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MEANS-TESTED SUBSIDIES AND ECONOMIC PERFORMANCE SINCE 2007

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ABSTRACT

The aggregate neoclassical growth model – with means-tested subsidies whose replacement rates began rising at the end of 2007 as its only impulse – produces time series for aggregate labor usage, consumption, investment, and real GDP that closely resemble actual U.S. time series. Despite having no explicit financial market, the model has investment fall steeply during the recession not because of any distortions with the supply of capital, but merely because labor is falling and labor is complementary with capital in the production function. Through the lens of the model, the fact that real consumption fell significantly below trend during 2008 suggests that labor usage per capita is expected to remain well below pre-recession levels for several years.

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The market economies of the U.S. and other countries have dramatically cut back their labor usage, measured both in absolute terms and relative to real output. Roughly coincident with the collapse of labor usage was a crisis in financial markets. To what degree was the financial market crisis a reaction to, and anticipation of, the labor collapse? Or did the financial crisis create unemployment? Answers to these questions are important because they go to the heart of the recession's causes, and speak to the role of government in affecting the economy's path.

Limiting the supply of aggregate investment is one mechanism for financial markets to impact the rest of the economy, and in fact real investment fell through the first year and a half of this recession. In this case, one notable effect of financial crisis would be to reduce aggregate investment below (and increase consumption above) what is efficient given the labor employed. Alternatively, financial crisis or something else could reduce labor usage more directly, and, given the complementarity of labor and capital in production, investment would respond to put the capital stock on a path that is consistent with the lesser amount of labor usage. To contribute to the bigger questions posed above, this paper considers whether a single "distortionary" or "wedge" shock (akin to a change the rate of labor income taxation) to the labor market in a simple aggregate model – without explicit financial markets – would produce dynamics that resemble this recession's measured dynamics and, if so, how such a "labor wedge" recession would end. The results offer one barometer for gauging the relative importance of various explanations for the actual recession.

The aggregate implications of shocks to the labor market are interesting in their own right given the history of business cycles, and the emergence during this recession of a number of public and private sector actions that are expected to distort the labor market.

The Great Depression of the 1930's cannot be entirely attributed to a labor wedge, but Ohanian (2010) makes the case that a peaking of union influence was an important factor. Some recessions are associated with little, if any, labor wedge, but an extensive labor wedge literature beginning with Barro and King (1984) and as recent as Shimer (2010) emphasize the significance of labor wedges over several previous business cycles.[1]

During this recession, a variety of private and public sector actions emerged that would likely distort the labor market. The federal minimum wage was hiked three times in and around this recession. Labor market search frictions are arguably greater than usual. Prices fell, and perhaps wages did not adequately adjust with them. A large number of homeowners owed more on their mortgage than their house was worth, and both private and public sector renegotiations of the mortgage contracts have served as a massive implicit tax on earning during the recession because borrowers can expect their earnings to affect the amount that lenders will forgive (Mulligan, 2009b). Renegotiations of business debts (Jermann and Quadrini, 2009), consumer loans (Han and Li, 2007), student loans, and tax debts present debtors with similar disincentives. A new home buyers' $8000 tax credit was made available, but phased out as annual family income varied from $80,000 to $120,000. Other parts of the 2009 "stimulus law" increased the generosity of mean-tested subsidies like food stamps, and employment-tested subsidies like unemployment insurance. Congress considered various legislation that would raise marginal income tax rates, and would present Americans with new health benefits that would be phased out as a function of income.

The quantitative incentive effects of many, if not all, of these events are complex and varied, and might therefore seem beyond the reach of aggregate analysis. My approach here is to (a) select the large subset of these events that can be characterized as means-tested transfers either from the government or from lenders, and model their combination as a single subsidy program whose aggregate expenditure closely follows the combined expenditure of the actual subsidy programs and (b) model the labor supply of persons ineligible for the subsidies separately from the labor supply of the rest of the population, recognizing the two groups would likely supply different amounts of labor even in the absence of subsidies, and that the causality between subsidies and labor

[1] See also Parkin (1988), McGrattan (1994), and further references cited below.

market outcomes is in both directions. My approach has the potential dangers that two groups might not be enough to capture the heterogeneity in the actual population, and that the programs selected for analysis are either not inclusive enough – some omitted programs also push incentives in the same direction as the included ones – are too inclusive because some parts of them might not be means-tested. Nevertheless, the results can suggest whether or not the combination of several means-tested subsidy programs were a major factor affecting employment and other aggregate outcomes.

Figure 1 displays inflation-adjusted quarterly time series for four types of means-tested personal receipts: receipts from unemployment insurance, receipts from other means-tested government transfer programs,[2] "home retention actions" (that is, loan modifications, short sales, and deeds-in-lieu of foreclosure), and consumer loan charge-offs by commercial banks. Before the recession, housing collapse, and expansion of means-tested programs, the combination of these benefits was only about $40 billion (constant dollars) per quarter. The combination (hereafter, "combined subsidies") reached over $100 billion per quarter by the second half of 2009, or about triple of unemployment benefits by themselves.[3] This paper shows that this large increase in the amount of means-tested subsidies could have been associated with work disincentives that were large enough to generate changes in the major macro aggregates that are in the direction and amounts resembling the actual changes.

In modeling the recession as a transition to an increasingly distorted labor market, I assume that production and capital markets are always efficient (or at least as efficient as they were before the recession), so that their dynamics are understood as mere reactions to the rising work disincentives stemming from expanding means-tested subsidy

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[2] The means-tested programs included are: food stamps, SSI, state and local family assistance, general assistance, and energy assistance. Social security, Medicare, Medicaid, education, veterans benefits, and various medical, retirement and pension transfers are excluded because some of them are either (a) not means-tested, (b) spending significant sums on the elderly (below I explain more about the model's two population groups), and/or (c) may (as in the case of medical spending) be valued by the beneficiary at a lot less than program cost. A large majority of the increase shown in the chart is from unemployment insurance and federal spending on food stamps. Appendix II details Figure 1's data sources.

[3] A long literature (Meyer, 1990, is one of the classics) has recognized that unemployment benefits are a work disincentive, and a few papers have offered back-of-the-envelope calculations of the unemployment rate effect of extending unemployment benefits during this recession (Elsby, Hobijn, and Sahin, 2010 is an example). The contribution of my paper is to consider this recession's extended and expanded unemployment benefits in combination with several other means-tested subsidies in an equilibrium framework for evaluating effects on consumption, investment, wages, etc.

programs. In the short run, consumption, investment, and labor all decline, albeit in different proportions. Labor declines in the short run, because (by construction) the sole impulse is the rising work disincentive. Consumption declines because of the permanent income effect. Thanks to the legacy of a capital stock accumulated prior to the rising work disincentives, the marginal product of capital falls with the workforce, which creates an intertemporal substitution effect on consumption partially offsetting the permanent income effect. Thus, consumption declines less than labor in the short run, regardless of whether the new work disincentives are temporary or permanent. For the same reason that the marginal product of capital falls, the marginal and average products of labor initially rise.

If the new work disincentives are long lasting, the labor reduction will be long lasting and investment will be low for long enough to eventually reduce the capital stock by the same proportion as labor. Once the work disincentives stabilize at a higher level, the marginal product of capital can rise again, and reduce the intertemporal substitution effect that had mitigated the consumption decline. After enough time has passed with the new work disincentives in place, labor, consumption, and capital have all been reduced in the same proportions.

The paths for consumption and investment that are efficient in the model given a long-lasting work disincentive are compared with the monthly and quarterly aggregate time series since December 2007, when this recession began. The model and data agree that investment expenditure would fall 20-30 percent, although the data show investment dropping about three quarters sooner than predicted by the model. If labor is expected to be depressed for a long time to come, consumption dynamics in the model are similar to those in the data. In this sense, a large majority of this recession's consumption dynamics and investment decline were a reaction to, and anticipation of, low levels of labor usage.

Consistent with the model, the percentage consumption decline has so far been much less than the percentage decline in work hours. Total factor productivity has not significantly deviated from prior trends. Real wages per hour and labor productivity have risen in absolute terms, and relative to prior trends. A large and obvious "labor wedge" has emerged during this recession (Mulligan, 2009a; Ohanian, 2010).

If work disincentives are expected to eventually return to their pre-recession values in the not-too-distant future, consumption and capital will never get as low as labor while the disincentives last. Thus, under the assumption the recession is solely caused by a labor market distortion, the consumption and investment responses indicate how long the labor distortion and the low levels of labor usage are expected to remain. Through the lens of the model, actual consumption behavior indicates that labor usage will average at least five percent below the previous trend for the indefinite future (now it is about 10 percent below trend), and will not return to the previous trend for quite a long time. Even if the model were known to be correct, it is more difficult to predict, on the basis of aggregate behavior so far, whether the average 5 percent below trend will be achieved through still further declines in the short run followed by a subsequent increase back to trend, or whether labor will always remain significantly below its previous trend.

I. The Neoclassical Growth Model with Targeted Means-Tested Subsidies

Consider an economy with many identical families, each with many family members. Family members will ultimately differ in terms of whether and how much labor they supply to the market, and in their expectation that they will be eligible for a means-tested subsidy in the event that they are not working. In order to simplify a complex reality in which there are many means-tested subsidy programs each with its own eligibility rules, I partition the family into just two groups: "prime" members who have a probability $p > 0$ of receiving a means-tested subsidy in the event they are not working (more below about this probability), and "others" who are never eligible for means-tested subsidies. The relative size of the two groups, which I assume to be constant over time, is not relevant for my qualitative results or even many of my aggregate quantitative findings, but in cases that it does matter I measure the prime group as persons aged 25-64 because they are likely to have a recent history of employment covered by unemployment insurance, be heads of households with children, and/or have home mortgages and consumer credit that may be partly forgiven by lenders on the basis of "ability to pay."

Time is continuous. Gross output is produced with capital and prime labor, and is (exhaustively) used for market consumption goods and gross investment.

$$A n_t^{\alpha} k_t^{1-\alpha} = c_t + \dot{k}_t + \delta k_t \tag{1}$$

where n denotes prime labor input, k denotes capital input, and c denotes market consumption, each relative to a constant exponential trend that reflects the constant exogenous growth rates for population and technology (Barro and Sala-i-Martin, 2003). t is the time subscript and $A > 0$ and $\alpha \in (0,1)$ are constant technology parameters. Dots denote time derivatives, and $\delta > 0$ reflects the rate of capital depreciation as well as population growth and rates of technical progress.

When date t the flow of market consumption is c_t, prime members supply n_t units of labor, and other members supply m_t units of labor, the representative household's flow of utility u_t is:

$$u_t = \tfrac{\sigma}{\sigma-1} c_t^{(\sigma-1)/\sigma} - \gamma_n \tfrac{\eta}{\eta+1} \left(n_t - \beta m_t\right)^{(\eta+1)/\eta} - \gamma_m \tfrac{\eta}{\eta+1} m_t^{(\eta+1)/\eta} \tag{2}$$

where the positive constants σ, η, β, γ_n, and γ_m denote preference parameters. σ is the constant elasticity of substitution of consumption over time, and η is the constant Frisch elasticity of labor supply. In order to rule out unrealistically large intertemporal substitution effects, I make the weak assumption that $\sigma \leq 1/(1-\alpha)$.[4] For simplicity, and because of my lack of emphasis on the composition of employment, equation (2) assumes that the utility function is homothetic in the two labor amounts, and that the two types of labor enter additively in utility.[5] Households discount the utility flows at constant rate $\rho > 0$, which may also reflect exogenous growth rates of population and technology.

Firms rent the labor of prime family members at rate w_t in the labor market (market consumption c is the numeraire good). Prime members who work at date t supply one unit of labor and receive wage rate w_t. Prime members who do not work at

[4] For example, with $\alpha = 0.7$, this upper bound on the intertemporal substitution elasticity σ is 3.3.

[5] As a result of these assumptions, the equilibrium relative supply prices of n and m will be constant (and equal to β) and the composition of the workforce will be constant, at values that are independent of technology and public policy parameters. More generally, the composition of the workforce could be impacted in either direction by those parameters if the utility function were non-homothetic or the two types of labor were substitutes in production.

date t receive subsidy b_t with probability p_t and no subsidy with probability 1 - p_t; their expected replacement rate from the time t subsidy is $\tau_t \equiv p_t b_t / w_t$, which is assumed to be less than one for all t. Other family members work, if at all, producing household services, as indicated by the utility function (2). Assuming that households own the capital and rent it to firms at gross rental rate r, a household's dynamic budget constraint is:

$$n_t w_t + (\Gamma - n_t)\tau_t w_t + r_t k_t = c_t + \dot{k}_t + \delta k_t + L_t \tag{3}$$

where L_t denotes date t lump sum taxes, and Γ is the prime worker time endowment.

Given values for the scalar taste and technology parameters, a value for the initial capital stock k_0, and a time path for the replacement rate, a market equilibrium is a list of time paths on $t \geq 0$ for utility flows, consumption flows, capital, both types of labor, wage rates, capital rental rates, and lump sum taxes such that: the subsidy program's budget constraint $L_t = (\Gamma - n_t)\tau_t w_t$ balances at each date;[6] and the paths for utility, consumption, labor, and capital (a) maximize profits $A n_t^{\alpha} k_t^{1-\alpha} - w_t n_t - r_t k_t$ at each date and (b) maximize the present discounted value of utility $\int_0^{\infty} e^{-\rho t} u_t dt$ subject to (2) taking as given the time paths for factor rental rates, replacement rates, lump sum taxes, to the household's sequence of dynamic budget constraints (3), and to a no-Ponzi condition on their capital ownership.

Equilibrium factor rental rates equal their marginal products, so the equilibrium time paths for consumption, prime labor, and capital are the solution to a two-dimensional system of differential equations (1) and (4), plus the algebraic equation (5), whose boundary conditions are the initial capital stock and the usual transversality condition:

$$\dot{c}_t = \sigma \left[(1-\alpha) A (n_t / k_t)^{\alpha} - \delta - \rho \right] c_t \tag{4}$$

[6] Alternatively, the tax could be a flat-rate labor income tax without affecting the results as long as the flat-rate were constant over time. In fact, there have been few changes in the federal personal income tax since 2007. The federal payroll tax was cut by 2 percentage points as of January 1, 2011, but this may be approximately offset by income and sales taxes by states and localities, and in any case most of my data predates 2011.

$$n_t^{1/\eta} = \frac{1-\tau_t}{\gamma} \frac{\alpha A}{c_t^{1/\sigma}} (k_t / n_t)^{1-\alpha} \tag{5}$$

where the constant $\gamma > 0$ is a combination of the preference parameters η, β, γ_n, and γ_m. Differential equation (4) is the usual consumption Euler equation equating the intertemporal marginal rate of substitution of consumption to the net marginal product of capital. Algebraic equation (5) equates the marginal rate of substitution between consumption and prime labor to the marginal product of prime labor net of subsidy replacement. A replacement rate $\tau_t > 0$ is a labor distortion in the sense that it causes the marginal product of labor to differ from the marginal rate of substitution.

Many studies have put a wedge in condition (1) by letting the productivity parameter vary over time. Other studies have also put a wedge in the consumption Euler equation (4), perhaps with a rate of time preference or price of investment goods that varies over time. But the thesis of this paper is that a large fraction of this recession can be understood as a consequence of a time-varying labor wedge alone, and that much of that wedge has to do with subsidies available to the unemployed and financially distressed,[7] so I have omitted those possible sources of time-variation.

The functional form (5) for the marginal rate of substitution function implies that consumption and leisure are normal goods and has been occasionally used in the macroeconomics literature, although for different reasons. My purposes here are simplicity and a maximum of analytic results and to have consumption, capital, and labor all change in the same proportions in the long run.[8]

[7] A labor distortion or "labor wedge" has also been used to model labor market regulations (Mulligan, 2002, 2005), other market distortions (Gali, Gertler, and Lopez-Salido, 2007; Shimer, 2009), as well as errors in the specification of the marginal rate of substitution function (Parkin, 1988; Hall, 1997). These alternative sources of labor distortions may also be relevant in this recession, but this paper does not attempt to quantify them except perhaps as a residual labor wedge changes that cannot be attributed to subsidies.

[8] Another functional form commonly used in the real business cycle literature – Cobb-Douglas in consumption and leisure – has constant proportional long run effects on consumption and *leisure*. I have also obtained numerical paths derived from the Cobb-Douglas consumption-leisure specification (not shown here), and arguably these fit the data well too.

II. Dynamics of the Stationary System

If the labor distortion were constant over time, the dynamical system (1), (4), (5) would be stationary and saddle path stable, with only the saddle path satisfying the transversality condition. The stationary state of the system (c_{ss},k_{ss},n_{ss}) has a closed form solution:

$$k_{ss}^{1/\eta+1/\sigma} = \frac{1-\tau}{\gamma} \frac{\alpha}{\left(\delta+\rho\right)^{(1/\eta+1)/\alpha-1}\left(\alpha\delta+\rho\right)^{1/\sigma}} A^{(1/\eta+1)/\alpha}\left(1-\alpha\right)^{(1/\eta+1)/\alpha-1+1/\sigma}$$

$$(1-\alpha)A\left(n_{ss}/k_{ss}\right)^{\alpha} = \delta+\rho \tag{6}$$

$$c_{ss} = \frac{\alpha\delta+\rho}{1-\alpha}k_{ss}$$

The last two equations in (6) determine the steady state ratio of market consumption and prime work to the capital stock, and do not depend on the value for the replacement rate τ. Thus, a permanent increase in the replacement rate reduces the long run capital stock, and reduces market consumption and work in the same proportion.

