NBER WORKING PAPER SERIES

THE SECOND BEST THEORY OF DIFFERENTIAL CAPITAL TAXATION

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Working Paper No. 1781

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
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December 1985

The research reported here is part of the NBER's research program in Taxation and project in Government Budgets. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

NBER Working Paper #1781
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ABSTRACT

An important proposition in the theory of efficient taxation is that, if capital income is taxed, all types of capital income should be taxed at the same rate. This conclusion has motivated extensive empirical analysis of the tax rates on different types of capital income. It has also been the basis for a variety of proposals to revise actual tax rules.

The present paper emphasizes that the comventional view must be modified in the very common situation in which some capital tax rate is politically constrained to something other than its optimal value, e.g., the zero rates on the imputed income on owner-occupied housing. The formal analysis of the paper examines the case in which there are three types of capital income and one of the tax rates is arbitrarily constrained to be zero.

Three general "rule of thumb" results emerge from the specific analysis: First, if the several types of capital can be regarded as independent in production, the optimal tax rates on the taxable types of capital income should depart from equality in the direction of an inverse elasticity rule. Second, in comparison to these rates, capital that is a complement to the untaxed capital should generally be taxed more heavily while capital that is a substitute for the untaxed capital should be taxed less heavily. Third, variations in the degree of complementarity or substitutability between the two types of capital should alter the two tax rates in a way that maintains a constant difference in the total taxes on each type of capital.

Although these rule-of-thumb results help to modify the conventional equal-tax-rates rule in an appropriate way, the most important implication of the present analysis is that any departure from optimal taxation makes it very difficult to set other capital tax rates optimally.

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THE SECOND BEST THEORY OF DIFFERENTIAL CAPITAL TAXATION

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A fundamental principle of optimal tax theory is that production efficiency should be maintained (e.g., Diamond and Mirrlees, 1971). If lump-sum taxation is not feasible, all taxes should be levied on factor incomes or on the consumption of different goods. Explicitly precluded is the differential taxation of the inputs of firms.

As specific applications of this principle, studies have estimated the welfare costs of taxing capital differently in different uses. Individual studies include the excess burden of the corporate income tax (Harberger, 1964; Shoven and Whalley, 1972), of the differential taxation of equipment, structures and inventories (King and Fullerton, 1984; Auerbach, 1979), and of the lack of taxation of the implicit income produced by owner-occupied housing (Laidler, 1969; Aaron, 1972). In the recent tax reform debate in the United States, the Treasury (U.S. Treasury, 1984, 1985) has emphasized the existing disparities in capital income tax rates among industries and types of capital investments and has proposed changes designed to reduce these disparities.

Experience nevertheless suggests that governments will continue to use suboptimal tax policies. It is significant that even the initial "radical reform" proposals of the U.S. Treasury (1984) did not suggest taxing the implicit income of owner-occupied housing capital or eliminating the corporate income tax or taxing the interest on general purpose bonds issued by state and

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local governments. Although it is useful to derive the optimal tax rules that should guide a benevolent and politically unconstrained government, we also need a firmer base for prescribing piecemeal improvements when tax policy is restricted by suboptimal constraints.[2] For example, if political constraints make it impossible to tax the implicit income on owner-occupied housing, should other types of capital income still be taxed at equal rates with the aim of assuring production efficiency in the rest of the economy? If not, what principle should guide the relative tax rates on the other types of capital income?

There has been surprisingly little attention to the issue of second best factor taxation in general or to the second best differential taxation of capital income in particular. The one noticeable exception is an important but little heeded paper by Auerbach (1979) in which he shows that in general it is not optimal to tax all types of capital equally if some other condition of optimal fiscal policy is not satisfied. Auerbach analyzes two significant cases in which different types of capital should be taxed at different rates: first, when the government does not have the instruments needed to bring the economy to the golden rule level of capital intensity[3] and second, when the tax rate on labor income is not set optimally.

Auerbach's analysis assumes that the government is free to set an optimal tax rate on each kind of capital income. His results are therefore not directly

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[2]On the equal theory of piecemeal reform, see Bruno (1972) and Guesnerie (1977).

