^j) = C_t^j \left(\frac{W_t^j}{C_t^j} \right)^{\frac{1}{1-(1/\psi)}}$$

for $W_t^j = C_t^j + P_t^{jc}$ where P_t^{jc} is the time t expected value of lifetime consumption for investor j. All prices are determined by the Euler equation (8) in the text:

$$E_t \left\{ \beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^P)^{(\theta-1)} R_{t+1}^\ell \right\} = 1$$

where $R_{t+1}^P = (C_{t+1}^j + P_{t+1}^{jc})/P_t^{jc}$ is the return on the asset that pays out consumption and R_{t+1}^ℓ is the return on any asset. Then clearly the Euler equation for the consumption asset can be written as in equation (13):

$$E_t \left\{ \beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^P)^\theta \right\} = 1$$

We next describe the solution for P_t^{jc} and the value function $V(C_t^j, W_t^j)$ for the case of transitory only risk before considering persistent risk.

B.1 The Transitory Only Case

When all consumption risk is transitory, consumption growth is given by:

$$g_{c,t+1}^j = \mu^j + \eta_{t+1}^j.$$

We then substitute $\exp(g_{c,t+1}^j) = \left(C_{t+1}^j / C_t^j \right)$ into the Euler equation for the consumption asset and use the definition of returns R_{t+1}^P to obtain:

$$E_t \left\{ \beta^\theta (\exp(g_{c,t+1}^j))^{\left(-\frac{\theta}{\psi}\right)} \left[\frac{C_{t+1}^j + P_{t+1}^{jc}}{P_t^{jc}} \right]^\theta \right\} = 1 \tag{50}$$

Note that since all risk is I.I.D. for the present case, the conditional expectations are the same as the unconditional expectations.

[33] We also checked our solution against the solution implied by the guess-and-verify approach substituting consumption growth in the utility function as in Lewis (2000).

B.1.1 Benchmark Economy Welfare

In the benchmark economy, consumption is trivially given as $Y_t^j = C_t^j$. Substituting this process into equation (50) and using properties of log normality yields the price of consumption in the benchmark economy given in the text as equation (15):

$$P_t^j = \frac{Y_t^j \beta M_j^{\left(1-\frac{1}{\psi}\right)}}{1 - \beta M_j^{\left(1-\frac{1}{\psi}\right)}}$$

where $M_j \equiv \exp\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^{j2}\right]$ for $\widetilde{\mu}^j = \mu^j + \frac{1}{2}\sigma^{j2}$ and we have dropped the superscript c for parsimony. Using the fact that in the benchmark economy $W_t^j = Y_t^j + P_t^j$, the value function can then be written:

$$V(C_t^{jB}, W_t^{jB}) = Y_t^j \left(1 + \frac{P_t^j}{Y_t^j}\right)^{\frac{1}{1-(1/\psi)}} = Y_t^j \left[\frac{1}{1 - \beta M_j^{\left(1-\frac{1}{\psi}\right)}}\right]^{\frac{1}{1-(1/\psi)}}$$

thereby verifying equation (16) in the text.

B.1.2 Risk Sharing Economy Welfare

In the risk sharing economy, consumption for country j is a weighted share of world output: $C_t^j = \varpi^j Y_t^w$ where $\varpi^j = \left(Y_0^j + P_0^{j*}\right) / (Y_0^w + P_0^{w*})$. Thus, solving for the value function requires determining the price of the world consumption asset, P_0^{w*}, as well as the value of country j output on world markets, P_0^{j*}.

We begin with the price of world consumption, P_0^{w*}. In this case, the common growth rate across countries is the weighted sum of the country growth rates:

$$g_{c,t+1}^* = \mu^* + \eta_{t+1}^* \tag{51}$$

where $\mu^* \equiv \frac{1}{J}\Sigma_{j=1}^J \mu^j$ and $\sigma^{*2} = \left(\frac{1}{J}\right)^2 \iota' \, \Sigma \, \iota$ for Σ, the variance-covariance matrix of consumption growth rates, and ι, a J-dimensional unit vector. Note that treating the world consumption process as the country-weighted sum of growth rates is an approximation since the sum of the log of consumption is not equal to the log of the sum of the country consumptions. In the Empirical Methods appendix below we describe Monte Carlo experiments that suggest our world process remains close to log normally distributed despite this approximation.

