NBER WORKING PAPER SERIES

A BEHAVIORAL MODEL OF DEMANDABLE DEPOSITS AND ITS IMPLICATIONS FOR FINANCIAL REGULATION

Julio J. Rotemberg

Working Paper 16620
http://www.nber.org/papers/w16620

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
December 2010

I wish to thank Robin Greenwood and workshop participants at the University of Montreal for helpful comments and suggestions. All errors are my own. The views expressed herein are those of the author and do not necessarily reflect the views of the National Bureau of Economic Research.

A Behavioral Model of Demandable Deposits and its Implications for Financial Regulation
Julio J. Rotemberg
NBER Working Paper No. 16620
December 2010
JEL No. D03,G21,G33

ABSTRACT

A model is developed which rationalizes contracts that give depositors the right to obtain funds on demand even when depositors intend to use these funds for consumption in the future. This is explained by depositor overoptimism regarding their own ability to collect funds in a run. Capitalized institutions serving depositors with such beliefs emerge in equilibrium even if depositors and bank owners have the same preferences and the same investment opportunities. Various government regulations of these institutions, including minimum capital levels, requirements concerning the assets they may hold, deposit insurance and compulsory clawbacks in bankruptcy can raise the average ex post welfare of depositors.

Julio J. Rotemberg
Graduate School of Business
Harvard University, Morgan Hall
Soldiers Field
Boston, MA 02163
and NBER
jrotemberg@hbs.edu

This paper shows that a simple cognitive bias that seems consistent with experimental evidence can account for the popularity of demandable deposits. The cognitive bias I consider is a form of overconfidence: people are too optimistic about their future position in line when they expect scarce resources to be distributed on a first-come first-served basis. For depositors, this would imply overoptimism concerning their chances to recover their funds in a bank run. This does not require depositors to consider themselves well informed. It is sufficient for them to believe that other depositors are even slower to react than they are.

The main result of the paper is that this overconfidence can rationalize demandable deposit contracts even when everyone derives the same utility from consumption in every state of nature, everyone has the same investment opportunities and everyone has the same beliefs regarding the payoffs of these investments. Overoptimistic agents deposit their endowment with other agents in exchange for the right to be paid on a first-come first-served basis. They do this even though they are risk averse and they rationally recognize that first-come first served contract are risky because they lead early withdrawers in a run to more consumption than late ones.

Because some agents are overoptimistic, it is possible to increase their average *ex post* utility with suitably designed policy interventions. This leads me to consider four different types of policies. The first is a policy of imposing "clawbacks" in bankruptcy so that depositors who arrive too late to receive anything from their bank can recover resources from those who withdrew earlier. The result is that all depositors end up being treated symmetrically at all times, as assumed by Allen and Gale (1998). In the U. S., bankruptcy law requires such clawbacks in the case of nonfinancial bankruptcies but exempts financial institutions.

The second policy I examine is the provision of deposit insurance financed by taxation. Like clawbacks, this avoids inefficient runs, which is particularly useful when runs lead to costly transfers of assets. When such transfers (or liquidations) are costless, an alternative policy that also tends to raise *ex post* welfare, and which may be administratively more expedient, is to force bank owners to put up more of their own capital. Finally, I demonstrate that it is possible to raise this welfare by forcing banks to hold assets that they would otherwise shun.

A central motivation for the analysis in this paper is that demandable liabilities, *i.e.*, liabilities with the property that claimants are entitled to receive predetermined quantities of a valuable asset whenever they want to, have been ubiquitous for a long time. Mueller (1997, p. 15-25) shows that they were important in XIV$^{\text{th}}$ century Venice, well before any government provided deposit insurance might have made such deposits safe. Indeed, demandable deposits have proven unsafe in a number of instances.[1] This was particularly true in the American Free Banking era where, as Hasan and Dwyer (1996) report, numerous banks closed without fulfilling all their contractual obligations to depositors.

In the current U.S. context, a different but equally remarkable feature of demandable liabilities is their enormous volume. According to the Flow of Funds Accounts, total financial assets of households were about \$ 31 trillion in 2000.[2] At the same time, Bucks *et al.* (2009) report that in the Survey of Consumer Finances Surveys of both 1998 and 2001, households held 11.4 % of their financial assets in "transactions accounts" (which are basically accounts that are immediately available). This means that the total held in these accounts was about \$ 3.5 trillion. For comparison, total annual consumer expenditure on goods and services in the U.S. equaled \$ 6.8 trillion in 2000. Most consumers are also receiving income from other sources with at least a monthly frequency, so that transactions balances of the same size as monthly consumer expenditures ought to be sufficient to pay for these expenditures in the vast majority of circumstances. It follows that many households have "transactions balances" that exceed the maximum they could possibly need for transactions over an horizon of, say, three months.

The enormous volume of demandable deposits suggests that the most widely used theoretical rationale for their existence, namely the rationale offered in Diamond and Dybvig (1983), may not suffice. Diamond and Dybvig (1983) let consumers be uncertain about

[1] Mueller (1997) also documents several instances of bank failures and notes their cost to "the unlucky depositors who failed to withdraw his money in time" (p. 121).

[2] This is the last year for which Supplemental table L100a lists the assets of nonprofit institutions. For subsequent years, only the total of households and nonprofit institutions is available.

whether they will need funds relatively quickly, and show that demandable deposits provide insurance against the risk of having such a need. This insurance has a price, however, so that consumers with funds that are intended for consumption beyond a short horizon would be better off holding long term assets.[3] In my analysis, by contrast, depositors who intend to consume far into the future hold demandable assets because these give them the opportunity to change their portfolio at will on terms that are determined in advance.

An essential component of the model is that depositors all seek to change their portfolio, and thus run on the institution that issued demand deposits, when there are bad news regarding the assets held by the institution. The model is thus consistent with the empirical literature on the incidence of banking panics. Gorton's (1988) pioneering paper studied runs in the U.S. National Banking Era (1863-1914) and showed that these occurred mostly at times of weak business conditions, when bank loans were presumably impaired. Similarly, Hasan and Dwyer (1996) show that bank runs in the Free Banking Era (1836-1863) mostly took place when banks held bonds that had fallen in value. Lastly, Schumacher (2000) shows that the banks that experienced runs in Argentina after the 1994 Mexico crisis tended to have relatively weak balance sheets.

The best studied runs involve deposits at commercial banks. Still, U.S. Money Market Mutual Funds seem to be subject to similar runs even though they do not explicitly specify how much their depositors will be able to withdraw in the future. They may do so implicitly, however. At least some of these funds have maintained the right of investors to redeem each "share" for one dollar when the assets underlying these shares were worth less. As an example, a fund called the "Reserve Primary Fund" suffered declines in the value of its claims on Lehman Brothers as Lehman was going bankrupt in 2008, and this led to a run. The fund's liabilities stood at \$62.6 billion on Friday, September 12, and a flood of redemptions reduced this to \$23 billion by Tuesday, September 16 at 3 p.m. Redemption requests received

[3]It is worth noting that Bryant (1980) models the risk of needing to consume early as being the risk of learning early in one's life that one will die imminently, and then wishing to consume all one's assets. I have interpreted this as a metaphor for short term changes in the desire to consume but it may apply more broadly. Bryant (1980) motivates his paper as providing a rationalization for deposit insurance and the current paper could be motivated similarly.

before this point were honored at $1 a share. At 3 p.m. redemptions were frozen, and the fund was then slowly liquidated. Investors who remained at 3 p.m. were paid off gradually, and received substantially less than $1 per share.

The bias considered in this paper is closely related to forms of overconfidence demonstrated that have been documented in the past. One oft-cited finding in the literature is that survey responses display an "optimistic bias."[4] Weinstein (1980) was one of the first to exhibit this phenomenon. He showed that survey respondents tend to say that the probability that they will obtain desirable outcomes (like having a mentally gifted child) is higher than the probability that their peers will obtain these outcomes. By contrast, respondents report that their own probability of a bad outcome such as a heart attack or a drinking problem is lower than that of their peers.[5] Weinstein (1980) also reports that this bias is larger for events that people judge to be controllable. The finding that perception of control is correlated with increased optimism about risky outcomes has been reproduced by a number of authors.[6]

One issue that arises at this point is how people come to have the perception that they control an outcome. Interestingly, this does not appear to require much actual influence. As Langer (1975) shows, numerous manipulations can give people an "illusion of control." One of the most striking pieces of evidence for this can be found in Strickland, Lewicki and Katz (1966) and Rothbart and Snyder (1970). These papers show that people express more confidence that they know the outcomes from rolls of dice, and that are willing to wager more of their own funds on these outcomes, when they place their bets before they roll the dice rather than afterwards. This is true even though the one would not expect the outcome of a dice throw to depend on whether a bet is placed on it beforehand. What is particularly useful about these experiments is that they do involve actual payoffs and therefore the

[4]In October 2010, Google Scholar lists about 5,000 works using the term.

[5]Harris and Hahn (2010) point out that these results could be a statistical artifact which results from the fact that many of the positive outcomes studied by Weinstein (1980) (like getting a job one likes) were common while many of the negative outcomes (like attempting suicide) were uncommon. Still, the base rates for several of the positive and negative effects demonstrated by Weinstein (1980) (including those I mention in the text) seem comparable.

[6]See Thompson *et al* (1998) for a valuable discussion.

interpretation of the results is not marred by the ambiguity of natural language.[7]

Obtaining a scarce product that is distributed on a first-come first-served basis does require one to take action, and people may well feel quite a bit of control over this outcome. Moreover, people who entrust a debtor with funds often have a strong desire to receive it back. This too may foster a false sense of control. As Thompson *et. al* (1998, p. 151) put it, "When people have a strong need for an outcome or a strong commitment to getting the outcome, they know that their intention to obtain the outcome is strong, and this may influence their judgment of control."

Related Literature. This paper is related to two behavioral literatures in economics. The first concerns overconfidence about one's own abilities. Bénabou and Tirole (2002), Compte and Postlewaite (2004), and Köszegi (2006) discuss methods agents use to manipulate their own beliefs that can lead to overconfidence. Yildiz (2003) analyzes the effect of overconfidence in bargaining games. Sandroni and Squintani (2004) propose a model in which overconfidence affects insurance markets. Manove and Padilla (1999) and Landier and Thesmar (2009) develop models of equilibrium contracts between rational lenders and overoptimistic entrepreneurs. Landier and Thesmar (2009) also present evidence that, consistent with their theory, entrepreneurs who respond with more excessive optimism in a survey also take on more short term debt. Also focusing on empirical differences in overconfidence across people, Grinblatt and Keloharju (2009) show that Finnish investors whose survey responses suggest that they are more overconfident trade stocks more frequently.

This last paper also belongs to a second literature that is related to the current paper. This second literature studies the effect of cognitive limitations on the behavior of financial market participants. This includes, notably, Daniel, Hirshleifer and Subrahmanyam (1998) and Odean (1998), who show that various puzzles in the behavior of stock prices can be rationalized by the existence of investors who are overconfident about their ability to predict

[7]Camerer and Lovallo (1999) report another experiment with financial payoffs in which subjects act as if they are overconfident. Their subjects express more willingness to participate in an "investment game," and thereby make it more likely that they will receive low payoffs, when the outcome of the game depends on their relative skill at a trivia task than when the outcome depends on a randomization device.

the future.[8] Incorrect predictions about the future play a role in many other papers concerned with asset prices, including Barberis, Shleifer and Vishny (1998) and Hong, Stein and Yu (2007). Gennaioli, Shleifer and Vishny (2010) show that incorrect beliefs can also lead agents to hold new asset classes that appear safe to them because they have not yet become aware of some of their incipient risks. One distinction between this literature and the current paper is that I focus on beliefs that people have about their own future actions rather than beliefs they have about external events.

The paper proceeds as follows. The next section introduces my formulation of overconfidence into a one period model with certainty. By showing how individuals trade off risk aversion, which makes demand deposits undesirable, with excessive optimism, this section lays the foundation for the subsequent analysis. Section 2 shows that overconfidence can make demand deposits emerge in equilibrium in a simple stochastic environment where everyone starts with an identical endowment whose payoff is stochastic. With two aggregate states of nature, depositors are fully paid off in the favorable state while there is a run in the unfavorable one. In Section 3, I let bank owners capitalize their banks in order to attract depositors and show that they quite generally do it in equilibrium. Section 4 studies policy interventions that have the potential for raising the average *ex post* utility of depositors. These include minimal capital requirements, mandatory clawbacks in bankruptcy, deposit insurance and the regulation of bank assets. Section 5 expands the model to include three periods so that it is possible to analyze costly withdrawals in the middle period. The section demonstrates that banks may offer contracts that induce these costly withdrawals in response to bad news. Finally, Section 6 concludes.

