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ABSTRACT

This paper studies optimal policy in a business-cycle setting in which firms have a blurry understanding of the state of the economy due to informational or cognitive constraints. The latter are not only the source of nominal rigidity but also an impediment in the coordination of production. The optimal allocation thus differs from familiar Ramsey benchmarks (Lucas and Stokey, 1983; Correia, Nicolini, and Teles, 2008) in manners that may be misinterpreted as a call for macroeconomic stabilization. Furthermore, conventional policy instruments serve new functions: they manipulate the firms' collection and use of information or their cognitive efforts. Despite these facts, the optimal taxes are similar to those in the aforementioned benchmarks and the optimal monetary policy replicates flexible-price allocations. On the other hand, the rationale for price stability falls apart: replicating flexible-price allocations and minimizing relative-price distortions become equivalent to a certain form of "leaning against the wind".

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1 Introduction

How should fiscal and monetary policy respond to business cycles? The literature has studied this question extensively, but typically only under strong assumptions about what economic agents know and how well they can comprehend what's going on in the economy.[1] In particular, the Ramsey and the New-Keynesian frameworks alike assume that agents have common knowledge of the underlying aggregate shocks and their consequences, of how deep or long a recession might be, and so on.

In this paper, we depart from this tradition by letting firms have a blurry understanding of the state of the economy due to informational, or cognitive, frictions. Crucially, we let such frictions interfere with both the firms' price-setting decisions and their input choices. We thus accommodate both a form of nominal rigidity and an imperfection in the coordination of production.

We then proceed to make two contributions: one methodological and one applied. On the methodological front, we extend the primal approach of the Ramsey literature, and the results of Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1994) and, especially, Correia, Nicolini, and Teles (2008), to settings that accommodate informational, cognitive, and coordination frictions. On the applied front, we highlight how conventional policy instruments can serve new functions; we study how the considered frictions affect the socially optimal allocation and the policies that implement it; and finally we explain why the rationale for the desirability of price stability is turned upside down once the information, or cognitive, constraints of the firms are taken into consideration.

Background. The frictions considered in this paper are easily motivated. Even if arbitrarily rich data is readily available in the public domain, agents may lack the time and the cognitive capacity needed for fully digesting all the available data. A number of authors have thus argued that the accommodation of noisy, and heterogeneous, information need not be interpreted too literally; instead, it can be the formal *representation* of the difficulty that the agents face in comprehending what's going on in the economy and in coordinating their behavior with that of others.[2]

Such frictions offer a compelling *substitute* to Calvo-like sticky prices (Mankiw and Reis, 2002; Woodford, 2003a; Mackowiak and Wiederholt, 2009), as well as a powerful *complement* to them (Nimark, 2008). They help explain the observed inertia in the response employment and output to technology shocks (Angeletos and La'O, 2010), the volatility of unemployment (Venkateswaran, 2014), and other salient features of the business-cycle data (Lorenzoni, 2009; Angeletos, Collard, and Dellas, 2015). They impede coordination (Morris and Shin, 2002, 2003), attenuate general-equilibrium effects (Angeletos and Lian, 2017), and open to door to forces that resemble animal spirits (Angeletos and La'O, 2013; Benhabib, Wang, and Wen, 2015; Huo and Takayama, 2015). They make the economy behave *as if* the agents were myopic (Angeletos and Lian, 2016a), offering a resolution to some of the paradoxical predictions of the New-Keynesian framework. Last but not least, the assumed frictions are consistent with the observed heterogeneity in forecasts and their response to shocks (Mankiw, Reis, and Wolfers, 2004; Coibion and Gorodnichenko, 2015).

[1] Ball, Mankiw, and Reis (2005), Adam (2007) and Paciello and Wiederholt (2014) are exemptions, which are discussed in due course.

[2] See, *inter alia*, Angeletos and Lian (2017, 2016b), Morris and Shin (2002, 2003), Sims (2003, 2010), Tirole (2015), and Woodford (2003a, 2009).

Despite these important advances, the normative implications of the considered frictions are less well understood. To the best of our knowledge, our paper is indeed the first to study how the policy lessons of the Ramsey and New-Keynesian paradigms are affected by letting firms face such frictions when making *both* their production and price-setting decisions.[3]

Framework. Our framework resembles those found in either the Ramsey literature on optimal fiscal policy (Lucas and Stokey, 1983; Chari, Christiano, and Kehoe, 1994) or the related New-Keynesian literature on optimal monetary policy (Correia, Nicolini, and Teles, 2008). It features a representative household, centralized markets, and a continuum of monopolistically competitive firms, each producing a differentiated commodity that enters the production of the final good. It also allows the planner to control two kinds of policy instruments: a rich set of linear taxes (set by the fiscal authority) and the nominal interest rate (set by the monetary authority).

There are, however, three key differences between our framework and the aforementioned works. First, the price of each firm is measurable in a noisy, private signal of the state of Nature. Second, some of the firm's input choices must also be measurable in the aforementioned signal. Third, the stochastic structure of that signal is flexible and can be endogenously chosen by the firm.

The first feature, which is common to Woodford (2003a), Mankiw and Reis (2002), and Mackowiak and Wiederholt (2009), represents a form of *nominal* rigidity. Although this feature offers an appealing alternative to sticky prices and menu costs, it does not alone upset the key normative lessons of the New-Keynesian paradigm: when the only modification is in the formalization of the nominal rigidity, the results of Correia, Nicolini, and Teles (2008) remain intact.

The second feature, which is novel *vis-□ -vis* all the aforementioned works, introduces a *real* friction. Because each firm must fix some of her inputs on the basis of a blurry and idiosyncratic understanding of the underlying state of Nature and of the equilibrium choices of the other firms, production can no more be perfectly coordinated across the firms, regardless of policy and regardless of whether nominal rigidity interferes with the workings of the price mechanism. As we explain in due course, it is this kind of imperfection in the coordination of production, and only this, which is responsible for the novelty of lessons we deliver in this paper.

The third feature is useful for two reasons. First, it allows us to nest a variety of information specifications that have appeared in prior work and to establish our results with a high degree of generality. Second, it facilitates the reinterpretation of the assumed private signals as the product of "rational inattention" (Sims, 2003, 2010), or as the "cognitive states" that represent how well the agents comprehend what's going on in the economy and how to best respond (Tirole, 2015).

Methods and Results. The richness of our framework precludes a closed-form solution of the equilibrium regardless of policy. Although such tractability has played a central role in prior work, it is neither necessary nor useful for our purposes. To the contrary, by allowing for an essentially arbitrary specification of the firms' signals and by showing how one can adapt the primal approach from the Ramsey literature to the environments of interest, we are able to not only bypass the need for tractability, but also deliver the key lessons with a high level of transparency.

[3] Note the emphasis on "both": Ball, Mankiw, and Reis (2005), Adam (2007) and Paciello and Wiederholt (2014) have studied optimal monetary policy in settings that allow the informational friction to impede the adjustment of nominal prices but assume it away from the firms' production decisions.

We thus start by characterizing the entire set of allocations that can be implemented as market-based equilibria with the help of the available policy instruments. For pedagogical reasons, we do so under two scenarios. The one allows the information, or cognitive, constraints to be the source of both nominal and real friction, in the sense explained above. The other shuts down the nominal friction by letting the price of each firm be contingent on the true state of Nature. Although the second scenario is less realistic and precludes monetary policy from having real effect, it is instrumental for understanding, not only the optimal taxes, but also the optimal monetary policy under the first scenario. Adopting, or perhaps paraphrasing, the terminology used in the New-Keynesian literature, we refer to the allocations that are implementable under the first scenario as "sticky-price allocations" and to the ones under the second scenario as "flexible-price allocations".

We next proceed to characterize the solution of a *relaxed* problem, which allows the planner to directly control how each firm maps her private signal to her actions. This relaxed problem resembles the one studied in Correia, Nicolini, and Teles (2008), except for one key difference: the heterogeneity of the signals and the associated imperfection in the coordination of production precludes the planner from attaining either the first best or the kind of second best that is relevant in that paper and in the Ramsey literature more generally (Lucas and Stokey, 1983; Chari, Christiano, and Kehoe, 1994). That said, by characterizing the solution to this relaxed problem and by showing that this solution belongs to the (appropriately redefined) sets of the flexible- and sticky-price allocations, we are able to shed ample light on the nature of the optimal allocation and of the combination of taxes and monetary policy that implement that allocations as an equilibrium.

The following lessons emerge:

- Familiar policy instruments serve new functions: they help the planner manipulate not only how firms act on the basis of their idiosyncratic knowledge of the state of economy but also how much attention they pay to the ongoing economic conditions or how much cognitive effort they put in understanding how they should respond.

- Because of the underlying friction, the observable properties of the optimal allocation differ from the relevant benchmarks identified in Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1994), and Correia, Nicolini, and Teles (2008). The difference is evident in both the cross section of the firms and the aggregate time series. In the cross section, the planner affords some dispersion in marginal products in order to allow each firm to utilize her own private information, or to do what is best given her cognitive abilities. In the time series, the planner allows the economy to vary with shocks that resemble "animal spirits" or "sentiments" as formalized in Angeletos and La'O (2013) and Benhabib, Wang, and Wen (2015).

- Despite the aforementioned novelties in the nature of the optimal allocation and in the functions of the tax instruments, the optimal tax policy remains the same as in Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994). For a familiar class of preferences and technologies, the optimal wedges, and hence also the optimal taxes, are indeed invariant with the state of the economy. This is because once the policy has been set so as to balance the underlying tax and monopoly distortions there is no further welfare gain—in fact, there is typically a strict welfare loss—from trying to manipulate how firms respond to their signals.

- The optimal monetary policy replicates flexible prices. This extends a result from Correia, Nicolini, and Teles (2008) to the environments we are interested in. As in that paper and contrary to what may be suggested by textbook treatments of the New-Keynesian framework, the optimality of replicating flexible-price allocations holds true despite the fact that the distortion relative to the first best is non-zero and may even vary with the business cycle. The reason is that, at least with the allowed tax instruments, the set of flexible-price allocations contains the solution to the relaxed planning problem described above.

- The optimal monetary policy does not induce price stability. Instead, it induces a negative relation between the price level and real economic activity. This result holds despite the fact that the underlying flexible-price allocations *can* and *should* be replicated. It is therefore orthogonal to the more conventional arguments that justify a departure from price stability either by preventing the replication of the optimal flexible-price allocation[4] or by letting monetary policy substitute for missing tax instruments.[5] Instead, it is a direct implication of letting the firms make the best possible use of their idiosyncratic knowledge or understanding of what's going on in the economy and of how much they should produce.

Our last result can thus be read as a *revision* of the so-called "divine coincidence." On the one hand, we preserve "divine coincidence" in the sense that, in our setting, the replication of flexible prices achieves two goals at once: first, it eliminates the output gap relative to an appropriate reference point; and second, it minimizes relative-price distortions or maximizes production efficiency, properly defined. On the other hand, we turn "divine coincidence" on its head by equating the second goal, and the replication of flexible prices, with a certain departure from price stability.

Relatedly, we also qualify the reference point relative to which the "output gap" has to be measured. It is neither the first best that appears in textbook treatments of the New-Keynesian framework, nor the type of second best studied in Lucas and Stokey (1983) and Correia, Nicolini, and Teles (2008). Instead, it is a third best that incorporates the underlying informational/cognitive friction and as a result may display "exotic" observable properties. The definition and the characterization of this reference point are integral parts of our contribution.

Layout. The rest of the paper is organized as follows. Section 2 sets up our framework and discusses the key assumptions that differentiate our paper from the previous literature. Section 3 defines the sticky-price and flexible-price scenarios that are appropriate to consider in our context. Section 4 characterizes and compares the set of allocations that can be implemented as market-based equilibria in each of these two scenarios. Section 5 defines and characterizes the relaxed planning problem that helps identify the optimal allocation; it also derives our key lessons regarding optimal taxes and optimal monetary policy. Section 6 endogenizes the information or cognitive friction. Section 7 concludes. The Appendix contains all proofs as well as a tractable example that illustrates some of the broader insights in a sharper form.

[4] E.g., by combining sticky prices with sticky wages, or by restricting the policy maker to follow a Taylor rule that is not sufficiently sophisticated (although perhaps more realistic).

[5] E.g., by introducing markup shocks (shortcuts for forces that trigger inefficient business cycles under flexible prices) and by preventing the planner from offsetting these shocks with the "right" tax instruments.

2 The Framework

In this section, we introduce our framework. We first describe the components of the environment that are invariant to the information structure. We next formalize the informational friction and its two facets (the nominal and the real). We finally comment on some of the distinct qualities of our framework, as well as on some of its limitations.

2.1 The underlying environment

Time is discrete and periods are indexed by $t \in \{0, 1, 2, ...\}$. There is a representative household, which pools all the income in the economy and makes the consumption, capital accumulation, and labor supply decisions. There is a continuum of monopolistically competitive firms, indexed by $i \in I = [0, 1]$. These firms produce differentiated goods, which are used by a competitive retail sector as intermediate inputs into the production of a final good. The latter in turn can be used for three purposes: as consumption; as investment into capital; or as an intermediate input in the production of the differentiated goods. Finally, there is a government, which lacks lump-sum taxation but can levy a variety of distortionary taxes and can issue both a contingent and non-contingent debt.

States of Nature. In each period t, Nature draws a random variable s_t from a finite set S_t. This variable is meant to contain not only innovations in aggregate TFP and government spending but also any other aggregate aggregate innovation in the cross-section of the signals upon which the firms can act (more on this below). The aggregate state of the economy, or the "state of Nature," in period t is comprised by the history of draws of s_τ for all $\tau \in \{0, ..., t\}$. The state is therefore an element of $S^t \equiv S_0 \times ... \times S_t$ and is henceforth denoted by $s^t \equiv (s_0, \dots, s_t)$. Its unconditional probability is denoted by $\mu(s^t)$.

Tax and debt instruments. The government lacks access to both lump-sum taxation and firm-specific taxes. It can nonetheless impose four kinds of economy-wide taxes: a proportional tax on consumption at rate τ_t^c; a proportional tax on labor income at rate τ_t^ℓ; a proportional tax on capital income, net of depreciation, at rate τ_t^k; and a 100% tax on distributed profits. In addition, the government can issue and trade with the representative household two kinds of debt instruments. The first is a one-period, non-contingent, debt instrument that costs 1 dollar in period t and pays out $1 + R_t$ in period t; R_t therefore denotes the nominal interest rate between t and $t+1$. The second is a complete set of state-contingent assets (or Arrow securities). These are indexed by $s \in S^{t+1}$, they cost $Q_{t,s}$ dollars in period t, and they pay out 1 dollar in period $t+1$ if state s is realized and 0 otherwise. Their corresponding quantities are denoted by $D_{t,s}$. The quantity of the non-contingent debt, on the other hand, is denoted by B_t. It follows that the nominal value of all debt issued at the end of period t is $B_t + \sum_{s \in S^{t+1}} Q_{t,s} D_{t,s}$, while the nominal liability of the government in the beginning of period $t+1$ is $(1 + R_t)B_t + D_{t,s^{t+1}}$.

The household. We adopt the following notation: K_t denotes the capital stock accumulated by the end of period t; L_t denotes the labor supply in period t; r_t and w_t denote the pre-tax real values of the rental rate of capital and the wage rate in period t, respectively; C_t and X_t denote the period-t real levels of consumption and investment, respectively; and finally P_t denotes the period-t price level (i.e., the nominal price of the final good). The household's period-t budget constraint can thus be expressed, in nominal terms, as follows:

$$(1+\tau_t^c)P_tC_t+P_tX_t+B_t+\sum_{s\in S(s^t)}Q_{t,s}D_{t,s}=(1-\tau_t^\ell)P_tw_tL_t+(1-\tau_t^k)P_tr_tK_{t-1}+(1+R_{t-1})B_{t-1}+D_{t-1,s^t}$$

The law of motion of the capital stock, on the other hand, is given by

$$K_t=(1-\delta)K_{t-1}+X_t,$$

where $\delta\in[0,1]$ is the depreciation rate of capital. Finally, the household's preferences are given by her expectation of

$$\sum_{t=0}^{\infty}\beta^tU(C_t,L_t,s^t)$$

where $\beta\in(0,1)$ and U is strictly increasing and strictly concave in $(C_t,-L_t)$.

The firms. The intermediate goods, and the monopolistic firms producing them, are indexed by $i\in[0,1]$. Take firm i, that is, the firm that produces variety i. Its output in period t is denoted by y_{it} and is given by

$$y_{it}=A(s^t)F\left(k_{it},h_{it},\ell_{it}\right),$$

where $A(s^t)$ is an exogenous aggregate productivity shock,[6] F is a CRS production function, k_{it} is the capital input, h_{it} is the final-good input (or "materials"), and ℓ_{it} is the labor input. The firm faces a proportional tax on revenue, at rate τ_t^r. Its nominal profit net of taxes is therefore given by

$$\Pi_{it}=(1-\tau_t^r)p_{it}y_{it}-P_tr_tk_{it-1}-P_th_{it}-P_tw_t\ell_{it},$$

where p_{it} denotes the nominal price of the intermediate good i, P_t denotes the nominal price of the final good (also, the price level), and r_t and w_t denote, respectively, the real rental rate of capital and the real wage rate. The final good, in turn, is produced by a competitive retail sector, whose output is a CES aggregator of all the intermediate varieties:

$$Y_t=\left[\int_I(y_{it})^{\frac{\rho-1}{\rho}}di\right]^{\frac{\rho}{\rho-1}},$$

where Y_t denotes the quantity of the final good and $\rho>1$ is the elasticity of substitution across the intermediate varieties.[7]

The government. The government's period-t budget constraint, in nominal terms, is given by

$$(1+R_{t-1})B_{t-1}+D_{t-1,s^t}+P_tG_t=B_t+\sum_{s\in S^{t+1}}Q_{t,s}D_{t,s}+T_t$$

[6]We rule out idiosyncratic productivity shocks mostly for expositional reasons; see Appendix B for an example that accommodates such shocks.

[7]Clearly, the (nominal) profits of the retail sector are given by $P_tY_t-\int_I p_{it}y_{it}di$ and are zero in equilibrium. Also, we could have introduced the aggregate productivity shock in

where $G_t = G(s^t)$ is the exogenous *real* level of government spending and T_t is the *nominal* level of tax revenue, given by

$$T_t = \tau_t^r P_t Y_t + \tau_t^c P_t C_t + \tau_t^\ell P_t w_t L_t + \tau_t^k P_t r_t k_t + \sum_i \Pi_{it} di$$

With some abuse of notation, we let $D_t = (D_{t,s})_{s\in S^{t+1}}$ and $Q_t = (Q_{t,s})_{s\in S^{t+1}}$. We thus identify the fiscal-policy instruments in period t with $(\tau_t^r, \tau_t^\ell, \tau_t^k, \tau_t^c, B_t, D_t)$, the taxes and the debt issuances, and the monetary-policy instrument with R_t, the nominal interest rate. To simplify the exposition and keep the analysis comparable to that of Correia, Nicolini, and Teles (2008), we abstract from the zero lower bound on the nominal interest rate. We finally bypass the issue studied in Straub and Werning (2014) and guarantee the validity of the optimality of a zero tax on capital income by allowing the government to tax fully the initial capital stock.

Market Clearing. Market clearing in the goods market (or, equivalently, the resource constraint of the economy) is given by

$$C_t + H_t + X_t + G_t = Y_t,$$

where $X_t \equiv \int_I x_{it} di$ denotes aggregate investment and $H_t \equiv \int_I h_{it} di$ denotes the aggregate quantity of the final good used as intermediate input. Market clearing in the labor market, on the other hand, is given by

$$\int_I \ell_{it} di = L_t.$$

2.2 The informational, or cognitive, friction

Throughout, we let the aggregate quantities $(C_t, L_t, X_t, K_t, Y_t)$, the wage w_t, the rental rate r_t, the asset prices Q_t, the aggregate price level P_t, and the policy instruments $(\tau_t^r, \tau_t^\ell, \tau_t^k, \tau_t^c, B_t, D_t, R_t)$ be measurable in s^t, for all t. We next define our "frictionless" or "complete-information" benchmark by the scenario in which the firm-specific variables $(p_{it}, k_{it}, h_{it}, \ell_{it}, y_{it})$ are also measurable in s^t, for all i and all t. This scenario, which is commonplace in the literature, is akin to letting the realized state of Nature not only be perfectly known to each of the firms but also common knowledge to them: when a firm acts, it knows that every firm knows s^t, it knows that every firm knows that every firm knows s^t, and so on. We finally depart from this benchmark and accommodate the sought-after friction by requiring that the firms must instead act on the basis of an noisy, and idiosyncratic, signal of s^t.

More specifically, the friction takes the following form. For every t, every realization of s^t, and every firm i, Nature draws a random variable ω_i^t from a finite set Ω^t according to some probability distribution φ. The joint probability of the pair (ω^t, s^t) is denoted by $\varphi(\omega^t, s^t)$, the probability of ω^t conditional on s^t is denoted by $\varphi(\omega^t|s^t)$, and the probability of s^t conditional on ω^t is denoted by $\varphi(s^t|\omega^t)$. Conditional on s^t, the draws of ω^t are i.i.d. across firms and a law of large number applies so that $\varphi\left(\omega^t|s^t\right)$ is also the realized fraction of the population that receives the signal ω^t when the underlying state is s^t.[8] Finally, while the variable ℓ_{it} is allowed to be measurable in both ω_i^t and s^t, the vector (p_{it}, k_{it}, h_{it}) is restricted to be measurable only in ω_i^t, for all i and all t.

[8]See Uhlig (1996) for an applicable law of large numbers with a continuum of draws.

Requiring that p_{it} be measurable in ω_i^t rather than s^t introduces the same kind of *nominal rigidity* as the one featured in Mankiw and Reis (2002), Woodford (2003a), Adam (2007), Mackowiak and Wiederholt (2009), Paciello and Wiederholt (2014), and a growing literature that replaces Calvo-like sticky prices with an informational friction.[9] Relative to this literature, the key innovation here is to add a *real friction* by requiring that (k_{it}, h_{it}) be also measurable in ω_i^t. As will become clear, our results depend on the interaction of the two rigidities. Finally, letting ℓ_{it} (and thereby also y_{it}) adjust to s^t guarantees that supply can meet demand for all realizations of uncertainty.

2.3 Discussion

By requiring that the firms make certain choices on the basis of dispersed private information about the underlying state of Nature, we connect to a long tradition in macroeconomics that studies "island economies", that is, economies in which information and trading is geographically segmented (Lucas, 1972; Townsend, 1983; Prescott and Rios-Rull, 1992).

There are, however, two subtle—and connected—differences. First, these earlier works maintain the assumption of the standard Arrow-Debreu framework that agents can condition their choices on the *true* prices. By contrast, our framework allows the firms to act on the basis of an imperfect observation, or understanding, of the input prices they transact at. Second, these earlier works formalized the informational friction as the product of restricted market participation. In particular, they prevented the endogenous aggregation of information that would have obtained in a complete Arrow-Debreu setting by letting only a small, and non-representative, sample of the population trade in any particular market at any point of time and by precluding the participants of one market to observe the outcomes of other markets. Accordingly, this allowed information to heterogeneous *across* markets but restricted it to be homogeneous *within* markets. By contrast, we allow markets to be centralized and information to be heterogeneous within markets; moreover, we entirely bypass the issue of the endogenous aggregation of information by recasting the information friction as a cognitive friction.

These modeling choices are not strictly needed for the policy lessons of this paper. An early incarnation of our paper (Angeletos and La'O, 2008) had obtained similar results for a variant economy that featured segmented markets and allowed firms to condition their choices on the actual input prices. The current formulation, however, permits us to connect to a growing literature that interprets the noisy signal that an agent receives about the state of the economy as the formal representation of the agent's bounded capacity to pay attention to available data (Sims, 2003; Woodford, 2003a; Mackowiak and Wiederholt, 2009), to comprehend what's going on around her, to form beliefs about the behavior of others, and to figure out her own course of action (Tirole, 2015; Angeletos and Lian, 2017). We find this interpretation to be appealing not only on conceptual grounds but also on empirical grounds: these days the most binding constraint seems to be limited time and cognitive capacities rather than the unavailability of data. This interpretation is also supported by experimental evidence (Khaw, Stevens, and Woodford, 2016).

To enhance this interpretation, Section 6 extends the analysis to the case in which each firm

[9] See also Chwe (1999) for an earlier, and overlooked, contribution that emphasizes how lack of common knowledge can rationalize monetary non-neutrarility.

chooses optimally the joint distribution of the signal ω_i^t with the state s^t, subject to a cost. This can be thought of as the choice of how much attention to pay to the available data or how much cognitive effort to exert towards understanding what's going on in the economy.[10] For the time being, however, we treat φ, the joint distribution of the signal and the state, as exogenous.

Our framework is otherwise fully flexible. For instance, the state s^t and the signal ω_i^t do not have to be Gaussian. Furthermore, s^t may become known at the end of each period, with any other finite lag, or never. Also, ω_i^t may, but does not have to, be measurable in ω_i^τ for all $\tau > t$: that is, firms can suffer from "imperfect recall" (Woodford, 2009; Pavan, 2016). Last but not least, s^t may contain all of the following: innovations to current fundamentals, news about future fundamentals (Beaudry and Portier, 2006; Jaimovich and Rebelo, 2009), correlated errors in beliefs of the fundamentals or "noise shocks" (Lorenzoni, 2009; Angeletos and La'O, 2010), or more exotic shocks to higher-order beliefs. The latter type of shock decouples variation in equilibrium expectations of economic outcomes from variation in fundamentals (or first-order beliefs thereof), and can thus be interpreted as a product of "sentiments" or "animal spirits" (Angeletos and La'O, 2013; Benhabib, Wang, and Wen, 2015; Huo and Takayama, 2015).

While not strictly needed, this flexibility is useful for two reasons. First, it helps clarify the precise nature and the robustness of our results. Second, it permits us to nest a plethora of more special information structures that have been used previously in the literature.

For instance, consider Woodford (2003a), Adam (2007), Nimark (2008), and Angeletos and La'O (2010). These papers study models in which each firm observes a new private signal of the underlying aggregate fundamental in each period, possibly in combination with a public signal. To nest these settings, we may abstract from the government spending shock and suppose that each firm receives a pair of signals (a_{it}, z_t) in each period t, where $a_{it} = \log A_t + \epsilon_{it}$ is the private signal of the underlying aggregate TFP, $z_t = \log A_t + u_t$ is the public signal, ϵ_{it} is idiosyncratic noise, and u_t is aggregate noise; and finally let $s^t = (\log A_\tau, u_\tau)_{\tau \leq t}$ and $\omega_i^t = (a_{i\tau}, z_\tau)_{\tau \leq t}$.[11]

Alternatively, consider models with "sticky information" as in Mankiw and Reis (2002) and Ball, Mankiw, and Reis (2005). These settings are directly nested in our framework by letting φ assign probability λ to $\omega_i^t = (\omega_i^{t-1}, s^t)$ and probability $1 - \lambda$ to $\omega_i^t = \omega_i^{t-1}$, where $\lambda \in (0, 1)$ is the probability with which a firm updates its information set with the perfect observation of the underlying state and $1 - \lambda$ is the probability with which the firm is stuck with her old information set.

Consider next the forms of "rational inattention" found in Sims (2003), Mackowiak and Wiederholt (2009), and Paciello and Wiederholt (2014), the variants proposed by Myatt and Wallace (2012) and Pavan (2016), the model of fixed observation costs found in Alvarez, Lippi, and Paciello (2011), and the model of "costly contemplation" considered in Tirole (2015). For our purposes, these settings boil down to endogenizing the joint distribution of the state s^t and the signal ω^t in a variety of ways, all

[10]Clearly, the accommodation of these ideas is another feature that distinguishes our framework, and the related works mentioned in this paragraph, from the earlier literature on "island economies".

[11]If the underlying TFP shock is persistent, this specification allows for gradual learning and persistent dynamics in higher-order beliefs, as in Woodford (2003a), Angeletos and La'O (2010), and Huo and Takayama (2015). Alternatively, one can assume that the underlying TFP becomes common knowledge at the end of each period and let $\omega_i^t = s^{t-1} \cup (a_{i\tau}, z_\tau)_{\tau \leq t}$. Finally, it is possible to recast a_{it} as firm-specific TFP, which itself serves as a noisy private signal of aggregate TFP; this was actually the case in earlier versions of this paper (Angeletos and La'O, 2008).

of which are nested in the extended framework of Section 6.

The following point is also worth emphasizing. By allowing firms to have not only noisy but also heterogeneous information about s^t, our framework accommodates higher-order uncertainty. By contrast, high-order uncertainty is *a fortiori* ruled out in the RBC and New-Keynesian frameworks because all firms are assumed to share the same information at all times. As emphasized elsewhere,[12] ruling out higher-order uncertainty is synonymous to imposing perfect coordination: it is *as if* all the economic agents can congregate in a room, talk to one another, and flawlessly coordinate their choices. Conversely, a key quality of our framework is that it allows for imperfection in the coordination of the firms' production and pricing choices. This imperfection turns out to be key to some of our policy lessons—most notably, the suboptimality of price stability.

Notwithstanding the aforementioned flexibility, our framework abstracts from markup shocks and from labor or capital market frictions. The rationale is the following. In the New Keynesian literature, such features are often introduced in conjunction with appropriate restrictions on the tax instruments so as to justify a monetary policy that deviates from replicating flexible-price allocations. Had we made the same assumptions as in that literature, we would have recovered the familiar argument that such a deviation is desirable only when monetary policy substitutes for missing tax instruments. By abstracting from markup shocks and the like, we instead ensure that the insights delivered in this paper are orthogonal to what is already known.[13]

3 Sticky Prices, Flexible Prices, and Feasibility: Definitions

We view the accommodation of the dual role of the informational friction—the nominal rigidity associated with the restriction that p_{it} be measurable in ω_i^t and the real rigidity associated with the restriction that k_{it} and h_{it} also be measurable in ω_i^t—as a defining feature of our framework. Accordingly, we are primarily interest in the scenario in which *both* roles are active. To understand the optimal policy under this scenario, it is nevertheless instrumental to study the alternative scenario in which the nominal rigidity is artificially shut down by letting p_{it} be measurable in s^t. Borrowing, and somewhat paraphrasing, the terminology of the New-Keynesian literature, we henceforth refer to the former scenario—the one of interest—as "sticky prices" and to the latter one as "flexible prices." In this section, we delineate the two roles of the informational friction and define the sets of allocations, prices, and policies that can be part of an equilibrium under each scenario.

3.1 Sticky-Price Equilibria

We henceforth represent an allocation by a sequence $\xi \equiv \{\xi_t(.)\}_{t=0}^{\infty}$, where

$$\xi_t(.) \equiv \{k_t(.), h_t(.), \ell_t(.), y_t(.); K_t(.), H_t(.), L_t(.), Y_t(.), C_t(.)\}$$

[12] See, among others, the discussions in Morris and Shin (2002, 2003), Angeletos and La'O (2013) and Angeletos and Lian (2016b, 2017).

