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SMALL COUNTRIES IN MONETARY UNIONS: A TWO-TIER MODEL

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Small Countries in Monetary Unions: A Two-Tier Model

ABSTRACT

In a previous analysis of the West African Monetary Union, Macedo (1985a), size is taken to be a major structural characteristic of a country in the sense that large countries are not affected by disturbances originating in small countries but small countries are affected by large countries' domestic disturbances. In this paper, we generalize some of the results and present the structure of the model in more detail.

Using a four-country, two-tier macroeconomic model, it is shown that the pseudo-exchange rate union of the two small countries with the large partner has no effect on their real exchange rates but affects their price levels, whereas a full monetary union requires in principle a transfer from the large partner in the union. The allocation of this transfer between the two small countries by their common central bank will have real effects when the allocation rule differs from the steady-state monetary distribution.

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I. Introduction

In connection with the celebrated controversy about the relative desirability of fixed and flexible exchange rates, a considerable analytic literature on monetary unions has emerged. It was surveyed ten years ago by Tower and Willet (1976). Since then, there have been contributions by Allen and Kenen (1980), Aoki (1983a and b), Marston (1984a and 1985), Melitz (1984), Huizinga (1984) and others.

According to this literature, the key factors on which the impact of a union depends are, first, the sources and types of economic disturbances giving rise to exchange rate fluctuations, second, the trade patterns of the country joining the union, and, third, wage and price behavior at home and abroad. The conditions under which a fixed exchange rate regime is superior to floating according to some social welfare criterion usually involve a complicated weighting of these key factors, making generalizations difficult.

The relative size of the partners is generally reflected in the source and types of disturbances as well as in the trade pattern. In our analysis of the West African Monetary Union, Macedo (1985a), though, size is the major structural characteristic of a country. Specifically, large countries are **not** affected by disturbances originating in small countries but small countries are affected by large countries's domestic disturbances. In this paper, we generalize some of the results and present the structure of the model in more detail.

It relies on standard aggregate demand and aggregate supply relationships, with trade and capital movements linking national economies. Account is taken of the unequal size of the potential partners by modelling two pairs of identical economies, large and small. These are two identical large economies whose bilateral exchange rate floats freely and two identical small economies who decide on whether they will float or fix their exchange rate with one of the large countries. In so doing, they also allow the union-wide central bank to decide on monetary allocations.

Due to the difference in size between the partners in the union, only the distribution of money between the two small countries is endogenously determined. Even there, it can be modified by the allocation of a monetary transfer from the large partner. In the terminology of Corden (1972), there is a pseudo-exchange rate union between one of the large countries and the small countries but full monetary integration between the two small countries.

Each national economy is highly stylized, and the focus of the model is on the interaction of the members of the monetary union, two small countries labelled country one and country two, who take as given the member of the pseudo-exchange rate union, labelled country star and the country outside the union, labelled country double-star. The model is recursive because the small-country tier is irrelevant to the large-country tier. The results can therefore be extended to three or more tiers.

The national economies are described by conventional aggregate relationships. Demand for domestic output (the IS curve) is a function of foreign outputs, relative prices or the real exchange rate, and the real interest rate and it can also be changed by an exogenous demand disturbance. Demand for real balances (the LM curve) is a function of domestic output and the nominal interest rate, as a measure of the return differential. By eliminating the nominal interest rate, we obtain an aggregate demand curve which relates domestic output to the real exchange rate, to foreign output and to the exogenous demand and monetary disturbances. A real depreciation increases the demand for domestic output along conventional foreign trade multiplier lines.

The supply of domestic output is derived from labor market equilibrium, where the supply of labor by workers responds to the wage deflated by a consumer price index and the demand for labor by firms responds to the wage deflated by price of the domestic good. Eliminating the nominal wage, we obtain an aggregate supply curve relating domestic output to the real exchange rate and an exogenous supply disturbance, which can be interpreted as an increase in productivity. A real depreciation lowers the supply of domestic output because it raises the product wage. Prices change as a proportion of the difference between demand for and supply of domestic output, so that a Phillips curve allowing for real wage rigidity is featured.

The model is closed by the assumption that domestic and foreign assets are perfect substitutes, so that interest rates are equalized in the steady-state. This determines recursively the real exchange rate and the price of domestic output, in terms of the exogenous real and monetary disturbances respectively. Then, under flexible exchange rates, the nominal exchange rate is given by monetary disturbances, whereas, under fixed rates, the nominal money stock is determined endogenously.

Size does not affect the interest-rate elasticities of money demand and aggregate demand, which are common to all four countries, and the other parameters are identical between the pairs of large and small countries. These assumptions are not essential for the results, but an analytical solution does require some symmetry between economic structures.

The model is used to assess the effect of fixing the bilateral exchange rates of the two small countries with one of the large countries. Under price flexibility, the exchange rate regime has no effect on the real exchange rate, since the effect on the nominal exchange rate and the price level offset each other. Nevertheless, a monetary union between one of the large countries and the two small countries may require a transfer from the large partner, which offsets internal and external disturbances. To that extent, the union allows the small countries' central bank to enforce an asymmetric monetary allocation rule. Then prices will not be adjusted to the nominal exchange rate and the real exchange rate will also have to change as a consequence of the price rigidity.

We write here the model with variables expressed as logarithmic deviations from the stationary state, leaving the derivation and interpretation of the parameters to Appendix 1. We also set to zero the rate of change in prices and exchange rates. This can be interpreted as the stationary state of a model with perfect foresight on these variables. The dynamic version of the model is solved in Appendix 2, where its stability is also discussed. Finally, we present the large country tier in terms of average sums and differences of variables and exogenous disturbances, as done by Aoki (1981). The more conventional presentation can also be seen in Appendix 2. Thus, a variable x_j, j = 1, 2 will be expressed as

(1) $x^d = (x_1 - x_2)/2$

(2) $x^s = (x_1 + x_2)/2$

It is obvious that:

(3) $x_1 = x^s + x^d$

(4) $x_2 = x^s - x^d$

II. Flexible Exchange Rates

The log-linear model consists of the equations listed in Table 1. The variables and parameters are defined in Table 2. We solve the large country tier first.

Equations (5)-(10) in Table 1 show that monetary disturbances have no effect on the real exchange rate, whereas they have one-to-one effects on the price level, as can be seen from equation (11). Indeed, the real exchange rate is given by a composite real disturbance which involves the difference between relative demand and supply shocks in the two large countries. The supply disturbance enters with a multiplier effect, given by ν:

(16) $$\theta^* = -\frac{1}{N} [{}^*u_A^d - (1 + \nu)\, {}^*u_\pi^d]$$

where $N = a + k\beta(1 + \nu)$

While relative real disturbances are channeled through the real exchange rate, global real disturbances are channeled through the real (= nominal) interest rates, which are equal in both countries in the stationary state:

(17) $$i^* = i^{**} = \frac{1}{b} [{}^*u_A^s - (1 - \nu)\, {}^*u_\pi^s]$$

Table 1

THE MODEL

IS equations

(5) $*y^{d}(1 + \nu) = a\theta* - b\ *i^{d} + *u_{A}^{d}$

(6) $*y^{s}(1 - \nu) = -b\ *i^{s} + *u_{A}^{s}$

(7) $y_{j} = (a* + a**)\theta_{j} - a\theta* + \nu*y* + \nu**y** - bi_{j} + u_{A}^{j}$ $\quad j = 1, 2$

Supply equations

(8) $*y^{d} = -\ k\ \beta\ \theta* + *u_{\pi}^{d}$

(9) $*y^{s} = *u_{\pi}^{s}$

(10) $y_{j} = -\ h(1 - \alpha)\ \theta_{j} + h\alpha_{*}\theta* + u_{\pi}^{j}$ $\quad j = 1, 2$