Substituting this process into equation (50) and using properties of log normality implies the price of world consumption described in the text is:

$$P_t^{w*} = \frac{Y_t^w \beta M_*^{\left(1-\frac{1}{\psi}\right)}}{1-\beta M_*^{\left(1-\frac{1}{\psi}\right)}}$$

where $M_* \equiv \exp\left[\mu^* + \frac{1}{2}(1-\gamma)\sigma^{*2}\right]$

We solve for the price of output in world markets in a similar manner. Specifically, we substitute the individual output growth process, $g_{y,t+1}^j = \mu^j + \eta_{t+1}^j$, into the Euler equation (8), using the fact that $\exp(g_{c,t+1}^*) = \left(C_{t+1}^*/C_t^*\right)$ and that $R_{t+1}^\ell = (Y_{t+1}^j + P_{t+1}^{j*})/P_t^{j*}$ where P_t^{j*} is the price of country j output in the open economy. Taking expectations, equating the Euler equation to one and solving for the price implies:

$$P_t^{j*} = \frac{Y_t^j \beta M_*^{-\frac{1}{\psi}} H_j}{1-\beta M_*^{-\frac{1}{\psi}} H_j} \tag{52}$$

where $H_j = \exp\left(\mu^j + \frac{1}{2}\sigma^{j2} + \frac{1}{2}\gamma\sigma^{*2} - \gamma Cov(\eta^j, \eta^*)\right)$ for $\eta^* \equiv \frac{1}{J}\Sigma_{j=1}^J \eta^j$, the shock from the portfolio of consumption growth rates. This factor reflects the hedging properties of the country's output in world markets. As its covariance with the world market decreases, country j's output has a better value as a hedge, inducing the price to increase.

Substituting $C_t^j = \varpi^j Y_t^w$ for $\varpi^j = \left(Y_0^j + P_0^{j*}\right)/\left(Y_0^w + P_0^{w*}\right)$ along with the solutions to the world and country prices implies the value function for country j in the open economy is:

$$V_t(C_t^{j*}, W_t^{j*}) = \varpi^j Y_t^w \left[1 + \frac{P_t^{w*}}{Y_t^w}\right]^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} = \varpi^j Y_t^w \left[\frac{1}{1-\beta M_*^{\left(1-\frac{1}{\psi}\right)}}\right]^{\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)}$$

corresponding to equation (7) in the text.

B.1.3 Welfare gains

Calculating the welfare gains are straightforward given the value functions. Welfare gains are determined by Δ^j in:

$$V_0((1+\Delta^j)C_0^{jB}, (1+\Delta^j)W_0^{jB}) = V_0(C_0^{j*}, W_0^{j*})$$

Substituting the value function in the benchmark economy and the counterpart for the risk sharing economy into the condition yields the solution in equation (18):

$$(1+\Delta^j) = \left\{ \frac{V_0(C_0^{j*}, W_0^{j*})}{V_0(C_0^{jB}, W_0^{jB})} \right\} = \left[\frac{1-\beta M_*^{\left(1-\frac{1}{\psi}\right)}}{1-\beta M_j^{\left(1-\frac{1}{\psi}\right)}} \right]^{-\left(\frac{1}{1-\left(\frac{1}{\psi}\right)}\right)} \frac{\varpi^j Y_0^w}{Y_0^j}$$

B.1.4 Implied Returns

Given the price solutions and the assumed output processes above, the returns follow directly from the definitions $R_{t+1}^{*w} = (C_{t+1}^w + P_{t+1}^{*w})/P_t^{*w}$, and $R_{t+1}^j = (Y_{t+1}^j + P_{t+1}^{j*})/P_t^{j*}$. Similarly, the risk-free rate is determined by solving the Euler equation for:

$$E_t \left\{ \beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^P)^{(\theta-1)} \right\} R_t^{Rfree} = 1$$

