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ABSTRACT

Do theoretical welfare gains from trade translate into aggregate measures of economic activity? We calculate the changes in real GDP and real consumption that result from changes in trade costs in a range of workhorse trade models, following the procedures outlined by statistical agencies in the United States. Our main findings are as follows: First, real GDP and measured aggregate productivity rise in response to reductions in variable trade costs if GDP deflators capture the decline in trade costs. Second, with balanced trade in each country, changes in world real consumption and changes in world real GDP (i.e.: weighting the change in each country by its nominal GDP) in response to changes in variable trade costs coincide, up to a first-order approximation, with changes in world theoretical (welfare-based) consumption. The equivalence between measured consumption and theoretical consumption holds country-by-country under stronger conditions. Third, for given trade shares and changes in variable trade costs, changes in real GDP and changes in world real consumption are approximately equal in magnitude across the models we consider.

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1 Introduction

What are the aggregate gains from reductions in the costs of international trade? There are two major approaches to address this question. A first approach uses structural models to infer unobservable welfare gains from changes in trade costs or in trade patterns (see e.g. Eaton and Kortum 2001, Alvarez and Lucas 2007, Arkolakis, Costinot and Rodriguez-Clare 2011, Donaldson 2010, and Waugh 2010). A second approach documents the empirical link between the level or the change in international trade and aggregate indicators of economic activity (see e.g. Frankel and Romer 1999, Rodriguez and Rodrik 2001, and Feyrer 2009a, 2009b).

This paper connects these two approaches by studying, within a range of workhorse models of international trade, the relationship between theoretical welfare gains from trade and aggregate measures of economic activity, namely real GDP and real consumption as constructed by national statistical agencies. In doing so, we shed light on the following questions. Should we expect measured aggregate productivity and real GDP to rise with trade? Are aggregate measures of economic activity informative of theoretical gains from trade? Do different models have common sufficient statistics for the impact of trade on aggregate measures of economic activity?

The models that we base our analysis on include Armington models with perfect competition and exogenous specialization in production (e.g. Anderson 1979), Ricardian models with endogenous specialization in production (e.g. Dornbusch, Fisher and Samuelson 1977 and Eaton and Kortum 2001), and monopolistically competitive firm models with heterogenous firms and constant markups (e.g. Krugman 1980 and Melitz 2003). We consider extensions of the model with multiple factors of production (but common factor intensities across producers) and with endogenous quality choice. We include international trade costs of the form of iceberg variable trade costs, fixed export costs (in the model with monopolistic competition), and import tariffs. In all of these models, reductions in international trade costs typically result in a rise in welfare for the representative consumer.

We calculate these models' implications of reductions in trade costs for real GDP and real consumption calculated following the procedures outlined by the Bureau of Labor Analysis to construct the National Income and Product Accounts (NIPA) in the United States. For many industries and components of GDP, comprehensive measures of physical quantities are difficult to obtain in practice. In such cases, real quantities are typically calculated by deflating current dollar measures of output or consumption with price indices — e.g. in most cases the producer price index (PPI) for output and the consumer price index (CPI) for consumption.

We first calculate the change in measured aggregate productivity and real GDP following a change in international trade costs. We show that, in response to a decline in variable trade costs, aggregate productivity and real GDP in any country rise only if trade costs are recorded in GDP and GDP deflators reflect the fall in trade costs. That is, measured productivity and real GDP rise when variable trade costs fall if the services and activities required to sell goods abroad (which include shipping services provided by the transportation industry and, more broadly, any other production, marketing, regulatory, and information costs that apply differentially to exported products)[1] are performed and recorded in the home country, as opposed to being performed abroad or not measured at all. This is because, under certain conditions, measured aggregate productivity in any given country only responds to shifts in its production possibility frontier as is, in principle, desirable for a measure of productivity.[2]

The response of real GDP also depends on the form of trade liberalization. In particular, changes in fixed trade costs (if these are expensed and hence not recorded in GDP) have no direct impact on GDP deflators and hence leave real GDP unchanged. Reductions in import tariffs increase real GDP from the expenditure side if tariff revenues at constant prices rise (which requires an increase in the physical quantity of imports).

Next, we compare changes in real GDP and in real consumption (in our baseline model, changes in consumption expenditures are equal to changes in total absorption). Real GDP and real consumption can differ even when trade is balanced due to movements in the price of exports relative to the price of imports (the terms of trade). We show, however, that if trade is balanced in each country, changes in world real GDP are equal, up to a first-order approximation, to changes in world real consumption (where each country is weighted by its current-dollar GDP). The equality holds at the world aggregate level because terms of trade improvements in one country are associated to terms of trade worsenings in another country. While changes in real GDP country-by-country depend critically on the patterns of specialization in the production of trade services, the equality between changes in world real GDP and world consumption does not.

Perhaps more importantly, we compare changes in measured real consumption with changes in theoretical (or welfare-based) consumption. Differences between theoretical and measured consumption arise from differences between consumption deflators and the theoretical price index. Consumption deflators in our model differ from the welfare-based price index in three respects. First, consumption deflators do not fully take into account substitu-

[1] Anderson and Van Wincoop (2004) argue that these additional costs are at least as important as narrowly-defined transportation costs.

[2] As shown in Kohli (2004) and Kehoe and Ruhl (2008), the value of production at constant prices does not respond, to a first-order approximation, to changes in international prices that leave the domestic production possibility frontier unchanged.

tion in consumption from changes in relative prices. Second, they do not take into account changes in the mass of consumed goods which, in the presence of love for variety, matter for theoretical consumption. Third, they do not take into account improvements in product quality if quality changes are measured inaccurately in consumption deflators. The report by the Boskin Commission (1996) examines in detail how these and other biases in the CPI lead to an understatement of real consumption growth in the U.S.

We show how, under certain conditions, these multiple biases in consumption deflators may not result in a mismeasurement of theoretical consumption. If the set of consumed goods and product quality are fixed (so that the second and third sources of the bias are absent), then in response to any type of trade cost movement, changes in theoretical consumption are bounded between measured real consumption calculated using initial base-year prices and real consumption using end base-year prices. This implies, as is well-known (see e.g. Hausman 2003), that the substitution bias is of second order: in each country, changes in real consumption equal changes in theoretical consumption, up to a first-order approximation.

When the set of consumed goods and product quality are not fixed, we establish the following result. In response to changes in variable trade costs, with trade balance in each country, changes in world real consumption equal changes in world theoretical consumption (defined analogously to world real consumption and world real GDP), up to a first-order approximation. That is, while changes in theoretical consumption and real consumption may differ country-by-country, these differences cancel-out when adding them across countries in the world. Under stronger assumptions (i.e. Pareto distribution of entering firms' productivity and fixed export costs paid in the destination market, as in Eaton, Kortum, and Kramarz 2010), the equality between measured consumption and theoretical consumption holds country-by-country, up to a first-order approximation, as in the model with a fixed set of consumed goods. We also show that, in response to large reductions in variable trade costs (for which we must solve the model numerically), the elasticities of theoretical consumption and real consumption can be quite close, country-by-country (and hence also at the world level), independently of whether fixed export costs are incurred domestically or abroad.

Finally, we ask whether the different models that we consider give rise to different sufficient statistics for measured gains from trade. We consider this question separately for our measures of real GDP and real consumption. Across our range of models, we obtain a common expression for the change in real GDP as an average of changes in variable trade costs weighted by export shares of continuing exporting producers. Conditional on this direct impact of changes in trade costs, reallocation of production from less productive to more productive producers, entry and exit into production and exporting, and changes in the mass of producers, have no additional effects on changes in measured aggregate productivity and real GDP.

We also show that across our range of models, changes in world real consumption and world real GDP are equal in magnitude, up to a first-order approximation, for given trade shares and for given changes in variable trade costs. This first-order equivalence in measured gains from trade in consumption across seemingly different models does not reflect an inadequacy of the aggregate measures of real consumption. Instead, this equivalence in measured gains from trade is consistent with the underlying equivalence in the welfare implications of these models under some restrictions, as demonstrated by Arkolakis, Costinot and Rodriguez-Clare (2011) and Atkeson and Burstein (2010). Note, however, that changes in fixed trade costs or foreign country size that increase trade shares (and also welfare, under the assumptions of Arkolakis, Costinot and Rodriguez-Clare 2011) may not result in measured gains from trade.

Our paper is related to a recent paper by Bajona, Gibson, Kehoe, and Ruhl (2010), who ask whether the increase in welfare following a trade liberalization translates into an increase in real GDP as measured in NIPA. They conclude, as summarized in Kehoe and Ruhl (2010), that "...standard trade models do not imply that opening to trade increases productivity or real GDP, but that it increases welfare". The two main differences of our paper relative to Bajona et. al. (2010) are as follows. First, while Bajona et. al. (2010) focus on the implications of trade liberalization on real GDP, we also study the effects on real consumption and provide conditions under which the response of real consumption to changes in trade costs equals that of theoretical consumption. Second, Bajona et. al. (2010) focus on cases in which price indices do not directly reflect changes in international trade costs, either because trade costs are fully incurred abroad or because countries are in autarky before the trade liberalization (in which case price indices of exported goods, as measured by the BLS, are not well defined since there are no continuing exported goods). In the class of models considered in both papers, this implies that measured real GDP is unchanged with trade liberalization (abstracting from changes in real tariff revenues). We show, however, that starting with positive trade levels, any reduction in trade costs that is reflected in price indices *does* result in an increase in real GDP.

Our work is also related to Feenstra (1994) and Broda and Weinstein (2006), who quantify the mismeasured growth in real consumption in the U.S due to the rise in the number of imported varieties that is not accounted for in the CPI, without taking a stand on the source of the growth in the number of imported varieties.[3] We show in our models that, in

[3]Relatedly, Feenstra, Reinsdorf, and Slaughter (2008) and Neiman and Gopinath (2011) argue that if export and import price indices are mismeasured (among other reasons, due to changes in import variety), changes in tariffs or in the terms of trade can result in changes in measured aggregate productivity.

response to a reduction in variable trade costs that results in a rise in the number of imported varieties, to a first-order approximation there is no bias in consumption deflators at the world aggregate level or, under stronger conditions, country-by-country, when simultaneously taking into account in general equilibrium other biases in the price indices. Hence, any underestimate of real consumption growth stemming from an increase in the mass of imported varieties that is not captured in the import price index is offset by the other biases in the CPI. Finally, our paper is related to the work of Pavcnik (2002) and others, that construct measures of aggregate productivity as weighted averages of productivity estimates across producers. While those measures of aggregate productivity may reflect the reallocation of production towards more productive producers induced by trade liberalization, we argue, using a range of models of trade and firm heterogeneity as a laboratory, that measures of aggregate productivity constructed from NIPA do not capture this reallocation.

The paper is organized as follows. Section 2 presents an overview of the measurement procedures that we use in our models. Section 3 presents our baseline Armington model with exogenous specialization in the set of goods that are produced and traded in each country. Section 4 derives our basic results on measured real GDP, real consumption, and theoretical consumption in the Armington model. Section 5 shows that these basic results apply in a Ricardian model with endogenous specialization and perfect competition. Section 6 extends the basic results to the version of the model with endogenous specialization and monopolistic competition. Section 7 considers two additional extensions: endogenous quality choice and multiple factors of production. Section 8 concludes. Various proofs and details are relegated to the Appendix.

2 Aggregate Measurement: Overview

In this section we provide a brief overview of the procedures that we use to calculate changes in aggregate quantities. We follow as closely as possible the procedures outlined by the Bureau of Economic Analysis in the United States to construct the National Income and Product Accounts (NIPA).[4]

To calculate aggregate measures of output such as real GDP, or aggregate measures of expenditures such as real consumption, we use a Fisher index, which is a geometric average of a Laspeyres and a Paasche quantity index. For example, real GDP in period t relative to

[4] See, e.g. Concepts and Methods of the U.S. National Income and Product Accounts (2009). The procedures that we consider are broadly consistent with the recommendations by the United Nations in their System of National Accounts.

period $t-1$ is given by

$$\frac{RGDP_t}{RGDP_{t-1}} = \left(\frac{\sum p_{t-1}q_t}{\sum p_{t-1}q_{t-1}}\right)^{0.5} \times \left(\frac{\sum p_t q_t}{\sum p_t q_{t-1}}\right)^{0.5}, \tag{1}$$

where p_t and q_t denote prices and quantities in period t of the detailed components of GDP, and where the sum is calculated across all of these components. The terms $p_{t-1}q_t$ and $p_t q_{t-1}$ represent "real" quantities of any given GDP component evaluated at constant prices. The first term in expression (1) is a Laspeyres quantity index (based on $t-1$ prices), while the second term is a Paasche quantity index (based on t prices).[5] Real GDP in period T relative to period 0 is given by

$$\frac{RGDP_T}{RGDP_0} = \prod_{t=1}^{T} \frac{RGDP_t}{RGDP_{t-1}}. \tag{2}$$

The detailed components of GDP in expression (1) can be industries, sectors, or groups of narrowly defined goods that jointly conform aggregate GDP or other aggregate measures of output and expenditures. While estimates of the current-dollar value of production, $p_t q_t$, are typically available for each of these individual components, data on physical quantities, q_t, are often not.

For those components of GDP for which data on physical output are available, real quantities are computed using either the direct valuation method (sum of quantities evaluated at constant prices) or the quantity extrapolation method (using a quantity indicator that approximates the movements of the component series). For those components of GDP for which estimates of physical quantities are not available, real quantities are estimated using the deflation method, dividing current-dollar values by appropriate price indices.[6] In particular, for any component of GDP, $p_{t-1}q_t = (p_t q_t) / (\mathcal{P}_t/\mathcal{P}_{t-1})$ and $p_t q_{t-1} = (p_{t-1}q_{t-1}) \times (\mathcal{P}_t/\mathcal{P}_{t-1})$, where $\mathcal{P}_t/\mathcal{P}_{t-1}$ denotes the change in the price index between periods $t-1$ and t. In our baseline calculations, we compute aggregate quantities using the deflation method.

To calculate real GDP from the production side using the deflation method, we deflate the current-dollar value added of production (including the value added of the activities performed at home to sell goods internationally) using the producer price index (PPI) as a

[5] The implicit GDP deflator is calculated as the ratio of current-dollar GDP to real GDP, $\left(\sum p_t q_t / \sum p_{t-1}q_{t-1}\right) / \left(RGDP_t/RGDP_{t-1}\right)$, which is equal to a geometric average of a Laspeyres and a Paasche *price* index.

[6] The direct valuation method is used, for example, to calculate real output of autos and light trucks, while quantity extrapolation is used to calculate real output of housing and utilities services. The majority of the other subcomponents of GDP are calculated using the deflation method since physical output is not recorded across producers (see "Summary of NIPA Methodologies", p.12 for a description of the method used to estimate each subcomponent of GDP).

deflator.[7] The change in the PPI between periods $t-1$ and t is a weighted average of price changes between these two periods across goods and services that are produced domestically to sell at home or to export abroad.[8]

We consider two alternative deflation procedures. The first procedure deflates the total value of production using a single aggregate price index. The second procedure deflates the value of output bound for each destination using a destination-specific price index. We show that, using disaggregated deflators by destination country, real GDP is equal to that obtained using the direct valuation method based on data on physical quantities of each commodity.

Export prices in the PPI and in the export price index (EPI) are typically measured at fob (i.e. free-on-board) values, and hence exclude shipping services incurred abroad. A critical assumption determining the impact of changes in international trade costs on measured real GDP is whether changes in measured prices in the PPI reflect, at least partly, these changes in trade costs. In addition to shipping costs (that are included in the transportation industry), international trade costs include production and marketing costs that apply differentially to exported goods, information costs, costs associated with the use of different currencies, contract enforcement costs, legal regulatory costs, and other time costs associated to international trade (see e.g. Anderson and Van Wincoop 2004). To understand the implications of the nature of trade costs on aggregate measurement, we consider two alternative specifications. In our baseline specification, we assume that the activities required to sell goods abroad are performed in the home country, and hence changes in the variable component of these trade costs are reflected in the home PPI. In an alternative specification, we assume that all export costs are incurred in foreign countries, in which case changes in trade costs are not reflected in the PPI.

We also calculate GDP from the expenditure side, defined as current-dollar absorption (which in our baseline model is equal to consumption), plus exports less imports. Real consumption is calculated analogously to real GDP (using expressions 1 and 2), but deflating

[7] This is the procedure used in the GDP by industry accounts published by the BEA. When intermediate inputs are used in production, real value added is calculated using the double deflation method. This consists of first deflating gross output and inputs separately (using their respective PPIs), and then computing real value added as the difference between real gross output and real intermediate inputs.

[8] To construct the PPI, the Bureau of Labor Statistics (BLS) collects prices for a sample of items that can be priced consistently through time. Price indices are then constructed by averaging price changes of individual items weighted by the the value of production in some base year. The set of sampled items and the weights are updated every few years (between 5 and 7 years for the typical good in the PPI). Price changes from product replacements tend to be dropped from the index, which is equivalent to attributing to discontinued goods the rate of change in the overall price index. For more details on the construction of producer price indices and international price indices in the US, see Chapters 14 and 15 of the BLS Handbook of Methods.

each component of nominal consumption (when physical quantities are not available) by its consumer price index (CPI) instead of the PPI. The change in the CPI is a weighted average of consumer price changes of domestic and imported goods consumed in both time periods.[9]

In the presence of import tariffs, current-dollar GDP from the expenditure side (defined as the sum of final expenditures including tariffs) is not equal to current-dollar GDP from the production side (defined as the sum of firm value added excluding tariffs). In order to reconcile estimates of GDP from the production and expenditure sides, the BEA adds import taxes to factor payments when computing value added by industry.[10] To be consistent with this procedure, in the model with tariffs we calculate real GDP from the expenditure side. In deflating consumption expenditures, the CPI is constructed using prices inclusive of tariffs. In deflating imports, the import price index (IPI) is constructed using prices exclusive of import tariffs.

3 Model with Exogenous Specialization and Perfect Competition

In this section we present an Armington version of our model with exogenous specialization and perfect competition. The extensions of the model that follow build upon this basic setup.

