CAPITAL MOBILITY AND ASSET PRICING

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ABSTRACT

We present a model for the equilibrium movement of capital between asset markets that are distinguished only by the levels of capital invested in each. Investment in that market with the greatest amount of capital earns the lowest risk premium. Intermediaries optimally trade off the costs of intermediation against fees that depend on the gain they can offer to investors for moving their capital to the market with the higher mean return. Those fees also depend on the bargaining power of the investor, in light of potential alternative intermediaries. In equilibrium, the speeds of adjustment of mean returns and of capital between the two markets are increasing in the degree to which capital is imbalanced between the two markets.

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1 Introduction

We present a model for the equilibrium movement of capital between markets. Equilibrium conditional mean rates of return vary across markets according to the levels of capital invested in the respective markets. As a matter of supply and demand within each market, that market with the greater amount of capital earns lower conditional mean returns. Given a sufficient disparity in the capital levels in the markets, intermediaries find it optimal to search for investors in the market with "surplus" capital and offer them the opportunity to move their capital to the other market, which offers higher risk premia. Intermediaries charge investors a fee that is based on their gain from the move and based on the degree of competition in the market for intermediation. The equilibrium behavior of intermediaries is solved analytically, and characterized. Competition among intermediaries can in some cases *reduce* intermediation in equilibrium, relative to the monopolistic case.

This paper is motivated by empirical evidence, some of which is reviewed in the last section, that supply or demand shocks in asset markets, in addition to causing an immediate price response, also lead to adjustments over time in the distribution of capital across markets and adjustments over time in relative conditional mean asset returns, in a way that reflects delays in the adjustments of investors' portfolios. We are particularly interested in how those adjustments are affected by the endogenous behavior of intermediaries.

In our equilibrium model, the greater the relative difference in capital levels across the markets, the more intensive are intermediaries' efforts to re-balance the distribution of capital across the markers, and the greater is the rate of convergence of the mean rates of return of different assets toward a common level. We study the impact on capital mobility of search costs, discounting, asset volatility, and other parameters.

An example is the limited mobility of capital into reinsurance markets, documented by Froot and O'Connell (1999), who write: "Our results suggest that capital market imperfections are more important than shifts in actuarial valuation for understanding catastrophe reinsurance pricing. Supply, rather than demand, shifts seem to explain most features of the market in the aftermath of a loss." In subsequent work, Froot (2001) continues: "We ... find the most compelling (evidence) to be supply restrictions associated with capital market imperfections and market power exerted by traditional

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reinsurers."

We are particularly interested in the impact of competition among intermediaries on the equilibrium degree of capital mobility, through two channels. First, an intermediary does not internalize the entire impact of its search activity on leveling the distribution of capital across markets because each intermediary gets only a fraction of aggregate intermediation fees. This prompts intermediaries to search more as the number of intermediaries increases, all else equal. Competition among intermediaries has a second and potentially offsetting effect on capital mobility through the impact of fee bargaining on incentives to intermediate. In the simplest setting that we analyze, the second effect can in dominate, so that in some cases increasing the number of intermediaries reduces capital mobility.

A significant body of theory examines the implications of search frictions for asset pricing. For example, differences in search frictions across different asset markets are treated by Weill (2008) and Vayanos and Wang (2007). Duffie, Gârleanu, and Pedersen (2005) study the implications of search frictions in a single asset market with market making. In the context of a single asset market, Duffie, Gârleanu, and Pedersen (2007) and Lagos, Rocheteau, and Weil (2008) model recoveries in mean returns after a shock to the preferences of investors, corresponding to a gradual re-allocation of the asset to more suitable investors, rather than by cross-market capital dynamics as here. Earlier search-based models of intermediation include those of Rubinstein and Wolinsky (1987), Bhattacharya and Hagerty (1987), Moresi (1991), Gehrig (1993), and Yavaş (1996).

Related work on the implications of capital market frictions for asset pricing dynamics includes the models of Basak and Croitoru (2000) and He and Krishnamurthy (2007). In terms of some objectives and model features, independent work by Gromb and Vayanos (2009) is closely related to ours. Common to our models, local hedgers are immobile, while arbitrageurs can work across markets, driving returns toward fundamental levels, subject to frictions that prevent them from perfectly equating returns in the two markets. Our respective approaches, however, are quite different. We focus on the dynamics of intermediation, capital movements, and risk premia.

Section 2 describes the market setting. Sections 3 and 4 analyze the monopolistic and oligopolistic intermediation cases, respectively. Section 5 summarizes the implications of our model for asset-price dynamics and provides some evidence regarding the premise

or implications of our model. Proofs and several extensions of the model are found in Appendices. Appendices F through K are located in Duffie and Strulovici (2011), a supplement to this paper.

2 The Market Setting

This section presents a stylized model for the endogenous adjustment of capital and risk premia across markets. There are three types of agents: (i) local hedgers; (ii) investors who provide risk-bearing to hedgers in each of two local markets; and (iii) intermediaries (such as asset managers) who provide the fee-based service to investors of moving capital from the "over-capitalized" market to that market with less capital, thereby allowing them to earn a higher premium for the same risk.

We fix a probability space $(\Omega, \mathcal{F}, P)$ and a common information filtration $\{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions.[1]

In each of two financial markets, labeled a and b, a continuum (a non-atomic measure space) of local risk-averse agents own short-lived risky assets that they are willing to sell at or above their respective reservation prices. Equivalently, they are willing to buy insurance contracts against the risks to which they are exposed. These "hedgers" are not mobile across markets. They can be viewed in this respect as relatively unsophisticated in the use of cross-capital-market transactions, or as having high transactions costs for trading outside of their local markets. A continuum of investors that supply capital have access to cross-market trading, subject to intermediation frictions to be described. These suppliers of capital are risk-neutral, offering to bear the risk that hedgers desire to shed in return for any strictly positive risk premium. In an insurance context, one might think of these suppliers of capital as stylized versions of the "Names" that supply risk bearing capacity to the insurance market known as "Lloyd's of London."

The total levels of capital available in the two markets at time t are X_{at} and X_{bt}, respectively. Capital can be reinvested continually at the discretion of each investor, that is, "rolled over" in the short-lived assets that are continually made available for sale by hedgers. Each unit of capital that is currently invested in market i at time t is paid cash

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[1]See, for example, Protter (2004) for the usual conditions and for other standard properties of stochastic processes to which we refer.

at the equilibrium dividend rate $\pi(X_{it})$, where $\pi(\cdot)$ is a strictly decreasing continuous function. The dividend rate $\pi(X_{it})$ is continually reset in double auctions at which the supply and demand for the asset in market i are matched at each point in time. As the amount of capital available to invest in the asset is increased, the market-clearing dividend rate declines. In Appendix F, we provide an example in which $\pi(x)$ is the equilibrium insurance premium in a market with x units of insurance capital.

In return for the dividend rate $\pi(X_{it})$, the provider of each unit of capital in market i agrees to absorb losses of capital in proportion to the increments of a Lévy process ρ_i. (That is, ρ_i has independently and identically distributed increments over non-overlapping time periods of the same length. Examples include Brownian motions, Poisson processes, compound Poisson processes, and linear combinations of these.) The idea is that the short-lived risky asset promises $1 + d\rho_{it} + \pi(X_{it}) \, dt$ at time $t + dt$ per unit of capital invested at time t, in the instantaneous sense. More precisely, each unit of capital invested in market i at any time s, and rolled over continually in that market until some time $\tau > t$ accumulates to W_τ units of capital according to the stochastic differential equation $dW_t = W_{t-} \, d\rho_{it}$, and in the meantime generates cash flows at the rate $W_{t-} \, \pi(X_{it})$. (The notation "W_{t-}" means the left limit of the path of W at time t, that is, the level just before any jump at time t.)

In the illustrative case of an insurance market, we can take ρ_i to be a compound Poisson process that jumps down at the arrival times of loss events, and is otherwise constant. In this case, one unit of capital invested at time t pays the supplier of capital $1 + \pi(X_{it}) \, dt$ at time $t + dt$ (in the above instantaneous sense) if there is no loss event, and if there is a loss event, has a recovery value of $1 + \Delta\rho_{it}$, where $\Delta\rho_{it}$ is the jump size. The jumps of ρ_i are bounded below by -1, preserving limited liability. If the loss events have mean arrival rate η and a loss-size distribution ν with mean $\overline{\nu}$, then the mean loss rate is $\eta\overline{\nu}$. The risk-neutral investors therefore optimally supply all of their local capital inelastically so long as the mean rate of return $\pi(x) - \eta\overline{\nu}$ is strictly larger than their time preference rate r. This necessary condition on an equilibrium cash payout function $\pi(\cdot)$ is satisfied in the cases that we examine, as indicated in Appendix A.

As with typical asset-management contracts used by private-equity partnerships, cash payouts are not re-invested into the capital pool. For us, this assumption is merely a modeling convenience.

We assume that $\rho_a = \epsilon_a + \epsilon_c$ and $\rho_b = \epsilon_b + \epsilon_c$, where the market-specific processes ϵ_a and ϵ_b as well as the common component ϵ_c are independent Lévy processes. We assume that ϵ_a and ϵ_b have the same distribution, so that the two markets have identically and symmetrically distributed risks. This symmetry simplifies the calculation of an equilibrium and has the further illustrative advantage that any differences in the conditional expected returns in the two markets are due solely to differences in the capital levels of the markets. We briefly discuss the asymmetric case in Section 5.

If there were no capital-market frictions, investors would instantly move capital between the markets so as to obtain the higher dividend rate, and in doing so would equate the dividend rates $\pi(X_{at})$ and $\pi(X_{bt})$, and thereby equate X_{at} and X_{bt} at all times. Indeed, given the symmetrically distributed returns of the two markets, investors would do so even if they were risk-averse, provided that they have no other hedging motives.

Frictions in the movement of capital may, however, lead to unequal levels of capital in the two markets. If, for example, $X_{at} < X_{bt}$, then the conditional excess mean rate of return of the risky asset in market a exceeds that in market b by $\pi(X_{at}) - \pi(X_{bt})$, despite the identical idiosyncratic and systematic risks of the two assets. Whichever market has "too much capital" receives the lower risk premium.

An investor chooses how to deploy re-invested capital between the two markets, subject to the available trading technology. Letting C_t denote the net cumulative amount of capital moved by a particular investor from market a into market b through time t, this investor's capital levels, W_{at}^C in market a and W_{bt}^C in market b, thus satisfy

$$dW_{at}^C = W_{at-}^C \, d\rho_{at} - dC_t$$

and

$$dW_{bt}^C = W_{bt-}^C \, d\rho_{bt} + dC_t.$$

Capital can be moved only when in contact with an intermediary, as will be explained. A model for a proportional intermediation-fee process K will be determined in equilibrium. An investor is infinitely-lived, and thus has a utility of

$$E\left(\int_0^\infty e^{-rt} \left([W_{at}^C \pi(X_{at}) + W_{bt}^C \pi(X_{bt})] \, dt - K_{t-} \, d|C|_t\right)\right),$$

where $|C|_t$ denotes the total variation of C up to time t. A minor alteration of the model

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that allows for randomly timed exit and entrance of investors would be equally tractable.[2] For simplicity, we have assumed that transactions costs are paid directly by investors, and not deducted from the capital moved from market to market.

Because there is a continuum of investors, each takes as given the total capital processes X_a and X_b of the respective markets.

Intermediaries contact investors in order to profit from fees for moving their capital from one market to another. In equilibrium, at any time, only investors in that market with greater capital agree to have any of their capital moved to the other market. Because an investor has linear preferences, it is optimal when contacted to move either no capital or to move all capital to the other market.[3]

We let $W_{ij}(t)$ denote the level of capital in market i of investor j at time t. Conditional on the intensity process λ for contacts of investors by intermediaries, investors are contacted pairwise independently[4] at the conditional mean rate λ_t. In a manner similar to that of Weill (2007), the exact law of large numbers allows us to calculate the aggregate rate of movement of capital. Letting $m(\cdot)$ denote the non-atomic measure over the space of investors, the total rate at which capital is moved from market a to market b is almost surely[5]

$$
\begin{aligned}
\int \lambda_t 1_{\{X_{at} > X_{bt}\}} W_{aj}(t)\, dm(j) &= \lambda_t 1_{\{X_{at} > X_{bt}\}} \int W_{aj}(t)\, dm(j) \\
&= \lambda_t 1_{\{X_{at} > X_{bt}\}} X_{at}.
\end{aligned}
$$

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[2]For this, investors would exit at exponentially distributed times that are pairwise independent, and consume their capital at exit. New investors would appear in proportion to the current levels of capital. Any difference between exit and entrance rates would thus be subtracted from the proportional drifts of the capital accumulation processes X_a and X_b.

[3]If he or she has any capital in the market with more total capital, then all of this investor's capital will be moved, provided the proportional transaction-costs process K is not too large, and this is the case in any equilibrium for our model, as we shall see once the model is completely specified. Thus, although we allow that a given supplier of capital may initially have non-zero capital in both markets, all of his or her invested capital will optimally be held in just one of the two markets at any time after the first time of contact with an intermediary.

