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ABSTRACT

We study Ramsey-optimal fiscal policy in an economy in which product varieties are the result of forward-looking investment decisions by firms. There are two main results. First, depending on the particular form of variety aggregation in preferences, firms' dividend payments may be either subsidized or taxed in the long run. This policy balances monopoly incentives for product creation with consumers' welfare benefit of product variety. In the most empirically relevant form of variety aggregation, socially efficient outcomes entail a substantial tax on dividend income, removing the incentive for over-accumulation of capital, which takes the form of variety. Second, optimal policy induces dramatically smaller, but efficient, fluctuations of both capital and labor markets than in a calibrated exogenous policy. Decentralization requires zero intertemporal distortions and constant static distortions over the cycle. The results relate to Ramsey theory, which we show by developing welfare-relevant concepts of efficiency that take into account product creation.

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1 Introduction

A growing literature studies the importance of product creation and turnover for welfare and macroeconomic dynamics. This research program has recently received impetus from the availability of micro-level data sets and the development of macroeconomic frameworks that incorporate richer micro-level product dynamics than in standard macro models.[1] Thus far, however, there has been little work on developing the implications of endogenous product variety for optimal macroeconomic policy. This paper is an early step toward that goal.[2]

   We characterize the long-run and short-run properties of optimal fiscal policy in an economy in which monopolistically-competitive firms make forward-looking decisions regarding developing differentiated products based on the prospect of earning long-lived streams of monopoly profits. Product development is thus an investment activity. The starting point of the analysis is the general equilibrium model of Bilbiie, Ghironi, and Melitz (2007), who study the business-cycle implications of an endogenous, time-varying stock of differentiated product varieties. The Bilbiie, Ghironi, and Melitz (2007) framework — hereafter, BGM — generates many empirically relevant features of fluctuations, including the ability to match well the cyclical dynamics of profits, net product creation, and goods market markups. Taken together, the BGM framework portrays well the microeconomic underpinnings of product turnover and has become the basis for models studying a growing number of macro questions.[3] We first extend the BGM framework to incorporate realistic aspects of long-run and short-run fiscal policy assuming that policy is set exogenously, which itself contributes to the development of the BGM class of models as a positive description of U.S. business cycles. We then endogenize tax policy using the standard Ramsey, or second-best, approach.[4]

   There are two main results from the Ramsey analysis. First, in the long run, optimal dividend-income taxation can be zero, positive, or negative, depending on the form of variety aggregation in preferences. However, in the most empirically relevant and intuitively appealing version of the model, socially efficient outcomes entail a positive dividend income tax rate in the long run — 50 percent, if the model is taken literally. Dividend taxation, which is a form of capital income taxation, discourages inefficiently high product development.[5] Second, in the short run, the optimal

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[1]Notable contributions to the recent empirical literature include Bernard, Redding, and Schott (2010) and Broda and Weinstein (2010). A more complete list of references is in Bilbiie, Ghironi, and Melitz (2007).

[2]Other early steps are Bilbiie, Ghironi, and Melitz (2008a), Bilbiie, Fujiwara, and Ghironi (2009), Faia (2009) and Lewis (2009a), which are monetary policy applications, and Lewis (2009b), which is a fiscal policy application.

[3]In addition to those noted above, other recent studies that have built on the BGM framework include Stebunovs (2008), Shao and Silos (2009), and Colciago and Etro (2011).

[4]This distinguishes our approach from the pure Pigovian, or first-best, fiscal policy analysis of Bilbiie, Ghironi, and Melitz (2008b).

[5]Our analysis of long-run optimal policy abstracts from product creation as an engine of growth. Inclusion of long-run, endogenous growth in the BGM framework with general preferences that we use presents challenges that

labor income tax rate is constant (or, depending on how product varieties aggregate, very nearly so) at all points along the business cycle. The cornerstone Ramsey insight of the optimality of *tax smoothing* thus remains intact when product dynamics are modeled in a way consistent with micro evidence. The Ramsey government uses tax smoothing to implement sharply smaller fluctuations of capital markets and labor markets than in the benchmark exogenous policy equilibrium. Moreover, low volatility of tax rates keeps distortions constant over the business cycle.

While the goal of optimal policy is to "smooth wedges" in equilibrium conditions just as in standard Ramsey models, the very nature of "wedges" *does* depend on the nature of product dynamics. Another contribution of our work is thus to develop a welfare-relevant notion of efficiency for models based on the BGM framework. Efficiency concerns lie at the heart of any model studying policy. The welfare-relevant concept of efficiency we develop is based on only the primitives of the environment, independent of any optimization problem. This concept of efficiency is grounded in the elementary concepts of marginal rates of substitution and corresponding, model-consistent, marginal rates of transformation, and it makes transparent the basic Ramsey forces at work. This clear characterization of efficiency should be helpful in interpreting other results in the literature. It also allows us to connect easily the optimal policy results to the classic Chamley (1986) and Judd (1985) results on capital income taxation.[6]

While it turns out that the basic Ramsey principles of wedge-smoothing and zero intertemporal distortions apply in this framework, it is not obvious that they must. Albanesi and Armenter (2007) recently provided a unified framework with which to think about capital taxation in a variety of environments. Their central result, a set of sufficient conditions for the optimality of zero intertemporal distortions, unfortunately does not apply to our model. The failure of the Albanesi-Armenter sufficient conditions is due to the equilibrium increasing returns to scale in product varieties that are inherent in standard models of product differentiation with endogenous varieties. Application of the Albanesi-Armenter sufficient conditions requires constant returns in production both at the level of the firm and in the aggregate. As a contribution to Ramsey theory, then, it is important to know that empirically-appealing dynamic macro frameworks richer than "first-generation" constant-returns, complete-markets Ramsey models also prescribe zero intertemporal distortions as part of optimal policy.[7]

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are beyond the scope of this paper. In a model with growth, optimal long-run policy would balance the forces at work in this paper against the externalities at the heart of variety-driven growth. See Croce, Nguyen, and Schmid (2011) for an early Ramsey analysis of optimal labor income taxation in a stochastic model with endogenous, variety-driven growth and recursive Epstein-Zin (1989) preferences.

[6]We also discuss below the relation between the optimality of taxing capital in our environment with Judd's (1997, 2002) result that it is optimal to *subsidize* capital accumulation when firms have monopoly power.

[7]Our paper is also related to the complete-markets Ramsey literature that began with Lucas and Stokey (1983) and Chari, Christiano, and Kehoe (1991). We do not consider incomplete markets, a distinct branch of the Ramsey

Related to this broadening of the scope of Ramsey principles, our work also contributes to a recent branch of the optimal policy literature, examples of which are the monetary policy studies in frictional labor markets by Faia (2008), Thomas (2007), and Arseneau and Chugh (2008), in frictional monetary markets by Aruoba and Chugh (2010), and the study of labor income taxation in frictional labor markets by Arseneau and Chugh (2010). The unifying idea of these "second-generation" complete-markets Ramsey models is forward-looking private-sector behavior in markets richer in detail than portrayed in standard real business cycle (RBC) or New Keynesian models. This literature has shown that forward-looking behavior richer in micro detail than tangible capital accumulation and pricing decisions can offer new insights on some classic questions about optimal policy.

Finally, by placing the spotlight on fiscal policy, our paper contributes to the literature on efficiency in product creation.[8] Regulatory policy is the tool often thought to be most natural to address inefficiencies in product development.[9] However, historical evidence suggests that regulators are usually concerned only with product-development inefficiencies caused by very large companies. By applying to the entire universe of firms regardless of size, fiscal policy can be a very effective tool to address distortions in new product development.[10]

The rest of the paper is organized as follows. Section 2 describes the economic environment. Section 3 calibrates a non-Ramsey version of the model to document its basic cyclical properties. Section 4 studies the Ramsey equilibrium using the calibrated model. Section 5 formalizes static and intertemporal notions of marginal rates of transformation and efficiency to parse the optimal-policy results. Section 6 shows which features of the decentralized economy disrupt efficiency. Section 7 uses these concepts of efficiency and distortions to inspect several aspects of the model and results. Section 8 concludes.

2   The Model

The model features an endogenously evolving stock of differentiated product varieties that are costly to develop and bring to market. As described above, the model is based on BGM, into which we incorporate several realistic aspects of fiscal policy.

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literature, prominent examples of which are Aiyagari, Marcet, Sargent, and Seppala (2002) and Fahri (2009).

[8] An incomplete list of references is Benassy (1996), Bilbiie, Ghironi, and Melitz (2008b), Chamberlin (1950), Dixit and Stiglitz (1977), Grossman and Helpman (1991), Judd (1985), Mankiw and Whinston (1986), and Spence (1976).

[9] A standrd reference on regulation policy is Laffont and Tirole (1993).

[10] We thank Jeffrey Campbell for suggesting this point. See also Auerbach and Hines (2002) on optimal taxation and producer entry.

2.1 Product Turnover

To introduce some basic notation of the model, suppose that a pre-determined measure N_t of a continuum of product varieties exists at the beginning of period t. These N_t varieties are produced and sold on monopolistically competitive consumer markets during period t. Firms also develop new product varieties during period t, of which there is an aggregate measure N_{Et}. Because innovation takes time, newly-developed varieties can only be brought to market in the subsequent period. There is thus a time-to-build aspect of product development. Before period $t+1$ begins, a fraction $\delta \in (0,1)$ of both pre-existing and newly-developed varieties are hit by an exogenous exit shock.[11] Thus, because not all newly-developed products actually make it to the consumer market, the total measure of product varieties available in period $t+1$ is $N_{t+1} = (1-\delta)(N_t + N_{Et})$. Figure 1 summarizes the timing of the model.

The representative household obtains utility from consuming a symmetric, homothetic variety aggregator C_t. The aggregate C_t is defined over the set Ω of all the varieties to which the household would like to have access. Costly product entry implies that, in equilibrium, only the subset $\Omega_t \subset \Omega$ is available for purchase in period t; N_t is the mass of the subset Ω_t.[12]

2.2 Households

For periods $t = 0, 1, ...$, the representative household chooses state-contingent decision rules for consumption C_t, hours worked H_t, end-of-period holdings of a complete set of state-contingent government bonds B_{t+1}^j (j indexes the possible states in period $t+1$), and end-of-period holdings x_{t+1} of a mutual fund that finances firm activity in order to maximize expected lifetime discounted utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t), \tag{1}$$

subject to a sequence of flow budget constraints:

$$C_t + v_t x_{t+1}(N_t + N_{Et}) + \sum_j \frac{1}{R_t^j} B_{t+1}^j = (1 - \tau_t^H)w_t H_t + B_t + \left[v_t + (1 - \tau_t^D)d_t \right] x_t N_t. \tag{2}$$

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[11] Specifically, the probability that a given product is hit by the exit shock is assumed to be δ, independent of whether the product is a newly-developed or an incumbent one, or, in the case of incumbent products, how long the product has been in the market. Exit shocks are thus a Poisson process. The simplifying assumption of exogenous exit captures in a parsimonious, aggregative way the idea of product life cycles and is consistent with the relative acycliacality of product destruction in Broda and Weinstein (2010) and plant exit rates in Lee and Mukoyama (2007).

[12] Bundling in household preferences is the formalism we use. Alternatively, one could think of a "final goods" sector in which perfectly competitive firms bundle differentiated products into a homogenous final good, which is then sold to consumers. In this alternative formalism, differentiated products would be labeled "intermediate goods," but the equilibrium of the model would be identical. We follow the consumption aggregator approach only to make interpretation of results as similar as possible to BGM and the literature that has used the same approach.

The household's subjective discount factor is $\beta \in (0,1)$, and $u(.)$ is a standard within-period utility function that is strictly increasing and strictly concave in C_t, strictly decreasing and strictly convex in H_t, and satisfies standard Inada conditions. The notation u_{Ct} and u_{Ht} will be used to denote the marginal utility functions, evaluated at time-t arguments.

At the start of period t, the household owns x_t shares of a mutual fund of the N_t product lines that produce in period t, each of which pays a dividend d_t. The period-t market value of the household's start-of-period share holdings is thus $v_t x_t N_t$, with v_t denoting the per-share price. During period t, the household purchases x_{t+1} shares in a fund of these N_t product lines as well as the N_{Et} new product lines created during period t, to be carried into period $t+1$. Total stock-market purchases are thus $v_t x_{t+1}(N_t + N_{Et})$. By the time period $t+1$ begins, a fraction δ of these varieties $(N_t + N_{Et})$ disappears from the market. Due to the Poisson nature of exit shocks, the household does not know which product lines will disappear from the market, so it finances continued operations of all incumbent products as well as entry of all new products.

Following production and sales of the N_t varieties in the monopolistically competitive goods markets, firms remit the dividend d_t required by the terms of stock ownership. The household's total dividend income is thus $D_t \equiv d_t x_t N_t$, which is taxed at the rate τ_t^D.

The rest of the notation is standard: w_t is the market real wage, which is taxed at the rate τ_t^H; the household's holdings of the state-contingent one-period real government bond that pays off in period t are B_t; and B_{t+1}^j are end-of-period holdings of government bonds that pay off in state j in period $t+1$, which has purchase price $1/R_t^j$ in period t. Finally, because this is a Ramsey taxation model, there are no lump-sum taxes or transfers between the government and the private sector.[13]

2.2.1 Household Optimality Conditions

A standard labor supply condition

$$-\frac{u_{Ht}}{u_{Ct}} = (1 - \tau_t^H)w_t \tag{3}$$

and standard bond Euler conditions

$$u_{Ct} = \beta R_t^j u_{C_{t+1}^j}, \ \forall j \tag{4}$$

result from household optimization. As usual, the complete set of bond Euler conditions (4) defines the one-period-ahead stochastic discount factor, $E_t \Xi_{t+1|t} \equiv \beta E_t u_{Ct+1}/u_{Ct}$. The other household optimality condition is the stock demand equation:

$$v_t = (1-\delta)E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1}^D)d_{t+1} + v_{t+1} \right] \right\}. \tag{5}$$

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[13]When we consider how the model economy responds to exogenous fiscal policy in Section 3, we do temporarily allow for lump-sum taxation because there we are not studying government financing issues. For the Ramsey analysis in Section 4, lump-sum taxes are again fixed to zero.

Forward iteration implies that the share price is equal to the expected present discounted value of after-tax dividend payments, adjusted for the risk of exit.

Having optimally chosen the consumption index C_t, the household then chooses a quantity $c_t(\omega)$ of each product variety ω to minimize the total cost of purchasing C_t. With a symmetric and homothetic aggregator over a continuum of varieties, the demand function for each variety ω is

$$c_t(\omega)\,d\omega = \frac{\partial P_t}{\partial p_t(\omega)}C_t. \tag{6}$$

The specifications for the variety aggregator are described below. The nominal price of the consumption index is P_t, and $p_t(\omega)$ is the nominal price of symmetric variety ω. From here on, we cast things in terms of the relative price, $\rho_t \equiv p_t/P_t$, of a variety, and, anticipating the symmetry of the equilibrium, we drop the argument ω; ρ_t is denominated in units of the consumption index C_t.[14]

2.3   Firms

There is a continuum of identical firms that produce and sell output, so we can restrict attention to a representative firm. The representative firm is modeled as being a "large firm" that produces "many" varieties.[15] This formulation facilitates interpretation of results and yields identical equilibrium conditions as the formulation of BGM, who do not use the large firm approach.[16]

Expressed in real terms (that is, in units of the consumption index C_t), the intertemporal profit function of the representative firm is:

$$E_0 \sum_{t=0}^{\infty} \Xi_{t|0}\left[(1-\tau_t^D)(\rho_t - mc_t)q_tN_t - (1-\tau_t^S)\frac{w_t}{Z_t}f_{Et}N_{Et}\right]. \tag{7}$$

[14]The assumption of complete asset markets in government bonds allows us to focus only on real variables below, with no concern for nominal prices — in particular, P_t.

[15]The firm is "large" in the sense that it produces multiple varieties, but the assumption of a continuum of firms ensures that each is small relative to the overall economy, and hence does not internalize the effects of its decisions on the economy's price index P_t. With a representative firm, P_t turns out to represent also the firm's price index in equilibrium. Thus, we are assuming that the firm's product creation decisions do not internalize the profit destruction externality of new products on *any* existing ones within the firm. This can be rationalized by assuming that new products are introduced by independent product line managers who communicate little with each other or are even encouraged to compete with each other. See Stebunovs (2008) for a model in which a discrete number of financial intermediaries can be reinterpreted as headquarters of multi-product firms that internalize the profit destruction externality of new product introduction.

[16]One can view our large-firm approach as analogous to many recent general equilibrium macro models that feature search and matching frictions in various markets. In such models, the "large firm" assumption (for example, a representative firm that has to search individually for the "many" employees it seeks to hire) also facilitates aggregation and ignores across- and within-firm strategic considerations.

Because households are the ultimate owners of firms, the intertemporal discount factor the firm applies to its profits is $\Xi_{t|0}$, the period-zero value to the representative household of period-t goods.[17] The profit function is written in such a way that it anticipates an equilibrium that is symmetric across all product varieties.[18] We now describe the rest of the components of the profit function (7).

There is an unbounded set of potential products. Developing a new product in period t entails a sunk cost f_{Et}, which is denominated in effective labor units and is identical across product varieties.[19] Measured in consumption units, the cost of developing a new product is $w_t f_{Et}/Z_t$, with Z_t denoting the effectiveness of labor in the economy. Like f_{Et}, Z_t is independent of any particular variety ω. The total number of new varieties developed in period t is N_{Et}.

Product development costs are subsidized by the government at the proportional rate τ_t^S. From a positive perspective, product development subsidies — for example, in the form of subsidies for research and development — are often elements of cyclical fiscal policy legislation — for example, to combat recessions. From a model-based perspective, allowing product development subsidies makes it easy to ensure that the tax system is *complete*, in the Ramsey sense that there is at least one independent tax instrument along each unique equilibrium margin of the model — this point is discussed further in Sections 4 and 7.

Sales of each product variety occur in a monopolistically competitive market. The demand for each symmetric variety is denoted q_t.[20] The flow profit that each variety generates is thus $(\rho_t - mc_t) q_t$, in which mc_t denotes the marginal cost of producing an existing variety, also assumed independent of any particular variety ω. As described below, the production technology of each variety (given that the development cost $w_t f_{Et}/Z_t$ is sunk) is constant returns, hence marginal and average costs of production coincide.[21] In equilibrium, $(\rho_t - mc_t) q_t$ is the dividend, d_t, that households receive on their stock holdings. Because households receive only the after-tax share $(1 - \tau_t^D)$ and because there is no principal-agent problem between firms and households, the firm discounts the dividends it disburses by the same after-tax rate $(1 - \tau_t^D)$.

The profit maximization problem is analyzed in three steps. We first characterize the new

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[17]Because $\Xi_{s|0} \equiv \frac{\beta^s u_{Cs}}{u_{C0}}$, we have that the one-period stochastic discount factor is $\frac{\Xi_{t+1|0}}{\Xi_{t|0}} = \Xi_{t+1|t} = \frac{\beta u_{Ct+1}}{u_{Ct}}$, which will appear in the firm's optimality conditions below.

[18]A priori, the profit function is $\sum_{t=0}^{\infty} \Xi_{t|0} \left[(1 - \tau_t^D) \int_{\Omega_t} (\rho_t(\omega) - mc_t) q(\rho_t(\omega)) d\omega - (1 - \tau_t^S) \frac{w_t}{Z_t} f_{Et} N_{Et} \right]$.

[19]To the extent that f_{Et} contains a policy-determined component, regulatory policy, in BGM's interpretation, would operate through f_{Et}.

[20]We write q_t to stand for total demand for a variety, rather than c_t, because the model also features (exogenous) government consumption. To facilitate aggregation, the bundle of differentiated varieties the government purchases is identical to the private consumption bundle of households. Hence, q_t subsumes both private and public demand for a (symmetric) variety. The units of q_t are physical units of differentiated variety.

[21]The assumptions of symmetric costs and a symmetric aggregator formally justify that the equilibrium will be symmetric across product varieties.

product creation decision, then derive the firm's optimal pricing function, and finally describe the production process and characterize the firm's choice of inputs.

2.3.1 Creation of New Varieties

The representative firm takes as constraint the sequence of laws of motion for the total number of varieties it produces and sells,

$$N_{t+1} = (1 - \delta)(N_t + N_{Et}). \tag{8}$$

Optimization of (7), subject to the sequence of constraints (8), with respect to N_{Et} and N_{t+1} yields:

$$(1 - \tau_t^S)\frac{w_t}{Z_t}f_{Et} = (1 - \delta)E_t\left\{\Xi_{t+1|t}\left[(1 - \tau_{t+1}^D)(\rho_{t+1} - mc_{t+1})q_{t+1} + (1 - \tau_{t+1}^S)\frac{w_{t+1}}{Z_{t+1}}f_{Et+1}\right]\right\}, \tag{9}$$

which we refer to as the product creation condition. Combining the product creation condition (9) with the stock demand equation (5), and recalling that all flow profits are distributed as dividends — i.e., $d_t = (\rho_t - mc_t)q_t$ — yields the equilibrium free entry condition

$$v_t = (1 - \tau_t^S)\frac{w_t}{Z_t}f_{Et} \tag{10}$$

assumed in BGM and the related literature (adjusted for the product creation subsidy). Thus, firms raise (on a per-variety basis) $(1 - \tau_t^S)\frac{w_t}{Z_t}f_{Et}$ on the stock market for product development activities. Combined with the per-variety subsidy $\tau_t^S\frac{w_t}{Z_t}f_{Et}$, the entirety of product creation costs are financed.

