COUNTERPARTY RISK EXTERNALITY:
CENTRALIZED VERSUS OVER-THE-COUNTER MARKETS

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Counterparty Risk Externality: Centralized Versus Over-the-counter Markets
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ABSTRACT

We model the opacity of over-the-counter (OTC) markets in a setup where agents share risks, but have incentives to default and their financial positions are not mutually observable. We show that this setup results in excess "leverage" in that parties take on short OTC positions that lead to levels of default risk that are higher than Pareto-efficient ones. In particular, OTC markets feature a "counterparty risk externality" that we show can lead to ex-ante productive inefficiency. This externality is absent when trading is organized via a centralized clearing mechanism that provides transparency of trade positions, or a centralized counterparty (such as an exchange) that observes all trades and sets prices competitively. While collateral requirements and subordination of OTC positions in bankruptcy can ameliorate the counterparty risk externality, they are in general inadequate in addressing it fully.

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``To restrain private people, it may be said, from receiving in payment the promissory notes of a banker for any sum, whether great or small, when they themselves are willing to receive them; or, to restrain a banker from issuing such notes, when all his neighbours are willing to accept of them, is a manifest violation of that natural liberty, which it is the proper business of law not to infringe, but to support. Such regulations may, no doubt, be considered as in some respects a violation of natural liberty. But those exertions of the natural liberty of a few individuals, which might endanger the security of the whole society, are, and ought to be, restrained by the laws of all governments; of the most free, as well as of the most despotical. The obligation of building party walls, in order to prevent the communication of fire, is a violation of natural liberty, exactly of the same kind with the regulations of the banking trade which are here proposed." – Adam Smith, *The Wealth of Nations*, 1776.[1]

1   Introduction and motivation

An important risk that needs to be evaluated at the time of financial contracting is the risk that a counterparty will not fulfill its future obligations. This counterparty risk is difficult to evaluate because the exposure of the counterparty to various risks is generally not public information. Contractual terms such as prices and collateral that affect a trade can be tailored to mitigate counterparty risk, but the extent to which this can be achieved, and how efficiently so, depends in general on how contracts are traded.

One possible trading infrastructure is an over-the-counter (OTC) market in which each party trades with another, subject to a bankruptcy code that determines how counterparty defaults will be resolved.[2] A key feature of OTC markets is their *opacity*. In particular, even within a set of specific contracts, for example, credit default swaps (CDS), no trading party has full knowledge of positions of others. We show theoretically that such opacity of exposures

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[1]The paragraph cited appears on Vol. 1, p. 289, of the J.M. Dent & Sons Publisher edition, London, 1910.

[2]The contract may adhere to a uniformly applicable corporate bankruptcy code, or when the contract is exempt from the code, the bankruptcy outcome may be specified in the contract.

in OTC markets leads to an important risk spillover – *a counterparty risk externality*[3] – that leads to excessive "leverage" in the form of short positions that collect premium upfront but default ex post. Such excessive leverage results in inefficient levels of risk-sharing, deadweight costs of bankruptcy, and productive inefficiency.

Counterparty risk externality is the effect that the default risk on one contract will be increased if the counterparty agrees to the same contract with another agent because the second contract increases the probability that the counterparty will be unable to perform on the first one. Put simply, the default risk on one deal depends on *what else is being done.* The intuition for our result concerning the inefficiency of OTC markets is that in OTC markets it is not at all transparent what else is being done. Hence, counterparties cannot charge price schedules that effectively penalize the creation of counterparty risk. This makes it likely that excessively large short positions will be built by some institutions without other market participants being able to discourage them through pricing or risk controls for these institutions.

For example, in September 2008, it became known that A.I.G.'s liquidity position was inadequate given that it had written credit default swaps (bespoke CDS) for many investors guaranteeing protection against default on mortgage-backed products. Each investor realized that the value of A.I.G.'s protection was dramatically reduced on its individual guarantee. Investors demanded increased collateral – essentially posting of extra cash – which A.I.G. was unable to provide and the Treasury had to take over A.I.G. The counterparty risks were so widespread globally that a default would probably have spurred many other defaults, generating a downward spiral. The A.I.G. example illustrates the cost that large OTC exposures can impose on the system when a large institution defaults on its obligations. But, more importantly, it also raises the question of whether A.I.G.'s true risk as a counterparty was subject to adequate risk controls in protections they sold. We argue that the opacity of the OTC markets in which these credit derivatives traded was at least in part responsible for allowing the build-up of such large exposures in the first place.[4]

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[3]The term "counterparty risk externality" is as employed by Acharya and Engle (2009). A part of the discussion below, especially related to A.I.G. is also based on that article.

[4]Traditionally, in economics, we have considered the moral hazard problem of insurance as being with respect to the hidden action of the *insured* party. In this paper, and as the A.I.G. example illustrates, the problem is flipped and the moral hazard is with respect to the hidden action (trades, contracts, etc.) of the *insurer*.

A number of financial innovations in fixed income, foreign exchange, and credit markets have traded until now in OTC markets, the (gross) global notional outstanding of such derivatives being close to $500 trillion in December 2009, as per the Global Financial Stability Report of the IMF (April 2010). In contrast, many derivative products linked to commodity and equities have traded successfully on centralized trading platforms such as exchanges. A distinguishing feature of an exchange relative to OTC trading is that even though individual agents still do not see each others' trades, there is a centralized counterparty – the exchange – that sees all trades (at least on all products traded on that particular exchange). This enables the exchange to offer individual parties pricing schedules for trades (in practice, collateral arrangements and exposure limits) that are contingent not just on observable or public characteristics (e.g., credit ratings) but also on its own knowledge of other trades (e.g., net positions in futures contracts). However, exchanges are often viewed as detrimental to the ease of search facilitated by bilateral OTC markets, especially for customized or non-standardized financial products. Hence, as an alternative to intermediating trades on a centralized platform or through a centralized counterparty, a centralized clearing mechanism has been proposed that registers all trades in OTC markets and then serves as a data repository providing transparency of these trades.

We show formally that when trading is organized in the form of a centralized clearing mechanism, transparency can enable market participants to condition contract terms for each counterparty based on its overall positions. Such conditioning is sufficient to get that party to internalize the counterparty risk externality of its trades achieve the efficient risk-sharing outcome. In other words, the moral hazard that a party wants to take on excessive leverage through short positions – collect premiums today and default tomorrow – is counteracted by the fact that they face a steeper price schedule by so doing. We show that a competitive centralized exchange or a centralized counterparty also would induce efficient risk-sharing, but in practice, this would be at the cost of restricting all trades, including those involving non-standardized financial assets, through a single intermediary.

1.1 Model and results

We derive these results in a competitive two-period general equilibrium (GE) model which allows for the possibility of default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005). There is a

single financial asset, which can be interpreted as a contingent claim on future states of the world, and agents can take long or short positions in the asset. Trades are backed by agents' endowments. When an agent has short positions that cannot be met by the pledgeable fraction of endowment, there is default. Default results in deadweight costs which are borne by the short position and are increasing in the size of short positions, e.g., due to a greater number of parties to deal with in a bankruptcy proceeding. Such costs may arise also due to loss of customers or franchise value in fully dynamic setups. We do not model the structure of bankruptcy costs but simply postulate their pecuniary equivalent in reduced form.

The possibility of default (the option to exercise limited liability, to be precise) implies that long and short positions do not necessarily yield the same payoff and indeed that there is counterparty risk in trading. We assume a natural bankruptcy rule that illustrates why counterparty risk potentially arises in such a setting. In particular, in any given state of the world, the payoff to long positions is determined pro-rata based on delivery from short positions. This rationing of payments implies that each trade imposes a payoff externality on other trades. This spillover is precisely what we refer to as a *counterparty risk externality.*

In this setup, we consider various trading structures and ask whether they can internalize the counterparty risk externality or whether they lead instead to inefficient risk-sharing. One structure, a centralized clearing mechanism with transparency, guarantees that all trades are observable and agents can set pricing schedules that are conditional on this knowledge. Another structure, a centralized exchange or a centralized counterparty, observes all trades and can set pricing schedules based on this knowledge. In contrast, in economies with OTC market structure, trades are not mutually observed and thus pricing schedules faced by agents are not conditional on their other trades (even though they might be conditioned on public information about their type, e.g., their level of endowment).

Our first result is that *competitive equilibria in economies with a transparent centralized clearing mechanism or a centralized exchange are constrained Pareto efficient.* This is true even allowing for market incompleteness so that the result is not simply a consequence of welfare theorems in case of complete markets. Our second result is that *competitive equilibria in economies with OTC markets are robustly constrained inefficient.*[5]

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[5]We study two different cases, one in which OTC markets operate with a *bilateral*

The inefficiency in the OTC setting manifests as excessively large short positions as counterparties taking long positions do not internalize the default risk they impose on other long positions. Intuitively, as long as there is a "risk premium" on the underlying contract (e.g., because the risk being insured in the contract is aggregate in nature) and the costs of defaulting are not excessively large, the short position (the insurer) perceives a benefit from collecting premiums upfront and defaulting ex post. We interpret this outcome as characterizing excessive "leverage".[6] Formally, we capture the resulting inefficiency in the form of deadweight costs of bankruptcy. More generally, the inefficiency could manifest as excessive systemic risk due to spillover on to other counterparties. Furthermore, in an extension, we clarify that the inefficiency of OTC markets extends beyond just inefficient risk-sharing. When we allow agents to alter their production schedules, the ability to hedge the production risk through financial contracts creates additional demand for long positions and the incentives to build excessive leverage through short OTC positions markets translate into a production inefficiency.[7]

In another extension, we consider the role of bilateral collateral arrangements in addressing the counterparty risk externality. We show that since bilateral arrangements cannot be conditioned on information about all other trades of counterparties, in general, they do not deliver constrained efficiency of equilibrium outcomes. In particular, there are economies in which a sufficiently "tight" collateral arrangement can preclude any default by a counterparty. This level of default risk may or may not be optimal. But even

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netting mechanism and one without bilateral netting. We show that OTC markets are robustly constrained inefficient in both cases. In other words, the counterparty risk externality is orthogonal to what could be called the "netting externality" – that default decision of one party depends on the default decision of its counterparties. This makes it precise that it is the *opacity* or lack of transparency of positions in the OTC markets (rather than differences with centralized trading in how bankruptcy is resolved) that leads to ex-ante inefficiency.

[6]Interestingly, this implies a lower unit cost of insurance since the realized insurance payoff is smaller when the insurer is more likely to default.

[7]As an example, suppose that there is insurance being provided on economy-wide mortgage defaults. This would carry a significant hedging premium due to demand from mortgage lenders, giving rise to perverse insurer incentives to default. Thus, in equilibrium, the insurer would take on large and inadequately-collateralized short-selling (of protection) on pools of mortgages and the insured lenders would feed the excessive creation of the housing stock backing such mortgages. This may be a partial explanation of the role played by credit default swaps, sold in large quantities by A.I.G. on corporate loan and mortgage pools, in fueling the credit boom preceding the crisis of 2007-09.

when it is optimal, the required collateral can alter productive efficiency of the economy by resulting in over-investment in the collateral assets, or alternately, if the collateral asset is limited in quantity, ensuring no default can induce too little risk-sharing.

Finally, we examine whether subordinating OTC positions in bankruptcy relative to centrally cleared ones (when both OTC and centralized markets co-exist) can eliminate counterparty risk externality. We show that in general this is not the case. On the one hand, conditioned on default, the size of OTC positions does not affect the payoff on centrally cleared positions. This limits the externality from OTC positions to centrally cleared ones. On the other hand, counterparty risk externality in OTC markets remains unaddressed. As a result, there may now be default in states of the world where there would be none under centrally cleared markets. Due to deadweight costs of default, this lowers the payoff on centrally cleared positions.

The remainder of the paper is structured as follows. Section 2 provides a simple example of the counterparty risk externality in OTC markets. Section 3 presents the general model, the various trading structures (OTC and centralized clearing with transparency), and the welfare analysis of competitive equilibrium under these structures. Section 4 discusses extensions of the model. Section 5 discusses the relationship between the competitive equilibrium in our model and the market microstructure of OTC and centralized trading structures in practice. This section also considers the policy implications of our model for OTC versus centralized clearing. Section 6 relates our work to existing literature. Section 7 concludes. The analysis of trading with a centralized exchange or counterparty structure is in the Appendix, which also contains proofs.

2   Counterparty risk externality: An example

Consider a two-period ($t = 0, 1$) competitive economy with three types of agents ($i = 1, 2, 3$). There are two states of the world at $t = 1$, denoted by Good (G) and Bad (B). The probabilities of these states are p and $(1 - p)$, respectively. Agents' endowments in the two states are denoted as $w^i(s)$, $i = 1, 2, 3$, and $s = G, B$. Their initial endowments are denoted w_0^i. We assume that initial endowments are large enough that there are no default considerations at $t = 0$. For simplicity, we also assume that

$$w^1(G) > w^2(G) > w^3(G) = 0,$$

and

$$w^1(B) = w^2(B) = 0 < w^3(B).$$

In other words, agents of type 1 and type 2 have endowment in the good state of the economy, but none in the bad state; agents of type 3 are endowed in the bad state but not in the good state.

Agents of each type have a mean-variance utility function:

$$E[u(x_0, x(s)] = x_0 + E(x(s)) - \frac{\gamma}{2} var(x(s)),$$

where x_0 is the residual endowment at $t = 0$, and $x(s)$ is the realized endowment at $t = 1$, both taking account of trades that are structured at $t = 0$ and materialize at $t = 1$.