When the replacement rate is constant, a phase diagram for saddle path stable systems describes the dynamics of the system from any initial capital stock. The $\dot{k}=0$ schedule in the [k,c] plane is implicitly defined by:

$$c = \left[AN(c,k;1-\tau)^{\alpha}-\delta\right]k$$

$$N(c,k;1-\tau)^{1-\alpha+1/\eta} \equiv \frac{1-\tau}{\gamma}\frac{\alpha A}{c^{1/\sigma}k^{1/\eta}} \tag{7}$$

where $N(c,k;1-\tau)$ is the labor-capital ratio that satisfies the labor market condition (5) for given values of consumption, capital, and the replacement rate. Along $\dot{k}=0$, market consumption equals net market output, so the schedule slopes up if and only if net market output increases with capital, taking into account the positive effect of capital on labor for a given marginal utility of consumption. The maximum of this schedule therefore occurs at a capital stock that exceeds the "golden rule" capital stock that maximizes net output for a given labor, which itself exceeds the steady state capital stock for which the marginal product of capital equals the rate of time preference ρ. A larger value for the replacement rate τ is associated with a $\dot{k}=0$ schedule that is lower at each value of k.

The $\dot{c}=0$ schedule is implicitly defined by:

$$N(c,k;1-\tau)^{\alpha}=\frac{\delta+\rho}{1-\alpha}\frac{1}{A} \tag{8}$$

Thus, the schedule slopes down and has elasticity equal to -σ/η. A larger value for the replacement rate τ is associated with a $\dot{c}=0$ schedule that is lower at each value of *k*.

Figure 2 shows $\dot{c}=0$ and $\dot{k}=0$ schedules and the implied dynamics of the system. When capital is below (above) its steady state value, there is an initial value for market consumption that is necessarily below (above) its steady state value so that the dynamics of the system asymtotically approach the steady state. Proposition 1 characterizes the stable manifold containing such paths.

Proposition 1. If $\sigma \in (0,(1-\alpha)^{-1})$, the stable manifold of the system slopes up in the [*k*,*c*] plane and crosses the ray from the origin from above.

Proof In the [*k*,*c*] plane, both the stable and unstable manifolds solve the ordinary differential equation

$$\begin{aligned} c'(k) &= \sigma\frac{(1-\alpha)AN(c(k),k;1-\tau)^{\alpha}-\delta-\rho}{AN(c(k),k;1-\tau)^{\alpha}-\delta-c(k)/k}\frac{c(k)}{k} \\ N(c(k),k;1-\tau)^{1-\alpha+1/\eta} &\equiv \frac{1-\tau}{\gamma}\frac{\alpha A}{c^{1/\sigma}k^{1/\eta}} \end{aligned} \tag{9}$$

The elasticity of these two manifolds in the neighborhood of the steady state can be found by using L'Hopital's rule and noting that the resulting quadratic equation is satisfied for both elasticities. One of the quadratic equation's solutions is in the interval (0,1),[9] while the other is in (-∞,1). The unstable manifold's elasticity corresponds to the smaller solution, which means that the elasticity of the stable manifold at the steady state is in (0,1). The steady state lies on the ray from the origin, which means that the stable manifold crosses that ray at the steady state from above.

For any capital stock below (above) its steady state value, the stable manifold is below (above) the $\dot{k}=0$ schedule, which implies that equation (9)'s denominator is positive (negative), respectively. At the steady state, the numerator is zero and the manifold slopes up. Because the net marginal product of capital term in the numerator

[9] To prove this, note that the large quadratic root increases with σ, is 0 at σ = 0, and is one at $\sigma = (1-\alpha)^{-1}$.

declines in both *c* and *k*, it would be positive (negative) even if $c'(k)$ were zero, which means that $c'(k)$ is positive for any capital stock less (greater) than the steady state, respectively.

A number of analytical results can be obtained for this model. Other of the results are better displayed numerically, in which case parameter values are assumed as in shown in Table 1 below (Appendix II has more on calibration).

III. Short Run Effects of a Permanent and Immediate Increase in the Replacement Rate

Proposition 2 A permanent increase in the replacement rate reduces the steady state capital stock, effort, and consumption in the same proportions.

Figure 3 shows the steady states and stable manifolds of the stationary system with replacement rate τ and the stationary system with higher replacement rate $\tau' > \tau$. Because the steady states lie on the same ray from the origin, and both stable manifolds cross that ray from above, the stable manifold corresponding to the lesser replacement rate lies above that corresponding to the greater replacement rate.[10]

Lemma If $\sigma \in (0,(1-\alpha)^{-1})$, the elasticity of labor with respect to capital is less than the elasticity of consumption with respect to capital along the stable manifold of the stationary system.

Proof See Appendix I.

[10] They cannot cross. To prove this, suppose not: they cross at (k,c) with $k > k_{ss}$ and the stable manifold corresponding to the lesser replacement rate crossing from above. The labor-capital ratio *N* must be greater on the lesser replacement rate manifold. From equation (9), this means that the stable manifold for the smaller distortion is steeper, which is a contradiction.

Proposition 3 If $\sigma \in (0,(1-\alpha)^{-1})$, the initial effects of a permanent and immediate increase in the replacement rate are to reduce labor and consumption, but consumption declines in a lesser proportion.

Proof As shown in Proposition 2, the steady state consumption and labor impacts are in the same proportion. Because the initial capital stock exceeds the steady state capital stock, initial consumption must exceed steady state consumption. Initial labor may (or may not, depending on parameter values) exceed steady state labor, but the Lemma guarantees that the percentage gap between initial and steady state is greater for consumption.

A closed-form, albeit cumbersome, formula for the ratio θ of the initial change in log consumption to the initial change in log labor can be calculated in the locality of the steady state. Figure 4 below displays values for θ as a function of the intertemporal substitution elasticity σ and the labor supply elasticity η, using assumed values for the parameters shown in Table 1.[11] The initial change in log consumption is not sensitive to the labor supply elasticity, but (as a percentage of the initial change in log labor) does range from 20 to 50 percent depending on the intertemporal substitution elasticity. Thus, the result that labor's initial decline is at least twice as much as consumption's when the shock comes from a greater replacement rate holds for a wide range of parameter values, despite the fact that in the long run both labor and consumption decline by the same percentage.

IV. A Gradual and Permanent Increase in the Replacement Rate

The sudden and immediate increase in the replacement rate is intellectually cumbersome because it combines two types of initial effects: the wealth effect of the news that the present value of output is less than previously thought, and the substitution effect of the initial replacement rate increase creating an increase in leisure and a reduction in consumption. More important, the actual replacement rates in the economy

[11] For small changes in the amount of labor market distortion, θ does not depend on the level of the distortion.

may accumulate continuously as, for example, the housing market continuously deteriorates, or various means-tested government subsidies are introduced or expanded at staggered dates.

For this reason, I consider a replacement rate that evolves continuously with time. At time T, the replacement rate reaches its long run value, which I assume exceeds its initial value. Two types of dynamics are possible, depending on whether, and how much, the peak replacement rate exceeds the long run replacement rate. I consider each case separately.

IV.A. A Gradual Monotone Transition

Suppose that the replacement rate increases continuously and monotonically with time for T years, and then remains forever after T at that higher level, as shown in Figure 5. As of time T, the system must be on the stable manifold corresponding to the long run value for the replacement rate, which is shown in Figure 3 as a black curve. Consumption cannot jump any time after time zero, so time paths for consumption and capital prior to time T satisfy the differential equations (1) and (4), satisfy the labor market condition (5) for the replacement rate amount assumed at each date, satisfy the given initial capital stock, and terminate at time T at an allocation $[k(T),c(T)]$ that is on the stable manifold of the stationary system with the high replacement rate. Furthermore, as shown in Figure 3's green curve, the time path in the $[k,c]$ plane approaches the stable manifold from above because the replacement rate is less before time T than at time T.[12]

Not surprisingly, the initial reduction in consumption is less than it would be if the entire replacement rate change occurred immediately at time 0, because the wealth and substitution effects on labor supply are both smaller in this case. Labor initially increases a bit, because of the adverse wealth effect and the fact that too little increase in the replacement rate is initially present to create a substitution effect. The initial labor increase raises the marginal product of capital and consumption growth (after consumption's initial jump down), which motivates some investment in the short run. As the replacement rate increases, labor, investment, and consumption growth fall. The

[12] For a formal proof, differentiate $c'(k)$ as shown in equation (9) with respect to the distortion τ, holding c and k constant, and note that this derivative is negative.

length of the initial high-labor period depends on the size of relative wealth and substitution effects on labor supply, and how quickly the replacement rate approaches its long run value.

Between time zero and time T the replacement rate has not yet hit its maximum, and an observer might wonder how much larger the replacement rate will get. The answer to this question is revealed by labor and consumption behavior: labor and consumption growth above (below) steady state values means that the replacement rate will (will not) get significantly larger. In other words, the fact that more labor is used in the face of a higher replacement rate reveals that agents are attempting to save in preparation for a still higher replacement rate.

The initial labor and investment increase turns out to be short-lived. To see this, consider numerical simulation of equilibrium time series using the benchmark parameter values shown in Table 1 and a replacement rate that linearly transitions from 13 percent to 26 percent over two years ($T = 2$) and then remains at that level forever.[13] The model's aggregate quantities are displayed as red curves in Figures 6b-6f (the other series shown in the Figures are explained below). The horizontal axis in the Figures is measured in years $t \in [0,7]$. The vertical axis normalizes each series relative to the low-replacement-rate-steady-state values, which are assumed to be the values that prevailed before time zero, when it, according to the model, became known the economy would be additionally distorted in the future (with dynamics shown in Figure 3). Because the aggregate production function is assumed to be Cobb-Douglas (marginal and average products are in fixed proportions), and the time series are shown relative to their low-replacement-rate-steady states, the same series that graphs the marginal product in Figure 6e would also graph the average product.

As noted above, labor is initially high, but Figure 6b shows that it is only about one percent above the low-replacement-rate-steady-state, and even then falls below the low-replacement-rate-steady-state within about a month. By the end of the second year, labor has fallen about 10 percent and consumption (Figure 6c) has fallen less than five percent. The marginal and average products of labor have risen three percent (Figure 6d).

[13] Appendix II explains the numerical simulation method, as well as the sensitivity of results to assumed parameter values. As explained further below, the 13 and 26 percent amounts are proxies for the average implicit marginal tax rates created by the programs shown in Figure 1 in 2007 and 2009, respectively.

IV.B. A Gradual Non-Monotone Transition

At least part of the subsidies shown in Figure 1, such as unemployment insurance that is available for up to 99 weeks, are likely to be temporary. Others, such as underwater mortgages in certain parts of the country, may be present for a number of years after the recession is over. Moreover, new taxes to pay for (and, sometimes, means-tests associated with the distribution of) growing public pensions and publicly financed health care, may replace some of the temporary marginal tax rate effects of the programs shown in Figure 1, and be present for a number of years thereafter. For these reasons, I also consider a gradual increase in the replacement rate that reverses itself before settling at a constant value that is at least as large as the pre-existing replacement rate.

Two kinds of dynamics are possible depending on how much the peak replacement rate exceeds its long run value. If the peak is not too much above the long run value (i.e., the replacement rate's time path is sufficiently close to monotonic), then the dynamics are like Figure 3's green curve except that the path in the $[k,c]$ plane approaches the stable manifold of the stationary system with the long run replacement rate from below. As for the path shown in Figure 3, this path has consumption and capital falling as time T approaches, and there is never a time during which capital is rising from a value below its initial one.

If the replacement rate's peak is high enough relative to its long run value, capital will drop below its long run value for some time sub-interval of $[0,T]$. In this case, consumption and capital reach their lows before time T and are rising thereafter.[14] Figure 7 shows a time path for this case in the $[k,c]$ plane.

The contrast between these two possibilities shows how aggregate time series for the first part of the recession can help an observer determine whether replacement rates are expected to get significantly smaller. In both cases, the economy reaches a point at which labor rises and the replacement rate falls. But capital rises and net investment is positive only if labor is anticipated to get significantly higher.

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[14] The path in the $[k,c]$ plane approaches the stable manifold of the stationary system with long run replacement rate from above, and thereby crosses that stable manifold at least twice.

IV.C. The Amount of Distortion Coming from Means-Tested Subsidies

The combined subsidies shown in Figure 1 potentially replace a lot of what prime-aged workers would have earned if they all had been working. Figure 8's "measured series" displays my calculation of an expected replacement rate $\tau_t = p_t b_t / w_t$ by dividing the combined subsidies by an estimate of the aggregate foregone full-time earnings of *all* non-working prime aged persons, which is itself the product of their numbers and a $700 per week estimate of the median earnings of prime-aged persons who did work full-time. This measured replacement rate increases from 12 or 13 percent before the recession to over 25 percent by late 2009.[15]

Conceptually the average replacement rate is just the ratio of what the average non-employed person gains in terms of subsidies as a consequence of not working, expressed as a ratio to what he would have earned if he had been lucky (or willing) enough to have a full-time job. Thus, the replacement rate is underestimated to the degree that the numerator excludes various means-tested subsidies and means-tested loan forgiveness, and to the degree that the denominator exaggerates foregone earnings or the size of the potentially eligible population. Nothing about Figure 8's measured replacement rate assumes that the most of the recession was caused by subsidy expansion – that depends on the degree to which labor supply is elastic to incentives (the preference parameter η is particularly important here) – and nothing about it denies the fact that subsidy programs automatically spend more when people become unemployed for other reasons.

Many of the subsidies featured in Figure 1 are temporary but nonetheless may last a number of years, and may be replaced in the future by distortions from income tax hikes or a more permanent expansion of the welfare state.[16] Therefore, I consider three replacement rate scenarios: (a) a permanent replacement rate increase, (b) a fully temporary replacement rate increase (the "full recovery" scenario), and a partially temporary replacement rate increase (the "partial recovery" scenario). All three scenarios

[15] For example, my estimate of the foregone earnings of the 45 million prime-aged people not employed in a typical week of the fourth quarter of 2010 is the product of their numbers and 13 times 700, or $410 billion. 25 percent of that is $102 billion, which is the combined quarterly subsidy shown in Figure 1.

[16] In this sense, my simulation results can be interpreted as an American transition from its 2007 welfare state, to a welfare state more like Europe's, as modeled by Prescott (2004).

have the same replacement rate series through the end of 2009, and differ in terms of what happens thereafter. The partial recovery scenario mimics the "theoretical" series intended to closely approximate the replacement rate measures in Figure 8, continuing the same downward trend from the beginning of 2011 to the end of 2013, at which time it remains constant at 21.7% (two thirds of the way between the permanent scenario's long run value of 26% and fully recovery scenario's long run value of 13%). For each replacement rate scenario, the paper then examines the equilibrium response of the rest of the economy, and compares that response to actual experience.

V. Monthly and Quarterly Indicators of Aggregate Economic Quantities

The model is especially simple in that output has only two uses: as consumption or investment. The national accounts offer more detail than this, so I aggregate nondefense government consumption, private nondurable consumption, and private service consumption into a single consumption aggregate.[17] Gross domestic private investment, government non-defense investment, and private purchases of consumer durables are summed together into a single investment expenditure aggregate, and then deflated by the consumption deflator. Labor is measured as the sum of aggregate private work hours (measured as the Aggregate Weekly Hours Index for all Private Industries) and aggregate public work hours (estimated as public sector employment times private work hours per private sector employee).

Model households provide their own household services, but in practice many household services like child care, or close substitutes for them like dining services, are traded in the marketplace. The model's best analogue for date *t* national accounts expenditure on nondurable consumption goods and services is therefore $c_t + \beta(1-\tau_t)w_t m_t$.

Investment and GDP are available only quarterly, although real consumer durables expenditure is measured monthly.[18] I also consider a second measure of

[17] The month-to-month log change in the real consumption index is the average of monthly log changes in each of the three components, weighted by their expenditure shares. A monthly real non-defense public consumption index is calculated by assigning the quarterly real non-defense public consumption index (which itself is constructed from two indexes: real public consumption and real defense consumption) to the middle month of each quarter, and interpolating the remaining months based on the within-quarter monthly pattern of public employment.

[18] Interestingly, detrended consumer durables (which are about a third of overall investment) purchases follow a similar time pattern as overall investment.

investment that excludes residential investment, because housing capital is likely less complementary with labor than business capital. The average product of labor is measured as the ratio of GDP to labor usage.

The model is expressed relative to constant exponential trends for population and labor productivity.[19] I therefore detrend the labor usage data by an assumed expected population growth rate of one percent per year. To match the other measured series with their model counterpart, each is detrended by expected population growth plus 0.4%: the average annual rate of TFP growth over the four years prior to the recession.[20]

Figures 6b-6e compare the three scenarios from the model to the measured time series for labor usage, real consumption, the average product of labor, and real investment. The Figures express model variables a ratio to their values in the steady state with the small replacement rate τ, and measured variables as a ratio to their value in December 2007 (or, in the case of quarterly data, 2007 Q4). The model paths are labeled "full recovery," "labor permanently 10 percent below trend," and "partial recovery" to reflect the assumed replacement rate time paths (see Figure 6a).

Figure 6b compares the model with measured labor data. The fact that all three model labor paths decline through year two is largely a result of Figure 8's finding that average replacement rates rose during that period. However, the fact that all three model labor paths ultimately decline about ten percent over those two years is a combination of Figure 8's finding that replacement rates rose about 13 percentage points, the benchmark assumption that the Frisch wage elasticity of labor supply is one, and the benchmark assumption that labor's share is 0.7.[21]

In the model, and in reality, the causality between employment and expenditures on means-tested subsidies goes in both directions. The model's subsidy expenditure is $(\Gamma - n_t) p_t b_t$: a reduction in prime labor for any reason would, holding constant the expected

[19] For example, δ in the model is not only capital's rate of economic depreciation, but also the rate at which capital must be augmented to keep up with population and exogenous technical change (for more on growth models with exogenous trends, see Barro and Sala-i-Martin, 2003).
[20] The Hodrik-Prescott filter is often used for this purpose, but the filter sometimes has unusual properties at the very end of a time series.
[21] A partial equilibrium back-of-the-envelope calculation illustrates the magnitudes: the change in log labor is the change in the log of the after tax share (i.e., log(1-0.26) – log(1-.13)) times the incidence parameter, which is the product of the ratio of the magnitude of the labor demand elasticity (1/(1-0.7)=3.33) and the labor supply elasticity (1) divided by their sum. The result of the partial equilibrium calculation is -0.12, as compared to -0.10 for the general equilibrium calculation which includes a wealth effect.

benefit $p_t b_t$, cause more spending on means-tested subsidies. However, in order for reverse causality to explain the entire increase in Figure 1 – the combined means-tested transfers almost tripled – the number of non-employed prime workers would have to increase by the same proportion. Instead, the number of non-employed persons aged 25-64 increased "only" 22 percent,[22] which is the main reason why Figure 8 – constructed as the ratio of Figure 1's amounts to estimates of the aggregate foregone earnings of non-employed persons aged 25-64 – shows a marked increase in expected replacement rates.

