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ABSTRACT

While the traditional view of financial innovation emphasizes the risk sharing role of new financial assets, belief disagreements about these assets naturally lead to speculation, which represents a powerful economic force in the opposite direction. This paper investigates the effect of financial innovation on portfolio risks in an economy when both the risk sharing and the speculation forces are present. I consider this question in a standard mean-variance framework. Financial assets provide hedging services but they are also subject to speculation because traders do not necessarily agree about their payoffs. I define the average variance of traders' net worths as a measure of portfolio risks for this economy, and I decompose it into two components: the uninsurable variance, defined as the average variance that would obtain if there were no belief disagreements, and the speculative variance, defined as the residual variance that results from speculative trades based on belief disagreements. Financial innovation always decreases the uninsurable variance because new assets increase the possibilities for risk sharing. My main result shows that financial innovation also always increases the speculative variance. This is true even if traders completely agree about the payoffs of new assets. The intuition behind this result is the hedge-more/bet-more effect: Traders use new assets to hedge their bets on existing assets, which in turn enables them to place larger bets and take on greater risks.

The net effect of financial innovation on portfolio risks depends on the quantitative strength of its effects on the uninsurable and the speculative variances. I consider a calibration of the model for new assets linked to national incomes of G7 countries, which were recommended by Athanasoulis and Shiller (2001) to facilitate risk sharing. For reasonable levels of belief disagreements, these assets would actually increase the average consumption risks of individuals in G7 countries. In addition, a profit seeking market maker would introduce a different subset of these assets than the ones proposed by Athanasoulis and Shiller (2001). The endogenous set of new assets would be directed towards increasing the opportunities for speculation rather than risk sharing.

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1 Introduction

According to the traditional view of financial innovation, new financial assets facilitate the diversification and the sharing of risks.[1] However, this view does not take into account that new assets are often associated with much uncertainty, especially because they do not have a long track record. Belief disagreements come as a natural by-product of this uncertainty and change the implications of risk taking in these markets. In particular, market participants' disagreements about how to value new assets naturally lead to speculation, which represents a powerful economic force that tends to increase risks.

An example is offered by the recent crisis. Assets backed by pools of subprime mortgages (e.g., subprime CDOs) became highly popular in the run-up to the crisis. One role of these assets is to allocate the risks to market participants who are best able to bear them. The safer tranches are held by investors that are looking for safety (or liquidity), while the riskier tranches are held by financial institutions who are willing to hold these risks at some price. While these assets (and their CDSs) should have served a stabilizing role in theory, they became a major trigger of the crisis in practice, when a fraction of financial institutions realized losses from their positions. Importantly, the same set of assets also generated considerable profits for some market participants,[2] which suggests that at least some of the trades on these assets were speculative. What becomes of the risk sharing role of new assets when market participants use them to speculate on their different views?

To address this question, this paper analyzes the effect of financial innovation on portfolio risks in a model that features both the risk sharing and the speculation motives for trade. Traders with income risks take positions in a set of financial assets, which enables them to share and diversify some of their background risks. However, traders have belief disagreements about asset payoffs, which induces them to take also speculative positions on assets. I assume traders have mean-variance preferences over net worth. In this setting, a natural measure of portfolio risk for a trader is the variance of her net worth (calculated according to her own beliefs). I define *the average variance* as an average of this risk measure across all traders. I further decompose the average variance into two components: *the uninsurable variance*, defined as the variance that would obtain if there were no belief disagreements, and *the speculative variance*, defined as the residual amount of variance that results from speculative trades based on belief disagreements. I model financial innovation as an expansion of the set of assets available for trade. My main result characterizes the effect of financial innovation on each component of the average variance. In line with the traditional view, financial innovation always decreases the uninsurable variance because new assets increase the possibilities for risk sharing. Theorem 1 shows that financial innovation also always increases the speculative variance. Moreover,

[1] Cochrane (2001) summarizes this view as follows: "Better risk sharing is much of the force behind financial innovation. Many successful new securities can be understood as devices to more widely share risks."

[2] Lewis (2010) provides a detailed description of investors that took a short position on housing related assets in the run-up to the recent crisis.

there exist economies in which this increase in the speculative variance is sufficiently strong that financial innovation increases the average variance (by an arbitrary amount). This result formalizes the sense in which financial innovation can increase portfolio risks.

My analysis identifies two main channels by which financial innovation increases the speculative variance. First, new assets lead to new disagreements because they are associated with new uncertainties. Second, and perhaps more importantly, new assets also amplify speculation on existing disagreements. To illustrate the second channel, Theorem 1 shows that new assets increase the speculative variance even if traders completely agree about their payoffs. This result is somewhat surprising because traders use new assets to hedge some of the speculative risks they have been undertaking from their bets on existing assets. In view of this direct hedging effect, one could expect new assets (on which there is complete agreement) to reduce the speculative variance. This view does not take into account a powerful amplification mechanism, *the hedge-more/bet-more* effect.

To illustrate this effect, consider the following example. Suppose two traders have differing views about the Swiss Franc, which is highly correlated with the Euro. The optimist believes the Franc will appreciate while the pessimist believes it will depreciate. Traders do not disagree about the Euro, perhaps because they disagree about the prospects of the Swiss economy but not about the Euro zone. First suppose traders can only take positions on the Franc and not the Euro. In this case, traders' positions in the Franc will be determined by a standard risk-return trade-off. Traders may not take too large positions on the Franc especially because the Franc is affected by multiple sources of risks, e.g., the shocks that affect the Swiss economy as well as the shocks to the Euro zone. To bet on their belief differences, traders must bear all of these risks, which might make them reluctant to take large positions. Suppose instead the Euro is also introduced for trade (which can be interpreted as "financial innovation" in this example). In this case, traders will complement their positions in the Franc by taking the opposite positions in the Euro. This is because the complementary positions enable traders to hedge the risks that also affect the Euro, which they don't disagree about, and to take purer bets on the Franc. With purer bets, traders bear less risk for each unit position on the Franc, which in turn enables them to take larger positions. Put differently, when traders are able to hedge more, they are induced to bet more. Theorem 1 shows that the hedge-more/bet-more effect is sufficiently strong that the introduction of the Euro in this example (and more generally, a new asset) increases the speculative variance.

My main result, Theorem 1, takes the new assets as exogenous and analyzes their impact on portfolio risks. In practice, new financial assets are endogenously introduced by economic agents with profit incentives. A sizeable literature emphasizes risk sharing as a major driving force in endogenous financial innovation [see, for example, Allen and Gale (1994) or Duffie and Rahi (1995)]. A natural question is to what extent the risk sharing motive for financial innovation is robust to the presence of belief disagreements. I address this question by introducing a profit seeking market maker that innovates new assets for which it subsequently

serves as the intermediary. The market maker's expected profits are proportional to traders' perceived surplus from trading new assets. Thus, traders' speculative trading motive, as well as their risk sharing motive, creates innovation incentives for the market maker. In particular, the optimal asset design (characterized in Theorem 2) depends on the size and the nature of belief disagreements, in addition to the risk sharing possibilities. When traders have common beliefs, the market maker innovates assets that minimize the average variance, as in Demange and Laroque (1995) and Athanasoulis and Shiller (2000). In contrast to these traditional results, Theorem 3 also characterizes the polar opposite case: When traders' belief disagreements are sufficiently large, the market maker innovates assets which *maximize* the average variance among all possible choices, completely disregarding the risk sharing motive for financial innovation.

These results show that belief disagreements, when they are sufficiently large, change the nature of financial innovation as well as its effect on portfolio risks. A natural question is how large belief disagreements must be to make these results practically relevant. To address this question, I consider a calibration of the model in the context of the national income markets proposed by Shiller (1993), and analyzed in detail by Athanasoulis and Shiller (2001). Assets whose payoffs are linked to (various combinations of) national incomes could in principle facilitate the sharing of income risks among different countries. Athanasoulis and Shiller (2001) characterize the optimal design of such assets. They also calibrate their model for G7 countries, and argue that the innovation of a couple of these assets would lead to large welfare gains in view of the reduction in individuals' consumption risks. I consider the effect of belief disagreements on their results about consumption risks. Using exactly their data and calibration, I find that reasonable amounts of belief disagreements imply that the new assets proposed by Athanasoulis and Shiller (2001) would actually increase the average consumption variance of individuals in G7 countries.

To illustrate this result, consider the following as a measure of belief disagreements on a random variable, X:

$$\delta^X = \frac{\text{cross-sectional standard deviation of (prior) beliefs for the mean of } X}{\text{standard deviation of } X}.$$

The measure, δ^X (which is independent of linear transformations of X), captures how dispersed individuals' prior beliefs are relative to the volatility of X. I show that if δ on the yearly per-capita income growth of a G7 country is at least 2%, then the new assets proposed by Athanasoulis and Shiller (2001) increase individuals' average consumption variance. Athanasoulis and Shiller (2001) estimate the standard deviation of yearly per-capita income growth to be 2.46% for G7 countries over the years they consider. Given this estimate, my result holds if two randomly chosen individuals' beliefs for the mean of the per-capita income growth of a G7 country differ on average by about 0.05%. Disagreements at this order of magnitude do not seem unreasonable. According to the Philadelphia Fed's Survey of Profes-

sional Forecasters, the interquartile range of forecasters' beliefs for the US yearly GDP growth averaged 0.70% between 1992 and 2011. Over the same period, the standard deviation of the US yearly GDP growth has been 2.08%. This suggests $\delta = 25\%$, which is an order of magnitude larger than necessary to overturn the risk sharing implications of Athanasoulis and Shiller (2001).

The intuition for the calibration result comes from the fact that the per-capita income risks in developed countries is small relative to their per-capita incomes. Moreover, income risks are correlated across developed countries. Thus, even if these risks are perfectly diversified, the reduction in the standard deviation of consumption amounts to a relatively small fraction of income. According to Athanasoulis and Shiller's (2001) estimates, completing the international risk sharing markets reduces the standard deviation of per-capita consumption growth in a G7 country from 2.46% to 2.13%. In terms of my variance decomposition, this implies that the reduction in uninsurable risks is small relative to average income. In contrast, with a typical calibration for the relative risk aversion parameter, $\theta^{relative} = 3$, individuals are willing to risk a greater fraction of their incomes in their pursuit for speculative gains. In particular, belief disagreements at the order of $\delta = 2\%$ are sufficient to ensure that the increase in the speculative variance dominates the relatively small decrease in uninsurable variance.

This calibration result concerns the new assets which were characterized by Athanasoulis and Shiller (2001) to be socially optimal absent belief disagreements. When there are no belief disagreements, these are the same assets that would be endogenously designed by a profit seeking market maker (characterized in Theorem 2). However, belief disagreements also change the nature of financial innovation in this setting. When $\delta = 2\%$ and $\theta^{relative} = 3$, the endogenous asset design is typically very different than in Athanasoulis and Shiller (2001), because new assets are directed towards increasing the opportunities for speculation rather than risk sharing. This result suggests that the speculation motive for financial innovation is likely to be important in practice.

The rest of the paper is organized as follows. The next subsection discusses the related literature. Section 2 introduces the basic environment. This section also uses simple examples to illustrate the two channels by which new assets increase traders' portfolio risks. Section 3 completes the description of the environment and characterizes the equilibrium. Section 4 defines the average variance and decomposes it into the uninsurable and the speculative components. Section 5 presents the main result, which characterizes the effect of financial innovation on the two components of the average variance. Section 6 analyzes endogenous financial innovation. Section 7 generalizes the earlier results to the case in which the average variance is defined using an empirical distribution (as opposed to traders' subjective beliefs). Section 8 presents the calibration results and Section 9 concludes. Appendix A contains proofs that are omitted from the main text.

1.1 Related Literature

My paper is related to a sizeable literature on financial innovation and security design [see, in addition to the above mentioned papers, Van Horne (1985), Miller (1986), Ross (1988), Merton (1989, 1992), Duffie and Jackson (1989), Cuny (1993), Tufano (2003)]. To my knowledge, this literature has not explored the implications of heterogenous beliefs for security design. For example, in their survey of the literature, Duffie and Rahi (1994) note that "one theme of the literature, going back at least to Working (1953) and evident in the Milgrom and Stokey (1982) no-trade theorem, is that an exchange would rarely find it attractive to introduce a security whose sole justification is the opportunity for speculation." The results of this paper show that this observation does not apply if traders have *heterogeneous prior beliefs* rather than heterogeneous information. The observation also does not apply if traders have heterogeneous information but security prices do not reveal information fully due to the presence of noise traders. The analogues of my results can be derived for this alternative setting. The important economic ingredient is that traders continue to have some disagreements after observing asset prices. In addition, the quantitative results of this paper suggest that a relatively small amount of belief disagreements of this type is sufficient to ensure that speculation is a significant factor in financial innovation.

Another strand of the literature has analyzed the implications of belief disagreements for asset prices or volume of trade. A very incomplete list includes Miller (1977), Harrison and Kreps (1978), Varian (1985, 1989), Harris and Raviv (1993), Kandel and Pearson (1995), Zapatero (1998), Chen, Hong and Stein (2003), Scheinkman and Xiong (2003), Geanakoplos (2009), Cao (2010), Simsek (2011). The main difference of my paper from this literature is the focus on the effect of belief disagreements on *the riskiness of traders' portfolios*, rather than the riskiness (and the level) of prices or the volume of trade.

My paper also contributes to a literature that analyzes the role of financial innovation in generating financial instability. Rajan (2005) and Calomiris (2008) emphasize the effect of financial innovation on agency problems. Gennaioli, Shleifer and Vishny (2010) investigate the neglected risks associated with new financial assets. My paper identifies the increase in traders' speculative variance as an additional channel through which financial innovation decreases financial stability. A related paper by Stein (1987) shows that speculation driven by financial innovation can reduce welfare through informational externalities. My paper differs from Stein (1987) by modeling speculation with heterogeneous prior beliefs rather than heterogeneous information. With this approach, I show that financial innovation increases the speculative variance of traders' net worths even without informational externalities. However, unlike Stein (1987), I do not make any welfare statements since the welfare analysis with heterogeneous prior beliefs raises some unresolved theoretical issues, which I discuss further in the concluding Section 9. The idea that speculation may create financial instability appears also in Stiglitz (1989), Summers and Summers (1991), and Stout (1995). However, these analyses are mostly informal and they do not derive any results similar to my theorems.

In related work, Brock, Hommes, and Wagener (2009) identify another source of instability brought about by financial innovation and the hedge-more/bet-more effect. Their main ingredient is reinforcement learning: That is, they assume traders choose their beliefs according to a fitness measure, such as past profits made by the belief. This ingredient implies that the steady-state corresponding to the fundamental asset price can be dynamically unstable. Brock et al. (2009) show that the introduction of new assets increases the range of parameters for which the steady-state is unstable. In view of the hedge-more/bet-more effect, new assets enable traders to take greater positions on their beliefs. Consequently, a belief that turns out to be correct yields a greater profit, and it is chosen by a greater number of traders in the next period. This in turn makes the steady-state more likely to be dynamically unstable. In contrast to this paper, I take traders' prior beliefs as given and I show that financial innovation increases the speculative variance of traders' net worths regardless of how those beliefs are formed. In particular, my results do not require the reinforcement learning ingredient.

2 Basic Environment and Main Channels

Consider an economy with two dates, $\{0, 1\}$, and a single consumption good (dollar). There are a finite number of traders denoted by $i \in I = \{1, 2, .., |I|\}$. Each trader is endowed with e dollars at date 0, which is constant. Trader i is also endowed with w_i dollars at date 1, which is a random variable that captures the trader's background risks. Traders only consume at date 1, and they can transfer resources to date 1 by investing in one of two ways. They can invest in cash which yields one dollar for each dollar invested. Alternatively, they can invest in risky assets denoted by $j \in J = \{1, .., |J|\}$. Asset j is in fixed supply, normalized to zero, and it pays a^j dollars at date 1, which is a random variable. At date 0, the asset is traded in a competitive market at price p^j. Assets' payoffs and prices are respectively denoted by column vectors $\mathbf{a} = \left(a^1, .., a^{|J|}\right)'$ and $\mathbf{p} = \left(p^1, .., p^{|J|}\right)'$. This vector notation will be used throughout the paper.