2.3.2 Optimal Pricing

Given a number of product varieties N_t, the first-order condition for profit maximization with respect to the relative price ρ_t of any given variety (see Appendix A) is:

$$q_t + \rho_t\frac{\partial q_t}{\partial \rho_t} - mc_t\frac{\partial q_t}{\partial \rho_t} = 0. \tag{11}$$

Defining $\zeta_t \equiv \frac{\rho_t}{q_t}\frac{\partial q_t}{\partial \rho_t}$ as the price elasticity of demand for a symmetric variety, the optimal pricing rule can be expressed in the familiar form:

$$\rho_t = \left(\frac{\zeta_t}{1 + \zeta_t}\right)mc_t, \tag{12}$$

which shows that the relative price of a variety is in general an endogenously time-varying markup over real marginal cost. Denoting the gross markup by $\mu_t \equiv \frac{\zeta_t}{1+\zeta_t}$, the optimal pricing rule can be expressed more compactly as:

$$\rho_t = \mu_t mc_t. \tag{13}$$

The precise expression for the markup μ_t depends on the specific form of the variety aggregator that we will use below.

2.3.3 Production, Choice of Inputs, and Labor Market Clearing

Production of each existing variety occurs using a linear-in-labor technology. Letting h_t denote labor used to produce y_t units of a particular variety, the existing-goods-producing technology is $y_t = Z_t h_t$, where Z_t is the exogenous level of labor productivity that is common across varieties.[22] Given unit production cost for an existing variety, $mc_t = w_t/Z_t$, the market clearing condition $y_t = q_t$ determines the quantity of labor hired for production of each variety.

Given the exogenous cost of product creation (in units of effective labor) f_{Et}, the technology for creation of new products is also linear and such that $h_{Et} = f_{Et}/Z_t$ units of labor are required for the development of each new product. With h_{Et} units of labor required to develop each new variety and h_t units of labor required to produce each existing variety, the total quantity of labor hired by the representative firm is $h_t N_t + h_{Et} N_{Et}$, which, in equilibrium, must be equal to the quantity H_t supplied by the representative household.

2.4 Government

The government finances an exogenous stream of spending $\{G_t\}_{t=0}^{\infty}$ by collecting labor income taxes, dividend income taxes, and issuing real state-contingent debt. As described above, it also provides product development subsidies. The period-t government budget constraint is

$$\tau_t^H w_t H_t + \tau_t^D d_t x_t N_t + \sum_j \frac{1}{R_t^j} B_{t+1}^j = G_t + B_t + \tau_t^S \frac{w_t}{Z_t} f_{Et} N_{Et}. \tag{14}$$

Government absorption G_t is of the same bundle of varieties as private consumption C_t, which facilitates aggregation.[23] The fact that the government is able to issue fully state-contingent real debt means that none of the optimal policy results is driven by incompleteness of debt markets or ad-hoc limits on government assets.

2.5 Competitive Equilibrium

Now that we are at the stage of constructing the equilibrium, we make explicit the equilibrium dependence of the markup and the relative price of a given product on the total stock of products in the economy — thus, we now explicitly write $\mu(N_t)$ and $\rho(N_t)$ instead of μ_t and ρ_t. The analytic forms of these functions depend on the form of the variety aggregator when we make specific assumptions below.

As shown in Appendix B, the definition of a symmetric competitive equilibrium can be expressed quite compactly. Specifically, a symmetric competitive equilibrium is a set of endogenous state-

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[22]Following BGM, we assume the same exogenous productivity for labor used in production of existing varieties and creation of new ones.

[23]Thus, for each variety ω, $g_t(\omega)\partial\omega = \frac{\partial P_t}{\partial p_t(\omega)}G_t$.

contingent processes $\{C_t, H_t, N_{t+1}, N_{Et}, v_t, B_{t+1}\}_{t=0}^{\infty}$ that satisfy six sequences of conditions: the labor optimality condition

$$-\frac{u_{Ht}}{u_{Ct}} = \frac{(1-\tau_t^H)}{\mu(N_t)} Z_t \rho(N_t), \tag{15}$$

the intertemporal product creation condition

$$(1-\tau_t^S)\rho(N_t)f_{Et} = (1-\delta)E_t\left\{\Xi_{t+1|t}\left[(1-\tau_{t+1}^D)\left(\mu(N_t) - \frac{\mu(N_t)}{\mu(N_{t+1})}\right)\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + (1-\tau_{t+1}^S)\frac{\mu(N_t)}{\mu(N_{t+1})}\rho(N_{t+1})f_{Et+1}\right]\right\}, \tag{16}$$

the equilibrium entry condition

$$v_t = (1-\tau_t^S)\frac{\rho(N_t)}{\mu(N_t)}f_{Et}, \tag{17}$$

the law of motion for the number of product varieties

$$N_{t+1} = (1-\delta)(N_t + N_{Et}), \tag{18}$$

the flow government budget constraint (14),[24] and the consumption resource constraint

$$C_t + G_t + \rho(N_t)f_{Et}N_{Et} = \rho(N_t)Z_tH_t. \tag{19}$$

The consumption resource constraint (19) is obtained by summing the flow household budget constraint (2) (after imposing the equilibrium condition $x_{t+1} = x_t = 1 \ \forall t$) and the flow government budget constraint (14), and then substituting several equilibrium conditions; a complete derivation appears in Appendix B. An important feature to note about this frontier is the appearance of $\rho(N_t)$, which represents a relative price in the decentralized economy. As is well understood in models of monopolistic competition with endogenous variety, the relative price $\rho(N_t)$ captures the welfare benefit of variety embedded in household preferences; as such, it is a primitive of the economy, which can be interpreted as a measure of increasing returns to variety.[25]

2.6 Welfare-Consistent versus Data-Consistent Concepts

The concepts of consumption, government expenditures, investment (in new product development), and GDP that appear in the model description are the welfare-relevant ones; however, they are not data-consistent concepts. As discussed in BGM, achieving comparability between the model and the data requires measuring consumption, government expenditures, investment, and GDP in the model as $\frac{C_t}{\rho(N_t)}$, $\frac{G_t}{\rho(N_t)}$, $\frac{v_tN_{Et}}{\rho(N_t)}$, and $\frac{w_tH_t+d_tN_t}{\rho(N_t)}$, respectively, which adjusts for the benefit of variety

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[24]Which can be expressed in terms of only the processes listed above by substituting the equilibrium expressions for the real wage, $w_t = Z_tmc_t = \frac{Z_t\rho(N_t)}{\mu(N_t)}$, and dividend payments, $d_t = \left(1 - \frac{1}{\mu(N_t)}\right)\left(\frac{C_t+G_t}{N_t}\right)$.

[25]As noted above, an alternative formalism of the model casts product varieties as intermediate goods in production of a homogenous final good. In this alternative setup, $\rho(N_t)$ captures equilibrium increasing returns to variety in production of the final good.

(required because the data do not adjust for it).[26] Thus, all results reported below are for these data-consistent measures; we indicate these data-consistent measures with a subscript "R," which denotes division by $\rho(N_t)$ to remove the variety effect.[27] Being precise about welfare-consistent versus data-consistent measures requires that the exogenous purchases component of fiscal policy is taken to be $G_{Rt} \equiv \frac{G_t}{\rho(N_t)}$, as described below.

3 Exogenous Fiscal Policy

Before studying the model's implications for optimal tax policy, we study its cyclical properties conditional on exogenous fiscal policy. Incorporating realistic features of fiscal policy adds to the literature's understanding of the BGM framework. The model's dynamics conditional on exogenous policy also provide a benchmark for understanding the optimal policy results in Section 4. In the exogenous policy experiments here, the product development subsidy, and the dividend tax are set to $\tau_t^S = 0 \ \forall t$ and $\tau_t^D = \tau^D > 0 \ \forall t$, respectively. To enhance comparability with the results of BGM, parameter values and/or calibration targets are taken from their study where possible.

3.1 Calibration

For utility, we adopt a standard functional form:

$$u(C_t, H_t) = \ln C_t - \frac{\zeta}{1 + 1/\nu} H_t^{1+1/\nu}. \tag{20}$$

Following BGM, the Frisch elasticity of labor supply is set to $\nu = 4$, and the scale parameter is set to deliver a steady-state fraction of time spent working of $H = 0.2$ (the required value is $\zeta = 6.8$, given all other parameter values). The model frequency is quarterly, so the subjective discount factor is set to $\beta = 0.99$, which delivers an annual real interest rate of approximately four percent.

We consider the variety aggregators studied by BGM: Dixit-Stiglitz (1977) preferences and the translog expenditure function proposed by Feenstra (2003). The baseline calibration is for the case of Dixit-Stiglitz aggregation because of its widespread use in macro models. In the Dixit-Stiglitz case, the final consumption index C_t is composed of the underlying varieties $c_t(\omega)$ according to $C_t = \left[\int_{\omega \in \Omega} c_t(\omega)^{(\theta-1)/\theta} d\omega \right]^{\frac{\theta}{\theta-1}}$. Dixit-Stiglitz aggregation implies a gross markup independent of the number of product varieties, $\mu = \frac{\theta}{\theta-1}$, and a relative price $\rho_t = N_t^{\mu-1}$ of a symmetric variety. As in BGM, we set $\theta = 3.8$ as a benchmark, which implies a 35-percent average net markup.

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[26]Note that we use the NIPA definition of GDP as total income, $w_t H_t + d_t N_t$, which equals the sum of private consumption expenditure, government expenditure, and total investment expenditure, $C_t + G_t + \frac{w_t}{Z_t} f_{Et} N_{Et}$. In our exercises, we focus on the private portion of investment, $v_t N_{Et}$, which differs from the economy's total investment because of the product development subsidy (when it differs from zero).

[27]Thus, we write GDP_{Rt} to indicate $\frac{w_t H_t + d_t N_t}{\rho(N_t)}$, C_{Rt} to indicate $\frac{C_t}{\rho(N_t)}$, G_{Rt} to indicate $\frac{G_t}{\rho(N_t)}$, I_{Rt} to indicate $\frac{v_t N_{Et}}{\rho(N_t)}$, and so on.

The sunk cost of creating a new product is fixed at $f_{Et} = 1$ and is assumed invariant along the business cycle. As noted in the Introduction, we focus on the efficiency implications of fiscal policy (in both the short run and the long run), rather than regulation policy, which justifies fixing f_{Et}. While regulation is likely to affect entry costs (for example, by reducing bureaucratic costs), it is unlikely to do so over the cycle. Furthermore, regulation policy is likely applied heterogeneously (as a long-run tool) across firms of different sizes, an issue beyond the scope of this paper.[28] The rate of destruction of product varieties is set to $\delta = 0.025$, following the calibration of BGM. Given the quarterly frequency of the model, this means roughly 10 percent of product varieties disappear from the market every year, independent of product age.

The three exogenous processes are productivity, government spending (which, as noted above, is measured in data-consistent units), and the labor income tax rate, each of which follows an AR(1) process in logs:

$$\ln Z_t = \rho_Z \ln Z_{t-1} + \epsilon_t^Z, \tag{21}$$

$$\ln G_{Rt} = (1 - \rho_{G_R}) \ln \bar{G}_R + \rho_{G_R} \ln G_{Rt-1} + \epsilon_t^{G_R}, \tag{22}$$

and

$$\ln \tau_t^H = (1 - \rho_{\tau^H}) \ln \bar{\tau}^H + \rho_{\tau^H} \ln \tau_{t-1}^H + \epsilon_t^{\tau^H}. \tag{23}$$

The innovations ϵ_t^Z, $\epsilon_t^{G_R}$, and $\epsilon_t^{\tau^H}$ are distributed $N(0, \sigma_{\epsilon^Z}^2)$, $N(0, \sigma_{\epsilon^{G_R}}^2)$, and $N(0, \sigma_{\epsilon^{\tau^H}}^2)$ respectively, and are independent of each other. Persistence parameters are set to $\rho_Z = 0.979$, which matches BGM and King and Rebelo (1999), and $\rho_{G_R} = 0.97$, as in the benchmark quantitative Ramsey models of Chari and Kehoe (1999). The magnitudes of innovations are set to $\sigma_{\epsilon^{G_R}} = 0.027$, also consistent with baseline Ramsey models, and $\sigma_{\epsilon^Z} = 0.0072$, which is the same value as in BGM and enables the benchmark exogenous policy model to generate GDP volatility in line with its magnitude in U.S. fluctuations. In the exogenous policy Dixit-Stiglitz benchmark, the steady-state level of government spending $\bar{G}_R$ is calibrated so that it absorbs 22 percent of steady-state GDP; the resulting value is $\bar{G}_R = 0.044$. However, this value is reset (to $\bar{G}_R = 0.074$) when we study the Ramsey equilibrium in order to keep the steady-state GDP share of government spending, and hence the revenue requirements of the government, constant at 22 percent.

The parameterization of the labor income tax process is taken from Arseneau and Chugh (2010), who use the methodology of Jones (2002) to construct an empirical measure of the average U.S. labor income tax rate from 1947:Q1-2009:Q4.[29] The mean labor income tax rate over this period is about 20 percent. In terms of its cyclical properties, the first-order autocorrelation is 0.66, and the

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[28]BGM discuss the consequences of "universal" deregulation (a permanent decline in f_{Et}). See also Cacciatore and Fiori (2009).

[29]The source data are the NIPA accounts of the U.S. Bureau of Economic Analysis, and the methodology to construct the tax rate series is described in detail in Appendix B of Jones (2002).

standard deviation of the cyclical component of the tax rate is 2.8 percent, which means that the standard deviation of the *level* of the tax rate is about 0.70 percentage points around its mean of 20 percent. Matching the persistence and volatility of this empirical tax rate series requires setting $\rho_{\tau^H} = 0.87$ and $\sigma_{\epsilon^{\tau^H}} = 0.037$.

Finally, the dividend income tax rate is assumed to be a constant $\tau^D = 0.30$ in every period, which is representative of the average U.S. corporate (including both federal and state) tax rate. For the exogenous policy experiments only, the government is assumed to have available a lump-sum tax/transfer vis-a-vis households in order to balance its budget, which allows us to ignore government financing issues. When we move to the Ramsey analysis, the lump-sum tax is dropped and the government instead has one-period state-contingent debt as a policy tool (in addition to its proportional tax instruments τ^H, τ^D, and τ^S).[30] In the Ramsey analysis, the steady-state government debt-to-GDP ratio (at an annual frequency) is calibrated to 0.5, in line with the average U.S. post-war government debt.

When we move to translog preferences, we adjust the calibration so that the model hits the same long-run targets. Doing so requires appropriately setting one new parameter the translog aggregator introduces and resetting only two parameters from above. The translog primitive is the expenditure function across varieties. BGM and Feenstra (2003) provide detailed analysis; here, we simply note that in the translog case, the markup is given by $\mu_t = 1 + \frac{1}{\sigma N_t}$, with $\sigma > 0$, and the relative price of a symmetric variety is $\rho(N_t) = \exp\left(-\frac{1}{2}\frac{\tilde{N} - N_t}{\sigma \tilde{N} N_t}\right)$, with the parameter $\tilde{N}$ interpreted as the mass of the potential set of varieties that ever could exist, N_t of which actually exist and are produced in period t.[31] As shown in BGM, it is possible to set σ so that the translog case results in the same steady-state markup and number of products as the Dixit-Stiglitz case — given our parameterization, this requires $\sigma = 1.9$[32] The long-run level of government absorption must be reset (to $\bar{G}_R = 0.035$) to keep its share in GDP fixed at 22 percent in the translog case — and, just as noted above for the Dixit-Stiglitz case, is reset again (to $\bar{G}_R = 0.087$) when we study the

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[30]In the exogenous policy experiments, the (endogenous and state-contingent) lump-sum tax allows us to ignore the nature and dynamics of government debt in the data, i.e., is it state-contingent debt? what are the fiscal rules by which debt is stabilized? etc. These are interesting questions, not only for our study but the broad fiscal policy literature, but beyond the scope of our paper.

[31]The translog specification has the intuitively appealing property that an increase in the number of varieties available in the economy is associated with an increase in the degree of substitutability between any given pair of varieties. This aspect of aggregation is absent in the most commonly used specification of the Dixit-Stiglitz aggregator, which assumes a constant elasticity of substitution across varieties even if their number is endogenous.

[32]The parameter $\tilde{N}$ is set very loosely, $\tilde{N} = 10^9$, which represents the idea that there is an unbounded number of varieties in the potential product space. BGM show that $\tilde{N}$ drops out of a *log*-linear approximation of the model's dynamics. As noted below, we compute dynamics using a *level*-linear approximation, in which the parameter $\tilde{N}$ does not drop out, hence our need to choose a numerical value; the value $\tilde{N} = 10^9$ is orders of magnitude larger than needed so that its precise setting does not affect the model's steady state or dynamics.

translog Ramsey equilibrium.

The deterministic steady-state equilibrium is computed using a nonlinear numerical solver. To study dynamics, we compute a first-order approximation of the equilibrium conditions around the deterministic steady state.[33] We use the first-order accurate decision rules to simulate time paths of the equilibrium in response to productivity, government spending, and labor tax realizations, the shocks to which we draw according to the parameters of the laws of motion described above. We conduct 500 simulations, each 200 periods long. For each simulation, we then HP filter (using quarterly smoothing parameter 1,600), compute second moments of interest, and report the medians of these moments across the 500 simulations.

3.2 Results

Figure 2 presents, for both Dixit-Stiglitz and translog preferences, impulse responses of GDP, product creation, markups, and aggregate profits (which are four key measures whose cyclical dynamics the baseline BGM model reproduces well) to one-time, one-standard-deviation positive shocks to productivity (first row), government spending (second row), and the labor income tax rate (third row). Conditional on productivity shocks, the impulse responses are similar to those in BGM. All differences (in magnitudes and persistence) compared to BGM are due to the presence of long-run distortions induced by fiscal policy, distortions that are absent in the BGM analysis.[34] Differences between the Dixit-Stiglitz and translog cases are due to the aggregator-specific behavior of product substitutability and markups. While markups are constant with Dixit-Stiglitz preferences, translog preferences generate procyclical substitutability and hence countercyclical markups. As a consequence, ceteris paribus, the benefit to consumers of additional variety and the profit incentive for firms to develop new products decrease (increase) over time during expansions (contractions). Thus, fluctuations in product entry are dampened in the translog case compared to the Dixit-Stiglitz case.

Regarding fiscal policy, the responses to a government spending shock (second row) are qualitatively similar to those to a productivity shock. Consumption (not shown) declines as it is crowded out by increased government absorption, a standard counterfactual prediction due to Ricardian consumer behavior.[35] A one-time increase in the labor income tax rate (third row) leads to a relatively large output contraction, due mainly to a sharp decline in new product development, which

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[33]Our numerical method is our own implementation of the perturbation algorithm described by Schmitt-Grohé and Uribe (2004).

[34]We have confirmed this result by also computing impulse responses to productivity for the parameter values $\bar{G}_R = \tau^H = \tau^D = 0$, a point noted again below.

[35]Recall that a lump sum tax is present for the exogenous policy experiments, which generates Ricardian equivalence. This counterfactual prediction would be easily fixed by introducing a set of non-Ricardian consumers, as is common in the literature, but this is beyond the scope of the paper.

falls roughly ten percent on impact under both forms of variety aggregation. A higher tax rate causes a roughly two percent decline in aggregate hours (not shown) for both forms of preferences. Because the setup cost f_E of developing new products requires labor, hours worked in the product development sector also fall sharply.

To provide more quantitative detail on the model's cyclical dynamics, especially those due to shifts in taxes, Table 1 presents simulation results. The upper panels display results when all three exogenous processes are active, and the lower panels display results conditional on shocks to only productivity and government spending, holding constant the labor income tax rate at 20 percent. Fluctuations in Z and G are the inputs to the dynamic Ramsey analysis in Section 4, hence the lower panels of Table 1 provide a benchmark.

Three main aspects of the simulation results are worth highlighting, each of which contributes to the development of the BGM class of models as a positive description of U.S. business cycles. First, regardless of the form of variety aggregation, the volatility of aggregate hours is virtually identical to the volatility of output, in line with the relative volatility of hours in U.S. macro data. However, if τ^H is constant over the business cycle (the lower panels of Table 1), the relative volatility of total hours is about 0.6, just as in the baseline BGM model without fiscal shocks. The results in Table 1 show that incorporating realistic tax fluctuations is a step in the right direction by substantially improving the model's relative volatility of hours. Successfully reproducing the dynamics of labor-market outcomes is a long-standing central issue in macroeconomic modeling.[36]

Second, the volatility of investment in product creation is about six times the volatility of GDP when fluctuations are driven by shocks to all three exogenous processes, which is roughly double the relative volatility of investment in U.S. data. When it is only shocks to productivity and government absorption that cause cycles (the lower panels of Table 1), this relative volatility falls to between four to five. However, note that overall volatility, as measured by the volatility of GDP, also falls quite sharply when τ^H is constant — from about 2.5 percent to less than 1.5 percent. Fluctuations in tax rates thus contribute quantitatively significantly to the magnitude of overall fluctuations, in both absolute and relative terms.