[8] These papers rely on miscalibration, *i.e.*, people's tendency to overstate the precision of their knowledge. For direct evidence of miscalibration by financial managers, see Ben-David *et al.* (2010). Ben-David *et al.* (2010) distinguish between miscalibration, which they see as a form of overconfidence, and "optimism" regarding the mean exogenous random variables. Following Weinstein (1980), I use the term "optimism" to describe a different kind of overconfidence, namely people's excessive confidence in their ability to affect their own outcomes favorably.

1 Queuing Optimism with Aggregate Certainty

To introduce the idea of optimism regarding one's position in a queue, consider first a general setting of rationing. Suppose a firm has given N individuals the right to obtain a good at a price that is lower than consumers' valuation of the good. Suppose further that the firm has only $Z < N$ units available and that it has promised to allocate its units to creditors on a first-come first-served basis, as is common in shortage situations. Then, if creditors are symmetric and rational, each of them should each expect to receive it with probability Z/N.

The simplest way to formalize this probabilistic outcome is to suppose that some random elements, however insignificant in size, affect creditors' position in line. Suppose then, that creditors' actual order of arrival depends on the realization they draw of a random variable d, with creditors who obtain a lower d arriving before those who obtain a higher one. One interpretation for this is that d is the distance between a creditor and the firm at the moment the creditor decides to obtain the good. An alternative interpretation would have all creditors start out at the same distance from the firm, while the outcome d would represent the inverse of the speed with which a particular creditor is able to cover this distance.

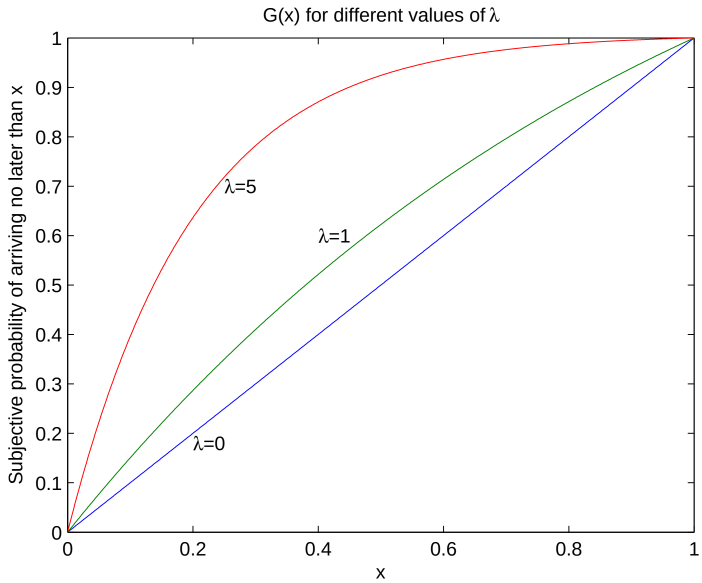
Let F be the cumulative density function for d. Then, creditors whose d is smaller than or equal to $F^{-1}(Z/N)$ receive the good while others do not. Since $F(d)$ is distributed uniformly between zero and one, the probability of obtaining such a d is Z/N. Since creditors are affected only by the order in which they arrive, one can equivalently think of them as drawing a realization for $x = F(d)$, where x has a standard uniform distribution.

Queuing optimism is captured as follows. Each creditor is assumed to correctly believe that the x's of other creditors are drawn from a standard uniform distribution. On the other hand, creditors believe that their own x is drawn from a truncated exponential distribution with cumulative density function $G(x)$ given by

$$G(x, \lambda) = \frac{1 - e^{-\lambda x}}{1 - e^{-\lambda}}. \qquad (1)$$

The combination of these beliefs leads creditors to view $G(x, \lambda)$ as being equal to their own probability of being among the first $100x\%$ to arrive. The limit of $G(x)$ when λ goes to zero

Figure 1:



is x so that this limit corresponds to individuals who are rational. At the other extreme, an individual with a λ equal to one is essentially certain to be the first in line. More generally, the Appendix shows that $G(x)$ is strictly increasing in λ for $0 < x < 1$. The parameter λ is thus a measure of optimism: creditors with higher λ's are strictly more optimistic about that the likelihood that they will be among the first $100x\%$ of creditors to ask for the good. The function $G(x)$ is displayed for a few values of λ in Figure 1

The first and second derivatives of $G(x)$ with respect to x are given by

$$G'(x) = \frac{\lambda e^{-\lambda x}}{1 - e^{-\lambda}} \qquad G''(x) = -\frac{\lambda^2 e^{-\lambda x}}{1 - e^{-\lambda}}, \tag{2}$$

so that $G'(x)$ declines as x rises. For low values of x, the slope of people's subjective probability of being among the first x rises more rapidly than the actual probability. This can be seen by noting that $G'(0)$ equals $\lambda/(1 - e^{-\lambda})$, which is greater than one for $\lambda > 0$. For high values of x, on the other hand, the subjective probability of being among the first x rises more slowly than the actual probability because, as I show below, the Einstein function $\lambda e^{-\lambda}/(1-e^{-\lambda})$ is smaller than one. There thus exists an interior value of x at which $G'(x) = 1$

and the overoptimistic bias $G(x) - x$ is the largest. This value of x plays a role in what follows.

So far, the analysis applies to any firm that is required to ration its customers.[9] This bias may be particularly relevant in situations where actual rationing is infrequent, so that people have not had an opportunity to learn about their actual tendency to be at the front of the line. As long as there are only occasional bank runs, the bias should be particularly pronounced for the rationing that takes place in these runs.

While queueing optimism affects people's subjective probability of receiving funds, it need not affect how creditors value the funds they receive. It is thus consistent with individuals having a standard expected utility function. For convenience, suppose that this utility function takes the CRRA form with relative risk aversion between 0 and -1. Creditors thus expect their welfare to be

$$U_c = \bar{E}(C^\gamma) \qquad 0 < \gamma \leq 1, \tag{3}$$

where $\bar{E}$ takes expectations using their own perceptions and C are the funds they receive.

To understand the implications of queueing optimism on financial contracts, it is worth starting with the simple case where a bank has signed contracts that entitle a continuum of creditors of mass one to receive a quantity of funds Y on a first-come first-served basis. The bank, meanwhile has $Z \leq Y$ funds available. The bank's owners are assumed to have limited liability so that the bank closes after Z has been paid out and only Z/Y arrive in time to receive funds from the bank. The amount that these creditors get to keep, and the amounts received by the rest, depend on the legal regime. Under laissez-faire, the government does not intervene so that the Z/Y creditors that arrive first keep the Y they obtained from the bank and the rest remain empty-handed.

An alternative is for the government to allow the last $1 - Z/Y$ creditors to clawback funds from those who were paid by the bank. In ordinary bankruptcy, payments made up to 90 days before the filing of bankruptcy (what is known as the "preference period") are

[9]This bias may also help explain why people often favor non-price rationing. For example, Frey and Pommerehne (1993) find that many respondents prefer to distribute a limited supply of water on a first-come first-serve rather than using high prices to discourage demand.

potentially subject to such clawbacks. Under many circumstances, creditors that are paid during the preference period can be forced to return these funds to a bankruptcy trustee, who then treats them like other claimants.[10]

Suppose first that creditors are rational so that, when $Z \leq Y$, they each expect to receive Y from the debtor with probability Z/Y. Their expected utility is then

$$\frac{Z}{Y} Y^{\gamma}. \tag{4}$$

The derivative of this expression with respect to Y is $(\gamma - 1)ZY^{\gamma-2}$, which is negative if $\gamma < 1$. This means that risk averse creditors would prefer lowering Y towards Z whenever Y exceeds Z. They thus strictly prefer a clawback regime that gives Z to all creditors. This result ought to generalize. Whenever $Y > Z$, laissez-faire leads to a gamble that pays zero with probability $1 - Z/Y$ and Y with the remaining probability. For risk-averse individuals, this is less desirable than the clawback regime, which pays the expected value of this gamble (Z/Y) with probability one.

With queueing optimism, creditors' expected utility from the promise of a payment of Y by a debtor who has $Z \leq Y$ funds per creditor is

$$U_c = \frac{1 - e^{-\lambda Z/Y}}{1 - e^{-\lambda}} Y^{\gamma}, \tag{5}$$

and the resulting gain from a slight increase in Y when $Y \geq Z$ is

$$\frac{dU_c}{dY} = \frac{Y^{\gamma-1}}{1 - e^{-\lambda}} \left[-\frac{\lambda Z}{Y} e^{-\lambda Z/Y} + \gamma(1 - e^{-\lambda Z/Y}) \right]. \tag{6}$$

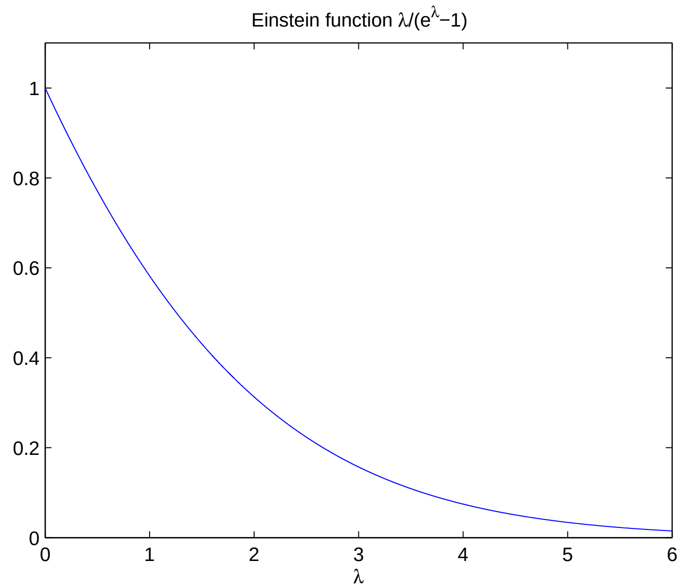
This is positive at $Y = Z$ if

$$\gamma > \frac{\lambda}{e^{\lambda} - 1} \equiv n(\lambda). \tag{7}$$

The right hand side of this inequality is an Einstein function, which is depicted in Figure 2 and which I denote by $n(\lambda)$. This function equals one in the limit where $\lambda = 0$ and is

[10] In the United States, these "preference actions" are governed by the U.S. Code Chapter 11, section 547. Section 546 of the same chapter contains exceptions that apply to firms in the financial sector. See Bliss and Kaufman (2007) for a discussion of this and other differences between the way that banks and other corporations are treated when they become insolvent.

Figure 2:



declining in λ for $\lambda > 0$. This means that, for every $\lambda > 0$, there is a critical $\gamma < 1$ such that creditors prefer setting Y above Z for any γ larger than this critical value and prefer setting Y equal to Z for values of γ below it. Risk aversion thus continues to act as a reason for people to prefer not to have their outcome be determined by their random position in line. However, there is now a counteracting force, namely that $\lambda > 0$ leads people to be confident that they will not be among the very last to arrive. People thus prefer a Y that exceeds Z if and only if their risk aversion is below a threshold that is increasing in λ. Put differently, increases in both γ and in λ induce a greater desire to have Y exceed Z.

When the degree of risk aversion is below this threshold, the optimal value of Y from the point of creditors, Y_o leads dU_c/dY in (6) to equal zero so that[11]

$$\frac{\lambda Z/Y_o}{e^{\lambda Z/Y_o} - 1} = n\left(\frac{\lambda Z}{Y_o}\right) = \gamma.$$

Therefore

$$Y_o = \frac{Z}{r_o(\gamma, \lambda)} \qquad \text{where} \qquad r_o(\gamma, \lambda) = \frac{n^{-1}(\gamma)}{\lambda}. \tag{8}$$

[11]While the function U_c need not be globally concave in Y, a straightforward calculation establishes that its second derivative with respect to Y is negative at the point where this equation is satisfied.

The optimum thus requires that the ratio of Z to Y_o, which is the probability that depositors are paid, be equal to a constant r_o that depends only on γ and λ. Since the n function is decreasing, r_o falls with both γ and λ.