Even though expected replacement rates increased dramatically, that doesn't prove that the increase is responsible for the labor usage decline, because in principle labor usage could be fairly insensitive to replacement rates. That possibility is embodied in my model's Frisch labor supply elasticity η, which was assumed to be one for the purposes of Figure 5b-e. As shown in Appendix II, actual labor declines more than model labor if η were assumed to be much less than one, which might suggest that rising replacement rates were not the only factor depressing labor usage.[23]

For the moment, consider only the scenario represented by the red series in each Figure 6a-6f, which assumes that the replacement rate follows the pattern shown in Figure 5 with $\tau = 13\%$, $\tau' = 26\%$, and $T = 2$. In words, the replacement rate stops rising in early 2010, but never falls. As in the model, measured real consumption drops the most early in the recession and continues to drop gradually thereafter, although measured consumption does not literally make the instantaneous jump that it does in the model. Whether measured to include residential investment (circles in Figure 6e) or not (triangles in Figure 6e), actual investment is somewhat lower than model investment, although both model and actual fall sharply and hit bottom between 73 and 83 percent of

[22] Using the CPS Merged Outgoing Rotation Groups made available by the National Bureau of Economic Research, I calculated the 22 percent increase as a change from 37.2 million in 2006 to 45.5 million in 2010.

[23] It is interesting, but beyond the scope of this paper, to explain why replacement rates increased since 2007, rather than some other time period. Part of the explanation may be the high debt burden accumulated by households prior to 2007. Perhaps national electoral victories by the Democratic party set the political stage for the expansion of means-tested programs, or reflected increased support for such programs among voters. The endogenous erosion of "welfare stigma" (or the diffusion of welfare information) along the lines of Moffitt (1983) and Lindbeck (1995) could be another story: as more people lose jobs and collect benefits, others feel less stigmatized by collecting them (or become aware that such benefits are available), which raises program expenditures for a given size of the eligible population, while that endogenous increased likelihood of program participation reduces the perceived cost of being unemployed and thereby expands the size of the eligible population to the extent that labor supply is elastic.

their pre-recession values through about two years. As in the model, measured labor productivity rises during the recession, and is about 4 percent about pre-recession values through about two years.

Given the noticeable quantitative resemblance between the data and such a simple model, it is worth considering what the model suggests about the causes of the recession, and where the economy is headed in the future. The next section calculates the consumption implications of alternative scenarios for the future of the replacement rate, finding that a "partial recovery" scenario fits the consumption data best. It concludes by offering some interpretations of the investment decline that has occurred so far during this recession.

VI. Indicators of the Labor Market's Future, and the Magnitude of Investment Distortions, From Current Consumption and Investment Behavior

The model says that consumption falls the most early in the recession, and that the magnitude of the drop depends on the time path for the replacement rate. In other words, consumption is a leading indicator of labor, and could be used with early-recession data to forecast the labor market thereafter.

Consider first the immediate and permanent replacement rate increase considered above. Figure 4 showed comparative statics of the ratio of the initial log consumption impact to the initial log labor impact, which was essentially the same as the long run log labor impact. For example, if that ratio were 32% (as it is for $\eta = 1$ and $\sigma = 1.35$), then the long run log labor impact would be about triple the initial log consumption impact.

VI.A. Consumption Data Indicates No Near-Term Labor Recovery

Figure 9 displays the magnitude of the initial log labor impact as a function of the intertemporal consumption substitution elasticity σ and the magnitude of the initial log consumption impact. The actual data in Figure 6c shows that detrended consumption fell about 2 percent before the recession was one year old (more below on the current recession's data), so Figure 9 is calibrated with initial consumption impacts of 1%, 2%, and 3%.[24] As shown in the Figure, the initial labor impact is about 5-6%, and maybe more if $\sigma > 1.5$.

Figures 3 and 9 display calculations for the immediate and permanent replacement rate increase. However, consumption in the model is determined by wealth – the sum of capital and the present value of labor income – regardless of the dynamics of the shock. The immediate impact on the value of capital is zero, so the immediate consumption impact is the impact on the present value of labor income.[25]

Figure 6c shows the measured consumption data, and the three corresponding model paths for consumption. "Full Recovery" implies relatively little consumption decline (about two percent) through two years because the present value of the labor decline is relatively low. As discussed above, the consumption data seem to suggest that about twice as much labor will be lost in present value than implied by the "full recovery" scenario. The other two paths show a more significant consumption decline. By the second year of the recession, both scenarios have real consumption's cumulative decline of four or five percent, and have real consumption continuing to fall beyond year two.

Figure 6d compares the labor productivity data to the three scenarios from the model. All three scenarios assume the same long run labor productivity and have very similar predictions for labor during the first two years of the recession. Given that all three obtain most of their short run labor productivity variation from variation in the amount of labor, it is no surprise that both have similar time paths for labor productivity during the first two years. Labor productivity after year two varies across scenarios largely because labor varies across scenarios. All three scenarios over-predict actual labor productivity in quarters four and five by about a percentage point because total factor productivity did not exactly follow the previous trend.

[24] The labor elasticity η is assumed to be one; Figure 4 showed that results are not sensitive to this parameter.

[25] When the replacement rate has richer dynamics, such as those considered in Section IV above, there is no single number for "labor impact" because labor varies significantly over time. However, the results in Figure 9 can be interpreted as a weighted average labor impact, in which the weighted average counts the present more heavily according to the present value formula.

VI.B. Much, but Not All, of the Investment Decline Appears Efficient Given the Rising Replacement Rate

As shown in Figure 6e, all three scenarios predict that real investment would drop sharply, to 75-83 percent of the pre-recession value by the end of the recession's second year. Through the middle of the second year, the data also show real investment about 75 percent of the prerecession value. Investment declines in the model because capital and labor are complements in production (in the sense that the marginal product of capital increases with the amount of labor), and the labor market becomes increasingly distorted with time.

The scenarios differ from each other, and from the data, in terms of investment during the first year and a half. The "full recovery" scenario fits the investment data the best of the three, and the discussion of labor productivity results (Figure 6d) suggests that this scenario's success with predicting investment may be due to offsetting errors: over-predicting consumption and (in some of the quarters) over-predicting output.

The two scenarios with heavy present value labor losses – "Labor permanently 10 percent below trend" and "partial recovery" – have a short period of somewhat higher investment followed by low investment during the second year. The early investment period reflects model consumers' desire to smooth consumption, in anticipation of low output and labor usage in the near future.[26] The models' investment is quite low early in the second year of the recession, although measured real investment is somewhat lower (regardless of whether the measure includes housing).

For the last two measured quarters – the middle of 2009 – the data show investment expenditure at 73% of its 2007 Q4 value (76% for non-housing investment), whereas the "partial recovery" model shows it at 85 or 86%.[27] In this sense, low labor usage explains more than half of the investment decline.

[26] The early investment period would be shorter if the intertemporal substitution elasticity σ were smaller.
[27] As shown in Figure 6e, the empirical results are fairly insensitive to the inclusion of housing investment because housing investment is less than one-fifth of total investment. For the same reason, modifying the model to have separate flows of housing and business investment would have little effect on the results. The housing stock, on the other hand, is a large share of the total stock, which is why housing debt can be associated with transfer flows as large as shown in Figure 1.

VI.C. An Investment Distortion by Itself does not Fit Actual Behavior

An alternative view of the recession is that labor usage fell, and safety net expenditures increased, as a consequence of an investment spending collapse stemming from the financial crisis, rather than being caused by an expansion of the safety net or by some other labor market distortion. My model has no investment friction, but its components indicate how the economy might evolve if the supply of funds for new investment had been curtailed. Suppose for example that gross investment were frozen at zero on the time interval [0,*T*]. As of time *T* the economy would be on the stable manifold of the stationary system shown in Figure 2, with a capital stock less than its steady state value. Prior to that date, labor would be low. Close to time zero, labor would be low due to an income effect: potential output remains high and none of it is spent on investment. Closer to time *T*, capital has fallen due to lack of investment, consumption would have fallen with capital, and labor would be low due to low labor productivity. In summary, the time paths for labor usage and productivity would, for a time, be similar to what they are with the sudden and permanent labor distortion studied in Section III above, but the time path for consumption would be very different.

Moreover, unlike this example, actual gross investment has not been anywhere close to zero, and my framework offers a straight-forward calculation of the effects of investment distortions (a wedge in the consumption Euler equation (4) that had no direct effect on the labor market condition (5) or the resource constraint (1)) that were in exactly the right amounts to replicate actual investment. Given an initial capital stock, a path for investment expenditure, and the taste and technology parameters, an investment-distortion equilibrium is a list of time paths on $t \geq 0$ for utility flows, consumption flows, both types of labor, wage rates, and rental rates, such that the paths for utility, consumption, and labor (a) maximize profits $An_t^{\alpha}k_t^{1-\alpha} - w_t n_t - r_t k_t$ at each date and (b) maximize the present discounted value of utility $\int_0^{\infty} e^{-\rho t} u_t dt$ subject to (2), to the household's sequence of dynamic budget constraints (3).[28]

[28] The equilibrium quantities for any period $t \geq 0$ are calculated as the solution $\{c_t,n_t\}$ to the two algebraic equations (5) and $c_t = An_t^{\alpha}k_t^{1-\alpha} - I_t$, where It is gross investment expenditure at date *t* and τ_t is set to zero.

Figure 10a's solid curve is the monthly time path for labor usage from December 2007 to April 2011 that is an investment distortion equilibrium given that model's investment path over that time is exactly equal to the actual investment path, and given the benchmark parameters.[29] The black circles indicate actual labor usage (the same data shown in Figure 6b). For the benchmark wage elasticity of labor supply, the model's labor usage declines in 2008 and 2009, with a minor recovery thereafter, but to a much lesser degree than actual labor usage. In the short run, an investment distortion has essentially no effect on the capital stock and thereby reduces labor solely because of an income effect. As the investment distortion persists, the capital stock is reduced: the income effect is smaller, and eventually in the other direction (increasing labor), while low wages tend to reduce labor. The dashed series shows that model predictions are similar if the elasticities η and σ are set to two times their benchmark values.

Figure 10b shows how the investment distortion model predicts reduced labor for the "wrong" reason – increased consumption. Contrary to the investment distortion model, consumption actually fell during the recession.

Figure 10c shows how the investment distortion model predicts rising expenditure $(\Gamma\text{-}n_t)\tau_t w_t$ on means-tested subsidies, but not nearly to the degree that actually occurred. Much of the model's subsidy under-prediction comes directly from its assumption that replacement rate τ_t was constant, whereas Figure 8 suggests that the replacement rate doubled. The rest of the subsidy under-prediction comes from its under-prediction of the labor decline, which, depending on the assumed labor supply elasticity, in part also derives from the constant replacement rate assumption.

A credit crisis might also directly reduce consumption spending, but in this case it is unclear why the labor usage reduction would be of such a great magnitude relative to the consumption spending drop. Figure 6f displays the ratio of the labor usage decline (as explained above, from a trend line determined by expected population growth) to the real consumption decline (from a trend line determined by expected population and productivity growth) in both the models and the data. Throughout the recession, the actual fall in labor usage has exceeded the fall in real consumption, and by a wide margin

[29] For the purposes of constructing Figures 10a-10c, monthly investment data are measured by assigning quarterly investment rates (as shown in Figure 6e's actual series) to the middle month of each quarter, and linearly interpolating the remaining months.

over the last two years. The data and two of the replacement rate scenarios agree on this general pattern.

Admittedly, Figure 6e shows deviations between the data and any one of the replacement rate scenarios. One possibility is that some of the measured investment decline was due to financial frictions. Or still other factors may explain part of the actual investment decline. For example, option value reasoning of Abel et al. (1996) suggests that, to the extent that additional uncertainty about the efficient amount or composition of capital arose in 2008 and 2009, investment might have fallen even if labor had been constant. Investors may have anticipated an investment tax credit in 2010, and therefore saw investment in 2008 or 2009 as too expensive (Lucas, 1976).

VII. Conclusions

In the aggregate, subsidies flowing to the unemployed and financially distressed households, in the forms of loan forgiveness and government transfers, almost tripled since 2007. Much of that increase cannot be explained by a mere increase in the number of people without jobs, but much of it reflects increasing receipts per unemployed person. For example, unemployment benefits were extended and made more generous on multiple occasions since 2007, and the collapse of housing collateral values has prevented lenders from fully collecting mortgage debts from borrowers who are unable to pay.

I estimate that, before the recession began, the combined receipts from unemployment insurance, other means-tested government transfer programs, "home retention actions," and consumer loan charge-offs by commercial banks, on average replaced less than 13 percent of a prime-aged unemployed worker's earnings. By 2009, the replacement rate had doubled. To the extent that labor supply is responsive to marginal tax rates, higher replacement rates will reduce labor usage in the economy.

Using one of the simplest versions of the neoclassical growth model in which all parameters except the replacement rate are constant, this paper simulates efficient responses to a replacement rate that grew steadily during 2008 and 2009 from 13 to 26 percent. Assuming that the replacement rate declines over the next several years (but not all of the way to its previous level of 13 percent), and assuming a Frisch wage elasticity of labor supply of one, the model's predictions for labor, consumption and productivity closely match the data from 2007 to the present.

The model also explains more than half of the actual investment decline as a response to the labor decline. Investment may be higher in the model than in the data for a couple of reasons: (a) consumption adjusts immediately in the model, whereas the actual consumption drop of 2008 took several months, and (b) actual productivity did grow during late 2008 and early 2009, but slightly below the trend assumed in the model. Since investment is a small fraction of total spending, these small deviations between model and data consumption and output create larger deviations between model and data investment.

Although my labor results for the aggregate of many means-tested programs are roughly in line with studies of unemployment benefits,[30] I cannot conclude that the entire labor decline since 2007 is the result of rising replacement rates, because the precise values for the replacement rate time series and the precise value for the labor supply elasticity could be somewhat different than are estimated and assumed in my benchmark parameterization. For example, my sensitivity analysis shows that a labor supply elasticity of 0.5, rather than 1.0, implies that a permanent replacement rate increase from 13 to 26 percent (the replacement rate series shown in red in Figure 6a) would reduce per capita labor usage by five percent by the end of 2009, or about half of the actual decline of ten percent. On the other hand, the available means-tested subsidies are not limited to the various subsidies that I combined for the purposes of estimating a replacement rate time series. Moreover, search frictions, rigid wages, and other labor market distortions could have many of the same effects as rising replacement rates.

The main features of this recession are that labor and consumption fell – but consumption fell by a much smaller percentage – and labor productivity rose. These observations alone suggest that a large and obvious "labor wedge" emerged during this recession (Mulligan, 2009a; Ohanian, 2010), even if some or most of that wedge cannot

[30] About 40 percent of the increase shown in Figure 1 comes from unemployment benefits. Elsby, Hobijn, and Sahin (2010) concluded that emergency unemployment benefits "account for as much as 15 to 40 percent of the rise in aggregate unemployment duration," suggesting that (as a back-of-the-envelope estimate) the combination of programs shown in Figure 1 might account for as much as 38 to 100 percent of the reduction in labor.

be attributed to rising replacement rates of means-tested transfers. For example, a labor supply elasticity of 0.5 and a labor wedge that gradually and permanently increases from 13 to 37 percent – an increase that may be too large to attribute entirely to the programs featured in my Figure 1 – can generate almost exactly the model time series as shown by the red curves in Figures 6b-f.

Given that some of the means-tested transfers are expected to be temporary, the "partial recovery" scenario shown in Figures 6b-6f can serve as predictions for the macro aggregates between now and 2014. In particular, Figure 6b showed that labor usage *per capita* will not return to pre-recession levels, or even half way to pre-recession levels, at any time between now and 2014. Given a population growth rate of one percent per year, the aggregate hours index (private and public sectors combined) will not decline below its low of late 2009, but will not return to pre-recession levels until sometime in 2014. With continuing population and productivity growth, real GDP will grow significantly over the next several years, but the recovery will be "jobless" in the sense that employment and hours per capita will not return to pre-recession levels for many years, and appear to be investment-driven in the sense that real GDP will grow faster than consumption through 2014 (compare the green series in Figure 6d with the green series in Figure 6e).

In order to illustrate the explanatory power of a labor distortion, I assume that all other fundamentals stayed exactly on trend during this period (and will continue to do so) and have assumed highly stylized dynamics for the labor distortions. Reasonable people can debate whether these assumptions are approximately accurate, but certainly they are not literally true,[31] and therefore even if the forecasts offered in my Figures 6b-6f are reasonably accurate, they are not the best forecasts possible given current information. For example, total factor productivity growth was somewhat low prior to the recession, so as of 2008 one might have reasonably expected total factor productivity growth to be somewhat low at first, and then increase a bit with time. But, if this paper's emphasis on

[31] Chari, Kehoe, and McGrattan (2007) offer a methodology (not yet applied to this recession) which would account for *all* of the changes during a recession as the combination of several shocks in the neoclassical growth model. Because their method includes the same labor market condition (5) as mine, and the same definition of total factor productivity, we would agree that changes in the labor distortion during this recession were substantial and TFP changes minor.

new and significant labor market distortions is correct, various other omitted factors will tend to be negligible at least in the near term.

VIII. Appendix I: Proof of Lemma

Lemma If $\sigma \in (0,(1-\alpha)^{-1})$, the elasticity of labor with respect to capital is less than the elasticity of consumption with respect to capital along the stable manifold of the stationary system.