B.2 Including Persistent Risk

When consumption includes persistent risk, consumption growth is given by equation (19), reproduced here:

$$\begin{aligned} g_{y,t+1}^j &= \mu^j + x_t^j + \eta_{t+1}^j \\ x_{t+1}^j &= \rho x_t^j + e_{t+1}^j \end{aligned}$$

We now substitute the new process for $\exp(g_{c,t+1}^j) = \left(C_{t+1}^j / C_t^j\right)$ into the Euler equation for the consumption asset as in equation (13), repeated here:

$$E_t \left\{ \beta^\theta (C_{t+1}^j / C_t^j)^{\left(-\frac{\theta}{\psi}\right)} (R_{t+1}^P)^\theta \right\} = 1$$

With persistent risk, it is no longer possible to solve the value function in closed form. Therefore, we follow Bansal and Yaron (2004) in assuming returns can be approximated using the Campbell-Shiller approximation[34]:

$$R_{t+1}^j = k_0^j + k_1^j z_{t+1}^j - z_t^j + g_{t+1}^j \tag{53}$$

where $z_t^j = \ln(P_t^j / D_t^j)$, the log of the price-to-payout ratio for the asset, and where k_0^j and k_1^j are approximating constants[35]. For the return on the consumption asset, for example, $z_t^j \equiv \ln(P_t^{cj} / C_t^j)$, the log of the price-to-consumption ratio while for the asset paying country j output,

[34] However, these approximations can lead to misleading conclusions. As pointed out by Hansen (forthcoming), the true returns from recursive preferences depend upon a nontrivial factorization.

[35] The constants are $k_1^j = \frac{exp(\bar{z}^j)}{1+exp(\bar{z}^j)}$ and $k_0^j = log(1 + exp(\bar{z}^j)) - k_1^j \bar{z}_j$, where $\bar{z}^j$ is the steady state log price to consumption ratio.

$z_t^j \equiv \ln(P_t^{*j}/Y_t^j)$ the log of the price-to-output ratio. We use these relationships below to determine the welfare in the benchmark and the risk sharing economy.

B.2.1 Benchmark Economy Welfare

Since the value function and the return process depends upon the price-to-payout ratio, it is necessary to solve for this ratio, $P_t^{cj}/C_t^j = \exp(z_t^j)$. Once again, in the benchmark economy, $Y_t^j = C_t^j$ so that $\exp(z_t^j) \equiv P_t^{cj}/Y_t^j$. Following Bansal and Yaron (2006), we conjecture that the log price-to-consumption ratio is linear in the persistent risk. Thus,

$$z_t^j = A_0^j + A_1^j x_t^j. \tag{54}$$

Substituting equation (54) into the consumption asset Euler equation above (eqn (13)) and taking expectations implies:

$$A_1^j = \frac{1-\frac{1}{\psi}}{1-k_1^j\rho} \tag{55}$$

$$A_0^j = \ln(\beta) + \left(1-\frac{1}{\psi}\right)\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^2\left(1+\frac{\varphi_e^2\, k_1^{j2}}{(1-k_1\rho)^2}\right)\right] + \frac{k_0^j}{1-k_1^j} \tag{56}$$

where $k_0^j = log(1+exp(\bar{z}^j)) - k_1^j\bar{z}_j$ and $k_1^j = exp(\bar{z}^j)/\left(1+exp(\bar{z}^j)\right)$. Note that the appproximating constants k_0^j, k_1^j depend upon the solution to the long run value of z_t^j so our solution solves for the fixed point between the z_t^j equation (54) and the constant A_0^j in equation (56).