[3]This is described by Auerbach and others as a limitation on government debt activity although in reality the necessary level of the government debt would be negative. That is, the government would have to be a creditor to increase the nation's total capital stock to the golden rule level.

relevant to the question of how capital tax rates should be set when owner-occupied housing or interest on state and local government securities is untaxed or corporate capital is subject to an additional tax. When such constraints are imposed, how should the tax rates be set on capital invested in different sectors or in different types of capital assets? When investment in owner-occupied residences is untaxed, should business investment in housing be taxed at the same rate as all other types of business investment? When structures used for owner-occupied housing bear no tax, should other structures be taxed at the same rate as equipment and inventories? Several studies of the inefficiency of existing tax rules[4] assume that the answer to these and other such questions is yes. The present paper will show that the opposite is true.

The analysis here is related to the studies of second-best excise taxation (see Green, 1961 and Atkinson and Stiglitz, 1980). In his classic study, Green (1961) derived the optimal tax rates on n-1 consumer goods when the tax rate on the n-th good is constrained to be some arbitrary value. Since the government in Green's analysis can also use lump-sum taxes and transfers, the optimal tax rates on all n goods would be zero if there were no constraint. Green showed that the constraint implies that the remaining n-1 tax rates should not be equal but should differ according to the complementarity or substitutability of the goods with leisure and with the n-th good. When the government does not have a lump sum tax as an option, the unconstrained optimum for the n tax rates (or for any available subset of rates) satisfies

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[4]Auerbach's (1983) analysis of the welfare cost of the differential taxation of different types of business capital ignores his own earlier conclusions about the inappropriateness of taxing all capital equally when the economy is not at the golden rule level of capital intensity. See also Fullerton and Henderson (1984) and U.S. Treasury (1984,1985).

the rules derived by Ramsey (1927), Diamond and Mirrlees (1971) and others.

In the present paper, I look at the problem of setting optimal tax rates on n-1 types of capital income when the tax rate on the n-th type of capital income is arbitrarily set equal to zero. To focus on the optimal allocation of the capital stock, I ignore the problem of labor supply and intertemporal capital accumulation.[5] I assume fixed supplies of labor and capital and a fixed amount of tax that must be raised by taxing capital income. In this context, if all tax rates could be set optimally, it would be optimal to tax all types of capital equally. The analysis shows that constraining the tax rate on one type of capital to be zero implies that in general the other tax rates should no longer be equal. An explicit expression for those tax rates is derived and interpreted.

1. Optimal Tax Rates on Capital Income

The economy produces aggregate output (X) using three types of capital (K_1, K_2, and K_3) and labor (L) according to the production function

(1.1) $$X = F(K_1, K_2, K_3, L).$$

Labor income is untaxed and the government must raise total revenue R by taxing the three types of capital income. If all types of capital income can be taxed, the government's budget constraint is

(1.2) $$R = t_t F_1 K_1 + t_2 F_2 K_t + t_3 F_3 K_3$$

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[5]This focus is essentially the same adopted by Auerbach (1983), King and Fullerton (1984), Fullerton and Henderson (1984), and the U.S. Treasury (1984,1985) in their analyses of existing tax distortions.

where F_i is the marginal product of capital of type i.

Private investors will allocate the fixed stock of capital ($\bar{K}$) among the three uses to equalize the after tax rates of return. Thus:

(1.3) $$(1-t_1)F_1 = (1-t_2)F_2$$

and

(1.4) $$(1-t_1)F_1 = (1-t_3)F_3.$$

The government's problem is to set tax rates to maximize aggregate output subject to the government's budget constraint and the constraint that the available capital will be allocated by investors to equalize net rates of return. It is immediately clear that the government can achieve this by setting all tax rates equal. With $t_1 = t_2 = t_3$, the net return equalization (equations 1.3 and 1.4) implies that gross returns are also equal: $F_1 = F_2 = F_3$. Since $F_1 = F_2 = F_3$ is the condition for maximizing output in the absence of a government budget constraint, the government achieves the first-best allocation by setting all tax rates equal. This is not at all surprising since with total capital fixed this is equivalent to a lump-sum tax on capital income.

Consider now the more general and realistic second-best problem in which one of the tax rates is arbitrarily fixed. Specifically, let $t_1 = 0$. What is the optimal relation between t_2 and t_3?