[4]That is, conditional on the path $\{\lambda_t : t \geq 0\}$ of the intermediation intensity process, the times of contacts of any distinct pair of investors, i and j, are the event times of independent Poisson processes N_i and N_j with the common time-varying intensity process λ.

[5]Sun (2006) shows that the probability space, the agent space, and the measurable subsets of their product can be constructed so as to satisfy the exact law of large numbers. With this, the total quantity of contacts of agents by intermediaries up to time t is $\int N_{it}\, dm(i) = \int_0^t \lambda_s\, ds$ almost surely.

Likewise, the rate at which capital moves from market b to market a is $\lambda_t 1_{\{X_{bt} > X_{at}\}} X_{bt}$.

Our model can be generalized by supposing that each investor also has a personal technology by which opportunities to move capital to the other market arrive at random times, independent across investors, with a constant mean arrival rate. This would cause only minor modifications to the structure and solution of our model. We avoid it for simplicity. Increasing the mean arrival rates of these alternative capital-shifting opportunities reduces the average degree of imbalance of capital and the difference in risk premia between the two markets, and thus reduces the profitability of intermediation. In Section 5, we review some evidence of inattention by investors that presents an opportunity for profitable contacts by intermediaries.

An intermediary's rate of cost for applying contact intensity λ_t is $c\lambda_t$, for some technological cost coefficient $c \geq 0$. For example, doubling the expected rate at which investors are contacted costs the intermediary twice as much.[6] The maximum feasible contact intensity of the market is some constant $\overline{\lambda} > 0$.

3 The Monopolistic Case

We begin with the case of a monopolistic intermediary. We restrict attention to the illustrative example of an insurance market in which each loss event affects only one of the two markets and results in a total loss of capital ($\overline{\nu} = 1$). Appendices treat more general cases, including partial recovery, loss events that can affect both markets simultaneously, as well as proportional losses and gains that are based on Brownian motion.

3.1 Equilibrium

We will define and characterize equilibria in which the intermediation intensity is of the symmetric Markov form $\lambda_t = \Lambda(X_t, Y_t)$, for some measurable policy function $\Lambda : \mathbb{R}_+^2 \to [0, \overline{\lambda}]$, where

$$
\begin{aligned}
X_t &= \max(X_{at}, X_{bt}) \\
Y_t &= \min(X_{at}, X_{bt}).
\end{aligned}
$$

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[6]This can be viewed as a contact technology in which the intermediary adjusts a "broadcast" intensity, for example adjusting the rate of purchase of advertisements or other forms of market-wide intermediation efforts. This differs from a model in which, for example, contacting twice as many individuals at a given intensity costs twice as much.

The monopolistic intermediation fee is assumed to be a fraction q of the gain in present value to an investor associated with redeploying the investor's capital from the over-capitalized market, that with X_t, to the undercapitalized market. This fraction is endogenized in Section 4 for the case of multiple intermediaries. The continuation value of an investor per unit of capital in the market with excess capital can be represented as $G(X_t, Y_t)$, for some $G : \mathbb{R}_+^2 \to [0, \infty)$, and likewise for the present value $H(X_t, Y_t)$ of each unit of capital in the under-capitalized market.

These values, defined from primitive stochastic processes in Appendix A, include the effects of future movements of capital to a market that is under-capitalized and, once that market becomes over-capitalized, back to the other market, and so on, net of fees. Assuming differentiability, which we will verify in equilibrium, Itô's formula implies that G and H are characterized as solutions to the coupled equations

$$
\begin{aligned}
rG(x,y) &= \pi(x) - G_x(x,y)x\Lambda(x,y) + G_y(x,y)x\Lambda(x,y) \\
&\quad + (1-q)\Lambda(x,y)(H(x,y) - G(x,y)) - \eta G(x,y) + \eta(G(x,0) - G(x,y)) \quad (1)
\end{aligned}
$$

$$
\begin{aligned}
rH(x,y) &= \pi(y) - H_x(x,y)x\Lambda(x,y) + H_y(x,y)x\Lambda(x,y) \\
&\quad + \eta(G(y,0) - H(x,y)) - \eta H(x,y), \quad\quad\quad\quad\quad\quad\quad\quad\quad (2)
\end{aligned}
$$

where subscripts denote partial derivatives. The first of these equations states that, in the over-capitalized market, the rate of loss in value due to time preference, $rG(x,y)$, is equal to the total expected rate of net profit to investment in that market. That rate of expected profit includes, first of all, the dividend payout rate $\pi(x)$. The next two terms are the rates of change of $G(X_t, Y_t)$ due to intermediated flows of capital out of the over-capitalized market and into the under-capitalized market, respectively. The following term is the expected rate of gain $(1-q)\Lambda(x,y)(H(x,y) - G(x,y))$, net of intermediation fees, associated with the chance to switch to the higher-premium under-capitalized market. The final two terms reflect the expected rate of impact of loss events. As the basic version of the model assumes no recovery value at these events, $\eta G(x,y)$ is the expected loss in value due to the occurrence of these loss events in the investor's market. The final term $\eta(G(x,0) - G(x,y))$ is the expected gain to the investor associated with a loss event in the other market. The second equation, for the total rate of investment return while in the undercapitalized market, is similarly explained.

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Fixing an intermediation policy Λ, the gain in present value per unit of redeployed capital at time t, before the associated fee, is

$$F^\Lambda(X_t, Y_t) = H(X_t, Y) - G(X_t, Y_t).$$

Thus, at bargaining power q, the intermediary's fee per unit of redeployed capital is $qF^\Lambda(X_t, Y_t)$.

If forced to accept the fees associated with a conjectured intermediation policy Γ, a monopolistic intermediary's optimal value for beginning with capital levels x and y in the respective markets is

$$V(x, y) = \sup_\Lambda E\left(\int_0^\infty e^{-rt}\Lambda(X_t, Y_t)(X_t qF^\Gamma(X_t, Y_t) - c)\, dt\right). \tag{3}$$

If a policy Λ that solves this problem is the same as conjectured policy Γ, then we say that Λ is an *equilibrium*. Thus, in equilibrium, fees are based on consistent conjectures by investors of the monopolist's future intermediation intensity process.

We assume, and later verify, that the intermediary's equilibrium initial value $V(x, y)$ is finite and differentiable. The associated Hamilton-Jacobi-Bellman (HJB) equation, when fees are determined by a conjectured intermediation policy Γ, is

$$0 = \sup_{\ell \in [0, \overline{\lambda}]} \{-rV(x, y) + \mathcal{U}(V, x, y, \ell, \Gamma)\}, \tag{4}$$

where, by Itô's formula,

$$\mathcal{U}(V, x, y, \ell, \Gamma) = -V_x(x, y)\ell x + V_y(x, y)\ell x + \eta[V(y, 0) + V(x, 0) - 2V(x, y)] + \ell(xqF^\Gamma(x, y) - c).$$

PROPOSITION 1 (HJB EQUATION) *Given an assumed intermediation policy Γ, suppose that $\hat{V}$ is a bounded differentiable function satisfying the HJB equation (4). Then $\hat{V}$ is the value function V of the optimization problem (3) and any measurable policy $(x, y) \mapsto \Lambda(x, y)$ which, for each (x, y), attains the supremum (4) is an optimal policy.*

The proof given in Appendix A.2 is by a traditional martingale verification argument.

From the HJB equation, it is optimal for a monopolistic intermediary to search at maximal intensity for opportunities to move capital whenever it is strictly beneficial to search at all, and this is precisely when

$$qF^{\Lambda}(X_t, Y_t) - [V_x(X_t, Y_t) - V_y(X_t, Y_t)] > \frac{c}{X_t}. \tag{5}$$

The left-hand side is the fee per unit of capital moved, net of the associated marginal loss in present value of future intermediation fees. This loss in future fees is caused by the associated reduction in heterogeneity of capital levels across the two markets, which always lowers the gain to investors of shifting their capital. The right-hand side of (5) is the cost of search per unit of capital that can be attracted.

In order to obtain the simplification associated with homogeneity,[7] we suppose that the inverse demand function $\pi(\cdot)$ is of the form $k_0 + k/x$ for positive constants k_0 and k. As explained in the insurance setting of Appendix F, this can be arranged by suitable assumptions on the cross-sectional distribution of hedgers' aversions to loss risk. Capital is therefore optimally reinvested locally, absent an opportunity to move capital to a different market, so long as the mean rate of return to investment exceeds the discount rate. This is always the case if $k_0 > r + \eta$, which we assume throughout. Because the constant k_0 is common to the two markets, however, it has no effect on benefits to switching capital and the analysis proceeds without loss of generality by ignoring k_0 (treating it as though equal to zero) from this point.[8]

Because the intermediary has linear time-additive preferences and because of the homogeneity of π, it is natural to look for equilibria for which the ratio $Z = X/Y$ of total capital in the over-capitalized market to total capital in the under-capitalized market determines the optimal intermediation intensity, and we focus on such equilibria from now on. A loss event causes the capital ratio X_t/Y_t to jump to $+\infty$. While we allow this formally, the analysis can be done similarly in terms of the ratio Y_t/X_t, which remains in $[0, 1]$ almost surely, and our results apply with only notational changes.[9]

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[7]In Appendix A, some results are shown for the more general case $\pi(x) = k_0 + kx^{-\gamma}$ for where γ is any positive constant.

[8]The present value of a marginal unit of capital includes the constant k_0/r, regardless of the intermediary's and investors' strategies.

[9]Provided the initial conditions include a strictly positive amount of capital in at least one market, the probability that X_t and Y_t ever reach zero at the same time is 0. The partial-recovery case that we analyze in Appendix I has

The homogeneity of the payout-rate function π and policy Λ implies that H and G are homogeneous of degree -1. As a result, $G(z, 0) = g_0 k z^{-1}$ for some positive constant g_0 to be determined. Letting $f(z) = F^\Lambda(z, 1)/k$ and $L(z) = \Lambda(z, 1)$, homogeneity of F^Λ and direct calculation from (1)-(2) implies that f solves the ordinary differential equation

$$
\begin{aligned}
0 = {} & -rf(z) + (1 - z^{-1}) - zL(z)f'(z) + (-\gamma f(z) - zf'(z))L(z)z \\
& - (1 - q)f(z)L(z) + \eta[g_0(1 - z^{-1}) - 2f(z)].
\end{aligned} \tag{6}
$$

The relevant boundary condition is $f(1) = 0$, corresponding to no gain from switching when the two markets have the same capital levels. Using (6), Appendix A.3 provides a proof of the following result that the switching gain $f(z)$ is strictly positive when capital levels are unequal.

PROPOSITION 2 *Given any intermediation policy Λ, $f(z)$ is strictly positive for $z > 1$. That is, given any Λ, investors in the over-capitalized market optimally accept the offer to move all of their capital out of the over-capitalized market whenever given the opportunity.*

Taking F^Λ as given, the optimal present value V of intermediation profits is homogeneous of degree 0, that is, $V(x, y) = V(x/y, 1)$ for $y > 0$. In particular, the policy Λ achieving the supremum of the HJB equation (4) must also be homogeneous of degree 0; that is, $\Lambda(x, y) = L(x/y)$ for some $L(\cdot)$. Because the switching-gain function f depends on the policy function L, the determination of equilibrium is reduced to a fixed-point problem: Find a pair (f, L) such that: (i) given f, the policy L is optimal, and (ii) given L, the marginal gain function f is that determined by L through (6).

In Appendix A.5 (Proposition 12), we show that any equilibrium must be of the "bang-bang" form:

$$
\begin{aligned}
\Lambda(x, y) &= 0, & x < Ty, \\
&= \overline{\lambda}, & x \geq Ty,
\end{aligned}
$$

strictly positive capital levels in both markets at all times after time zero, given a strictly positive level of capital in at least one of the markets at time zero. In that context, the assumption that loss events strike only one of the two markets at a time is without loss of generality because any common jump component would have no effect on the ratio of X to Y. The sole exception is a case of common jumps with a jump-size distribution that supports -1, in which case there is a non-zero probability that X_t and Y_t can be zero simultaneously. For the same reason, it is without loss of generality when characterizing equilibria that the return processes ρ_a and ρ_b have no drift, that is, a component that is linear in t. Any common Brownian component is likewise irrelevant to optimal intermediation behavior.

for some trigger ratio $T \geq 1$ of the capital level in the over-capitalized market to the capital level in the under-capitalized market. This is intuitive. Because the HJB equation is linear with respect the intensity chosen by the intermediary, we anticipate the optimality of switching from minimal to maximal intensity whenever there is sufficient marginal gain from moving capital from one market to the other. This occurs when the levels of capital in the two markets are sufficiently different. Our problem is reduced to finding an optimal trigger ratio T, which then completely determines equilibrium behavior.

