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ESTIMATES OF THE VALUE OF PATENT
RIGHTS IN EUROPEAN COUNTRIES
DURING THE POST-1950 PERIOD

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ABSTRACT

This paper examines the distribution of the values of patent rights in the United Kingdom, France, and Germany during the post-1950 period. These values are inferred from the behavior of patentees with respect to payment of renewal fees on their patents. A simple economic model of renewal decisions is combined with data on the proportion of patents renewed at alternative ages and the renewal fee schedules to produce estimates of the distribution (and the total) value of patent rights in these countries. Moreover, the data indicate that there have been changes in the value distribution, and we follow these changes over the period. The empirical results of particular interest concern: the total value of patent rights and the relationship between changes in it and changes in the quantity of patents, the skew in the distribution of patent values, and the rate of obsolescence on the returns to patents.

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The objective of this paper is to analyze empirically the private value of patent rights in the United Kingdom, France, and Germany during the post-1950 period. Since patent rights are seldom marketed, we cannot obtain direct evidence on their values. Instead, we infer the value of patent rights from the economic behavior of patentees. In particular, in most countries (including the three studied here) agents must pay an annual renewal fee in order to keep their patents in force. We use a simple economic model of the renewal decision to recover the distribution of the values of the patents in a cohort from the proportion of patents in that cohort renewed at different ages, and the renewal fee schedules faced by the cohort.

The model, together with the renewal data, suffice to provide estimates of the total value of the patents in a cohort, and its distribution among the members of that cohort. In addition, the data indicate that the parameters of the model differ by both year and cohort. We allow for these differences and then examine the implied movements in the mean and total value of cohorts of patents during the post-1950 period. Since there is little previous large sample evidence on changes in the value of patents over time, most aggregate intertemporal (and cross section) comparisons of patent output have focused on changes in quantities of patents (either applied for or granted; see, for example, the articles in Griliches ed., 1984). Our empirical results indicate that, at least during the post-1950 period in these three countries, changes in the quantity of patents are frequently inversely related to changes in the "quality" (or mean value) of these patents. Previous studies which rely exclusively on the quantity of patents as an indicator

of inventive activity miss the changes over time in the total value of patent rights, and as a result their conclusions may be misleading.

Section 1 presents the patent renewal model, which is a slightly extended version of a model developed by us in earlier work (Pakes and Schankerman 1984a; Schankerman and Pakes 1985). Section 2 describes in some detail the data set used for the empirical work. This new data set consists of patent renewal rates and fees for cohorts of patents at the aggregate level. It contains almost all patents taken out in the United Kingdom, France and Germany during the post-1950 period. Section 3 describes the empirical specification of the model and presents the estimates. In Section 4 we consider the implications of these estimates. This includes an empirical characterization of the distribution of the private value of patent rights for each country, and an examination of the movements over time in both the quality and quantity of annual patent applications. Brief concluding remarks close the paper.

1. A Model of Patent Renewal

Consider an agent who holds a patent. Let j denote the cohort of the patent and t be its age, so $t + j$ represents the year. In order to keep the patent in force the agent must pay an annual renewal fee. The renewal fee varies with the age and possibly the cohort of the patent, and we denote the sequence of renewal fees at different ages by $\{C_{tj}\}$. An agent who pays the renewal fee earns the implicit return to patent protection during the coming year, R_{tj} . We shall assume that the sequence $\{R_{tj}\}$ is known with certainty at the time the patent is applied for.¹ The agent's decision problem is to maximize the discounted value

of net returns accruing to the patent by choosing an optimal age at which to stop paying the renewal fee. Formally, the agent chooses the lifespan of the patent, T , to

$$(1) \quad \max_{T \in \{1, 2, \dots, \bar{T}\}} V(T) = \sum_{t=1}^T (R_{tj} - C_{tj})(1+i)^{-t}$$

where i is the discount rate and \bar{T} is the statutory limit to patent protection. Provided the sequence $\{R_{tj} - C_{tj}\}_{t=1}^{\bar{T}}$ is nonincreasing in t , the optimal lifespan T^* is the first age at which $R_{tj} - C_{tj} < 0$, or if no such $T^* \in \{1, 2, \dots, \bar{T}\}$ exists, then $T^* = \bar{T}$. Equivalently, in a world of certainty with nonincreasing net returns, the condition for renewal of the patent at age t is that the annual returns at least cover the cost of renewal, or

$$(2) \quad R_{tj} \geq C_{tj}.$$

Since the renewal fees are nondecreasing in age (see Section 2), a condition which insures that net revenues are nonincreasing is that the sequence $\{R_{tj}\}$ is nonincreasing, that is, that the returns to holding a patent do in fact decay over time.

The sequence of returns $\{R_{tj}\}$ reflects the initial returns R_{0j} and the sequence of the rates of decay of those returns $\{\delta_{tj}\}$. If the sequence of returns $\{R_{tj}\}$ were the same for all patents in a given cohort, then all patents would be cancelled at the same age and the time path of renewals would be degenerate. We allow patents in a given cohort to differ in their initial returns but assume that the sequence of decay rates $\{\delta_{tj}\}$ does not differ among patents.² The decay rates, however, will be allowed to vary in response to changes in the economic

environment (see Section 3). Under these assumptions, $R_{tj} = R_{0j} \prod_{\tau=1}^t d_{\tau j}$ where $d_{\tau j} = 1 - \delta_{\tau j}$, and the patent holder will renew at age t if and only if $R_{0j} \geq C_{tj} \prod_{\tau=1}^t d_{\tau j}^{-1}$. Let $f(R_{0j}; \theta_j)$ and $F(R_{0j}; \theta_j)$ be the density and associated distribution functions of initial revenues, where θ_j denotes a vector of parameters. Then the proportion of patents in cohort j renewed at age t , P_{tj} , is

$$(3) \quad P_{tj} = \int_{z_{tj}}^{\infty} f(R_{0j}; \theta_j) dR_{0j} = 1 - F(z_{tj}; \theta_j)$$

where $z_{tj} = C_{tj} \prod_{\tau=1}^t d_{\tau j}^{-1}$.

The estimation problem is to use data on the proportion of patents renewed and the costs of renewal to estimate the sequence of decay rates and the parameters characterizing the density function of initial revenues. These parameters will allow us to derive the distribution of the value of patent rights and characterize changes that have occurred in it over time (see Section 4). We begin with a description of the data.

2. Description of the Data

The data were obtained directly from the patent offices in the U.K., France and Germany.³ For each country we obtained information on the number of patents in each cohort which were renewed at different ages, the total number of patent applications in the cohort, and the renewal cost schedules in nominal terms during the post-1950 period.