The solution has an intuitive interpretation in light of the solution with transitory risk. The deviation of the long run price-to-consumption ratio from the effect of the long run constant, $\frac{k_0^j}{1-k_1^j}$ is here given by:

$$E\left[\frac{P_t^{cj}}{C_t^j}\right] = \exp(A_0^j) = \beta\widehat{M}_j^{\left(1-\frac{1}{\psi}\right)}$$

where $\widehat{M}_j = \exp\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^{j2}\left(1+\frac{\varphi_e^2\, k_1^{j2}}{(1-k_1\rho)^2}\right)\right]$. Recall that in the transitory risk case, we had $\beta M_j^{\left(1-\frac{1}{\psi}\right)}$ where $M_j \equiv \exp\left[\widetilde{\mu}^j - \frac{1}{2}\gamma\sigma^{j2}\right]$. Thus, the persistent case increases the certainty equivalent risk through increasing the variance by a factor of $\varphi_e^2\, k_1^{j2}/\left(1-k_1\rho\right)^2$. This factor is the proportion of persistent risk to transitory risk, φ_e^2, grossed up by the effect of the autoregressive coefficient, ρ, weighted by the approximating constant which is less than one, k_1.Defining $Z_t^j \equiv \exp(z_t^j)$, the value function can be found by substituting the solution for the price-to-consumption ratio into the wealth equation giving:

$$V(C_t^{jB}, W_t^{jB}) = Y_t^j \left(1 + \frac{P_t^j}{Y_t^j}\right)^{\frac{1}{1-(1/\psi)}} = Y_t^j \left(1 + Z_t^j\right)^{\frac{1}{1-(1/\psi)}}$$

B.2.2 Risk Sharing Economy Welfare

In the risk sharing economy, we follow the same steps to find the consumption for country j as a weighted share of world output: $C_t^j = \varpi^j Y_t^w$ where $\varpi^j = \left(Y_0^j + P_0^{j*}\right) / \left(Y_0^w + P_0^{w*}\right)$.

We begin with the price of world consumption, P_0^{w*}. In this case, the common growth rate across countries is the weighted sum of the country growth rates:

$$\begin{aligned} g_{c,t+1}^* &= \mu^* + x_t^* + \eta_{t+1}^* \\ x_{t+1}^* &= \rho x_t^* + e_{t+1}^* \end{aligned}$$

where $\mu^* \equiv \frac{1}{J}\Sigma_{j=1}^J \mu^j$, $x_t^* \equiv \frac{1}{J}\Sigma_{j=1}^J x_t^j$, $\eta_t^* \equiv \frac{1}{J}\Sigma_{j=1}^J \eta_t^j$ and $e_t^* \equiv \frac{1}{J}\Sigma_{j=1}^J e_t^j$ so that $\sigma^{*2} = \left(\frac{1}{J}\right)^2 \iota' \, \Sigma \, \iota$ and $\sigma_e^{*2} = \left(\frac{1}{J}\right)^2 \iota' \, \Sigma_e \, \iota$ for Σ and Σ_e, the variance-covariance matrix of transitory and persistent shocks, respectively, and ι, a J-dimensional unit vector. Note that this specification assumes the autocorrelation in persistent shocks ρ are common across countries. We also solved the model relaxing this assumption, though it significantly complicated the analysis without altering the results much.

With this process for world consumption, the log price-to-consumption process can be rewritten:

$$z_t^w = A_0^w + \sum_{i=1}^J A_j^w x_t^i \tag{57}$$

Substituting equation (57) and the world process $g_{c,t+1}^*$ into the consumption asset Euler equation above (eqn (13)) and taking expectations implies:

$$A_1^w = \frac{1 - \frac{1}{\psi}}{1 - k_1^* \rho}$$

$$A_0^w = \ln(\beta) + \left(1 - \frac{1}{\psi}\right) \left[\widetilde{\mu}^* - \frac{1}{2}\gamma\sigma^{*2}\left(1 + \frac{\varphi_e^{*2} \, k_1^{*2}}{(1 - k_1^*\rho)^2}\right)\right] + \frac{k_0^*}{1 - k_1^*}$$

where the approximating constants k_0^*, k_1^* are the same as before but correspond to the world price-to-consumption ratio. The solution to this fixed point problem determines $Z_t^w \equiv \exp(z_t^w)$.