The government's problem is to maximize the Lagrangian expression:

(1.5) $$Z = F(K_1,K_2,K_3,L) + \lambda(K_1+K_2+K_3-\bar{K}) + \mu(t_2F_2K_2+t_3F_3K_3-R)$$

subject to the further constraint that the investors will equalize after tax rates of return. With $t_1 = 0$, these extra constraints imply $F_1 = (1-t_2)F_2$ and

$F_1 = (1-t_3)F_3$. These conditions can be used to rewrite 1.5 without t_2 and t_3 since they imply that $t_2F_2 = F_2 - F_1$ and $t_3F_3 = F_3 - F_1$. Thus the government's problem is to maximize

$$(1.6) \qquad Z = F(K_1,K_2K_3,L) + \lambda(K_1+K_2+K_3-\bar{K}) + \mu[(F_2-F_1)K_2 + (F_3-F_1)K_3 - R].$$

Although the government does not control the allocation of capital directly, the choice of t_2 and t_3 uniquely determines K_1, K_2 and K_3. The government's problem can therefore be solved by choosing the values of K_1, K_2 and K_3 that maximize (1.6) and then noting the implications for t_2 and t_3. The three first order conditions are:

$$(1.7) \qquad F_1 + \lambda + \mu[K_2(F_{21}-F_{11}) + K_3(F_{31}-F_{11})] = 0,$$

$$(1.8) \qquad F_2 + \lambda + \mu[(F_2-F_1) + K_2(F_{22}-F_{12}) + K_3(F_{32}-F_{12})] = 0,$$

and

$$(1.9) \qquad F_3 + \lambda + \mu[K_2(F_{23}-F_{13}) + (F_3-F_1) + K_3(F_{33}-F_{13})] = 0$$

Using equation 1.7 to eliminate λ from (1.8) and (1.9) and noting again that $F_2 - F_1 = t_2F_2$ and $F_3 - F_1 = t_3F_3$ yields:

$$(1.10) \qquad (1+\mu)t_2F_2 = -\mu[(F_{11}+F_{22}-2F_{12})K_2 + (F_{11}+F_{32}-F_{31}-F_{12})K_3]$$

and

$$(1.11) \qquad (1+\mu)t_3F_3 = -\mu[(F_{11}+F_{23}-F_{13}-F_{21})K_2 + (F_{33}+F_{11}-2F_{13})K_3]$$

The optimal ratio of t_2 to t_3 therefore satisfies:

$$(1.12) \qquad \frac{t_2}{t_3} = \frac{F_3}{F_2} \cdot \frac{(F_{11}+F_{22}-2F_{12})K_2 + (F_{11}+F_{32}-F_{31}-F_{12})K_3}{(F_{11}+F_{32}-F_{31}-F_{12})K_2 + (F_{11}+F_{33}-2F_{13})K_3}$$

2. Interpreting the Second Best Optimum Conditions

It is immediately clear that the second-best tax rates on the two types of taxable capital will not in general be equal. To interpret (1.12), it is useful to begin with the simplest case in which the marginal product of each type of capital does not depend on the amounts of the other types of capital in use, i.e., $F_{ij} = 0$ for $i \neq j$. The more general case in which the different types of capital may be substitutes or complements will be considered in the next section.

If $F_{ij} = 0$, for $i \neq j$, equation (1.12) becomes:

$$(2.1) \qquad \frac{t_2}{t_3} = \frac{F_3}{F_2} \cdot \frac{F_{11}(K_2+K_3) + F_{32}K_2}{F_{11}(K_2+K_3) + F_{33}K_3}$$

Two special cases will point the way to the general implications of this equation. Consider first the case in which the marginal product of the untaxed capital is constant, i.e., $F_{11} = 0$. Equation (2.1) then immediately implies

$$(2.2) \qquad \frac{t_2}{t_3} = \frac{F_{22}K_2/F_2}{F_{33}K_3/F_3}.$$

If we write $\epsilon_{22} = F_{22}^{-1}F_2/K_2 = (\partial K_2/\partial F_2)F_2/K_2$ as the elasticity along the production function of type 2 capital with respect to its own marginal product, equation (2.2) yields the familiar inverse elasticity formula:

$$(2.3) \qquad \frac{t_2}{t_3} = \frac{\epsilon_{33}}{\epsilon_{22}}.$$

This rule tells us to tax capital incomes of type 2 and 3 in a ratio which is the inverse of the responsiveness of the two capital stocks to changes in the marginal product of capital.

