NBER WORKING PAPER SERIES

RATIONAL BUBBLES IN STOCK PRICES?

Behzad T. Diba

Herschel I. Grossman

Working Paper No. 1779

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 1985

National Science Foundation Grant SES-8210129 has supported this research. The research reported here is part of the NBER's research program in Financial Markets and Monetary Economics. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #1779
December 1985

Rational Bubbles in Stock Prices?

ABSTRACT

This paper reports empirical tests for the existence of rational bubbles in stock prices. The analysis focuses on a familiar model that defines market fundamentals to be the expected present value of dividends, discounted at a constant rate, and defines a rational bubble to be a self-confirming divergence of stock prices from market fundamentals in response to extraneous variables. The tests are based on the theoretical result that, if rational bubbles exist, time series obtained by differencing real stock prices do not have stationary means. Analysis of the data in both the time domain and the frequency domain suggests that the time series of aggregate real stock prices is nonstationary in levels but stationary in first differences. Applications of the time domain tests to simulated nonstationary time series that would be implied by rational bubbles indicates that the tests have power to detect relevant nonstationarity when it is present. Furthermore, application of the time-domain and frequency-domain tests to the time series of aggregate real dividends also indicates nonstationarity in levels but stationarity in first differences--suggesting that market fundamentals can account for the stationarity properties of real stock prices. These findings imply that rational bubbles do not exist in stock prices. Accordingly, any evidence that stock price fluctuations do not accord with market fundamentals (as specified above) is attributable to misspecification of market fundamentals.

Behzad Diba
Department of Economics
Brown University
Providence, RI 02912

Herschel I. Grossman
Department of Economics
Brown University
Providence, RI 02912

Popular commentators as well as professional economists--see, for example, Keynes (1936, pp. 154-155)--have long entertained the idea that movements in stock prices can involve "bubbles"--that is, psychologically based responses to extraneous factors. More recently, theorists using the assumption of rational expectations have analyzed formally the formation of asset prices, their incorporation of market fundamentals, and the possible influence of factors that are not part of market fundamentals. In an earlier paper--Diba and Grossman (1985)--we develop a general theoretical case, summarized briefly below, against the existence of rational bubbles. The present paper reports complementary empirical evidence that fluctuations in American stock prices do not incorporate rational bubbles.

The empirical analysis utilizes the familiar linear rational expectations model of stock price determination. This model assumes that the expected real return from holding stock--including expected dividends and expected capital gains or losses--equals a constant required real rate of return. One solution to this model, often referred to as the market fundamentals solution, equates the real stock price to the present value of rationally expected real dividends discounted at a constant rate. LeRoy and Porter (1981) and Shiller (1981) find that stock price indices exhibit more volatility than this solution can account for. Blanchard and Watson (1982) demonstrate that the model also possesses a solution that includes a stochastic rational bubble component--leading to a self-confirming divergence of stock prices from market fundamentals in response to extraneous factors--that can potentially account for the observed volatility of prices.

As Blanchard and Watson point out, the apparent excess volatility of stock prices, relative to what a particular specification of market fundamentals can explain, does not by itself prove that rational bubbles exist. Apparent excess volatility instead can result from such factors as time-varying discount rates (Grossman and Shiller, 1981), small sample bias (Flavin, 1983), and misspecification of the process generating the time series of dividends (Marsh and Merton, 1983, 1984).

West (1984a, 1984b) presents empirical evidence that he interprets as supporting the idea that stock prices incorporate rational bubbles. West (1984a) conducts a specification test that rejects the joint hypothesis that (a) the expected real rate of return from holding stock is constant, (b) rational expectations holds, (c) a specific autoregressive process generates aggregate real dividends, and (d) aggregate real stock prices conform to market fundamentals as specified above. West also reports diagnostic tests that do not reject hypotheses (a), (b), and (c) individually, and he concludes that the rejection of the joint hypothesis can only be attributed to the existence of rational bubbles.

West (1984b) shows that the data do not satisfy an upper bound on the conditional variance of stock prices that is implied by the hypothesis that stock prices conform to market fundamentals. He demonstrates that his test is immune to Marsh and Merton's objections to earlier volatility tests, and he also suggests that his test is not subject to the small sample bias problem discussed by Flavin. On the basis of these results, he argues that the contribution of rational bubbles to stock price fluctuations is quantitatively large.

West's conclusion that rational bubbles exist critically depends on the power of his diagnostic tests against misspecification of the market fundamentals component of stock prices. Other empirical studies--see, for example, Fama and Schwert (1977), Shiller (1981, pp. 432-433), and Campbell (1984)--reject the hypothesis that the ex ante required real rate of return from holding stock is constant. Engle and Watson (1985) analyze specifications of market fundamentals, bringing in factors that West did not consider, that seem to be consistent with the data and that do not require consideration of rational bubbles.

A general problem in testing for rational bubbles, which arises specifically in interpreting West's evidence, is that the econometrician cannot observe rational bubbles separately from the market fundamentals component of an asset's price. Consequently, any test of the hypothesis that an asset's price involves rational bubbles must formulate a joint hypothesis about variables that influence market fundamentals. Hamilton and Whiteman (1984) analyze the econometric consequences of this observation in the context of tests for the existence of rational bubbles in the price level proposed by Sargent and Wallace (1984) and implemented by Flood and Garber (1980), Burmeister and Wall (1982), and Flood, Garber, and Scott (1982). Hamilton and Whiteman demonstrate that rational bubbles and unobservable variables (e.g., money demand disturbances) influencing market fundamentals do not impose empirically distinguishable restrictions on moving average representations of observable variables. Accordingly, any evidence that can be interpreted as suggesting that rational bubbles exist can also simply reflect the fact that econometricians do not observe some variables that influence market fundamentals. Hamilton (1985) presents a clear discussion of this point in the context of the stock market model discussed below supplemented by an unobservable variable reflecting such factors as risk-premia or anticipated changes in tax laws.

The main innovation in the present paper is the implementation of a strategy for obtaining evidence against the existence of rational bubbles in stock prices that does not depend on accepting joint hypotheses about market fundamentals or about the factors generating bubbles. This strategy is based on the theoretical result that differencing the stochastic process that generates rational bubbles a finite number of times does not lead to a process with a stationary mean. This result does not mean that evidence of nonstationarity establishes the existence of rational bubbles, because such evidence could also be attributed to nonstationarity of a possibly unobservable variable in market fundamentals. The converse inference, however, is possible. Namely, in principle, evidence that the time series obtained by differencing stock prices n times, for any finite n, possesses a stationary mean would be evidence against rational bubbles.