While (7) implies that creditors prefer a payment of Y_o to one of Z, they do not prefer the laissez faire outcome for all Y higher than Y_o. This is seen in (5), which implies that the limit of U_c when Y goes to infinity equals zero whenever γ is strictly less than one. With any risk aversion at all, there exist values of Y large enough that creditors prefer the certain clawback payoff Z to the laissez faire lottery that pays Y with probability Z/Y. So, even when (7) is satisfied, there is a maximum value of Y such that people would find the laissez faire outcome acceptable relative to the clawback outcome. At this value of Y, denoted by Y_m, creditors are indifferent between having Z for sure or the laissez faire outcome. Therefore, using (5), $Y_m(Z)$ satisfies

$$\frac{1 - e^{-\lambda Z/Y_m}}{1 - e^{-\lambda}} (Y_m)^\gamma = Z^\gamma, \tag{9}$$

or

$$Y_m = \frac{Z}{r_m(\gamma, \lambda)} \qquad \text{where} \qquad \frac{1 - e^{-\lambda r_m}}{1 - e^{-\lambda}} r_m^{-\gamma} = 1. \tag{10}$$

The variable $r_m(\gamma, \lambda)$ represents the minimum probability of a distribution of Y that creditors require for them not to prefer a clawback. For rational creditors, this minimum equals 1, whereas it is lower if (7) is satisfied. This condition also implies that r_m is smaller than r_o, since r_o gives strictly more utility under laissez faire than under a clawback while the latter leads to indifference. Differentiating the second equation in (10), we have

$$(n(\lambda r_m) - \gamma)(dr_m/r_m) + (n(\lambda r_m) - n(\lambda))(d\lambda/\lambda) - \log(r_m)d\gamma = 0$$

When (7) is satisfied $r_m < r_o < 1$, and this equation implies that both increases in λ and γ lower the smallest tolerable ratio r_m. This makes sense since, as we discussed, both these changes in parameters induce a greater desire to participate in a queue.

Before closing this section, it is worth discussing the implicit assumption that the amount a creditor is paid depends only on whether the bank is able to pay the creditor with its existing funds, and does not otherwise depend on a creditor's position in line. This assumption turns out to be restrictive, in the sense that households would *ex ante* prefer a more flexible contract. To see this, imagine a contract that pays $y(x)$ to the household that arrives right after a fraction x of households have received their required payments from the bank. Since each household believes that its probability of being among the first x to claim funds is $G(x, \lambda)$, its expected utility from an arrangement that pays $y(x)$ is

$$\int_{x=0}^{1} \frac{y(x)^{\gamma} \lambda e^{-\lambda x}}{1 - e^{-\lambda}} dx. \tag{11}$$

If banks offered the contract preferred by creditors, they would maximize this subject to the constraint that $y(x) \geq 0$ and that

$$\int_{x=0}^{1} y(x) dx = Z,$$

where this equation takes into account that all x's are in fact equally likely and assures that the total amount the bank pays out equals its available funds Z. The maximization of (11) subject to this constraint implies that

$$\frac{y(x_1)}{y(x_2)} = \left(\frac{e^{-\lambda x_1}}{e^{-\lambda x_2}} \right)^{1-\gamma},$$

for any x_1 and x_2 such that $y(x_1)$ and $y(x_2)$ are strictly positive. Since $x_1 < x_2$ leads creditors to regard x_1 as having more density than x_2, creditors' expected utility is higher if the bank makes somewhat larger payments to the creditor that shows up in position x_1. I neglect such contracts for two related reasons. The first is that the contracts we observe do not appear to have the property that different people who request funds while the bank remains open are treated asymmetrically.

The second reason to neglect these more elaborate contracts is that their enforcement requires the dissemination of a great deal of information, and this is likely to be either costly or impossible. At the very least, they require that an individual who arrives in position x be able to demonstrate this position to the courts that enforce this contract. The first-come first-served contract requires less hard information. All that needs to be verifiable for this contract to be enforceable is whether the bank still has sufficient funds at the moment the individual shows up, and whether the individual is in fact paid the fixed quantity stipulated in the contract. One thing that is not required, in particular, is for people who are paid to either know or be able to prove to others the quantity of funds available to the bank. When the bank runs out of funds, it needs to prove that its funds have been properly used. This, however, is a single tally, which needs to be audited just once, as opposed to a running tally after each individual is paid.

One can rationalize a contract whose payment is constant with the following set of strong assumptions. Suppose it is impossible to keep track of the number of people who have been paid already. Suppose further that the firm must pay a fixed cost every time it verifies the current state of its assets. This verification process also checks whether all payments that have been made to date are legitimate repayments to creditors. This verification process leaves behind hard evidence that can be shown to an outside party. Moreover, once hard evidence exists that the bank has no further assets, it is common knowledge that the bank has zero assets from then on. Once a creditor arrives at the bank, the order of moves is the following. First, the bank makes a payment to the creditor. Then, if the creditor receives any less than the maximum amount he is entitled to under a contract, he forces the bank to verify its level of current assets (unless this is already known to be zero).[12] If the fixed cost of verification is sufficiently large, the bank avoids all but one verification by setting its contractual payment to a constant and waiting to verify until it runs out of funds.[13]

[12]This assumption fits broadly with the idea in Rotemberg (2008) that an agent who is "upset" lashes out at the individual he is upset with. In the current context, individuals who are disappointed at the amount they receive can then be thought of as going to a judge who, in turn, forces the bank to verify its assets.

[13]There is some similarity between these assumptions and those used in the costly state verification literature pioneered by Townsend (1975). Both have in common that no verification costs have to be paid if the firm had sufficient funds to fulfill all its obligations. One difference is that I have emphasized the difficulties in verifying a firm's current assets in a context where payments are made in sequence while the the costly state verification literature supposes that, at a cost, it is possible to know the total resources that the firm has available before it distributes any of them. As a result, the allocation in the case where the state is verified treats all debt claimants symmetrically. The resulting allocation often coincides with what I have termed the clawback outcome.

2 Deposit contracts with no bank equity

There are now two periods labeled 0 and 2 (a third period, period 1, will be added below). The people who are alive at time 0 have utility functions that depend only on the consumption in period 2. Each of these individuals has an endowment of one unit of an asset called K and this asset pays off in period 2. The payoff of each unit of K is random: it equals Z_H in the high state (which occurs with probability π) and equals Z_L in the low state.

There are two types of individuals. One group consists of potential creditors, and these creditors have the utility function given by (3) with risk aversion coefficient $(1-\gamma)$. Under autarky, their utility level is

$$U_{ca} = \pi Z_H^{\gamma} + (1-\pi) Z_L^{\gamma}. \tag{12}$$

The other group consists of potential bank owners. These have utility functions given by

$$U_b = \bar{E}(C_b^{\xi}) \qquad 0 < \xi \leq 1, \tag{13}$$

where C_b is their period 2 consumption and $\bar{E}$ takes expectations given their own beliefs. This is identical to the utility function of creditors if $\xi = \gamma$. The case where the two types have the same utility function is particularly interesting because, since they also have the same endowment, it leaves them without any standard motivation for trading with one another. Below, I also consider the more traditional case where bankers are risk neutral (so that $\xi = 1$) while creditors are risk averse γ.

Groups of $1/\omega$ bank owners can offer contracts to a unit mass of creditors. These contracts stipulate the amount Y that the bank will distribute in period 2 on a first-come first-served basis to any creditor who has "deposited" their unit of K in period 0. I consider only contracts where the contractual payment Y is independent of the state of nature. This assumption, which reduces the attractiveness of the contract relative to laissez-faire, can be justified by supposing that the state is not verifiable at the time that individuals start making withdrawals.

Bank owners compete for depositors and I capture this by supposing that the equilibrium contract maximizes the expected utility of creditors subject to the constraint that bank owners are no worse off than they are under autarky. As I show in the next section, this competition among potential bank owners can lead them to assign some of their own endowment of K as equity into their bank.

For simplicity, this section ignores such equity infusions. This can be rationalized by supposing that ω is very large so that bank owners have only a trivial endowment relative to the deposits at their bank. In terms of modern-day institutions, the analysis without bank capital corresponds most closely to the analysis of money market mutual funds, whose managers receive fees but do not invest in an equity cushion.

This section studies whether bank contracts give more expected utility to potential depositors than autarky. This is a nontrivial question because banks without capital do not change the aggregate that depositors consume in any given state of nature. Their only role is to redistribute funds from some creditors to others. Nonetheless, I demonstrate that creditors prefer to give their endowment to banks for broad ranges of parameters.

I consider two cases. In the first, banks are not allowed to set Y above Z_H. One reason this case is interesting is that a contract that offers a payment above Z_H in period 2 is in some sense fraudulent, since there is no state of nature where the bank would be able to fully honor it. Limiting the payment to Z_H eliminates this promissory fraud.

One can interpret this limitation as fitting broadly with the government's prohibition that money market mutual funds pay their shareholders more than the "net asset value" of the fund. Money market funds in the U.S. are regulated pursuant the 1940 Investment Company Act, which specifies both the assets that a mutual fund may hold and the maximum amounts that it can pay to investors who wish to redeem their shares. Rule 2a-7 of the Investment Company Act of 1940 stipulates that Money Market Funds are allowed to value their assets using the "Amortized Cost Method of Valuation." As Rule 2a-7 puts it, this method values assets at their "acquisition cost as adjusted for amortization of premium or accretion of discount rather than at their value based on current market factors." One can thus interpret this rule as saying that money market funds are allowed to make payments based on optimistic assessments of returns, which corresponds to Z_H. The result, of course, is that less favorable outcomes lead the market value of the funds' assets (in dollars) to be lower than the number of shares outstanding.

Within the context of the model itself, allowing payments of more than Z_H in period 2 is worse for *ex post* average depositor welfare than restricting them so they are no larger than Z_H. This could lead the government to prohibit contractual obligations that exceed the bank's maximum possible resources. Nonetheless, I also consider the case where banks are free to set any payment that they wish. Part of the interest in doing this is that, even when banks have this freedom, the equilibrium contract has a payment equal to Z_H under certain conditions.

A payment of Z_H gives the same utility in the high state as both the clawback outcome and autarky. For potential creditors to prefer bank contracts with required payments of Z_H, these contracts must give more expected utility in the low state. This requires that

$$\frac{Z_L}{Z_H} > r_m(\gamma, \lambda). \tag{14}$$

When the maximum payment Y equals Z_H, bank contracts emerge in equilibrium as long as $\lambda > 0$ and γ is large enough relative to λ. This is demonstrated in the following proposition:

Proposition 1. *Let Z_L and Z_H be arbitrary and define r by $r = Z_L/Z_H$. If $\lambda > 0$, banks are required to set $Y \leq Z_H$, and*

$$\max\left(n(\lambda), \log\left(\frac{1-e^{-\lambda r}}{1-e^{-\lambda}}\right) \Big/ \log(r)\right) < \gamma, \tag{15}$$

all potential creditors deposit their endowment in equilibrium.

Proof. Because (15) requires that $n(\lambda) < \gamma$, it implies that potential depositors prefer to be paid more than Z_L in the low state. If γ were equal to $\log((1-e^{-\lambda r})/(1-e^{-\lambda}))/\log(r)$, Z_H would equal Z_L/r_m. A necessary and sufficient condition for (14) to be true is thus that γ be greater than this. Thus, in the low state, depositors prefer a payment of Z_H to a payment of Z_L. With a payment of Z_H, depositors are indifferent in the high state between the bank contract and autarky. When the state is unknown at time 0, depositors are thus strictly better off with a bank contract that offers a payment Z_H than under autarky.