A second interpretation of (2.3) is also familiar from the theory of Ramsey tax rules: new taxes on capital incomes of type 2 and 3 should be levied in a ratio that causes the two capital stocks to shrink in the same proportion. To see this note that (2.3) can be rewritten as

$$(2.4) \qquad \frac{dK_2}{dF_2} \cdot \frac{F_2}{K_2} \cdot t_2 = \frac{dK_3}{dF_3} \cdot \frac{F_3}{K_3} \cdot t_3.$$

Since $F_1 = (1-t_2)F_2$ is a condition of investor equilibrium and we are studying the special case of $F_{11} = 0$, a change in t_2 causes $dF_2 = -F_2dt_2/(1-t_2)$. A new small tax (i.e., approximately $dt_2 = t_2$ at $t_2 = 0$) therefore implies $dF_2 = -F_2t_2$. Substituting into (2.4) shows

$$(2.5) \qquad -\frac{1}{K_2} \frac{\partial K_2}{\partial F_2} \cdot dF_2 = - \frac{1}{K_3} \frac{\partial K_3}{\partial F_3} dF_3.$$

The optimum conditions of (2.4) thus imply equiproportionate decreases in K_2 and K_3.

Why does $F_{11} = 0$ imply these results? With $F_{11} = 0$, the reduced capital in K_2 and K_3 can be absorbed as increased K_1 with constant productive value, F_1. The entire welfare loss therefore arises because each successive unit of K_2 and K_3 has a greater value. The tax rates t_2 and t_3 must therefore be set so that K_2 and K_3 are reduced in a mix that minimizes the aggregate loss. This requires taxing more heavily the capital for which any given tax induces a smaller reduction in the type of capital. The exact balancing is expressed by the inverse elasticity rule of (2.4).

Return now to the first order condition of (1.13) and, instead of assuming $F_{11} = 0$, consider the opposite special case in which $-F_{11}$ tends to infinity. As $-F_{11}$ increases, the values of F_{22} and F_{33} become relatively less important as determinants of the optimal tax ratio and t_2/t_3 tends to F_3/F_2. In the limit, $t_2/t_3 = F_3/F_2$ or $t_2F_2 = t_3F_3$. Combining this optimum condition on the tax rates with the investors' equilibrium condition that $(1-t_2)F_2 = (1-t_3)F_3$ implies that $F_2 = F_3$ and therefore that $t_2 = t_3$. So in this limiting case it is optimal to tax all of the taxable types of capital income equally. The reason for this is easy to see. In the limiting case in which $-F_{11}$ is infinite, the investor equilibrium that $F_1 = (1-t_2)F_2$ implies that K_1 cannot change at all (since any finite change in K_1 would cause an infinite change in F_1). With K_1 fixed, the total $K_2 + K_3$ is also fixed. The optimum allocation of a fixed total amount of capital between two different uses requires equal marginal products in both uses ($F_2 = F_3$) and therefore equal tax rates. Thus, with the capital in the untaxed sector fixed in quantity and with a separable production technology, the untaxed sector becomes irrelevant and the problem becomes equivalent to a first-best taxation question for the taxable sectors of the economy.

Between the two extremes of $F_{11} = 0$ and $-F_{11} = \infty$, the relative tax rates vary monotonically from the inverse elasticity condition of equation (2.3) when $F_{11} = 0$ to the equality of tax rates when $-F_{11} = \infty$.[6] To see this, note that (2.1) can be rewritten using $F_1 = (1-t_2)F_2 = (1-t_3)F_3$ as:

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[6]This assumes that the elasticities do not change in a way that reverses their relative magnitudes.