[13] This, however, does not mean that there are no *additional* insights to be derived from studying the interaction of the aforementioned features with informational frictions: see Paciello and Wiederholt (2014) and Angeletos, Iovino, and La'O (2016) for examples.

is a vector of functions that map the realizations of uncertainty to the quantities chosen by the typical firm (for the first four components of ξ_t) and the aggregate quantities (for the remaining five components). We similarly represent a price system by a sequence $\varrho \equiv \{\varrho_t(.)\}_{t=0}^{t}$, where

$$\varrho_t(.) \equiv \{p_t(.), P_t(.), r_t(.), w_t(.), Q_t(.)\}$$

is a vector of functions that map the realizations of uncertainty to the nominal price set by the typical firm, the aggregate price level, the real wage rate, the real rental rate of capital, and the nominal prices of the Arrow securities. We finally represent a policy with a sequence $\theta = \{\theta_t(.)\}_{t=0}^{t}$, where

$$\theta_t(.) \equiv \left\{\tau_t^r(.), \tau_t^\ell(.), \tau_t^k(.), \tau_t^c(.), B_t(.), D_t(.), R_t(.)\right\}$$

is a vector of functions that map the realizations of uncertainty to the various policy instruments,

Throughout our analysis, we let the domain of $K_t(.), H_t(.), L_t(.), Y_t(.)$, $C_t(.)$, $P_t(.)$, $r_t(.)$, $w_t(.)$, $Q_t(.)$, $\tau_t^r(.)$, $\tau_t^\ell(.)$, $\tau_t^k(.)$, $\tau_t^c(.)$, $B_t(.)$, $D_t(.)$, and $R_t(.)$ be S^t. This means that all the aggregate outcomes, the real wages, the real interest rate, the asset prices, and the policy instruments are measurable in s^t. We next embed the *real* aspect of the informational friction by assuming that the domain of the functions k_t and h_t is Ω^t; this simply means that k_{it} and h_{it} are restricted to be measurable in ω_i^t. By contrast, the domain of ℓ_t and y_t is $\Omega^t \times S^t$; this means that the labor input and the output of a firm are allowed to respond to the realized state. We finally embed a *nominal friction*, or let prices be (informationally) *sticky*, by requiring that p_{it} be measurable in ω_i^t or, equivalently, that the domain of $p_t(.)$ be Ω^t.

For future reference, we collect the relevant firm-level measurability restrictions in the following two properties.

Property 1. *The firm-level quantities satisfy*

$$h_{it} = h_t(\omega_i^t), \quad k_{it} = k_t(\omega_i^t), \quad \ell_{it} = \ell_t(\omega_i^t, s^t), \quad y_{it} = y_t(\omega_i^t, s^t),$$

for all i, all t, and all realizations of uncertainty.

Property 2. *The prices satisfy*

$$p_{it} = p_t(\omega_i^t)$$

for all i, all t, and all realizations of uncertainty.

Properties 1 and 2 are, in effect, a definition of the kind of allocations and prices that are "informationally feasible" under the scenario of interest. Note in particular that k_{it}, h_{it}, and p_{it} are prevented from being contingent on pieces of information that are contained in ω_j^t for some $j \neq i$ but are *not* contained in ω_i^t. In this sense, information cannot be transferred from one firm to another. This is the key restriction that distinguishes our analysis from Correia, Nicolini, and Teles (2008) and more generally from the Ramsey policy paradigm: in that paradigm, it is *as if* information can be transferred from one agent to another instantaneously and without any restriction.[14,15]

[14]Let us emphasize once again that the issue at stake is orthogonal to either the question of how precise the available information is at any given point, or the question of how information evolves over time. The Arrow-Debreu framework and the Ramsey paradigm can accommodate a lot of richness in these two dimensions, but do not allow for different agents to have different information and to face higher-order uncertainty.

[15]Properties 1 and 2 impose not only the relevant informational friction but also a certain symmetry: two firms can choose

In the rest of this section, we formulate the household's and the firm's problems and define the equilibrium of the economy. Throughout, we restrict attention to triplets (ξ, ϱ, θ) that satisfy Properties 1 and 2.

Consider first the household. The statement of her problem is standard.[16]

Household's Problem. *The household chooses* $\{C(.), L(.), K(.), B(.), D(.)\}$ *so as to maximize her expected utility,*

$$\mathcal{W} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right) \left[U\left(C\left(s^t\right), L\left(s^t\right), s^t\right)\right],$$

subject to her budget constraint,

$$\left(1+\tau^c\left(s^t\right)\right) C\left(s^t\right) + X(s^t) + \frac{1}{P\left(s^t\right)} \left\{ B\left(s^t\right) + \sum_{s^{t+1}} Q(s^{t+1}) D(s^{t+1}) \right\} = \left(1-\tau^\ell\left(s^t\right)\right) w\left(s^t\right) L\left(s^t\right) +$$

$$+ \left(1-\tau^k\left(s^t\right)\right) r\left(s^t\right) K\left(s^{t-1}\right) + \frac{1}{P\left(s^t\right)} \left\{\left(1+R\left(s^{t-1}\right)\right) B\left(s^{t-1}\right) + D(s^t)\right\} \quad \forall t, s^t,$$

and the law of motion for capital,

$$K\left(s^t\right) = (1-\delta) K\left(s^{t-1}\right) + X\left(s^t\right) \quad \forall t, s^t.$$

Consider next the typical monopolistic firm. Her (ex ante) valuation is given by

$$\mathcal{V} \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \mathcal{M}(s^t) \frac{\Pi(\omega_i^t, s^t)}{P(s^t)}\right] = \sum_{t=0}^{\infty} \sum_{\omega_i^t, s^t} \left\{\beta^t \mathcal{M}(s^t) \frac{\Pi(\omega_i^t, s^t)}{P(s^t)} \varphi\left(\omega_i^t, s^t\right)\right\},$$

where

$$\mathcal{M}(s^t) \equiv \frac{U_c\left(s^t\right)}{1+\tau^c\left(s^t\right)}$$

is the "pricing kernel", $U_c\left(s^t\right)$ is a shortcut for $\frac{\partial}{\partial c} U\left(C\left(s^t\right), L\left(s^t\right), s^t\right)$, and

$$\Pi(\omega_i^t, s^t) \quad \equiv \quad \left(1-\tau^r\left(s^t\right)\right) \frac{p\left(\omega_i^t\right)}{P\left(s^t\right)} y\left(\omega_i^t, s^t\right) - h\left(\omega_i^t\right) - w\left(s^t\right) \ell\left(\omega_i^t, s^t\right) - r(s^t) k(\omega_i^{t-1})$$

is the firm's real profit net of the revenue tax. The demand faced by the monopolist is given by[17]

$$y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho} Y\left(s^t\right). \tag{1}$$

We may thus express the monopolist's problem as follows.

different quantities and/or prices only if they have different types. This is without any loss of generality given the assumed convexity in technology and preferences.

[16] To ease the notation, we henceforth drop the subscript t from the functions $C_t(.), L_t(.)$, etc, except for few occasions in which it is useful to make explicit the dependence on t.

[17] As usual, condition (1) here as well as condition (3) in the sequel follow from optimality in the retail sector.

Monopolist's Problem. *The typical monopolist chooses the plan* $\{p, k, h, \ell, y\}$ *so as to maximize her valuation,*

$$\sum_t \sum_{\omega_i^t, s^t} \left\{ \beta^t \mathcal{M}(s^t) \left[\left(1 - \tau^r\left(s^t\right)\right) \frac{p(\omega_i^t)}{P(s^t)} y\left(\omega_i^t, s^t\right) - h\left(\omega_i^t\right) - w\left(s^t\right) \ell\left(\omega_i^t, s^t\right) - r(s^t) k(\omega_i^t) \right] \varphi\left(\omega_i^t, s^t\right) \right\},$$

subject to the technology,

$$y\left(\omega_i^t, s^t\right) = A\left(s^t\right) F\left(k_i\left(\omega_i^t\right), h_i\left(\omega_i^t\right), \ell_i\left(\omega_i^t, s^t\right)\right) \quad \forall t, s^t, \omega_i^t,$$

and the demand for her product,

$$y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho} Y\left(s^t\right) \quad \forall t, s^t, \omega_i^t.$$

Finally, since the cross-sectional distribution of the signal in period t and state s^t is given by $\varphi(.|s^t)$, the following properties are self-evident: aggregate output is given by

$$Y\left(s^t\right) = \left[\sum_{\omega \in \Omega^t} \left(y\left(\omega, s^t\right)\right)^{\frac{\rho-1}{\rho}} \varphi\left(\omega | s^t\right)\right]^{\frac{\rho}{\rho-1}} \quad \forall t, s^t; \tag{2}$$

the price level (the price of the final good) by

$$P(s^t) = \left[\sum_{\omega \in \Omega^t} \left(p\left(\omega\right)\right)^{\rho-1} \varphi\left(\omega | s^t\right)\right]^{\frac{1}{\rho-1}} \quad \forall t, s^t; \tag{3}$$

the market for the final good clears if and only if

$$C\left(s^t\right) + X(s^t) + G\left(s^t\right) + \sum_{\omega \in \Omega^t} h\left(\omega\right) \varphi\left(\omega | s^t\right) = Y\left(s^t\right) \quad \forall t, s^t; \tag{4}$$

the market for labor clears if and only if

$$\sum_{\omega \in \Omega^t} \ell\left(\omega\right) \varphi\left(\omega | s^t\right) = L\left(s^t\right) \quad \forall t, s^t; \tag{5}$$

and the market for capital clears if and only if

$$\sum_{\omega \in \Omega^t} k\left(\omega\right) \varphi\left(\omega | s^t\right) = K\left(s^t\right) \quad \forall t, s^t. \tag{6}$$

We can thus define an equilibrium as follows.

Definition 1. *A* ***sticky-price equilibrium*** *is a triplet* (ξ, ϱ, θ) *of allocations, prices, and policies that satisfy Properties 1 and 2 and are such that:*

(i) $\{C(\cdot), L(\cdot), K(\cdot), B(\cdot), D(\cdot)\}$ *solves the household's problem;*

(ii) $\{p(\cdot), k(\cdot), h(\cdot), \ell(\cdot), y(\cdot)\}$ *solves the firm's problem;*

(iii) the quantity of the final good is given by (2) and its price by (3);

(iii) the government's budget constraint is satisfied;

(iv) all markets clear, namely, conditions (4), (5), and (6) are satisfied.

3.2 Flexible-price Equilibria: Definition

We qualified the equilibria defined in the previous subsection as *sticky-price* equilibria in order to underscore the nominal friction that is embedded in Property 2. We next consider the alternative scenario in which this friction is shut down.

We say that prices are flexible, or that the nominal rigidity is absent, when p_{it} can be measurable in both ω_i^t and s^t. Formally, we identify this scenario by replacing Property 2 with the following.

Property 2′. *The prices satisfy*

$$p_{it} = p_t(\omega_i^t, s^t)$$

for all i, all t, and all realizations of uncertainty.

Accordingly, the monopolist's problem is reformulated with $p\left(\omega_i^t, s^t\right)$ in the place of $p(s^t)$. Similarly, condition (3) is adjusted as follows:

$$P(s^t) = \left[\sum_{\omega_i^t} \left(p\left(\omega_i^t, s^t\right)\right)^{\rho-1} \varphi\left(\omega_i^t | s^t\right)\right]^{\frac{1}{\rho-1}} \tag{7}$$

We therefore arrive at the the following definition.

Definition 2. *A* ***flexible-price equilibrium*** *is a triplet (ξ, ϱ, θ) of allocations, prices, and policies that satisfy Properties 1 and 2′ and are such that:*

(i) $\{C(\cdot), L(\cdot), K(\cdot), B(\cdot), D(\cdot)\}$ solves the household's problem;

(ii) $\{p(\cdot), k(\cdot), h(\cdot), \ell(\cdot), y(\cdot)\}$ solves the firm's problem;

(iii) the quantity of the final good is given by (2) and its price by (7);

(iii) the government's budget constraint is satisfied;

(iv) all markets clear.

In a nutshell, the definition of flexible-price equilibria is the same as that of sticky-price equilibria, except that we have replaced Property 2 with Property 2′.

Remark. Our flexible-price scenario preserves the real friction and removes the nominal one. The diametrically opposite scenario would preserve the nominal friction yet remove the real one; this can be accommodated by maintaining Property 2 and replacing Property 1 with a variant that allows $(k_{it}, h_{it}, \ell_{it})$ to be measurable in both ω_i^t and s^t. As will become clear in due course, this scenario helps reveal that the results of Correia, Nicolini, and Teles (2008) directly extend to settings in which the informational friction is the source of only nominal rigidity, which in turn underscores the significance of accommodating the real rigidity formalized in Property 1.

3.3 Feasibility

We conclude this section with one additional definition, whose meaning is self-evident.

Definition 3. *An allocation ξ is* ***feasible*** *if and only if it satisfies Property 1 and resource constraints (2), (4), (5) and (6).*

4 Sticky vs Flexible Prices: Characterization and Replication

What is the *entire* set of the allocations that can be implemented as part of an equilibrium with *some* policy? In this section we address this question under both the flexible-price and the sticky-price scenario. By allowing the policy to be arbitrary, the analysis in this section delivers three key insights which are instrumental to the lessons we develop in the following section about *optimal* policy. First, we highlight how the available policy instruments can serve a new function,[18] namely, how they can manipulate the manner in which each firm utilizes her idiosyncratic information, or responds to her "cognitive state", and thereby also influence the cross-sectional dispersion in production that originates from the underlying friction. Second, we shed light on which tax instruments are missing and on whether monetary policy can substitute for them once prices are sticky. Third, we show that, under a mild qualification, every allocation that is part of flexible-price equilibrium is also part of a sticky-price equilibrium.

4.1 Flexible-Price Allocations

Consider any flexible-price equilibrium. The characterization of the household's problem is standard. Its solution is pinned down by the combination of the usual transversality condition along with the following set of first-order conditions:

$$U_\ell\left(s^t\right) = U_c\left(s^t\right)\frac{1-\tau^\ell\left(s^t\right)}{1+\tau^c(s^t)}w\left(s^t\right) \quad \forall t, s^t \tag{8}$$

$$\mathcal{M}(s^t) = \beta\mathbb{E}\left[\mathcal{M}(s^{t+1})\left\{1-\delta+\left(1-\tau^k\left(s^{t+1}\right)\right)r\left(s^{t+1}\right)\right\}\middle| s^t\right] \quad \forall t, s^t \tag{9}$$

$$\mathcal{M}(s^t) = \beta\left(1+R\left(s^t\right)\right)\mathbb{E}\left[\mathcal{M}(s^{t+1})\frac{1}{1+\pi\left(s^{t+1}\right)}\middle| s^t\right] \quad \forall t, s^t \tag{10}$$

$$\mathcal{M}(s^t)Q(s^{t+1}) = \beta\mathcal{M}(s^{t+1})\frac{1}{1+\pi\left(s^{t+1}\right)} \quad \forall t, s^t, s^{t+1} \tag{11}$$

where $\mathcal{M}(s^t)\equiv\frac{U_c\left(s^t\right)}{1+\tau^c(s^t)}$ and $\pi(s^{t+1})\equiv\frac{P(s^{t+1})}{P(s^t)}-1$. The first condition is the optimality condition for labor; the second is the Euler equation for capital; the third is the Euler equation for the non-contingent bond; and the last is the Euler equation for the state-contingent securities.

The characterization of the monopolist's problem is slightly more exotic because of the noise and heterogeneity in the signal ω_i^t upon which the inputs k_{it} and h_{it} must be chosen. To conserve on notation, we henceforth let, for $z\in\{\ell,h,k\}$,

$$MP_z\left(\omega_i^t,s^t\right)\equiv\left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}A(s^t)\frac{\partial}{\partial z}F\left(k\left(\omega_i^t\right),h\left(\omega_i^t\right),\ell\left(\omega_i^t,s^t\right)\right).$$

In the eyes of the planner, MP_z represents the marginal product of input z in firm i, expressed in terms of the final good; in the eyes of the firm, it captures the corresponding marginal *revenue* product once

[18] By "new" we mean relative to the standard Ramsey paradigm, which rules out the kind of informational, or cognitive, frictions we have accommodated here.

it is multiplied by $\frac{\rho-1}{\rho}$, the reciprocal of one plus the monopoly markup. As shown in the Appendix, we can then express the first-order conditions of the firm as follows:

$$\begin{aligned}
\left(1-\tau^r\left(s^t\right)\right)\tfrac{\rho-1}{\rho}MP_\ell\left(\omega_i^t,s^t\right)-w(s^t) &= 0 \quad \forall t,\omega_i^t,s^t & (12)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left\{\left(1-\tau^r\left(s^t\right)\right)\tfrac{\rho-1}{\rho}MP_h\left(\omega_i^t,s^t\right)-1\right\}\Big|\,\omega_i^t\right] &= 0 \quad \forall t,\omega_i^t & (13)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left\{\left(1-\tau^r\left(s^t\right)\right)\tfrac{\rho-1}{\rho}MP_k\left(\omega_i^t,s^t\right)-r(s^t)\right\}\Big|\,\omega_i^t\right] &= 0 \quad \forall t,\omega_i^t. & (14)
\end{aligned}$$

These conditions have a simple interpretation. The firm seeks to equate the cost of each input with its after-tax marginal revenue product. The only difference among the three conditions is the extent to which this goal is achieved. Because labor is contingent on the realized state s^t, its marginal revenue product is equated with the real wage state-by-state. By contrast, the other two conditions hold only "on average," that is, in expectation conditional on the firm's signal.

This bears a similarity to models with time-to-build or adjustment costs: in those models, too, there is an input whose marginal product is equated to the user cost only in expectation. There is, however, a key difference: in those models, expectations are contingent on the same information set; in our setting, by contrast, expectations are contingent on *heterogeneous* information. It is this heterogeneity in information, and the resulting heterogeneity in input choices, that ushers in a coordination friction in production. This in turn ultimately drives our result regarding the suboptimality of price stability.

Moving on, note that the combination of the aforementioned optimality conditions, the market clearing conditions (4)-(6), and the government budget constraint is necessary and sufficient for a system of prices, allocations, and policies to constitute an equilibrium. Solving out for the prices and the policy instruments, we reach the following result.

Proposition 1. *A feasible allocation is part of a flexible-price equilibrium if and only if the following two properties hold.*

(i) The allocation satisfies

$$\sum_{t,s^t}\beta^t\mu\left(s^t\right)\left[U_c\left(s^t\right)C\left(s^t\right)+U_\ell\left(s^t\right)L\left(s^t\right)\right]=0. \quad (15)$$

(ii) For every t, there exist functions $\psi^r,\psi^\ell,\psi^c,\psi^k : S^t\to\mathbb{R}_+$ such that

$$\begin{aligned}
\psi^r\left(s^t\right)\tfrac{\rho-1}{\rho}MP_\ell\left(\omega_i^t,s^t\right)-\psi^\ell\left(s^t\right) &= 0 \quad \forall\,\omega_i^t,s^t & (16)\\
\mathbb{E}\left[\psi^r\left(s^t\right)\tfrac{\rho-1}{\rho}MP_h\left(\omega_i^t,s^t\right)-\psi^c\left(s^t\right)\Big|\,\omega_i^t\right] &= 0 \quad \forall\,\omega_i^t & (17)\\
\mathbb{E}\left[\psi^r\left(s^t\right)\tfrac{\rho-1}{\rho}MP_k\left(\omega_i^t,s^t\right)-\psi^k\left(s^t\right)\Big|\,\omega_i^t\right] &= 0 \quad \forall\,\omega_i^t & (18)
\end{aligned}$$

Necessity is straightforward. Condition (15) follows from combining the intertemporal budget constraint of the government with the optimality conditions of the household.[19] Conditions (16)-(18) follow from combining the optimality conditions of the household with those of the firms and letting

$$\psi^\ell\left(s^t\right)=\frac{-U_\ell(s^t)}{1-\tau^\ell(s^t)},\quad \psi^c(s^t)=\frac{U_c(s^t)}{1+\tau^c(s^t)},\quad \psi^k(s^t)=\frac{U_c(s^t)}{1+\tau^c(s^t)}\frac{\tilde{r}\left(s^t\right)}{1-\tau^k\left(s^t\right)}, \quad (19)$$

[19]Without serious loss of generality, we assume that $D_0=0$ and $B_0=K_0$, which explains why the right hand side of condition (15) is zero.

$$\text{and} \quad \psi^r\left(s^t\right) = \frac{U_c(s^t)(1-\tau^r(s^t))}{1+\tau^c(s^t)} = \psi^c(s^t)(1-\tau^r(s^t)), \tag{20}$$

where $\tilde{r}(s^t)$ denotes the net-of-taxes return to savings. The above equations reveal that the vector $(\psi^r, \psi^\ell, \psi^c, \psi^k)$ captures the "wedges" induced by the tax instruments.

To prove sufficiency, and to understand why these wedges are "free variables" under the planner's control, note the following. Pick any allocation ξ that is feasible and satisfies condition (15). Once such an allocation is fixed, the paths for $U_c(s^t)$ and $U_\ell(s^t)$ are also fixed. Still, the planner can induce any pair of values for the wedges ψ^c and ψ^ℓ by choosing appropriately the values of the taxes τ_c and τ_ℓ. Furthermore, the planner can trivially satisfy the household's optimality conditions by letting $\tilde{r}$, the net-of-taxes rental rate of capital, and $\tilde{w}$, the net-of-taxes wage rate, be such that

$$\psi^c(s^t) = \beta\mathbb{E}\left[\psi^c(s^{t+1})\left(1-\delta+\tilde{r}\left(s^{t+1}\right)\right)\middle|\, s^t\right] \quad \text{and} \quad \psi^\ell\left(s^t\right) = \tilde{w}(s^t)\psi^c(s^t) \quad \forall t, s^t.$$

Note next that any pair of values for ψ^r and ψ^k can be induced by setting appropriately the values for τ_r and τ_k, while the firm's optimality conditions are satisfied provided that conditions (16)-(18) hold. The argument is completed in the Appendix by "reverse-engineering" the entire price system and the asset portfolios that support the considered allocation in an equilibrium.

Let us now expand on the meaning of Proposition 1 and on its relation to existing results from the Ramsey literature. Condition (15) is fully familiar from that literature: it identifies the aggregate quantities that are consistent with the intertemporal budget balance for the government, optimality for the household and the firms, and market clearing. It can thus been read as an "on-the-equilibrium" representation of the intertemporal government budget, expressed in terms of the considered allocation alone. Importantly, this condition encapsulates the fact that taxation is distortionary: if lump-sum taxation were available, the aforementioned condition would be void.

Consider next conditions (16)-(18). When the information, or cognitive, friction is absent, as in the analyses of Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1994), and Correia, Nicolini, and Teles (2008), the firms can condition their input choices on the true underlying state of the economy. As a result, the aforementioned conditions reduce to the following:

$$MP_\ell\left(\omega_i^t, s^t\right) = \frac{\psi^\ell(s^t)}{\psi^r\left(s^t\right)}, \quad MP_h\left(\omega_i^t, s^t\right) = \frac{\psi^c(s^t)}{\psi^r\left(s^t\right)}, \quad \text{and} \quad MP_k\left(\omega_i^t, s^t\right) = \frac{\psi^k(s^t)}{\psi^r\left(s^t\right)}, \quad \forall t, \omega_i^t, s^t.$$

And since the ψ's are free variables, the above are satisfied if and only if the marginal product of each input is equated across all firms at all dates and all states of nature. This defines what we call "perfect coordination" in the production side of the economy. It also means that the sole role of the available tax instruments under complete information is to control the wedges between the *common* MRTs of the firms and the corresponding MRSs of the household.

When instead the informational friction is present, each firm must condition her optimal choice of certain inputs on a noisy and idiosyncratic knowledge, or understanding, of what's going on in the economy (that is, on the private signal ω^t of the state s^t). As a result, the marginal products of these inputs need not be equated in the cross section of firms. This dispersion in marginal products and in the underlying heterogeneity in input choices are indications of the mis-coordination of production across firms. The corresponding hallmark at the macro level is an aggregate TFP loss: for given aggregate quantities of capital and labor, the aggregate quantity of the final good that goes to consumption and investment is depressed relative to the benchmark characterized in Lucas and Stokey (1983), Chari, Christiano, and Kehoe (1994), and Correia, Nicolini, and Teles (2008). This aggregate TFP loss is the result of the misallocation of resources induced by firms' inability to condition their choices on the same information set.

Under such circumstances, the available tax instruments start playing a new role. Because the aggregate state is correlated with the signal received by the typical firm, the contingency of the taxes on the former influences how the optimal choices of the firm respond to the latter. This enables the planner to control not only the macro level business cycle (i.e., the covariation of aggregate output with the underlying state) but also the micro-level mis-coordination (i.e., the aforementioned heterogeneity in input choices and the resulting dispersion in marginal product). It is this new role of the taxes that is encoded into conditions (16)-(18).

An example. We illustrate the preceding insights in the Appendix with the help of an example that admits a closed-form characterization of the log-linearized flexible-price allocations. In this example, we abstract from capital accumulation, shut down any shocks to government spending, and impose homothetic preferences and Cobb-Douglas technology. We also let the tax system be such that the relevant wedges are log-linear functions of aggregate productivity and aggregate output only. We finally assume that the information structure is Gaussian.

To be concrete, let us herein make the additional assumption that the information contained in ω_i^t about A_t can be summarized in two sufficient statistics, one given by $a_{it} = \log A_t + \xi_{it}$ and another given by $z_t = \log A_t + u_t$, where ξ_{it} is idiosyncratic noise and u_t is common noise, both orthogonal to $\log A_t$. As in Morris and Shin (2002) and Angeletos and La'O (2010), one can then think of a_{it} and z_t as, respectively, private and public signals about the underlying fundamental. For our purposes, however, it is best to think of z_t more broadly as a proxy for correlated errors in the firms' equilibrium beliefs of aggregate economic outcomes. For instance, z_t could be the limit of a private signal that has a vanishing idiosyncratic error and a non-vanishing common error.[20]

It is then easy to show the following result. First, for any tax structure, there exists scalars $\gamma_0, \gamma_a, \gamma_u \in \mathbb{R}$ such that equilibrium GDP is given by

$$\log GDP\left(s^t\right) = \gamma_0 + \gamma_a \log A_t + \gamma_u u_t. \tag{21}$$

Second, the coefficients γ_a and γ_u, which measure the elasticities of aggregate output to the underlying TFP and to the noise, can take a wide range of values in $\mathbb{R}^2$; different values for these elasticities are supported by different contingencies of the taxes on aggregate productivity and output.

This result illustrates how the planner can use taxes to influence the extent to which the business cycle is driven by fundamental or non-fundamental forces. As shown in the Appendix, this insight extends to a larger class of information structures, which allows the firms to observe an essentially arbitrary set of Gaussian signals not only about the underlying fundamental but also about one another's information. The result stated above continues to hold, except that now u_t has to be re-interpreted

[20] It is also possible to re-cast a_{it} as firm-specific TFP, which itself serves as a private signal of aggregate TFP.

as a proxy for all aggregate variation in the equilibrium expectations of Y_t that is orthogonal to the underlying variation in A_t. Such variation in equilibrium expectations of Y_t reflects correlated movements in either first- or higher-order beliefs of A_t. It can thus capture not only the "noise shocks" studied in Lorenzoni (2009), Angeletos and La'O (2010), and Barsky and Sims (2011), but also the "sentiment shocks" studied in Angeletos and La'O (2013), Benhabib, Wang, and Wen (2015), and Huo and Takayama (2015).

In fact, there exists a tax policy that insulates the economy from such "exotic", beliefs-driven fluctuations and that also induces the same covariation between aggregate output and aggregate TFP as the one that is optimal according to Lucas and Stokey (1983) and Correia, Nicolini, and Teles (2008). And yet, as will be shown in the next section, such a policy is not optimal once the underlying friction is properly accounted for in the planner's calculation of social welfare.

The basic intuition is the following. To insulate aggregate output from such beliefs shocks, the firms would have to disregard any signal that is correlated with these shocks, such as the signal z_t in the example given above. But this would mean disregarding socially valuable information. In particular, by letting firms condition their choices on the aforementioned signal, the planner can attain a higher degree of coordination in production; that is, she can reduce the dispersion in the cross-sectional allocation of resources, the resulting dispersion in marginal products, and the associated TFP loss at the aggregate level.

To sum up, the considered example illustrates, not only the novel roles that conventional tax instruments can play in the presence of informational/cognitive frictions, but why the optimal plan may feature more exotic fluctuations than those familiar from the standard Ramsey and New-Keynesian paradigms. This, of course, raises the question of how exactly the optimal plan is determined. We address this question in the next section; in the remainder of the current section, we characterize the set of allocations that can be implemented as part of a sticky-price equilibrium and compare it to its flexible-price counterpart.

4.2 Sticky-Price Allocations

We now add back the nominal friction (Property 2) and study how this modifies the set of implementable allocations. Clearly, the addition of the nominal friction does not alter the optimality conditions of the household, the budget constraints, and the market-clearing conditions. The implementability constraint in part (i) of Proposition 1 therefore remains intact. Part (ii), on the other hand, has to be modified so as to take into account how the nominal friction interferes with firm optimality.

A detailed characterization of the firm's problem can be found in the Appendix. The key difference from the flexible-price scenario is that the realized monopoly markup can fluctuate around the ideal one insofar as the policy instruments and the associated allocations respond to contingencies not contained in the information upon which the firm conditions her price. As a result, there now exists a random variable $\chi(\omega_i^t, s^t)$ such that the following conditions hold:

$$\begin{aligned}
\chi(\omega_i^t, s^t)\left(1-\tau^r\left(s^t\right)\right) MP_\ell\left(\omega_i^t, s^t\right) - w(s^t) &= 0 \quad \forall\, \omega_i^t, s^t &(22)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left\{\chi(\omega_i^t, s^t)\left(1-\tau^r\left(s^t\right)\right) MP_h\left(\omega_i^t, s^t\right) - 1\right\}\middle|\ \omega_i^t\right] &= 0 \quad \forall\, \omega_i^t &(23)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left\{\left(1-\tau^r\left(s^t\right)\right)\chi(\omega_i^t, s^t) MP_k\left(\omega_i^t, s^t\right) - r(s^t)\right\}\ \middle|\ \omega_i^t\right] &= 0 \quad \forall\, \omega_i^t &(24)\\
\mathbb{E}\left[\mathcal{M}(s^t) Y\left(s^t\right)^{1/\rho} y\left(\omega_i^t, s^t\right)^{1-1/\rho}\left(1-\tau^r\left(s^t\right)\right)\left\{\chi(\omega_i^t, s^t) - \chi^*\right\}\ \middle|\ \omega_i^t\right] &= 0 \quad \forall\, \omega_i^t &(25)
\end{aligned}$$

where $\chi^* \equiv \frac{\rho-1}{\rho} \in (0,1)$.

To interpret these conditions, note that $\chi(\omega_i^t, s^t)$ represents the reciprocal of the *realized* markup, while χ^* captures the *ideal* markup. When the nominal rigidity is absent, the realized markup coincides with the ideal one for all realizations of uncertainty; equivalently, conditions (22)-(24) reduce to conditions (12)-(14). When instead the nominal rigidity is present, the realized markup may differ from the ideal one in some, or even all, realizations of uncertainty. Nevertheless, the optimal price-setting behavior of the firm requires that that the *average* value of the markup across all possible realizations of uncertainty coincides with the ideal one, in the sense of condition (25). This condition therefore captures the optimal price-setting behavior of the firm, whereas the remaining three conditions capture cost minimization.

Similar to before, we can combine conditions (22)-(24) with the optimality conditions of the household in order to obtain a set of joint restrictions on the allocation and the relevant wedges. This gives us the following result.