Next, we require the price of country j output in world markets. For this purpose, we solve for the price-to-output ratio given by:

$$z_t^{*j} = A_0^{*j} + \sum_{i=1}^J A_{i,1}^{*j} x_t^i$$

Substituting this price-to-output equation into the Euler equation and taking expectations yields:

$$A_{i,1}^{*j} = \frac{(\theta - 1 - \frac{\theta}{\psi}) + (\theta - 1)(k_1^w \rho - 1)A_j^w}{1 - k_1^{*i}\rho},$$

$$A_0^{*i} = \frac{[\theta \ln \beta + (\theta - 1 - \frac{\theta}{\psi})\sum_j w_j \mu^j + (\theta - 1)(k_0^w - A_0^w(1 - k_1^w)) + k_0^{*i} + \mu^i] + \frac{1}{2}\sigma^{*2} + \frac{1}{2}\sigma_e^{*2}}{1 - k_1^{*i}}.$$

Solving the fixed point for the new approximating constants, k_0^{*i} and k_1^{*i} determine the equilibrium $Z_t^{*j} = \exp(z_t^{*j})$. Then the country share is given by:

$$\varpi^j = \frac{\left(Y_0^j + P_0^{j*}\right)}{(Y_0^w + P_0^{w*})} = \frac{Y_0^j\left(1 + Z_0^{*j}\right)}{Y_0^w\left(1 + Z_0^w\right)} \tag{58}$$

B.2.3 Welfare gains

We can now calculate the welfare gains as before using the Z solutions. The general form for the welfare gains is given by Δ^j in:

$$V_0((1 + \Delta^j)C_0^{jA}, (1 + \Delta^j)W_0^{jA}) = V_0(C_0^{j*}, W_0^{j*})$$

$$(1 + \Delta^j) = \frac{\varpi^j Y_0^w}{Y_0^j}\left[\frac{1 + \frac{P_0^{w*}}{Y_0^w}}{1 + \frac{P_0^j}{Y_0^j}}\right]^{\left(\frac{1}{1-\frac{1}{\psi}}\right)} = \frac{\varpi^j Y_0^w}{Y_0^j}\left[\frac{1 + Z_0^w}{1 + Z_0^j}\right]^{\left(\frac{1}{1-\frac{1}{\psi}}\right)}$$

where the share ϖ^j is given above in equation (58).

B.2.4 Implied Returns

Given the price-to-consumption solutions for benchmark economy, Z_t^j, and the world economy, Z_t^w, we then calculate the returns using the Campbell-Shiller equation (53) as well as the risk-free rate.

C Appendix: Empirical Methods

In this appendix we describe the empirical methods used in our analysis.

C.1 Data Description

Our analysis requires data for consumption, asset returns, and dividends. Moreover, our framework considers risk from variations in a common good. Therefore, we must adjust all consumption, returns, and dividends to insure they are valued in units of this common good. For consumption, we use per capita consumption from the Penn World Tables National Accounts measured with a common Purchasing Power Parity (PPP) price deflator from 1950 to 2009. As such, real exchange rate variations appear as through PPP deviations that add to the variability in our consumption data.

For dividend and equity return data, we use quarterly data from the Total Market Indices in Datastream-Thomson Financial from 1970 to 2009. For the risk-free rates we update the series in Campbell (2003) obtained from the IMF's International Financial Statistics. To be consistent with the annual consumption data, we first aggregate the quarterly data to annual. We then use the common good deflator from Penn World Tables to form real annual equity returns, risk free rates, and dividend growth rates. Therefore, as with our consumption measures, the real value of these asset returns incorporate real exchange rate risk through PPP deviations.

C.2 Solutions and Simulated Method of Moments

We solve for the consumption process parameters in our model by fitting target moments from a reduced Simulated Method of Moments (SMM). We conduct this analysis for both versions of our model: (a) equity as the "consumption asset"; and (b) equity as the "dividend asset".

To generate the parameters values, we first calibrate the monthly growth rates μ and μ_d to the annual means of consumption growth and dividend growth. For this purpose, we calculate the mean annual growth rates from the data and divide by 12. In trial runs of the SMM procedure described below, we find that this change makes little difference in the estimation of the remaining parameters and greatly decreases the computation time.