LM equations

(11) $u_{m}^{j} - p_{j} = y_{j} - ci_{j}$ $\quad j = *, **, 1, 2$

Interest parity

(12) $i_{j} = i_{\ell}$ $\quad \ell = *, **, 1, 2 \neq j$

Real exchange rates

(13) $\theta* = e + p** - p*$

(14) $\theta_{j} = e_{j}^{**} + p** - p_{j}$ $\quad j = 1, 2$

Triangular arbitrage

(15) $e_{j}^{*} = e_{j}^{**} - e$ $\quad j = 1, 2$

Table 2

LIST OF VARIABLES

y_j = real output of country j, j = *, **, 1, 2

p_j = price of output of country j

i_j = nominal (and real) interest rate in country j

$e^*_\ell (e^{**}_\ell)$= price of currency of large country star (double star) in terms of the currency of small country ℓ, ℓ = 1, 2

e = price of the currency of country doublt star (the numeraire) in terms of the currency of country star

u^j_A = exogenous increase in the demand for the output of country j

u^j_π = exogenous increase in the supply of the output of country j

u^j_m = exogenous increase in the money stock of country j

ν = large countries' foreign output multiplier

ν^j = small countries' output multiplier with large country j, j = *, **

a = term involving the large countries' foreign trade elasticities

a^j = terms involving the small countries' foreign trade elasticities with large country j

α^j = share of large country j in small countries' consumer price index

$\alpha = 1 - \alpha_* - \alpha_{**}$ share of domestic goods in small countries' consumer price index

β = share of foreign goods in large countries' consumer price index

k(h) = large (small) countries' real exchange rate elasticity of aggregate supply

b(c) = interest semi-elasticity of aggregate demand (money demand)

Note the dampened effect of a global supply disturbance on the interest rate, in contrast with the magnified effect of a relative supply disturbance on the real exchange rate. A positive supply shock in the starred country raises output and therefore requires a depreciated real exchange rate and (if $\nu < 1$) a lower interest rate to raise domestic and foreign demand for the starred country's output. A proportional increase in the relative demand for and supply of output in both countries ($*u_A^d = *u_\pi^d$) still depreciates the real exchange rate by ν/N but now raises the interest rate by ν/b.

Given (16), we obtain y* and y** from (8) and (9). Given (17) and real outputs, we get their price from (11). Finally, we obtain the nominal exchange rate from the definition of the real exchange rate and the solution for the prices of domestic output. It is useful to write those in terms of composite disturbances:

$$p^i = u_m^i \pm \tilde{U}_*^d + ci^* - *u_\pi^s \qquad i = *, ** \tag{18}$$

where $\tilde{U}_*^d = \frac{k\beta}{N} *u_A^d + \frac{a}{N} *u_\pi^d$

Note that $y^* = \tilde{U}_*^d + *u_\pi^s$ and $y^{**} = -\tilde{U}_*^d + *u_\pi^s$, so that we get the LM curve immediately. Also, the effect of a global supply shock is magnified on the price level because, aside from the interest rate effect, given by $(1 - \nu)c/b$, there is a distinct one-to-one effect. In (18) this effect is made up of a global part ($*u_\pi^s$) and a relative part ($*u_\pi^d$). The latter is included in the composite relative real disturbance $\tilde{U}_*^d$, which enters negatively for p* and positively for p**. If the multiplier term included in N were zero, this would become a weighted average of demand and supply disturbances. The demand disturbance is weighted by the supply elasticity ($k\beta/N$) and the supply disturbance is weighted by the demand elasticity (a/N). Therefore if output is demand-determined (k = 0) and there are no supply shocks ($u_{\pi}^{i} = 0$), this composite disturbance vanishes. If, in addition, there are no global shocks ($*u_i^s = 0$), the interest rate effect also vanishes and prices are proportional to monetary expansion. Of course, the interest rate effect always cancels in the expression for relative prices:

$$(19) \qquad p^* - p^{**} = 2{*u}_m^d - 2\,\tilde{U}_*^d$$

Let us now consider the effect of the disturbances in turn. Monetary disturbances have no effect on the real exchange rate and offsetting one-to-one effects on the nominal exchange rate and on the own price level as can be seen from (18). Negatively correlated real disturbances (such that $*u_i^d = u_i^*$ and $*u_i^s = 0$, $i = A, \pi$) leave interest rates unchanged ($i^* = i^{**} = 0$). The effect on the price level is given by the first term in (18). For example, relative demand expansion in country star appreciates the real exchange rate by 1/N, raises the home price level by $k\beta/N$, lowers the foreign price level by the same amount and leads to a nominal appreciation of $2k\beta/N$. Unless the supply elasticity is very small, therefore, there will be a magnification of the effect on the real exchange rate. Note that when $k = u_{\pi}^{j} = 0$, $j = *, **$, the price level is independent of real disturbances, and the real money stock $u_m^j - p^j$ is fixed.

Positively correlated real disturbances (such that $*u_i^s = u_i^*$ and $*u_i^d = 0$, $i = A, \pi, m$) leave exchange rates unchanged ($\theta^* = e = 0$). The effect on the price level is given by the second term in (18). The effect of a demand shock differs from the effect of a supply shock by a factor of $1 - \nu + b/c$.

Coming to the small countries' model, the interest rate is still given by (17) but the real exchange rate now reflects two factors in addition to the relative real disturbances featured in (16) for the real exchange rate of the two large countries. These are the choice of the numeraire and the nature of the trade patterns between the domestic economy and the two large countries. We write the solution as:

$$(20) \qquad \theta = \zeta\, \theta^* + \frac{U^{\nu}}{H} - \frac{1}{H}\,(u_A^d - u_\pi^d)$$

$$U^{\nu} = (\nu - \nu^* - \nu^{**})\ {}^*u_\pi^s - (\nu^* - \nu^{**})\ \tilde{U}_*^d$$

and $$u_i^d = u_i - {}^*u_i^s \qquad\qquad i = A,\ \pi$$

The first term measures the relative sensitivity of the domestic economy to the two large countries in terms of the demand elasticities (a^* and a^{**}) and the share of foreign goods in the consumer price index (α_* and α_{**}). We will return to it below. The second term (in square brackets) captures the effect of trade patterns. If the trade multipliers with the two large countries are the same ($\nu^* = \nu^{**}$), relative real disturbances there have no effect. Global supply disturbances in the two large countries still have an effect, however, as long as the small country trade multiplier ($2\nu^*$) differs from the large countries trade multiplier (ν). Thus, if $\nu/2 > \nu^*$, a favorable supply disturbance in the large countries will lead to a real depreciation in the small countries and conversely. When trade patterns are strongly symmetric ($\nu/2 = \nu^* = \nu^{**}$), this term drops out. The effect of relative real disturbances featured in (16) above is captured by the third term in (20), where the cyclical position of the domestic economy is measured relative to the world average. For example, if domestic demand increases by more ($u_A >$ $*u^s_A$), the real exchange rate depreciates.