The world economy is composed of I countries. The utility of the representative consumer in country n is

$$U_n = \sum_{t=0}^{\infty} \beta^t u\left(C_{nt}\right),\ \beta \leq 1\ ,$$

where C_{nt} denotes *theoretical* consumption of the final good at time t, given by

$$C_{nt} = \left[\int_{\Omega_{nt}} q_{nt}\left(\omega\right)^{\frac{\rho-1}{\rho}} \mathrm{d}\omega\right]^{\frac{\rho}{\rho-1}} . \tag{3}$$

Here, $q_{nt}\left(\omega\right)$ denotes the consumption of good ω and Ω_{nt} denotes the set of available differentiated goods in country n. The parameter ρ denotes the elasticity of substitution across

[9] See *"Updated Summary of NIPA Methodologies"*, for details on the deflator used in each expenditure component of GDP. See McCully, Moyer, and Stewart (2007) for a detailed comparison of the CPI and the implicit deflator for personal consumption (where the latter is constructed as the ratio of nominal and real consumption). See Feenstra, Heston, Timmer, and Deng (2009) for a detailed discussion of the relation between real GDP from the production side and real GDP from the expenditure side as measured in the Penn World Tables.

[10] In particular, in the "Gross Domestic Product by Industry Accounts" computed by the BEA, value added is defined as the sum of: "Compensation of employees", "Taxes on production and imports less subsidies" and "Gross operating surplus". For a detailed description of the transactions that are included in value added, see "Concepts and Methods of the U.C. Input-Output Accounts", Chapter 6, under "Value-added transactions".

varieties. In the model with monopolistic competition below we assume $\rho > 1$. Demand for each good is $q_{nt}(\omega) = [p_{nt}(\omega)/P_{nt}]^{-\rho} C_{nt}$, where $p_{nt}(\omega)$ denotes the consumer price of good ω in country n, and $P_{nt} = \left[\int_{\Omega_{nt}} p_{nt}(\omega)^{1-\rho} \mathrm{d}\omega\right]^{\frac{1}{1-\rho}}$ is the welfare-based price index in country n. We assume that consumption of the final good C_{nt} and the welfare-based price index P_{nt} cannot be directly observable (or similarly, that the final good is not a physically traded commodity). If C_{nt} and P_{nt} were directly observable, then measuring the gains from trade would be straightforward.

Each producer specializes in the production of a single differentiated good. Production uses labor according to the production function $y = zl$, where y and l denote output and labor of a producer with productivity z (multiple inputs are introduced in Section 7). We denote by $M_{it}(z)$ the distribution of producers, indicating the mass of producers with productivity z in country i at time t. Given the symmetry of goods in the production function of the final good (3), we interchangeably index goods by ω, or by their productivity z and source country i. For example, $q_{int}(z)$ and $p_{int}(z)$ denote the consumption quantity and price, respectively, in country n of good produced by z producers in country i. We assume that all prices are already expressed in a common currency (which we refer to as dollars).

Goods can be internationally traded subject to a technology described below. We denote by Ω_{int} the set of producers (indexed by their productivity) from country i that sell a *positive* quantity to country n at time t. In the absence of international trade between countries i and n at time t, the set Ω_{int} is empty.

In the *model with exogenous specialization*, we assume that the distribution of producers, $M_{it}(z)$, is exogenously given and constant over time. We also assume that the set of goods that are internationally traded, Ω_{int}, is exogenously given and that, as long as there is any trade between countries i and n, it is constant over time. We do not make assumptions on how the set of goods Ω_{int} varies across destinations, hence not all goods sold domestically need to be exported, and vice-versa. For example, only goods with high productivity z might be traded. The case of $\Omega_{int} = \Omega_{iit}$ corresponds to the Armington model in which all goods are internationally traded (unless countries are in autarky).

Goods can be shipped across countries subject to iceberg variable international trade costs. In our baseline specification, we assume that international trade costs are incurred in each source country, as is typically assumed in the literature.[11] In particular, each unit of a good produced in country i with productivity z shipped to country n at time t requires $(\tau_{int} - 1)/z$ units of labor from country i, where $\tau_{int} \geq 1$ and $\tau_{iit} = 1$. International

[11] While in this formulation we assume that trade costs use factors of production in the exporting country, we can instead assume that they use factors from the importing country (or from both). This would complicate the notation without changing substantially the results.

trade services could be provided by the same producer of the good, or by some third-party intermediary.[12]

Summing-up production and shipping costs, the total amount of country i labor required to deliver a unit of country i's good in country n is τ_{int}/z. Equivalently, this technology transforms 1 unit of a good produced in country i into $1/\tau_{int}$ unit of the good for consumption in country n. Country i's resource constraint is

$$\sum_n \int_{\Omega_{int}} \tau_{int} q_{int}/z \mathrm{d}M_{it} = \bar{L}_i,$$

where $\bar{L}_i$ denotes the labor supply in country i, integrals are evaluated with respect to z, and the dependence of q_{int} on the argument z is omitted.

In the model with perfect competition, producer prices for goods manufactured in country i and sold in country n equal $\bar{p}_{int} = W_{it}/z$, where W_{it} denotes the wage in country i. Prices for the services to sell goods from country i to country n equal $\bar{p}^s_{int} = (\tau_{int} - 1) W_{it}/z$. Consumer prices in country n equal $p_{int} = \bar{p}_{int} + \bar{p}^s_{int} = \tau_{int} W_{it}/z$. Consumption expenditures at final prices in country n are given by $E_{nt} = P_{nt}C_{nt} = \sum_n \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it}$. GDP in current dollars from the production side (the sum of value added across all producers), is equal to GDP from the income side (total wage payments plus profits), and to GDP from the expenditure side (consumption expenditures plus exports less imports). This three-way equivalence can be expressed as:

$$\begin{aligned} GDP_{it} &= \sum_n \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it} = W_{it}\bar{L}_i + \Pi_{it} \\ &= E_{it} + \sum_{n \neq i} \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it} - \sum_{n \neq i} \int_{\Omega_{nit}} p_{nit} q_{nit} \mathrm{d}M_{nt}, \end{aligned} \tag{4}$$

The variable Π_{it} denotes aggregate profits, which equal zero under perfect competition and constant returns to scale.

We denote by λ_{int} the share of country i's GDP accounted for by production sold to country n,

$$\lambda_{int} = \frac{GDP_{int}}{GDP_{it}}, \tag{5}$$

where $GDP_{int} = \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it}$. Note that $1 - \lambda_{iit}$ indicates the share of total exports in

[12] The assumption of iceberg variable international trade costs implies that producers that are more efficient at production are also more efficient at selling goods abroad. Consider an alternative formulation of the model in which goods vary by quality (as discussed in Section 7) instead of productivity. If production of higher quality goods entail higher marginal costs, the assumption of iceberg trade costs implies that higher quality goods are more expensive to sell abroad.

country i's GDP.

4 Results: Exogenous Specialization and Perfect Competition

In this section, we present our results in the basic model with exogenous specialization and perfect competition. We first calculate changes in real GDP in response to changes in variable trade costs. We then show how changes in real GDP vary if we assume that the production of international trade services is specialized in one country. We then compare changes in real consumption and theoretical consumption, and next compare changes in world real GDP and world real consumption. Finally, we calculate the response of real GDP and consumption to changes in tariffs. We conclude this section by summarizing the results.

Real GDP

We first construct real GDP from the production side. In order to apply expressions (1) and (2), we must specify how goods are grouped into components of GDP. We consider two cases. First, we aggregate production by all producers to all destinations into a single component, and construct real quantities by deflating current-dollar GDP using a single, aggregate deflator. Second, we decompose total production by destination country, and calculate real quantities by deflating destination-specific production values using destination-specific price indices.[13] We show that real GDP under the second case is equal to real GDP constructed using the direct valuation method in which data on physical quantities and prices of individual producers (i.e. q_{int}, p_{int}) is used.

Real GDP using aggregate deflators: We construct real quantities by deflating the total current-dollar value of production with the aggregate PPI. The PPI is a weighted average of changes in producer prices of continuing goods, based on production weights in period t_0. We do not make assumptions on what the base-year t_0 is or how frequently it is updated with the exit of existing products or the entry of new products. The PPI in country i in

[13] In both cases, in defining these detailed components of GDP, we are implicitly assuming that in the model there is a representative sector or industry composed of differentiated goods which aggregate according to (3). Extending the model to allow for heterogeneous industries or sectors, aggregated into the final good with an outer CES technology, is straighforward at the expense of extra notation, and does not substantially alter our results.

period t relative to period $t-1$ is given by[14]

$$\frac{PPI_{it}}{PPI_{it-1}} = \frac{\sum_n \int_{\Omega^c_{int}} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}}\right) \mathrm{d}M_{it_0}}{\sum_n \int_{\Omega^c_{int}} p_{int_0} q_{int_0} \, \mathrm{d}M_{it_0}} = \sum_n \bar{\lambda}_{int} \frac{\tau_{int}}{\tau_{int-1}} \frac{W_{it}}{W_{it-1}}, \tag{6}$$

where $\Omega^c_{int} = \Omega_{int_0} \cap \Omega_{int-1} \cap \Omega_{int}$ is the set of goods sold from country i to country n with positive sales at time t_0, $t-1$ and t, and $\bar{\lambda}_{int}$ is the share of country i's revenues to country n at time t_0 of these continuing goods,

$$\bar{\lambda}_{int} = \frac{\int_{\Omega^c_{int}} p_{int_0} q_{int_0} \mathrm{d}M_{it_0}}{\sum_n \int_{\Omega^c_{int}} p_{int_0} q_{int_0} \mathrm{d}M_{it_0}}. \tag{7}$$

In deriving (6), we have used the fact that, with iceberg variable trade costs, the percentage change in prices is independent of productivity z.

Note that, if countries i, n do not trade at time t_0, $t-1$ or t, then $\Omega^c_{int} = \varnothing$ and the PPI excludes price changes from this pair of countries. Hence, if a country is in autarky at time t_0, $t-1$ or t then the PPI only takes into account changes in domestic prices.

Real GDP in period t relative to period $t-1$, using expression (1) with a single aggregate component, is given by

$$\begin{aligned} \frac{RGDP_{it}}{RGDP_{it-1}} &= \left(\frac{GDP_{it}/\left(PPI_{it}/PPI_{it-1}\right)}{GDP_{it-1}}\right)^{0.5} \left(\frac{GDP_{it}}{GDP_{it-1} \times \left(PPI_{it}/PPI_{it-1}\right)}\right)^{0.5} \\ &= \frac{1}{\sum_n \frac{\tau_{int}}{\tau_{int-1}} \bar{\lambda}_{int}}. \end{aligned} \tag{8}$$

Note from (8) that, using a single aggregate deflator, the Laspeyres and the Paasche quantity indices between periods $t-1$ and t are equal.

From expression (8), we can see that if the share of exports in GDP is positive at times t_0, $t-1$ and t (i.e. $\sum_{n \neq i} \bar{\lambda}_{int} > 0$) and variable trade costs in country i fall between time $t-1$ and time t (i.e. $\tau_{int} \leq \tau_{int-1}$ for $n \neq i$ with at least one strict inequality), then real GDP rises.

If trade costs are unchanged between any two consecutive periods, $\tau_{int} = \tau_{int-1}$, then real GDP remains unchanged. Therefore, if trade costs change permanently between $t = 0$ and $t = 1$, then chained real GDP in any period $T \geq 1$ relative to period $t = 0$ (using expression

[14] Here we are assuming that producer prices in the PPI are the sum of manufacturing and shipping prices, $\bar{p}_{int}$ and $\bar{p}^s_{int}$ respectively. Alternatively, we could assume that producer prices and shipping prices are entered separately instead of summed into the PPI (because these activities are performed by distinct producers or industries). The PPIs under both assumptions are equivalent up to a first-order approximation.

2) is given by

$$\frac{RGDP_{iT}}{RGDP_{i0}} = \frac{RGDP_{i1}}{RGDP_{i0}}. \tag{9}$$

Intuitively, a reduction in variable trade costs entails an improvement of domestic technologies, which lowers producer prices relative to the wage, and increases real GDP. This rise in real GDP shows up as a rise in aggregate productivity. If the PPI does not take into account changes in trade costs (or if a country is initially in autarky), then $PPI_{int} = W_{it}/W_{it_0}$, and $RGDP_t/RGDP_{t-1} = 1$.

Note that any reallocation in production towards more productive producers (due to, for example, a higher productivity of exporters relative to non-exporters) does not result in larger changes in measured aggregate productivity. To understand this implication of the model, we can rewrite the ratio of real GDP in period t to relative to period $t-1$, using (4) and (8), as

$$\frac{RGDP_{it}}{RGDP_{it-1}} = \frac{\sum_n \int_{\Omega_{int}} \frac{l_{int}}{L_{it}} \times \frac{p_{int} q_{int}}{l_{int}} \mathrm{d}M_{it}}{\sum_n \int_{\Omega_{int-1}} \frac{l_{int-1}}{L_{it-1}} \times \frac{p_{int-1} q_{int-1}}{l_{int-1}} \mathrm{d}M_{it-1}} \frac{L_{it}}{L_{it-1}} \frac{1}{PPI_{it}/PPI_{it-1}}, \tag{10}$$

where $l_{int}(z)$ denotes production labor used by country i producers with productivity z to sell in country n, and L_{it} denotes the aggregate quantity of labor used for production in country i (equal to $\bar{L}_i$ in this model). Note that value added per worker by individual producers, $p_{int}(z)\, q_{int}(z)/l_{int}(z)$, is equal to the wage, W_{it}, for all producers independent of their productivity z. Using $\sum_n \int_{\Omega_{int}} \frac{l_{int}}{L_{it}} \mathrm{d}M_{it} = 1$ and (6) we obtain expression (8). Therefore, any reallocation of labor towards more productive producers does not result in any further increase of aggregate productivity beyond the direct effect from a reduction in variable trade costs.[15]

Real GDP using disaggregated deflators: We now compute real quantities by deflating destination-specific production values using destination-specific PPIs. The PPI in period t relative to period $t-1$ for goods produced in country i and shipped to country n is

$$\frac{PPI_{int}}{PPI_{int-1}} = \frac{\int_{\Omega^c_{int}} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}}\right) \mathrm{d}M_{it_0}}{\int_{\Omega^c_{int}} p_{int_0} q_{int_0} \mathrm{d}M_{it_0}} = \frac{\tau_{int}}{\tau_{int-1}} \frac{W_{it}}{W_{it-1}}. \tag{11}$$

Here we used the fact that percentage changes in producer prices are equal for all goods bound to a given destination. The disaggregated deflator PPI_{int}/PPI_{int-1} is well defined only when the set of continuing goods is non-empty.

[15] Note that if the PPI were calculated as a change in average prices (instead of an average change in prices), then reallocation of production towards more productive producers would result in a larger decline in the PPI and a higher increase in real GDP.

Real GDP in period t relative to period $t-1$, using equation (1) with destination-specific GDP components, is given by

$$
\begin{aligned}
\frac{RGDP_{it}}{RGDP_{it-1}} &= \left(\frac{\sum_n \frac{GDP_{int}}{PPI_{int}/PPI_{int-1}}}{\sum_n GDP_{int-1}}\right)^{0.5} \left(\frac{\sum_n GDP_{int}}{\sum_n GDP_{int-1} \times PPI_{int}/PPI_{int-1}}\right)^{0.5} \\
&= \left(\frac{\sum_n \lambda_{int} \frac{\tau_{int-1}}{\tau_{int}}}{\sum_n \lambda_{int-1} \frac{\tau_{int}}{\tau_{int-1}}}\right)^{0.5}.
\end{aligned} \tag{12}
$$

Note that, in contrast to the measures of real GDP based on aggregate deflators, the Laspeyres and the Paasche quantity indices of real GDP are not equal when we use disaggregated deflators. From expression (12), if trade costs fall between time $t-1$ and t, then real GDP rises. If trade costs change permanently between $t=0$ and $t=1$, then chained real GDP in any period $T \geq 1$ relative to period $t=0$, is equal to $RGDP_{i1}/RGDP_{i0}$, as in expression (9).

The expressions for changes in real GDP based on aggregated and disaggregated deflators, given by (8) and (12), differ in terms of the base-year in which trade shares are calculated. However, up to a first-order approximation (i.e. around $\tau_{int}/\tau_{int-1} \simeq 1$), the two measures of changes in real GDP are equal and given by

$$
d \log RGDP_{it} = d \log W_{it} - \sum_n \lambda_{int} d \log PPI_{int} = - \sum_n \lambda_{int} d \log \tau_{int}. \tag{13}
$$

Note that we can re-write the change in real GDP based on disaggregated deflators in the first line of expression (12) as

$$
\frac{RGDP_{it}}{RGDP_{it-1}} = \left(\frac{\sum_n \int_{\Omega_{int}} p_{int-1} q_{int} \mathrm{d}M_{it}}{\sum_n \int_{\Omega_{int-1}} p_{int-1} q_{int-1} \mathrm{d}M_{it-1}}\right)^{0.5} \left(\frac{\sum_n \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it}}{\sum_n \int_{\Omega_{int-1}} p_{int} q_{int-1} \mathrm{d}M_{it-1}}\right)^{0.5}.
$$

Here we used the fact that the term $GDP_{int}/\left(PPI_{int}/PPI_{int-1}\right)$ in (12) is equal to $\sum_n \int_{\Omega_{int}} p_{int-1} q_{int} \mathrm{d}M_{it}$, and the term $GDP_{int-1} \times PPI_{int}/PPI_{int-1}$ is equal to $\sum_n \int_{\Omega_{int-1}} p_{int} q_{int-1} \mathrm{d}M_{it-1}$. This expression corresponds to the change in real GDP calculated according to the direct valuation method (a geometric average of Laspeyres and Paasche quantity indices), evaluating using production values at constant prices. While this procedure requires data on physical quantities and prices of individual commodities (which is typically not available in many industries and subset of goods), what we showed is that the implied change in real GDP is equal to that using the deflation method with country specific price deflators.