We then use the reduced SMM to fit the remaining parameters for each country: $\left[\sigma^j, \sigma_e^j, \rho^j\right]$ for the "consumption asset" case, and $\left[\sigma^j, \sigma_e^j, \sigma_d^j, \rho^j\right]$ for the "dividend asset" case. Implementing the SMM procedure involves the following steps. For every set of parameter values, we first solve the model using the analytical solutions for returns in the benchmark economy. We choose a set of targeted moments to best represent both consumption and asset pricing data[36]. We then compute

[36] See Gallant and Tauchen (1999) for a discussion on efficient method of moments and problems with moment selection.

a weighted difference between a targeted set of model generated moments and the data moments using a weighting matrix. To treat all targets equally, we report the estimates using the identity matrix.[37] The set of parameter values that minimizes this difference is the SMM estimate.

For the "consumption asset" model, we choose the following set of target data moments for each country: the standard deviation of log consumption growth (σ_{gc}), the first order auto-correlation of log consumption growth (ρ_{gc}), the mean equity premium ($E(R^p - R^{rf})$), the mean risk free rate ($E(R^{rf})$), the standard deviation of the market return ($\sigma(R^p)$), and the standard deviation of the risk free rate ($\sigma(R^{rf})$). Using these six moments per country, we estimate the three parameters capturing the transitory risk, σ^j, persistent risk, σ_e^j, and degree of persistence, ρ^j. As a practical matter, we find that estimates of ρ^j are quite similar across countries so we equate them in the analysis reported in the paper.

For the "dividend asset" model, we first follow Bansal and Yaron (2004) in setting the sensitivity of dividends to persistent risk $\phi^j = 3$. For this version of the model, we augment the number of targets over the six moment "consumption asset" set to include the standard deviation of dividend growth (σ_{gd}), and the first order auto-correlation of log consumption growth (ρ_{gd}). Using these eight moments per country, we estimate the same three consumption parameters along with the standard deviation of monthly dividend growth σ_d^j. Once again, the ρ^j estimates are similar across countries so that we set them equal in the reported results.

Our estimation requires a set of preference parameters. For this purpose, we use parameter estimates that have been found to fit asset returns best in the US. We therefore take the parameters from Bansal and Yaron (2004) of IES = 1.5, $\gamma = 10$, and monthly $\beta = .998$ or annualized $\beta = .985$. As is required from our model, these parameters are the same across all countries.

The model is estimated at the monthly level and therefore the simulated data from the model must be time-aggregated to match the annual data moments. To time-aggregate, we compute the growth between the levels at t and $t + 12$, given the realizations of 12 monthly growth rates.[38] To match our annual consumption, dividend growth and asset return moments, we then time-aggregate the model-generated data from monthly to annual frequency. Parameter estimates and simulated model moments are the averages of 500 simulations, each with 840 time-aggregated monthly observations.

[37] We also implemented the reduced SMM procedure using a diagonal matrix with typical components equal to the sample variance. This procedure gain qualitatively similar results.

[38] By comparison, we multiply monthly rates times 12 when we annualize as opposed to time aggregate.

C.3 Monte Carlo Experiments

As noted in the Appendix B, the solution for the world equilibrium approximates the aggregate world consumption growth rate as the weighted sum of the individual country growth rates. This approximation treats the log growth rate of the sum of outputs as the sum of the log growth rates of output. For example, in the equally-weighted model, the world consumption growth rate is assumed to follow: $g^w_{c,t} \equiv \frac{1}{J}\Sigma^J_{j=1}\ g^j_{c,t}$ Since each of the processes are conditionally log normal and the solution to the Euler equation assumes log normality, this approximation may render the solution approach invalid.

To evaluate this approximation, we conducted a Monte Carlo experiment First we used the processes for the individual log-normally distributed growth rates $g^j_{c,t}$ to generate 1000 draws. We then constructed the resulting world growth rate process $g^w_{c,t}$. On this simulated series, we calculated the skewness and kurtosis moments. We found that these moments matched closely the normal distribution, suggesting that the approximated world growth rate is close to being log-normally distributed.