The real effective exchange rate of the small country can be defined as a weighted average using the shares in the consumer price index as weights:

$$(21) \qquad {}^E\theta = \bar{\zeta}\ (e^* + p^* - p) + (1 - \bar{\zeta})\ (e^{**} + p^{**} - p)$$

$$= \theta - \bar{\zeta}\ \theta^*$$

where $\bar{\zeta} = \alpha_*/(1 - \alpha)$

Using (21) in (20) we get

$$(22) \qquad {}^E\theta = \frac{1}{H}\ [A^*\theta^* + U^\nu - u^d_A + u^d_\pi)$$

where $\qquad A^* = (a^*\alpha_{**} - a^{**}\alpha_*)/(1 - \alpha)$

Note that the choice of the numeraire continues to play a role unless the trade elasticities are proportional to the weights in the consumer price index, i.e. unless $\zeta = \bar{\zeta}$ or $A^* = 0$. This case, emphasized by Marston (1984a) in the context of his model, may be called the case of "balanced sensitivities". It is a useful benchmark, but (22) shows clearly that the real exchange rate between the two large countries will have a positive effect on the small countries' real effective exchange rate is trade with the partner country is relatively more sensitive than reflected on the share of partner countries' goods in the consumer price index. Conversely, if the share is large relative to the trade elasticities, a real depreciation of the starred countries' currency will imply a real appreciation of the small country's currency.

Another way to look at this effect is in terms of "optimal weights". Then (20) shows that the weighted sum of demand and supply elasticities with each trading partner, $\tilde{a}^j$, should be used to offset the effect of the choice of the numeraire. With these weights, and in the absence of asymmetries between the two partners ($U^\nu = 0$), the real effective exchange rate is solely a function of the cyclical position of the small country relative to the rest of the world.

The price of domestic output is again obtained from the supply equation, (10) in Table 1, and money-market equilibrium, equation (11) in Table 1. Leaving the separate effect of the interest rate as given from (17) above and using (20), we get:

$$(23) \qquad p = u_m + \chi(A^*\theta^* + U^\nu) - \tilde{U}^d + ci^* - {}^*u_\pi^s$$

where $\quad \chi = h(1 - \alpha)/H$

and $\quad \tilde{U}^d = \chi\, u_A^d + (1 - \chi)\, u_\pi^d$

The weighted average of domestic demand and supply disturbances (relative to foreign global disturbances), denoted by $\tilde{U}^d$, is now a proper one since, for the small open economy, we neglect the multiplier effect with repercussion present in $\tilde{U}^d_*$. In fact, when $A^* = 0$ and $U^\nu = 0$, we get an expression for y of the same form as above, $y = \tilde{U}^d + {}^*u_\pi^s$, except for the role of foreign global disturbances.

Using (18), the difference between the domestic price and the price in the double starred country can be written as:

$$(24) \qquad p - p^{**} = u_m - u_m^{**} + \chi(A^*\theta^* + U^\nu) - \tilde{U}^d_* - \tilde{U}^d$$

Adding (20) and (24), we obtain the nominal exchange rate with the numeraire. Using triangular arbitrage, we obtain the exchange rate with the starred country. These can be written respectively as:

$$(25) \qquad e^{**} = u_m - u_m^{**} + \zeta\theta \; + \xi(A^*\theta^* + U^{\nu}) - \tilde{U}_*^d - U^d$$

$$(26) \qquad e^{*} = u_m - u_m^{*} - (1 - \zeta)\theta^* + \xi(A^*\theta^* + U^{\nu}) + \tilde{U}_*^d - U^d$$

where $\qquad \xi = \chi + \frac{1}{H} \lessgtr 1$

and $\qquad U^d = \xi \, u_A^d + (1 - \xi) \, u_\pi^d$

Note that the direction of the effect of supply shocks depends on demand elasticities. If $a^* + a^{**} < 1$, then $\xi > 1$ and the effect will be negative on the composite disturbance U^d.

As mentioned, when sensitivities are balanced ($A^* = 0$), the real effective exchange rate of the domestic country is given by a ζ-weighted average. The same is of course true for the nominal effective exchange rate. In general, using (19), we can write the nominal effective exchange rate and the effective relative price ratio respectively as:

$$(27) \qquad {}^{E}e = u_m - {}^{*}u_m^{\zeta} + \xi(A^*\theta^* + U^{\nu}) - (1 - 2\zeta)\, \tilde{U}_*^d - U^d$$

$$(28) \qquad {}^{E}p = u_m - {}^{*}u_m^{\zeta} + \chi\,(A^*\theta^* + U^{\nu}) - (1 - 2\zeta)\, \tilde{U}_*^d - \tilde{U}^d$$

where $\qquad {}^{*}u_m^{\zeta} = \zeta \, u_m^{*} + (1 - \zeta)\, u_m^{**}$

When trade patterns are strongly symmetric, $U^{\nu} = 0$ and $A^* = 0$ and in addition $\zeta = \frac{1}{2}$, the expressions simplify considerably. In particular, ${}^{*}u_m^{\zeta} = u_m^{s}$ and the effective exchange rate in (27) becomes:

(29) $\quad {}^{E}e = u_m^d - U^d$

where $\quad u_m^d = u_m - {}^{*}u_m^s$ as before.

To sum up the results of the three-country model under flexible exchange rates, we see that, with complete symmetry, disturbances enter as differences between the domestic and average foreign value, which we have denoted as u_i^d, $i = A, \pi, m$. This is due to the neglect of foreign repercussions for the small open economy. Monetary disturbances have one-to-one effects on nominal variables and zero effects on real variables. Real disturbances have less than one-to-one effects on the price level (the smaller the lower the supply elasticity relative to the demand elasticity, as captured by χ) but the effect on the real exchange rate depends on the size of both, as captured by H. The effect of demand disturbances on the nominal exchange rate will be more than one-to-one if elasticities are low ($\xi > 1$), in which case a favorable supply shock will involve a depreciation.

III. Two-tier Monetary Unions

1. A two-country union

If the domestic economy enters into an exchange-rate union with the starred country so that $e^* = 0$ (taking an initial value of unity), then $e^{**} = e$. In this case, the foreign model determines the exchange rate and the prices of foreign outputs but the domestic money stock adjusts to keep real money balances at the level required by domestic and foreign real disturbances.

The irrelevance of the exchange rate union for real variables reflects the classical dichotomy between the real and monetary sides of the economy. Either a small country lets the exchange rate float and determines its money stock or it fixes the exchange rate, and the money stock becomes endogenous. We can thus write:

(30) $e - e^{**} = \tilde{p} - p = m - u_m$

where $\tilde{p}$ is the price level under the union

and m is the (endogenous) money stock under the union.

According to (30), if the fixed exchange rate e is lower than the one prevailing before the agreement, the money stock and the price of domestic output will fall by the same amount (relative to their previous values) under flexible exchange rates. The fall in the money stock is brought about by a capital outflow which would increase in magnitude if the government attempted to increase the supply of domestic assets to the public. As long as real output does not change, the real money stock remains fixed and the fall in money balances is transmited to prices. Only by increasing demand for real output could the government enforce a different nominal income. Alternatively, as we will see, the loss in reserves could be offset by a transfer from abroad.

To obtain the small country's money stock under the union, set the left-hand-side of (26) equal to zero and solve for m:

(31) $m = u_m^* + (1 - \zeta)\theta^* - \xi(A^*\theta^* + U^\nu) - \tilde{U}_*^d + U^d$

Substituting for θ^* so as to show the nominal exchange rate, we get:

(32) $m = {}^*u_m^\zeta + (1 - \zeta)e - \xi(A^*\theta^* + U^\nu) + (1 - 2\zeta)\tilde{U}_*^d + U^d$

This is nothing but the definition of the nominal effective exchange rate in (27) solved for the money stock. In fact, we can write the simplified equilibrium condition in (29) in this regime as:

(33) $$m = {}^{*}u_m^s + {}^{E}\tilde{e} + U^d$$

where ${}^{E}\tilde{e} = (1 - \zeta)e$, the effective exchange rate under the union.

The two countries which have formed the exchange-rate union have to agree on how to defend their common parity with the double-starred country. The union-wide money stock is still exogenous and, together with the money stock of the double-starred country, it determines the flexible exchange rate between the two large countries. Any exogenous increase in the union-wide money stock is endogenously allocated between the two partners on the basis of their steady-state shares:

(34) $$\eta^{*}m + (1 - \eta^{*})m^{*} = t^{*}$$

where t^{*} is an exogenous increase in the union-wide money stock

and η^{*} is the share of the small country's money stock.