There was no available information either on individual patents or on a breakdown of cohorts by industrial sector or type of patent. Table 1 summarizes some basic characteristics of the data.

In each of these countries, maintenance of patent protection requires payment of an annual renewal fee which begins some years after the patent is applied for and continues until the statutory limit to patent protection. The range of patent ages varies across countries (see row 1 in Table 1) and represents the span of ages over which data on patent renewals and fees are available. The data contain (at least partial) information on the renewals of between twenty-seven and twenty-nine cohorts of patents applied for between 1950 and 1979, depending on the country. The range of years in which we observe the renewals of the cohorts still in force is 1955-1981 in the U.K. and Germany, but only 1970-1981 for France.

The renewal fee schedules are published by the respective patent offices and are changed periodically, but the most recent schedule applies to all patents regardless of the year of initial patent application. Hence the renewal fee at a particular age depends on the year in which the patent reaches that age. The nominal renewal fees in domestic currency were converted to real costs using the country's implicit GDP deflator and then to 1980 U.S. dollars using the official exchange rates in 1980.

We designate a cohort of patents as all those patents applied for in a given year, and distinguish different cohorts within a country by the index j . In the U.K. and France, the proportion of patents in cohort j which is renewed at age t , P_{tj} , is calculated as the ratio of

Table 1. Characteristics of the Data

	United Kingdom	France	Germany
1. Patent ages ^a	5-16	2-20	3-18
2. Range of Cohorts	1950-1976	1951-1979	1952-1978
3. Range of Years	1955-1981	1970-1981	1955-1981
4. Number of Observations	258	209	312
5. Mean Number of Patents per Cohort	37,286	36,865	21,273
6. Ratio of Patent Grants to Applications ^b	.83	.93	.35
7. Ratio of Between Age to Total Variance in P_{tj} ^c	.984	.999	.987
8. Ratio of Between Age to Total Variance in C_{tj}	.864	.976	.993
9. Ratio of Between Age to Total Variance in $P_{t-1,j} - P_{tj}$.560	.517	.506

^aIn 1976 in Germany and in 1980 in the U.K. changes in the patent laws extended the statutory limit to patent lives to twenty years.

^bThis ratio is computed directly from the German data, and averaged over cohorts. The data for the U.K. and France do not associate grants with cohorts. We approximate the ratio as $T^{-1} \sum_{t=1}^T [\sum_{\tau=1}^4 .25 \tilde{N}_{t+\tau}] / N_t$ where \tilde{N}_t is the number of patents granted and N_t is the number of applications in year t .

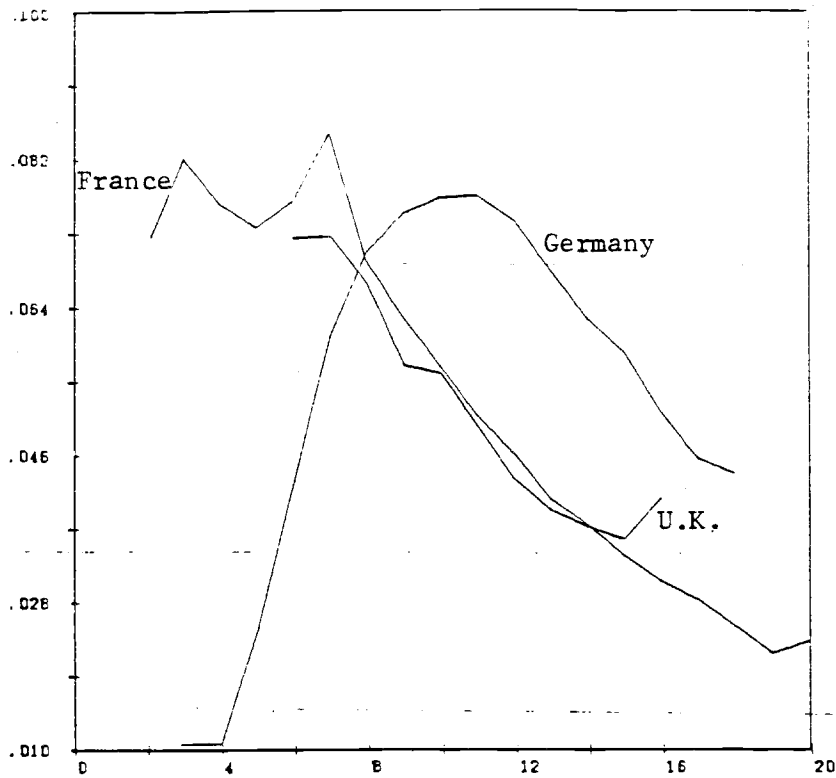
^cThe ratio of the between age to total variance in X_{tj} is the ratio of the variance in $X_{t.} - X_{..}$ to the variance in $X_{tj} - X_{..}$, after correcting both terms for their degrees of freedom. Here $X_{t.}$ is the average (across cohorts) value of X_{tj} at age t , and $X_{..}$ is the grand mean of X .

renewals to patent applications (actually "completed specifications" in the U.K., which denotes completed applications). Therefore, some of the patent dropouts in the early ages may be "involuntary," reflecting their failure to be granted rather than the patentee's comparison of returns to costs of renewal. This is particularly true for France where patents are recorded from age two. In the U.K. the first observed renewal age is five and we were informed by patent office personnel that virtually all grants occur within five years after the patent application. In Germany the renewal data are recorded only for patents which have already been granted, but the age of the patent is based on the application date as in the U.K. and France. We compute the renewal proportion P_{tj} as the ratio of renewals at age t to the patents from cohort j which are eventually granted. The German renewal data avoid the problem of "involuntary attrition," but they sample only from the population of patents granted (in contrast to the U.K. and France, where the population consists of all patents applied for). The countries also differ substantially in the fraction of patent applications which is granted. The German data allow us to compute this fraction directly, but in the U.K. and France we have to approximate it (see notes to Table 1) because the data on grants are not identified with specific cohorts of patents. As row 6 in Table 1 shows, the patent screening process in the U.K. and France does not weed out many patents (83 and 93 percent of applications are granted, respectively). However, only about one-third of all applications are granted in Germany.

