NBER WORKING PAPER SERIES

FINANCIAL CONNECTIONS AND SYSTEMIC RISK

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Working Paper 16177
http://www.nber.org/papers/w16177

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2010

We are grateful to Piero Gottardi, Iftekhar Hasan, John Kambhu, Fernando Vega Redondo and participants at presentations at the NBER Meetings in July 2009, the NBER Financial Institutions and Market Risk Conference in June 2010, our discussants there Mark Carey and Mark Flannery, the Federal Reserve Bank of New York, the Einaudi Institute for Economics and Finance, the European University Institute, the Huntsman School of Business, the Thammasat Business School, Tilburg University, the University of Naples Federico II, the University of South Florida and the University of Pennsylvania for helpful comments. We also thank the Sloan Foundation for financial support. The corresponding author is Franklin Allen with e-mail allenf@wharton.upenn.edu. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 16177
July 2010
JEL No. G01,G11,G2,G21

ABSTRACT

We develop a model where institutions form connections through swaps of projects in order to diversify their individual risk. These connections lead to two different network structures. In a clustered network groups of financial institutions hold identical portfolios and default together. In an unclustered network defaults are more dispersed. With long term finance welfare is the same in both networks. In contrast, when short term finance is used, the network structure matters. Upon the arrival of a signal about banks' future defaults, investors update their expectations of bank solvency. If their expectations are low, they do not roll over the debt and there is systemic risk in that all institutions are early liquidated. We compare investors' rollover decisions and welfare in the two networks.

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1 Introduction

Understanding the nature of systemic risk is key to understanding the occurrence and propagation of financial crises. The term usually refers to a situation where many (if not all) financial institutions fail as a result of a common shock or a contagion process. Herring and Wachter (2001) and Reinhart and Rogoff (2009) find evidence that a collapse of residential or commercial real estate values is the main cause for system wide failures of financial institutions during many financial crises. Allen and Gale (2000), Freixas, Parigi and Rochet (2000) and numerous other subsequent papers (see Allen, Babus and Carletti, 2009, for a survey) analyze the risk of contagion where the failure of one financial institution leads to the default of other financial institutions through a domino effect. This type of systemic risk is often used by central banks as the justification for intervening and bailing out institutions that are "too big to fail".

The recent developments in financial markets and the crisis that started in 2007 have highlighted the importance of another type of systemic risk related to the structure of connections among financial institutions and their funding maturity. The emergence of financial instruments in the form of credit default swaps and other credit derivative products, loan sales and collateralized loan obligations has improved the possibility for financial institutions to diversify risk. However, it has also led to more overlap and more similarities among their portfolios. This has increased the probability that the failure of one institution is likely to coincide with the failure of other similar institutions. Combining this with a greater reliance on wholesale short term finance has increased rollover risk for financial institutions. When a bank is in difficulty, investors may fear that other banks with similar portfolios will also be in trouble and hence may refuse to reinvest their funds. Financial markets can dry up and push all banks into difficulties.

In this paper we focus on the interaction between financial connections and funding maturity in generating systemic risk. We develop a simple two-period model, where each bank invests in a risky project and needs external funds to finance it. Investors provide the funds to the banks in exchange for a debt contract. We initially consider the case of long term debt and subsequently that of short term debt. As projects are risky, banks may default at the final date. When this occurs, investors recover the return of the bank's project net of bankruptcy costs, while the bank does not receive anything. When default does not occur, investors obtain the repayment specified in the debt contract and the bank retains any surplus. As project returns are independently distributed, each bank has an incentive to diversify by exchanging shares of its own project with other banks. This lowers banks' individual default probabilities and bankruptcy costs thus allowing them to promise investors a lower repayment. However, exchanging projects is costly. Banks incur a due diligence cost for each project they exchange. In equilibrium, banks trade off the advantages of diversification with the due diligence costs.