Proof From the labor market condition (5) the elasticity of labor with respect to *k* along the stable manifold satisfies:

$$\left((1-\alpha)\eta+1\right)\left(\ln n(k)-\ln k\right)=\eta\ln\left(\frac{1-\tau}{\gamma}\alpha A\right)-\frac{\eta}{\sigma}\ln c(k)-\ln k$$

$$\left((1-\alpha)\eta+1\right)\left(1-\frac{d\ln n(k)}{d\ln k}\right)=\frac{\eta}{\sigma}\varepsilon(k)+1$$

$$\varepsilon(k)-\frac{d\ln n(k)}{d\ln k}=\frac{\frac{\eta}{\sigma}\varepsilon(k)+1}{(1-\alpha)\eta+1}+\varepsilon(k)-1=\frac{\frac{\eta}{\sigma}+(1-\alpha)\eta+1}{(1-\alpha)\eta+1}\varepsilon(k)-\frac{(1-\alpha)\eta}{(1-\alpha)\eta+1}$$

The consumption elasticity exceeds the labor elasticity if and only if:

$$\varepsilon(k)>\frac{(1-\alpha)\eta}{\frac{\eta}{\sigma}+(1-\alpha)\eta+1}$$

To confirm that this is satisfied for $k > k_{ss}$ (the reader can confirm that the proof for $k < k_{ss}$ is quite similar), note that Proposition 1 implies:

$$\frac{c}{k}+\delta<\frac{\delta+\rho}{1-\alpha}<\frac{\delta+\rho}{1-\alpha}+\frac{\sigma}{(1-\alpha)\eta}\left[1+(1-\alpha)\eta\right]\left[\delta+\rho-(1-\alpha)AN^{\alpha}\right]$$

$$(1-\alpha)\eta\left(\frac{c}{k}+\delta\right)<(\delta+\rho)\eta+\sigma\left[1+(1-\alpha)\eta\right]\left[\delta+\rho-(1-\alpha)AN^{\alpha}\right]$$

$$(1-\alpha)\eta\left(\frac{c}{k}+\delta-(1-\alpha)AN^{\alpha}\right)<(\delta+\rho)\eta+\sigma\left[1+(1-\alpha)\eta\right]\left[\delta+\rho-(1-\alpha)AN^{\alpha}\right]-(1-\alpha)\eta AN^{\alpha}$$

$$(1-\alpha)\eta\left(\frac{c}{k}+\delta-(1-\alpha)AN^{\alpha}\right)<\left[\delta+\rho-(1-\alpha)AN^{\alpha}\right]\eta+\sigma\left[1+(1-\alpha)\eta\right]\left[\delta+\rho-(1-\alpha)AN^{\alpha}\right]$$

$$\frac{(1-\alpha)\eta}{\eta+\sigma\left[1+(1-\alpha)\eta\right]}<\frac{\delta+\rho-(1-\alpha)AN^{\alpha}}{\frac{c}{k}+\delta-(1-\alpha)AN^{\alpha}}$$

$$\varepsilon(k)=\sigma\frac{\delta+\rho-(1-\alpha)AN^{\alpha}}{\frac{c}{k}+\delta-(1-\alpha)AN^{\alpha}}>\frac{(1-\alpha)\eta}{\eta/\sigma+1+(1-\alpha)\eta}$$

where the final equality is the elasticity of the stable manifold in the [k,c] plane, calculated from equation (9).

IX. Appendix II: Calibration and Simulation

As described in the text, the Figure 8's replacement rate time series is calculated as the ratio of Figure 1's combined program expenditure to an estimate of the foregone earnings of persons aged 25-64 who are not employed. The UI expenditure in Figure 1 is from line 7 of BEA Table 3.12U. The other means-tested transfers are the sum of lines 21, 23, and 35-39. Home retention action amounts are the number of home retention transactions (reported quarterly by the Office of Thrift Supervision) times the average present value of each transactions (an average monthly payment reduction of $400[32] for 60 months, discounted at 7% per year, is $20,319), divided by 0.63 to reflect the fact that the metrics report includes only 63 percent of home mortgages. Consumer loan charge-offs are the product of the amount of consumer loans at commercial banks and the charge-off rates for those loans (i.e., the product of St. Louis Fed FRED series ACLACB and CORCACBS).

Aside from the replacement rate time path, the model has seven parameters to be calibrated (two of which are irrelevant for most of the calculations): labor's share α, the adjusted depreciation rate δ, the adjusted time preference rate ρ, and wage elasticity of labor supply η, and the consumption elasticity σ, prime labor's share of pre-recession labor income $1-\left[2+(\gamma_m/\gamma_n)^\eta/\beta\right]^{-1}$, and the amount Γ of the prime labor time endowment.[33] Labor's share is taken as 0.7, in order to coincide with measured values of the share of employee compensation in non-proprietor's private national income.[34]

The adjusted depreciation rate is taken as a pure depreciation rate minus population growth minus (TFP growth)/α: the rate of gross investment expenditure on a balanced growth path. TFP growth (specifically, real GDP growth adjusted for hours of labor usage and real non-residential private fixed assets) varied considerably prior to the recession: it averaged 0.0%/yr in 2006 and 2007, 0.1%/yr in 2005-7, 0.4%/yr 2004-7, and 1.3%/yr 2003-7 (which was also close to its average over longer periods). I use the value

[32] Office of Thrift Supervision (March 2011, Table 24) reports an average monthly payment reduction for mortgage modifications of $405.25 in 2010, when most of the modifications were occurring.
[33] Productivity and leisure preference are normalized so that low replacement rate steady state labor and capital are one.
[34] Capital and labor income and not separately measured for proprietors, or in the public sector.

of 0.4% per year to reflect the likely possibility that, by 2008, TFP growth was expected to be positive, but less that it has been in the 1990s and early 2000s. I also consider alternate values of 0.0%/yr and 1.0%/yr.

Population growth is taken as 1.0 percent per year. The annual pure depreciation rate is taken as 5.5% plus 0.5%, where the 5.5% is the ratio of private fixed asset depreciation per dollar of real private fixed assets in the national accounts. The model's investment is produced with the same technology as consumption goods, so it is best interpreted as investment expenditure deflated with the consumption deflator, and as such is expected to "depreciate" an additional amount according to the expected rate of decline of the real price of investment goods, which I take to be 0.5% per year. The benchmark adjusted depreciation rate is therefore 7.6%/yr, with alternative values of 7.0%/yr and 8.4%/yr depending on the alternative assumed value for expected TFP growth.

Prime labor's share of pre-recession labor income, which only matters for inferring total consumption expenditure [c + $(1-\tau)w\beta m$] from market consumption expenditure c, is assumed to be 0.9. The amount Γ of the prime time endowment matters only for simulating the equilibrium size of the subsidy budget, and is taken to be 1.32, because 76 percent (=1/1.32) of persons aged 25-64 were employed during any given week in the years before the recession.

The "consumption elasticity" is more precisely the elasticity of consumption growth with respect to the marginal product of capital. Some of the macroeconomics and consumption literature has assumed that this elasticity is necessarily the same as the elasticity of consumption growth with respect to the real return on safe short term loans, which in some models is closely related to the marginal product of capital. However, the asset pricing literature has shown that the marginal product of capital and the risk free interest rate are often change quite differently, so I stick with the more literal interpretation. Mulligan (2004) finds an elasticity of consumption growth with respect to the (after-tax) marginal product of capital to be about 1.35. This paper also reports sensitivity analysis using values of $\sigma = 0.7$ and $\sigma = 2.0$.

The model's steady state ratio of consumption to output is $(\alpha\delta+\rho)/(\delta+\rho)$. I choose the rate of adjusted rate of time preference ρ so that the consumption output ratio is 73 percent as in the data. This implies that the benchmark ρ is 1.0%/yr, with alternative values of 0.9%/yr and 1.1%/yr depending on the alternative assumed value for expected TFP growth.

My benchmark value for the labor elasticity is one. With this value, a two year increase of the replacement rate by about 10 percentage points explains much of the labor decline over the first two years of the recession (see Figures 6a and 6b). If the labor elasticity were assumed instead to be, say, 0.5, then the same labor dynamics could be explained, but with a replacement rate that increases by about 20 percentage points instead of 10. The red and green series in Figure 11a illustrate this point. The red series is simulated for the baseline parameters (all series assume a "partial recovery" scenario for replacement rates), and the green series is for elasticities η and σ assumed to be half of their baseline values. Both follow very similar paths through 2009 because the baseline parameters are simulated with replacement rate dynamics that are about half those simulated for the green series.

The smaller elasticities are consistent with a somewhat better recovery (Figures 11a-c) because more of the 2008 consumption drop can be inferred as consumption smoothing rather than expectations of relatively large replacement rates in the distant future.

Figures 11b-c show that results are more sensitive to assumptions about TFP growth. The blue series are based on an assumed 1.6% annualize TFP growth rate, as compared to zero for the black series and 0.6% for the baseline. Greater TFP growth is consistent with greater replacement rates in the future because the 2008 consumption seems especially large when compared against TFP and population growth. Ultimately, higher TFP growth means more consumption and real GDP growth.

Time paths were simulated in three steps. First, the stable manifold for the stationary system having the lesser (long run) replacement rate was calculated by numerically solving the ordinary differential equation in k (9) evaluated at the lesser (long run) replacement rate, respectively, using the steady state as the boundary condition. Second, I guessed a time T point on the stable manifold for the stationary system having the long run replacement rate and numerically solved the dynamical system (1) and (4), which is non-stationary because the replacement rate varies with time, backwards in time until the capital stock k corresponded to the initial steady state capital stock.[35] I then checked whether *T* units of time have evolved and, if not, repeated the procedure after taking as my guess another point on the stable manifold for the stationary system having the long run replacement rate. Third, dynamics in the time dimension after time *T* were simulated by numerically integrating the capital accumulation equation (1), imposing that consumption be on the stable manifold for the stationary system having the long run replacement rate.

[35] Time *T* is the calendar time when the replacement rate stops changing.

Table 1. Parameter Values Assumed for the Purposes of Numerical Results

	Parameter	Value(s)	Units	Comments
α	labor's share	0.7	share	
ρ	time preference rate, adjusted	1.0%	per year	chosen to produce a steady-state investment consumption ratio of 0.34
δ	capital depreciation rate, adjusted	7.6%	per year	5.5% pure depreciation, adjusted for 1% populaton growth and 0.6% technical change, and 0.5 percent expected investment price trend
σ	intertemporal consumption elasticity	[0.5,2]	elasticity	benchmark value of 1.35 (Mulligan, 2005)
η	labor substitution elasticity	[0.5,2]	elasticity	benchmark value of 1
A	productivity level	normalized		normalized so that the low-distortion steady state capital and prime labor are one
γ	leisure preference	normalized		
N	prime-worker time endowment	1.32		low-distortion steady state has 76% prime labor usage
	prime-worker's share of labor income	0.9	share	a function of the preference parameters

Figure 1. Transfers to, and Loan Forgiveness of, the Unemployed and Financially Distressed

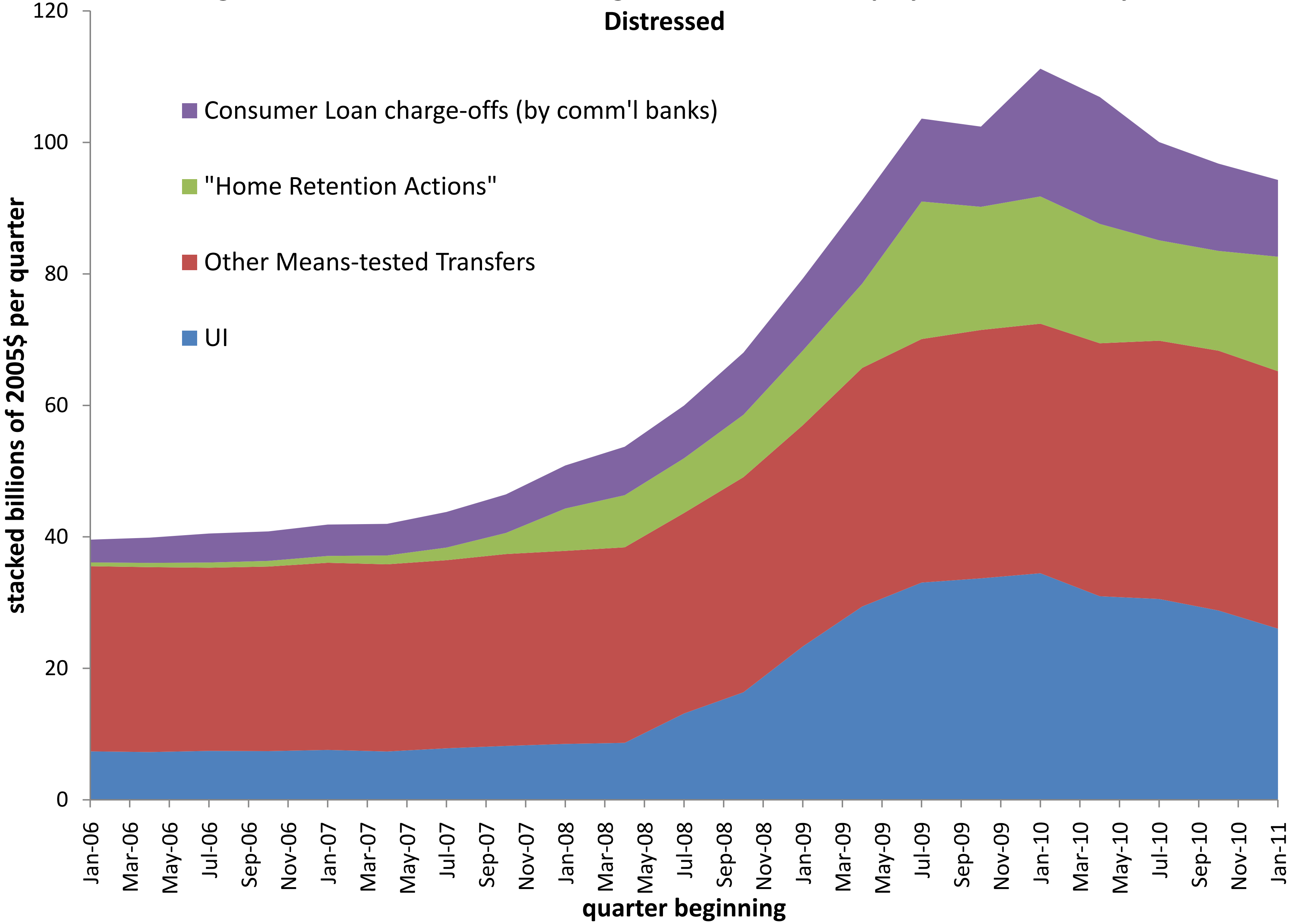


Fig 2. Capital-Consumption Phase Diagram for the Stationary System

The Figure shows the stationary system's steady state, dynamics, and stable manifold.

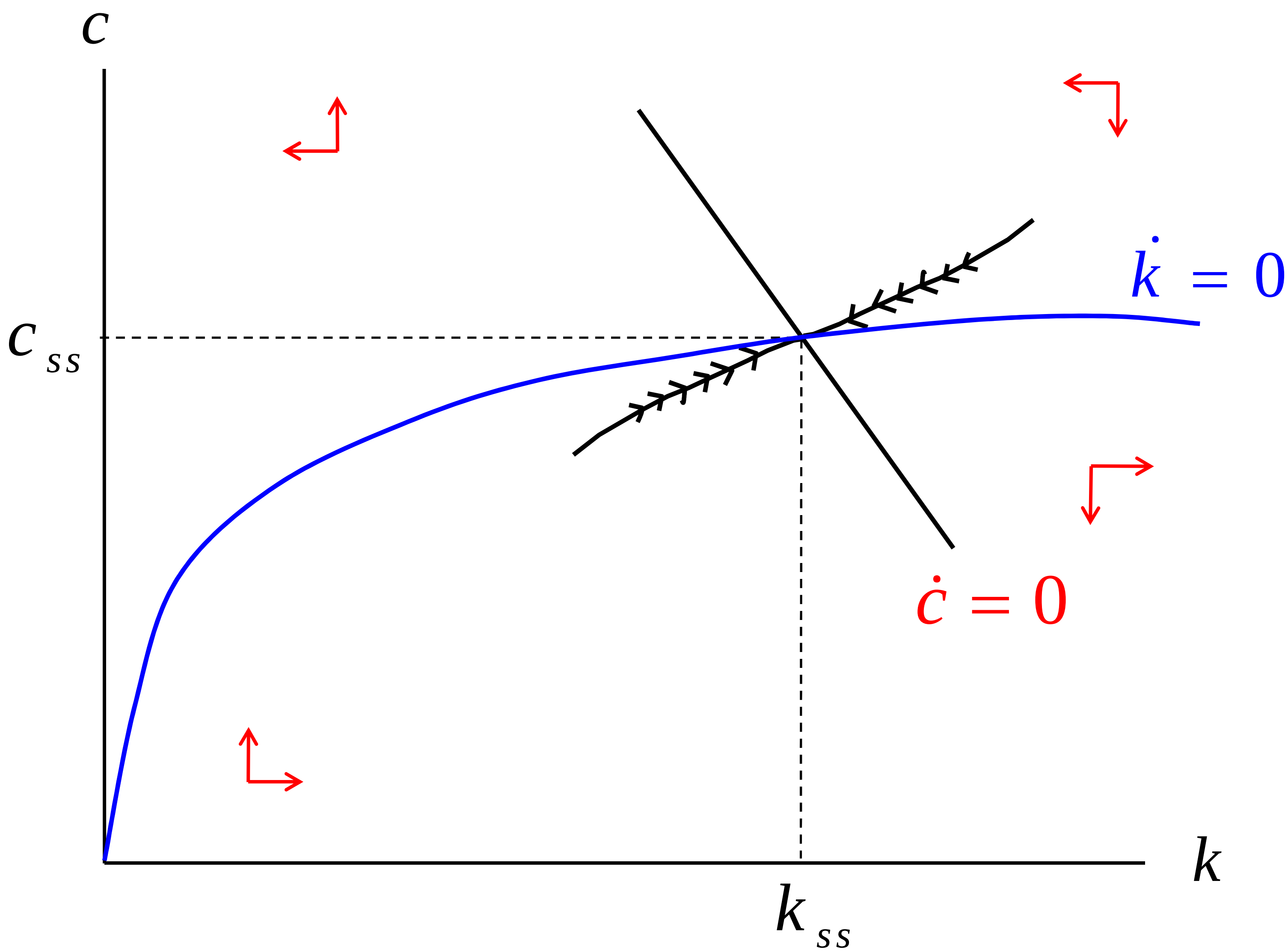


Fig 3. Stable Manifolds for High and Low Replacement Rates

The Figure shows the system's dynamics and stable manifold. The dynamics shown by the red arrows correspond to the low replacement rate that prevailed before date 0. When the new replacement rate path is first anticipated at date 0, consumption falls, but not as far as it will fall in the long run. The new stable manifold (shown as a black path) describes dynamics once the replacement rate has reached its higher long run value.

