NBER WORKING PAPER SERIES

STANDARD-RATE WAGE SETTING, LABOR QUALITY, AND UNIONS

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Working Paper No. 1717

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 1985

I am grateful to seminar participants at Columbia University and the National Bureau of Economic Research for very helpful comments. Specific suggestions by J. Altonji, L. Katz, G. Solon and A. Weiss have been incorporated; remaining shortcomings are mine to claim. Gunther Elert helped with the tabulations in Section 2. The National Science Foundation and the Computer Science Center at the University of Maryland (where this paper was begun) for supporting the computer work in the Appendix. The research reported here is part of the NBER's research program in Labor Studies. Any opinions expressed are those of the author and not those of the National Bureau of Economic Research.

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ABSTRACT

"Standard rate" wage policies, under which all workers in a particular job receive the same wage, are common for blue-collar workers, especially those covered by collective bargaining agreements and those who work for large employers. This paper analyzes the impact of standard-rate wage setting.

There are two important conclusions. First, a standard-rate rule which leaves the employer free to set the rate can either increase or reduce the quality of labor hired. Given empirically likely distributions of alternative wages for workers, it pushes employers toward the middle of the quality distribution. Second, union standard-rate policies allow union-nonunion differences in wages for workers of a given quality to exist even when union employers are free to alter the quality of their workforces.

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Standard-rate wage determination, under which all workers in a particular job receive the same wage, is an empirically important policy which has received little theoretical attention. One study of workers in metropolitan areas found that 37 percent of the plant workers were paid under "single-rate" plans which "provide the same rate to all experienced workers in the same job classification," and an additional 16 percent were paid under a system whereby one's progression through a range of wage rates in a job classification is "based on automatic advancement according to length of service" (Cox, 1971, p. 54). Thus, over half of the plant workers were in compensation systems where wages do not vary directly with performance within a job classification.

Buckley's (1985) study of 28 blue collar occupations focused on actual wages paid rather than on wage-setting systems. In each occupation studied, an establishment could fall into one of three groups: only one worker in the job; several workers in the job receiving the same wage; several workers in the job paid at two or more wage rates. In 13 of the 28 jobs categories, more workers were in the second (single wage) group than in the multiple-wage group. For those in the multiple-wage group, within-establishment, within-job wage spreads were calculated (percent difference between high and low wage). In half of the job categories, the mean within-establishment within-job wage spread was 18 percent or less. Both Cox and Buckley found much less rate standardization among while collar workers.

Freeman and Medoff (1984, p. 80) found that unionized establishments are much more likely than are nonunion establishments in the same industry to use both single-rate and automatic progression wage determination (54% vs 27% and 14% vs 8%, respectively). Brown and Medoff (1984) found a similar tendency for larger establishments to adopt single-rate policies (holding unionization constant). While Freeman and Medoff highlighted the implications of their finding for wage inequality, there has been little theoretical discussion of other implications of standard-rate policies. In this paper, I analyze the effect of such policies by themselves and their effect when combined with a union-imposed wage floor.

In section 1, I consider a firm which can hire as many workers of a given quality as it wishes, so long as it is willing to pay the going wage for such workers. In this case, the firm has no incentive to have unequal quality workers in a given job (so standard rates are irrelevant). If unionized such a firm simply upgrades quality until its workers receive no higher wage than similar workers receive elsewhere. In section 2, I assume a firm faces a limited number of potential workers of varying qualities, so a homogeneous workforce (within a job category) is impossible. Such a firm will hire workers of differing qualities and pay each quality its reservation wage. Thus, a standard-rate requirement that all workers (in a job classification) receive the same wage is a real constraint, even if the employer is free to choose the wage. In section 3, a union which imposes a wage floor *and* a standard-rate requirement is analyzed. Concluding comments are presented in section 4.

There are two important conclusions. First, a standard-rate rule which leaves the employer free to set the rate can either increase or reduce the quality of labor hired. Given empirically likely alternative-wage functions, it will raise the quality of labor of employers who would otherwise hire low quality workers and reduce it for employers who would otherwise hire high quality labor. Second, union standard-rate policies allow union-nonunion differences in wages *for workers of a given quality* to exist even when union employers are free to alter the quality of their workforces.