Real GDP under international specialization of shipping services

We now consider an alternative specification on the nature of trade costs in which a subset of countries specializes in producing shipping services for all other countries. For concreteness (but without loss of generality for our results), we assume that the world-wide production of shipping services is concentrated in country i_s. That is, shipping one unit of a good produced in country i with productivity z to country n at time t, for all countries i and n, requires $(\tau_{nit} - 1)/z$ units of *country* i_s*'s labor*. Prices received by producers of these services in country i_s for the services to sell goods from country i to country n equal $\bar{p}^s_{int} = (\tau_{int} - 1) W_{i_s t}/z$, and consumer prices in country n equal $p_{int} = (W_{it} + (\tau_{int} - 1) W_{i_s t})/z$.

The resource constraint in country i is

$$\sum_n \int_{\Omega_{int}} q_{int}/z \; \mathrm{d}M_{it} = \bar{L}_i \text{ for } i \neq i_s,$$

while in country i_s it is

$$\sum_n \int_{\Omega_{i_s nt}} \tau_{i_s nt} q_{i_s nt}/z \; \mathrm{d}M_{i_s t} + \sum_{i \neq i_s} \sum_n \int_{\Omega_{int}} (\tau_{int} - 1) q_{int}/z \; \mathrm{d}M_{it} = \bar{L}_{i_s}.$$

GDP in current dollars from the production side in country $i \neq i_s$ is $GDP_{it} = \sum_n \int_{\Omega_{int}} \bar{p}_{int} q_{int} \mathrm{d}M_{it}$, while in country $i = i_s$ it is

$$GDP_{i_s t} = \sum_n \int_{\Omega_{i_s nt}} \left(\bar{p}_{i_s nt} + \bar{p}^s_{i_s nt}\right) q_{i_s nt} \mathrm{d}M_{i_s t} + \sum_{i \neq i_s} \sum_n \int_{\Omega_{int}} \bar{p}^s_{int} q_{int} \mathrm{d}M_{it}.$$

In countries $i \neq i_s$ that do not specialize in shipping services, the PPI is simply $PPI_{it}/PPI_{it-1} = W_{it}/W_{it-1}$, and the ratio of real GDP in periods t and $t-1$ is $RGDP_{it}/RGDP_{it-1} = 1$.

In contrast to the previous specification, changes in trade costs now leave domestic technologies unchanged for those countries that do not specialize in shipping services. Hence, in these countries changes in the PPI are equal to changes in the wage, so real GDP remains unchanged. This result holds more generally, if changes in foreign trade costs also change relative prices faced by different domestic producers (as in the Hecksher-Ohlin model, for example) but not their technologies. To see this, recall that when calculating real quantities by deflating the value of production with destination specific PPIs, real GDP is equal to the value of production evaluated at constant-year prices. From revealed production choices, the value of production falls (rises) between $t-1$ and t when evaluated at $t-1$ (t) prices. To a first-order approximation, real GDP remains constant. This line-of-argument cannot be used when changes in trade costs change domestic technologies, as in our baseline specification.

In Appendix B we derive the change in real GDP in country i_s that specializes in the production of shipping services. Reductions in trade costs (across any pair of countries) do improve domestic technologies in country i_s. Hence, in response to any reduction in trade costs, the PPI falls relative to the wage, and real GDP rises.

Measured real consumption and theoretical consumption in each country

We now calculate changes in real consumption, and compare them to changes in theoretical (or welfare-based) consumption of individual countries. As we did for real GDP, we use the deflation method, first using an aggregate deflator and then using country-specific deflators.

Real consumption using aggregate deflators: We calculate real consumption using the deflation method, deflating consumption expenditures with a consumer price index (CPI). We construct the CPI as a weighted average of ratios of final prices between two periods (of goods that are consumed in both periods) using t_0 weights. The CPI in country n at time t relative to time $t-1$ is given by

$$\frac{CPI_{nt}}{CPI_{nt-1}} = \frac{\sum_i \int_{\Omega_{int}^c} \left(p_{int_0} q_{int_0}\right) \left(\frac{p_{int}}{p_{int-1}}\right) \mathrm{d}M_{it_0}}{\sum_i \int_{\Omega_{int}^c} p_{int_0} q_{int_0} \mathrm{d}M_{it_0}}. \tag{14}$$

Real consumption in country n at time t relative to $t-1$, using expression (1) with a single aggregate component, is given by

$$\begin{aligned} \frac{RC_{nt}}{RC_{nt-1}} &= \left(\frac{E_{nt}/\left(CPI_{nt}/CPI_{nt-1}\right)}{E_{nt-1}}\right)^{0.5} \left(\frac{E_{nt}}{E_{nt-1} \times \left(CPI_{nt}/CPI_{nt-1}\right)}\right)^{0.5} \\ &= \frac{E_{nt}/\left(CPI_{nt}/CPI_{nt-1}\right)}{E_{nt-1}}. \end{aligned} \tag{15}$$

The ratio of theoretical consumption in periods t and $t-1$ is equal to $C_{nt}/C_{nt-1} = (E_{nt}/E_{nt-1})/(P_{nt}/P_{nt-1})$, where P_{nt} is the welfare-based price index defined above. Hence, differences between changes in real consumption and theoretical consumption stem only from differences between the CPI and the theoretical CES price index. It is straightforward to show that, to a first-order approximation, the log change in the CES price index with a fixed set of goods is equal to an expenditure-weighted average of log price changes of individual goods, as it is for the CPI defined in expression (14). Hence, for marginal changes in prices, changes in real consumption coincide with changes in theoretical consumption, country-by-country.

Large changes in prices give rise to the well-known substitution bias. However, if the CPI is evaluated using $t_0 = t-1$ or $t_0 = t$ weights, we can bound this substitution bias. In particular, in Appendix A we show that, if the set of goods consumed in each period is unchanged, then the CPI with initial (final) period weights, $t_0 = t-1$ ($t_0 = t$) overstates (understates) changes in the welfare-based price index between periods $t-1$ and t. That is,

$$\left.\frac{CPI_{nt}}{CPI_{nt-1}}\right|_{t_0=t} \leq \frac{P_{nt}}{P_{nt-1}} \leq \left.\frac{CPI_{nt}}{CPI_{nt-1}}\right|_{t_0=t-1}. \tag{16}$$

Hence, changes in theoretical consumption are bounded above (below) by real-consumption calculated with the CPI based on final (initial) period weights.[16] These results hold under both specifications of international trade costs.

Real consumption using disaggregated deflators: We now calculate changes in real consumption by deflating source–country specific consumption expenditures using their respective CPIs. Country specific expenditures and CPIs are defined, respectively, as $E_{int} = \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it}$ and

$$\frac{CPI_{int}}{CPI_{int-1}} = \frac{\int_{\Omega^c_{int}} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}}\right) \mathrm{d}M_{it_0}}{\int_{\Omega^c_{int}} p_{int_0} q_{int_0} \mathrm{d}M_{it_0}}.$$

Real consumption in period t relative to period $t-1$, using equation (1) with country-specific expenditure components, is given by

$$\begin{aligned} \frac{RC_{nt}}{RC_{nt-1}} &= \left(\frac{\sum_i \frac{E_{int}}{CPI_{int}/CPI_{int-1}}}{\sum_i E_{int-1}}\right)^{0.5} \left(\frac{\sum_i E_{int}}{\sum_i E_{int-1} \times CPI_{int}/CPI_{int-1}}\right)^{0.5} \\ &= \left(\frac{\sum_i \int_{\Omega_{int}} p_{int-1} q_{int} \mathrm{d}M_{it}}{\sum_i \int_{\Omega_{int-1}} p_{int-1} q_{int-1} \mathrm{d}M_{it-1}}\right)^{0.5} \left(\frac{\sum_i \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it}}{\sum_i \int_{\Omega_{int-1}} p_{int} q_{int-1} \mathrm{d}M_{it-1}}\right)^{0.5}, \end{aligned} \tag{17}$$

where we used the fact that percentage changes in all prices for goods coming from a common source country are equal. This expression coincides with a geometric average of Laspeyres and Paasche quantity indices using the direct-valuation method.

Up to a first-order approximation, changes in real consumption based on aggregate deflators and disaggregated deflators (as well as theoretical consumption) are equal and given

[16] Inequality (16) does not hold in the case in which there is no trade between countries i and n at time $t-1$ or t, so that $\Omega^c_{int} = \varnothing$. In this case, inequality (16) would hold if the CPI to incorporated price changes for all goods, including those that are not consumed, and assumed that unavailable goods have an infinite price. However, this is not the approach taken by the national statistics when calculating the CPI.

by

$$d \log RC_{nt} = d \log E_{nt} - \sum_i \frac{E_{int}}{E_{nt}} d \log CPI_{int}. \tag{18}$$

World real GDP, consumption, and theoretical consumption

As can be observed by comparing expressions (13) and (18), differences between changes in real GDP and real consumption, country-by-country, arise from (1) differences between the current-dollar value of consumption and GDP (in the presence of trade imbalances) and (2) differences between changes in the PPI and in the CPI due to movements in relative wages and relative trade costs that change the price of exports relative to imports (i.e. the terms of trade) in each country.

We now show that, if trade is balanced in each country, a weighted-average (based on each country's current-dollar GDP) of changes in real consumption across countries is equal to the same weighted average of changes in real GDP across countries, up to a first-order approximation. Here we consider the baseline specification of trade costs in which these are incurred using labor in each exporting country. In the Appendix we show that the equivalence between changes in world real GDP and world real consumption also holds in the model in which a subset of countries specializes in the production of shipping services.

Define s_{it} to be country i's share in total current-dollar GDP across all countries in period t: $s_{it} = GDP_{it} / \sum_i GDP_{it}$. From expression (13), the world change in real GDP is, to a first-order approximation,

$$\sum_i s_{it} d \log RGDP_{it} = \frac{1}{\sum_i GDP_{it}} \left[\sum_i GDP_{it} d \log W_{it} - \sum_i \sum_n GDP_{int} d \log PPI_{int} \right]. \tag{19}$$

From expression (18), assuming balanced trade in each country (so that, as can be seen in 4, GDP and expenditures in current-dollars are equal, $GDP_{it} = E_{it}$), the world change in real consumption is, to a first-order approximation,

$$\sum_i s_{it} d \log RC_{it} = \frac{1}{\sum_i GDP_{it}} \left[\sum_i GDP_{it} d \log GDP_{it} - \sum_i \sum_n E_{nit} d \log CPI_{nit} \right]. \tag{20}$$

The first term in expression (19) is equal to the first term in expression (20) because $d \log W_{it} = d \log GDP_{it}$. The second term in expression (19) is equal to the second term in expression (20) because $GDP_{int} d \log PPI_{int} = E_{int} d \log CPI_{int}$. Intuitively, for any pair of trading countries, an improvement in the bilateral terms of trade for one country implies a worsening in the terms of trade for the other country. Hence, changes in the world CPI are equal to changes in the world PPI, and so are world real consumption and world real

GDP.[17]

Note that, from our results on the equality of changes in real consumption and theoretical consumption country-by-country, it follows immediately that changes in world real GDP and changes in world real consumption are both equal, to a first-order approximation, to a weighted average of the change in theoretical consumption across countries:

$$\sum_i s_{it} d \log RGDP_{it} = \sum_i s_{it} d \log RC_{it} = \sum_i s_{it} d \log C_{it}. \tag{21}$$

We can solve explicitly for the change in world real GDP and world real consumption in response to changes in variable trade costs. In particular, from (13) and (21), it follows that, to a first-order approximation,

$$\sum_i s_{it} d \log RGDP_{it} = \sum_i s_{it} d \log RC_{it} = -\frac{1}{\sum_i GDP_{it}} \times \sum_i \sum_n \text{Exports}_{int} \times d \log \tau_{in}, \tag{22}$$

where $\text{Exports}_{int} = \int_{\Omega_{int}} (\bar{p}_{int} + \bar{p}^s_{int}) \, q_{int} \mathrm{d}M_{it}$. Changes in world real GDP and real consumption in response to changes in variable trade costs are, to a first-order approximation, equal to a weighted average of changes in bilateral variable trade costs, where the weights are simply the shares of bilateral exports in world GDP.

Tariffs and real GDP from the expenditure side

We now introduce ad-valorem import tariffs. We denote by $d_{int} \geq 1$ the gross tariff set by country n at time t for imports from country i (with $d_{iit} = 1$). Consumer prices in country n are $p_{int} = d_{int} (\bar{p}_{int} + \bar{p}^s_{int})$. Tariffs revenues are rebated back to consumers. To simplify the notation, we calculate our aggregate statistics only for the case in which trade costs are incurred in each exporting country, but it is straightforward to extend the results to the case in which country i_s specializes in the production of shipping services.

The local equivalence, country-by-country, between changes in real consumption and theoretical consumption is immediate because, to a first-order approximation, the CPI is equal to the welfare-based price index (both of which are calculated using final prices inclusive of import tariffs). For large price changes, we still obtain the bound stated in inequality (16).

The relation between current-dollar GDP from the production side and current-dollar GDP from the expenditure side, provided in (4), must be modified by the presence of tariffs.

[17] The equivalence between world changes in real GDP and real consumption also holds for large changes in trade costs if real GDP and real consumption are calculated using either Laspeyres or Paasche quantity indices (instead of using a geometric average of both, as stated in expression 1) based on disaggregated deflators.

For example, in country $i \neq i_s$, we have

$$\begin{aligned} GDP_{it} &= \sum_n \int_{\Omega_{int}} \left(\bar{p}_{int} + \bar{p}^s_{int}\right) q_{int} \mathrm{d}M_{it} + \Upsilon_{it} = W_{it}\bar{L}_i + \Pi_{it} + \Upsilon_{it} \\ &= E_{it} + \sum_{n \neq i} \int_{\Omega_{int}} \left(\bar{p}_{int} + \bar{p}^s_{int}\right) q_{int} \mathrm{d}M_{it} - \sum_{n \neq i} \int_{\Omega_{nit}} \left(\bar{p}_{nit} + \bar{p}^s_{nit}\right) q_{nit} \mathrm{d}M_{nt}, \end{aligned} \tag{23}$$

where $\Upsilon_{it} = \sum_n (d_{nit} - 1) \int_{\Omega_{nit}} (\bar{p}_{nit} + \bar{p}^s_{nit}) q_{nit} dM_{nt}$ denotes tariff revenues collected in country i.

Real GDP calculated from the production side excluding import tariffs from both current-dollar GDP and from price deflators is unchanged to changes in tariffs for the same reasons that real GDP in the model with only trade costs is unchanged to changes in trade costs if these are excluded from price indices.

We now calculate real GDP from the expenditure side by separately deflating each country-specific expenditure component of GDP. The export price index (EPI) for goods sold by country i to country n is given by

$$\frac{EPI_{int}}{EPI_{int-1}} = \frac{\int_{\Omega^c_{int}} \left(\bar{p}_{int_0} + \bar{p}^s_{int_0}\right) q_{int_0} \left(\frac{\bar{p}_{int} + \bar{p}^s_{int}}{\bar{p}_{int-1} + \bar{p}^s_{int-1}}\right) \mathrm{d}M_{it_0}}{\int_{\Omega^c_{int}} \left(\bar{p}_{int_0} + \bar{p}^s_{int_0}\right) q_{int_0} \mathrm{d}M_{it_0}}. \tag{24}$$

The imports price index (IPI) in country i for goods imported from country n (inclusive of trade costs incurred abroad but exclusive of tariffs) is given by $IPI_{nit}/IPI_{nit-1} = EPI_{nit}/EPI_{nit-1}$.

The Laspeyres real GDP index in country i is given by

$$\frac{RGDP_{it}}{RGDP_{it-1}} = \frac{\sum_n \frac{E_{nit}}{CPI_{nit}/CPI_{nit-1}} + \sum_{n \neq i} \left[\frac{\text{Exports}_{int}}{EPI_{int}/EPI_{int-1}} - \frac{\text{Exports}_{nit}}{EPI_{nit}/EPI_{nit-1}}\right]}{GDP_{it-1}} = \tag{25}$$

$$= \frac{\sum_n \int_{\Omega_{int}} \left(\bar{p}_{int-1} + \bar{p}^s_{int-1}\right) q_{int} \mathrm{d}M_{it} + \sum_n \int_{\Omega_{nit}} \left(d_{nit-1} - 1\right) \left(\bar{p}_{nit-1} + \bar{p}^s_{nit-1}\right) q_{nit} \mathrm{d}M_{nt}}{\sum_n \int_{\Omega_{int-1}} \left(\bar{p}_{int-1} + \bar{p}^s_{int-1}\right) q_{int-1} \mathrm{d}M_{it-1} + \sum_n \int_{\Omega_{nit-1}} \left(d_{nit-1} - 1\right) \left(\bar{p}_{nit-1} + \bar{p}^s_{nit-1}\right) q_{nit-1} \mathrm{d}M_{nt-1}}.$$

The first term in expression (25) indicates the change in the constant-price value of production, and the second term represents the change in the constant price value of tariffs. The Paasche real GDP index is calculated analogously to the Laspeyres real GDP index, but using constant period t prices and tariffs instead of period $t-1$ prices and tariffs. The change in real GDP between period $t-1$ and t is a geometric average of the Laspeyres and Paasche indices, as defined in expression (1).

Note that, in the absence of tariffs, real GDP from the expenditure side coincides with real GDP from the production side using disaggregated deflators.[18] In the presence of tariffs, there is an additional source of changes in real GDP. Specifically, real GDP rises if the value of tariff revenues evaluated at base-prices and base-tariffs, $\sum_{n\neq i}\int_{\Omega_{nit}}(d_{nit_0}-1)\left(\bar{p}_{nit_0}+\bar{p}^s_{nit_0}\right)q_{nit}\mathrm{d}M_{nt}$ (with $t_0=t-1$ or $t_0=t$), increases. That is, real GDP rises if imported physical quantities weakly increase.