The domestic economy being small relative to its partner means we can make $\eta^{*} = 0$, so that the money stock of the starred country is still exogenous, $m^{*} = t^{*} = u_m^{*}$.

2. A monetary union of two small countries

Suppose now that there are two, rather than one, small countries which peg their exchange rates to one of the large countries and pool their reserves together. We assume that the total money stock of these two countries -- which we now refer to as the union-wide money stock -- is small relative to the money stock of the large country. Since the two small countries are of similar size, however, we need to track the money between them. In that case, the mechanisms of monetary allocation within the exchange rate union may have real effects in the two small countries.

We still neglect the trade multiplier as well as the role of relative prices in determining the pattern of trade between the two small countries. Links are exclusively monetary.

Like (34), a monetary union between the two small countries involves an allocation of their money stocks. Unlike then, though, the two countries are of similar size and both money stocks are endogenous:

(35) $$\eta m_1 + (1 - \eta)\, m_2 = t$$

where t is an exogenous increase in the union-wide money stock.

and η is the share of country one's money stock.

Since the two small countries fix their bilateral exchange rate, we can equate their effective exchange rates so that, making ${}^{E}e_1 = {}^{E}e_2$ in (28):

(36) $$m_1 = t + (1 - \eta)(U^1 - U^2)$$

(37) $$m_2 = t - \eta(U^1 - U^2)$$

(38) $${}^{E}e = t - {}^{*}u_m^{\zeta} - (1 - 2\zeta)\, \tilde{U}_{*}^{d} + {}^{*}U^{s} - U^{\eta}$$

where $U^j = \xi\, u_A^j + (1 - \xi)\, u_\pi^j$ $\qquad j = 1, 2$

$${}^{*}U^{s} = \xi\, {}^{*}u_A^{s} + (1 - \xi)\, {}^{*}u_\pi^{s}$$

and $$U^{\eta} = \eta\, U^{1} + (1 - \eta)U^{2}$$

Given the unchanged real exchange rates, equations (36) through (38) are the solution of the exchange-rate union between two small countries. If t = 0, the money stocks are unchanged when demand and supply disturbances are perfectly correlated ($U^{1} = U^{2}$). In that case, from (38), the effective exchange rate appreciates by $U^{\eta} = U^{1}$.

Clearly, when the two countries are identical in steady state and $\eta = \frac{1}{2}$, we can express the solutions in terms of differences and sums of the relative domestic disturbances. In particular $U^{\eta} = (U^{1} + U^{2})/2$, so that $U^{\eta} - {}^{*}U^{s}$ is simply a ξ-weighted average of the difference in global disturbances in small and large countries.

3. A three-country exchange-rate union

The effect of fixing the exchange rate with country star can now be seen from (38). In that case, the effective exchange rate is fixed by the monetary arrangement and some other variable has to adjust. Solving (38) for the required increase in the small-countries union money stock, denoted by $\tilde{t}$, we get:

$$(39) \qquad \tilde{t} = U^{\eta} - {}^{*}U^{s} + {}^{E}\tilde{e} + (1 - 2\zeta)\tilde{U}^{d}_{*} + {}^{*}u^{\zeta}_{m}$$

$$= U^{\eta} - {}^{*}U^{s} + (1 - \zeta)\theta^{*} - \tilde{U}^{d}_{*} + u^{*}_{m}$$

The first equation in (39) shows the nominal effective exchange rate under the union, whereas the second substitutes for the floating exchange rate to express the determinants of the required transfer by the large partner.

The difference in small countries composite global disturbances U^j (weighted by η) and the large countries is captured by the first two terms in (39). For example, the effect of demand expansion is positive and given by $\xi\{[\eta\, u_A^1 + (1 - \eta)\, u_A^2] - (u_A^* + u_A^{**})/2\}$. The supply effect hinges again on whether $\xi \gtrless 1$. The next two terms capture relative real disturbances in the large countries. The effect of relative demand expansion in the partner ($*u_A^d > 0$) is negative and given by $(1 - \zeta + k\beta)/N$ whereas the supply effect is ambiguous. It will also be negative if $a > (1 + \nu)(1 - \zeta)$, that is if the trade is biased toward the partner country ($\zeta = 1$). The last term captures monetary expansion in the partner country and it has the one-to-one positive effect expected from (31). Even if all foreign variables are at their stationary-state values, the required transfer will be positive if there is fiscal expansion in either one of the small countries. Conversely, any transfer in these conditions will require fiscal expansion $\tilde{u}_A^j$ given by:

(40) $$\eta\, \tilde{u}_A^1 + (1 - \eta)\, \tilde{u}_A^2 = t/\xi$$

where $u_\pi^j = 0,\ j = 1, 2$

and all foreign variables are zero.

In (40), it is assumed that the transfer has no negative effect on the money stock of country star because of the size difference ($\eta^* = 0$).

Therefore, as long as the distribution of money among the small partners is done according to (35), there is no induced change in real variables. The situation changes with the establishment of a common central bank or in the presence of a nominal rigidity.

Taking the latter point first, if the price of domestic output is fixed, changes in the real exchange rate are transmitted to the nominal exchange rate, which must be flexible. When both the nominal exchange rate and the price of domestic output are fixed, then, the government must use fiscal policy to offset the effects of real disturbances. In particular, if the domestic country pegs its exchange rate and prices do not adjust, the real exchange rate will have to change accordingly. The monetary union has a real effect due to the combination of the price rigidity and the change in monetary regime. If, at a price given by p_j^T the real exchange rate is θ and the price does not change after joining the union, the difference in real exchange rates equals the difference in nominal rates, so that, from (30), dropping j subscripts, we get:

$$\theta^T - \theta = \tilde{p} - p_T \tag{41}$$

The difference between the price prevailing in the neutral situation and the rigid situation can be decomposed further into the difference in real outputs and in money stocks. The latter, in turn, can derive from an increase in the foreign money stock. Assuming that $\theta = 0$ under the "neutral" union (i.e. $u_A = u_\pi = 0$), we get:

$$\begin{aligned} \tilde{p} - p_T &= m - m_T - y + y_T \\ &= -\frac{1}{H} u_A^T - \bar{u}_m \end{aligned} \tag{42}$$

where m_T is given by (33) with $U^d = \xi u_A^T$ and $*u_m^s = \bar{u}_m$

A demand expansion u_A^T, perhaps in the form of fiscal expansion, appreciates the real exchange rate by 1/H, whereas a monetary transfer from abroad has a one-to-one effect.

When account is taken of the induced real appreciation, the demand expansion increases output by $\chi < 1$. Given monetary policy, this expansion would reduce prices by the same amount it expands output, so that the nominal appreciation would be given by $\chi - 1/H = \xi$. Ruling out the exchange rate change and the fall in prices requires an increase in the money stock by the same factor ξ, which will be less than one if the trade elasticities are high enough. The real appreciation is accompanied by a rise in prices in the amount 1/H. To keep the nominal exchange rate constant, demand expansion must therefore be consistent with the increase in the money stock or $u_A = \bar{u}_m/\xi$. Of the equivalent rise in nominal income, a proportion χ/ξ goes to real output expansion and the remainder $(1 - \chi)/\xi$ goes to the rise in prices and fall in the real exchange rate.

In sum, the effects of a fixed exchange rate regime are confined to nominal variables unless there is a price rigidity, an induced demand for domestic output, as a consequence of fiscal expansion, or a transfer from abroad. The latter possibility becomes quite relevant when there is a common central bank for involving the two small countries.