1. Homogeneous Workforce

In this and subsequent sections, I assume that the firm has a fixed number of vacancies per period (in a particular job) to fill, and that the "quality" of each potential worker, q, is easily observed. It is convenient to measure the quality of potential workers so that q is uniformly distributed between 0 and 1. The value of marginal product of a worker of quality q is v(q). All potential workers of quality q are assumed to have alternative wage a(q). This abstracts from problems measuring q and from commuting costs or other factors which would lead to different reservation wages among workers of the same quality. I assume $v'(q) > 0$ and $a'(q) > 0$, and $v''(q) - a''(q) < 0$.

In this section, I analyze a firm which can hire as many workers of quality q as it wants by offering to pay them a(q).

The firm's problem is

$$\max_{q} \pi = v(q) - a(q)$$

and it choses q so that $v'(q) = a'(q)$. The second-order condition $v''(q) - a''(q) < 0$ guarantees a unique choice of q. Each worker receives a wage equal to a(q).

Let q_n and $a(q_n)$ be the firm's choice of quality level and its wage in the absence of unionization. If the firm is unionized and required to pay wage $W_u > a(q_n)$, the firm's problem becomes

$$\max_{q} \pi = v(q) - W_u \qquad \text{subject to} \qquad a(q) \leq W_u.$$

The firm simply upgrades its quality until $a(q) = W_u$. It is forced to buy a higher quality than it would choose in the absence of unions, but its workers receive their alternative wage elsewhere. This upgrading until union workers receive no higher wages than comparable nonunion workers underlies Pettengill's (1980) general equilibrium analysis of unionization. Empirical studies of individual workers who change union status confirm the conclusion of simpler studies that, long after unionization first reached its current level, union status raises wages. Reconciling the clear incentives to upgrade with the apparently incomplete upgrading found empirically has remained something of a puzzle.[1]

2. Heterogeneous Workforce

Now assume one firm faces a limited set of potential workers, again with a uniform (0,1) quality distribution and alternative wages a(q). Let r be the ratio of vacancies to potential workers per period.

The firm's problem can now be written

$$\max_{H,L} \pi = \int_{L}^{H} [v(q) - a(q)]\, dq \quad \text{subject to } H - L = r.$$

here H and L are, respectively, the highest and lowest quality of workers the firm hires, and the simplified form of the constraint follows from the convention that q is "measured" so as to have a uniform distribution. The firm's problem can be written more simply by substituting out the constraint:

$$\max_{H} \pi = \int_{H-r}^{H} [v(q) - a(q)]\, dq.$$

If H_o is the firm's optimal[2] value of H, H_o satisfies

$$v(H_o) - v(H_o - r) = a(H_o) - a(H_o - r)$$

This is very similar to the first-order-condition for the firm in section 1: equality of slopes over the range from H_o-r to H_o replaces equality of slopes at the single quality q.

Now suppose that the firm is constrained - or chooses - to pay the same wage to all workers (in this job category).[3] The firm's problem becomes

$$\max_{W,H,L} \pi = \int_L^H [v(q)-W]\, dq \quad \text{subject to} \quad H - L = r \text{ and } a(H) \leq W.$$

Since v'(q) is always positive, the last constraint holds as an equality.

Again substituting out the constraints, we have

$$\max_{H} \pi = \int_{H-r}^{H} [v(q) - a(H)]\, dq$$

The first - and second - order conditions are

$$v(H) - v(H-r) = r\, a'(H)$$

and

$$v'(H) - v'(H-r) - r\, a''(H) < 0.$$

A natural question is whether the standard-rate rule has predictable effects on the quality of labor which a firm hires. The simplest way of answering this question is to formulate a slightly more general wage setting policy. Let the wage paid to workers of quality q be a weighted average of the flexible wage and standard-wage regimes; i.e., $\alpha\, a(H) + (1-\alpha)\, a(q)$. The firm's problem is

$$\max_{H} \pi = \int_{H-r}^{H} [v(q) - (1-\alpha)\, a(q)]\, dq - \alpha r a(H)$$

The first-order condition is

$$\frac{\partial \pi}{\partial H} = v(H) - v(H-r) - (1-\alpha)\, [a(H) - a(H-r)] - \alpha\, r a'(H) = 0.$$

Increases in α can then be interpreted as moves from a flexible-wage to a standard-rate policy. Differentiating the first-order condition with respect to α and H, and setting the result equal to zero yields:

$$\frac{dH}{d\alpha} = -\frac{\partial^2\pi/\partial H\partial\alpha}{\partial^2\pi/\partial H^2}$$