Finally, consider the equivalence between world real consumption and world real GDP. Suppose that each country is under balanced trade (exclusive of tariffs), i.e. $\sum_n\int_{\Omega_{int}}(\bar{p}_{int}+\bar{p}^s_{int})\,q_{int}\mathrm{d}M_{it}=\sum_n\int_{\Omega_{nit}}(\bar{p}_{nit}+\bar{p}^s_{nit})\,q_{nit}\mathrm{d}M_{nt}$. In this case, from (23), current-dollar GDP (inclusive of import-tariffs) is equal to current-dollar expenditures. Define country-specific weights based on current-dollar GDP (inclusive of import-tariffs), $s_{it}=GDP_{it}/\sum_i GDP_{it}$. It is straightforward to show, following the steps above in the model without tariffs, that the change in world real GDP is equal, to a first-order approximation, to the world change in real consumption, as indicated in expression (21).

Summary of Results

Our central results on the implications of changes in trade costs on measures of real GDP and real consumption in our model with exogenous specialization and perfect competition can be summarized as follows:

Result 1: *In response to reductions in variable international trade costs incurred in country i that are captured in GDP and its deflators, real GDP in country i rises. If changes in variable international trade costs are not captured in country i's GDP nor its deflators (either because producer prices in price indices exclude trade costs, or because country i starts in autarky, or because international trade services are produced in other countries), real GDP in country i is unchanged;*

Result 2: *In response to changes in physical trade costs or tariffs, the change in theoretical (welfare-based) consumption in each country lies between the changes in real consumption calculated using consumption deflators with pre- and post-trade liberalization base-year weights. To a first-order approximation, changes in real consumption and in theoretical consumption coincide country-by-country;*

Result 3: *In response to changes in import tariffs that raise the value of country i's tariff revenues at constant prices, real GDP from the expenditure side in country i rises;*

Result 4: *With balanced trade in each country, the change in world real consumption is equal, up to a first-order approximation, to the change in world real GDP (defined as*

[18] If we use single aggregate deflators, real GDP from the production and from the expenditure side are equal up to a first-order approximation.

cross-country weighted averages of changes in real consumption and GDP, respectively, using current-dollar GDP weights).

Combining Results 2 and 4, we obtain the corollary that if each country is under balanced trade, changes in world real GDP equal, to a first-order approximation, changes in world theoretical consumption, independently of where are trade services produced.

5 Endogenous Specialization and Perfect Competition

In the model studied in the previous section, we assumed that the sets Ω_{int}, indicating the range of goods that are produced and sold in each country, were exogenously given. In this section, we briefly discuss how our previous results hold in a model that endogeneizes the set of traded goods, while keeping the assumption of perfect competition. Specifically, we consider a Ricardian version of our model, as in e.g. Dornbusch, Fisher and Samuelson (1977) and Eaton and Kortum (2001).

Instead of assuming that each country produces its own differentiated goods, we assume that every good ω can be produced by all countries. To incorporate this assumption in our general framework, the notation must be slightly modified as follows (see e.g. Alvarez and Lucas 2007). Each good is indexed by the vector z of productivities for this good in all countries, and $M(z)$ denotes the exogenous distribution of goods in the world. We do not make any parametric assumptions on $M(z)$. Every period, countries purchase each good from the source country with lowest marginal cost of delivering the good. These sourcing choices determine the sets Ω_{int}. With perfect competition, the final price of good z in country n is $p_{nt}(z) = \min_i \{\bar{p}_{int}(z) + \bar{p}^s_{int}(z)\}$, where $\bar{p}_{int}(z)$ and $\bar{p}^s_{int}(z)$ are equal to the marginal cost to produce and deliver, respectively, good $\mathbf{z}$ from country i to n. We focus on the specification in which trade costs are incurred in each exporting country, but the results extend to the specification in which the production of shipping services is concentrated in a subset of countries.

Real GDP

In constructing the PPI and EPI, goods for which the identity of the producer changes over time are discontinued and hence are not included in the respective price index, as can be seen in expressions (6) and (24) with M_{it} substituted for M. All continuing producers included in the price index (i.e. those in the set Ω^c_{int}) change prices by the same percentage. Following the steps used above, we obtain the same expressions for the change in real GDP (based on aggregate deflators) as in (8). Hence, Result 1 remains unchanged.

Note that, while the expression for changes in real GDP is the same in the model with endogenous and in the model with exogenous specialization, the actual change in real GDP in both models can differ, for given levels of trade shares (λ_{int-1} and λ_{int}), and for given changes in trade costs (τ_{int}/τ_{int-1}). This is because changes in real GDP depend on trade shares for continuing producers $\bar{\lambda}_{int}$, which can differ from overall trade shares λ_{int} in the presence of switching in the country of origin of individual products.

The measures of real GDP based on country-specific deflators are derived in exactly the same form as in the model with exogenous specialization. Changes in real GDP are again given by expression (12), and are unaffected by the extent of changes over time in the source country of producers (as long as they are well defined in the sense that there is a non-zero mass of continuing producers).[19]

Real GDP using aggregate and disaggregated deflators now differ not only in terms of the base-year in which trade shares are calculated (as in the model with exogenous specialization), but also because the former uses trade shares for continuing producers ($\bar{\lambda}_{int}$) while the latter uses trade shares for all producers (λ_{int-1} and λ_{int}). For marginal changes in trade costs ($\tau_{int}/\tau_{int-1} \simeq 1$), however, differences between the measures $\bar{\lambda}_{int}$, λ_{int-1} and λ_{int} have no first-order effects on real GDP (we establish this formally in the proof of Result 5 in the Appendix). Therefore, changes in real GDP based on aggregate and disaggregate deflators are equal and given by expression (13).

Establishing Results 3 in the model with endogenous specialization is straightforward since it was derived above using measures of real GDP based on disaggregated deflators, which are equivalent in the two models. Establishing Result 4 in this model is also straightforward since it was derived above using first-order changes in real GDP and real consumption, each of which is equal in the two models.

Real consumption

Constructing the CPI is straightforward since all goods in Ω are consumed every period. If the identity of the producer selling any given good in a particular country changes over time, we substitute the price charged by the new producer for that of the old (using the logic that the BLS looks for close substitutes if the original good is not available). That is, the CPI between periods $t-1$ and t is given by

$$\frac{CPI_{nt}}{CPI_{nt-1}} = \frac{\int_{\Omega} \left(p_{nt_0} q_{nt_0}\right) \left(\frac{p_{nt}}{p_{nt-1}}\right) \mathrm{d}M}{\int_{\Omega} p_{nt_0} q_{nt_0} \mathrm{d}M}. \tag{26}$$

Given that all good are consumed every period, even under autarky, Result 2 on the local

[19] To obtain an equivalence between real GDP using disaggregated deflators and real GDP calculated using the direct valuation method, we must assume that the imputed price change for newly produced (or exported) goods in a country is equal to the change in the country-specific PPI.

equivalence between real consumption and theoretical consumption applies immediately, and the counterpart of inequality (16) holds even if a country starts in autarky.

6 Endogenous Specialization and Monopolistic Competition

In this section we return to our baseline model with product differentiation, with the following two modifications. First, we assume monopolistic competition. In particular, each good is produced by a single producer that, with our CES demand, sets price as a constant markup $\rho/(\rho-1)$ over marginal cost. Assuming that iceberg trade costs τ_{int} are incurred by the producers in their home country, and abstracting from tariffs, producer prices and final prices of goods with productivity z produced in country i and sold in country n are[20]

$$\bar{p}_{in}(z)+\bar{p}^{s}_{in}(z)=p_{in}(z)=\frac{\rho}{\rho-1}\frac{W_{it}\tau_{int}}{z}. \tag{27}$$

Second, we endogeneize the distribution of producers $M_{it}(z)$ in country i, and the set of producers (indexed by their productivity z) from country i that sell in country n, Ω_{int}. To do so, we modify the technology as follows. In addition to iceberg variable trade costs, we assume that producers from country i are subject to fixed labor costs f_{int} when selling any positive amount in country n. In our baseline model, we assume that these fixed labor costs are incurred in the home country. We also consider an extension in which they are incurred in the importing country.

Every period there is an unbounded mass of potential entrants that can pay a fixed cost f_{Ei} to enter and produce a differentiated good. A measure M_{Eit} of new producers enter with a given productivity level z that remains constant throughout their life. The initial productivity is drawn from the distribution $G_i(z)$. For some of our results, we assume that $G_i(z)$ is Pareto.

Every period, producers die with probability $\delta>0$. The distribution of producers in country i, $M_{it}(z)$, is determined by the mass of entrants, exit decisions, and the death rate. The free-entry condition implies that expected discounted profits at entry (including the fixed cost of entry) are non-positive. We assume that each period the mass of entrants is positive, $M_{Eit}>0$, so that expected discounted profits at entry are equal to zero. Under two

[20] This expression for final prices also results if producers and intermediaries are vertically integrated and maximize joint profits. If producers and intermediaries are not vertically integrated, then producers do not face a constant elasticity of demand (since final prices are $\bar{p}+\bar{p}^s$ and the producer chooses $\bar{p}$) so markups vary across producers and over time. We abstract from these complications by assuming that the producer and intermediary are vertically integrated. If the producer is vertically integrated with a foreign intermediary, and the PPI includes all costs incurred by the domestic producer (including foreign trade costs), then our results carry-through for Gross National Product, which includes profits earned abroad.

special cases of our model described below, our results also hold if we assume that entry is restricted so that the mass of entering firms is exogenously fixed (as in Chaney 2008).

The equivalence between GDP from the production, income, and expenditure side, in the absence of import tariffs, is given by (4). Current-dollar GDP from the production side is equal to aggregate revenues across all destination markets. Note that we are assuming that entry costs and fixed costs are expensed, and hence do not show up as output or investment in GDP. Aggregate profits Π_{it} are equal to aggregate revenues by country i producers across all destinations net of production labor, fixed labor, and entry costs:

$$\Pi_{it} = \sum_n \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it} - W_{it} \left[\sum_n \int_{\Omega_{int}} \left(l_{int} + f_{int} \right) \mathrm{d}M_{it} + f_{Ei} M_{Eit} \right]. \tag{28}$$

In what follows, we consider trade liberalization of the following form. The economy is in a steady-state at $t = 0$. Between $t = 0$ and $t = 1$, there is a permanent, unexpected change in variable and/or fixed trade costs.

We further assume that in the initial steady-state ($t = 0$) and in at least one period after the trade-liberalization ($t = T \geq 1$), aggregate profits in country i, Π_{it}, represent a constant share of aggregate revenues by country i producers. That is,

$$\Pi_{it} = \kappa_i \sum_n \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it} \text{ , for } t = 0 \text{ and } t = T \geq 1. \tag{29}$$

Note from (4) that (29) also implies that aggregate profits represent a constant share current-dollar GDP. This assumption is similar to assumption R2 in Arkolakis et. al. (2011).

There are three simple cases, derived in Appendix C, in which condition (29) is satisfied in the steady-state of our model. First, if there are no fixed costs of selling in each market (i.e. $f_{int} = 0$) so that all entering producers sell in all countries. Second, if the discount factor approaches zero ($\beta \rightarrow 1$), with or without fixed costs. In this case, aggregate profits in steady-state equal the expected discounted value of profits at entry, which are equal to zero due to the free-entry condition. Hence, $\kappa_i = 0$ in steady-state. In this case, the steady-state of our model is analogous to the equilibrium in static models with free-entry such as the ones considered in Melitz (2003) and Arkolakis et al. (2011), in which aggregate profits are zero. Third, if the productivity distribution of entering producers is Pareto.

In the first and third special cases, condition (29) also applies if we assume that entry is restricted so that the mass of firms is exogenously fixed. Moreover, with endogenous entry, in the first and third special cases the mass of entrants M_{Eit} does not respond to permanent changes in variable or fixed trade costs. Hence, there are no transition dynamics in response to permanent trade liberalization, and condition (29) holds for any time period $T \geq 1$. In all other cases with aggregate transition dynamics between steady-states, the share of profits in revenues κ_i need not be constant along the transition paths. In these cases, our results hold across steady-states.

Using (4) and (29), current-dollar GDP at time $t = 0$ and any time period $t = T$ in which condition (29) holds is given by

$$GDP_{it} = \frac{W_{it}\bar{L}_i}{1-\kappa_i}. \tag{30}$$

We now calculate changes in real GDP and real consumption between $t = 0$ and any time period $t = T$ in which condition (29) holds.

Real GDP

We first calculate changes in real GDP based on aggregate deflators. The ratio of real GDP between periods $t = 0$ and $t = T$ is given by

$$\begin{aligned} \frac{RGDP_{iT}}{RGDP_{i0}} &= \prod_{t=1}^{T}\left(\frac{GDP_{it}/GDP_{it-1}}{PPI_{it}/PPI_{it-1}}\right) = \frac{GDP_{iT}}{GDP_{i0}}\prod_{t=1}^{T}\left(\frac{1}{PPI_{it}/PPI_{it-1}}\right) \\ &= \frac{1}{\sum_n \frac{\tau_{in1}}{\tau_{in0}}\bar{\lambda}_{in1}}, \end{aligned} \tag{31}$$

which coincides with expression (9) in the previous models. In deriving expression (31), the first step uses (2) and (8), the second step factors-out the ratios of current-dollar GDPs, and the last step uses (6), (27), and (30). The expression for the change in real GDP using disaggregated deflators (which, recall, is also the one resulting from using the direct valuation method) is derived in a similar fashion, and coincides with expression (12) in the previous models.

Note that, for given levels of trade shares by continuing producers, $\bar{\lambda}_{int}$ (which might differ from overall trade shares λ_{int} due to entry and exit by firms into individual countries) and for given changes in variable trade costs, τ_{int}/τ_{int-1}, the change in real GDP in the model with endogenous specialization and monopolistic competition is the same as in the previous models. For given values of $\bar{\lambda}_{int}$ and τ_{int}/τ_{int-1}, reallocation of production from less productive to more productive producers (including exit by less productive producers and entry into exporting by more productive producers) does not result in an additional source of changes in aggregate productivity and real GDP. This is because value-added per production worker of individual producers, which is related to real GDP by expression (10), is equal to the ratio of the wage and the constant markup, independent of productivity z of individual producers.

Consider now changes in fixed costs or in the size of foreign countries when variable costs are unchanged. While these can induce changes in the volume and revenue share of trade, the ratio of PPIs is equal to W_{iT}/W_{i0} and hence does not directly reflect the changes in fixed costs. Real GDP from expression (9) is unchanged: $RGDP_{iT}/RGDP_{i0} = 1$. This result is summarized in the following corollary to Result 1.[21]

Corollary to Result 1: *In response to changes in fixed international trade costs between any pair of countries, real GDP in each country is unchanged.*

Real consumption and theoretical consumption

The expressions for changes in real consumption are the same as those in our baseline model: (15) with aggregate deflators or (17) with disaggregated deflators. Together with the fact that the expressions for changes in real GDP are also the same as in the previous models, Result 4 on the equivalence, to a first-order approximation, between changes in world real consumption and world GDP under trade balance holds.

What differs in this model is the comparison between real consumption and theoretical consumption, country-by-country. Changes over time in the set of consumed varieties produces differences between real consumption and theoretical consumption beyond the standard substitution bias. In particular, while the CPI between any two time periods only includes changes in prices of goods that are available for consumption in both periods, the theoretical price index also reflects changes in the mass of consumed goods.

This implies that in the model with endogenous specialization and monopolistic competition, inequality (16) that bounds the difference between real consumption and theoretical consumption does not apply since it is derived under the assumption that the set of available goods for consumption is unchanged between time periods.[22] Moreover, with changes in the mass of consumed varieties, either from changes in the set of goods supplied domestically or from changes in the set of goods imported from abroad, changes in the theoretical price index are not equal to the CPI as defined in (14), even to a first-order approximation. Therefore, Result 2, establishing the equality between changes in real consumption and theoretical consumption country-by-country, does not apply immediately in this version of the model. For example, an increase in the mass of consumed goods from abroad lowers the welfare-based price index (and hence increases theoretical consumption), but does not directly change the

[21] There are interactions effects from changes in variables costs and changes in fixed costs on real GDP. For example, a reduction in variable trade costs between countries i and n that is accompanied by a reduction in fixed export costs f_{int} can result in a larger trade share by continuing exporters at time t_0 and hence lead to a larger increase in real GDP.

[22] Inequality (16) would hold if the CPI attributed a price equal to infinite to goods that are not available for consumption.

CPI (and hence does not affect measured real consumption).

We show, however, that the equivalence between changes in real consumption and theoretical consumption in response to marginal changes in variable trade costs holds at the world level. This result, which is derived in Appendix D, is summarized as follows:

Result 5: *If each country has balanced trade, then steady-state changes in world real consumption and theoretical consumption (defined as cross-country weighted averages of changes in real consumption and theoretical consumption, respectively, using current-dollar GDP weights) in response to changes in variable trade costs are equal, up to a first-order approximation, and both are given by expression* (22)*.*

Results 4 and 5 combined imply that, up to a first-order approximation, steady-state changes in world real GDP and in world theoretical consumption in response to changes in variable trade costs are equal, up to a first-order approximation.

Note that, given that expression (22) holds in all the models that we consider, we have that for given trade shares and given marginal changes in variable trade costs, steady-state changes in world real GDP, real consumption, and theoretical consumption are all equal across these models up to a first-order approximation. This equivalence does not require any parametric assumption on the productivity distribution of entering firms, $G_i(z)$, as long as our restriction (29) holds.

Result 5 can be understood as follows. Note that when countries are symmetric, this result states that changes in real consumption equal changes in theoretical consumption in response to marginal changes in variable trade costs. This is because, as discussed in Atkeson and Burstein (2010), when countries are symmetric the indirect effect of a change in trade cost on consumption through its effect on the set of consumed goods (due to changes in the mass of entering firms and changes in exit and export thresholds, which are not captured in the CPI) is zero up to a first-order-approximation. Hence, in each country changes in the theoretical price index are approximately equal to changes in the CPI. With asymmetric countries, changes in relative country sizes alter the equivalence between real consumption and theoretical consumption, country-by-country, due to changes in the relative market size of countries. This effect, however, washes-out across countries (i.e. the gain in one country is a loss for another) when comparing steady-state changes in world real consumption and world theoretical consumption.[23]

To establish the equality between real consumption and theoretical consumption, country-

[23]For this result to hold, it is important that fixed and entry costs are denominated in terms of labor. If these costs entail a combination of labor and final good, then changes in the relative wage can result in additional indirect effects from changes in the mass of consumed varieties on the welfare-based price index that are not captured in the CPI (see the related discussion for welfare in Arkolakis et. al. 2011 and Atkeson and Burstein 2010).

by-country, in response to changes in variable trade costs (as in Result 2), we must impose two additional assumptions. First, fixed export costs are paid in the importing country. Second, the distribution of productivities of entering firms, $G_i(z)$ is Pareto. These assumptions are made in Eaton, Kortum, and Kramarz (2010) and for some results in Arkolakis et. al. (2011).[24] Under these assumptions, we obtain the following result that we prove in Appendix E.

