THE ALLOCATION OF CREDIT
AND FINANCIAL COLLAPSE

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ABSTRACT

This paper examines the allocation of credit in a market in which borrowers have greater information concerning their own riskiness than do lenders. It illustrates (1) the allocation of credit is inefficient and at times can be improved by government intervention, and (2) small changes in the exogenous risk-free interest rate can cause large (discontinuous) changes in the allocation of credit and the efficiency of the market equilibrium. These conclusions suggest a role for government as the lender of last resort.

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I. INTRODUCTION

In this paper I examine the allocation of credit in a market in which borrowers have greater information on the probability of default than do lenders. My purpose is to illustrate two propositions. First, the equilibrium resulting in an unfettered market is inefficient; government intervention of a sort often observed may at times be able to enhance market efficiency, even if the government has no informational advantage over lenders. Second, the unfettered market equilibrium is precarious: small changes in the exogenous risk-free interest rate can cause large and inefficient changes in the allocation of credit.

Many recent studies note the importance of asymmetric information for credit markets.[1] The two results emphasized here, while natural consequences of asymmetric information, often escape unnoticed. Understanding these conclusions, however, is critical to evaluating the impact of various government policies.

Government intervention into the allocation of credit is substantial. Federal loan guarantees to the Chrysler Corporation and to New York City are among the most publicized examples. On a continuing basis, the government plays a central role in the markets for loans to students, farmers, and homeowners. Economists often criticize this role on the grounds that the market can best allocate credit.[2] The model presented here shows that this conclusion is not generally correct. While credit programs are frequently justified on distributional grounds, I show that a social planner concerned solely with economic efficiency may often endorse the type of policy

currently effective in many credit markets.

The model presented here is close in spirit to those of Stiglitz and Weiss [1981] and Ordover and Weiss [1981]. The common theme is that changes in the interest rate alter the riskiness of the pool of borrowers. While these previous two papers note the possibility that the equilibrium is socially inefficient, the policy interventions they propose do not correct the market failure discussed here. Stiglitz and Weiss suggest a usury law (an interest rate ceiling) as one solution. In the model of this paper, a usury law does not improve on the market allocation; instead, it causes the market for these loans to disappear.[3] Ordover and Weiss propose the policy of forcing banks to lend to all borrowers at some interest rate. The equilibrium in this model, however, can be inefficient even if no borrower is credit rationed in the sense of being excluded at any interest rate; even when such credit rationing does occur, the Ordover-Weiss policy merely induces banks to charge a prohibitively high interest rate. In neither case is this policy effective. Nonetheless, a credit subsidy, such as a loan guarantee, can at times improve on the market allocation.

The model also has macroeconomic implications. As noted above, in the absence of government intervention, an increase in the exogenous risk-free interest rate can cause the collapse of the credit market. A market that was efficient at the initial interest rate can disappear, driving out socially profitable investments. In other words, the total surplus derived for a particular credit market can be a discontinuous function of the interest rate. In models without asymmetric information, restrictive monetary policy moves the economy smoothly along the marginal efficiency of capital schedule;

in this model, restictive policy is potentially more costly, as it can precipitate a financial crisis.

While the model in this paper is general, I present it in the context of a specific credit market. In particular, I discuss loans from banks to students. There are two reasons that student loans are a useful prototype for studying credit market imperfections. First, only a limited number of financial instruments are available to students. A corporation can fund investments with either debt or equity. It has, in addition, more complex options: preferred stock, convertable bonds, callable bonds. Imperfections in the market for one instrument may be less important if there exist other financing methods. In contrast, a student faces a much simpler problem. He must borrow if he is to invest; he cannot issue equity on his human capital. In principle, we could attempt to explain the paucity of financial instruments available to the student. For this paper, though, it is both reasonable and realistic to assume that his only option is debt finance.

The second reason for discussing the market for student loans is that it has evoked substantial government intervention. The OECD reported in 1978 that Canada, France, Germany, Japan, the Netherlands, Norway, Sweden, and the United States all provide assistance to students in the form of loans or loan guarantees. Of course, there are many reasons for public support of education. Nonetheless, it is instructive that this support so often takes the form of credit market intervention. The pervasiveness of public student loan programs suggests at least the perception of imperfections in the market for credit.

II. THE MODEL

This section presents a simple model of a market for loans to a particular group of students. To the banks, who provide the loans, the students are indistinguishable. The students, though, differ by the expected return on their education and by their probability of repaying the loan. Each student knows his own expected return and repayment probability, even though they are not observable by the banks or by the government.