The exchange of project shares forms links among banks that lead to overlaps in their portfolios. Banks choose the number of links but not the network structure that emerges in equilibrium. For ease of exposition, we focus on the case of six banks with each of them optimally forming two connections with other banks. This leads to two possible network structures. In one, which we call clustered, banks are connected in two clusters of three banks each. Within each cluster all banks hold the same portfolio, but the two clusters are independent of each other. In the second network, which we call unclustered, banks are connected in a circle. Each of them exchanges projects only with the two neighboring banks so that none of the banks holds identical portfolios.

We show that with long term debt the structure of the network does not matter for welfare. The reason is that in either network each bank's portfolio is formed by three independently distributed projects with the same distribution of returns. Thus, the number of bank defaults and the expected costs of default are the same in the two structures and so is total welfare.

In contrast, the structure of the network plays an important role in determining systemic risk and welfare when banks use short term debt. The main difference is that at the intermediate date investors decide whether to roll over their investments conditional on a signal concerning banks' future solvency. The signal indicates whether all banks will be solvent in the final period (good news) or whether at least one of them will default and will not be able to repay investors the promised repayment (bad news). Upon observing the signal, investors update the probability that their bank will be solvent at the final period and roll over the debt if they expect to be able to recover their opportunity cost. They always roll over the debt when there is a good signal but not when there is a bad one. When rollover does not occur, all banks are forced into early liquidation. This source of systemic risk is the focus of our analysis. Investors' rollover decisions depend on the structure of the network, investors' opportunity cost and the magnitude of bankruptcy costs.

We show that, upon the arrival of bad news, rollover occurs less often in the clustered than in the unclustered network. When investors recover enough in the case of default or have a low opportunity cost, debt is rolled over in both networks. As the amount they recover decreases and their opportunity cost increases, debt is still rolled over in the unclustered network but not in the clustered one. The reason is that defaults are more concentrated in the clustered network than in the unclustered network. Investors infer that the probability of default conditional on the bad signal is high and thus decide not to roll over. In the unclustered network defaults are less concentrated and the arrival of the bad signal indicates a lower probability of a rash of bank defaults. When investors obtain little after banks default because of high bankruptcy costs or have a high opportunity cost, banks are early liquidated in both networks.

The welfare properties of the two network structures with short term finance depend on the investors' rollover decisions, the proceeds from early liquidation and the bankruptcy costs. When banks continue and offer investors a repayment of the same magnitude in either network, total welfare is the same in the two network structures. When the debt rollover requires a higher promised repayment in the clustered than in the unclustered network, welfare is higher in the latter as it entails lower bankruptcy costs. When the debt is not rolled over in the clustered network only, the comparison of total welfare becomes ambiguous. Initially, when neither the bankruptcy costs nor the proceeds from early liquidation are too high, total welfare remains higher in the unclustered network. However, as investors recover little in the case of bankruptcy and a large amount in the case of early liquidation, welfare becomes higher in the clustered network, and remains so even when early liquidation occurs in both network structures.

Our paper is related to several strands of literature. Concerning the effects of diversification on banks' portfolio risk, Shaffer (1994) argues that while diversification is good for each bank individually, it can lead to greater systemic risk as banks' investments become more similar. Wagner (2010) shows in a model with two banks that diversification can increase the likelihood of systemic crises and thus be undesirable. Ibragimov, Jaffee and Walden (2010) identify conditions under which it may be socially optimal to have financial intermediaries hold less diversified portfolios in order to have a lower probability of widespread collapses. In these papers, banks always have the same portfolios and social welfare is non-linearly decreasing in the number of bank failures in the system. We consider a framework where the degree of diversification, the network structure and the funding structure of financial institutions interact in determining systemic risk and welfare.