Third, volatility falls further if it is only productivity shocks that are active, as Table 2 shows. The model's GDP volatility conditional on shocks to only Z is smaller than found in BGM, which is due to the long-run distortionary effects of fiscal policy. Indeed, if we assume no distortions whatsoever by setting $\bar{G}_R = \bar{\tau}^H = \tau^D = 0$, the dynamics of the model conditional on only productivity shocks (shown in the lower panels of Table 2) are identical to those in BGM.

Overall, Tables 1 and 2 document that the business cycle properties of the BGM model are

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[36]BGM show that inclusion of physical capital as a factor of production also improves the model's performance along this dimension. Shao and Silos (2009) and Cacciatore and Fiori (2009) introduce unemployment in the BGM framework by incorporating matching frictions in the labor market.

noticeably different once realistic features of fiscal policy are incorporated. At the center of the mechanism is the dynamic behavior of the within-period deviation, or "wedge," between the household's marginal rate of substitution between consumption and labor and the "effective" marginal product of labor in producing consumption, $\rho(N_t)Z_t$, that appears in the consumption resource frontier (19). Figure 3 illustrates this point with an impulse response of the within-period wedge (defined from the labor optimality condition (15) as $1 - \frac{-u_{Ht}/u_{Ct}}{Z_t \rho(N_t)}$) to a positive shock to the labor tax rate. The wedge fluctuates sharply and, together with the results shown in (the third row of) Figure 2, is clearly countercyclical. The dynamics of the wedge conditional on exogenous tax policy are important for understanding the Ramsey equilibrium.[37]

4   Optimal Fiscal Policy

With the baseline calibration and dynamics established, we now discard the exogenous process (23) for the labor income tax rate and instead endogenize tax policy (labor income taxes, dividend taxes, and product development subsidies).[38] While taxes are now optimally chosen by the Ramsey government, government purchases continue to follow the exogenous process (22).[39]

4.1   Ramsey Problem

A standard approach in Ramsey models based on neoclassical markets is to capture in a single, present-value implementability constraint (PVIC) all equilibrium conditions of the economy apart from the resource frontier. The PVIC is the key constraint in any Ramsey problem because it governs the welfare loss of using non-lump-sum taxes to finance government expenditures.[40]

We can construct a PVIC starting from the household flow budget constraint (2) and using the household optimality conditions (3), (4), and (5). However, because of the forward-looking aspects of *firm* optimization, the PVIC does not capture all of the model's equilibrium conditions.[41] Derived

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[37]Table 1 reports also the properties of the model-generated, data-consistent price of capital in the model, which will also be useful to understand the Ramsey equilibrium.

[38]We also return to the case of zero lump-sum taxes, required for a Ramsey analysis.

[39]Thus, we follow the standard convention in Ramsey analysis that spending is exogenous but the revenue side of fiscal policy is determined optimally.

[40]See, for example, Ljungqvist and Sargent (2004, p. 494) for more discussion. The PVIC is the household (equivalently, government) budget constraint expressed in intertemporal form with all prices and policies substituted out using equilibrium conditions. In relatively simple models, the PVIC encodes all the equilibrium conditions that must be respected by Ramsey allocations in addition to feasibility. In complicated environments that deviate substantially from neoclassical markets, however, such as Schmitt-Grohé and Uribe (2005), Chugh (2006), and Arseneau and Chugh (2008), it is not always possible to construct such a single constraint.

[41]A very similar, in form, construction of the Ramsey problem arises in Arseneau and Chugh (2010), who study optimal fiscal policy in a model with labor market frictions.

in Appendix C is the PVIC:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u_{Ct} C_t + u_{Ht} H_t \right) = u_{C0}[v_0 + (1 - \tau_0^D) d_0] N_0 + u_{C0} B_0. \tag{24}$$

Because the number of product varieties is a state variable, the household's ownership, via share holdings, of the initial stock of varieties, N_0, is part of its time-zero assets, as the right-hand side of (24) shows. In this sense, the initial stock of varieties acts like the initial stock of physical capital in a Ramsey analysis of the baseline RBC model.

However, unlike in a standard model, the PVIC (24) does not capture all equilibrium conditions, so the Ramsey problem cannot be cast in the standard pure "primal" form. In particular, Ramsey allocations must also respect the intertemporal product creation condition (16) and the entry condition (17). The appearance of (expectations of) future tax rates in the product creation condition prevents formulation in pure primal form because there is no way to eliminate the future tax rates from the Ramsey problem. Hence, we directly compute Ramsey first-order conditions with respect to the product creation subsidy and (with a caveat discussed next) the dividend income tax to characterize their optimal settings.

Two novel issues regarding the nature of available tax instruments and how they can be used to decentralize Ramsey allocations require discussion. First, as just noted, Ramsey first-order conditions with respect to the product creation subsidy and the dividend income tax directly must be computed. However (considering the period-t competitive equilibrium), it is *only* the period-$t+1$ dividend tax that appears in the period-t equilibrium conditions. The *realized* period-t dividend tax does *not* directly affect the period-t competitive equilibrium due to the forward-looking nature of product development decisions.[42] In principle, this requires computing a Ramsey first-order condition with respect to τ_{t+1}^D as part of the period-t Ramsey first-order conditions. This would pose no problem if the environment were deterministic. However, with uncertainty, τ_{t+1}^D is indeterminate with respect to the period-t information set of the economy. This indeterminacy requires setting up the Ramsey problem in a novel way.

We resolve the indeterminacy by assuming that the Ramsey government chooses a state-contingent schedule of one-period-ahead dividend tax rates, one for each of the possible realized states. We use the notation $\tau_{t+1|t}^D$ to denote this state-contingent schedule, which *is* in the private sector's period-t information set. Thus, in conducting the Ramsey optimization, we replace τ_{t+1}^D with $\tau_{t+1|t}^D$ in the product creation condition (16), along with the auxiliary assumption that the Ramsey government always implements its one-period-ahead state-contingent "announcements" of dividend taxes.[43] That is, the Ramsey government optimally chooses the schedule $\tau_{t+1|t}^D$ in period

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[42]Inspecting (14)-(19) shows that only τ_{t+1}^D appears in the period-t equilibrium conditions.

[43]Formally, this means that the period $t+1$ dividend tax rate can be taken out of the expectation operator in the product creation condition (16); note, however, that this does *not* make the product creation condition deterministic.

t, and then implements with certainty the *particular* value of $\tau_{t+1|t}^D$ that the schedule specificies as the *actual* τ_{t+1}^D in period $t+1$. This is a novel type of "one-period commitment" on the part of the Ramsey government, but we view it as compatible with (and weaker than) the maintained assumption of commitment to policy functions from period zero onwards that is a defining feature of Ramsey analysis. From here on, we use the phrase "optimal dividend income tax" when discussing the Ramsey equilibrium, recognizing that, outside the deterministic steady state, what the Ramsey government chooses is a state-contingent one-period-ahead schedule.[44]

The second novel issue is also one of indeterminacy, although between tax instruments in a given time period rather than for a given tax instrument across time periods. The product development decision (16) is affected by both development subsidies and (the state-contingent schedule of) dividend taxes. Because neither policy instrument appears in any other period-t private-sector equilibrium condition, an infinite combination of pairs $(\tau_t^S, \tau_{t+1|t}^D)$ induces identical product development decisions. This is a standard form of Ramsey indeterminacy, and the Ramsey equilibrium can endogenously pin down only one, but not both, of the instruments $\tau_{t+1|t}^D$ and τ_t^S; this point is elaborated further in Section 7 in the context of a broader discussion of the nature of the assumed tax system. In the Ramsey results reported below, the indeterminacy is resolved by fixing, in turn, one of the tax instruments to zero and optimizing with respect to the other; we refer to the former as the "inactive" instrument and the latter as the "active" instrument.[45]

The Ramsey problem is thus to choose state-contingent processes for $\{C_t, H_t, N_{t+1}, N_{Et}\}_{t=0}^{\infty}$ and either $\{\tau_t^S\}_{t=0}^{\infty}$ or $\{\tau_{t+1|t}^D\}_{t=0}^{\infty}$ to maximize (1) subject to the PVIC (24), the product creation condition (16), the entry condition (17), the law of motion for the measure of product varieties (18), and the resource constraint (19). Finally, as is standard in Ramsey taxation problems and implicit in the discussion above, the Ramsey government is assumed to fully commit to time-invariant policy functions as of period zero. Thus, none of the results is driven by the use of a discretionary policy.[46]

4.2 The Timeless Perspective and Computational Issues

The first-order conditions of the Ramsey problem are assumed to be necessary and sufficient, and all allocations are assumed to be interior. As in the exogenous policy baseline, a nonlinear numerical solution algorithm is used to compute the deterministic Ramsey steady-state equilibrium. As is

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[44]We thank Marco Bassetto for suggesting this approach. In the context of an incomplete-markets Ramsey model, Fahri (2009) uses a similar approach in choosing the one-period-ahead (non-state-contingent) capital income tax rate; doing so retains the incomplete-markets nature of his analysis. Analogously, allowing the choice of the one-period-ahead state-contingent dividend income tax retains the complete-markets nature of our analysis.

[45]Alternatively, we could fix the inactive instrument to any arbitrary value, both in the long run and along the stochastic fluctuations of the Ramsey equilibrium, but there is little basis for preferring one decentralization over another.

[46]The stock nature of product varieties is what allows scope for use of a discretionary policy.

common in the Ramsey literature, when characterizing asymptotic policy dynamics (that is, the dynamics implied by the Ramsey $t > 0$ first-order conditions), we also make the auxiliary assumption that the initial state of the economy is the asymptotic Ramsey steady state, which is tantamount to adopting the "timeless perspective" common in Ramsey-based quantitative analysis.[47]

More precisely, to study dynamics, we compute a first-order approximation of the Ramsey first-order conditions for period $t > 0$ around the deterministic steady state of these conditions. We then use the first-order accurate decision rules to simulate the Ramsey equilibrium in the face of productivity and government spending realizations. The productivity and government spending realizations used to conduct the Ramsey simulations are the same as those in the exogenous policy experiments in Section 3, which means that any differences between the Ramsey equilibrium and exogenous policy equilibrium are attributable entirely to the dynamics of tax policy.

4.3   Long-Run Optimal Policy

The first main result is that the long-run Ramsey equilibrium achieves efficiency along the product creation margin. Efficiency can be decentralized by an appropriate dividend income tax or product creation subsidy, depending on which instrument is active. Regardless of which instrument is active, its precise setting depends on the particular form of variety aggregation in preferences.

Before presenting results, it is useful to define the welfare benefit of variety in elasticity form:

$$\epsilon(N_t) = \rho'(N_t)\frac{N_t}{\rho(N_t)}. \tag{25}$$

As noted above, the relative price $\rho(N_t)$ measures the (welfare) return to product variety, to which we refer as the "variety effect."[48] The elasticity $\epsilon(N_t)$ turns out to be a convenient way of characterizing the variety effect.[49]

4.3.1   Dividend Taxation

First suppose that dividend income taxes are active, and product development subsidies are inactive ($\tau^S \equiv 0$).

Proposition 1. *Optimal Long-Run Dividend Income Tax. In the deterministic steady state of the Ramsey equilibrium in which only dividend income taxes are active, the optimal dividend income tax rate is characterized by:*

$$1 - \tau^D = \frac{\epsilon(N)}{\mu(N) - 1}, \tag{26}$$

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[47]Among other references, see Khan, King, and Wolman (2003).

[48]Symmetry across varieties implies $C_t + G_t = \rho(N_t)Z_t h_t N_t$ (recall that h_t is the labor used to produce y_t units of a particular variety). Abstracting from G_t, $\rho(N_t)$ captures the additional welfare gain of consuming the output $Z_t h_t$ of each of the N_t varieties. This role of $\rho(N_t)$ was also made apparent in the resource constraint (19).

[49]The notation $\rho'(N_t)$ recognizes that, for all preference specifications we use, ρ_t is indeed a function only of N_t.

and this tax supports long-run efficiency of product creation.

Proof. See Appendix D. □

For the BGM environment, the pure social planning allocations and the corrective *Pigovian* taxes needed to support them were developed by Bilbiie, Ghironi, and Melitz (2008b). Their results provide the analytical basis for the results we obtain regarding long-run *Ramsey* taxation. Of particular importance for our work here is that Bilbiie, Ghironi, and Melitz (2008b) — hereafter, BGM2 — determined the constellations of conditions for the markup incentives governing product development and the variety effect on welfare that are important for efficiency. It is the tradeoff of these two forces that shapes the long-run optimal dividend income tax.

A striking aspect of the Ramsey-optimal long-run dividend income tax is that it is identical to the Pigovian tax derived by BGM2. In particular, the goal of dividend taxation is to align the beneficial effects of product variety with net monopoly markups. As (26) shows, this alignment is accomplished with no need for taxation if and only if $\epsilon(N) = \mu(N) - 1$. The analysis in BGM2 is about efficiency (Pigovian) taxes because it abstracts from public finance considerations by assuming the availability of lump-sum taxation. Proposition 1 shows that endogenizing public finance considerations does not affect this normative result.

Further discussion of the result that the Ramsey equilibrium achieves efficient product creation is deferred until Section 7. In the rest of this section, we consider the implications of Proposition 1 for the precise value of τ^D. Given the normalization $\tau^S = 0$ for the analysis here, the detail of the economic environment that matters for the precise value of τ^D is the form of variety aggregation. As described above, we study the Dixit-Stiglitz and translog aggregators for the quantitative analysis. For the analytical result here, however, we also consider the Benassy (1996) generalization of the Dixit-Stiglitz aggregator, which disentangles the variety effect from the monopoly markup. Table 3 presents functional forms for markups and variety effects for each of the three aggregators; for the intuitive discussion here of the Benassy aggregator, let κ govern the variety effect, and θ continue to govern the markup as in the standard Dixit-Stiglitz aggregator.[50]

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[50]Formally, the Benassy aggregator is

$$C_t = N_t^{\kappa+1-\frac{\theta}{\theta-1}} \left[\int_{\omega \in \Omega} c_t(\omega)^{(\theta-1)/\theta} \, d\omega \right]^{\frac{\theta}{\theta-1}}, \tag{27}$$

with $\kappa \geq 0$. With Benassy aggregation, the markup of a symmetric variety is $\mu = \frac{\theta}{\theta-1}$, just as in the Dixit-Stiglitz case, but the relative price of a symmetric variety is given by $\rho_t = N_t^\kappa$. The Dixit-Stiglitz specification is recovered if $\kappa = \frac{\theta}{\theta-1} - 1$. The Benassy case is omitted from the dynamic stochastic analysis below due to lack of reliable ways of calibrating κ, and because the quantitative implications are very similar to the Dixit-Stiglitz case for all scenarious we tried (results are available upon request). We discuss the Benassy case for the steady state, however, because it yields qualitatively different results regarding optimal policy than Dixit-Stiglitz aggregation. Further details are in BGM2 and Benassy (1996).

With these three aggregators, the optimal dividend income tax can be positive, negative, or zero. Specifically, based on the functional forms of $\epsilon(N)$ and $\mu(N)$ in Table 3, the optimal long-run dividend income tax rate in the Dixit-Stiglitz aggregation is

$$\tau_{DS}^D = 1 - \frac{\frac{\theta}{\theta-1} - 1}{\frac{\theta}{\theta-1} - 1} = 0; \tag{28}$$

in the Benassy aggregation is

$$\tau_{BENASSY}^D = 1 - \frac{\kappa}{\frac{\theta}{\theta-1} - 1} = 1 - \kappa(\theta - 1); \tag{29}$$

and in the translog aggregation is

$$\tau_{TRANSLOG}^D = 1 - \frac{\frac{1}{2\sigma N}}{\frac{1}{\sigma N}} = 0.5. \tag{30}$$

The intuition for why the variety aggregator clearly matters for the optimal long-run dividend income tax is that with *zero* dividend taxation and either translog aggregation or Benassy aggregation featuring a sufficiently small variety effect, the monopoly incentives governing product development are stronger than the beneficial effects of increased product variety on welfare. Too many products are thus developed in equilibrium. A dividend income tax, which effectively taxes monopoly profits, corrects this distortion by reducing household incentives to finance product creation. In the Dixit-Stiglitz case, the product development incentive of profits and the variety effect exactly balance each other, which thus calls for a *zero* dividend tax. In the Benassy aggregation, optimal dividend income taxes can be either positive or negative, depending on which of the two effects is stronger. Taken together, the results suggest that the optimal dividend income tax in the long run is *not* likely to be zero, unless one is committed to the Dixit-Stiglitz knife-edge case.

Unless one believes literally in the translog aggregator, it is difficult to offer a precise numerical target for the long-run dividend income tax because there is little empirical evidence about the magnitude of the variety effect. Nonetheless, based on the success of the basic BGM model in reproducing a number of business cycle facts with translog aggregation, one may lean toward that as the most favored model with which also to consider optimal taxation. Furthermore, as shown in Table 3, translog aggregation has a-priori appeal because it captures the idea that the larger the mix of available products, the closer substitutes they are, an idea captured by neither the Dixit-Stiglitz nor Benassy specification.

4.3.2   Product Development Subsidies

Suppose instead that dividend taxes are inactive ($\tau^D \equiv 0$), and product creation subsidies are active.

Proposition 2. *Optimal Long-Run Product Creation Subsidy.* *In the deterministic steady state of the Ramsey equilibrium in which only product creation subsidies are active, the optimal product creation subsidy is characterized by:*

$$1 - \tau^S = \frac{\mu(N) - 1}{\epsilon(N)}, \tag{31}$$

and this tax supports long-run efficiency of product creation.

Proof. See Appendix D. $\qquad\qquad\square$

This result is also identical to BGM2. With only τ^S active, Dixit-Stiglitz aggregation requires $\tau^S = 0$ in the long run, while translog aggregation requires $\tau^S = -1$ (a 100 percent *tax* on the entry cost) in the long run. Intuitively, the optimal τ^S achieves the same objective as the optimal τ^D of aligning the welfare benefit of variety (and the associated household incentive to finance entry) with the markup (and the associated firm incentive to create products). With translog aggregation, a dividend tax achieves the objective by cutting in half the dividends received by households, while a *tax* on product creation does so by doubling firms' creation costs. We comment further on the redundancy of τ^D and τ^S with respect to each other in Section 7.

4.4 Short-Run Optimal Policy

For the rest of the analysis, we return to considering only the Dixit-Stiglitz and translog cases. Table 4 presents short-run optimal policy results. As discussed above and further in Section 7, only one of the two instruments, $\tau^D_{t+1|t}$ or τ^S_t, can be uniquely determined in the Ramsey equilibrium. Given this indeterminacy, Table 4 divides results for each form of variety aggregation into those conditional on an optimally chosen time-varying dividend income tax or an optimally-chosen product development subsidy. There are three main results to highlight regarding the Ramsey dynamics.

First, the volatility of optimal tax rates is very small. The labor income tax rate is constant in the Dixit-Stiglitz case and very nearly constant in the translog case. Regardless of whether it is $\tau^D_{t+1|t}$ or τ^S_t that is the active instrument, it also has zero volatility in the Dixit-Stiglitz case and near-zero volatility in the translog case. Tax smoothing is thus the optimal policy, as in baseline Ramsey models. Slightly different from baseline Ramsey models, however, is the "joint" nature of tax smoothing, in which *both* the labor income tax and the instrument operating on the intertemporal margin ($\tau^D_{t+1|t}$ or τ^S_t) have zero or near-zero volatility; in baseline Ramsey models, "tax smoothing" entails only the former.

Second, labor market fluctuations are much smaller in the Ramsey equilibrium than in the exogenous policy equilibrium: The relative volatility of total hours is about one third smaller, as comparison of Table 4 with the lower panels of Table 1 shows.

Third, in the translog case, the relative volatility of the stock price v_R is smaller in the Ramsey equilibrium than in the exogenous policy equilibrium. For Dixit-Stiglitz aggregation, v_R does not fluctuate in any equilibrium, Ramsey or non-Ramsey, as implied by rearranging the entry condition (17). Recall from Section 2 that v_R is the data-consistent measure of the stock price. The welfare-relevant stock price v (not shown), however, *does* fluctuate in equilibrium, and fluctuations in v are much smaller in magnitude in the Ramsey equilibrium than in the non-Ramsey equilibrium. Stock prices govern investment in new product development, and the Ramsey equilibrium also displays smaller fluctuations in investment. At face value, an objective of the Ramsey government appears to be to implement much more stable capital markets in terms of both prices and quantities.