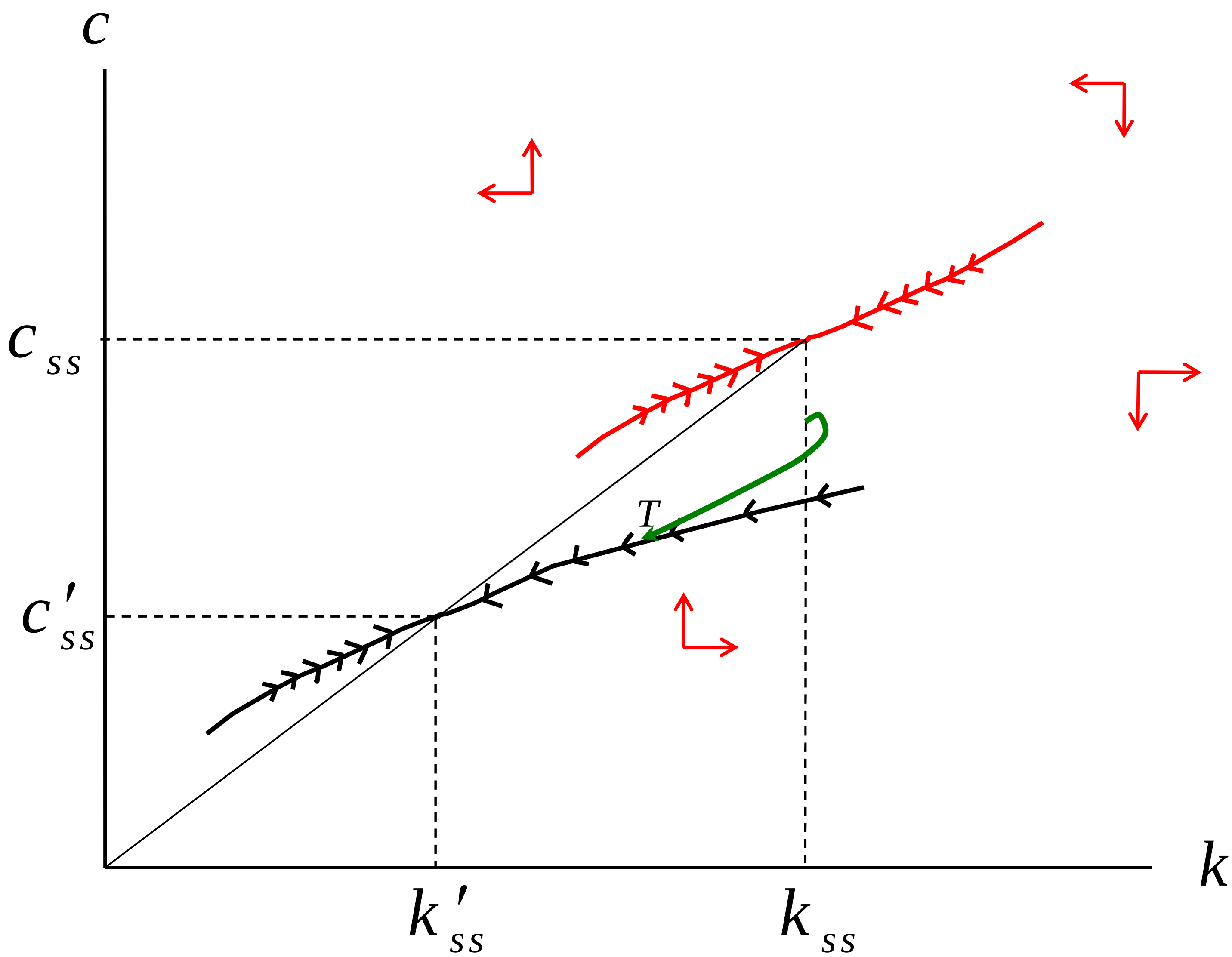


Figure 4: Initial Log Consumption Impact of a Permanent Replacement Rate Increase

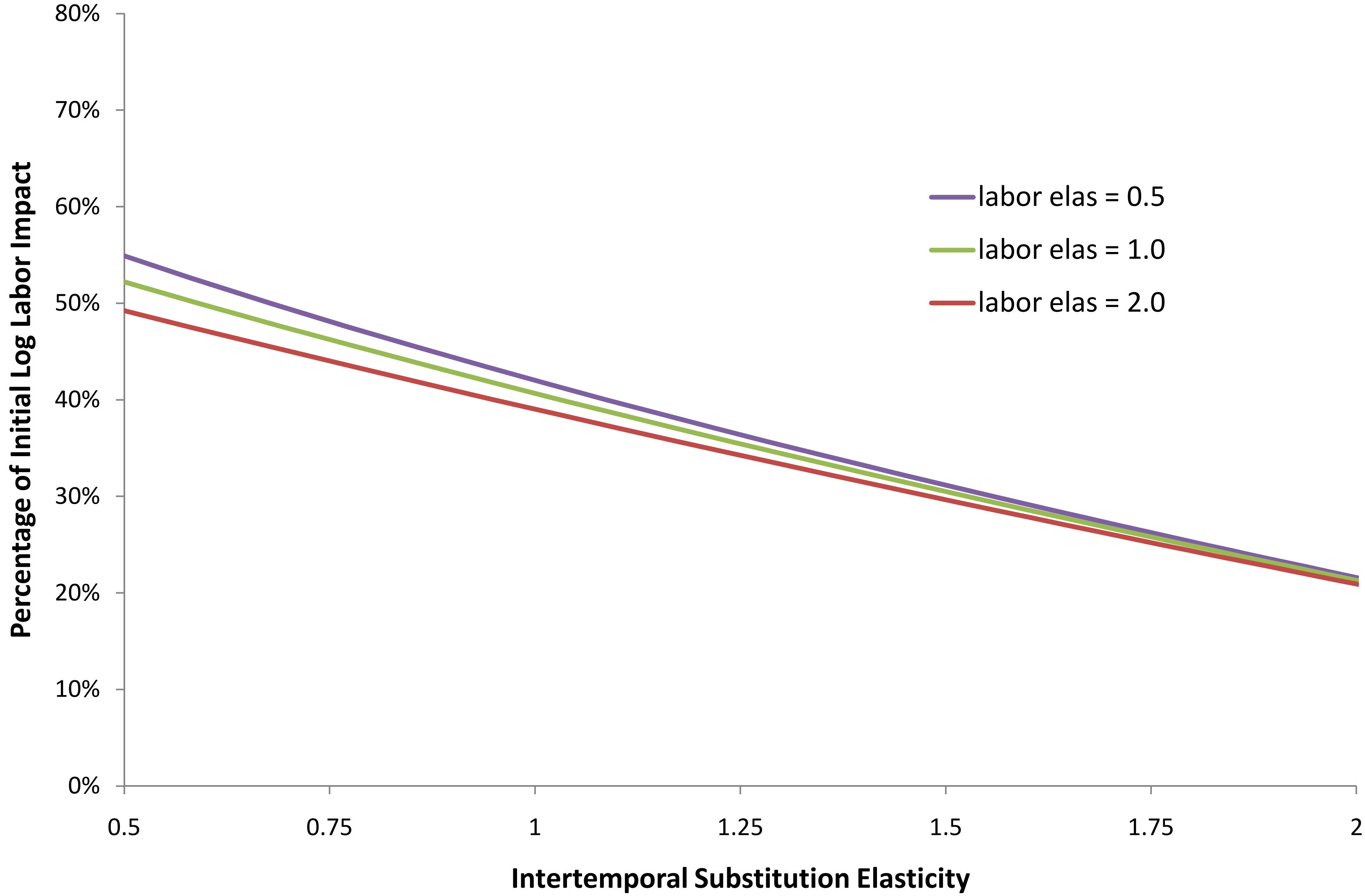
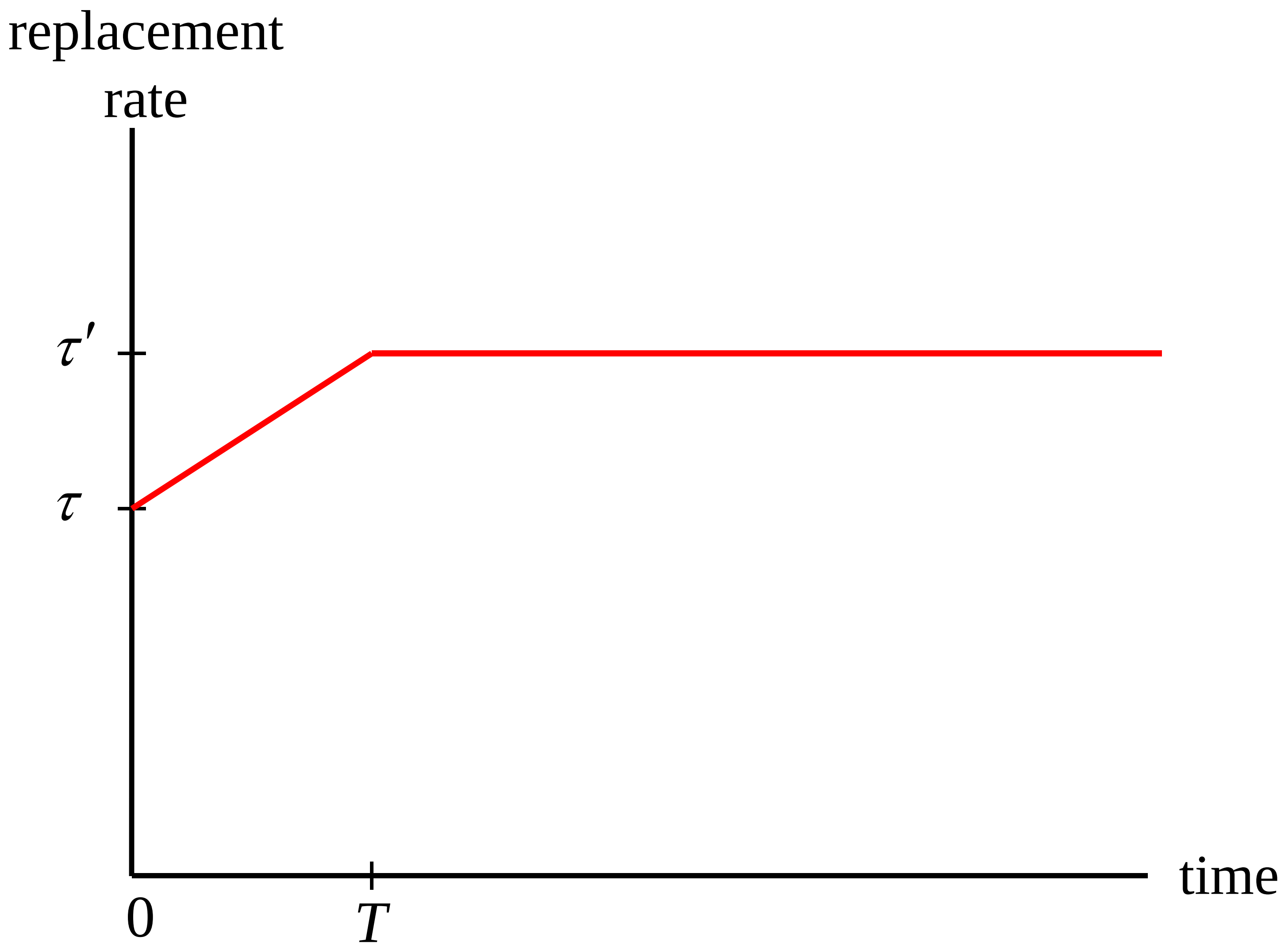


Fig 5. Time Path for the Gradually Increasing Replacement Rate



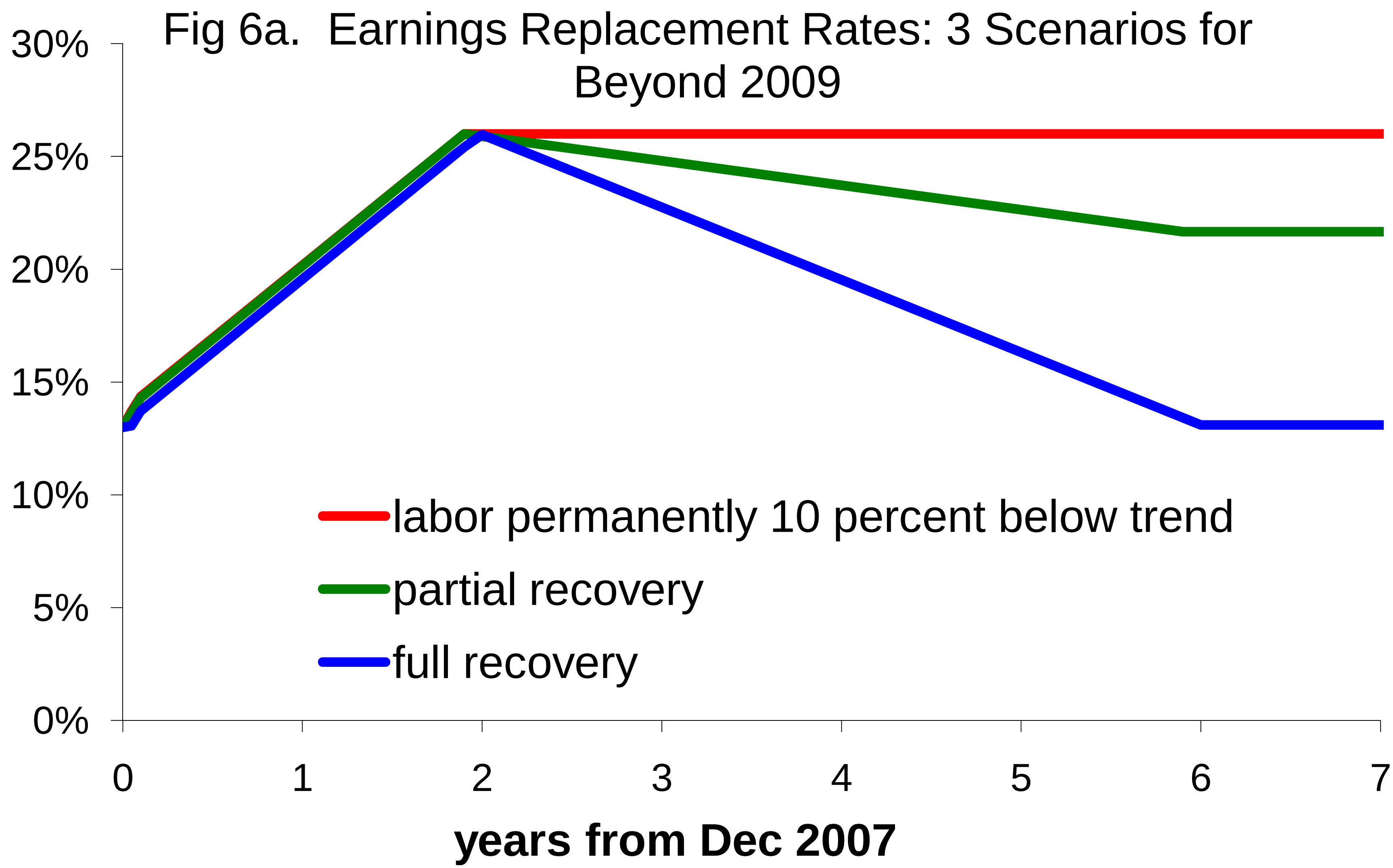


Fig 6a. Earnings Replacement Rates: 3 Scenarios for Beyond 2009
30%
25%
20%
15%
10%
5%
0%
0
1
2
3
4
5
6
7
years from Dec 2007
labor permanently 10 percent below trend
partial recovery
full recovery