In terms of the rollover risk entailed by short term finance, Acharya, Gale and Yorulmazer (2009) explain market freezes in the presence of rollover risk based on incoming information and transaction costs. He and Xiong (2009) show that rollover risk leads to dynamic bank runs. Concerning liquidity risk more generally, Diamond and Rajan (2009) find that liquidity dry-ups can arise from the fear of fire sales; while Bolton, Santos and Scheinkman (2009) look at maturity mismatch and its impact on liquidity demand when there is asymmetric information. All these studies use a representative bank/agent framework. By contrast, we analyze how different network structures affect the rollover risk resulting from short term finance.

More generally, our paper is also related to a strand of literature stressing the importance of externalities among banks as a source of systemic risk (see Allen and Babus, 2009, for a survey on contagion in financial networks). For example, Boyson, Stahel and Stulz (2008) provide evidence of such externalities within the hedge fund sector, while

Billio et al. (2010) measure the interconnectedness among hedge funds, banks, brokers, and insurance companies and their impact on systemic risk. Adrian and Brunnermeier (2009) and Danielsson, Shin and Zigrand (2009) point out that designing regulation on banks' individually optimal risk management may not be appropriate. Our paper relates to this literature in that it analyzes how the individual choice of the optimal degree of diversification may lead to multiple network structures with very different properties in terms of systemic risk and welfare.

Some other papers study the extent to which banks internalize the negative externalities that arise from contagion. For instance, Babus (2009) proposes a model where banks share the risk that the failure of one bank propagates through contagion to the entire system. Castiglionesi and Navarro (2010) show that an agency problem between shareholders and debt holders of a bank leads to fragile financial networks. Zawadowski (2010) takes a different approach to show that banks that are connected in a network of hedging contracts fail to internalize the negative effect of their own failure. Banks funded with short-term debt hold insufficient capital to prevent lenders from running. All these papers rely on a domino effect as a source of systemic risk. By contrast, we focus on diversification and overlaps in banks' portfolios as a source of systemic risk in the presence of information externalities.

The rest of the paper proceeds as follows. Section 2 lays out the basic model when banks use long term debt. Section 3 describes the equilibrium that emerges in this case in terms of the individually optimal degree of diversification and the multiple network structures that can arise from it. Section 4 introduces short term debt. It analyzes investors' decision to roll over the debt in response to information about banks' future solvency and the welfare properties of the different network structures. Section 5 discusses a number of extensions of the basic model. Finally, Section 6 concludes.

2 The basic model with long term finance

Consider a three-date ($t = 0, 1, 2$) economy with six banks, denoted by $i = 1, ..., 6$, and a continuum of small, risk-neutral investors. Each bank i has access at date 0 to an investment project that yields a stochastic return $\theta_i = \{R_H, R_L\}$ at date 2 with probability p and $1-p$, respectively, and $R_H > R_L > 0$. The returns of the projects are independently distributed across banks.

Banks raise one unit of funds each from investors at date 0 and offer them, in exchange, a long term debt contract that specifies an interest rate r to be paid at date 2. Investors provide finance to one bank only and are willing to do so if they expect to recover at least their two period opportunity cost $r_F^2 < E(\theta_i)$.

We assume that $R_H > r_F^2 > R_L$ so that a bank can pay r only when the project yields a high return. When the project yields a low return R_L, the bank defaults at date 2 and investors recover a fraction $\alpha \in [0, 1]$ of the project return. The remaining fraction $(1 - \alpha)$ is lost as bankruptcy costs. Thus, investors will finance the bank only if their participation constraint as given by

$$pr + (1 - p)\alpha R_L \geq r_F^2$$

is satisfied. The first term on the left hand side represents the expected payoff to the investors when the bank repays them in full. The second term represents investors' expected payoff when the bank defaults at date 2. The right hand side is the investors' opportunity cost.