We assume that the only traded contract is an "insurance" that resembles a put option on the bad state of the economy. The contractual payoff of the contract is $R(G) = 0$ and $R(B) > 0$. For simplicity, we will refer to $R(B)$ simply as R. Importantly, the economy will allow for default so that the actual payoff on the contract in the bad state may be less than R. The insurance contract must be paid for at $t = 0$ and we denote its price as q.

To highlight our main point, we consider agents 1 and 2 purchasing insurance contract from agents 3.[8] We denote the long positions of agents 1 and 2 as $z^i \geq 0$, $i = 1, 2$, and the short position of agents 3 as $z^3 \geq 0$. Note that the only agents that can default given our assumptions are agents 3. We assume that in case they default, they suffer a linear non-pecuniary penalty as a function of the positions defaulted upon, whose pecuniary equivalent in the bad state is given by ϵz^3. Broadly speaking, this penalty can be interpreted as loss of continuation (or franchise) value in a multi-period setting. Hence, the equilibrium cash flow of agents 3 will be negative in the bad state if they default, reflecting this deadweight cost.

2.1 OTC markets

We consider the case of over-the-counter (OTC) trading: agents do not observe the size of the trades put on by other agents and hence prices cannot be conditioned on these. In other words, all agents take the price per unit of

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[8]It would suffice to simply consider in this example economy two types of agents. Nevertheless, for sake of clearer exposition of counterparty risk externality, we consider three types of agents.

insurance as a given constant (and not a schedule depending on total insurance sold by agents 3 in the economy). Agents are fully rational, however, and anticipate correctly the likelihood of default, and its consequent effect on the realized payoff on the insurance contract (R^+) relative to the promised payoff (R), with $R^+ \leq R$. Then, the $t = 0$ payoffs to the three agents are

$$(x_0^1, x_0^2, x_0^3) = (w_0^1 - z^1 q, w_0^2 - z^2 q, w_0^3 + z^3 q),$$

and $t = 1$ payoffs in good and bad states are given respectively as

$$[x^1(G), x^2(G), x^3(G)] = [w^1(G), w^2(G), w^3(G)], \text{ and}$$

$$[x^1(B), x^2(B), x^3(B)] = [R^+ z^1, R^+ z^2, w^3(B) - R^+ z^3 - \epsilon z^3 1_D],$$

where 1_D is an indicator variable which takes on the value of one if there is default $(R^+ < R)$ and zero otherwise. Equivalently, we will show below that $x^3(B) = \max(w^3(B) - Rz^3, -\epsilon z^3)$.

Then, equilibrium in the economy is characterized by the trading positions, the payoff on the insurance contract (involving the possibility of default), and the cost of insurance, denoted as (z^1, z^2, z^3, R^+, q), such that:

1. Each agent maximizes its expected utility by choosing its trade positions (as we describe below);

2. Market for insurance clears: $z^3 = z^1 + z^2$; and,

3. In case of default, (we assume that) there is pro-rata sharing of agents 3's total endowment between the long positions of agents 1 and 2:

$$R^+ = \begin{cases} \frac{w^3(B)}{z^1 + z^2} & \text{if } 1_D = 1 \\ R & \text{else} \end{cases}$$

Now, consider agent 1's maximization problem:

$$max_{z^1} \; w_0^1 - z^1 q + p w^1(G) + (1 - p) R^+ z^1 - \frac{\gamma}{2} var(x^1(s)),$$

where

$$var(x^1(s)) = p(1 - p)[w^1(G) - R^+ z^1]^2.$$

Then, the first-order condition for agent 1 implies that:

$$z^1(R^+, q) = \frac{1}{R^+} \left[w^1(G) - \frac{(q - (1 - p)R^+)}{\gamma p(1 - p)R^+} \right]. \tag{1}$$

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Similarly, we obtain for agent 2's long position that:

$$z^2(R^+, q) = \frac{1}{R^+}\left[w^2(G) - \frac{(q - (1-p)R^+)}{\gamma p(1-p)R^+}\right]. \qquad (2)$$

In other words, all else equal, agents 1 and 2 purchase more insurance if they have greater endowment in the good state and less so if the cost of insurance rises. The crucial observation is that even though the payoff R^+ is affected by each agent's long position in equilibrium, agents are competitive and do not internalize this effect. This is the source of counterparty risk externality in the model. In a GE model without default, R^+ is guaranteed to be R so that the externality would not arise.

Next, we will show that agents of type 3 have incentives to default in state B whenever the parameter governing the deadweight cost of default, ε, is not too high. To clarify agent 3's choice with regard to default, consider first the case in which it cannot default. In this case, its problem is

$$max_{z^3}\ w_0^3 + z^3 q + (1-p)[w^3(B) - Rz^3] - \frac{\gamma}{2}p(1-p)[w^3(B) - Rz^3]^2,$$

which yields

$$z_{ND}^3 = \frac{1}{R}\left[w^3(B) + \frac{(q - (1-p)R)}{\gamma p(1-p)R}\right]. \qquad (3)$$

In the limit where there are no default costs, that is, $\epsilon = 0$, agent 3 with position z_{ND}^3 will not default in equilibrium only if

$$w^3(B) \geq Rz_{ND}^3,$$

which turns out to be equivalent to requiring that $q \leq (1-p)R$. This condition has the intuitive interpretation that the insurer has incentives not to default ex post only if the price of insurance is smaller than or equal to the expected payoff on the insurance, or in other words, that there is no "risk premium" in the insurance price. This will, however, not hold in equilibrium in general, whenever the insurance is against a risk that is aggregate in nature and cannot be fully diversified away, e.g., if $w^1(G) + w^2(G) > w^3(B)$.[9]

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[9]A large component of default risk is driven by macroeconomic risks. This explains why there is the moral hazard of default on part of insurers selling credit default swaps (CDS): CDS effectively insure at least some portion of aggregate risk contained in the default risk of the underlying entity. In contrast, there is less risk of such a moral hazard on the part of insurers selling traditional insurance products such as policies on death, accidents, etc. These risks are easily diversified away across agents in the economy, so that insurers simply earn the actuarially fair premium, or in other words, do not earn a significant risk premium.

Consider then the problem of agent 3, the insurer, when we explicitly allow for default at proportional cost of default $\epsilon > 0$:

$$max_{z^3} \; w_0^3 + z^3 q - (1-p)\epsilon z^3 - \frac{\gamma}{2}p(1-p)(\epsilon z^3)^2.$$

Clearly, the insurer pledges the entire endowment in the bad state at $t = 1$ in order to collect as much insurance premium as possible at $t = 0$.[10] Thus, from the first-order condition, we obtain that

$$z^3 = \frac{q - (1-p)\epsilon}{\gamma p(1-p)\epsilon^2}. \tag{4}$$

Thus, the lower the cost of default ϵ and greater the price of insurance q, the greater is the quantity of insurance supplied by the insurers.

Substituting for (z^1, z^2, z^3) in the market-clearing and bankruptcy conditions of the equilibrium yields two equations in the realized insurance payoff R^+ and insurance price q which can be solved to characterize the equilibrium:

$$R^+(q) = \frac{w^3(B)\gamma p(1-p)\epsilon^2}{q - (1-p)\epsilon}, \tag{5}$$

$$w^3(B) = w^1(G) + w^2(G) + \frac{2}{\gamma p} - \frac{2q}{\gamma p(1-p)R^+}. \tag{6}$$

To get intuition, we define as "risk premium":

$$\Delta p = \frac{q}{R^+} - (1-p), \tag{7}$$

which is, the difference between the "risk-neutral" probability of state B and its actual or statistical probability. Then, solving the system in Δp and R^+ yields as the solution:

$$\Delta p = \frac{1}{2}\gamma p(1-p)\left[w^1(G) + w^2(G) - w^3(B)\right], \tag{8}$$

implying there is a risk premium whenever agents are risk-averse $(\gamma > 0)$, there is risk $(0 < p < 1)$, and this risk cannot be diversified away across

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[10]Note that the no-default condition now takes the form:

$$w^3(B) \geq (R - \epsilon)z^3.$$

agents $(w^1(G) + w^2(G) > w^3(B))$, and

$$R^+ = \frac{(1-p)\varepsilon + \sqrt{(1-p)^2\varepsilon^2 + 4w^3(B)\gamma p(1-p)\left[\Delta p + (1-p)\right]\varepsilon^2}}{2\left[\Delta p + (1-p)\right]} \qquad (9)$$

which is increasing in ε. In other words, the higher the bankruptcy costs, the lower is the equilibrium default rate on the contract. It follows then that the contract price $q = \left[\Delta p + (1-p)\right]R^+$ is also increasing in ε. In turn, there is default in equilibrium $(R^+ < R)$ if and only if bankruptcy costs are sufficiently small (ε smaller than some threshold level $\bar{\varepsilon}$).

2.2 Numerical example

We parametrize the above economy with $w^1(G) = 10$, $w^2(G) = 5$, and $w^3(B) = 10$ so that state B is aggregate risky in nature. We set $\gamma = 1$, $p = 0.9$ and vary ϵ in the range $[0.1, 1.0]$ (a subset of the entire possible range $\epsilon > 0$). Figures 1, 2 and 3 plot respectively the equilibrium quantity of insurance sold (z^3), its realized payoff (R^+), and its price (q), all as a function of ϵ, the proportional deadweight cost of default.

There is a critical value of ϵ below which defaults take place and this value is around 0.548. Above this value, there is no default. Interestingly, for all ϵ smaller than this threshold value, the equilibrium is effectively the same as far as risk-sharing is concerned: agents of type 3 transfer all their endowment in the bad state at $t = 1$ to agents 1 and 2. To be precise, the equilibrium utilities (relative to $t = 0$ endowments) are $(U^1, U^2, U^3) = (-1.97, -0.84, 1.35)$ regardless of ϵ in the default range. However, this is not true of the equilibrium quantity of insurance contracts sold and the unit price of insurance.

For example, when $\epsilon = 0.5$, the quantities traded are $(z^1, z^2) = (8.22, 2.74)$ with $z^3 = z^1 + z^2$; there is 9% default on the contract $(R^+ = 0.91)$; and, insurance price is $q = 0.30$. In turn, the risk premium Δp equals 0.23.

In contrast, with $\epsilon = 0.01$, the quantities traded become much larger: $(z^1, z^2) = (410.95, 136.98)$; there is 98% default on the contract $(R^+ = 0.02)$; and, insurance price is much lower at $q = 0.0067$.

To summarize, as the default incentives for agents of type 3 become stronger, there is greater quantity of insurance sold, greater default, and greater deadweight costs suffered by these agents. In turn, the equilibrium insurance price is smaller too. Since the payoff on the contract is rationally

anticipated by those purchasing insurance to be smaller: the quality of insurance has gone down given the insurer's default risk. Interestingly, there is no effect of default risk on the risk premium, which is constant and is given by equation (8).

2.3   Inefficiency of OTC markets

The inefficiency of equilibrium in the example above when $\varepsilon < 0.548$ stems from excessive deadweight costs of agent 3's bankruptcy. This can be seen in Figure 4 which plots the sum of utilities of all three agents and also separately of agents of type 3. Agents 1 and 2 enjoy the same equilibrium utility as ϵ varies. However, for $\epsilon < 0.548$, default leads to deadweight costs borne by agents of type 3 and their equilibrium utility is substantially lower compared to the case where $\epsilon \geq 0.548$. The result of counterparty risk externality is that there is too much demand for insurance in equilibrium, which gives insurers the incentive to default ex post, for which they pay ex ante.

It is clear then that in the example the planner can improve upon the OTC case when ϵ is smaller than 0.548. Essentially, the planner needs to enforce a "position limit" that restricts agents of type 3 from selling a quantity of insurance z^3 that is beyond their endowment in the bad state $w^3(B)$. One way in which this position limit can be implemented is through a non-linear pricing schedule: $q(z^3) = 0$ if $z^3 > w^3(B)$, and $q(z^3)$ determined by the markets otherwise. While in this example, it is efficient for insurance to be fully collateralized so that any default is ruled out in equilibrium, this is in general not true. What is however true, and we show below, is that the OTC markets always feature (weakly) greater likelihood of default in equilibrium compared to its (Pareto) efficient level.

3   The general model

We now build on the above example to construct a general model of an OTC market with default risk. In particular, we allow for an arbitrary number of agent types, with arbitrary structure of endowments, and the possibility of each of the agents taking long and short positions with each other (requiring us to also introduce some additional notation). Without much further complication, we also allow only a part of each agent's endowment to be pledgeable in honoring its short positions. Deadweight costs of bankruptcy

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could be interpreted as (partly) arising from loss of the non-pledgeable endowment. For sake of simplicity, we continue to restrict attention to a single financial contract. After completing the analysis of OTC markets, we consider financial markets with a centralized clearing mechanism that provides transparency and compare its equilibrium outcome with that under the OTC markets.

Formally, we extend the two-period General Equilibrium (GE) exchange economy with default (Geanakoplos, 1997, Geanakoplos and Zame, 1998, Dubey, Geanakoplos and Shubik, 2005) to allow for different mechanisms for financial market trading. In the interest of pedagogical clarity, we state the optimization programs in each setting fully even though some parts are common across different programs.