Both students and banks are risk neutral. I make this assumption to simplify the analysis. There is no reason to suppose that the market failure discussed here would disappear if some agents were risk averse. A major advantage of the risk neutrality assumption is that it makes clear that the market failure is not attributable to the underprovision of insurance to risk averse agents.

Each potential student is considering investing in some human capital, say, a college degree. The project is discrete, has unit cost, and has expected future payment R. (All return variables I use are expressed as the return factor. That is, if the expected return is 5 percent, then R = 1.05) The other characteristic of each student is the probability P that he will repay the loan. The values of R and P vary across students. Each student knows his own R and P, but these characteristics are not observable by banks. These two characteristics are distributed throughout the population with the density function f(P,R), which is public knowledge.

The model takes each student's parameters P and R as primitive. One could construct a more complete model in which the student's default behaviour is endogenous. For example, one could model the students as having

varying degrees of honesty; certain students get greater disutility from dishonest acts. Default probabilities vary because a less honest student is more likely to avoid repayment illegitimately. Alternatively, one could model all the students as well-meaning. A student then defaults when his return leaves him unable to repay his loan ex post; the probability of this state occuring is then private information.[4] Either such model might suggest that each student's repayment probabilty depends on the market interest rate. I maintain the assumption that P is exogenous for each student to simplify the exposition.

A bank can invest in a safe asset, such as a Treasury bill, and obtain the certain future payment ρ. Alternatively, a bank can lend to one of the above group of students. Let r be the interest rate the bank charges these students. It is the same for all students, since they are indistinguishable to the bank.

If a student defaults, the bank receives no payment on the loan. Including a default payment of Δ ($\Delta < \rho$), such as collateral, is certainly possible. In particular, in such a world, one could consider the student as taking out a loan of Δ/ρ that is repaid with certainty and a loan of $1 - \Delta/\rho$ that is repaid with probability P and and fully defaulted with probability $1 - P$. It is straightforward to carry the extra terms throughout the analysis and show that the existence of the risk-free loan does not substantially affect the market for the risky loan. Thus, I set the default payment to zero without loss of generality.[5]

Let Π be the average probability of repayment; that is, Π is the average of P for those potential students who in fact borrow. The expected payment

to the bank on a student loan is Πr. This expected payment must equal the safe payment p if the bank is to make any student loans. Hence, the first equilibrium condition is

(1) $\Pi r = p$.

This equation describes the locus of market loan rates and repayment probabilities that provide the lenders the required rate of return.

Each potential student must decide whether to borrow at the market rate r and invest in additional human capital. The expected return on his investment is R, while the expected cost of borrowing is Pr. Hence, he borrows and invests if and only if R > Pr. It is useful to examine this investment condition graphically. Figure I shows the area in (R,P) space for which the investment is made. Those students in areas A and B borrow and invest. Those in areas C and D do not. An increase in the loan rate from r_0 to r_1 unambiguously reduces the areas A and B, and thus reduces the number of loans and investments. Given any expected return R, the students driven out by the increase in the interest rate are those with relatively high repayment probabilities.

Even at this early stage, we can show the market allocation is not fully efficient. An investment should be made, from the viewpoint of the social planner, if and only if the expected return R exceeds the opportunity cost p. Those investments in area B are socially efficient and are undertaken, while those in C are socially inefficient and are not undertaken. Yet those investments in area D are socially efficient but not undertaken, and those in area A are socially inefficient and are undertaken. No loan rate r can

make both areas A and D disappear. Thus, no loan rate, including the market equilibrium rate, can in general reach the first best allocation.

The assumption of asymmetic information plays a key role here. As already mentioned, a student invests if R > Pr. Using the equilibrium condition (1), the investment condition is R > (P/Π)ρ. If there is no information asymmetry regarding the default probability, then P = Π and the student invests if and only if his return is socially profitable (R > ρ). In this case, the market reaches the fully efficient allocation even though R is not publicly observable. If there is information asymmetry regarding P, then low P investments are overly encouraged and high P investments are overly discouraged.

The repayment probability Π as seen by banks is the average of P for those students who invest, that is, for those in areas A and B. Thus, the function relating r to Π is

(2) $\Pi(r) = E[P \mid R > Pr]$.

For any density f(P,R), the function $\Pi(r)$ is a well-defined conditional expectation.

III. MARKET EQUILIBRIUM

Equations (1) and (2) are the equilibrium two conditions. They simultaneously determine the market loan rate r, from which we can infer the decision of each potential student and, thereby, the average repayment probability Π. It is useful to illustrate the equilibrium conditions graphically. Equation (1) defines a rectangular hyperbola, which is labelled

LL in Figure II since it is determined by the required return of lenders.