For any finite sample, however, there is always a value of n large enough to induce the appearance of stationarity even in a time series truly generated by a rational bubble. Consequently, the choice of n is of considerable importance in implementing this strategy. The empirical analysis reported below uses a conservative procedure, justified by Hamilton and Whiteman (1984), of setting n equal to the smallest number of times the time series of observable variables entering market fundamentals--in the present context, dividends--must be differenced before they appear stationary. In the present case, n turns out to equal one.

The empirical evidence on stationarity of means is based on inspection of estimated autocorrelations and spectra and on Dickey-Fuller tests for presence of unit roots in autoregressive models fitted to the relevant time series. Each of these procedures strongly suggests that the time series of aggregate real stock prices and dividends are nonstationary in levels but stationary in first differences. Moreover, inspection of estimated autocorrelations and application of Dickey-Fuller tests to nonstationary time series generated by simulating rational bubbles indicates that, given the actual sample size, this analysis is able to detect the relevant nonstationarity when it is present. These findings imply that rational bubbles do not exist in American stock prices.

In what follows, Section 1 sets up and solves the model. Section 2 reviews the theoretical case against rational bubbles. Section 3 reports empirical analysis of the stationarity properties of the time series of aggregate real stock prices and dividends. Section 4 analyzes simulated rational bubbles. Section 5 summarizes the analysis and conclusions.

1. The Model and Its Solution

The theoretical model consists of a single equation that assumes that the expected real rate of return from holding stock, including expected dividends and expected capital gains or losses, equals a constant required real rate of return--namely,

(1) $$(1+r)P_t = E_t(d_{t+1} + P_{t+1})$$

where

- r is the constant required real rate of return,
- P_t is the stock price at date t, relative to a general index of prices of goods and services,
- d_{t+1} is the real dividend paid to the owner of the stock at date $t+1$, and
- E_t is the conditional expectations operator.

The information set at date t on which E_t is based contains at least the current and past values of P_t and d_t. The variable d_t is stochastic and its innovations are independent of past stock prices.

Equation (1) is a first order expectational difference equation. Because the eigenvalue, $1+r$, is greater than unity, the forward-looking solution to this equation involves a convergent sum, as long as expected real dividends, $E_t d_{t+j}$, for any t do not grow with j at a geometric rate equal to or greater than $1+r$. This forward-looking solution, denoted by F_t and referred to in the literature as market fundamentals, is given by

$$(2) \qquad F_t = \sum_{j=1}^{\infty} (1+r)^{-j} E_t d_{t+j}.$$

Equation (2) says that market fundamentals equal the present value of expected real dividends discounted at the constant rate $1+r$.

The general solution to equation (1) is the sum of F_t and the general solution to the homogeneous expectational difference equation

$$(3) \qquad E_t B_{t+1} - (1+r) B_t = 0.$$

Solutions to equation (3) other than $B_t = 0$, for all t, represent rational bubbles. Any solution to equation (1) can be expressed as

$$(4) \qquad P_t = B_t + F_t$$

for some B_t satisfying equation (3).

Solutions to equation (3) satisfy the stochastic difference equation

$$(5) \qquad B_{t+1} - (1+r) B_t = z_{t+1},$$

where z_{t+1} is a random variable (or combination of variables) generated by a stochastic process that satisfies

$$(6) \qquad E_{t-j} z_{t+1} = 0 \quad \text{for all} \quad j \geq 0.$$

The key to the relevance of equation (5) for the general solution of P_t is that equation (3) relates B_t to $E_t B_{t+1}$, rather than to B_{t+1} itself as would be the case in a perfect-foresight model.

The random variable z_{t+1} is an innovation, comprising new information available at date t+1. This information can be intrinsically irrelevant--that is, unrelated to F_{t+1}--or it can be related to truly relevant variables, like d_{t+1}, through parameters that are not present in F_{t+1}. The only critical property of z_{t+1}, given by equation (5), is that its expected future values are always zero. (In the model of bursting bubbles discussed by Blanchard and Watson (1982) the analog to z_{t+1} satisfies equation (6) even though it is not covariance stationary.)

The solution to equation (5), for any date t, $t \geq 0$, is

$$(7) \qquad B_t = \sum_{\tau=1}^{t} (1+r)^{t-\tau} z_\tau + (1+r)^t B_0$$

where date zero is the date of inception of the stock market. (Note that the specification of date zero as a point in the finite past is necessary for B_t to be finite, for finite t.) Equation (7) relates B_t, the rational-bubbles component of stock price at date t, to B_0, the value of the rational-bubbles component at date zero, and to realizations of the random variable z between dates 1 and t. Since the eigenvalue 1+r exceeds unity, the contribution of z_τ to B_t increases exponentially with the difference between t and τ. For example, a past realization z_τ, $1 < \tau < t$, contributes only the amount z_τ to B_τ, but contributes $(1+r)^{t-\tau} z_\tau$ to B_t.

2. Theoretical Arguments Against Rational Bubbles

Although linear rational expectations models appear to permit rational bubbles, deeper theoretical analysis suggests that such models fail to capture important economic considerations that would affect demand for assets at extremely low and/or extremely high prices and that would preclude rational bubbles. Equation (3) implies that for any $j > 0$, the expected rational-bubbles component of stock price is related to the current value of the rational-bubbles component by

$$(8) \qquad E_t B_{t+j} = (1+r)^j B_t, \quad j > 0.$$

Accordingly, if B_t differs from zero, market participants must expect the rational-bubbles component either to increase (if $B_t > 0$) or to decrease (if $B_t < 0$) without bound geometrically at the rate $1+r$.

As several authors (e.g., Blanchard and Watson) have observed, equation (8) rules out negative rational bubbles because a positive probability that stock prices will be negative at a finite date in the future would be inconsistent with free disposal of stocks. Moreover, as Tirole (1982) demonstrates, equation (8) rules out positive rational bubbles in a model with finitely many immortal agents. In this model, it is not rational to expect real stock prices to grow without bound because such growth would require indefinite postponement of consumption. In addition, as Weil (1984) and Tirole (1985) show, equation (8) also rules out positive rational bubbles in an overlapping-generations model with a growth rate less than the real rate of interest. In such a model, the expected growth rate of a rational bubble would have to equal the real rate of interest, but it is not rational to expect stock prices to grow faster than the growth rate of the economy because such an expectation would imply that the value of the existing stock eventually would outgrow the endowments of the young generation.

Diba and Grossman (1985) show that, even if the growth rate of the economy exceeds the rate of interest and, therefore, positive rational bubbles could be sustained, the impossibility of negative bubbles restricts the inception of positive bubbles. Consider the possible inception of a positive rational bubble at date t, $t \geq 1$, assuming $B_{t-1} = 0$. The restriction

$B_t \geq 0$ together with equation (5) implies $z_t \geq 0$. Because, by condition (6), z_t must have a mean of zero, this nonnegativity restriction implies $z_t = 0$ with probability one.