The differential equation (6) for f now reduces to

$$(r + 2\eta + \overline{\lambda}[(1 - q) + z])f(z) + \overline{\lambda}(1 + z)zf'(z) = (1 + \eta g_0)\left(1 - \frac{1}{z}\right), \quad z \geq T, \quad (7)$$

and

$$(r + 2\eta)f(z) = (1 + \eta g_0)\left(1 - \frac{1}{z}\right), \quad z \in [1, T]. \quad (8)$$

For $z \in [1, T]$, the solution is trivial:

$$f(z) = \frac{1 + \eta g_0}{r + 2\eta}\left(1 - \frac{1}{z}\right). \quad (9)$$

In particular, we verify that $f(1) = 0$, consistent with the observation that the net present value of moving capital from one market to the other is 0 when the levels of capital in the two markets are the same.

We can re-write (7) as

$$(a + z)f(z) + z(1 + z)f'(z) = \left(1 - \frac{1}{z}\right)b, \quad z \geq T, \quad (10)$$

where $a = (r + 2\eta + (1 - q)\overline{\lambda})/\overline{\lambda}$ and $b = (1 + \eta g_0)/\overline{\lambda}$.

Letting $v(z) = V(z, 1)/k$, the HJB equation reduces to

$$0 = \sup_{\ell \in [0, \overline{\lambda}]} \left\{-rv(z) - \ell z v'(z) - \ell z^2 v'(z) + 2\eta[v_0 - v(z)] + \left(qzf(z) - \frac{c}{k}\right)\ell\right\}, \quad (11)$$

where $v_0 = V(y, 0)/k = V(x, 0)/k$. Therefore

$$v(z) = v_1, \quad z \in [1, T], \quad (12)$$

where

$$v_1 = \frac{2\eta}{r + 2\eta}v_0 < v_0, \quad (13)$$

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and

$$\kappa v(z) + v'(z)z(1+z) = d + qzf(z), \quad z \geq T, \tag{14}$$

where $\kappa = (r + 2\eta)/\overline{\lambda}$ and

$$d = \frac{2\eta v_0}{\overline{\lambda}} - \frac{c}{k}.$$

Appendix A.4 contains a proof of the following monotonicity and regularity of $v(\cdot)$. Monotonicity of the value $v(z)$ in the capital heterogeneity measure z is not an obvious result, in particular because the switching gain $f(z)$ is not in general monotonic. That is, fixing the capital level $y = 1$ in the market with less capital, the marginal gain $f(x)$ from switching capital from the over-capitalized market to the undercapitalized market need not be monotone in x even though the increase in payout rate $\pi(y) - \pi(x)$ is strictly monotone in x. This is the case because, as x gets large, both $g(x)$ and $h(x) \equiv H(x,1)/k$ go to zero, and so must therefore $f(x) = h(x) - g(x)$ go to zero. Intuitively, as x gets large, the global amount of capital is large, and although it can be intermediated from one market to another, the value of being a capitalist is not attractive when there is "too much" capital relative to the demand by hedgers to lay off risk.

The intuition for monotonicity of $v(z)$, however, is that, for any assumed trigger ratio T, optimal or not, the total rate of activated intermediation fees $q\overline{\lambda}zf(z)$, per unit of capital in the over-capitalized market, is not relevant on $[0, T]$ by definition of T, and is strictly increasing in z above T. In particular, even though $f(z)$ need not be monotone in z, $zf(z)$ is monotone in z, as shown in Appendix A.4.

PROPOSITION 3 (VALUE FUNCTION MONOTONICITY) *For any trigger capital ratio T, the solution v of (11)-(14) is bounded, increasing, and strictly increasing on $[T, \infty)$.*

The smooth-pasting condition $v'(T) = 0$ implies the trigger capital ratio

$$T = 1 + \frac{c(r + 2\eta)}{(1 + \eta g_0)qk}. \tag{15}$$

In order to identify the constant g_0, we use a conservation equation: The sum of the value functions of all investors and of the intermediary must be equal to the present value of all cash dividend payments net of the search costs incurred by the intermediary. After calculations shown in Appendix A.3, this conservation principle is equivalent to

$$kg_0 = \frac{2}{r} - \frac{c\overline{\lambda}}{r}\left(1 - e^{-(2\eta+r)a(T)}\right) - V(1,0), \tag{16}$$

14

where $a(T) = \log(1 + 1/T)/\bar{\lambda}$.

A proof of the following result guaranteeing the existence and uniqueness of a trigger strategy is found in Appendices A.6 (existence) and A.7 (uniqueness).

PROPOSITION 4 (EXISTENCE AND UNIQUENESS) *There exists a unique trigger capitalization ratio T satisfying* (16), (7), (8), *and* (15).

This analysis leads to the following characterization of equilibrium, which includes the result that in the absence of search costs, the intermediary does not exploit his position to restrict movement of capital, but rather provides maximal intermediation, nevertheless generating fee income from his or her imperfect ability to instantaneously move capital from one market to the other due to the upper bound $\bar{\lambda}$ on contact intensity. As $\bar{\lambda}$ becomes large, the capital levels will be nearly equated across the two markets at all times, as in a completely frictionless market, and in the limit there would be no intermediation rents.

PROPOSITION 5 *Suppose that the payout-rate function π is of the form $\pi(x) = k_0 + k/x$. Then there exists a unique equilibrium. In equilibrium, there is no intermediation ($\lambda_t = 0$) whenever the ratio of capital levels in the two markets is between $1/T$ and T, for a uniquely determined capital-ratio trigger T. Otherwise, intermediation is at full capacity ($\lambda_t = \bar{\lambda}$). The trigger ratio T is given by* (15), *where the constant g_0 is given by* (16). *If there is no intermediation cost ($c = 0$), then the intermediary always works at full capacity (that is, $T = 1$).*

Relation (15) also provides an upper bound on the equilibrium capital-ratio trigger level:

$$T \leq 1 + \frac{c(r + 2\eta)}{qk}.$$

This bound is useful for computing numerical solutions to the optimization problem. An algorithm for computing the constant g_0, and thus T, is given in Appendix G.

3.2   How Intermediation Depends on Market Parameters

We turn to comparative statics, focusing on the behavior of the threshold capital ratio T. A higher trigger ratio T corresponds to less intermediation, because the intermediary waits until X_t/Y_t exceeds T before becoming maximally active. We therefore define

capital mobility to be increasing whenever the equilibrium threshold T is decreasing. We will show that these comparative statics carry over to the oligopolistic case.

PROPOSITION 6 (COMPARATIVE STATICS FOR c, k, η, AND r.) *Capital mobility is decreasing in the intermediation cost coefficient c, increasing in the payout-rate coefficient k, and decreasing in the discount rate r. Fixing the dividend payout rate function $\pi(\cdot)$, capital mobility is decreasing in the loss-event intensity η.*

A proof is provided in Appendix H. It is intuitive that increasing the costs of intermediation, represented by c, reduces the amount of intermediation provided in equilibrium. Once the trigger T is chosen, the costs of search intensity are borne entirely by the intermediary. Other things equal, raising c therefore lowers the desire to search. The coefficients c and k affect the trigger T only through the ratio c/k, explaining the comparative static for k. The comparative statics for η and r are more subtle because, for a given trigger ratio T, both η and r have an effect on the value functions of investors. Equation (15) is the key to understand these comparative statics. Other things equal, a higher after-shock continuation value for investors (that is, a higher g_0) results in a lower T and thus a higher capital mobility. As the discount rate r goes up, the present value of the intermediary's future fees for moving capital are lower, fixing the fees, which increases the incentive to wait before expending the costs of intermediation. However, a higher r also reduces investors' gain from switching capital and hence the fees that they pay to the intermediary, with an unclear impact on their value function. As we show in our proof, the former effect dominates, so T is decreasing in r.

Our proof likewise shows that T is increasing in η, holding π fixed. Of course, increasing the mean loss frequency η would naturally raise the equilibrium loss insurance rate $\pi(X_{it})$, which on its own would tend to increase intermediation (lower T). To analyze this effect, we can write $\pi(x) = k_0(\eta) + k(\eta)/x$ in order to show the dependence of the coefficients $k_0(\eta)$ and $k(\eta)$ on the mean loss rate η. The coefficient $k_0(\eta)$ plays no role in intermediation gains. The impact on intermediation intensity of replacing η with some $\eta' > \eta$ is thus equivalent to the effect of leaving π unchanged and replacing the cost coefficient c with $c' \leq ck(\eta)/k(\eta')$. This effect can be small or large, depending on the sensitivity of $k(\eta)$ to η, reflecting how the elasticity of hedging demand varies with the expected loss frequency. Thus, the overall impact on intermediation intensity of a given

increase in the loss-event intensity η is to lower intermediation incentives (raise T) precisely when the impact on $k(\eta)$ is sufficiently small, and otherwise to raise intermediation intensity.

Increasing the bargaining power q of the intermediary increases the fraction of the gains to trade that goes to the intermediary, prompting the intermediary to search for more investor capital to move, thus setting a lower trigger ratio T, holding constant the gains to investors for moving capital. Obviously, however, raising q lowers the present value of investors associated with future movements of capital, thus lowering the amount of gain they have to share with the intermediary. The proof of the following result given in Appendix H demonstrates that the direct effect dominates the indirect effect on the investors' values, provided that q remains below $1/2$.

PROPOSITION 7 (COMPARATIVE STATICS FOR q) *Capital mobility is increasing in the bargaining power q of the intermediary, for $0 < q < 0.5$.*

The impact on capital mobility of the capacity $\overline{\lambda}$ for search intensity depends on other parameters, and particularly on the discount rate. There are situations in which lowering $\overline{\lambda}$ lowers the trigger ratio T, leading intermediaries to work more often, albeit with a lower capacity when they do work. We will provide examples and some intuition for the fact that, depending on the discount rate r, the threshold capital ratio T can either increase or decrease with capacity.

We will argue by continuity from the case of $\eta \simeq 0$. Taking $\pi(x) = k_0(\eta) + k(\eta)/x$, we can create a family of economies that keep $k(\eta)$ fixed at some $\bar{k}$ as η varies near zero by increasing the loss aversion of hedgers. With this, as η approaches zero, the equilibrium intermediation policy converges to a policy that moves capital until the threshold T is reached (with no changes in capital levels afterward), so the associated trigger ratio converges to

$$T = 1 + \frac{cr}{\bar{k}q}, \tag{17}$$

which is independent of the capacity $\overline{\lambda}$. Moreover, the associated gain functions converge to a gain function f with

$$f(T) = \frac{1}{r}\left(1 - \frac{1}{T}\right).$$

Suppose first that agents are patient (have a low discount rate r). As $\overline{\lambda}$ goes up, capital heterogeneity goes down, hence the value of moving capital is lower. Moreover,

a higher-capacity intermediary will more quickly run out of capital to be moved, and thus stop receiving fees earlier. Overall, this implies by continuity that we can construct example economies with small η such that, as $\overline{\lambda}$ is reduced and other parameters are held constant, the intermediary receives more fees, and for longer, and hence the value function of investors is lower, resulting in a higher threshold.[10]

Now, we consider the opposite case of nearly myopic investors (high r). The previous argument breaks down: Investors do not care much about future heterogeneity; they care mainly about the immediate gain from switching, which depends on *current* heterogeneity. Moreover, the immediate fees are increasing with capacity. A higher-capacity intermediary therefore receives higher immediate fees, which reduces investors' overall value of switching capital and increases the threshold.

Numerical examples support this intuition. For $q = 0.5$, $c = \bar{k}$, and $r = 0.04$, for η sufficiently small and $k(\eta) = \bar{k}$ sufficiently insensitive to η, the capital trigger T decreases with $\overline{\lambda}$ for $0.01 < \overline{\lambda} < 0.5$. For the same q and c but taking $r = 10$, however, the trigger ratio T increases with the intermediation capacity $\overline{\lambda}$, over the same interval.

4   Intermediary Competition

We now solve for equilibria with oligopolistic or perfectly competitive markets for intermediation.

There are two channels through which intermediary competition affects the equilibrium level of intermediation offered by the market. First, an intermediary internalizes the impact of intermediation intensity on the heterogeneity of capital levels across the two markets, and thus the impact on the intermediary's future profits. A given intermediary does not, however, internalize the effect of increasing it's intermediation on reducing the profit opportunities of *other* intermediaries. Through this first channel, increasing the number of intermediaries should therefore weakly increase the total amount of intermediation. In the benchmark case in which a loss event destroys all capital in the affected market, there is nothing to internalize, because the after-shock situation is independent of the pre-shock heterogeneity level, and our upcoming Proposition 8 states that the threshold is independent of the number of intermediaries. In Appendix C, we explain why the

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[10]See Equation (15), which shows that T is decreasing in investors's after-shock continuation value g_0.

increase is strict whenever shocks are partial, so that post-shock heterogeneity depends on pre-shock heterogeneity.