If depositors prefer a payment below Z_H, banks would offer such a payment. Depositors would still prefer this contract to autarky even though it would lead bank owners to make positive profits in the high state. □

The restriction that $Y \leq Z_H$ generally reduces the attractiveness of deposit contracts. Under some additional conditions, however, banks would want to offer a contract with a payment of Z_H even when they do not have to. To see this, note first that creditors who accept contracts with liabilities per creditor Y greater than or equal to Z_H, have a subjective expected utility of

$$U_c^+ = \left[\pi \frac{1 - e^{-\lambda Z_H/Y}}{1 - e^{-\lambda}} + (1-\pi)\frac{1 - e^{-\lambda Z_L/Y}}{1 - e^{-\lambda}}\right] Y^\gamma. \tag{16}$$

A contract with $Y \leq Z_H$ pays Y to all depositors in the high state. As long as $Y > Z_L$, the expected utility of creditors who accept such a contract is

$$U_c^- = \left[\pi + (1-\pi)\frac{1 - e^{-\lambda Z_L/Y}}{1 - e^{-\lambda}}\right] Y^\gamma. \tag{17}$$

These two utility functions are different functions of Y, Z_L and Z_H because a reduction in the required payment below Z_H reduces the expected value of payments while an increase in the required payment above Z_H leaves this expected value unchanged. As result, there is a range of parameters for which Z_H is the level of payment that creditors most prefer.
In particular

Proposition 2. *For any $\lambda > 0$, a required payment of Z_H equal to Z_L/r with $r < 1$ is more desirable than any other required payment as well as being more desirable than autarky, if and only if γ and π obey*

$$\max\left(n(\lambda), \log\left(\frac{1 - e^{-\lambda r}}{1 - e^{-\lambda}}\right) \Big/ \log(r)\right) < \gamma < n(\lambda r) \tag{18}$$

and

$$\frac{\psi}{\psi+\gamma} < \pi < \frac{\psi}{\psi+\gamma-n(\lambda)} \qquad \text{where} \qquad \psi = (n(\lambda r)-\gamma)\frac{1-e^{-\lambda r}}{1-e^{-\lambda}}. \tag{19}$$

Proof. I first show that (18) defines a nontrivial region. Because the n function decreases in its argument, $n(\lambda) < n(\lambda r)$. If the second inequality in (18) were to hold as an equality, r would be equal to $n^{-1}(\gamma)/\lambda$. Creditors would then get more utility from Z_L/r than from receiving a payment of Z_L in the low state. It would then be the case that

$$\frac{1-e^{-\lambda r}}{1-e^{-\lambda}} r^{-\gamma} > 1.$$

Taking logarithms on both sides, and noticing that $r < 1$, so that the logarithm of r is negative, establishes that γ is then strictly larger than $\log((1-e^{-\lambda r})/(1-e^{-\lambda}))/\log(r)$. The left hand side of (18) is thus strictly smaller than the right hand side, so that values of γ that satisfy (18) can always be found. As we saw in Proposition (1), the first inequality in (18) implies that creditors prefer the contract that pays Z_H to autarky. When this inequality is violated, a contract with Z_H is less desirable than autarky because it gives lower utility in the low state and the same utility in the high state.

The second inequality in (18) implies that the numerator of ψ is positive so that the left hand side of (19) is between zero and one. In addition, (7) implies that $\gamma - n(\lambda)$ is positive so that the right hand side is both larger than the left hand side and between zero and 1. As a result, there exists a nonempty set of values of π satisfying both inequalities in (19).

Now consider the desirability of setting Y above Z_H. The derivative of (16) with respect to Y is

$$\begin{aligned}\frac{dU_c^+}{dY} \;=\; & \Big[\gamma\Big(\pi(1-e^{-\lambda Z_H/Y}) + (1-\pi)(1-e^{-\lambda Z_L/Y})\Big) \\ & -\frac{\lambda}{Y}\Big(\pi Z_H e^{-\lambda Z_H/Y} + (1-\pi) Z_L e^{-\lambda Z_L/Y}\Big)\Big]\frac{Y^{\gamma-1}}{1-e^{-\lambda}}.\end{aligned}$$

Evaluated at $Y = Z_H$, this derivative is

$$\frac{dU_c^+}{dY} = \left[\pi\Big(\gamma - n(\lambda)\Big) + (1-\pi)\frac{(1-e^{-\lambda r})}{(1-e^{-\lambda})}\Big(\gamma - n(\lambda r)\Big)\right] Z_H^{\gamma-1}.$$

For creditors not to wish to be paid more than Z_H, this has to be smaller than or equal to zero. Given the first inequality in (18), it is both necessary and sufficient for this to be true that both the second inequality in (18) and the second inequality in (19) hold.

Now turn to the desirability of setting Y above Z_L but below Z_H. The derivative of (17) with respect to Y is

$$\frac{dU_c^-}{dY} = \left[\gamma\left(\pi + (1-\pi)\frac{1-e^{-\lambda Z_L/Y}}{1-e^{-\lambda}}\right) - (1-\pi)\frac{\lambda Z_L}{Y}\frac{e^{-\lambda Z_L/Y}}{1-e^{-\lambda}}\right] Y^{\gamma-1}.$$

Evaluated at $Y = Z_H$, this derivative is

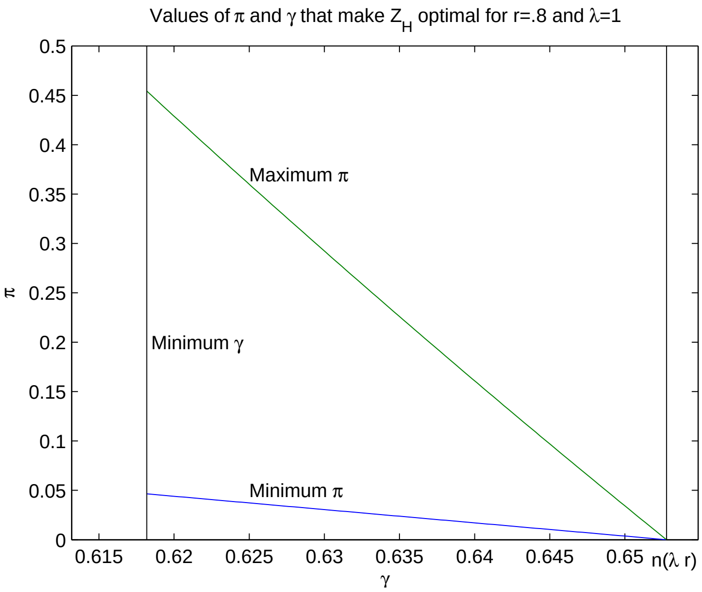
$$\frac{dU_c^-}{dY} = \left[\pi\gamma + (1-\pi)\frac{(1-e^{-\lambda r})}{(1-e^{-\lambda})}\Big(\gamma - n(\lambda r)\Big)\right] Z_L^{\gamma-1}.$$

For creditors not to wish to be paid less than Z_H, this has to be nonnegative. The first inequality in (19) is necessary and sufficient for this. □

The second inequality in (18) requires that r be smaller than $n^{-1}(\gamma)/\lambda$ so that Z_H has to be greater than Z_L/r_o. The reason this is needed is that (7) implies that creditors prefer a payment larger than Z_H in the high state. If Z_H were lower than Z_L/r_o, they would prefer it in the low state as well, and the result would be that Z_H would not be the optimal payment level. The fact that, at the equilibrium contract, Z_H can be "too large" in the low state plays a key role in some of the results below.

When γ is close to the upper limit defined by (18), ψ is close to zero so that π has to be close to zero as well. The reason is that Z_H is nearly ideal for Z_L so that, if the probability of Z_H were substantial, creditors would prefer a higher payment. As γ is reduced below its upper limit, ψ rises and the acceptable values of π rise. There is a limit, however, to the size of π, Since creditors would like the payment to exceed Z_H when the state is high, the probability of the high state cannot be large. This is illustrated in Figure 3, which depicts the ranges of γ and π that satisfy (18) and (19) when $\lambda = 1$ and $r = .8$. In this case, the maximum value of π is about .45.

Figure 3:



3 Bank capital in the two period model

Before committing to deposit their funds at a bank, potential creditors are assumed to observe the bank's equity.[14] This equity consists of $k \geq 0$ units of K per depositor that the bank owners have "invested" in their bank. The proceeds from these investments are available to pay depositors in period 2. A bank with equity k whose contract offers Y is promising total payouts to creditors in state i that equal the minimum of Y and $Z_i(1+k)$. If anything is left over, the difference between $Z_i(1+k)$ and Y is distributed to the bank owners in proportion to their initial contributions. If $(1+k)Z_L = Y$, depositors are fully insured against the state. I restrict attention to contracts with $(1+k)Z_L \leq Y$. This is without loss of generality because increasing k beyond this point does not have any effect on the consumption of depositors (who receive Y) or bank owners (who receive the returns from the additional units of K).

[14]Because each bank ends up with a volume of creditors equal to one, creditors can determine the amount of capital that each bank has per creditor (or deposit). An alternative formulation would have banks announce their capital per unit of deposits and impose a mechanism that ensures compliance with this announcement.

The equilibrium contract is the pair $\{Y, k\}$ that maximizes the utility of creditors U_c subject to the constraint that banker utility be the no lower than under autarky. Under autarky, each individual keeps her equity position in K so that bankers obtains an expected utility of

$$U_{ba} = \pi Z_H^{\xi} + (1 - \pi) Z_L^{\xi}.$$

Bank owners have a concave utility function so they benefit from sharing the investment of k equally among themselves. This means that the owners of a bank with equity k have to give up ωk units of their own endowment. In exchange, they receive ω times the difference between the bank's revenue of $(1+k)Z_i$ and its obligation Y. With $(1+k)Z_L \leq Y$, owners receive a distribution from their bank only in the high state. Denoting the consumption of bank owners in state i by C_{bi}, we have

$$C_{bL} = Z_L(1 - \omega k) \qquad \text{and} \qquad C_{bH} = Z_H(1 - \omega k) + \omega(Z_H(1+k) - Y). \tag{20}$$

The capital infusion k thus lowers consumption in the low state while having no effect on the high state for given Y. By the same token, creditors benefit from a higher k in the low state whenever there is less than full insurance so $Y > Z_L(1+k)$. This implies that, when capital infusions are possible, the equilibrium utility of bank owners cannot exceed their utility under autarky. The reason is that it would then be possible to increase the utility of depositors by raising k while still giving bank owners U_{ba}. As a result, the equilibrium must satisfy

$$\pi(Z_H(1+\omega) - \omega Y)^{\xi} + (1 - \pi)(Z_L(1 - \omega k))^{\xi} = U_{ba}. \tag{21}$$

This equation gives the amount by which owners who put up equity k need to be compensated by being allowed to pay out a Y lower than Z_H.[15] It is convenient to recast (21) as an equation giving k as a function of Y

$$k = \phi(Y) \qquad \text{with} \qquad \phi' = -\frac{\pi}{Z_L(1-\pi)} \left(\frac{Z_L(1 - \omega k)}{Z_H(1+\omega) - \omega Y} \right)^{1-\gamma} < 0. \tag{22}$$

[15] Notice that, since the bank must have funds left over in the high state to make payments to its owners when $k > 0$, equilibrium deposit contracts cannot involve bank runs in this state.

While depositors are assured of receiving Y in the high state, they receive it in the low state only with probability $(1+k)Z_L/Y$. From their point of view, their expected utility is thus

$$U_c = \left(\pi + (1-\pi)\frac{1-e^{-\lambda(1+k)Z_L/Y}}{1-e^{-\lambda}}\right) Y^{\gamma}.$$

Taking into account the need to vary k as Y varies in accordance with (22), the change in U_c as Y varies is given by

$$dU_c = U_{cY}dY + U_{ck}\phi' dY, \tag{23}$$

where

$$\begin{aligned} U_{cY} &= \left[\gamma\left(\pi + (1-\pi)\frac{1-e^{-\lambda(1+k)Z_L/Y}}{1-e^{-\lambda}}\right) - \frac{(1-\pi)\lambda(1+k)Z_L e^{-\lambda(1+k)Z_L/Y}}{Y(1-e^{-\lambda})}\right] Y^{\gamma-1} \\ U_{ck} &= \left[\frac{(1-\pi)\lambda Z_L e^{-\lambda(1+k)Z_L/Y}}{1-e^{-\lambda}}\right] Y^{\gamma-1}. \end{aligned}$$

At an interior equilibrium, dU_c/dY in (23) must be zero.

I now show that, even when $\xi = \gamma$, bank owners may choose to capitalize their banks.