There are two dimensions to the data, the age and the cohort (date of application) of the patent. The last three rows in Table 1 present

the ratio of the between age to total variance in the cost of renewal C_{tj} , the renewal proportion P_{tj} , and the proportion of dropouts (mortality rate) $P_{t-1,j} - P_{tj}$. It is clear that almost all of the variance in renewals and costs of renewals is between age variance, implying that the age paths of the renewal proportion and renewal fees do not vary much among cohorts in a given country. For the renewal proportion, however, this is largely a result of looking at the levels (and hence the accumulation of dropouts over ages). The total variance in the mortality rate is divided about equally between the age and cohort dimensions. Figure 1 provides the average mortality rates at each age for each country (averaged over the available observations on cohorts for that age). The mortality rates vary substantially both over ages and across countries. This is particularly noticeable for the first few ages in France, where part of the inter-age variance may be a result of the patent granting procedure. Figures 2 and 3 present the age paths of the renewal fees and the proportion of patents renewed for each country, averaged over the available observations on cohorts. Note that the renewal fees rise monotonically and the renewal proportion declines monotonically in age in all countries. The renewal fees start at low levels and rise much more steeply in Germany after age six. The age paths of renewals are strikingly similar in the U.K. and France after age five, and there is a large fraction which does not pay the fifth renewal in both countries. More than half of the patents are cancelled by age eight and only 25 percent survive past age thirteen. In Germany the proportion renewed is higher than in the other countries at all ages, but it declines at a significantly faster pace after age six.

Figure 1. Age Paths of Mortality Rate of Renewals*



*The mortality rate at age two in France and age three in Germany represents the cumulative dropouts until that age. The mortality rate at age five in the U.K. is not illustrated, but equals .305.

Figure 2. Age Paths of Renewal Costs

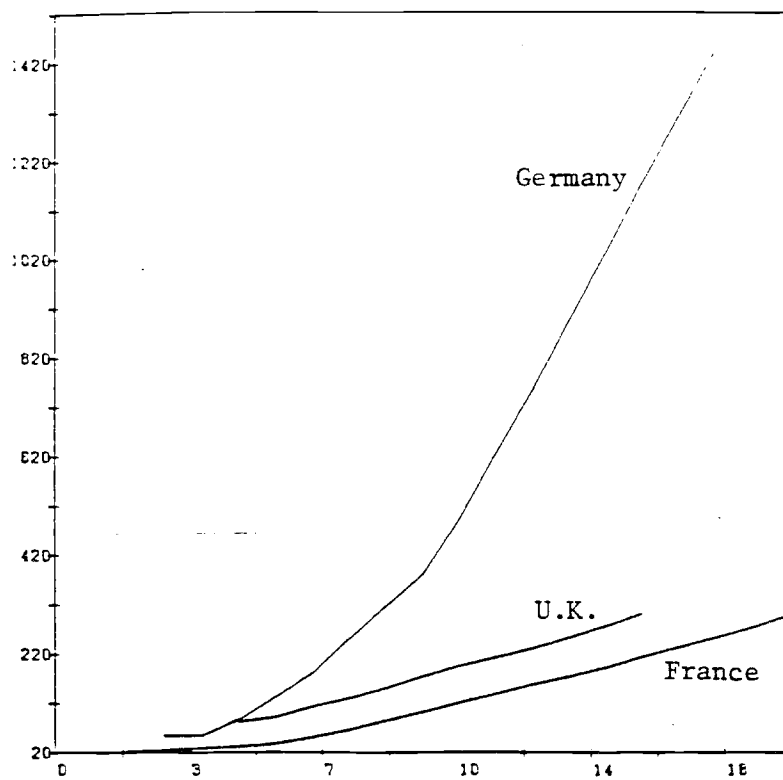
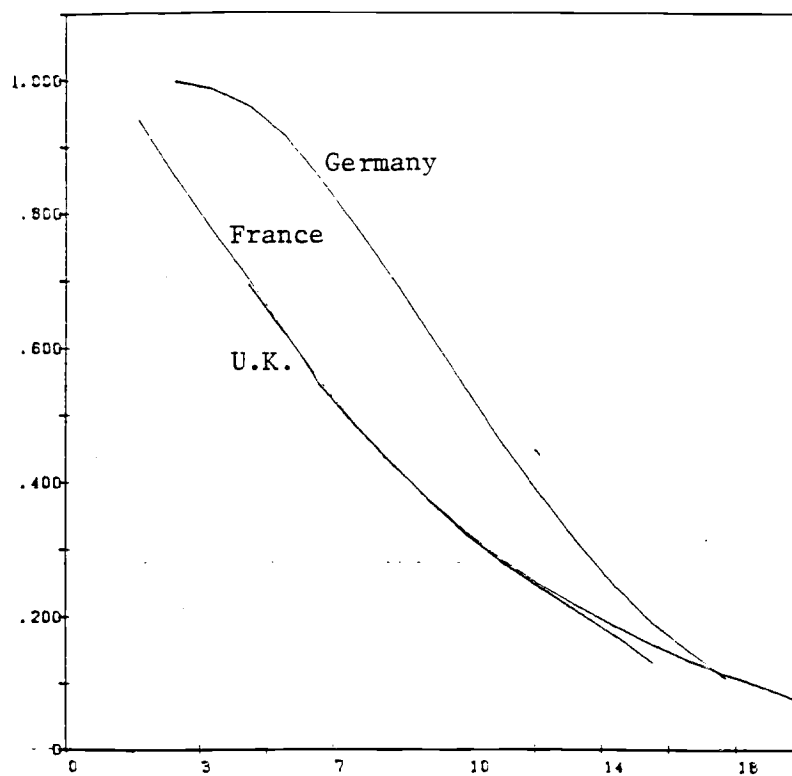


Figure 3. Age Paths of Renewal Proportions



These characteristics are consistent both with our prior information on the German data and with the model of renewal behavior in the previous section. Since the German renewal data are based on granted patents, if the German patent office is successful in weeding out relatively unprofitable patents one would expect fewer dropouts in the German data in the early ages when the renewal fees in that country are still relatively small and comparable to the renewal fees in the other countries. After age six, however, the renewal fees in Germany increase at a distinctly faster pace than in the U.K. and France. The model of renewal behavior implies, all else equal, that this should result in a faster rate of decline in the proportion of patents renewed in Germany, and this is in fact what we observe.

Two other conclusions emerge from the evidence in Figures 2 and 3. First, if the model of patent renewal in Section 1 is correct, the age path of renewals indicates that there is a concentration of patents with very little private economic value. The reason is that the model implies that patents are renewed if the annual revenues exceed the renewal cost, yet the bulk of patents are cancelled even at rather modest levels of renewal fees. Second, again conditional on our model, it may be difficult with these data to estimate the tail of the distribution of the value of patent rights very precisely. Since renewal fees are never very large in absolute terms, a proportion of patents will be maintained for the entire period, and the renewal data will not be very informative on their values. (We return to this point below.)