Second, when in contact with an investor, an intermediary considers the ability of the investor to compare the intermediation fee offered with the fees offered by other intermediaries. This plays a role, in an extension of the model that is offered later in this section, in determining the effective bargaining power of the intermediary, and through that channel, has an impact on the profitability of intermediation. We start by taking bargaining power as fixed and then endogenize the fees received by the intermediary.

4.1 Intermediary Competition At Fixed Bargaining Power

For a given bargaining power q, we show that equilibrium trigger policies for the oligopolistic case can be translated directly from the case of monopolistic intermediation through a simple inspection of the associated Hamilton-Jacobi-Bellman equations. From this, we also obtain comparative statics directly from those of the monopolistic case.

For the oligopolistic case, we take n identical intermediaries, each with an upper bound $\overline{\lambda}/n$ on intermediation intensity, and with the same proportional cost c of intermediation. The monopolistic case $(n = 1)$ is the special case considered in the previous section. Thus, all cases have the same feasible market dynamics and costs.

We consider only Markov time-homogeneous equilibria. Equilibrium incorporates the degree to which intermediaries internalize the impact of their intermediation intensity on the heterogeneity of capital levels across markets. For an oligopolistic equilibrium in trigger strategies, each of the n intermediaries has a reduced value function v, with $v(z) = V(z, 1)/k$, that solves the reduced HJB equation, extending the monopolistic version (11):

$$
\begin{aligned}
0 = \sup_{\ell \in [0, \overline{\lambda}/n]} \Bigg\{ & -rv(z) + \left(-\frac{(n-1)}{n}\overline{\lambda}1_{\{z \geq T\}} - \ell \right) zv'(z) \\
& - \left(\frac{(n-1)}{n}\overline{\lambda}z1_{\{z \geq T\}} + \ell z \right) zv'(z) + 2\eta[v_0 - v(z)] + \left(qzf(z) - \frac{c}{k} \right)\ell \Bigg\}, \quad (18)
\end{aligned}
$$

reflecting the presumption by the given intermediary that the $n - 1$ other intermediaries have adopted a specific trigger capital ratio T. The equilibrium condition is that the same trigger policy is optimal for the given intermediary. Verification of the HJB solution as the value function and of the optimality of the associated candidate policy function is as

for the monopolistic case.

Thus, an equilibrium for the n-intermediary problem is again given by bang-bang control for all intermediaries, each exerting no effort when $Z_t < T$ and maximal intermediation intensity $\overline{\lambda}/n$ whenever $Z_t \geq T$, for a trigger capital ratio T. We will show that optimality implies that there is no intermediation at or below the capital ratio T satisfying the smooth pasting condition $v'(T) = 0$. This, along with (18), implies that

$$qTf(T) - \frac{c}{k} = 0. \tag{19}$$

From (19), we see that an intermediary's optimization problem in a setting with n intermediaries is equivalent to that of a monopolistic intermediary with maximum intermediation intensity $\overline{\lambda}$. Indeed, for a given threshold T, the monopolistic and oligopolistic cases yield the same function f determining proportional intermediation fees, and hence the same smooth-pasting condition (19). In fact, this is actually the unique equilibrium, even allowing for the possibility of non-trigger strategies! To see this, consider any Markov equilibrium, not necessarily of the trigger-ratio form, and let f denote the function determining the associated gain from switching. An intermediary's HJB equation is of the form (18), except that *(i)* the aggregate of other intermediaries' contact intensities may be almost arbitrary, and *(ii)* the value functions may vary across intermediaries. Owing, however, to the form of the HJB equation, the indifference condition is nevertheless given by (19), and thus is the same for all intermediaries. This shows that any Markov equilibrium must be symmetric and of the trigger form.[11] In fact, repeating arguments from the monopolistic case leads to the following proposition.

PROPOSITION 8 *With n intermediaries, there is a unique Markov, homogeneous equilibrium. This equilibrium is symmetric and determined by a trigger capital ratio equal to that of a monopolistic intermediary with the oligopolistic maximal contact intensity $\overline{\lambda}$.*

When shocks lead to a partial recovery of capital, however, the intermediation trigger capital ratio T is strictly decreasing in the number n of intermediaries, as shown in Appendix C.

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[11]The trigger form comes from showing, as in the monopoly case (Lemma 2), that the function $z \mapsto zf(z)$ is increasing.

4.2 Endogenous Bargaining Power

Competition to supply intermediation may also have an impact on an intermediary's share of gains from trade when in contact with an investor. We next consider the implications of market structure for the intermediary bargaining power q. With $n > 1$ intermediaries, we suppose that some fraction ψ_n of investors are "well connected," meaning that as they prepare to switch capital from one market to another, they are in simultaneous contact with more than one intermediary. The number of intermediaries with whom a given investor is in contact could also be random, exploiting the law of large numbers, in which case ψ_n can be taken to be the probability that an investor, when contacted, is in contact with more than one intermediary. Intuitively, a well-connected investor has more bargaining power than a "captive" investor, one who is in contact with only one intermediary.

When modeling this intuition with a bargaining game, an issue is whether the contacted intermediary is assumed to know whether the investor is in contact with other intermediaries. We take this case.[12] Another modeling approach is a multilateral bargaining game with complete information, as in Stole and Zwiebel (1996).

We consider a bargaining procedure à la Rubinstein (1982), in which the investor and a particular intermediary alternate offers. In our continuous-time setting, the times between offer rounds can be treated as arbitrarily small, so the inter-round discount factor can be taken to be 1. In that case, the investor and intermediary agree immediately to split the surplus according to the Nash bargaining solution. The investor's share depends on his outside option. If the investor is captive, his outside option is simply $G(x, y)$, the value of remaining in the over-capitalized market. Thus, the normalized Nash product associated with a proportional fee of s is

$$[v(z) + s - v(z)][h(z) - s - g(z)],$$

which is maximal at $s = f(z)/2$, corresponding to $q = 1/2$, meaning an equal splitting of the gains with the intermediary.

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[12]It would be possible to allow for one-sided information. The fees derived could be obtained as equilibrium outcomes of a bargaining process, although there may be additional equilibria. See, for example, Sutton (1986). For an alternative approach to treating uncertainty about the degree to which an intermediary's customer is in contact with other intermediaries, see Green (2007).

For a well-connected investor, the normalized Nash product is

$$[v(z) + s - v(z)][h(z) - s - g(z) - (1 - q_0)f(z)],$$

where q_0 is the conjectured proportion of the gain from trade that the investor would pay to another intermediary if this first round of bargaining were to break down. The Nash product is maximized at $s = 0$, for a proportional intermediary share of $q = 0$, corresponding to the extraction of all surplus by the well-connected investor.[13]

If the number of intermediaries in contact with the investor is known only by the investor, then q is similarly obtained, and depends on the probability that the investor is captive.

Assuming pairwise independence of the connectedness of individual investors, the average of an intermediary's share of gains across the infinite population of investors is almost surely

$$q(n) = 0 \times \psi_n + \frac{1}{2}(1 - \psi_n) = \frac{1 - \psi_n}{2}. \tag{20}$$

In particular, $q(n)$ is decreasing in n if ψ_n is increasing in n. Obviously, $\psi_2 \geq \psi_1$. Going beyond the case of $n = 2$, it is somewhat intuitive that an investor is more likely to be well connected as the number of intermediaries increases. Appendix D briefly outlines a model with this natural feature.

Noting from (20) that $q(n) < 1/2$, Proposition 7 hints that lowering $q(n)$ reduces an intermediary's incentive to search, all else equal, because, for given capital dynamics, lowering $q(n)$ reduces intermediation profits, and therefore lowers the marginal benefit of raising intermediation intensity. We will next illustrate the second channel through which oligopolistic intermediation affects capital mobility: By reducing each intermediary's bargaining power, the incentive to intermediate is lowered.

Endogenous bargaining leads to complex dynamics, in which the number of intermediaries *actively* searching for capital varies over time. In order to see this, consider a candidate equilibrium in which n intermediaries search at full capacity whenever $z > T$, and no intermediary searches when $z \leq T$. If a single intermediary deviates by searching for capital when z is in a left neighborhood of T, then his fee per unit of capital switched is that of a monopolist, not that of the n-intermediary case. This increases the value of this

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[13]Another way to obtain this prediction is to assume that intermediaries connected to a given intermediary post prices and engage in Bertrand competition.

deviation. Despite this added complexity, we now show that oligopolistic intermediation may reduce capital mobility.

4.3 Reduced Capital Mobility With More Intermediaries

A Markov strategy profile for n intermediaries consists of functions $L_1, L_2, \ldots, L_n$ on $[1, \infty)$ into $[0, \bar{\lambda}/n]$. Here, $L_i(z)$ denotes the search intensity of intermediary i when the heterogeneity of capital across the two markets is $z = x/y$. The associated aggregate capital mobility is

$$L(z) = \sum_{i=1}^{n} L_i(z).$$

In order to exploit the fee share $q(n)$ derived above, we focus on simple strategies, for which $L_i(z)$ is either 0 or $\bar{\lambda}/n$. With this restriction,[14] we can associate with any strategy profile an increasing sequence $T_0, T_1, \ldots, T_J$ of capital-ratio thresholds with the property that, whenever the capital ratio Z_t is in $[T_j, T_{j+1})$, a particular set N_j of intermediaries is active. We let $n_j = |N_j|$ denote the number of intermediaries in N_j.

Using our previous analysis of the oligopolistic case with fixed bargaining power, we say that a profile of simple strategies is a *Markov equilibrium* if, for all j and $z \in [T_j, T_{j+1})$,

$$q(n_j)zf^L(z) - \frac{c}{k} \geq 0, \quad i \in N_j, \tag{21}$$

and

$$q(n_j + 1)zf^L(z) - \frac{c}{k} \leq 0, \quad i \notin N_j, \tag{22}$$

where $f^L(z)$ denotes the marginal gain to a investor from switching to the market with less capital, given an aggregate intensity policy L.

The first inequality means that any intermediary searching at capital ratio z does so optimally, given equilibrium fee share $q(n_j)$. The second equation states that any intermediary not searching at capital ratio z does so optimally, given the equilibrium fee share $q(n_j + 1)$ that he would get if he searched. We let $\overline{T} = \inf\{\bar{z} : L(z) = \bar{\lambda}, z \geq \bar{z}\}$, the smallest level of capital heterogeneity above which intermediaries search at full capacity. We denote by T_1 the monopolistic threshold. For the result to follow, recall that η is the mean arrival rate of loss events and that $\overline{T}$ depends, through L, on the particular Markov

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[14]Extending the analysis to general Markov strategies would be possible if one computes, for any possible strategy, the expected fee for each intermediary as a function of his search intensity and of the aggregate search intensity.

equilibrium under consideration. It is possible to show that in any equilibrium, $zf^L(z)$ is increasing in z. (For this, see the proof of Proposition 11.) This and Equations (21) and (22) imply that when $q(n)$ is decreasing in n (that is, a more connected investor pays a lower fee), the number of active intermediaries is increasing in z in any equilibrium.

PROPOSITION 9 (MONOTONICITY) *Suppose that $q(n)$ is decreasing in n. Then, n_j is increasing in j.*

The following result applies for all equilibria.

PROPOSITION 10 *Suppose the number n of intermediaries is at least 2. There exists some $\bar{\eta} > 0$ such that for any loss event intensity $\eta \in (0, \bar{\eta})$ and any associated Markov equilibrium trigger capital ratio T, we have $T_1 < \overline{T}$.*

This results states that for sufficiently infrequent loss events, the reduced bargaining power caused by oligopolistic competition reduces the domain of maximal capital mobility relative to that of the monopolistic case. Our assumption of a sufficiently small mean arrival rate η of loss events exploits the fact that intermediation fees are generated from the time of each loss event until capital is sufficiently equalized across the markets. Although there are technical steps in the proof of this proposition, found in Appendix E, the argument relies on a bound on improvements in the present value of intermediation fees as one changes from one market setting to another. A simple way to provide such a bound is thus to control the speed with which new fee-generating loss events occur.

Proposition 10 shows that oligopolistic competition results in less intermediation than achieved by a monopolist, for some range of market heterogeneity. This does not, however, rule out intermediation by oligopolists at capital ratios below the monopolistic trigger level. The next result shows that, provided that loss events are not expected too frequently, oligopolistic and monopolistic settings lead to a cessation of intermediation at approximately the same levels of market heterogeneity.

For any n-intermediary Markov equilibrium with aggregate intermediation strategy L, let

$$S_n = \inf\{z : L(z) > 0\},$$

the smallest heterogeneity level above which capital is mobile. A proof of the next proposition may be found in Appendix E. We will rely on a sufficiently small loss event intensity

for the same bounding effect explained after the statement of Proposition 10.