Accordingly, in order for a positive rational bubble to exist at any date t, $t \geq 1$, it must have existed at all previous dates τ, $0 \leq \tau \leq t$. In particular, if at any date t, $t \geq 0$, an existing rational bubble were to vanish--an event that, for example, has constant probability in the model of bursting bubbles discussed by Blanchard and Watson (1982)--then a rational bubble cannot exist at any subsequent date τ, $\tau > t$.

The specification of equation (1) assumes that demand for stocks is infinitely elastic at a constant required rate of return. The theoretical case against positive rational bubbles would be even stronger in alternative models--for example, models with a log-linear specification of demand--in which, because a positive bubble would increase the fraction of equity in the real value of asset portfolios, portfolio balance would require the expected rate of return from holding stock to rise as the bubble grew. In such models, positive rational bubbles would imply that equity holders expect stock prices to grow at an <u>accelerating</u> rate. If the economy's output does not grow at a compatible accelerating rate, positive rational bubbles would not be consistent with the economy's output constraint.

In a log-linear setting negative rational bubbles would imply asymptotic convergence of the expected stock price to zero, as its logarithm tends to minus infinity. Hence, the standard argument against negative rational bubbles based on nonnegativity of stock prices, discussed above, does not apply. Nevertheless, it is also not rational to expect stock prices to converge to zero if stocks entitle their holders to positive streams of real dividends. The proof by Obstfeld and Rogoff (1983) that a negative rational bubble cannot exist in the value of money that is convertible to some real asset uses an analogous argument.

Diba and Grossman (1985) contains a more detailed discussion of the theoretical case for ruling out rational bubbles. The following section presents empirical evidence that complements this theoretical analysis.

3. Evidence Based on Stationarity Properties

If the excess volatility found by West (1984b) is attributable to rational bubbles, the innovations in the rational-bubbles component contribute much more than those of the market-fundamentals component to stock price fluctuations. West claims that "about 75 to 95 percent of the variance of the error in forecasting the following year's stock price is attributable to bubbles" (1984b, p. 22). If this claim is correct, it seems reasonable to expect that the time series properties of stock prices would closely resemble those of rational bubbles.

An important property of rational bubbles reflected in equation (8) is their explosive conditional expectations. For most specifications of the generating process of z_{t+1} the exploding conditional expectations property implies that time series of rational bubbles do not possess a stationary (unconditional) mean. The only exceptions discussed in the literature involve stochastic rational bubbles that can burst in any given period with nonzero probability. Quah (1985) demonstrates that such rational bubbles possess a stationary unconditional mean despite their exploding conditional expectations property. As was pointed out in the preceding section, however, the impossibility of negative rational bubbles implies that if (positive) rational bubbles ever burst, they cannot restart. Therefore, the only possible rational bubble that tests of the stationarity properties of stock prices could not detect is one that started at the inception of the stock market and did not last long enough (relative to sample size) to induce the appearance of nonstationarity in the mean of stock prices. The following empirical analysis abstracts from this possibility.

The explosive conditional expectations property associated with rational bubbles is not peculiar to the model discussed in Section 1. Mussa (1984) shows that various examples of attempts to construct alternative models in which potential rational bubbles are convergent all preclude a forward-looking market-fundamentals solution for some relevant price variable and, therefore, are not economically interesting.

Quah (1985) develops a stock market model in which, although stock prices are equal to the present value of expected future dividends, convergent rational bubbles can affect both stock prices and dividends. However, this model is based on a backward-looking specification of the process generating dividends. Specifically, Quah assumes that firms disregard information about current and future earnings and other relevant information about future events in choosing their dividend stream. Moreover, even if convergent rational bubbles are possible, their empirical implications, as Quah (1985, p. 43) recognizes, are not distinguishable from those of unobservable variables--e.g., expectations of a change in tax-laws--that may impinge on market fundamentals. Accordingly, the following empirical analysis focuses on the hypothesis that explosive rational bubbles exist in stock prices.

Differencing equation (5) n times yields

$$(9) \qquad [1 - (1+r)L](1-L)^n B_t = (1-L)^n z_t,$$

where L denotes the lag operator. If z_t is white noise, an ARMA process that is neither stationary (i.e., the autoregressive polynomial has a root inside the unit circle) nor invertible (i.e., the moving average polynomial has unit roots) generates the n^{th} difference of B_t, $(1-L)^n B_t$. More generally, equation (9) implies that, if differencing the time series of stock prices n times, for any finite n, yields a time series with stationary mean, stock prices do not contain rational bubbles.

Implementing the above approach to testing for rational bubbles involves two difficulties. First, even in the absence of rational bubbles, the (differenced) time series of stock prices may be nonstationary because the (differenced) time series of some variables appearing in market fundamentals--including dividends as well as other variables possibly left out of the model--may be nonstationary. This problem is a reflection of the general ambiguity of any evidence suggesting the existence of rational bubbles, discussed in the introduction and in Hamilton and Whiteman (1984) and Hamilton (1985). Second, even if rational bubbles exist, given a finite sample, differencing the time series of stock prices a sufficient number of times, will always induce the appearance of stationarity. Therefore, the choice of n is, in practice, quite important.

A conservative response to both of these problems, justified by Hamilton and Whiteman, is to set n equal to the smallest number of times the time series of observable variables in market fundamentals--in the present context, dividends--must be differenced before they appear stationary. To check that the analysis has power, given sample size, to detect rational bubbles when they exist, we also examine time series obtained by differencing simulated rational bubbles n times.

The data used for the present study, supplied by Robert Shiller, are the same as Data Set 1 in Shiller (1981). West also used this same data. The observations are annual from 1871 to 1980. The price series is Standard and Poor's Composite Stock Price index for January of each year divided by the wholesale price index. The dividend series is total dividends accruing to this portfolio of stocks for the calendar year divided by the average wholesale price index for the year.

Table 1

Sample Autocorrelations of Real Stock Prices, Dividends, and their First-differences

Number of Lags Series	1	2	3	4	5	6	7	8	9	10
P_t	0.95	0.89	0.86	0.82	0.76	0.70	0.64	0.57	0.50	0.43
d_t	0.95	0.89	0.83	0.78	0.74	0.70	0.65	0.61	0.57	0.54
ΔP_t	0.06	-0.27	0.11	0.18	0.01	-0.08	0.12	0.01	0.01	-0.05
Δd_t	0.22	-0.16	-0.08	-0.04	0.00	-0.02	-0.18	-0.11	0.07	0.14

Note: The price (P_t) and dividend (d_t) series contain 109 observations. Their first-differences (ΔP_t and Δd_t) contain 108 observations.