3. Empirical Specification and Results

The model of patent renewal in Section 1 assumes a known, fixed distribution of initial revenues which then decay over time. There are several pieces of evidence which indicate that this may not be a good assumption for the early ages. We reported in the previous section that the mortality rate of renewals behaves quite irregularly during the first few ages in France (see Figure 1). For the U.K. we do not observe any renewals until age five. However, in Germany, where our data contain the renewals of those patents already granted, the mortality rate at age three is much smaller than that for the subsequent ages, even though the costs are essentially the same. Part of the reason underlying the behavior of the French data in the early ages is likely to be the involuntary attrition resulting from patent applications which were not granted--a phenomenon not accounted for by our model. A second cause of the inappropriateness of the deterministic decay assumption in the early ages is that in those ages future returns to the patent may be highly uncertain, and agents may hold the patent until more information on its value accumulates. In a separate paper, Pakes (1984) constructs a more complicated model of patent renewal which allows both for agents to uncover more profitable uses for the ideas embodied in their patents, and for the effects of the patent-granting process. Estimates of that model indicated that these processes are important in the early ages, but their combined effects are negligible in France and the U.K. after age four, and in Germany after age three. In view of this evidence and the fact that the first observed renewal for the U.K. is for age five, we base our empirical work on renewals after age five in France and the

U.K., and on renewals after age three in Germany. The universe of patents is taken as those patents which survive to these initial ages, and hence all renewal proportions P_{tj} are normalized by the value of P_{tj} at the initial age for the associated cohort.⁴

To complete the specification of the model one requires a parameterization of the distribution of initial revenues, $f(R_{0j}; \theta_j)$, where θ_j is the vector of parameters for cohort j . We experimented empirically with three alternative specifications, the Weibull, Pareto-Levy and the lognormal distributions. The lognormal consistently fit the data better than either the Weibull or the Pareto-Levy.⁵ Assuming, then, that R_{0j} distributes lognormally and letting lower case letters denote the logarithms of upper case ones, we have $r_{0j} \sim N(\mu_j, \sigma_j)$ where $N(\cdot, \cdot)$ designates the normal distribution. In logarithmic form, the decision rule in Section 1 is to renew a patent at age t if and only if

$$r_{0j} \geq c_{tj} - \sum_{\tau=1}^t \ln d_{\tau j} \text{ or, equivalently,}$$

$$(4) \quad \frac{r_{0j} - \mu_j}{\sigma_j} \geq \frac{c_{tj} - \mu_j - \sum_{\tau=1}^t \ln d_{\tau j}}{\sigma_j} .$$

Noting that $(r_{0j} - \mu_j)/\sigma_j$ has a standardized normal distribution, the equation for the proportion of patents in cohort j which have dropped out by age t is given by

$$1 - P_{tj} = \Phi \left[\frac{c_{tj} - \mu_j - \sum_{\tau=1}^t \ln d_{\tau j}}{\sigma_j} \right] ,$$

which implies that

$$(5) \quad y_{tj} \equiv \Phi^{-1}(1 - P_{tj}) = -\frac{\mu_j}{\sigma_j} + \frac{1}{\sigma_j} c_{tj} - \frac{\sum_{\tau=1}^t \ln d_{\tau j}}{\sigma_j},$$

where $\Phi(\cdot)$ is the standardized normal distribution function.

Given only the proportion renewing in each cohort/age cell, we cannot estimate separate decay rates for each cell and a separate lognormal distribution for each cohort. However, we do want to allow for some variation in both the decay rate and the initial distributions over the approximately three decades covered by our data. Given the lognormal specification, the mean level of initial revenues in a cohort is given by $e^{\mu + \frac{1}{2}\sigma^2}$, and the coefficient of variation is σ . We will allow for cohort-specific values of μ but maintain a common value of σ across cohorts. This is equivalent to letting cohorts of patents differ by a proportional rescaling of the initial revenues of all patents in a given cohort. Second, we want to permit the decay rates to vary over time. The initial revenues of a patent R_{0j} will decay as the patent ages because of competitive pressures, but they may also experience some growth as the relevant market expands. The net decay rate will reflect both of these factors. To allow for this, we use the specification $d_{tj} \equiv (1 - \delta_{tj}) = (1 - \delta)\exp\{\beta_0 g_{t+j} + \beta_1 D_1 + \beta_2 D_2\}$ where g_{t+j} is the rate of growth of aggregate demand (GDP) in year $t + j$ and we expect $\beta_0 > 0$, $D_1 = 1$ if $1960 \leq t + j \leq 1969$ and zero elsewhere, and $D_2 = 1$ if $t + j \geq 1970$ and zero otherwise. The time dummy variables D_1 and D_2 are included to capture broad differences in decay rates across decades which are not reflected in annual movements in aggregate demand. Note that positive values for β_1 or β_2 indicate a decline in the rate of decay during the 1960's or 1970's relative to the 1950's.

Finally, in writing down the model in (5) we have ignored any sampling error in the observations on P_{tj} . The variance of the sampling error is given by $P_{tj}(1 - P_{tj})/N_j$ where N_j is the number of patents in cohort j . For cohorts as large as those in the sample (see row 5 in Table 1), this variance is essentially zero and will not affect the results. In order to allow for discrepancies between the actual and predicted values from the model, we follow Amemiya (1981) by specifying an error term in the renewal rule (4), ϵ_{tj} . Incorporating these various specifications, the model we actually estimate is

$$(6) \quad y_{tj} = -\frac{\mu_j}{\sigma} + \frac{1}{\sigma} c_{tj} - \frac{\ln(1 - \delta)}{\sigma} t - \frac{\beta_0}{\sigma} \sum_{\tau=1}^t g_{\tau+j} \\ - \frac{\beta_1}{\sigma} \sum_{\tau=1}^t D_1 - \frac{\beta_2}{\sigma} \sum_{\tau=1}^t D_2 + \epsilon_{tj}$$

where (conditional on t and j) ϵ_{tj} is assumed to have mean zero and variance σ_ϵ^2 .⁶ Equation (6) is estimated by nonlinear least squares.