Equation (2) is labelled BB, since it is determined by the optimizing reaction of borrowers. The shape of the BB curve is more ambiguous. As r goes to zero, Π approaches the unconditional expectation $E(P) < 1$, since everyone borrows. As r goes to infinity, Π goes to zero as long as R is bounded from above and $f(P,R)$ is non-zero everywhere else. If R were constant across borrowers, the BB curve would be monotonic; as r rises, high P borrowers drop out of the market. In general both R and P vary and the BB curve need not be monotonic.[6]

The LL curve and the BB curve might not intersect, as in Figure III. In this case, there is no equilibrium in which loans are made. At any interest rate, the pool of students who seek loans is too risky to give the banks their required return. I call this a "collapsed" credit market.

The two curves may intersect more than once, as they do in Figure II. If they do cross more than once, it seems reasonable to restrict attention to the first intersection and to rule out any additional equilibrium as not stable. To see why, consider point y in Figure II. If the economy were at this point, both equilibrium conditions would be satisfied. But a bank would have an incentive to lower its interest rate, say to r_1. At r_1, the BB curve lies above the LL curve. The repayment probability Π is thus greater than necessary to give the bank its required return ρ. A bank can therefore make a higher expected return by charging r_1, which is below the "market" rate r_y. Similar reasoning shows that point x is a stable equilibrium. At interest rates just above r_x, lenders earn a rate of return above ρ, which causes a capital inflow and lowers the interest rate.

Conversely, at interest rates below r_X, the repayment probability is too low to give banks a return of ρ, which causes a capital outflow and raises the interest rate. For these reasons, only the first crossing appears to be an interesting equilibrium.

It is possible that there are more than two crossing of the two curves. If so, at the third (and every odd) crossing the BB curve cuts the LL curve from below. Thus, the intersection is locally stable; that is, a bank could not make a profit by a small reduction in its interest rate. The further intersections, however, are not globally stable, since a bank could make a large reduction in its interest rate and make positive profits. Hence, even if there are multiple crossings, we should expect the economy to locate at the first one.

Strong unequivocal statements regarding the welfare properties of the market equilibrium are impossible. As discussed above, if there is no information asymmetry regarding P, the market reaches the first best allocation. At other times, however, the market is grossly inefficient and government intervention can enhance efficiency. To illustrate this possibility, I examine a special case.

Example: Uniform Expected Return

Suppose that the expected return R is constant across students. The only unobserved heterogeneity is the repayment probability P. In this case, the market fails to reach the first best allocation.

Since R is constant, either all the investments are socially efficient or all are socially inefficient. If an equilibrium exists, then the invest-

ments are socially efficient. That is, if the BB curve and the LL curve interesect, then the expected return R exceeds the opportunity cost of capital ρ. This proposition is easy to prove. As discussed earlier, a student borrows if and only if R > Pr. Averaging this inequality over the investments undertaken shows R > Πr. Since Πr = ρ, we know R > ρ. Thus, if R is an observable characteristic, then the unfettered market equilibrium allows only socially productive investments.

On the other hand, investments may be socially productive but not undertaken in equilibrium. That is, it is possible the projects are productive in the sense that R > ρ, but not all investments are undertaken. An example most easily shows this proposition. Suppose P is uniformly distributed from zero to one. Then equilibrium condition (2) becomes: Π = $\frac{1}{2}$ for r<R, Π = R/2r for r>R. The LL curve lies above the BB curve at all r, unless R > 2ρ. In this example, no equilibrium exists unless the expected payment is twice the required payment. The unfettered market equilibrium may leave profitable investment opportunities unrealized.

IV. GOVERNMENT CREDIT POLICY

I now discuss the potential for efficient government intervention. Imagine the market begin in the unregulated equilibrium. Let us consider the effects of a small credit subsidy, which would reduce the market interest rate r and shift leftward the upward sloping line in Figure I.

This reduction in the interest rate has two effects. First, it reduces the area D; some of those students with high returns and high repayment probabilities who were previously not investing are now induced to invest.

Second, it increases area A; some more students with low returns and low repayment probabilities are induced to invest. The first effect is socially beneficial while the second is socially harmful.

A government loan guarantee is a special case of a subsidy. In particular, under a guarantee program, the market rate becomes the risk-free rate ($r = \rho$); that is, a loan guarantee is equivalent to a government loan at the risk-free interest rate. At $r = \rho$, area D disappears, implying that all socially productive investments are undertaken.