Table 1 presents sample autocorrelations of real stock prices, dividends, and their first-differences, for one through ten lags. The autocorrelations of the undifferenced price and dividend series both drop off very slowly as lag length increases--suggesting nonstationary means. Their patterns correspond closely to what would be expected for integrated moving average processes according to a formula presented by Wichern (1973). In contrast, autocorrelations of the differenced series, both for prices and dividends, are consistent with the assumption that these series have stationary means. Thus, the autocorrelation patterns suggest that the nonstationarity of the time series of real stock prices is attributable to their market fundamentals component and that rational bubbles do not exist in stock prices.

The Dickey-Fuller test yields further evidence on stationarity properties of the stock price and dividend time series. The Dickey-Fuller procedure looks for stochastic drift in the mean of a time series $\{x_t\}$ by testing the null hypothesis that the autoregressive representation of x_t, which is assumed to exist, has a unit root against the alternative hypothesis that all the roots of the autoregressive polynomial lie outside the unit circle. For more discussion, see Fuller (1976, pp. 366-382) or Nelson and Plosser (1982).

The test is based on estimating the OLS regression

$$x_t = \mu + \gamma t + \rho x_{t-1} + \sum_{i=1}^{k} \beta_i \Delta x_{t-i} + \text{residual}, \tag{10}$$

where t denotes time and Δ is the difference operator. The null hypothesis to be tested is that $\gamma = 0$ and $\rho = 1$. Under this null hypothesis, Δx_t is generated by an AR(k) process. Therefore, we can select the lag length k in equation (10) by applying Box-Jenkins identification procedures to choose the appropriate AR model for Δx_t. The test proceeds to calculate the conventional t-ratio for testing $\rho = 1$ in the OLS estimate of equation (10). This statistic is not t-distributed but its empirical percentiles have been tabulated--see, for example, Fuller (1976, p. 373).

If the bubble innovations, z_{t+1} in equation (5), are white noise, the process that generates rational bubbles is AR(1) with a root inside the unit circle. (Equation (5) is a special case of equation (10) with $\mu = \gamma = \beta_1 = \ldots = \beta_k = 0$ and $\rho = 1+r$.) Therefore, if rational bubbles exist, the Dickey-Fuller test should not reject the unit-root hypothesis in favor of the alternative hypothesis that the root is outside the unit circle.

Although rejection of the unit-root hypothesis would be evidence against rational bubbles, failure to reject the unit root hypothesis would not necessarily imply that rational bubbles exist. The tests reported by Nelson and Plosser fail to reject the unit root hypothesis for time series (such as nominal and real GNP, wage and price indices, and money stock) that presumably do not reflect rational bubbles. As in interpreting the autocorrelation patterns, we attribute the presence of a unit root in the time series of stock prices to market fundamentals rather than to rational bubbles as long as the test also suggests the presence of a unit root in the autoregressive process fitted to the time series of dividends.

A possible problem for the applicability of the Dickey-Fuller test is that, even if bubble innovations are white noise, first-differences of rational bubbles follow an ARMA process that is neither stationary nor invertible. Setting n equal to one in equation (9) leads to

$$(11) \qquad [1 - (1+r)L]b_t = (1-L)z_t,$$

where $b_t = (1-L)B_t$. The non-invertibility (i.e., the unit root of the moving average polynomial) precludes the existence of the pure AR representation on which the Dickey-Fuller test is based.

As a practical matter, however, time series generated by this process will resemble those generated by the process

$$[1 - (1+r)L]b_t = (1-\lambda L)z_t,$$

for λ close to but less than unity. This latter process has the AR representation

$$[1 - (1+r)L](1-\lambda L)^{-1}b_t = z_t. \quad (12)$$

The autoregressive polynomial in equation (12) has a root, $(1+r)^{-1}$, inside the unit circle--implying $\rho > 1$ in the counterpart of equation (10) with k set to infinity. The simulations presented in the next section show that the Dickey-Fuller test is relevant, as this argument suggests, for finding evidence against rational bubbles.

Table 2 reports OLS estimates of equation (10) for real stock prices, real dividends, and their first-differences. The first few observations on each series have been dropped to adjust sample size to 100 because Fuller (1976, p. 373) tabulates the critical values of the test statistic for a sample of this size. For each series, the lag length k was selected by choosing the appropriate AR(k) model for its first-differences on the basis of Box-Jenkins identification procedures.

For the undifferenced stock price and dividend time series, OLS estimates of the parameter ρ are below unity. However, the OLS estimator of this parameter is biased towards zero under the null hypothesis $\rho = 1$--see, for example, Nelson and Plosser's Table 1. The test statistic $\tau(\hat{\rho})$, reported in the last row of Table 2 below, is calculated as the conventional t-ratio for testing $\rho = 1$, i.e., $\tau(\hat{\rho}) = (\hat{\rho}-1)/S_{\hat{\rho}}$. Its 5% critical value, for a sample of 100 observations, is -3.45, with the rejection region given by smaller values of $\tau(\hat{\rho})$. Since the values of this statistic for both undifferenced series are larger than the

Table 2

Tests for Unit Root in the Autoregressive Representations

x_t:	p_t	d_t	Δp_t	Δd_t
$\hat{\mu}$	0.0101 (0.0147)	0.0008 (0.0004)	0.0091 (0.0153)	0.0002 (0.0004)
$\hat{\gamma}$	0.0004 (0.0003)	0.00003 (0.00001)	-0.0001 (0.0002)	-0.000001 (0.000006)
$\hat{\rho}$	0.9077 (0.0448)	0.8545 (0.0514)	0.0069 (0.1801)	0.0532 (0.1239)
$\hat{\beta}_1$	0.1826 (0.1024)	0.3233 (0.0971)	0.1190 (0.1348)	0.2175 (0.0993)
$\hat{\beta}_2$	-0.2390 (0.0978)	-0.1263 (0.0996)	-0.1685 (0.1018)	
$\hat{\beta}_3$	0.2076 (0.1015)			
$\tau(\hat{\rho})$	-2.06	-2.83	-5.51	-7.64

Note: Regressions are of the form:

$$x_t = \mu + \gamma t + \rho x_{t-1} + \sum_{i=1}^{k} \beta_i \Delta x_{t-i} + \text{residual}.$$

Standard errors are in parentheses below coefficients. Sample size is 100 in all cases. The statistic $\tau(\hat{\rho})$ is the conventional t-ratio for testing $\rho = 1$, i.e., $\tau(\hat{\rho}) = (\hat{\rho}-1)/S_{\hat{\rho}}$, but is not t-distributed under the null hypothesis. Its empirical percentiles are tabulated in Fuller (1976). The 0.05 critical value is -3.45, with the rejection region given by smaller values of $\tau(\hat{\rho})$.

critical value, we cannot reject the hypothesis $\rho = 1$ for either time series.