(2.6) $$\frac{t_2F_2}{t_3F_3} = \frac{F_{11}(K_2+K_3)(1-t_2)F_2F_1^{-1} + F_{22}K_2F_2F_2^{-1}}{F_{11}(K_2+K_3)(1-t_3)F_3F_1^{-1} + F_{33}K_3F_3F_3^{-1}}$$

Factor out F_2 from the numerator and F_3 from the denominator, multiply numerator and denominator of the righthand side by K_1, and rewrite in elasticity form as:

(2.7) $$\frac{t_2}{t_3} = \frac{\epsilon_{11}^{-1}(K_2+K_3)(1-t_2) + \epsilon_{22}^{-1}}{\epsilon_{11}^{-1}(K_2+K_3)(1-t_3) + \epsilon_{33}^{-1}} .$$

Solving explicitly for t_2/t_3 yields:

(2.8) $$\frac{t_2}{t_3} = \frac{\epsilon_{11}^{-1}(K_2+K_3) + \epsilon_{33}^{-1}}{\epsilon_{11}^{-1}(K_2+K_3) + \epsilon_{22}^{-1}}$$

when $F_{11} = 0$, $\epsilon_{11} = -\infty$ and $t_2/t_3 = \epsilon_{22}/\epsilon_{33}$ as noted previously. Similarly, as $-F_{11}$ tends to ∞, ϵ_{11} tends to zero and t_2/t_3 tends to 1. To show the monotonicity of t_2/t_3 with respect to ϵ_{11}, note that 2.8 implies

(2.9) $$\text{sign}\left[\frac{d(t_2/t_3)}{d\epsilon_{11}}\right] = \text{sign}[\epsilon_{33}^{-1} - \epsilon_{22}^{-1}].$$

Since the sign of $\epsilon_{33}^{-1} - \epsilon_{22}^{-1}$ also determines whether $t_2/t_3 < 1$ or $t_2/t_3 > 1$, equation (2.9) implies that t_2/t_3 moves monotonically from $t_2/t_3 = \epsilon_{22}/\epsilon_{33}$ if $F_{11} = 0$ to $t_2/t_3 = 1$ as $-F_{11}$ tends to infinity.

3. Substitutes and Complements in Production

The simple optimal tax formula of equation (2.1) and the conclusion that the ratio of the tax rates on the taxable sources of capital income lies between one and the inverse elasticity ratio depend on the simplifying assumption that the different types of capital are independent in production, i.e., $F_{ij} = 0$ for $i \neq j$. This section of the paper analyzes how relaxing this assumption alters the second best pattern of taxes.

Note first that even with no restriction on F_{ij}, the two tax rates tend toward equality as $-F_{11}$ tends to infinity. This result, which is directly apparent in equation (1.12), occurs for the same reason that it did in the simpler context in which $F_{ij} = 0$ for $i \neq j$. At $-F_{11} = \infty$, K_1 is effectively constant and can therefore be ignored. The problem is then equivalent to setting tax rates t_2 and t_3 as if K_2 and K_3 were the only types of capital. In this context, with no restriction on the relevant tax rates, t_2 and t_3 should be equalized.

More generally, however, the sign and magnitude of the F_{ij} terms influence the optimal rates of tax on the two types of taxable capital income. The analysis in this section shows that the effect of a change in the production function cross-product terms, F_{12} and F_{13}, can be decomposed into two components, a direct "allocation effect" and a secondary "budget effect." The direct "allocation effect" of a change in F_{12} or F_{13} on the optimal relative taxation of the two types of taxable capital income is unambiguous: the relative tax rate on a particular type of capital rises if that type of capital becomes more of a complement with the untaxed good and falls if that type of capital becomes more of a substitute with the untaxed capital. The indirect "budget effect" reflects the fact that a change in F_{12} or F_{13} alters the shadow value of the government's budget constraint, μ. A change in μ can either reinforce the direct allocation effect or shift the optimal tax rates in the opposite direction. This section derives the explicit expressions for the allocation effect and the budget effect and discusses the conditions under which they are reinforcing and those in which they are opposing. The section also considers the implication of these results for two examples: the nontaxation of housing and the differential taxation of equipment and structures.