4. A monetary allocation rule

Consider now that the required transfer is allocated by the union's central bank based on an administrative rule given by:

$$(43) \qquad \tilde{m}_1 = \frac{\omega}{\eta}\tilde{t}$$

where $\tilde{m}_1$ is the money stock of country one under the ω-rule.

Under this rule, the money stock of country two becomes:

(44) $$\tilde{m}_2 = \frac{1 - \omega}{1 - \eta} \tilde{t}$$

Contrasting (43) with (36), it is clear that these two money stocks will not be the same, so that prices will also differ in the two situations. Given that the nominal exchange rate is tied by the monetary arrangement, the real exchange rates and hence real outputs will also be different. Denoting the real exchange rate of country one under the ω-rule for the transfer by θ_1^T, we have:

(45) $$\theta_1^T - \theta_1 = m_1 - \frac{\omega}{\eta} \tilde{t}$$

(46) $$\theta_2^T - \theta_2 = m_2 - \frac{1-\omega}{1-\eta} \tilde{t}$$

Using (36), (37) and (39), we get:

(47) $$\theta_1^T - \theta_1 = (\frac{\eta-\omega}{\eta})z + (1 - \omega) U^1 - \omega(\frac{1-\eta}{\eta}) U^2$$

(48) $$\theta_2^T - \theta_2 = - (\frac{\eta-\omega}{1-\eta})z + \omega U^1 - (1 - \omega) \frac{\eta}{1-\eta} U^2$$

where $z = {}^*U^s + (1 - \alpha)\theta^* - \tilde{U}_*^d + u_m^*$.

Several features of (47) and (48) deserve notice. If $\eta = \omega$, the gap in the real exchange rates is entirely determined by the relative cyclical positions of the two small countries, weighted by their monetary shares in stationary state. If those are equal ($\eta = \frac{1}{2}$) we get:

(49) $$\theta_1^T - \theta_1 = -(\theta_2^T - \theta_2) = (U^1 - U^2)/2$$

If the whole transfer goes to the home country ($\omega = 1$), domestic disturbances have no effect on the gap. When $\omega = 0$, on the other hand, the same happens to partner country disturbances:

$$\theta_1^T - \theta_1 = -\left(\frac{1-\eta}{\eta}\right)(z + U^2) \quad \text{if } \omega = 1 \tag{50}$$

$$\theta_1^T - \theta_1 = z + U^1 \quad \text{if } \omega = 0 \tag{51}$$

The effect of foreign disturbances can be easily analyzed. It is also possible to combine the monetary rule with a price rigidity.

V. Conclusion

The implications of the two-tier model described in this paper roughly confirm the importance of the factors traditionally mentioned in the literature in monetary unions. Their impact hinges on the type of disturbance shocking the macroeconomic equilibrium, the trade pattern of the various countries and wage and price behavior. Except for the special Keynesian case mentioned at the end of Appendix 1, no emphasis was placed here on the latter factor. The impact of the other two, particularly of the first, was extensively analyzed.

We now list the major results, mentioning the relevant parameters for ease of reference. We begin with the dynamic stability of the large countries block, discussed in Appendix 2. The condition for the two economies to reach the stationary state under perfect foresight, given initial conditions, are that the share of domestic goods in consumption be greater than the share of foreign goods ($\beta < \frac{1}{2}$) and that there be some sluggishness in price adjustment. The latter requirement ensures saddle-point stability under perfect foresight insofar as it compensates the instability of price and exchange-rate expectations (whose strength is c and b respectively) by a trade-off between price inflation and the output gap (whose slope is γ). The condition is $\gamma c < 1$.

Looking at the long-run solution of the model, the real and nominal exchange rate between the two large countries (θ^* and e respectively) as well as the price level (p^{**}, the price of the numeraire currency country was chosen) in one of them are simultaneously determined. They are expressed in terms of relative demand supply, and monetary disturbances in both countries. Thus, if the disturbances are perfectly correlated ($u_A^* = u_A^{**} = u_A$ etc.), the nominal and real exchange rates remain at their long-run equilibrium values and the price level increases (less than proportionately) with demand expansion and decreases (more than proportionately) with a productivity improvement.

The same result holds for the difference between the price level of the small domestic economy under flexible exchange rates and under the monetary union. For example, demand expansion will be more inflationary under flexible exchange rates if foreign demand disturbances are perfectly correlated and conversely.

The effect of domestic real disturbances is always more inflationary under the monetary union when the trade-elasticities are large (the composite domestic disturbance term is a weighted average when $\xi < 1$).

The model of the pseudo-exchange rate union relies on the automatic adjustment of the balance of payments of the partner to solve for the real exchange rate or the price level of the partner countries. The money stock of the domestic economy becomes an endogenous rather than a policy variable. This is the classic counterpart to making the exchange rate between the partners a policy variable, which is precisely fixed by monetary partners intervention.

In a full monetary union, however, the monetary allocation between member states becomes endogenous. In that case, the union may change the real exchange rate of the countries tied by the monetary rule. If steady-state shares are given by η, then the rule is denoted by the share of money, ω/η allocated to country one. Then monetary expansion in the large country in the pseudo-exchange rate union ($u_m^* > 0$) will increase the real exchange rate gap (and thus depreciate the real rate of the domestic economy) if the monetary allocation rule is less than the "natural distribution" given by $\omega = \eta$. If $\omega > \eta$, on the other hand, the real rate of country one will depreciate and the real rate of country two will appreciate. When the natural distribution is maintained, demand expansion in the two large countries has no effect on the real rate gap in the two small countries.

The focus of the analysis was on the allocation of a given transfer between the two small countries, because -- as shown in Macedo (1985a) -- this is an important feature in the recent experience of the West African Monetary Union. Nevertheless, the transfer mechanism is likely to be present whenever small countries peg their exchange rates with large ones. By emphasizing the size difference, the model presented in this paper was able to handle a 3-country exchange rate union with a central bank between two of its members. An analysis of the strategic interaction between the members of the union is a natural application of this set-up.

APPENDIX I

DERIVATION OF A LOG LINEAR MODEL

This appendix derives the log-linear model used in the text for one small country. It can easily be adapted to the large countries. The supply side is an extension of the three-country model in Marston (1984a) which introduces domestic supply disturbances and an endogenous labour supply. The wage contract set-up is left out. Supply disturbances are featured in the two-country model of Marston (1984b). The demand side is adapted from Macedo (1983).

1. Supply

Consider a Cobb-Douglas technology for domestic output, subject to a random productivity disturbance. For a given stock of capital, set to one by choice of units, we have

(1) $$Y = \tilde{U}_{\pi} L^{\lambda}$$

where Y is domestic output

L is employment

$\tilde{U}_{\pi}$ is a supply disturbance

λ is the share of labor (a constant)

By marginal productivity pricing, we have:

(2) $\frac{WL}{PY} = \lambda$

where W is the wage rate

P is the price of domestic output

Substituting for Y in (2), we get labor demand as a function of the real product wage and the disturbance term:

(3) $L^{d} = [\tilde{U}_{\pi}/(W/P)]^{1/(1-\lambda)}$

We assume that the supply of is a function of the wage measured in terms of the consumer price index, defined as a geometric average of the domestic currency prices of the goods produced in the three countries:

(4) $P_{c} = P^{\alpha} \ (P^{*}E^{*})^{\alpha^{*}} \ (P^{**}E^{**})^{\alpha^{**}}$

where P*(P**) is the foreign currency price the good produced in the partner (non-partner) country;

E*(E**) is the price of the partner's (non-partner's) currency in units of domestic currency;

and $\alpha + \alpha_{*} + \alpha_{**} = 1$

The price of the partner's currency in terms of the non-partner's is determined by triangular arbitrage:

(5) $E = E^{**}/E^{*}$

Using the definition of the two relevant real exchange rates, we have another expression for P_{c}:

(6) $$P_c = P\theta^{(1 - \alpha)}/\theta*^{\alpha*}$$

where $\theta* = P**E/P*$

and $\theta = P**E**/P$

According to (6), proportional changes in P^c and P require that the real exchange rate efffects offset each other or $\theta*^{\alpha*} = \theta^{(1 - \alpha)}$. If the domestic real exchange rate depreciates, the real exchange rate of the partner will have to depreciate by a smaller amount. The larger the bias in trade toward the partner, measured by $\alpha_*/(1 - \alpha)$, the smaller this dampening effect.