Table 2 presents the empirical results for various versions of the model. Regression (1) in each panel refers to the model with a constant decay rate and no cohort-specific variation in μ (i.e., $\mu_j = \mu$ for all j ; we call this the no-effects model), while regression (2) allows for a free sequence of $\{\mu_j\}_{j=1}^J$ (we call this the fixed-effects model). The null hypothesis that there are no cohort effects ($\mu_j = \mu$ for all j) is rejected in all three countries. The computed F statistics are $F(26,203) = 3.99$ for the U.K., $F(25,257) = 12.10$ for Germany and $F(25,157) = 3.95$ for France, compared to a critical value at the .05 level of 1.52. This implies that significant changes have occurred in

Table 2. Parameter Estimates for the Patent Renewal Model^a

	Panel A: United Kingdom				Panel B: Germany				Panel C: France		
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(1)	(2)	(3)
$\bar{\mu}^b$	6.61 (.24)	6.40 (.20)	6.38 (.19)	6.44 (.20)	6.87 (.11)	7.26 (.14)	7.30 (.15)	7.35 (.15)	6.27 (.16)	5.14 (.23)	5.11 (.23)
σ	1.73 (.19)	1.56 (.16)	1.49 (.15)	1.57 (.16)	1.24 (.05)	1.43 (.05)	1.45 (.06)	1.46 (.06)	2.87 (.18)	1.93 (.24)	1.93 (.24)
δ	.20 (.03)	.18 (.03)	.26 (.05)	.24 (.05)	.07 (.01)	.10 (.01)	.12 (.04)	.15 (.04)	.25 (.02)	.10 (.04)	.11 (.04)
β_0			1.48 (.67)				-.65 (.36)				.22 (.54)
β_1			.10 (.05)	.11 (.05)			.06 (.04)	.07 (.04)			-
β_2			.08 (.05)	.07 (.05)			.02 (.04)	.04 (.04)			-
δ_{60}^c			.15 (.01)	.15 (.03)			.09 (.01)	.09 (.01)			-
δ_{70}^d			.17 (.02)	.18 (.03)			.11 (.01)	.12 (.01)			.10 (.04)
σ_ϵ	.136	.118	.114	.115	.148	.106	.103	.103	.089	.074	.074
R^2	.957	.971	.974	.973	.981	.991	.992	.992	.978	.987	.987
df	228	203	200	201	281	257	254	255	151	128	127

^aEstimated standard errors are in parentheses.

^b $\bar{\mu}$ is the average of the estimated cohort-specific μ_j 's, except in the no-effects model (regression 1 in each country).

^c $\delta_{60} = 1 - (1 - \delta)\exp(\beta_0\bar{g} + \beta_1)$ where \bar{g} is the average growth rate of GDP for 1960-1969.

^d $\delta_{70} = 1 - (1 - \delta)\exp(\beta_0\bar{g} + \beta_2)$ where \bar{g} is the average growth rate of GDP for 1970-1975.

the distribution of the value of patent rights in the post-1950 period in all countries. A more detailed investigation of these changes is provided in Section 4.⁷

Regression (3) in each panel presents the estimates for the model which allows both for fixed effects in μ and variations in the decay rate. Note first that the basic parameters of the model (μ , σ and δ) all have the right sign and are statistically significant in all three countries. The estimates of σ indicate that the distribution of initial revenues exhibits substantial dispersion and skewness in all three countries. The degree of skewness is illustrated by the ratio of the mean to the median value of initial revenues, which for the lognormal is given by $e^{\sigma^2/2}$ (Johnson and Kotz 1970, Chapter 14). This ratio varies from 2.86 in Germany to 6.44 in France, indicating a rather sharp skewness to the right. Intercountry differences in μ are inversely correlated with those in σ , so that countries with higher mean returns have lower coefficients of variation in those returns.⁸

The estimates of the rate of decay in the returns from holding patents in the 1950's is lower in Germany (0.12) than in the U.K. (0.26); there is no information on renewals in the 1950's for France (see Table 1). Together with the higher mean and lower coefficient of variation in Germany, this seems to indicate that the relatively stringent German patent granting procedures are, on the whole, successful in selecting patents with higher initial returns and lower subsequent decay in those returns. The estimated coefficients on the decadal dummy variables (β_1 and β_2) provide some tentative evidence that the rate of decay declined in the 1960's and 1970's. The implied estimate of the rate of decay in

the 1970's is 0.17 for the U.K. and 0.11 for Germany, which is closer to the estimate for France which is also for the 1970's. The hypothesis we advanced that the rate of decay depends inversely on the rate of growth of the market ($\beta_0 > 0$) receives mixed support from the data. The point estimates of β_0 have the expected sign in the U.K. and France though not in Germany, but the null hypothesis that $\beta_0 = 0$ can be rejected only for the U.K. Moreover, given the magnitudes of β_0 and the market growth rates, the implied quantitative impact of GDP growth on the decay rate is small, at least at the economy-wide level of aggregation. On the whole, the estimates of the decay rate do indicate a fairly rapid decline in the private returns from holding patents, higher than the rate of decay generally assumed for the physical productivity of traditional capital goods. This result is not surprising since the decay in the returns earned from holding patents is not due to any decline in the physical productivity of the knowledge embodied in them, but rather to the two related points concerning the market valuation of the innovations they represent--namely, that it is difficult to establish and maintain effective proprietary rights over the knowledge and that new inventions are developed which displace the original one (see Arrow 1962; Pakes and Schankerman 1984 summarize related evidence on the rate of obsolescence in the returns to innovation).

4. The Value of Patent Rights

In this section we use the parameter estimates in Table 2 to derive the distribution of the value of patent rights and to examine empirically changes that have occurred in both the size and quality of

cohorts of patents since 1955. The present value of patent protection for a single patent, denoted by V , is

$$V = \sum_{t=1}^{T^*} (R_t - C_t)(1+i)^{-t} = \sum_{t=1}^{T^*} [R_0(1-\delta)^t - C_t](1+i)^{-t}$$

where $R_t - C_t$ is the net revenue from holding the patent during age t , i is the discount rate, δ is the appropriate decay rate, and T^* is the optimal lifespan of the patent as defined in Section 1

The lognormal distribution on R_0 induces a distribution of these values. The estimates of the parameters μ , σ , and δ are used to generate the quantiles of the distribution of V and their standard errors by simulating the value distribution.⁹

Table 3 presents the distribution of the value of patent rights for the 1970 cohort in each country. The distributions for the U.K. and France are based on the returns that accrue to patents which survive until age five, measured from age five until the patent is allowed to lapse. Two sets are generated for Germany, one based on age three and a second (for comparative purposes) rebased at age five. This is done by treating patents which are cancelled (in the simulation procedure) before age five as having zero value and adjusting downward the revenue stream for all surviving patents by the factor $(1-\delta)^2$.