The basic results and the decomposition into allocation effects and budget effects follows directly from equations (1.10) and (1.11). Dividing both sides of these equations by $1 + \mu$ and subtracting (1.11) from (1.10) yields an expression for the difference between the tax per unit of type 2 capital ($\theta_2 = t_2F_2$) and the tax per unit of type 3 capital ($\theta_3 = t_3F_3$):

$$(3.1) \qquad \theta_2 - \theta_3 = -\frac{\mu}{1+\mu}[(F_{11}+F_{22}-2F_{12})K_2 + (F_{11}+F_{32}-F_{31}-F_{12})(K_3-K_2) - (F_{33}+F_{11}-2F_{13})K_3].$$

The derivative of this tax difference with respect to F_{12} is therefore

$$(3.2) \qquad \frac{d(\theta_2-\theta_3)}{dF_{12}} = \frac{\mu}{1+\mu}(K_2+K_3) + \frac{1+\mu}{\mu}(\theta_2-\theta_3)\frac{\partial[\mu/(1+\mu)]}{\partial F_{12}}$$

The first term on the right side of equation (3.2) is unambiguously positive since μ is the shadow cost of the government budget constraint and therefore $\mu > 0$. The second term reflects the effect of a change in the shadow cost of the government budget constraint. Since it will be shown that $\partial[\mu/(1+\mu)]/\partial F_{12} < 0$, the sign of the "budget effect" is the opposite of the sign of $\theta_2 - \theta_3$. The implications of this and the explicit derivation of $\partial[\mu(1+\mu)]/\partial F_{12} < 0$ will be presented below. But first I derive the effect of a change in F_{13}.

It follows immediately from (3.1) that

$$\text{(3.3)} \qquad \frac{d(\theta_2-\theta_3)}{dF_{13}} = -\frac{\mu}{1+\mu}(K_2+K_3) + \frac{1+\mu}{\mu}(\theta_2-\theta_3)\frac{\partial[\mu/(1+\mu)]}{\partial F_{13}}$$

The direct allocation effect of an increase in F_{13} is to reduce $\theta_2 - \theta_3$. This corresponds exactly to equation (3.2) since a decrease in $\theta_2 - \theta_3$ means that $\theta_3 - \theta_2$ rises with F_{13}.

The economic interpretation of the direct allocation effect is clear. Consider first the effect of variations in F_{12}. In comparison to the optimal tax rates when $F_{12} = 0$, the direct allocation effect implies that θ_2 rises relative to θ_3 if type 2 capital is a complement to the untaxed type 1 capital. Conversely, θ_2 falls relative to θ_3 if capital of types 1 and 2 are substitutes ($F_{12} < 0$). For example, if the three types of capital are owner occupied housing (K_1), rental housing (K_2) and manufacturing capital (K_3), it is reasonable to posit $F_{12} < 0$ and $F_{13} = F_{23} = 0$. Since owner-occupied housing and rental housing are substitutes, the optimal tax on rental housing is lower relative to the tax on manufacturing capital than it would be if $F_{12} = 0$. One way of stating the rationale for this is that a lower tax on rental housing capital helps to balance the specific distortion in favor of owner-occupied housing.

As a second example, assume that the three types of capital are manufacturing equipment (K_1), manufacturing structures (K_2), and housing (K_3). By assumption, let the effective tax rate on manufacturing equipment capital be zero. It is reasonable to posit that manufacturing structures and equipment are complements in production ($F_{12} > 0$) and that the contribution of housing capital is independent of both ($F_{13} = F_{23} = 0$). It now follows that manufacturing structures should be taxed more heavily than housing capital (relative to the optimal tax rates when $F_{12} = 0$ as well). A higher tax on manufacturing structures raises the overall taxation on manufacturing capital and thereby reduces the distortion that would otherwise exist between the manufacturing sector and the housing sector.

It is clear from equation (3.2) and (3.3) that the direct allocation effect of variations in F_{12} and F_{13} depends on the relative degrees of complementarity and substitutability of K_2 and K_3 with K_1. Combining these two equations implies

$$(3.4) \qquad d(\theta_2-\theta_3) = \frac{\mu}{1+\mu}(K_2+K_3)(dF_{12}-dF_{13})$$

$$+ \frac{1+\mu}{\mu}(\theta_2-\theta_3)\left[\frac{\partial(\mu/1+\mu)}{dF_{12}}dF_{12} + \frac{\partial(\mu/1+\mu)}{\partial F_{13}}dF_{13}\right]$$

Thus, if $dF_{12} = dF_{13}$, there is no direct allocation effect.