Trader i's position in asset j is denoted by $x_i^j \in \mathbb{R}$. Given the price vector $\mathbf{p}$, the trader chooses an asset portfolio, $\mathbf{x}_i$, and invests the rest of her budget, $e - \mathbf{x}_i'\mathbf{p} \in \mathbb{R}$, in cash.[3] With these investment decisions, her net worth at date 1 is given by:

$$n_i = e - \mathbf{x}_i'\mathbf{p} + w_i + \mathbf{x}_i'\mathbf{a}.$$

Trader i's preferences are captured by a CARA utility function over net worth at date 1. In particular, she chooses her portfolio to maximize:

$$E_i\left[-\exp\left(-\theta_i n_i\right)\right],$$

[3] Note that traders are allowed to take unrestricted negative positions in risky assets or cash, that is, both short selling and leverage are allowed. Similarly, the asset payoffs can take negative values because the environment is frictionless. In particular, there is no limited liability and repayment is enforced by contracts.

where θ_i denotes her coefficient of absolute risk aversion. I make assumptions below which ensure that the asset payoffs, $\left\{a^j\right\}_j$, and the trader's income shock, w_i, are jointly Normally distributed according to the trader's beliefs. In view of this Normality assumption and CARA preferences, the trader's optimization reduces to the usual mean-variance problem:

$$\max_{\mathbf{x}_i} E_i\left[n_i\right] - \frac{\theta_i}{2} var_i\left[n_i\right]. \tag{1}$$

Here, $E_i\left[\cdot\right]$ denotes the mean and $var_i\left[\cdot\right]$ denotes the variance according to trader i's beliefs.

Remark 1. *The only role of the CARA preferences and the Normality assumption is to generate the mean-variance optimization in* (1)*. In particular, the results of this paper apply as long as traders' portfolio choice can be reduced to the form in* (1) *over net worth. An important special case is the continuous-time model in which traders have time-separable expected utility preferences (which are not necessarily CARA), and asset returns and background risks follow diffusion processes. In this case, the optimization problem of a trader at any date can be reduced to the form in* (1) *(see Ingersoll, 1987). The only caveat is that the reduced form coefficient of absolute risk aversion, θ_i, is endogenous since it depends on the trader's value function. Thus, in the continuous trading environment, the results of this paper apply at a trading date conditional on traders' coefficients of absolute risk aversion, $\{\theta_i\}_i$.*

The equilibrium in this economy is a collection of asset prices, $\mathbf{p}$, and portfolios, $\left(\mathbf{x}_1, .., \mathbf{x}_{|I|}\right)$, such that each trader i chooses her portfolio to solve problem (1) and prices clear asset markets, that is,

$$\sum_i x_i^j = 0 \text{ for each } j \in J.$$

I will capture financial innovation in this economy as an expansion of the set of traded assets, J. The main goal of this paper is to characterize the effects of financial innovation on portfolio risks. Before I turn to the general characterization, I use two simple examples to illustrate respectively the two channels by which financial innovation increases portfolio risks.

Example 1: New assets generate new disagreements

This example illustrates that new assets generate speculation when traders disagree about their payoffs. Moreover, this speculation can be sufficiently strong that financial innovation increases traders' portfolio risks despite the fact that new assets also provide their traditional risk sharing benefits.

Suppose there are two traders with the same coefficient of risk aversion, i.e., $I = \{1, 2\}$ and $\theta_1 = \theta_2 \equiv \theta$. Traders' date 1 endowments are perfectly negatively correlated. In particular, let $v \sim N\left(0, 1\right)$ denote a standard normal random variable and suppose:

$$w_1 = v \text{ and } w_2 \equiv -v.$$

First consider the case in which there are no assets, i.e., $J = \emptyset$. In this case, traders' date 1 net worths are given by

$$n_1 = e + v \text{ and } n_2 = e - v. \tag{2}$$

Traders' net worths are risky because they are unable to hedge their endowment risks.

Next suppose a new asset is introduced with payoff:

$$a^1 = v.$$

Suppose traders have common beliefs about the asset payoff, given by $N(0, 1)$. In this case, traders' equilibrium portfolios are given by:

$$x_1^1 = -1 \text{ and } x_2^1 = 1$$

(and the equilibrium price is $p^1 = 0$). Traders' net worths are constant and given by

$$n_1 = n_2 = e.$$

With common beliefs, financial innovation enables traders to hedge and diversify their idiosyncratic risks.

Next suppose traders have *heterogeneous prior beliefs* for the payoff of the new asset. In particular, trader 1's prior belief for the random variable v is denoted by $N(\varepsilon, 1)$, while trader 2's prior belief is $N(-\varepsilon, 1)$, for some $\varepsilon > 0$. The parameter ε captures the level of belief disagreements. Note that trader 1 is optimistic about the asset payoff while trader 2 is pessimistic. Their equilibrium portfolios can be calculated as:

$$x_1^1 = -1 + \frac{\varepsilon}{\theta} \text{ and } x_2^1 = 1 - \frac{\varepsilon}{\theta}.$$

In this case, traders' positions are influenced by their belief differences as well as their hedging demands. Traders' net worths are given by:

$$n_1 = e + \frac{\varepsilon}{\theta} v \text{ and } n_2 = e - \frac{\varepsilon}{\theta} v.$$

If $\varepsilon > \theta$, then traders' net worths are even riskier than the case in which no new asset is introduced [cf. Eq. (2)]. With these beliefs, trader 1 is so optimistic about the asset's payoff that she takes a positive position, despite the fact that her endowment is positively correlated with the asset payoff. Consequently, the new asset increases the riskiness of her net worth. Hence, when traders' disagreements about new assets are sufficiently large, financial innovation increases traders' portfolio risks.

Example 2: New assets amplify speculation on existing disagreements

This example illustrates that financial innovation increases portfolio risks through a second channel: By amplifying traders' speculation on existing disagreements. In view of this channel, new assets increase portfolio risks even if traders completely agree about their payoffs.

As in Example 1, suppose there are two traders with the same coefficient of risk aversion, i.e., $I = \{1, 2\}$ and $\theta_1 = \theta_2 \equiv \theta$. But this time suppose traders' date 1 endowments are constant, $w_1 = w_2 = 0$, which ensures that there is no risk sharing motive for trade. Suppose also that the fundamental risk in this economy is captured by 2 random variables, v_1, v_2, which are i.i.d. with standard Normal distribution, $N(0, 1)$. Traders have common beliefs for v_2 given by the distribution, $N(0, 1)$. However, they disagree about v_1. In particular, trader 1's belief for v_1 is given by $N(\varepsilon, 1)$ while trader 2's belief is given by $N(-\varepsilon, 1)$.

First suppose there is a single asset for trade (corresponding to the Franc in the Introduction) with payoff:

$$a^1 = v_1 + \alpha v_2.$$

In particular, the asset is affected by both sources of fundamental risk, with the weight, α, capturing the relative importance of the second risk. By symmetry, the equilibrium price is given by $p^1 = 0$. Substituting this expression, trader 1's mean-variance problem [cf. Eq. (1)] can be written as:

$$\max_{x_1^1} x_1^1 \varepsilon - \frac{\theta}{2}\left(1 + \alpha^2\right)\left(x_1^1\right)^2. \tag{3}$$

The first term in this expression is the trader's expected payoff in equilibrium. The second term is the trader's expected cost from variance of her net worth. Trader 2's mean-variance problem takes a similar form. Traders' equilibrium portfolios can be solved as:

$$x_1^1 = \frac{\varepsilon}{\theta}\frac{1}{1+\alpha^2} \text{ and } x_2^1 = -\frac{\varepsilon}{\theta}\frac{1}{1+\alpha^2}, \tag{4}$$

and their net worths are given by:

$$n_1 = e + \frac{\varepsilon}{\theta}\frac{v_1 + \alpha v_2}{1+\alpha^2} \text{ and } n_2 = e - \frac{\varepsilon}{\theta}\frac{v_1 + \alpha v_2}{1+\alpha^2}. \tag{5}$$

Note that traders do not necessarily take large speculative positions because the asset's payoff is influenced by the risk, v_2, as well as the risk, v_1. In particular, traders' positions (and the riskiness of their net worths) are decreasing in α, and they converge to zero as α limits to infinity. Intuitively, the ability to trade asset 1 enables traders to take only *impure bets* because the asset's payoff also responds to risks traders do not disagree about. To bet on their belief disagreements, traders must also hold these additional risks [as formally captured by the α term in problem (3)], which makes them reluctant to take large positions.

Next suppose a new asset is introduced (corresponding to the Euro in the Introduction)

with payoff:

$$a^2 = v_2.$$

Straightforward calculations show that traders' equilibrium portfolios are given by:

$$\text{Trader 1} \quad : \quad x_1^1 = \frac{\varepsilon}{\theta}, \qquad x_1^2 = -\alpha\frac{\varepsilon}{\theta}, \tag{6}$$

$$\text{Trader 2} \quad : \quad x_2^1 = -\frac{\varepsilon}{\theta}, \qquad x_2^2 = \alpha\frac{\varepsilon}{\theta},$$

and their net worths are given by:

$$n_1 = e + \frac{\varepsilon}{\theta}v_1 \text{ and } n_2 = e - \frac{\varepsilon}{\theta}v_1. \tag{7}$$

Note that the magnitude of traders' positions on asset 1 is greater than the earlier setting in which asset 2 was not available [cf. Eqs. (6) and (4)]. Importantly, traders' net worths are also riskier [cf. Eqs. (7) and (5)]. Put differently, the innovation of asset 2, about which traders do not disagree, enables traders to take greater speculative positions on asset 1 and increases their portfolio risks.

To understand these results, first consider the portfolios in (6). Note that traders complement their speculative positions in asset 1 by taking the opposite positions in asset 2. These complementary positions enable the traders to hedge the risk, v_2, which they do not disagree about. This in turn enables traders to take *purer bets* on the risk, v_1. In fact, traders' net worths (7) are identical to those that would obtain if they could trade an alternative asset that pays:

$$a^{syn} = a^1 - \alpha a^2 = v_1. \tag{8}$$

Traders "create" this synthetic asset by simultaneously investing in multiple assets.

Next consider why traders increase their positions on asset 1 and why their net worths become riskier. To understand these effects, it is useful to consider the analogue of the mean-variance problem (3) in this case. In terms of the synthetic asset in (8), trader 1 solves:

$$\max_{x_1^{syn}} x_1^{syn}\varepsilon - \frac{\theta}{2}\left(x_1^{syn}\right)^2. \tag{9}$$

Note that problems (3) and (9) are very similar, except that the former problem has an additional factor of $\left(1+\alpha^2\right)$ multiplying the cost term. This difference captures the *hedging effect*: The fact that traders use new assets to hedge the risks on their speculative positions tends to reduce the riskiness of their net worths and the associated costs. In fact, controlling for a trader's speculative position on risk v_1, i.e., assuming $x_1^{syn} = x_1^1$, the hedging effect leads to a lower variance of net worth. In view of this observation, a naive view could suggest that new assets on which there is complete agreement should reduce traders' portfolio risks.

However, the naive view misses an important amplification mechanism: the *hedge-more/bet-*

more effect. When traders are able to take purer bets, they also take larger bets. In this example, the marginal cost (in terms of additional variance) of taking an additional speculative position on v_1 is lower when the second asset is available. This induces the trader to take a larger speculative position in that case, i.e., $x_1^1 < x_1^{syn}$ for the respective optima of problems (3) and (9).

The amplification of speculative positions through the hedge-more/bet-more effect tends to increase the riskiness of traders' net worths. Recall also that the direct hedging effect tends to reduce the riskiness of traders' net worths. A priori, it is not clear that the amplification effect should be sufficiently strong to overcome the direct hedging effect. However, this is always the case for the standard mean-variance setting. In particular, since problems (3) and (9) are linear-quadratic, their cost terms at optimum satisfy:

$$\left(1+\alpha^2\right)\frac{\theta}{2}\left(x_1^1\right)^2 < \frac{\theta}{2}\left(x_1^{syn}\right)^2.$$

That is, the reduction in the marginal cost of taking speculative positions generates such a large portfolio reaction that traders' total costs (and net worth variances) increase. Consequently, financial innovation increases traders' portfolio risks even if traders agree about their payoffs.

3 Environment and Equilibrium

The examples in the previous section illustrated the two channels by which financial innovation tends to increase portfolio risks. As illustrated by Example 1, there are also the traditional risk sharing channels by which financial innovation tends to decrease portfolio risks. The rest of the paper considers a general mean-variance economy in which all of the channels of the previous section are present. This section characterizes the equilibrium and shows that the traders' complete portfolios can be decomposed into risk sharing and speculative portfolios which represent the two motives for trade in this economy. The subsequent sections consider the effect of financial innovation on this equilibrium.

The economy in this section builds upon the environment in Section 2 by specifying the uncertainty and the agents' beliefs. The uncertainty in this economy is captured by the m dimensional random variable, $\mathbf{v} = (v_1, .., v_m)'$. Traders have potentially heterogeneous prior beliefs about $\mathbf{v}$. They agree about the variance of $\mathbf{v}$, which simplifies the analysis, but they might disagree about the mean of $\mathbf{v}$. In particular:

Assumption (A1). Trader i's belief for the random variable $\mathbf{v}$ is given by the Normal distribution, $N\left(\boldsymbol{\mu}_i^{\mathbf{v}}, \Lambda^{\mathbf{v}}\right)$, where $\boldsymbol{\mu}_i^{\mathbf{v}} \in \mathbb{R}^m$ is the mean vector and $\Lambda^{\mathbf{v}}$ is an $m \times m$ covariance matrix with full row rank.

Traders' date 1 endowment can be written in terms of $\mathbf{v}$ as:

$$w_i = \left(\mathbf{W}_i\right)' \mathbf{v},$$

for some $\mathbf{W}_i \in \mathbb{R}^m$. Asset j's payoff can also be written in terms of $\mathbf{v}$ as:

$$a^j = \left(\mathbf{A}^j\right)' \mathbf{v},$$

for some $\mathbf{A}^j \in \mathbb{R}^m$. The vectors, $\left\{\mathbf{A}^j\right\}_j$, are linearly independent, which ensures that the assets are not redundant. An economy is formally denoted by $\mathcal{E}(J) = \left(J;\, \left\{\mathbf{A}^j\right\}_{j\in J};\, \{\mathbf{W}_i\}_i;\, \{\boldsymbol{\mu}_i^{\mathbf{v}}, \Lambda^{\mathbf{v}}\}_i\right)$.

Note that I have not specified the empirical (or realized) distribution for $\mathbf{v}$. This distribution does not matter for much of the analysis in this paper. In particular, the mean of the empirical distribution does not play a role in any of the results. This is because the main goal of this paper is to characterize traders' portfolio risks, for which it is not necessary to take a position on who is right on average. In addition, the variance of the empirical distribution plays only a limited role. This is because traders' portfolio risks could be defined by using their *perceived* variance, $\Lambda^{\mathbf{v}}$, without reference to the empirical variance. This is the approach that will be taken until Section 7. However, the empirical variance becomes of interest when the model is taken to the data. Section 7 generalizes the main results of this paper to the case in which portfolio risks are defined with the empirical variance.

3.1 Characterization of equilibrium

Given the above specification, trader i believes the asset payoffs are Normally distributed, $N(\boldsymbol{\mu}_i, \Lambda)$, with:

$$\boldsymbol{\mu}_i \equiv \mathbf{A}'\boldsymbol{\mu}_i^{\mathbf{v}} \text{ and } \Lambda \equiv \mathbf{A}'\Lambda^{\mathbf{v}}\mathbf{A}.$$

Note that, in view of assumption (A1), traders agree about the variance of asset payoffs while they may disagree about their mean. In addition, trader i believes that her endowment is Normally distributed, and that the covariance of her endowment with the asset payoffs is given by:

$$\boldsymbol{\lambda}_i = \mathbf{A}'\Lambda^{\mathbf{v}}\mathbf{W}_i.$$

Given traders' beliefs, the mean variance problem in (1) can be solved in closed form. In particular, trader i's portfolio demand is given by:

$$\mathbf{x}_i = \Lambda^{-1}\left[\frac{\boldsymbol{\mu}_i - \mathbf{p}}{\theta_i} - \boldsymbol{\lambda}_i\right]. \tag{10}$$

This expression illustrates the two sources of demand. First, the trader tends to hold a positive position on an asset if its payoff is negatively correlated with her endowment, as captured by the term, $\boldsymbol{\lambda}_i$. Second, she also tends to hold a positive position if she thinks the asset is underpriced, as captured by the term, $\frac{\boldsymbol{\mu}_i - \mathbf{p}}{\theta_i}$.