Proposition 3. *Suppose that $\xi = \gamma$, and λ, γ, and r satisfy (18). If either π satisfies (19) or Y is required to be smaller than or equal to Z_H, the equilibrium contract features $k > 0$ and $Y < Z_H$.*

Proof. The conditions of the proposition imply that, with k set equal to zero, $Y \leq Z_H$ and creditors prefer this contract to autarky. If the equilibrium contract with $k = 0$ has $Y < Z_H$, bank owners obtain more utility than at autarky when $k = 0$ so they compete by setting $k > 0$. To demonstrate that $k > 0$ when the equilibrium contract with $k = 0$ has $Y = Z_H$, it is sufficient to show that $dU_c/dY < 0$ in (23) when $Y = Z_H$ and $k = 0$. Letting r denote Z_L/Z_H and using (22) in (23), dU_c/dY when $Y = Z_H$ and $k = 0$ is

$$\begin{aligned} \frac{dU_c}{dY} &= \left[\gamma\pi + \frac{1-\pi}{1-e^{-\lambda}}\left(\gamma(1-e^{-\lambda r}) - \lambda r e^{-\lambda r}\right) - \frac{(1-\pi)\lambda e^{-\lambda r}}{1-e^{-\lambda}}\frac{\pi}{1-\pi}r^{1-\gamma}\right] Y^{\gamma-1} \\ &= \left[\frac{1-\pi}{1-e^{-\lambda}}\left(\gamma(1-e^{-\lambda r}) - \lambda r e^{-\lambda r}\right) + \pi\left(\gamma - \frac{\lambda r e^{-\lambda r}}{1-e^{-\lambda}}r^{-\gamma}\right)\right] Y^{\gamma-1} \\ &< \left[\frac{1-\pi}{1-e^{-\lambda}}\left(\gamma(1-e^{-\lambda r}) - \lambda r e^{-\lambda r}\right) + \pi\left(\gamma - \frac{\lambda r e^{-\lambda r}}{1-e^{-\lambda r}}\right)\right] Y^{\gamma-1} < 0, \end{aligned}$$

where the first inequality follows from the fact that (18) implies that $(1-e^{-\lambda r})r^{-\gamma}/(1-e^{-\lambda})$ is less than one while the second inequality follows from the fact that (18) requires that $\gamma < n(\lambda r)$. □

This proposition may be surprising because it gives fairly general conditions under which bank owners offer insurance to depositors in spite of the fact that they both have the same risk tolerance. When Z_H is the depositor's favorite value of Y, as implied by the combination of (18) and (19), the insurance is offered because creditors in the low state would prefer a lower value of Y. This motivation for insurance remains valid when banks are not allowed to make a payment of more than Z_H, as long as the second inequality in (18) holds. Creditors are then willing to give up Y in exchange for a higher value of k.

As might be expected, this willingness of bankers to insure depositors by infusing equity into their banks carries over to the case where $\xi = 1$ and $\gamma < 1$. One question that arises in this case is whether bank owners would then fully insure their depositors by placing sufficient equity in their bank to pay Y to all depositors in the low state. Full insurance requires that Y be set equal to the expected value of Z_i, $\pi Z_H + (1-\pi)Z_L$, and that depositors receive in this in both states of nature. Since banks only have $(1+k)Z_L$ in the low state, $(1+k)Z_L$ must equal the expected value of Z_i as well. This is possible only if bank owners have sufficient resources. When bank owners stake their entire endowment as equity, k equals $1/\omega$. Therefore, bank owners have sufficient resources to provide full insurance if

$$\omega \leq \frac{\pi(Z_H - Z_L)}{Z_L}. \tag{24}$$

It turns out, however, that this condition is insufficient for full insurance to be observed in equilibrium.

Proposition 4. *Suppose that $\xi = 1$ and $\gamma < 1$ and (24) holds. If (7) is violated, the equilibrium contract features $(1+k)Z_L < Y$ whereas $(1+k)Z_L = Y$ if (7) is satisfied.*

Proof. Since $\xi = 1$ and (24) holds, bank owners are willing to offer the full insurance contract with $Y = \pi Z_H + (1-\pi)Z_L = (1+k)Z_L$. Moreover, since $\gamma < 1$, potential depositors strictly prefer to deposit their endowment and receive this full insurance payment to remaining in autarky. Lowering Y below $\pi Z_H + (1-\pi)Z_L = (1+k)Z_L$ implies that depositors receive less in both states regardless of k, which is bad for them.

Using (23), the change in depositor utility from raising Y starting at $Y = (1+k)Z_L$ is

$$dU_c = \left[\left(\gamma - \frac{(1-\pi)\lambda e^{-\lambda}}{1-e^{-\lambda}}\right) dY + \frac{(1-\pi)\lambda Z_L e^{-\lambda}}{1-e^{-\lambda}} \phi' dY\right] Y^{\gamma-1} = \left(\gamma - \frac{\lambda e^{-\lambda}}{1-e^{-\lambda}}\right) Y^{\gamma-1} dY,$$

where the second equality follows from the fact that, with $\xi = 1$, (22) implies that $\phi' = -\pi/[(1-\pi)Z_L]$. It follows that dU_c/dY is positive if and only if (7) is satisfied. $\square$

Condition (7) leads depositors to wish to "gamble" with bank runs, and thus prevents full insurance from being offered in equilibrium even though bankers are risk neutral and depositors are risk averse. The ensuing outcome does not maximize true average depositor welfare *ex post*, since this would be accomplished by letting bankers fully insure depositors. In the next section, I consider how policy can affect these *ex post* outcomes more generally.

4 Regulatory Policy in the Two Period Model

Even in democracies, political discourse often involves elements of paternalism. Such paternalism is easier to understand if people are biased in the pursuit of their own objectives, as I have assumed here. In this case, outside observers who care for depositors but do not agree with the depositors' self-assessment of their competence would be inclined to institute policies that raise the average *ex post* utility of depositors. This true average *ex post* creditor utility, which I denote by $\tilde{U}_c$, would equal their expected *ex ante* utility in the absence of bias. This section is devoted to studying the effect of various financial regulations on $\tilde{U}_c$, which is given by

$$\tilde{U}_c = \left(\pi + (1-\pi)\frac{(1+k)Z_L}{Y}\right) Y^{\gamma}. \tag{25}$$

Since actual policy is probably also guided by other considerations, including creditor's own perceived utility as well as the repercussions of regulation on other variables, this paper's analysis of policy is necessarily partial.

4.1 Capital Requirements

I continue to suppose that owners are compensated for increases k by reductions in Y that keep them indifferent, as in (21) and (22). The change in $\tilde{U}_c$ when one changes Y while varying k in this manner is

$$d\tilde{U}_c = \tilde{U}_{cY} dY + \tilde{U}_{ck} \phi' dY,$$

where

$$\begin{aligned} \tilde{U}_{cY} &= \left[\gamma \left(\pi + \frac{(1-\pi)(1+k)Z_L}{Y} \right) - \frac{(1-\pi)(1+k)Z_L}{Y} \right] Y^{\gamma-1} \\ \tilde{U}_{ck} &= (1-\pi) Z_L Y^{\gamma-1}. \end{aligned}$$

It follows that:

Proposition 5. *Suppose that (18) holds and that either banks are prevented from paying more than Z_H or that (19) holds. Further, suppose that (24) holds. Then, a sufficient condition for* ex post *average utility to rise from its equilibrium level when k is increased while Y is reduced so as to keep bank owners indifferent is*

$$\frac{(1+k)Z_L}{Y} \geq \frac{\log(\lambda) - \log(1-e^{-\lambda})}{\lambda}. \tag{26}$$

Proof. The conditions of this proposition imply that potential creditors deposit their endowment and that $k > 0$ in equilibrium. Since (24) holds, the equilibrium is interior and dU_c/dY in (23) equals zero. Therefore $U_{cY} + U_{ck}\phi' = 0$, where U_{cY} and U_{ck} are both positive. *Ex post* average utility would increase by raising k if $\hat{U}_{cY} + \tilde{U}_{ck}\phi'$ were negative. The first term of $\tilde{U}_{cY}$ is smaller than the first term of U_{cy} because $(1 - e^{-\lambda(1+k)Z_L/Y})/(1 - e^{-\lambda})$ is larger than $(1+k)Z_L/Y$. Thus, a sufficient condition for $\tilde{U}_{cY} + \tilde{U}_{ck}\phi'$ to be negative is that

$$\frac{\lambda e^{-\lambda(1+k)Z_L/Y}}{1-e^{-\lambda}} < 1.$$

□

The ratio $(1+k)Z_l/Y$ gives the actual probability that depositors receive Y in the low state. This proposition thus says that, if this probability is above the value given by the right hand side of (26), *ex post* utility is increased by raising the capital of banks above its equilibrium value. The value of $(1+k)Z_L/Y$ that satisfies (26) with equality is the value of x that leads $G'(x)$ in (2) to equal one. In other words, it is the value of x such the subjective probability of being among the first x has a slope of one with respect to x, just as does the true probability. The second equation in (2) implies that the slope of the subjective probability is larger for smaller values of x and smaller for larger ones.

This result has the following interpretation: One important benefit of raising k while reducing Y is that depositors have an increased probability of receiving Y rather than nothing. For $(1+k)Z_l/Y$ above the value on the RHS of (26), however, the subjective probability of receiving Y rises less than the objective probability so that depositors perceive this benefit as being smaller than it is. As a result, they end up better off on average if k is increased (and Y reduced) relative to the contract that they find most attractive.

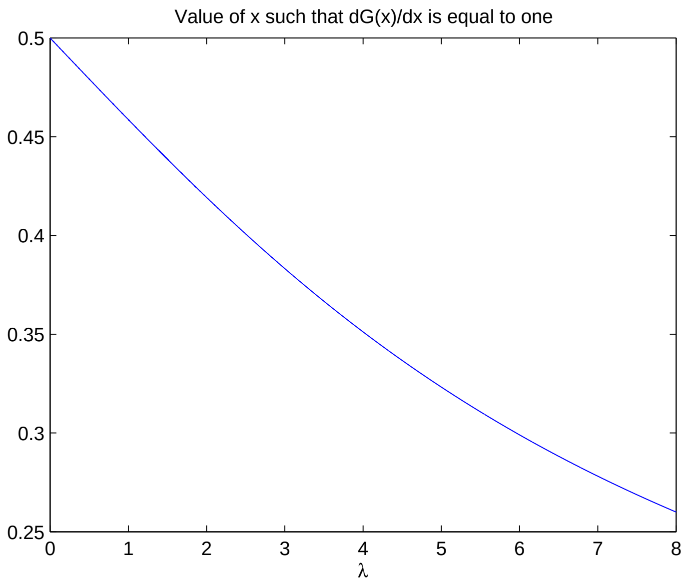
Since k and Y are determined in equilibrium, condition (26) does not apply directly to the exogenous parameters. However, because $k > 0$ and $Y < Z_H$, $(1+k)Z_L/Y$ is necessarily larger than the exogenous ratio Z_L/Z_H so a stronger sufficient condition than (26) involves putting Z_L/Z_H on the left hand side. Even in this stronger form, condition (26) is not very demanding. Figure 4 demonstrates this by depicting the value on the RHS of (26) for various values of λ. Its limit is .5 for λ arbitrarily low and then falls steadily as λ rises. This is consistent with the fact that, for high values of λ the probability of being among the first x is close to one even if x is small, so that $G'(x)$ becomes smaller than one for quite small values of x.

Even when (26) is satisfied, depositors oppose capital requirements that raise k, since they obtain the contract they most like under laissez faire.

4.2 Clawbacks and deposit insurance

A second policy choice is to treat financial contracts like non-financial ones and subject them to clawback in bankruptcy. This would mean that depositors who arrive after all the funds at their bank are exhausted could sue and end up receiving the same resources as those who

Figure 4:



arrived while the bank still had funds. The utility of depositors from a contract with $\{k, Y\}$ would then be

$$U_d = \pi Y^\gamma + (1-\pi)((1+k)Z_L)^\gamma, \tag{27}$$

so that the change in utility when Y and k change is

$$dU_d = \gamma \left[\pi Y^{\gamma-1} dY + (1-\pi) Z_L^\gamma (1+k)^{\gamma-1} dk \right].$$

When $\xi = \gamma$, the ratio of dk/dY that keeps depositors indifferent at $k = 0$ and $Y = Z_H$ is equal to ϕ' in (22). This implies that bank owners set $k = 0$ and $Y = Z_H$. The outcome is thus the same as under autarky even if bank owners are allowed to add capital to their banks. The end result of this is that a policy of clawbacks essentially eliminates the incentive to deposit in a bank, though this result obviously depends crucially on my assumption that everyone is endowed with the same asset to begin with.

Now consider a policy of offering deposit insurance, which is financed by lump sum taxes on those individuals who are potential depositors. Since creditors take taxes as given, outlays from the deposit insurance scheme raise the attractiveness of deposits. Indeed, deposits with insurance are more attractive than either autarky or the outcome with clawbacks as long as their official payout exceeds the certainty equivalent amount $(\pi Z_H^\gamma + (1-\pi)Z_L^\gamma)^{1/\gamma}$.

All the taxes that pay for the insurance are assumed to come from the depositors themselves, however. Depositor consumption in the high state is thus equal to Y, where Y is the amount bank owners pay out in this state. Similarly, depositor consumption in the low state remains $(1+k)Z_L$ where k is the amount of K that owners pledge in period 0. Thus, the expected utility of depositors is once again given by (27) as long as Y is interpreted as the amount the government takes from the bank in the high state.

If the government wants to keep owners willing to operate their banks, which is consistent with my assumption that only potential depositors are taxed, (21) must still hold. When $\xi = \gamma$, this implies that the changes in k and Y that leave bank owners indifferent leave depositors indifferent as well. In other words, adding capital to banks does not improve welfare. When financed in this way, the best deposit insurance outcome is the autarkic Pareto Optimum.