Consider now the effect that works through changes in μ. To be specific, I will examine equation (3.2). The value of $\partial(\mu/1+\mu)/\partial F_{12}$ can be evaluated from the budget constraint:

$$(3.5) \qquad R = t_2F_2K_2 + t_3F_3K_3.$$

Using (1.10) and (1.11) to substitute for t_2F_2 and t_3F_3 yields

$$(3.6)\qquad R = -(\frac{\mu}{1+\mu})[(F_{11}+F_{22}-2F_{12})K_2^2 + 2(F_{11}+F_{23}-F_{13}-F_{21})K_2K_3 + (F_{11}+F_{33}-2F_{13})K_3^2]$$

Since R must remain constant at the required level of revenue, it follows that

$$(3.7)\qquad \frac{\partial[\mu/(1+\mu)]}{\partial F_{12}} = -\frac{\partial R/\partial F_{12}}{\partial R/\partial[\mu/(1+\mu)]} = -2[\frac{\mu}{1+\mu}]^2\frac{K_2(K_2+K_3)}{R}$$

Substituting this expression into (3.2) and writing $R = \theta_2K_2 + \theta_3K_3$ yields

$$(3.8)\qquad \frac{d(\theta_2-\theta_3)}{dF_{12}} = \frac{\mu}{1+\mu}(K_2+K_3) - \frac{1+\mu}{\mu}(\theta_2-\theta_3)2(\frac{\mu}{1+\mu})^2\frac{K_2(K_2+K_3)}{R}$$

$$= \frac{\mu}{1+\mu}\frac{K_2+K_3}{\theta_2K_2+\theta_3K_3}[\theta_3(K_2+K_3) + (\theta_3-\theta_2)K_2].$$

It is immediately clear that $\theta_3 > \theta_2$ implies $d(\theta_2-\theta_3)/dF_{12} > 0$. If $\theta_3 < \theta_2$, the effect of F_{12} on $\theta_2 - \theta_3$ is ambiguous and depends on the relative magnitudes of the tax rates and capital stocks.

The source of the ambiguity can be explained as follows: An increase in F_{12} reduces the output loss associated with any given budget requirement because the tax on K_2 is to some extent also an indirect tax on the untaxable complement K_1. In the extreme, if K_2 and K_1 had to be used in fixed proportions, the inability to tax K_1 would be irrelevant and the tax rates on K_2 and K_3 could be set to avoid any excess burden. This explains why $\partial[\mu(1+\mu)/\partial F_{12} < 0$.

Equations (1.10) and (1.11) show that θ_2 and θ_3 respond to variations in $\mu/(1+\mu)$ with elasticities equal to one. For example, since equation (1.10) can be written

$$(3.9)\qquad \theta_2 = -\frac{\mu}{1+\mu}[(F_{11}+F_{22}-2F_{12})K_2 + (F_{11}+F_{32}-F_{31}-F_{12})K_3],$$

it follows immediately that

(3.10) $$\frac{\mu/(1+\mu)}{\theta_2}\,\frac{d\theta_2}{d\mu/(1+\mu)} = 1$$

Thus if $\theta_2 = \theta_3$, a change in μ does not alter that equality. But if $\theta_2 > \theta_3$, a decline in μ causes θ_2 to decline by more than θ_3. Since the direct allocation effect of an increase in F_{12} is to raise θ_2 relative to θ_3, the indirect effect that works through a reduction in μ has an offsetting effect if θ_2 is initially greater than θ_3. Conversely, if $\theta_2 < \theta_3$, the induced decline in μ reduces θ_3 by more than θ_2 and therefore reinforces the direct allocation effect of an increase in F_{12}. The effects on $\theta_2 - \theta_3$ of decreases in F_{12} and of variations in F_{13} can be explained in the same way.