Lemma 1. *A feasible allocation is part of a sticky-price equilibrium only if the following two properties hold.*

(i) The allocation satisfies

$$\sum_{t,s^t} \beta^t \mu\left(s^t\right)\left[U_c\left(s^t\right) C\left(s^t\right) + U_\ell\left(s^t\right) L\left(s^t\right)\right] = 0. \tag{26}$$

(ii) For every t, there exists functions $,\psi^r, \psi^\ell, \psi^k, \psi^c : \mathcal{S}^t \to \mathbb{R}_+$ and $\chi : \Omega^t \times \mathcal{S}^t \to \mathbb{R}_+$ such that the following conditions hold:

$$\begin{aligned}
\chi(\omega, s^t)\psi^r\left(s^t\right) MP_\ell\left(\omega_i^t, s^t\right) - \psi^\ell(s^t) &= 0 \quad \forall\, \omega_i^t, s^t &(27)\\
\mathbb{E}\left[\chi(\omega, s^t)\psi^r\left(s^t\right) MP_h\left(\omega_i^t, s^t\right) - \psi^c(s^t)\middle|\omega_i^t\right] &= 0 \quad \forall\, \omega_i^t &(28)\\
\mathbb{E}\left[\chi(\omega, s^t)\psi^r(s^t) MP_k\left(\omega_i^t, s^t\right) - \psi^k(s^t)\middle|\omega_i^t\right] &= 0 \quad \forall\, \omega_i^t &(29)\\
\mathbb{E}\left[Y\left(s^t\right)^{1/\rho} y\left(\omega_i^t, s^t\right)^{1-1/\rho}\psi^r\left(s^t\right)\left\{\chi(\omega_i^t, s^t) - \chi^*\right\}\middle|\ \omega_i^t\right] &= 0 &(30)
\end{aligned}$$

Lemma 1 resembles Proposition 1, except for two differences. First, the conditions seen in part (ii) entail a new variable, namely $\chi(\omega_i^t, s^t)$. Second, whereas Proposition 1 provides a set of conditions that is *both* necessary and sufficient for a feasible allocation to be part of an equilibrium, the result stated above establishes only necessity; sufficiency requires an additional condition.

Consider the first difference, the one regarding $\chi(\omega_i^t, s^t)$. As already noted, this variable captures the realized monopoly markup of the firm. Through the lens of Lemma 1, this variable emerges as a firm-specific wedge in conditions (27)-(29). Relative to Proposition 1, this wedge represents an *additional* variable under the control of the planner: it encapsulates the power that monetary policy acquires over real allocations once the nominal friction (as per Property 2) is accommodated.

This power is restrained by condition (30): the realized markup can vary away from the ideal one only in a manner that cannot be predicted on the basis of the signal upon which the firm condition her price. This power can nevertheless be quantitatively substantial.[21] It is also more subtle than the one familiar from the New-Keynesian framework: as with taxes, monetary policy can influence how firms utilize their idiosyncratic knowledge about the state of the economy and can therefore also influence the coordination of production among them.

Consider now the second difference, the one regarding necessity versus sufficiency. It is straightforward to check that the conditions seen in Lemma 1 are necessary for a feasible allocation to be part of a sticky-price equilibrium. To establish sufficiency, however, we must add one more condition. To this goal, we first introduce the following definition.

Definition 4. *An allocation is **log-separable** if and only if there exist positive-valued functions Ψ^ω and Ψ^s such that*

$$\log y\left(\omega_i^t, s^t\right) = \log \Psi^\omega(\omega_i^t) + \log \Psi^s(s^t) \quad \forall \omega_i^t, s^t. \tag{31}$$

The property defined above requires that the output of a firm can be expressed as the logarithmic sum of two components: one that depends only on the firm's information set, ω_i^t, and another that depends only on the true aggregate state, s^t. We refer to the former as the "ω-component" and to the latter as the "s-component". At first glance, this restriction may appear exotic. However, as the next lemma shows, this restriction is a direct implication of the nominal rigidity.

Lemma 2. *Every sticky-price allocation is log-separable.*

To see why, fix a period t and a state s^t, and take an arbitrary pair of firms (i, j), with $j \neq i$. In any sticky-price equilibrium, it must be that the *nominal* price set by firm i is contingent at most on ω_i^t, and similarly the nominal price set by firm j must be contingent at most on ω_j^t. At the same time, the *relative* price of the two firms is pinned down, from the consumer's side, by their relative output. Putting the two properties together, we infer that *any* sticky-price allocation must satisfy the following restriction between the nominal prices and the relative output of the two firms:

$$\log p(\omega_i^t) - \log p(\omega_j^t) = -\rho \left[\log y(s^t, \omega_i^t) - \log y(s^t, \omega_j^t)\right] \tag{32}$$

Clearly, the above condition can hold for all realizations of ω_i^t, ω_j^t and s^t only if the right-hand side is independent of s^t conditional on the pair (ω_i^t, ω_j^t). For this to be the case, the dependence of y_{it} on any component of s^t that is not measurable in ω_i^t must cancel with the corresponding dependence of y_{jt}. This is precisely where the log-separability restriction kicks in: the above holds if and only if the allocation is log-separable in the sense of Definition 4.

We can thus state the sought-after result as follows.

[21] See, *inter alia*, Nimark (2008), Melosi (2014), Mackowiak and Wiederholt (2009, 2015), Mankiw and Reis (2002, 2011), and the references therein.

Proposition 2. *A feasible allocation is part as a sticky-price equilibrium if and only if it satisfies conditions (26)-(30) and it is log-separable in the sense of Definition 4.*

Log-separability is therefore the "missing" condition that must added in Lemma 1 in order to complete the characterization of the set of sticky-price allocation.

We conclude this subsection by considering the following question: Can monetary policy substitute for missing tax instruments? And if so, what exactly are these tax instruments? In light of Lemma 1 and Proposition 2, the answer to this question is straightforward: the real effects of monetary policy under sticky prices are equivalent to those of a certain class of firm-specific taxes under flexible prices. These taxes are restricted to be zero "on average" in the sense of condition (30), but are otherwise free to vary not only with the aggregate state of the economy but also with the private signal of the firm. It is this kind of tax instruments that have been ruled out in our setting and that can be mimicked by monetary policy once the nominal friction is switched on.[22]

4.3 Replication

By substituting for missing tax instruments, monetary policy enables the planner to attain allocations that are *outside* the set of flexible-price allocations. Whether this is valuable or not will be addressed in the next section. Here, we consider a different issue, namely whether monetary policy enables the planner to attain allocations *inside* the aforementioned set.

Let $\mathcal{X}^f$ denote the set of flexible-price allocations; this set is characterized by the conditions stated in Proposition 1. Let $\tilde{\mathcal{X}}^f$ denote the set of flexible-price allocations that are log-separable. By definition, $\tilde{\mathcal{X}}^f \subseteq \mathcal{X}^f$. Finally, let $\mathcal{X}^s$ denote the set of sticky-price allocations; this set is characterized by the conditions stated in Proposition 2. By comparing the aforementioned sets of conditions, the following is immediate.

Corollary 1. $\tilde{\mathcal{X}}^f \subseteq \mathcal{X}^s$. *That is, a flexible-price allocation can be replicated under sticky prices if and only if it is log-separable.*

This result extends a similar result from Correia, Nicolini, and Teles (2008). In that paper, *every* flexible-price allocation can be replicated as a sticky-price allocation. Here, the result is qualified by leaving out flexible-price allocations that fail to be log-separable.

The reason is essentially the same as the one that explains Lemma 2. By requiring that the *nominal* price of any given firm i is measurable in her idiosyncratic signal, the nominal rigidity also restricts how the *relative* price of any two firms can vary with any difference in their information. This boils down to requiring that the relative quantity of the two firms does not vary with the state s^t conditional on the joint information of the two firms. By contrast, a flexible-price allocation is not bound by this restriction: in general, it is possible that the relative price of two firms varies with s^t when information is incomplete.

[22] Needless to say, monetary policy can substitute for *additional* fiscal instruments if we further restrict the set of such instruments. For instance, if neither state-contingent consumption taxes nor state-contingent debt are available, monetary policy can substitute for them by varying the real value of the interest payments on the non-contingent nominal debt. This point is well understood. Here, we have focused on a novel aspect, namely on how monetary policy can substitute for a specific kind of tax instrument that are relevant only when information is heterogeneous.

It is useful then to note two special cases in which this possibility can be ruled out and, therefore, the log-separability restriction becomes vacuous. The first is when we shut down the real friction, that is, when we allow k_{it} and h_{it} to be measurable in s^t. This case nests, not only the incomplete-information settings of Woodford (2003a), Mankiw and Reis (2002), and others, but also the New-Keynesian framework of Correia, Nicolini, and Teles (2008). This in turn explains why the *entire* set of flexible-price allocations can be replicated in these earlier works—and hence why the log-separability restriction is new to the literature.

The second case is when the real friction of interest is present but technology takes a commonly-used specification.

Proposition 3. *Suppose that the production function is Cobb-Douglas or, more generally, iso-elastic in labor, the input that can adjust to the realized state:*

$$F(k,h,\ell) = \ell^{\alpha} F(k,h,1) \tag{33}$$

for all (k,h,ℓ) *and some* $\alpha \in (0,1)$. *Then,* all *flexible-price allocations are log-separable and can therefore be replicated under sticky prices:*

$$\mathcal{X}^f = \tilde{\mathcal{X}}^f \subseteq \mathcal{X}^s.$$

On the basis of this result, it seems justified to view the log-separability requirement largely as a technicality: in the most commonly used class of economies, every flexible-price allocation is log-separable and can therefore be replicated by an appropriate monetary policy.

Remark. An obvious reason why the monetary authority may be unable to replicate certain flexible-price allocations is that it is unable to observe—or respond appropriately to—the state of the economy. This possibility is relevant regardless of whether the firms are themselves informationally constrained (as in our paper) or not (as in the standard New-Keynesian framework). In this paper, we abstract from this issue.[23]

5 The Ramsey Optimum

We now turn to optimality. As standard in the Ramsey literature, we assume that the policy maker has full commitment. To fix language, we introduce the following definitions.

Definition 5. *The* ***Planner's Problem*** *is to maximize welfare over* $\mathcal{X}^s$*, the set of sticky-price allocations. An* ***optimal allocation*** *is a solution to this problem. An* ***optimal policy*** *is a combination of taxes and monetary policy that support the optimal allocation in a sticky-price equilibrium.*

Note that that the planner's problem is herein formulated in terms of allocations as opposed to policy instruments. This is the hallmark of the so-called "primal approach" used in much of the

[23] Such a policy friction is, instead, at the core of Baeriswyl and Cornand (2010). It can also rationalize the kind of demand shocks formalized in Lorenzoni (2009), or the kind of monetary shocks featured in Woodford (2003a), Hellwig (2005), and Mackowiak and Wiederholt (2009): both kinds of shocks represent "policy mistakes", that is, deviations from the (unconstrained) optimal monetary policy.

Ramsey literature but never before in economies featuring the kind of informational or cognitive frictions accommodated herein. An integral part of our contribution, which culminates in this section, is to show how this method may be applied to such economies.

In what follows we take for granted the existence of a solution to the planner's problem. To simplify the exposition, and without loss of generality, we also assume that the optimal allocation is unique; accordingly, we henceforth talk about *the* optimal allocation as opposed to *an* optimal allocation.[24]

5.1 A Relaxed Problem and the Optimal Allocation

Our ultimate goal is to solve the problem of maximizing welfare over the set of sticky-price allocations, $\mathcal{X}^s$. As a step in this direction, we first solve an auxiliary *relaxed* planning problem in which we maximize welfare over an enlarged set of allocations.

Definition 6. *Let the **relaxed set** $\mathcal{X}^R$ denote the set of all feasible allocations that satisfy condition (26). The **relaxed optimal allocation** is the one that maximizes welfare over $\mathcal{X}^R$ and is henceforth denoted by ξ^*.*

Similar to the sets $\mathcal{X}^f$ and $\mathcal{X}^s$, the set $\mathcal{X}^R$ contains only allocations that satisfy the familiar technology and resource constraints, the requirement that the pair (k_{it}, h_{it}) is measurable in ω_i^t for all i and all t, and the solvency constraint for the government, namely condition (26). However, relative to $\mathcal{X}^f$ and $\mathcal{X}^s$, what differentiates $\mathcal{X}^R$ from these sets is that it drops the implementability constraints appearing in part (ii) of Proposition 1 and Lemma 1. As a result, $\mathcal{X}^f$ and $\mathcal{X}^s$ are subsets of $\mathcal{X}^R$: they require allocations to satisfy additional constraints.

In fact, as long as information is heterogeneous, $\mathcal{X}^f$ and $\mathcal{X}^s$ are *proper* subsets of $\mathcal{X}^R$, in a manner that is of substance for our purposes. To see this more clearly, put aside the solvency constraint, which applies equally to all three sets.[25] Note then that $\mathcal{X}^R$ allows the planner to make the production choices of each firm *arbitrary* functions of their private information. To implement a typical allocation in $\mathcal{X}^R$ as a market-based outcome, the wedges faced by the firm would thus have to be functions of, not only the aggregate state s^t, but also the firm's idiosyncratic signal ω_i^t. As already explained, such wedges are unavailable in $\mathcal{X}^f$ and can only partially be achieved within $\mathcal{X}^s$. In a nutshell, while the set $\mathcal{X}^R$ gives the planner *complete* control over each firm's use of her information, the sets $\mathcal{X}^f$ and $\mathcal{X}^s$ impose certain constraints on this control.

But are these constraints binding? The answer to this question is, essentially, no. We establish this point in two steps: first for the scenario in which the nominal friction is assumed away and then for the scenario for which it is present.

Proposition 4. $\xi^* \in \mathcal{X}^f$ *always.*

Proposition 5. $\xi^* \in \mathcal{X}^s$*, and therefore* ξ^* *identifies the true optimal allocation (as defined in Definition 5), if and only if* ξ^* *is log-separable.*

[24]Note, however, that uniqueness of the optimal allocation does not mean uniqueness of an optimal policy: there may exist multiple policies that implement the same allocation.

[25]Alternatively, allow for lump sum taxation so as to drop constraint (26) entirely.

Proposition 4 states that ξ^* can *always* be implemented as part of a flexible-price equilibrium. This means that, although $\mathcal{X}^f$ precludes the planner from using taxes that are contingent on the private information of each firm, this restriction is without any loss of optimality: it is sufficient that the taxes faced by a firm depend *at most* on the aggregate state s^t. Proposition 5 adds that, insofar as ξ^* is log-separable, it can be implemented under sticky prices, too. Along with the fact that $\mathcal{X}^s \subseteq \mathcal{X}^R$, this means that ξ^* identifies the optimal sticky-price allocation.

Proposition 5 follows directly from combining Proposition 4 and Corollary 2, namely, our earlier finding that a flexible-price allocation can be replicated under sticky prices if and only if it is log-separable. What remains is to prove Proposition 5. We do so with the help of the following result, which characterizes of the relaxed optimum.

Proposition 6. *There exists a constant* $\Gamma \geq 0$ *such that* ξ^* *is given by the feasible allocation that satisfies the following conditions:*

$$\tilde{U}_c\left(s^t\right) MP_\ell\left(\omega_i^t, s^t\right) + \tilde{U}_\ell\left(s^t\right) = 0 \quad \forall \omega_i^t, s^t \tag{34}$$

$$\mathbb{E}\left[\tilde{U}_c(s^t)\left\{MP_h\left(\omega_i^t, s^t\right) - 1\right\} \,\middle|\, \omega_i^t\right] = 0 \quad \forall \omega_i^t \tag{35}$$

$$\mathbb{E}\left[\tilde{U}_c(s^t)\left\{MP_k\left(\omega_i^t, s^t\right) - \kappa\left(s^t\right)\right\} \,\middle|\, \omega_i^t\right] = 0 \quad \forall \omega_i^t \tag{36}$$

for some function $\kappa : \mathcal{S}^t \to \mathbb{R}_+$ *that satisfies*

$$\tilde{U}_c(s^t) = \beta \mathbb{E}\left[\tilde{U}_c(s^{t+1})\left\{1 + \kappa\left(s^{t+1}\right) - \delta\right\} \,\middle|\, s^t\right] \quad \forall s^t, \tag{37}$$

where $\tilde{U}_c(s^t)$ *and* $\tilde{U}_\ell\left(s^t\right)$ *are shortcuts for* $\frac{\partial}{\partial C}\tilde{U}\left(C\left(s^t\right), L\left(s^t\right), s^t; \Gamma\right)$ *and* $\frac{\partial}{\partial L}\tilde{U}\left(C\left(s^t\right), L\left(s^t\right), s^t; \Gamma\right)$, *respectively, and where*

$$\tilde{U}\left(C, L, s; \Gamma\right) \equiv U\left(C, L, s\right) + \Gamma C \frac{\partial}{\partial C} U\left(C, L, s\right) + \Gamma L \frac{\partial}{\partial L} U\left(C, L, s\right).$$

To interpret this result, note that Γ is the Lagrange multiplier on constraint (26) and $\tilde{U}$ is the per-period Lagrangian of the planner's problem, that is, the per-period welfare adjusted for the shadow value of the government budget. The above conditions resemble those that characterize the optimal allocation in Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994). There is only one subtle difference *vis-□ -vis* those benchmarks: in our setting, certain quantities and expectations are conditioned on heterogeneous, firm-specific signals of the underlying state of Nature.

To see this more clearly, consider the case with no information frictions, i.e. let $\omega_i^t = s^t$ for all states. In this case, the marginal product of every input is equated across all firms and the conditions (34)-(36) reduce to the following:

$$MP_\ell\left(s^t\right) = \frac{\tilde{U}_\ell\left(s^t\right)}{\tilde{U}_c\left(s^t\right)} \qquad MP_h\left(s^t\right) = 1$$

$$\tilde{U}_c\left(s^t\right) = \beta \mathbb{E}\left[\tilde{U}_c\left(s^{t+1}\right)\left(1 - \delta + MP_k\left(s^{t+1}\right)\right) \middle| s^t\right]$$

where $MP_z\left(s^t\right)$ now denotes the *aggregate* marginal product of input z which is *common* across all firms. The first condition is identical to the one found in Lucas and Stokey (1983) and identifies the optimal tax on labor. The second condition implies that the tax on the intermediate input is zero, an example of the result in Diamond and Mirrlees (1971): taxes should not interfere with productive efficiency. The last condition is identical to that found in Chari, Christiano, and Kehoe (1994) and relates to the celebrated Chamley-Judd result about the optimality of zero taxes on capital income.

Consider now how this familiar benchmark is modified once the informational friction is taken into consideration. Because the quantities h_{it} and x_{it} are constrained to be measurable in the firm's private information, the aforementioned complete-information benchmark is typically no more attainable. Instead, the planner finds it worthwhile to afford some cross-sectional dispersion in marginal products in order for firms to act on the basis of their idiosyncratic information in a manner that is best for society.

In short, the optimal allocation features a positive level of cross-sectional dispersion in the realized marginal products. Seen through the lens of the complete information benchmark, this property would signal the need for policy intervention. But once one takes into account that firms are informationally or cognitively constrained, the optimal dispersion in marginal products—and the associated dispersion in input choices, production levels, and relative prices—is no more zero.

For similar reasons, the optimal allocation may also feature seemingly exotic time-series properties. To illustrate this point, we again consider the example introduced in Section 4. The optimal level of GDP in this example is given by

$$\log GDP\left(s^t\right) = \gamma_0^* + \gamma_a^* \log A_t + \gamma_u^* u_t, \tag{38}$$

where the scalars γ_0^*, γ_a^*, and γ_u^* are pinned down by the primitives of the environment and where u_t captures the variation in first- and higher-order beliefs of A_t. This shock therefore captures variation in equilibrium expectations of economic outcomes that is not spanned by variation in A_t. The planner may therefore let the economy fluctuate with "sentiment shocks" as featured in Angeletos and La'O (2013), Benhabib, Wang, and Wen (2015), and Huo and Takayama (2015), despite the fact that the available tax instruments empower the planner to insulate the economy from such forces. This is because, as explained above, the planner finds it worthwhile to let firms utilize all of their available information when making decisions, including pieces of information that cause correlated movements in higher-order beliefs and resemble "animal spirits".

Having described the nature and the likely observable properties of the relaxed optimum, let us now return to, and complete, the proof of Proposition 4. The statement that $\xi^* \in \mathcal{X}^f$ follows immediately from the characterization of ξ^* in Proposition 6 along with the characterization of $\mathcal{X}^f$ in Proposition 1: if we take conditions (16)-(18) and let

$$\psi^c(s^t) = \frac{U_c\left(s^t\right)}{\tilde{U}_c\left(s^t\right)},\ \psi^\ell\left(s^t\right) = \frac{V_\ell\left(s^t\right)}{\tilde{V}_\ell\left(s^t\right)},\ \psi^r(s^t) = \tfrac{\rho-1}{\rho}\psi^c(s^t),\ \psi^k(s^t) = \psi^c(s^t)\tilde{r}(s^t) \quad \text{for all } s^t, \tag{39}$$

we reach conditions (34)-(36), which implies that the flexible-price allocation associated with the above wedges coincides with ξ^* and therefore $\xi^* \in \mathcal{X}^f$, as claimed.[26]

[26]From condition (39), one can also read the taxes that implement ξ^R as part of a flexible-price equilibrium; we postpone this for the next subsection.

5.2 Optimal Fiscal and Monetary Policy

Without serious loss of generality, suppose ξ^* is log-separable.[27] From Proposition 5, we know that ξ^* can be implemented as part of a sticky-price equilibrium with *some* taxes and *some* monetary policy (provided that it is log-separable). We now identify a policy combination that accomplishes this task.

Theorem 1. *Provided that it is log-separable, ξ^* identifies the optimal allocation and is implemented under sticky prices with the combination of*

(i) a monetary policy that replicates flexible prices; and

(ii) the following taxes:

$$\frac{1-\tau^{\ell}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)}=\frac{U_{\ell}\left(s^{t}\right)/U_{c}\left(s^{t}\right)}{\tilde{U}_{\ell}\left(s^{t}\right)/\tilde{U}_{c}\left(s^{t}\right)},\quad 1-\tau^{k}\left(s^{t}\right)=1,\quad 1-\tau^{r}(s^{t})=\tfrac{\rho}{\rho-1},\tag{40}$$

$$1+\tau^{c}\left(s^{t}\right)=\delta\frac{U_{c}\left(s^{t}\right)}{\tilde{U}_{c}\left(s^{t}\right)}\tag{41}$$

where U_c, U_ℓ, $\tilde{U}_c$, and $\tilde{U}_\ell$ are evaluated at ξ^* and where $\delta>0$ is any *state-invariant scalar.*

Part (ii) of Theorem 1 generalizes the optimal taxation results of Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994) to the class of economies under consideration. This part holds regardless of whether the nominal friction is present or assumed away. Adding the nominal rigidity, however, yields part (i). This part generalizes a key result of Correia, Nicolini, and Teles (2008) to the class of economies under consideration: it is optimal for monetary policy to replicate the flexible-price allocations that are associated with the taxes characterized in part (ii).

Consider the taxes identified in part (ii). These taxes—and their corresponding wedges—are similar to those found in the aforementioned papers. There are, however, two subtle differences. The first is that the relevant wedges are evaluated at an allocation whose observable business cycle properties may differ substantially from those characterized in Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1994), for the reasons explained in the previous subsection. This opens the door to the possibility that although the "tax formula" is the same, the cyclical properties of the taxes may be different. The second subtle difference regards the consumption tax. In the preceding papers, τ^c is typically restricted to be zero and this restriction is without loss of optimality. Here, instead, it is generally necessary to let τ^c vary with the state of Nature.

To illustrate why, abstract from monopoly power and capital (and therefore also set $\tau^r=\tau^k=0$). If we switch off the informational friction, our setting reduces essentially to that of Lucas and Stokey (1983). In this case, the optimal allocation is implemented as a flexible-price equilibrium by *any* pair (τ^c,τ^ℓ) that satisfies the first equation in condition (40), that is:

$$\frac{\tilde{U}_{\ell}\left(s^{t}\right)}{\tilde{U}_{c}\left(s^{t}\right)}=\frac{1-\tau^{\ell}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)}\cdot\frac{U_{c}\left(s^{t}\right)}{U_{\ell}\left(s^{t}\right)}.$$

The reason is quite simple: the above condition is necessary and sufficient for the labor-supply decision of the representative consumer to align with the solution to the planner's problem. But since

[27] Recall that ξ^* is *necessarily* log-separable when the technology is Cobb-Douglas. But even if ξ^* is not log-separable, the lesson presented in the sequel regarding the sub-optimality of price stability survives.

there is only one margin to take care of and two tax instruments that can be used for this purpose, the precise combination of τ^c and τ^l is indeterminate. This explains why Lucas and Stokey (1983) can restrict $\tau^c = 0$ in all states without loss of optimality.

Let us now switch back on the informational friction. In this case, there is an *additional* implementability constraint to consider: the one captured in condition (35), that is, the one regarding the firm's input choice under incomplete knowledge or understanding of the aggregate state. Because this friction introduces risk in the eyes of the firm, the "risk appetite" of the firm—as captured by the covariation of the relevant pricing kernel, $\mathcal{M}(s^t) = \frac{U_c(s^t)}{1+\tau^c(s^t)}$, with the firm's marginal returns—matters. For the firm to make the "right" choices, this kernel must be aligned with that of the planner. Instead, it is sufficient—and in general necessary—that $\mathcal{M}(s^t)$ is a constant multiple of $\tilde{U}_c\left(s^t\right)$. This explains the form of the consumption tax seen in condition (41) above.

Notwithstanding these subtleties, the take-home message of Theorem 1 is that the *essence* of the optimal taxes remains the same as in the extant Ramsey literature. To reinforce this message, we next identify a special case in which there is no loss at all in ignoring the aforementioned subtleties.

Lemma 3. *Suppose preferences are given by*

$$U\left(C, L\right) = \frac{C^{1-\gamma}}{1-\gamma} - \eta \frac{L^{1+\epsilon}}{1+\epsilon} \tag{42}$$

for some $\gamma, \epsilon, \eta > 0$*. Then, the optimal allocation is implemented with a zero tax on capital* ($\tau^k = 0$)*, a zero tax on consumption* ($\tau^c = 0$)*, and a time- and state-invariant tax on labor given by*

$$1 - \tau^\ell = \frac{1 + \Gamma\left(1-\gamma\right)}{1 + \Gamma\left(1+\epsilon\right)},$$

where Γ *is the Lagrange multiplier.*

That is, once we impose the most commonly used specification for preferences, the optimal taxes in our setting are *exactly* the same as those predicted by Lucas and Stokey (1983), Chamley (1986), and Chari, Christiano, and Kehoe (1994). This is despite the fact that, as already noted, the underlying optimal allocation accommodates "exotic"beliefs-driven aggregate fluctuations and cross-sectional dispersion in realized marginal products.

Consider, now part (i) of Theorem 1 which states that it is optimal for the monetary authority to replicate flexible-price allocations; as noted above, this extends a key result from Correia, Nicolini, and Teles (2008). There is, however, a crucial difference: in that paper, and more generally in the New-Keynesian framework, replicating flexible-price allocations is equivalent to implementing price stability. As we explain in the subsequent section, this equivalence breaks down in the class of economies under consideration—indeed, it breaks as soon as firms lack common knowledge of the state of the economy

Remark 1. Theorem 1 allows for a certain indeterminacy in the optimal tax policy: there is a whole continuum of (different) consumption taxes that are consistent with implementing the (same) optimal allocation. This comes together with a certain indeterminacy in the optimal monetary policy: as we vary the scalar δ in Theorem 1, we vary, not only the consumption tax, but also the nominal interest rate that helps support the optimal allocation as a sticky-price equilibrium. The basic logic behind this kind of indeterminacy is the same as in Correia, Nicolini, and Teles (2008) and Correia et al. (2013): the consumption tax and the nominal interest rate are completely interchangeable instruments for controlling consumer spending.[28]

Remark 2. Notwithstanding the previous remark, the optimal monetary policy is determinate in the following respect: *any* monetary policy that implements ξ^* *has* to replicate flexible prices. This is a direct implication of the fact that ξ^* belongs in $\mathcal{X}^f$. That is, the property stated in part (i) of Theorem 1 is *necessary* for optimality. The same applies for the results obtained in the following subsection, regarding the (sub)optimality of price stability.

5.3 On the Optimal Cyclicality of the Price Level

Within the New-Keynesian framework, the logic in favor of price stability is that it minimizes relative-price distortions or, more precisely, maximizes productive efficiency (i.e., efficiency in the use of resources in the cross-section of firms). We now explain why this logic is upset once the informational or cognitive limitations of the firms are taken into consideration.

Consider the optimal allocation, as per Definition 5. This may differ from the *relaxed* optimum studied in Subsection 5.1 if and only if the latter fails to be log-separable. But even if that happens to be the case (which, as already discussed, is of little practical relevance), the optimal allocation *must* be log-separable, simply because *every* sticky-price allocation is log-separable. It follows that there exist functions Ψ^ω and Ψ^s such that, along the optimal allocation, $y(\omega_i^t, s^t) = \log \Psi^\omega(\omega_i^t) + \log \Psi^s(s^t)$.

Consider now any two firms i and j. For all realizations of their signals and of the underlying state, their nominal prices must satisfy the following joint restriction:

$$\begin{aligned} \log p(\omega_i^t) - \log p(\omega_j^t) &= -\tfrac{1}{\rho}\left[\log y(\omega_i^t, s^t) - \log y(\omega_j^t, s^t)\right] \\ &= -\tfrac{1}{\rho}\left[\log \Psi^\omega(\omega_i^t) - \log \Psi^\omega(\omega_j^t)\right] \end{aligned}$$

The relative price of any two firms is therefore inversely related to the relative belief of these firms, as measured by $\Psi^\omega(\omega_i^t)$. Intuitively, if optimistic firms are to produce more than pessimistic ones, they must also set lower relative prices. But, as long as firm i does not know ω_j^t and, symmetrically, firm j does not know ω_i^t, this is possible if and only if the nominal price of firm i is itself negatively related to $\Psi^\omega(\omega_i^t)$, and similarly for j. That is, for *every* firm i, it must be that $\log p(\omega_i^t) = z - \frac{1}{\rho} \log \Psi^\omega(\omega_i^t)$, for some variable z that is common knowledge to the firms. Aggregating this property across all the firms, and letting

$$\mathcal{B}(s^t) \equiv \left[\int \Psi^\omega\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}} d\mu(\omega_i^t | s^t)\right]^{\frac{\rho}{\rho-1}}, \tag{43}$$

gives the following result.

[28] This logic is complicated in our context because of the extra role of the consumption tax described earlier on. Yet, the essence remains. For instance, suppose we allow the scalar δ in part (ii) of Theorem 1 to be an arbitrary function of time (or of any other variable that happens to be common knowledge to the firms). Then, the optimal allocation is still implemented, although with a different path for the nominal interest rate.

Theorem 2. *Along any sticky-price equilibrium that implements the optimal allocation, there must exist a commonly known variable z_t such that, for every $s \in \mathcal{S}^t$,*

$$\log P_t(s) = z_t - \tfrac{1}{\rho} \log \mathcal{B}(s) \tag{44}$$

To interpret this result, note that $\Psi^\omega(\omega_i^t)$ captures the variation in a firm's output that is driven by the firm's signal. We can thus think of $\Psi^\omega(\omega_i^t)$ as a proxy for how the output of firm i depends on her information, her level of understanding of what's going on in the economy, and her overall "sentiment" about how much she should produce. By the same token, we can think of $\mathcal{B}(s^t)$ as a measure of the average belief or sentiment in the economy. Furthermore, from Proposition 3 we know that $\Psi^\omega(\omega_i^t)$ is an increasing function of k_{it} and h_{it}, which suggests that $\mathcal{B}(s^t)$ inherits the cyclical properties of aggregate capital and intermediate good purchases—a point we make precise in the sequel. With this point in mind, we may interpret Theorem 2 as follows: along the optimal allocation, the price level is inversely related to real economic activity.