To evaluate the net social impact of such a subsidy, one only needs to know the distribution of attributes, $f(P,R)$. It is not necessary for the government to be able to distinguish the high return students from the low return students. As the example below illustrates, it is possible that the extra investment generated is on net socially optimal but is not undertaken in the market equilibrium because it requires that $\Pi r < \rho$.

Of course a government credit subsidy has a budgetary cost. While the return from students to banks is Πr, banks still require return ρ. The difference is made up by the government. If the government must raise money using distortionary taxes, then the deadweight losses are an additional cost of the credit program. As with all expenditure programs, the marginal benefit must exceed the marginal deadweight losses if the program is to be socially efficient.

Before turning to the example, a few general propositions regarding the optimal interest rate $r*$ can be established. First, $r*$ is never below the risk-free return ρ; charging a lower rate would only induce inefficient investment. Second, $r*$ is generally strictly above the risk-free rate. To see this, note that social welfare (ignoring the cost of raising revenue) is:

(3)   Social Welfare = $\int_0^1 \int_{Pr}^\infty$ (R - ρ) f(P,R) dR dP

The derivative of social welfare with respect to the interest rate r is:

(4) $\dfrac{d\ SW}{d\ r}$ = $\int_0^1$ -P(Pr - ρ) f(P,Pr) dP

Evaluated at r = ρ, this derivative is non-negative and is strictly positive as long as f(P,R) is everywhere non-zero. Thus, an efficient loan program generally charges a loan rate greater than the risk-free rate.

Third, depending on the density f(P,R), it is possible that the optimal interest rate r* exceeds the unregulated equilibrium interest rate r^e. In this case, the government would tax student loans to drive out borrowers with low R and low P.[7] While it is difficult to derive general conditions under which r* > r^e, it is possible to examine the effect on social welfare of small changes in the interest rate around r^e. In particular, at r^e the sign of the dSW/dr is the same as the sign of dΠ/dr.[8] Hence, if the BB curve is upward sloping at the equilibrium, then a small _increase_ in the interest rate is welfare-enhancing; conversely, if the BB curve is downward sloping at the equilibrium, then a small _decrease_ in the interest rate is welfare-enhancing. In other words, if a small subsidy or tax is to increase social welfare, it must increase the average repayment probability.

Example, continued

Consider again the example of uniform expected returns. Suppose the government provides a loan guarantee. The vertical line r = ρ replaces the LL curve, as in Figure IV. This program clearly changes the nature of the equilibrium. In particular, it is possible that the guarantee program

creates a market, whereas without the program, no market existed, as in Figure III.

As already noted, under a guarantee program, all socially profitable investments are undertaken. It is possible, though, that socially unproductive investments are undertaken once the guarantee is provided. That is, even if $R < \rho$, those students for whom $P < R/\rho$ choose to borrow and invest. Of course, since R is known in this example, the government can avoid this inefficiency by providing guarantees only if $R > \rho$.

The reason the government can so effectively correct the market failure is that the government requires a different information set than private lenders. To make a socially profitable loan, we only need to know that the expected return R exceeds the required return ρ. The probability of repayment P _per_ _se_ is irrelevant to a social planner. (Remember that the government program is not required to be revenue neutral.) Yet to private lenders, the expected return on a project R _per_ _se_ is irrelevant, and the repayment probability P is critically important. Hence, this example of constant R may be the case in which the government can most easily improve on the private allocation of credit.

Under what conditions is the unfettered market least efficient? To answer this question, I specialize the example further by supposing P is uniformly distributed from P_0 to P_1. From (2) and straightforward calculation, we can compute the equation for the BB curve. It is

(5) $\Pi = (P_0 + P_1)/2$ for $0 < r < R/P_1$

       $= (P_0 + R/r)/2$ for $R/P_1 < r < R/P_0$.

The intersection of (1) and (5) determines the interest rate in an unfettered equilibrium. When the solution is interior, the number of loans made is

(6) $\text{Loans} = \dfrac{2-2(\rho/R)}{2(\rho/R)-1} \cdot \dfrac{1}{(P_1/P_0)-1}$.

Note that the level of risk--P_0 and P_1--is not relevant to the number of loans made in equilibrium. Instead, the ratio P_1/P_0 is the crucial determinant. As P_1/P_0 increases, the number of loans made decreases. The more heterogeneous are the borrowers in terms of their repayment probabilities, the more severe is the market failure and the greater is the benefit of government intervention.