Next consider the determination of the equilibrium price vector, $\mathbf{p}$. Aggregating Eq. (10)

and using market clearing, prices can be solved in closed form as:

$$\mathbf{p} = \frac{1}{|I|} \sum_{i \in I} \left(\frac{\bar{\theta}}{\theta_i} \boldsymbol{\mu}_i - \bar{\theta} \boldsymbol{\lambda}_i \right), \tag{11}$$

where $\bar{\theta} \equiv \left(\sum_{i \in I} \theta^{-1} / |I| \right)^{-1}$ is the Harmonic mean of traders' absolute risk aversion coefficients. Intuitively, the price of an asset is high either if the asset is negatively correlated with traders' endowments (captured by the $\boldsymbol{\lambda}_i$ term) or if traders on average believe that the asset will yield a high payoff (captured by the $\boldsymbol{\mu}_i$ term). The beliefs of more risk averse traders have a smaller effect on the price since they bet relatively less on their opinions.

Using the price expression (11), trader i's portfolio in (10) can also be solved in closed form. In addition, trader i's portfolio can be decomposed into two components that capture the two motives for trade in this economy. In particular,

$$\begin{aligned} \mathbf{x}_i &= \mathbf{x}_i^R + \mathbf{x}_i^S, \quad \text{where} \\ \mathbf{x}_i^R = -\Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \qquad &\text{and} \qquad \mathbf{x}_i^S = \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}. \end{aligned} \tag{12}$$

Here,

$$\tilde{\boldsymbol{\lambda}}_i = \boldsymbol{\lambda}_i - \frac{\bar{\theta}}{\theta_i} \frac{1}{|I|} \sum_{\bar{\imath} \in I} \boldsymbol{\lambda}_{\bar{\imath}} \tag{13}$$

denotes the relative covariance of trader i's endowment, and

$$\tilde{\boldsymbol{\mu}}_i = \boldsymbol{\mu}_i - \frac{1}{|I|} \sum_{\bar{\imath} \in I} \frac{\bar{\theta}}{\theta_{\bar{\imath}}} \mu_{\bar{\imath}} \tag{14}$$

denotes the relative optimism of trader i. Note that $\mathbf{x}_i^R$ would be the trader's equilibrium portfolio if there were no belief differences (i.e., if $\tilde{\boldsymbol{\mu}}_i = 0$ for each i). Hence, I refer to $\mathbf{x}_i^R$ as the *risk sharing portfolio* of trader i. On the other hand, the residual portfolios, $\left\{ \mathbf{x}_i^S \right\}_i$, are purely driven by belief differences. Thus, I refer to $\mathbf{x}_i^S$ as the *speculative portfolio* of trader i. To understand the economic forces that operate in this economy, it is useful to investigate in more detail the determinants of the risk sharing and the speculative portfolios.

Economic determinants of the risk sharing portfolios. Eqs. (12) and (13) show that the risk sharing portfolios allocate risks efficiently through two distinct channels: (i) they diversify idiosyncratic risks, (ii) and they transfer aggregate risks to those traders who are best able to bear them. To understand the first (diversification) channel, consider the special case in which $\theta_i = \theta$ for each i. In this case, Eq. (13) illustrates that all risk sharing trades are driven by *differences* in the covariances of traders' endowments with asset payoffs. If an asset covaries equally with all traders' endowments, then this asset earns a risk premium [as

captured by Eq. (11)], but it does not generate trade. If instead the asset covaries more with some traders' endowments than others, then this asset generates trade which facilitates the diversification of risks. To understand the second (risk-transfer) channel, consider the special case in which there are two traders with $\theta_i < \theta_{\bar{\imath}}$. Suppose also that traders have perfectly correlated endowment risks so there is no scope for diversification. Finally, suppose that there is an asset which is correlated with traders' endowments, so that $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{\bar{\imath}} \neq 0$. In this case, the weighted average in Eq. (13) illustrates that $\tilde{\boldsymbol{\lambda}}_i$ and $\tilde{\boldsymbol{\lambda}}_{\bar{\imath}}$ are non-zero even though $\boldsymbol{\lambda}_i = \boldsymbol{\lambda}_{\bar{\imath}}$. This in turn generates trade which facilitates the transfer of risks from trader $\bar{\imath}$ who has a higher risk aversion to trader i who has a lower risk aversion.

Economic determinants of the speculative portfolios. The channels captured by the risk sharing portfolios correspond to the traditional benefits of financial innovation. The novel effects in this economy are captured by the speculative portfolios, $\left\{\mathbf{x}_i^S\right\}_i$. Eq. (14) shows that that the speculative portfolios are determined by traders' belief disagreements. If the trader i is more optimistic for some assets than the (weighted) average belief of other traders, then she holds a non-zero speculative portfolio. The opposite side of these trades are taken by those traders that are less optimistic than the average. Put differently, the speculative portfolios enable the traders to bet on their different views. As implied by the decomposition, $\mathbf{x}_i = \mathbf{x}_i^R + \mathbf{x}_i^S$, these speculative portfolios distort the traders' complete portfolios relative to a benchmark in which the only motive for trade is risk sharing.

4 The Average Variance and Its Decomposition

Eqs. $(11) - (14)$ complete the characterization of equilibrium in this economy. The main goal of this paper is to analyze the effect of financial innovation on portfolio risks. To this end, this section defines the average variance of traders' net worths as an appropriate measure of portfolio risks for this economy. It then shows that the average variance can be decomposed into uninsurable and speculative components. The main result in the next section characterizes the effect of financial innovation on each of these components.

Given the mean-variance framework, a natural measure of portfolio risk for a trader i is the variance of her net worth, $var_i(n_i)$. I consider an average of this measure across all traders, *the average variance*, defined as follows:

$$\begin{aligned} \Omega &= \frac{1}{|I|}\sum_{i\in I}\frac{\theta_i}{\bar{\theta}} var_i(n_i), \\ &= \frac{1}{|I|}\sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\left(\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i + 2\mathbf{x}_i'\boldsymbol{\lambda}_i + \mathbf{x}_i'\Lambda\mathbf{x}_i\right). \end{aligned} \tag{15}$$

A couple comments about this definition are in order. First, note that the portfolio risk of a trader is calculated according to her own belief. Section 7 generalizes the main results to the

case in which portfolio risks are defined by using the empirical distribution for the underlying risks. Second, note that traders that are relatively more risk averse are given a greater weight in the average.

There are at least two justifications for considering the average variance, Ω, as the appropriate measure of portfolio risks for this economy. For a first justification, consider the certainty equivalent aggregate net worth in this economy according to the belief of an arbitrary trader $\bar{\imath}$, given by:

$$\begin{aligned} N_{\bar{\imath}} &= \sum_{i \in I} \left(E_{\bar{\imath}}\left[n_i\right] - \frac{\theta_i}{2} var_{\bar{\imath}}\left(n_i\right) \right), \\ &= \sum_{i \in I} E_{\bar{\imath}}\left[e - \mathbf{x}_i' \mathbf{p} + w_i + \mathbf{x}_i' \mathbf{a} \right] - \frac{\theta_i}{2} var_i\left(n_i\right), \\ &= E_{\bar{\imath}}\left[\sum_{i \in I} e + w_i \right] - \sum_{i \in I} \frac{\theta_i}{2} var_i\left(n_i\right). \end{aligned}$$

Here, the second line replaces $var_{\bar{\imath}}\left(n_i\right)$ with $var_i\left(n_i\right)$ in view of the assumption that traders agree on the variances and the last line uses the market clearing condition, $\sum_{i \in I} \mathbf{x}_i = 0$. This expression illustrates that the certainty equivalent aggregate net worth can be decomposed into two components: An expected endowment component which does not depend on traders' portfolios, and a variance loss component which depends on the portfolios. Moreover, the variance loss component is a constant scaling of the average variance, Ω. Consequently, choosing portfolios to maximize the certainty equivalent aggregate net worth *according to any trader's beliefs* is equivalent to choosing them to minimize the average variance, Ω. Intuitively, the portfolio allocations in this economy do not generate expected net worth since they simply redistribute wealth across traders. Hence, the portfolios affect the certainty equivalent aggregate net worth only through their effect on the variance. For each trader, the certainty equivalent loss from variance is proportional to her coefficient of absolute risk aversion, which justifies the form of Ω in (15).

A second justification for the average variance, Ω, is provided by the following lemma.

Lemma 1. *The risk sharing portfolios, $\left\{\mathbf{x}_i^R\right\}_i$, minimize the average variance, Ω, among all feasible portfolios:*

$$\min_{\left\{\mathbf{x}_i \in \mathbb{R}^{|J|}\right\}_i} \Omega \qquad s.t. \ \sum_i \mathbf{x}_i = 0. \tag{16}$$

Equivalently, they maximize the certainty equivalent aggregate net worth, $N_{\bar{\imath}}$, according to the belief of any trader, $\bar{\imath}$.

If there are no belief disagreements, i.e., $\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} = 0$ for each i, then the complete portfolios and the risk sharing portfolios coincide, i.e., $\mathbf{x}_i = \mathbf{x}_i^R$ for each i. Thus, Lemma 1 shows that without belief disagreements the equilibrium portfolios, $\{\mathbf{x}_i\}_i$, minimize the average variance, Ω (equivalently, they maximize the certainty equivalent aggregate net worth). This result further

illustrates that Ω is the natural measure of portfolio risks in this economy. In particular, this is the measure of risks that would be minimized in equilibrium absent belief disagreements.

In view of Lemma 1, I let Ω^R denote the minimum for problem (16) and refer to it as the *uninsurable variance*. The extent to which Ω deviates from Ω^R (and the certainty equivalent aggregate net worth deviates from its maximum possible value) could be viewed as the effect of speculation based on belief disagreements. I thus define $\Omega^S = \Omega - \Omega^R$ and refer to it as the *speculative variance*. This provides a decomposition of the average variance into the uninsurable and the speculative components,

$$\Omega = \quad \Omega^R \quad + \quad \Omega^S.$$

The next lemma characterizes the two components of average variance in terms of the exogenous parameters of the model.

Lemma 2. *The uninsurable variance is given by:*

$$\Omega^R = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\overline{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i - \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right), \tag{17}$$

and the speculative variance is given by:

$$\Omega^S \equiv \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\overline{\theta}} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}' \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} \right). \tag{18}$$

The forms of Ω^R and Ω^S are intuitive. Eq. (17) illustrates that the uninsurable variance is lower when the assets provide better risk sharing opportunities, captured by larger $\tilde{\boldsymbol{\lambda}}_i$. Similarly, Eq. (18) illustrates that the speculative variance is greater when the assets feature greater belief disagreements, captured by larger $\tilde{\boldsymbol{\mu}}_i$. The next sections characterize the effect of financial innovation on Ω^R and Ω^S.

5 Financial Innovation and Portfolio Risks

I model financial innovation as an expansion of the set of traded assets. For this purpose, it is useful to define economies in which only a subset of the assets in J are traded. In particular, given $\hat{J} \subset J$, let $\mathcal{E}\left(\hat{J}\right) = \left(\hat{J};\ \{\mathbf{A}^j\}_{j \in \hat{J}};\ \{\mathbf{W}_i\}_i;\ \{\boldsymbol{\mu}_i^{\mathbf{v}}, \Lambda^{\mathbf{v}}\}_i\right)$ denote the economy in which the asset set is given by $\hat{J}$. Where it does not create confusion, I also use the notation, $z\left(\hat{J}\right)$, to refer to the equilibrium variable z for the economy $\mathcal{E}\left(\hat{J}\right)$.

To capture financial innovation, suppose J can be broken down into a set of old assets, J_O, and a set of new assets, J_N (formally, $J = J_O \cup J_N$ where J_O and J_N are disjoint sets). The differences between the economies $\mathcal{E}(J_O)$ and $\mathcal{E}(J_O \cup J_N)$ can be attributed to financial innovation. I next present the main result.

Theorem 1 (Financial Innovation and Portfolio Risks). *Consider the average variance and its components respectively for the economies $\mathcal{E}(J_O)$ and $\mathcal{E}(J_O \cup J_N)$.*

(i) Financial innovation always reduces the uninsurable variance, that is:

$$\Omega^R(J_O \cup J_N) \leq \Omega^R(J_O).$$

(ii) Financial innovation always increases the speculative variance, that is:

$$\Omega^S(J_O \cup J_N) \geq \Omega^S(J_O).$$

(ii) For any $\eta > 0$, there exist economies in which the increase in the speculative variance is sufficiently large that financial innovation increases the average variance by at least η, that is, $\Omega(J_O \cup J_N) > \Omega(J_O) + \eta$.

The first part of this theorem is a corollary of Lemma 1, and it shows that financial innovation always provides some risk sharing benefits. This part formalizes the traditional view of financial innovation in the context of this model. On the other hand, the second part of the theorem identifies a second force which always operates in the opposite direction. In particular, when there are belief disagreements, financial innovation also always increases the speculative variance. Hence, the net effect of financial innovation on average variance is ambiguous, and it depends on the relative strength of the two forces.

Most of the literature on financial innovation considers the special case without belief disagreements. Theorem 1 shows that the common-beliefs assumption is restrictive, as it shuts down an important economic force by which financial innovation always has a positive effect on portfolio risks. Furthermore, the third part of the theorem shows that it is indeed possible for the force from belief disagreements to dominate the traditional force.

It is also worth emphasizing the generality of the second part of Theorem 1. The result applies for all sets of existing and new assets, J_O and J_N, with no restrictions on the joint distribution of asset payoffs or traders' beliefs for $\mathbf{v}$ [except for the relatively mild Assumption (A1)]. For example, Theorem 1 shows that financial innovation increases the speculative variance even if there are no belief disagreements about new assets. This is surprising because, as illustrated by Example 2, new assets are used to hedge (to some extent) the risks from traders' speculation on their existing disagreements. Put differently, the direct *hedging effect* of new assets tends to reduce Ω^S. However, as illustrated by Example 2, there is also the *hedge-more/bet-more effect* which tends to increase Ω^S. Theorem 1 shows that, in the standard mean-variance framework, the hedge-more/bet-more effect is sufficiently strong that new assets always increase Ω^S.

The rest of this section provides a sketch proof and a complementary intuition for the second part of Theorem 1 (the proofs for the first and the third parts relegated to the Appendix). The proof proceeds in four steps. First, the form of $\mathbf{x}_i^S$ in Eq. (12) implies that the speculative

portfolio, $\mathbf{x}_i^S$, solves the following version of the mean-variance problem:

$$\max_{\mathbf{x}_i \in \mathbb{R}^{\hat{J}}} (\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i - \frac{\theta_i}{2} \mathbf{x}_i' \Lambda \mathbf{x}_i. \tag{19}$$

Moreover, the speculative variance, Ω^S, is found by averaging the variance costs for each trader at the solution to this problem:

$$\Omega^S = \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \left(\mathbf{x}_i^S\right)' \Lambda \mathbf{x}_i^S. \tag{20}$$

Intuitively, problem (19) is the traders' mean-variance problem in a hypothetical economy that is identical except that traders have no background risks (i.e., $\mathbf{W}_i = 0$ for all $i \in I$), so that the only motive for trade is speculation. The solution to this problem gives the speculative portfolio in the actual economy, and also determines the speculative variance as captured by (20).

Second, note that financial innovation relaxes the constraint set of problem (19), which in turn increases the optimum value of the problem. That is, when the asset set is $\hat{J} = J^O \cup J^N$, traders are able to make all the speculative trades they could make when the asset set is $\hat{J} = J^O$, and some more. Put differently, new assets expand the "betting possibilities frontier" for traders, which in turn increases their certainty-equivalent payoffs from betting. This observation, which is central for the result, further reinforces the messages of Examples 1 and 2. In particular, financial innovation increases the betting possibilities frontier through two distinct channels. As emphasized by Example 1, new assets are likely to generate new disagreements. In addition, as emphasized by Example 2, even if new assets do not generate disagreements themselves, they enable the traders to take purer bets on existing disagreements. Both of these channels manifest themselves as an expansion of the constraint set of problem (19).

Third, since problem (19) is a quadratic optimization problem, at the optimum traders' expected payoffs, $(\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i$, are proportional to their variance costs, $\frac{\theta_i}{2} \mathbf{x}_i' \Lambda \mathbf{x}_i$. In particular:

$$(\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i^s = 2 \frac{\theta_i}{2} \left(\mathbf{x}_i^s\right)' \Lambda \mathbf{x}_i^s. \tag{21}$$

Consequently, financial innovation increases not only the certainty-equivalent payoff but also both of its components: the expected payoff, $(\tilde{\boldsymbol{\mu}}_i)' \mathbf{x}_i^s$, and the variance, $\left(\mathbf{x}_i^s\right)' \Lambda \mathbf{x}_i^s$. For intuition, consider the introduction of an asset for which trader i is optimistic. The introduction of this asset naturally increases the expected return for this trader. However, the trader responds by increasing her investments to a point that she also takes greater risks than before. At the optimal portfolio, higher expected returns go hand-in-hand with higher risks as captured by Eq. (21).