The most prominent feature of these distributions is their sharp skewness. There is a dense concentration of patent rights with very little economic value. The median value of patent protection is only \$1861 in the U.K., \$897 in France, and \$5710 in Germany. Despite a substantial rise in the third quantile, only ten percent of all patent

Table 3. Distribution of the Value of Patent Rights in 1970^{a,b}

Quantile	United Kingdom (Age five)	France (Age five)	Germany (Age five)	Germany (Age three)
.25	461 (40)	109 (13)	1,580 (619)	2,534 (646)
.50	1,861 (271)	847 (158)	5,710 (690)	8,022 (1,068)
.75	5,959 (1,401)	4,022 (1,016)	17,329 (3,469)	23,169 (4,561)
.90	16,125 (5,203)	13,682 (5,153)	45,370 (7,856)	59,006 (11,429)
.95	28,435 (10,887)	27,479 (12,720)	77,029 (14,789)	100,180 (21,471)
.975	45,859 (19,891)	49,450 (26,526)	120,322 (25,039)	156,485 (36,293)
.99	82,475 (40,963)	101,743 (63,679)	208,061 (47,476)	270,594 (68,661)
Mean	6,963 (2,626)	6,656 (3,323)	19,124 (3,630)	25,278 (5,232)

^aThe value of patent rights is the discounted sum of net returns from age five until the patent lapses, for patents which survive until age five, in 1980 U.S. dollars. The discount rate is 0.10 and the parameter estimates from Table 2 are used for (μ, σ, δ) --that is, (6.36, 1.57, .19) for the U.K., (7.51, 1.47, .12) for Germany, and (5.39, 1.93, .10) for France.

^bEstimated standard errors are in parentheses.

rights are worth more than \$16,125 in the U.K., \$13,682 in France and \$45,370 in Germany. Most of the value of the stock of patent rights is concentrated in the tail of the distribution (especially the upper five percent). The quantiles are estimated with reasonable precision (standard errors less than half of point estimates), even in the tail. The general picture of a sharply skewed distribution of the value of patent rights emerges clearly in all three countries. We noted earlier that the patent renewal data are the only direct source of information on the distribution of the value of patent protection. However, the limited amount of information available on the related distribution of the value of patented innovations is similar in this respect. The survey evidence in Sanders, Rossman and Harris (1958), and Grabowski and Vernon (1983), and the larger-sample econometric evidence in Pakes (1985) suggest that the distribution of values of patented innovations is extremely skewed.

The mean of the discounted sum of returns from age five, among those patents still in force at age five, is \$6,963 in the U.K., \$6,656 in France, and \$19,124 in Germany. These differences are only partly a result of intercountry differences in the proportion of patents that survive until age five (a proportion determined in part by patent granting procedures). The mean returns from age five among all patents applied for (in contrast to just those surviving) are \$4,735 in the U.K., \$4,792 in France, and \$6,502 in Germany.¹⁰ Note that these latter values are almost perfectly proportional to the levels of GDP in the various countries (\$435.2, \$457.8, and \$623.8 million in the U.K., France, and Germany, respectively). The evidence, therefore, is

consistent with a form of Schmookler's (1966) demand inducement hypothesis in which differences in the returns from holding patents are primarily determined by differences in market size.

An estimate of the total discounted (at ten percent) value of patent protection from age five for all those patents in the 1970 cohort can be obtained by multiplying the means provided in the last paragraph by the number of patents applied for in the 1970 cohort. These estimates are: \$234 million for the U.K., \$217.6 million for France, and \$381.7 million for Germany. As measures of the total value of patent protection, these figures are biased downwards since they ignore both the value of patent protection for those patents which do not survive until age five, and the value of the first five years of protection for those patents which do survive. Nevertheless, it would be of interest to compare them to the R&D costs of producing the patents in the 1970 cohort. Though the desired R&D figures are not available, we do have the R&D expenditures made by the business enterprises in each country in 1970 (OECD 1982). The ratio of the value of the patent rights from age five in these countries to the R&D expenditures of their business enterprises is .057 in the U.K., .068 in France, and .056 in Germany. This comparison of our estimates of the total value of patent rights in a country to that country's R&D costs ignores, in addition to the value of patent protection prior to age five, various balance of trade and timing effects.¹¹ Even so, the figures do suggest two conclusions. First, the returns that result from the proprietary rights created by the patent laws seem to be over 5.5 percent of total R&D expenditures. Of course, whether a six percent increase in the value of inventions

represents a quantitatively important stimulus to R&D effort depends on the response elasticity of R&D investment to such incentives--a topic beyond the scope of this paper. The second conclusion is that the bulk of the returns from R&D investments do not seem to result from the ability of R&D performers to obtain patents. It is worth emphasizing, however, that these are aggregate results and the relative importance of patent protection may differ across sectors of the economy.¹²

One further caveat is in order here. We noted that the results indicate that much of the total value of the patent rights in a cohort of patents is concentrated in the tail of the value distribution. Since the patents in this tail are those which are renewed until the statutory limit to patent lives, our only nonparametric information on their values results from the behavioral assumption that they would not be renewed at the statutory limit unless their current returns were greater than the costs of renewal. Our model imputes exact values to these patents by extrapolating the estimated lognormal distribution. As noted earlier, the lognormal distribution did fit the observed renewal data better than the other distributions we tried (see note 5), but we used a global measure of fit which may not reflect closeness of fit in the tail. Of course we can never compare fits for that part of the tail we do not observe, but we can examine the robustness of our conclusions to dropping the renewal information in the neighborhood of the expiration date. This corresponds to asking what would have happened if we had less information on the tail than we did in fact have.¹³ Dropping the last one, two, and three ages results in a change of the estimate of the mean value of the U.K. distribution from \$6,963, to \$6,686, \$7,296, and

\$9,176, respectively. The corresponding changes for the French means are from \$6,656, to \$6,301, \$6,401, and \$6,696; and for Germany from \$19,124 to \$17,363, \$17,176, and \$18,094. None of these differences alters the qualitative nature of the conclusions drawn above.

Table 4 summarizes the secular changes that have occurred in the characteristics of cohorts of patents between 1955 and 1975. It provides index number values, at five year intervals, for the number of patents applied for (PA), the number of patents surviving until age five (P5), the estimates of the mean value of patent protection from age five for patents that survive until age five (\bar{V}), and the estimates of the total value of patent protection from age five (V). This table makes it clear that there are striking differences between the changes that occur over time in the number of patents in a cohort, and those that occur in the value of the patent rights embodied in those cohorts. Regardless of whether PA or P5 is used as a measure of quantity, we find that in all countries (though to varying degrees) the number of patents increased between 1955 and 1965, but then decreased between 1965 and 1975. In contrast, the value of the cohorts of patents increased in both ten-year intervals--and the increase was greatest in the later period in the U.K. and Germany. The point to stress here is that our estimates imply that one cannot make inferences on changes in the value of cohorts of patents during this period from changes in the quantity of patents in those cohorts, for there have been large (and largely offsetting) changes in the "quality" (or mean values) of the patents in the cohorts. We return to this point below.