4.3 Regulations on the asset-side of banks' balance sheets

Since the economy considered so far has a single productive asset, it has to be modified to discuss the possibility that regulators would force banks to change the composition of their assets. In this subsection, I examine a particularly simple modification for this purpose. I introduce an alternate asset that is similar to K in the sense that it has a payoff of $Z_H + dZ_U$ with probability $\pi + d\pi$ and a payoff $Z_L + dZ_L$ with probability $1 - \pi - d\pi$ where dZ_H, dZ_L and $d\pi$ are all small. Asset K can be converted into this alternative asset, but there are some restrictions. First, individuals who manage their own asset can only convert either their entire endowment or none of it. Second, banks can convert their K into the alternative asset but can only do it if they convert all the assets in their possession. Moreover, I require depositors to either hand over their entire endowment to a bank or to keep it all themselves.

These rather strong assumptions ensure that no one can hold a diversified portfolio, and thereby simplify the analysis considerably. In particular, they make it easy to state a condition under which the alternative asset would be undesirable to depositors if they were to hold it directly. Using the autarkic utility (12), and supposing that dZ_H, dZ_L and $d\pi$ are small enough that they affect utility only to first order, this condition is

$$dU_{ca} = \left[\left(1 - \left(\frac{Z_L}{Z_H}\right)^\gamma\right) d\pi + \gamma\pi\frac{dZ_H}{Z_H} + \gamma(1-\pi)\left(\frac{Z_L}{Z_H}\right)^\gamma \frac{dZ_L}{Z_L}\right] Z_H^\gamma < 0. \tag{28}$$

The objective of this subsection is to prove that combinations of dZ_H, dZ_L and $d\pi$ can be found such that (28) holds and depositors are nonetheless better off on average if banks are forced to hold the alternate asset. I further show that some of these combinations reduce the perceived expected utility of depositors from depositing their endowment at banks that hold this alternate asset, so that the alternate asset would only be held if the government mandated it.

These results are demonstrated in a setting that is purposefully kept simple, and I abstract from bank capital in this subsection. As already discussed, this fits with Money Market Funds, which are indeed heavily regulated in terms of the assets that they may hold. Without bank equity and with $Y = Z_H$, the average *ex post* utility of depositors whose bank holds their original asset K is given by (25) with $Y = Z_H$ and $k = 0$. Differentiating this, one obtains the change in *ex post* utility if the bank replaces K with the alternate asset:

$$d\tilde{U}_c = \left[\left(1 - \frac{Z_L}{Z_H}\right) d\pi + \left(\pi\gamma - (1-\pi)(1-\gamma)\frac{Z_L}{Z_H}\right)\frac{dZ_H}{Z_H} + (1-\pi)\frac{Z_L}{Z_H}\frac{dZ_L}{Z_L}\right] Z_H^\gamma. \tag{29}$$

Without bank equity and with $Y = Z_H$, the expected utility of creditors is given by (16) with Y given by Z_H. Differentiating this equation, one obtains the change in perceived utility if the bank holds the alternate asset instead:

$$\begin{aligned} dU_c \;=\; & \Bigg[(e^{-\lambda Z_L/Z_H} - e^{-\lambda})d\pi + \gamma(1 + \pi(e^{-\lambda Z_L/Z_H} - e^{-\lambda}))\frac{dZ_H}{Z_H} \\ & + \frac{(1-\pi)\lambda Z_L e^{-\lambda Z_L/Z_H}}{Z_H}\left(\frac{dZ_H}{Z_H} - \frac{dZ_H}{Z_H}\right)\Bigg]\frac{Z_H^\gamma}{1 - e^{-\lambda}}. \end{aligned} \tag{30}$$

It follows that:

Proposition 6. *Suppose that $k = 0$ and that either (18) and (19) hold or that Y must be no larger than Z_H, (15) holds and the first inequality in (19) holds. There exist alternate* *assets with $dZ_H = 0$, $dZ_L < 0$, $d\pi > 0$ and*

$$\frac{1}{r^{-1}-1}\left|\frac{dZ_L}{Z_L}\right| < \frac{d\pi}{1-\pi} < \min\left(\frac{\gamma}{r^{-\gamma}-1}, \frac{(\lambda r)e^{-\lambda r}}{e^{-\lambda r}-e^{-\lambda}}\right)\left|\frac{dZ_L}{Z_L}\right|, \tag{31}$$

where $r = Z_L/Z_H$. These satisfy (28) and lead dU_c in (30) to be negative while making (29) positive so that $\tilde{U}$ rises relative to K when banks hold these alternate assets.

There also exist alternate assets with $dZ_L = 0$, $dZ_H < 0$, $d\pi > 0$ and

$$\frac{\pi\gamma-(1-\pi)(1-\gamma)r}{1-r}\left|\frac{dZ_H}{Z_H}\right| < d\pi < \frac{\pi\gamma}{1-r^{\gamma}}\left|\frac{dZ_H}{Z_H}\right|. \tag{32}$$

These too satisfy (28) while making (29) positive so that ex post *average utility $\tilde{U}$ rises when these assets are held at depositors' banks. There are parameter ranges consistent with (32) such that banks holding these alternate assets give less perceived utility U_c to their depositors than banks holding the original asset K.*

Proof. According to Proposition 2, $Y = Z_H$ when (18) and (19) hold. The reasoning in the Proposition's proof also establishes that, assuming (15) holds, the first inequality in (19) implies that Y is no lower than Z_H. Thus, the conditions of the proposition imply that (29) and (30) give the change in average *ex post* utility and expected utility respectively.

The first inequality in (31) implies that $d\tilde{U}_c > 0$. The inequality with the first term of the min operator ensures that (28) holds while the one with the second term leads dU_c to be negative. I first demonstrate that the left hand side of (31) is always smaller than the first term of the min operator. The former is actually the same as the latter with γ set to equal one. For the left hand side of (31) to be smaller, the derivative of $\gamma/(r^{-\gamma}-1)$ with respect to γ be negative. The expression tends to $1/\log(1/r)$ when γ tends to zero while it equals $1/(1/r-1)$ when $\gamma = 1$ with the latter being smaller than the former. The derivative itself is

$$\frac{r^{-\gamma}-1-\gamma\log(r)r^{-\gamma}}{(r^{-\gamma}-1)^2}.$$

The numerator of this expression is zero at $\gamma = 0$. For the derivative to change sign between $\gamma = 0$ and $\gamma = 1$, this numerator would have to have another zero in this range. But, this is impossible because the derivative of the numerator with respect to γ equals $-\gamma^2(\log(r))^2 r^{-\gamma}$, which is negative for $r > 0$.

I now prove that the left hand side of (31) is smaller than the second term of the min operator. Since the denominators of both expressions are positive, this inequality is true if

$$e^{-\lambda r} - e^{-\lambda} < \lambda(1-r)e^{-\lambda r} \qquad \text{or} \qquad 1 - e^{\lambda(r-1)} < \lambda(1-r).$$

Since $\lambda(1-r)$ is an arbitrary positive number, this inequality requires that $x/(1-e^{-x})$ be greater than one for all positive x. Its limit when $x = 0$ is 1 while it equals infinity for x infinite. The derivative of this expression with respect to x is

$$\frac{1 - e^{-x} - xe^{-x}}{(1-e^{-x})^2}.$$

The numerator of this is zero at $x = 0$ and the derivative of this numerator with respect to x is $x^2 e^{-x}$ which is always positive. Thus, there is no other value of x for which the numerator equals zero. As a result, $x/(1-e^{-x})$ is monotone increasing from $x = 0$ onwards.

The first inequality of (32) implies that alternate assets with this property lead to $d\tilde{U}_c > 0$ while the second inequality ensures that they satisfy (28). The numerator of the left hand side is smaller than the numerator of the right hand side while the denominator of the right hand side is smaller. Thus, the left hand side is indeed smaller than the right hand side.

Using (29), an increase in π accompanied by a reduction in Z_H lowers U_c if

$$d\pi < \gamma\pi + \frac{\gamma - (1-\pi)(\lambda r)e^{-\lambda r}}{e^{-\lambda r} - e^{-\lambda}}.$$

For this change to increase $\tilde{U}_c$, it must satisfy the first inequality in (32). Values of $d\pi$ that satisfy both simultaneously can be found if and only if the left hand side of (32) is smaller than the right hand side of the above inequality. In the numerical experiments I carried out, this turned out to be true whenever $(1-e^{-\lambda r})r^{-\gamma} > (1-e^{-\lambda})$ as required by (15). This condition is readily verified when $\lambda = 1$ and $r = .8$ while π and γ belong to the set depicted in Figure 3 that satisfies (18) and (19). □

Proposition 6 shows that there are two broad ways of raising *ex post* welfare by changing the characteristics of bank assets. They both involve increasing the probability π of realizing a favorable outcome. In the first, depositors give up some payoff in the low state by lowering Z_L, while in the latter they give up Z_H. If the rise in π is sufficiently modest, utility would fall if the assets were held directly by depositors and perceived utility from having the alternate assets at the bank would fall as well. It is nonetheless possible to raise *ex post* average welfare because lowering the probability of the low state reduces an inefficiency. This inefficiency is the result of the fact that depositors are not all treated the same in a bank run. Depositors, of course, are attracted to this difference in treatment even though it is bad for them *ex post*.

5 A three period model with the potential for costly asset transfers

I maintain the assumption that creditors can deposit their endowment at a bank in period 0 and that they care only about consumption in period 2. The key issue I take up in this section is the effect of permitting the asset K to be reassigned, at a cost, before it yields Z_i. This transfer (or liquidation) is allowed to occur in period 1. As in Allen and Gale (1998), this is the period where uncertainty about the realization of Z_i in period 2 is resolved. Even though I suppose that no depositor wishes to consume in this period, I show that depositors may choose bank contracts that allow them to withdraw from their bank account in period 1.

To make the cost of these withdrawals transparent, I introduce a new class of agents which I call arbitrageurs. For simplicity, arbitrageurs live only in period 1 and 2. They are born with a substantial endowment M of a good that they can consume in period 1. This good is called good 1, with good 2 representing the good that is produced by K. The utility function of arbitrageurs is assumed to be

$$U_a = C_{1a} + C_{2a}, \tag{33}$$

where C_{ia} represents their consumption of good i in period i. Arbitrageurs are thus willing to buy asset K in period 1 for a price that equals the yield they expect to receive from this asset in period 2. When controlled by arbitrageurs, this yield is ρZ_i in state i. The constant $\rho \leq 1$ captures the idea that the transfer of control between a bank and an arbitrageur is costly, perhaps because some knowledge about how to manage the asset is lost in the process.

Arbitrageurs are also assumed to be able to credibly pledge asset K to agents that give them units of good 1. The existence of arbitrageurs makes it possible for banks to give depositors units of good 1 that banks obtain by selling units of asset K. These depositors can then turn to arbitrageurs, and offer them the units of good 1 they obtained from their bank in exchange for promises to receive good 2. Competition among arbitrageurs ensures that the price in terms of good 1 for a unit of good 2 is one. The result is that, if banks offer deposit contracts with a promised payment of Y_1 in period 1, those depositors who successfully withdraw their funds in period 1 obtain Y_1 units of consumption in period 2.

The critical question is whether equilibrium contracts offered by banks allow depositors to withdraw funds in period 1. It is a nontrivial question because, when $\rho < 1$, period 1 withdrawals costs resources. It turns out that there are two different reasons for equilibrium deposit contracts to be demandable in period 1.

First, depositors may have more queue optimism in period 1, right after information about the state becomes available, than in period 2. The idea here is not that depositors are particularly well informed, only that they might see themselves as reacting more promptly than others to public information. The example of the 2007 run on Northern Rock, a UK bank dedicated to mortgages, may be useful for illustration. Northern Rock financed many of its mortgages in public securities markets, and the cost of this funding rose through 2007 with the result that its stock price halved between January and September 1, 2007. Lines of depositors wishing to withdraw funds did not form until September 15, the day after the Bank of England first pledged that it would support Northern Rock with its own funds. This does not demonstrate that depositors would not continue to be optimistic about withdrawals if their deposits did not allow them to obtain their funds on demand at all times. It is, however, consistent with the possibility that depositors may be particularly optimistic regarding their capacity to react quickly to public events.