When the project returns R_H, the bank acquires the surplus $(R_H - r)$. Otherwise, it receives 0. The bank's expected profit is then given by

$$\pi_i = p(R_H - r).$$

Given projects are risky and returns are independently distributed, banks can reduce their default risk through diversification. This reduces expected bankruptcy costs $(1 - p)(1 - \alpha)R_L$ and investors' promised repayment r. Each bank exchanges shares of its own project with ℓ_i other banks and connections are bilateral. That is, bank i exchanges a share of its project with bank j if and only if bank j exchanges a share of its project with bank i. When this happens, there is a link between banks i and j denoted as ℓ_{ij}. Then each bank i ends up with a portfolio of $1+\ell_i$ projects with a return equal to

$$X_i = \frac{\theta_{i1} + \theta_{i2} + ... + \theta_{i1+\ell_i}}{1+\ell_i}.$$

Exchanging shares of projects with other banks entails a due diligence cost c per link. The idea is that banks know their own project, but they do not know those of the other banks. Thus they need to exert costly effort to check that the projects of the banks they want to form links with are bona fide as well.

The exchange of project shares creates linkages among banks. The collection of all linkages can be described as a network g. In any network, each bank has shares of $1+\ell_i$ independently distributed projects in its portfolio. The banks' portfolios now overlap in the sense that they hold not only their own project but those of other banks too. The degree of overlap depends on the number of links ℓ_i that each bank has with other banks and on the structure of links among banks. For a given ℓ_i there may be multiple network structures as discussed below.

3 Long term finance

We model banks' portfolio decisions as a network formation game. We first derive the participation constraint of the investors and banks' profits when each bank i has ℓ_i links with other banks and holds a portfolio of $1+\ell_i$ projects. An equilibrium network structure is one where banks maximize their expected profits and do not find it worthwhile to sever or add a link.

We denote as $r \equiv r(g, \ell_i)$ the interest rate that bank i promises investors in a network g where banks have ℓ_i links and $1+\ell_i$ projects. Investors receive r at date 2 when the return of bank i's portfolio, X_i, is $X_i \geq r$, while they receive a fraction α of the bank's portfolio return when $X_i < r$. The participation constraint of the investors is then given by

$$\Pr(X_i \geq r)r + \alpha E(X_i < r) \geq r_F^2, \tag{1}$$

where $\Pr(X_i \geq r)$ is the probability that the bank remains solvent at date 2 and $E(X_i < r) = \sum_{x<r} x \Pr(X_i = x)$ is the bank's expected portfolio return when it defaults at date 2. The equilibrium r is the lowest interest rate that satisfies (1) with equality. Diversification increases the probability $\Pr(X_i \geq r)$ that investors receive their promised return r thus reducing bankruptcy costs and allowing the banks to offer a lower rate of return to investors.

Banks receive the surplus $X_i - r$ whenever $X_i \geq r$ and 0 otherwise. The expected profit of a bank i in a network g is

$$\pi_i(g) = E(X_i \geq r) - \Pr(X_i \geq r)r - c\ell_i, \tag{2}$$

where $E(X_i \geq r) = \sum_{x \geq r} x \Pr(X_i = x)$ is the expected return of the bank's portfolio, $\Pr(X_i \geq r)r$ is the expected repayment to investors when the bank remains solvent at date 2, and $c\ell_i$ are the total due diligence costs. Substituting the equilibrium interest rate r from (1) with equality into (2), the expected profit of bank i becomes

$$\pi_i(g) = E(X_i) - r_F^2 - (1-\alpha)E(X_i < r) - c\ell_i. \tag{3}$$

The bank's expected profit is given by the expected return of its portfolio $E(X_i)$ minus the investors' opportunity cost r_F^2, the expected bankruptcy costs $(1-\alpha)E(X_i < r)$, and the total due diligence costs $c\ell_i$. As (3) shows, greater diversification involves a trade-off between lower bankruptcy costs and higher total due diligence costs.

Banks choose the number of links ℓ_i in order to maximize their expected profits. The choice of ℓ_i determines the (possibly multiple) equilibrium network structure(s). A network g is an equilibrium if it satisfies the notion of *pairwise stability* introduced by Jackson and

Wolinsky (1996). This is defined as follows.