The denominator must be negative if the initial equilibrium maximized profits. The numerator equals a(H) - a(H-r) - ra'(H). Since [a(H) - a(H-r)]/r is the slope of the chord joining H and H-r while a'(H) is the slope of the tangent at H, this expression is positive when a(q) is concave and negative when a(q) is convex. Therefore, labor quality is increased by moves toward a standard rate policy when a(q) is concave and reduced when a(q) is convex.[4]

The intuition behind this result is similar to that offered by Weiss and Landau (1984, p. 483) for a related problem. When a(q) is concave (convex) an increase (reduction) in q represents a move toward a q where the frequency distribution of alternative wages is denser, and so (for fixed r) a more homogeneous workforce is being chosen. It is intuitively sensible that a firm which pays all workers (in a job category) the same wage as its best worker will move toward recruiting a more homogeneous workforce.

Under the assumption that alternative (market) wages are an exact function of quality, the a(q) function is just the cumulative distribution function of the market wage, but with the wage plotted on the vertical axis. If wages follow a unimodal distribution, a(q) will be concave at "low" values of q and convex at "high" values. For a symmetric distribution such as the normal the change from concave to convex will occur at q=.5; if the median exceeds the mode (as would be true, for example, for a lognormal distribution), the change from concave to convex will occur at $q < .5$, and $a(q)$ will be convex for most of its range.

Wage distributions for individual occupations in particular metropolitan areas are available from the Bureau of Labor Statistics Area Wage Surveys. While AWS excludes small establishments, it is in other respects an ideal source of data for blue collar wages, offering both local-market detail and reasonably narrow occupations.

The AWS contains data for 30 separate plant occupations such as maintenance electrician, tool and die maker, boiler tender, tractor-truck driver, and shipping packer. Using the surveys for the 10 largest SMSAs studied by AWS in 1980, there were 274 city-occupations (30 times 10, minus 26 not reported due to too-small sample size). For each occupation, the wage rate at the 25th, 50th, and 75th percentiles of the wage distribution is available. If the wage distribution is symmetric, $W(75)-W(50)=W(50)-W(25)$, where $W(p)$ is the wage rate at the pth percentile. This implies that

$$S = \frac{W(75)+W(25)}{2\ W(50)}$$

is a simple measure of skewness, with $S=1$ if the distribution is symmetric. If $W(q)$ is ("predominantly") concave, S is less than one, while S is greater than one if $W(q)$ is ("predominantly") convex.

Of the 274 city-occupation observations, S was less than 1.0 in 140 -- i.e., in almost exactly half of the cases. Using an alternative measure -- the ratio of mean to median wage -- produced a nearly identical result (143 values below 1.0) Thus, there is no evidence that strong positive or negative skewness of within-occupation wage distributions is the rule.

If wage distributions are unimodal and more or less symmetric, so that a(q) switches from concave to convex near the median, the analysis of this section implies that standard-rate policies move firms toward the middle of the quality distribution. Firms initially hiring low quality workers face a locally concave a(q) function and raise q, while those hiring high quality workers face (locally) convex a(q) and reduce q.

3. Unions

While Freeman and Medoff (1984) have argued that the standard rate is an empirically important part of unionism, the preceding section does not really characterize union wage setting. Perhaps the most satisfactory simple characterization is that unions impose both a minimum wage constraint and a standard rate rule. Formally, the firm's problem is

$$\max_{W,H,L} \int_L^H [v(q)-W]\, dq \text{ subject to } H - L = r,\ a(H) = W, \text{ and } W \geq W_u.$$

There are two cases to consider, depending on whether the final constraint is binding.

(1) If the final constraint holds as an equality, the constraints determine the choice variables: $W = W_u$, $a(H) = W_u$ determines H, and $L = H-r$. The highest-quality worker is paid his alternative wage, but the standard-rate policy allows the remaining workers to earn more than their alternative wage. There may be quality upgrading ($H>H_o$) but it does not result in elimination of a wage premium for being unionized.

(2) If the constraint $W > W_u$ does not hold as an equality, the firm pays more than the union-mandated wage but maintains the standard rate policy. This case seems empirically unlikely but is not impossible. The model of section 2 would be appropriate here. Once again, workers whose quality is less than the highest receive more than their alternative wage.

Thus, to the question "why doesn't employers' workforce-upgrading in response to unionization eliminate any true wage differential?" an answer is "standard rate policies prevent it."