Table 4. Indices of Quantity, Quality, and Total Value of Cohorts of Patents at Five-Year Intervals Between 1955 and 1975^a

Year	United Kingdom					France					Germany				
	PA	P5	\bar{V}	V	V	PA	P5	\bar{V}	V	V	PA	P5	\bar{V}	V	V
1955	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
1960	1.24	1.35	.80	1.08	1.08	1.26	1.39	1.14	1.59	1.59	1.04	1.08	1.01	1.10	1.10
1965	1.56	1.67	.86	1.42	1.42	1.67	1.78	1.49	2.64	2.64	1.21	1.04	.99	1.03	1.03
1970	1.72	1.78	.84	1.49	1.49	1.67	1.83	1.68	3.08	3.08	1.21	1.01	1.32	1.33	1.33
1975	1.45	1.58	1.40	2.18	2.18	1.55	1.65	1.86	3.06	3.06	1.10	.99	1.93	1.91	1.91

^aPA and P5 are indices of the number of patents applied for and the number of patents which survive until age 5, respectively. \bar{V} and V are indices of the mean and total of the estimated discounted present value of patent protection from age five for the patents surviving until age five. The index numbers refer to the cohorts applied for in the year row. These indices are computed from simulations of the value distribution for each cohort using the parameter estimates from regression (4) for the U.K. and Germany, and regression (2) for France (see Table 2).

Before going into country specific detail, it should be noted both that PA and P5 move in a similar fashion in all countries (so that either could be used to follow changes in the quantity of patents), and that changes in V over time in each country are quite highly (and positively) correlated with changes in GDP in the respective countries. The squared correlation coefficient between changes in log V and those in log GDP are .92, .55, and .91 for the U.K., Germany and France, respectively. Thus, the Schmooklerian hypothesis that differences in the benefits from holding patents depend primarily on differences in market size receives some support in the time series (differences within a country over time), as well as in the cross section (differences across countries), dimensions.

Our estimates imply that in the U.K. the total value of a cohort of patents approximately doubled over the twenty year period (GDP increased by a factor of 1.65 over the same time span). The increase in the total value index resulted from about equal increases in the quantity (PA or P5), and the quality (\bar{V}), indices. A more detailed look at the underlying data, however, reveals two distinct subperiods for each index. Between 1955 and 1969 the quantity index moved up rapidly, while the quality index actually declined somewhat. In 1969 there was a clear structural break which reversed the direction of change of both indices. The quantity of patents declined and their quality increased thereafter. In Germany the total value of a cohort of patents also doubled over the twenty-year period (GDP increased by a factor of 2.37), but almost the entire increase resulted from increases in the quality index (the quantity of patents in a cohort barely changed over the period). There are also two distinct subperiods in the German data. From 1955 to 1964

the quantity of patents increased slightly and there was a barely perceptible drop in the quality index. After 1964 there was a sharp increase in the quality index and a small decrease in quantity. In France the total value of a cohort tripled over the twenty-year period (GDP increased by a factor of 2.67), and this was a result of roughly equal increases in the quality and quantity indices. There is also some indication of two subperiods in the French results, though the break between them seems to be much less sharp than for the U.K. or Germany. Both quantity and quality indices move upward fairly smoothly until 1966, at which point the quantity index begins a slow downward trend and the quality index moves upward more sharply (though in a somewhat choppy fashion).

We conclude from this evidence that there appears to be some kind of structural break in the mid to late 1960's in all three countries. This shift is characterized by a marked decline in the number of patents in a cohort and a simultaneous sharp increase in the mean value of the patent rights of patents in a cohort in the U.K. and Germany (and possibly in France). A number of authors have noted an apparent worldwide decline in patenting activity during the 1970's (e.g., Evenson 1984; Griliches 1984). Evenson (1984) also documents a pervasive decline in patenting per unit of inventive input since the late 1960's (including the U.K., France and Germany). He interprets this trend as evidence of an exhaustion of technological potential, that is, as a decline in the "real invention per unit of inventive input" (Evenson 1984, p. 108). However, the evidence in this paper on the rise in the mean value of patent rights since the late 1960's suggests an alternative, though not

mutually exclusive, hypothesis. Part of the decline in patenting per unit of inventive input may reflect a shift away from "more patents" to patents of "higher quality."

Concluding Remarks

This paper provides an empirical investigation of the private value of patent protection and its changes over time. The approach is based on a simple model of patent renewal behavior, originally developed in Pakes and Schankerman (1984). The patent holder makes a decision each year, until the statutory expiration, whether to renew the patent on the basis of a comparison of the cost of renewal and the contemporaneous revenues which accrue to holding the patent. Given an assumed distribution function for initial revenues and observations on the actual time path of patent renewals and fees, the model delivers empirical estimates of the parameters of the distribution function of initial revenues and the rate of decay of these revenues. These parameters can then be used to generate the distribution of the value of patent rights. The empirical application of the model is based on a lognormal distribution of initial revenues and a comprehensive data set covering essentially all patent applications made during the post-1950 period in the U.K., France and Germany.

The qualitative characteristics of the empirical results that emerge from this study can be summarized quite succinctly. First, the distribution of the value of patent rights is sharply skewed in all three countries. There is a concentration of patent rights with very little private economic value, but the tail of the distribution contains highly

valuable patent rights. Second, the private rate of decay in revenues is quite high, and there is some evidence that it declined during the 1960's and 1970's. Third, the aggregate value of patent rights, though large in absolute terms, appears to be less than ten percent of the domestic R&D expenditures of the business enterprises in these countries. Though this finding suggests that at the aggregate level patent protection is a relatively small component of the incentive structure underlying private R&D investments, it does not necessarily imply that patent protection is an ineffective stimulus to R&D.

Finally, there have been substantial changes over time in both the number of patents applied for annually, and in the mean value of the patent rights that accrue to them. Moreover, the variation in the quantity of patents in different cohorts tended to be negatively related to the variation in their mean values, implying that exclusive reliance on patent counts as an indicator of inventive output can be quite misleading. In particular, there appears to have been a sharp structural break during the mid to late 1960's, after which the number of patents began to fall but the "quality" of the patents that were applied for rose substantially. Our estimates indicate that once the movements in quality are accounted for, the total value of patent rights increased rather than declined.

The methodology and empirical results provided here suggest that, at least in countries with renewal fees, it is both feasible and important to incorporate explicit measures of the "quality" of patents in measures of inventive output based on patent variables. The next important step is to analyze the empirical characteristics and the

theoretical determinants of variation in the quality dimension at a more disaggregated level, among different industries, and between different types of patents.