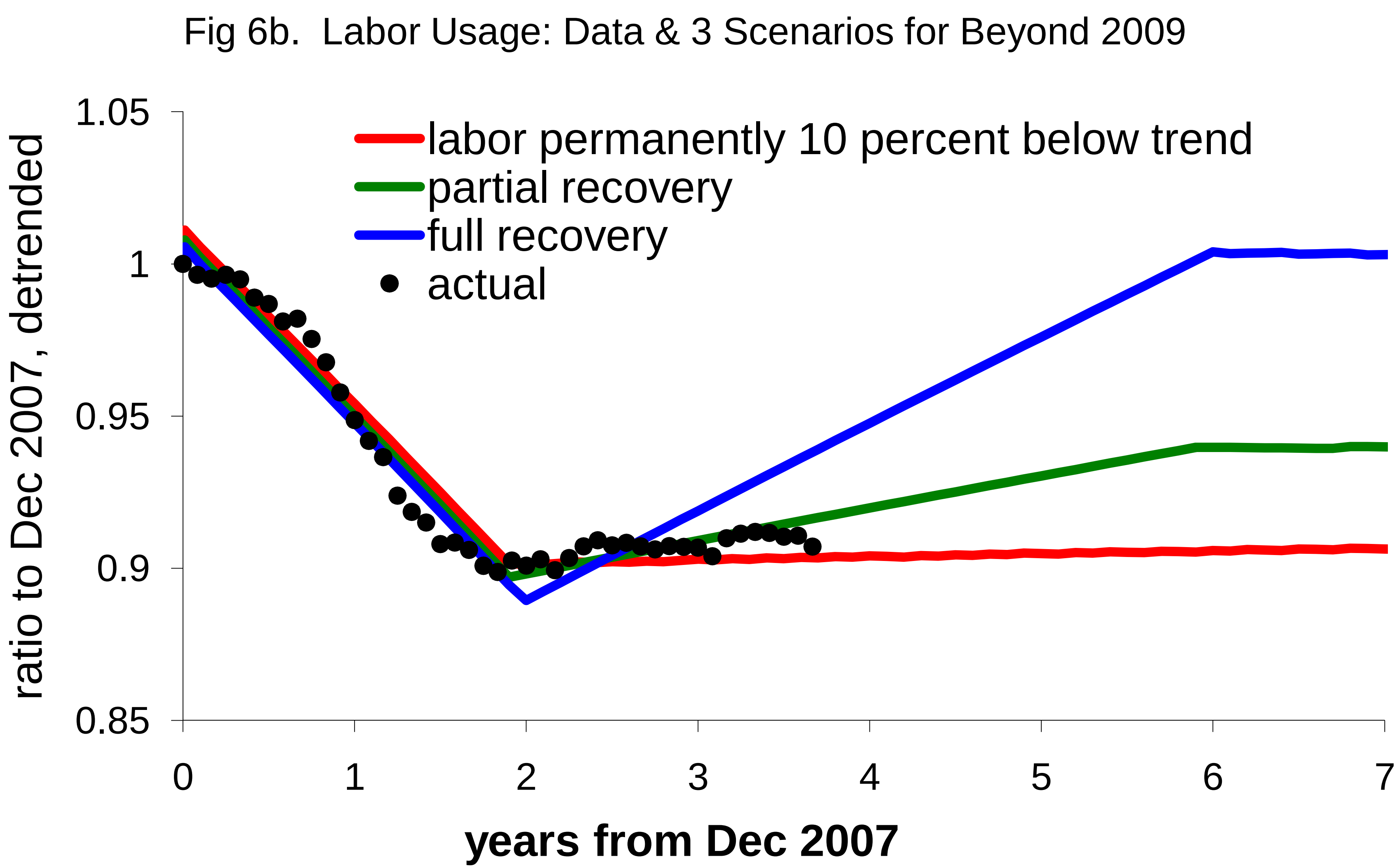


Fig 6b. Labor Usage: Data & 3 Scenarios for Beyond 2009
labor permanently 10 percent below trend
partial recovery
full recovery
actual
ratio to Dec 2007, detrended
1.05
1
0.95
0.9
0.85
0
1
2
3
4
5
6
7
years from Dec 2007

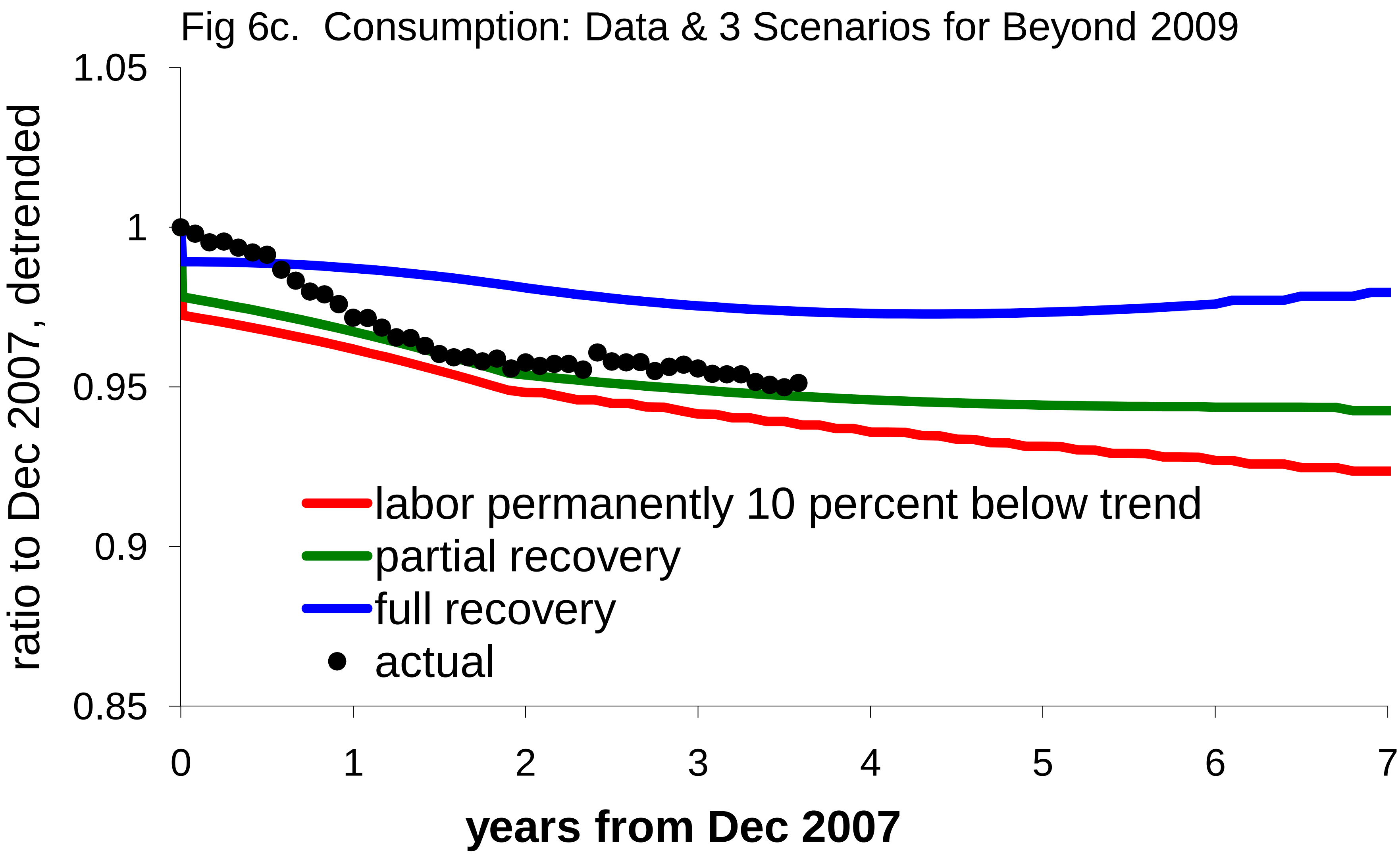


Fig 6c. Consumption: Data & 3 Scenarios for Beyond 2009
1.05
1
0.95
0.9
0.85
ratio to Dec 2007, detrended
0
1
2
3
4
5
6
7
years from Dec 2007
labor permanently 10 percent below trend
partial recovery
full recovery
actual

Fig 6d. Labor Productivity: Data & 3 Scenarios for Beyond 2009

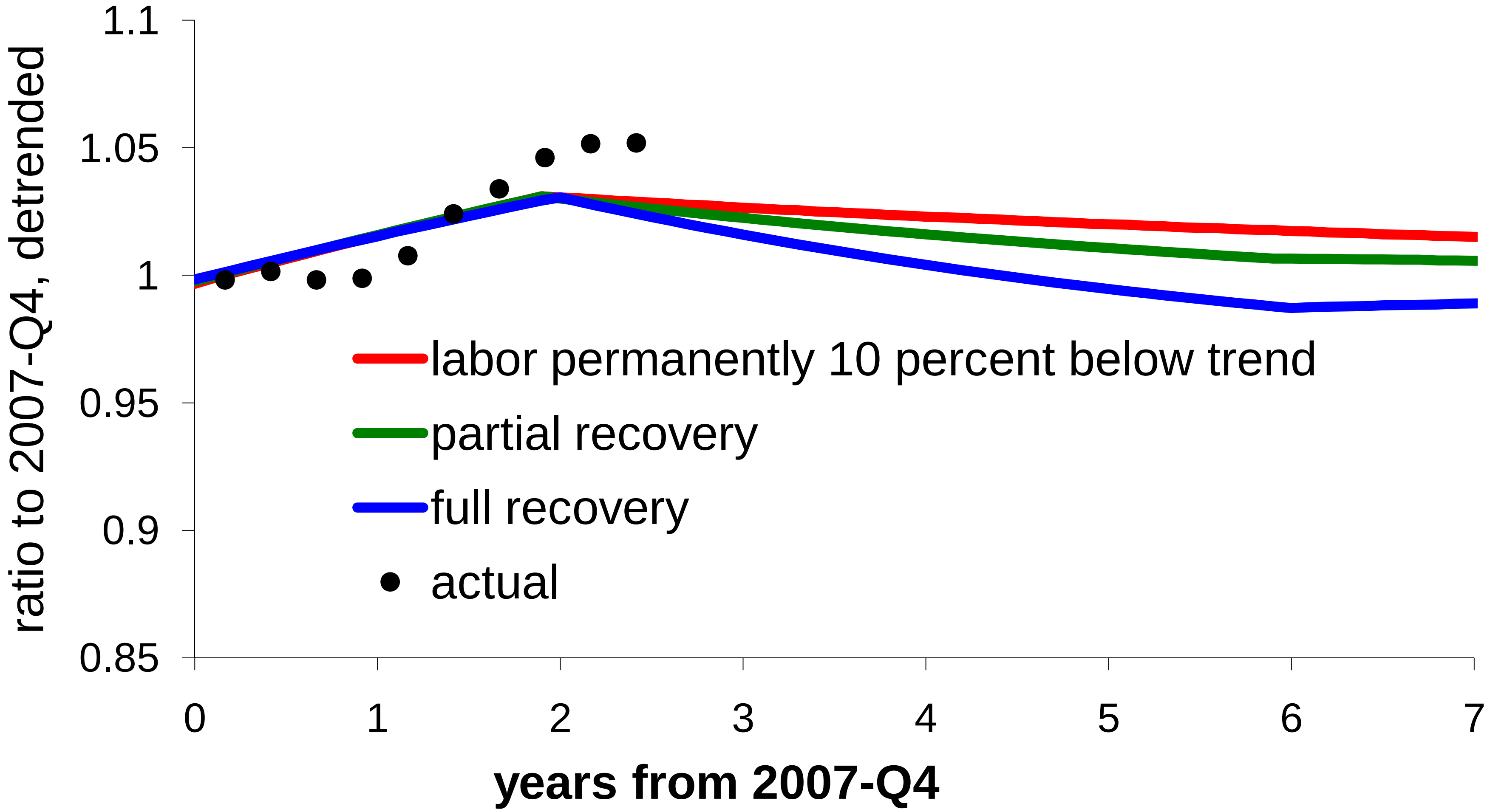


Fig 6e. Investment: Data & 3 Scenarios for Beyond 2009

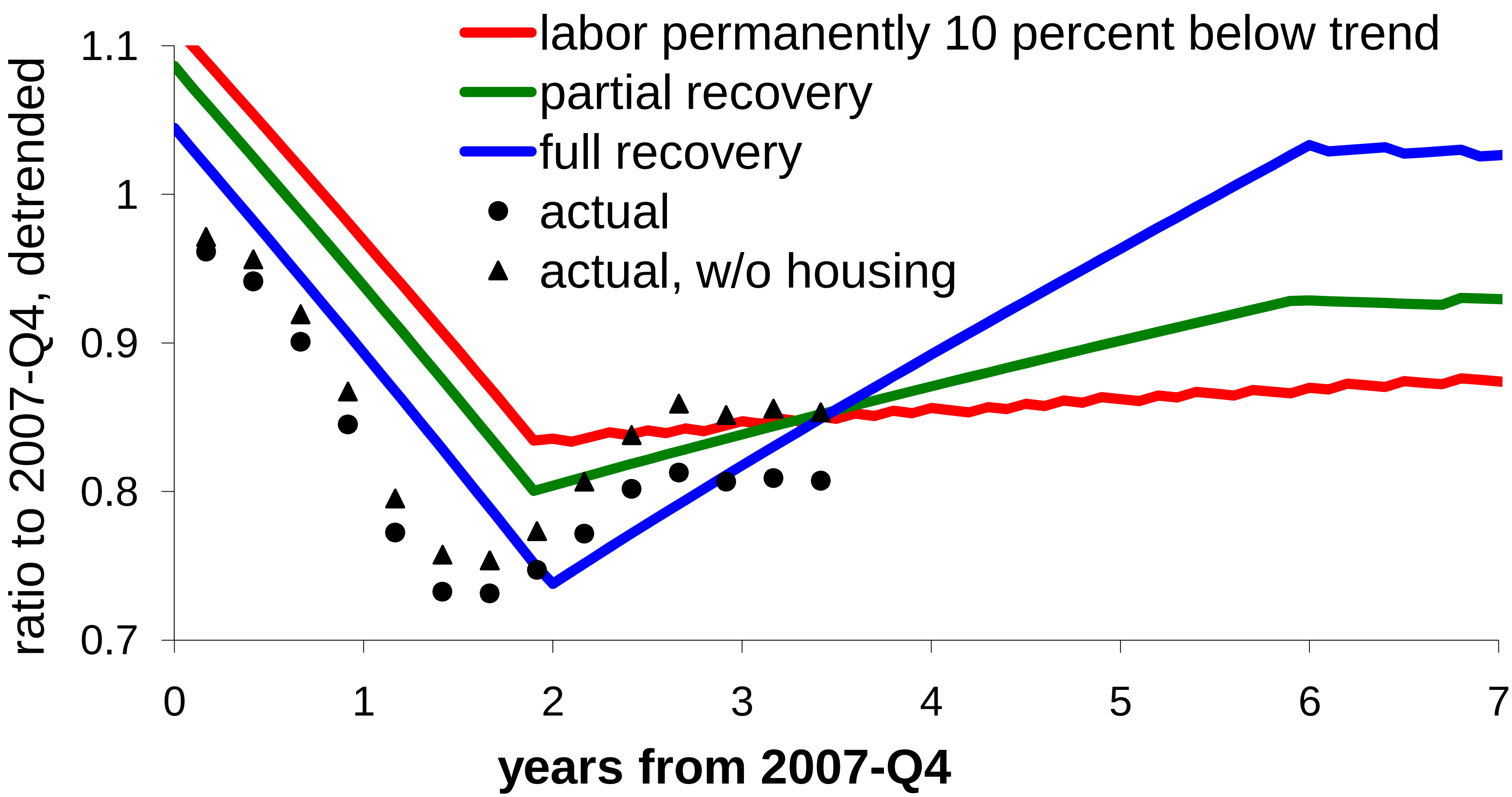


Fig 6f. Relative Impact: Data & 3 Scenarios for Beyond 2009

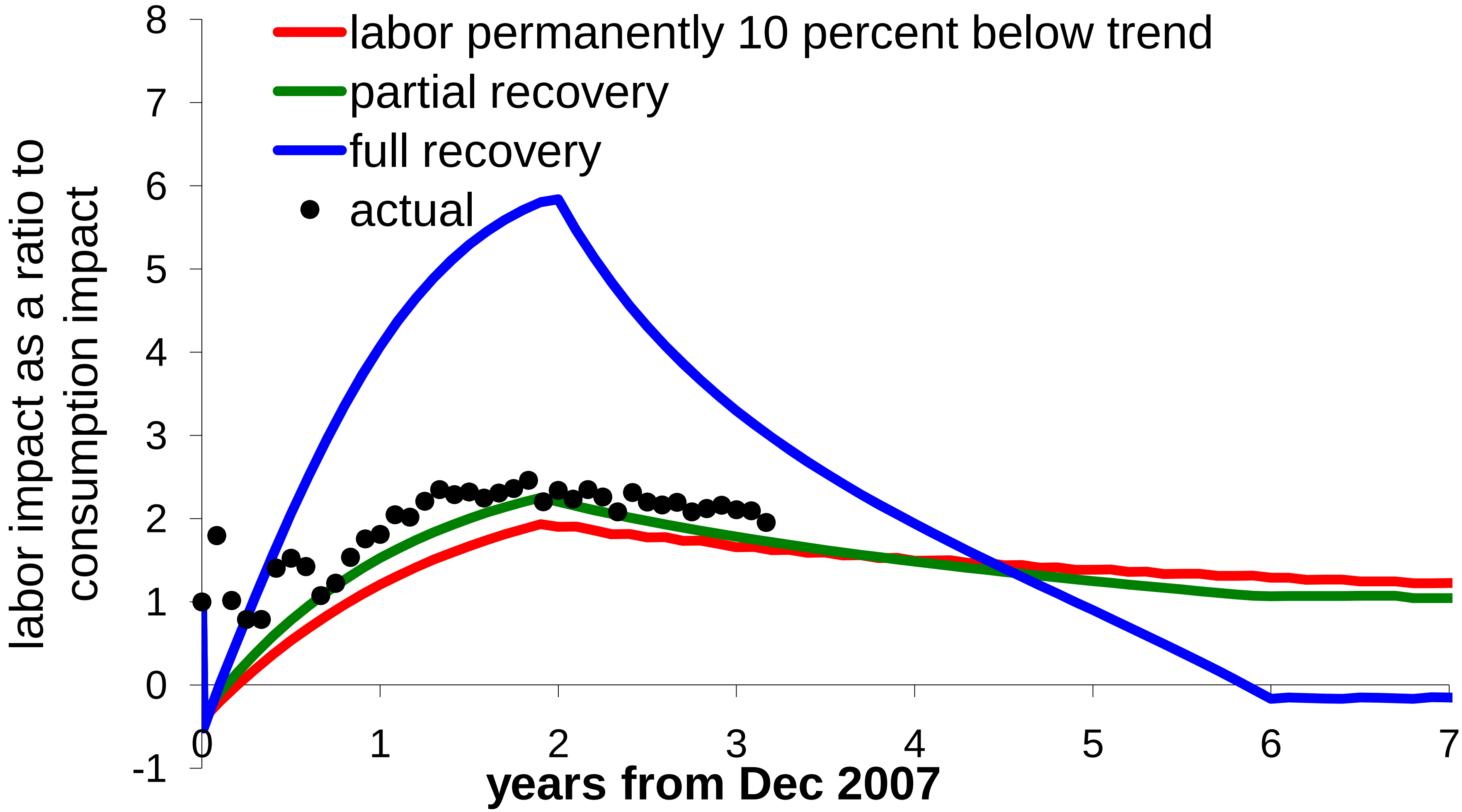


Fig 7. Dynamics when the Rep. Rate Increase is Largely Temporary

The Figure shows the system's dynamics and stable manifold. The dynamics shown by the red arrows correspond to the low replacement rate that prevailed before date 0. When the new replacement rate path is first anticipated at date 0, consumption falls, but not as far as it will fall in the long run. The new stable manifold (shown as a black path) describes dynamics once the replacement rate has reached its higher long run value.