We are now in a position to express labor supply as a positive function of the real wage, with elasticity n:

(7) $$L^S = N_o \, (W/P^c)^n$$

Using (6) in (7) we get

(8) $$L^S = N_o \, [(W/P)\theta*^{\alpha*}\theta^{-(1 - \alpha)}]^n$$

In equilibrium, demand for labor equals supply of labor except for a frictional unemployment pool. Equating (8) to (3), we obtain the equilibrium product wage as a function of the terms of trade. Denoting logarithmic deviations by lower case letters we get:

(9) $$[1+n(1-\lambda)](w - p) = -n(1-\lambda)[\alpha_*\theta* - (1-\alpha)\theta] + \tilde{u}_\pi$$

Using (2) to substitute for L in (1), we get aggregate supply as a function of the product wage, which, upon substitution from (9), yields:

(10) $$y = -h(1 - \alpha)\theta + h\alpha_* \theta^* + u_\pi$$

where $$h = \frac{\lambda n}{1 + n(1 - \lambda)}$$

and $$u_\pi = [\frac{1}{1-\lambda} - \frac{1}{1+n(1-\lambda)}] \tilde{u}_\pi$$

2. Demand

The demand side is obtained from the open-economy income identity which defines aggregate demand:

(11) $$Y = A(Y, R, U_A) + \sum_i X^i(Y^i, \frac{E^iP^i}{P}) - \sum_i \frac{E^iP^i}{P} M^i(Y, \frac{E^iP^i}{P})$$

where $A = C + I + G$ is real absorption

X^i (M^i) are exports (imports) to (from) country i, i = *, **

R is real interest rate

U_A is a demand disturbance

In (11) the trade balance is expressed in units of the domestic good and the effects of foreign (domestic) income on exports (imports) are to be interpreted in common units, not made explicit to avoid cluttering. To linearize (11), log differentiate, denote again logarithmic deviations by lower case letters and define r = dR, to obtain:

(12) $$\bar{y} = \sum_i \left[a^i(p^i + e^i - p) + \nu^i y^i\right] - br + u_A$$

where $$a^* = \frac{1}{\Delta}\frac{X^*}{Y}\left[\frac{M^*\Theta}{X^*\Theta^*}(\eta_X^* - 1) + \eta_M^*\right]$$

$$a^{**} = \frac{1}{\Delta}\frac{X^{**}}{Y}\left[\frac{M^{**}\Theta}{X^{**}}(\eta_X^{**} - 1) + \eta_M^{**}\right]$$

$$\nu^i = \frac{1}{\Delta}\frac{Y^i}{Y}\frac{\partial X^i}{\partial Y^i}$$

$$b = \frac{1}{\Delta Y}\frac{\partial A}{\partial R}$$

$$U_A = \frac{1}{\Delta}\frac{\partial A}{\partial U_A}dU_A$$

$$\Delta = 1 - \frac{\partial A}{\partial Y} + \sum_i \frac{\partial M^i}{\partial Y}$$

$$\eta_Z^i = -\frac{\partial Z^i/Z^i}{\partial(E^iP^i/P)/(E^iP^i/P)} \qquad Z = M,\ X$$

APPENDIX 2

STABILITY

1. The two large countries

The large countries model, whose stationary-state version is solved in the text, consists of the following set of equations, where variables are defined as logarithmic deviations from their steady-state level and precise definitions were given in Appendix 1:

IS equations

(1) $y^* = \nu y^{**} + a\theta^* - br^* + u_A^*$

(2) $y^{**} = \nu y^* - a\theta^* - br^{**} + u_A^{**}$

LM equations

(3) $u_m^* - p^* = y^* - ci^*$

(4) $u_m^{**} - p^{**} = y^{**} - ci^{**}$

Price adjustment rules

(5) $\dot{p}^* = \gamma[y^* + k\beta\theta^* - u_\pi^*]$

(6) $\dot{p}^{**} = \gamma[y^{**} - k\beta\theta^* - u_\pi^{**}]$

(7) $i^* = i^{**} + \dot{e}$ — interest parity

(8) $\theta^* = e + p^{**} - p^*$ — real exchange rate

(9) $r^j = i^j - \dot{p}_c^j \quad j = *, **$ — real interest rate

(10) $p_c^* = p^* + \beta\theta^*$ — consumer price indexes

(11) $p_c^{**} = p^{**} - \beta\theta^*$

The model includes five differential equations, respectively the two IS curves, the two price adjustment rules and interest parity. The state of the system is described by the real and nominal exchange rates and the price of the output of the double-starred country, θ^*, e and p^{**} respectively. To reduce the system to three differential equations, we first eliminate the interest rates by using (9) to eliminate the real rates and then using (3) and (4) to eliminate the nominal rates:

$$-b\beta\dot{\theta}^* - b\dot{p}^* + (1 + \frac{b}{c})y^* - \nu y^{**} - a\theta^* + \frac{b}{c}p^* = u_A^* + (1 + \frac{b}{c})u_m^*$$

(12)

$$b\beta\dot{\theta}^* - b\dot{p}^{**} + (1 + \frac{b}{c})y^{**} - \nu y^* + a\theta^* + \frac{b}{c}p^{**} = u_A^{**} + (1 + \frac{b}{c})u_m^{**}$$

Using (3) and (4) again to substitute for nominal interest rates in (7), we get:

(13) $$c\dot{e} = y^* - y^{**} + p^* - p^{**} - u_m^* + u_m^{**}$$

Using the difference of the two equations in (5) and (6) to eliminate the difference in outputs from (10), we have:

(14) $$-\frac{1}{\gamma}\dot{\theta}^* + (\frac{1}{\gamma} - c)\dot{e} = (1 + 2k\beta)\theta^* - e - u_\pi^* + u_\pi^{**} + u_m^* - u_m^{**}$$

Similarly, we use (5) and (6) to eliminate outputs from (12). For the starred country we get:

(15) $$-b\beta\dot{\theta}^* - [\frac{1}{\gamma}(1 + \frac{b}{c}) - b]\dot{p}^* - \frac{\nu}{\gamma}\dot{p}^{**} - [a+(1 + \phi + \nu)k\beta]\theta^* + \frac{b}{c}p^* =$$

$$u_A^* - (1 + \frac{b}{c})u_\pi^* + \nu u_\pi^{**} + \frac{b}{c}u_m^*$$

Finally, we eliminate $\dot{p}^*$ and p^* using the definition of the real exchange rate in (8). The system can then be written as:

(16)

$$\frac{1}{\gamma}\begin{bmatrix} -\Phi - \beta b\gamma & \Phi - \nu & \Phi \\ \nu + \beta b\gamma & \Phi - \nu & -\nu \\ -1 & 0 & 1-c\gamma \end{bmatrix}\begin{bmatrix} \dot{\theta}^{*} \\ \dot{p}^{**} \\ \dot{e} \end{bmatrix} =$$

$$+\begin{bmatrix} -N-(1+k\beta)\phi & \phi & \phi \\ N+k\beta\phi & \phi & 0 \\ -(1+2k\beta) & 0 & 1 \end{bmatrix}\begin{bmatrix} \theta^{*} \\ p^{**} \\ e \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & -(1+\phi) & \nu & \phi & 0 \\ 0 & 1 & \nu & -(1+\phi) & 0 & \phi \\ 0 & 0 & -1 & 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} u_A^{*} \\ u_A^{**} \\ u_\pi^{*} \\ u_\pi^{**} \\ u_m^{*} \\ u_m^{**} \end{bmatrix}$$

where $\Phi = 1 + \phi(1 - c\gamma)$

$N = a + (1 + \nu)k\beta$

and $\phi = b/c$

To write (16) more compactly, we define matrices and vectors as follows:

(17) $J_{*}\dot{x}^{*} + D_{*}\, x^{*} = Z_{*}\, u^{*}$

where $x^{*} = (\theta^{*}\ p^{**}\ e)'$

We now solve the homogeneous system, in order to ascertain its dynamic properties. Inverting J_{*} in (17), and multiplying its inverse by minus D_{*}, we obtain the system in the form:

(18) $\dot{x}^* = A_* x^*$

where $A_* = J_*^{-1} D_*$.

Since $a_{12} = a_{32} = 0$ in A_*, the characteristic equation only involves seven of the nine coefficients and can be written as:

(19) $$(a_1 + \lambda)(\lambda^2 - a_2\lambda - a_3) = 0$$

where $$a_1 = \frac{\phi\gamma^3}{\phi-\nu}$$

$$a_2 = \frac{\gamma^2 k\beta(\phi+\nu)-b\gamma^3(1-2\beta)}{(\phi-\nu)(\phi+\nu-2c\gamma\beta)c}$$

$$a_3 = 2N\gamma^3$$

and λ is an eigenvalue of A_*.

We therefore have the solutions:

(20)
$$\lambda = -a_1$$
$$2\lambda = a_2 \pm \sqrt{a_2^2 + 4a_3}$$

The signs of a_1 and a_2 are ambiguous. They basically depend on whether $\gamma c \gtrless 1$. A preference for domestic goods is usually assumed, so that $\beta < \frac{1}{2}$ (the "transfer condition"). Not surprisingly, a crucial parameter is the speed of price adjustment. Recalling that c would be one if the interest elasticity were equal to the level of the nominal interest rate (say 15%), then $\gamma c < 1$, a low speed of adjustment, implies $\Phi > 1 > \nu$ (ν is given by the marginal propensity to import divided by the sum of the marginal propensities to import and not to spend). This means not only that $a_1 > 0$ so that we have one negative eigenvalue, but also that the other two roots alternate in sign because a_3 is unambiguously positive. Therefore, when prices adjust slowly, the system has two directions of stability and one direction of instability, given by the nominal exchange rate. When prices adjust fast, on the other hand, the real exchange rate also has a positive eigenvalue and the system has two directions of instability, associated with the two jump variables. In the limiting case where prices adjust infinitely fast to excess demand for goods ($\gamma \to \infty$), all three state variables are forward looking. Then dynamics come solely from expectations and we have:

$$J_* = \begin{bmatrix} b(1 - \beta) & -b & -b \\ b\beta & -b & 0 \\ 0 & 0 & -c \end{bmatrix}$$

Performing the same operation as before, we obtain the eigenvalues of the new A_* matrix as

$$\lambda_1 = 2N/b(1 - 2\beta)$$

(21)

$$\lambda_2 = \lambda_3 = 1/c$$

The fact that $\lambda_2 = \lambda_3$ is a reflection of the recursive nature of the new system, evident from the structure of J_*. The important point, though, is that the three eigenvalues are positive unless the "transfer condition" $\beta < \frac{1}{2}$ does not hold, in which case $\lambda_1 < 0$. Note that this condition was not crucial in the previous analysis because of the irrelevance of the sign of a_2.

The structure of the two large economies simplifies considerably when the price of their domestic output is assumed to be fixed at some identical level ($\gamma = 0$ and $p^* = p^{**} = u_p^*$). This reduces the model in (16) to one differential equation in the nominal (and real) exchange rate. Since the remaining aggregate demand and supply equations cannot be solved uniquely for outputs in the two countries, we concentrate on the case where output is demand-determined so that we do not use (5) and (6). The case where output is supply-determined could be handled by neglecting (1) and (2).

The neglect of supply side effects makes the model in (1)-(11) a conventional Keynesian model with perfect foresight and (16) reduces to:

(22) $$c[1 + \phi(1 - 2\beta)]\dot{e} + 2ae = 2({}^*u_m^d - {}^*u_A^d)$$

where $${}^*u_i^d = \frac{\overset{*}{u}_i - \overset{**}{u}_i}{2} \qquad i = m,\ A$$

Consider now that aggregate demand and monetary policy in the large countries fluctuate according to:

(23) $$\dot{u}_i^j = -\psi u_i^j$$

where $j = *, **$ $i = A, m$

Then the non-homogenous system (22) and (23) will have one positive root associated with the jump variable e and one negative root associated with the composite forcing variable describing the relative cyclical position of the two countries. The equilibrium solution where $\dot{e} = 0$ and $\dot{u}_i^j = 0$ will be a saddlepoint. Along the stable path, we will have:

$$(24) \qquad e = \{2a + \psi c[1 + \phi(1 - 2\beta)]\}^{-1}\, 2({}^{*}u_m^d - {}^{*}u_A^d)$$

Note that the size of the effect of a particular disturbance is smaller the larger the speed of adjustment ψ and the preference for domestic goods (the smaller β). The stationary solution when disturbances are permanent is obtained by making $\psi = 0$ in (24). This special case is worked out in Macedo (1985b).

To take a particular solution to (17) given by $\dot{x}^* = 0$, we solve for the stationary state of the endogenous variables, $\bar{x}^*$, in terms of the exogenous disturbances. We rewrite here in compact form the system solved in the text:

$$(25) \qquad \bar{x}^* = -D_*^{-1} Z_* u^*$$

$$\text{where } -D_*^{-1} Z_* = \frac{1}{2N\phi} \begin{array}{lll} -\phi & \phi & \phi(1+\nu) \\ N+k\beta\phi & N-k\beta\phi & -(1-\nu)N-k\beta\phi(1+\nu) \\ -\phi(1+2k\beta) & \phi(1+2k\beta) & \phi(1+\nu)-2a\phi \end{array}$$

$$\begin{array}{lll} -\phi(1+\nu) & 0 & 0 \\ -(1-\nu)N+k\beta\phi(1+\nu)-2N\phi & 0 & 2N\phi \\ -(1+\nu)\phi+2a\phi & 2N\phi & -2N\phi \end{array}$$

2. <u>The two small economies</u>

We present here the model of what we will call the domestic economy. It is easy to modify it for the other (identical) small country, as was shown in the text:

(26) $y = \sum_j a^j(p^j + e^j - p) + \nu^j y^j - br + u_A$ IS equation

(27) $m - p = y - ci$ LM equation

(28) $\dot{p} = \gamma[y - h\alpha_* \theta^* + h(1 - \alpha)\theta - u_\pi]$ price adjustment rule

(29) $i = i^* + \dot{e}^*$ interest parity

(30) $e^* = e^{**} - e$ triangular arbitrage

(31) $\theta = e^{**} + p^{**} - p$ real exchange rate

(32) $r = i - \dot{p}_c$ real interest rate

(33) $p_c = \alpha p + \alpha_* (p^* - e^*) + \alpha_{**} (p^{**} + e^{**})$ consumer price index