Definition 1 *A network g is pairwise stable if*

(i) for any pair of banks i and j that are linked in the network g, neither of them has an incentive to unilaterally sever their link ℓ_{ij}. That is, the expected profit each of them receives from deviating to the network $(g - \ell_{ij})$ is not larger than the expected profit that each of them obtains in the network g ($\pi_i(g - \ell_{ij}) \leq \pi_i(g)$ and $\pi_j(g - \ell_{ij}) \leq \pi_j(g)$);

(ii) for any two banks i and j that are not linked in the network g, at least one of them has no incentive to form the link ℓ_{ij}. That is, the expected profit that at least one of them receives from deviating to the network $(g + \ell_{ij})$ is not larger than the expected profit that it obtains in the network g ($\pi_i(g + \ell_{ij}) \leq \pi_i(g)$ and/or $\pi_j(g + \ell_{ij}) \leq \pi_j(g)$).

To make the analysis more tractable, we impose a condition to ensure that for any $\ell_i = 0, .., 5$ the bank is bankrupt and is unable to repay r to investors at date 2 only when all projects in its portfolio pay off R_L. When this is the case, the probability of the bank defaulting at date 2 is $\Pr(X_i < r) = (1-p)^{1+\ell_i}$ and the probability of the bank being solvent at date 2 is $\Pr(X_i \geq r) = 1 - (1-p)^{1+\ell_i}$. As shown in the Appendix, a sufficient condition to ensure this is

$$(1 - (1-p)^6)\frac{5R_L + R_H}{6} + (1-p)^6 \alpha R_L \geq r_F^2. \tag{4}$$

Condition (4) guarantees that there exists an interest rate r in the interval $[r_F^2, \frac{\ell_i R_L + R_H}{1+\ell_i}]$ that satisfies the investors' participation constraint (1) for any $\ell_i = 0, .., 5$, where $\frac{\ell_i R_L + R_H}{1+\ell_i}$ is the next smallest return realization of a bank's portfolio after all projects return R_L.

Given (4), the bank's expected profit (3) can be written as

$$\pi_i(g) = E(X_i) - r_F^2 - (1-p)^{1+\ell_i}(1-\alpha)R_L - c\ell_i. \tag{5}$$

It is easy to show that (5) is concave in ℓ as the second derivative with respect to ℓ is negative.

In what follows we will concentrate on the case where in equilibrium banks find it optimal to have $\ell = 2$ links and only symmetric networks are formed. The reason is that this is the minimum number of links such that there are multiple network structures. We have the following.

Proposition 1 *For any $c \in [p(1-p)^3(1-\alpha)R_L, p(1-p)^2(1-\alpha)R_L]$ a network g^* where all banks have $\ell^* = 2$ links is pairwise stable and Pareto dominates equilibria with $\ell^* \neq 2$.*

Proof. See Appendix. ■

In equilibrium banks trade off the benefit of greater diversification in terms of lower expected bankruptcy costs with higher total due diligence costs. Proposition 1 identifies the parameter space for the cost c such that this trade off is optimal at $\ell^* = 2$.

Banks choose the number of links but not the network structure so that multiple networks can emerge, for a given number of links. With $\ell^* = 2$ there are two equilibrium networks g^* as shown in Figure 1. In the first network, that we define as clustered ($g = C$), banks are connected in two clusters of three banks each. Within each cluster, banks hold identical portfolios but the two clusters are independent of each other. In the second network, denoted as unclustered ($g = U$), banks are all connected in a circle. Each of them exchanges projects only with the two neighboring banks so that none of the banks holds identical portfolios. In this sense, risk is more concentrated in the clustered than in the unclustered network.

Both networks are pairwise stable if the due diligence cost c is in the interval $[p(1-p)^3(1-\alpha)R_L, p(1-p)^2(1-\alpha)R_L]$. No bank has an incentive to deviate by severing or adding a link as it obtains higher expected profit in equilibrium. Given that the bank's expected profit function is concave in ℓ_i and that investors always recover their opportunity cost, the restriction on c in Proposition 1 also guarantees that the equilibrium with $\ell^* = 2$ is the best achievable.