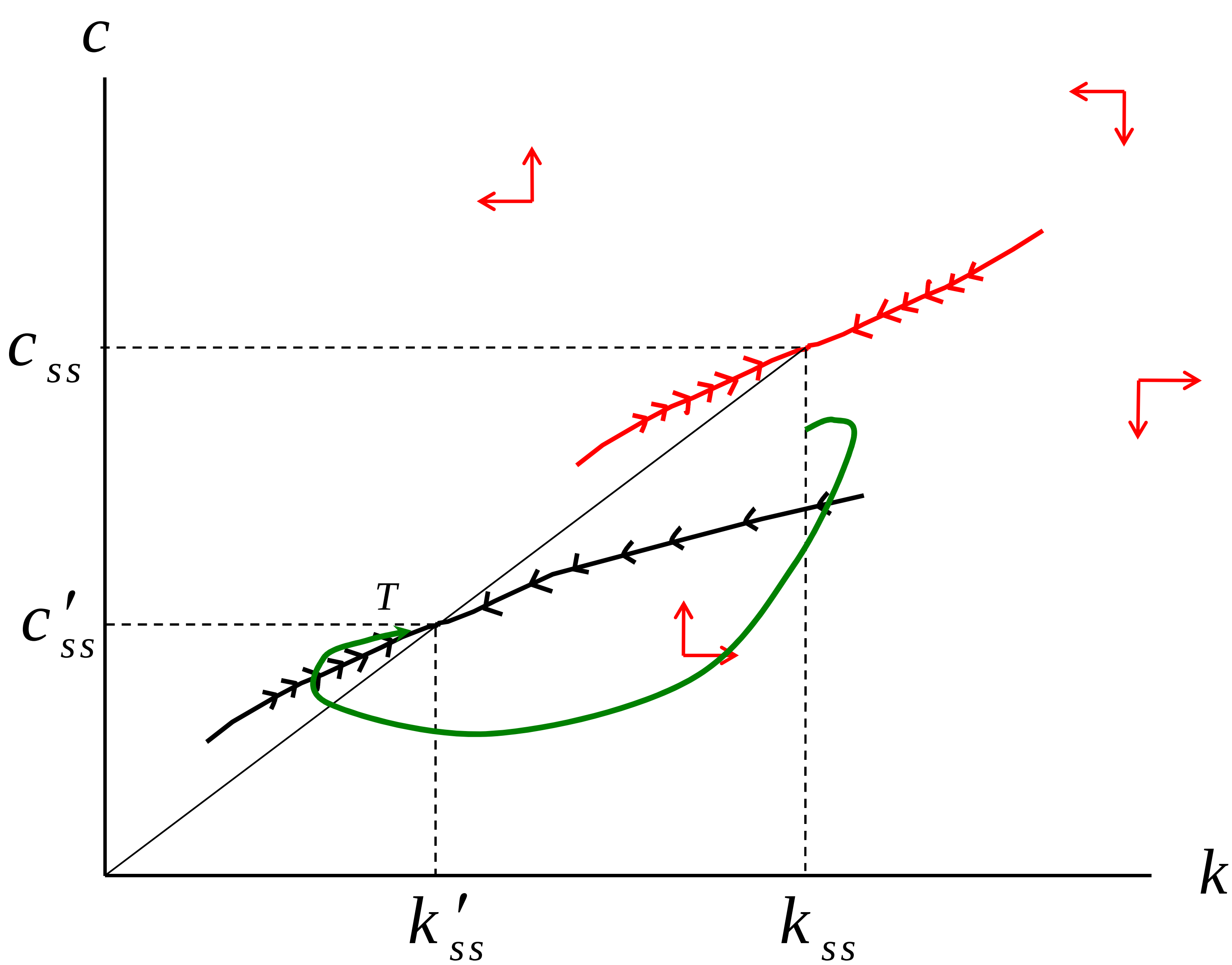


Figure 8. Average Earnings Replacement for Non-Employed Prime-aged Persons
(assumes that non-employed would have median full-time earnings)

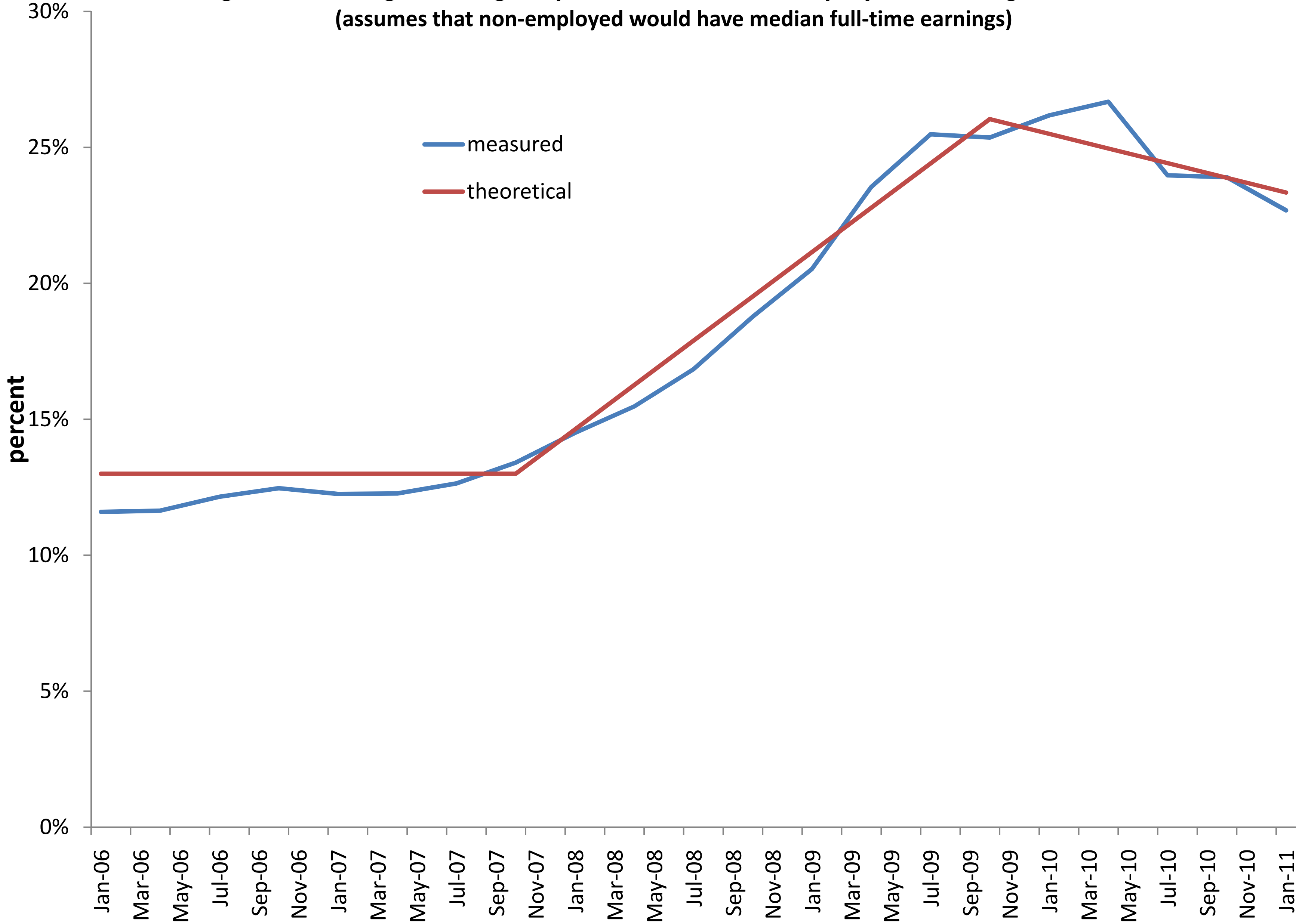
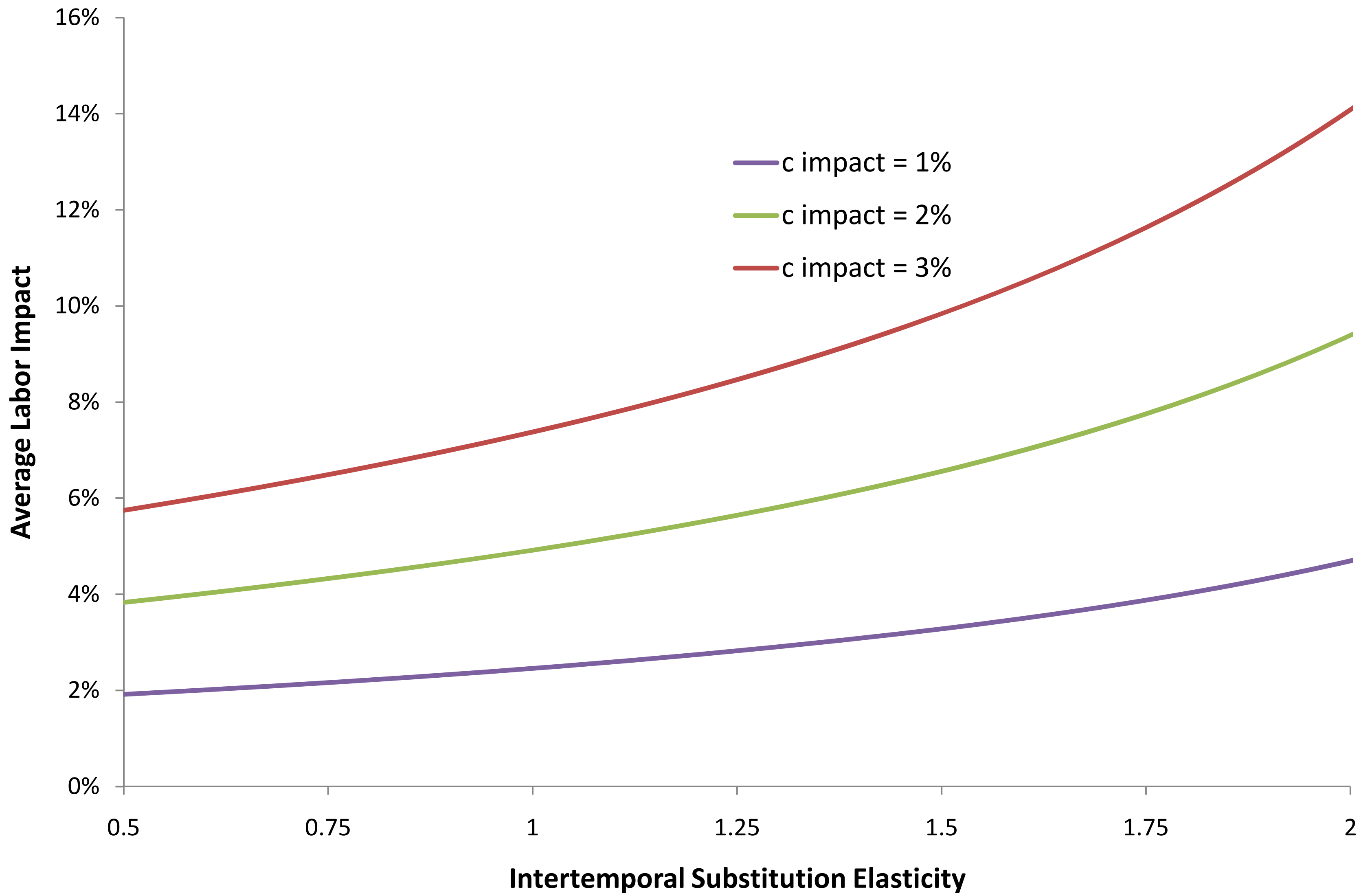


Figure 9: Inferring Average Labor Impact from the Initial Consumption Impact



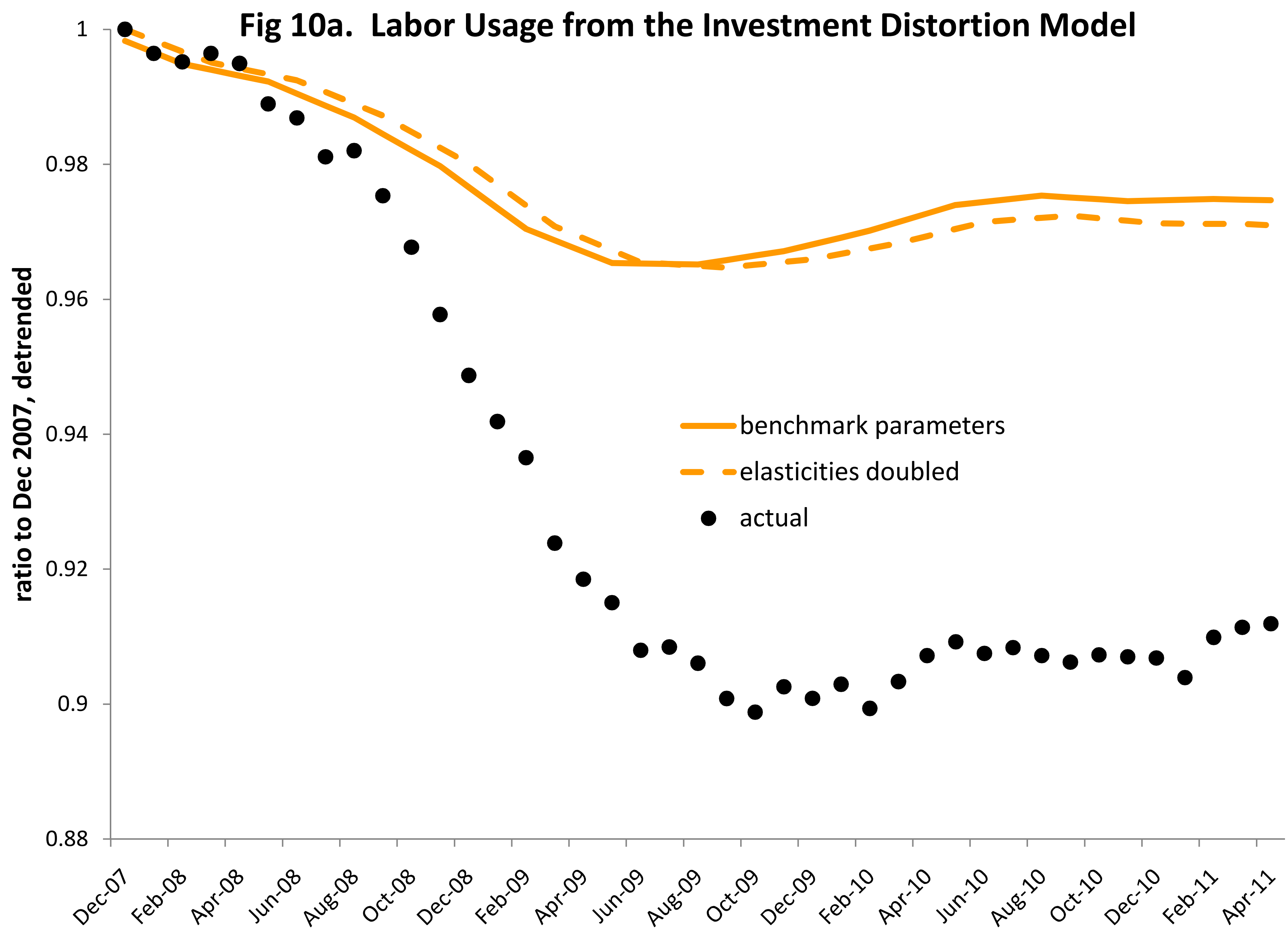


Fig 10a. Labor Usage from the Investment Distortion Model
ratio to Dec 2007, detrended
1
0.98
0.96
0.94
0.92
0.9
0.88
benchmark parameters
elasticities doubled
actual
Dec-07
Feb-08
Apr-08
Jun-08
Aug-08
Oct-08
Dec-08
Feb-09
Apr-09
Jun-09
Aug-09
Oct-09
Dec-09
Feb-10
Apr-10
Jun-10
Aug-10
Oct-10
Dec-10
Feb-11
Apr-11