PROPOSITION 11 *For any $\varepsilon > 0$, there exists a strictly positive $\bar{\eta}$ such that for any mean loss event rate $\eta \in (0, \bar{\eta})$ and any associated Markov equilibrium with n players, we have $S_n \geq T_1 - \varepsilon$.*

Propositions 10 and 11 together show that capital mobility is lower, at any levels of capital, with oligopolistic intermediation than with monopolistic intermediation, provided that loss events are sufficiently infrequent.[15]

5   Discussion and Concluding Remarks

In a neoclassical model of asset markets, investors continually adjust their portfolios so as to achieve the highest possible mean return for a given type of risk. In equilibrium, an investor bearing a given type of risk is therefore compensated by a unique associated excess mean rate of return, no matter the asset that carries the risk. In practice, however, investors make portfolio adjustments with delays. In our model, the mobility of capital across different investments is increased through the equilibrium efforts of intermediaries. Our model has several implications for asset-price dynamics:

1. With unexpected changes in the amount of capital that is available to bear the risk represented by an asset, risk premia adjust more severely to capital shocks than in a neoclassical (perfect-mobility) setting, and then revert somewhat over time as capital is redeployed. This time signature, present in essentially any setting with slow moving capital, is dampened to the extent that intermediaries are active. Consider for example our simplest setting in which capital levels change only at the times at which all capital in a given asset market is lost. Perfect capital mobility ($c = 0, \overline{\lambda} = \infty$) implies that $X_{at} = X_{bt}$ for all t. A loss event at time τ in Market a would cause half of the capital from Market b to move instantly to Market a, so risk

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[15]For the last two results, we have held constant the dependence of the dividend rate function $\pi(x)$ on the capital level x as the mean rate η of loss events is varied. We have the freedom, however, of varying the population of hedgers as η changes, so as to offset the impact of variation of η on $\pi(x)$, thereby satisfying the stated comparison between monopolistic and oligopolistic intermediation for each fixed economy.

premia in both markets jump up by

$$\pi(X_{a\tau}) - \pi(X_{a\tau-}) = \frac{k}{X_{a\tau-}},$$

and then remain constant until the next loss event. With imperfect mobility, however, the risk premium in market a would jump immediately to $+\infty$ and then decline at the rate $\overline{\lambda} Z_t \frac{k}{X_{at}}$ until the capital heterogeneity ratio $Z_t = X_{bt}/X_{at}$ drops to T or until another loss event occurs.

2. The degree to which risk premia vary across assets, after controlling for other determinants of risk premia, is increasing in the degree to which capital is heterogeneously distributed across assets.

3. The speed of reversion of risk premia across assets toward common levels (after adjustment for other determinants of risk premia) is decreasing in the cost of intermediation. Increasing intermediation capacity, however, can either increase or reduce capital mobility, depending on the setting, as explained in Section 3.2.

4. All else equal, increasing the fraction of gross gains from moving capital that accrue to intermediaries increases the speed of adjustment of capital and risk premia.

5. Lowering time discount rates increases the mobility of capital.

6. Increasing the volatility of asset returns, represented in our model by the mean frequency η of loss events, can either increase or decrease the mobility of capital through intermediation, depending on the relative magnitudes of the effects of: (i) raising neoclassical risk premia (thus increasing the rents available to intermediaries) and (ii) increasing the volatility of capital heterogeneity at a given level of intermediation, which lowers the incentive to intermediate.

7. Increasing the scope for intermediary competition by splitting the market among more intermediaries can increase or decrease the equilibrium provision of intermediation, depending on the relative magnitudes of the effects of (i) reducing the concern of an intermediary regarding the impact of its own activity on lowering capital heterogeneity, and (ii) lowering the bargaining power of an intermediary vis-avis its customers because of increased competition with other intermediaries. As the number of intermediaries increases, the former effect raises intermediation incentives, while the latter effect can lower intermediation incentives.

Our introduction uses the example of the market for catastrophe risk reinsurance. Froot and O'Connell (1999), Zanjani (2002), and Born and Viscusi (2006) explain how premiums for catastrophe risk insurance typically increase dramatically when insurance and re-insurance firms suffer significant damage claims after natural disasters, such as Hurricane Andrews in 1992. Then, over many months, premiums drop toward "soft-market" levels (absent other shocks to the capital of insurers) because the replacement of insurance capital is delayed by institutional barriers to capital raising, including the time spent searching for suitable new investors. According to Enz (2001), premiums swing up and down by as much as 50% over multi-year periods, and are closely linked with changes in the capital levels of insurers, regardless of whether these changes in capital are caused by damage claims or by unexpected returns to the asset portfolios of insurers. From this, we know that the dynamics of insurance premiums after a major natural catastrophe are not caused mainly by inference concerning the arrival rate of future such events. We also know that most of the observed price impacts are not caused by inference about losses because major changes over time in insurance premiums following shocks to capital levels are highly correlated across all major lines of property insurance covered by the same pools of capital covering catastrophe risk.[16] These other lines cover, for example aviation, marine, motor, and proportional property. The link tying premium dynamics across the various lines of insurance is the level of capital commonly available to bear losses. Froot and O'Connell (1999) emphasize the slow speed of capital replacement as the major cause of slow premium adjustments.

That there is scope for intermediaries to mobilize dormant capital is apparent from a significant body of evidence that, when left on their own, many individual investors adjust their portfolios remarkably infrequently. For example, Ameriks and Zeldes (2004) report that over a 10-year period, 44% of investors in their sample made no changes to their portfolio allocations, and that an additional 17% of these investors made a single re-allocation during this period. Mitchell, Mottola, Utkus, and Yamaguchi (2006) find that, of 1.2 million U.S. employees covered by over 1,500 401(k) investment plans, approximately 80% initiated no trades over a two-year period, while an additional 10% made only a single trade.[17] Based on our theoretical results, one presumes that investor inattention

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[16]See, for example, Enz (2001), page 5, Figure 1.

[17]For further evidence on the slowness of individual portfolio adjustments, see Lusardi (1999), Lusardi (2003), Brunnermeier and Nagel (2008), and Bilias, Georgarakos, and Haliassos (2009).

is more evident in the data when the cost of contacting individuals and deploying their capital is large relative to the potential associated intermediation profits. For example, high-net-worth individual investors and institutional investors are likely to receive more attention from intermediaries because of the amounts of capital they can deploy and the associated higher total intermediation fees, than are most of the smaller investors whose inattention is documented in this literature. This is consistent with evidence provided by Bilias, Georgarakos, and Haliassos (2009) and Feldhütter (2009). Further, variation across investors in financial sophistication (not captured in our model) may lead to a negative correlation between the cost of achieving a given level of contact intensity and the level of deployable capital. In our model, intermediaries cannot differentiate among sub-classes of investors.

Delays in processing information for purposes of investment decisions are also in evidence from "price momentum" following fundamental news, as documented empirically by Chan (2003), Engelberg, Reed, and Ringgenberg (2010), Dellavigna and Pollet (2009), and Cohen and Lou (2010), among others. Given that most financial intermediaries are themselves likely to receive and processes fundamental news quickly relative to the time for price reactions documented in this literature, one presumes that there are limits on the average speed with which they can attract potential investors to such investment opportunities. This inference is also consistent with numerous examples of slow price adjustments to supply shocks in equity markets, including those of Holthausen and Mayers (1990), Scholes (1972), Coval and Stafford (2007), Andrade, Chang, and Seasholes (2008), Kulak (2008); with respect to supply shocks caused by index re-compositions, Shleifer (1986), Harris and Gurel (1986), Kaul, Mehrotra, and Morck (2000), Chen, Noronha, and Singhal (2004), and Greenwood (2005); and with respect to the expiration of commodity futures contracts, Mou (2011).

In corporate bond markets, which are not traded on a central exchange, one observes large price drops and delayed price recovery in connection with major downgrades or defaults, as described by Hradsky and Long (1989) and Chen, Lookman, and Schürhoff (2008), when certain classes of investors have an incentive or a contractual requirement to sell their holdings. Mitchell, Pedersen, and Pulvino (2007) document the effect on convertible bond hedge funds of large capital redemptions in 2005. Convertible bond prices dropped and rebounded over several months. A similar drop-and-rebound pattern

was observed in connection was the LTCM collapse in 1998. Newman and Rierson (2003) show that large issuances of European Telecom bonds during 1999-2002 temporarily raise credit spreads throughout the sector, evidence that it takes time for intermediaries to locate long-term investors.

In these examples, the time pattern of returns or prices after a supply or demand shock reveals that the friction at work is not merely a transaction cost for trade. If that were the nature of the friction, then all investors would immediately adjust their portfolios, or not, optimally. The new market price and expected return would be immediately established, and remain constant until the next change in fundamentals. In all of the above examples, however, after the immediate price response, whose magnitude reflects the size of the shock and the supply of immediately available capital, there is a relatively lengthy period of time over which the price reverts in mean toward its new fundamental level. In the meantime, additional shocks can occur, with overlapping consequences. The typical pattern suggests that the initial price response is larger than would occur with perfect capital mobility, and reflects the demand curve of the limited pool of investors that are quickly available to absorb the shock. The speed of adjustment after the initial price response is a reflection of the time that it takes more investors to realign their portfolios in light of the new market conditions, or for the initially responding investors to gather more capital.

In our setting, as in practice, there can be substantial differences in mean returns across assets that are due not only to cross-sectional differences in "fundamental" cash-flow risks, but are also due to unbalanced distributions of capital, relative to a market without intermediation frictions. Empirical "factor" models of asset returns do not often account for factors related to the distribution of ownership of assets, or related to likely changes in the distribution of ownership. Exceptions include the recent work of Coval and Stafford (2007) and Lou (2009), who note that the conditional mean returns of an equity tend to be lower due to price pressure when mutual funds owning that equity are experiencing liquidation-motivated outflows, and that the conditional mean returns recover as price pressure abates. Similarly, Bartram, Griffin, and Ng (2010) show that divergent levels of ownership by national domiciles play a role in equity returns.

In our model, delays in portfolio adjustments are due to the time that it takes for intermediaries to locate suitable investors. This is only an abstraction, which can also

proxy for other forms of delay, including time to educate investors about assets with which they have limited familiarity (awareness), time for contracting, and time for investors to dispose of their current positions, which could involve similar delays and price shocks, as suggested by Chaiserote (2008).

We could extend our model so as to treat asymmetric markets. Provided that the local inverse demand function $\pi(\cdot)$ of each market satisfies similar homogeneity assumptions, intermediation would be characterized by two distinct thresholds of capital ratios, one for movement of capital from market a to market b, and another for the reverse movement. For example, if returns in market a are riskier than those in market b, then, all else equal, capital will be less mobile toward market a than toward market b. Asymmetry, for example, would allow a consideration of capital mobility from a low-risk "money market" into a high-risk market such as that for private equity. Many of the qualitative features of our symmetric model, such as the dynamics of capital mobility and the impact of intermediation competition, are anticipated to carry over to asymmetric settings, at least under regularity conditions.

Another natural extension concerns the case of three or more markets. Consider, for example, three symmetric markets differing only in their capital levels, and satisfying our homogeneity conditions. We conjecture that capital will flow exclusively to the highest premium market, with more mobility from the lowest-premium market than from the mid-premium market one.

Appendices

Appendices F through K are located in Duffie and Strulovici (2011), a supplement to this paper.

A Equilibrium Analysis

Here, we provide a stochastic analysis of Markov equilibrium that is more complete and general than that provided in the main text.

Given an intermediation contact intensity process λ and initial conditions for capital in each market, we let X_{it}^λ denote the total capital in market i at time t. Given an associated transaction-cost process K, the marginal value to a supplier of one additional unit of capital in market i at time t is

$$\theta_{it}^\lambda = E\left(\int_t^\infty e^{-r(s-t)}\left[W_s\pi\left(X_{D(s),s}^\lambda\right)\,ds - W_{s-}\,K_{s-}\,dN_s\right]\,\Big|\,\mathcal{F}_t\right), \tag{23}$$

where N_s is the cumulative number of switches back and forth between the two markets through time s by the holder of this unit of capital, and the market indicator $D(s)$ is a or b, depending on whether, at time s, the accumulated capital W_s is currently located in market a or b. This capital thus accumulates according to

$$dW_s = W_{s-}\,d\rho_{D(s-)}(s),$$

with initial condition $W_t = 1$. The market-indicator process D is a marked point process with an initial condition at time t of $D(t) = i$, and with an intensity of jumping from market i to market j at time s of $\lambda_s 1_{\{X_{is}^\lambda > X_{js}^\lambda\}}$. In the equilibrium that we shall describe, the value of switching from market i to market j is strictly positive if and only if $X_{it}^\lambda > X_{jt}^\lambda$. The marginal value of moving capital is thus

$$\phi_t^\lambda = \max(\theta_{at}^\lambda, \theta_{bt}^\lambda) - \min(\theta_{at}^\lambda, \theta_{bt}^\lambda).$$

At each time t, intermediaries charge investors some fraction $q \in [0,1]$ of the gain ϕ_t^λ from switching each unit of capital. That is, the proportional intermediation fee is $K_t^\lambda = q\phi_t^\lambda$.