$= p + (1 - \alpha)\theta - \alpha_* \theta^*$

The model is solved differently depending on whether there is a floating regime, with one exchange rate, e^{**}, endogenous, or whether there is a fixing of one bilateral rate, with the domestic money stock, m, being endogenous. We begin by solving the model for the flexible exchange rate case, by reducing it to two dynamic equations, one for the real exchange rate and the other for the domestic price level since the respective nominal exchange rate can be obtained by (31). Using a notation consistent with (16), the two supplementary equations can be written as:

$$\frac{1}{\gamma}\begin{bmatrix} \alpha_* b\gamma+\nu^* & -\nu^*-\nu^{**} & -\nu^* & -(1-\alpha)b\gamma & \phi \\ 0 & 1-c\gamma & 0 & c\gamma & -1+c\gamma \end{bmatrix}\begin{bmatrix} \dot{\theta}^* \\ \dot{p}^{**} \\ \dot{e} \\ \dot{\theta} \\ \dot{p} \end{bmatrix} =$$

$$\begin{bmatrix} -H^* & 0 & 0 & H(1+\phi\chi) & -\phi \\ h\alpha_* - k\beta & -1 & 0 & -h(1-\alpha) & 1 \end{bmatrix}\begin{bmatrix} \theta^* \\ p^{**} \\ e \\ \theta \\ p \end{bmatrix}$$

$$+\begin{bmatrix} 0 & 0 & \nu^* & \nu^{**} & 0 & 0 & 1 & -(1+\phi) & \phi \\ 0 & 0 & 0 & -1 & 0 & 1 & 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} u_A^* \\ u_A^{**} \\ u_\pi^* \\ u_\pi^{**} \\ u_m^* \\ u_m^{**} \\ u_A \\ u_\pi \\ u_m \end{bmatrix}$$

where $\tilde{a}^{j} = a^{j} + h\alpha_{j} \qquad j = *, **$

$$H = \tilde{a}^{*} + \tilde{a}^{**}$$

$$\chi = h(1-\alpha)/H$$

and $H^{*} = \tilde{a}^{*} + \phi h\alpha_{*} + (\nu^{*}-\nu^{**})k\beta$

It is clear from (34) that the system is to be solved in terms of the foreign variables, exogenous to the domestic economy. In fact, in obvious notation, it can be written as

(35) $$J_{**}\dot{x}^{*} + J\dot{x} = D_{**}x^{*} + Dx + Z_{**}u^{*} + Zu$$

The similarity of (36) and (17) is apparent. Note also that H is the three-country equivalent of N. The share of the supply effect, χ, is the small country equivalent of $k\beta$. It will be of use below.

We proceed to analyze the homogenous solution. We set x^{*} (and thus $\dot{x}^{*}$) and u to zero so that we only have to compute $J^{-1}D$ in order to solve the characteristic equation. In this two-by-two case, it is sufficient to compute the determinant and the trace of that matrix. The sign of the determinant depends again on whether $c\gamma \gtrless 1$:

(37) $$\text{Det}\ (J^{-1}D) = -\gamma H/c(1 - \alpha + \alpha\Phi)$$

where $J^{-1} D = \frac{1}{c(1-\alpha+\alpha\Phi)} \begin{bmatrix} a^{*} + a^{**} - Hc\gamma & 1 \\ H(1-\phi\chi)c\gamma & -\alpha b\gamma \end{bmatrix}$

Recalling that $\Phi > 0$ when $c\gamma < 1$, we see that, if prices do not adjust too fast, Det < 0 and the two roots will be of opposite sign. The negative root will be larger in absolute value if the trace is negative. Under the same condition about γ, this requires that the ratio of the trade elasticities to the average of ϕ (the ratio of the interest elasticities), and h (the aggregate supply elasticity), weighted by α, be small enough, or

(38) $$\frac{a^* + a^{**}}{\alpha\phi+(1-\alpha)h} < \frac{c\gamma}{1-c\gamma}$$

If $c\gamma > 1$, the trace is of course positive. In fact, when γ is infinite the determinant is equal to $H/\alpha bc$ and the system is again unstable. The matrix J becomes then

$$J = \begin{bmatrix} -(1 - \alpha)b & -b \\ -c & c \end{bmatrix}$$

Now we know from above that $\dot{x}^* = A_* x^*$ when all exogenous variables are at their steady-state levels (so that u remains at zero). We therefore rewrite (36) as

(39) $$\dot{x} = J^{-1} Dx + Mx^*$$

where $M = -J^{-1} [J_{**}J_{*}^{-1}D^* - D_{**}]$.

If the dynamics of x* are sufficiently stable, then, the system in (39) can be stable even if the homogenous system is not. But the solution for the dynamics of x* in terms of the exogenous variables would have to be substituted for before we make some assumption about their dynamics. For example, if each disturbance is given by (23) above, then, for sufficiently low speeds of adjustment, a stable (or saddlepoint-stable) solution to the system will exist, even if γ were infinite.

We now concentrate on a particular solution to (36) given by $\dot{x}^* = \dot{x} = 0$:

(40) $$x = -D^{-1}(Bu^* + Zu)$$

where $B = [D_{**}D_*^{-1}Z_* + Z_{**}] = \frac{1}{2N\phi}$

$$\begin{array}{cccc} & \phi H^* & & -\phi H^* \\ & -N+\phi h\alpha_* & & -N+\phi h\alpha_* \\ \phi[2N\nu^*-H^*(1+\nu)] & \phi[2N\nu^{**}+H^*(1+\nu)] & 0 & 0 \\ N(1-\nu)+\phi(1+\nu)h\alpha_* & N(1-\nu)-\phi(1+\nu)h\alpha_* & 0 & 0 \end{array}$$

Under the assumption that the domestic economy is small relative to country star ($y^* = 0$), , we solve the system in (34) - which collapses into a single equation - for the new exchange rate between the domestic country and the double starred country, θ:

(41) $$\frac{1}{\gamma}\{-[\phi(1-\alpha\ c\gamma)-\nu^*]\dot{\theta}^*+(\Phi-\nu^*-\ \nu^{**})\dot{p}^{**}+(\Phi-\ \nu^*)\dot{e}+(b\alpha\gamma-1)\dot{\theta}\}$$

$$= -[\tilde{a}^*+(\nu^*\ -\ \nu^{**})k\beta\ -\ \phi(1+k\beta)]\theta^*-\phi p^{**}-\phi e$$

$$+\ \ H\theta\ +\ (\nu^*-\phi)u_\pi^*+\nu^{**}u_\pi^{**}+\phi u_m^*+u_A-u_\pi.$$

The homogenous solution to (41) is simply:

(42) $$\dot{\theta} = \frac{H}{b\alpha\gamma - 1}\,\theta$$

If $b\alpha\gamma > 1$, the root is positive and the system is unstable with the proviso discussed in connection with (39) above.

Again we concentrate on the particular solution obtained by setting $\dot{x}^* = \dot{\theta} = 0$. Under fixed rates, the B matrix in (40) becomes a vector, which we denote as $\tilde{B}$. Setting up domestic disturbances to zero, we obtain:

(43) $$\theta = -\frac{\tilde{B}}{H}\,u^*$$

Consider a country exactly identical to the domestic economy described. above and index both of these small countries by 1 and 2. If the links between the two small countries are negligible, we have instead of the right-hand-side of (35):

(44) $$D_{**}x^* + D\,x_1 + Z_{**}u^* + Z\,\tilde{u}_1 = 0$$

(45) $$D_{**}x^* + D\,x_2 + Z_{**}u^* + Z\,\tilde{u}_2 = 0$$

where $x_j = (\theta_j\ p_j)'$

and $\tilde{u}_j = (u_A^j\ u_\pi^j\ u_m^j)'$

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