Agents and endowments   The economy is populated by $i = 1, ..., I$ types of agents. Let x_0^i be consumption of agent i at time 0. Let $s = 1, ..., S$ denote the states of uncertainty in the economy, which are realized at time 1. State s occurs with probability p_s, and $\sum_s p_s = 1$. Let x_1^i be agent i's consumption at time 1, a random variable over the state space S: $x_1^i(s)$, for $s \in S$. Let w_0^i be the endowment of agent i at time 0; and $w_1^i(s)$ her endowment at time 1 in state s . The utility of agent i over consumption in state s is denoted as $u^i(x_0^i, x_1^i(s))$ and belongs to the von-Neumann Morgenstern class of expected utility functions.

Financial markets and default   We assume, for simplicity, that only one financial asset is traded in this economy, an asset whose payoff is an exogenous non-negative S-dimensional vector R. We can imagine it representing a derivative contract, e.g., a credit default swap.

Agents selling the asset might default on their required payments. In particular, agent i's short positions are effectively backed by the pledgeable fraction α of her endowment at time 1. In other words, in the event of default, creditors (counterparties holding long positions on the asset with the defaulting party) have recourse only to a fraction $\alpha \in [0, 1]$ of agent i's endowment $w_1^i(s)$. Other than the defaulting agent simply losing her pledgeable endowment to counterparties, default is assumed to have a direct deadweight cost that is proportional to the size of the position defaulted upon. Deadweight costs of default will serve the formally convenient purpose of providing a bound on short positions on the asset.

Agents are assumed to trade bilaterally in financial markets. Even though one single asset is traded ex ante, the asset pay-off ex post depends on the type of the agent shorting it, as that agent's default decision also depends on the type. Let z_+^{ij} be long positions of agents of type i sold by agents of type j. Let $\mathbf{z}_+^i = \left(z_+^{ij} \right)_{j \in I}$ denote the long portfolio vector of agents of type i (with $z_+^{ii} = 0$, by construction). Let z_-^i be the short position of agents of type i. As we will explain shortly, all short positions are symmetric for the agents shorting the asset, independently of the counterparty, so that there is no need to index short positions of an agent by the counterparty. Then, in case of its default, agent i suffers a deadweight cost of default whose pecuniary equivalent is assumed to be εz_-^i, with $\varepsilon > 0$.

3.1 OTC markets

Consider first the case in which trading is intermediated in over-the-counter (OTC) markets. We model OTC markets as standard competitive markets with no centralized clearing or centralized counterparty (such as an exchange). We assume that no creditor has privileged recourse to a debtor's endowment in case of default. Nonetheless, a bankruptcy mechanism operates to distribute the cash flow delivered on the short positions (full cash flow or endowment recovered in case of default) pro-rata amongst the long positions. To be precise, consider an agent of type i shorting the asset. At equilibrium, the total repayment cash flow from an agent of type i is distributed pro-rata among the holders of long positions against counterparty i.[11]

The default condition   An agent of type i with (long, short) portfolio position $(\mathbf{z}_+^i, z_-^i)$ will default in period 1 in state s if and only if her income after assets have paid off is smaller than the non-pledgeable fraction of her endowment net of the bankruptcy costs. Let $R^j(s)$ denote the payoff in state s of agent i's long asset portfolio with counterparty $j \in I \backslash \{i\}$. The payoff $R^j(s)$ is taken as given by each agent, though it will be endogenously determined, depending on the equilibrium default rate of agents of type j in the economy.

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[11]Given the competitive nature of the model, the bankruptcy mechanism pools all repayments of *all* agents of type i and redistributes them pro-rata to all their counterparties. This is without loss of generality, as we concentrate on symmetric equilibria.

Consider an agent i with a net short position $z_-^i > 0$. She will default on her short position in state s iff:

$$w_1^i(s) + \sum_j R^j(s)z_+^{ij} - R(s)z_-^i < (1 - \alpha)\, w_1^i(s) - \varepsilon z_-^i\ . \tag{10}$$

Note that in general we allow for an agent to maintain at the same time both short and long positions on the asset: z_-^i and $z_+^{ij} > 0$, for some j. In other words, we assume that the clearing mechanism provided by OTC markets does not necessarily include *bilateral netting*. We shall study netting later on in the section. Let $I^d(\mathbf{z}_+, z_-; i, s)$ be an indicator variable taking on value 1 if agent i with position $(\mathbf{z}_+, z_-)$ will default at equilibrium in state s, and zero otherwise. Finally, let $I^{nd}(\mathbf{z}_+, z_-; i, s) = 1 - I^d(\mathbf{z}_+, z_-; i, s)$. Clearly, $I^d(\mathbf{z}_+, 0; i, s) = 0$.

Equilibrium payoffs on long and short positions: Since all long positions share pro-rata the payments from defaulting and non-defaulting short positions, the equilibrium payoff of the asset shorted by agent j, denoted $R^j(\mathbf{z}_+^j, z_-^j; s)$, is given by

$$R^j(\mathbf{z}_+^j, z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} & if\ I^d(\mathbf{z}_+^j, z_-^j; j, s) = 1 \\ R(s) \text{ otherwise} \end{cases} \tag{11}$$

where $(\mathbf{z}_+^j, z_-^j)$ is the portfolio of agents of type j at equilibrium.

Opacity In OTC markets, there is no centralized clearing and transparency, nor any centralized counterparty that sees all trades. Thus, the trades of each agent i, $(\mathbf{z}_+^i, z_-^i)$, are *not* observed in OTC markets by other agents.

Prices and budget constraints Long and short bilateral positions will in general be traded at a unitary price q^j, where the apex j denotes the type of the agent in the short position. Note that the price depends on the short agent's type j, as the type determines the agent's endowment which is public knowledge and affects her probability of default. Importantly though, the price is not a schedule contingent on overall trades of agent j, that is, does *not* depend on her portfolio, since it is not observed.

The budget constraints of agent i in the OTC market are thus given by:

$$x_0^i + \sum_j q^j z_+^{ij} - q^i z_-^i = w_0^i,$$
$$x_1^i(s) = \max\left\{ w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i, (1-\alpha) w_1^i(s) - \varepsilon z_-^i \right\} \quad (12)$$

where $z_+^{ij}, z_-^i \geq 0$, for any j.

Competitive equilibrium   In equilibrium, financial markets clear:

$$\sum_i z_+^{ij} - z_-^j = 0, \text{ for any } j. \quad (13)$$

Furthermore, the equilibrium payoffs $R^j(s)$ satisfy the condition:

$$R^j(s) = R^j(\mathbf{z}_+^j, z_-^j; s) \quad (14)$$

Let

$$m^i(s) = MRS^i(s) \equiv p_s \frac{\frac{\partial u^i\left(x_0^i, x_1^i(s)\right)}{\partial x_1^i}}{\frac{\partial u^i\left(x_0^i, x_1^i(s)\right)}{\partial x_0^i}} \quad (15)$$

denote the marginal rate of substitution between date 0 and state s at date 1 for agents of type i at equilibrium; that is, the *stochastic discount factor* of agents of type i. The equilibrium price of an asset is then simply equal to the discounted value of asset payoffs, where the discount rate is adjusted for risk according to the stochastic discount factor of any agent with a long position in the asset. More precisely, agents with a long position in the asset are those who have the highest marginal valuation for the asset's return, and hence at equilibrium, prices q^j satisfy:

$$q^j = \max_i E\left(m^i R^j\right), \text{ for any } j.^{12} \quad (16)$$

3.2   OTC markets with netting

In the OTC markets modeled in the previous section, an agent i is allowed to go both short and long on the asset, and in equilibrium it might be that

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[12]Alternatively, but equivalently, the equilibrium price for any j can be written as follows: $q^j = E\left(m^i R^j\right)$, for any i s.t. $z_+^{ij} > 0$ and $q^j = \max_i E\left(m^i R^j\right)$, if $z_+^{ij} = 0$ for any i.

$z^i_- > 0$ and, at the same time, $z^{ij}_+ > 0$ with some counterparty j. In this context, an ex-post mechanism for state-by-state bilateral netting might have welfare consequences. Hence, we also consider an economy with *OTC markets and netting* so as to be able to better distinguish the welfare effects of various distinct components of OTC and centralized market clearing mechanisms.

We model *bilateral netting* by requiring that agents are (without loss of generality) on only one side of the market, that is, for an agent of type i:

$$z^{ij}_+ z^i_- = 0, \text{ for any } j. \tag{17}$$

As a consequence, an agent of type i with a short position $z^i_- > 0$ will default in state s iff:

$$w^i_1(s) - R(s)z^i_- < (1 - \alpha) w^i_1(s) - \varepsilon z^i_- . \tag{18}$$

Therefore, with bilateral netting, the default decision of any agent i is independent of $\mathbf{z}^i_+$, which is constrained to be equal to 0 whenever $z^i_- > 0$. Let the default indicator of agents of type i be now denoted as $I^d(z_-; i, s)$, taking on value 1 if agent i with short position z_- will default at equilibrium in state s, and zero otherwise. Agent j's short position payoffs are now written as

$$R^j(z^j_-; s) = \begin{cases} \frac{\alpha w^j_1(s)}{z^j_-} & if \ I^d(z^j_-; j, s) = 1 \\ R(s) \text{ otherwise} \end{cases} \tag{19}$$

and budget constraints of agents i are restricted by $z^{ij}_+ z^i_- = 0$, for any j. Finally, at a competitive equilibrium of an economy with OTC markets and netting, financial markets clear, the consistency condition $R^j(s) = R^j(z^j_-; s)$ is satisfied, and equilibrium prices satisfy

$$q^j = \max_i E\left(m^i R^j\right), \text{ for any } j. \tag{20}$$

3.3 Centralized clearing

In the previous section, we formalized the competitive equilibrium of an economy in which financial market trades are intermediated by an OTC market. In this section we model instead the operation of a centralized clearing mechanism. We model centralized clearing mechanisms as being composed of two fundamental functions: *bilateral netting* and *transparency*. Transparency is obtained because a centralized clearing mechanism is assumed to aggregate

all the information about trades and disseminate it to market participants. Two points are in order before we proceed. One, in the model, transparency provided by centralized clearing mechanism is coincident with submission and execution of trades. Our equilibrium setup cannot deal with the timing or market micro-structure issues associated with *when* trades are submitted and *when* they are made transparent. We discuss this issue in some detail in Section 5.2. Second, we stress that a centralized clearing mechanism need not centrally intermediate the trades as, for instance, a centralized exchange would do (which we discuss later).

Regarding bankruptcy resolution, we continue to assume that no creditor has direct privileged recourse to a debtor's collateral in case of default; and that, at equilibrium, the sum total of cash flows received by the debtor is distributed pro-rata among the holders of long positions against the debtor. Because of *bilateral netting*, an agent i with a short position $z_-^i > 0$ will default in state s iff

$$w_1^i(s) - R(s)z_-^i < (1 - \alpha) w_1^i(s) - \varepsilon z_-^i; \qquad (21)$$

and the equilibrium payoff of the asset shorted by agent j is thus given by

$$R^j(z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} \; if \; I^d(z_-^j; j, s) = 1 \\ R(s) \; \text{otherwise} \end{cases}. \qquad (22)$$

Because of *transparency*, each agent in the economy has access to detailed information about all trades and can condition contract terms on this information. We assume that prices are set in a competitive manner. Specifically, agents are price-takers. However, the payoff on the short position of agent j depends on the position itself, z_-^j, and prices will in general reflect such dependence. Different agents will face different prices, reflecting the probability of default implied by their characteristics: their type (e.g., level of endowment) as well as their trading positions. This requires us to modify the price-taking assumption for short positions in an important manner (that is similar in spirit to modifications in Acharya and Bisin, 2008, and Bisin, Gottardi and Ruta, 2009).

Specifically, an agent of type j with short position z_-^j will face an ask price map

$$q^j(z_-^j) = \max_i E\left(m^i R^j(z_-^j)\right). \qquad (23)$$

That is, an agent of type j understands that the price it will face for a short position depends on the total short positions it sells, z_-^j. Furthermore, an

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agent of type j understands that the price it will face for a short position will reflect a risk adjustment according to the stochastic discount factor of the agents who would hold such a short position, that is, of those agents who share the highest marginal valuation for the payoff associated to its position, $R^j(z^j_-)$. Price taking is then represented by the fact that agents take the vector of stochastic discount factors $\left(m^1, \ldots, m^i, \ldots, m^I\right)$ as given.[13] On the other hand, regarding long positions, the payoff $R^j(s)$ is taken as given by each agent, and so is the price q^j.

The budget constraints of agent i are thus given by:

$$
\begin{aligned}
&x_0^i + \sum_j q^j z_+^{ij} - q^i(z_-^i)z_-^i = w_0^i, \\
&x_1^i(s) = \max\left\{ w_1^i(s) + \sum_j R^j(s)z_+^{ij} - R(s)z_-^i, (1-\alpha)\, w_1^i(s) - \varepsilon z_-^i \right\}
\end{aligned}
\tag{24}
$$

where $z_+^{ij}, z_-^i \geq 0, z_+^{ij}z_-^i = 0,$ for any j.