Compared with those working for flexible wage nonunion employers (who earn their alternative wage), union members earn a wage premium because all those with $q < H$ are earning $a(H)$ instead of $a(q)$. Union workers receive a standard rate $a(H)$ which in general exceeds their alternative wage $a(q)$ so that the average wage premium beyond wages available elsewhere is

$$M = a(H) - (1/r) \int_{H-r}^{H} a(q)\, dq.$$

To get a rough sense of the magnitudes involved, suppose a nonunion employer has a within-job wage spread of 20 percent[5] and that alternative wages are uniformly distributed ($a(q)$ is linear) in this region. A union establishment which ended up with the same quality distribution of workers would pay its best worker his alternative wage but its worst worker 20 percent more than he could get elsewhere; on average, this firm's workers would have $M = .10$, more or less in line with empirical estimates of the union wage effect.

When a union organizes many firms in the same labor market and negotiates the same standard rate at different firms, the effect may be more dramatic. If, for example, half the workers are organized and the wage is "taken out of competition" across firms, the least-qualified union worker would earn the same as someone fifty percentile points higher in the quality distribution. Notice that no reshuffling of workers across firms eliminates this (perhaps very substantial) differential. This suggests an alternative explanation for the proposition that union wage effects will be higher as the proportion organized is increased--increasing the share of the market which is organized and achieving rate standardization across firms increases the disparity of qualities of worker receiving the union wage, and hence the true differential.

A further implication of the model is a comparison of those working for nonunion standard-rate employers to those in the union sector. The expression for M presented above holds for each type of firm. The difference is that the union employer must pay a higher wage, and thus obtains higher q workers--i.e., M is evaluated at a greater H in the union firm. The consequent effect on M is given by

$$\frac{dM}{dH} = a'(H) - (1/r)\ [a(H) - a(H-r)]$$

As noted above, this expression is negative when a(q) is concave and positive when a(q) is convex. Since standard rates are more common among larger employers, who hire higher quality labor (Brown and Medoff, 1984), a concave-then-convex a(q) function would mean that workers at large unionized employers receive a premium compared to workers at large standard-rate nonunionized employers. However, the union-nonunion differential for large employers should be smaller than the union-nonunion differential for smaller employers (whose nonunion sector follows flexible wage rather than standard-rate policies). Most available evidence supports this prediction (Freeman and Medoff, 1984.)

4. Conclusions

The model presented in this paper shows how an employer who introduces a standard rate policy would alter the quality of labor employed, and it argues that standard-rating explains the apparent inability of employers to take full advantage of the upgrading opportunity which a union wage floor would seem to present.

Three possible directions for future work are to attempt a general-equilibrium analysis, to introduce imperfections in firms' knowledge of worker quality, and to make the size of the firm (i.e., the number of vacancies and/or the size of the available worker pool) endogenous in the long run. I would expect the very simple relationship between standard rating and labor quality would be complicated by these extensions; whether it would be altered in a fundamental way is less certain. The effect of standard rating on a unionized firm's attempt to upgrade would also be complicated by such extensions, but I would expect the fundamental point to survive.

I have begun to work on the third generalization, by allowing the firm to hire as many or as few workers as it finds profitable, while maintaining the same general structure to the problem in other respects.[6] I have not established a simple analytic relationship between the quality of labor hired in the flexible-rate and standard rate environments. However, for the case where a(q) and v(q) are both quadratic, a large number of numerical examples have failed to uncover a counterexample to the result in section 2, that standard rates increase the quality of labor hired if and only if a(q) is concave. (These examples are discussed in the Appendix.) Of course, the firm finds it profitable to hire fewer workers, so it increases L and reduces H, and profits are lower in the standard rate environment. If one is willing to assume fixed capital costs per worker, such costs can be accommodated in a(q) (and in W), and the model then characterizes long run behavior.

FOOTNOTES

1. For example, Pencavel (1981) gives three possible explanations: (1) studies which find a union wage premium have controlled inadequately for worker quality; (2) employers are not free to hire the best labor available at the union wage; (3) if the employer upgrades, unions will simply negotiate a wage which represents a premium over the new workers' opportunity wage. The first explanation is less convincing now that studies which compare particular workers who change union status are available. The second explanation applies to referral unions, but most unions (e.g. in manufacturing) are not referral unions. The third explanation requires that firms avoid hiring the best applicants for fear that attracting better applicants will stimulate further wage-raising. This seems implausible -- especially in the case of multi-employer contracts, where the individual employer would have an upgrading incentive even if the employer group did not. Mincer (1984, p. 324) argues that screening costs and "technological constraints" limit firms' ability to upgrade enough to eliminate the union wage premium.