Footnotes

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1. For a more complicated model which allows agents to be uncertain about the $\{R_{tj}\}$ sequence see Pakes (1984). It should be noted, however, that our model does not assume that the rate of decay in the sequence $\{R_{tj}\}$ is exogenous to the firm's decision making process. In a dynamic context, a firm possessing an innovation has to choose between increasing present revenues and inducing entry, and charging smaller royalties to forestall entry. This choice is the basis of Gaskins' (1971) dynamic limit pricing analysis of situations involving temporary monopoly power. Gaskins' model can be used to show that the optimal revenue stream declines in age, a condition assumed below.

2. One interesting generalization of this model would be to allow for differences in decay rates across patents and to estimate the parameters of the joint distribution of the value of initial returns and the decay rates. The assumption of a common decay rate on the output of inventive activity, however, is used frequently in the empirical literature, and one advantage of our approach is that one can compare the estimates provided in this paper to the values assumed in that literature.

3. The raw data appear in the Annual Report of the Comptroller General of Patents, Designs and Trademarks (United Kingdom), the Bulletin Officiel de la Propriété Industrielle (France), and the Blatt für Patent, Muster and Zeichenwesen (Germany). We are grateful for the assistance of the respective patent office personnel for sending the data to us and answering our subsequent queries.

4. The French data do not contain the renewal proportion at age five for the cohorts 1951-1965, since they cover only the later ages. To obtain estimates of P_{5j} for these cohorts, we first ran a linear regression of P_{5j} against the exogenous variables included in the model (see equation (6) below) using the remaining cohorts 1966-1976. The estimated parameters were then used to generate predictions of P_{5j} , \hat{P}_{5j} , for cohorts 1950-1965, using the actual values of the exogenous variables for those cohorts. The normalization was then conducted with \hat{P}_{5j} . Given the nonlinear form of the dependent variable in the final estimating equation (see (6) below), this two-stage procedure is not fully consistent but there is no alternative if we wish to exploit the French data.

5. The comparison between the alternative specifications was based on two different measures of statistical fit suggested by Amemiya (1981): the sum of squared differences between P_{tj} and \hat{P}_{tj} , where \hat{P}_{tj} is the estimate of P_{tj} implied by the estimates of the parameters of the relevant model; and the weighted sum of squares using as a weight the binomial sampling variance of P_{tj} about its true value $[P_{tj}(1 - P_{tj})/N_j]$, where N_j is the number of patents on cohort j . The comparisons were made both for the fixed effects and the no-effects model (see the

discussion below). The lognormal specification fit the data best in all three countries.

6. Two points should be noted. First, Amemiya's (1981) suggestion of superimposing the error ε_{tj} which is presumed to be independently distributed is not, strictly speaking, consistent with the model since the sequence $\Phi^{-1}(1 - P_{tj})$ is (by construction) nondecreasing in t . We use Amemiya's procedure and ignore this problem in the results we present because the estimate of the variance of ε_{tj} is too small for one to think correction for the problem would have a significant effect on the parameter estimates. Second, note that since we have data only on the cohorts rather than on individual patents, any patent-specific disturbance (such as an optimization error in the renewal rule) is subsumed in the distribution function.

7. Two remarks are in order. First, we tested a random effects specification of the μ 's by comparing the within-cohort and between-cohort parameter estimates for the model with a constant decay rate (see Hausman and Taylor, 1981). The hypothesis of random effects is rejected decisively in all three countries. Second, we tried summarizing the full set of cohort fixed effects as a linear or quadratic trend over time, i.e., $\mu_j = \mu + \alpha_0 j + \alpha_1 j^2$ where μ represents the value for the initial cohort. This specification is rejected formally by the data (computed F statistics about six), but the overall trends in the individual μ 's are reflected by the estimates. In the U.K. and Germany the quadratic fits much better than the linear trend. The point estimates (standard errors) for the U.K. are $\mu = 6.47$ (.22), $\alpha_0 = -.048$ (.013) and $\alpha_1 = .0022$ (.0005); for Germany, $\mu = 7.01$ (.13), $\alpha_0 = -.0073$

(.011), $\alpha_1 = .0016$ (.0004). In both countries the implied μ 's first decline and then rise. In France one cannot reject the linear trend as against the quadratic. The estimates are $\mu = 4.53$ (.22) and $\alpha_0 = .044$ (.01).

8. The estimate of $\bar{\mu}$ for Germany is not directly comparable to those for the U.K. and France since it is based on renewals from age three. Making the adjustment to age five (i.e., $\bar{\mu} + \ln(1 - \delta)^2$) yields an estimate of 7.04, which does not alter our conclusion. Note also that we can estimate the response of patent renewals to the costs of renewal. The estimate of $1/\sigma$ provides the response of $y_{tj} = \Phi^{-1}(1 - P_{tj})$ to increases in costs. The estimates in the table imply that a one percent increase in renewal fees decreases the proportion renewed by about .02 percentage points. (This is based on derivatives evaluated at sample means, but the figure does not vary much across ages or countries.) Of course, to obtain the elasticity of P with respect to costs we have to divide .02 by P , and this will vary with age.

9. We draw 50,000 pseudo-random variables from a lognormal distribution with the estimated values of μ and σ , calculate V for each of them, and then derive the quantiles of the implied distribution of V . The process is repeated three more times, each time perturbing one of the estimated parameters (μ , σ , and δ) by one percent. This provides numerical estimates of the derivatives of each of the quantiles with respect to the parameters, which are used with the estimated covariance matrix of the parameter estimates to calculate the (asymptotic) standard errors of each of the quantiles.

10. These means are measured by multiplying the mean value of patents surviving to age five by the proportion of all applications which reach age five.

11. Not all of the returns to the patents in force in a given country accrue to nationals of that country, and business enterprises in a given country also earn returns from patents in force elsewhere. Moreover, there is some lag between R&D expenditures and patentable output (though the mean lag does not appear to be long; see Pakes and Schankerman 1984). It is worth noting that the ratios presented in the text are insensitive to the choice of cohort. The figures for the U.K., France and Germany in 1965 are .062, .087 and .067 respectively; for 1975 they are .085, .055 and .071.

12. In an important study of the British patent system based on extensive survey data, Taylor and Silberston (1973) conclude that patent protection is for the most part not an important component of the incentive structure inducing R&D investment, but there are some notable exceptions (e.g., chemicals and pharmaceuticals). Also see Mansfield, Schwartz and Wagner (1981).

13. We are grateful to Robin Sickles for this suggestion.

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