At competitive equilibrium, all markets clear:

$$
\sum_i z_+^{ij} - z_-^j = 0, \text{ for any } j,
\tag{25}
$$

and the price maps and returns are rationally anticipated by agents:

$$
\begin{aligned}
q^j &= q^j(z_-^j) = \max_i E\left(m^i R^j(z_-^j)\right), &\tag{26} \\
R^j(s) &= R^j(z_-^j; s)\,. &\tag{27}
\end{aligned}
$$

3.4 Welfare

How does the competitive equilibrium under OTC markets compare in terms of efficiency properties to the competitive equilibrium under centralized clearing with transparency? To answer this question, we write down the constrained Pareto efficient outcome as the solution to the following problem:

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[13]Our definition of competitive price maps can be thought of as capturing the same consistency condition required by Perfect Nash equilibrium in strategic environments: every agent understands that the ask price she will face for any (possibly out-of-equilibrium) short position z^j_- will depend on the willingness to pay of agents on the long side of the market. In a competitive equilibrium, however, all deviations from equilibrium are necessarily "small," and hence such willingness to pay coincides with the highest marginal valuation at equilibrium.

$$\max_{(x_0^i, x_1^i, z_+^{ij}, z_-^i)_{i,j}} \quad \sum_i \lambda^i E\left(u^i(x_0^i, x_1^i)\right)$$

$$s.t.$$

$$\sum_i x_0^i - w_0^i \;=\; 0,$$

$$\sum_i x_1^i(s) - w_1^i(s) \;=\; 0, \text{ for any } s$$

$$x_1^i(s) \;=\; \max\left\{ w_1^i(s) + \sum_j R^j(s) z_+^{ij} - R(s) z_-^i, (1-\alpha)\, w_1^i(s) - \varepsilon z_-^i \right\}, \tag{28}$$

$$R^j(z_-^j; s) \;=\; \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} & \text{if } I^d(\mathbf{z}_+^j, z_-^j; j, s) = 1 \\ R(s) & \text{otherwise} \end{cases} \tag{29}$$

and where $z_+^{ij}, z_-^i \geq 0$, and λ^i is the Pareto weight associated to agents of type i.

This is the standard constrained efficiency problem for a GE economy once it is assumed that default is not controlled by the planner. The constraint (28) serves two purposes: (i) it restricts the planner's allocations to those that can be achieved with the limited financial instruments available in the economy; and (ii) it accounts for the fact that each agent can choose to default or not, in each state s: consumption in default state s is $(1-\alpha)\, w_1^i(s) - \varepsilon z_-^i$, the non-pleadgeable fraction of endowment net of the deadweight costs.[14]

3.5 Results

We can derive the following results on the constrained efficiency of the economy with centralized clearing and transparency, in contrast to the (generic) constrained inefficiency of the economy with OTC markets.

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[14]Formally, the constraint includes the incentive compatibility constraint for each agent's choice of default:

$$u^i(x_0^i, x_1^i(s)) \geq u^i(x_0^i, (1-\alpha)\, w_1^i(s) - \varepsilon z_-^i). \tag{30}$$

Proposition 1. *Any competitive equilibrium of an economy with a centralized clearing mechanism is constrained Pareto optimal.*

The intuition for efficiency of the economy with centralized clearing and transparency is that each agent j that is short on the asset faces a price $q^j(z^j_-) = \max_i E\left(m^i R^j(z^j_-)\right)$ that is conditioned on her positions. Consequently, she internalizes the effect of her default on the payoff of long positions on the asset. The observability of all trades allows for such conditioning of prices and internalization of any externality that trading and default choices impose on other agents. Importantly, note that an *economy with a centralized clearing mechanism,* as we have defined it, is characterized by both a *bilateral netting mechanism* and *transparency.* Both these components are needed for efficiency.

We can show that the opacity of OTC markets induces inefficiencies through the counterparty risk externality, independently of the netting mechanism in place. Conversely, it can be shown that the lack of a bilateral netting mechanism is associated with an externality that induces inefficiencies in equilibrium even when transparency is guaranteed.

First of all, consider an economy with OTC markets *without bilateral netting.* As we noted, in this case, an agent of type i with a short position $z^i_- > 0$ will default in state s iff:

$$w^i_1(s) + \sum_j R^j(s)z^{ij}_+ - R(s)z^i_- < (1-\alpha)\, w^i_1(s) - \varepsilon z^i_- . \qquad (31)$$

The default decision of an agent of type i, therefore, depends on $R^j(s)$, that is, on agents j's default decisions, which in turn depend on i's default decisions, introducing a *netting externality* at equilibrium. Then, it is the case that:

Competitive equilibria of economies with OTC markets without netting are robustly not Pareto efficient.

The proof of this statement, however, requires some complex differential computations and is omitted. It is an adaptation of that in Bisin, Geanakoplos, Gottardi, Minelli, and Polemarchakis (2001).

Next, in OTC markets with bilateral netting, an agent i with a short position $z^i_- > 0$ will default in state s iff:

$$w^i_1(s) - R(s)z^i_- < (1-\alpha)\, w^i_1(s) - \varepsilon z^i_- . \qquad (32)$$

No netting externality arises in this case. Nevertheless, we shall show that equilibria of an economy with *OTC markets and netting* are also typically constrained inefficient. In other words, the transparency provided by centralized clearing mechanism (but not provided by OTC market economies, with or without netting) is necessary for constrained efficiency.

Proposition 2. *Competitive equilibria of economies with a centralized clearing mechanism cannot be robustly supported as equilibria in economies with OTC markets, with or without netting.[15] More specifically, any competitive equilibrium of the economy with centralized clearing mechanism in which default occurs with positive probability cannot be supported in the economy with OTC markets, with or without netting.*

The intuition is that in OTC markets, with or without netting, each agent j that is short on the asset faces a price q^j that is *not* conditioned on her position z_-^j. Consequently, she does not internalize the effect of her default on the payoff of long positions on the asset. This is a *counterparty risk externality* . More generally, when the *counterparty risk externality* interacts with the *netting externality*, it is also the case that:

Competitive equilibria of economies with OTC markets and netting are robustly not constrained Pareto efficient.[16]

Finally, let the leverage of agent j, L^j, be defined as the value of her short positions' contractual payoff (promised debt payment) divided by the value of her endowment (asset value).

$$L^j \equiv \frac{E(m^j R z_-^j)}{E(m^j w_1^j)}. \tag{33}$$

Then,

Proposition 3. *For deadweight costs ε that are small enough, competitive equilibria of economies with OTC markets, with or without netting, are characterized by weakly greater (and robustly by strictly greater) leverage and*

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[15]Formally, by robustly we mean: *for an open set of economies parametrized by agents' endowments and preferences.*

[16]Once again, we omit the proof of this statement to avoid some complex differential computations.

default risk compared to equilibria of the same economy with a centralized clearing mechanism.

Since ask prices in economies with OTC markets, with or without netting, do not penalize the short positions for their own incentives to default, agents have incentives to exceed the Pareto efficient short positions. Indeed, the proof of these main propositions in the Appendix shows that as long as (i) the underlying asset has some aggregate risk, its price will robustly carry a risk premium that is positive (as explained in the example economy of Section 2), and (ii) bankruptcy costs are not too high (ε is small), then agents with endowments in the aggregate risky states have an incentive to go excessively short. This increases the equilibrium default rate and leads to inefficient risk-sharing.[17] For efficient risk-sharing, it is in general necessary to be able to commit to future payoffs on financial assets, but in OTC markets, such commitment cannot be ensured through prices.

Opacity and counterparty risk externality   When combined together, Propositions 1, 2, and 3 imply that a centralized clearing mechanism with transparency is an efficient response to counterparty risk externality. Our analysis, especially in Propositions 2 and 3, makes it precise that it is the *opacity* or lack of transparency of the OTC markets that leads to ex ante inefficiency in terms of excessively large short positions or leverage. In equilibrium, agents anticipate the lowering of payoff on long positions due to counterparty risk and the price of insurance falls. However, this is not sufficient to preclude the insurers from selling large quantities of insurance and defaulting ex post, as the risk premiums they earn (which depend on the ratio of price to the payoff) remain unaffected.

Centralized exchange economy   In the Appendix, we study an economy in which all asset trades are operated by a *competitive centralized exchange.* In essence, the exchange is a centralized counterparty that observes all trades and conditions contract terms for individual agents on these trades. In practice, this could be thought of as capturing a setting with a specialist that sees all trades and sets price schedules, or an exchange that sees all trades and

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[17]If $\varepsilon = 0$, z_-^i is unbounded and, strictly speaking, the economy has no equilibrium. This is just an extreme case, which is of interest to identify the "force" towards borrowing and default built into our model of opaque OTC markets. Positive deadweight costs, $\varepsilon > 0$, guarantee the existence of equilibrium.

imposes exposure limits on traders based on their overall positions. It can be shown that the competitive equilibrium allocations of economies with such a centralized exchange coincide with those of economies with a centralized clearing mechanism. Therefore, by Proposition 1, competitive equilibrium allocations of economies with a centralized exchange are constrained efficient.

4 Extensions

4.1 Collateral constraints

We have not yet analyzed the welfare properties of a commonly employed risk control and policy instrument, namely bilateral collateral constraints.[18]

Consider our example OTC economy of Section 2 with bilateral netting in which selling one unit of the asset short requires posting k units of the date-0 commodity as "collateral" to the counterparty. We assume that, when posted as collateral, one unit of the date-0 commodity pays an exogenous constant return r. To start with, we will assume r is equal to one. The collateral is "segregated" for each counterparty in that it has privileged access to its collateral in case of default on the contract.

Then, agents of type 3 do not default in state B provided

$$w^3(B) + kz^3 - Rz^3 \geq -\varepsilon z^3 \, ,$$

which can be expressed as

$$kz^3 \geq Rz^3 - \varepsilon z^3 - w^3(B) \, , \tag{34}$$

a condition that provides a lower bound on the required collateral constraint to deter default. However, not all collateral constraints are feasible for posting by agents of type 3 at date 0. This date-0 budget constraint is

$$w_0^3 + qz^3 \geq kz^3 \, , \tag{35}$$

which yields an upper bound on the feasible collateral constraint.

Since in our example economy, efficiency is achieved when there is no default and $Rz_{ND}^3 = w^3(B)$, efficiency can be attained with a collateral

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constraint if and only if the lower bound above is smaller than available resources for $z = z_{ND}^3$:

$$(R - \varepsilon)z_{ND}^3 - w^3(B) \leq w_0^3 + q_{ND}z_{ND}^3, \qquad (36)$$

where q_{ND} is the equilibrium price of insurance absent any default.

It follows then that in this example, collateral constraints can in general achieve efficiency if and only if ε, the deadweight cost of bankruptcy for the insurer, is not too small. Intuitively, the incentive to default is rather strong when ε is small, so that counteracting it requires the insurer to post high levels of collateral; such high levels might, however, not be feasible given insurer's limited endowment.

Put another way, collateral adds to the insurer's cost of default since it is seized by the counterparty in case of insurer's default, but it is released for the insurer otherwise. Thus, collateral increases the insurer's liability from default. The extent of such increased liability is limited by insurer's ability to post collateral in the first place, in other words by the starting endowment.

Indeed, when ε is too small, collateral constraint k that rules out default needs to be so large that it restricts z^3, as given by $z^3 \leq \frac{w_0^3}{(k-q)}$, to a level that is smaller than z_{ND}^3, limiting the extent of risk-sharing in the economy to below efficient levels (even though insurer's default is averted). The supply of hedging by the insurer, z^3, is now decreasing in the extent of collateral constraint, k, whereas the price of insurance, q, is rising in k (but at a rate that is smaller than one). The insurers are rendered funding-constrained in a bid to avoid their default but this restricts equilibrium provision of insurance to inefficient levels.

In the general economy with collateral constraint k, an agent of type i with a short position $z_-^i > 0$ will default in state s iff:

$$w_1^i(s) - R(s)z_-^i + kz_-^i < (1 - \alpha)\,w_1^i(s) - \varepsilon z_-^i\ ,$$

that is,

$$w_1^i(s) - R(s)z_-^i < (1 - \alpha)\,w_1^i(s) - (\varepsilon + k)\,z_-^i\ , \qquad (37)$$

confirming that the collateral constraints affect the default choice analogously to how the bankruptcy cost ε does. In the example economy we just studied, efficiency was associated with no default, and we saw that collateral constraints can induce the optimal no-default pattern, provided ε is sufficiently large and the level of constraint k is chosen appropriately as a policy variable.

More generally, however, controlling k, or even a type-dependent collateral constraint k^i, is *not* enough to induce optimal default.

To see this, recall that efficiency requires in general that prices for short sales of the asset be of the form $q^i(z^i_-)$, so that at equilibrium $q^i(z^i_-) = \max_j E\left(m^j R^i(z^i_-)\right)$. It follows then from (37) that efficiency requires collateral constraints that depend on the shorting agent type i and on her position z^i_-. But collateral constraints of the form $k^i(z^i_-)$ require the observability of z^i_-, that is, a centralized clearing mechanism on the part of the regulator imposing the constraints, or the transparency of overall short positions to counterparties setting the constraints bilaterally. By implication, collateral constraints do not suffice in OTC markets to achieve efficiency of allocations.

Next, as our example illustrated, a bilateral collateral constraint in the OTC market is limited in its effectiveness when it requires the insurer to hold large quantities of collateral asset. We argue now that even in the case where holding such large quantities of collateral asset is feasible, it might not in general be possible to obtain efficient allocations if the return on the collateral asset r is not adequately large. The key observation is that collateral constraints can now impose a mis-allocation cost on the economy as some agents are required to hold sub-optimal asset portfolios, specifically, a position in an asset that induces excessive consumption at date 1 for those agents who are shorting the asset.