D Appendix: Persistent Consumption Risk Correlation

In this appendix, we detail the identification of correlation in the shock to persistent risk, e^i_t.

D.1 Consumption Asset

The consumption process with persistent risk is given by:

$$\begin{aligned} g^j_{y,t+1} &= \mu^j + x^j_t + \eta^j_{t+1} \\ x^j_{t+1} &= \rho x^j_t + e^j_{t+1} \end{aligned}$$

where $\eta^j_{t+1} \backsim N(0,\sigma^j)$ and $e^j_{t+1} \backsim N(0,\sigma^j_e)$.

Then clearly the covariance of consumption across any two countries i and j is given by equation (21) in the text:

$$Cov(g^i_c, g^j_c) = \sigma^i\sigma^j Corr(\eta^i,\eta^j) + \frac{\sigma^i_e\sigma^j_e}{1-\rho^2}Corr(e^i,e^j)$$

In order to identify the correlations from η separately from e, we require an independent observation of these correlations. Recall that the Campbell-Shiller approximation of returns in equation (53) states:

$$R^j_{t+1} = k^j_0 + k^j_1 z^j_{t+1} - z^j_t + g^j_{t+1}$$

where here $z_t^j \equiv \ln(P_t^{c,j}/C_t^j)$, since equity is assumed to payout consumption. Moreover, we have solved above in Appendix B for the the log price-to-consumption ratio z_t^j as

$$z_t^j = A_0^j + A_1^j x_t^j$$

where A_0^j and A_1^j are given by equations (56) and (55), respectively. Substituting these solutions into the equation for z_t^j and the result into the Campbell-Shiller approximation in (53) and rearranging implies that equity returns for country i can be written in the form:

$$R_{t+1}^i = a_0^i + a_1^i x_t^i + a_2^i e_{t+1}^i + \eta_{t+1}^i$$

where a_0^i, a_1^i, a_2^i are given by:

$$\begin{aligned} a_0^i &= k_0^i + k_1^i A_0^i - A_0^i + \mu^i \\ a_1^i &= k_1^i A_1^i \rho - A_1^i + 1 \\ a_2^i &= k_1^i A_1^i \end{aligned}$$

Calculating the covariance of equity returns between any two countries i and j using this solution yields:

$$Cov(R_{t+1}^i, R_{t+1}^j) = \sigma^i \sigma^j Corr(\eta_{t+1}^i, \eta_{t+1}^j) + \left[\frac{a_1^i a_1^j}{1-\rho^2} + a_2^i a_2^j\right] \sigma_e^i \sigma_e^j Corr(e_{t+1}^i, e_{t+1}^j)$$

given as equation (23) in the text.

Combining the consumption covariances in equation (21) with the equity covariance in equation (23), we solve for the correlation in the persistent shock as:

$$Corr(e^i, e^j) = D_o \frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} \left[Corr(R^i, R^j) - \frac{\sigma_c^i \sigma_c^j}{\sigma_R^i \sigma_R^j} Corr(g_c^i, g_c^j)\right]$$

where $D_o \equiv \left[\frac{a_1^i a_1^j - 1}{1-\rho^2} + a_2^i a_2^j\right]^{-1}$. Substituting the solutions for A_1^i for the $a_1^i, a_1^j, a_2^i, a_2^j$ parameters and using the fact that $\psi > 1$ and k_1^i and k_1^j in our analysis verifies that $D_o > 0$. Since the data implies $\sigma_R^i \sigma_R^j \gg \sigma_c^i \sigma_c^j > \sigma_e^i \sigma_e^j$ and since $Corr(R^i, R^j) > Corr(g_c^i, g_c^j)$, the correlation on the persistent risk, $Corr(e^i, e^j)$, must be high.