V.   FINANCIAL COLLAPSE

Let us now return to the unfettered market equilibrium and consider the effects of an increase in the required rate of return ρ. This change shifts the LL curve upward and to the right, as in Figure V. Not surprisingly, the interest rate charged to these borrowers increases. As shown in Section II, the number of students taking out loans declines. The effect on Π is in general ambiguous, as the BB curve need not be downward sloping.

An increase in the interest rate can have far more serious effects. It is possible that a shift in the LL curve can make the equilibrium disappear. Whereas at the lower interest rate, the economy is modeled as in Figure II, at the higher interest rate, Figure III is the more appropriate representation. Remember that the investment projects may still be socially profitable at the new higher interest rate. Nonetheless, none of the investors is able to raise the necessary capital.

An inward shift in the BB curve has the same effects as an increase in the interest rate. A small increase in the riskiness of some of the potential borrowers can cause the credit market for all of them to collapse, even though there may be no change in the expected return of investment projects R. Hence, small changes in risk perception can have large effects upon the allocation of credit.

One of the previous examples can usefully illustrate the model's potential for financial collapse. Let R be constant and let P be uniformly distributed from zero to one. Section III showed that no equilibrium exists when $\rho > R/2$. At $\rho < R/2$, all students borrow in the equilibrium. Figure VI displays the surplus received from this market as a function of the safe interest rate. At $\rho < R/2$, the surplus received is $R - \rho$, while at $\rho > R/2$, no surplus is received, as no loans are made. Thus, at $\rho = R/2$, there is a severe discontinuity. A small increase in the interest rate can cause the disappearance of market for loans to these borrowers. The social cost of this sudden financial collapse is potentially great and could reasonably motivate the government to act as the lender of last resort.

This potential for financial collapse has important macroeconomic implications. In the textbook IS-LM model, restrictive monetary policy (or any contractionary shift in the LM curve) reduces aggregate demand by increasing the real interest rate. At this greater required rate of return, some investments are no longer profitable. Thus, in the textbook model, a monetary contraction precludes marginally productive investments. In this alternative model of the allocation of credit, however, restictive monetary policy can have more dramatic effects. The higher interest rate can cause

the collapse of the market to some borrowers, even though their projects may remain socially productive at the higher interest rate. A monetary contraction can therefore have a large impact on the efficiency of the market allocation of credit. It is possible that when the monetary authority induces or allows a "credit crisis," government intervention on behalf of certain borrowers is efficiency enhancing, even though these borrowers may require no assistance under normal credit market conditions.

VI. CONCLUSIONS

The Federal government has played a central role in the allocation of credit among competing uses. This paper illustrates that this sort of government program can under plausible conditions improve on the unfettered market allocation. A necessary condition for efficient government intervention is unobservable heterogeneity among would-be borrowers regarding the probability of default. The greater is such heterogeneity, the greater is the potential for efficient intervention.

Historical examinations of financial markets (e.g., Kindleberger [1978]) emphasize their propensity for instability and collapse. Our models should therefore reflect this instability. If we are to understand the effects of alternative monetary policies, for example, we must appreciate the potential for financial crisis. At times, it may be efficient for the government to remove some risk from the private sector by guaranteeing certain financial arrangements or, equivalently, by acting as a lender of last resort.

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NOTES

1.   See, for example, Akerloff [1970], Jaffee and Russell [1976], Stiglitz and Weiss [1981], Ordover and Weiss [1981], Bernanke [1983], and Blinder and Stiglitz [1983].

2.   For example, the 1982 Economic Report of the President [p. 94], after noting that the Federal Government was involved in 21.4 percent of all funds advanced in U.S. credit markets in fiscal year 1982, presents the standard evaluation of this credit activity:

> Increasingly, therefore, political judgements, rather than
> marketplace judgements, have been responsible for allocating
> the supply of credit. As the discipline of the marketplace
> is replaced by the political process, less efficient economic
> activities are financed, and productivity in the economy
> declines.

3.   I assume that lenders can freely enter from and exit into a risk-free asset, such as Treasury bills.

4.   In such a model, R is the expected payout while 1-P is the probability that the payout is in the tail of the distribution in which default occurs. Without further structure, there is no necessary connection between the two variables.

5.   The discussion in the text assumes Δ is the same for all students. If Δ varies and is not observable by the bank, then this uncertainty would enter the analysis as does uncertainty regarding P. If Δ varies, is observable, and is potentially informative regarding P and R, then we could consider a continuum of credit markets indexed by Δ; the analysis of each would proceed as in the text.