Formally, let $z^{i*}_- > 0$ denote the efficient portfolio allocation of an agent i shorting the asset, and $(x^{i*}_0, x^{i*}_1(s))$ her consumption allocations. Let also k^* denote the (minimal) collateral constraint which guarantees that agent i, has no incentive to default in the collateral constraint economy, when she holds the optimal portfolio z^{i*}_-. The budget constraints of agent i, in the collateral constraint economy k^*, are

$$x^i_0 - (q - k^*)z^i_- = w^i_0 \,,$$
$$x^i_1(s) = \max\left\{w^i_1(s) - R(s)z^i_- + rk^*z^i, (1 - \alpha)w^i_1(s)\right\}.$$

It is clear then that any optimal allocation such that $z^{i*}_- > 0$ (and hence $x^{i*}_0 > w^i_0$) can be decentralized with collateral constraints only if at equilibrium $q > k^*$, which does not necessarily have to hold. If instead $q \leq k^*$ the agent is constrained to consume an amount smaller or equal to her endowment at date 0.[19] Consider the case in which at equilibrium $q > k^*$. In

_____

[19]When $q < k^*$, the optimal portfolio might even be infeasible for the agent; that is, $(k^* - q)z^{i*}_- > w^i_0$.

this case, any allocation $x_0^{i*} > w_0^i$ can in fact be decentralized with collateral constraints, by choosing

$$z_-^i = \frac{q}{q-k} z_-^{i*} \ .$$

However, consumption at date 1 is now not optimal, unless $r = \frac{R(s)}{q}$. If $r < \frac{R(s)}{q}$, for all s, the collateral constraint is costly in terms of efficiency in that it requires agent i to hold an asset whose return is dominated. Note that if the collateral storage technology is *not* dominated, that is, if $\frac{R(s)}{q} < 1$, for some s, then the storage technology is a new asset in the economy and welfare comparisons are not meaningful, unless we introduce storage also in the baseline economy.

To summarize, bilateral collateral constraints by themselves cannot generally restore efficiency in OTC markets, without the transparency guaranteed by a centralized clearing mechanism:

Proposition 4. *Competitive equilibria of economies with a centralized clearing mechanism cannot be robustly supported as equilibria in economies with OTC markets and collateral constraints, not even with type-dependent collateral constraints.*

4.2 Bankruptcy design

We now analyze environments in which a trasparent clearing mechanism co-exists with OTC markets. In practice, trading mechanisms that require a centralized clearing for all trades to guarantee the necessary transparency are in some cases considered demanding. OTC markets might represent an effective outlay to trade non-standardized financial products. OTC markets might also exist as a form of "regulatory arbitrage" to seek leverage outside of centralized clearing markets.

In this case, our analysis suggests that to address excessive leverage and the counterparty risk externality of OTC markets, a regulatory mechanism penalizing access into OTC markets might be necessary. One such regulatory mechanism is a bankruptcy rule imposing seniority of centrally cleared positions over OTC positions. Such subordination has in fact been proposed as a possible regulatory tool in discussions at the International Monetary Fund and Financial Stability Board (Basel) for containing contingent risks linked to derivatives. It would seem that with such subordination, junior OTC positions would not dilute the senior centrally cleared positions, for

which counterparties would face appropriate incentives and risk controls.[20] In the context of our example, for instance, it is easy to show that a junior OTC market alongside a centralized clearing markets would remain inactive at equilibrium, as all hedging opportunities are efficiently exhausted by the centralized clearing market. This is however not the case in general.

Formally, consider our general economy with a centralized clearing mechanism. An agent i can trade a long position of an asset with nominal payoff $R(s)$ in state s, with counterparty j at price q^j; agent j, in turn, faces a price schedule $q(z_-^j)$ for observable short position z_-^j on the asset. Suppose now that the same asset can also be shorted by agent j in OTC markets, and let $z_-^{j,OTC}$ denote such a short position. The price of the short position in OTC markets will depend on the agent's trading position in the centralized mechanism, which is transparently observable, but not on her position in OTC markets: we shall denote the price of the short position in OTC markets $q^{j,OTC}(z_-^j)$.

An agent j with short positions $\left(z_-^j, z_-^{j,OTC}\right)$, respectively in the centralized clearing mechanism and in OTC markets, will default in state s if and only if

$$w_1^j(s) - R(s)\left(z_-^j + z_-^{j,OTC}\right) < (1-\alpha)\,w_1^j(s) - \varepsilon\left(z_-^j + z_-^{j,OTC}\right). \qquad (38)$$

The default decision of agent j, therefore, will depend on her total position in centralized and OTC markets, $z_-^j + z_-^{j,OTC}$. Of course, at equilibrium, agents trading long positions in OTC markets will take into account of their counterparties' incentives to default. As a consequence, the equilibrium price in OTC markets will account for the equilibrium default rate of short positions. As in our model with only OTC markets, not surprisingly, this is cause for inefficiency: a counterparty risk externality exists within the OTC market. It is important, however, to understand whether this externality extends to the centralized clearing markets. That is, does an OTC market alongside a centralized clearing market have a negative externality on an otherwise efficient market mechanism, even if bankruptcy law guarantees the seniority of trades in the centralized clearing market?

We answer this question in the affirmative. To illustrate this answer, consider an economy with no OTC markets (that is, with only a centralized

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[20]Similar mechanisms are common in civil law countries, in the form of seniority rules favoring (transparent) notarized transactions over (opaque) bilateral ones. Thanks to Sabino Patruno for pointing this out.

clearing market) in which an agent j who shorts the asset, at equilibrium, does not default in state s. Consider the case, in particular, in which at equilibrium agent j shorts the asset as much as possible, without defaulting. Her position z_-^j is therefore determined by the default condition, that is, it is the short position which makes her indifferent with regards to defaulting:

$$w_1^j(s) - R(s)z_-^j = (1 - \alpha)\, w_1^j(s) - \varepsilon z_-^j.^{21} \tag{39}$$

The return in state s for agents with long positions against agent j is then $R(s)$, if j does not default, and $\frac{\alpha w_1^j(s)}{z_-^j} = R(s) - \varepsilon$, if agent j defaults. Any larger short position of agent j, than that implied by equation (39), would induce her to default.

Suppose now that we allow agents to trade in OTC markets also and that agent j indeed trades there to increase her short position. In this case, if the centralized clearing market is senior in bankruptcy, then whenever agent j defaults; her counterparties in the centralized clearing market will obtain returns equal to $R(s) - \varepsilon$, independent of the size of her position in the OTC markets but nonetheless lower than $R(s)$. The opening of OTC markets, even if junior in bankruptcy with respect to the centralized clearing market, will impose a negative externality on the latter, as long as agent j will have an incentive to trade in the OTC market. This is in fact the case if the price she obtains for a short position in that market, $q^{j,OTC}(z_-^j)$, is greater than ε, which occurs robustly.[22]

To summarize, seniority of centrally cleared positions over OTC positions cannot generally restore efficiency:

Proposition 5. *Competitive equilibria of economies with a centralized clearing mechanism cannot be robustly supported as equilibria in economies with centralized clearing mechanism and OTC markets, not even when seniority of centrally cleared positions over OTC positions in bankruptcy is imposed.*

OTC markets will induce excessive leverage and higher probability of default in equilibrium with respect to the equilibrium in the economy with

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[21]Note that this equilibrium configuration is robust as the price map at equilibrium $q^j(z_-^j)$ will generally have a discontinuity at such z_-^j.

[22]Note that $q^{j,OTC}(z_-^j)$ is not the equilibrium price of the economy with both centralized clearing markets and OTC markets but rather the price at the equilibrium of the economy with no OTC markets, that is, the maximal marginal valuation across counterparties of agent j's short position in OTC.

only a centralized clearing mechanism. By trading in OTC markets, agents do impose an externality on the counterparties of their positions on both the OTC markets themselves as well as the centralized clearing mechanism, and equilibria will be inefficient. Of course, in practice, this form of inefficiency of OTC markets needs to be traded off with the efficiency gain due to the creation of OTC markets with non-standardized financial products, a feature not accounted for in our model.

4.3   Production risk

In the analysis so far, the aggregate endowment of the economy, $\sum_{i \in I} w_0^i$ at time 0 and $\sum_{i \in I} w_s^i$ in each state $s \in S$, has been kept constant. We showed that with regard to assets that insure bad aggregate states, the presence of an equilibrium risk premium in the price creates incentives for insurers to take on excessively short positions and default ex post. This effect can in fact arise even in the absence of aggregate risk in initial endowments if we allow for production in the economy and consider assets that help insure the risk of production. The "hedging premium" on such assets then serves the same purpose as the risk premium on assets insuring aggregate risk of exogenously given endowments.

Suppose each agent is endowed with a production function f which transforms consumption goods at time 0 into consumption goods at time 1. More precisely, consider the following technology. Let $K = \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_A \end{pmatrix}$ denote a capital allocation vector over A activities (e.g., projects), so that $k_1 + k_2 + ...k_A = k$. The production function can then be defined as the output in state s given capital allocation K: $f(s, K), \forall s.$[23] Note that, by allowing for multiple technological activities ($A \geq 2$), this formulation allows for some control of the agents over the distribution of capital across activities and hence over the probability distribution of outcomes, that is, over production risk.[24]

In this extension, the equilibrium analysis of centralized clearing and OTC economies can be extended to production. For instance, the budget

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[23]We assume f is continuously differentiable, strongly increasing, and strictly quasi-concave.

[24]While, for simplicity, we restrict to an economy with "backyard production" on the part of agents, the analysis directly extends to a firm-level production economy.

constraints in the economy with a centralized clearing mechanism become:

$$x_0^i + \sum_j q^j z_+^{ij} - q^i(z_-^i)z_-^i = w_0^i - k^i \ ,$$

$$x_1^i(s) = \max \left\{ w_1^i(s) + \sum_j R^j z_+^{ij} - R(s)z_-^i + f(s, K^i), (1 - \alpha)\, w_1^i(s) - \varepsilon z_-^i \right\} \ , \tag{40}$$

and agent i chooses a non-negative portfolio $(\mathbf{z}_+^i, z_-^i)$, s.t. $z_+^{ij} z_-^i = 0$ for any j, as well as a non-negative capital allocation K^i. Budget constraints in the OTC economies are similarly formulated.

It can be shown (proofs available upon request) that in the production economy, (1) A centralized clearing mechanism with transparency continues to decentralize constrained Pareto efficient allocations; and (2) The generic inefficiency due to opacity of OTC markets manifests itself as excessive shorting of assets that help hedge production risk, which in equilibrium leads to deadweight costs from default of providers of these hedges.

An example of this inefficient risk-taking is the possible effect of credit default swaps sold by A.I.G. to a large number of financial firms in the United States and the Europe, effectively insuring the tail risk of corporate bond and loan portfolios and mortgage-backed securities, which was tantamount to selling insurance on assets "produced" by the banking sector.[25]

To see this in a transparent manner, we revisit our example economy of Section 2. We suppress agent 2 and suppose that $w^1(G) = w^3(B)$, so that there is no aggregate risk in starting endowments. But we suppose that agents 1 have access to a production technology, e.g., making mortgages, that yields per unit of investment $f(G)$ in the good state and $f(B)$ in the bad state, where the investment contains aggregate risk in that $f(G) > f(B)$. We define average cash flow produced per unit of investment as $\overline{f} = pf(G) + (1-p)f(B)$. Then denoting the level of investment as k, and the cost of investment as $c(k)$, where $c'(k) > 0$ and $c''(k) > 0$, agent 1's maximization problem is:

$$max_{z^1,k}\ w_0^1 - c(k) - z^1 q + p\left[w^1(G) + f(G)k\right] + (1-p)\left[R^+ z^1 + f(B)k\right] - \frac{\gamma}{2}var(x^1(s)),$$

where

$$var(x^1(s)) = p(1-p)[w^1(G) + f(G)k - R^+ z^1 - f(B)k]^2.$$

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[25]If we cleared the market for the produced assets, e.g., housing stock, then our model could potentially generate a (housing) "bubble."

Then, for a given level of insurance z^1, the first-order condition for agent 1's investment decision implies that

$$c'(k) = \overline{f} - \gamma p(1-p)\left[f(G) - f(B)\right]\left[w^1(G) + f(G)k - R^+z^1 - f(B)k\right] \ .$$

Intuitively, the investment is more attractive the greater is its average return ($\overline{f}$), the lower is its risk ($p(1-p)\left[f(G) - f(B)\right]$)), and greater is the extent of insurance and its quality (R^+z^1) since insurance lowers the attendant risk of the investment.

Agent 1's demand for the asset insuring state B reflects a hedging motive:

$$z^1 = \frac{1}{R^+}\left[w^1(G) + \left[f(G) - f(B)\right]k - \frac{\Delta p}{\gamma p(1-p)}\right] \ ,$$

where Δp is now the hedging premium in the price of the asset, defined as in Section 2 as $\left[\frac{q}{R^+} - (1-p)\right]$.

Assuming the investment is indeed positive net present value, the availability of insurance for the producers is desirable up to an extent. Hedging with such insurance reduces risk of the producer (agent 1) and facilitates greater investment in the economy. However, as is clear from the above maximization problem, agent 1 does not take account of the fact that in case insurance is associated with default of the insurer (agent 3), there may be deadweight costs (ϵz^3 whenever $z^3 > w^3(B)$) to facilitating production in the economy in this manner. On the other side, the insurer is unable to commit not to default given the attraction of collecting hedging premium upfront and defaulting on the insurance ex post.