2. This formulation assumes the firm opts for a single range of worker quality; it excludes, for example, hiring both low- and high-q workers but not offering high enough wages to in-between quality workers to attract them. The obvious motivation for this assumption is analytical convenience.

3. In this paper, I take the decision to pay standard rates as given. A more general model would explain why some firms not constrained by union contracts opt for standard-rate policies. One way of achieving

such generality is to postulate a fixed per-worker cost of administering a flexible-wage system, identified with administrative costs (or morale costs if workers prefer standard rates). Comparisons of flexible-wage and standard-rate firms presented below could then be interpreted as illustrating differences between firms whose wage policies differ because of different costs of a flexible-wage system. My reason for not putting greater emphasis on this approach is the belief that a really satisfactory model of why some firms choose standard-rate policies would require introducing costs of ascertaining worker quality (and effort), which greatly complicates the analysis. The most similar paper in the existing literature (Weiss and Landau, 1984) also takes the standard-rate decision as exogenous.

4. This conclusion does depend to some extent on the assumption that the pool of potential workers is given. An alternative assumption would be this pool P can be expanded by "advertising costs" C(P), and the firm hires r/P of the pool, where P is now a choice variable. (Measuring P so that C(1) = 0 leaves the model in the text as a special case.) In such a model, the effect of standard rates on average quality (or average alternative wage) is quite complicated. First, holding P fixed, increasing α changes H as described in the text. Second, holding H fixed, increasing α increases P. The intuition here is that increasing α increases the incentive to hire "more similar" workers, which can be done by expanding P. This increase in P raises L (since L = H-r/P), raising average quality. Third, the interaction between H and P is ambiguous in sign.

5. Buckley (1985) reports that the average within-establishment, within-job spread in nonunion establishments was 25% for the median blue-collar

occupation in his study. This overstates the true size of the spread to the extent that some of it reflects a pure-seniority differential for which all workers eventually qualify, and because it excludes non-union establishments paying all workers the same wage.

6. See also footnote 4 for a discussion of allowing the size of the available worker pool to vary, holding the number of workers to be hired constant.

REFERENCES

Brown, Charles, and Medoff, James. "The Employer Size Wage Effect," xeroxed, 1984.

Buckley, John. "Wage Differences Among Workers in the Same Job and Establishment," Monthly Labor Review, vol. 108, no. 3, March 1985, pp. 11-16.

Cox, John. "Time and Incentive Pay Practices in Urban Areas," Monthly Labor Review, vol. 94, no. 12, December 1971, pp. 53-56.

Freeman, Richard and Medoff, James. What Do Unions Do? (New York: Basic Books, 1984)

Mincer, Jacob. "The Economics of Wage Floors", in Ronald Ehrenberg, ed., Research in Labor Economics, vol. 6 (Greenwich CT: JAI Press, 1984), pp. 311-333.

Pencavel, John. Review of Labor Unions and the Inequality of Earned Income, by John Pettengill, Journal of Economic Literature vol. 18, no. 4, December 1981, pp. 1636-1637.

Pettengill, John. Labor Unions and the Inequality of Earned Income (New York: North Holland, 1980).

U.S. Bureau of Labor Statistics, Area Wage Survey, B.L.S. Bulletin No. 3000, 1980-81.

Weiss, Andrew and Landau, Henry. "Wages, Hiring Standards, and Firm Size," Journal of Labor Economics, vol. 2, no. 4, October 1984, pp. 477-499.

Appendix

This appendix summarizes numerical solutions to the firm's choice of labor quality problem when the firm has no constraint on the number of workers it must hire.

Both a(q) and v(q) are assumed to be quadratic functions:

$$a(q) = a_2q^2 + a_1q + a_0$$
$$v(q) = v_2q^2 + v_1q + v_0.$$

If we normalize v(q) so that v(0) = 0 and v(1) = 1, this implies $v_0 = 0$ and $v_2 = 1-v_1$. Since $v'(q) > 0$, $v'(0) = v_1 > 0$ and $v'(1) = 2v_2 + v_1 = 2-v_1 > 0$, so $v_1 < 2$. Similarly, $a'(0) = a_1 > 0$ and $a'(1) = 2a_2 + a_1 > 0$, so $a_2 > - a_1/2$.