For the differenced time series of stock prices and dividends, estimates of ρ are not significantly different from zero. Moreover, values of $\tau(\hat{\rho})$ are well below the critical value of -3.45 (i.e., in the rejection region). Therefore, for both of these time series we can comfortably reject $\rho = 1$ in favor of $\rho < 1$.

The results of Dickey-Fuller tests for the original and differenced time series confirm the conclusion based on inspection of sample autocorrelations. The mean of real stock prices exhibits nonstationarity, but we can explain this nonstationarity without invoking rational bubbles because real dividends also possess a nonstationary mean. First-differences of real dividends are stationary and so are first-differences of real stock prices--contrary to what the existence of rational bubbles would imply.

We can also study the stationarity properties of the relevant time series by analyzing the data in the frequency domain. Although estimation of the spectrum presumes a stationary mean, estimated spectra are, in practice, helpful for detecting non-stationarity--see, for example, Jenkins and Watts (1968, pp. 7-8). Working with estimated spectra, rather than sample auto-correlations, avoids the potential problem that correlation among neighboring values can distort the sample autocorrelation function. In contrast, the estimated spectrum would isolate the effects of a nonstationary mean at the low frequencies--suggesting the presence of a "spike" at the zero frequency. In other words, the estimated spectrum rises sharply as it approaches the low frequencies and stays flat over a band near the zero frequency.

Figures 1 to 4 depict the logarithms of the estimated spectra of real stock prices, dividends, and their first-differences. The reported spectra were estimated with 128 ordinates and a tent window of width 11. Variations of the number of ordinates and the type (i.e., tent or flat) and width of the window did not appear to have a major effect on the features of estimated spectra that are discussed below.

For both stock prices and dividends, the spectra of the undifferenced series suggest the presence of a spike at the zero frequency (corresponding to cycles with infinite periodicity) but the spectra of first-differenced series do not. (The differenced time series of stock prices seems to exhibit a spike at the periodicity of 3.88 years--presumably associated with business cycles--but not at the zero frequency. As was pointed out above, the relevant features of the spectrum, as far as stationarity of the mean is concerned, are concentrated around the zero frequency.)

The frequency domain results warrant the same inferences as the time domain results discussed above. The nonstationary mean of the undifferenced time series of stock prices seems attributable to market fundamentals. The stationary mean of the differenced time series of stock prices suggests that rational bubbles do not exist.

4. Stationarity Properties of Simulated Rational Bubbles

This section presents evidence that the time domain tests discussed in the preceding section have power against the no-bubbles hypothesis when that hypothesis is, by construction, false. The tests applied to real stock price and real dividend time series are applied to 50 simulated time series, containing 109 observations each, of rational bubbles.