The final step combines the third and the first steps to prove Theorem 1. As financial inno-

vation increases the speculative variance of each trader, it also increases the average speculative variance in Eq. (20) (see Appendix A.2 for an alternative proof based on matrix algebra).

A complementary intuition for Theorem 1 can be provided by characterizing traders' speculative risks in terms of the Sharpe ratios of their speculative portfolios. To this end, consider the hypothetical economy in the above proof in which there are no background risks. Define the *speculative Sharpe ratio* of a trader as the Sharpe ratio of her portfolio in this hypothetical economy.[4] The speculative Sharpe ratio of a trader i can be calculated as:

$$Sharpe_i^S = \frac{\left(\mathbf{x}_i^S\right)'\left(\boldsymbol{\mu}_i - \mathbf{p}\right)}{\sqrt{\left(\mathbf{x}_i^S\right)'\Lambda\mathbf{x}_i^S}} = \sqrt{\tilde{\boldsymbol{\mu}}_i'\Lambda^{-1}\tilde{\boldsymbol{\mu}}_i}, \tag{22}$$

where the second equality follows from Eqs. (11) − (14). In addition, the standard deviation of the trader's net worth in the hypothetical economy can be written as $\sqrt{\left(\mathbf{x}_i^S\right)'\Lambda\mathbf{x}_i^S} = \frac{1}{\theta_i}\sqrt{\tilde{\boldsymbol{\mu}}_i'\Lambda^{-1}\tilde{\boldsymbol{\mu}}_i}$. Dividing this by the initial net worth, e, the standard deviation of the trader's *portfolio return* can be written as:

$$\sigma_i^S \equiv std\left(\frac{n_i}{e}\right) = \frac{1}{\theta_i e}\sqrt{\tilde{\boldsymbol{\mu}}_i'\Lambda^{-1}\tilde{\boldsymbol{\mu}}_i}. \tag{23}$$

Note that the ratio, $\theta_i e$, provides a measure of the coefficient of relative risk aversion for trader i. Thus, combining Eqs. (22) and (23) gives the familiar result that the standard deviation of the portfolio return is equal to the Sharpe ratio of the optimal portfolio divided by the coefficient of relative risk aversion (see Campbell and Viceira, 2002). This textbook result applies also in this model when there are no background risks.

Theorem 1 can then be understood from the lenses of Eqs. (22) and (23). Financial innovation increases the traders' speculative Sharpe ratios by expanding the betting possibilities frontier through the two channels emphasized before. Once traders are able to obtain higher Sharpe ratios, they also undertake greater speculative risks, providing a complementary intuition for Theorem 1.

6 Endogenous Financial Innovation

The analysis so far has taken the set of new assets as exogenous. In practice, many financial products are introduced endogenously by economic agents with profit incentives. A large literature emphasizes risk sharing as a major driving force for endogenous financial innovation [see, for example, Allen and Gale (1994), Duffie and Rahi (1995), Demange and Laroque (1995), Athanasoulis and Shiller (2000, 2001)].[5] A natural question, in view of the results in

[4] Recall that the Sharpe ratio of a portoflio is defined as the expected portfolio return in excess of the risk-free rate (which is normalized to 0 in this model) divided by the standard deviation of the portfolio return.

[5] Risk sharing is one of several drivers of financial innovation emphasized by the previous literature. Other factors include mitigating agency frictions, reducing asymmetric information, minimizing transaction costs, and

the earlier sections, is to what extent the risk sharing motive for financial innovation is robust to the presence of belief disagreements. To address this question, this section endogenizes the asset design by introducing a profit seeking market maker and obtains two main results. First, the optimal asset design depends on the size and the nature of traders' belief disagreements, in addition to the possibilities for risk sharing. Second, when traders' belief disagreements are sufficiently large, the market maker designs assets that *maximize* traders' average portfolio risks among all possible choices, completely disregarding the risk sharing motive for financial innovation.

The main feature of the model in this section is that the assets, J, are introduced by a market maker. The market maker is constrained to choose $|J| < m$ assets, but is otherwise free to choose the asset design, $\mathbf{A}$. Here, recall that the matrix, $\mathbf{A} = \left[\mathbf{A}^1, \mathbf{A}^2, .., \mathbf{A}^{|J|}\right]$, captures the asset payoffs which are given by $a^j = \left(\mathbf{A}^j\right)' \mathbf{v}$ for each j. Thus, the market maker's choice of $\mathbf{A}$ affects the belief disagreements and the relative covariances according to [cf. Eqs. (14) and (13)]:

$$\tilde{\boldsymbol{\mu}}_i(\mathbf{A}) = \mathbf{A}'\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} \text{ and } \tilde{\boldsymbol{\lambda}}_i(\mathbf{A}) = \mathbf{A}'\Lambda^{\mathbf{v}}\tilde{\mathbf{W}}_i,$$

where the deviation terms are defined as:

$$\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} = \boldsymbol{\mu}_i^{\mathbf{v}} - \frac{1}{|I|}\sum_{\bar{\imath}\in I}\frac{\bar{\theta}}{\theta_{\bar{\imath}}}\boldsymbol{\mu}_{\bar{\imath}}^{\mathbf{v}} \text{ and } \tilde{\mathbf{W}}_i = \mathbf{W}_i - \frac{\bar{\theta}}{\theta_i}\frac{1}{|I|}\sum_{\bar{\imath}\in I}\mathbf{W}_{\bar{\imath}}.$$

Once the market maker chooses the asset design, $\mathbf{A}$, the assets are traded in a competitive market similar to the previous sections. The market maker intermediates these trades which enables it to extract some of the surplus from traders. To keep the analysis simple, suppose the market maker can extract the full surplus.[6] In particular, the market maker sets a fixed membership fee, π_i, for each trader i and makes a take it or leave it offer. If trader i accepts the offer, then she can trade the available assets in the competitive market. Otherwise, trader i is out of the market, and her net worth is given by her endowment, $e + \mathbf{W}_i'\mathbf{v}$.

The equilibrium of this economy can be characterized backwards. First consider the competitive equilibrium after the market maker has chosen $\mathbf{A}$ and traders decided whether or not to participate in the market. Assume that all traders have accepted the offer, which will be the case in equilibrium. In view of the mean-variance framework, traders' portfolio choices are independent of the fixed fees they have paid. In particular, the equilibrium is characterized as in the earlier sections.

Next consider the fixed fees the market maker charges for a given choice of $\mathbf{A}$. If trader i rejects the offer, she receives the certainty equivalent payoff from her endowment. Otherwise,

sidestepping taxes and regulation (see Tufano, 2004, for a recent survey). These other factors, while clearly important, are left out of the analysis in this paper to focus on the effect of belief disagreements on the risk sharing motive for innovation.

[6] The results below remain unchanged under the less extreme (reduced form) assumption that the market maker extracts a constant fraction, $\zeta \in (0, 1]$, of the surplus regardless of the choice of $\mathbf{A}$.

she receives the certainty equivalent payoff from her equilibrium portfolio net of the fixed fee, $\pi_i(\mathbf{A})$. The market maker sets $\pi_i(\mathbf{A})$ so that the trader is just indifferent to accept the offer. Straightforward calculations (relegated to Appendix A.3) show that the market maker's expected total profits are given by:

$$\sum_{i\in I}\pi_i(\mathbf{A}) = \sum_i \frac{\boldsymbol{\theta}_i}{2}\left(\frac{\tilde{\boldsymbol{\mu}}_{\mathbf{i}}(\mathbf{A})}{\boldsymbol{\theta}_{\mathbf{i}}} - \tilde{\boldsymbol{\lambda}}_{\mathbf{i}}(\mathbf{A})\right)' \Lambda^{-1}\left(\frac{\tilde{\boldsymbol{\mu}}_{\mathbf{i}}(\mathbf{A})}{\boldsymbol{\theta}_{\mathbf{i}}} - \tilde{\boldsymbol{\lambda}}_{\mathbf{i}}(\mathbf{A})\right). \tag{24}$$

This expression reflects the two motives for trade in this economy. Traders are willing to pay to trade assets that facilitate better risk sharing [i.e., larger $\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})$], or to trade assets that generate greater belief disagreements [i.e., larger $\tilde{\boldsymbol{\mu}}_i(\mathbf{A})$].

The market maker chooses an asset design, $\mathbf{A}$, that maximizes the expected profits in (24). Note that many choices of $\mathbf{A}$ represent the same trading opportunities over the space of the underlying risks, $\mathbf{v}$ (and thus, also generate the same profits). Thus, suppose without loss of generality that the market maker's choice is subject to the following normalizations:

$$\Lambda = \mathbf{A}'\Lambda^{\mathbf{v}}\mathbf{A} = \mathbf{I}_{|J|}, \qquad \text{and} \qquad \left((\Lambda^{\mathbf{v}})^{1/2}\mathbf{A}\right)_1^j \geq 0 \text{ for each } j \in J. \tag{25}$$

Here, $(\Lambda^{\mathbf{v}})^{1/2}$ denotes the unique positive definite square root of the matrix, $\Lambda^{\mathbf{v}}$. The first condition in (25) normalizes the variance of assets to be the identity matrix, $\mathbf{I}_{|J|}$. This condition determines the column vectors of the matrix for normalized asset payoffs, $(\Lambda^{\mathbf{v}})^{1/2}\mathbf{A}$, up to a sign. The second condition resolves the remaining indeterminacy by adopting a sign convention for these column vectors.

Theorem 2 (Optimal Asset Design). *Suppose the matrix*

$$\frac{1}{|I|}\sum_i \frac{\theta_i}{\overline{\theta}}\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)' \tag{26}$$

is non-singular. Then, an asset design is optimal if and only if the columns of the matrix for normalized asset payoffs, $(\Lambda^{\mathbf{v}})^{1/2}\mathbf{A}$, correspond to the eigenvectors corresponding to the $|J|$ largest eigenvalues of the matrix in (26)*. If the eigenvalues are distinct, then the asset design is uniquely determined by this condition along with the normalizations in* (25)*. Otherwise, the asset design is determined up to a choice of the $|J|$ largest eigenvalues.*

This result generalizes the results in Demange and Laroque (1995) and Athanasoulis and Shiller (2000) to the case with belief disagreements, $\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}} \neq \mathbf{0}$. Importantly, the expressions (24) and (26) show that financial innovation is partly driven by the size and the nature of traders' belief disagreements. The size of the belief disagreements, $\left|\left|(\Lambda^{\mathbf{v}})^{-1/2}\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}\right|\right|$, (along with the risk aversion coefficients, θ_i) determine to what extent endogenous innovation is driven by the speculation motive for trade as opposed to risk sharing. Assuming that this term is significant, the nature of the belief disagreements, $\frac{(\Lambda^{\mathbf{v}})^{-1/2}\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{||(\Lambda^{\mathbf{v}})^{-1/2}\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}||}$, bias the choice of assets towards those

that maximize the opportunities for speculation.

The next result characterizes the optimal asset design further in two extreme cases: when traders have common beliefs, and when their belief disagreements are very large.

Theorem 3 (Optimal Asset Design and Portfolio Risks). *Consider a collection of economies, $\{\mathcal{E}_K\}_{K\in\mathbb{R}_+}$, which are identical except for beliefs given by $\boldsymbol{\mu}^{\mathbf{v}}_{i,K} = K\boldsymbol{\mu}^{\mathbf{v}}_i$ for all i. For each economy $\mathcal{E}_K$, suppose the matrix in* (26) *is non-singular with distinct eigenvalues.*[7] *Let $\Omega_K(\cdot)$ denote the average variance and $\mathbf{A}_K$ denote the optimal asset design (characterized in Theorem 2) for economy $\mathcal{E}_K$.*

(i) If $K = 0$, then the market maker innovates assets that minimize *the average variance:*

$$\mathbf{A}_0 \in \arg\min_{\hat{\mathbf{A}}} \Omega_0\left(\hat{\mathbf{A}}\right) \text{ subject to } (25).$$

For the next two parts, suppose there exists at least two traders with different beliefs, i.e., $\boldsymbol{\mu}^{\mathbf{v}}_i \neq \boldsymbol{\mu}^{\mathbf{v}}_{\bar{\imath}}$ for some $i, \bar{\imath} \in I$. Let $\Omega_K(\emptyset)$ denote the average variance without any assets.

(ii) As $K \to \infty$, the market maker innovates assets that maximize *the average variance:*[8]

$$\lim_{K\to\infty} \mathbf{A}_K \in \arg\max_{\hat{\mathbf{A}}} \left(\lim_{K\to\infty} \frac{1}{K^2}\Omega_K\left(\hat{\mathbf{A}}\right)\right) \text{ subject to } (25). \tag{27}$$

(iii) For any $\eta > 0$, there exists $\overline{K}_\eta$ such that if $K > \overline{K}_\eta$, then endogenous financial innovation increases the average variance at least by η:

$$\Omega_K(\mathbf{A}_K) \geq \Omega_K(\emptyset) + \eta.$$

Without belief disagreements, the market maker innovates assets that minimize average portfolio risks in this economy, as illustrated by the first part of the theorem. The second part provides a sharp contrast to this traditional view. When traders' belief disagreements are large, the market maker innovates assets that *maximize* average portfolio risks, completely disregarding the risk sharing motive for innovation. Thus, belief disagreements change the nature of financial innovation as well as its impact on portfolio risks.

The third part considers the intermediate cases in which both the risk sharing and speculation considerations play a role in financial innovation. As long as traders' belief disagreements are sufficiently large, the speculation force dominates and the endogenous innovation increases the average variance by an arbitrary amount. This part complements the main result, Theorem 1, by identifying sufficient conditions under which assets that increase portfolio risks endogenously emerge in this economy.

[7] The assumption of distinct eigenvalues can be relaxed at the expense of additional notation.

[8] The scale factor, $\frac{1}{K^2}$, in (27) ensures that the objective function remains bounded in the limit, so that the optimization problem is well defined.

7 Financial Innovation and Empirical Portfolio Risks

The analysis so far described the effect of financial innovation on traders' portfolio risks calculated according to their own beliefs [cf. Eq. (15)]. However, to take the model to the data, it is necessary to consider an *empirical* version of traders' portfolio risks, that is, the risks that would be reflected ex-post in the data. This section shows that, under a slight strengthening of assumption (A1), the main result, Theorem 1, continues to hold after replacing the perceived variances in its statement with empirical variances.

In this section, suppose that the underlying risks, $\mathbf{v}$, have an empirical (or ex-post realized) distribution denoted by $N\left(\boldsymbol{\mu}^{\mathbf{v}}_{emp}, \Lambda^{\mathbf{v}}_{emp}\right)$. Suppose also that traders' beliefs are related to the empirical distribution. In particular, traders know the empirical variance, $\Lambda^{\mathbf{v}}_{emp}$, but they do not know the empirical mean, $\boldsymbol{\mu}^{\mathbf{v}}_{emp}$. Trader i has a prior belief for the empirical mean parameter, $\boldsymbol{\mu}^{\mathbf{v}}_{emp}$, given by the Normal distribution, $N\left(\boldsymbol{\mu}^{\mathbf{v}}_{i}, \Lambda^{\mathbf{v}}_{i}\right)$. The following assumption about beliefs simplifies the analysis in this section:

Assumption (A1$^{\mathrm{S}}$). For each i, $\Lambda^{\mathbf{v}}_{i} = \tau^{-1}\Lambda^{\mathbf{v}}_{emp}$ for some constant $\tau > 0$.

This assumption implies the earlier assumption (A1), because the trader's marginal belief for $\mathbf{v}$ can be written as:

$$N\left(\boldsymbol{\mu}^{\mathbf{v}}_{i}, \Lambda^{\mathbf{v}}\right), \text{ where } \Lambda^{\mathbf{v}} = \Lambda^{\mathbf{v}}_{emp} + \Lambda^{\mathbf{v}}_{i} = \left(1 + \frac{1}{\tau}\right)\Lambda^{\mathbf{v}}_{emp}.$$

The stronger assumption (A1$^{\mathrm{S}}$) is useful because it ensures that there is a linear wedge between traders' (common) perceived variance, $\Lambda^{\mathbf{v}}$, and the empirical variance, $\Lambda^{\mathbf{v}}_{emp}$. The size of the wedge is controlled by the parameter, τ, which captures the precision of traders' beliefs. Intuitively, traders' perceived uncertainty for $\mathbf{v}$ is greater than the empirical uncertainty of $\mathbf{v}$ because they face additional parameter uncertainty.