Result 6: *Suppose fixed export costs are paid in the importing country, and that the distribution of entering firms is Pareto. If each country has balanced trade, then steady-state changes in real consumption and theoretical consumption in response to changes in variable trade costs are equal country-by-country, to a first-order approximation.*

Result 6 implies that, in response to marginal changes in variable trade costs, changes in the mass and in the composition of consumed domestic and exported goods (due to changes in exit and export thresholds) offset each other in each country's theoretical price index. Hence, changes in the CPI and in the theoretical price index coincide, up to a first-order approximation. Note that this result does not require that the mass of consumed varieties remains unchanged in each country (even though the mass of entering firms in each country does). Indeed, reductions in marginal trade costs typically result in an increase in the mass of consumed goods (which, however, does not affect the theoretical price index).

Numerical example

We illustrate how changes in real GDP, real consumption, and theoretical consumption compare in a quantitative example of our model with monopolistic competition. We consider small and large reductions in variable trade costs to evaluate the accuracy of some of our equivalence results derived using first-order approximations. We consider a two-country version of our model with trade balance, symmetric trade costs ($\tau_{12t} = \tau_{21t} = \tau_t$ and $f_{12t} = f_{21t}$), Pareto productivity distribution of entering firms with slope parameter of 5 (as in Eaton, Kortum and Kramarz 2010, implying a trade elasticity equal to 5), and elasticity of substitution ρ equal to 3. Variable trade costs are fully incurred in each exporting country, and fixed export costs are incurred in either the exporting country or the importing country (in the latter case, the economy satisfies the assumptions in Result 6). We choose the initial level of variable trade costs $\tau_0 = 1.47$, and relative country sizes $\bar{L}_1/\bar{L}_2 = 2.05$, so that the goods' trade share in country 1 is $\lambda_{120} = 7\%$ and the trade share in country 2

[24] These assumptions are required for the "ex-ante" result of Proposition 2 in Arkolakis et. al. (2011). Under these assumptions, their model responds to any global change in variable trade costs like an Armington model. Given that the welfare-based prices in the Armington model behaves, to a first-order approximation, like the CPI, we obtain the equivalence between real consumption and consumption-based welfare, country by country.

is $\lambda_{210} = 15\%$. The share of each country in world GDP is $s_{10} = 0.68$ and $s_{20} = 0.32$, respectively. The unchanged level of fixed costs do not affect our reported results. Recall that in this specification, entry remains unchanged, so the economy immediately transits to the new steady-state (at time $t = 1$).

We consider reductions in variable trade costs, ranging from very small (corresponding to our first-order approximations) to quite large (τ falls from roughly 1.47 to 1.23 so that the trade share more than doubles). Figure 1 considers the case in which fixed export costs are paid in the exporting country and Figure 2 the case in which fixed export costs are paid in the importing country. Based on the results in Arkolakis et. al. (2010), the specification in which fixed export costs are incurred in the importing country is exactly equal to the Armington version of our model with perfect competition and exogenous specialization and to the Krugman version of our model with monopolistic competition but no fixed costs, both parameterized with $\rho = 5$.

In each figure, the x-axis displays the ratio of trade shares in the post- and pre-liberalization periods, $\lambda_{in1}/\lambda_{in0}$ and the y-axis displays the negative of the elasticity of real GDP, real consumption, and theoretical consumption with respect to the change in variable trade costs (e.g.. $-\log(RGDP_1/RGDP_0)/\log(\tau_1/\tau)$). We report the measures of real GDP and real consumption calculated based on disaggregated deflators, which minimize the standard substitution bias in response to large changes in trade costs. We report separately the responses in each country and at the world level.

From Figures 1 and 2 we can observe that the higher order terms can be quite large. That is, the elasticities of each aggregate variable are largely increasing in the size of the reduction in trade costs. This implies that, for example, expression (21) is not a very accurate approximation for large reductions in trade costs: the elasticity of world real GDP and world real consumption is $s_{10} * \lambda_{120} + s_{20} * \lambda_{210} \simeq 0.09$ in response to a marginal reduction in trade costs, and roughly 0.15 in response to a large reduction in trade costs that doubles the trade share.

However, quite remarkably, theoretical and measured gains from trade are fairly close even for large reductions in trade costs that result in large increases in trade shares. In particular, first, the elasticity of world real GDP and the elasticity of world real consumption are almost exactly equal for any size of the reduction in trade costs (Result 4).[25] Second, for large reductions in trade costs, the elasticity of world real consumption is only slightly higher than the elasticity of world theoretical consumption (Result 5). Third, in each country (and

[25] For any change in trade costs, the increase in real GDP in country 1 (country 2) is slightly larger (smaller) than the increase in real consumption in that country, reflecting the fact that the wage in country 1 rises relative to the wage in country 2.

especially in country 2), for any size of the reduction in trade costs the elasticity of real consumption is quite close to the elasticity of theoretical consumption. This is not only the case when fixed export costs are incurred in the importing country (Result 6) but also when fixed export costs are incurred in the exporting country (for which we do not have an analytic result). Finally, comparing the elasticity of each variable in Figures 1 and 2 for any given change in variable trade costs, both specifications have very similar quantitative implications for both theoretical and measured aggregate gains from trade.

7 Two Extensions

In this section, we consider two extensions of our model. The first extension adds endogenous quality choice by firms. The second extension introduces multiple factors of production. We introduce these extensions in our model with monopolistic competition. We provide conditions under which our previous results on the response of aggregate productivity to changes in trade costs, and on the first-order equivalence between changes in real GDP, real consumption, and theoretical consumption at the world level (or country-by-country for real and theoretical consumption under stronger conditions) hold in the extended model. Details are provided in Appendices F and G.

Endogenous quality choice

The final good is given by

$$C_{nt} = \left[\int_{\Omega_{nt}} a_{nt}\left(\omega\right)^{\frac{1}{\rho}} \left(\omega\right) q_{nt}\left(\omega\right)^{\frac{\rho-1}{\rho}} \mathrm{d}\omega \right]^{\frac{\rho}{\rho-1}},$$

where $a_{nt}\left(\omega\right)$ denotes the quality of differentiated good ω in country n. The theoretical price index is given by $P_{nt} = \left[\int_{\Omega_{nt}} a_{nt}\left(\omega\right) p_{nt}\left(\omega\right)^{1-\rho} \mathrm{d}\omega \right]^{\frac{1}{1-\rho}}$. Higher levels of quality decrease the price index.

Demand in country n for good z produced in country i is given by $q_{int}\left(z\right) = a_{int}\left(z\right) \left(p_{int}\left(z\right)/P_{nt}\right)^{-\rho} C_{nt}$. Higher quality increases demand, given prices. We assume that each period, individual producers from country i with productivity z must employ $h\left(z; a_{int}\right)$ units of labor in the home country to set quality a_{int} for sales in country n, where $h\left(z; .\right)$ is increasing and convex in a.[26] We assume that these costs are expensed, so they are not included in GDP. Given that quality costs are independent of the volume

[26] We assume throughout that $h\left(z; a\right)$ is such that the level of a for active products is positive and bounded, and so that in steady-state there is positive entry and a stationary size distribution. All our results hold if a_{int} is constrained to be equal across destination countries, with the exception of the equivalence between real and theoretical consumption country by country, which requires that a_{int} be destination specific.

of production, reductions in trade costs that raise the scale of exporters typically induce a higher investment in quality by exporters relative to non-exporters.

The share of profits in GDP is constant in the steady-state (condition 29) under the two following alternative assumptions. First, if the discount factor approaches zero ($\beta \to 1$). As $\beta \to 1$, aggregate profits, which now include the costs of quality choice, become zero from the free-entry condition, so $\kappa_i = 0$ in steady-state. Second, if $h(z; a)$ takes the form $h(z; a) = \frac{\gamma_0}{\gamma} \bar{h}(z) a^{\gamma}$ and either (i) there are no fixed costs of supplying individual markets or (ii) the productivity distribution of entering producers is Pareto. In Appendix F we derive κ_i for this case.

Prices set by individual producers are given by expression (27) as in our baseline model. A key consideration that determines the aggregate measured gains from trade is whether deflators are constructed using prices adjusted for quality (i.e. $p_{int}(z) / a_{int}(z)$) or non-adjusted for quality (i.e. $p_{int}(z)$).[27]

If prices in the PPI do not adjust for quality changes, then the expression for changes in real GDP is equivalent to that in our baseline model without endogenous quality (expressions 8 and 12), derived using condition (29). If prices in the PPI do adjust for quality changes, then if average quality rises in response to a reduction in trade costs, the PPI falls relative to the scenario in which prices are not adjusted for quality changes. In this case, the increase in real GDP (conditional on trade shares and changes in trade costs) is larger than the one in expressions (8) and (12).

Consider now the response of real consumption. In Appendix F we establish the following result. If prices in the CPI do not reflect changes in product quality, then changes in world real consumption and world theoretical-consumption in response to marginal changes in variable trade costs are equal, to a first-order approximation, and given by expression (22). This equality also applies to world real GDP if GDP deflators do not adjust for quality changes. Intuitively, the effects on the world welfare-based price index from changes in the set of consumed goods (changes in the mass of entering firms and changes in exit and export thresholds) and endogenous quality changes add up to zero, up to a first-order approximation. If prices in the CPI do not capture any of these margins (i.e. prices are not adjusted for quality changes), then the CPI coincides with the welfare-based price index.

Suppose instead that prices in the CPI do adjust for quality changes. If average quality rises in response to a reduction in trade costs, the CPI falls relative to the baseline scenario in which prices are not adjusted for quality changes, and measured gains in world real

[27] Product quality in this setup can be re-interpreted as producer productivity. In this case, producers innovate to improve productivity rather than product quality. This re-interpretation does not change any of the model's implications for theoretical consumption. Note, however, that changes in productivity are more likely to be captured in price indices, as when prices are adjusted for quality.

consumption exceed those in world theoretical consumption.

In Appendix F we show that if the productivity distribution of entering producers is Pareto, $h(z;a)$ takes the form $h(z;a) = \frac{\gamma_0}{\gamma} z^{\mu} a^{\gamma}$, and both fixed costs and innovation costs are incurred using labor in the importing country, then in response to marginal changes in variable trade costs the equivalence between changes in real and theoretical consumption (when prices in the CPI do not reflect changes in product quality) holds not only at the world level but also country-by-country.

Multiple factors of production

We now consider multiple factors of production, which can be accumulated or in fixed supply. The production of intermediate goods uses labor and J additional inputs, denoted by k_j, according to:

$$y = z l^{\alpha_L} \prod_{j=1}^{J} k_j^{\alpha_j} \ , \tag{32}$$

where we assume constant returns to scale, so $\alpha_L + \sum_{j=1}^{J} \alpha_j = 1$. All producers are subject to a production function with the same factor shares α_j. Fixed costs of supplying individual markets and entry costs are all denominated in terms of labor.

Without loss of generality, we assume that inputs $j \leq J_F$ can be accumulated at the aggregate level (e.g. capital), while inputs $j > J_F$ are exogenously supplied and constant over time. None of our results depend on the choice of J_F. Consumption and accumulable inputs are both produced using a final non-tradeable good defined in (3). The final good resource constraint in country i is $C_{it} + \sum_{j=1}^{J_F} K_{j,it} = Q_{it}$, were $K_{j,it}$ denotes the aggregate stock of input j in the economy, and Q_{it} denotes the quantity of the final good used in country i. The assumption that accumulable inputs fully depreciate every period is without loss of generality for our results.

Letting $R_{j,it}$ denote the price of input j in country i in period t, cost minimization implies

$$\frac{R_{j,it}}{W_{it}} = \frac{\alpha_j}{\alpha_L} \frac{l_{int}}{k_{j,int}} = \frac{\alpha_j}{\alpha_L} \frac{L_{it}}{K_{j,it}}, \tag{33}$$

where L_{it} denotes the aggregate quantity of labor used for production in country i. The second equality follows from the assumption that factor shares and factor prices are common across firms. The optimal price of a country i producer with productivity z selling in country n is given by $p_{int}(z) = \frac{\rho}{\rho-1} \frac{\tau_{int} c_{it}}{z}$, were $c_{it} = \hat{\alpha} W_{it} \prod_{j=1}^{J} [R_{j,it}/W_{it}]^{\alpha_i}$ is the cost of the input bundle in country i. Using (33), we can rewrite c_{it} as

$$c_{it} = \hat{\alpha} W_{it} \prod_{j=1}^{J} \left[\frac{\alpha_j}{\alpha_L} \frac{L_{it}}{K_{j,it}} \right]^{\alpha_i}. \tag{34}$$

Real GDP: GDP includes output used for both consumption and accumulable inputs. We calculate real GDP using aggregate deflators. We first calculate the aggregate PPI. Note that, given that consumption and accumulable inputs use the same production technology, there is a single PPI for final goods, given by (6), which can be written as:

$$\frac{PPI_{it}}{PPI_{it-1}} = \frac{\sum_n \int_{\Omega_{int}^c} p_{int_0} q_{int_0} \left(\frac{p_{int}}{p_{int-1}} \right) \mathrm{d}M_{it_0}}{\sum_n \int_{\Omega_{int}^c} p_{int_0} q_{int_0} \mathrm{d}M_{it_0}} = \frac{c_{it}}{c_{it-1}} \sum_n \frac{\tau_{int}}{\tau_{int-1}} \bar{\lambda}_{int}. \tag{35}$$

In the Appendix, we show that in this version of the model, current dollar GDP is proportional to aggregate labor payments. Hence, the ratio of real GDP in time T to time $t = 0$ in response to a permanent trade liberalization at time $t = 1$ is

$$\begin{aligned} \frac{RGDP_{iT}}{RGDP_{i0}} &= \frac{GDP_{iT}}{GDP_{i0}} \prod_{t=1}^{T} \left(\frac{1}{PPI_{it}/PPI_{it-1}} \right) = \frac{W_{iT}\bar{L}_i}{W_{i0}\bar{L}_i} \prod_{t=1}^{T} \left(\frac{1}{\frac{c_{it}}{c_{it-1}} \sum_n \frac{\tau_{int}}{\tau_{int-1}} \bar{\lambda}_{int}} \right) \\ &= \prod_{j=1}^{J} \left[\frac{K_{j,iT}}{K_{j,i0}} \right]^{\alpha_i} \frac{1}{\sum_n \frac{\tau_{in1}}{\tau_{in0}} \bar{\lambda}_{in1}}, \end{aligned} \tag{36}$$

where the last step follows from equation (34). Given trade shares of continuing producers and given changes in variable trade costs, the change in measured aggregate productivity coincides with that in our baseline model with a single factor of production (i.e. expression (8). Of course, growth in aggregate quantities of non-labor factors of production contributes to growth in real GDP.

Note that accumulable inputs may also be interpreted as intermediate goods. In this case, GDP differs from gross output as it excludes the use of intermediate inputs. However, our assumptions imply that the share of value added in firms' gross-output is constant, so the expression for real GDP remains unchanged.[28]

World real GDP, consumption, and theoretical consumption: In Appendix G we derive the equivalence between world theoretical consumption, world consumption, and world GDP, up

[28] We can also calculate real GDP using the double deflation method and obtain the same expression. The key is that intermediate inputs are produced using the same technology as final goods, so they are deflated using the same deflator (35) as that used to deflate gross output.

to a first order approximation, in response to marginal changes in variable trade costs (if the set of consumed products is unchanged or if the distribution of entering firms is Pareto and fixed costs are incurred in the importing country, the first-order equivalence between real consumption and theoretical consumption holds country-by-country). A key step in the analysis is that, under our assumptions, changes in trade costs do not change the steady-state ratio of consumption to final output, C/Q, in each country. The actual magnitudes of changes in world aggregates (for given trade shares and changes in trade costs) differ from those in the baseline model due to endogenous changes in aggregate quantities of non-labor factors of production.

8 Conclusions

In this paper we have studied the implications of trade liberalization for aggregate measures of economic activity in a widely-used class of workhorse models of international trade. We have characterized how in these models real GDP and real consumption, as calculated by statistical agencies in the United States, respond to changes in variable trade costs, fixed trade costs, and tariffs.

For the class of models that we consider, our conclusions can be broadly summarized as follows. First, aggregate output measured by real GDP and aggregate productivity constructed using data on real GDP increase in response to reductions in trade costs insofar as prices used to construct deflators reflect these changes in trade costs. Real GDP and aggregate productivity, however, do no capture the reallocation of production towards more productive producers resulting from trade liberalization. Second, gains in theoretical (welfare-based) consumption from reductions in variable international trade costs translate into measures of real consumption when aggregating these measures across countries. Under stronger but common assumptions in the literature, the equivalence between theoretical and measured consumption also holds country-by-country. Differences between consumption deflators and welfare-based price indices in response to changes in variable trade costs, that may arise from changes in the set of consumed varieties or changes in the quality of individual products wash-out when treated jointly across all countries (or country-by-country under stronger assumptions). Third, conditional on trade shares (of continuing producers) and changes in variable trade costs, all the models we consider deliver approximately the same measured aggregate gains from trade. The equivalence in measured gains from trade arises due to the equivalence in the welfare implications of these models.

Our results establish a benchmark to understand how the extensive empirical evidence on the link between trade and aggregate measures of economic activity can be interpreted through the lens of workhorse trade models, and how the theoretical link between trade and welfare in these models translates into observable aggregates. Our results should be, however, treated with caution to the extent that the measurement procedures in individual countries differ from those carried out in the United States and recommended by the United Nations. Finally, the extent to which our results carry over to richer models featuring additional sources of gains from trade to the ones we considered, such as the endogenous response of markups, remains an open research question.