We restrict λ to be a progressively-measurable process so that, at each time, the intermediary chooses a contact intensity that depends only on information that is currently available.

A monopolistic intermediary's total rate of fee revenue is $\lambda_t \max(X_{at}, X_{bt})q\Phi_t$, where $\Phi_t = \phi_t^\lambda$ is the gain from switching capital under policy λ. Given the initial conditions $X_{a0}^\lambda = x_a$ and $X_{b0}^\lambda = x_b$, and given a gain-from-switching process Φ, the intermediary's utility for a contact intensity process λ is

$$U(x_a, x_b, \Phi, \lambda) = E\left(\int_0^\infty e^{-rt} \lambda_t [q\Phi_t \max(X_{at}^\lambda, X_{bt}^\lambda) - c] \, dt \right).$$

We assume that the parameters are such that this utility is finite, which is the case in the equilibria that we analyze. We restrict attention to intermediation policies that depend on the available information only through the current capital levels (X_{at}, X_{bt}). The intermediary might otherwise prefer to commit once and for all time to a path-dependent intensity policy that could, at some future time, be dominated by another policy available at that time, given the current capital market conditions at that time.

The inability to commit to an intermediation strategy may in principle be overcome by sophisticated punishment threats, as in Ausubel and Deneckere (1989) and Mailath and Samuelson (2006). In such equilibria, if the intermediary deviates, investors would update their beliefs about the intermediary's strategy in a way that harms the intermediary. Such equilibria are based on sophisticated off-equilibrium-path investor beliefs, which are not in the spirit of our assumption that investors are less sophisticated than intermediaries. Another possible justification for our focus on Markov equilibria is the fact that more sophisticated equilibria unravel in finite-horizon models where (possibly state-dependent) stage games have a unique Nash equilibrium.

Given the symmetry of the two markets, it suffices to characterize equilibrium behavior in terms of

$$X_t = \max(X_{at}, X_{bt})$$
$$Y_t = \min(X_{at}, X_{bt}).$$

The payoff processes to investments in the "larger" and "smaller" markets are, respectively,

$$d\rho_t^X = 1_{\{X_{at} > X_{bt}\}} \, d\rho_{at} + 1_{\{X_{at} \leq X_{bt}\}} \, d\rho_{bt}$$
$$d\rho_t^Y = 1_{\{X_{at} \leq X_{bt}\}} \, d\rho_{at} + 1_{\{X_{at} > X_{bt}\}} \, d\rho_{bt}.$$

From the Lévy property, (ρ^X, ρ^Y) has the same joint distribution as the primitive payoff processes (ρ_a, ρ_b).

Because we restrict attention to an intermediation intensity process λ that depends only on current capital levels, and because of symmetry, we can suppose that $\lambda_t = \Lambda(X_t, Y_t)$ for some measurable policy function $\Lambda : \mathbb{R}_+^2 \to [0, \overline{\lambda}]$ with the property that there is a solution to the associated stochastic differential equation

$$dX_t = -\Lambda(X_t, Y_t)X_t \, dt + X_t \, d\rho_t^X \tag{24}$$

$$dY_t = \Lambda(X_t, Y_t)X_t \, dt + Y_t \, d\rho_t^Y. \tag{25}$$

Letting $\mathcal{L}$ denote the space of intermediation intensity processes of this form, and given an assumed gain-from-switching process Φ, the intermediary solves the problem

$$\sup_{\lambda \in \mathcal{L}} U(x, y, \Phi, \lambda). \tag{26}$$

An equilibrium is a pair (λ, Φ) consisting of an intermediation intensity process λ that attains the supremum (26) given Φ, and such that $\Phi_t = \phi_t^\lambda$. This definition includes consistency with the optimality for investors to move their capital, in exchange for the marginal fee determined by Φ, when contacted by the intermediary, and includes consistency between the conjectured and actual dynamics for capital movements and search intensity.

A.1 Homogeneous Case

Allowing somewhat more generality than in the main text, we take the inverse demand function $\pi(\cdot)$ to be of the form $k_0 + kx^{-\gamma}$ for positive constants k_0, k, and γ. Also without loss of generality, in the following we take $k_0 = 0$ and, by re-scaling, we take $k = 1$. That is, the equilibrium behavior for (k, c) is the same as that for $(1, c/k)$. Because the intermediary has linear time-additive preferences and because of the homogeneity of π, and therefore of ϕ^λ, the ratio $Z = X/Y$ of total capital in the over-capitalized market to total capital in the under-capitalized market determines the optimal intermediation intensity. Thus, we can further assume the independence of ρ_a and ρ_b without loss of generality because any common Lévy component would have no effect on the ratio of X to Y. (The sole exception is a case of common jumps with a jump-size distribution that supports -1, in which case there is a non-zero probability that X_t and Y_t can be zero simultaneously. We rule out this exception.)

Consistent with the insurance example, we suppose that ρ_a and ρ_b are of the form $\rho_{it} = \mu t + \epsilon_{it}$, where μ is a constant and ϵ_a and ϵ_b are independent compound Poisson processes with common jump intensity η and a given jump-size probability distribution ν. The proportional payoff processes processes ρ_a and ρ_b could also be given a common Brownian component without affecting our analysis, for this also has no effect on the relative proportions of capital in the two markets. Cases with market-specific Brownian components are analyzed in Appendix K. Likewise, the constant drift rate μ plays no role in the analysis of optimal intermediation, and can be taken to be zero without loss of generality for purposes of determining equilibrium intermediation policies. The effect of non-zero μ on actual capital levels can be reintroduced later with the scaling by $e^{\mu t}$ of both X_t and Y_t.

The marginal gain from switching capital is

$$\phi_t^\lambda = F^\Lambda(X_t, Y_t) \equiv H(X_t, Y_t) - G(X_t, Y_t), \tag{27}$$

where, under our regularity, H and G satisfy the coupled equations (1)-(2). For general γ, letting $f(z) = F^\Lambda(z, 1)$ and $L(z) = \Lambda(z, 1)$, the ODE (6) for f generalizes to

$$\left(r + 2\eta + L(z)(\gamma z + (1 - q))\right) f(z) + z(1 + z)L(z)f'(z) = (1 + \eta g_0)(1 - z^{-\gamma}). \tag{28}$$

A.2 Verification of Optimality of HJB Solution

This appendix provides a proof that the HJB equation (4) characterizes optimality, allowing for a general gain function F^Γ. For this, given an arbitrary intensity process λ, let

$$S_t = e^{-rt}\hat{V}(X_t^\lambda, Y_t^\lambda) + \int_0^t e^{-rs}\lambda_s[X_s^\lambda q F^\Gamma(X_s^\lambda, Y_s^\lambda) - c]\, ds.$$

By Itô's Formula, a local martingale is defined by

$$\hat{V}(X_t^\lambda, Y_t^\lambda) - \int_0^t w(s)\, ds.$$

where

$$w_s = -\hat{V}_x(X_s^\lambda, Y_s^\lambda)\lambda_s X_s^\lambda + \hat{V}_y(X_s^\lambda, Y_s^\lambda)\lambda_s X_s^\lambda + \eta[\hat{V}(X_s^\lambda, 0) + \hat{V}(X_s^\lambda, 0) - 2\hat{V}(X_s^\lambda, Y_s^\lambda)].$$

Because λ and $\hat{V}$ are bounded, this local martingale is in fact a martingale. From this and the implication of the HJB equation that

$$-r\hat{V}(X_t^\lambda, Y_t^\lambda) - \mathcal{U}(\hat{V}, X_t^\lambda, Y_t^\lambda, \lambda_t, \Gamma) \leq 0,$$

another application of Itô's formula implies that S is the sum of a decreasing process and a martingale. Thus, S is a supermartingale. Because $\hat{V}$ is bounded, we have the "transversality" condition that for any intermediation intensity process λ,

$$\lim_{t \to \infty} E[e^{-rt}\hat{V}(X_t^\lambda, Y_t^\lambda)] = 0. \tag{29}$$

Thus, for any intermediation intensity process λ,

$$\hat{V}(x, y) \geq \mathcal{V}(x, y, \lambda, \Gamma) \equiv E\left(\int_0^\infty e^{-rt}\lambda_t[X_t^\lambda qF^\Gamma(X_t^\lambda, Y_t^\lambda) - c]\, dt\right). \tag{30}$$

Let Λ be a policy such that, for each (x, y), $\Lambda(x, y)$ attains the supremum (4). For each t, let $\lambda_t^* = \Lambda(X_t, Y_t)$. Then, the fact that

$$-r\hat{V}(X_t, Y_t) - \mathcal{U}(\hat{V}, X_t, Y_t, \lambda_t^*, \Gamma) = 0$$

implies that S is a martingale. Thus

$$\hat{V}(x, y) = \mathcal{V}(x, y, \lambda^*, \Gamma). \tag{31}$$

Thus, for any intermediation intensity process λ,

$$\hat{V}(x, y) = \mathcal{V}(x, y, \lambda^*, \Gamma) \geq \mathcal{V}(x, y, \lambda, \Gamma),$$

proving the result.

A.3   Nonnegativity of the Gain From Switching f

We now prove Proposition 2, allowing for general γ. Because the righthand side of (28) is strictly positive, f or f' must be strictly positive. This implies that f cannot cross 0 from above. Hence, f must be strictly positive on some interval of the form (z, ∞), and is non-positive on $[1, z]$ for some level z. It remains to show that $z = 1$. Because $f(1) = 0$, the intermediary does not search when the markets have equal levels of capital, given that $c > 0$. That is, $L(z)$ vanishes on a neighborhood of 1. From (28), this implies that f is positive on that neighborhood, which concludes the proof.

The total-present-value conservation equation is

$$V(x, y) + xG(x, y) + yH(x, y) = R(x, y) - P_T(x, y),$$

where $R(x, y)$ is the present value of the total future cash flows at rate $X_t \pi(X_t) + Y_t \pi(Y_t)$, to be divided among the intermediaries and the investors, and $P_T(x, y)$ is the the intermediary's expected discounted search costs over the infinite horizon, given a trigger T.

Because π is homogeneous of degree -1, we have $R(x, y) = 2/r$. The search-cost present value $P_T(1, 0)$ solves

$$P_T(1, 0) = p + E[e^{-r\tau} P_T(1, 0)], \tag{32}$$

where p is present value of search costs from time zero to the exponentially distributed time τ of the next loss event. We now show that, for the case of no recovery at loss event,

$$P_T(1, 0) = \frac{c\bar{\lambda}}{r} \left(1 - e^{-(2\eta + r)a(T)}\right), \tag{33}$$

where $a(T) = \log(1 + 1/T)/\bar{\lambda}$.

Starting with $X_0 = 1$ and $Y_0 = 0$, for $t < \tau$ we have

$$dX_t = -\bar{\lambda} X_t 1_{\{Z_t > T\}} \, dt$$

and

$$dY_t = \bar{\lambda} X_t 1_{\{Z_t > T\}} \, dt.$$

This yields $X_t = e^{-\bar{\lambda}t}$ and $Y_t = 1 - e^{-\bar{\lambda}t}$, for $t < \tau$. The intermediary will stop searching at that time $a(T)$ at which $Z_{a(T)} = T$, so

$$\frac{e^{-\bar{\lambda}a(T)}}{1 - e^{-\bar{\lambda}a(T)}} = T.$$

This yields

$$a(T) = \frac{1}{\bar{\lambda}} \log\left(\frac{1+T}{T}\right).$$

The present value of search costs until the next loss event is

$$p = E\left[\int_0^{\min(a(T), \tau)} e^{-rt} \bar{\lambda} c \, dt\right] = \frac{\bar{\lambda}c}{r} \left(1 - E[e^{-r\min(a(T), \tau)}]\right).$$

Because τ is exponentially distributed with parameter 2η,

$$E(e^{-r\tau}) = \frac{2\eta}{2\eta + r}$$

and

$$E[e^{-r\min(a(T), \tau)}] = \frac{2\eta + re^{-r(2\eta + r)a(T)}}{2\eta + r}.$$

Substitution of these into (32) yields the result (33).

A.4 Proof of Proposition 3

That v is bounded follows from the fact that it is dominated by $2/r$. The monotonicity result is based on two intermediate lemmas.

First, given the function f determining intermediation fees, let

$$\varrho(z) = \left(1 - z^{-\gamma}\right)\left(\frac{1 + \eta g_0}{r + 2\eta}\right) - f(z).$$

The first term of $\varrho(z)$ is the present value of switching capital to the under-capitalized market if the intermediary arrests intermediation efforts from the point at which the capital ratio Z_t is at z until the next loss event occurs, given g_0. (See (9).) Suppose in particular, a given reduced policy $L(z) = \Lambda(z, 1)$, and a particular z at which $L(z) = 0$. Then $\varrho(z) = 0$. As a special case, $\varrho(1) = 0$ (which can also be checked directly from the definition of ϱ and the fact that $f(1) = 0$). We note that, since the first term defining ϱ is strictly increasing in z, $\varrho'(z)$ must be positive whenever $f'(z)$ is negative. Given a policy L, we will show that ϱ is nonnegative. In order to see this, we observe that for $z \geq 1$, (28) can be re-written as

$$L(z)\left[((1-q) + z\gamma)f + z(1+z)f'\right] = (r + 2\eta)\varrho(z). \tag{34}$$

We already know that $\varrho(1) = 0$. Since f is positive from Proposition 2, this implies that $f'(z)$ is negative whenever $\varrho(z) \leq 0$, and hence that $\varrho' > 0$ whenever $\varrho \leq 0$. Therefore, ϱ cannot cross 0 from above, which proves our first lemma.