If we let $c(k) = \frac{1}{2}\mu k^2$, then the equilibrium investment is

$$k^* = \frac{\overline{f}}{\left[\mu + \gamma p(1-p)\left[f(G) - f(B)\right]\right]},$$

and the hedging premium is

$$\Delta p = \frac{\gamma p(1-p)\overline{f}\left[f(G) - f(B)\right]}{\mu + \gamma(1-p)\left[f(G) - f(B)\right]^2} \ .$$

Interestingly, equilibrium R^+ satisfies the same expression as in the solution of the example economy in Section 2 (given Δp). And unsurprisingly, we again obtain that there is inefficiency due to counterparty risk externality and deadweight default costs whenever ε is sufficiently small.

As explained in the last remarks of Section 2, default should be restricted, and if it be Pareto efficient, even eliminated. Doing so would restrict leverage in the economy and ensure productive efficiency in that investments take account of deadweight costs involved in potential default on hedging contracts. It also follows then that centralized clearing with transparency can again achieve constrained efficiency in the production economy.

5 Discussion

5.1 Strategic default and bankruptcy costs

A central feature of our modeling technology is the strategic nature of default – sell insurance today and default tomorrow – and the ex-post costs associated with such a default. It is useful to interpret this feature in view of the practical settings in which financial firms trade. Again, the example with A.I.G. as the protagonist is useful.

A.I.G. had traders (specifically, at A.I.G. Financial Products) who were engaged in the business of selling insurance through synthetic credit default swaps on portfolios of mortgages and corporate loans. It seems with the benefit of hindsight that their incentive to sell a large quantity of such swaps was (as in the model) to collect premiums upfront and get paid salaries and bonuses based on these premiums. The result was a highly levered bet of A.I.G. on the tail risk of the economy, that is, the likelihood of default of A.I.G. in the case where aggregate risk materialized was rather large. Indeed, A.I.G. as an enterprise itself suffered the substantial costs of the resulting default on these swaps.

These costs can be interpreted as ϵz^3 in our example economy. Due to limited liability, these costs were not borne by traders at A.I.G.FP who sold the swaps but in fact were borne by the rest of A.I.G.'s businesses (such as life and property insurance) whose franchise value was at least in part being deployed to pay off A.I.G.'s (non bailed-out) positions. In other words, the traders' perception of ϵ was small from their private standpoint, giving rise to the incentives to default.

This suggests that one mechanism to deal with counterparty risk externality is to raise ϵ, or in other words, weaken the incentives to default by increasing the bankruptcy costs suffered by defaulting financial firms, e.g., by arranging tougher resolution of their financial distress and requiring

co-investments from significant risk-takers. To the extent that such penalties are restricted by limited liability, our analysis highlights that improving trading infrastructure of markets can play an important complementary role in regulatory design to contain excessive leverage. Also, there is the time-consistency problem which is that ex post, that is, once default has realized, it is in society's interest to *lower* the bankruptcy costs, for instance, through regulatory forbearance towards the defaulting entity. This would, however, only lower the perceived ϵ, aggravate the strategic incentives to default, and raise ex-ante leverage, in what can be interpreted as a form of too-big-to-fail or too-interconnected-to-fail problem (as arose with A.I.G.FP).

5.2   Implementing centralized clearing

Our model highlights that the crucial aspect of centralized clearing and transparency is that agents can condition the terms of the contracts they trade on the total financial position of the counterparty and not just on bilateral positions. This is a natural reduced-form trading mechanism in the context of competitive equilibrium modeling we adopted. The issue of its implementation in actual financial markets remains open, especially considering that financial positions in practice are contracted sequentially.

What is required to implement competitive pricing and centralized clearing with transparency is a trading mechanism that allows prices and other contractual terms to adjust *continuously* with each agent's total position. Such a mechanism could look much like a margin or collateral arrangement. However, currently such arrangements are based on mark-to-market valuation of positions and an overall assessment of counterparty risk (e.g., through a credit rating). Hence, they are not exactly equivalent to continuously observing each agent's total position and conditioning price on that information. In particular, such arrangements cannot preclude institutions from positions beyond a certain size, that is, cannot implement non-linear pricing schedules - or "position limits" - as often employed on clearinghouses and exchanges.

To allow conditioning of trades on overall positions of a counterparty, post-trade transparency - in which trades are conducted during the day, reconciled and registered with a centralized clearing agency at the end of the day, and transparency provided to market participants on these trades thereafter - is necessary. However, if economic behavior of institutions is not stationary, then even pre-trade transparency may be necessary because in absence of information about trades an institution plans to undertake, it

is not possible for counterparties to charge an appropriate pricing schedule. Needless to say, such pre-trade transparency may be perceived to be too intrusive in some markets, which may thus continue to remain OTC.

An alternative approach in such cases, as we examined, would be to specify a bankruptcy rule whereby centrally cleared positions have seniority over OTC positions. While not perfect, as we discussed, it can help partly ameliorate the counterparty risk externality of OTC markets.

Finally, some recent changes in OTC markets, especially in contract terms of standardized credit default swaps (the so-called "Big Bang" protocol laid out in April 2009), require counterparties to exchange a part of their exchanged risk in pre-funded terms. This effectively amounts to requiring a high upfront or initial margin from the short position, or in other words, to reducing the leverage that can be built through a short position. As we have argued, such collateral requirements may be desirable if there are limitations to implementing centralized clearing, transparency or central counterparties, but they may be harmful in terms of limiting risk-sharing if they are designed to be too steep.

5.3   Proposals to reform the OTC markets

The financial crisis of 2007-09 has led to several reform proposals for OTC markets.[26] Our theoretical analysis can help provide a normative framework for evaluation of these proposals.

Acharya, Engle, Figlewski, Lynch and Subrahmanyam (2009) divide the reform proposals into requiring a (i) centralized registry with no disclosure to market participants; (ii) centralized clearing with disclosure of aggregate trade information to market participants; and (iii) centralized counterparty or exchange with full public disclosure of prices and volumes. Our theoretical analysis makes it clear that a centralized registry by itself is *not* sufficient as it only gives regulators ex-post access to trade-level information but does not counteract the ex-ante moral hazard of institutions wanting to take on excessive leverage. Both centralized clearing and exchange improve on this ground but it is transparency that is crucial. In our model, it is sufficient that centralized clearing disseminates trade positions to market participants and they themselves set price schedules and risk controls conditional on that

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[26]See Stulz (2009) for a summary of the dimensions along which OTC markets for credit derivatives likely contributed, and did not contribute, to the crisis.

information. In particular, requiring all trades to take place through a centralized exchange is not necessary though in that case there would be no need to disclose information on all trades to individual agents.

Regulatory reforms under the Dodd-Frank Act (July 2010) in the United States, and similar reform proposals in the U.K. and the Europe, require that mature and standardized derivatives such as the plain vanilla interest rate derivatives and single-name and index credit default swaps (CDS) be centrally cleared; there is no proposal to mandate that these be traded on an exchange. Complex products will continue to trade in OTC markets. Regulators will gain unfettered access to information on prices, volumes and exposures for all contracts, but the proposals do not require that exposure-level information be made public, not even with a delay.While some aggregate information will be disseminated to all market participants, such as the recent data published by the Depository Trust and Clearing Corporation (DTCC) on all live positions in credit derivatives, full transparency of positions of a given counterparty is being required only for regulatory usage.

Our results suggest that these proposed changes are unlikely to be fully adequate, though they take some steps in the right direction. Importantly, without exposure-level transparency for markets at large, counterparties cannot adopt "position limit" style restrictions. While centralized clearing platforms could impose such limits on standardized products, for OTC markets the onus would be on regulators to get capital and collateral requirements "right". Besides the limits on regulatory capacity to do so for complex products, a purely collateral-based approach – as we have argued – may be excessively costly in terms of locking up economy's resources. Exposure-level transparency to markets might be a cheaper alternative, in our view. However, if such transparency requirements should be seen as too restrictive, perhaps because they would limit generation of proprietary information which is costly to acquire, then there may be a role for seniority rule in bankruptcy in favor of centrally cleared positions over OTC positions. This would partly, but not fully, address the counterparty risk externality due the OTC positions.

6   Related Literature

The bilateral nature of contracts in the OTC markets has been stressed in the recent literature on the subject. Duffie, Garleanu and Pedersen (2005, 2007)

focus on search frictions, dynamic bargaining and valuation in OTC markets; Caballero and Simsek (2009) analyze the role of complexity introduced by bilateral connections and their role in causing financial panics and crises; and, Golosov, Lorenzoni, and Tsyvinski (2009) examine what kind of bilateral contracts will get formed when agents have private information about their endowment shocks. Our paper, while focused on OTC markets, is concerned primarily with issues of opacity and resulting inefficiencies.

The literature on insurance provision through financial contracts (e.g., Duffee and Zhou, 2001, Acharya and Johnson, 2007, Parlour and Winton, 2008) has largely focused on moral hazard on part of the insured due to the presence of information frictions. In contrast, our paper is concerned with moral hazard on part of the insurer, and how OTC markets contribute to it. Some of these aspects feature in the work of Allen and Carletti (2006), Thompson (2009) and Zawadowski (2009).

Allen and Carletti (2006) consider contagion from the insurance sector to the financial sector when there is credit risk transfer, but they do not consider agency-theoretic issues. In contrast, we allow for default incentives of the insurer and model credit risk transfer more generally as risk-sharing through financial contracts in a GE setting. Zawadowski (2009) analyzes counterparty risk in entangled financial systems. The system is "entangled' because banks hedge risks using bilateral OTC contracts but do not internalize the cost of their own failure on other banks (through counterparty risk exposures). As a result of this network externality, banks purchase less insurance against low probability events. Thus, there is less insurance in his model whereas due to moral hazard on part of the insurer, there is in fact excessive insurance in our setup but it is of low quality and entails default by the insurer. Thompson (2009) considers the moral hazard of default on part of the insurer when there is credit risk transfer in the financial sector. His focus is on analyzing how this moral hazard provides incentives to the insured parties to reveal information about their type, so that the two agency problems interact and reduce each others' adversity. Our paper, in contrast, is concerned with opacity of the insurers' positions.

More specifically on the benefits of OTC versus centralized markets, our analysis did not consider practical issues relating to the *extent* of netting of positions that is possible under different market structures. Duffie and Zhu (2009) explain that for a centralized counterparty to reduce counterparty risk more than in one OTC products setting, it would require netting across a large number other products. In our model, the primary role of the cen-

tralized clearing mechanism or centralized counterparty is not necessarily to reduce or eliminate counterparty risk but to improve its price by aggregating information on trades. We conjecture that if there were a centralized registry of *all* positions that is observed by different clearing platforms or exchanges and disseminated to market participants, then the pricing would be efficient ex ante, and so would be the levels of ex-post default risk. This is related to Leitner (2009)'s result that a clearinghouse-style mechanism, by allowing each party to declare its trades and revealing publicly those that hit pre-specified position limits, can prevent agents from promising the same asset to multiple counterparties and then defaulting.

From a pedagogical standpoint, our primary contribution lies in a formal modeling and analysis of opacity in OTC markets. We considered competitive equilibria of economies with moral hazard, where the moral hazard is induced by the opacity of agents' positions and their strategic default decisions. In the terminology of existing literature, we compared competitive equilibria in *exclusive contractual environments* to competitive equilibria in *non-exclusive contractual environments*. Exclusive contractual environments are by definition those in which one party in a contractual relationship can constrain all of the counterparty's trades with third parties. Therefore, in exclusive contractual environments counterparty risks play no role, as in our economies with a centralized clearing mechanism or with a centralized exchange. In non-exclusive contractual environments, on the contrary, agents cannot be restricted from engaging in multilateral contractual obligations which are not observable by the counterparties, as in the OTC markets.[27]

We focused on symmetric information about states of the world in our analysis. However, there could be adverse selection, e.g., in the form of unobservable probability distributions over S, the uncertain state at date 1. Modeling adverse selection in our setup would require combining features of Rothschild and Stiglitz (1976) and Akerlof (1970).[28] We conjecture that there

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[27]The distinction between exclusive and non-exclusive contracts is central in the theory of competitive economies with moral hazard; see e.g., Bisin and Gottardi (1999) and Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (2001). In the context of principal agent models, see the early work of Arnott and Stiglitz (1993) and Hellwig (1983). In finance, several papers have exploited the distinction between exclusive and non-exclusive markets in different contexts: e.g., Bizer and DeMarzo (1992) in a sequential model of banking, Parlour and Rajan (2001) in a model of credit card loans, and Bisin and Rampini (2006) in a model of bankruptcy.

[28]Santos and Scheinkman (2001) have adverse selection as well in their model of competition of exchanges.

would be separating equilibria in the economy with centralized clearing or exchanges, and excessive *lemons* trading (in the form of risky short positions) in the case of OTC markets. In turn, we conjecture that the inefficiency of OTC markets will be exacerbated in a setting with adverse selection.

7 Conclusion

In this paper, we formalized an important market failure arising due to opacity of over-the-counter (OTC) markets, in particular that the payoff on each position depends in default on other positions sold by the defaulting party, but there is no way for market participants to condition their trades or prices based on knowledge of these other positions. We showed that this counterparty risk externality can lead to excessive default and production of aggregate risk, and more generally, inefficient risk-sharing. Centralized clearing, by enabling transparency of trades, and exchanges, by creating a centralized counterparty to all trades, can help agents fully internalize the counterparty risk externality. Our model provides one explanation – based on incentives to excessively sell short, collect risk premiums and default ex post – for the substantial buildup of OTC positions in credit default swaps in the period leading up to the crisis of 2007-09. The model also helps understand the likely contribution of OTC trading to over-extension of credit in the economy, and effectiveness of proposed remedies for limiting this excess such as position-level transparency, centralized clearing or counterparty, collateral requirements, and subordination of OTC claims relative to centrally cleared ones.