Note that the empirical and perceived variances coincide when $\tau = \infty$, i.e., when traders' beliefs are very precise. Thus, in this special case, all of the results of earlier sections apply for the empirical variance as well as the perceived variance of net worths. When $\tau < \infty$, it is not immediately clear that the analogues of earlier results would hold for the empirical variance. The rest of this section shows that this is the case. The following result establishes that, assuming $\tau = \infty$ is without loss of generality as long as the risk aversion coefficients, $\{\theta_i\}_i$, are appropriately adjusted. To state the result, define the *empirical average variance* as:

$$\Omega_{emp} = \frac{1}{|I|}\sum_{i \in I}\frac{\theta_i}{\overline{\theta}} var_{emp}\left(n_i\right). \tag{28}$$

Define also the *empirical uninsurable variance*, Ω^{R}_{emp} as the solution to problem (16) with Ω in the optimization replaced by Ω_{emp}. Finally, define the *empirical speculative variance* as the residual, $\Omega^{S}_{emp} = \Omega_{emp} - \Omega^{R}_{emp}$.

Lemma 3. *Consider an economy $\mathcal{E}(J)$ that satisfies assumption (A1^S). Consider an alternative economy, $\mathcal{E}_{emp}(J)$, which is identical to $\mathcal{E}(J)$ except for two aspects: (i) traders' precision parameter is adjusted according to $\tau_{emp} = \infty$, and (ii) traders' risk aversion coefficients are also adjusted according to:*

$$\theta_{emp,i} = \left(1 + \frac{1}{\tau}\right)\theta_i \text{ for each } i. \tag{29}$$

Then,

(i) The equilibrium, $\left(\mathbf{p}, \left(\mathbf{x}_1, .., \mathbf{x}_{|I|}\right)\right)$, is the same in economies $\mathcal{E}(J)$ and $\mathcal{E}_{emp}(J)$.

(ii) The empirical average variance and its components, $\Omega_{emp}, \Omega^R_{emp}, \Omega^S_{emp}$, are the same in economies $\mathcal{E}(J)$ and $\mathcal{E}_{emp}(J)$.

For a proof of this result, consider the demand expression in Eq. (10). Inspecting this equation for both economies, $\mathcal{E}_{emp}(J)$ and $\mathcal{E}(J)$, shows that the demand functions are the same. It follows that the equilibrium portfolios are also identical, proving part (i). Intuitively, traders in economy $\mathcal{E}(J)$ are reluctant to take risky positions not only because they are risk averse but also because they face additional parameter uncertainty. This makes them effectively more risk averse, as captured by the adjustment in (29). In fact, $\theta_{emp,i}$ could be viewed as trader i's effective coefficient of risk aversion.

Next note that the empirical variance of a trader's net worth is also the same in economies $\mathcal{E}(J)$ and $\mathcal{E}_{emp}(J)$, because the differences between these economies do not concern the empirical distributions. In addition, the adjustment of the risk aversion coefficient in (29) does not affect the relative weights used in averaging the variances of net worths [cf. Eq. (28)]. It follows that the empirical average variance and its components are the same in the two economies, proving part (ii).

Lemma 3 is useful because in the alternative economy, $\mathcal{E}_{emp}(J)$, the empirical and the perceived variances coincide. Thus, applying the earlier results for this economy characterizes the effect of financial innovation on the empirical variances in the original economy, $\mathcal{E}(J)$. In particular, the following result follows as a corollary of Lemma 3 and Theorem 1.

Theorem 4 (Financial Innovation and Empirical Portfolio Risks). *Consider an economy $\mathcal{E}(J)$ that satisfies assumption (A1^S), and let J_O and J_N respectively denote the set of old and new assets. Financial innovation always decreases the empirical uninsurable variance and increases the empirical speculative variance, that is:*

$$\Omega^R_{emp}(J_O \cup J_N) \leq \Omega^R_{emp}(J_O) \text{ and } \Omega^S_{emp}(J_O \cup J_N) \geq \Omega^S_{emp}(J_O).$$

A similar argument implies that the analogues of the results in Section 6 hold for empirical variances. As suggested by these results, under assumption (A1^S), the effects of financial innovation can be analyzed by assuming $\tau = \infty$ (and appropriately adjusting the risk aversion coefficients) so that the perceived and empirical variances coincide.

8 A Quantitative Exploration

The results in the previous sections have theoretically established that belief disagreements, when they are sufficiently large, change the nature of financial innovation as well as its effect on portfolio risks. A natural question is how large belief disagreements should be to make these results practically relevant. To address this question, this section considers a calibration of the model in the context of the national income markets, first proposed by Shiller (1993), and analyzed in detail by Athanasoulis and Shiller (2001).

Assets whose payoffs are linked to (various combinations of) national incomes could in principle facilitate the sharing of income risks among different countries. Athanasoulis and Shiller (2001) characterize the optimal design of such assets. They also calibrate their model for G7 countries, and argue that the innovation of a couple of these assets would lead to large welfare gains in view of the reduction in individuals' income and consumption risks. I first replicate their empirical results by mapping their model (and calibration) to this framework. I then show that, with reasonable amounts of belief disagreements, the new assets proposed by Athanasoulis and Shiller (2001) would have the unintended consequence of *increasing* individuals' income and consumption risks. Finally, I consider endogenous financial innovation in this setting and illustrate that, with belief disagreements, new assets that would be designed by a profit seeking market maker are different than those proposed by Athanasoulis and Shiller (2001).

8.1 Replicating Athanasoulis and Shiller's (2001) results in this framework

Athanasoulis and Shiller (2001) consider a dynamic risk sharing model with CARA preferences and Normal shocks. Individuals with uncertain incomes are allowed to trade a fixed number of assets whose payoffs are linear combinations of their incomes. Athanasoulis and Shiller (2001) characterize the optimal design of these assets and the effect of their introduction on risk sharing.

In view of the CARA preferences, certain aspects of Athanasoulis and Shiller's (2001) dynamic model are isomorphic to a static model. In particular, the equilibrium portfolios of risky assets are identical to the portfolios in a static model in which individuals face the same income shocks. Moreover, the CARA preferences ensure that the variance of each individual's consumption is the same as the variance of her net income, that is, her income excluding capital gains from her asset holdings (but including dividend gains). Given the equivalence for the portfolios of risky assets, the variance of net income in the dynamic model is equal to the variance of net worth in the static model. Thus, the variance of net worth in the static model accurately describes both income and consumption risks in the dynamic model.[9] In the rest

[9] The dynamic aspects of the model are useful to analyze the precautionary savings motive and to determine the equilibrium interest rates, but they do not play a role in analyzing consumption risks. The proof of the equivalence results between the dynamic and the static models (along with a complete solution of the dynamic model with belief disagreements) is in Appendix B which is available on request.

of this section, I present the results in terms of consumption risks.

To make the mapping between models more precise, consider the empirical part of Athanasoulis and Shiller (2001) in which they analyze the income shocks in G7 countries. They assume that the per-capita income of a country $c \in C = \{1, .., |C|\}$ evolves exogenously according to:

$$y_t(c) - y_{t-1}(c) = \alpha_{t-1}(c) + v_t(c).$$

Here, $\alpha_{t-1}(c)$ is predetermined at date $t-1$, and $v_t(c)$ is a zero-mean random variable which captures the income shock for country c between dates $t-1$ and t. In addition, the income shocks, $\mathbf{v}_t = \{v_t(c)\}_c$, are assumed to be i.i.d. and uncorrelated over time (but shocks to different countries are allowed to be correlated). Let $N\left(\mathbf{0}, \mathbf{\Lambda}^{\mathbf{v}}_{emp}\right)$ denote the empirical variance of $\mathbf{v}_t$.

To map this analysis to my framework, consider an economy in which the underlying risks correspond to the income shocks, $\mathbf{v}$ (where the time index is dropped to simplify the notation). Denote the individuals in country c with $I(c)$, so that $I = \cup_{c=1}^{|C|} I(c)$ corresponds to the set of all individuals. Athanasoulis and Shiller (2001) assume that individuals have the same coefficients of risk aversion, i.e., $\theta_i = \theta$ for each i. They also simplify the analysis by assuming that individuals in the same country experience the same income shocks.[10] This implies that the income shock of an individual $i(c)$ in economy c is given by $\left(\mathbf{W}_{i(c)}\right)' \mathbf{v}$, where

$$\mathbf{W}_{i(c),m} = \begin{cases} 1, & \text{if } c = m, \\ 0, & \text{otherwise.} \end{cases}$$

It remains to specify the beliefs in this economy. Athanasoulis and Shiller (2001) assume that individuals have homogeneous (and rational) beliefs for the income shocks, $\mathbf{v}$. In contrast, I consider the setup in the previous section in which traders know the variance, $\mathbf{\Lambda}^{\mathbf{v}}_{emp}$, but they may disagree about the mean, $\boldsymbol{\mu}^{\mathbf{v}}_{emp}$. I also adopt assumption (A1$^{\mathrm{S}}$), which ensures that the precision of traders' beliefs is captured by the single parameter, τ. To simplify the exposition, I assume $\tau = \infty$ so that the empirical and the perceived variances coincide, i.e., $\mathbf{\Lambda}^{\mathbf{v}}_{emp} = \mathbf{\Lambda}^{\mathbf{v}}$. In view of Lemma 3, this is without loss of generality as long as the parameters, $\{\theta_i\}_i$, are interpreted as effective coefficients of risk aversion [cf. Eq. (29)].

In addition, given any individual i and country c, I assume that the mean of the individual's belief for the country's income shock is an i.i.d. draw from a Normal random variable:

$$\mu^{\mathbf{v}}_i(c) \sim N\left(0, \left(\sigma^{\mu(c)}\right)^2\right).$$

The assumption that a trader's beliefs for different countries are independent simplifies the analysis, but otherwise does not play an important role. With this specification, the amount

[10] While clearly counterfactual, this assumption enables them to focus on international risk sharing and to take advantage of the higher quality national income data.

of belief disagreements for country c is captured by the parameter, $\sigma^{\mu(c)}$: the cross sectional standard deviation of the beliefs for the mean of country c's income shock. It is useful to normalize this parameter with the standard deviation of the income shock,

$$\delta^{v(c)} = \frac{\sigma^{\mu(c)}}{\sigma^{v(c)}}.$$

With this normalization, $\delta^{v(c)}$ provides a measure of belief disagreements that is independent of the linear transformations of $v(c)$. I also simplify the exposition by assuming that the amount of belief disagreements are the same for all G7 countries, that is:

$$\delta^{v(c)} = \delta^{v} \text{ for some constant } \delta^{v} \geq 0.$$

The model in Athanasoulis and Shiller (2001) corresponds to the special case, $\delta^{v} = 0$. I analyze the robustness of their results by considering the cases in which δ^{v} is positive but small.

For the other parameters of the model, I use exactly the estimates and calibration of Athanasoulis and Shiller (2001). To estimate the variance matrix, $\Lambda^{\mathbf{v}}$, they propose a spatial correlation model which yields the following structural form:

$$(\Lambda^{\mathbf{v}})^{c,c'} = \begin{cases} \exp\left(\alpha^{world}\right) + \exp\left(\alpha^{country}\right) + \exp\left(\alpha^{spatial}\right), & \text{if } c = c' \\ \exp\left(\alpha^{spatial}\right)\exp\left(-\gamma d\left(c,c'\right)\right), & \text{if } c \neq c' \end{cases}.$$

Here, $\alpha^{world}, \alpha^{country}, \alpha^{spatial}$ capture standard deviations of respectively world-wide shocks, country specific shocks, and spatial shocks that are partially correlated across countries. The strength of this spatial correlation depends on the parameter, γ, and the geographic distance, $d(c,c')$, between the two countries. Athanasoulis and Shiller (2001) estimate the parameters, $\alpha^{world}, \alpha^{country}, \alpha^{spatial}, \gamma$, using per-capita gross domestic products in 1985 U.S. dollars from the Penn World Table over the years 1950-1992. I use their parameter estimates along with data on distances to reconstruct their estimate for the variance matrix, $\Lambda^{\mathbf{v}}$. Finally, Athanasoulis and Shiller (2001) consider a relative risk aversion coefficient of three, $\theta^{relative} = 3$, as representing "a consensus by many who work in this literature." This enables them to calibrate the absolute risk aversion coefficient as $\theta = \frac{\theta^{relative}}{\overline{y}} = 0.000203$, where $\overline{y} = \$14783.43$ denotes the average per-capita income of G7 countries in 1992. As a benchmark, I choose the same calibration as the (effective) coefficient of absolute risk aversion in my setting. I discuss the effect of alternative calibrations of this parameter later in this section.

Finally, consider the asset design, $\mathbf{A}$, in this economy. Athanasoulis and Shiller (2001) assume that the assets are designed by a planner to maximize a social welfare function. The top panel of Figure 1 illustrates this optimal asset design when $|J| = 2$. The most important asset to create resembles an income swap between the US and Japan. Intuitively, this asset enables risk sharing between the individuals in the US and Japan. The model picks the US and Japan because these are large countries whose income shocks are relatively less correlated

	$A^1_c(\times 10^{-4})$	$A^2_c(\times 10^{-4})$	$x^1_{i(c)}$	$x^2_{i(c)}$	$A^1_c/Pop_c(\times 10^{-9})$	$A^2_c/Pop_c(\times 10^{-9})$
Canada	1.66	-0.14	-58.58	3.54	6.05	-0.53
the US	37.22	-11.50	-141.24	30.32	14.60	-4.51
Japan	-20.41	-29.77	159.26	161.45	-16.46	-24.01
France	-4.23	10.17	71.28	-119.19	-7.37	17.73
Germany	-5.43	11.41	80.64	-117.84	-8.33	17.53
Italy	-4.80	9.97	80.34	-115.90	-8.30	17.24
the UK	-4.02	9.87	67.24	-114.69	-6.95	17.06

	Risk exposure of individuals in the US before innovation	With two assets	With complete markets
Beta_Canada	0.00	-0.02	0.04
Beta_US	1.00	0.44	0.40
Beta_Japan	0.00	0.20	0.19
Beta_France	0.00	0.09	0.09
Beta_Germany	0.00	0.11	0.10
Beta_Italy	0.00	0.10	0.09
Beta_UK	0.00	0.09	0.09
Std of cons. (1985 dollars)	363.99	315.41	314.36

Figure 1: The top table illustrates the asset design and the equilibrium portfolios for the benchmark without belief disagremeents (with two assets). The last two columns display the asset design normalized by the country populations, $|I_c|$, for comparison with Table 1 of Athanasoulis and Shiller (2001). The bottom table shows the effect of financial innovation on consumption risks in the US. The columns display the slope coefficients in the following regression, $Consumption = \alpha + \sum_{c=1}^{7} \beta_c v_c$, which has a perfect fit in the model.

(since they are geographically far), which increases the benefits from risk sharing. The second most important asset also resembles an income swap, this time between Japan and the core EU region, for a similar reason.

When there are no belief disagreements, i.e., $\delta^v = 0$, my analysis replicates the results in Athanasoulis and Shiller (2001). First, the endogenous asset design characterized in Section 6 is the same as in Athanasoulis and Shiller (2001). In particular, without belief disagreements, a profit seeking market maker would choose the same set of assets as the social planner in Athanasoulis and Shiller (2001).

Second, the individuals' equilibrium holdings of risky assets are identical in both settings. The top table of Figure 1 illustrates these equilibrium portfolios (cf. Table 1 of Athanasoulis and Shiller, 2001). Much of the trade in the first asset is among the individuals in Japan and the US who take the opposite positions to diversify their income risks. Similarly, much of the trade in the second asset is among the individuals in Japan and the core Euro region.