Figure 1

Log-Spectrum of p

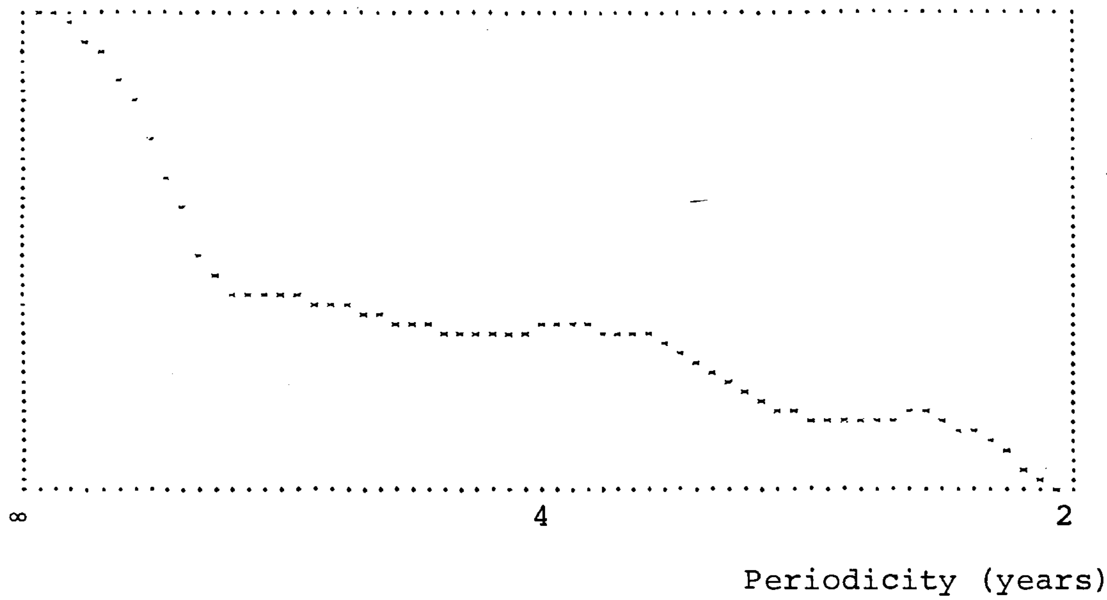


Figure 2

Log-Spectrum of d

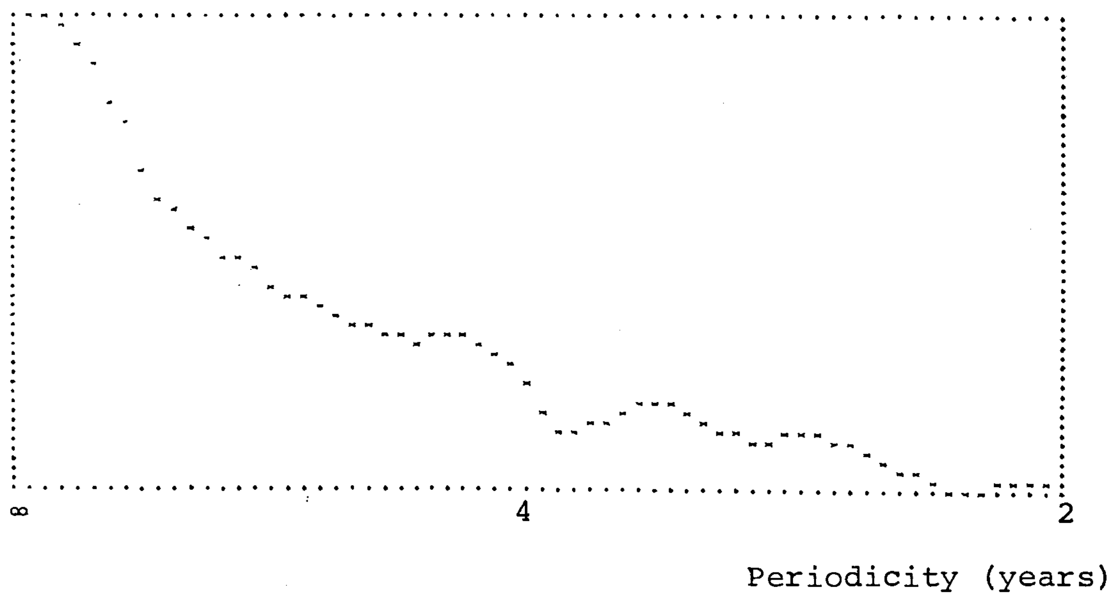


Figure 3

Log-Spectrum of Δp

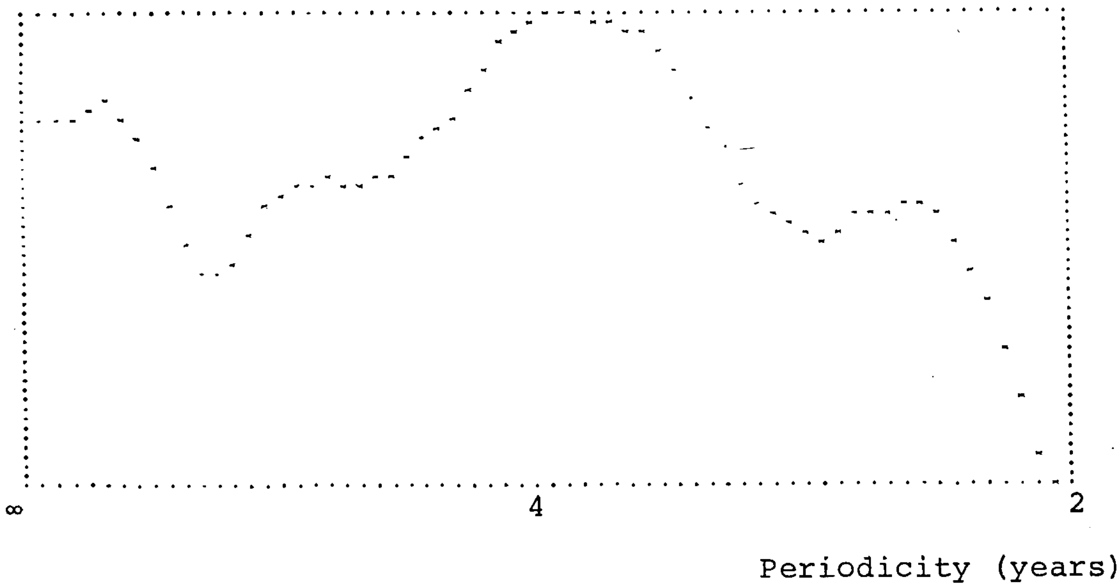
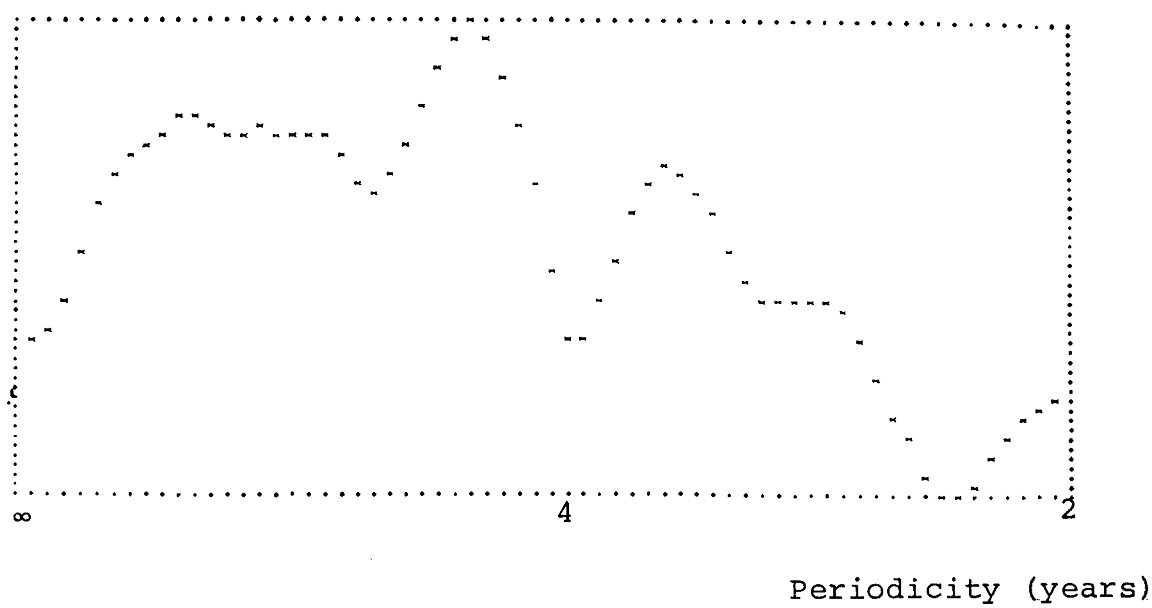


Figure 4

Log-Spectrum of Δd



In the simulations the bubble innovations, z_t in equation (7), are assumed to have the standard normal distribution, and B_0 is set equal to zero. Since interest at this point is focused on stationarity properties of time series of rational bubbles, the simulations ignore the theoretical impossibility of negative bubbles and questions related to inception of rational bubbles. The simulations require an assumption about the value of r, the ex ante required real rate of return from holding equity. This assumption is important because, as inspection of equation (11) reveals, first-differenced time series of rational bubbles can be empirically indistinguishable from white noise if r is close to zero. The mean of the ex post real rate of return for the data discussed in the preceding section is 0.081. However, to emphasize the ability of the tests to detect rational bubbles for lower values of r, the simulations set this parameter equal to 0.05, which is the value assumed by Shiller (1981).

For each simulated time series of rational bubbles and its first-difference, autocorrelation coefficients, for one through ten lags, were calculated. The first few observations on each untransformed and first-differenced series were then dropped to adjust the number of observations to 100, equation (10) was estimated, and for the simulations with $\hat{\rho} < 1$, the $\tau(\hat{\rho})$ statistic (discussed in the preceding section) was evaluated. The lag length k was set equal to zero when estimating equation (10) for undifferenced series. For the differenced series, the autoregressive approximation given by equation (12) is not finite. The lag length k suggested by t-ratios of estimated coefficients ranged between three and six in most cases. Because leaving out relevant terms could bias the results, whereas inclusion of irrelevant ones would only reduce efficiency, k was set equal to six for all of the different series.

The results for the undifferenced series are not of much interest and are, therefore, not reported. The sample autocorrelations dropped off very slowly in all cases. Point estimates of the parameter ρ in equation (10) were in all cases above unity, making the Dickey-Fuller test redundant.