Several extensions of our basic setup are worthy of detailed modeling in future. One, we focused on competitive markets. It is interesting to consider bilateral OTC markets in the presence of a "large" individual agent that effectively observes the trades of all others but whose trades are not seen by others. Such an agent would enjoy monopoly rents in the OTC setting, which in turn would reduce private incentives in the economy to coordinate on a transparent, centralized trading platform and achieve Pareto improvement. Second, our model suggests that excessive leverage and excessive production arising due to the OTC nature of trading can lead to a "bubble" in the market for goods (e.g., the housing stock), a subsequent crash upon realization of adverse shocks, and a breakdown of risk transfer (credit or insurance markets) in those states. Finally, our analysis also suggests that the possibility of

regulatory forbearance of "too big to fail" positions can result in ex-ante inefficiencies even with transparency and centralized exchange trading.

References

Acharya, V.V. and A. Bisin (2009) "Managerial Hedging, Equity Ownership, and Firm Value," *Rand Journal of Economics*, 40(1), 47–77.

Acharya, V.V. and R. Engle (2009) "Derivatives Trades Should All Be Transparent," *Wall Street Journal*, 15 May 2009.

Acharya, V. V., R. Engle, S. Figlewski, A. Lynch and M. Subrahmanaym (2009) "Centralized Clearing for Credit Derivatives," Chapter 9 in *Restoring Financial Stability: How to Repair a Failed System*, edited by Acharya, V. V. and M. Richardson, John Wiley & Sons, 2009.

Acharya, Viral and Timothy Johnson (2007) "Insider Trading in Credit Derivatives", *Journal of Financial Economics*, 84(1), 110–141.

Akerlof, G. (1970) "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism," *Quarterly Journal of Economics,* 84(3), 488-500.

Allen, Franklin and Elena Carletti (2006) "Credit Risk Transfer and Contagion," *Journal of Monetary Economics*, 53, 89–111.

Arnott R. and J. Stiglitz (1993) "Equilibrium in Competitive Insurance Markets with Moral Hazard," mimeo, Boston College.

Bisin, A., J. Geanakoplos, P. Gottardi, E. Minelli, and H. Polemarchakis (2001) "Markets as Contracts," Brown University, mimeo; available at

http://www.econ.nyu.edu/user/bisina/gea.pdf.

Bisin, A. and Piero Gottardi (1999) "General Competitive Analysis with Asymmetric Information," *Journal of Economic Theory,* 87(1), 1-48.

Bisin, A., P. Gottardi, and G. Ruta (2009) "Equilibrium Corporate Finance and Macroeconomics," mimeo, New York University.

Bisin, A. and A. Rampini (2006) "Exclusive Contracts and the Institution of Bankruptcy," *Economic Theory*, 27(2), 277-304.

Bizer, D. and P. DeMarzo (1992) "Sequential Banking," *Journal of Political Economy*, 100, 41–61.

Bizer, D. and P. DeMarzo (1999) "Optimal Incentive Contracts When Agents Can Save, Borrow, and Default," *Journal of Financial Intermediation*, 8(4), 241–269.

Caballero, R. and A. Simsek (2009) "Complexity and Financial Panics," NBER Working Paper 14997.

Dubey, P., J. Geanakoplos and M. Shubik (2005) "Default and Punishment in General Equilibrium," *Econometrica*, 73(1), 1–37.

Duffee, Greg and Chunsheng Zhou (2001) "Credit Derivatives in Banking: Useful Tools For Managing Risk?," *Journal of Monetary Economics*, 48, 25–54.

Duffie, D., N. Garleanu and L. H. Pedersen (2005), "Over-the-Counter Markets," *Econometrica*, 73(6), 1815-1847.

Duffie, D., N. Garleanu and L. H. Pedersen (2005), "Valuation in Over-the-Counter Markets," *The Review of Financial Studies*, 20(5), 1865-1900.

Duffie, D. and H. Zhu (2009) "Does a Central Clearing Counterparty Reduce Counterparty Risk?," Working Paper, Stanford University.

Geanakoplos, J. (1997) "Promises, Promises" In *The Economy as an Evolving Complex System II,* ed. W. Brian Arthur, Steven Durlauf, and David Lane, 285–320. Reading, MA: Addison-Wesley.

Geanakoplos, J., and W. Zame (1998) "Default, Collateral, and Crashes.' Unpublished paper, Yale University.

Golosov, M., G. Lorenzoni, A. Tsyvinski (2009) "Decentralized Trading with Private Information," mimeo slide presentation, at

http://www.econ.yale.edu/faculty1/tsyvinski/presentation4winedt.pdf.

Hellwig M. (1983) "On Moral Hazard and Non-price Equilibria in Competitive Insurance Markets," mimeo, University of Bonn.

International Monetary Fund (2010) "Making Over-The-Counter Derivatives Safer: The Role of Central Counterparties," Chapter 3, Global Financial Stability Report, April 2010.

Leitner, Yaron (2009) "Inducing Agents to Report Hidden Trades: A Theory of an Intermediary", Working Paper, Federal Reserve Bank of Philadelphia.

Parlour, Christine and Guillaume Plantin (2008) "Loan Sales and Relationship Banking," *Journal of Finance*, 63, 1291–1314.

Parlour, C. and U. Rajan (2001) "Competition in Loan Contracts," *American Economic Review*, 91(5), 1311–1328.

Rothschild, M. and J. Stiglitz (1976) "Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information," *Quarterly Journal of Economics,* 90(4), 630-49.

Santos, T. and J.A. Scheinkman (2001) "Competition Among Exchanges," *Quarterly Journal of Economics*, 116(3), 1027-62.

Stulz, R. (2009) "Credit Derivatives and the Credit Crisis," Working Paper, Ohio State University.

Thompson, James (2009) "Counterparty Risk in Financial Contracts: Should the Insured Worry about the Insurer?", *forthcoming, Quarterly Journal of Economics.*

Zawadowski, Adam (2009) ""Entangled Financial Systems", Working paper, Princeton University.

Appendix

Centralized exchange economy. To model a centralized exchange economy, we continue to assume that no creditor has direct privileged recourse to a debtor's endowment, in case of default. The centralized exchange, on the other hand, has full recourse to the debtors' pledgeable endowment. Furthermore, the exchange operates as a bankruptcy mechanism, by distributing the cash flow of the short positions pro-rata with respect to the long positions.

An agent i, taking a long position $z_+^{ij} > 0$ with short counterparty j, takes the return R^j as well as the price q^j as given. The equilibrium payoff of the asset shorted by agent j, denoted $R^j(z_-^j; s)$, is given by

$$R^j(z_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{z_-^j} \ if \ I^d(z_-^j; s) = 1 \\ R(s) \ \text{otherwise} \end{cases} \tag{41}$$

Consequently, an agent of type j with short position z_-^j faces an ask price map

$$q^j(z_-^j) = \max_i E\left(m^i R^j(z_-^j)\right). \tag{42}$$

Price taking is represented by the fact that agents take the pricing kernels $\left(m^1, \ldots, m^i, \ldots, m^I\right)$ as given.

We turn next to the decision problem of the competitive centralized exchange, which controls the supply of the asset to agents. Let the supply offered by the exchange to agent i for long and short positions be denoted (θ_+^i, θ_-^i), where $\theta_+^i = \left(\theta_+^{ij}\right)_{j \in I}$ and $\theta_+^{ij}\theta_-^i = 0$ for any j.[29]

Then, given the supplies, the exchange can compute the cash flow of the short positions of agents:

$$R^j(\theta_-^j; s) = \begin{cases} \frac{\alpha w_1^j(s)}{\theta_-^j} \ if \ I^d(\theta_-^j; s) = 1 \\ R(s) \ \text{otherwise} \end{cases}. \tag{43}$$

The exchange prices a unitary short position of agent j as $\max_i E\left(m^i R^j(\theta_-^j)\right)$, taking as given the stochastic discount factor of the agent i who values it the most at the margin, that is the agent who would acquire it if offered.

To summarize, a competitive exchange takes as given the stochastic discount factors $\left(m^1, \ldots, m^i, \ldots, m^I\right)$. Crucially, the exchange anticipates the

---

[29] By construction, $\theta_+^{ii} = 0$.

compositional effects on default risk of portfolios of different agent types, that is, it recognizes how each agent i's incentives to default are affected by her positions (θ_+^i, θ_-^i). Thus, the exchange maximizes its profits as per the following problem:

$$\max_{\left(\theta_+^i, \theta_-^i\right)_i} \left[\sum_j \max_k E\left(m^k R^j(\theta_-^j)\right)\left(\theta_+^{ij} - \theta_-^j\right) \right] \tag{44}$$

s.t.

$$\sum_i \theta_+^{ij} - \theta_-^j = 0, \text{ for any } j. \tag{45}$$

At competitive equilibrium, the portfolios demanded by the agents are offered by the competitive exchange and markets clear:

$$\theta_+^{ij} = z_+^{ij}, \ \theta_-^i = z_-^i, \ \forall i, j \ , \tag{46}$$

and the price maps and returns anticipated by agents are consistent with those perceived by the exchange:

$$q^j(z_-^j) = \max_i E\left(m^i R^j(z_-^j)\right), \tag{47}$$

$$q^j = q^j(z_-^j) = \max_i E\left(m^i R^j(z_-^j)\right), \text{ and} \tag{48}$$

$$R^j(s) = R^j(z_-^j; s). \tag{49}$$

It is straightforward to show (proof available upon request) that the competitive equilibrium allocations of economies with such a centralized exchange coincide with those of economies with a centralized clearing mechanism. Therefore, by Proposition 1, competitive equilibrium allocations of economies with a centralized exchange are constrained efficient. Note however that a centralized exchange which intermediates all financial market trades may for some products be a much more invasive institution than a centralized clearing mechanism which allows trading to remain decentralized but requires all trades to be registered and made transparent to market participants.

Proof of Proposition 1. The proof proceeds by contradiction. Let $\left(x_0^i, x_1^i, \mathbf{z}_+^i, z_-^i\right)$, $\forall i$, denote an equilibrium of the economy with centralized clearing mechanism and let $\left(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{\mathbf{z}}_+^i, \widehat{z}_-^i\right)$ denote a constrained Pareto optimal allocation which dominates the equilibrium allocation. Assume that both allocations satisfy netting, for any i and j:

$$z_+^{ij} z_-^i = 0, \quad \widehat{z}_+^{ij} \widehat{z}_-^i = 0.$$

Recall that $z^{ii} = 0$, for any i, by construction. Then, the allocation $\left(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{\mathbf{z}}_+^i, \widehat{z}_-^i\right)$ must not have been budget feasible at equilibrium prices $\{q^j\}$. That is,

$$\widehat{x}_0^i + \sum_j q^j \widehat{z}_+^{ij} - q^i(\widehat{z}_-^i)\widehat{z}_-^i - w_0^i \geq$$
$$x_0^i + \sum_j q^j z_+^{ij} - q^i(z_-^i)z_-^i - w_0^i ,$$

with $>$ for at least one agent i. Summing over i, we obtain that

$$\sum_i (\widehat{x}_0^i - x_0^i) +$$
$$\sum_i \left(\sum_j q^j \widehat{z}_+^{ij} - q^i(\widehat{z}_-^i)\widehat{z}_-^i \right) - \sum_i \left(\sum_j q^j z_+^{ij} - q^i(z_-^i)z_-^i \right) > 0 .$$

But, since market clearing must hold for both $\left(x_0^i, x_1^i, \mathbf{z}_+^i, z_-^i\right)$ and $\left(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{\mathbf{z}}_+^i, \widehat{z}_-^i\right)$, $\sum_i (\widehat{x}_0^i - x_0^i) = 0$, we obtain:

$$\sum_i \left(\sum_j q^j \widehat{z}_+^{ij} - q^i(\widehat{z}_-^i)\widehat{z}_-^i \right) - \sum_i \left(\sum_j q^j z_+^{ij} - q^i(z_-^i)z_-^i \right) > 0.$$

Furthermore, at equilibrium, $q^i = q^i(z_-^i)$. Hence,

$$\sum_j q^j(\widehat{z}_-^j) \sum_i \left(\widehat{z}_+^{ij} - \widehat{z}_-^i \right) - \sum_j q^j(z_-^j) \sum_i \left(z_+^{ij} - z_-^i \right) > 0.$$

But market clearing at equilibrium implies $\sum_i \left(z_+^{ij} - z_-^i \right) = 0$, for any j, and hence $\sum_j q^j(z_-^j) \sum_i \left(z_+^{ij} - z_-^i \right) = 0$ for non-negative prices $q^j(z_-^j)$. Therefore,

$$\sum_j q^j(\widehat{z}_-^j) \sum_i \left(\widehat{z}_+^{ij} - \widehat{z}_-^i \right) > 0,$$

a contradiction with feasibility of $\left(\widehat{x}_0^i, \widehat{x}_1^i, \widehat{\mathbf{z}}_+^i, \widehat{z}_-^i\right)$. $\blacksquare$

Proof of Proposition 2. We only prove the statement regarding economies with OTC markets and netting. The proof in the case of OTC markets without netting only requires straightforward modifications. We restrict attention to the case when ε is small so that there is default in the economy. Furthermore, assume to start with that $\varepsilon = 0$. Let $(\mathbf{z}_+^i, z_-^i)$ be the equilibrium portfolio for agent i in an economy with centralized clearing mechanism. Let $S(i) \subseteq S$ denote the subset of the states of uncertainty in which, at equilibrium, an agent i will default. Then, $S(i)$ is robustly non-empty. Furthermore,

if $S(i)$ is non-empty, then $z_-^i > 0$. For any economy such that $S(i)$ is non-empty (and $z_-^i > 0$) for some i, at equilibrium of the centralized clearing mechanism, we must have

$$q^i(z_-^i) = \sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z_-^i} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s).$$

Suppose, by contradiction, that such a competitive equilibrium of the centralized exchange economy can be supported in an economy with OTC markets and netting. Then it is necessarily supported by price $q^i = \sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z_-^i} +$
$\sum_{s \in S \setminus S(i)} p_s m^i(s) R(s)$, such that $q^i = q^i(z_-^i)$ at the equilibrium portfolio $(\mathbf{z}_+^i, z_-^i)$.