D.2 Dividend Asset

The consumption process with persistent risk in dividends is given by:

$$\begin{aligned} g^j_{c,t+1} &= \mu^j + x^j_t + \eta^j_{t+1} \\ x^j_{t+1} &= \rho^j x^j_t + e^j_{t+1} \\ g^j_{d,t+1} &= \mu^j_d + \phi^j x^j_t + u^j_{t+1} \end{aligned}$$

where $\eta^j_{t+1} \sim N(0, \sigma^j)$, $e^j_{t+1} \sim N(0, \sigma^j_e)$, $u^j_{t+1} \sim N(0, \sigma^j_u)$, $u^j_{t+1} \perp \eta^j_{t+1} \perp e^j_{t+1}$ and μ^j_d is the growth rate of dividends.

The covariance of the consumption process across countries is the same as before as given by equation (21). However, now equity pays out dividends so we must solve for the price-dividend ratio. Defining the log price-to-dividend ratio as $z^j_{mt} \equiv \ln(P^{;j}_t / D^j_t)$, we conjecture the form of the process:

$$z^j_{mt} = A^j_{0,m} + A^j_{1,m} x^j_t \tag{59}$$

Substituting the return process into the Euler equation and solving for the constants implies[39]:

$$A^j_{1,m} = \frac{\phi - \frac{1}{\psi}}{1 - k^j_{1,m}\rho}$$

where k^j_{1m} is the approximating constant for the dividend paying asset. Substituting the solutions for $A^j_{1,m}$ into equation (59) and the resut into the Campbell-Shiller equation (53) generates equity returns of the form:

$$R^i_{t+1} = b^i_0 + b^i_1 x^i_t + b^i_2 e^i_{t+1} + u^i_{t+1}$$

where b^i_0, b^i_1, b^i_2 are given by:

$$\begin{aligned} b^i_0 &= k^i_{0,m} + k^i_{1,m} A^i_{0,m} - A^i_{0,m} + \mu^i_d \\ b^i_1 &= k^i_{1,m} A^i_{1,m}\rho - A^i_{1,m} + \phi^i \\ b^i_2 &= k^i_{1,m} A^i_{1,m} \end{aligned}$$

where k^j_{1m} is the approximating constant counterpart to k^j_1 for the dividend paying asset. Calculating the covariance of equity returns between i and j implies:

$$Cov(R^i, R^j) = \sigma^i_u \sigma^j_u Corr(u^i, u^j) + \left[\frac{b^i_1 b^j_1}{1-\rho^2} + b^i_2 b^j_2\right] \sigma^i_e \sigma^j_e Corr(e^i, e^j) \tag{60}$$

[39] The correlation does not depend upon A^j_{0m} so its solution is ommitted to save space.

Using the expression for dividend growth in equation (25), the covariance between dividend growth in country i and j can be written:

$$Cov(g_d^i, g_d^j) = \sigma_u^i \sigma_u^j Corr(u^i, u^j) + \phi^i \phi^j \frac{\sigma_e^i \sigma_e^j}{1-\rho^2} Corr(e^i, e^j) \tag{61}$$

Given the covariance in equity returns (equation (60)) and the covariance in dividends (equation (61)), we can now solve for the correlation in persistent consumption risk, $Corr(e^i, e^j)$, in terms of the equity return and dividend growth cross-country correlations:

$$Corr(e^i, e^j) = B_o \frac{\sigma_R^i \sigma_R^j}{\sigma_e^i \sigma_e^j} \left[Corr(R^i, R^j) - \frac{\sigma_d^i \sigma_d^j}{\sigma_R^i \sigma_R^j} Corr(g_d^i, g_d^j) \right]$$

where $B_o \equiv \left[\frac{b_1^i b_1^j - \phi^i \phi^j}{1-\rho^2} + b_2^i b_2^j \right]^{-1}$. Given our parameterization, $B_o > 0$ when $\phi^i \phi^j > 1$, a condition that is satisfied by the BY assumptions that $\phi = 3$. As with the consumption asset case, the data relationships imply high correlations in persistent risk, e^i. That is, empirically we find $\sigma_R^i \sigma_R^j > \sigma_d^i \sigma_d^j > \sigma_e^i \sigma_e^j$. Moreover, $Corr(R^i, R^j) > Corr(g_d^i, g_d^j)$. As a result, $Corr(e^i, e^j)$ is high and near one.