Achieving smaller (relative) fluctuations of both labor and capital markets are only reduced-form "objectives," however, not the primitive objective of the Ramsey equilibrium. A precise explanation of the incentives that shape Ramsey outcomes, as well as how they are decentralized, requires introducing several new concepts, which is done in Sections 5 and 6. Section 7 then uses these concepts to explain the optimal policy results in a way that connects naturally to the Ramsey literature.

5 Efficiency

Ramsey allocations trade off efficiency against market decentralization. Characterizing efficient allocations is thus a necessary first step for understanding the optimal policy results. As proven in Appendix E, efficient allocations $\{C_t, H_t, N_{Et}, N_{t+1}\}_{t=0}^{\infty}$ are characterized by four (sequences of) conditions:

$$-\frac{u_{Ht}}{u_{Ct}} = Z_t \rho(N_t), \tag{32}$$

$$\rho(N_t) f_{Et} = (1-\delta) E_t \left\{ \frac{\beta u_{Ct+1}}{u_{Ct}} \left[\epsilon(N_{t+1}) \left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}} \right) + \rho(N_{t+1}) f_{Et+1} \right] \right\}, \tag{33}$$

$$C_t + G_t + \rho(N_t) f_{Et} N_{Et} = \rho(N_t) Z_t H_t, \tag{34}$$

and

$$N_{t+1} = (1-\delta)(N_t + N_{Et}). \tag{35}$$

The efficiency conditions (32) and (33) are obtained by maximizing household welfare, given by (1), subject to the technological frontier defined by the sequence of consumption resource constraints (34) and laws of motion for variety (35).

Condition (32) is a static dimension of efficiency and is analogous to static consumption-leisure efficiency in the RBC model. Condition (33) is an intertemporal dimension of efficiency, and it corresponds to the RBC model's Euler equation for efficient capital accumulation. Even though the model does not have "physical capital" in the strict RBC sense, the creation of new product

varieties is an investment activity that yields a long-lasting asset, as BGM emphasize. Taken together, conditions (32) and (33) define the two "zero-wedge" benchmarks for Ramsey allocations.

To highlight this "zero-wedges" aspect, it is useful to restate efficiency in terms of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT).[51] For the intertemporal condition, this restatement is most straightforward for the non-stochastic case, which allows an informative disentangling of the preference and technology terms inside the expectation operator in (33).

Proposition 3. *Efficient Allocations.* *The MRS and MRT for the pairs (C_t, H_t) and (C_t, C_{t+1}) are defined by:*

$$MRS_{C_t,H_t} \equiv -\frac{u_{Ht}}{u_{Ct}}, \qquad MRT_{C_t,H_t} \quad \equiv \quad Z_t \rho(N_t),$$

$$IMRS_{C_t,C_{t+1}} \equiv \frac{u_{Ct}}{\beta u_{Ct+1}}, \qquad IMRT_{C_t,C_{t+1}} \quad \equiv \quad \frac{(1-\delta)\left(\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + \rho(N_{t+1})f_{Et+1}\right)}{\rho(N_t)f_{Et}}.$$

Static efficiency (32) is characterized by $MRS_{C_t,H_t} = MRT_{C_t,H_t}$, and (for the non-stochastic case) intertemporal efficiency (33) is characterized by $IMRS_{C_t,C_{t+1}} = IMRT_{C_t,C_{t+1}}$.

Proof. See Appendix E. □

Each MRS in Proposition 3 has the standard interpretation as a ratio of marginal utilities. By analogy, each MRT has the interpretation as a ratio of the marginal products of an appropriately defined transformation frontier.[52] Elementary economic theory prescribes that efficient allocations are characterized by an MRS = MRT condition along each of the static and intertemporal optimization margins, implying zero distortion on each. These efficiency conditions are the welfare-relevant ones for the Ramsey government. However, rather than taking the efficiency conditions as prima facie justification that the expressions in Proposition 3 *are* properly to be understood as MRTs, each can be described conceptually from first principles, independent of the characterization of efficiency. Formal details of the following mostly intuitive discussion appear in Appendix E.

5.1   Static MRT

To understand the static MRT, MRT_{C_t,H_t}, in Proposition 3, consider how the economy can transform a unit of leisure in period t into a unit of output, and hence consumption, in period t. By construction, this within-period transformation holds fixed all allocations beyond period t. The

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[51]This approach of casting efficiency and optimal-policy results in models with fundamental frictions in terms of appropriately defined MRS and MRT concepts was first developed by Aruoba and Chugh (2010).

[52]We have in mind a very general notion of transformation frontier as in Mas-Colell, Whinston, and Green (1995, p. 129), in which every object in the economy can be viewed as either an input to or an output of the technology to which it is associated. Appendix E provides formal details.

transformation is described in terms of leisure because leisure is a good (and hence gives positive utility), while labor effort is a bad (and gives disutility); we proceed by describing transformation as occurring between goods.

A unit reduction in household leisure allows a unit increase in aggregate hours H_t, which can be devoted to production of existing varieties ($N_t h_t$) or creation of new ones ($N_{Et} h_{Et}$). The technology frontier (34) implies that labor is transformed into consumption-unit resources at the rate $\rho(N_t)Z_t$, where $\rho(N_t)$ captures the return to variety. Hence, the overall within-period MRT between leisure and consumption-unit output is $\rho(N_t)Z_t$, as shown in Proposition 3, and efficiency requires $MRT_{C_t,H_t} = \rho(N_t)Z_t$.

5.2 Intertemporal MRT

Now consider the intertemporal MRT (IMRT) in Proposition 3. The IMRT measures how many additional units of C_{t+1} the economy can achieve if one unit of C_t is foregone. By construction, this transformation across periods t and $t+1$ holds fixed all allocations *beyond* period $t+1$.

If C_t is reduced by one unit, $\frac{1}{\rho(N_t)f_{Et}}$ additional new varieties can be produced, holding fixed total consumption-unit output, as (34) shows. Due to product destruction, this addition to the flow of period-t new product development increases the stock of existing varieties in period $t+1$, N_{t+1}, by $\frac{1-\delta}{\rho(N_t)f_{Et}}$.

In period $t+1$, the additional $\frac{1-\delta}{\rho(N_t)f_{Et}}$ varieties can be transformed into consumption-unit output through two channels. First, they yield consumption units directly at the rate $\rho(N_{t+1})f_{Et+1}$, as shown by the technology frontier (34) (this is simply the inverse of the transformation that occurred in period t).

Second, each of the additional $\frac{1-\delta}{\rho(N_t)f_{Et}}$ varieties in period $t+1$ further increases period $t+1$ consumption by a net $\rho'(N_{t+1})\left(Z_{t+1}H_{t+1} - f_{Et+1}N_{Et+1}\right)$ units, based on the period $t+1$ consumption resource constraint. This expression can be rewritten in several steps,

$$
\begin{aligned}
\rho'(N_{t+1})\left(Z_{t+1}H_{t+1} - f_{Et+1}N_{Et+1}\right) &= \rho'(N_{t+1})\left(Z_{t+1}H_{t+1} - Z_{t+1}h_{Et+1}N_{Et+1}\right) \\
&= \rho'(N_{t+1})Z_{t+1}h_{t+1}N_{t+1} \\
&= \rho'(N_{t+1})N_{t+1}q_{t+1} \\
&= \rho'(N_{t+1})N_{t+1}\left(\frac{C_{t+1}+G_{t+1}}{\rho(N_{t+1})N_{t+1}}\right) \\
&= \epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right),
\end{aligned}
\tag{36}
$$

in which the first line uses the definition $h_{Et} = f_{Et}/Z_t$; the second line uses the labor market equilibrium condition $H_t = h_t N_t + h_{Et}N_{Et}$; the third line uses the variety-level equilibrium condition $q_t = Z_t h_t$; the fourth line uses the condition $C_t + G_t = \rho(N_t)N_t q_t$; and the fifth line uses the definition

$\epsilon(N_t) = \frac{\rho'(N_t)N_t}{\rho(N_t)}$. The overall addition to period-$t+1$ consumption through this second channel is thus $\frac{(1-\delta)\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right)}{\rho(N_t)f_{Et}}$.

Putting together this logic leads to the IMRT shown in Proposition 3. The fully stochastic intertemporal efficiency condition can thus be represented as:

$$1 = E_t\left\{\frac{\beta u_{Ct+1}}{u_{Ct}}\left[\frac{(1-\delta)\left(\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + \rho(N_{t+1})f_{Et+1}\right)}{\rho(N_t)f_{Et}}\right]\right\} = E_t\left\{\frac{IMRT_{C_t,C_{t+1}}}{IMRS_{C_t,C_{t+1}}}\right\}.$$
(37)

In the deterministic steady state, intertemporal efficiency is characterized by:

$$\frac{1}{\beta} = (1-\delta)\left[\frac{\epsilon(N)\left(\frac{C+G}{N}\right) + \rho(N)f_E}{\rho(N)f_E}\right].$$
(38)

6 Equilibrium Wedges

With the model-appropriate characterizations of static and intertemporal efficiency just developed, equilibrium wedges are defined as the deviations of MRS from MRT that arise in the decentralized economy. These wedges measure inefficiencies. Understanding the determinants and consequences of these inefficiencies provides the foundation for understanding optimal policy.

6.1 Static Distortion

Proposition 4. *Static Wedge.* *In the decentralized economy, the within-period (static) equilibrium margin can be expressed as*

$$-\frac{u_{Ht}}{u_{Ct}} = \left(\frac{1-\tau_t^H}{\mu(N_t)}\right)Z_t\rho(N_t).$$
(39)

The term in parentheses measures the static distortion.

Proof. Compare the efficiency condition (32) with the equilibrium condition (15). □

From Proposition 4, it is clear that a sufficient condition for the decentralized economy to achieve static efficiency is $\tau_t^H = 1 - \mu(N_t)$. This is a standard result in models of monopolistic competition with endogenous labor supply — efficiency requires a subsidy to labor income to offset markup distortions.[53] However, with government spending that requires financing and no lump-sum taxes, as in our Ramsey analysis, static efficiency cannot be achieved.[54]

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[53] As discussed in BGM2, monopoly power implies no static distortion if labor supply is inelastic.

[54] In a Ramsey taxation problem, $\tau^H \leq 0$ can only occur if the initial assets of the government are so large, either by assumption or via an effective initial lump-sum levy on existing private assets, that the government never needs to impose distortionary taxes. As usual in the Ramsey literature, we rule out these possibilities because they assume away the nature of the Ramsey problem.

6.2 Intertemporal Distortion

Proposition 5. *Intertemporal Wedge.* *In the decentralized economy, the intertemporal equilibrium margin can be expressed as*

$$1 = E_t \left\{ \frac{\beta u_{Ct+1}}{u_{Ct}} \left[\frac{(1-\delta)\left((1-\tau^D_{t+1|t})\left(\mu(N_t) - \frac{\mu(N_t)}{\mu(N_{t+1})}\right)\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + (1-\tau^S_{t+1})\frac{\mu(N_t)}{\mu(N_{t+1})}\rho(N_{t+1})f_{Et+1}\right)}{(1-\tau^S_t)\rho(N_t)f_{Et}} \right] \right\}.$$

(40)

Comparing the term in square brackets with the term in square brackets in the intertemporal efficiency condition (37) implicitly defines the intertemporal distortion.

Proof. Rewrite the equilibrium condition (16). □

Substituting the optimal long-run dividend tax (26) from Proposition 1 (along with $\tau^S = 0$) in the deterministic steady-state version of (40) confirms that the Ramsey equilibrium achieves long-run intertemporal efficiency.[55]

Regarding stochastic fluctuations, however, we cannot prove analytically that the optimal trade-off between static and dynamic distortions will result in intertemporal efficiency along the business cycle. The numerical results presented next show that zero intertemporal wedges are indeed achieved by the Ramsey equilibrium at all points along the business cycle.

7 Discussion

Based on the welfare-relevant concepts of efficiency and wedges developed in Sections 5 and 6, it is now straightforward to explain the optimal policy results through the lens of Ramsey theory as well as discuss a few other related points.

7.1 Short-Run Optimal Policy

7.1.1 Wedge Smoothing

A basic result in dynamic Ramsey analysis is that the least distortionary way for a government to collect a present value of revenue through proportional taxes is to maintain *low volatility of distortions* — "wedge smoothing" — across time periods. Keeping distortions constant (or nearly constant) over time is the basic insight behind Barro's (1979) partial equilibrium tax-smoothing result, which carries over to quantitative general equilibrium models, as first shown by Chari, Christiano, and Kehoe (1991) and recently by Werning (2007).

This basic Ramsey insight also applies to our model. Table 5 compares the exogenous policy case of Section 3 to optimal policy and shows that the latter maintains *zero* volatility of both

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[55]The same result is also achieved by substituting $\tau^D = 0$ and τ^S that satisfies $1 - \tau^S = \frac{\mu(N)-1}{\epsilon(N)}$.

static and intertemporal distortions. In the translog case especially, distortions are fairly volatile under exogenous policy: the volatility of the intertemporal wedge conditional on exogenous policy is 20 times larger than in the Dixit-Stiglitz case, which itself is non-zero.[56] Whether in the long run or in the short run, intertemporal distortions reduce welfare. We proved in Section 4 that the Ramsey equilibrium eliminates long-run intertemporal distortions. Table 5 shows that even seemingly "small" fluctuations of intertemporal distortions are completely eliminated in the Ramsey equilibrium. Fluctuations of static wedges are simultaneously also completely eliminated. This latter result connects back to the impulse responses presented in Figure 3 of the static wedge to a labor income tax shock in the exogenous policy analysis; as we anticipated there, the heart of the Ramsey equilibrium is in designing the behavior of wedges.

Supporting perfect wedge stabilization along both the static and dynamic margins requires no adjustment in tax rates at all in the Dixit-Stiglitz aggregation. With translog aggregation, the dynamics of labor and dividend tax rates that support perfect wedge smoothing are shown in Figure 4. In response to positive productivity and government spending shocks, the labor tax rate displays a slow (albeit small) rise that mirrors the slow decline in the markup shown in Figure 5. This dynamic response is intuitive: The only way for the wedge in the within-period equilibrium condition (15) to remain constant following a shock is if the labor tax rate perfectly offsets movements in the markup. Confirming this, the simulation-based correlation between the Ramsey-optimal labor tax rate and the induced markup is indeed -1 with translog aggregation.

7.1.2 Dynamics of Product Development, Markups, and Profits

One of the most appealing features of the baseline BGM framework is its ability to reproduce quantitatively the business cycle properties of not only standard macro quantities such as GDP, consumption, and investment in response to productivity shocks, but also of procyclical product entry, procyclical profits, and (in the translog case) countercyclical goods markups. Figure 2 showed that the introduction of distortionary fiscal policy does not disrupt these central predictions of the model. Figure 5 confirms that the Ramsey equilibrium also preserves these predictions: The impulse responses in Figure 5 (which are plotted assuming the dividend tax is active and the product creation subsidy is inactive, but the results are very similar for the opposite case) have very similar profiles as those in Figure 2, but, as suggested by the discussion above, are smaller in magnitude than in the non-Ramsey equilibrium.

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[56]The unit of measure in Table 5 is consumption because both the static and intertemporal MRSs and MRTs are in units of consumption.

7.2 Long-Run Optimal Policy

7.2.1 Relation to Capital Taxation Literature

Proposition 1 stated that the long-run optimal dividend tax supports long-run efficiency in product creation, which is the economy's intertemporal margin. As noted in the Introduction, Albanesi and Armenter (2007) recently generalized the well known zero-capital-taxation results of Chamley (1986) and Judd (1985) by developing a set of sufficient conditions for a wide class of models that guarantee the optimality of zero intertemporal distortions. Their sufficient conditions require constant returns to scale in production. The aggregate production function of the BGM model displays increasing returns to scale in product variety, thus the Albanesi-Armenter sufficient conditions do not apply. Moreover, existing analytical results regarding zero intertemporal distortions apply only to the steady state, as does our Proposition 1. The preceding numerical results showed, however, that Ramsey optimal policy achieves intertemporal efficiency not only in the long run, but also at all points along the business cycle.

Our model does not include physical capital in the strict sense, but intertemporal efficiency is nonetheless a primary concern of policy due to the asset nature of product varieties. In the aggregate, variety is a form of capital. As Proposition 3 implies, product development is in fact the means by which consumption is transformed across time and hence the means by which the economy saves. The intertemporal efficiency insight of Ramsey analysis is thus not limited to a narrow notion of physical capital, but instead applies to any accumulation decision.[57] Thus, the analogy offered by BGM and BGM2 that the stock of varieties is akin to the stock of capital in an RBC economy is helpful not only for understanding positive business cycle analysis (as in BGM), but also for understanding normative taxation analysis.

7.2.2 Relation to Optimal Investment in Monopolistic Models

With the preceding analogy, our results on long-run taxation of accumulation decisions in models of monopolistic competition can be tightly related to the analysis of the optimal quantity of research and development (R&D) by Benassy (1998) and the optimality of subsidizing capital accumulation in Judd (1997, 2002). Benassy (1998) applied his own (Benassy, 1996) variety aggregator to the Romer (1990) endogenous growth model to ask whether too much or too little R&D occurs in the decentralized economy relative to the social optimum. The answer was that it depends on whether the variety effect is stronger than or weaker than the markup effect, and he concluded that there

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[57]An early example of the generality of zero intertemporal distortions is Jones, Manuelli, and Rossi (1997), who show that the insight also applies to human capital accumulation. Another recent example in which intertemporal efficiency is a central goal of policy, despite the absence of physical capital, is the search-and-matching model of labor markets in Arseneau and Chugh (2010).

is no basis for offering normative prescriptions due to lack of empirical evidence about the variety effect. The BGM framework is based on the Romer (1990) endogenous growth environment, with zero long-run growth. So, although Benassy (1998) does not go all the way to drawing policy prescriptions and does not consider a business cycle analysis, our long-run results can be viewed as a detrended version of his. If we were limited to the Benassy aggregation in forming our conclusions, we would agree that there is no basis for recommending even a sign for the optimal dividend income tax because the sign of $\tau_{BENASSY}^D = 1 - \kappa(\theta - 1)$ depends on parameters. While plausible values for θ can be pinned down by data, no such evidence exists for κ.

Judd (1997, 2002) finds that it is optimal to *subsidize* capital accumulation when firms have monopoly power. This prescription seems to conflict with our result that it is likely optimal to *tax* accumulation of product varieties by monopolists. Judd's finding is a consequence of the familiar result that monopoly power implies a mark-down of the marginal q of capital relative to the perfectly competitive outcome. A monopolistic firm has an incentive to underaccumulate capital to reduce output supply and increase its price relative to perfect competition (Hayashi, 1982). In the BGM model, accumulation of products can exceed its welfare benefit, requiring a tax to correct the distortion. Optimal policy may be turned in the direction of a subsidy if we assumed a discrete set of firms that internalize the effect of their product creation on the price index (i.e., if each firm internalized the profit destruction externality of new products). Much as in the capital accumulation story, this would imply a mark-down in the valuation of additional products to the firm (see Stebunovs, 2008). Because firms would, however, not also internalize the welfare benefit of products, this would push results towards the optimality of a subsidy as in Judd (1997, 2002).[58]

7.2.3 Static Distortion

Much of our focus has been on the ability of the Ramsey government to implement intertemporal efficiency, with particular emphasis on achieving efficient *fluctuations*. This does not mean that Ramsey equilibria achieve the efficient *level* of activity. Figure 6 plots a few indicators of the long-run inefficiency of Ramsey equilibria, which is unavoidable because the Ramsey government must raise revenue using distortionary taxes. For brevity, results are shown only for the Dixit-Stiglitz aggregation. The long-run outcomes in Figure 6 are traced out as the parameter θ varies between 3 and 20 (recall the benchmark setting was $\theta = 3.8$), which achieves variation in the markup between 50 percent and 5 percent.

Consistent with the preceding analysis, the upper left and upper middle panels of Figure 6 show that long-run inefficiencies are loaded entirely on the static margin. This amounts to a distortion in the long-run equilibrium quantity of labor; the inefficiently large quantity of labor in the Ramsey

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[58]For instance, a subsidy (rather than $\tau^D = 0$) would become optimal in the Dixit-Stiglitz case.

equilibrium (upper right panel) causes inefficient overproduction of varieties (lower middle panel). However, the investment-to-GDP ratio (lower left panel) in the Ramsey equilibrium *is* efficient, and this is the essence of maintaining zero distortions along the intertemporal product creation margin. Finally, for completeness, the lower right panel of Figure 6 shows the Ramsey optimal labor income tax as a function of θ.

7.3 Optimal Taxation Issues

7.3.1 Completeness of Tax System

An important issue in models of optimal taxation is whether or not the available tax instruments constitute a *complete* tax system. The tax system is complete in our model. Establishing this is important for two reasons. First, at a technical level, proving completeness reaffirms that the Ramsey problem as formulated in Section 4 is indeed correct. As shown by Chari and Kehoe (1999, p. 1680), Correia (1996), Armenter (2008), and many others, incompleteness of the tax system requires imposing additional constraints that reflect the incompleteness. Second, it is well understood in Ramsey theory that incomplete tax systems can lead to a wide range of "unnatural" policy prescriptions in which the use of some instruments (in either the short run or the long run) proxy for other, perhaps more natural, instruments. Demonstrating completeness therefore establishes that none of our results is due to any policy instrument serving as an imperfect proxy for other, unavailable, instruments.