Although the optimal tax rates must in principle be evaluated explicitly in each case, the expression for $d(\theta_2-\theta_3)/dF_{12}$ in equation (3.8) suggests that the direct allocation effect is likely to dominate the indirect budget effect. That will not be true and $d(\theta_2-\theta_3)/dF_{12} < 0$ only if $\theta_3(K_2+K_3) + (\theta_3-\theta_2)K_2 < 0$. This requires not only that $\theta_2 > \theta_3$ but also that the tax that would be collected if the lower of the two optimal tax rates were applied to the entire capital stock $(\theta_3(K_2+K_3))$ be less than the extra tax collected on the complementary stock by the differential tax rate $[(\theta_2-\theta_3)K_2]$. Reversal of the direct allocation effect therefore requires that the difference in the tax rates $(\theta_2-\theta_3)$ must be larger than the lower of the tax rates $[(\theta_2-\theta_3) > \theta_3]$ and that this differential must be proportionately greater than the ratio of the total taxable capital stock to the stock of the complementary capital:

(3.11) $$\frac{\theta_2-\theta_3}{\theta_3} > \frac{K_2+K_3}{K_2}$$

Although this inequality could in principle be satisfied, in general it would not be and the direct allocation effect would dominate.

A final word is appropriate about the effect of the production interdependence of K_2 and K_3. It follows from (3.1) that

$$(3.12)\qquad \frac{d(\theta_2-\theta_3)}{dF_{23}} = -\frac{\mu}{1+\mu}(K_3-K_2) + \frac{(1+\mu)}{\mu}(\theta_2-\theta_3)\frac{\partial\mu/(1+\mu)}{\partial F_{23}}$$

$$= \frac{\mu(K_2+K_3)}{(1+\mu)(\theta_2K_2+\theta_3K_3)}(\theta_2K_2-\theta_3K_3)$$

Thus an increase in F_{23}, the complementarity in production between the two taxable types of capital, increases $\theta_2 - \theta_3$ if more revenue is initially collected from type 2 capital. The nature of this result is clear if we focus on the effect of F_{23} on the two amounts of taxes collected $(\theta_2K_2-\theta_3K_3)$ rather than the two tax rates. Multiplying equation (1.10) by K_2 and equation (1.11) by K_3 and subtracting (1.11) from (1.10) yields

$$(3.13)\qquad \theta_2K_2 - \theta_3K_3 = -\frac{\mu}{1+\mu}[(F_{11}+F_{22}-2F_{12})K_2^2 - (F_{33}+F_{11}-2F_{13})K_3^2].$$

Since F_{23} does not appear on the righthand side of this equation, it follows immediately that the degree of complementarity or substitutability between K_2 and K_3 does not alter the optimal difference in taxes collected on these two types of capital.

4. Concluding Remarks

This paper has emphasized that the conventional view that all tax rates should be set equal to each other must be modified in the very common situation in which some tax rate is politically constrained to be at something other than its optimal value. Although the present analysis has focused on the implications of not taxing one type of capital income, the results here could easily be extended to the case where one type of capital income is taxed at an arbitrary non-zero rate.

It would be useful to extend the current analysis to explore the implications of the existence of a corporate income tax or the favorable tax rates on extractive industries and on the capital used by state and local governments. It would also be desirable to analyze an economy with more than three kinds of capital so that two or more tax rates could be fixed arbitrarily.

Three general "rules of thumb" type results emerged from the specific analysis of the present paper. First, if the several types of capital can be regarded as independent in production, the optimal tax rates on the taxable types of capital income should depart from equality in the direction of an inverse elasticity rule. Second, in comparison to these rates, capital that is a complement to the untaxed capital should generally be taxed more heavily while capital that is a substitute to the untaxed capital should be taxed less heavily. Third, variations in the degree of complementarity or substitutability between the two types of taxed capital should alter the two tax rates in a way that maintains a constant difference in the total taxes on each type of capital income.

Although these rule-of-thumb results may help to modify the conventional equal-tax-rates rule in an appropriate way, it is important to recognize that they are only rough approximations to the optimal second best taxation of capital. Perhaps the most important implication of the present anlaysis is that any departure from optimal capital taxation makes it very difficult to set other capital tax rates optimally. That is a further reason for seeking to overcome political constraints that prevent setting all tax rates optimally. But as long as such political constraints remain, economists should recognize the limitation of the simple equal-tax-rates rule and should try to point to the optimal second-best differential taxation of capital income.

Cambridge, MA
June 1985

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