Before expanding on the economic substance of this result, we wish to comment on the nominal indeterminacy allowed by Theorem 2. As stated in the theorem, the optimal price level is determined only up to a variable z_t that is itself common knowledge to the firms. The reason is that as long as z_t is common knowledge firms can *perfectly* coordinate their response to z_t; as a result, any variation in z_t moves the nominal price level without affecting either the relative prices across any two firms or any of the real quantities in the economy.[29]

Note, however, that this kind of indeterminacy hinges on the variable z_t being common knowledge. If, instead, any two firms have different beliefs about z_t, then these firms will also have different beliefs about their nominal marginal costs and will therefore not find it optimal to set the same nominal prices in response to z_t. It follows that letting the price level depend on z_t can not be neutral *vis-□ -vis* the real allocations. In a nutshell, the kind of nominal indeterminacy allowed by Theorem 2 can be removed by a "refinement" that requires that no variable be commonly known. But even without such a refinement, this nominal indeterminacy is of no real consequence and does not affect the essence of Theorem 2: if the planner wishes the firms to utilize information that is commonly available to them, the planner must induce a negative correlation between the price level and real economic activity as measured by $\mathcal{B}(s^t)$.

With these points in mind, we now return to our earlier claim that $\mathcal{B}(s^t)$ can be interpreted as a proxy for real economic activity.

Lemma 4. *Suppose that technology is Cobb-Douglas, let $K(s^t)$ denote the aggregate capital stock, and let $H(s^t)$ denote the aggregate intermediate good purchases. Then, up to a first-order log-linear approximation,*

$$\log \mathcal{B}(s^t) = \zeta_K \log K(s^t) + \zeta_H \log H(s^t), \tag{45}$$

for some constants $\zeta_K > 0$ and $\zeta_H > 0$.

[29] An extreme version of this kind of indeterminacy emerges when s^t is itself common knowledge, that is, when the friction of interest is assumed away. Then, clearly, the price level can be an arbitrary function of s^t. It follows that Theorem 2 is of substance *only* when s^t is not common knowledge.

By considering a log-linear approximation, we effectively abstract from any cyclical variation in the cross-sectional dispersion of firm-level outcomes. We can then express $\log \mathcal{B}(s^t)$ as a linear combination of the aggregate quantities of capital and intermediate good purchases and can therefore reach the following result.

Corollary 2. *Suppose that along the optimal allocation the aggregate quantities of capital and intermediate goods are procyclical. Then, up to a log-linear approximation, $\mathcal{B}(s^t)$ is procylical, and therefore $P(s^t)$ is countercyclical.*

To sum up, the empirically relevant scenario appears to be the one in which monetary policy ought to induce a negative correlation between the price level and real output—a property that resembles "nominal GDP targeting". Importantly, this is true regardless of whether the variation in output is due to actual innovations in the underlying fundamentals or to correlated noise in the firms' information, or sentiment, about the state of the economy.

To illustrate, we may again consider the tractable example introduced in Section 4. In this example, the aggregate level of GDP that obtains along the optimal allocation is given by condition (38). The associated price level, on the other hand, is given by

$$\log P(s^t) = \delta_0^* - \delta_a^* \log A_t - \delta_u^* u_t, \tag{46}$$

where the scalars δ_a^* and δ_u^* are pinned down by the primitives of the environment and measure the elasticities of the price level to the underlying TFP and belief shocks. Furthermore, for *any* primitives, these elasticities and the corresponding elasticities of the optimal GDP level satisfy

$$\frac{\delta_a^*}{\gamma_a^*} > 0 \qquad \text{and} \qquad \frac{\delta_u^*}{\gamma_u^*} > 0.$$

It follows that the optimal policy targets a *negative* relation between the price level and aggregate output, regardless of whether the business cycle is triggered by innovations in fundamentals, by correlated errors in first-order beliefs, or even by the more exotic beliefs shocks considered in Angeletos and La'O (2013), Benhabib, Wang, and Wen (2015), and Huo and Takayama (2015).

Our findings should not, however, be misinterpreted as a case against price stability *per se*. Theorem 2 requires the price level and aggregate output co-vary *only* in response to shocks that themselves affect the optimal allocation. If we instead consider shocks that move the equilibrium allocation under *some* policies but do not move the optimal allocation, then it is optimal to stabilize both the level of output and the price level *vis-□ -vis* these particular shocks. If we shut down capital accumulation, examples of such shocks include discount-rate shocks and news about future TFP; more generally, they can be monetary shocks or pure sunspots.[30]

[30] By the latter we mean random variables that are unrelated, not only to the underlying payoff-relevant fundamentals, but also the entire belief hierarchy about the fundamentals. By monetary shocks, on the other hand, we mean mean either shocks to the nominal interest rate set by the monetary authority, or shocks to the demand for real money balances in an appropriate extension of our framework. In particular, suppose the per-period utility is given by $U(C, L, s^t) + V(m, s^t)$, where $m \equiv M/P$ denotes real money balances and M denotes the nominal supply of money, and consider a shock that moves V_m without affecting U, F, A, G, and the entire belief hierarchy about these objects.

Finally note that the *strict* optimality of stabilizing the price level against such shocks requires that these shocks not be commonly known, otherwise our earlier comment regarding the indeterminacy of the price level applies. We conclude that that the friction we have accommodated in this paper offers a joint rationale for letting the price level vary inversely with the optimal level of output and for stabilizing the former against shocks that do not justify variation in the latter.

5.4 Discussion

To understand the precise meaning of our results regarding the optimal monetary policy, and to place them within the literature, it is useful to address the following questions.

What is the right optimality benchmark? In textbook treatments of the New-Keynesian framework (e.g., Woodford, 2003b; Galí, 2008), the optimality of replicating flexible-price allocations is often tied to the question of whether the first best is attainable. In particular, the typical argument, which goes back to Rotemberg and Woodford (1997) and Goodfriend and King (2001), is based on the elementary observation that insofar as the planner can attain the first best under flexible prices with the use of appropriate taxes, the planner can also attain the first best under sticky prices with the use of the same taxes and a monetary policy that replicates flexible prices.

More generally, however, the benchmark relative to which one must assess the optimality of replicating flexible prices does *not* have to be the first best. This point was elegantly highlighted in Correia, Nicolini, and Teles (2008): in that paper, the appropriate reference point is not the first best but rather the kind of second best characterized in Lucas and Stokey (1983).

Things are even more subtle in our setting: the right gauge for monetary policy is the third best defined and characterized in Subsection 5.1. Recognizing this basic point and characterizing the relevant optimum are integral parts of our contribution.

Is replicating flexible prices the same as targeting price stability? The answer is no in our setting, whereas it is yes in the baseline New-Keynesian framework. Importantly, the latter is true *regardless* of whether the underlying flexible-price allocations are themselves optimal or not. This explains why the applied New-Keynesian literature has justified a deviation from price stability only by letting such a deviation partially correct the inefficiency of the underlying flexible-price allocations. By contrast, we have justified a certain departure from price stability while preserving the optimality of the underlying flexible-price allocations.

Is replicating flexible prices the same as minimizing relative-price distortions? Yes, in both our setting and the New-Keynesian framework. However, this statement is meaningful only subject to an appropriate definition of what "minimizing relative-price distortions" means. This underscores the importance and subtlety of distinguishing the appropriate optimality benchmark.

Is there a trade off between minimizing relative-price distortions and minimizing the output gap? Answering this question, too, requires a definition of what the "output gap" is. In Goodfriend and

King (2001) and the typical textbook treatment of the New-Keynesian framework, the output gap is measured relative to the first best; in Correia, Nicolini, and Teles (2008), the appropriate reference point is instead the second best characterized in Lucas and Stokey (1983); and in our setting it is the third best studied in Subsection 5.1. With these definitions in place, the answer to the above question is the same in our setting as in Goodfriend and King (2001), and Correia, Nicolini, and Teles (2008): there is no trade off.

Does monetary policy substitute for missing tax instruments? This is effectively the same question as the previous one. The applied New-Keynesian literature has sought to formalize a trade off between the aforementioned two goals (or between output and price stabilization) by introducing markup shocks and other market distortions, precluding the planner from correcting these distortions with tax instruments, and letting monetary policy substitute for the missing tax instruments. While this possibility may be important in practice, it is entirely orthogonal to the point we have made here. Indeed, Theorem 1 implies that the optimal monetary policy *should not* substitute for any of the missing tax instruments, despite the fact that it *could*.[31]

Does "divine coincidence" hold in our setting? This depends on what this fussy notion means. On the one hand, we preserve divine coincidence in the sense that replicating flexible prices helps achieve two goals at once: the goal of minimizing the output gap (properly defined); and the goal of minimizing relative-price distortions (properly defined). On the other hand, we turn divine coincidence on its head by equating these goals and replicating flexible prices with a certain departure from price stability.

Does monetary policy "lean against the wind"? Yes in the sense of letting the nominal price level move in the opposite direction than real output. But not in the sense of pushing the allocation away from its flexible-price counterpart (or equivalently of substituting for missing tax instruments).

What is the role of the real friction? We conclude this section by highlighting how our result regarding the sub-optimality of price stability hinges on allowing the informational friction to be a real friction, as opposed to merely a form of nominal rigidity. To formalize this point, we maintain Property 2 but replace Property 1 with the following variant.

Property 1′. *The allocation ξ is such that firm-level quantities satisfy*

$$k_{it} = k_t(\omega_i^t, s^t), \quad h_{it} = h_t(\omega_i^t, s^t), \quad \ell_{it} = \ell_t(\omega_i^t, s^t), \quad y_{it} = y_t(\omega_i^t, s^t),$$

This property removes the real friction by allowing all inputs of a firm to vary with the true state of the economy. We may thereby consider equilibria in which allocations satisfy Property 1′ while prices continue to be rigid in the sense of Property 2.

[31] Recall the discussion from Subsection 4.2 on how monetary policy could mimic certain kind of firm-specific taxes.

Proposition 7. *Suppose we maintain Property 2 (which means that the nominal friction is preserved) but replace Property 1 with Property 1′ (which means that the real friction is assumed away). Then, the optimal allocation is implemented by targeting price stability.*

The different micro-foundations of the nominal rigidity proposed by Mankiw and Reis (2002), Woodford (2003a), and Mackowiak and Wiederholt (2009) therefore do not alone upset the lesson of Correia, Nicolini, and Teles (2008) that price stability is optimal as long as monetary policy does not have to substitute for missing tax instruments. Instead, what upsets that lesson is the accommodation of the real friction formalized in Property 1. For it is this feature alone that makes the underlying flexible-price allocations sensitive to the private information of each firm.

This elementary point also explains the key difference between our contribution and those of Ball, Mankiw, and Reis (2005), Adam (2007), and Paciello and Wiederholt (2014). Similar to our work, these papers study optimal monetary policy in settings in which firms set their prices on the basis of incomplete information. Yet, by precluding the kind of real friction we have accommodated herein, these papers share the prediction of the standard New-Keynesian framework that price stability is synonymous to minimizing relative-price distortions (or maximizing productive efficiency). To keep the analysis interesting, these papers add markup shocks, constrain the available tax instruments, and introduce a trade off between relative-price distortions and output-gap stabilization. This trade-off may be important in practice. It is also a staple of much of the applied New-Keynesian literature. Yet, it is orthogonal to the point we have made here.

6 Endogenous Information/Attention

In the preceding analysis, we have treated φ, the distribution from which a firm's signal is drawn conditional on the underlying state of Nature, as an exogenous object. We now allow each firm to choose her φ optimally, subject to some cost. One can think of this either as costly acquisition of information or the firm's decision of how much attention to pay to the available data (Sims, 2003) or how much cognitive effort to put into comprehending what's happening around her and how to best respond (Tirole, 2015). The key result of this section is that the policies that are optimal in our baseline framework remain optimal in the extended framework. This means that these policies implement not only the optimal allocation taking the stochastic process of the signals as given but also the socially optimal choice of this process itself.

Remark. The analysis in this section is most closely connected to Paciello and Wiederholt (2014). Like them, we endogenize the signal structure. Unlike them, we do not require that monetary policy substitute for missing tax instruments. Most crucially, we let the informational friction be the source of a real friction (in the sense of Property 1). We also allow for a more general formulation of rational inattention (namely, arbitrary Φ and arbitrary κ).

6.1 Set up

We extend our baseline framework as follows. For any i, let $\varphi_i \equiv \{\varphi_i^t\}_{t=0}^{\infty}$, where φ_i^t denotes the distribution from which ω_i^t is drawn conditional on s^t. Note that φ_i represents a complete description of how the information or the cognitive state of firm i evolves over time and over the different realizations of the underlying state of Nature. So far, φ_i was restricted to be the same across all i and was exogenously fixed. We now let each each firm choose her own φ_i, at the beginning of time, from some set Φ, subject to a cost represented by a function $\kappa : \Phi \to \mathbb{R}_+$.

To simplify the exposition, we shut down capital[32] and assume that the aforementioned cost is in terms of utility or "cognitive effort".[33] As will become evident, the arguments we develop in this section do not hinge on these simplifications. We also bypass the technical issue of the existence of an equilibrium or existence of a Ramsey optimum by requiring that all maximization and fixed-point problems defined henceforth admit a solution. We finally impose that for every $\varphi \in \Phi$, the firm learns the realization of an extrinsic random variable that is independent of s^t for all t, is i.i.d. across firms, and is drawn from a uniform distribution over $[0, 1]$. This guarantees that it is without loss of generality to concentrate on equilibria and optima in which all firms end up choosing the same distribution and the same strategies.[34]

More crucially, no restriction of economic substance is imposed on the set Φ nor on the function κ. For instance, there is no need to order the elements of Φ in terms of more or less information or to model κ in terms of relative (Shannon) entropy or Kullback–Leibler divergence. There is also no need to take a stand on whether firms can recall their past signals effortlessly or suffer from partial amnesia, nor specify whether the cost κ is separable across time or signals. We can thus nest, *inter alia*, the specifications considered in Sims (2003), Myatt and Wallace (2012), Paciello and Wiederholt (2014), and Pavan (2016). Last but not least, since the domains of s^t and of ω^t are allowed to be arbitrary, the economy can be understood as a "cognitive game" in the sense of Tirole (2015). This refers to a class of two-stage games such that: in stage 2, the players play a standard game with a fixed distribution for their Harsanyi types; in stage 1, the players jointly choose the distribution of their stage-2 Harsanyi types. It follows that the choice of φ can capture not only how much information the firms possess about the *exogenous* fundamentals but also how well they can grasp the *endogenous* behavior of one another. In short, choosing φ is like choosing how much to know about everything that is going on in the economy.

6.2 Equilibria, Implementability, and Optimality

We now proceed to define and characterize the equilibria and the Ramsey optimum of the economy with endogenous information (or endogenous cognition). To simplify, we concentrate on the case with flexible prices; the case with sticky prices is analogous.

Consider the problem faced by an arbitrary firm i. This problem can be split into two subproblems: the "outer" problem of choosing a φ_i; and the "inner" problem of choosing the optimal input

[32] That is, we set $k_{it} = 1$ and $x_{it} = 0$ for all i, all t, and all realizations of uncertainty.

[33] This assumption guarantees that, whenever $\varphi_i = \varphi$ for all i and for some ψ, the definition and the characterization of the sets of feasible, flexible-price, sticky-price, and optimal allocations *conditional* on ψ remain exactly the same as in our baseline model. If, instead, we had specified the cost in terms of final good (or, say, labor), we would have to adjust appropriately all the earlier analysis: the cost would show up in firm profits and in the resource constraint.

[34] This is because any asymmetric equilibrium (or optimum) can be replicated by a symmetric one that let's each firm condition her production choices on the aforementioned extrinsic variable.

and output strategies for given φ. Recall that any given triplet (ξ, ρ, θ) contains a unique collection $\{Y_t(\cdot), C_t(\cdot), W(\cdot), \theta_t(\cdot)\}_t^\infty$, that is, it is associated with a unique stochastic process for aggregate output, aggregate consumption, the wage rate, and taxes. With this in mind, we can represent the firm's inner problem as follows:

$$\Pi(\varphi; \xi, \rho, \theta) = \max_{y,\ell,h} \sum_t \sum_{\omega,s} \beta^t \mathcal{M}(s^t) \pi\left(\omega^t, s^t\right) \varphi^t\left(\omega^t \middle| s^t\right) \mu^t(s^t) \tag{47}$$

$$s.t. \quad y\left(\omega^t, s\right) = A(s^t) F\left(h\left(\omega^t\right), \ell\left(\omega^t, s^t\right)\right),$$

where $\mathcal{M}(s^t) = \frac{U_c(C(s^t))}{1+\tau^c(s^t)}$ and

$$\pi\left(\omega^t, s^t\right) \equiv \left(1 - \tau^r\left(s^t\right)\right) \left(\frac{y(\omega^t, s)}{Y(s)}\right)^{-\frac{1}{\rho}} y\left(\omega^t, s^t\right) - h\left(\omega^t\right) - W\left(s^t\right) \ell\left(\omega^t, s^t\right).$$

We can then represent the solution to the outer problem as follows:

$$\varphi \in \Gamma(\xi, \rho, \theta) \equiv \arg\max_\phi \{\Pi(\phi; \xi, \rho, \theta) - \kappa(\phi)\} \tag{48}$$

To interpret these representations, note that the first problem takes φ as given but lets the firm optimize her input and output choices. The second problem then describes the optimal choice of φ.

The above determines the firm's optimal choice of φ for *any* triplet (ξ, ρ, θ). But not every such triplet is relevant: (ξ, ρ, θ) can be part of an equilibrium of the "overall game" in which firms choose both their information structures and their input/output strategies only if it is also an equilibrium of the "continuation game" that obtains once the firms' information structures have been fixed. We therefore define an equilibrium as follows.

Definition 7. *In the economy with endogenous information, a flexible-price equilibrium is a collection* $(\varphi, \xi, \rho, \theta)$ *such that: (i)* $(\xi, \rho, \theta) \in \mathcal{E}^{flex}(\varphi)$*; and (ii)* $\varphi \in \Gamma(\xi, \rho, \theta)$.

An equilibrium now contains not only the triplet (ξ, ρ, θ) that describes the allocation (or the firm strategies), the price system, and the government policy, but also the information structure φ. Part (i) requires that, taking φ as given, the triplet (ξ, ρ, θ) constitutes an equilibrium in the sense of Definition 2. Part (ii) on the other hand requires that φ is itself a solution to the optimal information/cognition problem that the typical firm faces when the rest of the economy is described by (ξ, ρ, θ). An equilibrium of the economy with endogenous information is therefore a fixed point between the mapping $\mathcal{E}$, which was studied earlier (see especially Proposition 1), and the mapping Γ, which is defined by condition (48) above.

Consider next the planner's problem. By manipulating the available policy instruments, the planner can now influence not only the equilibrium allocation in the "continuation game" that obtains once φ is fixed but also the optimal choice of φ in the first place. To understand how this modifies the planner's problem relative to the one studied earlier on, pick an arbitrary $\hat{\varphi}$ and let $\hat{\xi}$ be the allocation that is optimal in the sense of Definition 5 (that is, when treating $\hat{\varphi}$ as exogenous). Relative to this benchmark, the planner's problem has been eased by the introduction of the option to choose a $\varphi \neq \hat{\varphi}$. However, the planner's problem has also been worsened by the introduction of an additional implementability constraint: namely the requirement that the pair (φ, ξ) must be consistent with the individually optimal information/cognition problem the firms.

To formalize this point, we first adapt the notion of implementability as follows.

Definition 8. *A pair (φ, ξ) of an information or cognition structure and an allocation is implementable (under flexible prices) if there exists a policy θ and a price system ρ such that the collection $(\varphi, \xi, \rho, \theta)$ is an equilibrium in the sense of Definition 7*

We then state the following result, which can be proved following similar steps as in the proof of Proposition 1.

Proposition 8. *A pair (φ, ξ) is implementable if and only if the following properties hold.*

(i) The following constraint is satisfied at the aggregate level:

$$\sum_{t,s^t} \beta^t \mu\left(s^t\right)\left[U_c\left(s^t\right) C\left(s^t\right) + U_\ell\left(s^t\right) L\left(s^t\right)\right] = 0; \tag{49}$$

(ii) There exist wedges ψ=($\psi^c, \psi^\ell \psi^r$) $: \mathcal{S}^{3t} \to \mathbb{R}$ such that the following conditions hold at the firm level:

$$\begin{aligned} \psi^r\left(s^t\right) \tfrac{\rho-1}{\rho} MP_\ell\left(\omega^t, s^t\right) - \psi^\ell\left(s^t\right) \quad &= \quad 0 \quad \forall\, \omega_i^t, s^t & (50) \\ \sum_{s^t} \left\{\psi^r\left(s^t\right) \tfrac{\rho-1}{\rho} MP_h\left(\omega^t, s^t\right) - \psi^c\left(s^t\right)\right\} \varphi\left(s^t \middle| \omega^t\right) \quad &= \quad 0 \quad \forall\, \omega_i^t & (51) \end{aligned}$$

$$\varphi \;\in\; \arg\max_{\phi} \left\{\tilde{\Pi}(\phi; \xi, \psi) - \kappa(\varphi)\right\} \tag{52}$$

where

$$\tilde{\Pi}(\phi; \xi, \psi) \equiv \tag{53}$$

$$\max_{y,\ell,h} \sum_t \sum_{\omega,s} \beta^t \left\{\psi^r\left(s^t\right)\left(\frac{y\left(\omega^t, s\right)}{Y(s)}\right)^{-\frac{1}{\rho}} y\left(\omega_i^t, s^t\right) - \psi^c\left(s^t\right) h\left(\omega^t\right) + \psi^\ell(s^t)\ell\left(\omega^t, s^t\right)\right\} \phi\left(\omega^t \middle| s^t\right) \mu^t(s^t)$$

$$s.t. \quad y\left(\omega_i^t, s\right) = A(s^t) F\left(h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right)\right),$$

Comparing this result to Proposition 1 makes clear that the option to choose φ adds an extra degree of freedom to the planner's problem, whereas condition (52) adds an extra implementability constraint. This constraint reflects the lack of a certain class of policy instruments, namely instruments that would permit the planner to manipulate the equilibrium value of φ while holding ξ constant. Think, in particular, of a direct Pigouvian tax or subsidy on the firm's acquisition of information or cognition effort. If the planner had access to such an instrument, condition (52) would drop out of Proposition 8, and the planner would be free to control the equilibrium allocation ξ without having to worry how this affects the firms' choice of φ. Now, by contrast, the planner must take into account the feedback effect from the equilibrium value of ξ to that of φ. In other words, the planner faces a potential trade off between influencing the *use* of information and influencing the *collection* of information.[35]

[35] We have qualified the trade off as a *potential* one because it remains to be seen whether this trade off is relevant for understanding the solution to the planner's problem.

With slight abuse of notation, let $\mathcal{X}^{flex}$ denote the set of the pairs (φ, ξ) that are implementable in the sense of Definition 7. The planner's problem is to maximize welfare (defined as the ex ante utility of the representative agent, net of the cost κ) over the set $\mathcal{X}^{flex}$.

Definition 9. *The Ramsey optimum is given by a pair (φ, ξ) that maximizes welfare over $\mathcal{X}^{flex}$.*

We characterize the Ramsey optimum again by adapting the methods developed in Section 5 to the endogeneity of φ. In particular, we let $\mathcal{X}^{relax}$ denote the set of the pairs (φ, ξ) that satisfy *only* condition (49) and note that, trivially, $\mathcal{X}^{flex} \subset \mathcal{X}^{relax}$. We then consider the following object.

Definition 10. *The relaxed optimum is given by a pair (φ^*, ξ^*) that maximizes welfare over $\mathcal{X}^{relax}$.*

The next lemma provides two necessary conditions for a pair (φ^*, ξ^*) to a be relaxed optimum.

Lemma 5. *If (φ^*, ξ^*) is a relaxed optimum, the following two properties must hold.*
(i) taking φ^* as given, ξ^* is optimal in the sense of Definition 5; and
(ii) taking ξ^* as given, φ^* satisfies

$$\varphi^* \in \arg\max_{\varphi} \left\{ \mathcal{Z}\left(\varphi; \xi^*\right) - \kappa\left(\varphi\right) \right\}, \tag{54}$$

where

$$\mathcal{Z}\left(\varphi; \xi\right) \equiv \max_{y,\ell,h} \sum_t \sum_{\omega^t, s^t} \beta^t \left[\tilde{U}_c\left(s^t\right) \left(\int_0^{y(\omega,s)} \left(\frac{z}{Y(s)}\right)^{-\frac{1}{\rho}} dz - h\left(\omega^t\right) \right) + \tilde{U}_\ell\left(s^t\right) \ell\left(\omega_i^t, s^t\right) \right] \varphi\left(\omega^t \middle| s^t\right) \mu^t(s^t).$$

Part (i) states that ξ^* is optimal whether the planner takes into account the endogeneity of φ or treats φ as fixed at φ^*. This is trivially true because the relaxed problem has dropped the implementability constraint (52): the aforementioned trade off between the collection and the use of information has been removed by assumption. To understand part (ii), note that, because each firm is infinitesimal, the planner can vary *both* a firm's production choices *and* her information structure without affecting the aggregate outcomes. It follows that the contribution of any firm to social welfare is captured by $\mathcal{Z}\left(\varphi; \xi\right)$; this measures the social surplus generated by the optimal production choices of the firm, when her information structure is fixed at φ. By the same token, the socially optimal choice of φ maximizes the aforementioned surplus net of the information cost, which is what part (ii) states.

We next prove that any pair (φ^*, ξ^*) that satisfies the aforementioned two properties belongs to the set $\mathcal{X}^{flex}$. This guarantees that the solution to the relaxed problem coincides with the solution to the actual Ramsey problem, a property that mirrors the one encountered in Section 5.2. We furthermore prove that the same taxes that are optimal in the baseline economy in which φ is exogenously fixed at φ^* permit the planner to implement the pair (φ^*, ξ^*) as an equilibrium of the (extended) economy in which φ is endogenously chosen.

Proposition 9. *Let (φ^*, ξ^*) be a relaxed optimum. This can be implemented as part of a flexible-price equilibrium (in the sense of Definition 7) with the same taxes as in Theorem 1.*

This follows directly from Lemma 5 together with the following argument. Let θ^* be the taxes identified in Theorem 1 and let ρ^* be the associated price system. From our earlier analysis, we know that $(\xi^*, \theta^*, \rho^*)$ is an equilibrium of the (restricted) economy in which the φ is exogenously fixed at φ^*. What remains to show is that, when the firm faces $(\xi^*, \theta^*, \rho^*)$, she finds it individually optimal to pick $\varphi = \varphi^*$.

To establish that this is indeed true, consider the firm's market valuation, as given in condition (47). At $(\xi, \theta, \rho) = (\xi^*, \theta^*, \rho^*)$, this reduces to the following:

$$\Pi\left(\varphi; \xi^*, \theta^*, \rho^*\right) = \\ \max_{y,\ell,h} \sum_t \sum_{\omega^t, s^t} \beta^t \left[\tilde{U}_c\left(s^t\right)\left(\tfrac{\rho}{\rho-1} Y^*(s^t)^{\frac{1}{\rho}} y\left(\omega^t, s^t\right)^{1-\frac{1}{\rho}} - h\left(\omega^t\right)\right) + \tilde{U}_\ell\left(s^t\right) \ell\left(\omega^t, s^t\right)\right] \varphi\left(\omega^t, s^t\right).$$

Next, evaluating the innermost integral of (54), the social surplus generated by firm i can be expressed as follows:

$$\mathcal{Z}\left(\varphi; \xi^*\right) = \max_{y,\ell,h} \sum_t \sum_{\omega^t, s^t} \beta^t \left[\tilde{U}_c\left(s^t\right)\left(\tfrac{\rho}{\rho-1} Y^*(s^t)^{\frac{1}{\rho}} y\left(\omega^t, s^t\right)^{1-\frac{1}{\rho}} - h\left(\omega^t\right)\right) + \tilde{U}_\ell\left(s^t\right) \ell\left(\omega^t, s^t\right)\right] \varphi\left(\omega^t, s^t\right).$$

It follows that $\Pi\left(\varphi; \xi^*, \theta^*, \rho^*\right) = \mathcal{Z}\left(\varphi; \xi^*\right)$ for *every* φ. Combining this property with part (ii) of Lemma 5, we conclude that

$$\varphi^* \in \Gamma\left(\xi^*, \theta^*, \rho^*\right),$$

which verifies the claim that φ^* is optimal in the eyes of the typical firm and completes the proof of Proposition 9.

To understand this result, it is useful to build an analogy. Consider a neoclassical growth model in which a monopolist can choose her production technology (e.g., as in Romer, 1990) and ask the following question: can a uniform subsidy on firm sales induce both the efficient level of output for given technology and the efficient choice of technology? The answer to this question is positive as long as one maintains the usual Dixit-Stiglitz specification for intermediate good demand and abstract from any knowledge spillovers. These conditions suffice for the aforementioned subsidy to equate both the marginal revenue of the firm with the marginal utility of the consumer and the total profit made from any given technology with the corresponding social surplus.[36] Our result can thus be understood as a variant of this observation: the choice of an information structure in our context is the analogue of the choice of technology in the growth context, the Dixit-Stiglitz specification has been maintained, and spillovers are ruled out—the cost κ faced by each firm is independent of the choices of other firms.[37] One subtlety with our result is that the appropriate notion of social surplus takes into account both the measurability constraint that precludes the firm from conditioning its choices on the true state as well as the shadow value of the government budget constraint.

[36] Without the aforementioned conditions, the planner may need a *non-linear* subsidy, as in a two-part tariff, in order to hit both goals.

[37] The latter condition can be violated if the firms have access to, and can digest with little or no cognitive effort, the information of other firms either directly (e.g., by sharing information with one another) or indirectly (e.g., by observing for free macroeconomic statistics or the choices of other firms). These possibilities amount to introducing informational externalities; see the remark at the end of this section.

It is straightforward to extend the above arguments to the more general case that allows for capital accumulation and for nominal rigidity (in the sense of Property 2). We conclude that the policy lessons provided in the earlier sections of our paper are robust to endogenous acquisition of information, rational inattention, and the like.

Theorem 3. *Theorems 1 and 2 continue to hold in the extended framework described in this section, despite the influence that the policy instruments can exert on the information acquisition, or the cognitive effort, of the firms and thereby on the severity of the considered friction.*

7 Conclusion

In the last few years, a rapidly growing literature has renewed interest in the macroeconomic implications of rational inattention (Sims, 2003) and other related forms of informational frictions (Woodford, 2003a; Mankiw and Reis, 2002). Such frictions seem *a priori* plausible, they are consistent with survey evidence, and they can justify significant nominal rigidity at the macro level even if prices change frequently at the micro level (Mackowiak and Wiederholt, 2009). Moreover, the introduction of such frictions in macroeconomic models can also be seen as a modeling substitute for "bounded rationality" (Angeletos and Lian, 2017, 2016a).

The goal of this paper was to study how such frictions affect the optimal design of taxes and monetary policy over the business cycle. In our setting, each firm makes its pricing and production decisions on the basis of an imperfect and heterogeneous understanding of state of the economy. Under such circumstances, familiar policy instruments serve new functions: they enable the planner to control the extent of coordination among firms as well as manipulate how much information the typical firm collects, how much attention is paid to available data, or how much cognitive effort is put into comprehending what's going on in the economy.

Despite this property, we find that the optimal taxes are similar to those in the standard Ramsey paradigm. This is because these taxes guarantee the alignment of private and social incentives regardless of whether firms share the same knowledge about the state of the economy and regardless of how well they coordinate their behavior.

We also find that the optimal monetary policy replicates flexible-price allocations (properly defined). As in the New-Keynesian paradigm, this is true because monetary policy does not have to substitute for missing tax instruments and, equivalently, there is no trade off between minimizing relative-price distortions and minimizing the output gap (once again, properly defined). Unlike that paradigm, however, replicating flexible-price allocations and minimizing relative-price distortions do not imply price stability. Instead, they imply a particular kind of "leaning against the wind," namely, a negative correlation between the price level and aggregate output along the optimal plan. This property is necessary in order to ensure that firms do not discard valuable private knowledge about what needs to be done in response to the underlying shocks.