Imagine, then, that depositors have a subjective probability that they will be among the first x to withdraw in the low state equal to $G(x)$ in period 1 and equal to x in period 2. Suppose, further, that we restrict attention to banks that put up no capital and to contracts that promise a payment of Z_H in period 2. If the contracts does not give a depositor access to funds in period 1, the depositors expected utility from the contracts is

$$\left(\pi + (1-\pi)\frac{Z_L}{Z_H}\right) Z_H^{\gamma}. \tag{34}$$

Consider now the alternative of offering the depositor a contract with one added feature, namely the right to withdraw ρZ_H in period 1 in exchange for extinguishing the bank's obligation. It follows that

Proposition 7. *Suppose that either (18) and (19) hold or that Y cannot exceed Z_H and that both (15) and the first inequality in (19) holds. There exist values of $\rho < 1$ such that*

$$\rho^{\gamma} > \frac{Z_L}{Z_H} \Bigg/ \frac{1 - e^{-\lambda Z_L/Z_H}}{1 - e^{-\lambda}} . \tag{35}$$

Suppose (for this proposition only) that depositors believe that their probability of being among the first x to withdraw is x in period 2 but $G(x, \lambda)$ in period 1. Then, if ρ satisfies (35), the contract that allows withdrawals of ρZ_H in period 1 in lieu of the right of Z_H in period 2 is preferred by depositors to the contract that gives only the latter right.

Proof. The Appendix proves that $G(x)$ is strictly increasing in λ, which implies that the right hand side of (35) is smaller than one. As a result, there exist values of $\rho < 1$ satisfying this inequality.

The conditions of the proposition imply that, if banks were unable to offer payments in period 1, the equilibrium contract would have a payment of Z_H. The modified contract that allows withdrawals of ρZ_H in period 1 does not create runs in period 1 if the state is high because depositors have nothing to lose by waiting until period 2. The only equilibrium in the low state with the modified contract involves a run. If a depositor expects all others to withdraw, he should run too because otherwise he gets nothing. Even if no one else withdraws, an individual withdrawing in the low state gets ρZ_H while he would only expect to get Z_H with probability Z_L/Z_H by waiting until period 2. The utility of the former is $(\rho Z_H)^\gamma$ while that of the latter is $(Z_L/Z_H)Z_H^\gamma$ so the former is larger than the latter if (35) holds.

Since the modified contract gives the same utility as the original one in the high state, it is preferable if it gives more utility in the low state. This occurs if

$$\frac{1-e^{\lambda Z_L/Z_H}}{1-e^{-\lambda}}(\rho Z_H)^\gamma > \frac{Z_L}{Z_H}Z_H^\gamma,$$

which is implied by (35). □

This proposition thus shows that, if optimism exists only about withdrawals in period 1, certain contracts that allow withdrawals at time 1 will be preferred to contracts that are otherwise identical but allow withdrawals only in period 2. This idea should also extend to more elaborate contracts than those considered in the proposition. I do not consider the issue further because it turns out that there is also a subtler reason for contracts that allow withdrawals in period 1 to dominate those that do not. Allowing for these withdrawals has the advantage of allowing contractual terms to be modified to suit the queuing bias of depositors. Instead of being forced to make a single contractual payment in both states of nature, banks can offer a payment in period 1 that is tailored to the low state.

To show that this is valuable on its own, I now let depositors be equally optimistic about being among the first x to withdraw in period and in period 2.

Proposition 8. *If (18) holds, there exists values of $\rho < 1$ such*

$$\rho^\gamma > \frac{(1-e^{-\lambda Z_L/Z_H})Z_H^\gamma}{(1-e^{-n^{-1}(\gamma)})(\lambda Z_L/n^{-1}(\gamma))^\gamma}. \tag{36}$$

Suppose that (18) and the first inequality of (19) hold, that k is zero and that banks are prevented from offering a payment of more than Z_H in period 2. Then, for values of $\rho < 1$ that satisfy (36), the equilibrium contract offers depositors a choice between withdrawing Z_H in period 2 or $\rho Z_L/r_o$ in period 1, where r_o is defined in (8). This equilibrium contract leads to a run if and only if the state is low.

Proof. The right hand side of (36) gives the ratio of U_c in the low state when resources equal Z_L and the payments equals Z_H to the level of U_c when resources equal Z_L and payments equal Z_L/r_o. This ratio is therefore smaller than one unless $Z_L/r_o = Z_H$. Condition (18), however, implies that $\gamma < n(\lambda Z_L/Z_H)$ so that $Z_L/r_o < Z_H$. Therefore, values of $\rho < 1$ that satisfy (36) can be found.

The conditions of the proposition imply that banks offer a payment of Z_H in period 2 if they do not offer to make payments in period 1. For any value of ρ satisfying (36), the utility of depositors when resources equal ρZ_L and the payment is set at $\rho Z_L/r_o$ equals

$$\frac{1-e^{-\frac{\lambda\rho Z_L}{\lambda\rho Z_L/n^{-1}(\gamma)}}}{1-e^{-\lambda}}\left(\frac{\lambda\rho Z_L}{n^{-1}(\gamma)}\right)^{\gamma} = \frac{1-e^{-n^{-1}(\gamma)}}{1-e^{-\lambda}}\left(\frac{\lambda\rho Z_L}{n^{-1}(\gamma)}\right)^{\gamma},$$

and (36) implies that this exceeds the utility of being paid Z_H when resources equal Z_L. The contract that pays $\rho Z_L/r_o$ in period 1 does in fact yield this level of utility in the low state because depositors run to the bank in period 1. The reason they withdraw their funds in this state is that, whether others withdraw their funds or not, they thereby guarantee themselves a higher level of utility than they obtain by waiting.

They do not run in the high state because the required payment, $\rho Z_L/r_o$ is less than ρZ_H which is less than the amount that depositors obtain by waiting. Waiting nets depositors Z_H whether others withdraw in period 1 or not because depositors who withdraw in period 1 get less than ρZ_H. Relative to a contract that only allows Z_H to be withdrawn in period 2, the proposed contract therefore gives the same utility in the high state and higher utility in the low state.

Lastly, the derivation of (8) implies that $\rho Z_L/r_o$ is indeed the depositor's favorite payment when the bank's resources equal ρZ_L. □

The advantage of the contract that gives depositors the right to withdraw $\rho Z_L/r_o$ in period 1 is that it lets banks make a payment that depositors prefer to Z_H when the state is low. Note that, if banks were free to adjust their second period payment to raise the utility of their depositors, condition (7) would lead them to set it equal to Z_H/r_o, which exceeds Z_H. This would raise utility in the high state as well and would make contracts that allow deposits to withdraw funds in period 1 even more desirable. Moreover, since the payment and the utility obtained in the high state would be even larger, the arguments in Proposition 8 would still establish that the optimal payment in period 1 is $\rho Z_L/r_o$. As a consequence, there would still be a run in the low state.

I now turn to the analysis of bank capital in the three period model. As far as bank owners are concerned, the addition of period 1 does not matter. For the bank to be able to make payments based on $(1+k)$ units of K per depositor, each bank owner must give up ωk units. Since the proceeds from this equity injection are paid out to depositors in the low state, owners' consumption in the low state remains $Z_H(1-\omega k)$. Owner utility thus remains equal to the left hand side of (21), where Y equals the payments made to depositors in the high state. Since bank owners are only allowed to keep the bank resources that remain after all depositors have been paid their contractual obligations, Y must equal the amount that depositors are entitled to collect in period 2 (if they do not withdraw in period 1). For bankers to be willing to provide positive quantities of capital, (21) and (22) must hold with this interpretation of Y.

As long as $(1+k)Z_L/r_o$ is smaller than Y, the argument made in Proposition 8 implies that depositors prefer to have the choice of $\rho(1+k)Z_L/r_o$ in period 1 or Y in period 2 rather than having only the right of demanding Y in period 2. Indeed, the contract that allows depositors to withdraw $\rho(1+k)Z_L/r_o$ in period 1 is the one that depositors prefer among all those in which the firm pays out Y in period 2 and has $(1+k)$ units of K with which to pay off depositors in the low state. For low levels of capital, conditions (18) and (19) thus ensure that this is the equilibrium contract.

Because the equilibrium contract sets the payment in period 1 to $\lambda\rho(1+k)Z_L/n^{-1}(\gamma)$ and the firm has $\rho(1+k)Z_L$ units of good 1 available in the low state of period 1, the actual probability that depositors are paid in this state equals $\lambda/n^{-1}(\gamma)$. This probability is independent of k because the period 1 payment is adjusted upwards when owners inject more capital into their bank.

The perceived utility of depositors under this contract is

$$U_c = \pi Y^\gamma + (1-\pi)\frac{1-e^{-n^{-1}(\gamma)}}{1-e^{-\lambda}}\left(\frac{\lambda\rho(1+k)Z_L}{n^{-1}(\gamma)}\right)^\gamma. \tag{37}$$

This equation shows that a reduction in ρ makes k less attractive relative to Y. As the next Proposition shows, this implies that bank owners will capitalize their banks in equilibrium for some values of $\rho < 1$ but not for others.

Proposition 9. *Suppose that (18) and the first inequality of (19) hold, and that banks are prevented from offering a payment of more than Z_H in period 2. There exists values of $\rho < 1$ such*

$$\rho^\gamma > \frac{1-e^{-\lambda}}{1-e^{-n^{-1}(\gamma)}}\left(\frac{n^{-1}(\gamma)}{\lambda}\right)^\gamma. \tag{38}$$

For certain values of ρ that satisfy (38), the equilibrium value of k is positive and banks offers to let depositors withdraw $\rho(1+k)Z_L/r_o$ in the first period. At interior equilibria of this kind, k falls with ρ.

Condition (38) implies that (36) holds but the opposite is not true. There thus exist values of ρ that satisfy the latter but not the former. In this case, the equilibrium has $k = 0$.

Proof. The expression on the right hand side of (38) is equal to one when $\lambda = n^{-1}(\gamma)$. Denoting the right hand side of (38) by Ψ and differentiating with respect to λ we have

$$\frac{d\Psi}{\Psi} = \left(\frac{\gamma}{\lambda} - \frac{e^{-\lambda}}{1-e^{-\lambda}}\right)d\lambda.$$

Condition (18) requires that $\gamma > n(\lambda)$. Since $\Psi > 0$, $d\Psi/d\lambda > 0$. As a result, the right hand side of (38) is smaller than one. Values of ρ smaller than one that obey (38) can thus be found.

By setting $Z_L = rZ_H$, the right hand side of (36) can be rewritten as

$$\frac{\Psi}{1-e^{-\lambda}}(1-e^{-\lambda r})r^{-\gamma}.$$

This would equal Ψ if r were equal to one. Denoting the expression on the right hand side of (36) by $\tilde{\Psi}$ and differentiating it with respect to r, we obtain

$$\frac{\tilde{\Psi}}{r}\left[\frac{\lambda r}{e^{\lambda r}-1} - \gamma\right].$$

Condition (18) implies that this is positive. Since $Z_L < Z_H$, the expression on the right hand side of (36) is smaller than the expression on the right hand side of (38).

Differentiating (37), while using (22), the gain in utility from raising Y while keeping bank owners indifferent is

$$\frac{dU_c}{dY} = \gamma\pi Y^{\gamma-1} + \gamma(1-\pi)\frac{1-e^{-n^{-1}(\gamma)}}{1-e^{-\lambda}}\left(\frac{\lambda\rho(1+k)Z_L}{n^{-1}(\gamma)}\right)^{\gamma}\frac{\phi'}{1+k}.$$

Evaluating this at the point where $k = 0$ and $Y + Z_H$, we have

$$\frac{dU_c}{dY} = \left[1 - \frac{1-e^{-n^{-1}(\gamma)}}{1-e^{-\lambda}}\left(\frac{\lambda\rho}{n^{-1}(\gamma)}\right)^{\gamma}\right]\gamma\pi Z_H^{\gamma-1}. \tag{39}$$

It follows that the derivative of U_c with respect to Y is positive when $k = 0$, $Y = Z_H$, and ρ satisfies (36) but not (38). Since k cannot be negative, the equilibrium for these values of ρ involves setting $k = 0$. For values of ρ slightly above the level that is needed for (38) to be satisfied, Y must be slightly smaller than Z_H, so that k must be slightly positive. The second order conditions then require that the equilibrium value of Y fall when ρ rises so that k must be rising in ρ. Lastly, the fact that Y is near Y_H and k is near zero implies that $(1 + k)Z_L/r_o$ is smaller than Y. Given that (36) holds, the equilibrium gives depositors a choice between withdrawing $(1 + k)Z_L/r_o$ in period 1 or withdrawing Y in period 2. □

This proposition implies that, as long as the cost of liquidation is modest, the result that bank owners provide some insurance to depositors extends to the case where depositors can also withdraw funds in the first period. Here, bank owners who add capital also increase the maximum amount that depositors are contractually allowed to withdraw in period 1. This is so attractive to depositors that they are willing to give up more resources in the high state than the owners need as compensation.