As Chari and Kehoe (1999, pp. 1679-1680) describe, an incomplete tax system is in place if, for at least one pair of goods in the economy, the government has *no* policy instrument that, in the decentralized economy, uniquely creates a wedge between the MRS of those goods and the corresponding MRT. Based on the model-appropriate concepts of MRTs and wedges developed in Sections 5 and 6, it is trivial to show that the set of instruments $(\tau_t^H, \tau_{t+1|t}^D, \tau_t^S)$ constitutes a complete tax system. Indeed, they constitute an "overcomplete" tax system.

The argument is as follows: Proposition 3 proved that there are two margins of adjustment in the economy. Completeness thus requires at least two policy instruments whose *joint* setting induces a unique wedge in each of the two margins. The labor tax τ_t^H coupled with *either* the state-contingent one-period-ahead schedule $\tau_{t+1|t}^D$ *or* τ_t^S do exactly this. The labor tax appears only in the static wedge (39), hence it uniquely creates a static distortion. Stated instead in terms of the inverse mapping, τ_t^H is uniquely determined given the Ramsey allocation. The two instruments $\tau_{t+1|t}^D$ and τ_t^S *both* appear *only* in the intertemporal wedge (40), hence an infinite number of pairs of values for the two create a given intertemporal distortion. Stated instead in terms of the inverse mapping, one of the two must be fixed arbitrarily in order for the other to be uniquely determined by the Ramsey allocation.

A consequence of this "over-completeness" of the tax system is that the introduction of any additional tax instruments into the environment necessarily implies (further) indeterminacy of the decentralization of Ramsey allocations. From the point of view of theory, and putting aside positive considerations, this may raise the question of why both the (state-contingent, one-period-ahead) dividend tax and the development subsidy were both included in the first place. We allowed for both as a check on the novel way in which we conducted the Ramsey optimization with respect to dividend taxes, in which the ex-ante schedule $\tau_{t+1|t}^D$ was technically the policy instrument, rather than an ex-post value for τ_t^D itself. As Table 4 showed, the Ramsey allocations were identical in the Dixit-Stiglitz case. However, for translog preferences, even though the main insights carry over from the Dixit-Stiglitz case, the Ramsey allocations were not identical under the two active instruments. We discuss this point next.

7.3.2 Taxation of Initial Wealth and Dimensions of Transformation

Despite the completeness of the tax system, the fact that two distinct instruments are allowed to tax (one at a time) the creation margin raises another subtle optimal taxation issue. As discussed in Section 4, we adopted the timeless perspective, in which the deterministic Ramsey allocation before period one (i.e., in periods zero and earlier) is assumed to be identical to the limiting ($t \to \infty$) Ramsey allocation. The practical consequence of this, which is standard when applying the timeless concept, is that the initial wealth of households that appears in the PVIC (24) is endogenous to the Ramsey solution,

However, for the case of translog preferences, the long-run Ramsey allocation is not invariant to which of the two instruments, τ^D or τ^S, is active. The two instruments affect the initial value of wealth in different ways, which in turn leads to tax-specific values of the multiplier on (24) in the Ramsey solution. If the dividend tax is active, it affects initial wealth by reducing the dividend flow that households receive (provided $\tau^D > 0$, as we showed in Section 4.3 for the translog Ramsey equilibrium). If the product creation instrument is active, it affects initial wealth by increasing (ceteris paribus) the market value of stock (provided $\tau^S < 0$, as is the case in the translog Ramsey equilibrium) because $v = (1 - \tau^s)\frac{\rho(N)}{\mu(N)}f_E$.

These two channels for affecting initial wealth are not isomorphic. Intuitively, one (τ^S) influences the value of owning a *stock*, whereas the other (τ^D) influences a *one-time* (period zero) *flow* generated by the stock. These two channels by which the Ramsey government can influence initial wealth imply different long-run Ramsey allocations, which is the reason why the results presented in the lower panels of Table 4 depend on which instrument is active.

An alternative to the timeless perspective would be to suppose that the government confiscates ownership of the entire initial variety stock N_0. Such a one-time confiscation is indeed optimal from

the capital taxation perspective, the connections to which we emphasized above. If we use this as the initial setting for policy (that is, effectively set $N_0 = 0$ in the PVIC (24)), rather than the timeless setting, the long-run allocations for the translog case are *identical* under either τ^D or τ^S being active, and both Propositions 1 and 2 remain intact. Moreover, the fluctuations of Ramsey allocations are then also identical under the two alternative active instruments in this case.

This issue is *not* about an insufficiently rich set of tax instruments in a completeness sense. As argued above, the tax system is complete in the usual sense understood in the Ramsey literature that there is at least one unique tax instrument per independent margin of adjustment in the private economy. Instead, the crux of the issue is that the initial wealth of the economy is not a margin of adjustment for the private economy. However, initial wealth *is* a margin of adjustment for the Ramsey government when it chooses the best equilibrium. Quantitatively, our main results are affected very little by this issue. Nonetheless, a broader issue this observation raises for Ramsey analysis of models that feature primitive frictions is that it may not be just *whether* an equilibrium margin is affected uniquely by a given tax, but also *how* an equilibrium margin is affected by a given tax that matters.

Digging into this a bit based on the efficiency analysis in Section 5, it may be informative to think in terms of "multidimensionality" in the IMRT. To see this, note how $\tau^D_{t+1|t}$ and $\left\{ \tau^S_t, \tau^S_{t+1} \right\}$ affect differently the components of the expression in square brackets in equation (40). This defines the market-valued transformation of current consumption into future consumption. At market values, foregoing one unit of consumption today yields $\frac{(1-\delta)\mu(N_t)}{(1-\tau^S_t)\rho(N_t)f_{Et}}$ new market-valued products, which can be transformed directly back into consumption at the rate $(1 - \tau^S_{t+1})\frac{\rho(N_{t+1})}{\mu(N_{t+1})}f_{Et+1}$ in period $t + 1$. Moreover, the monopoly market valuation of additional consumption in $t + 1$ generated by these products happens at rate $(1 - \tau^D_{t+1|t})\left(1 - \frac{1}{\mu(N_{t+1})}\right)\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right)$. These different ways in which policy instruments affect different dimensions of the IMRT (the investment flow to create the stock and the dividend generated by the stock) determine their different impacts on the PVIC. Further exploration of the policy implications of "multidimensional" IMRTs is an interesting topic for future research.[59]

8 Conclusion

In this paper, we studied optimal fiscal policy in an environment in which product varieties are the result of purposeful, forward-looking investment decisions by firms. One main result is that the long-run optimal dividend income tax rate is positive in the most empirically relevant and intuitively appealing version of the model. Depending on the form of variety aggregation, it is also possible

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[59]Note that such "multidimensionality" does not arise in the basic RBC model, in which the only endogenous component of intertemporal transformation is the marginal product of capital.

that a long-run dividend income *subsidy* is instead optimal; however, the optimality of a strictly *zero* dividend tax is non-generic. In all cases, dividend taxes support zero intertemporal distortions. The second main result is that keeping labor income tax rates constant (or virtually constant) at all points along the business cycle is optimal. The Ramsey policy keeps static distortions completely constant and intertemporal distortions exactly zero over time. Thus, the Ramsey principles of tax and wedge smoothing apply, in ways that we established analytically and quantitatively. Together, these results extend basic Ramsey principles beyond "first-generation" complete-markets Ramsey models.

A methodological contribution of the analysis was to develop precise characterizations of static and intertemporal efficiency for models based on the framework in Bilbiie, Ghironi, and Melitz (2007). As this framework continues to be applied to a wider array of macro questions, especially policy questions, the efficiency templates we developed should help guide understanding of the results that emerge.
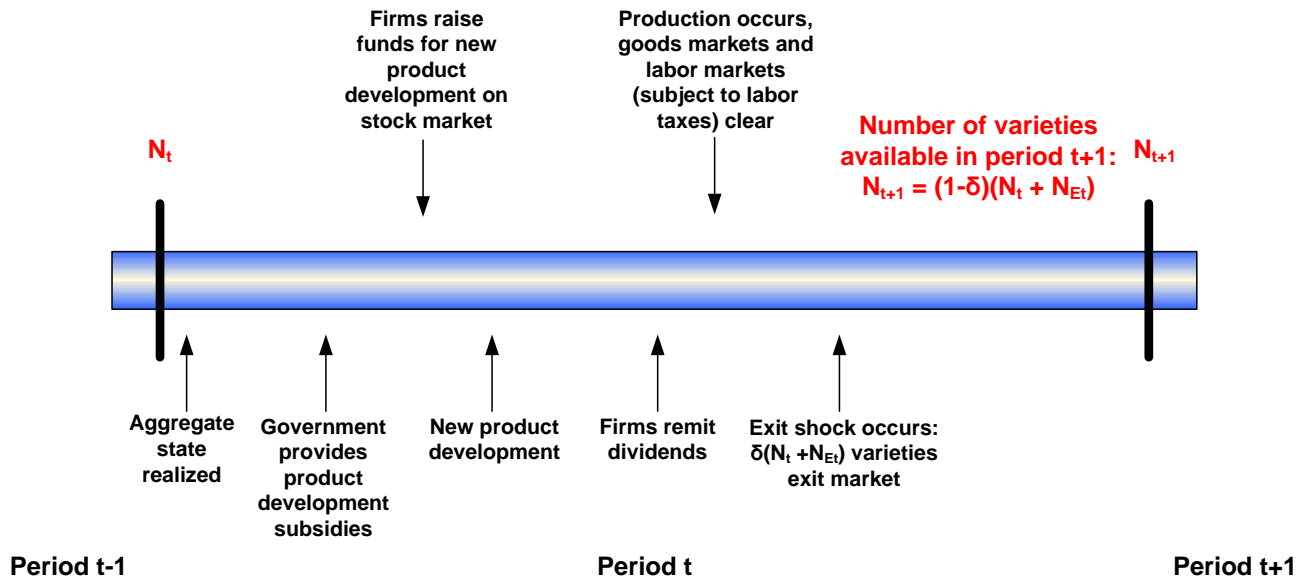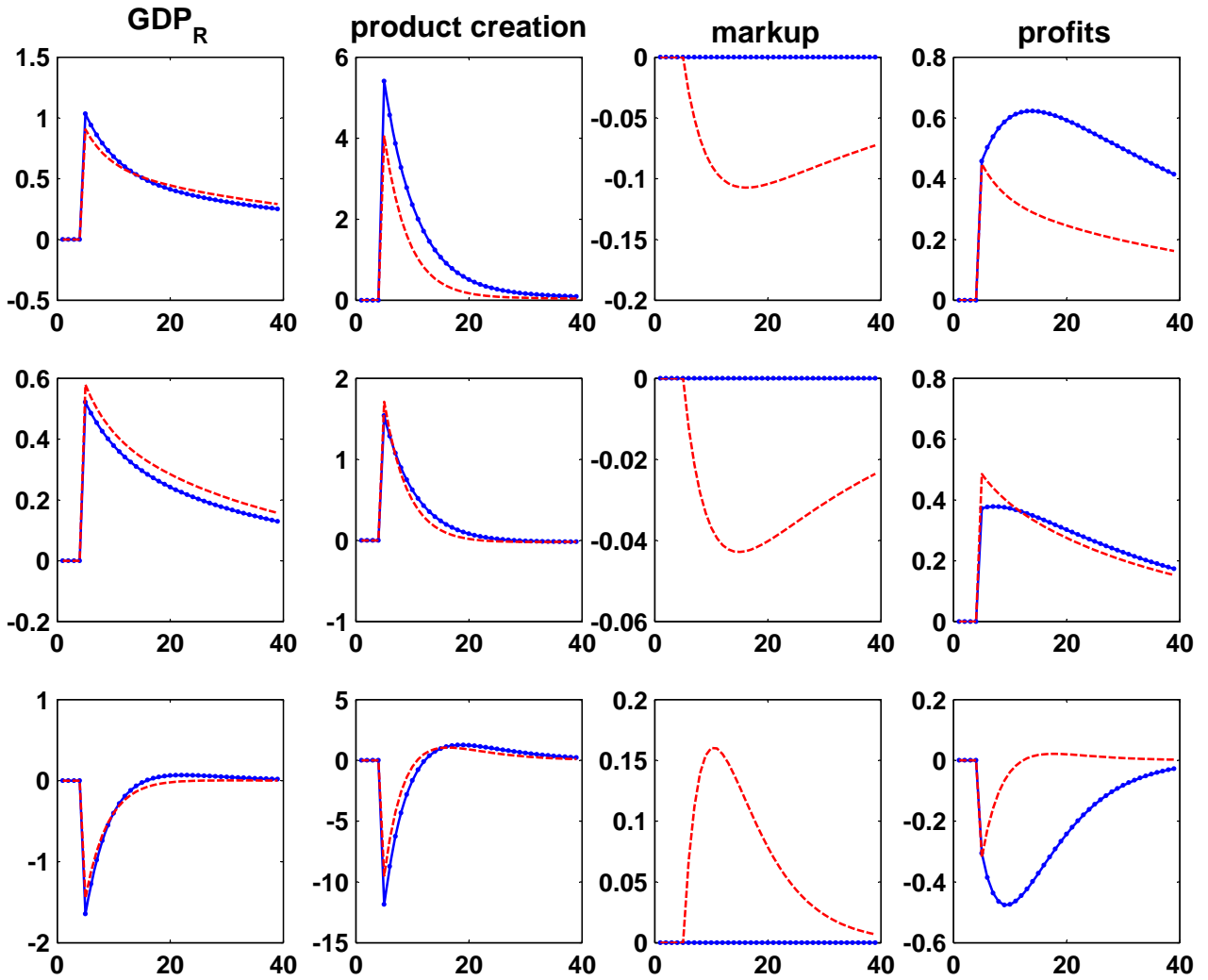
Figure 1: Timing of events.

Figure 2: Impulse responses in exogenous policy model. First row: positive shock to productivity. Second row: positive shock to government spending. Third row: positive shock to labor income tax rate. Dotted lines denote Dixit-Stiglitz preferences, dashed lines denote translog preferences. Horizontal axes plot number of quarters. Vertical axes plot percentage deviations from respective long-run allocation.

	τ^H	GDP_R	C_R	N	N_E	v_R	I_R	H
			All shocks					
			Dixit-Stiglitz aggregation					
Mean	20%	0.19	0.13	1.20	0.03	0.74	0.02	0.20
Volatility (SD%)	0.56%	2.44	1.00	1.12	16.09	0	16.09	2.63
Relative Volatility	—	1	0.41	0.46	6.60	0	6.60	1.08
Autocorrelation	0.65	0.63	0.74	0.93	0.61	—	0.61	0.60
Correlation with GDP_R	0.78	1	0.74	0.03	0.97	—	0.97	0.94
			Translog aggregation					
Mean	20%	0.20	0.13	1.30	0.03	0.72	0.02	0.20
Volatility (SD%)	0.56%	2.15	1.20	0.81	12.39	0.23	12.36	2.32
Relative Volatility	—	1	0.56	0.38	5.76	0.11	5.74	1.08
Autocorrelation	0.65	0.62	0.79	0.92	0.57	0.92	0.57	0.59
Correlation with GDP_R	0.77	1	0.70	0.14	0.93	0.14	0.94	0.92
			Shocks to Z and G_R					
			Dixit-Stiglitz aggregation					
Mean	20%	0.19	0.13	1.20	0.03	0.74	0.02	0.20
Volatility (SD%)	0	1.41	0.82	0.53	6.89	0	6.89	0.93
Relative Volatility	0	1	0.58	0.37	4.87	0	4.87	0.66
Autocorrelation	—	0.66	0.73	0.94	0.64	—	0.64	0.66
Correlation with GDP_R	—	1	0.72	0.11	0.96	—	0.96	0.88
			Translog aggregation					
Mean	20%	0.20	0.13	1.30	0.03	0.72	0.02	0.20
Volatility (SD%)	0	1.30	0.92	0.37	5.19	0.11	5.18	0.86
Relative Volatility	0	1	0.70	0.28	3.98	0.08	3.97	0.66
Autocorrelation	—	0.66	0.76	0.93	0.60	0.93	0.60	0.65
Correlation with GDP_R	—	1	0.66	0.24	0.92	0.24	0.93	0.82

Table 1: Business cycle dynamics in exogenous policy model. The "R" subscript denotes division by ρ to remove the variety effect. Volatilities computed as standard deviation of cyclical components of HP-filtered simulated data, except for tax rates, for which volatilities are reported in percentage points. Top panels: shocks to productivity, government absorption, and labor income tax rate. Bottom panels: shocks only to productivity and government absorption.

	τ^H	GDP_R	C_R	N	N_E	v_R	I_R	H
Shocks only to Z								
Dixit-Stiglitz aggregation								
Mean	20%	0.19	0.13	1.20	0.03	0.74	0.02	0.20
Volatility (SD%)	0	1.26	0.80	0.51	6.62	0	6.62	0.61
Relative Volatility	0	1	0.64	0.40	5.26	0	5.26	0.48
Autocorrelation	—	0.66	0.73	0.94	0.64	—	0.64	0.64
Correlation with GDP_R	—	1	0.95	0.11	0.98	—	0.98	0.93
Translog aggregation								
Mean	20%	0.19	0.13	1.30	0.03	0.72	0.02	0.20
Volatility (SD%)	0	1.10	0.90	0.34	4.77	0.10	4.77	0.42
Relative Volatility	0	1	0.82	0.31	4.37	0.09	4.37	0.39
Autocorrelation	—	0.66	0.76	0.93	0.60	0.93	0.60	0.61
Correlation with GDP_R	—	1	0.94	0.24	0.94	0.24	0.95	0.84
Shocks only to Z, with $\bar{G}_R = 0$, $\tau^H = 0$, $\tau^D = 0$								
Dixit-Stiglitz aggregation								
Mean	0	0.19	0.16	1.60	0.04	0.74	0.03	0.20
Volatility (SD%)	0	1.60	0.70	0.54	6.74	0	6.74	0.99
Relative Volatility	0	1	0.44	0.33	4.21	0	4.21	0.62
Autocorrelation	—	0.67	0.74	0.95	0.66	—	0.66	0.66
Correlation with GDP_R	—	1	0.95	0.09	0.98	—	0.98	0.98
Translog aggregation								
Mean	0	0.19	0.16	1.53	0.04	0.75	0.03	0.20
Volatility (SD%)	0	1.42	0.82	0.40	5.40	0.10	5.39	0.73
Relative Volatility	0	1	0.57	0.28	3.79	0.07	3.78	0.51
Autocorrelation	—	0.67	0.77	0.94	0.62	0.94	0.62	0.62
Correlation with GDP_R	—	1	0.93	0.22	0.95	0.22	0.95	0.95

Table 2: Business cycle dynamics in exogenous policy model conditional on shocks only to productivity. The "R" subscript denotes division by ρ to remove the variety effect. Volatilities computed as standard deviation of cyclical components of HP-filtered simulated data, except for tax rates, for which volatilities are reported in percentage points. Top panels: $\bar{G}_R$, τ^H, and τ^D held constant at their long-run values. Bottom panels: $\bar{G}_R = \tau^H = \tau^D = 0$.
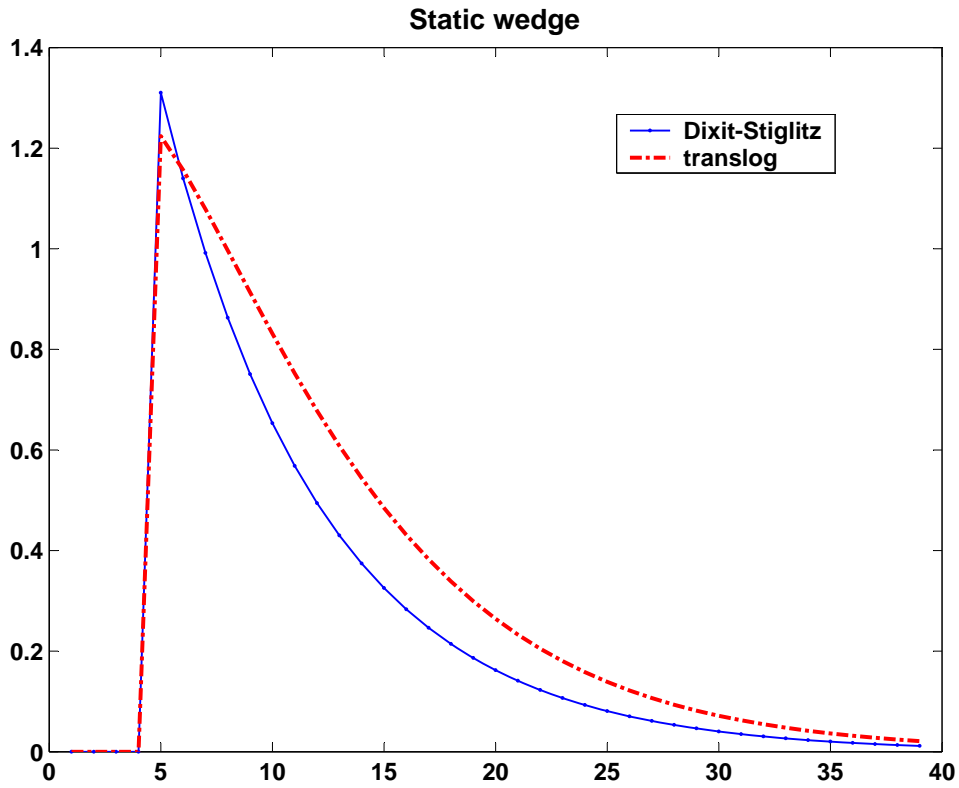
Figure 3: Response of within-period wedge (defined as $1 - \frac{-u_{Ht}/u_{Ct}}{Z_t \rho(N_t)}$) to one-time, one-standard-deviation positive shock to labor income tax rate. Dotted lines denote Dixit-Stiglitz preferences, dashed lines denote translog preferences. Horizontal axes plot number of quarters. Vertical axes plot percentage deviations from respective long-run allocation.