It is straightforward to see that in this case, at price q^i agent i prefers a portfolio $(\mathbf{z}_+^i, z_-^i + dz)$, for some $dz > 0$. This is because the marginal valuation of the discounted repayment of a unitary extra short portfolio dz, $\sum_{s \in S(i)} p_s m^i(s) \frac{\alpha w^i(s)}{z_-^i} + \sum_{s \in S \setminus S(i)} p_s m^i(s) R(s)$, depends negatively on z_-^i; while the price obtained at time 0 from the same unitary extra short portfolio, dz, q^i, does not. Since the portfolio $(\mathbf{z}_+^i, z_-^i + dz)$ is budget feasible, a contradiction is reached. This is the case for any equilibrium of the centralized exchange economy such that $S(i)$ is non-empty, for some i, and hence the contradiction holds robustly.

The proof extends by continuity to ε sufficiently small. ■

Proof of Proposition 3. Once again, we only prove the statement regarding economies with OTC markets and netting. The proof in the case of OTC markets without netting only requires straightforward modifications. Once again, assume $\varepsilon = 0$ and the proof below extends by continuity to ε sufficiently small. Finally, the "weakly greater" part of the statement is straightforward. We turn to prove the robustly "strictly greater" part.

Consider the robust subset of economies for which, with centralized clearing at equilibrium, $S(i)$ is non-empty. An argument analogous to the one in the proof of Proposition 2 guarantees that, for these economies, when ε is small enough, at an equilibrium of the economy with OTC markets and netting, $S(i) = S$. Agents i, in other words, default in all states $s \in S$. This proves that default is robustly strictly greater at equilibria of the economy with OTC markets and netting than with centralized clearing.

Consider such an equilibrium with OTC markets and netting, to study leverage. Consider now the general case in which $\varepsilon > 0$. At equilibrium it

must be that $q^i > 0$. Suppose on the contrary that $q^i \leq 0$. In this case, we claim agents i would rather choose $z^i_- = 0$ and hence would trivially not default. In fact, if $S(i) = S$, and $q^i \leq 0$, agents i would consume

$$
\begin{aligned}
x^i_0 &= w^i_0 + q^i z^i_- \\
x^i_1(s) &= (1-\alpha)\, w^i_1(s) - \varepsilon z^i_-,
\end{aligned}
$$

(recall that, because of netting, $\sum_j q^j z^{ij}_+ = 0$). But then

$$
\begin{aligned}
x^i_0 &\leq w^i_0 + q^i z^i_- \leq w^i_0 \\
x^i_1(s) &= (1-\alpha)\, w^i_1(s) - \varepsilon z^i_-
\end{aligned}
$$

By resorting to autarchy, $z^i_- = z^{ij}_+ = 0$, instead agents i would guarantee themselves

$$
\begin{aligned}
x^i_0 &= w^i_0 \\
x^i_1(s) &= w^i_1(s)
\end{aligned}
$$

which they prefer. Prices such that $q^i \leq 0$ therefore imply no default. This is the case for all agents of all types i. But then $R_+(s) = R(s)$, for all $s \in S$ and z^{ji}_+ is robustly > 0, for some j, a contradiction with market clearing. At an equilibrium of the economy with OTC markets and netting, therefore, it must be that $q^i > 0$. In this case z^i_- grows unbounded as $\varepsilon \to 0$. This proves that leverage is robustly strictly greater in the economy with OTC markets and netting than with centralized exchange for ε small enough. ∎

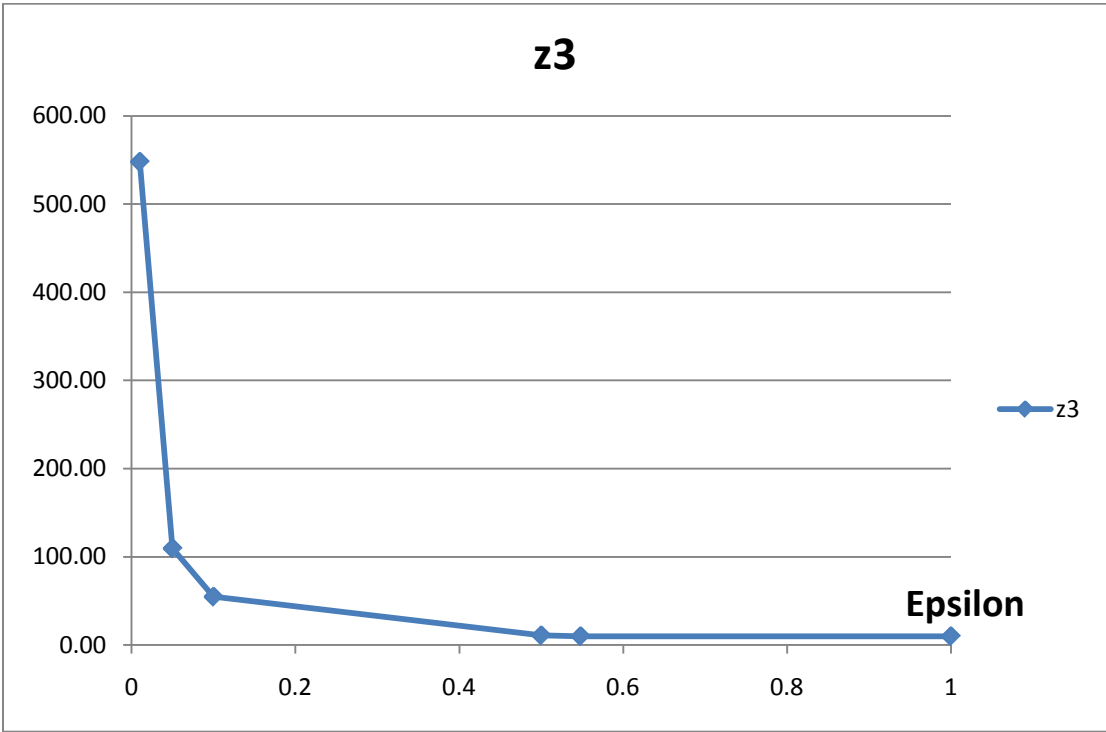Figure 1: The quantity of insurance sold (z^3) as a function of the deadweight cost of default (ε)

Figure 2: The realized payoff on the insurance (R^+) as a function of the deadweight cost of default (ε)
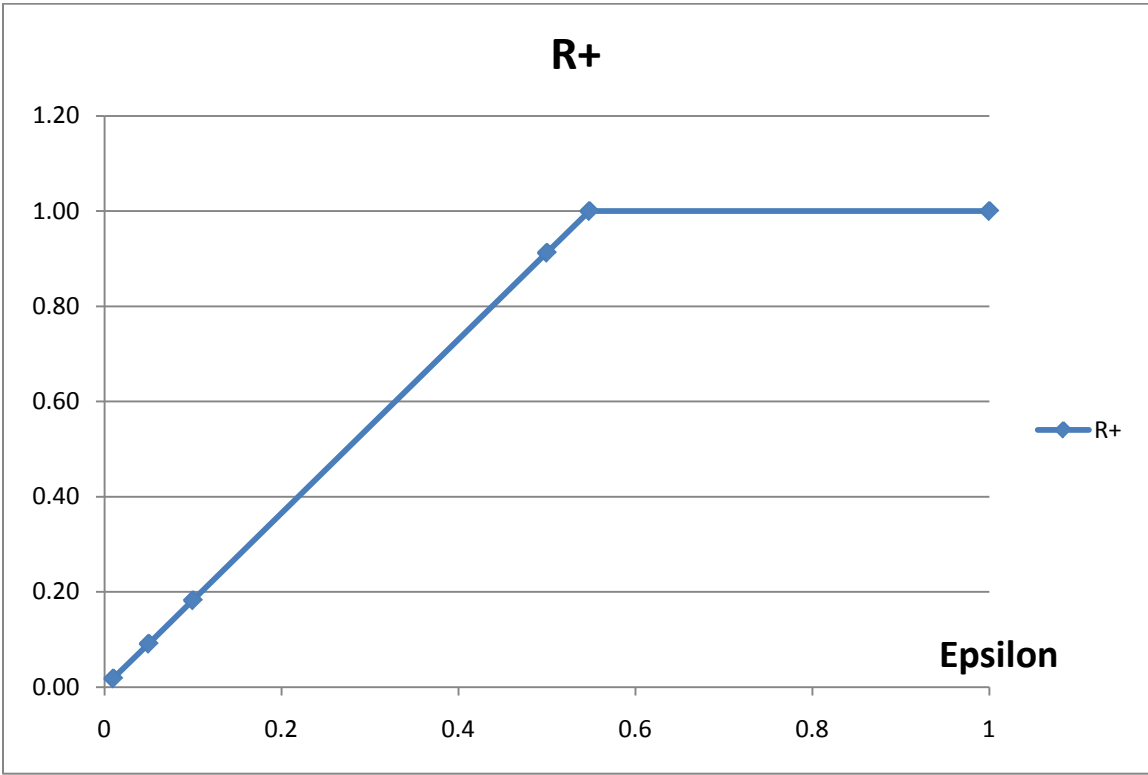
Figure 3: The equilibrium price of insurance (q) as a function of the deadweight cost of default (ε)
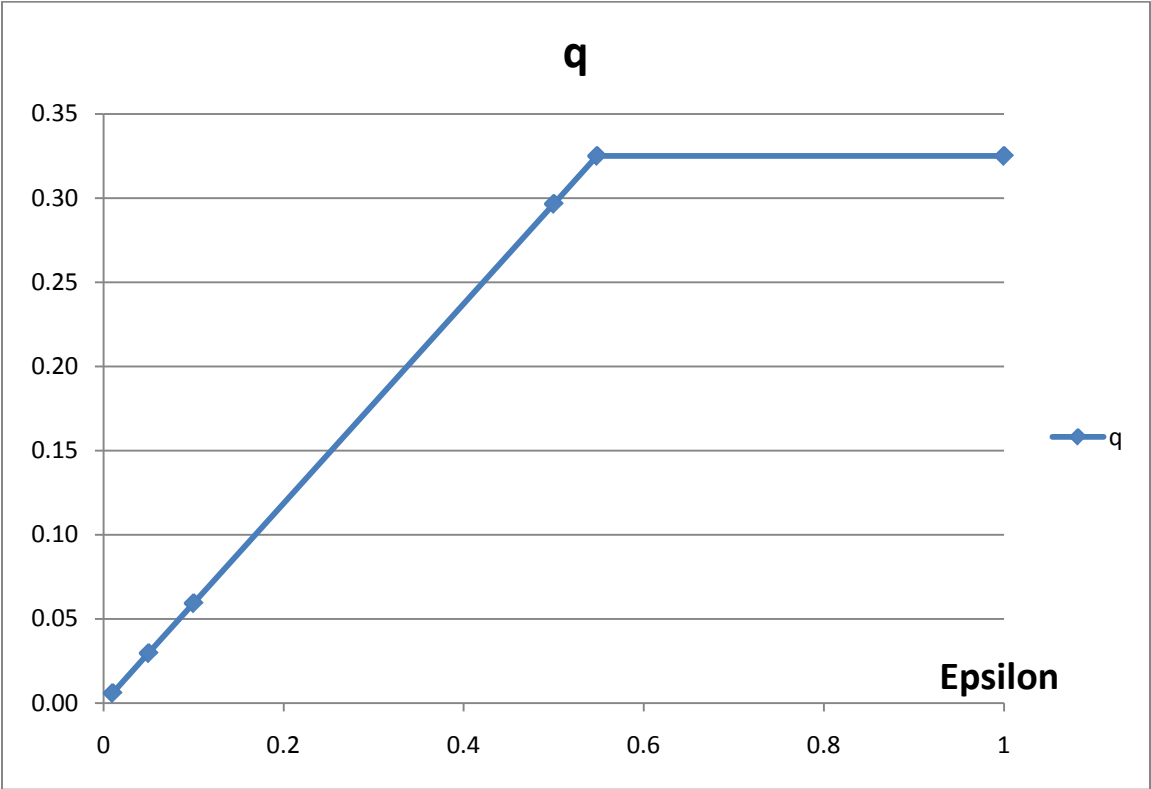
Figure 4: The equilibrium utilities as a function of the deadweight cost of default (ε)