Third, the portfolio risks of individuals in this setting is the same as the consumption risks of individuals in Athanasoulis and Shiller (2001). The second panel of Figure 1 illustrates these consumption risks for a sample country, the US. Before financial innovation, the consumption of individuals in the US has an exposure of one to the US income shock, v_{US}, and an exposure of zero to the income shocks of other countries. Trading the new financial assets enables the individuals to reduce their exposure to the US income shock by taking on some exposure to the

	Std of cons. in 1985 dollars before innovation	After innovation of two assets	With complete markets
Canada	363.99	351.87	314.36
the US	363.99	315.41	314.36
Japan	363.99	314.64	314.36
France	363.99	342.16	314.36
Germany	363.99	341.83	314.36
Italy	363.99	342.58	314.36
the UK	363.99	344.11	314.36

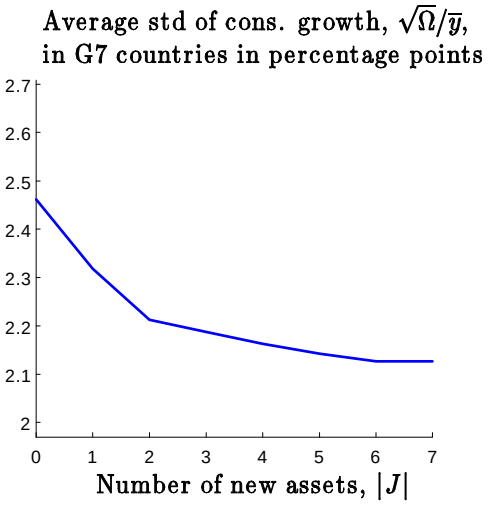

Figure 2: The left table illustrates the effect of financial innovation on consumption risks in different countries. The right table plots the average standard deviation of consumption growth, $\sqrt{\Omega}/\overline{y}$, as a function of the number of new assets.

income shocks of other countries (in particular, Japan). Consequently, the individuals in the US are able to diversify and reduce their consumption risks. According to Athanasoulis and Shiller's (2001) estimates, the individuals are able to reduce the standard deviation of their consumption from about \$364 to \$315.4 (in 1985 dollars) by trading two assets. Introducing additional assets reduces risks further but there are diminishing returns. Introducing all seven assets, which would complete the international markets, would reduce this measure to \$314.4.

The left table of Figure 2 illustrates the effect of the introduction of new assets on the standard deviation of consumption in each country. With two assets, the US and Japan gain the most but consumption risks decline in all countries. Guided by the analysis in earlier sections, I also consider an average measure for consumption risks, $\sqrt{\Omega}/\overline{y}$: The quadratic average standard deviation of consumption divided by the per-capita income, $\overline{y} = \$14783.43$. Note that $\sqrt{\Omega}/\overline{y}$ is an average of the standard deviation of consumption *growth* over individuals in G7 countries. The right panel of Figure 2 illustrates that $\sqrt{\Omega}/\overline{y}$ declines from 2.46% to 2.21% with two assets, and it declines further to 2.13% with complete markets.

I next consider the robustness of these results to the presence of belief disagreements, $\delta^v > 0$. As a first step, I take the asset design in Athanasoulis and Shiller (2001) as exogenously given, which is useful to see the direct impact of belief disagreements on consumption risks. I then analyze the effect of belief disagreements, $\delta^v > 0$, on the endogenous asset design.

	A^1_c ($\times 10^{-4}$)	A^2_c ($\times 10^{-4}$)	$x^{R,1}_{i(c)}$	$x^{R,2}_{i(c)}$	$x^{S,1}_{c'\text{-optimist}}$	$x^{S,2}_{c'\text{-optimist}}$
Canada	1.66	-0.14	-58.58	3.54	5.96	-0.52
the US	37.22	-11.50	-141.24	30.32	133.53	-41.25
Japan	-20.41	-29.77	159.26	161.45	-73.22	-106.81
France	-4.23	10.17	71.28	-119.19	-15.16	36.49
Germany	-5.43	11.41	80.64	-117.84	-19.47	40.94
Italy	-4.80	9.97	80.34	-115.90	-17.22	35.75
the UK	-4.02	9.87	67.24	-114.69	-14.42	35.40

	Risk exposure of individuals in the US before innovation	With two assets, for a moderate	With two assets, for a US–optimist	With complete markets, for a US–optimist
Beta_Canada	0.00	-0.02	-0.00	-0.57
Beta_US	1.00	0.44	0.98	1.38
Beta_Japan	0.00	0.20	0.05	0.12
Beta_France	0.00	0.09	-0.01	0.03
Beta_Germany	0.00	0.11	-0.01	0.06
Beta_Italy	0.00	0.10	-0.01	0.05
Beta_UK	0.00	0.09	-0.01	0.01
Std cons. (1985 $s)	363.99	315.41	360.86	402.94

Figure 3: The top table illustrates the equilibrium portfolios with belief disagreements ($\delta^v = 0.02$). The third and the fourth columns illustrate the risk sharing portfolios in country c. These are also the complete portfolios of *moderates*, who have the mean belief for each country. The last two columns illustrate the speculative portfolios of c'-*optimists*, whose beliefs for country c' are one standard deviation above the mean and whose beliefs for all other countries are equal to the mean. The bottom table illustrates the consumption risks in the US for moderates and US-optimists (see Figure 1 for a detailed explanation).

8.2 Effect of belief disagreements on consumption risks

Suppose there are belief disagreements parameterized by $\delta^v = 0.02$. Recall that $\delta^v = \frac{\sigma^{\mu(c)}}{\sigma^{v(c)}}$ is the cross sectional standard deviation of beliefs for the mean of the income shock relative to the standard deviation of the same shock. Note also that δ^v provides a measure of disagreements that is independent of linear transformations of the income shock. Thus, δ^v is the same as δ on the growth rate of per-capita income.[11] Athanasoulis and Shiller (2001) estimate the standard deviation of the growth rate of yearly per-capita income (before financial innovation) as 2.46%. Hence, assuming $\delta^v = 0.02$ implies a cross-sectional standard deviation of beliefs for the same variable given by 0.0492%. Thus, this assumption is satisfied when two randomly chosen individuals' beliefs for the growth rate of per-capita income of a G7 country differ by about 0.05%. Belief disagreements at this order of magnitude do not seem unreasonable. I next show that this level of belief disagreements is sufficient to overturn the risk sharing implications of Athanasoulis and Shiller (2001). I then turn to a more systematic calibration of δ^v.

To illustrate the effect of belief disagreements, I start by considering the portfolio allocations

[11] More specifically, note that the growth rate of income per-capita in year t can be written as

$$g_{t-1,t}^{\text{per-capita income}}(c) = \frac{y_t(c) - y_{t-1}(c)}{y_{t-1}(c)} = \frac{\alpha_{t-1}(c)}{y_{t-1}(c)} + \frac{1}{y_{t-1}(c)} v_t(c),$$

which is a linear transformation of $v_t(c)$.

for a couple individuals whose beliefs are specified exactly and who are representative of a larger class of individuals with similar beliefs. Let a *moderate* be someone whose belief for the income shocks of all countries is the same as the mean belief. In contrast, define a c'-*optimist* as someone whose belief for the income shock of country c' is exactly one standard deviation above the mean belief, and whose belief for all other income shocks is equal to the mean belief. The top table of Figure 3 shows the portfolio allocations for moderates and c'-optimists when there are two new assets. A moderate holds exactly the risk sharing portfolio in her respective country, which is illustrated in the third and the fourth columns. Thus, her portfolio is unaffected by the presence of belief disagreements. In contrast, a c'-optimist who lives in country c combines the risk sharing portfolio for country c with her speculative portfolio, which is illustrated in the last two columns. Note that, for a US-optimists and a Japan-optimist, the speculative portfolio is comparable in magnitude to (and often larger than) the risk sharing portfolios. Consequently, the complete portfolio of a US-optimist or a Japan-optimist is significantly influenced by the speculation motive for trade.

The bottom table in Figure 3 shows the consumption risks for moderates and US-optimists who live in the US. Moderates continue to diversify their risks in this case, as illustrated by the second column. However, the third column illustrates that a US-optimist does not diversify her risks. The risk sharing considerations would require this individual to take a short position in the first asset. However, her optimism about the US induces her to take a long position. When there are two assets, the two forces almost perfectly balance for this individual (cf. the top table), who remains exposed to the US income shock (cf. the bottom table). The last column illustrates the case with complete markets, in which case the speculation motive for trade dominates for a US-optimist. In this case, this individual has a greater exposure to the US income shock, and consequently greater consumption risks, than before financial innovation.

This analysis illustrates that, with belief disagreements, financial innovation has a different qualitative effect on the income risks of moderates and optimists. Guided by the earlier analysis, I assess the overall effect by considering $\sqrt{\Omega}$, which provides a quadratic average of the standard deviation of consumption over individuals in G7 countries. The left table in Figure 4 shows that financial innovation increases this average for each country, $\sqrt{\Omega_c}$. The right panel of Figure 4 plots the overall average standard deviation of consumption growth, $\sqrt{\Omega}/\overline{y}$. In contrast with the case without belief disagreements (the dashed line), financial innovation increases $\sqrt{\Omega}/\overline{y}$ (the solid line).[12]

8.2.1 The intuition for the quantitative results

As illustrated by Figure 4, the main result of this section is equivalent to saying that the decrease in the uninsurable portfolio risks, $\sqrt{\Omega^R(\emptyset) - \Omega^R(J)}/\overline{y}$, is quantitatively smaller than

[12] In fact, an even smaller level of belief disagreements, $\delta^v = 0.015$, is sufficient to offset completely the effect of two new assets on the overall average, $\sqrt{\Omega}$. A greater level, e.g., $\delta^v = 0.02$, is sufficient to offset the effect separately for each country average, $\sqrt{\Omega_c}$.

	Average std of cons. in 1985 dollars before innovation	After innovation of two assets	With complete markets
Canada	363.99	408.42	596.38
the US	363.99	377.46	596.38
Japan	363.99	376.82	596.38
France	363.99	400.08	596.38
Germany	363.99	399.80	596.38
Italy	363.99	400.44	596.38
the UK	363.99	401.75	596.38

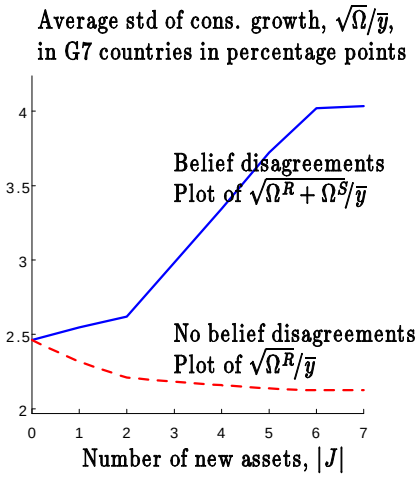

Figure 4: The left table illustrates the effect of financial innovation on consumption risks in different countries, for the case with belief disagreements parameterized by $\delta^v = 0.02$. The right table plots the average standard deviation of consumption growth, $\sqrt{\Omega}/\overline{y}$, as a function of the number of new assets.

the increase in the speculative portfolio risks, $\sqrt{\Omega^S(J) - \Omega^S(\emptyset)}/\overline{y} = \sqrt{\Omega^S(J)}/\overline{y}$. To understand this result, it is useful to analyze the determinants of each term in turn.

Note that the reduction in the uninsurable portfolio risks, $\sqrt{\Omega^R(\emptyset) - \Omega^R(J)}/\overline{y}$, is the same as the reduction in portfolio risks, $\sqrt{\Omega(\emptyset) - \Omega(J)}/\overline{y}$, in the benchmark case without belief disagreements. From Figure 2, the standard deviation of consumption growth, $\sqrt{\Omega(J)}/\overline{y}$, decreases from 2.46% to 2.21% for the case with two assets, and to 2.13% for the complete markets case. This implies a reduction in the uninsurable portfolio risks given by:

$$\frac{\sqrt{\Omega^R(\emptyset) - \Omega^R(J)}}{\overline{y}} = \begin{cases} 1.08\%, & \text{with two assets,} \\ 1.24\%, & \text{with complete markets,} \end{cases} \tag{30}$$

which is small in magnitude. Intuitively, the income risks in developed countries are small relative to their average incomes. Moreover, income risks are correlated across developed countries. Thus, even if these risks are perfectly diversified, the reduction in the standard deviation of consumption growth, 2.46% to 2.13%, is small in magnitude. Consequently, the reduction in the uninsurable portfolio risks, $\frac{\sqrt{\Omega^R(\emptyset) - \Omega^R(J)}}{\overline{y}}$, is also small.

In view of Eq. (30), the main result of this section holds as long as the increase in speculative portfolio risks, $\sqrt{\Omega^S(J)}/\overline{y}$, is greater than 1.24%. To understand when this is the case, recall from Eqs. (22) – (23) that the speculative portfolio risks of an individual i can be written in

terms of her speculative Sharpe ratio:

$$\sigma_i^S = \frac{1}{\theta^{relative}} \sqrt{\tilde{\boldsymbol{\mu}}_i \Lambda^{-1} \tilde{\boldsymbol{\mu}}_i} = \frac{1}{\theta^{relative}} Sharpe_i^S. \tag{31}$$

Here, recall that $Sharpe_i^S$ is the Sharpe ratio that trader i would perceive in a hypothetical scenario in which there are no background risks. In this scenario, a textbook result of mean-variance analysis applies and characterizes the standard deviation of the speculative portfolio as in (31). Recall also that the term, $\sqrt{\Omega^S(J)}/\overline{y}$, is a (quadratic) average of the expression in (31) over all individuals. Thus, given $\theta^{relative} = 3$ and the threshold 1.24%, it suffices to have that individuals' speculative Sharpe ratios "on average" exceed 3.72%.

Sharpe ratios at this order of magnitude do not seem unreasonable. To see this, it is useful to characterize the Sharpe ratio for a particular individual, the US-optimist. To this end, first consider a benchmark case in which there is only one asset whose payoff is equal to v_{US}. In this case, the Sharpe ratio of a US-optimist has a simple expression:

$$Sharpe_i^S = \frac{\tilde{\mu}^{v(US)}_{\text{US-optimist}}}{\sigma^{v(US)}} = \frac{\sigma^{\mu(US)}}{\sigma^{v(US)}} = \delta^v.$$

Here, the first equality uses Eq. (31), the second equality uses the definition of a US-optimist, and the last equality uses the definition of δ^v. Intuitively, the expected excess payoff perceived by the US-optimist is equal to one cross-sectional standard deviation, $\sigma^{\mu(US)}$, while the risk of the payoff is equal to the standard deviation, $\sigma^{v(US)}$. Thus, the Sharpe ratio, $\frac{\sigma^{\mu(US)}}{\sigma^{v(US)}}$, in this case is exactly equal to the parameter, $\delta^v = 2\%$. For the assets proposed by Athanasoulis and Shiller (2001), the Sharpe ratio of a US-optimist is even greater than in this benchmark, as illustrated by the following table:

	$Sharpe_i^S$ for a US-optimist
Benchmark case with single asset that pays v_{US}	$\delta^v = 2\%$
Athanasoulis and Shiller (2001) assets, $\lvert J\rvert = 1$	2.71%
Athanasoulis and Shiller (2001) assets, $\lvert J\rvert = 2$	2.84%
Athanasoulis and Shiller (2001) assets, $\lvert J\rvert = 7$	3.80%

Intuitively, the first asset, the US-Japan swap, not only enables the US-optimist to bet on her belief, but it also provides an endogenous hedge since the income shocks of the US and Japan are positively correlated. In view of the hedge-more/bet-more effect, the Sharpe ratio of the speculative portfolio in this case exceeds the benchmark case, as illustrated by the above table. Increasing the number of assets provides the US-optimist with increasingly higher speculative Sharpe ratios. With complete markets, the US-optimist is able to obtain a Sharpe ratio of 3.80%. This is greater than 3.72%, which is the threshold required for speculative risks to dominate the reduction in uninsurable risks (on average). This observation provides an intuition for the earlier result illustrated in Figure 3: For a US-optimist living in the US,

complete markets generate higher consumption risks than before financial innovation.

The speculative Sharpe ratio exceeds the required threshold, 3.72%, not just for the US-optimist but also many other similarly optimistic (or pessimistic) individuals: Individuals whose beliefs for the US income shock are more than one standard deviation above mean, individuals who are sufficiently pessimistic about the US income shock, individuals who are sufficiently optimistic or pessimistic about other countries' income shocks, and so on. Consequently, $\delta^v = 2\%$ is sufficient to ensure that the speculative Sharpe ratios exceed 3.72% on average. This in turn provides the intuition for why the increase in speculative portfolio risks, $\sqrt{\Omega^S(J)}/\overline{y}$, dominate the reduction in the uninsurable portfolio risks, $\frac{\sqrt{\Omega^R(\emptyset)-\Omega^R(J)}}{\overline{y}}$.