6.   Even if P and R are independently distributed, the BB curve can be upward sloping in parts. For example, suppose R takes on the values 1.0 and 3.0 each with probability $\frac{1}{2}$, and P takes on the values 0.5 and 1.0 each with probability $\frac{1}{2}$. Then the equation for the BB curve is:

$$\Pi = 3/4 \quad \text{for} \quad 0 < r < 1$$
$$2/3 \quad \text{for} \quad 1 < r < 2$$
$$3/4 \quad \text{for} \quad 2 < r < 3$$
$$1/2 \quad \text{for} \quad 3 < r < 6$$

When r increases from just below 2 to just above 2, Π increases as well.

7.   Suppose $f(P,R)$ is $(1, 2p)$ with probability $\frac{1}{2}$ and $(1/3, 2p/3)$ with probability $\frac{1}{2}$. The equilibrium interest rate is $r = 3p/2$, in which case both types borrow. An increase in the interest rate would drive out the low return students before the high return students.

8.   This result is established by evaluating (4) at $r = p/\Pi$ and comparing to the derivative of $\Pi(r)$. The intuition is that $\Pi'(r)$ conveys the P for the marginal borrowers relative to Π and therefore whether they have too little or too great an incentive to borrow.
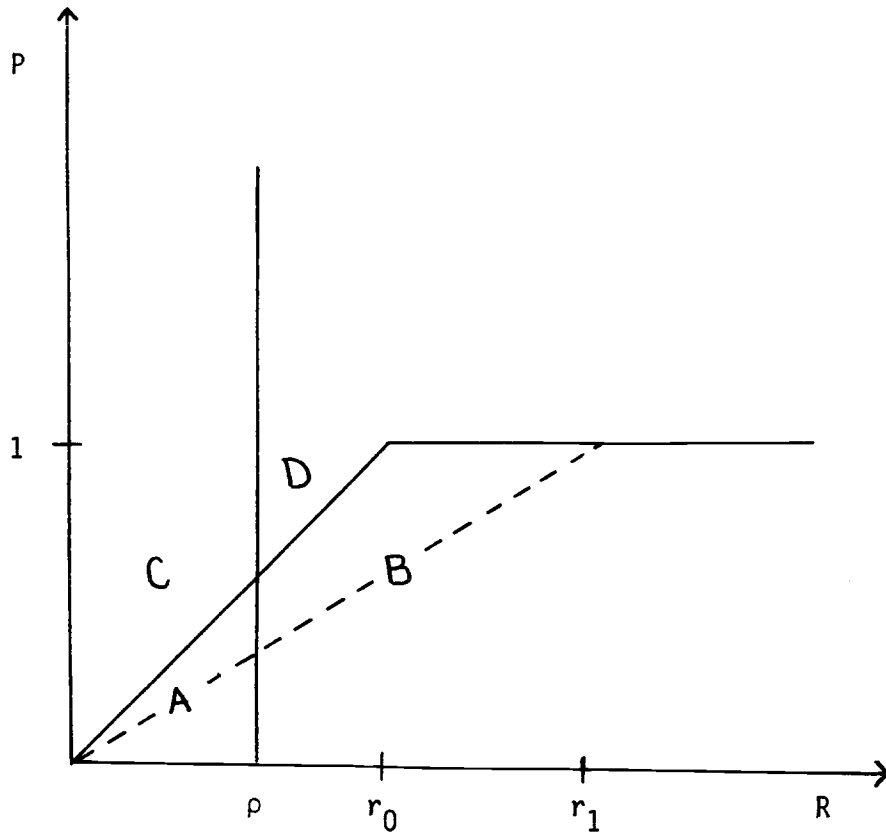
Figure I