We next consider welfare in the two networks. For either of them, the welfare per bank is the sum of a representative bank i's expected profit and its investors' expected returns.

Given that the investors always recover their opportunity cost, from (5) the welfare per bank is simply given by

$$W(g) = E(X_i) - (1-\alpha)E(X_i < r) - c\ell_i. \tag{6}$$

Expression (6) indicates that in the case of long term financing total welfare per bank is just equal to the sum of each bank's expected portfolio return $E(X_i)$ net of the expected bankruptcy costs $(1-\alpha)E(X_i < r)$ and the total due diligence costs $c\ell_i$. In either equilibrium network each bank's portfolio is formed by $1+\ell^*$ independently distributed projects with the same distribution of returns. This implies that in both networks all banks offer the same interest rate to investors and have the same bankruptcy probability. This gives the following result.

Proposition 2 *Total welfare is the same in the clustered and unclustered networks.*

4 Short term finance

In the previous sections we have assumed that the maturity of the financing matches the maturity of the assets. Now we analyze the case where banks use short term finance and investors have per period opportunity cost r_f. As with long term finance, we continue focusing on the clustered and unclustered networks with $\ell^* = 2$ and on the range $R_L < r_f^2 < \frac{5R_L+R_H}{6}$. We show that the structure of the network matters for systemic risk and total welfare when short term finance is used.

The main difference with short term finance is that it needs to be rolled over every period. If adverse information arrives, investors may refuse to roll over the debt thus forcing the bank into early liquidation. To capture this, we assume that a signal on the banks' future portfolio returns arrives at date 1. The signal can either indicate the good news that all banks will be solvent at date 2 ($S = G$) or the bad news that at least one bank will default ($S = B$). The idea is that investors hear of a bank failure and then have to infer the prospects of their own bank. For simplicity, we assume that the signal does not reveal any information about any individual bank. As far as individual investors are concerned, all banks look alike and have an equal probability of default once the signal arrives.

Figure 2 shows the sequence of events in the model with short term finance. At date 0 each bank in network $g = C, U$ raises one unit of funds and promises investors an interest rate $r_{01}(g)$ at date 1. Investors know the network structure, but do not know the position of any particular bank in the network. At the beginning of date 1, before investors are repaid $r_{01}(g)$, the signal $S = \{G, B\}$ arrives. With probability $q(g)$ the signal $S = G$ reveals the good news that all banks will be solvent at date 2. With probability $1 - q(g)$ the signal $S = B$ reveals the bad news that at least one bank will default at date 2. Upon observing the signal, investors decide whether to retain $r_{01}(g)$ or roll it over for a total promised repayment of $\rho_{12}^S(g)$ at date 2. If rollover occurs, the bank continues till date 2. Investors receive $\rho_{12}^S(g)$ and the bank $X_i - \rho_{12}^S(g)$ if it remains solvent. Otherwise, when the bank goes bankrupt, investors receive αX_i and the bank 0. If rollover does not occur, the bank is forced into early liquidation at date 1. Investors receive the proceeds from early liquidation, which for simplicity we assume to be equal to r_f, and the bank receives 0. We discuss the case where early liquidation pays off less than r_f in Section 5 below.

The interest rate $r_{01}(g)$ promised to investors at date 0 must be such that they recover their per period opportunity cost r_f at date 1. Given that the proceeds from early liquidation are equal to r_f, investors always recover their opportunity cost at date 1, irrespective of whether the bank is continued or liquidated at date 1. This implies that they will always finance the bank initially and that $r_{01}(g) = r_f$.