8.2.2 Calibrating the level of belief disagreements

How reasonable is it to assume $\delta^v = 2\%$? For a systematic calibration of δ^v, I consider the Philadelphia Fed's *Survey of Professional Forecasters* (SPF) on macroeconomic forecasts in the US. This survey provides (among other things) quarterly data for forecasters' beliefs about the growth rate of the US gross domestic product (GDP) in the subsequent one year period, $g^{\text{GDP}}_{t-1,t}(\text{US})$, where $t-1$ denotes the forecast date. To use this data, suppose the population growth between $t-1$ and t is a constant that is known at date $t-1$.[13] With this assumption, δ^v can be calibrated by using data on beliefs about the growth rate of GDP (as opposed to GDP per capita). The SPF data shows that the cross-sectional interquartile range of forecasts of $g^{\text{GDP}}_{t-1,t}(\text{US})$ is on average given by 0.70% between the first quarter of 1992 and the third quarter of 2011. This implies a cross-sectional standard deviation of beliefs for the mean of $g^{\text{GDP}}_{t-1,t}(\text{US})$ given by $\frac{0.70\%}{1.35} = 0.52\%$ (under a Normality assumption for beliefs). Over the same period, the historical standard deviation of $g^{\text{GDP}}_{t-1,t}(\text{US})$ is given by 2.08%. Thus, the belief disagreements between professional forecasters imply:

$$\delta^v = \frac{0.52\%}{2.08\%} = 0.25.$$

In particular, the implied δ^v is an order of magnitude larger than the required level, 2%.

A caveat is order at this point. Note that the traders in the model agree to disagree about their beliefs since belief differences are modeled with heterogeneous priors. Thus, the δ^v that is relevant for the model concerns traders' beliefs after they learn about the beliefs of all other traders. However, the δ^v calibrated from the SPF does not necessarily correspond to this measure. The forecasters' beliefs arguably reflect a combination of forecasters' interpretation of publicly available data as well as their private information. The disagreements based on interpretation are similar to having different priors, but disagreements based on private information are not. To the extent that the disagreements are driven by private information,

[13] This assumption is reasonable in this context because the growth rate of population is much less volatile than the growth rate of GDP. The yearly population growth rate of the US between 1992 and 2009 averaged 1.05% with a standard deviation of 0.14% (cf. Penn World Tables). In contrast, the yearly growth rate of GDP between 1992 and 2011 averaged 2.58% with a standard deviation of 2.08% (cf. NIPA tables).

the forecasters would update their beliefs after learning about the forecasts of others. For example, a forecaster with an extremely optimistic belief might revise her forecast downward after learning about the forecasts of others. If this is the case, then the measure of belief disagreements calibrated from the SPF overestimates the true δ^v. While it is difficult to adjust δ^v for forecasters' private information, the fact that the unadjusted δ^v is an order of magnitude larger than necessary suggests that the results would continue to hold for reasonable amounts of private information.

8.2.3 Calibrating the coefficient of relative risk aversion

As suggested by the analysis in Section 8.2.1, the calibration for the coefficient of relative risk aversion, $\theta^{relative} = 3$, plays an important role for the quantitative results. Increasing the parameter, $\theta^{relative}$, reduces the speculative risks [cf. Eqs. (18) and (31)], but it does not affect the uninsurable variance [cf. Eq. (17)]. Intuitively, when individuals are more risk averse, they speculate less but they continue to share their income risks. For an extreme case, suppose $\theta^{relative} = 50$ which is the level of the relative risk aversion parameter that is necessary to rationalize the equity premium puzzle in a CRRA environment, but which is also considered to be implausibly high (Campbell, 2000). In this case, if belief disagreements are calibrated to be as high as implied by the SPF data, $\delta^v = 0.25$, then the average standard deviation of consumption growth, $\sqrt{\Omega}/\overline{y}$ remains roughly unchanged for two assets and it increases for a greater number of assets, illustrating the robustness of the results.

8.3 Effect of belief disagreements on endogenous financial innovation

The analysis so far took the assets in Athanasoulis and Shiller (2001) as exogenous. However, as emphasized in Section 6, belief disagreements also influence the asset design. The left table of Figure 5 illustrates the first two assets which a profit seeking market maker would introduce according to the characterization in Theorem 3. Note that the endogenous asset design with $\delta^v = 0.02$ (and $\theta^{relative} = 3$) is very different than the case without belief disagreements. Intuitively, the market maker's incentives are driven not only by individuals' surplus from risk sharing but also from their (perceived) surplus from speculation. Moreover, $\delta^v = 0.02$ is sufficient to ensure that the speculation force dominates. More specifically, increasing δ^v further leaves the optimal asset design in Figure 5 qualitatively unchanged.

The endogenous asset design in this case is determined mainly by the nature of individuals' belief disagreements. To keep the analysis simple, I have assumed that an individual's beliefs for different countries are independent and that the level of disagreements on each country is the same, i.e., $\delta^v(c) = \delta^v$ for all c. These assumptions about the nature of belief disagreements are admittedly arbitrary. Nonetheless, it is a useful exercise to think about what they imply for the endogenous asset design. Figure 5 illustrates that the most important two assets in this case also resemble income swaps, albeit different ones than the case without belief disagreements.

	Planner A^1_c (x10^{-4})	Planner A^2_c (x10^{-4})	Market A^1_c (x10^{-4})	Market A^2_c (x10^{-4})
Canada	1.66	-0.14	-43.85	-4.59
the US	37.22	-11.50	49.48	3.97
Japan	-20.41	-29.77	-2.33	-0.62
France	-4.23	10.17	3.65	-50.54
Germany	-5.43	11.41	0.64	-2.61
Italy	-4.80	9.97	-2.80	9.65
the UK	-4.02	9.87	-4.57	44.79

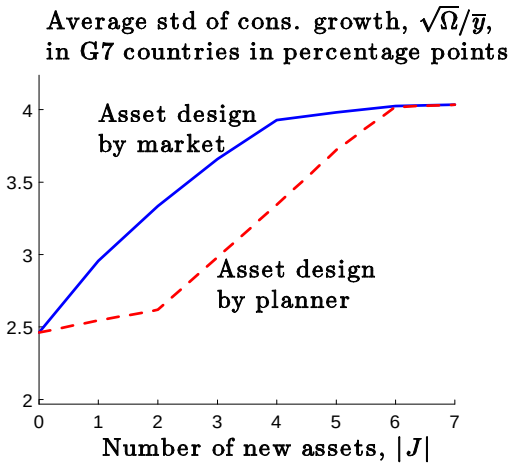

Figure 5: The left table illustrates the optimal asset design without belief disagreements (planner columns) and with belief disagreements parameterized by $\delta^v = 0.02$ (market column). The right table plots for each of these cases the average standard deviation of consumption growth, $\sqrt{\Omega}/\overline{y}$, as a function of the number of new assets.

In particular, the market maker prefers to introduce the US-Canada swap and the UK-France swap.

To see the intuition behind these innovations, consider the first swap between the US and Canada. Among the G7 countries, the US and Canada have the highest correlation of income shocks according to the Athanasoulis and Shiller's (2001) estimates. A swap between these countries is attractive to speculators because it endogenously provides a hedge for their bets. For example, consider the US-optimist who is optimistic about the US income shock but who has the mean belief for the income shock of Canada. By investing in the US-Canada swap, this individual is able to take a relatively pure bet on her optimistic view about the US. When she is able to take purer bets, she also takes larger bets in view of the hedge-more/bet-more effect. This in turn implies larger profits for the market maker who intermediates these trades, inducing it to innovate the US-Canada swap first. Similarly, the second introduced swap is between France and the UK, whose income shocks are the second most correlated among the G7 countries according to the Athanasoulis and Shiller's (2001) estimates.

Recall that, without belief disagreements, the optimal assets are swaps between large countries or regions whose income shocks are the least correlated (e.g., Japan and the US). In contrast, the optimal assets in this section are swaps between countries whose income shocks are the *most* correlated. While this result is driven by somewhat arbitrary assumptions about the nature of belief disagreements, it illustrates how belief disagreements could fundamentally

change the nature of financial innovation.

Finally, consider the effect of the endogeneity of financial innovation on consumption risks. The right panel of Figure 5 plots the average consumption variance as a function of new assets when the asset design is endogenous. Note that new assets lead to a greater consumption variance in this case compared to the earlier case in which assets are exogenously set as those considered by Athanasoulis and Shiller (2001). Intuitively, the fact that new assets are directed towards maximizing the opportunities for speculation provides an additional force that tends to increase risks. When this force is present, financial innovation has a greater quantitative impact on portfolio and consumption risks.

9 Conclusion

This paper theoretically analyzed the effect of financial innovation on portfolio risks in a standard mean-variance setting in which both the speculation and risk sharing forces are present. In this framework, I have defined the average variance of traders' net worths as a natural measure of portfolio risks. I have also decomposed the average variance into two components: the uninsurable variance, defined as the variance that would obtain if there were no belief disagreements, and the speculative variance, defined as the residual amount of variance that results from speculative trades based on belief disagreements. My main result characterized the effect of financial innovation on both components of the average variance. Financial innovation always reduces the uninsurable variance through the traditional channels of diversification and the efficient transfer of risks. However, financial innovation also *always* increases the speculative variance, through two distinct economic channels. First, new assets generate new disagreements. Second, new assets amplify traders' speculation on existing disagreements. The second channel stems from an important economic force, the hedge-more/bet-more effect: Traders use new assets to hedge their bets on existing assets (i.e., to take purer bets), which enables them to take larger bets. In view of this effect (and the second channel), my main result shows that new assets increase the speculative variance even if traders completely agree about their payoffs.

I have also analyzed endogenous financial innovation by considering a profit seeking market maker who introduces the new assets for which it subsequently serves as the intermediary. The market maker's profits are proportional to traders' perceived surplus from trading the new assets. Consequently, traders' speculative motive for trade as well as their risk sharing motive for trade creates innovation incentives for the market maker. In particular, the endogenous set of assets depends on the size and the nature of belief disagreements, in addition to the risk sharing possibilities emphasized by the previous literature.

A natural question is how large belief disagreements should be to make these results practically relevant. I considered a calibration of the model in the context of the national income markets for G7 countries analyzed by Athanasoulis and Shiller (2001). For reasonable levels

of belief disagreements, the assets proposed by Athanasoulis and Shiller (2001) would *increase* the consumption risks of individuals in G7 countries. This is because income risks constitute a relatively small fraction of income in G7 countries. Moreover, income risks are correlated across the G7 countries. Hence, even if these risks were perfectly diversified, the reduction in the standard deviation of consumption is a relatively small fraction of income. In contrast, for reasonable levels of the coefficient of relative risk aversion and belief disagreements, individuals are willing to bet a larger fraction of their incomes in their pursuit of speculative gains. I have also shown that the endogenous asset design is typically very different than in Athanasoulis and Shiller (2001) because new assets are directed towards increasing the opportunities for speculation rather than risk sharing.

A number of avenues for future research are opened by this paper. The first open question concerns the policy implications of the results. This paper characterized the positive effects of belief disagreements on portfolio risks and financial innovation, but it has been quiet about the normative aspects. This is because the equilibrium in this paper is Pareto efficient despite the fact that trade in new securities may increase the average variance of traders' net worths. In view of belief disagreements, each trader perceives a large expected payoff from her speculative portfolio that justifies the additional risks that she is taking. Despite the Pareto efficiency of equilibrium, it is important to analyze policy implications for at least two reasons. First, while this paper illustrates the results in a standard mean-variance framework without externalities, the main mechanisms apply also in richer environments that may feature externalities. For example, if the traders are financial intermediaries that do not fully internalize the social costs of their losses (or bankruptcies), then an increase in speculation may lead to a Pareto inefficiency. I develop a model along these lines in a companion paper. Second, the notion of Pareto efficiency with heterogeneous priors is somewhat unsatisfactory. This is because while all traders perceive a large expected return, at most one of these expectations can be correct.[14] The analysis of the appropriate welfare notion in these settings is a fascinating topic which I leave for future work.

A second avenue of new research concerns the evolution of belief disagreements. This paper analyzed financial innovation in a model in which traders' beliefs are exogenously fixed. In a companion paper, I consider financial innovation in a model in which traders' beliefs evolve over time. The novel feature of this dynamic setting is that traders learn from past observations of asset payoffs. Under appropriate assumptions, traders' belief disagreements on a given set of new assets disappear in the long run. Thus, in these environments, there is a tension between the short run and the long run effects of new assets on portfolio risks. The resolution of this tension has important implications for the optimal regulation of financial innovation, which I leave for future research.

[14] This point was also noted by Stiglitz (1989), who wrote: "there are real difficulties in interpreting the welfare losses associated with impeding trades based on incorrect expectations."

A Appendix: Omitted Proofs and Derivations

A.1 Omitted proofs for Section 3

Proof of Lemma 1. Recall that the objective function for Problem (16) is given by

$$\Omega = \frac{1}{|I|}\sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\left(\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i + \mathbf{x}_i'\Lambda\mathbf{x}_i + 2\mathbf{x}_i'\boldsymbol{\lambda}_i\right). \tag{A.1}$$

The first order conditions are given by:

$$\Lambda\mathbf{x}_i + \boldsymbol{\lambda}_i = \boldsymbol{\gamma}\frac{\bar{\theta}}{\theta_i} \text{ for each } i \in I,$$

where $\boldsymbol{\gamma} \in \mathbb{R}^{|J_E|}$ is a vector of Lagrange multipliers. Note that $\mathbf{x}_i^R = -\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i$ satisfies these first order conditions for the Lagrange multiplier $\boldsymbol{\gamma} = \left(\sum_{i\in I}\boldsymbol{\lambda}_i\right)/|I|$. This shows that $\left\{\mathbf{x}_i^R\right\}_i$ is the unique solution to Problem (16).

Proof of Lemma 2. Plugging in $\mathbf{x}_i^R = -\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i$ into the objective function $(A.1)$, the optimal value, Ω^R, is given by:

$$\begin{aligned}
\Omega^R &= \frac{1}{|I|}\sum_i\frac{\theta_i}{\bar{\theta}}\left(\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i + \tilde{\boldsymbol{\lambda}}_i'\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i\right) - \frac{2}{|I|}\sum_i\tilde{\boldsymbol{\lambda}}_i'\Lambda^{-1}\frac{\theta_i\boldsymbol{\lambda}_i}{\bar{\theta}},\\
&= \frac{1}{|I|}\sum_i\frac{\theta_i}{\bar{\theta}}\left(\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i + \tilde{\boldsymbol{\lambda}}_i'\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i\right) - \frac{2}{|I|}\sum_i\tilde{\boldsymbol{\lambda}}_i'\Lambda^{-1}\frac{\theta_i\tilde{\boldsymbol{\lambda}}_i}{\bar{\theta}}\\
&= \frac{1}{|I|}\sum_i\frac{\theta_i}{\bar{\theta}}\left(\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i - \tilde{\boldsymbol{\lambda}}_i'\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i\right).
\end{aligned}$$

Here, the second line uses the fact that $\sum_i\tilde{\boldsymbol{\lambda}}_i = 0$ to replace $\frac{\theta_i\boldsymbol{\lambda}_i}{\bar{\theta}}$ with its deviation from average, $\frac{\theta_i\tilde{\boldsymbol{\lambda}}_i}{\bar{\theta}}$. This completes the derivation of Eq. (17).

To derive Eq. (18), first consider the expression $|I|\left(\Omega - \frac{1}{|I|}\sum_i\frac{\theta_i}{\bar{\theta}}\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i\right)$. Using the definition of the average variance in (15), this expression can be written as:

$$\begin{aligned}
\sum_i\frac{\theta_i}{\bar{\theta}}\mathbf{x}_i'\Lambda\mathbf{x}_i + 2\sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\mathbf{x}_i'\boldsymbol{\lambda}_i &= \sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\right)'\Lambda^{-1}\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\right) + 2\sum_{i\in I}\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\right)'\Lambda^{-1}\frac{\theta_i\boldsymbol{\lambda}_i}{\bar{\theta}}\\
&= \sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\left[\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\right)'\Lambda^{-1}\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\right) + 2\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\right)'\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i\right]\\
&= \sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\left[\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i\right)'\Lambda^{-1}\left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} + \tilde{\boldsymbol{\lambda}}_i\right)\right]\\
&= \sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}'\Lambda^{-1}\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \sum_{i\in I}\frac{\theta_i}{\bar{\theta}}\tilde{\boldsymbol{\lambda}}_i'\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_i.
\end{aligned}$$

Here, the first line substitutes for the portfolio demands from (12); the second line replaces $\frac{\theta_i \boldsymbol{\lambda}_i}{\theta}$ with its deviation from average, $\frac{\theta_i \tilde{\boldsymbol{\lambda}}_i}{\theta}$ (as in the first part of the proof); and the next two lines follow by simple algebra. Next, using the fact that the last line equals $|I| \left(\Omega - \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\theta} \mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i \right)$, the average variance can be written as:

$$\Omega = \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\overline{\theta}} \left(\mathbf{W}_i' \Lambda^{\mathbf{v}} \mathbf{W}_i - \tilde{\boldsymbol{\lambda}}_i' \Lambda^{-1} \tilde{\boldsymbol{\lambda}}_i \right) + \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\overline{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}' \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}.$$

Using the definition of Ω^R in (17), it follows that the speculative variance is given by the expression in (18).