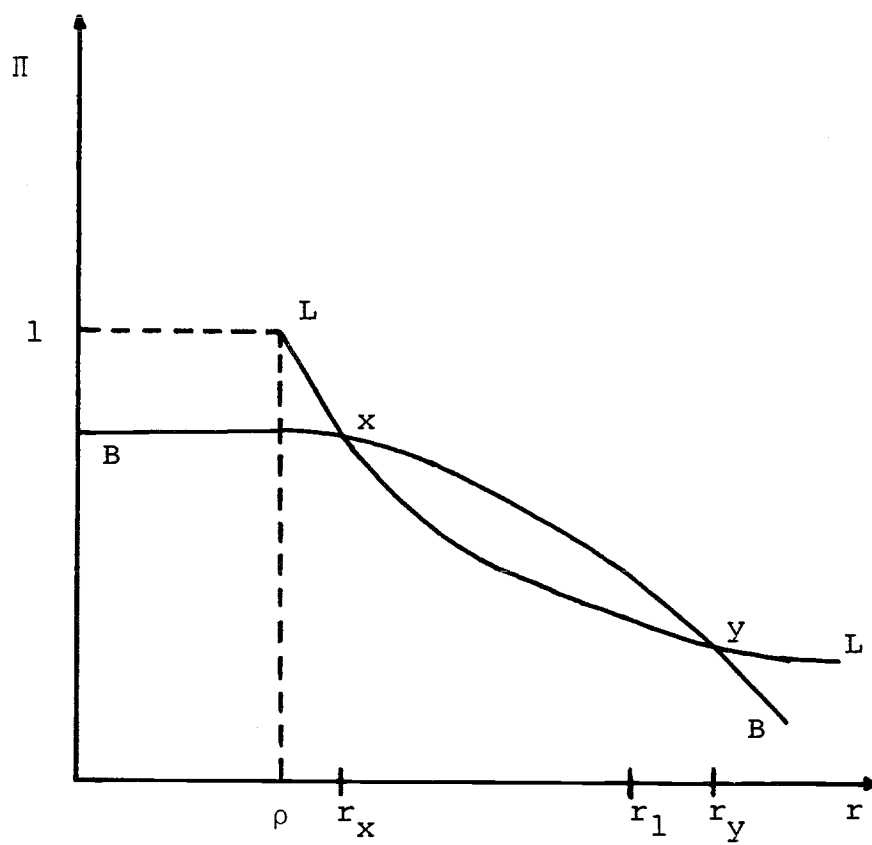Area of Investment

Figure II

Market Equilibrium

Figure III

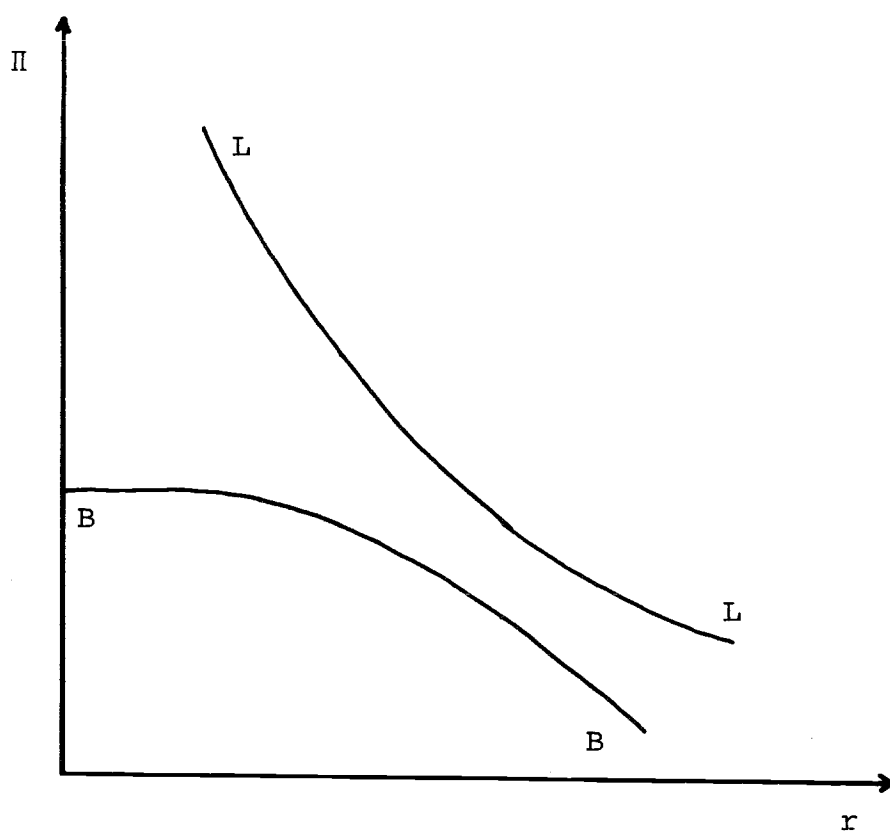A Collapsed Credit Market

Figure IV