We conclude with possible directions for future research. One, which is topical, is to study how the frictions under consideration interact with the zero-lower bound on interest rates. Wiederholt (2015) and Angeletos and Lian (2016a) have already made some progress in this direction; they have not addressed optimality, however.

Another possibility, already mentioned, is to introduce markup shocks or labor-market distortions, constrain the tax instruments, and let monetary policy substitute for the missing tax instruments. Adam (2007) and Paciello and Wiederholt (2014) have moved in this direction but only while abstracting from the real friction that has been at the core of our analysis.

A third possibility is to explore the robustness of our policy lessons to the introduction of informational/cognitive frictions on the household side. In an early incarnation of this paper (Angeletos and La'O, 2008), we had allowed the workers to face a certain kind of informational friction but had sidestepped the possibility that this translates into uninsurable idiosyncratic consumption risk by requiring that all workers belong to the same "big family". Under this simplification, we obtained essentially the same results as in the present draft. We suspect that this logic extends to the more general specification of informational/cognitive frictions considered in the present version of our paper as long as one abstracts from incomplete markets. The alternative scenario, which allows the considered frictions to generate uninsurable idiosyncratic risk, seems more challenging.

Last but not least, one may step back from the macroeconomic context of this paper with and, instead, address the following more elementary question: do the two fundamental theorems of welfare economics apply in the presence of the considered kind of frictions? We have hinted to such a possibility by showing that the flexible-price equilibria of our setting maximize production efficiency, properly defined. We suspect that a reformulation of the two fundamental welfare theorems is possible along similar lines, subject, however, to the caveat that cognitive frictions can open the door to uninsurable idiosyncratic risk.

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Appendix A: Proofs

In this Appendix, we first state and prove two auxiliary lemmas, which do not appear in the main text and which offer a complete characterization of the sets of sticky- and flexible-price equilibria. We then proceed with the proofs of the results that appear in the main text.

Lemma 6. *An allocation ξ, a policy θ, and a price system ϱ are part of a sticky-price equilibrium if and only if the following four properties hold.*

(i) The following household optimality conditions are satisfied:

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)} = \beta\left[\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}\left(1+R\left(s^t\right)\right)\middle|\ s^t\right] \tag{55}$$

$$-U_\ell\left(s^t\right) = U_c\left(s^t\right)\frac{\left(1-\tau^\ell\left(s^t\right)\right)}{\left(1+\tau^c\left(s^t\right)\right)}w\left(s^t\right) \tag{56}$$

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)} = \beta\left[\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)}\left(1+\tilde{r}\left(s^{t+1}\right)-\delta\right)\middle|\ s^t\right] \tag{57}$$

$$Q\left(s^{t+1}\right) = \beta\frac{\mu\left(s^{t+1}\right)}{\mu\left(s^t\right)}\frac{U_c\left(s^{t+1}\right)}{U_c\left(s^t\right)}\frac{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)} \tag{58}$$

where

$$\tilde{r}\left(s^t\right) = \left(1-\tau^k\left(s^t\right)\right)r\left(s^t\right) \tag{59}$$

is the net-of-taxes return on savings.

(ii) The following firm optimality conditions are satisfied:

$$\lambda\left(\omega_i^t, s^t\right)A(s^t)f_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) = 0 \tag{60}$$

$$\mathbb{E}\left[\mathcal{M}(s^t)\left(\lambda(\omega_i^t, s^t)A(s^t)f_h\left(\omega_i^t, s^t\right)-1\right)\middle|\ \omega_i^t\right] = 0 \tag{61}$$

$$\mathbb{E}\left[\mathcal{M}(s^t)\left(\lambda(\omega_i^t, s^t)A(s^t)f_k\left(\omega_i^t, s^t\right)-r\left(s^t\right)\right)\middle|\ \omega_i^t\right] = 0 \tag{62}$$

$$\mathbb{E}\left[\mathcal{M}(s^t)y\left(\omega_i^t, s^t\right)\left\{\left(1-\tau^r(s^t)\right)\left(\tfrac{\rho-1}{\rho}\right)\frac{p(\omega_i^t)}{P(s^t)}-\lambda\left(\omega_i^t, s^t\right)\right\}\middle|\ \omega_i^t\right] = 0 \tag{63}$$

with $\mathcal{M}(s^t) \equiv \frac{U_c\left(s^t\right)}{1+\tau^c\left(s^t\right)}$, along with the intermediate-good demand condition, namely,

$$y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho}Y\left(s^t\right). \tag{64}$$

.

(iii) The household's and the government's budget constraints are satisfied.

(iv) All markets clear, namely, conditions (4), (5), and (6) are satisfied.

Lemma 7. *An allocation ξ, a policy θ, and a price system ϱ, are part of a flexible-price equilibrium if and only if the following four properties hold.*

(i) The household optimality conditions in (55)-(58) are all satisfied;

(ii) The following firm optimality conditions are satisfied:

$$\begin{aligned}
\left(1-\tau^r(s^t)\right)\tfrac{\rho-1}{\rho}\frac{p(\omega_i^t)}{P(s^t)}A(s^t)f_\ell\left(\omega_i^t,s^t\right)-w\left(s^t\right) &= 0 \qquad (65)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left(\left(1-\tau^r(s^t)\right)\tfrac{\rho-1}{\rho}\frac{p(\omega_i^t)}{P(s^t)}A(s^t)f_h\left(\omega_i^t,s^t\right)-1\right)\middle|\,\omega_i^t\right] &= 0 \qquad (66)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left(\left(1-\tau^r(s^t)\right)\tfrac{\rho-1}{\rho}\frac{p(\omega_i^t)}{P(s^t)}A(s^t)f_k\left(\omega_i^t,s^t\right)-r\left(s^t\right)\right)\middle|\,\omega_i^t\right] &= 0 \qquad (67)
\end{aligned}$$

where $\mathcal{M}(s^t)\equiv\frac{U_c(s^t)}{1+\tau^c(s^t)}$, *along with the intermediate-good demand condition (64).*

(iii) The household's and the government's budget constraints are satisfied.

(iv) All markets clear, namely, conditions (4), (5), and (6) are satisfied.

Proof of Lemma 6. We first derive the household's optimality conditions. Following this we derive the firm's optimality conditions.

Household. Consider the Household's problem stated in Section 3. Let $\Lambda\left(s^t\right)$ be the Lagrange multiplier on the Household's budget constraint in history s^t. The Lagrangian for the household's problem is given by

$$\begin{aligned}
L &= \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu\left(s^t\right)\left[U\left(C\left(s^t\right),L\left(s^t\right),s^t\right)\right]\\
&-\sum_{t=0}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\begin{array}{c}\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)C\left(s^t\right)+B\left(s^t\right)+\sum_{s^{t+1}}Q(s^{t+1})D(s^{t+1})\\ +P\left(s^t\right)\left(K\left(s^t\right)-(1-\delta)K\left(s^{t-1}\right)\right)\end{array}\right]\\
&+\sum_{t=0}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\begin{array}{c}\left(1-\tau^\ell\left(s^t\right)\right)P\left(s^t\right)w\left(s^t\right)L\left(s^t\right)+\left(1-\tau^k\left(s^t\right)\right)P\left(s^t\right)r\left(s^t\right)K\left(s^{t-1}\right)\\ +\left(1+R\left(s^{t-1}\right)\right)B\left(s^{t-1}\right)+D(s^t)\end{array}\right]
\end{aligned}$$

The household's first order conditions for consumption, labor, bonds, and state-contingent securities are given by

$$\begin{aligned}
\beta^t\mu\left(s^t\right)U_c\left(s^t\right)-\Lambda\left(s^t\right)\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right) &= 0,\text{ for all } s^t \qquad (68)\\
\beta^t\mu\left(s^t\right)U_\ell\left(s^t\right)+\Lambda\left(s^t\right)\left(1-\tau^\ell\left(s^t\right)\right)P\left(s^t\right)w\left(s^t\right) &= 0,\text{ for all } s^t \qquad (69)\\
-\Lambda\left(s^t\right)+\sum_{s^{t+1}|s^t}\Lambda\left(s^{t+1}\right)\left(1+R\left(s^t\right)\right) &= 0,\text{ for all } s^t \qquad (70)\\
-Q\left(s^{t+1}\right)\Lambda\left(s^t\right)+\Lambda\left(s^{t+1}\right) &= 0,\text{ for all } s^{t+1} \qquad (71)
\end{aligned}$$

By combining (68) and (70) we derive the household's Euler equation,

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)}=\beta\sum_{s^{t+1}|s^t}\mu\left(s^{t+1}|s^t\right)\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}\left(1+R\left(s^t\right)\right). \qquad (72)$$

And by combining (68) and (69) we derive the household's intratemporal condition,

$$-U_\ell\left(s^t\right)=U_c\left(s^t\right)\frac{\left(1-\tau^\ell\left(s^t\right)\right)}{\left(1+\tau^c\left(s^t\right)\right)}w\left(s^t\right)$$

Thus we obtain optimality conditions for the household stated in (55) and (56). From (71), we have that the state-contingent price satisfies:

$$Q\left(s^{t+1}\right)=\frac{\Lambda\left(s^{t+1}\right)}{\Lambda\left(s^{t}\right)}=\beta\frac{\mu\left(s^{t+1}\right)}{\mu\left(s^{t}\right)}\frac{U_c\left(s^{t+1}\right)}{U_c\left(s^{t}\right)}\frac{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}.$$

Next, the household's optimality condition for capital is given by

$$-\Lambda\left(s^t\right)P\left(s^t\right)+\sum_{s^{t+1}|s^t}\left[\Lambda\left(s^{t+1}\right)\left(1-\tau^k\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)r\left(s^{t+1}\right)+\Lambda\left(s^{t+1}\right)P\left(s^{t+1}\right)(1-\delta)\right]=0$$

which may be rewritten as

$$\Lambda\left(s^t\right)P\left(s^t\right)=\sum_{s^{t+1}|s^t}\Lambda\left(s^{t+1}\right)P\left(s^{t+1}\right)\left[1+\left(1-\tau^k\left(s^{t+1}\right)\right)r\left(s^{t+1}\right)-\delta\right] \tag{73}$$

Using (68) to replace $\Lambda\left(s^t\right)P\left(s^t\right)$ and $\Lambda\left(s^{t+1}\right)P\left(s^{t+1}\right)$ in the above equation, we get

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)}=\beta\sum_{s^{t+1}|s^t}\mu\left(s^{t+1}|s^t\right)\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)}\left[1+\left(1-\tau^k\left(s^{t+1}\right)\right)r\left(s^{t+1}\right)-\delta\right].$$

Thus we obtain the household optimality condition stated in (57).

Firms. Turning attention now to the firms, we first consider the final-good retail sector. Its optimal input choices satisfy

$$y\left(\omega_i^t,s^t\right)=\left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho}Y\left(s^t\right). \tag{74}$$

This gives the demand function faced by the typical intermediate-good monopolistic firm. Then the monopolist follows the problem stated in Section 3.

The demand function (74) implies that we may write monopolistic firm's real revenue as

$$\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}y\left(\omega_i^t,s^t\right)=\left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{1-\rho}Y\left(s^t\right).$$

We can thus state the monopolistic firm's pricing and production problem as follows:

$$\max\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\mathcal{M}(s^t)\left\{\left(1-\tau^r\left(s^t\right)\right)\left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{1-\rho}Y\left(s^t\right)-h\left(\omega_i^t\right)-w\left(s^t\right)\ell\left(\omega_i^t,s^t\right)-r(s^t)k(\omega_i^{t-1})\right\}\middle|\omega_i^t\right]$$

subject to

$$A\left(s^t\right)F\left(k_i\left(\omega_i^t\right),h_i\left(\omega_i^t\right),\ell_i\left(\omega_i^t,s^t\right)\right)=\left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho}Y(s^t)\quad\forall\omega_i^t,s^t$$

The first constraint is simply the law of motion for capital. The second constraint, which follows from combining condition (74) with the production function, dictates how labor adjusts so as to meet the realized demand, whatever that might be.

Let $\beta^t \mathcal{M}(s^t)\lambda(\omega_i^t, s^t)$ be the Lagrange multiplier on the second constraint. The first order conditions with respect to labor, intermediate inputs, and investment are given by the following:

$$\lambda\left(\omega_i^t, s^t\right) A(s^t) f_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) \quad = \quad 0 \tag{75}$$

$$\mathbb{E}\left[\mathcal{M}(s^t)\left(\lambda(\omega_i^t, s^t) A(s^t) f_h\left(\omega_i^t, s^t\right) - 1\right) \middle| \omega_i^t\right] \quad = \quad 0 \tag{76}$$

$$\mathbb{E}\left[\mathcal{M}(s^t)\left(\lambda(\omega_i^t, s^t) A(s^t) f_k\left(\omega_i^t, s^t\right) - r\left(s^t\right)\right) \middle| \omega_i^t\right] \quad = \quad 0 \tag{77}$$

The first-order condition with respect to the price $p\left(\omega_i^t\right)$, on the other hand, can be stated as follows:

$$\mathbb{E}\left[\mathcal{M}(s^t) y\left(\omega_i^t, s^t\right)\left\{\left(1 - \tau^r(s^t)\right)\left(\tfrac{\rho-1}{\rho}\right)\frac{p(\omega_i^t)}{P(s^t)} - \lambda\left(\omega_i^t, s^t\right)\right\} \middle| \omega_i^t\right] = 0 \tag{78}$$

Thus we obtain optimality conditions for the firm stated in (60)-(62) and (63). **QED.**

Proof of Lemma 7. The household's problem is the same as in the sticky price equilibrium, and hence follows the proof of Lemma 6. On the firm's side, the demand for intermediate goods from the final-good retail sector continues to satisfy (64).

Thus, the only difference between the sticky-price and flexible-price equilibria are the intermediate good firms' problem. We may state the monopolistic firm's production problem as follows:

$$\max \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \mathcal{M}(s^t)\left\{\left(1 - \tau^r\left(s^t\right)\right) y\left(\omega_i^t, s^t\right)^{\frac{\rho-1}{\rho}} Y\left(s^t\right)^{\frac{1}{\rho}} - h\left(\omega_i^t\right) - w\left(s^t\right)\ell\left(\omega_i^t, s^t\right) - r(s^t) k(\omega_i^{t-1})\right\} \middle| \omega_i^t\right]$$

subject to the production function

$$y\left(\omega_i^t, s^t\right) = A\left(s^t\right) F\left(k_i\left(\omega_i^t\right), h_i\left(\omega_i^t\right), \ell_i\left(\omega_i^t, s^t\right)\right)$$

The FOCs of this problem are given by

$$\left(1 - \tau^r(s^t)\right)\tfrac{\rho-1}{\rho}\left(\frac{y\left(\omega_i^t, s^t\right)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}} A(s^t) f_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) \quad = \quad 0 \tag{79}$$

$$\mathbb{E}\left[\mathcal{M}(s^t)\left(\left(1 - \tau^r(s^t)\right)\tfrac{\rho-1}{\rho}\left(\frac{y\left(\omega_i^t, s^t\right)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}} A(s^t) f_h\left(\omega_i^t, s^t\right) - 1\right) \middle| \omega_i^t\right] \quad = \quad 0 \tag{80}$$

$$\mathbb{E}\left[\mathcal{M}(s^t)\left(\left(1 - \tau^r(s^t)\right)\tfrac{\rho-1}{\rho}\left(\frac{y\left(\omega_i^t, s^t\right)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}} A(s^t) f_k\left(\omega_i^t, s^t\right) - r\left(s^t\right)\right) \middle| \omega_i^t\right] \quad = \quad 0 \tag{81}$$

Combining these with the intermediate good demand in (64) yields equations (65)-(67). **QED.**

Equipped with the aforementioned auxiliary results, in the remainder of this appendix we offer the proofs of the results that appear in the main text.

Proof of Proposition 1. *Necessity.* We first prove necessity. First, take equation (79). This may be rewritten as

$$\left(1-\tau^{r}\left(s^{t}\right)\right)\tfrac{\rho-1}{\rho}MP_{\ell}\left(\omega_{i}^{t},s^{t}\right)-w(s^{t})=0\quad\forall t,\omega_{i}^{t},s^{t}$$

Combining this with the household's intratemporal condition (56), we obtain

$$\frac{U_{c}\left(s^{t}\right)}{\left(1+\tau^{c}\left(s^{t}\right)\right)}\left(1-\tau^{r}(s^{t})\right)\tfrac{\rho-1}{\rho}MP_{\ell}\left(\omega_{i}^{t},s^{t}\right)-\frac{-U_{\ell}\left(s^{t}\right)}{\left(1-\tau^{\ell}\left(s^{t}\right)\right)}=0$$

thereby proving necessity of (16) with

$$\psi^{r}\left(s^{t}\right)\equiv\frac{U_{c}(s^{t})(1-\tau^{r}(s^{t}))}{1+\tau^{c}(s^{t})},\ \chi^{*}\equiv\frac{\rho-1}{\rho}\quad\text{and}\quad\psi^{\ell}\left(s^{t}\right)\equiv\frac{-U_{\ell}(s^{t})}{1-\tau^{\ell}(s^{t})}$$

Next, take equation (80). This may be rewritten as

$$\mathbb{E}\left[\frac{U_{c}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)}\left\{\left(1-\tau^{r}\left(s^{t}\right)\right)\tfrac{\rho-1}{\rho}MP_{h}\left(\omega_{i}^{t},s^{t}\right)-1\right\}\middle|\,\omega_{i}^{t}\right]=0\quad\forall t,\omega_{i}^{t}$$

We thereby prove necessity of (17) with

$$\psi^{c}\left(s^{t}\right)\equiv\frac{U_{c}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)}$$

Next, take equation (81). This may be rewritten as follows

$$\mathbb{E}\left[\frac{U_{c}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)}\left(\left(1-\tau^{r}\left(s^{t}\right)\right)\tfrac{\rho-1}{\rho}MP_{k}\left(\omega_{i}^{t},s^{t}\right)-r(s^{t})\right)\middle|\,\omega_{i}^{t}\right]=0\quad\forall t,\omega_{i}^{t}$$

We thereby prove necessity of (18) with

$$\psi^{k}\left(s^{t}\right)=\frac{U_{c}(s^{t})}{1+\tau^{c}(s^{t})}r\left(s^{t}\right)=\frac{U_{c}(s^{t})}{1+\tau^{c}(s^{t})}\frac{\tilde{r}\left(s^{t}\right)}{1-\tau^{k}\left(s^{t}\right)}.$$

So far we have established the necessity of conditions 79)-(18). The necessity of the resource constraint follows from the combination of budgets and market clearing. What remains is to prove the necessity of the implementability condition (15).

To obtain this condition, we multiply the household's budget constraint at s^t by $\Lambda\left(s^{t}\right)$ and then sum over s^t and t. This gives us the following

$$\begin{aligned}&\sum_{t,s^{t}}\Lambda\left(s^{t}\right)\left[\begin{array}{c}\left(1+\tau^{c}\left(s^{t}\right)\right)C\left(s^{t}\right)+B\left(s^{t}\right)+\sum_{s^{t+1}}Q(s^{t+1})D(s^{t+1})\\+\left(K\left(s^{t}\right)-\left(1-\delta\right)K\left(s^{t-1}\right)\right)\end{array}\right]\\=&\sum_{t,s^{t}}\Lambda\left(s^{t}\right)\left[\begin{array}{c}\left(1-\tau^{\ell}\left(s^{t}\right)\right)w\left(s^{t}\right)L\left(s^{t}\right)+\left(1-\tau^{k}\left(s^{t}\right)\right)r\left(s^{t}\right)K\left(s^{t-1}\right)\\+\left(1+R\left(s^{t-1}\right)\right)B\left(s^{t-1}\right)+D(s^{t})\end{array}\right]\end{aligned}$$

Substituting in the FOCs for debt (70) and state contingent bonds (71) we get that

$$\begin{aligned}\sum_{t,s^{t}}\Lambda\left(s^{t}\right)\left[\left(1+\tau^{c}\left(s^{t}\right)\right)C\left(s^{t}\right)+K\left(s^{t}\right)\right]=&\sum_{t,s^{t}}\Lambda\left(s^{t}\right)\left[\left(1-\tau^{\ell}\left(s^{t}\right)\right)w\left(s^{t}\right)L\left(s^{t}\right)\right]\\&+\sum_{t,s^{t}}\Lambda\left(s^{t}\right)\left(1+\left(1-\tau^{k}\left(s^{t}\right)\right)r\left(s^{t}\right)-\delta\right)K\left(s^{t-1}\right)\end{aligned}$$

where we have used the fact that $B_0 = D_0 = 0$. Next, substituting in the FOC for capital (73), we get

$$\sum_{t,s^t} \Lambda\left(s^t\right)\left(1+\tau^c\left(s^t\right)\right) C\left(s^t\right) = \sum_{t,s^t} \Lambda\left(s^t\right)\left(1-\tau^\ell\left(s^t\right)\right) w\left(s^t\right) L\left(s^t\right)$$

Now, using the household's FOCs for consumption and employment, (68) and (69), to substitute out all prices, we obtain

$$\sum_{t,s^t} \beta^t \mu\left(s^t\right) U_c\left(s^t\right) C\left(s^t\right) = -\sum_{t,s^t} \beta^t \mu\left(s^t\right) U_\ell\left(s^t\right) L\left(s^t\right)$$

which we may re-write as follows

$$\sum_{t,s^t} \beta^t \mu\left(s^t\right)\left[U_c\left(s^t\right) C\left(s^t\right) + U_\ell\left(s^t\right) L\left(s^t\right)\right] = 0$$

We thus obtain condition in (15) and complete the proof of necessity.

Sufficiency. Consider now sufficiency. Take any allocation ξ_t that satisfies (15)-(18). We now prove that there exists a set of tax rates

$$\left\{\tau^c\left(s^t\right), \tau^\ell\left(s^t\right), \tau^k\left(s^t\right), \tau^r(s^t)\right\},$$

a real wage $w\left(s^t\right)$, relative prices $\left(p\left(\omega_i^t, s^t\right)\right)_{i\in I}$, a real rental rate $r\left(s^t\right)$, an interest rate function $R\left(s^t\right)$ and a path for nominal debt holdings $B\left(s^t\right)$ that implement this allocation as an equilibrium. We construct the equilibrium prices and policies as follows.

First, relative prices satisfy

$$\frac{p(\omega_i^t, s^t)}{P(s^t)} = p(\omega_i^t, s^t) = \left(\frac{y(\omega_i^t, s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}$$

where we normalize the aggregate price level to one: $P(s^t) = 1$. With these prices we satisfy the equilibrium conditions (64) for intermediate good demand.

Let us propose the following tax rates τ^ℓ, τ^c, and τ^r.

$$1+\tau^c\left(s^t\right) = \frac{U_c(s^t)}{\psi^c(s^t)},\ 1-\tau^\ell\left(s^t\right) = \frac{-U_\ell(s^t)}{\psi^\ell\left(s^t\right)},\ \text{and}\ 1-\tau^r(s^t) = \frac{\psi^r\left(s^t\right)}{\psi^c(s^t)} \tag{82}$$

We then satisfy the household's necessary optimality condition for labor (56) with the following real wage:

$$w\left(s^t\right) = \frac{\psi^\ell\left(s^t\right)}{\psi^c(s^t)} = \frac{-U_\ell\left(s^t\right)}{U_c\left(s^t\right)\frac{\left(1-\tau^\ell(s^t)\right)}{\left(1+\tau^c(s^t)\right)}} \tag{83}$$

Next, take condition (16). We may replace this with the wage from (83) and obtain

$$\chi^* \psi^r\left(s^t\right) MP_\ell\left(\omega_i^t, s^t\right) - \psi^c(s^t) w\left(s^t\right) = 0$$

Substituting in for ψ^r and ψ^c from (82) gives us:

$$\left(1-\tau^r(s^t)\right)\frac{\rho-1}{\rho} MP_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) = 0$$

This satisfies the firm's optimality condition for labor in (79).

Next, take implementability condition (17). Again substituting in for ψ^r and ψ^c from (82) gives us the following:

$$\mathbb{E}\left[\frac{U_c(s^t)}{1+\tau^c(s^t)}\left((1-\tau^r(s^t))\frac{\rho-1}{\rho}MP_h\left(\omega_i^t,s^t\right)-1\right)\middle|\omega_i^t\right]=0$$

This satisfies the firm's optimality condition for the intermediate good (80).

Next take implementability condition (18). Again substituting in for ψ^r from (82) gives us:

$$\mathbb{E}\left[\frac{U_c(s^t)(1-\tau^r(s^t))}{1+\tau^c(s^t)}\frac{\rho-1}{\rho}MP_k\left(\omega_i^t,s^t\right)-\psi^k\left(s^t\right)\middle|\omega_i^t\right]=0$$

This satisfies the firm's optimality condition for capital (81) as long as we set the real rental rate on capital be equal to

$$r\left(s^t\right)=\psi^k\left(s^t\right)\left(\frac{U_c(s^t)}{1+\tau^c(s^t)}\right)^{-1} \tag{84}$$

This implies that we may satisfy the household's Euler condition (57) with the following capital-income tax rate

$$1-\tau^k\left(s^t\right)=\frac{\tilde{r}\left(s^t\right)}{r\left(s^t\right)} \tag{85}$$

with $r\left(s^t\right)$ given by (84). Moreover, given the allocation, the following interest rate function

$$1+R\left(s^t\right)=\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)}\left\{\beta\mathbb{E}\left[\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)}\middle|\ s^t\right]\right\}^{-1}$$

ensures that condition (55) holds.

Finally we construct bond holdings such that the household's Euler equation (55) holds. We multiply by $\Lambda\left(s^t\right)$ the household's budget constraint and sum over all periods and states following s^r.

$$\begin{aligned}
&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\begin{array}{c}\left(1+\tau^c\left(s^t\right)\right)C\left(s^t\right)+B\left(s^t\right)+\sum_{s^{t+1}}Q(s^{t+1})D(s^{t+1})\\+\left(K\left(s^t\right)-(1-\delta)K\left(s^{t-1}\right)\right)\end{array}\right]\\
=\;&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\begin{array}{c}\left(1-\tau^\ell\left(s^t\right)\right)w\left(s^t\right)L\left(s^t\right)+\left(1-\tau^k\left(s^t\right)\right)r\left(s^t\right)K\left(s^{t-1}\right)\\+\left(1+R\left(s^{t-1}\right)\right)B\left(s^{t-1}\right)+D(s^t)\end{array}\right]
\end{aligned}$$

Substituting in the FOCs for debt (70) and state contingent bonds (71) we get that

$$\begin{aligned}
&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\left(1+\tau^c\left(s^t\right)\right)C\left(s^t\right)+K\left(s^t\right)\right]\\
=\;&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\left(1-\tau^\ell\left(s^t\right)\right)w\left(s^t\right)L\left(s^t\right)+\left(1+\left(1-\tau^k\left(s^t\right)\right)r\left(s^t\right)-\delta\right)K\left(s^{t-1}\right)\right]\\
&+\sum_{s^{r+1}|s^r}\Lambda\left(s^{r+1}\right)\left(1+R\left(s^r\right)\right)B\left(s^r\right)
\end{aligned}$$

Next, substituting in the FOC for capital (73), we get

$$\Lambda\left(s^r\right)B\left(s^r\right)=\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\left(1+\tau^c\left(s^t\right)\right)C\left(s^t\right)-\left(1-\tau^\ell\left(s^t\right)\right)w\left(s^t\right)L\left(s^t\right)\right]$$

Next, using the household's focs for consumption and labor (68) and (69) gives us

$$\frac{\beta^r\mu\left(s^r\right)U_c\left(s^r\right)}{\left(1+\tau^c\left(s^r\right)\right)}B\left(s^r\right)=\sum_{t=r+1}^{\infty}\sum_{s^t}\beta^t\mu\left(s^t\right)\left[U_c\left(s^t\right)C\left(s^t\right)+U_\ell\left(s^t\right)L\left(s^t\right)\right]$$

which we may rewrite as follows

$$\frac{U_c\left(s^r\right)}{\left(1+\tau^c\left(s^r\right)\right)}B\left(s^r\right)=\sum_{t=r+1}^{\infty}\sum_{s^t}\beta^{t-r}\mu\left(s^t|s^r\right)\left[U_c\left(s^t\right)C\left(s^t\right)+U_\ell\left(s^t\right)L\left(s^t\right)\right]$$

Therefore real bond holdings are given by

$$B\left(s^r\right)=\left(\frac{U_c\left(s^r\right)}{1+\tau^c\left(s^r\right)}\right)^{-1}\sum_{t=r+1}^{\infty}\sum_{s^t}\beta^{t-r}\mu\left(s^t|s^r\right)\left[U_c\left(s^t\right)C\left(s^t\right)+U_\ell\left(s^t\right)L\left(s^t\right)\right]$$

for any period r, state s^r. **QED.**

Proof of Lemma 1. Feasibility follows from the combination of budgets and market clearing.

Using the intermediate demand equation in (74), we may rewrite (63) as

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)}y\left(\omega_i^t,s^t\right)\left\{\left(1-\tau^r(s^t)\right)\left(\tfrac{\rho-1}{\rho}\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}-\lambda\left(\omega_i^t,s^t\right)\right\}\middle|\,\omega_i^t\right]=0$$

We re-write this condition as

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)}y\left(\omega_i^t,s^t\right)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\left\{\chi(\omega_i^t,s^t)-\chi^*\right\}\middle|\,\omega_i^t\right]=0$$

with $\chi^*=\frac{\rho-1}{\rho}$ and

$$\chi(\omega_i^t,s^t)\equiv\frac{\lambda\left(\omega_i^t,s^t\right)}{\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}}\tag{86}$$

Using the definition of $\psi^r\left(s^t\right)$ in (20) we obtain

$$\mathbb{E}\left[\psi^r\left(s^t\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}y\left(\omega_i^t,s^t\right)\left\{\chi(\omega_i^t,s^t)-\chi^*\right\}\middle|\,\omega_i^t\right]=0$$

thereby proving necessity of (30).

Next, we combine the intratemporal optimality conditions of the household and of the firm for labor. Substituting (56) into the firm's condition (60) to replace the real wage, we obtain:

$$\lambda\left(\omega_i^t, s^t\right) \frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)} A(s^t) f_\ell\left(\omega_i^t, s^t\right) - \left(\frac{-U_\ell\left(s^t\right)}{1-\tau^\ell\left(s^t\right)}\right) = 0. \tag{87}$$

From our definition of $\chi(\omega_i^t, s^t)$ in (86), we have that

$$\lambda\left(\omega_i^t, s^t\right) = \chi(\omega_i^t, s^t)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}$$

Substituting this into (87) we obtain

$$\chi(\omega_i^t, s^t)\left(1-\tau^r(s^t)\right) \frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)} \left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_\ell\left(\omega_i^t, s^t\right) - \left(\frac{-U_\ell\left(s^t\right)}{1-\tau^\ell\left(s^t\right)}\right) = 0.$$

We may write this as

$$\chi(\omega_i^t, s^t)\psi^r\left(s^t\right) MP_\ell\left(\omega_i^t, s^t\right) - \psi^\ell\left(s^t\right) = 0.$$

where $\psi^r\left(s^t\right)$ and $\psi^\ell\left(s^t\right)$ are given by (19) and (20), thereby proving necessity of (27).