Table 3 reports the results for first-differences of simulated rational bubbles. The patterns of autocorrelation coefficients in all but six cases (simulations numbered 10, 14, 21, 29, 33 and 35) strongly suggest nonstationarity. The autocorrelation function starts at a value of 0.8 or higher and drops off very slowly. For simulations 10, 14, 21, 29, and 35 the starting values are lower, but the autocorrelation functions still drop off slowly. (Wichern's results indicate that the latter criterion is a more reliable sign of nonstationarity.) Only for simulation number 33 does the pattern of autocorrelations resemble those of differenced time series of stock prices and dividends reported in Table 1 above.

Point estimates of ρ are below unity for simulations numbered 10, 14, 21, 29, 33, and 35. However, the test statistic $\tau(\hat{\rho})$ in all six cases is above the 0.05 critical value of -3.45, and the hypothesis $\rho = 1$ cannot be rejected in favor of $\rho < 1$. Point estimates of ρ are above unity in the remaining 44 cases, making the Dickey-Fuller test unnecessary.

Overall, the simulations strongly suggest that the pattern of autocorrelations and the Dickey-Fuller test have power to detect the nonstationary mean of first-differenced rational bubble series. Therefore, the results presented in the preceding section, that the first-differenced time series of real stock prices does not exhibit this type of nonstationarity is relevant evidence against the existence of rational bubbles in stock prices.

Table 3

Stationarity Properties of First-differences of Simulated Rational Bubble Series

Simulation Number	r_1	r_2	r_3	r_4	r_5	r_6	r_7	r_8	r_9	r_{10}	$\hat{\rho}$	$\tau(\hat{\rho})$
1	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.63	0.59	0.56	1.23	
2	0.93	0.89	0.83	0.79	0.75	0.71	0.67	0.63	0.60	0.57	1.18	
3	0.92	0.87	0.80	0.77	0.73	0.70	0.65	0.62	0.57	0.55	1.14	
4	0.93	0.87	0.82	0.79	0.75	0.72	0.67	0.63	0.59	0.56	1.23	
5	0.91	0.85	0.78	0.74	0.70	0.66	0.63	0.60	0.56	0.53	1.16	
6	0.95	0.90	0.85	0.80	0.76	0.71	0.67	0.64	0.60	0.56	1.21	
7	0.94	0.89	0.84	0.80	0.76	0.72	0.68	0.63	0.59	0.56	1.19	
8	0.93	0.88	0.84	0.79	0.76	0.72	0.68	0.64	0.60	0.57	1.19	
9	0.90	0.85	0.82	0.77	0.73	0.70	0.64	0.61	0.58	0.55	1.20	
10	0.65	0.65	0.62	0.56	0.53	0.45	0.51	0.43	0.42	0.36	0.92	-0.60
11	0.94	0.88	0.83	0.78	0.75	0.71	0.67	0.62	0.58	0.55	1.24	
12	0.91	0.85	0.82	0.76	0.73	0.68	0.64	0.61	0.56	0.53	1.17	
13	0.92	0.87	0.82	0.78	0.74	0.70	0.67	0.63	0.60	0.56	1.21	
14	0.62	0.64	0.55	0.60	0.51	0.46	0.44	0.43	0.39	0.40	0.87	-0.91
15	0.80	0.80	0.73	0.71	0.65	0.65	0.58	0.52	0.53	0.48	1.02	
16	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.63	0.59	0.56	1.22	
17	0.90	0.86	0.81	0.76	0.72	0.68	0.65	0.61	0.59	0.55	1.16	
18	0.93	0.89	0.84	0.79	0.75	0.70	0.66	0.62	0.59	0.56	1.18	
19	0.94	0.89	0.84	0.80	0.76	0.72	0.68	0.64	0.60	0.56	1.18	
20	0.94	0.89	0.85	0.80	0.76	0.72	0.67	0.64	0.60	0.57	1.17	
21	0.46	0.42	0.41	0.29	0.35	0.31	0.27	0.22	0.24	0.29	0.54	-2.82
22	0.93	0.88	0.83	0.78	0.74	0.70	0.66	0.62	0.58	0.54	1.21	
23	0.93	0.88	0.83	0.79	0.74	0.70	0.66	0.62	0.59	0.56	1.29	
24	0.94	0.90	0.85	0.80	0.75	0.71	0.67	0.63	0.60	0.56	1.20	
25	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.64	0.61	0.57	1.20	
26	0.93	0.88	0.83	0.78	0.74	0.70	0.66	0.63	0.59	0.55	1.19	
27	0.83	0.78	0.75	0.70	0.68	0.67	0.62	0.58	0.53	0.49	1.12	
28	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.64	0.60	0.56	1.19	
29	0.56	0.53	0.48	0.51	0.47	0.40	0.41	0.35	0.35	0.35	0.87	-0.80
30	0.94	0.89	0.84	0.80	0.75	0.71	0.68	0.64	0.60	0.56	1.15	
31	0.89	0.84	0.80	0.75	0.72	0.69	0.65	0.61	0.55	0.53	1.11	
32	0.93	0.88	0.83	0.79	0.74	0.70	0.66	0.61	0.58	0.55	1.20	
33	0.11	0.17	0.21	0.17	0.19	0.12	0.23	0.02	0.18	0.06	0.49	-1.86
34	0.94	0.89	0.85	0.80	0.75	0.71	0.67	0.63	0.59	0.56	1.21	
35	0.40	0.30	0.30	0.25	0.24	0.30	0.31	0.16	0.27	0.20	0.71	-1.30
36	0.93	0.89	0.84	0.79	0.75	0.70	0.66	0.63	0.59	0.56	1.15	
37	0.94	0.90	0.85	0.80	0.75	0.71	0.67	0.64	0.60	0.56	1.18	
38	0.94	0.89	0.84	0.80	0.76	0.72	0.68	0.64	0.60	0.56	1.17	
39	0.95	0.90	0.85	0.81	0.76	0.72	0.68	0.64	0.60	0.57	1.16	
40	0.94	0.89	0.84	0.80	0.76	0.71	0.68	0.64	0.60	0.56	1.27	
41	0.95	0.90	0.85	0.80	0.76	0.72	0.68	0.64	0.60	0.57	1.22	
42	0.94	0.89	0.84	0.80	0.75	0.71	0.67	0.64	0.60	0.57	1.20	
43	0.90	0.86	0.82	0.77	0.73	0.70	0.67	0.61	0.57	0.54	1.19	
44	0.95	0.90	0.85	0.81	0.76	0.72	0.68	0.64	0.60	0.56	1.18	
45	0.84	0.81	0.74	0.71	0.66	0.61	0.58	0.56	0.51	0.49	1.13	
46	0.94	0.89	0.85	0.81	0.76	0.72	0.68	0.64	0.61	0.57	1.20	
47	0.94	0.89	0.84	0.80	0.76	0.71	0.67	0.63	0.60	0.57	1.21	
48	0.93	0.89	0.84	0.79	0.76	0.72	0.68	0.64	0.60	0.57	1.16	
49	0.93	0.89	0.84	0.79	0.74	0.70	0.66	0.62	0.59	0.56	1.18	
50	0.90	0.85	0.81	0.76	0.73	0.69	0.64	0.58	0.56	0.53	1.17	