Dixit-Stiglitz	Benassy	Translog
$\mu(N_t) = \mu = \frac{\theta}{\theta-1}$	$\mu(N_t) = \mu = \frac{\theta}{\theta-1}$	$\mu(N_t) = 1 + \frac{1}{\sigma N_t}$
$\rho(N_t) = N_t^{\mu-1} = N_t^{\frac{1}{\theta-1}}$	$\rho(N_t) = N_t^{\kappa}$	$\rho(N_t) = \exp\left(-\frac{1}{2}\frac{\tilde{N}-N_t}{\sigma\tilde{N}N_t}\right), \tilde{N} \equiv \text{Mass(potential products)}$
$\epsilon(N_t) = \mu - 1$	$\epsilon(N_t) = \kappa$	$\epsilon(N_t) = \frac{1}{2\sigma N_t} = \frac{1}{2}(\mu(N_t) - 1)$

Table 3: The markup, relative price of symmetric variety, and love of variety as functions of the number of product varieties for the Dixit-Stiglitz, Benassy, and translog variety aggregators.

	τ^H	τ^D	τ^S	GDP_R	C_R	N	N_E	v_R	I_R	H
Dixit-Stiglitz aggregation (τ^D active, τ^S inactive)										
Mean	28.2%	0	—	0.34	0.21	2.87	0.07	0.74	0.05	0.36
Volatility (SD%)	0	0	—	0.99	0.38	0.15	3.57	0	3.57	0.34
Relative Volatility	0	0	—	1	0.38	0.15	3.58	0	3.58	0.34
Autocorrelation	—	—	—	0.69	0.72	0.95	0.68	—	0.68	0.69
Correlation with GDP_R	—	—	—	1	0.79	0.06	0.96	—	0.96	0.82
Dixit-Stiglitz aggregation (τ^S active, τ^D inactive)										
Mean	28.2%	0	—	0.34	0.21	2.87	0.07	0.74	0.05	0.36
Volatility (SD%)	0	0	—	0.99	0.38	0.15	3.57	0	3.57	0.34
Relative Volatility	0	0	—	1	0.38	0.15	3.58	0	3.58	0.34
Autocorrelation	—	—	—	0.69	0.72	0.95	0.68	—	0.68	0.69
Correlation with GDP_R	—	—	—	1	0.79	0.06	0.96	—	0.96	0.82
Translog aggregation (τ^D active, τ^S inactive)										
Mean	14.7%	50%	—	0.36	0.25	1.54	0.04	0.75	0.03	0.37
Volatility (SD%)	0.43%	0.32%	—	0.93	0.48	0.15	3.76	0.04	3.76	0.42
Relative Volatility	—	—	—	1	0.52	0.16	4.05	0.04	4.05	0.45
Autocorrelation	0.74	0.64	—	0.69	0.73	0.94	0.64	0.94	0.64	0.70
Correlation with GDP_R	0.58	0.82	—	1	0.58	0.21	0.86	0.21	0.86	0.57
Translog aggregation (τ^S active, τ^D inactive)										
Mean	19.8%	—	-100%	0.35	0.24	1.52	0.04	1.50	0.06	0.36
Volatility (SD%)	0.41%	—	0.14%	0.93	0.49	0.15	3.72	0.05	3.78	0.43
Relative Volatility	—	—	—	1	0.52	0.156	4.00	0.06	4.06	0.46
Autocorrelation	0.74	—-	0.74	0.69	0.72	0.94	0.64	0.64	0.64	0.70
Correlation with GDP_R	0.56	—	0.49	1	0.56	0.21	0.85	0.79	0.85	0.57

Table 4: Optimal policy. The "R" subscript denotes division by ρ to remove the variety effect. Volatilities computed as standard deviation of cyclical components of HP-filtered simulated data, except for tax rates, for which volatilities are reported in percentage points. Shocks are to productivity and government purchases.

| Aggregation | SD(%) of static wedge | | SD(%) of intertemporal wedge | | Optimal tax dynamics | |
	Exog. policy	Opt. policy	Exog. policy	Opt. policy	Vol. of τ_t^H	Vol. of $\tau_{t+1|t}^D$ (τ_t^S)
Dixit-Stiglitz	1.13	0	0.004	0	0	0 (0)
Translog	1.20	0	0.083	0	0.41%	0.32% (0.14%)

Table 5: Volatility of static and intertemporal wedges in exogenous policy equilibria and Ramsey equilibria, and volatility of taxes in Ramsey equilibria. Volatility of taxes reported in percentage points, volatility of wedges reported as percentage deviation from long-run level. For exogenous policy results, shocks are to productivity, government purchases, and labor income tax rate. For optimal policy results, shocks are to productivity and government purchases.
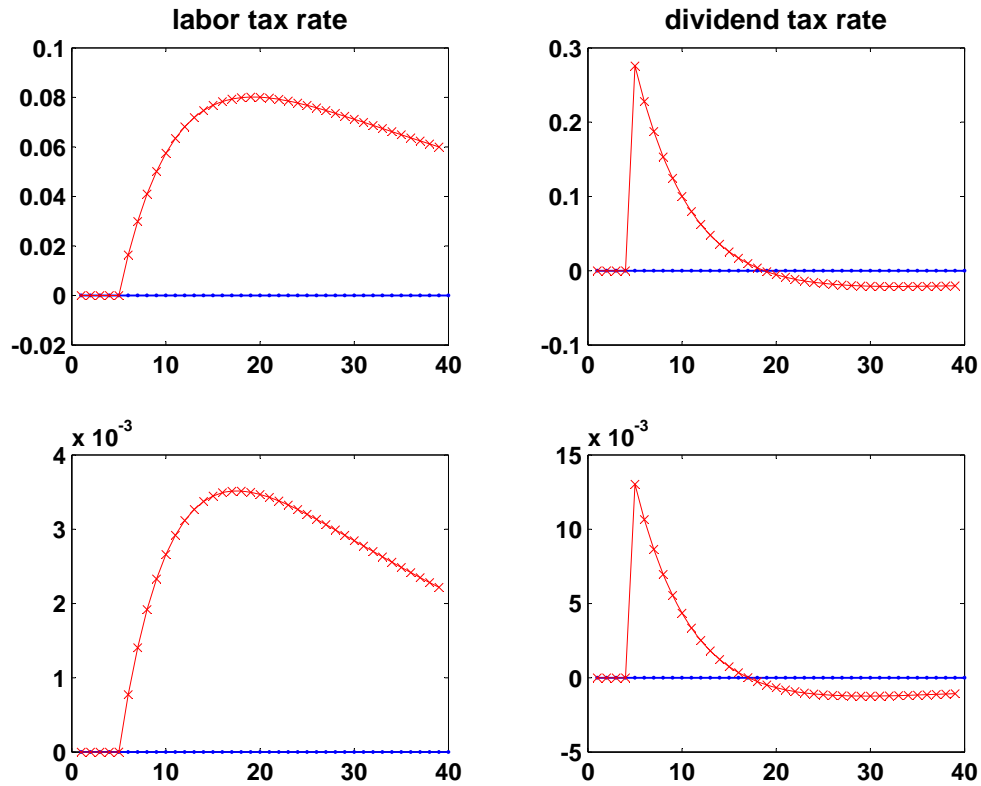
Figure 4: Impulse responses of Ramsey-optimal labor tax rate and dividend tax rate. First row: positive shock to productivity. Second row: positive shock to government spending. Dotted lines denote Dixit-Stiglitz preferences, crossed lines denote translog preferences. Horizontal axes plot number of quarters. Vertical axes plot percentage point deviations from respective long-run policy rates.
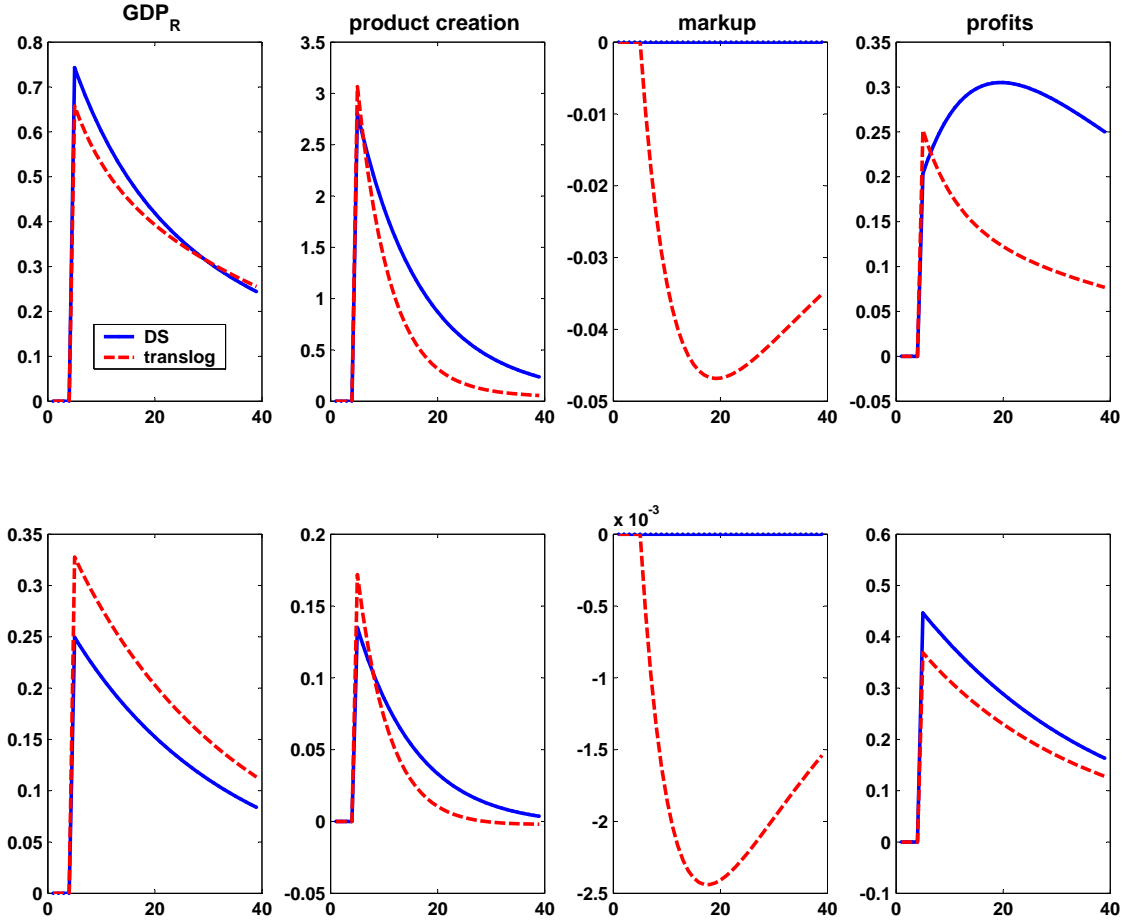
Figure 5: Impulse responses in Ramsey equilibrium with dividend tax active and product creation subsidy inactive. First row: positive shock to productivity. Second row: positive shock to government spending. Dotted lines denote Dixit-Stiglitz preferences, crossed lines denote translog preferences. Horizontal axes plot number of quarters. Vertical axes plot percentage deviations from respective long-run allocation.
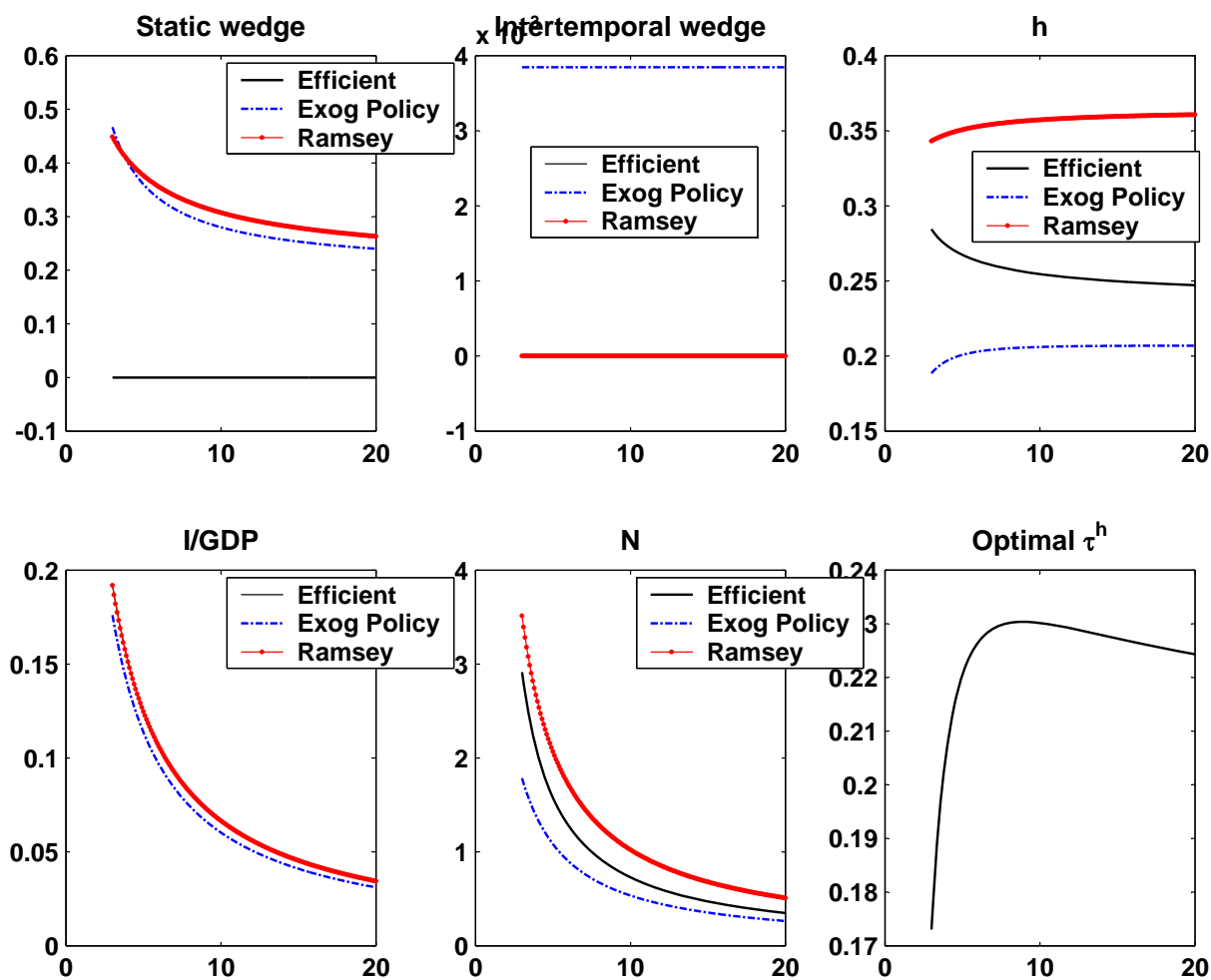
Figure 6: Dixit-Stiglitz aggregation. Long-run outcomes as parameter θ varies in three different allocations: efficient allocation, baseline exogenous policy model, and Ramsey equilibrium.

References

AIYAGARI, S. RAO, ALBERT MARCET, THOMAS J, SARGENT, AND JUHA SEPPALA. 2002. "Optimal Taxation without State-Contingent Debt." *Journal of Political Economy,* Vol. 110, pp. 1220-1254.

ARMENTER, ROC. 2008. "A Note on Incomplete Factor Taxation." *Journal of Public Economics,* Vol. 92, pp. 2275-2281.

ARSENEAU, DAVID M. AND SANJAY K. CHUGH. 2008. "Optimal Fiscal and Monetary Policy with Costly Wage Bargaining." *Journal of Monetary Economics,* Vol. 55, pp. 1401-1414.

ARSENEAU, DAVID M. AND SANJAY K. CHUGH. 2010. "Tax Smoothing in Frictional Labor Markets." University of Maryland.

ARSENEAU, DAVID M., SANJAY K. CHUGH, AND ANDRE KURMANN. 2008. "Optimal Capital Taxation in an Economy with Capital Allocation Frictions." University of Maryland.

ARUOBA, S. BORAGAN AND SANJAY K. CHUGH. 2010. "Optimal Fiscal and Monetary Policy When Money is Essential." *Journal of Economic Theory,* Vol. 145, pp. 1618-1647.

AUERBACH, ALAN J. AND J. R. HINES. 2002. "Taxation and Economic Efficiency. In *Handbook of Public Economics,* edited by Alan J. Auerbach and Martin Feldstein, Vol. 3. Elsevier.

BARRO, ROBERT J. 1979. "On the Determination of the Public Debt." *Journal of Political Economy,* Vol. 87, pp. 940-971.

BENASSY, JEAN-PASCAL. 1996. "Taste for Variety and Optimum Production Patterns in Monopolistic Competition." *Economics Letters,* Vol. 52, pp. 41-47.

BENASSY, JEAN-PASCAL. 1998. "Is There Always Too Little Research in Endogenous Growth With Expanding Product Variety?." *European Economic Review,* Vol. 42, pp. 61-69.

BERNARD, ANDREW B., STEPHEN REDDING, AND PETER SCHOTT. 2010. "Multi-Product Firms and Product Switching." *American Economic Review,* Vol. 100, pp. 70-97.

BILBIIE, FLORIN O., IPPEI FUJIWARA, AND FABIO GHIRONI. 2009. "Optimal Monetary Policy with Endogenous Entry and Product Variety." Mimeo, Paris School of Economics, Bank of Japan, and Boston College.

BILBIIE, FLORIN O., FABIO GHIRONI, AND MARC J. MELITZ. 2007. "Endogenous Entry, Product Variety, and Business Cycles." NBER Working Paper 13646.

BILBIIE, FLORIN O., FABIO GHIRONI, AND MARC J. MELITZ. 2008a. "Monetary Policy and Business Cycles with Endogenous Entry and Product Variety." NBER Macroeconomics Annual 2007.

BILBIIE, FLORIN O., FABIO GHIRONI, AND MARC J. MELITZ. 2008b. "Monopoly Power and Endogenous Product Variety: Distortions and Remedies." NBER Working Paper 14383.

BRODA, CHRISTIAN AND DAVID E. WEINSTEIN. 2010. "Product Creation and Destruction: Evidence and Price Implications." *American Economic Review,* Vol. 100, pp. 691-723.

CACCIATORE, MATTEO AND GIUSEPPE FIORI. 2009. "Macroeconomic Effects of Product and Labor Market (De)Regulation: The Long and Short of Reforming Europe." Mimeo, HEC Montreal and Bank of Italy.

CHAMLEY, CHRISTOPHE. 1986. "The Welfare Cost of Capital Income Taxation in a Growing Economy." *Econometrica,* Vol. 54, pp. 607-622.

CHARI, V.V., LAWRENCE CHRISTIANO, AND PATRICK KEHOE. 1991. "Optimal Fiscal and Monetary Policy: Some Recent Results." *Journal of Money, Credit, and Banking,* Vol. 23, pp. 519-539.

CHARI V. V., AND PATRICK J. KEHOE. 1999. "Optimal Fiscal and Monetary Policy. In *Handbook of Macroeconomics,* edited by John B. Taylor and Michael Woodford, Vol. 1C. Elsevier.

CHAMBERLIN, EDWARD. 1950. "Product Heterogeneity and Public Policy." *American Economic Review,* Vol. 40, pp. 85-92.

CROCE, MAX, T. NGUYEN, AND LUCAS SCHMID. 2011. "The Market Price of Fiscal Uncertainty." Mimeo, North Carolina University and Duke University.

DIXIT, AVINASH K. AND JOSEPH E. STIGLITZ. 1977. "Monopolistic Competition and Optimum Product Diversity." *American Economic Review,* Vol. 67, pp. 297-308.

EPSTEIN, LAWRENCE, AND STANLEY E. ZIN. 1989. "Substitution, Risk Aversion, and the Temporal Behavior of Consumption Growth and Asset Returns I: A Theoretical Framework." *Econometrica,* Vol. 57, pp. 937-969.