Next, we have the firm's optimality condition for intermediate goods given by (61). We again substitute for $\lambda\left(\omega_i^t, s^t\right)$ from (86) into (61) and obtain

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)}\left(\chi(\omega_i^t, s^t)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_h\left(\omega_i^t, s^t\right) - 1\right)\middle|\, \omega_i^t\right] \quad = 0$$

We may write this as

$$\mathbb{E}\left[\chi(\omega_i^t, s^t)\psi^r\left(s^t\right) MP_h\left(\omega_i^t, s^t\right) - \psi^c\left(s^t\right)\middle|\, \omega_i^t\right] \quad = 0$$

where $\psi^r\left(s^t\right)$ and $\psi^c\left(s^t\right)$ are given by (19) and (20), thereby proving necessity of (28).

Similarly we have the firm's optimality condition for capital investment given by (62). We again substitute for $\lambda\left(\omega_i^t, s^t\right)$ from (86) into (62) and obtain

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)}\left(\chi(\omega_i^t, s^t)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_k\left(\omega_i^t, s^t\right) - r\left(s^t\right)\right)\middle|\, \omega_i^t\right] \quad = 0$$

We may write this as

$$\mathbb{E}\left[\chi(\omega_i^t, s^t)\psi^r\left(s^t\right) MP_k\left(\omega_i^t, s^t\right) - \psi^k\left(s^t\right)\middle|\, \omega_i^t\right] \quad = 0$$

where $\psi^r\left(s^t\right)$ and $\psi^k\left(s^t\right)$ are given by (19) and (20), thereby proving necessity of (29). **QED.**

What remains is the implementability condition (26). To obtain this necessary condition, we follow the exact same steps used to obtain condition (15) in the proof of Proposition 1. **QED.**

Proof of Lemma 2. In any sticky-price equilibrium, prices must satisfy the intermediate good demand equation (64). Consider then the relative prices between two firms. Fix a period t and a state s^t, and take an arbitrary pair of firms (i, j), with $j \neq i$. From the consumer demand equation (64), the *relative* price of the two firms is pinned down by their relative output:

$$\frac{p\left(\omega_i^t\right)}{p\left(\omega_j^t\right)} = \left[\frac{y\left(\omega_i^t, s^t\right)}{y\left(\omega_j^t, s^t\right)}\right]^{-1/\rho}$$

Clearly, the above condition can hold for all realizations of ω_i^t, ω_j^t and s^t only if the right-hand side of this condition is independent of s^t conditional on the pair (ω_i^t, ω_j^t). This can be true if and only if y is log-separable. **QED.**

Proof of Proposition 2 Necessity has been established by Lemma 1. Consider now sufficiency. Take any log-separable allocation ξ_t that satisfies (26)-(30). We now prove that there exists a set of tax rates

$$\left\{\tau^c\left(s^t\right), \tau^\ell\left(s^t\right), \tau^k\left(s^t\right), \tau^r\left(s^t\right)\right\},$$

a real wage $w\left(s^t\right)$, nominal prices $\left(p\left(\omega_i^t\right)\right)_{i\in I}, P\left(s^t\right)$, a real rental rate $r\left(s^t\right)$, a nominal interest rate function $R\left(s^t\right)$, and a path for nominal debt holdings $B\left(s^t\right)$ that implement this allocation as an equilibrium. We construct the equilibrium prices and policies as follows.

First, because the allocation is separable, we have that $y\left(\omega_i^t, s^t\right) = \Psi^\omega(\omega_i^t)\Psi^s(s^t)$ for some functions Ψ^ω and Ψ^s. Let us then propose the following nominal prices:

$$p(\omega_i^t) = \Psi^\omega\left(\omega_i^t\right)^{-\frac{1}{\rho}},$$

which are by construction measurable in ω_i^t. It follows that the price level satisfies

$$P(s^t) = \left[\sum_{\omega\in\Omega^t} p\left(\omega_i^t\right)^{1-\rho}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}} = \left[\sum_{\omega\in\Omega^t}\Psi^\omega\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}},$$

while aggregate output satisfies

$$Y\left(s^t\right) = \Psi^s\left(s^t\right)\left[\sum_{\omega\in\Omega^t}\Psi^\omega\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}},$$

and therefore relative prices satisfy

$$\frac{p(\omega_i^t)}{P(s^t)} = \frac{\Psi^\omega\left(\omega_i^t\right)^{-\frac{1}{\rho}}}{\left[\sum_{\omega\in\Omega^t}\Psi^\omega\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}}} = \left(\frac{y(\omega_i^t, s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}$$

That is, we can find nominal prices that implement the right relative prices while being measurable in ω_i^t. These prices satisfy the equilibrium necessary condition (64) for intermediate good demand.

We propose tax rates τ^ℓ, τ^c, and τ^r as in (82). We then satisfy the household's necessary optimality condition for labor (56) with the real wage proposed in (83).

Next, take implementability condition (27). We may replace this with the wage from (83) and obtain

$$\chi(\omega, s^t)\psi^r\left(s^t\right) MP_\ell\left(\omega_i^t, s^t\right) - \psi^c(s^t) w\left(s^t\right) = 0.$$

Substituting in for ψ^r and ψ^c from (82) gives us:

$$\chi(\omega, s^t)\left(1 - \tau^r(s^t)\right) MP_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) = 0$$

This satisfies the firm's optimality condition for labor (60) as long as we let

$$\lambda\left(\omega_i^t, s^t\right) \equiv \chi(\omega_i^t, s^t)\left(1 - \tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}. \tag{88}$$

Next, take implementability condition (28). Again substituting in for ψ^r and ψ^c from (82) gives us:

$$\mathbb{E}\left[\frac{U_c(s^t)}{1+\tau^c(s^t)}\left(\chi(\omega, s^t)(1-\tau^r(s^t)) MP_h\left(\omega_i^t, s^t\right) - 1\right)\middle|\, \omega_i^t\right] = 0$$

This satisfies the firm's optimality condition for the intermediate good (61) with $\lambda\left(\omega_i^t, s^t\right)$ given by (88).

Next take implementability condition (29). Substituting in for ψ^r from (82) gives us:

$$\mathbb{E}\left[\frac{U_c(s^t)(1-\tau^r(s^t))}{1+\tau^c(s^t)}\chi(\omega, s^t) MP_k\left(\omega_i^t, s^t\right) - \psi^k(s^t)\middle|\, \omega_i^t\right] = 0$$

This satisfies the firm's optimality condition for capital (62) with $\lambda\left(\omega_i^t, s^t\right)$ given by (88) and with a real rental rate on capital given by (84). This implies further that we may satisfy the household's Euler condition (57) with the a capital-income tax rate τ^k as in (85).

Next, take implementability condition (30). Substituting in for ψ^r from (82) gives us:

$$\mathbb{E}\left[Y\left(s^t\right)^{1/\rho} y\left(\omega_i^t, s^t\right)^{1-1/\rho}\frac{U_c(s^t)(1-\tau^r(s^t))}{1+\tau^c(s^t)}\left\{\chi(\omega_i^t, s^t) - \chi^*\right\}\middle|\, \omega_i^t\right] = 0$$

which we may rewrite as

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)} y\left(\omega_i^t, s^t\right)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\left\{\chi(\omega_i^t, s^t) - \chi^*\right\}\middle|\, \omega_i^t\right] = 0$$

Substituting $\chi(\omega, s^t)$ from (86) gives us

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)} y\left(\omega_i^t, s^t\right)\left\{\lambda(\omega_i^t, s^t) - \tfrac{\rho-1}{\rho}\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\right\}\middle|\, \omega_i^t\right] = 0$$

Using the optimality for intermediate good demand (64) we may rewrite this as

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)} y\left(\omega_i^t, s^t\right)\left\{\lambda(\omega_i^t, s^t) - \tfrac{\rho-1}{\rho}\left(1-\tau^r(s^t)\right)\frac{p(\omega_i^t)}{P(s^t)}\right\}\middle|\, \omega_i^t\right] = 0$$

and therefore the firm's optimality condition for its nominal price (63) is satisfied.

Finally, given the allocation and the path for the nominal price level, the following nominal interest rate function

$$1+R\left(s^{t}\right)=\frac{U_{c}\left(s^{t}\right)}{\left(1+\tau^{c}\left(s^{t}\right)\right)P\left(s^{t}\right)}\left\{\beta\mathbb{E}\left[\left.\frac{U_{c}\left(s^{t+1}\right)}{\left(1+\tau^{c}\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}\right| s^{t}\right]\right\}^{-1}$$

ensures that condition (55) holds.

Finally what remains is to construct bond holdings such that the household's Euler equation (55) holds. For this we follow the exact same steps used to obtain bond holdings in the sufficiency proof of Proposition 1. Following these steps, real bond holdings are given by

$$\frac{B\left(s^{r}\right)}{P\left(s^{r}\right)}=\left(\frac{U_{c}\left(s^{r}\right)}{1+\tau^{c}\left(s^{r}\right)}\right)^{-1}\sum_{t=r+1}^{\infty}\sum_{s^{t}}\beta^{t-r}\mu\left(s^{t}|s^{r}\right)\left[U_{c}\left(s^{t}\right)C\left(s^{t}\right)+U_{\ell}\left(s^{t}\right)L\left(s^{t}\right)\right]$$

for any period r, state s^{r}. **QED.**

Proof of Proposition 3. Since technology is Cobb-Douglas (33), output may be written as

$$y_{i}\left(\omega_{i}^{t},s^{t}\right)=A\left(s^{t}\right)\ell\left(\omega_{i}^{t},s^{t}\right)^{\alpha}g\left(k\left(\omega_{i}^{t}\right),h\left(\omega_{i}^{t}\right)\right). \tag{89}$$

Take any flexible-price equilibrium: $\ell\left(\omega_{i}^{t},s^{t}\right)$ is pinned down by condition (16). Given technology (89), condition (16) may be expressed as

$$\chi^{*}\frac{\psi^{r}\left(s^{t}\right)}{\psi^{\ell}\left(s^{t}\right)}\left(\frac{y\left(\omega_{i}^{t},s^{t}\right)}{Y\left(s^{t}\right)}\right)^{-\frac{1}{\rho}}\alpha\frac{y\left(\omega_{i}^{t},s^{t}\right)}{\ell\left(\omega_{i}^{t},s^{t}\right)}=1 \tag{90}$$

We can solve (89) and (90) simultaneously for $y\left(\omega_{i}^{t},s^{t}\right)$ and $\ell\left(\omega_{i}^{t},s^{t}\right)$. We thereby get that equilibrium output is given by

$$y\left(\omega_{i}^{t},s^{t}\right)=\left[\alpha\chi^{*}\frac{\psi^{r}\left(s^{t}\right)}{\psi^{\ell}\left(s^{t}\right)}Y(s^{t})^{\frac{1}{\rho}}A\left(s^{t}\right)^{\frac{1}{\alpha}}g\left(k\left(\omega_{i}^{t}\right),h\left(\omega_{i}^{t}\right)\right)^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$$

Thus, output $y\left(\omega_{i}^{t},s^{t}\right)$ and labor $\ell\left(\omega_{i}^{t},s^{t}\right)$ are log-separable in ω_{i}^{t} and s^{t}

$$\begin{aligned} y\left(\omega_{it},s^{t}\right) &= \Psi^{\omega}(\omega_{i}^{t})\Psi^{s}(s^{t}) \\ \ell\left(\omega_{i}^{t},s^{t}\right) &= \Psi^{\omega}(\omega_{i}^{t})^{\frac{\rho-1}{\rho}}\left(\frac{\Psi^{s}(s^{t})}{A\left(s^{t}\right)}\right)^{\frac{1}{\alpha}} \end{aligned}$$

with

$$\Psi^{\omega}(\omega_{i}^{t}) = g\left(k\left(\omega_{i}^{t}\right),h\left(\omega_{i}^{t}\right)\right)^{\frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{91}$$

$$\Psi^{s}(s^{t}) = \left[Y(s^{t})^{\frac{1}{\rho}}A\left(s^{t}\right)^{\frac{1}{\alpha}}\frac{\psi^{r}\left(s^{t}\right)}{\psi^{\ell}\left(s^{t}\right)}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{92}$$

where we abstract from the constant scalar $(\alpha\chi^{*})^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$. This confirms that, with the assumed specification for the production function F, *every* flexible-price equilibrium allocation is log-separable, and can therefore be replicated as a sticky-price equilibrium. **QED.**

Proof of Proposition 6. Given the definition of the relaxed Ramsey problem, the *Relaxed Ramsey optimal allocation* solves the following problem

$$\max_{\xi_t} \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right) \left[U\left(C\left(s^t\right), L\left(s^t\right), s^t\right)\right]$$

subject to the implementability condition

$$0 \leq \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right) \left[U_c\left(s^t\right) C\left(s^t\right) + U_\ell\left(s^t\right) L\left(s^t\right)\right] \tag{93}$$

and the resource constraints,

$$C\left(s^t\right) + K\left(s^{t+1}\right) - (1-\delta)K\left(s^t\right) + G\left(s^t\right) + \sum_{\omega\in\Omega^t} h\left(\omega\right)\varphi\left(\omega|s^t\right) \tag{94}$$

$$= \left[\sum_{\omega\in\Omega^t} \left(A\left(s^t\right) F\left(k\left(\omega_i^t\right), h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right)\right)\right)^{\frac{\rho-1}{\rho}} \varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}} \quad \forall t, s^t;$$

$$\sum_{\omega\in\Omega^t} \ell\left(\omega\right)\varphi\left(\omega|s^t\right) = L\left(s^t\right), \quad \forall t, s^t; \tag{95}$$

$$\sum_{\omega\in\Omega^t} k\left(\omega\right)\varphi\left(\omega|s^t\right) = K\left(s^t\right), \quad \forall t, s^t; \tag{96}$$

Let Γ be the Lagrange multiplier on the implementability constraint (93). Let $\beta^t\mu\left(s^t\right)\zeta\left(s^t\right)$ and $\beta^t\mu\left(s^t\right)\zeta\left(s^t\right)\gamma\left(s^t\right)$ and $\beta^t\mu\left(s^t\right)\zeta\left(s^t\right)\kappa\left(s^t\right)$ be the multipliers on the feasibility constraints (94), (95), and (96), respectively. The relaxed Ramsey problem in Lagrangian form is then given by

$$\begin{aligned} L =& \sum_{t,s^t} \beta^t \mu\left(s^t\right) \tilde{U}\left(C\left(s^t\right), L\left(s^t\right), s^t\right) \\ & - \sum_{t,s^t} \beta^t \mu\left(s^t\right) \zeta\left(s^t\right) \left\{C\left(s^t\right) + K\left(s^{t+1}\right) - (1-\delta)K\left(s^t\right) + G\left(s^t\right) + \sum_{\omega\in\Omega^t} h\left(\omega\right)\varphi\left(\omega|s^t\right)\right\} \\ & + \sum_{t,s^t} \beta^t \mu\left(s^t\right) \zeta\left(s^t\right) \left[\sum_{\omega\in\Omega^t} \left(A\left(s^t\right) F\left(k\left(\omega_i^t\right), h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right)\right)\right)^{\frac{\rho-1}{\rho}} \varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}} \\ & - \sum_{t,s^t} \beta^t \mu\left(s^t\right) \zeta\left(s^t\right) \gamma\left(s^t\right) \left\{\sum_{\omega\in\Omega^t} \ell\left(\omega\right)\varphi\left(\omega|s^t\right) - L\left(s^t\right)\right\} \\ & - \sum_{t,s^t} \beta^t \mu\left(s^t\right) \zeta\left(s^t\right) \kappa\left(s^t\right) \left\{\sum_{\omega\in\Omega^t} k\left(\omega\right)\varphi\left(\omega|s^t\right) - K\left(s^t\right)\right\} \end{aligned}$$

where

$$\tilde{U}\left(C\left(s^t\right), L\left(s^t\right), s^t\right) \equiv U\left(C\left(s^t\right), L\left(s^t\right), s^t\right) + \Gamma\left(U_c\left(s^t\right) C\left(s^t\right) + U_\ell\left(s^t\right) L\left(s^t\right)\right)$$

The FOCs with respect to $C\left(s^t\right)$, $L\left(s^t\right)$, and $K\left(s^{t+1}\right)$ of this problem are

$$\begin{aligned}\tilde{U}_c(s^t) - \zeta\left(s^t\right) &= 0, \\ \tilde{U}_\ell(s^t) + \zeta\left(s^t\right)\gamma\left(s^t\right) &= 0\end{aligned}$$

and

$$-\beta^t \mu\left(s^t\right)\zeta\left(s^t\right) + \sum_{t,s^t} \beta^{t+1}\mu\left(s^{t+1}\right)\left[\zeta\left(s^{t+1}\right)\kappa\left(s^{t+1}\right) + \zeta\left(s^{t+1}\right)(1-\delta)\right] = 0.$$

The last of these conditions may be written as

$$\zeta\left(s^t\right) = \sum_{s^{t+1}} \beta\mu\left(s^{t+1}|s^t\right)\zeta\left(s^{t+1}\right)\left[1 + \kappa\left(s^{t+1}\right) - \delta\right]$$

Combining this with FOCs for $C\left(s^t\right)$ and $C\left(s^{t+1}\right)$, we get

$$\tilde{U}_c(s^t) = \beta\mathbb{E}\left[\tilde{U}_c(s^{t+1})\left\{1 + \kappa\left(s^{t+1}\right) - \delta\right\} \;\middle|\; s^t\right]$$

thereby obtaining equation (37) of the proposition.

Second, the FOCs with respect to $\ell\left(\omega_i^t, s^t\right)$ are given by

$$\beta^t\mu\left(s^t\right)\varphi\left(\omega|s^t\right)\zeta\left(s^t\right)\left\{\left[\sum_{\omega\in\Omega^t} y\left(\omega_i^t, s^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}-1} y\left(\omega_i^t, s^t\right)^{\frac{\rho-1}{\rho}-1} A(s^t) f_\ell\left(\omega_i^t, s^t\right) - \gamma\left(s^t\right)\right\} = 0$$

for all ω_i^t, s^t, which reduces to

$$\zeta\left(s^t\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_\ell\left(\omega_i^t, s^t\right) - \zeta\left(s^t\right)\gamma\left(s^t\right) = 0$$

Combining these with the FOCs for $C\left(s^t\right)$ and $L\left(s^t\right)$, we get

$$\tilde{U}_c(s^t)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_\ell\left(\omega_i^t, s^t\right) - \left(-\tilde{U}_\ell(s^t)\right) = 0$$

thereby obtaining equation (34) of the proposition.

Third, the FOCs with respect to $h\left(\omega_i^t\right)$ are given by

$$\sum_{s^t}\zeta\left(s^t\right)\mu\left(s^t\right)\varphi\left(\omega|s^t\right)\left\{\left[\sum_{\omega\in\Omega^t} y\left(\omega_i^t, s^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}-1} y\left(\omega_i^t, s^t\right)^{\frac{\rho-1}{\rho}-1} A(s^t) f_h\left(\omega_i^t, s^t\right) - 1\right\} = 0$$

for all ω_i^t, s^t. Next, by using

$$\mu\left(s^t\right)\varphi\left(\omega|s^t\right) = \varphi\left(s^t|\omega_i^t\right)\varphi\left(\omega_i^t\right)$$

we have that

$$\sum_{s^t}\zeta\left(s^t\right)\varphi\left(s^t|\omega_i^t\right)\varphi\left(\omega_i^t\right)\left\{\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_h\left(\omega_i^t, s^t\right) - 1\right\} = 0$$

or, equivalently,

$$\mathbb{E}\left[\tilde{U}_c(s^t)\left\{\left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_h\left(\omega_i^t,s^t\right)-1\right\}\,\middle|\,\omega_i^t\right]=0$$

We thereby obtain equation (35) of the proposition.

Fourth, the FOCs with respect to $k\left(\omega_i^t\right)$ are given by

$$\sum_{s^t}\zeta\left(s^t\right)\mu\left(s^t\right)\varphi\left(\omega|s^t\right)\left\{\left[\sum_{\omega\in\Omega^t} y\left(\omega_i^t,s^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}-1} y\left(\omega_i^t,s^t\right)^{\frac{\rho-1}{\rho}-1} A(s^t) f_k\left(\omega_i^t,s^t\right)-\kappa\left(s^t\right)\right\}=0$$

for all ω_i^t, s^t. Next, by using

$$\mu\left(s^t\right)\varphi\left(\omega|s^t\right)=\varphi\left(s^t|\omega_i^t\right)\varphi\left(\omega_i^t\right)$$

we have that

$$\sum_{s^t}\zeta\left(s^t\right)\varphi\left(s^t|\omega_i^t\right)\varphi\left(\omega_i^t\right)\left\{\left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_k\left(\omega_i^t,s^t\right)-\kappa\left(s^t\right)\right\}=0$$

or, equivalently,

$$\mathbb{E}\left[\tilde{U}_c(s^t)\left\{\left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_k\left(\omega_i^t,s^t\right)-\kappa\left(s^t\right)\right\}\,\middle|\,\omega_i^t\right]=0$$

We thereby obtain equation (36) of the proposition. **QED.**

Proof of Proposition 4. To show this, note that aside from the implementability condition (26) which holds in the relaxed Ramsey optimum, three conditions must be satisfied in order for an allocation to be implementable under flexible prices. These are:

$$\begin{aligned}
\chi^*\psi^r\left(s^t\right)MP_\ell\left(\omega_i^t,s^t\right)-\psi^\ell\left(s^t\right) &= 0 && (97)\\
\mathbb{E}\left[\chi^*\psi^r\left(s^t\right)MP_h\left(\omega_i^t,s^t\right)-\psi^c\left(s^t\right)\middle|\,\omega_i^t\right] &= 0 && (98)\\
\mathbb{E}\left[\chi^*\psi^r\left(s^t\right)MP_k\left(\omega_i^t,s^t\right)-\psi^k\left(s^t\right)\middle|\,\omega_i^t\right] &= 0 \quad \forall\,\omega_i^t && (99)
\end{aligned}$$

For this proof we need to show that there exists functions $\psi^c,\psi^\ell,\psi^k,\psi^r : \mathcal{S}^t\to\mathbb{R}_+$ such that the relaxed ramsey optimal allocation ξ^* satisfies these conditions.

First, consider condition (34) in the relaxed Ramsey optimal allocation,

$$\tilde{U}_c(s^t)MP_\ell\left(\omega_i^t,s^t\right)-\left(-\tilde{U}_\ell(s^t)\right)=0$$

Let us choose $\psi^r\left(s^t\right)$ and $\psi^\ell\left(s^t\right)$ such that

$$\psi^\ell\left(s^t\right)=-\tilde{U}_\ell(s^t)\quad\text{and}\quad\chi^*\psi^r\left(s^t\right)=\tilde{U}_c(s^t) \tag{100}$$

Then (34) along with our chosen functions $\psi^r\left(s^t\right)$ and $\psi^\ell\left(s^t\right)$ in (100) ensures that the flexible-price implementability condition (97) holds.

Second, consider condition (35) in the relaxed Ramsey optimal allocation,

$$\mathbb{E}\left[\tilde{U}_c(s^t)\left\{MP_h\left(\omega_i^t, s^t\right) - 1\right\} \,\middle|\, \omega_i^t\right] = 0$$

Let us choose $\psi^c\left(s^t\right)$ such that

$$\psi^c\left(s^t\right) = \tilde{U}_c(s^t). \tag{101}$$

Then (35) along with our chosen functions $\psi^r\left(s^t\right)$ and $\psi^c\left(s^t\right)$ in (100) and (101) ensures that the flexible-price implementability condition (98) holds.

Third, consider condition (36) in the relaxed Ramsey optimal allocation,

$$\mathbb{E}\left[\tilde{U}_c(s^t)\left\{MP_k\left(\omega_i^t, s^t\right) - \kappa\left(s^t\right)\right\} \,\middle|\, \omega_i^t\right] = 0$$

Let us choose $\psi^k\left(s^t\right)$ such that

$$\psi^k\left(s^t\right) = \tilde{U}_c(s^t)\kappa\left(s^t\right) \tag{102}$$

where the function $\kappa\left(s^t\right)$ satisfies (37). Then (36) along with our chosen functions $\psi^r\left(s^t\right)$ and $\psi^k\left(s^t\right)$ in (100) and (102) ensures that the flexible-price implementability condition (99) holds.

Therefore, there exists functions $\psi^c, \psi^\ell, \psi^k, \psi^r : \mathcal{S}^t \to \mathbb{R}_+$ given specifically by

$$\psi^c\left(s^t\right) = \tilde{U}_c(s^t),\;\; \psi^\ell\left(s^t\right) = -\tilde{U}_\ell(s^t),\;\; \psi^r\left(s^t\right) = \tilde{U}_c(s^t)/\chi^*, \text{ and } \psi^k\left(s^t\right) = \tilde{U}_c(s^t)\kappa\left(s^t\right) \tag{103}$$

such that the flexible price implementabiliy conditions (97)-(99) are all satisfied at the relaxed Ramsey optimal allocation. Thus the relaxed Ramsey optimal allocation may be implemented as an equilibrium under flexible prices. **QED.**

Proof of Proposition 5. Proposition 4 establishes that $\xi^* \in \mathcal{X}^f$ always. Suppose that ξ^* is log-separable; then $\xi^* \in \tilde{\mathcal{X}}^f$ and $\tilde{\mathcal{X}}^f \subseteq \mathcal{X}^s$ implies $\xi^* \in \mathcal{X}^s$. In particular, the optimal allocation ξ^* is implemented as an equilibrium under sticky prices with functions $\psi^c, \psi^\ell, \psi^k, \psi^r : \mathcal{S}^t \to \mathbb{R}_+$ given by (103), and a function $\chi : \Omega^t \times \mathcal{S}^t \to \mathbb{R}_+$ given by

$$\chi(\omega_i^t, s^t) = \chi^* \quad \forall\, \omega_i^t, s^t \tag{104}$$

If instead ξ^* is not log-separable, then $\xi^* \in \mathcal{X}^f$ but $\xi^* \notin \mathcal{X}^s$. **QED.**

Proof of Theorem 1. The result follows from Propositions 4 and 5. To see this, recall that the functions given in (103) for $\psi^c, \psi^\ell, \psi^k, \psi^r : \mathcal{S}^t \to \mathbb{R}_+$ implement the optimal allocation ξ^* as a flexible-price equilibrium. Combining these with the tax functions in (82), gives us the following tax rates consistent with this allocation:

$$1 + \tau^c\left(s^t\right) = \delta\frac{U_c(s^t)}{\tilde{U}_c(s^t)},\;\; 1 - \tau^\ell\left(s^t\right) = \delta\frac{-U_\ell(s^t)}{-\tilde{U}_\ell(s^t)},\;\text{ and }\; 1 - \tau^r(s^t) = (\chi^*)^{-1}$$

where $\delta > 0$ is a scalar. Finally, note that the optimal ψ^k is given by

$$\psi^k\left(s^t\right) = \tilde{U}_c(s^t)\kappa\left(s^t\right)$$

where the function $\kappa\left(s^t\right)$ satisfies (37). From (84), this implies an equilibrium rental rate of

$$r\left(s^t\right)=\kappa\left(s^t\right) \tag{105}$$

at the optimum. Recall that while $\kappa\left(s^t\right)$ satisfies (37), the equlibrium rental rate $r\left(s^t\right)$ must satisfy the household's Euler condition (57). Therefore, in order for these two conditions to coincide, it must be the case that $r\left(s^t\right)=\tilde{r}\left(s^t\right)$ which further implies

$$1-\tau^k\left(s^t\right)=1$$

as dictated by equation (85). We therefore obtain conditions (40) and (41) for the optimal taxes.

What remains to be shown is that there is a nominal interest rate that replicates the flexible price allocation, i.e. one that satisfies condition (104) for $\chi:\Omega^t\times\mathcal{S}^t\to\mathbb{R}_+$ and hence implements ξ^*. Recall that the equilibrium nominal interest rate is pinned down by equation (55). At the optimum, consumption taxes satisfy $1+\tau^c\left(s^t\right)=\delta U_c\left(s^t\right)/\tilde{U}_c\left(s^t\right)$. Substituting these taxes into (55) results in the following expression

$$\frac{\tilde{U}_c\left(s^t\right)}{P\left(s^t\right)}=\beta\mathbb{E}\left[\left.\frac{\tilde{U}_c\left(s^{t+1}\right)}{P\left(s^{t+1}\right)}\left(1+R\left(s^t\right)\right)\right|s^t\right].$$

By Theorem 2, the price level that implements flexible price allocations is given by $P(s^t)=e^{z_t}\mathcal{B}(s^t)^{-\frac{1}{\rho}}$ where z_t is commonly known. It follows that the nominal interest rate that implements the optimal allocation is given by

$$1+R\left(s^t\right)=\frac{\tilde{U}_c\left(s^t\right)}{\exp\left(z_t-z_{t-1}\right)\mathcal{B}(s^t)^{-\frac{1}{\rho}}}\left\{\beta\mathbb{E}\left[\left.\frac{\tilde{U}_c\left(s^{t+1}\right)}{\mathcal{B}(s^{t+1})^{-\frac{1}{\rho}}}\right|s^t\right]\right\}^{-1}$$

QED.

Proof of Lemma 3. With the homothetic preferences in (42),

$$\tilde{U}\left(C,L\right)=\frac{C^{1-\gamma}}{1-\gamma}-\eta\frac{L^{1+\epsilon}}{1+\epsilon}+\Gamma\left[\left(1-\gamma\right)\frac{C^{1-\gamma}}{1-\gamma}-\left(1+\epsilon\right)\eta\frac{L^{1+\epsilon}}{1+\epsilon}\right]. \tag{106}$$

This implies

$$\frac{U_c\left(s^t\right)}{\tilde{U}_c\left(s^t\right)}=\frac{1}{1+\Gamma\left(1-\gamma\right)},\quad\text{and}\quad\frac{U_\ell\left(s^t\right)}{\tilde{U}_\ell\left(s^t\right)}=\frac{1}{1+\Gamma\left(1+\epsilon\right)}$$

Consider the implementation scheme proposed; a zero tax rate on consumption implies that in order to obtain the optimal labor tax given in (40), it must satisfy

$$1-\tau^\ell\left(s^t\right)=\frac{U_\ell\left(s^t\right)}{\tilde{U}_\ell\left(s^t\right)}\left(\frac{U_c\left(s^t\right)}{\tilde{U}_c\left(s^t\right)}\right)^{-1}=\frac{1+\Gamma\left(1-\gamma\right)}{1+\Gamma\left(1+\epsilon\right)}$$

The tax rate on capital follows directly from Theorem 1. **QED.**

Proof of Theorem 2. Following the proof of Proposition 3, for any arbitrary common-knowledge process z_t, nominal prices are given by

$$p(\omega_i^t) = e^{z_t}\Psi^{\omega}\left(\omega_i^t\right)^{-\frac{1}{\rho}}$$

It follows that the aggregate price level satisfies

$$P(s^t) = \left[\sum_{\omega\in\Omega^t} p\left(\omega_i^t\right)^{1-\rho}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}} = e^{z_t}\left[\sum_{\omega\in\Omega^t}\Psi^{\omega}\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}},$$

We may thus express the aggregate price level in terms of $\mathcal{B}(s^t)$ as follows

$$P(s^t) = e^{z_t}\mathcal{B}(s^t)^{-\frac{1}{\rho}},$$

thereby obtaining condition (44). **QED.**

Proof of Lemma 4. From the proof of Proposition 3, when technology is iso-elastic in labor as in (33), along any equilibrium $y\left(\omega_{it}, s^t\right)$ is log-separable with $\Psi^{\omega}(\omega_i^t)$ given by (91) and $\Psi^{s}(s^t)$ given by (92). If technology is furthermore Cobb-Douglas, then we may write (91) as

$$\Psi^{\omega}(\omega_i^t) = \left[k\left(\omega_i^t\right)^{1-\eta}h\left(\omega_i^t\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$$

In this case, $\mathcal{B}(s^t)$ is given by

$$\mathcal{B}(s^t) = \left[\sum_{\omega\in\Omega^t}\left[k\left(\omega_i^t\right)^{1-\eta}h\left(\omega_i^t\right)^{\eta}\right]^{\frac{1-\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}\left(\frac{\rho-1}{\rho}\right)}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}}$$

This implies that up to a first-order log-linear approximation $\mathcal{B}(s^t)$ may be written as in (45) with scalars ζ_K, ζ_H given by

$$\zeta_K \equiv (1-\eta)\frac{1-\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)} > 0 \quad \text{and} \quad \zeta_H \equiv \eta\frac{1-\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)} > 0.$$

QED.