Note: Table reports the stationarity properties of first-differences of simulated rational bubble series:

$$B_t = 1.05 \; B_{t-1} + z_t,$$

where z_t is normally distributed white noise, and B_0 is set equal to zero. r_k, $k = 1, \ldots, 10$, is the autocorrelation coefficient at lag k. $\hat{\rho}$ is the OLS estimate of ρ in equation (10) in the text. **The key question is whether the hypothesis** $\rho = 1$ can be rejected in favor of $\hat{\rho} < 1$. For the simulations with $\hat{\rho} < 1$, the $\tau(\hat{\rho})$ statistic, discussed in Section 3, is reported. Its 0.05 critical value is -3.45, with the rejection region given by smaller values of $\tau(\hat{\rho})$.

5. Summary and Conclusions

Ignoring the possibility of a positive rational bubble that started at the inception of the stock market and vanished shortly after (relative to sample size), the existence of rational bubbles would imply nonstationarity of the means of (differenced) time series of stock prices. The empirical analysis looked for such nonstationarity. To avoid problems of drawing inferences from time series obtained by differencing stock prices an arbitrary number of times, we used two inferential strategies. First, if rational bubbles exist, stock prices should exhibit higher order nonstationarity than observable variables in their market fundamentals--e.g., dividends. Second, if rational bubbles do not exist, stock prices should exhibit lower order nonstationarity than time series of simulated rational bubbles.

Inferences about stationarity of means were based on inspection of sample autocorrelations and estimated spectra of the relevant time series and on Dickey-Fuller tests for unit roots in their autoregressive representations. The results strongly suggest that stationarity properties of aggregate real stock prices accord with those of aggregate real dividends. Both time series appear nonstationary in levels but stationary in first-differences. Moreover, first-differenced time series of simulated rational bubbles exhibit clear signs of nonstationarity--implying that the tests have power to detect rational bubbles if they existed in stock prices. These findings indicate that rational bubbles do not exist in stock prices.

REFERENCES

O.J. Blanchard and M.W. Watson, "Bubbles, Rational Expectations, and Financial Markets," in Crises in the Economic and Financial Structure, P. Wachtel, ed. (Lexington Books, 1982).

E. Burmeister and K. Wall, "Kalman Filtering Estimation of Unobserved Rational Expectations with an Application to the German Hyperinflation," Journal of Econometrics, 20, 1982, 255-284.

J.Y. Campbell, "Stock Returns and the Term Structure," Princeton University Working Paper, October 1984.

B.T. Diba and H.I. Grossman, "The Impossibility of Rational Bubbles," NBER Working Paper No. 1615, Revised December 1985.

R.F. Engle and M.W. Watson, "Applications of Kalman Filtering in Econometrics," unpublished, October 1985.

E.F. Fama and G.W. Schwert, "Asset Returns and Inflation," Journal of Financial Economics, 5, November 1977, 115-146.

M.A. Flavin, "Excess Volatility in the Financial Markets: A Reassessment of the Empirical Evidence," Journal of Political Economy, 91, December 1983, 929-956.

R. Flood and P. Garber, "Market Fundamentals Versus Price Level Bubbles: The First Tests," Journal of Political Economy, 88, August 1980, 745-770.

R. Flood, P. Garber, and L. Scott, "Further Evidence on Price Level Bubbles," NBER Working Paper No. 841B, January 1982.

W.A. Fuller, Introduction to Statistical Time Series (New York: John Wiley & Sons, 1976).

S.J. Grossman and R.J. Shiller, "The Determinants of the Variability of Stock Market Prices," American Economic Review, 71, May 1981, 222-227.

J.D. Hamilton, "On Testing for Self-Fulfilling Speculative Price Bubbles," University of Virginia Working Paper, July 1985.

J.D. Hamilton and C.H. Whiteman, "The Observable Implications of Self-Fulfilling Expectations," University of Virginia Working Paper, May 1984.

G.M. Jenkins and D.W. Watts, Spectral Analysis and Its Applications (San Francisco: Holden-Day, 1968).

J.M. Keynes, The General Theory of Employment Interest and Money (Macmillan, 1936).

S.F. LeRoy and R.D. Porter, "The Present Value Relation: Tests Based on Implied Variance Bounds," Econometrica, 49, May 1981, 555-574.

T.A. Marsh and R.C. Merton, "Aggregate Dividend Behavior and its Implications for Tests of Stock Market Rationality," Sloan School of Management Working Paper, 1983.

T.A. Marsh and R.C. Merton, "Dividend Variability and Variance Bounds Tests for the Rationality of Stock Market Prices," Sloan School of Management Working Paper, October 1984.

C.R. Nelson and C.I. Plosser, "Trends and Random Walks in Macroeconomic Time Series: Some Evidence and Implications," Journal of Monetary Economics, 10, 1982, 139-162.

M. Obstfeld and K. Rogoff, "Speculative Hyperinflations in Maximizing Models: Can We Rule Them Out?" Journal of Political Economy, 91, August 1983, 675-687.

D. Quah, "Estimation of a Nonfundamentals Model for Stock Price and Dividend Dynamics," Massachusetts Institute of Technology Working Paper, September 1985.

T.J. Sargent and N. Wallace, "Exploding Inflation," University of Minnesota Working Paper, January 1984.

R.J. Shiller, "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends," American Economic Review, 71, June 1981, 421-436.

J. Tirole, "On the Possibility of Speculation under Rational Expectations," Econometrica, 50, September 1982, 1163-1181.

J. Tirole, "Asset Bubbles and Overlapping Generations," Econometrica, 53, September 1985, 1071-1100.

P. Weil, "Confidence and the Real Value of Money in an Overlapping Generations Economy," Harvard University Working Paper, November 1984.

K.D. West, "A Specification Test for Speculative Bubbles," Princeton University Working Paper, July 1984 (a).

K.D. West, "Speculative Bubbles and Stock Price Volatility," Princeton University Working Paper, December 1984 (b).

D.W. Wichern, "The Behavior of the Sample Autocorrelation Function for an Integrated Moving Average Process," Biometrika, 60, 1973, 235-239.