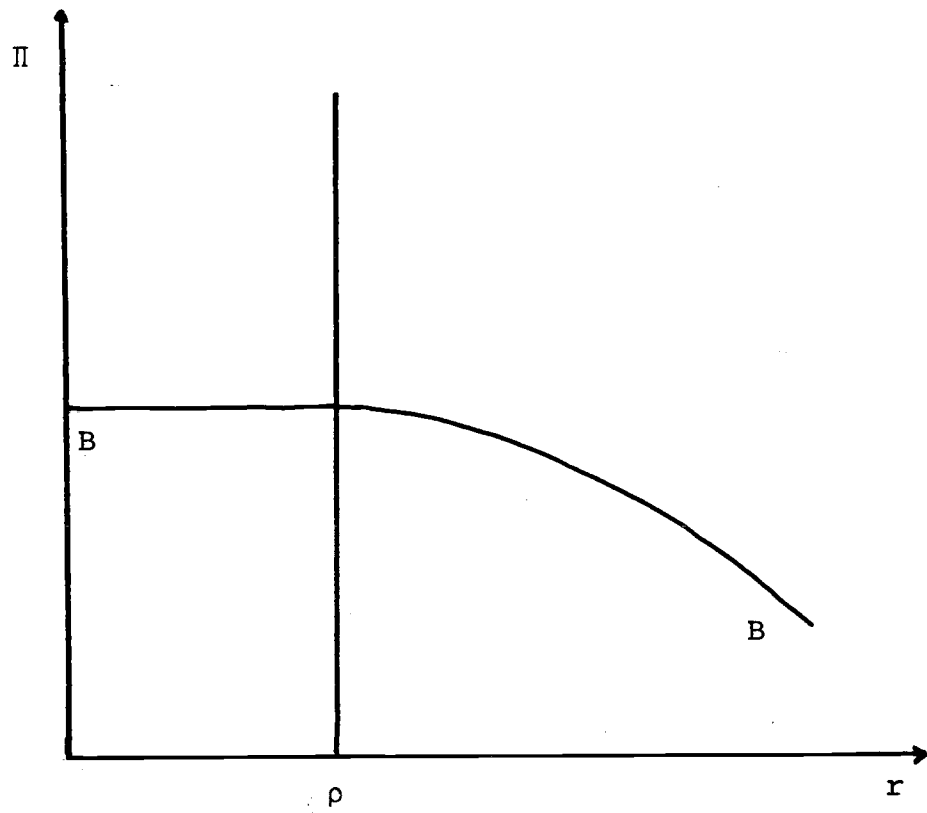Equilibrium with Government Loan at Risk-free Rate
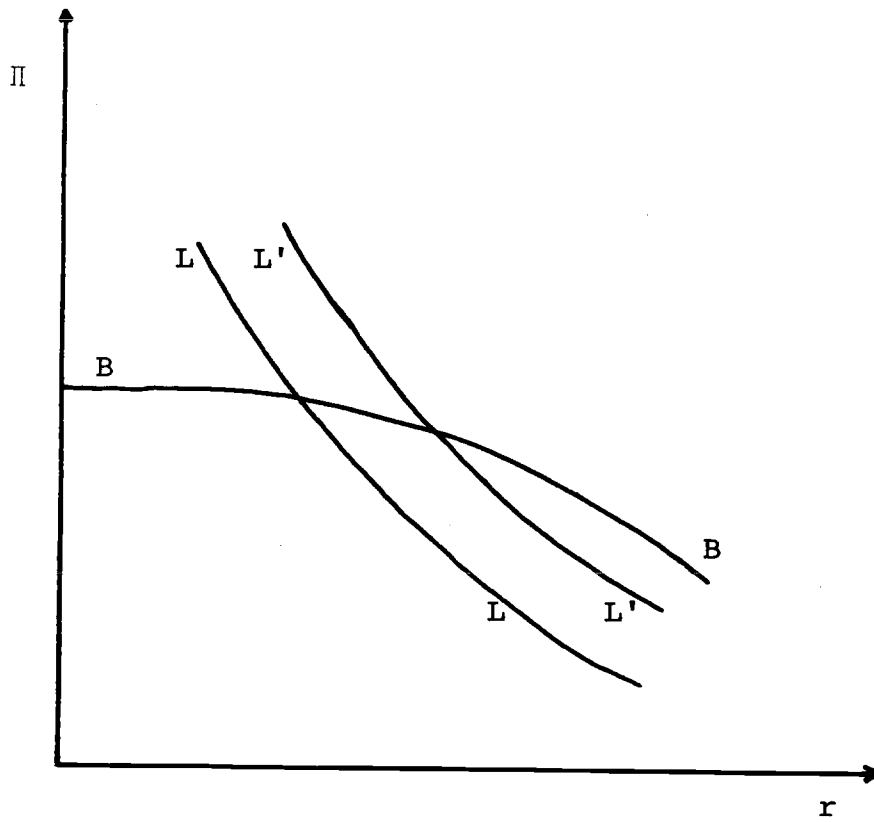
Figure V

An Increase in the Interest Rate

Figure VI

Surplus as a Function of the Interest Rate