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Figure 1: Gains from reductions in variable trade costs, Fixed costs in exporting country

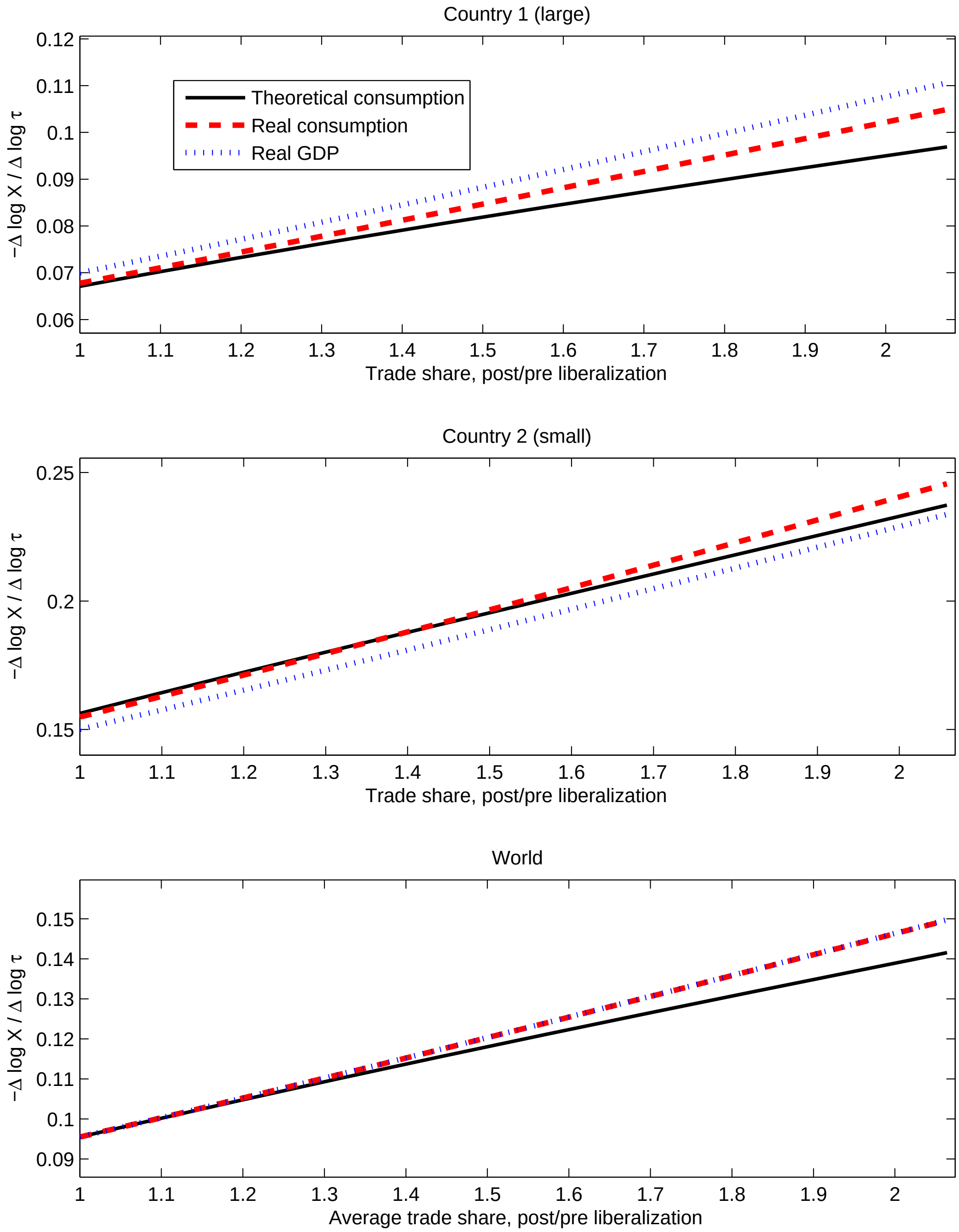
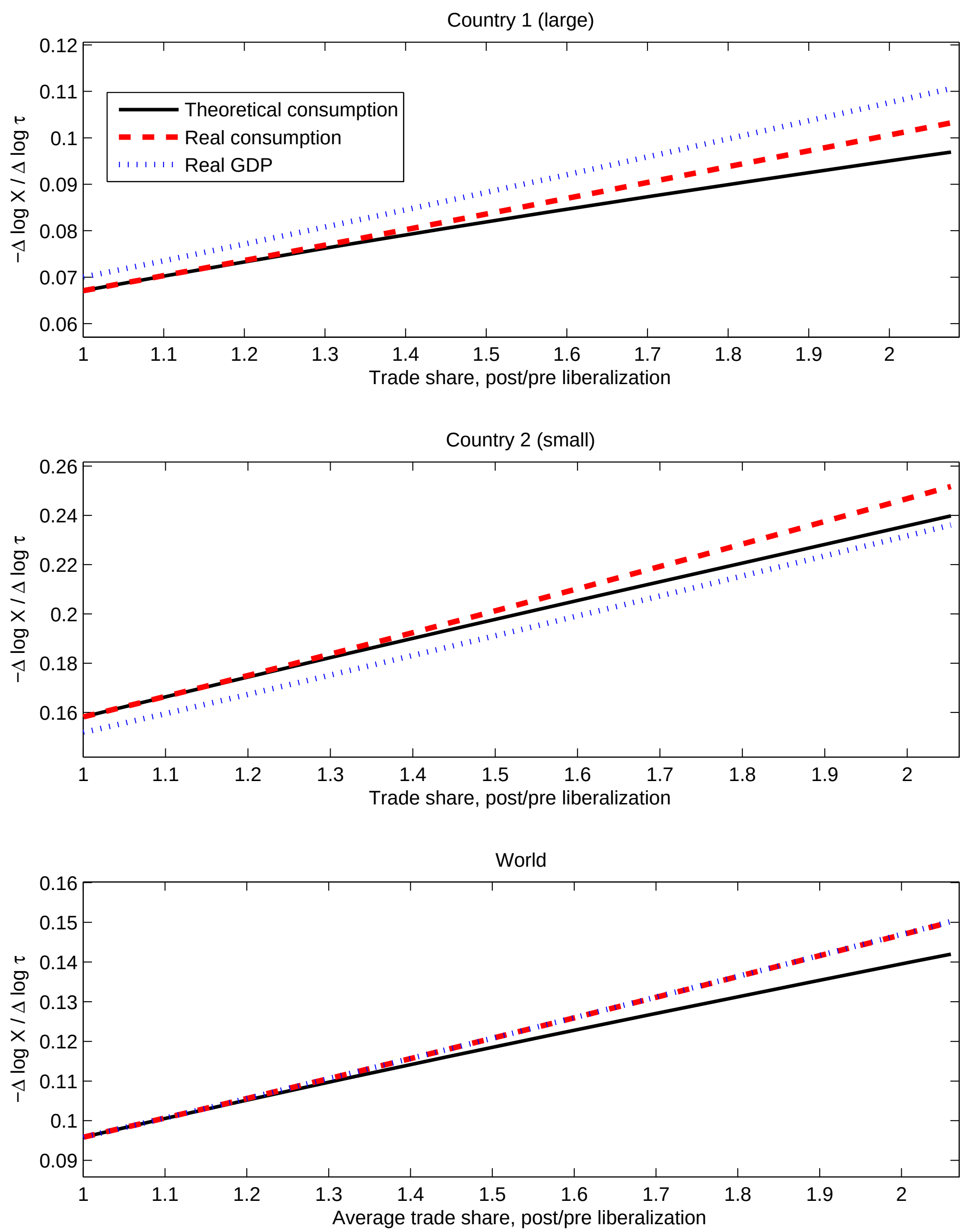


Figure 2: Gains from reductions in variable trade costs, Fixed costs in importing country



Appendix A: Substitution bias in the CPI

In this appendix we derive the well-know substitution bias on the CPI, establishing that the Laspeyres (Paasche) price index overstates (understates) changes in the welfare-based price index. We assume through this section that the same set of goods is consumed in all periods, $\Omega_{int} = \Omega_{in}$ and $M_{it} = M_i$. Note that we can write the Laspeyres price index as:

$$\left.\frac{CPI_{nT}}{CPI_{n0}}\right|_{t_0=0} = \sum_i \Lambda_{in0} \left(\frac{CPI_{inT}}{CPI_{in0}}\right),$$

where $\Lambda_{int} = \int_{\Omega_{in}} \left(p_{int} q_{int} \mathrm{d}M_i\right) / \sum_i \int_{\Omega_{in}} p_{int} q_{int} \mathrm{d}M_i$ is country $n's$ share of expenditures on goods produced in country i at date t. Similarly, we can re-write the Paasche price index as:

$$\left.\frac{CPI_{nT}}{CPI_{n0}}\right|_{t_0=T} = \left[\sum_i \Lambda_{inT} \left(\frac{CPI_{in0}}{CPI_{inT}}\right)\right]^{-1}.$$

The welfare-based price index is defined as:

$$P_{nt} = \min_{q_{int}} \sum_i \int_{\Omega_{in}} p_{int} q_{int} \mathrm{d}M_i : \left[u\left(C_{nt}\right) \geq \bar{u}\right].$$

Let q^*_{int} denote the solution to this problem when prices are p_{int}. The change in the welfare-based price index is given by:

$$\frac{P_{nT}}{P_{n0}} = \frac{\sum_i \int_{\Omega_{in}} p_{inT} q^*_{inT} \mathrm{d}M_i}{\sum_i \int_{\Omega_{in}} p_{in0} q^*_{in0} \mathrm{d}M_i} \leq \frac{\sum_i \int_{\Omega_{in}} p_{inT} q^*_{in0} \mathrm{d}M_i}{\sum_i \int_{\Omega_{in}} p_{in0} q^*_{in0} \mathrm{d}M_i} = \sum_i \Lambda^*_{in0} \left(\frac{CPI_{inT}}{CPI_{in0}}\right)$$

where the inequality follows from the definitions of q^*_{inT} and $\Lambda^*_{int} = \frac{\int_{\Omega_{in}} p_{int} q^*_{int} \mathrm{d}M_i}{\sum_i \int_{\Omega_{in}} p_{int} q^*_{int} \mathrm{d}M_i}$. Similarly:

$$\frac{P_{nT}}{P_{n0}} = \frac{\sum_i \int_{\Omega_{in}} p_{inT} q^*_{inT} \mathrm{d}M_i}{\sum_i \int_{\Omega_{in}} p_{in0} q^*_{in0} \mathrm{d}M_i} \geq \frac{\sum_i \int_{\Omega_{in}} p_{inT} q^*_{inT} \mathrm{d}M_i}{\sum_i \int_{\Omega_{in}} p_{in0} q^*_{inT} \mathrm{d}M_i} = \left[\sum_i \Lambda^*_{inT} \left(\frac{CPI_{in0}}{CPI_{inT}}\right)\right]^{-1}. \qquad \text{(A1)}$$

If u is homothetic (so that expenditure shares only depend on relative prices and do not depend on income), then, $\Lambda^*_{int} = \Lambda_{int}$, and $(A1)$ implies (16).

Appendix B: International specialization of shipping services

We first calculate the change in real GDP (using aggregate deflators) in country i_s. The PPI in period t relative to period $t-1$ is given by

$$\frac{PPI_{i_st}}{PPI_{i_st-1}} = \frac{W_{i_st}}{W_{i_st-1}} \left[\sum_n \bar{\lambda}_{i_snt} \left(\frac{\tau_{i_snt}}{\tau_{i_snt-1}} \right) + \sum_{i \neq i_s} \sum_n \left(\frac{\tau_{int} - 1}{\tau_{int-1} - 1} \right) \bar{\lambda}^s_{int} \right]$$

where

$$\bar{\lambda}_{i_snt} = \frac{\int_{\Omega^c_{i_snt}} \left(\bar{p}_{i_snt_0} + \bar{p}^s_{i_snt_0} \right) q_{i_snt_0} \mathrm{d}M_{i_st_0}}{\sum_n \int_{\Omega^c_{i_snt}} \left(\bar{p}_{i_snt_0} + \bar{p}^s_{i_snt_0} \right) q_{int_0} \mathrm{d}M_{it_0} + \sum_{i \neq i_s} \sum_n \int_{\Omega^c_{int}} \bar{p}^s_{int_0} q_{int_0} \mathrm{d}M_{it_0}}$$

and, for $i \neq i_s$,

$$\bar{\lambda}^s_{int} = \frac{\int_{\Omega^c_{int}} \bar{p}^s_{int_0} q_{int_0} \mathrm{d}M_{it_0}}{\sum_n \int_{\Omega^c_{i_snt}} \left(\bar{p}_{i_snt_0} + \bar{p}^s_{i_snt_0} \right) q_{int_0} \mathrm{d}M_{it_0} + \sum_{i \neq i_s} \sum_n \int_{\Omega^c_{i_snt}} \bar{p}^s_{int_0} q_{int_0} \mathrm{d}M_{it_0}} \ ,$$

with $\sum_n \bar{\lambda}_{i_snt} + \sum_{i \neq i_s} \sum_n \bar{\lambda}^s_{int} = 1$. The ratio of real GDP in time period t relative to $t-1$ is

$$\begin{aligned} \frac{RGDP_{i_st}}{RGDP_{i_st-1}} &= \left(\frac{\frac{GDP_{i_st}}{PPI_{i_st}/PPI_{i_st-1}}}{GDP_{i_st-1}} \right)^{0.5} \left(\frac{GDP_{i_st}}{\frac{GDP_{i_st-1}}{PPI_{i_st}/PPI_{i_st-1}}} \right)^{0.5} \\ &= \frac{1}{\sum_n \bar{\lambda}_{i_snt} \left(\frac{\tau_{i_snt}}{\tau_{i_snt-1}} \right) + \sum_{i \neq i_s} \sum_n \bar{\lambda}^s_{int} \left(\frac{\tau_{int}-1}{\tau_{int-1}-1} \right)} \ . \end{aligned} \tag{A2}$$

Clearly, $RGDP_{i_st}/RGDP_{i_st-1} > 1$ if trade costs fall.

We now derive the change in world real GDP and real consumption under balanced trade. Log-differentiating $(A2)$,

$$d \log RGDP_{i_st} = -\sum_n \lambda_{i_snt} d \log \tau_{i_snt} - \sum_{i \neq i_s} \sum_n \lambda^s_{int} \frac{\tau_{int}}{\tau_{int} - 1} d \log \tau_{int}.$$

Together with $d \log RGDP_{it} = 0$ for $i \neq i_s$, and using the definitions of $\bar{\lambda}_{i_snt}$ and $\bar{\lambda}_{int}$, we obtain expression (22), where $Exports_{int}$ for $i \neq i_s$ is evaluated at prices inclusive of trade services provided by country i_s.

With balanced trade in each country, the world change in real consumption is, to a first-order approximation, given by expression (20), where country-specific expenditures E_{int} are calculated inclusive of trade costs provided by country i_s, and changes in country specific

CPIs are given by

$$d\log CPI_{nit}=\frac{W_{nt}d\log W_{nt}+W_{i_st}\left(\tau_{nit}-1\right)\left[d\log W_{i_st}+\frac{\tau_{nit}}{\tau_{nit}-1}d\log\tau_{nit}\right]}{W_{nt}+W_{i_st}\left(\tau_{nit}-1\right)}\text{ for }n\neq i_s,$$

and

$$d\log CPI_{nit}=d\log W_{i_st}+d\log\tau_{nit}\text{ for }n=i_s.$$

Substituting E_{nit} and $d\log CPI_{nit}$into (20), we obtain expression (22).

Appendix C: Deriving the share of profits in total revenues

We now show that our assumption in equation (29) that aggregate profits represent a constant share of total revenues ($\Pi_{it}=\kappa_i Y_{it}$, where $Y_{it}=\sum_n\int_{\Omega_{int}}p_{int}q_{int}\mathrm{d}M_{it}$), is satisfied in the remaining two special cases of our model described in Section 5. In the first case, there are no fixed costs of selling into individual countries so that all firms sell in each country. In the second case, there are positive fixed costs of selling in individual countries (incurred in either the exporting or importing country) and productivities are Pareto distributed. We derive equation (29) for the general case in which a fraction ϕ of these fixed costs are incurred in the exporting country and a fraction $1-\phi$ of these fixed costs are incurred in the importing country. The baseline model in the body of the paper assumes $\phi=0$. We consider the case of $\phi=1$ in Result 6. We also show that, in these two cases, the mass of firms is unchanged following a trade liberalization. Remember that in the third special case described in Section 5, when $\beta\to1$, it is straightforward to show that the free entry condition implies that $\kappa_i=0$ in steady-state.

We start by deriving some preliminary equations of the model: first, note that combining (27) with the demand function we obtain that firm's revenues are proportional to firm's variable costs,

$$p_{int}q_{int}=\frac{\rho}{\rho-1}W_{it}l_{int}\ , \tag{A3}$$

variable labor demand is:

$$l_{int}\left(z\right)=z^{\rho-1}\tau_{int}^{1-\rho}\left[\frac{\rho}{\rho-1}W_{it}\right]^{-\rho}P_{nt}^{\rho}C_{nt}\ , \tag{A4}$$

and variable profits are:

$$\pi_{int}\left(z\right)=\frac{z^{\rho-1}\tau_{int}^{1-\rho}}{\rho^{\rho}\left(\rho-1\right)^{1-\rho}}W_{it}^{1-\rho}P_{nt}^{\rho}C_{nt}. \tag{A5}$$

In an equilibrium with selection by firms to sell in each country, there exists a threshold $\bar{z}_{int}$ such that only firms with $z \geq \bar{z}_{int}$ operate in destination n. That is, $\Omega_{int} = \{z : z \geq \bar{z}_{int}\}$. This threshold satisfies:

$$\pi_{int}\left(\bar{z}_{int}\right) = W_{it}^{\phi} W_{nt}^{1-\phi} f_{int} \ . \tag{A6}$$

Aggregate profits in country i in period t net of fixed labor costs and entry costs are given by:

$$\Pi_{it} = Y_{it} - W_{it} L_{it} - \sum_{n} W_{it}^{\phi} W_{nt}^{1-\phi} f_{int} \int_{\Omega_{int}} \mathrm{d}M_{it} - W_{it} M_{\mathrm{Ei}\,t} f_{Ei} \ ,$$

where L_{it} denotes aggregate variable labor used in production, $L_{it} = \sum_n \int_{\Omega_{int}} l_{int} \mathrm{d}M_{it}$. Note that from expression $(A3)$, aggregate revenues are proportional to variable labor payments:

$$Y_{it} = \frac{\rho}{\rho - 1} W_{it} L_{it} \ . \tag{A7}$$

If condition (29) holds, then in combination with (30), we obtain

$$\frac{1}{1-\kappa_i} W_{it} \bar{L}_i = Y_{it} = \frac{\rho}{\rho - 1} W_{it} L_{it} \ ,$$

which implies that variable production labor is a constant share of total labor:

$$\left(1 - \kappa_i\right) \frac{\rho}{\rho - 1} L_{it} = \bar{L}_i \ . \tag{A8}$$

Hence, if aggregate profits represent a constant share of aggregate revenues, then aggregate variable labor represents a constant fraction of total labor.