Proof of Proposition 7. Suppose that k and h may be conditioned on s^t. Then by symmetry, firm optimality conditions imply

$$y\left(\omega_i^t, s^t\right) = Y\left(s^t\right) \text{ for all } \omega_i^t$$

Therefore, along any equilibrium $y\left(\omega_{it}, s^t\right)$ is log-separable with $\Psi^{\omega}(\omega_i^t) = 1$. This implies that along any equilibrium (including the optimal one), $\mathcal{B}(s^t) = 1$ is a constant and as a result the equilibrium allocation is implemented by targeting price stability. **QED.**

Appendix B: A Tractable Example

In this Appendix, we use a tractable example to offer a sharp illustration of some of the results of our paper, as well as to shed light on some subtleties that may have been obscured by the level of generality.

The economy

We abstract from capital and assume constant government spending. We let preferences be homothetic as in (42) and technology be Cobb-Douglas. Moreover, we add idiosyncratic TFP shocks. The production function is thus given by

$$y_{it} = A_{it} \left(h_{it}^{\eta}\right)^{1-\alpha} \ell_{it}^{\alpha}, \quad \text{with } \alpha \in (0,1) \text{ and } \eta \in (0,1) \tag{107}$$

where A_{it} denotes the productivity of firm i in period t.[38] This is comprised of both an aggregate and a firm-specific component, namely,

$$A_{it} = A_t \exp v_{it}.$$

where A_t is the aggregate component and v_{it} is the idiosyncratic one.

The processes of A_t and v_{it} are assumed to be Gaussian and orthogonal to one another. In addition, v_{it} is assumed to be i.i.d. *across* firms, but can be correlated over time *within* a firm. We make no specific assumptions concerning the law of motion of aggregate productivity A_t other than it be an exogenous Gaussian process. We finally assume that each firm knows her own A_{it} but otherwise allow each firm to have arbitrary Gaussian information about aggregate TFP and about the information of other firms. We can thus think of ω_i^t as a vector that contains A_{it} along with any arbitrary set of Gaussian signals of some underlying aggregate state s^t. The latter in turn may contain not only the realized A_t but also any aggregate noise or correlated shocks to firms' first- and higher-order beliefs about A_t. It can also contain shocks that are unrelated to the entire belief hierarchy about A_t, such as pure sunspots or monetary shocks.

Although we impose that information is Gaussian, we allow for arbitrary persistence in either the underlying TFP shock or the noise in the information of the firms. The analysis that follows can therefore nest, inter alia, the kind of learning dynamics considered in (**?**) and (Angeletos and La'O, 2010). Furthermore, the analysis is agnostic as to whether information is exogenous (as in our baseline framework) or endogenously acquired (as in Section 6). As we move from one case to the other, we only have to adapt what the term "information parameters" means: in the first, this term refers to the exogenous volatilities of the various noises; in the latter, it also refers to the cost function κ.

Remark 1. To fix ideas, the reader may find it useful to restrict attention to the following special case. Suppose that $\log A_t$ is i.i.d. over time, drawn from a Normal distribution with mean 0 and variance 1. Suppose next that ω_i^t is given by ω_i^{t-1} along with the triplet $(A_{it}, z_{it}^{priv}, z_t^{pub})$, where z_{it}^{priv} and z_{it}^{pub} are, respectively, a exogenous private and an exogenous public signal about A_t. The private signal is given by $z_{it}^{priv} = \log A_t + \sigma_\epsilon \epsilon_{it}$, and the public one by $z_{it}^{pub} = \log A_t + \sigma_u u_t$, where ϵ_{it} and u_t are, respectively, idiosyncratic and aggregate noises, independent of one another, as well as of

[38] This production function satisfies the iso-elastic technology condition in (33).

A_t, and drawn from Normal distributions with means zero and variance 1, and where σ_ϵ and σ_u are exogenous scalar that parameterize the precision of the two signals.

Remark 2. The case considered in Remark 1 above resembles the one considered in (Morris and Shin, 2002) and Angeletos and Pavan (2007). At first glance, this case looks *too* special. It is indeed unappealing for positive purposes, as it rules out persistence in either TFP or the noise. For our purposes, however, it is not that special. In the Online Appendix, we effectively show that the normative results that are obtained in this special case can readily be extended to the more general case. This done by leveraging on our earlier result that the optimal policies are the same whether information by exogenous or endogenous; by adapting the methods of Bergemann and Morris (2015) to our example; and finally by re-interpreting the noise u_t as a metaphor for all the variation in GDP that is caused by the underlying first- and higher-order uncertainty about A_t. The latter means that, although u_t has to be uncorrelated with the *current* TFP, it may be correlated with *past* TFP; this is because, in settings with persistent TFP shocks and dispersed information, past TFP acts as a correlated error in the information about current TFP.

Remark 3. For any $h \geq 1$, let $\bar{E}_t^h[\log A_t]$ denote the h-th order average forecast of $\log A_t$.[39] Because we abstract from shocks to government spending and henceforth focus on log-linearized equilibria, the sequence $\left\{\bar{E}_t^h[\log A_t]\right\}_{h=1}^{\infty}$ is a sufficient statistic for the realized belief hierarchy about the underlying fundamentals. Furthermore, because we impose a Gaussian information structure, the aforementioned sequence and the true realization of $\log A_t$ are jointly Normal. To simplify the exposition, we henceforth rule out degenerate cases in which the relevant higher-order beliefs are *perfectly* correlated with the underlying fundamental by imposing that the regression of any weighted average of $\left\{\bar{E}_t^h[\log A_t]\right\}_{h=1}^{\infty}$ on $\log A_t$ has a non-zero residual. This is trivially satisfied in all the aforementioned examples.

The decentralized use of information and the Ramsey optimum

One policy lesson that emerges in our paper is that the policy instruments already familiar from the Ramsey literature can play *novel* roles once one relaxes the strong assumption that all information is commonly shared: by appropriately designing the contingency of these instruments to the underlying state of Nature, the planner can manipulate how firms use any private information at their disposal.

To illustrate this point, it suffices to allow the revenue tax to be contingent on the realized TFP and on the realized level of output. In particular, we impose that

$$-\log\left(1-\tau^r\left(A_t, Y_t\right)\right) = \hat{\tau}_0 + \hat{\tau}_A \log A_t + \hat{\tau}_Y \log Y_t$$

for some scalars $\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y \in \mathbb{R}$. We then let the remaining tax rates satisfy $\tau^k(s^t) = \tau^c(s^t) = 0$ and $1+\tau^\ell(s^t) = 1/\left(1-\tau^r(s^t)\right)$. The scalars $(\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y)$ can be thought of as the policy coefficients: they are the levers that may be used to influence the equilibrium use of information. We allow $\hat{\tau}_A$ to take any value in $\mathbb{R}$ but bound $\hat{\tau}_Y$ from below so as to guarantee the equilibrium is unique holding the tax

[39] This is defined recursively by letting $\bar{E}_t^1[\log A_t] \equiv \sum_{\omega^t\in\Omega^t} \mathbb{E}\left[\log A_t \middle| \omega^t\right] \phi(\omega^t|s^t)\ \left(= \int_i E_{it}[\log A_t]di\right)$ and, for any $h \geq 2$, $\bar{E}_t^h[\log A_t] \equiv \sum_{\omega^t\in\Omega^t} \mathbb{E}\left[\bar{E}_t^h[\log A_t]\middle|\omega^t\right]\phi(\omega^t|s^t)\ \left(= \int_i E_{it}\left[\bar{E}_t^h[\log A_t]\right]di\right)$.

system fixed. For the rest of this section, we finally consider the log-linearized approximation of the set of flexible-price allocations around the steady state in which $A = 1$.[40]

Proposition 10. *Consider the economy and the taxes described above.*

(i) In any flexible-price equilibrium, GDP satisfies, up to a log-linear approximation,

$$\log GDP\left(s^{t}\right) = \gamma_0 + \gamma_A \log A_t + \gamma_u u_t, \tag{108}$$

where the scalars $(\gamma_0, \gamma_A, \gamma_u)$ are pinned down by the policy coefficients $(\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y)$ and where u_t is a random variable that satisfies the following restriction: it is orthogonal to $\log A_t$ and, conditional on the latter, it is correlated with $\left\{\bar{E}_t^h[\log A_t]\right\}_{h=1}^{\infty}$ and is Normally distributed with mean 0 and variance 1.

(ii) The exists an equilibrium in which (108) holds with the pair (γ_A, γ_u) taking any *value in the set Υ, where*

$$\Upsilon \equiv \left\{(\gamma_A, \gamma_u) \in \mathbb{R}^2 : \text{ either } \gamma_u > 0 \text{ and } \gamma_A > \hat{\gamma} + \gamma_u\text{, or } \gamma_u < 0 \text{ and } \gamma_A < \hat{\gamma} + \gamma_u\right\} \tag{109}$$

and where $\hat{\gamma}$ is constant.[41]

To understand this result, note the variation in u_t has to be orthogonal to the underlying variation in $\log A_t$, but also correlated with the residual variation in the belief hierarchy about $\log A_t$. This allows room for the kind of noise- and sentiment-driven fluctuations considered in (Angeletos and La'O, 2013; Benhabib, Wang, and Wen, 2015; Huo and Takayama, 2015), but rules out the kind of "pure" sunspot fluctuations considered in Cass and Shell (??) and Benhabib and Farmer (???): for every t, the period-t equilibrium level of output is invariant to any random variable that is itself orthogonal the *entire* period-t belief hierarchy about the underlying TFP.

With these points in mind, the result can be read as follows. By choosing the coefficients $\hat{\tau}_A$ and $\hat{\tau}_Y$, the planner can control how sensitive a firm's expectation of her net-of-taxes returns are to her beliefs about *both* the underlying fundamental (aggregate TFP) and the actions of other firms (aggregate output). The planner can thereby influence the incentives the typical firm faces when contemplating how to react to different pieces of information or to different perceptions of what's going on in the economy. By the same token, the planner can influence the response of macroeconomic activity, not only to the underlying TFP shocks, but also to any correlated noise in the first- or higher-order beliefs about it, and therefore to the error u_t.

For instance, by setting a sufficiently large *negative* value for $\hat{\tau}_Y$ and a sufficiently large *positive* value for $\hat{\tau}_A$, the planner can induce the firms to disregard any noisy public signal and more generally any signal that is subject to correlated noise and, instead, pay close attention to their respective productivities and to any private information that is insulated from correlated noise. In so doing, the planner can eliminate the fluctuations driven by noise or by exotic "sentiments" and can replicate the business cycle that would have been optimal in the absence of the informational friction. Formally:

[40] The reason we cannot obtain an *exact* log-linear solution to our model despite is due to the fact that the resource constraint is additive in consumption and intermediate goods. If we restrict $\gamma = 0$, we can in fact obtain an exact log-linear relation between P and Y but not between P and C.

[41] This constant depends on the underlying preference, technology, and information parameters, but is invariant the policy and the implemented allocation. Also, if there is not equilibrium in which

Corollary 3. *There is a flexible-price equilibrium in which the process for GDP coincides with, or approximates arbitrarily well, the process that is optimal in the absence of the information friction.*

We henceforth refer to the allocation that would have been optimal in the absence of the information friction as the Lucas-Stokey benchmark. The level of GDP that obtains in this benchmark is nested in (108) by setting $\gamma_u = 0$ and $\gamma_A = \gamma_A^{LS}$ for some scalar γ_A^{LS} that is pinned down by preference and technology parameters along with the level of government spending (equivalently, the tightness of the government budget). We now contrast this benchmark to what is optimal in the presence of the informational friction.

Proposition 11. *In any equilibrium that implements the optimal allocation, GDP is given by*

$$\log GDP_t = \gamma_0^* + \gamma_A^* \log A_t + \gamma_u^* u_t, \tag{110}$$

where u_t satisfies the restrictions stated in Proposition 10[42] *and where the scalars γ_A^* and γ_u^* are uniquely determined by the underlying preference, technology, and information parameters. Furthermore,*

$$0 < \gamma_A^* < \gamma_A^{LS} \qquad \textit{and} \qquad \gamma_u^* > 0. \tag{111}$$

There are therefore two salient differences between the GDP path that is optimal in our setting and the one that is optimal in the Lucas-Stokey benchmark. First, GDP features a lower sensitivity to the underlying technology shock than in Lucas-Stokey; that is, the informational friction dampens the response of the optimum to the technology shocks. Second, GDP varies with the variable u_t; that is, the business cycle can be driven by noise shocks, sentiments, and the like.

The quantitative significance of the aforementioned differences is beyond the scope of our paper.[43] It is worth iterating, however, the following point: any attempt to guide policy by inspecting the volatility properties of the observe business cycle and by comparing them to familiar benchmark is likely to be misleading. Instead, our earlier results imply that the policy maker can safely "ignore" the observable properties of the business cycle and, instead, use the same principles as in the extant Ramsey literature to set the available tax instruments.

Leaning against the wind

We now turn attention to monetary policy. Theorems 1 and 2, of course, apply. The goal is to illustrate the particular form of "leaning against the wind" that obtains in this simple environment. This is accomplished in the following.

Proposition 12. *In any sticky-price equilibrium that implements the optimal allocation, the aggregate price level satisfies*

$$\log P(s^t) = \delta_0^* - \delta_A^* \log A\left(s^t\right) - \delta_u^* u_t, \tag{112}$$

for some scalars $\delta_0^*, \delta_A^*, \delta_u^*$ that are determined by the underlying preference, technology, and information parameters and satisfy $\delta_A^* > 0$ and $\delta_u^* > 0$.

[42] I.e., u_t is orthogonal to $\log A_t$ and, conditionally on the latter, u_t is correlated with the belief hierarchy about $\log A_t$ and its mean and variance are normalized to, respectively, 0 and 1.

[43] See Angeletos, Collard and Dellas (2014) and Huo and Takayama (2015) for recent attempts in this direction.

Corollary 4. *The optimal price level is negatively correlated with the optimal level of GDP both unconditionally and conditionally on the realized TFP.*

These findings illustrate the nature of the optimal monetary policy. As optimal output reacts to both aggregate productivity and to noise/sentiments, so does the optimal price level. The optimal policy therefore targets a negative relation between the aggregate price level and real economic activity regardless whether the fluctuations are driven by true innovations in TFP or by noise, sentiments, and the like.

We conclude by highlighting what happens when monetary policy fails to implement the optimal allocation. Clearly, the response of GDP to either A_t or u_t may deviate from the corresponding optimal levels; that is, (108) may hold with $\gamma_A \neq \gamma_A^*$ and/or $\gamma_u \neq \gamma_u^*$. In addition, GDP can now very with shocks that are unrelated, not only to the underlying TFP shock, but also to the entire belief hierarchy of it. Such shocks may pure monetary shocks, shocks to the discount factor, or shocks to the belief hierarchy about these objects. Had monetary policy been optimal, GDP would have been insulated from such shocks, and so would the price level. This iterates our earlier point that price stability is actually optimal vis-a-vis shocks that do not themselves move the optimal allocation.

Remark. The noise- or sentiment-driven fluctuations accommodated in Propositions 11 and 12 are tightly connected to the type of "non-fundamental" fluctuations formalized in (Angeletos and La'O, 2013; Benhabib, Wang, and Wen, 2015; Huo and Takayama, 2015), but less so to the one formalized in Lorenzoni (2009), Barsky and Sims (2011), and Blanchard et al (2013). The latter type rests on nominal rigidity and on the failure of monetary policy to replicate the optimal flexible-price allocation.

Proofs for Simple Illustration

Proof of Proposition 10 Take any flexible-price equilibrium. For any realization of $\left(\omega_i^t, s^t\right)$, the following two equations must hold:

$$1 = \chi^* \frac{\psi^r\left(s^t\right)}{\psi^\ell\left(s^t\right)} \left(\frac{y\left(\omega_i^t, s^t\right)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}} \alpha \frac{y\left(\omega_i^t, s^t\right)}{\ell\left(\omega_i^t, s^t\right)} \tag{113}$$

$$y\left(\omega_i^t, s^t\right) = A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)} \ell\left(\omega_i^t, s^t\right)^\alpha \tag{114}$$

Note that the main difference between these two equations and those stated previously in (89) and (90) is that firm specific productivity A is now measurable in ω_i^t. Following the proof for log-separability, we can solve (113) and (114) simultaneously for $y\left(\omega_i^t, s^t\right)$ and $\ell\left(\omega_i^t, s^t\right)$. We thus find that in any flexible-price equilibrium, output $y\left(\omega_i^t, s^t\right)$ and labor $\ell\left(\omega_i^t, s^t\right)$ are log-separable in ω_i^t and s^t and satisfy

$$y\left(\omega_{it}, s^t\right) = \Psi^\omega(\omega_i^t)\Psi^s(s^t) \tag{115}$$

$$\ell\left(\omega_i^t, s^t\right) = \Psi^\omega(\omega_i^t)^{\frac{\rho-1}{\rho}} \Psi^s(s^t)^{\frac{1}{\alpha}} \tag{116}$$

with

$$\Psi^{\omega}(\omega_i^t) = \left[A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{117}$$

$$\Psi^{s}(s^t) = \left[\alpha\chi^*\frac{\psi^r\left(s^t\right)}{\psi^\ell\left(s^t\right)}Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{118}$$

Next, consider the proposed tax policy. The revenue tax (and the associated wedges) takes the following form,

$$\log\left(1-\tau^r\left(A_t,Y_t\right)\right) = \hat{\tau}_0 - \hat{\tau}_A\log A_t - \hat{\tau}_Y\log Y_t \tag{119}$$

so that it is log-normally distribued for some scalars $\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y \in \mathbb{R}$, and the remaining tax rates satisfy $\tau^k(s^t) = \tau^c(s^t) = 0$, and $1+\tau^\ell(s^t) = 1/\left(1-\tau^r(s^t)\right)$.

Combining this last condition with the tax expressions in (82) implies

$$\frac{\psi^r\left(s^t\right)}{\psi^c(s^t)} = \frac{\psi^\ell\left(s^t\right)}{-U_\ell(s^t)}.$$

Rearranging and combining it with the expression for $\psi^c(s^t)$ in (82) gives us

$$\frac{\psi^r\left(s^t\right)}{\psi^\ell\left(s^t\right)} = \frac{U_c(s^t)}{-U_\ell(s^t)}$$

Using the above expression to replace the wedges in (118) gives us

$$\Psi^{\omega}(\omega_i^t) = \left[A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{120}$$

$$\Psi^{s}(s^t) = \left[\frac{U_c(s^t)}{-U_\ell(s^t)}Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{121}$$

where we abstract from the constant scalar $(\alpha\chi^*)^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$.

Aggregate output may be expressed as

$$Y\left(s^t\right) = \left[\sum_{\omega\in\Omega^t} y\left(\omega_{it},s^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}} = \Psi^s(s^t)\mathcal{B}(s^t) \tag{122}$$

Similarly using (116), aggregate labor may be expressed as

$$L\left(s^t\right) = \sum_{\omega\in\Omega^t}\ell\left(\omega_i^t,s^t\right)\varphi\left(\omega|s^t\right) = \Psi^s(s^t)^{\frac{1}{\alpha}}\mathcal{B}(s^t)^{\frac{\rho-1}{\rho}}. \tag{123}$$

Finally, the assumed specification for $U\left(C,L\right)$ in (42) allows us to rewrite $\Psi^s(s^t)$ in (121) as

$$\Psi^s(s^t) = \left[\frac{C(s^t)^{-\gamma}}{L\left(s^t\right)^{\epsilon}}Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}. \tag{124}$$

Taking logs of equations (122), (123), and (124) produces the following three equations.

$$\begin{aligned}\log Y\left(s^t\right) &= \log \Psi^s(s^t) + \log \mathcal{B}(s^t) &(125)\\ \log L\left(s^t\right) &= \frac{1}{\alpha}\log \Psi^s(s^t) + \frac{\rho-1}{\rho}\log \mathcal{B}(s^t) &(126)\\ \log \Psi^s(s^t) &= \alpha\zeta\left[\frac{1}{\rho}\log Y(s^t) - \gamma \log C(s^t) - \epsilon \log L\left(s^t\right)\right] &(127)\end{aligned}$$

where $\zeta \equiv \frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}$.

We combine these three equations as follows. Substituting (126) into (127) for $L\left(s^t\right)$ yields

$$\log \Psi^s(s^t) = \alpha\zeta\left[\frac{1}{\rho}\log Y(s^t) - \gamma \log C(s^t) - \epsilon\left(\frac{1}{\alpha}\log \Psi^s(s^t) + \frac{\rho-1}{\rho}\log \mathcal{B}(s^t)\right)\right].$$

We can solve this for $\Psi^s(s^t)$ and get

$$\log \Psi^s(s^t) = \frac{\alpha\zeta}{(1+\epsilon\zeta)}\left[\frac{1}{\rho}\log Y(s^t) - \gamma \log C(s^t) - \epsilon\frac{\rho-1}{\rho}\log \mathcal{B}(s^t)\right] \tag{128}$$

Combining this expression with equation (125) yields

$$\log Y\left(s^t\right) = \frac{\alpha\zeta}{(1+\epsilon\zeta)}\left[\frac{1}{\rho}\log Y(s^t) - \gamma \log C(s^t) - \epsilon\frac{\rho-1}{\rho}\log \mathcal{B}(s^t)\right] + \log \mathcal{B}(s^t)$$

Solving the above equation for $\mathcal{B}(s^t)$ gives us

$$\log \mathcal{B}(s^t) = \frac{1}{1-\epsilon\frac{\rho-1}{\rho}\frac{\alpha\zeta}{1+\epsilon\zeta}}\left[\left(1 - \frac{\alpha\zeta}{1+\epsilon\zeta}\frac{1}{\rho}\right)\log Y\left(s^t\right) + \gamma\frac{\alpha\zeta}{1+\epsilon\zeta}\log C(s^t)\right]. \tag{129}$$

Finally, from the definitions of $\mathcal{B}(s^t)$ and $\Psi^\omega\left(\omega_i^t\right)$, we have the following equation.

$$\mathcal{B}(s^t) = \left[\sum_{\omega\in\Omega^t}\left[A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}}$$

If we log-linearize our model about the complete information equilibrium, the previous equation becomes[44]

$$\log \mathcal{B}(s^t) = \zeta \log A\left(s^t\right) + \eta\left(1-\alpha\right)\zeta \log H\left(s^t\right). \tag{130}$$

In summary, thus far to describe the flexible price equilibrium we have a system of two equations, (129) and (130), in four unknowns: $Y\left(s^t\right), C\left(s^t\right), H\left(s^t\right)$, and $\mathcal{B}\left(s^t\right)$.

Complete Information Case. Our solution for the incomplete-information equilibrium will be a log-linear approximation around the complete-information Ramsey optimum. Without yet solving for the complete-information optimum, we characterize it below.

[44]Alternatively, we would obtain equation (130) in an exact version of our model if we assume that the information and shock structure are jointly log-Normal.

Lemma 8. *In the complete information optimum, aggregate intermediate good purchases and aggregate consumption are log-linear in aggregate productivity:*

$$\log H^{LS}\left(s^t\right) = \phi_A^{LS}\log A\left(s^t\right) + const \tag{131}$$
$$\log C^{LS}\left(s^t\right) = \gamma_A^{LS}\log A\left(s^t\right) + const \tag{132}$$

where ϕ_A^{LS} and γ_A^{LS} are scalar constants.

Thus, the complete information optimum is log-linear in the aggregate productivity shock, with a coefficient γ_A^{LS} on productivity for all aggregate variables. The scalar γ_A^{LS} is pinned down by preference and technology parameters along with the level of government spending (equivalently, the tightness of the government budget). For now, we take this allocation as given. We will prove Lemma 8 later when we consider the Ramsey planner's problem in the proof of Proposition 11.

Incomplete Information Log-Linearization. We now return to characterizing the equilibrium under incomplete information. First, we log-linearize the resource constraint around the complete information equilibrium characterized in Lemma 8; this gives us

$$\log Y\left(s^t\right) = (1-\varsigma)\log C\left(s^t\right) + \varsigma\log H\left(s^t\right) \tag{133}$$

where $\varsigma = \eta(1-\alpha)$ is the proportion of output that goes to intermediate good use under complete information. Substituting (133) for $Y\left(s^t\right)$ into equation (129) produces the following expression for $\mathcal{B}(s^t)$:

$$\log\mathcal{B}(s^t) = \zeta\left(\Gamma_C\log C\left(s^t\right) + \Gamma_H\log H\left(s^t\right)\right) \tag{134}$$

where

$$\Gamma_H \equiv \frac{1+\epsilon-\alpha}{1+\epsilon}\varsigma \in (0,1)\,, \quad \text{and}$$
$$\Gamma_C \equiv \frac{1+\epsilon-\alpha}{1+\epsilon}(1-\varsigma) + \frac{\alpha\gamma}{1+\epsilon} > 0.$$

Note that the coefficients Γ_H and Γ_C depend only on the parameters $(\alpha,\gamma,\epsilon,\eta)$ and are both strictly positive. Next, we combine (130) with (134) to obtain

$$\Gamma_C\log C\left(s^t\right) = \log A\left(s^t\right) + (\varsigma-\Gamma_H)\log H\left(s^t\right) \tag{135}$$

We thus reach an expression for aggregate GDP (consumption) in terms of $\log A\left(s^t\right)$ and $\log H\left(s^t\right)$.

Derivation of Beauty Contest. What remains to be characterized is the equilibrium behavior of intermediate good purchases $H\left(s^t\right)$. We show that there exists a fixed point in $h\left(\omega_{i,t}\right)$ and $H\left(s^t\right)$ which pins down their joint solution. To do this, we use the optimality condition for intermediate good purchases given in (17). With our specification of preferences, technology, and the proposed tax scheme, this condition may be written as follows:

$$\mathbb{E}\left[U_c(s^t)\left(\left(1-\tau^r(s^t)\right)\frac{\rho-1}{\rho}\left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\eta(1-\alpha)\frac{y\left(\omega_i^t,s^t\right)}{h\left(\omega_i^t\right)}-1\right)\middle|\,\omega_i^t\right]=0$$

where $1-\tau^r(s^t)$ satisfies (119). Next, the log-separability of $y\left(\omega_{it}, s^t\right)$ implies that this condition may be further expressed as

$$\mathbb{E}\left[U_c(s^t)\left(\left(1-\tau^r(s^t)\right)\bar{\chi}Y(s^t)^{\frac{1}{\rho}}\Psi^\omega(\omega_i^t)^{\frac{\rho-1}{\rho}}\Psi^s(s^t)^{\frac{\rho-1}{\rho}}-h\left(\omega_i^t\right)\right)\middle|\,\omega_i^t\right]=0$$

where $\bar{\chi}\equiv\left(\frac{\rho-1}{\rho}\right)\eta(1-\alpha)$. Next, substituting in for $\Psi^\omega(\omega_i^t)$ from (120) gives us

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\left(1-\tau^r(s^t)\right)\bar{\chi}Y(s^t)^{\frac{1}{\rho}}\Psi^s(s^t)^{\frac{\rho-1}{\rho}}\left[A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}-h\left(\omega_i^t\right)\right)\,\middle|\,\omega_i^t\right]=0$$

Solving the above equation for h, we obtain the following equation characterizing the firm's optimal choice of intermediate good purchases

$$h\left(\omega_i^t\right)^{1-\eta(1-\alpha)\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}=\bar{\chi}A\left(\omega_i^t\right)^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}\frac{\mathbb{E}\left[U_c\left(s^t\right)\left(1-\tau^r(s^t)\right)Y(s^t)^{\frac{1}{\rho}}\Psi^s(s^t)^{\frac{\rho-1}{\rho}}\,\middle|\,\omega_i^t\right]}{\mathbb{E}\left[U_c\left(s^t\right)\mid\omega_i^t\right]}$$

We may re-write this in logs as follows:

$$\log h\left(\omega_i^t\right)=\frac{1}{1-\eta(1-\alpha)\zeta\left(\frac{\rho-1}{\rho}\right)}\left\{\begin{array}{c}\zeta\frac{\rho-1}{\rho}\log A\left(\omega_i^t\right)+\frac{1}{\rho}\mathbb{E}_i\log Y(s^t)\\+\frac{\rho-1}{\rho}\mathbb{E}_i\log\Psi^s(s^t)+\mathbb{E}_i\log\left(1-\tau^r(s^t)\right)\end{array}\right\}$$

where we have abstracted from the constant scalar and used $\mathbb{E}_i$ as shorthand for the conditional expectation operator: $\mathbb{E}_i x=\mathbb{E}\left[x|\,\omega_i^t\right]$. Finally, we substitute in for the tax $1-\tau^r(s^t)$ from (119), giving us

$$\log h\left(\omega_i^t\right)=\frac{1}{1-\eta(1-\alpha)\zeta\left(\frac{\rho-1}{\rho}\right)}\left\{\begin{array}{c}\zeta\frac{\rho-1}{\rho}\log A\left(\omega_i^t\right)-\hat{\tau}_A\mathbb{E}_i\log A\left(s^t\right)\\+\left(\frac{1}{\rho}-\hat{\tau}_Y\right)\mathbb{E}_i\log Y(s^t)+\frac{\rho-1}{\rho}\mathbb{E}_i\log\Psi^s(s^t)\end{array}\right\}. \tag{136}$$

Next, using the fact that $\Psi^s(s^t)$ and $\mathcal{B}(s^t)$ simultaneously satisfy equations (128) and (134), we combine these to obtain

$$\log\Psi^s(s^t)=\frac{\alpha\zeta}{(1+\epsilon\zeta)}\left[\frac{1}{\rho}\log Y(s^t)-\gamma\log C(s^t)-\epsilon\frac{\rho-1}{\rho}\zeta\left(\Gamma_C\log C\left(s^t\right)+\Gamma_H\log H\left(s^t\right)\right)\right]. \tag{137}$$

Replacing $\Psi^s(s^t)$ in (136) with (137) gives us the following representation

$$\log h\left(\omega_i^t\right)=G_1\left(\log A\left(\omega_i^t\right),\ \mathbb{E}_i\log A(s^t),\ \mathbb{E}_i\log Y(s^t),\ \mathbb{E}_i\log C(s^t),\ \mathbb{E}_i\log H\left(s^t\right)\right) \tag{138}$$

where G_1 is a linear function of five variables. Next, using the log-linearized resource constraint (133) to replace $Y\left(s^t\right)$, equation (138) may be reduced to

$$\log h\left(\omega_i^t\right)=G_2\left(\log A\left(\omega_i^t\right),\ \mathbb{E}_i\log A(s^t),\ \mathbb{E}_i\log C(s^t),\ \mathbb{E}_i\log H\left(s^t\right)\right) \tag{139}$$

where G_2 is a linear function of four variables. Note that from (135) we may write aggregate consumption as follows:

$$\log C\left(s^t\right)=\Gamma_C^{-1}\log A\left(s^t\right)+\Gamma_C^{-1}\left(\varsigma-\Gamma_H\right)\log H\left(s^t\right). \tag{140}$$

Using this expression to replace $C(s^t)$ in (139) gives us the following result.