LEMMA 1 *For any policy, ϱ is everywhere nonnegative.*

This result is intuitive: other things equal, the expected gain from moving one's capital is larger if the intermediary immediately stops switching capital after that last movement, since the difference between capital levels, and hence between premia, is larger in that case. Lemma 1 has a crucial consequence for the case $\gamma = 1$: The rate at which fees are paid to the intermediary when he searches is strictly increasing in z. The more heterogeneous the markets, the higher is the intermediary's immediate profit from switching. Since this rate of fee payment, net of search costs, is $qzf(z) - c$, we must show that $zf(z)$ is strictly increasing in z. We can re-write (34) when $\gamma = 1$ as

$$L(z)(1 + z)(f(z) + zf'(z)) = (r + 2\eta)\varrho(z) + qL(z)f(z).$$

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Since f is positive and ϱ is nonnegative, this implies that $f(z)+zf'(z)$ is positive whenever $L(z) > 0$, hence that $zf(z)$ is strictly increasing in z. On any interval on which $L(z) = 0$, we have $f(z) = (1 + \eta g_0)/(r + 2\eta)(1 - 1/z)$, so f is strictly increasing, and, a fortiori, so is $zf(z)$.

LEMMA 2 *For $\gamma = 1$ and any policy, the revenue rate $zf(z)$ is strictly increasing in z.*

We can now show monotonicity of v for any trigger policy. From (13), v is constant for $z \leq T$. Starting with some capital ratio $Z_0 = z > T$,

$$v(z) = E\left[\int_0^\tau e^{-rt}[qf(Z_t)Z_t - c]1_{\{Z_t>T\}}\,dt + e^{-r\tau}v_0\right],$$

where τ is the time of the next loss event. The function $z \mapsto [qf(z)z - c]1_{z>T}$ is non-decreasing in z from Lemma 2, and strictly increasing for $z > T$. For $T < z < z'$, this implies that $v(z) < v(z')$ (because the event time τ has a distribution that does not depend on z or z'). This proves Proposition 3.

A.5 Optimality of a Trigger Policy

This appendix shows that for any equilibrium pair (f, L), the reduced policy function L must be a trigger policy. In fact, we will show that for any switching-gain function f that can arise as the result of an admissible intermediation policy, equilibrium or otherwise, the optimal policy must be of the trigger form.

From Appendix A.2, we know that, for a given f, any bounded solution of the HJB equation is the value to the intermediary of an optimal policy. We also know that f is continuous (and, in fact, differentiable) from (10). From Lemma 2, we also know that for any admissible policy, $zf(z)$ must be increasing. Finally f must be such that the value function v is bounded by $2/r$. These conditions define what we call "admissibility" of f. In particular, these conditions must be satisfied in any equilibrium.

We first show that there exists a solution to the HJB equation that is achieved by a trigger policy. Then we verify that any policy that achieves the value function that solves the HJB equation must be of the trigger form.

For any equilibrium, the function f is bounded, because

$$f(z) = |h(z) - g(z)| \leq h(z) + g(z) \leq h(z) + zg(z) \leq \frac{2}{r}.$$

Therefore, given any candidates for the capital trigger ratio T and the constant v_1, one can integrate the HJB equation (14) on $[T, \infty)$. The smooth-pasting condition is satisfied if $v'(T) = 0$, and this is equivalent to the condition that $qTf(T) = c$. (For this, see (11).) Given f, this uniquely determines T, because $Tf(T)$ is strictly increasing in T by Lemma 2. The only difficulty is to show the consistency condition $v_1 = (2\eta/2\eta + r)v_0$ (see (13)), where $v_0 = \lim_{z \to \infty} v(z)$, noting that v_0 enters as a coefficient of ODE 14 (in the constant d). In order to show this, we exploit the linearity of the ODE (14). Making the change of variables $u(z) = v(z) - v_1$, we have $u(T) = 0$. The dynamics of u do not depend on v_0, in that

$$u(z) + \alpha z(1 + z)u'(z) = \beta(z), \tag{35}$$

where $\beta(z) = \bar{\lambda}(qzf(z) - c)/(r + 2\eta)$ and $\alpha = \bar{\lambda}/(r + 2\eta) > 0$ is positive on (T, ∞). Moreover, the limit u_∞ is by construction equal to $v_0 - v_1$. This allows us to re-express the consistency condition as $u_\infty = (r/2\eta + r)v_0$. Therefore, having integrated u over $[T, \infty)$, one may simply read off the values v_0 and v_1. The resulting function $v(t) = u(t) + v_1$ solves the initial HJB equation with a v_0-dependent coefficient, and also satisfies the smooth pasting condition.

Thus, for any admissible f, there is an optimal policy of the trigger form. To conclude, we will show that there are no policies solving the HJB equation that are not of the trigger form. This follows from the linearity in ℓ of the HJB equation, implying a bang-bang solution, which is strict because indifference is characterized by the equation $qzf(z) = c$, which has a unique solution by Lemma 2. This analysis is summarized as follows.

PROPOSITION 12 *Suppose that the payout-rate function π is of the form $\pi(x) = k_0 + k/x$. Then any equilibrium intermediation policy Λ corresponds to a trigger capital ratio T. That is $\Lambda(x, y) = \bar{\lambda}1_{\{x/y > T\}}$.*

A.6 Existence of Equilibrium

So far, we have shown that any equilibrium must be of the trigger form. In this appendix we show that there exists such an equilibrium. Appendix A.7 shows uniqueness of such equilibria.

For any candidate trigger capital ratio T, let $f(z \,|\, T)$ be the net expected gain from switching capital across markets under the policy with trigger T, given current market

heterogeneity z. We need to show that there exists some T such that $qTf(T \mid T) = c$, that is, such that the intermediary ceases intermediation, given the switching gain function $f(\cdot) = f(\cdot \mid T)$, exactly when $z = T$. It suffices to show that $Tf(T \mid T)$ takes all values between 0 and ∞ as T varies from 1 to ∞.

Because $zf(z)$ is increasing, equation (9) implies that

$$f(z \mid T) \geq \frac{(T-1)}{T(r+2\eta)}, \quad z \geq T.$$

This implies that $Tf(T \mid T) \geq (T-1)/(r+2\eta)$. We note that the lower bound grows linearly with T. Because $Tf(T \mid T) = 0$ for $T = 1$, we know that $T \mapsto Tf(T \mid T)$ goes from 0 to ∞ as T goes from 0 to ∞. This function is continuous, so there exists some T^* such that $T^* f(T^* \mid T^*) = c/q$.

PROPOSITION 13 *Suppose that the payout-rate function π is of the form $\pi(x) = k_0 + k/x$. Then, there exists an equilibrium with a trigger policy.*

A.7 Trigger Uniqueness

PROOF OF PROPOSITION 4.   Suppose that trigger levels S and T, with $S < T$, both satisfy the equations of the proposition. Let $\phi(z) = f^T(z) - f^S(z)$ denote the difference between the gains from switching capital under policies S and T, as a function of z. (Throughout, we use superscripts to denote dependence on S or T.) From (15), $S < T$ implies that $g_0^S > g_0^T$. Optimality of S (respectively T) with respect to f^S (respectively, f^T) implies that, for any z in $(S, T]$,

$$qzf^S(z) - c - z(1+z)(v^S)'(z) > 0$$

and

$$qzf^T(z) - c - z(1+z)(v^T)'(z) \leq 0.$$

Because $(v^T)'(z) = 0$ for z in this interval $(S < T]$, while $(v^S)'(z) \geq 0$ by Proposition 3, we know that $\phi(T) < 0$. Subtracting the version of equation (7) for T from the version of the same equation for S yields

$$(a+z)\phi + z(1+z)\phi' = \alpha\left(1 - \frac{1}{z}\right), \quad z > T, \tag{36}$$

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where

$$a = \frac{r + 2\eta}{\overline{\lambda}} + (1 - q) > 0$$

and

$$\alpha = \frac{\eta(g_0^T - g_0^S)}{\overline{\lambda}} < 0.$$

Because $\phi(T) < 0$, this[18] implies that $\phi < 0$ for $z > T$, so that ϕ is everywhere negative.

By definition, g_0 is the marginal value of capital held by investors in the overcapitalized market, when $x = 1$ and $y = 0$ (that is, when no investor is initially present in the small market). Therefore,

$$g_0 = \frac{2}{r} - \Phi_0, \tag{37}$$

where Φ_0 is the expected discounted value of all future fees that investors will pay to the intermediary. (Recall that $2/r$ is the expected discounted stream of dividends paid on both markets. We have seen that $\phi < 0$, that is, $f^S(z) > f^T(z)$ for all $z > T$. This means that investors pay, for any z, more fees with S than with T for $z > T$. Moreover, for $z \in [S, T]$, investors pay fees (which are positive, from Proposition 2) for trigger S, whereas they pay nothing for trigger T. Therefore, $\Phi_0^S > \Phi_0^T$, which implies from (37) that $g_0^S < g_0^T$, a contradiction. ∎

B Numerical Illustration with Partial Recovery

We provide an illustrative example of equilibrium for the case of partial recovery, which is analyzed in Appendix I. We take the parameters $r = 0.04$, $\eta = 1.5$, $c = 0.04$, $\overline{\lambda} = 0.1$, $q = 1/30$. We assume beta-distributed recovery (one minus proportion lost) on $(0, 1)$, with parameters $(5, 1)$. The equilibrium intermediation trigger ratio T of capital in the over-capitalized market to capital in the under-capitalized market is found numerically to be 1.465.

Figure 1 shows simulated sample paths of the capitalization ratio $Z_t = X_t/Y_t$ and the immediate return $f(Z_t)/g(Z_t)$ to a supplier of capital, before transactions fees, associated with switching capital into the under-capitalized market. Figure 2 shows the present values, with one unit of capital in the under-capitalized market, of future cash flows to a provider of one unit capital in the over-capitalized market (net of fees), to a provider of

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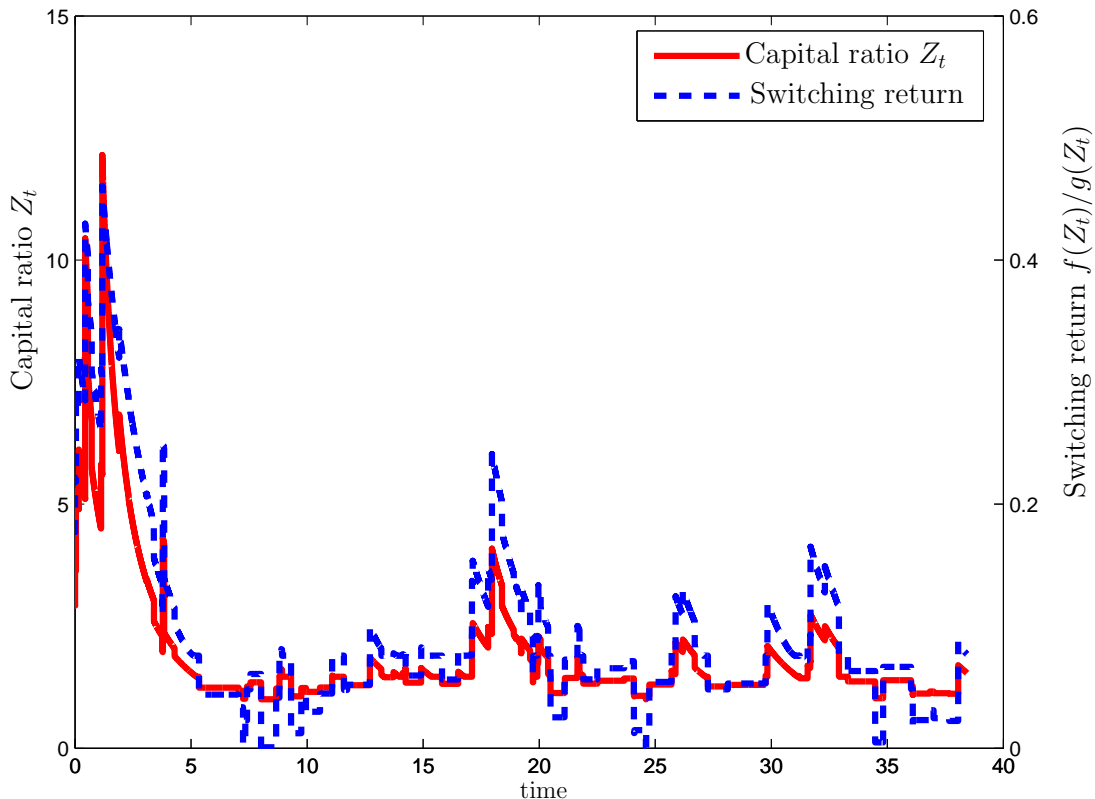[18]Indeed, $\phi(z) = 0$ implies that $\phi'(z) < 0$, so ϕ cannot cross zero from below.