Suppose we are on a steady-state equilibrium in which aggregate variables are constant. In steady-state, the interest rate is given by $1/\beta$ and the distribution of firms is given by $M_i\left(z\right) = \frac{M_{Ei}}{\delta} G_i\left(z\right)$ (we omit time subscripts for the reminder of this section to simplify notation). The aggregate free-entry condition in steady-state is:

$$W_i f_{Ei} M_{Ei} = \frac{\beta\delta}{1 - \beta\left[1 - \delta\right]} \left[Y_i - W_i L_i - \sum_{n} W_i^{\phi} W_n^{1-\phi} \left[1 - G_i\left(\bar{z}_{in}\right)\right] f_{in} \right] . \tag{A9}$$

In what follows, we solve for the constant of proportionality $\Pi_i / Y_{it} = \kappa_i = \kappa$ in steady-state under two special cases of our model. We then show that, in these two special cases, the aggregate response to a change in variable or fixed trade costs is immediate (i.e. there are no transition dynamics), so that κ remains constant over time.

Case 1: No fixed costs

Assume that there are no fixed costs of selling in individual countries, i.e. $f_{ii} = f_{in} = 0$, so that there is no selection. In this case, the aggregate free entry condition $(A9)$ is:

$$W_i f_{Ei} M_{Ei} = \frac{\beta\delta}{1-\beta\left[1-\delta\right]}\left[Y_i - W_i L_i\right] \ ,$$

and using $(A7)$,

$$W_i f_{Ei} M_{Ei} = \frac{\beta\delta}{1-\beta\left(1-\delta\right)}\frac{1}{\rho}Y_i \ . \tag{A10}$$

Aggregate profits are:

$$\begin{aligned}
\Pi_i &= Y_i - W_i L_i - W_i \frac{M_{Ei}}{\delta} f_{Ei} \\
&= Y_i - \frac{\rho-1}{\rho}Y_i - \frac{\beta}{1-\beta\left[1-\delta\right]}\frac{1}{\rho}Y_i \\
&= \frac{1-\beta}{\rho\left[1-\left(1-\delta\right)\beta\right]}Y_i \ ,
\end{aligned}$$

so $\kappa = \frac{1-\beta}{\rho(1-(1-\delta)\beta)}$. Note that if $\beta < 1$, aggregate cross-sectional profits are positive even though discounted profits at entry are zero.

The steady-state mass of entering firms is given by:

$$M_{Ei} = \frac{\beta\delta}{1-\beta\left(1-\delta\right)}\frac{\bar{L}_i}{\rho f_{Ei}\left(1-\kappa\right)} \ ,$$

where we used $(A7)$, $(A10)$, and equation $(A8)$. Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state, and $\kappa_{it} = \kappa$.

Finally, aggregate variable profits gross of entry costs are: $\Pi_i + W_i M_{Ei} f_{Ei} = \rho^{-1} Y_i$. Hence, with restricted entry (so that there are no costs incurred in entry), equation (29) holds with $\kappa = 1/\rho$.

Case 2: Pareto distributed productivities

Assume that there are positive fixed costs of selling in individual countries and that the distribution of entering firms G_i is Pareto with shape parameter θ, i.e. $G_i = 1 - z^{-\theta}$ for $z \geq 1$. We also assume that the productivity cutoffs are interior, $\bar{z}_{in} > 1$.

We first show that aggregate fixed labor costs are proportional to aggregate revenues. Using the Pareto form, we can rewrite the expression $(A6)$ that defines the cutoff $\bar{z}_{in}$ as:

$$\frac{\bar{z}_{in}^{\rho-1-\theta}\tau_{in}^{1-\rho}W_i^{1-\rho}}{\rho^{\rho}\left(\rho-1\right)^{1-\rho}}P_n^{\rho}C_n = W_i^{\phi}W_n^{1-\phi}f_{in}\bar{z}_{in}^{-\theta}. \tag{A11}$$

Fixed labor costs to sell in destination n are given by:

$$\frac{M_{Ei}}{\delta}W_i^{\phi}W_n^{1-\phi}f_{in}\bar{z}_{in}^{-\theta}=\frac{\theta+1-\rho}{\rho\theta}Y_{in} \tag{A12}$$

where $Y_{in} = \int_{\Omega_{in}} p_{in}q_{in}\mathrm{d}M_i$ denotes revenues from sales in country n. Summing across countries we obtain:

$$\frac{M_{Ei}}{\delta}\sum_n W_i^{\phi}W_n^{1-\phi}f_{in}\left[1-G_i\left(\bar{z}_{in}\right)\right]=\frac{\theta+1-\rho}{\rho\theta}Y_i\ , \tag{A13}$$

Using $(A7)$ and $(A13)$, we can write the aggregate free entry condition $(A9)$ as:

$$M_{Ei}W_if_{Ei}=\frac{\delta\beta}{1-\beta\left[1-\delta\right]}\frac{\rho-1}{\rho\theta}Y_i, \tag{A14}$$

Finally, combining $(A7)$, $(A13)$ and $(A14)$, aggregate profits are

$$\begin{aligned}\Pi_i &= Y_i-W_iL_i-\frac{M_{Ei}}{\delta}\sum_n W_i^{\phi}W_n^{1-\phi}\left[1-G_i\left(\bar{z}_{in}\right)\right]f_{in}-\frac{M_{Ei}}{\delta}W_if_{Ei}\\ &= \frac{\rho-1}{\theta\rho}\frac{1-\beta}{1-\beta\left[1-\delta\right]}Y_i\end{aligned}$$

so $\kappa=\left(\frac{\rho-1}{\theta\rho}\right)\left(\frac{1-\beta}{1-\beta[1-\delta]}\right)$. The steady-state mass of entering firms is given by:

$$M_{Ei}=\frac{\delta\beta\left(\rho-1\right)}{1-\beta\left(1-\delta\right)}\frac{1}{f_{Ei}\left(1-\kappa\right)\rho\theta}\bar{L}_i\ ,$$

where we used $(A7)$, $(A8)$, and $(A14)$. Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state.

Finally, aggregate variable profits gross of entry costs are $\Pi_i+W_iM_{Ei}f_{Ei}=\frac{\rho-1}{\theta\rho}Y_i$. Hence, in the model with restricted entry (in which there are no entry costs), equation (29) holds with $\kappa=\left(\rho-1\right)/\left(\theta\rho\right)$.

Appendix D: Proof of Result 5

We show that the steady-state change in theoretical consumption, real GDP and real consumption in response to marginal changes in variable trade costs in the model with heterogenous firms and monopolistic competition is given by expression (22). We assume here that fixed costs are incurred in the exporting country (i.e. $\phi=0$ using the notation of Appendix

C).

Note first that we can re-express variable profits relative to the wage in equation $(A5)$ as

$$\begin{aligned}\frac{\pi_{int}(z)}{W_{it}} &= \frac{z^{\rho-1}\tau_{int}^{1-\rho}}{\rho^{\rho}(\rho-1)^{1-\rho}}W_{it}^{-\rho}P_{nt}^{\rho}C_{nt} \\ &= \frac{z^{\rho-1}\tau_{int}^{1-\rho}}{\rho^{\rho}(\rho-1)^{1-\rho}}\left(\frac{\bar{L}_i}{1-\kappa_i}\right)^{\rho}C_{it}^{1-\rho}S_{int}\end{aligned} \tag{A15}$$

where $S_{int} = \frac{P_{nt}^{\rho}C_{nt}}{P_{it}^{\rho}C_{it}}$. In deriving this expression, we have used $\frac{W_{it}}{P_{it}} = \frac{C_{it}}{\bar{L}_i}(1-\kappa_i)$ from (30) and balanced trade.

Expected variable profits (relative to the wage) per entering firms are

$$\sum_n \int_{\Omega_{int}} \frac{\pi_{int}(z)}{W_{it}} \mathrm{d}G_i(z) = \frac{1}{\rho^{\rho}(\rho-1)^{1-\rho}}\left(\frac{\bar{L}_i}{1-\kappa_i}\right)^{\rho}C_{it}^{1-\rho}\sum_n \tau_{int}^{1-\rho}S_{int}Z_{int}, \tag{A16}$$

where $Z_{int} = \int_{\Omega_{int}} z^{\rho-1}\mathrm{d}G_i$. Free-entry in steady-state implies:

$$\hat{\beta}\sum_n \int_{\Omega_{int}} \frac{\pi_{int}(z)}{W_{it}} \mathrm{d}G_i(z) = f_{Ei} + \hat{\beta}\sum_n \left[1 - G_i(\bar{z}_{int})\right] f_{int} \ ,$$

where $\hat{\beta} = \frac{\beta}{1-\beta(1-\delta)}$. Log-differentiating this expression with respect to changes in $\boldsymbol{\tau}$ around the initial steady-state at time t, and using $(A16)$ yields

$$d\log C_{it} = -\frac{\sum_n \left[d\log \tau_{int} - \frac{1}{\rho-1}d\log S_{int}\right]\tau_{int}^{1-\rho}S_{int}Z_{int}}{\sum_n \tau_{int}^{1-\rho}S_{int}Z_{int}}. \tag{A17}$$

Here we have used an envelope condition to obtain that changes in cutoffs $\bar{z}_{int}$, defined by $(A6)$, have no first-order effects on expected profits at entry. Using $\lambda_{int} = \frac{\tau_{int}^{1-\rho}S_{int}Z_{int}}{\sum_n \tau_{int}^{1-\rho}S_{int}Z_{int}}$, we can re-write this expression as

$$d\log C_{it} = -\sum_n \lambda_{int} d\log \tau_{int} + \frac{1}{\rho-1}\sum_n \lambda_{int} d\log S_{int}.$$

The change in world theoretical consumption using weights $s_{it} = Y_{it}/\sum_i Y_{it}$, and using $P_{it}C_{it} = Y_{it}$ from trade balance, is given by

$$\sum_i s_{it} d\log C_{it} = \sum_i s_{it}\left[-\sum_n \lambda_{int} d\log \tau_{int} + \frac{1}{\rho-1}\sum_n \lambda_{int} d\log S_{int}\right].$$

Using balanced trade (which implies $s_{it}\sum_n \lambda_{int} = s_{nt}\sum_n \lambda_{nit}$), and $d\log S_{int} = -d\log S_{nit}$, we have $\sum_i s_{it} \sum_n \lambda_{int} d \log S_{int} = 0$, so

$$\sum_i s_{it} d \log C_{it} = -\sum_i s_{it} \sum_n \lambda_{int} d \log \tau_{int}.$$

Substituting the definition of s_{it}, we obtain expression (22).

We now calculate the change in world real GDP to marginal changes in variable trade costs. The aggregate PPI defined in (6) between $t-1$ and t, using (11), is given by

$$\frac{PPI_{it}}{PPI_{it-1}} = \sum_n \bar{\lambda}_{int} \frac{PPI_{int}}{PPI_{int-1}}.$$

Log-differentiating around $\boldsymbol{\tau} = \boldsymbol{\tau}_0$,

$$\begin{aligned} d \log PPI_{it} &= \sum_n \lambda_{int} d \log PPI_{int} + \sum_n d\bar{\lambda}_{int} \\ &= \sum_n \lambda_{int} d \log PPI_{int} \end{aligned}$$

where we used $\sum_n \bar{\lambda}_{int} = 1$ (which implies $\sum_n d\bar{\lambda}_{int} = 0$). Changes in trade shares by continuing producers have no first-order effects on the PPI. Hence, the change in the PPI is to a first-approximation equal to that in the model with a fixed set of producers selling in each country. Following the steps used in the model with exogenous specialization, the change in world real GDP is given by expression (22).

Finally, consider changes in real consumption. From equation (14), and the definition of CPI_{int}/CPI_{int-1}, the aggregate CPI in country i between period $t-1$ and t is given by:

$$\frac{CPI_{it}}{CPI_{it-1}} = \sum_n \bar{\Lambda}_{nit} \frac{CPI_{nit}}{CPI_{nit-1}},$$

where $\bar{\Lambda}_{nit} = \int_{\Omega^c_{nit}} p_{nit_0} q_{nit_0} \mathrm{d}M_{it_0} / \left[\sum_n \int_{\Omega^c_{nit}} p_{nit_0} q_{nit_0} \mathrm{d}M_{it_0} \right]$ is the date t_0 share of country i's expenditures on goods from country n for goods that are consumed in both periods. Note that with a constant set of consumed goods, $\bar{\Lambda}_{nit} = E_{nit_0}/E_{it_0}$. Log-differentiating around $\boldsymbol{\tau} = \boldsymbol{\tau}_0$,

$$\begin{aligned} d \log CPI_{it} &= \sum_n \frac{E_{nit}}{E_{it}} d \log CPI_{nit} + \sum_n d\bar{\Lambda}_{nit} \\ &= \sum_n \frac{E_{nit}}{E_{it}} d \log CPI_{nit}, \end{aligned}$$

where we used $\sum_n \bar{\Lambda}_{nit} = 1$ (which implies $\sum_n d\bar{\Lambda}_{nit} = 0$). Changes in expenditure shares due to changes in the set of consumed goods have no first-order effects on the CPI. Hence, the change in the CPI is to a first-approximation equal to that in the model with a fixed set of consumed goods. Following the steps used in the model with exogenous specialization, the change in world real consumption under balanced trade is given by expression (22).

Appendix E: Proof of Result 6

We now show our Result 6 on the equivalence between real consumption and theoretical consumption, country-by-country, when fixed cost of exporting are paid in the destination country ($\phi = 1$ in the notation of Appendix C), and the productivity distribution of entering firms is Pareto ($G_i(z) = 1 - z^{-\theta}$ for $z \geq 1$). We assume that trade is balanced every period, taking into account the export of goods and the export of fixed trade costs that foreign firms incurred in the domestic economy.

We start by showing that with Pareto distributed productivities, balanced trade in any country implies balanced trade both in fixed export cost services and in goods in that country. The condition of balanced trade in country i is:

$$\sum_{n \neq i} Y_{int} + \sum_{n \neq i} \frac{M_{Ent}}{\delta} W_{it} \left[1 - G_{it}(\bar{z}_{nit})\right] f_{nit} = \sum_{n \neq i} Y_{nit} + \sum_{n \neq i} \frac{M_{Eit}}{\delta} W_{nt} \left[1 - G_{it}(\bar{z}_{int})\right] f_{int}. \tag{A18}$$

Substituting $(A12)$ into $(A18)$ implies:

$$\sum_{n \neq i} Y_{int} = \sum_{n \neq i} Y_{nit} \ , \tag{A19}$$

which is the condition of balanced trade in goods.

We now derive Result 6. Balanced trade in services implies $Y_{it} = GDP_{it}$. Then, log-differentiating equation (30) with respect to changes in $\boldsymbol{\tau}$ around the initial steady-state at time t yields:

$$d \log GDP_{it}/W_{it} = \sum_n Y_{int} d \log Y_{int}/W_{it} = 0. \tag{A20}$$

Using $\frac{W_{it}}{P_{it}} = \frac{C_{it}}{\bar{L}_i}(1 - \kappa_i)$ from (30) and balanced trade in goods we can re-express Y_{int} as:

$$\begin{aligned} Y_{int} &= \int_{\Omega_{int}} p_{int} q_{int} \mathrm{d}M_{it} \\ &= \bar{\varphi} \frac{M_{Eit}}{\delta} W_{it} \tau_{int}^{1-\rho} S_{int} Z_{int} C_{it}^{1-\rho} \ , \end{aligned}$$

where $\bar{\varphi} \equiv \left(\frac{\bar{L}_i}{1-\kappa_i}\right)^\rho / \left[\rho^{\rho-1}\left[\rho-1\right]^{1-\rho}\right]$ and $S_{int} = \frac{P_{nt}^\rho C_{nt}}{P_{it}^\rho C_{it}}$ as in Appendix D. Log-differentiating we obtain:

$$\begin{aligned} d\log Y_{int}/W_{it} &= (1-\rho)\, d\log\tau_{int} + d\log S_{int} \\ &\quad + d\log Z_{int} + (1-\rho)\, d\log C_{it} \ , \end{aligned} \tag{A21}$$

substituting into $(A20)$, we can write the change in welfare based consumption as:

$$d\log C_{it} = -\sum_n \frac{Y_{int}}{Y_{it}}\left[d\log\tau_{int} + \frac{d\log S_{int}}{1-\rho} + \frac{d\log Z_{int}}{1-\rho}\right] \ .$$

Log-differentiating $(A19)$, substituting $(A21)$, and some algebra gives:

$$\sum_n Y_{int}\left[d\log\tau_{int} + \frac{d\log S_{int}}{1-\rho} + \frac{d\log Z_{int}}{1-\rho}\right] = \sum_n Y_{nit}\left[d\log\tau_{nit} + d\log\frac{W_{nt}}{W_{it}} + \frac{d\log Z_{nit}}{1-\rho}\right] \ ,$$

then:

$$d\log C_{it} = -\sum_n \frac{Y_{nit}}{Y_{it}}\left[d\log\tau_{nit} + d\log\frac{W_{nt}}{W_{it}} + \frac{d\log Z_{nit}}{1-\rho}\right] \ . \tag{A22}$$

Finally, using the Pareto form for G, we have:

$$Z_{int} = \frac{\theta}{\theta+1-\rho}\bar{z}_{int}^{\rho-1-\theta} \ , \tag{A23}$$

log differentiating $(A11)$ and $(A23)$ we obtain:

$$d\log Z_{nit} = (\rho-1-\theta)\left[d\log\tau_{nit} + d\log W_{nt}/W_{it} + d\log C_{it}\right] \ . \tag{A24}$$

Substituting $(A24)$ into $(A22)$ and using balanced trade in goods:

$$d\log C_{it} = -\sum_n \frac{E_{nit}}{E_{it}}\left[d\log\tau_{nit} + d\log W_{nt}/W_{it}\right] \ ,$$

where $E_{int} = Y_{int}$.