Lemma 9. *Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases satisfy the fixed point*

$$\log h\left(\omega_i^t\right) = m_\omega \log A\left(\omega_i^t\right) + m_A\left(\hat{\tau}\right)\mathbb{E}_i \log A(s^t) + m_H\left(\hat{\tau}\right)\mathbb{E}_i \log H(s^t) \tag{141}$$

with $H\left(s^t\right) = \sum h\left(\omega_i^t\right)\varphi\left(\omega|s^t\right)$*, where* m_ω *is a constant given by*

$$m_\omega = \frac{\frac{\rho-1}{\rho}}{1-\left(\alpha+\eta\left(1-\alpha\right)\right)\left(\frac{\rho-1}{\rho}\right)} > 0 \tag{142}$$

and $m_A\left(\hat{\tau}\right)$ *and* $m_H\left(\hat{\tau}\right)$ *are the following linear functions of the tax coefficients* $\hat{\tau} = \left(\hat{\tau}_A, \hat{\tau}_Y\right)$*:*

$$\begin{aligned} m_A\left(\hat{\tau}\right) &= \delta_A + \delta_{AA}\hat{\tau}_A + \delta_{AY}\hat{\tau}_Y, \\ m_H\left(\hat{\tau}\right) &= \delta_H + \delta_{HY}\hat{\tau}_Y. \end{aligned}$$

The coefficients δ_A*,* δ_H*,* δ_{AA}*,* δ_{AY}*, and* δ_{HY} *are scalars that are functions only of the primitive parameters* $(\alpha, \gamma, \epsilon, \eta, \rho)$*.*

$$\delta_A = \frac{\alpha^2\epsilon\eta\left(\rho-1\right)-\alpha\left(\gamma\left(1-\rho\right)+\epsilon+\eta-\epsilon\rho\left(1-\eta\right)\right)-\left(1+\epsilon\right)\left(1-\eta\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)} \tag{143}$$

$$\delta_H = \frac{\left(1-\alpha\right)\eta\left(\alpha^2\left(\gamma+\epsilon\eta\right)\left(\rho-1\right)-\alpha\left(\gamma+\epsilon+\eta-\epsilon\rho\left(1-\eta\right)\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)} \tag{144}$$

$$\delta_{AA} = \frac{-\left(\rho-\alpha\left(\rho-1\right)\right)}{\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)}$$

$$\delta_{AY} = \frac{\left(1+\epsilon\right)\left(1-\eta\left(1-\alpha\right)\right)\left(\rho-\alpha\left(\rho-1\right)\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)}$$

$$\delta_{HY} = \frac{\eta\left(1-\alpha\right)\left(\left(1+\epsilon\right)\left(1-\eta\right)+\alpha\left(\gamma+\left(1+\epsilon\right)\eta\right)\right)\left(\rho-\alpha\left(\rho-1\right)\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)}$$

The fixed-point representation in (141) pins down the flexible-price alllocation $h\left(\omega_i^t\right)$ and $H\left(s^t\right)$ for any Gaussian information structure. Given the linear structure of $m_A\left(\hat{\tau}\right)$ and $m_H\left(\hat{\tau}\right)$ the following corollary is immediate.

Corollary 5. *The tax elasticities* $\left(\hat{\tau}_A, \hat{\tau}_Y\right)$ *form a spanning set of* $\left(m_A\left(\hat{\tau}\right), m_H\left(\hat{\tau}\right)\right)$*.*

Morevoer, note that one may use $\hat{\tau}_Y$ to pin down any value for m_H, and given this, one may use $\hat{\tau}_A$ to pin down any value for m_A.

Fixed Point Solution to Beauty Contest. We now solve the fixed point described in Lemma 9. We take the beauty contest formulation given in (141) and transform it as follows. Let us define $\tilde{h}\left(\omega_i^t\right)$ as follows

$$\log \tilde{h}\left(\omega_i^t\right) \equiv \log h\left(\omega_i^t\right) - m_\omega \log A\left(\omega_i^t\right) \tag{145}$$

Then combining this with (141) implies

$$\log \tilde{h}\left(\omega_i^t\right) = m_A\left(\hat{\tau}\right)\mathbb{E}_i \log A(s^t) + m_H\left(\hat{\tau}\right)\mathbb{E}_i \log H(s^t) \tag{146}$$

Next, aggregating over (145) gives us

$$\log H\left(s^t\right) = \log \tilde{H}\left(s^t\right) + m_\omega \log A(s^t) \tag{147}$$

Finally, substituting the above expression into (146) we get

$$\log \tilde{h}\left(\omega_i^t\right) = \left(m_A\left(\hat{\tau}\right) + m_H\left(\hat{\tau}\right) m_\omega\right) \mathbb{E}_i \log A(s^t) + m_H\left(\hat{\tau}\right) \mathbb{E}_i \log \tilde{H}\left(s^t\right)$$

From this formulation the following result is immediate.

Lemma 10. *Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases satisfy the fixed point*

$$\log \tilde{h}\left(\omega_i^t\right) = \left(1 - \tilde{\alpha}\right) \tilde{\chi} \mathbb{E}_i \log A(s^t) + \tilde{\alpha} \mathbb{E}_i \log \tilde{H}\left(s^t\right) \tag{148}$$

with $\tilde{H}\left(s^t\right) = \sum \tilde{h}\left(\omega_i^t\right) \varphi\left(\omega | s^t\right)$ *and*

$$\tilde{\alpha} = m_H\left(\hat{\tau}\right) \qquad \text{and} \qquad \tilde{\chi} \equiv \frac{m_A\left(\hat{\tau}\right) + m_H\left(\hat{\tau}\right) m_\omega}{1 - m_H\left(\hat{\tau}\right)} \tag{149}$$

Morevoer, any pair $(\tilde{\alpha}, \tilde{\chi}) \in \mathbb{R}^2$ *can be attained by an appropriate choice of the pair* $(\hat{\tau}_A, \hat{\tau}_Y)$

Proof of Lemma 10. Equation (148) follows from the above analysis. As for the last claim in Lemma 10, the proof is straightforward. For any $(\tilde{\alpha}, \tilde{\chi}) \in \mathbb{R}^2$, choose $m_H = \tilde{\alpha}$ and $m_A = \tilde{\chi}\left(1 - m_H\right) - m_H m_\omega$. This is the pair (m_A, m_H) that attains $(\tilde{\alpha}, \tilde{\chi})$ given (149). Next recall that that for any pair $\left(m_A\left(\hat{\tau}\right), m_H\left(\hat{\tau}\right)\right) \in \mathbb{R}^2$ there exists a pair $(\hat{\tau}_A, \hat{\tau}_Y)$ that implements these coefficients. Therefore, there exists a pair $(\hat{\tau}_A, \hat{\tau}_Y)$ that attains $(\tilde{\alpha}, \tilde{\chi})$. QED.

Although any value of $\tilde{\alpha} \in \mathbb{R}$ can be achieved with appropriate tax instruments, from now on we restrict attention to $\tilde{\alpha} \in (-\infty, 1)$ so as to ensure a unique equlibrium. Equivalently, $m_H\left(\hat{\tau}\right) < 1$. With this qualification, next we note that the game in (148) is the same as in Bergemann Morris (200?) and hence can be spanned by a private and public signal. Thus suppose the agent gets two Gaussian signals, a private and public signal, call these (x, z) with mean zero and precisions (κ_x, κ_z). Then the solution to this system is given by

Lemma 11. *Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases are given by*

$$\log \tilde{h}\left(x, z\right) = \phi_0 + \phi_x x + \phi_z z \tag{150}$$

where

$$\phi_x = \frac{\left(1 - \tilde{\alpha}\right) \kappa_x}{\kappa_0 + \left(1 - \tilde{\alpha}\right) \kappa_x + \kappa_z} \tilde{\chi} \tag{151}$$

$$\phi_z = \frac{\kappa_z}{\kappa_0 + \left(1 - \tilde{\alpha}\right) \kappa_x + \kappa_z} \tilde{\chi} \tag{152}$$

Let $r_\phi \equiv \phi_z / \phi_x$ *be the ratio of these coefficients, so that* $\phi_z = r_\phi \phi_x$. *Any pair* $(\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+$ *can be attained by an appropriate choice of the pair* $(\hat{\tau}_A, \hat{\tau}_Y)$.

Proof of Lemma 11. Choose any pair $(\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+$. First, note that

$$r_\phi = \frac{\phi_z}{\phi_x} = \frac{1}{(1-\tilde{\alpha})} \frac{\kappa_z}{\kappa_x} \tag{153}$$

One may choose any $\tilde{\alpha}$ to satisfy (153). However, recall there is an upper bound on $\tilde{\alpha} \in (-\infty, 1)$. This imposes certain bounds on the ratio r_ϕ as follows.

$$\lim_{\alpha \to -\infty} r_\phi = 0 \quad \text{and} \quad \lim_{\alpha \to 1} r_\phi = \infty$$

Therefore the ratio r_ϕ must be weakly positive. Next given the $\tilde{\alpha}$ that satisfies (153), one need only choose the $\tilde{\chi}$ that implements ϕ_x in equation (151). Finally, recall that from Lemma 10 we know that any pair $(\tilde{\alpha}, \tilde{\chi})$ can be attained by an appropriate choice of the pair $(\hat{\tau}_A, \hat{\tau}_Y)$. This implies that for any pair $(\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+$ can be attained by an appropriate choice of $(\hat{\tau}_A, \hat{\tau}_Y)$. QED.

This implies that the ratio between ϕ_x and ϕ_z must be weakly positive. This is intuitive: if actions are increasing in the fundamental under complete information, then also under incomplete information agents will put a positive weight on both the private and public signal; conversely if actions are decreasing in the fundamental under complete information, then under incomplete information agents will put a negative weight on both the private and public signal. Thus in either case the pair ϕ_x, ϕ_z are of the same sign.

Equilibrium Aggregate Intermediated Good Purchases and Consumption (GDP). Next we compute aggregate intermediate good purchases. Equation (150) in Lemma 11 implies that the aggregate intermediate good purchases satisfies

$$\log \tilde{H}\left(s^t\right) = \phi_0 + \left(\phi_x + \phi_z\right) a_t + \phi_z u_t$$

We may transform this back into the true $H\left(s^t\right)$ from (147) as follows

$$\begin{aligned} \log H\left(s^t\right) &= \log \tilde{H}\left(s^t\right) + m_\omega \log A(s^t) \\ &= \phi_0 + \left(\phi_x + \phi_z\right) a_t + \phi_z u_t + m_\omega a_t \end{aligned}$$

We thus obtain the following result.

$$\log H\left(s^t\right) = \left(\phi_x + \phi_z + m_\omega\right) \log A\left(s^t\right) + \phi_z u_t + const \tag{154}$$

This is the solution to the original beauty contest game in (141). Equation (154) characterizes the equilibrium behavior of intermediate good purchases $H\left(s^t\right)$ as a function of the aggregate productivity shock and the common noise u_t.

Equilibrium Aggregate Consumption (GDP). Finally, we compute aggregate consumption (GDP). Using the expression in (154) to replace $H\left(s^t\right)$ in equation (140) gives us

$$\log C\left(s^t\right) = \Gamma_C^{-1} \log A\left(s^t\right) + \Gamma_C^{-1} \varsigma \left(\frac{\alpha}{1+\epsilon}\right) \left(\left(\phi_x + \phi_z + m_\omega\right) \log A\left(s^t\right) + \phi_z u_t\right)$$

where we have used the fact that $\varsigma - \Gamma_H = \varsigma\left(\frac{\alpha}{1+\epsilon}\right)$. Therefore

$$\log C\left(s^t\right) = \Gamma_C^{-1}\left(1 + \varsigma\frac{\alpha}{1+\epsilon}\left(\phi_x + \phi_z + m_\omega\right)\right)\log A\left(s^t\right) + \Gamma_C^{-1}\varsigma\frac{\alpha}{1+\epsilon}\phi_z u_t + const$$

We thus obtain the following characterization of aggregate consumption:

$$\log C\left(s^t\right) = \log GDP\left(s^t\right) = \gamma_0 + \gamma_a \log A\left(s^t\right) + \gamma_u u_t$$

where $(\gamma_0, \gamma_A, \gamma_u)$ are constants. The coefficients (γ_a, γ_u) satisfy

$$\gamma_a = \hat{\gamma} + \upsilon\left(\phi_x + \phi_z\right), \text{ and} \tag{155}$$

$$\gamma_u = \upsilon\phi_z \tag{156}$$

where $(\hat{\gamma}, \upsilon)$ are strictly positive scalars given by

$$\hat{\gamma} = \Gamma_C^{-1}\left(1 + \varsigma\frac{\alpha}{1+\epsilon}m_\omega\right) > 0 \quad \text{and} \quad \upsilon = \Gamma_C^{-1}\varsigma\frac{\alpha}{1+\epsilon} > 0. \tag{157}$$

We have thus derived equation (108) in Proposition 10. What remains to be derived are the values of (γ_a, γ_u) that may be spanned with the appropriate tax instruments. To do so, we use $\phi_z = r_\phi\phi_x$ to rewrite (155) and (156) as follows

$$\gamma_a = \hat{\gamma} + \upsilon\left(1 + r_\phi\right)\phi_x \quad \text{and} \quad \gamma_u = \upsilon r_\phi \phi_x \tag{158}$$

This implies that for any γ_u, the following relation must hold

$$\phi_x = \frac{\gamma_u}{\upsilon r_\phi}$$

where $\upsilon r_\phi > 0$. This implies that γ_u, ϕ_x, ϕ_z must all have the same sign. Plugging this into (158) gives us

$$\gamma_a = \hat{\gamma} + \upsilon\left(1 + r_\phi\right)\frac{\gamma_u}{\upsilon r_\phi} = \hat{\gamma} + \left(1 + \frac{1}{r_\phi}\right)\gamma_u$$

Recall that r_ϕ can take any positive number. Therefore the pair (γ_A, γ_u) may take *any* value in the set Υ defined in (109). **QED.**

Proof of Proposition 11. For any realization of $\left(\omega_i^t, s^t\right)$, at the Ramsey Optimum the following two equations must hold:

$$\frac{-\tilde{U}_\ell(s^t)}{\tilde{U}_c(s^t)} = \left(\frac{y\left(\omega_i^t, s^t\right)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}} \alpha\frac{y\left(\omega_i^t, s^t\right)}{\ell\left(\omega_i^t, s^t\right)} \tag{159}$$

$$y\left(\omega_i^t, s^t\right) = A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)} \ell\left(\omega_i^t, s^t\right)^\alpha \tag{160}$$

The first is the labor-optimality condition of the Ramsey planner and the second is the production function. Note that the only main difference between (159) and the corresponding labor-optimality condition for the flexible price equilibrium, (113), is that (159) holds specifically at the Ramsey optimum. Thus in (159), $-\tilde{U}_\ell(s^t)/\tilde{U}_c\left(s^t\right)$ is the Ramsey planner's marginal rate of substitution between consumption and labor and there are no tax wedges.

However, recall that with homothetic preferences, the function $\tilde{U}\left(s^t\right)$ is given by (106). We thereby replace (159) with the following equation:

$$-\left(\frac{1+\Gamma\left(1+\epsilon\right)}{1+\Gamma\left(1-\gamma\right)}\right)\frac{U_\ell\left(s^t\right)}{U_c\left(s^t\right)}=\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\alpha\frac{y\left(\omega_i^t,s^t\right)}{\ell\left(\omega_i^t,s^t\right)} \tag{161}$$

Following the proof of Proposition 10, we can solve (160) and (161) simultaneously for $y\left(\omega_i^t,s^t\right)$ and $\ell\left(\omega_i^t,s^t\right)$. We find that output at the Ramsey optimum must satisfy

$$y\left(\omega_i^t,s^t\right)=\left[\frac{U_c\left(s^t\right)}{-U_\ell\left(s^t\right)}Y(s^t)^{\frac{1}{\rho}}\left(A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$$

where we have abstracted from the constant scalar $\left(\alpha\frac{1+\Gamma(1-\gamma)}{1+\Gamma(1+\epsilon)}\right)^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$. Thus, output $y\left(\omega_i^t,s^t\right)$ and labor $\ell\left(\omega_i^t,s^t\right)$ are log-separable in ω_i^t and s^t and satisfy

$$y\left(\omega_{it},s^t\right) = \Psi^\omega(\omega_i^t)\Psi^s(s^t) \tag{162}$$

$$\ell\left(\omega_i^t,s^t\right) = \Psi^\omega(\omega_i^t)^{\frac{\rho-1}{\rho}}\Psi^s(s^t)^{\frac{1}{\alpha}} \tag{163}$$

with

$$\Psi^\omega(\omega_i^t) = \left[A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{164}$$

$$\Psi^s(s^t) = \left[\frac{U_c\left(s^t\right)}{-U_\ell\left(s^t\right)}Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{165}$$

Comparing (164) and (165) to the corresponding equations for Ψ^ω and Ψ^s in the flexible price allocation with the proposed tax scheme, (120) and (121), it is clear that these are identical up to a scalar multiple. This implies that we may write aggregate output as (122) and aggregate labor as in (123). Following the exact same steps as in the proof of Proposition 10, we may describe the Ramsey optimum with equations (133) for the resource constraint, (134) for aggregate sentiment, and (135) for aggregate consumption. We reproduce equation (140) here:

$$\log C\left(s^t\right)=\Gamma_C^{-1}\log A\left(s^t\right)+\Gamma_C^{-1}\left(\varsigma-\Gamma_H\right)\log H\left(s^t\right). \tag{166}$$

We thus reach the same expression for aggregate GDP (consumption) in terms of $\log A\left(s^t\right)$ and $\log H\left(s^t\right)$, abstracting from all constants.

Derivation of Planner's Beauty Contest. What remains to be characterized is the optimal behavior of intermediate good purchases $H\left(s^t\right)$. As in the proof for the flexible price allocaiton, we show that there exists a fixed point in $h\left(\omega_{i,t}\right)$ and $H\left(s^t\right)$ which pins down their joint solution for the Ramsey optimum. To do this, we use the optimality condition for intermediate good purchases given by (35).

With our specification of preferences and technology, this optimality condition may be written as follows:

$$\mathbb{E}\left[\tilde{U}_c\left(s^t\right)\left(\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\eta\left(1-\alpha\right)\frac{y\left(\omega_i^t,s^t\right)}{h\left(\omega_i^t\right)}-1\right)\,\middle|\,\omega_i^t\right]=0. \tag{167}$$

Recall that with homothetic prefences, the function $\tilde{U}\left(s^t\right)$ satisfies (106). We thereby rewrite equation (167) as follows:

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\eta\left(1-\alpha\right)Y(s^t)^{\frac{1}{\rho}}y\left(\omega_i^t,s^t\right)^{\frac{\rho-1}{\rho}}-h\left(\omega_i^t\right)\right)\,\middle|\,\omega_i^t\right]=0.$$

The log-separability of $y\left(\omega_{it},s^t\right)$ implies that this condition may be further expressed as

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\eta\left(1-\alpha\right)Y(s^t)^{\frac{1}{\rho}}\Psi^{\omega}(\omega_i^t)^{\frac{\rho-1}{\rho}}\Psi^{s}(s^t)^{\frac{\rho-1}{\rho}}-h\left(\omega_i^t\right)\right)\,\middle|\,\omega_i^t\right]=0.$$

Next, plugging in the definition of $\Psi^{\omega}(\omega_i^t)$ from (164) gives us

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\eta\left(1-\alpha\right)Y(s^t)^{\frac{1}{\rho}}\Psi^{s}(s^t)^{\frac{\rho-1}{\rho}}\left[A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}-h\left(\omega_i^t\right)\right)\,\middle|\,\omega_i^t\right]=0$$

Solving the above equation for h, we obtain the following equation characterizing the firm's optimal choice of intermediate good purchases

$$h\left(\omega_i^t\right)^{1-\eta(1-\alpha)\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}=\eta\left(1-\alpha\right)A\left(\omega_i^t\right)^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}\frac{\mathbb{E}\left[U_c\left(s^t\right)Y(s^t)^{\frac{1}{\rho}}\Psi^{s}(s^t)^{\frac{\rho-1}{\rho}}\,\middle|\,\omega_i^t\right]}{\mathbb{E}\left[U_c\left(s^t\right)\mid\omega_i^t\right]} \tag{168}$$

We may re-write this in logs as follows:

$$\log h\left(\omega_i^t\right)=\frac{1}{1-\eta\left(1-\alpha\right)\zeta\left(\frac{\rho-1}{\rho}\right)}\left[\zeta\frac{\rho-1}{\rho}\log A\left(\omega_i^t\right)+\frac{1}{\rho}\mathbb{E}_i\log Y(s^t)+\frac{\rho-1}{\rho}\mathbb{E}_i\log\Psi^{s}(s^t)\right] \tag{169}$$

where we have abstracted from the constant scalar and again used $\mathbb{E}_i$ as shorthand for the conditional expectation operator: $\mathbb{E}_i x=\mathbb{E}\left[x|\ \omega_i^t\right]$.

We use (137) to replace $\Psi^{s}(s^t)$ in (169), as the former holds true also in the Ramsey optimal allocation (with different constants). This gives us the following representation

$$\log h\left(\omega_i^t\right)=G_1^*\left(\log A\left(\omega_i^t\right),\ \mathbb{E}_i\log Y(s^t),\ \mathbb{E}_i\log C(s^t),\ \mathbb{E}_i\log H\left(s^t\right)\right) \tag{170}$$

where G_1^* is a linear function of four variables. Next, using the log-linearized resource constraint (133) to replace $Y\left(s^t\right)$, equation (170) may be reduced to

$$\log h\left(\omega_i^t\right)=G_2^*\left(\log A\left(\omega_i^t\right),\ \mathbb{E}_i\log C(s^t),\ \mathbb{E}_i\log H\left(s^t\right)\right) \tag{171}$$

where G_2^* is a linear function of three variables. Finally, using (166) to replace $C(s^t)$ in (171) yields the following result.

Lemma 12. *The Ramsey optimal level of intermediate good purchases satisfy the fixed point*

$$\log h\left(\omega_i^t\right) = m_\omega \log A\left(\omega_i^t\right) + m_A^* \mathbb{E}_i \log A(s^t) + m_H^* \mathbb{E}_i \log H(s^t) \tag{172}$$

with $H\left(s^t\right) = \sum h\left(\omega_i^t\right) \varphi\left(\omega|s^t\right)$*, where* $m_\omega > 0$ *is as defined in (142) and the coefficients* (m_A^*, m_H^*) *are scalars given by*

$$m_A^* = m_A(0) = \delta_A, \text{ and } \; m_H^* = m_H(0) = \delta_H,$$

with δ_A, δ_H *as defined in (143) and (144).*

The fixed-point representation in (172) pins down the Ramsey optimal $h\left(\omega_i^t\right)$ and $H\left(s^t\right)$ for any information structure. Note that this is the same fixed-point representation as in (141) of Lemma 9, but with the tax instruments set at $\hat{\tau}_A = 0$ and $\hat{\tau}_Y = 0$.

Fixed Point Solution to Beauty Contest. We now solve the fixed point described in Lemma 12. Following the exact same steps as in the previous derivation of Lemma 10, we may take the beauty contest formulation given in (172) and transform it as in (145). We thus reach the following result

Lemma 13. *Suppose managers have Gaussian information about the aggregate state. Then the optimal level of intermediate good purchases satisfy the fixed point*

$$\log \tilde{h}\left(\omega_i^t\right) = (1-\alpha^*)\,\tilde{\chi}^* \mathbb{E}_i \log A(s^t) + \alpha^* \mathbb{E}_i \log \tilde{H}\left(s^t\right) \tag{173}$$

with $\tilde{H}\left(s^t\right) = \sum \tilde{h}\left(\omega_i^t\right) \varphi\left(\omega|s^t\right)$ *and*

$$\alpha^* = m_H^* \qquad \text{and} \qquad \tilde{\chi}^* \equiv \frac{m_A^* + m_H^* m_\omega}{1 - m_H^*} \tag{174}$$

Morevoer, any pair $(\tilde{\alpha}, \tilde{\chi}) \in \mathbb{R}^2$ *can be attained by an appropriate choice of the pair* $(\hat{\tau}_A, \hat{\tau}_Y)$

Without serious loss of generality, we henceforth impose that $\tilde{\chi}^* > 0$, which simply means that the optimal Ht comoves positively with A_t in the frictionless benchmark. Given the above characterization and the previous analysis that followed Lemma 10, it is immediate that the solution to the fixed point described in Lemma 13 is given by

$$\log \tilde{h}^*(x,z) = \phi_0^* + \phi_x^* x + \phi_z^* z \tag{175}$$

where

$$\phi_x^* = \frac{(1-\alpha^*)\,\kappa_x}{\kappa_0 + (1-\alpha^*)\,\kappa_x + \kappa_z}\tilde{\chi}^* \tag{176}$$

$$\phi_z^* = \frac{\kappa_z}{\kappa_0 + (1-\alpha^*)\,\kappa_x + \kappa_z}\tilde{\chi}^* \tag{177}$$

Equation (175) thus gives the optimal level of intermediate good purchases. Furthermore, aggregating over (175) and again transforming back into the true $H\left(s^t\right)$ using (147), the optimal level of aggregate intermediate good purchases satisfies

$$\log H\left(s^t\right) = (\phi_x^* + \phi_z^* + m_\omega) \log A\left(s^t\right) + \phi_z^* u_t + const \tag{178}$$

Equation (178) characterizes the optimal behavior of intermediate good purchases $H\left(s^t\right)$ as a function of the aggregate productivity shock and the common noise u_t.

Finally, we compute optimal aggregate consumption (GDP). Using the expression in (178) to replace $H\left(s^t\right)$ in equation (166) gives us the following characterization for aggregate consumption:

$$\log C\left(s^t\right) = \log GDP\left(s^t\right) = \gamma_0^* + \gamma_a^* \log A\left(s^t\right) + \gamma_u^* u_t \tag{179}$$

where $(\gamma_0^*, \gamma_A^*, \gamma_u^*)$ are constants. The coefficients (γ_a^*, γ_u^*) satisfy

$$\gamma_a^* = \hat{\gamma} + \upsilon\left(\phi_x^* + \phi_z^*\right) \quad \text{and} \quad \gamma_u^* = \upsilon\phi_z^*$$

where $(\hat{\gamma}, \upsilon)$ are strictly positive scalars as defined in (157).

Finally, what remains to be shown is $0 < \gamma_A^* < \gamma_A^{LS}$ and $\gamma_u^* > 0$, where γ_A^{LS} is the coefficient on aggregate productivity in the complete-information Ramsey optimum. First note that

$$\gamma_a^* = \hat{\gamma} + \upsilon\left(\frac{(1-\alpha^*)\,\kappa_x + \kappa_z}{\kappa_0 + (1-\alpha^*)\,\kappa_x + \kappa_z}\right)\tilde{\chi}^* \tag{180}$$

with $\hat{\gamma}, \upsilon > 0$. Thus $\tilde{\chi}^* > 0$ is sufficient for $\gamma_a^* > 0$ and $\gamma_u^* > 0$. Now, to compare γ_A^* to γ_A^{LS} we finally solve for the complete information optimum and offer the proof of Lemma 8 as promised previously.

Proof of Lemma 8. The optimal allocation under complete information is the same allocation as in (178) and (179), except with $\kappa_x \to \infty$. In this limit,

$$\phi_x^* + \phi_z^* \to \tilde{\chi}^* \quad \text{and} \quad \phi_z^* \to 0$$

Therefore at the complete information optimum,

$$\begin{aligned} \log H^{LS}\left(s^t\right) &= \phi_A^{LS} \log A\left(s^t\right) + const \\ \log C^{LS}\left(s^t\right) &= \gamma_A^{LS} \log A\left(s^t\right) + const \end{aligned}$$

as in (131) and (132), where ϕ_A^{LS} and γ_A^{LS} are scalar parameters given by

$$\phi_A^{LS} = \tilde{\chi}^* + m_\omega \quad \text{and} \quad \gamma_A^{LS} = \hat{\gamma} + \upsilon\tilde{\chi}^* \tag{181}$$

QED.

We now take the difference between γ_A^{LS} and γ_A^*; using the expressions in (180) and (181), this difference is given by

$$\gamma_A^{LS} - \gamma_a^* = \upsilon\tilde{\chi}^* - \upsilon\left(\frac{(1-\alpha^*)\,\kappa_x + \kappa_z}{\kappa_0 + (1-\alpha^*)\,\kappa_x + \kappa_z}\right)\tilde{\chi}^*$$

which implies

$$\gamma_A^{LS} - \gamma_a^* = \upsilon\left[\frac{\kappa_0}{\kappa_0 + (1-\alpha^*)\,\kappa_x + \kappa_z}\right]\tilde{\chi}^*$$

Therefore $\tilde{\chi}^* > 0$ is sufficient for $\gamma_A^{LS} - \gamma_a^* > 0$, and as a result, $0 < \gamma_A^* < \gamma_A^{LS}$. **QED.**

Proof of Proposition 12. Following the proof of Theorem 2, for any arbitrary common-knowledge process z_t, the optimal aggregate price level is given by

$$P(s^t) = e^{z_t}\mathcal{B}(s^t)^{-\frac{1}{\rho}}$$

Taking logs and combining this with expression (134) for $\mathcal{B}(s^t)$, we may express the optimal aggregate price level as

$$\log P(s^t) = -\frac{1}{\rho}\log\mathcal{B}(s^t) = -\frac{1}{\rho}\zeta\left(\Gamma_C\log C\left(s^t\right) + \Gamma_H\log H\left(s^t\right)\right)$$

where we abstract from the common-knowledge process z_t. Next, by substitution of $H\left(s^t\right)$ and $C\left(s^t\right)$ from (178) and (179), we may express the aggregate price level as a log-linear function of A_t and u_t as follows

$$\log P(s^t) = -\frac{1}{\rho}\zeta\left\{\left(\Gamma_C\gamma_a^* + \Gamma_H\left(\phi_x^* + \phi_z^* + m_\omega\right)\right)\log A\left(s^t\right) + \left(\Gamma_C\gamma_u^* + \Gamma_H\phi_z^*\right)u_t\right\}$$

This yields the following expression for the aggregate price level at the Ramsey optimum:

$$\log P(s^t) = -\delta_A^*\log A\left(s^t\right) - \delta_u^* u_t + const$$

as in (112) where δ_A^* and δ_u^* are constants given by

$$\delta_A^* \equiv \frac{1}{\rho}\zeta\left(\Gamma_C\gamma_a^* + \Gamma_H\left(\phi_x^* + \phi_z^* + m_\omega\right)\right) \text{ and } \delta_u^* \equiv \frac{1}{\rho}\zeta\left(\Gamma_C\gamma_u^* + \Gamma_H\phi_z^*\right).$$

Finally, note that

$$\frac{\delta_A^*}{\gamma_a^*} = \frac{1}{\rho}\zeta\left[\Gamma_C + \Gamma_H\left(\frac{\phi_x^* + \phi_z^* + m_\omega}{\gamma_a^*}\right)\right] = \frac{1}{\rho}\zeta\left[\Gamma_C + \Gamma_H\left(\frac{\phi_x^* + \phi_z^* + m_\omega}{\hat{\gamma} + \upsilon\left(\phi_x^* + \phi_z^*\right)}\right)\right] > 0$$

and

$$\frac{\delta_u^*}{\gamma_u^*} = \frac{1}{\rho}\zeta\left(\Gamma_C + \Gamma_H\frac{\phi_z^*}{\gamma_u^*}\right) = \frac{1}{\rho}\zeta\left(\Gamma_C + \Gamma_H\frac{1}{\upsilon}\right) > 0$$

Therefore, the ratios δ_A^*/γ_u^* and δ_u^*/γ_u^* are strictly positive. This, along with $\gamma_a^* > 0$ and $\gamma_u^* > 0$, imply that δ_A^* and δ_u^* are strictly positive. **QED.**