Interestingly, the following proposition shows that bank capital is not attractive from the point of view of average *ex post* welfare. When firms let depositors choose between a withdrawal of $Y_o(\rho(1 + k)Z_L$ in period 1 and a withdrawal of Y, this welfare equals

$$\tilde{U}_c = \pi Y^{\gamma} + (1-\pi)\frac{\lambda}{n^{-1}(\gamma)}\left(\frac{\lambda\rho(1+k)Z_L}{n^{-1}(\gamma)}\right)^{\gamma}. \tag{40}$$

It follows that:

Proposition 10. *When ex post consumer welfare is given by (40), and the combinations of k and Y must satisfy (22), this welfare is maximized by setting $k = 0$.*

Proof. Differentiating (40) while taking into account that changes in k and Y must satisfy (22), we have

$$d\tilde{U}_c = \left[\pi Y^{\gamma-1} + (1-\pi)\frac{n^{-1}(\gamma)}{\lambda}\left(\frac{\lambda\rho(1+k)Z_L}{n^{-1}(\gamma)}\right)^{\gamma}\frac{\phi'}{1+k}\right]\gamma dY.$$

Using (22) and evaluating at $k = 0$ and $Y = Z_H$ yields

$$d\tilde{U}_c = \left[1 - \rho^{\gamma}\left(\frac{n^{-1}(\gamma)}{\lambda}\right)^{1-\gamma}\right]\pi\gamma Z_H^{\gamma-1}dY.$$

$d\tilde{U}_c/dY$ thus rises with ρ and, because $n^{-1}(\gamma) < \lambda$, is positive even when $\rho = 1$. Since k cannot be negative, the optimum is to set $k = 0$. □

This result of proposition 10 is quite different from the one in Proposition 5, which stated that increasing capital above its equilibrium level raised *ex post* depositor welfare under fairly weak conditions. In both cases, raising capital requirements has the effect of increasing the total amount that banks pay out in the low state while compensating bank owners by reducing their obligations in the high state. The difference is that, in the two period model considered above, the amount obtained by successful withdrawers in the low state also fell alongside the high state payment when capital was increased. This resulted in a benefit that depositors often did not value sufficiently, namely that it increased their probability of succeeding in obtaining a payment.

In the three period model studied here, by contrast, the amount obtained by successful withdrawers in the low state payment rises when capital is increased, while their probability of being successful stays constant. As a result, there is no social gain that is being insufficiently valued by depositors. In fact, the opposite is true. Depositors with queuing bias find it attractive to be given a larger payment in the low state when they are successful. However, from the point of view of average *ex post* welfare, this introduces undesirable consumption volatility.

Before closing this section it is worth showing that clawbacks and deposit insurance can also reinstate the autarkic Pareto Optimum in the three period model. Consider clawbacks first. If depositors were to accept a contract that offered a payment of more than Z_L in period 1, there would still be a period 1 bank run in the low state, and they would all end up with ρZ_L. Such a contract would not attract depositors because they would do better under autarky. Banks could, instead, offer a contract with a lower period 1 payment. However, this would again yield the autarkic allocation.

Now consider deposit insurance. As we saw earlier, deposit insurance eliminates any incentive for banks owners to contribute capital to their banks, so I set $k = 0$. In their competition for depositors, banks would set a second period payment no lower than Z_H, since lower payments would reduce expected depositor utility. Because of my assumption of lump sum taxes, setting a higher payment does not change the equilibrium. For concreteness, suppose this equals Z_H. If the promised period 1 payment is even slightly smaller, there are no runs in period 1. While banks do not have any preference regarding this payment, the government would gain by making sure it is below Z_H. As in Diamond and Dybvig (1983), one important benefit of deposit insurance is that it eliminates inefficient liquidations by giving depositors confidence in their deposits. One importance difference, however, is that here the government has to collect taxes in the low state to make good on this insurance.

6 Conclusion

This paper has provided a justification for various policies that governments use to regulate financial institutions, including a deposit insurance policy where the government makes actual losses in bad states of nature as well as regulations that control the assets of uninsured institutions. The benefit of these policies is their ability to help depositors who are too optimistic about how they will fare in a run.

Both to simplify the analysis and to preserve its comparability with important earlier studies like Diamond and Dybvig (1983) and Allen and Gale (1998), the structure of the model I have presented is extremely sparse. Unfortunately, this raises the question of how the policy implications of the model would carry over to a richer environment. The model implies, for example, that clawbacks in bankruptcy and deposit insurance both eliminate inefficient runs and achieve the same final allocation.

In practice, these two forms of regulation differ in other ways. Clawbacks, for example, lead to more uncertainty for individuals who withdraw funds since the withdrawal can be reversed. Such uncertainty may be more costly when withdrawals are more frequent, and this could be a reason why non-financial bankruptcies are subject to clawbacks in the U.S. while financial institutions are not. It does remain uncertain, however, whether there should also exist financial institutions such as money market funds that are subject to neither clawbacks nor government insurance.

The model suggests that the advisability of requiring financial institutions to hold more capital than they would on their own depends crucially on the contracts that they write. If increased capital leads them to reduce their contractual obligations in panics so that more depositors are able to withdraw, the model provides a rationale for increasing this capital. Capital is advantageous in this case because it reduces the volatility of consumption induced by depositors' unwarranted optimism. If, however, banks with more capital find a way to increase the amount that they pay out to successful withdrawers in a panic, forcing banks to have more capital may be counterproductive. In the model, banks ability to do this depends on the timing of withdrawals, consumption and the resolution of uncertainty. It would thus be valuable to learn how banks react to increases in capital in a richer dynamic model.

References

Allen, Franklin and Douglas Gale, "Optimal Financial Crises," *The Journal of Finance,* 53, August 1998, 1245–1284.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, "A model of investor sentiment," *Journal of Financial Economics,* 49, September 1998, 307–43.

Bénabou, Roland and Jean Tirole, "Self-Confidence and Personal Motivation," *Quarterly Journal of Economics,* 117, August 2002, 871–915.

Ben-David, Itzhak, John R. Graham and Campbell R. Harvey, "Managerial Miscalibration," mimeo, 2010.

Bliss, Robert R. and George G. Kaufman, "U.S. Corporate and Bank Insolvency Regimes: A Comparison and Evaluation," *Virginia Law & Business review,* 2, Spring 2007, 144–77.

Boyd, John H., Chung Chang and Bruce D. Smith, "Deposit Insurance: a Reconsideration," *Journal of Monetary Economics,* 49, September 2002, 1325–260.

Bryant, John, "A Model of Reserves, Bank Runs, and Deposit Insurance," *Journal of Banking and Finance,* 4, December 1980, 335–44.

Camerer, Colin and Dan Lovallo, "Overconfidence and Excess Entry: An Experimental Approach," *American Economic Review,* 89, March 1999, 306–18.

Daniel, Kent, David Hirshleifer, and Avanidhar Subrahmanyam, "Investor Psychology and Security Market Under- and Overreactions," *Journal of Finance,* 53, December 1998, 839–85.

Diamond, Douglas W. Phillip H. Dybvig, "Bank Runs, Deposit Insurance and Liquidity." *Journal of Political Economy,* 91, June 1983, 401–19.

Frey, Bruno S. and Werner W. Pommerehne, "On the fairness of pricing - An empirical survey among the general population," *Journal of Economic Behavior and Organization,* 20, April 1993, 295–307.

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, "Neglected Risks, Financial Innovation, and Financial Fragility," Working paper, 2010.

Grinblatt, Mark and Matti Keloharju, "Sensation Seeking, Overconfidence, and Trading Activity," *Journal of Finance,* 64, April 2009, 549–78.

Hasan, Iftekhar and Gerald P. Dwyer, “Bank Runs in the Free Banking Period,” *Journal of Money, Credit and Banking,* 26, May 1994, 271–88.

Hong, Harrison, Jeremy C. Stein, and Jialin Yu, “Simple Forecasts and Paradigm Shifts” *Journal of Finance,* 62, June 2007, 1207–42.

Köszegi, Botond, “Ego Utility, Overconfidence, and Task Choice,” *Journal of the European Economic Association,* 4, June 2006, 673–707.

Landier, Augustin, and David Thesmar, “Financial Contracting with Optimistic Entrepreneurs,” *Review of Financial Studies,* 22, January 2009, 117–50.

Manove, Michael and A. Jorge Padilla, “Banking (Conservatively) with Optimists,” *RAND Journal of Economics,* 30, Summer 1999, 324–50.

Mueller, Reinhold C. (1997) The Venetian Money Market, Banks, Panics, and the Public Debt, 1200-1500, Baltimore: Johns Hopkins University Press.

Odean, Terrence, “Volume, volatility, price, and profit when all traders are above average,” *Journal of Finance,* 53, December 1998, 1887–934.

Rotemberg, Julio J., “Minimally Acceptable Altruism and the Ultimatum Game,” *Journal of Economic Behavior and Organization,* 66, June 2008, 457–76.

Sandroni, Alvaro and Francesco Squintani, “Overconfidence, Insurance, and Paternalism,” *American Economic Review,* 97, December 2007, 1994–2004.

Schumacher, Liliana, “Bank runs and currency run in a system without a safety net: Argentina and the tequila shock,” *Journal of Monetary Economics,* 46, 2000, 257–277

Thompson, Suzanne C, Wade Armstrong, and Craig Thomas, “Illusions of Control, Underestimations and Accuracy: A Control Heuristic Explanation,” *Psychological Bulletin,* 123, 1998, 143–61.

Yildiz, Muhamet, “Bargaining without a Common Prior: An Immediate Agreement Theorem,” *Econometrica,* 71, May 2003, 793–811.

Appendix

Proof that $G(x)$ is strictly increasing in λ for $0 < x < 1$ and $\lambda > 0$

Using (5), the derivative of $G(x)$ with respect to λ is

$$\frac{dG(x,\lambda)}{d\lambda} = \frac{xe^{-x\lambda}(1-e^{-\lambda}) - e^{-\lambda}(1-e^{-x\lambda})}{(1-e^{-\lambda})^2}.$$

This is zero at both $x = 0$ and $x = 1$. To demonstrate that it is positive for $0 < x < \lambda$, I first prove that, for $\lambda > 0$, $dG/d\lambda$ is increasing in x at $x = 0$ and decreasing in x at $x = 1$. I then prove that the $dG/d\lambda$ has no other zeros between 0 and 1 by showing that $dG/d\lambda$ is concave in x at $x = 0$ and either remains concave or becomes convex in x as x rises. Since it never turns back from being convex to being concave, $dG/d\lambda$ remains above zero for $0 < x < 1$.

To demonstrate these properties, note that the derivatives of $dG/d\lambda$ with respect to x satisfy

$$\begin{aligned}(1-e^{-\lambda})^2\frac{d^2G}{d\lambda dx} &= e^{-x\lambda}(1-e^{-\lambda})(1-x\lambda) - \lambda e^{-\lambda(1+x)} \\ (1-e^{-\lambda})^2\frac{d^3G}{d\lambda dx^2} &= \lambda e^{-x\lambda}[-(1-e^{-\lambda})(1-x\lambda) - (1-e^{-\lambda}) + \lambda^2 e^{-\lambda}].\end{aligned}$$

At $x = 0$, $d^2G/d\lambda dx$ has the same sign as $1 - e^{-\lambda} - \lambda e^{-\lambda}$ when λ is positive. This is positive because $\lambda/(e^{\lambda} - 1)$ is an Einstein function that is smaller than one for $\lambda > 0$. At $x = 1$, $d^2G/d\lambda dx$ has the same sign as $e^{-\lambda}(1-\lambda-e^{-\lambda})$. This is negative because the function $e^{-\lambda}$ is tangent to the function $1 - \lambda$ at $\lambda = 0$ and is above $1 - \lambda$ for higher values of λ.

The function $d^3G/d\lambda dx^2$ is increasing in x for $\lambda > 0$. At $x = 0$, it has the same sign as $(2 + \lambda)e^{-\lambda} - 2$. This expression equals zero for $\lambda = 0$ and has a negative derivative with respect to λ. It follows that $d^3G/d\lambda dx^2 < 0$ at $x = 0$ for $\lambda > 0$ so that the function $dG/d\lambda$ is concave in x at $x = 0$. As x is increased, $d^3G/d\lambda dx^2$ rises. If it becomes positive, $dG/d\lambda$ becomes convex in x. It cannot, however, become concave once again for higher values of x.