ETRO, FEDERICO AND ANDREA COLCAGO. 2011. "Endogenous Market Structures and Business Cycles." *The Economic Journal.* Forthcoming.

FAHRI, EMMANUEL. 2009. "Capital Taxation and Ownership when Markets are Incomplete.." *Journal of Political Economy.* Forthcoming.

FAIA, ESTER. 2009. "Oligopolistic Competition and Optimal Monetary Policy." Goethe University.

FEENSTRA, ROBERT C. 2003. "A Homothetic Utility Function for Monopolistic Competition Models, Without Constant Price Elasticity." *Economics Letters,* Vol. 78, pp. 79-86.

GROSSMAN, GENE M. AND ELHANAN HELPMAN. 1991. *Innovation and Growth in the Global Economy.* MIT Press.

HAYASHI, FUMIO. 1982. "Tobin's Marginal q and Average q: A Neoclassical Interpretation." *Econometrica,* Vol. 50, pp. 213-224.

JONES, JOHN B.. 2002. "Has Fiscal Policy Helped Stabilize the Postwar U.S. Economy?." *Journal*

of Monetary Economics, Vol. 49, pp. 709-746.

JONES, LARRY E., RODOLFO E. MANUELLI, AND PETER E. ROSSI. 1997. "On the Optimal Taxation of Capital Income." *Journal of Economic Theory,* Vol. 73, pp. 93-117.

JUDD, KENNETH L. 1985. "Redistributive Taxation in a Simple Perfect Foresight Model." *Journal of Public Economics,* Vol. 28, pp. 59-83.

JUDD, KENNETH L. 1997. "The Optimal Tax Rate for Capital Income is Negative." NBER Working Paper 6004.

JUDD, KENNETH L. 2002. "Capital Income Taxation with Imperfect Competition." *American Economic Review,* Vol. 92, pp. 417-421.

KHAN, AUBHIK, ROBERT G. KING AND ALEXANDER L. WOLMAN. 2003. "Optimal Monetary Policy." *Review of Economic Studies,* Vol. 70, pp. 825-860.

KING, ROBERT G. AND SERGIO T. REBELO. 1999. "Resuscitating Real Business Cycles. In *Handbook of Macroeconomics,* edited by John B. Taylor and Michael Woodford, Vol. 1B. Elsevier.

LAFFONT, JEAN-JACQUES M. AND JEAN M. TIROLE. 1993. *A Theory of Incentives in Regulation and Procurement.* MIT Press.

LEE, YOONSEE AND TOSHIHIKO MUKOYAMA. 2007. "Entry, Exit, and Plant-Level Dynamics over the Business Cycle." Federal Reserve Bank of Cleveland Working Paper 07-18.

LEWIS, VIVIEN. 2009a. "Optimal Monetary Policy and Firm Entry." Mimeo, University of Ghent.

LEWIS, VIVIEN. 2009b. "Product Diversity, Market Structure, and Optimal Taxation." Mimeo, University of Ghent.

LUCAS, ROBERT E. JR. AND NANCY L. STOKEY. 1983. "Optimal Fiscal and Monetary Policy in an Economy Without Capital." *Journal of Monetary Economics,* Vol. 12, pp. 55-93.

MANKIW, GREGORY N. AND MICHAEL D. WHINSTON. 1986. "Free Entry and Social Inefficiency." *RAND Journal of Economics,* Vol. 17, pp. 48-58.

MAS-COLELL, ANDREW, MICHAEL D. WHINSTON, AND JERRY R. GREEN. 1995. *Microeconomic Theory.* Oxford University Press.

ROMER, PAUL M. 1990. "Endogenous Technological Change." *Journal of Political Economy,* Vol. 98, pp. S71-102.

SCHMITT-GROHÉ, STEPHANIE AND MARTIN URIBE. 2004. "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function." *Journal of Economic Dynamics and Control,* Vol. 28, pp. 755-775.

SCHMITT-GROHÉ, STEPHANIE AND MARTIN URIBE. 2005. "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model." NBER Macroeconomics Annual 2005.

SHAO, ENCHUAN AND PEDRO SILOS. 2009. "Firm Entry and Labor Market Dynamics." Federal
    Reserve Bank of Atlanta.

SIU, HENRY E. 2004. "Optimal Fiscal and Monetary Policy with Sticky Prices." *Journal of
    Monetary Economics,* Vol. 51, pp. 576-607.

SPENCE, MICHAEL. 1976. "Product Selection, Fixed Costs, and Monopolistic Competition."
    Review of Economic Studies, Vol. 43, pp. 217-235.

STEBUNOVS, VIKTOR. 2008. "Finance as a Barrier to Entry: U.S. Bank Deregulation and Business
    Cycles." Mimeo, Federal Reserve Board. 13646.

WERNING IVAN. 2007. "Optimal Fiscal Policy with Redistribution." *Quarterly Journal of Eco-
    nomics,* Vol. 122, pp. 925-967.

A Derivation of Pricing Equation

Denominated in units of consumption, the representative firm's profit function is:

$$\max_{\rho_t, N_{t+1}, N_{Et}} E_0 \sum_{t=0}^{\infty} \Xi_{t|0} \left[(1 - \tau_t^D)(\rho_t - mc_t)N_t q_t - (1 - \tau_t^S)mc_t f_{Et} N_{Et} \right], \tag{41}$$

in which ρ_t is the relative price of a symmetric variety, q_t is the demand function, and we used $mc_t = w_t/Z_t$. In period t, the firm faces the law of motion for the number of varieties it produces and sells:

$$N_{t+1} = (1 - \delta)(N_t + N_{Et}). \tag{42}$$

The first-order condition with respect to the relative price ρ_t of a symmetric variety is

$$(1 - \tau_t^D)N_t q_t + \rho_t(1 - \tau_t^D)N_t \frac{\partial q_t}{\partial \rho_t} - mc_t(1 - \tau_t^D)N_t \frac{\partial q_t}{\partial \rho_t} = 0, \tag{43}$$

from which a simple representation of the optimal pricing condition for a symmetric variety can be obtained. Canceling the $(1 - \tau_t^D)N_t$ terms and rearranging gives

$$\rho_t \frac{\partial q_t}{\partial \rho_t} = mc_t \frac{\partial q_t}{\partial \rho_t} - q_t. \tag{44}$$

Isolating ρ_t,

$$\rho_t = mc_t - \frac{1}{\frac{1}{q(\rho_t)} \frac{\partial q_t}{\partial \rho_t}}. \tag{45}$$

Multiplying and dividing the denominator of the second term on the right hand side by ρ_t,

$$\rho_t = mc_t - \frac{1}{\frac{\rho_t}{q(\rho_t)} \frac{\partial q_t}{\partial \rho_t} \frac{1}{\rho_t}}. \tag{46}$$

Rewriting,

$$\rho_t = mc_t - \frac{\rho_t}{\frac{\rho_t}{q(\rho_t)} \frac{\partial q_t}{\partial \rho_t}}. \tag{47}$$

Defining $\zeta_t \equiv \frac{\rho_t}{q(\rho_t)} \frac{\partial q_t}{\partial \rho_t}$ as the price elasticity of demand for a symmetric variety, we have

$$\rho_t \left(1 + \frac{1}{\zeta_t} \right) = mc_t. \tag{48}$$

The optimal relative price of a symmetric variety is thus

$$\rho_t = \left(\frac{\zeta_t}{1 + \zeta_t} \right) mc_t, \tag{49}$$

which is in general an endogenously time-varying markup over real marginal cost. Denoting by μ_t the gross markup, $\mu_t \equiv \frac{\zeta_t}{1+\zeta_t}$,

$$\rho_t = \mu_t mc_t. \tag{50}$$

B   Competitive Equilibrium

The most straightforward definition of equilibrium is that it is a set of 15 endogenous equilibrium processes $\{C_t, H_t, h_{Et}, h_t, N_{t+1}, N_{Et}, w_t, v_t, \mu_t, \rho_t, \xi_t, mc_t, q_t, d_t, B_{t+1}\}_{t=0}^{\infty}$, for given processes $\{Z_t, G_t, \tau_t^H, \tau_t^D, \tau_t^S, f_{Et}\}$, that satisfy the conditions listed below. The equilibrium conditions are the consumption-leisure optimality condition:

$$-\frac{u_{Ht}}{u_{Ct}} = (1 - \tau_t^H)w_t; \tag{51}$$

the relation between the marginal cost of production and the real wage:

$$mc_t = \frac{w_t}{Z_t}; \tag{52}$$

the stock demand condition:

$$v_t = (1 - \delta)E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1}^D)d_{t+1} + v_{t+1} \right] \right\}, \tag{53}$$

where $\Xi_{t+1|t} \equiv \frac{\beta u_{Ct+1}}{u_{Ct}}$; the optimal pricing condition for a symmetric variety:

$$\rho_t = \mu_t mc_t; \tag{54}$$

the relation between the gross markup and the price elasticity of demand:

$$\mu_t = \frac{\xi_t}{1 + \xi_t}; \tag{55}$$

the product creation condition:

$$(1 - \tau_t^S)\frac{w_t}{Z_t}f_{Et} = (1 - \delta)E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1}^D)(\rho_{t+1} - mc_{t+1})q_{t+1} + (1 - \tau_{t+1}^S)\frac{w_{t+1}}{Z_{t+1}}f_{Et+1} \right] \right\}; \tag{56}$$

the law of motion for the number of product varieties:

$$N_{t+1} = (1 - \delta)(N_t + N_{Et}); \tag{57}$$

total consumption output:

$$C_t + G_t = N_t \rho_t q_t; \tag{58}$$

the aggregate consumption-units resource constraint:

$$C_t + G_t + \rho_t N_{Et} f_{Et} = \rho_t Z_t H_t; \tag{59}$$

the condition that pins down hours worked in the product development sector:

$$h_{Et} = \frac{f_{Et}}{Z_t}; \tag{60}$$

labor-market clearing:

$$H_t = h_{Et}N_{Et} + h_tN_t; \tag{61}$$

per-variety dividends:

$$d_t = (\rho_t - mc_t)\, q_t; \tag{62}$$

and the government budget constraint (in which the market clearing condition $x_t = 1$ is substituted):

$$\tau_t^H w_t H_t + \tau_t^D d_t N_t + B_{t+1} = G_t + R_t B_t + \tau_t^S \frac{w_t}{Z_t} f_{Et} N_{Et}. \tag{63}$$

Assumptions on exogenous processes were described in Section 3 in the text, and the parametric forms adopted for the variety aggregator, which determine the functional relationship between N_t and ρ_t and μ_t, appear in Section 4.3. From here on, we emphasize this relationship by writing $\rho(N_t)$ and $\mu(N_t)$.

B.1 Compact Representation of Equilibrium

To characterize the equilibrium in the compact form presented in Section 2.5, combine the above conditions as follows. Conditions (53) and (56) imply $v_t = (1 - \tau_t^s)\frac{w_t}{Z_t} f_{Et}$, which from here on replaces (53) in our analysis; this justifies the inclusion of condition (17) as part of the definition of competitive equilibrium in the text.

Next, substitute $q_t = \frac{C_t + G_t}{N_t \rho(N_t)}$ in the product creation condition to express it as:

$$(1 - \tau_t^S)\frac{\rho(N_t)}{\mu(N_t)} f_{Et} = (1 - \delta)E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1}^D)\left(\rho(N_{t+1}) - \frac{\rho(N_{t+1})}{\mu(N_{t+1})} \right)\left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}\rho(N_{t+1})} \right) + (1 - \tau_{t+1}^S)\frac{\rho(N_{t+1})}{\mu(N_{t+1})} f_{Et+1} \right] \right\}, \tag{64}$$

in which we have also made the substitution $mc_t = \frac{\rho(N_t)}{\mu(N_t)}$. Canceling $\rho(N_{t+1})$ terms on the right-hand side, we have

$$(1 - \tau_t^S)\frac{\rho(N_t)}{\mu(N_t)} f_{Et} = (1 - \delta)E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1}^D)\left(1 - \frac{1}{\mu(N_{t+1})} \right)\left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}} \right) + (1 - \tau_{t+1}^S)\frac{\rho(N_{t+1})}{\mu(N_{t+1})} f_{Et+1} \right] \right\}. \tag{65}$$

Multiplying by $\mu(N_t)$, we have a compact representation of the product creation condition

$$(1 - \tau_t^S)\rho(N_t) f_{Et} = (1 - \delta)E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1}^D)\left(\mu(N_t) - \frac{\mu(N_t)}{\mu(N_{t+1})} \right)\left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}} \right) + (1 - \tau_{t+1}^S)\frac{\mu(N_t)}{\mu(N_{t+1})}\rho(N_{t+1}) f_{Et+1} \right] \right\}, \tag{66}$$

which is condition (16) in the text.

To obtain the static equilibrium condition, use the relation $w_t = Z_t mc_t$ in the consumption-leisure optimality condition, and then use the relation $mc_t = \frac{\rho(N_t)}{\mu(N_t)}$ to eliminate marginal cost. The resulting expression for the equilibrium consumption-leisure margin is

$$-\frac{u_{Ht}}{u_{Ct}} = \frac{(1 - \tau_t^H)}{\mu(N_t)} Z_t \rho(N_t), \tag{67}$$

which is condition (15) in the text.

B.2 Consumption Resource Constraint

To derive the representation of the aggregate consumption-units resource constraint above and presented in (19), sum the flow household budget constraint and the flow government budget constraint, which gives

$$C_t + G_t + \tau_t^S \frac{w_t}{Z_t} f_{Et} N_{Et} + v_t N_{Et} = w_t H_t + d_t N_t. \tag{68}$$

Substitute into this expression the equilibrium expression for (per-product) dividends, $d_t = (\rho_t - mc_t)q_t$,

$$C_t + G_t + \tau_t^S \frac{w_t}{Z_t} f_{Et} N_{Et} + v_t N_{Et} = w_t H_t + (\rho_t - mc_t)q_t N_t. \tag{69}$$

Next, use $q_t = \frac{C_t + G_t}{N_t \rho_t}$; canceling terms leaves

$$C_t + G_t + \tau_t^S \frac{w_t}{Z_t} f_{Et} N_{Et} + v_t N_{Et} = w_t H_t + (\rho_t - mc_t)\left(\frac{C_t + G_t}{\rho_t}\right). \tag{70}$$

Next, using the condition $v_t = (1 - \tau_t^S)\frac{w_t}{Z_t} f_{Et}$,

$$C_t + G_t + \frac{w_t}{Z_t} f_{Et} N_{Et} = w_t H_t + (\rho_t - mc_t)\left(\frac{C_t + G_t}{\rho_t}\right); \tag{71}$$

and substituting $w_t/Z_t = mc_t = \rho(N_t)/\mu(N_t)$:

$$C_t + G_t + \frac{\rho(N_t)}{\mu(N_t)} f_{Et} N_{Et} = w_t H_t + \left(\rho(N_t) - \frac{\rho(N_t)}{\mu(N_t)}\right)\left(\frac{C_t + G_t}{\rho(N_t)}\right). \tag{72}$$

Canceling terms on the right hand-side:

$$C_t + G_t + \frac{\rho(N_t)}{\mu(N_t)} f_{Et} N_{Et} = w_t H_t + \left(1 - \frac{1}{\mu(N_t)}\right)(C_t + G_t); \tag{73}$$

and canceling the $(C_t + G_t)$ that appears on both sides:

$$\frac{1}{\mu(N_t)}(C_t + G_t) + \frac{\rho(N_t)}{\mu(N_t)} N_{Et} f_{Et} = w_t H_t. \tag{74}$$

Next, recognize that $w_t = Z_t mc_t = Z_t \frac{\rho(N_t)}{\mu(N_t)}$, which gives:

$$\frac{1}{\mu(N_t)}(C_t + G_t) + \frac{\rho(N_t)}{\mu(N_t)} N_{Et} f_{Et} = Z_t \frac{\rho(N_t)}{\mu(N_t)} H_t. \tag{75}$$

Finally, multiplying by $\mu(N_t)$ gives:

$$C_t + G_t + \rho(N_t) N_{Et} f_{Et} = \rho(N_t) Z_t H_t, \tag{76}$$

which emphasizes that $\rho(N_t)$ is a primitive of the economy.

C   Derivation of Present-Value Implementability Constraint

The derivation of the present-value implementability constraint (PVIC) follows that laid out in Lucas and Stokey (1983) and Chari and Kehoe (1999). Start with the household flow budget constraint:

$$C_t + v_t x_{t+1}(N_t + N_{Et}) + \sum_j \frac{1}{R_t^j} B_{t+1}^j = (1 - \tau_t^H)w_t H_t + B_t + [v_t + (1 - \tau_t^D)d_t]x_t N_t. \tag{77}$$

Multiply each term by $\beta^t u_{Ct}$ (which, in equilibrium, is the shadow value to the household at time zero of a unit of period-t wealth) and, conditional on the information set at time zero, sum the sequence of budget constraints from $t = 0...\infty$ to arrive at:

$$E_0 \sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{Ct} v_t x_{t+1}(N_t + N_{Et}) + E_0 \sum_{t=0}^{\infty} \sum_j \beta^t u_{Ct} \frac{1}{R_t^j} B_{t+1}^j$$

$$= E_0 \sum_{t=0}^{\infty} \beta^t u_{Ct}(1 - \tau_t^H)w_t H_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{Ct} R_t B_t + E_0 \sum_{t=0}^{\infty} \beta^t u_{Ct}[v_t + (1 - \tau_t^D)d_t]x_t N_t.$$

Now begin to impose equilibrium conditions on this present-value budget constraint. For ease of notation, drop the E_0 term, but it is understood that all terms are conditional on the information set at time zero. First impose the sequence of stock-market clearing conditions $x_s = 1 \, \forall s$, which gives:

$$\sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + \sum_{t=0}^{\infty} \beta^t u_{Ct} v_t(N_t + N_{Et}) + \sum_{t=0}^{\infty} \sum_j \beta^t u_{Ct} \frac{1}{R_t^j} B_{t+1}^j$$

$$= \sum_{t=0}^{\infty} \beta^t u_{Ct}(1 - \tau_t^H)w_t H_t + \sum_{t=0}^{\infty} \beta^t u_{Ct} B_t + \sum_{t=0}^{\infty} \beta^t u_{Ct}[v_t + (1 - \tau_t^D)d_t]N_t.$$

Next, in the third summation on the left-hand side, substitute the sequence of state-contingent bond Euler equations, $u_{Cs} = \beta R_s^j u_{C_{s+1}^j}$, $\forall j, s$:

$$\sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + \sum_{t=0}^{\infty} \beta^t u_{Ct} v_t(N_t + N_{Et}) + \sum_{t=0}^{\infty} \sum_j \beta^{t+1} u_{C_{t+1}^j} B_{t+1}^j$$

$$= \sum_{t=0}^{\infty} \beta^t u_{Ct}(1 - \tau_t^H)w_t H_t + \sum_{t=0}^{\infty} \beta^t u_{Ct} B_t + \sum_{t=0}^{\infty} \beta^t u_{Ct}[v_t + (1 - \tau_t^D)d_t]N_t.$$

The term $\sum_j u_{C_{t+1}^j} B_{t+1}^j$ can be expressed as the payoff of a synthetic risk-free bond, $u_{Ct+1}B_{t+1}$, which then allows canceling terms in the third summation on the left-hand side with their counterpart terms in the second summation on the right-hand side, leaving only the time-zero bond-return term:

$$\sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + \sum_{t=0}^{\infty} \beta^t u_{Ct} v_t(N_t + N_{Et}) = \sum_{t=0}^{\infty} \beta^t u_{Ct}(1 - \tau_t^H)w_t H_t + \sum_{t=0}^{\infty} \beta^t u_{Ct}[v_t + (1 - \tau_t^D)d_t]N_t + u_{C0}B_0. \tag{78}$$

Next, in the first summation on the right-hand side, use the sequence of consumption-leisure opti-mality conditions, $-u_{Hs} = u_{Cs}(1 - \tau_s^H)w_s$, $\forall s$, and move this summation to the left-hand side:

$$\sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + \sum_{t=0}^{\infty} \beta^t u_{Ht} H_t + \sum_{t=0}^{\infty} \beta^t u_{Ct} v_t (N_t + N_{Et}) = \sum_{t=0}^{\infty} \beta^t u_{Ct}[v_t + (1 - \tau_t^D)d_t]N_t + u_{C0}B_0. \tag{79}$$

Next, use the sequence of stock demand conditions, $v_s = (1-\delta)E_s\left\{\frac{\beta u_{Cs+1}}{u_{Cs}}\left[(1 - \tau_{s+1}^D)d_{s+1} + v_{s+1}\right]\right\}$, $\forall s$, to substitute out the $u_{Cs}v_s$ terms in the third summation on the left-hand side, which yields:

$$\sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + \sum_{t=0}^{\infty} \beta^t u_{Ht} H_t$$

$$+(1-\delta)\sum_{t=0}^{\infty} \beta^{t+1} u_{Ct+1}\left[v_{t+1} + (1 - \tau_{t+1}^D)d_{t+1}\right](N_t + N_{Et}) = \sum_{t=0}^{\infty} \beta^t u_{Ct}[v_t + (1 - \tau_t^D)d_t]N_t + u_{C0}B_0.$$

Substituting the sequence of equilibrium laws of motion $\frac{N_{s+1}}{1-\delta} = N_s + N_{E,s}$, $\forall s$, in the third sum-mation on the left-hand side gives:

$$\sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + \sum_{t=0}^{\infty} \beta^t u_{Ht} H_t + \sum_{t=0}^{\infty} \beta^{t+1} u_{Ct+1}\left[v_{t+1} + (1 - \tau_{t+1}^D)d_{t+1}\right]N_{t+1} = \sum_{t=0}^{\infty} \beta^t u_{Ct}[v_t + (1 - \tau_t^D)d_t]N_t + u_{C0}B_0. \tag{80}$$

Canceling terms in the third summation on the left-hand side with their counterpart terms in the summation on the right-hand side leaves only the time-zero stock-payoff term:

$$\sum_{t=0}^{\infty} \beta^t u_{Ct} C_t + \sum_{t=0}^{\infty} \beta^t u_{Ht} H_t = u_{C0}[v_0 + (1 - \tau_0^D)d_0]N_0 + u_{C0}B_0. \tag{81}$$

Re-introducing the expectation E_0 operator, the PVIC is:

$$E_0 \sum_{t=0}^{\infty} \beta^t (u_{Ct} C_t + u_{Ht} H_t) = u_{C0}[v_0 + (1 - \tau_0^D)d_0]N_0 + u_{C0}B_0, \tag{82}$$

which is condition (24) in the main text.