Further restrictions on the a's can be derived from the requirement that L and H lie between zero and one in the flexible-wage equilibrium. The firm's problem is

$$\max_{H,L} \pi = \int_L^H [v(q) - a(q)]\, dq.$$

The first-order conditions are

$$v(H) - a(H) = 0$$
$$v(L) - a(L) = 0$$

This means that L and H are the two solutions to the quadratic equation v(q) - a(q) = 0. Letting $b_i = a_i-v_i$, i = 0,1,2, these solutions are

$$\frac{-b_1 \pm r}{2b_2}$$

where

$$r = (b_1^2 - 4b_0b_2)^{1/2}.$$

Assume for now that $b_2 > 0$. Then the two roots will be real and distinct if and only if $b_0 < b_1{}^2/4b_2$. Both roots are positive and distinct if and only if $b_1 < 0$ and $r < -b_1$, which in turn implies $b_o > 0$. Finally, the larger root is less than one if and only if $-b_1+r < 2b_2$, which can be manipulated to show $b_0 > -b_1-b_2$. Moreover, since $r \geq 0$, b_2 must be greater than $-b_1/2$.

The second-order condition for the problem is that

$$v'(H) - a'(H) < 0$$
$$v'(L) - a'(L) > 0$$

This means that $v'(L) - a'(L) - [v'(H) - a'(H)] = 2\ (a_2-v_2)\ (H-L) > 0$, or $a_2-v_2 = b_2 > 0$, as asserted earlier.

Collecting results, we have

$v_0 = 0$

$0 < v_1 < 2$

$v_2 = 1-v_1$

$0 < a_1 < v_1$ (The latter since $b_1 = a_1-v_1 < 0$)

$a_2 > v_2 + (v_1-a_1)/2.$ (Since $b_2 > -b_1/2$)

$a_0 < (a_1-v_1)^2/[4(a_2-v_2)]$

$a_0 > 0$ and $a_0 > (v_1-a_1) + (v_2-a_2)$ (either could be binding).

Let $N(v_1)$ be the number of values of v_1 to be considered. Evenly-spaced alternatives in the interval from zero to two were chosen:

$$v_1(i) = 2i/[N(v_1)+1].$$

For each $v_1(i)$, $N(a_1)$ values of a_1 were chosen, according to the rule

$$a_1(j) = v_1(i)/[N(a_1) + 1]$$

Given v_1 and a_1, $N(a_2)$ values of a_2 were chosen according to the rule

$$a_2(k) = 1-v_1 + (v_1-a_1)/\{2k/[N(a_2)+1]\}$$

Placing the index k in the denominator is a somewhat ad hoc response to the fact that no true upper bound on a_2 could be derived. Finally, given v_1, a_1, and a_2, $N(a_0)$ values of a_0 were chosen according to the rule

$$a_0(\ell) = \frac{\ell}{N(a_2)+1} \max\,(0,\ v_1-a_1 + v_2-a_2)$$

$$+ \left[1 - \frac{\ell}{N(a_2+1)}\right] \frac{(a_1-v_1)^2}{4(a_2-v_2)}$$

In addition to finding the solution to the firm's problem in the flexible-wage case, the solution to the standard-rate problem was also determined for each set of parameters. Here, the firm seeks to

$$\max_{H,L} \pi = \int_L^H [v(q)-a(H)]\, dq = \int_L^H v(q)dq - (H-L)a(H).$$

The first-order conditions for this problem are

$$v(H) - a(H) - (H-L)\, a'(H) = 0$$

$$-v(L) + a(H) = 0$$

Rewriting the second equation as

$$-[v(H)-a(H)] + v(H)-v(L) = 0$$

Adding this to the first equation gives

$$v(H)-v(L) - (H-L)\, a'(H) = 0$$

In the quadratic case, this is

$$v_2(H^2-L^2) + v_1\, (H-L) - (H-L)\, (2a_2H+a_1) = 0$$

or

$$(H-L)\, [v_2(H+L) + v_1 - (2a_2H+a_1)] = 0$$

Since $H-L > 0$, the term in brackets is zero, and this is a linear equation which can be solved for L:

$$L = -H + (2a_2H+a_1-v_1)/v_2$$

This can be substituted into the first first-order condition to obtain a quadratic equation in H alone.

In the first set of experiments, I chose five values for v_1, a_1, and a_2, and three values for a_0 according to the above rules. There were therefore $3 \cdot 5^3 = 375$ sets of parameters. When this produced very few negative values of a_2, the procedure was revised to allow nine values of v_1, running from 1.1 to 1.9, with the same number of values (but, obviously, different numerical values) of a_1, a_2, and a_0 as before. In no case did average quality (measured by either q or a(q)) rise when a_2 was positive or fall when a_2 was negative.