As shown in Appendix D, the change in the CPI is given by $d\log CPI_{it} = \sum_n \frac{E_{nit}}{E_{it}} d\log CPI_{nit}$, so the change in real consumption is given by expression (18). Substituting $d\log CPI_{int}$ in equation (18), we obtain:

$$d\log RC_{it} = \sum_n \frac{E_{nit}}{E_{it}}\left[-d\log\tau_{nit} - d\log W_{nt}/W_{it}\right], \tag{A25}$$

which coincides with $d\log C_{it}$.

Appendix F: Endogenous quality choice

In this appendix we consider the extended model with endogenous quality choice under endogenous specialization and imperfect competition. We first derive the result that, if prices in the deflators are not adjusted for changes in quality, then changes in world real consumption and theoretical consumption are equal, to a first-order approximation, in response to marginal changes in trade costs. The logic to obtain this result is very similar to that used to obtain Result 5 in Appendix D. Next, we derive condition (29) in this version of our model.

Following the same steps as those used to derive expression ($A15$), variable profits (relative to the wage) for a firm from country i with productivity z selling in country n are given by

$$\frac{\pi_{int}(z)}{W_{it}} = \frac{a_{int}(z)\, z^{\rho-1}\tau_{int}^{1-\rho}}{\rho^{\rho}(\rho-1)^{1-\rho}}\left(\frac{\bar{L}_i}{1-\kappa_i}\right)^{\rho} C_{it}^{1-\rho} S_{int},$$

where $a_{int}(z)$ denotes the quality choice of a firm in country i with productivity z selling in country n in period t. In an interior equilibrium with selection, the cutoff $\bar{z}_{int}$ is given by $\frac{\pi_{int}(\bar{z}_{int})}{W_{it}} - f_{int} - h(z; a_{int}(\bar{z}_{int})) = 0$. Profits (relative to the wage) in period t across all destinations, inclusive of fixed costs and quality costs are given by

$$\sum_n \mathbb{I}(z \geq \bar{z}_{int})\left[\frac{a_{int}(z)\, z^{\rho-1}\tau_{int}^{1-\rho}}{\rho^{\rho}(\rho-1)^{1-\rho}}\left(\frac{\bar{L}_i}{1-\kappa_i}\right)^{\rho} C_{it}^{1-\rho} S_{int} - f_{int} - h(z; a_{int}(z))\right],$$

where $\mathbb{I}(z \geq \bar{z}_{int}) = 1$ if $z \geq \bar{z}_{int}$ and zero otherwise. The static first-order condition for $a_{int}(z)$ is given by

$$\frac{a_{int} z^{\rho-1}\tau_{int}^{1-\rho}}{\rho^{\rho}(\rho-1)^{1-\rho}}\left(\frac{\bar{L}_i}{1-\kappa_i}\right)^{\rho} C_{it}^{1-\rho} S_{int} - h_2(z; a_{int}) = 0,$$

where $h_2(z; a_{int})$ denotes the derivative of h with respect to the second argument.

The free-entry condition in steady-state is given by

$$\hat{\beta}\sum_n \int_{\Omega_{int}} \frac{\pi_{int}(z)}{W_{it}} dG_i(z) = f_{Ei} + \hat{\beta}\sum_n [1 - G_i(\bar{z}_{int})]\, f_{int} + \int_{\Omega_{int}} h(z; a_{int}(z))\, dG_i(z). \quad \text{(A26)}$$

Log-differentiating the free-entry condition in the steady-state, using the first-order conditions for $\bar{z}_{int}$ and $a_{int}(z)$, we obtain the same expression for the change in theoretical consumption, ($A17$), as in the model without quality choice, where $Z_{int} = \int_{\Omega_{int}} a_{int}(z)\, z^{\rho-1} \mathrm{d}G_i$. That is, from the envelope conditions, changes in cutoffs and quality choices have no first-order effects on expected profits of entering firms. From expression $(A17)$, we use the same steps as those used in Appendix D to obtain expression (22).

The extension of Result 6 (the equivalence between real consumption and theoretical consumption, country-by-country) under stronger assumptions, is derived in the Online Appendix.

Deriving the share of profits in total revenues

Deriving assumption (29) when $\beta \longrightarrow 1$ is straightforward. We now show that this assumption holds if $\beta < 1$ when productivites are Pareto distributed, quality is destination-country specific, the cost of choosing quality a for a firm with productivity z is $h(z,a) = \frac{\gamma_0}{\gamma}\bar{h}(z)\,a^{\gamma}$, and a fraction ε of the innovation costs are incurred in the source country and the remaining fraction $(1-\varepsilon)$ are incurred in the destination country. We omit time subscripts to simplify notation.

Under these assumptions, the optimal quality choice a_{in} for a firm with productivity z satisfies:

$$\pi_{in}(z) = \gamma_0 W_i^{\varepsilon} W_n^{1-\varepsilon}\bar{h}(z)\,a_{in}^{\gamma}(z)\,.$$

Aggregate innovation costs, using the optimality condition and $\pi_{in} = \frac{1}{\rho-1}W_i l_{in}$, are:

$$\begin{aligned}
\frac{M_{Ei}}{\delta}\sum_n \int_{\Omega_{in}} \frac{\gamma_0}{\gamma} W_i^{\varepsilon} W_n^{1-\varepsilon}\bar{h}(z)\,a_{in}^{\gamma}(z)\,dG_i(z) &= \frac{1}{\gamma(\rho-1)}\frac{M_{Ei}}{\delta}\sum_n \int_{\Omega_{in}} W_i l_{in}(z)\,dG_i(z) \\
&= \frac{1}{\gamma(\rho-1)} W_i L_i.
\end{aligned}$$

The relation between variable and fixed labor costs is still given by equation $(A13)$. Aggregate entry costs, calculated using $(A26)$, $(A7)$ and $(A13)$, are

$$\begin{aligned}
W_i f_{Ei} M_{Ei} &= \frac{\beta\delta}{1-\beta(1-\delta)}\left[\begin{array}{c} Y_i - W_i L_i - \sum_n \frac{\gamma_0}{\gamma} W_i^{\varepsilon} W_n^{1-\varepsilon}\int_{\Omega_{in}} a_{in}^{\gamma}(z)\,\bar{h}(z)\,dG_i(z) \\ -\sum_n W_i^{\phi} W_n^{1-\phi}\left[1-G_i(\bar{z}_{in})\right] f_{in} \end{array}\right] \\
&= \frac{\beta\delta}{1-\beta(1-\delta)}\frac{\gamma(\rho-1)-\theta}{\gamma\theta}\frac{1}{\rho-1}W_i L_i. \qquad \text{(A27)}
\end{aligned}$$

Finally, combining $(A7)$, $(A13)$ and $(A27)$, aggregate profits are

$$\begin{aligned}
\Pi_i &= Y_i - W_i L_i - W_i M_{Ei} f_{Ei} \\
&\quad -\frac{M_{Ei}}{\delta}\left[\sum_n \frac{\gamma_0}{\gamma} W_i^{\phi} W_n^{1-\phi}\left[1-G_i(\bar{z}_{in})\right] f_{in} + \sum_n W_i^{\varepsilon} W_n^{1-\varepsilon}/\gamma \int_{\Omega_{in}} a_{in}^{\gamma}(z)\,\bar{h}(z)\,dG_i(z)\right] \\
&= \frac{\gamma(\rho-1)-\theta}{\gamma\theta\rho}\frac{1-\beta}{1-\beta(1-\delta)}Y_i
\end{aligned}$$

Therefore, $\kappa_i = \left(\frac{\gamma(\rho-1)-\theta}{\gamma\theta\rho}\right)\left(\frac{1-\beta}{1-\beta(1-\delta)}\right)$. The steady-state mass of entering firms is given by

$$M_{Ei} = \frac{\beta\delta}{1-\beta\left[1-\delta\right]}\frac{\gamma\left[\rho-1\right]-\theta}{\gamma\theta}\frac{\bar{L}_i}{f_{Ei}\left(1-\kappa_i\right)\rho}$$

where we used $(A7)$, $(A8)$, and $(A27)$. Hence, the mass of entrants M_{Ei} does not change in response to permanent changes in variable or fixed trade costs. Therefore, there are no transition dynamics to the new steady-state.

Finally, aggregate variable profits gross of entry costs are $\Pi_i + W_i f_{Ei} M_{Ei} = \frac{\gamma(\rho-1)-\theta}{\gamma\theta} Y_i$. Hence, in the model with restricted entry (so that there are no entry costs), equation (29) holds with $\kappa_i = \left[\gamma\left(\rho-1\right)-\theta\right]/\gamma\theta$.

Appendix G: Multiple factors of production

In this appendix we derive some results in the extension of the model that allows for multiple factors of production. We first show that GDP is proportional to total labor payments, and we then derive the equivalence between world real GDP, real consumption, and theoretical consumption. As in the baseline model, we assume that condition (29) is satisfied – it is straightforward to extend the proofs in Appendix C to this extension–.

We first show that production labor is proportional to aggregate labor supply, and that current-dollar GDP is proportional to aggregate labor payments. We also show that theoretical consumption is proportional to the aggregate production of the final good. We can write a modified version of equation $(A3)$ as

$$p_{int}q_{int} = \frac{\rho}{\rho-1}\alpha_L^{-1} W_{it} l_{int} \ ,$$

where we used (33) and the production function. Total revenues of active firms in country i are given by $Y_{it} = \frac{\rho}{\rho-1} L_{it} W_{it}/\alpha_L$, where L_{it} denotes aggregate labor used in variable production defined above. In the presence of intermediate inputs, total revenues are given by:

$$Y_{it} = W_{it}\bar{L}_i + \Pi_{it} + \sum_{j=1}^{J} R_{j,it} K_{it} \ .$$

In combination with (29), this implies, $Y_{it} = \frac{1}{1-\kappa_i}\left[W_{it}\bar{L}_i + \sum_{j=1}^{J} R_{j,it} K_{it}\right]$, or

$$Y_{it} = \frac{L_{it}W_{it}}{1-\kappa_i}\left[\frac{\bar{L}}{L_{it}} + \sum_i \frac{R_{j,it}K_{j,it}}{W_{it}L_{it}}\right] = \frac{L_{it}W_{it}}{1-\kappa_i}\left[\frac{\bar{L}}{L_{it}} + \frac{1-a_L}{\alpha_L}\right].$$

In combination with $Y_{it} = \frac{\rho}{\rho-1} L_{it} W_{it} / \alpha_L$, we obtain that variable production labor is a constant share of total labor,

$$\frac{\bar{L}}{L_{it}} = \frac{\rho\left(1-\kappa_i\right)-\left(1-\alpha_L\right)(\rho-1)}{\alpha_L(\rho-1)},$$

and that revenues are proportional to aggregate wages,

$$Y_{it} = \frac{L_{it} W_{it}}{1-\kappa_i}\left[\frac{\bar{L}_i}{L_{it}}+\frac{1-a_L}{\alpha_L}\right] = \frac{\rho}{\left[1-\kappa_i\right] \rho-\left(1-a_L\right)(\rho-1)} W_{it} \bar{L}_i. \tag{A28}$$

Note that intermediate inputs are also proportional to aggregate revenues:

$$\left[1-\kappa_i\right] Y_{it}-W_{it} \bar{L}_i = \sum_{j=1}^J R_{j,it} K_{it}$$

so $Y_{it} = \frac{\rho}{\left(1-a_L\right)(\rho-1)} \sum_{j=1}^J R_{j,it} K_{it}$. Finally, note that together with balanced trade this implies that consumption expenditures are proportional to aggregate labor payments and to expenditures in intermediate inputs:

$$Y_{it} = P_{it} Q_{it} = P_{it} C_{it}+\sum_{j=1}^J R_{j,it} K_{it} = \frac{\rho}{\rho-\left(1-\alpha_L\right)(\rho-1)} P_{it} C_{it} ,$$

so

$$Q_{it} = \frac{\rho}{\rho-\left(1-\alpha_L\right)(\rho-1)} C_{it} . \tag{A29}$$

We now show the equivalence, to a first-order approximation, between world GDP, measured real consumption, and theoretical consumption. Variable profits are given by

$$\pi_{int}(z) = \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho[\rho-1]^{1-\rho}} c_{it}\left[\frac{c_{it}}{P_{nt}}\right]^{-\rho} Q_{nt}. \tag{A30}$$

The threshold $\bar{z}_{int}$ satisfies $\pi_{int}\left(\bar{z}_{int}\right) = W_{it} f_{int}$. Expected profits at entry, using equation $(A30)$ are given by:

$$\begin{aligned}
\sum_n \int_{\Omega_{int}} \pi_{int}(z) \mathrm{d} G_i(z) & = c_{it} \sum_n \int_{\Omega_{int}} \frac{z^{\rho-1} \tau_{int}^{1-\rho}}{\rho^\rho[\rho-1]^{1-\rho}} c_{it}^{-\rho} \Gamma_{nt} \mathrm{d} G_i(z) \\
& = \left[c_{it} / W_{it}\right]^{1-\rho} \frac{W_{it} Q_{it}}{\rho^\rho[\rho-1]^{1-\rho}}\left[\frac{W_{it}}{P_{it}}\right]^{-\rho} \sum_n \tau_{int}^{1-\rho} \int_{\Omega_{int}} z^{\rho-1} S_{int} \mathrm{d} G_i(z) ,
\end{aligned}$$

where $\Gamma_{nt} = P_{nt}^{\rho} Q_{nt}$, and $S_{int} = \frac{\Gamma_{nt}}{\Gamma_{it}}$. Using $(A28)$ and $(A29)$ we can write this as:

$$\sum_n \int_{\Omega_{int}} \pi_{int} \mathrm{d}G_i(z) = W_{it} \frac{[c_{it}/W_{it}]^{1-\rho} \bar{\psi} C_{it}^{1-\rho} \bar{L}_i^{\rho}}{\rho^{\rho} [\rho-1]^{1-\rho}} \sum_n \tau_{int}^{1-\rho} \int_{\Omega_{int}} z^{\rho-1} S_{int} \mathrm{d}G_i(z) \ , \tag{A31}$$

where $\bar{\psi} = \frac{\rho}{\rho-1} \frac{\left[[1-\kappa_i]\frac{\rho}{\rho-1} - 1 + a_L\right]^{-\rho}}{[1/(\rho-1)+\alpha_L]^{1-\rho}}$ is a constant. Free-entry in steady-state implies:

$$\hat{\beta} \sum_n \int_{\Omega_{int}} \pi_{int}(z) \mathrm{d}G_i(z) = W_{it} f_{Ei} + \hat{\beta} \sum_n W_{it} [1 - G_i(\bar{z}_{int})] f_{int},$$

which can be rewritten as

$$[c_{it}/W_{it}]^{1-\rho} \frac{\bar{\kappa} C_{it}^{1-\rho} \bar{L}_i^{\rho}}{\rho^{\rho} [\rho-1]^{1-\rho}} \sum_n \tau_{int}^{1-\rho} S_{int} Z_{int} = \frac{f_{Ei}}{\hat{\beta}} + \sum_n [1 - G_i(\bar{z}_{int})] f_{int}.$$

Log-differentiating this expression in steady-state at time t and using the envelope condition for the cutoffs yields

$$d \log C_{it} = -\frac{\sum_n \left[d \log \tau_{int} - \frac{1}{\rho-1} d \log S_{int}\right] \tau_{int}^{1-\rho} S_{int} Z_{int}}{\sum_n \tau_{int}^{1-\rho} S_{int} Z_{int}} - d \log [c_{it}/W_{it}] \ .$$

Note that $\lambda_{int} = \frac{\tau_{int}^{1-\rho} S_{int} Z_{int}}{\sum_n \tau_{int}^{1-\rho} S_{int} Z_{int}}$, so the change in world theoretical consumption using the weights s_{it} defined above is:

$$\sum_i s_{it} d \log C_{it} = \sum_i s_{it} \left[\begin{array}{c} -\sum_n \lambda_{int} d \log \tau_{int} \\ +\frac{1}{\rho-1} \sum_n \lambda_{int} d \log S_{int} - d \log c_{it}/W_{it} \end{array} \right] .$$

Following the steps in Appendix C, we can show that $\sum_i s_{it} \sum_n \lambda_{int} d \log S_{int} = 0$, which implies:

$$\sum_i s_{it} d \log C_{it} = -\sum_i s_{it} \left[\sum_n \lambda_{int} d \log \tau_{int} + d \log c_{it}/W_{it} \right] .$$

Log differentiating the expression in (36) and doing the weighted sum across countries we obtain the equivalence, to a first-order approximation, between world theoretical consumption and world real GDP. To show the equivalence between world real GDP and world real consumption we follow the same steps used in our baseline model, together with the fact that current-dollar GDP is proportional to current-dollar consumption.

Note that the expression for the change in world real GDP, real consumption, and theoretical consumption differs from that in the model with only labor as a factor of production, in the presence of changes in the marginal cost to wage ratio, c_{it}/W_{it}. From expression (34), changes in this price ratio are driven by changes in aggregate quantities of non-labor factors of production.

The extension of Result 6 (the equivalence between real consumption and theoretical consumption, country-by-country) under stronger assumptions, is derived in an Online Appendix.