Figure 1: Simulated sample paths of the capitalization ratio, $Z_t = X_t/Y_t$, and the return from switching, $f(Z_t)/g(Z_t)$.

one unit of capital in the under-capitalized market (net of fees), and to the intermediary (in the form of fees net of search costs). These are, respectively, $g(z)$, $h(z)$, and $v(z)$, and depend on the ratio $z = x/y$ of the level of capital x in the over-capitalized market to the level y of capital in the under-capitalized market.

C   Intermediary Competition with Partial Recovery

Here, we discuss the case of oligopolistic competition with partial recovery. Recall from (59) the smooth-pasting condition for the monopolistic case:

$$qTf(T) - c = T(1+T)v'(T). \tag{38}$$

One can see that the trigger capital ratio T is determined not only by the function f determining the marginal gain from moving capital, but also by the derivative $v'(T)$ of the intermediary's value function. In order to understand the impact of oligopolistic
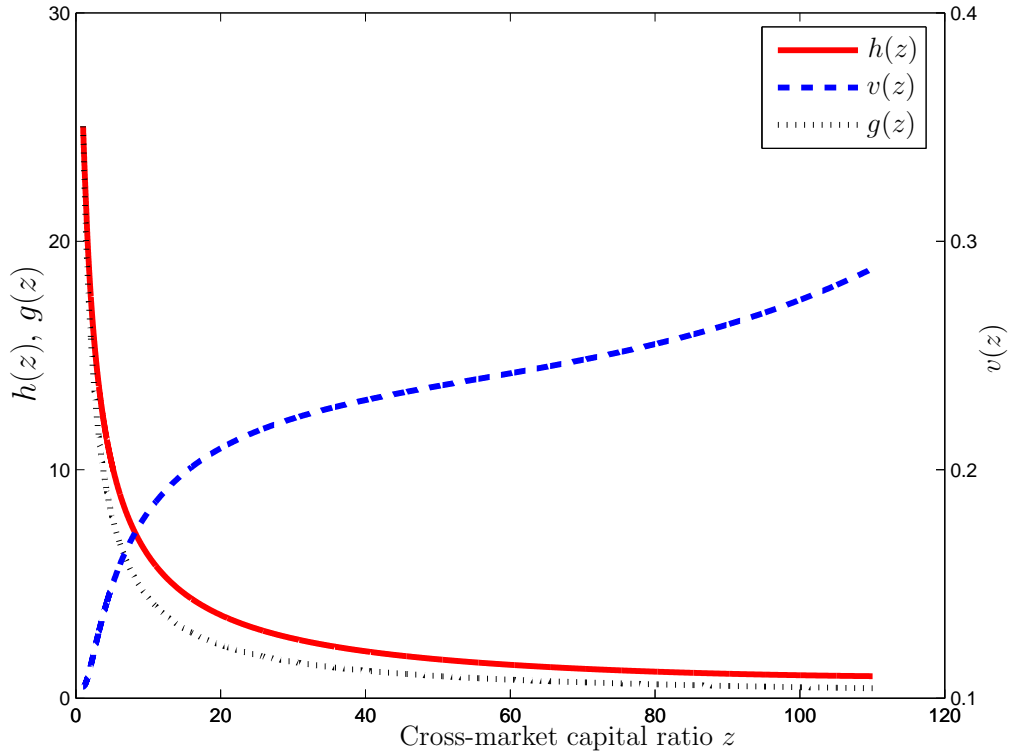
Figure 2: Value function $v(z)$ of the intermediary and the marginal values $g(z)$ and $h(z)$ of capital held in the over-capitalized and undercapitalized markets, respectively.

intermediation, suppose that intermediaries were to use, instead of the optimal trigger ratio T, the equilibrium trigger ratio of a monopolist with the same aggregate capacity for intermediation. In that case, f would be unchanged. Each intermediary, however, would receive only a fraction $1/n$ of the total intermediation fees. The righthand side of (38) is thus lowered, implying that intermediaries prefer to continue intermediating after the capital ratio exceeds the monopolistic trigger. This is the first channel through which oligopolistic competition matters: Because an oligopolistic intermediary does not internalize the full impact of his search on intermediation fees, he has a greater incentive to intermediate. More precisely, an intermediary does not work for opportunities to move capital when the immediate net marginal benefit of doing so, $qzf(z) - c$, is below the marginal value $z(1 + z)v'(z)$ associated with future capital heterogeneity. For a given trigger ratio T, an intermediary's value function v declines in direct proportion to the number n of intermediaries, and, hence, so does the derivative v'. This implies that the term $z(1 + z)v'(z)$ diminishes with n, while the immediate marginal benefit $qzf(z) - c$

is unchanged, keeping T constant. Thus, as n increases, the incentive to intermediate at the given trigger ratio T becomes strictly positive, prompting intermediaries to search more.[19]

As n goes to infinity, an intermediary's value function goes to zero (because the size of the pie to be shared among intermediaries is uniformly bounded above by $2/r$), and the derivative $v'(T)$ also goes to 0. The limit as n diverges is the competitive equilibrium, in which the trigger capital ratio T is determined by

$$qTf(T) - c = 0.$$

With perfect competition, an intermediary has no impact on aggregate search activity, and thus cares only about the immediate net benefit from switching.

D   Connectedness

In this appendix, we outline a model with the natural feature that an investor is increasingly likely to be in contact with multiple intermediaries at the point of bargaining as the total number of intermediaries is increased.

Suppose that there is an advertising medium handling intermediary ads. An intermediary's effort corresponds to the probability p that its advertisement will place the intermediary in contact with an investor at the time at which the investor checks the medium. We assume that p is bounded by some capacity constraint $\bar{p} < 1$. Each investor, pairwise independently across investors, has some exogenous intensity χ for the times of monitoring his capital and observing the advertising medium.[20] This is consistent with the framework of our main model: The intensity of times at which an investor is contacted by least one intermediary is $\chi\phi_n(p)$, where

$$\phi_n(p) = 1 - (1 - p)^n.$$

Then, $\overline{\lambda} = \chi\phi_n(\bar{p})$ is the intermediation capacity parameter of the basic model. Assuming that a well-connected investor initiates bargaining with a randomly selected intermediary

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[19]When there is zero recovery from a loss event, the after-event heterogeneity (which is infinite) does not depend of the pre-event heterogeneity. In that case, intermediaries already ignore the impact of their search activity on heterogeneity and the monopolistic solution coincides with the competitive one.

[20]At such times, the investor observes the medium and plays a bargaining game with advertised intermediaries. If bargaining breaks down, the investor leaves his capital in the large market, until the next monitoring time.

from among those contacted, each intermediary has maximal contact intensity $\bar{\lambda}/n$. The probability that, when in contact with an intermediary, an investor is in contact with at least two intermediaries is

$$\psi_n(p) = 1 - (1-p)^n - np(1-p)^{n-1}.$$

For a fixed $\overline{\phi} \in [0, 1]$, let $\bar{p}_n$ solve $\phi_n(\bar{p}_n) = \overline{\phi}$, so that $\bar{\lambda}$ is independent of n, as in our basic model. One may easily check that $\bar{p}_n$ is decreasing in n. Moreover, using that

$$\psi_n(p_n) = \bar{\phi} - np_n(1-p_n)^{n-1} = \bar{\phi} - \frac{(1-\bar{\phi})np_n}{1-p_n},$$

one can show that $\psi_n(p_n)$ is increasing in n.[21] Therefore, keeping constant the flow of investors being contacted at any given time, the average number of intermediaries in contact with any given investor is increasing in n. As the number of intermediaries goes to infinity, the probability that investor is well connected is:

$$\lim_{n \to \infty} \psi_n(p_n) = \bar{\phi} + (1-\bar{\phi})\log(1-\bar{\phi}).$$

The second term is negative. This specification can be generalized to an arbitrary number of media, with the same result that ψ_n is increasing in n.

E   Proofs of Results in Section 4.3

Proof of Proposition 10.   As before, we let $g_0^L = G(1, 0)$, under strategy L. For any equilibrium with aggregate mobility $z \mapsto L(z)$ and fee $z \mapsto q(z)$, one can easily modify the proof of Lemmas 1 and 2 to show that

$$\varrho^L(z) \equiv \left(\frac{1 + \eta g_0^L}{r + 2\eta}\right)\left(1 - \frac{1}{z}\right) - f^L(z)$$

is nonnegative and that $zf^L(z)$ is increasing in z. If $\overline{T} \leq T_1$, we have

$$f^1(\overline{T}) = \left(\frac{1 + \eta g_0^1}{r + 2\eta}\right)\left(1 - \frac{1}{\overline{T}}\right),$$

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[21] In order to verify this, one is to show that $np_n/(1-p_n)$ is decreasing. Expressing p_n in terms of $\alpha = (1-\bar{\phi})^{-1} > 1$ and letting $x = 1/n$, this is equivalent to showing that $(\alpha^x - 1)/x$ is increasing in x. This is easily done by checking the positivity of the derivative, whose numerator is increasing in $u = \alpha^x$ and vanishes for $u = 1$.

where f^1 and g_0^1 denote the corresponding quantities for the monopolistic case, since the intermediary does not search at $\overline{T}$. Further,

$$f^L(\overline{T}) \leq \left(\frac{1 + \eta g_0^L}{r + 2\eta}\right)\left(1 - \frac{1}{\overline{T}}\right),$$

from the nonnegativity of $\varrho^L(\overline{T})$. Therefore,

$$\overline{T}\left(f^1(\overline{T}) - f^L(\overline{T})\right) \geq \eta\left(\frac{g_0^1 - g_0^L}{r + 2\eta}\right)(\overline{T} - 1). \tag{39}$$

Since $g_0^L \leq 2/r$ for any policy, there exists, for any $\varepsilon > 0$, some $\bar{\eta}$ such that for all $\eta < \bar{\eta}$, the righthand side of (39) is bounded in norm by ε whenever $\overline{T} \leq T_1$, since we also have an upper bound on T_1 from (15). Choosing ε below $(1/q(n) - 1/q(1))c$ and setting $\bar{\eta}$ accordingly, we have for any $\overline{T} \leq T_1$,

$$q(1)\overline{T}f^1(\overline{T}) \geq \frac{q(1)}{q(n)}\left(q(n)\overline{T}f^L(\overline{T}) - q(n)\varepsilon)\right) \geq \frac{q(1)}{q(n)}(c - q(n)\varepsilon) > c, \tag{40}$$

which shows that it is strictly optimal for the monopolist to search at $\overline{T}$, contradicting the assumption that $\overline{T} \leq T_1$.

Proof of Proposition 11. At S_n, it cannot be strictly profitable for an intermediary to deviate by continuing to search and receive the net payoff $q(1)S_n f^L(S_n) - c$ per unit of effort, but it was profitable to some intermediaries to search at a capital heterogeneity just above S_n. This implies that S_n must satisfy the equation

$$q(1)S_n f^L(S_n) = c.$$

In words, there is a single active intermediary just before S_n is reached. We recall from the monopolistic case that T_1 satisfies the equation

$$q(1)T_1 f^1(T_1) = c.$$

Therefore, it suffices to show that the roots of these two equations are arbitrarily close if η is arbitrarily small. We have

$$f^L(S_n) = \left(\frac{1 + \eta g_0^L}{r + 2\eta}\right)\left(1 - \frac{1}{S_n}\right)$$

and

$$f^1(T_1) = \left(\frac{1 + \eta g_0^1}{r + 2\eta}\right)\left(1 - \frac{1}{T_1}\right).$$

Therefore, S_n and T_1 must satisfy

$$\left(\frac{1 + \eta g_0^L}{r + 2\eta}\right)(S_n - 1) - \left(\frac{1 + \eta g_0^1}{r + 2\eta}\right)(T_1 - 1) = 0,$$

which may be rewritten as

$$\left(\frac{1 + \eta g_0^L}{r + 2\eta}\right)(S_n - T_1) = \eta \left(\frac{g_0^L - g_0^1}{r + 2\eta}\right)(1 + T_1).$$

Because T_1 is uniformly bounded from (15) and because both g_0^L and g_0^1 are bounded by $2/r$, the righthand side is less than ε if η is chosen small enough. The first factor of the lefthand side is equivalent to $1/r$ when η is small enough. Combining these observations shows that $|S_n - T_1| \leq \varepsilon$ for any arbitrary $\varepsilon > 0$, provided that η is small enough.

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