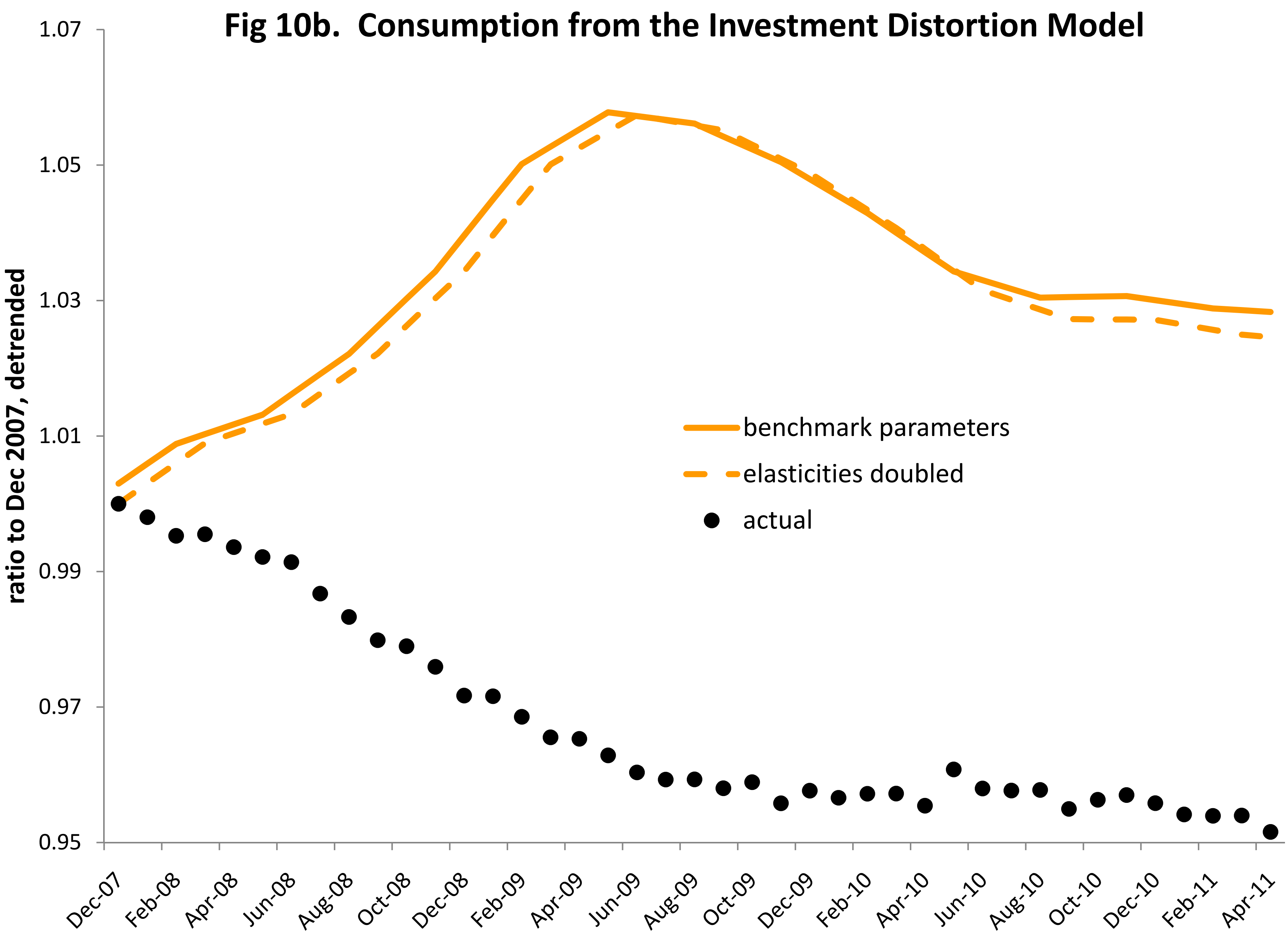
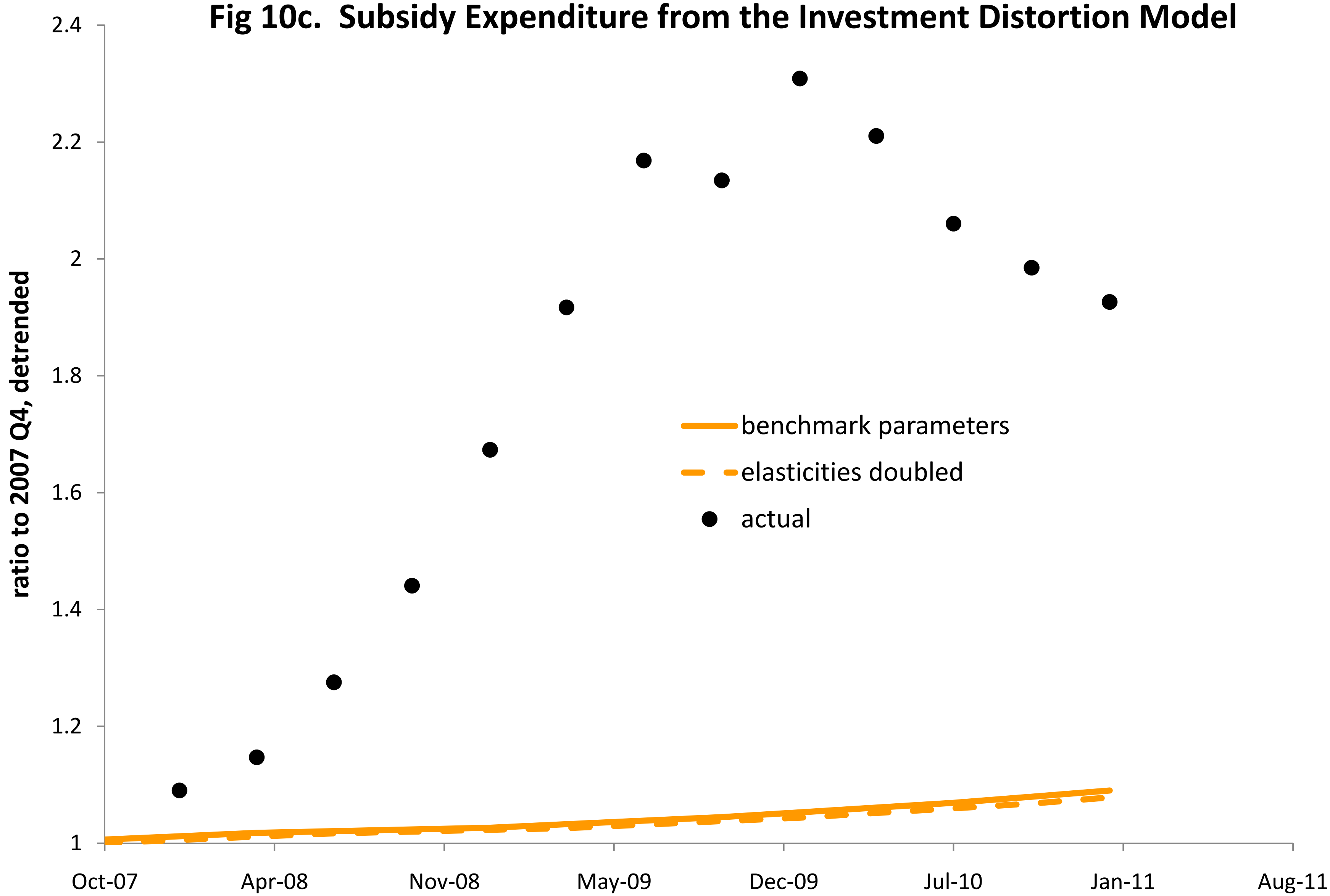


Fig 10b. Consumption from the Investment Distortion Model
ratio to Dec 2007, detrended
1.07
1.05
1.03
1.01
0.99
0.97
0.95
Dec-07
Feb-08
Apr-08
Jun-08
Aug-08
Oct-08
Dec-08
Feb-09
Apr-09
Jun-09
Aug-09
Oct-09
Dec-09
Feb-10
Apr-10
Jun-10
Aug-10
Oct-10
Dec-10
Feb-11
Apr-11
benchmark parameters
elasticities doubled
actual

Fig 10c. Subsidy Expenditure from the Investment Distortion Model



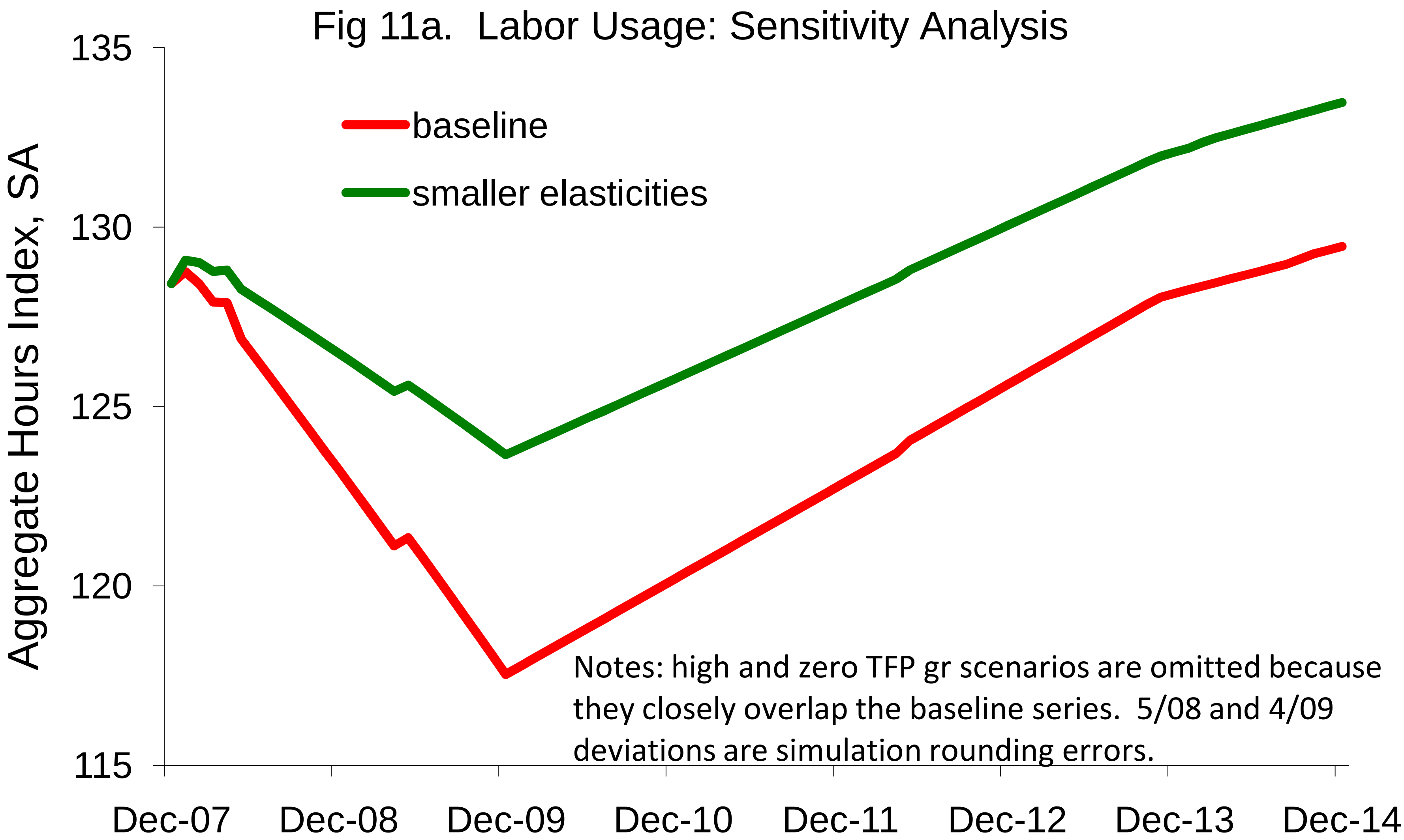
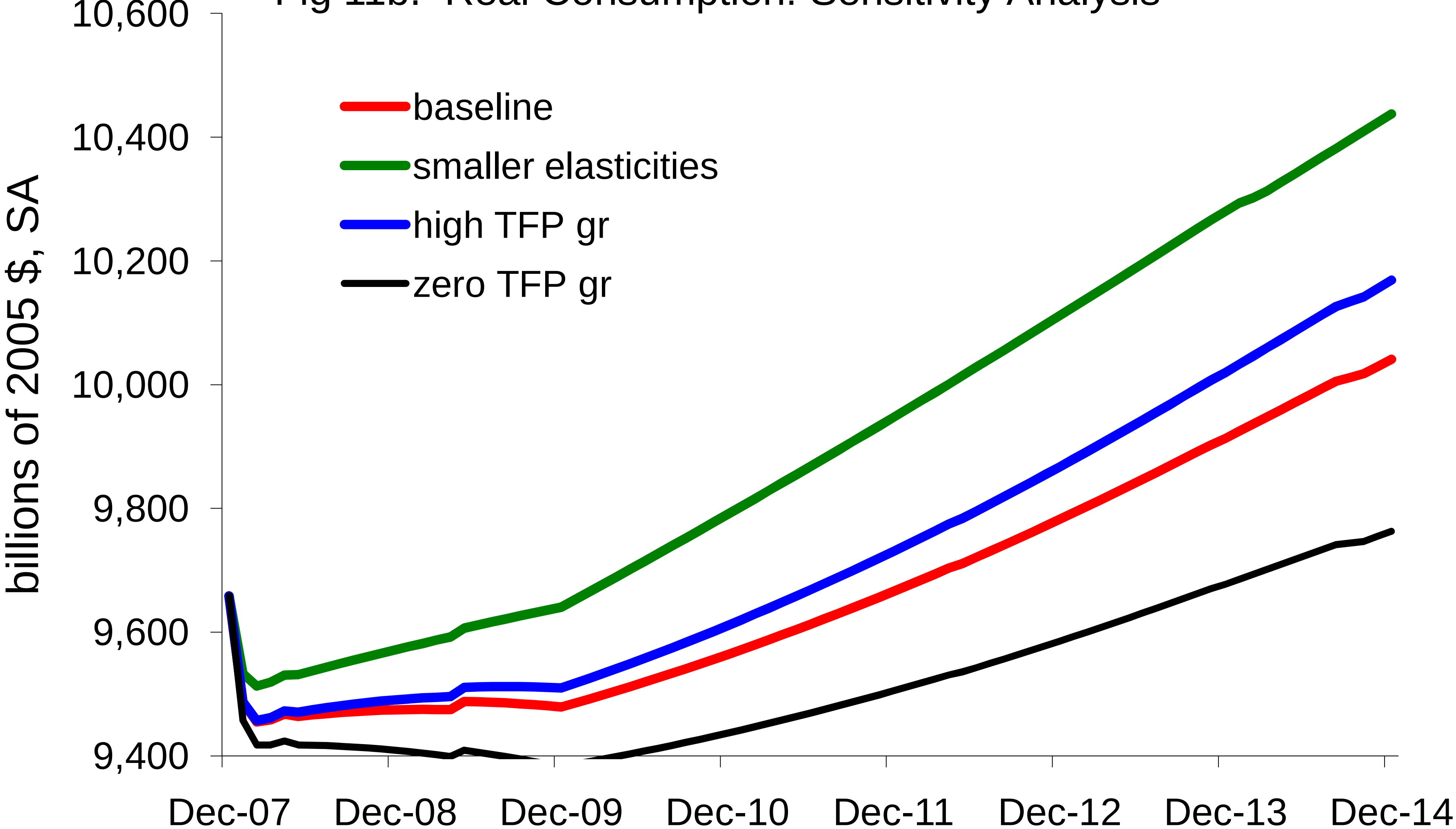


Fig 11a. Labor Usage: Sensitivity Analysis
135
130
125
120
115
Aggregate Hours Index, SA
baseline
smaller elasticities
Notes: high and zero TFP gr scenarios are omitted because they closely overlap the baseline series. 5/08 and 4/09 deviations are simulation rounding errors.
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12
Dec-13
Dec-14

Fig 11b. Real Consumption: Sensitivity Analysis



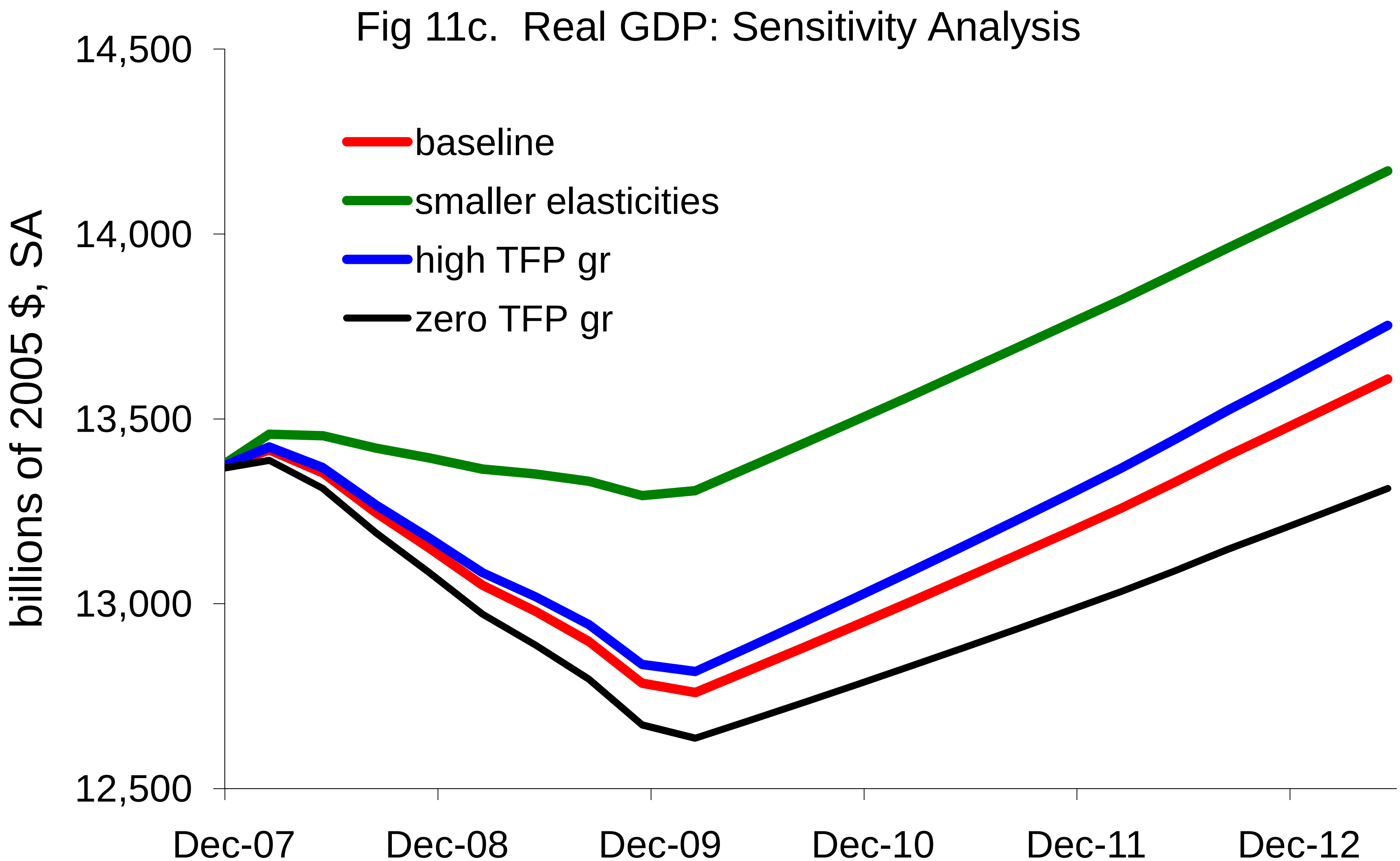


Fig 11c. Real GDP: Sensitivity Analysis
14,500
14,000
13,500
13,000
12,500
billions of 2005 $, SA
baseline
smaller elasticities
high TFP gr
zero TFP gr
Dec-07
Dec-08
Dec-09
Dec-10
Dec-11
Dec-12

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