D Optimal Long-Run Policy

Here we prove Propositions 1 and 2. As stated in Section 4.1, the Ramsey problem is to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t) \tag{83}$$

subject to the sequence of consumption resource constraints:

$$C_t + G_t + \rho(N_t)N_{Et}f_{Et} = \rho(N_t)Z_t H_t, \tag{84}$$

laws of motion for the measure of product varieties:

$$N_{t+1} = (1 - \delta)(N_t + N_{Et}), \tag{85}$$

the sequence of equilibrium product creation conditions:

$$(1 - \tau_t^S)\rho(N_t)f_{Et} = (1-\delta)E_t \left\{ \Xi_{t+1|t} \left[(1 - \tau_{t+1|t}^D) \left(\mu(N_t) - \frac{\mu(N_t)}{\mu(N_{t+1})} \right) \left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}} \right) + (1 - \tau_{t+1}^S) \frac{\mu(N_t)}{\mu(N_{t+1})} \rho(N_{t+1})f_{Et+1} \right] \right\}, \tag{86}$$

and the PVIC:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u_{Ct}C_t + u_{Ht}H_t \right) = u_{C0}[v_0 + (1 - \tau_0^D)d_0]N_0 + u_{C0}R_0 B_0. \tag{87}$$

The Ramsey choice variables are C_t, H_t, N_{t+1}, N_{Et}, and either $\tau_{t+1|t}^D$ (refer to the discussion in Section 4.1) or τ_t^S for $t > 1$. Associate the sequences of multipliers $\lambda_{1,t}$, $\lambda_{2,t}$, $\lambda_{3,t}$ with the first three sequences of constraints, and the multiplier ξ with the PVIC. Although we of course must consider the fully dynamic Ramsey problem to consider any aspect of the Ramsey equilibrium, our analytical results are only for the deterministic Ramsey steady state. Thus, here we can suppose the environment is deterministic and drop all expectation operators.

The first-order condition with respect to either $\tau_{t+1|t}^D$ or τ_t^S (again recall that only one of these two instruments can be active) immediately implies that $\lambda_3 = 0$ in the deterministic Ramsey steady state. This is a very useful result because it greatly simplifies the analysis of the rest of the Ramsey steady state. Intuitively, the result $\lambda_3 = 0$ says that in the Ramsey equilibrium (though, note, not in any arbitrary equilibrium), the product creation condition does not constrain the allocation. Stated another way, the Ramsey government ensures efficiency in the long run along the product creation margin. We rely on the long run result that $\lambda_3 = 0$ in what follows.

To prove results for the long-run optimal dividend income tax and product creation subsidy, we need to consider only the Ramsey first-order conditions with respect to N_{t+1} and N_{Et}. These first-order conditions are, respectively:

$$-\lambda_{2,t} + \beta \left[\lambda_{1,t+1}\rho'(N_{t+1}) \left(Z_{t+1}H_{t+1} - N_{Et+1}f_{Et+1} \right) + (1 - \delta)\lambda_{2,t+1} \right] = 0 \tag{88}$$

and:

$$-\lambda_{1,t}\rho(N_t)f_{Et} + (1-\delta)\lambda_{2,t} = 0. \tag{89}$$

We have ignored any derivatives through the product creation condition because, as just shown, the Lagrange multiplier on this constraint is zero in the deterministic steady state. Then, using exactly the same set of algebraic manipulations as in Appendix E, these two conditions can be expressed as:

$$\lambda_{1,t}\rho(N_t)f_{Et} = \beta(1-\delta)\left\{\lambda_{1,t+1}\left[\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + \rho(N_{t+1})f_{Et+1}\right]\right\}. \tag{90}$$

In the deterministic steady state, we have that the Ramsey-optimal level of product creation is characterized by:

$$\frac{1}{\beta} = (1-\delta)\left[\frac{\epsilon(N)\left(\frac{C+G}{N}\right) + \rho(N)f_E}{\rho(N)f_E}\right], \tag{91}$$

which is the long-run efficiency condition (38) that appears in the main text. Thus, the Ramsey equilibrium achieves the Pareto optimum along the product creation margin in the long run.

To decentralize this, refer to the deterministic steady-state version of the product creation condition:

$$\frac{1}{\beta} = (1-\delta)\left[\frac{(1-\tau^D)(\mu(N)-1)\left(\frac{C+G}{N}\right)}{(1-\tau^S)\rho(N)f_E} + 1\right]. \tag{92}$$

If the product creation subsidy is inactive ($\tau^S = 0$), comparison of these last two expressions implies that the long-run optimal dividend income tax rate is characterized by

$$1 - \tau^D = \frac{\epsilon(N)}{\mu(N)-1}, \tag{93}$$

or

$$\tau^D = 1 - \frac{\epsilon(N)}{\mu(N)-1}. \tag{94}$$

This proves Proposition 1.

Alternatively, if the dividend tax is inactive ($\tau^D = 0$), comparison of the two expressions implies that the long-run product creation subsidy rate is characterized by

$$1 - \tau^S = \frac{\mu(N)-1}{\epsilon(N)}, \tag{95}$$

or

$$\tau^S = 1 - \frac{\mu(N)-1}{\epsilon(N)}. \tag{96}$$

This proves Proposition 2.

E   Efficient Allocations

The social planning problem is to choose state-contingent functions for $\{C_t, H_t, N_{t+1}, N_{Et}\}$ to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, H_t) \tag{97}$$

subject to

$$C_t + G_t + \rho(N_t)N_{Et}f_{Et} = \rho(N_t)Z_tH_t \tag{98}$$

and

$$N_{t+1} = (1-\delta)(N_t + N_{Et}). \tag{99}$$

The social planner internalizes the effect of the number of varieties on the relative price.

Let ϕ_t denote the Lagrange multiplier on the consumption-units resource constraint and μ_t denote the Lagrange multiplier on the law of motion for the number of product varieties. The first-order conditions with respect to C_t, H_t, N_{Et}, and N_{t+1} are, respectively,

$$u_{Ct} - \phi_t = 0, \tag{100}$$

$$u_{Ht} + \phi_t\rho(N_t)Z_t = 0, \tag{101}$$

$$-\phi_t\rho(N_t)f_{Et} + (1-\delta)\mu_t = 0, \tag{102}$$

and

$$-\mu_t + \beta E_t \left\{ \phi_{t+1}\rho'(N_{t+1})\left[Z_{t+1}H_{t+1} - N_{Et+1}f_{Et+1}\right] + (1-\delta)\mu_{t+1} \right\} = 0. \tag{103}$$

Conditions (100) and (101) imply

$$-\frac{u_{Ht}}{u_{Ct}} = Z_t\rho(N_t). \tag{104}$$

This is the efficiency condition (32) that appears in the main text.

Solving condition (102) for μ_t and substituting $\phi_t = u_{Ct}$ from condition (100), we have

$$\mu_t = \frac{u_{Ct}\rho(N_t)f_{Et}}{1-\delta}. \tag{105}$$

Using the time-t and time-$t+1$ versions of this expression in condition (103) gives

$$u_{Ct}\rho(N_t)f_{Et} = (1-\delta)\beta E_t \left\{ u_{Ct+1}\left[\rho'(N_{t+1})\left(Z_{t+1}H_{t+1} - N_{Et+1}f_{Et+1}\right) + \rho(N_{t+1})f_{Et+1}\right] \right\}. \tag{106}$$

Next, apply several definitions and identities to simplify this expression. Using $h_{Et} = f_{Et}/Z_t$, this can be re-written as

$$u_{Ct}\rho(N_t)f_{Et} = (1-\delta)\beta E_t \left\{ u_{Ct+1}\left[\rho'(N_{t+1})\left(Z_{t+1}H_{t+1} - N_{Et+1}Z_{t+1}h_{Et+1}\right) + \rho(N_{t+1})f_{Et+1}\right] \right\}. \tag{107}$$

Next, by the condition $H_t = h_t N_t + h_{Et} N_{Et}$, this can be re-written as

$$u_{Ct}\rho(N_t)f_{Et} = (1-\delta)\beta E_t \left\{ u_{Ct+1} \left[\rho'(N_{t+1})Z_{t+1}N_{t+1}h_{t+1} + \rho(N_{t+1})f_{Et+1} \right] \right\}. \tag{108}$$

in which, recall, h_t is the labor hired *per product* in the goods producing sector. Next, use the goods production technology and market clearing, $q_t = Z_t h_t$, to express this as

$$u_{Ct}\rho(N_t)f_{Et} = (1-\delta)\beta E_t \left\{ u_{Ct+1} \left[\rho'(N_{t+1})q_{t+1}N_{t+1} + \rho(N_{t+1})f_{Et+1} \right] \right\}. \tag{109}$$

Next, using the per-variety relationship $q_t = \frac{C_t + G_t}{N_t \rho(N_t)}$, the preceding can be expressed as

$$u_{Ct}\rho(N_t)f_{Et} = (1-\delta)\beta E_t \left\{ u_{Ct+1} \left[\rho'(N_{t+1}) \left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}\rho(N_{t+1})} \right) N_{t+1} + \rho(N_{t+1})f_{Et+1} \right] \right\}. \tag{110}$$

The variety effect expressed in elasticity form is $\epsilon(N_t) \equiv \rho'(N_t)\frac{N_t}{\rho(N_t)}$; using this, we can again re-express the preceding as

$$u_{Ct}\rho(N_t)f_{Et} = (1-\delta)\beta E_t \left\{ u_{Ct+1} \left[\epsilon(N_{t+1}) \left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}} \right) + \rho(N_{t+1})f_{Et+1} \right] \right\}. \tag{111}$$

Dividing by u_{Ct}, we have

$$\rho(N_t)f_{Et} = (1-\delta)E_t \left\{ \frac{\beta u_{Ct+1}}{u_{Ct}} \left[\epsilon(N_{t+1}) \left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}} \right) + \rho(N_{t+1})f_{Et+1} \right] \right\}. \tag{112}$$

which is the intertemporal efficiency condition (33) that appears in the main text.

E.1 MRS-MRT Representation of Efficiency

The efficiency conditions (104) and (112) can be described in terms of appropriately defined concepts of marginal rates of substitution (MRS) and corresponding marginal rates of transformation (MRT). Defining MRS and MRT in a model-appropriate way allows us to describe efficiency in terms of the basic principle that efficient allocations are characterized by MRS = MRT conditions along all optimization margins.

Consider the static efficiency condition (104). The left-hand side is clearly the within-period MRS between consumption and labor (leisure) in any period t. The right-hand side is thus the corresponding MRT between consumption and labor.

We can similarly define MRS and MRT relevant for intertemporal efficiency. To do so, first restrict attention to the non-stochastic case because it makes clearer the separation of components of preferences from components of technology (due to endogenous covariance terms implied by the expectation operator). The non-stochastic intertemporal efficiency condition can be expressed as

$$\frac{u_{Ct}}{\beta u_{Ct+1}} = \frac{(1-\delta)\left(\epsilon(N_{t+1}) \left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}} \right) + \rho(N_{t+1})f_{Et+1} \right)}{\rho(N_t)f_{Et}}. \tag{113}$$

The left-hand side of (113) is clearly the intertemporal MRS (abbreviated IMRS) between C_t and C_{t+1}. We claim that the right-hand side is the corresponding intertemporal MRT (abbreviated IMRT). Applying this definition to the fully stochastic condition (112), we can thus express in-tertemporal efficiency as

$$1 = E_t \left\{ \frac{\beta u_{Ct+1}}{u_{Ct}} \left[\frac{(1-\delta)\left(\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + \rho(N_{t+1})f_{Et+1}\right)}{\rho(N_t)f_{Et}} \right] \right\} = E_t \left\{ \frac{IMRT_{C_t,C_{t+1}}}{IMRS_{C_t,C_{t+1}}} \right\}. \tag{114}$$

Rather than take the efficiency conditions (104) and (113) as prima facie evidence that the right-hand sides *must be*, respectively, the static MRT and intertemporal MRT, these MRTs can be derived from the primitives of the environment (i.e., independent of the context of any optimization), to which we now turn.

E.2   Proof of Proposition 3: Transformation Frontier and Derivation of MRTs

Based only on the primitives of the environment — that is, independent of the context of any optimization — we now prove that the right-hand sides of (104) and (113) are, respectively, the model-appropriate concepts of the static MRT and deterministic IMRT. Doing so thus proves Proposition 3 in the main text.

Consider the period-t consumption resource constraint and law of motion for variety: $C_t + G_t + \rho(N_t)f_{Et}N_{Et} = \rho(N_t)Z_tH_t$ and $N_{t+1} = (1-\delta)(N_t + N_{Et})$. Solving the former for the number of new products created, $N_{Et} = \frac{\rho(N_t)Z_tH_t - C_t - G_t}{\rho(N_t)f_{Et}}$, and substituting in the latter gives

$$\Upsilon(C_t, H_t, N_{t+1}; .) \equiv N_{t+1} - (1-\delta)N_t - \frac{(1-\delta)\left(\rho(N_t)Z_tH_t - C_t - G_t\right)}{\rho(N_t)f_{Et}} = 0, \tag{115}$$

which is defined as the period-t transformation frontier. The function $\Upsilon(.)$ is a more general notion of a transformation, or resource, frontier than either the goods resource constraint or the law of motion for variety alone because $\Upsilon(.)$ *jointly* describes *two* technologies in the economy: the technology that transfers variety over time and, conditional on the stock of varieties, the technology that creates output, in the form of existing goods and new ones. The dependence of $\Upsilon(.)$ on (among other arguments) C_t and H_t is highlighted because the period-t utility function is defined over C_t and H_t.

By the implicit function theorem, the static MRT between consumption and leisure is thus

$$-\frac{\Upsilon_{H_t}}{\Upsilon_{C_t}} = Z_t\rho(N_t), \tag{116}$$

which formalizes, independent of the social planning problem, the notion of the static MRT on the right-hand side of the efficiency condition (104) and presented in Proposition 3.

For use in deriving the IMRT below, note that the implicit function theorem also allows us to compute $\frac{\partial N_{t+1}}{\partial C_t} = -\frac{\Upsilon_{C_t}}{\Upsilon_{N_{t+1}}}$. The partials are $\Upsilon_{C_t} = \frac{1-\delta}{\rho(N_t)f_{Et}}$ and $\Upsilon_{N_{t+1}} = 1$. Thus,

$$\begin{aligned}
\frac{\partial N_{t+1}}{\partial C_t} &= -\frac{\Upsilon_{C_t}}{\Upsilon_{N_t}} \\
&= -\frac{1-\delta}{\rho(N_t)f_{Et}},
\end{aligned} \tag{117}$$

which gives the marginal effect on the period-$t+1$ stock of varieties of a change in period-t consumption. This effect has intertemporal consequences because N_{t+1} is the stock of varieties entering period $t+1$; because (115) cannot be solved explicitly for N_{t+1}, the effect must be accounted for implicitly.

Next, define the transformation frontier that links period t and period-$t+1$

$$\Gamma(C_{t+1}, N_{t+2}, C_t, N_{t+1}; .) = N_{t+2} - (1-\delta)N_{t+1} - \frac{(1-\delta)\left(\rho(N_{t+1})Z_{t+1}H_{t+1} - C_{t+1} - G_{t+1}\right)}{\rho(N_{t+1})f_{Et+1}} = 0. \tag{118}$$

In form, the function $\Gamma(.)$ is the same as the function $\Upsilon(.)$, but, for the purpose at hand, it is useful to view it as a generalization of $\Upsilon(.)$ in that $\Gamma(.)$ is explicitly viewed as a function of period t *and* period $t+1$ allocations.[60] The two-period transformation frontier $\Gamma(.)$ has partials with respect to C_{t+1} and C_t

$$\Gamma_{C_{t+1}} = \frac{1-\delta}{\rho(N_{t+1})f_{Et+1}} \tag{119}$$

and

$$\begin{aligned}
\Gamma_{C_t} &= -(1-\delta)\frac{\partial N_{t+1}}{\partial C_t}\left[1 + \frac{\rho'(N_{t+1})Z_{t+1}H_{t+1}}{\rho(N_{t+1})f_{Et+1}} - \frac{\rho(N_{t+1})Z_{t+1}H_{t+1} - C_{t+1} - G_{t+1}}{\rho(N_{t+1})f_{Et+1}}\left(\frac{\rho'(N_{t+1})f_{Et+1}}{\rho(N_{t+1})f_{Et+1}}\right)\right] \\
&= -(1-\delta)\frac{\partial N_{t+1}}{\partial C_t}\left[1 + \frac{\rho'(N_{t+1})}{\rho(N_{t+1})f_{Et+1}}\left(Z_{t+1}H_{t+1} - \frac{\rho(N_{t+1})Z_{t+1}H_{t+1} - C_{t+1} - G_{t+1}}{\rho(N_{t+1})}\right)\right] \\
&= -(1-\delta)\frac{\partial N_{t+1}}{\partial C_t}\left[1 + \frac{\rho'(N_{t+1})}{\rho(N_{t+1})f_{Et+1}}\left(\frac{C_{t+1} + G_{t+1}}{\rho(N_{t+1})}\right)\right] \\
&= -(1-\delta)\frac{\partial N_{t+1}}{\partial C_t}\left[1 + \frac{\rho'(N_{t+1})N_{t+1}}{\rho(N_{t+1})}\left(\frac{C_{t+1} + G_{t+1}}{N_{t+1}}\right)\frac{1}{\rho(N_{t+1})f_{Et+1}}\right] \\
&= -(1-\delta)\frac{\partial N_{t+1}}{\partial C_t}\left[1 + \frac{\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right)}{\rho(N_{t+1})f_{Et+1}}\right] \\
&= -(1-\delta)\frac{\partial N_{t+1}}{\partial C_t}\left[\frac{\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + \rho(N_{t+1})f_{Et+1}}{\rho(N_{t+1})f_{Et+1}}\right] \\
&= (1-\delta)\left(\frac{1-\delta}{\rho(N_t)f_{Et}}\right)\left[\frac{\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + \rho(N_{t+1})f_{Et+1}}{\rho(N_{t+1})f_{Et+1}}\right]
\end{aligned}$$

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[60] Rather than as a function of only period-t allocations, as we viewed $\Upsilon(.)$. Note also that, as must be the case, we could use $\Gamma(.)$, rather than $\Upsilon(.)$, to define the within-period MRT between consumption and labor. By the implicit function theorem, the within-period MRT (for period $t+1$) is $-\frac{\Gamma_{H_{t+1}}}{\Gamma_{C_{t+1}}} = Z_{t+1}\rho(N_{t+1})$, obviously identical to the static MRT (116) derived above.

the fifth line makes use of the definition of the variety effect expressed in elasticity form, $\epsilon(N_t) \equiv \frac{\rho'(N_t)N_t}{\rho(N_t)}$, and the last line follows from substituting (117).

By the implicit function theorem, the IMRT between C_t and C_{t+1} is thus

$$\frac{\Gamma_{C_t}}{\Gamma_{C_{t+1}}} = \frac{(1-\delta)\left(\epsilon(N_{t+1})\left(\frac{C_{t+1}+G_{t+1}}{N_{t+1}}\right) + \rho(N_{t+1})f_{Et+1}\right)}{\rho(N_t)f_{Et}}, \tag{120}$$

which formalizes, independent of the social planning problem, the notion of the IMRT on the right-hand side of the (deterministic) efficiency condition (113) and presented in Proposition 3.

With the static MRT and IMRT defined from the primitives of the environment, the efficiency conditions (104) and (113) are indeed interpretable as appropriately-defined MRS = MRT conditions.