A.2 Omitted proofs for Section 5

Proof of Theorem 1. Part (i). By definition, Ω^R, is the optimal value of the minimization problem (16). Financial innovation expands the constraint set of this problem. Thus, it also decreases the optimal value, proving $\Omega^R (J_O \cup J_N) \leq \Omega^R (J_O)$.

Part (ii). The proof is provided in the text. Here, to demonstrate an alternative approach, I provide a second proof using matrix algebra. First note that the definition of Ω^S in (18) implies that

$$\begin{aligned} \Omega^S (J) - \Omega^S (J_O) &= \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\overline{\theta}} \left(\frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}' \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i} - \frac{\tilde{\boldsymbol{\mu}}_i (J_O)}{\theta_i}' \Lambda (J_O)^{-1} \frac{\tilde{\boldsymbol{\mu}}_i (J_O)}{\theta_i} \right) \\ &= \frac{1}{|I|} \sum_{i \in I} \frac{\theta_i}{\overline{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}' \left(\Lambda^{-1} - \begin{bmatrix} \Lambda (J_O)^{-1} & 0 \\ 0 & 0 \end{bmatrix} \right) \frac{\tilde{\boldsymbol{\mu}}_i}{\theta_i}. \end{aligned} \tag{A.2}$$

I next claim that the matrix in the parenthesis,

$$\Lambda^{-1} - \begin{bmatrix} \Lambda (J_O)^{-1} & 0 \\ 0 & 0 \end{bmatrix}, \tag{A.3}$$

is positive semidefinite. In view of this claim, Eq. $(A.2)$ implies $\Omega^S (J) \geq \Omega^S (J_O)$, providing an alternative proof for this part.

This claim follows from Lemma 5.16 in Horn and Johnson (2007). This lemma considers a positive definite matrix partitioned into submatrices of arbitrary dimension, $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix}$. It shows that the matrix A^{-1} is weakly greater than the matrix $\begin{bmatrix} (A_{11})^{-1} & 0 \\ 0 & 0 \end{bmatrix}$ in positive semidefinite order. This in turn implies that the matrix, $A^{-1} - \begin{bmatrix} (A_{11})^{-1} & 0 \\ 0 & 0 \end{bmatrix}$, is positive semidefinite. Invoking this lemma for $A = \Lambda$ and $A_{11} = \Lambda (J_O)$ shows that the matrix in Eq. $(A.3)$ is positive semidefinite, completing the alternative proof.

Part (iii). Consider an economy, $\mathcal{E} (J)$, with two properties: (i) There is no disagreement

on existing assets, i.e., $\mu_i^j = \mu_{\bar{\imath}}^j$ for each $j \in J_O$ and $i, \bar{\imath} \in I$, and (ii) There is some disagreement about new assets, i.e., $\mu_i^j \neq \mu_{\bar{\imath}}^j$ for some $j \in J_N$ and $i, \bar{\imath} \in I$. Let $\mathcal{E}_K(J)$ denote the economy which is identical except that traders' beliefs for the underlying risks, $\mathbf{v}$, are scaled by the factor K, that is: $\boldsymbol{\mu}_{i,K}^{\mathbf{v}} = K\boldsymbol{\mu}_i^{\mathbf{v}}$ for each i. I claim that there exists $K > 0$ such that the result holds for the economy $\mathcal{E}_K(J)$, that is:

$$\Omega_K(J_O \cup J_N) > \Omega_K(J_O) + \eta. \tag{A.4}$$

To show this claim, first note that by assumption:

$$\tilde{\boldsymbol{\mu}}_i^{J_O} = \mathbf{0} \text{ and } \tilde{\boldsymbol{\mu}}_i^{J_N} \neq \mathbf{0}. \tag{A.5}$$

Here $\mathbf{z}^{\hat{J}}$ denotes the vector $\left(z^1, .., z^{\hat{J}}\right)$ and $\mathbf{0}$ denote the zero vector. Next note that traders' beliefs for asset payoffs in economy $\mathcal{E}_K(J)$ are scaled by a factor of K, i.e., $\boldsymbol{\mu}_{i,K} = K\boldsymbol{\mu}_i$. Using $(A.5)$ and (18), this implies:

$$\Omega_K^S(J_O) = K^2\Omega^S(J_O) = 0 \text{ and } \Omega_K^S(J) = K^2\Omega^S(J) > 0.$$

These expressions further imply:

$$\lim_{K\to\infty} \Omega_K^S(J) - \Omega_K^S(J_O) = \infty.$$

Finally, note from Eq. (17) that K does not affect the uninsurable variance Ω^R. In particular:

$$\Omega_K^R(J_O) - \Omega_K^R(J) = \Omega^R(J_O) - \Omega^R(J).$$

Using the last two displayed equations, it follows that there exists a sufficiently large $K > 0$ such that the inequality in $(A.4)$ holds, proving the claim.

A.3 Omitted proofs for Section 6

Derivation of the Market Maker's Profit. First note that trader i's payoff from rejecting the market maker's offer is the certainty equivalent payoff from her endowment:

$$e + \mathbf{W}_i'\boldsymbol{\mu}_i^{\mathbf{v}} - \frac{\theta_i}{2}\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i. \tag{A.6}$$

Next consider trader i's certainty equivalent payoff after trading the assets. Using Eq. (12), traders' net worth, n_i, can be written as:

$$n_i = e - \mathbf{x}_i'\mathbf{p} + \left[\mathbf{W}_i + \mathbf{A}\Lambda^{-1}\left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A})\right)\right]'\mathbf{v}.$$

The certainty equivalent of this expression is given by:

$$e - \mathbf{x}_i'\mathbf{p} + \mathbf{W}_i'\boldsymbol{\mu}_i^{\mathbf{v}} + \left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A})\right)' \Lambda^{-1}\boldsymbol{\mu}_i \tag{A.7}$$
$$-\frac{\theta_i}{2}\mathbf{W}_i'\Lambda^{\mathbf{v}}\mathbf{W}_i - \frac{\boldsymbol{\theta}_i}{2}\left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A})\right)' \Lambda^{-1}\left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A})\right) - \theta_i\left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \tilde{\boldsymbol{\lambda}}_i(\mathbf{A})\right)\Lambda^{-1}\boldsymbol{\lambda}_i(\mathbf{A}).$$

Since the fixed fee makes the trader indifferent, it is equal to the difference of the expression in $(A.7)$ from the expression in $(A.6)$. That is:

Proof of Theorem 2. To prove the result it is useful to consider the market maker's optimization problem in terms of a linear transformation of assets, $\hat{\mathbf{A}} = (\Lambda^{\mathbf{v}})^{1/2}\mathbf{A}$, where $(\Lambda^{\mathbf{v}})^{1/2}$ is the unique positive definite square root matrix of $\Lambda^{\mathbf{v}}$. Note that choosing $\hat{\mathbf{A}}$ is equivalent to choosing $\mathbf{A}$. The normalizations in (25) can be written in terms of $\hat{\mathbf{A}}$ as:

$$\hat{\mathbf{A}}'\hat{\mathbf{A}} = \mathbf{I}_{|J|}, \qquad \text{and} \qquad \tilde{\mathbf{A}}_1^j \geq 0 \text{ for each } j. \tag{A.8}$$

After using the normalization $\Lambda = \mathbf{I}_{|J|}$ and substituting $\hat{\mathbf{A}}$ for $\mathbf{A}$, the expected profit in (24) can also be written as:

$$\begin{aligned}
\sum_i \pi_i\left(\hat{\mathbf{A}}\right) &= \sum_i \frac{\theta_i}{2}\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)'\hat{\mathbf{A}}\hat{\mathbf{A}}'\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right), \\
&= tr\left(\hat{\mathbf{A}}'\mathbf{M}\hat{\mathbf{A}}\right) = \sum_j \left(\hat{\mathbf{A}}^j\right)'\mathbf{M}\hat{\mathbf{A}}^j, \\
\text{where } \mathbf{M} &= \left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)\left((\Lambda^{\mathbf{v}})^{-1/2}\frac{\tilde{\boldsymbol{\mu}}_i^{\mathbf{v}}}{\theta_i} - (\Lambda^{\mathbf{v}})^{1/2}\tilde{\mathbf{W}}_i\right)'.
\end{aligned} \tag{A.9}$$

Here, the second line uses the matrix identity $tr(XY) = tr(YX)$ and the linearity of the trace operator, and the last line defines the $m \times m$ matrix, $\mathbf{M}$. Thus, the market maker's problem reduces to choosing $\hat{\mathbf{A}} = (\Lambda^{\mathbf{v}})^{1/2}\mathbf{A}$ to maximize $(A.9)$ subject to the normalizations in $(A.8)$.

Next note that the first normalization in (25) implies:

$$\left(\hat{\mathbf{A}}^j\right)'\hat{\mathbf{A}}^j = 1 \text{ for each } j. \tag{A.10}$$

Consider the alternative problem of choosing $\hat{\mathbf{A}}$ to maximize the expression in $(A.9)$ subject to the relaxed constraint in $(A.10)$. The first order conditions for this problem are given by

$$\mathbf{M}\hat{\mathbf{A}} = \gamma^j\hat{\mathbf{A}}^j \text{ for each } j,$$

where $\gamma^j \in \mathbb{R}_+$ are Lagrange multipliers. From this expression, it follows that $\left\{\hat{\mathbf{A}}^j\right\}_j$ correspond to eigenvectors of the matrix, $\mathbf{M}$, and $\left\{\gamma^j\right\}_j$ correspond to eigenvalues. Plugging the

first order condition into Eq. $(A.9)$, the expected profit can be written as:

$$\sum_i \pi_i\left(\hat{\mathbf{A}}\right) = \sum_j \gamma^j \left(\hat{\mathbf{A}}^j\right)' \hat{\mathbf{A}}^j = \sum_j \gamma^j.$$

It follows that the objective value will be maximized if and only if $\left\{\gamma^j\right\}_j$ correspond to the $|J|$ largest eigenvalues of the matrix, $\mathbf{M}$. If the $|J|$ largest eigenvalues are unique, then the optimum vectors, $\hat{\mathbf{A}} = \left\{\hat{\mathbf{A}}^j\right\}_j$, are uniquely characterized as the corresponding eigenvectors which have length 1 [cf. Eq. $(A.10)$] and which satisfy the sign convention in $(A.8)$. If the $|J|$ largest eigenvalues are not unique, then the same argument shows that the vectors, $\left\{\hat{\mathbf{A}}^j\right\}_j$, are uniquely determined up to a choice of these eigenvalues.

Finally, consider the original problem of maximizing the expression in $(A.9)$ subject to the stronger condition, $\hat{\mathbf{A}}'\hat{\mathbf{A}} = \mathbf{I}$. Since $\mathbf{M}$ is a symmetric matrix, its eigenvectors are orthogonal. This implies that the solution, $\left\{\hat{\mathbf{A}}^j\right\}_j$, to the alternative problem is in the constraint set of the original problem. Since the latter problem has a stronger constraint, it follows that the solutions to the two problems are the same, completing the proof.

Proof of Theorem 3. Part (i). Note that $\tilde{\boldsymbol{\mu}}_{i,0}(\mathbf{A}) = \mathbf{A}'\tilde{\boldsymbol{\mu}}^{\mathbf{v}}_{i,0} = 0$ for any $\mathbf{A}$. This implies that the expected profit in (24) is given by $\sum_i \frac{\boldsymbol{\theta}_i}{2}\tilde{\boldsymbol{\lambda}}_{\mathbf{i}}(\mathbf{A})'\Lambda^{-1}\tilde{\boldsymbol{\lambda}}_{\mathbf{i}}(\mathbf{A})$. From Eq. (17), this expression is equal to $c_1 - c_2\Omega^R(\mathbf{A})$ for some constant c_1 and positive constant c_2. Thus, maximizing $\sum_i \pi_i(\mathbf{A})$ is equivalent to minimizing $\Omega^R(\mathbf{A})$. Finally, note from Eq. (18), that $\Omega_0^S(\mathbf{A}) = 0$ for any $\mathbf{A}$. This further implies $\Omega(\mathbf{A}) = \Omega^R(\mathbf{A})$, proving that the market maker innovates assets that minimize $\Omega(\mathbf{A})$.

Part (ii). Consider the following objective function:

$$\frac{1}{K^2}\sum_i \pi_{i,K}(\mathbf{A}), \tag{A.11}$$

which is just a scaling of the expected profit in (24). In particular, maximizing this expression is equivalent to maximizing the expected profit. In view of Theorem 2, the optimal asset design, $\mathbf{A}_K$, is uniquely determined. This also implies that $\mathbf{A}_K$ is a continuous function. Since $\mathbf{A}_K$ is bounded [from the normalization (25)], it follows that $\lim_{K\to\infty}\mathbf{A}_K$ exists.

Note also that the limit of the objective function in $(A.11)$ can be calculated as:

$$\begin{aligned}\lim_{K\to\infty}\frac{1}{K^2}\sum_i \pi_{i,K}(\mathbf{A}) &= \lim_{K\to\infty}\sum_i \frac{\boldsymbol{\theta}_i}{2}\left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \frac{\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})}{K}\right)'\Lambda^{-1}\left(\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i} - \frac{\tilde{\boldsymbol{\lambda}}_i(\mathbf{A})}{K}\right)\\ &= \sum_i \frac{\theta_i}{2}\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})'}{\theta_i}\Lambda^{-1}\frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i},\end{aligned} \tag{A.12}$$

where the first line uses $\tilde{\boldsymbol{\mu}}_{i,K}(\mathbf{A}) = K\tilde{\boldsymbol{\mu}}_i(\mathbf{A})$ and the second line uses $\tilde{\boldsymbol{\mu}}_i(\mathbf{A}) = \mathbf{A}'\tilde{\boldsymbol{\mu}}^{\mathbf{v}}_i$. In

particular, the objective function remains bounded as $K \to \infty$. Thus, Berge's Maximum Theorem applies and implies that $\mathbf{A}_K$ is upper hemicontinuous in K over the extended set $\mathbb{R}_+ \cup \{\infty\}$. In particular, $\lim_{K\to\infty} \mathbf{A}_K$ maximizes the limit objective function in $(A.12)$ subject to the normalization, (25).

Finally, consider the limit of the average variance

$$\lim_{K\to\infty} \frac{\Omega_K\left(\hat{\mathbf{A}}\right)}{K^2} = \lim_{K\to\infty} \left(\frac{\Omega_K^S\left(\hat{\mathbf{A}}\right)}{K^2} + \frac{\Omega_K^R\left(\hat{\mathbf{A}}\right)}{K^2} \right) = \frac{1}{|I|} \sum_i \frac{\theta_i}{\bar{\theta}} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})'}{\theta_i} \Lambda^{-1} \frac{\tilde{\boldsymbol{\mu}}_i(\mathbf{A})}{\theta_i},$$

where the second equality follows from Eqs. (18) and (17). In view of Eq. $(A.12)$, it follows that $\lim_{K\to\infty} \mathbf{A}_K$ maximizes $\lim_{K\to\infty} \frac{1}{K^2}\Omega_K\left(\hat{\mathbf{A}}\right)$ subject to the normalization, (25), completing the proof.

Part (iii). The assumption that there are two traders with different beliefs implies that the optimum value of the problem in (27) is strictly positive. This further implies that $\lim_{K\to\infty} \Omega_K(\mathbf{A}_K) = \infty$. By the definition of the limit, there exists $\overline{K}_\eta \in \mathbb{R}_+$ such that $\Omega_K(\mathbf{A}_K) \geq \Omega_K(\emptyset) + \eta$ for each $K \geq \overline{K}_\eta$, completing the proof.

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