At date 1, after the signal S is realized, the bank offers investors a promised repayment $\rho_{12}^S(g)$. Investors roll over the debt if $\rho_{12}^S(g)$ is such that they can recover $r_{01}(g)r_f = r_f^2$ at date 2. When $S = G$ all banks will be solvent at date 2. Investors infer that the probability $\Pr(X_i \geq \rho_{12}^B(g)|G)$ of receiving $\rho_{12}^G(g)$ at date 2 is equal to 1 as shown in Figure 2. Thus, they roll over the debt and $\rho_{12}^G(g) = r_f^2$.

When $S = B$, at least one bank will default at date 2. Investors' probability of receiving the promised repayment $\rho_{12}^B(g)$ at date 2 becomes $\Pr(X_i \geq \rho_{12}^B(g)|B)$. Rollover occurs if there exists a value of $\rho_{12}^B(g)$ that satisfies investors' date 1 participation constraint

$$\Pr(X_i \geq \rho_{12}^B(g)|B)\rho_{12}^B(g) + \alpha E(X_i < \rho_{12}^B(g)|B) \geq r_f^2. \tag{7}$$

The first term is the expected payoff to investors when $X_i \geq \rho_{12}^B(g)$ and the bank remains solvent at date 2 conditional on $S = B$. The second term is the expected payoff to investors conditional on $S = B$ when $X_i < \rho_{12}^B(g)$ and the bank defaults at date 2. In this case investors receive a fraction α of the bank's portfolio expected return $E(X_i < \rho_{12}^B(g)|B)$ $= \sum_{x<\rho_{12}^B(g)} x \Pr(X_i = x|B)$. The equilibrium value of $\rho_{12}^B(g)$ if it exists, is the minimum promised repayment that satisfies (7) with equality and minimizes the probability of bank default conditional on $S = B$. As we discuss below, the terms $\Pr(X_i \geq \rho_{12}^B(g)|B)$ and $E(X_i < \rho_{12}^B(g)|B)$ in (7) depend on the network g. As a result, investors' rollover decision may differ in the two networks.

The expected profit of bank i at date 0 depends on the realization of the signal and on the investors' rollover decision at date 1. When rollover occurs and the bank continues at date 1, the expected profit is given by

$$\pi_i(g) = q(g)\left[E(X_i \geq r_f^2|G) - r_f^2\right] + (1-q(g))\left[E(X_i \geq \rho_{12}^B(g)|B) - \Pr(X_i \geq \rho_{12}^B(g)|B)\rho_{12}^B(g)\right] - 2c. \tag{8}$$

The first term represents the expected profit when with probability $q(g)$ the good signal $S = G$ occurs. Investors receive r_f^2 at date 2 and the bank retains the expected surplus $E(X_i \geq r_f^2|G) - r_f^2$, where $E(X_i \geq r_f^2|G) = \sum_{x \geq r_f^2} x \Pr(X_i = x|G)$ is the bank's expected portfolio return conditional on $S = G$ when $X_i \geq r_f^2$. The second term is the expected profit when with probability $1 - q(g)$ the bad signal $S = B$ occurs. With probability $\Pr(X_i \geq \rho_{12}^B(g)|B)$ the bank remains solvent. It pays $\rho_{12}^B(g)$ to investors and retains the remaining $E(X_i \geq \rho_{12}^B(g)|B) - \Pr(X_i \geq \rho_{12}^B(g)|B)\rho_{12}^B(g)$, where $E(X_i \geq \rho_{12}^B(g)|B) =$ $\sum_{x \geq \rho_{12}^B(g)} x \Pr(X_i = x|B)$ is the bank's expected portfolio return conditional on $S = B$ when $X_i \geq \rho_{12}^B(g)$. The last term $2c$ is the total due diligence costs with $\ell^* = 2$.

Substituting the promised repayment $\rho_{12}^B(g)$ from (7) with equality into (8), this simplifies to

$$\pi_i(g) = E(X_i) - r_f^2 - (1-q(g))(1-\alpha)E(X_i < \rho_{12}^B(g)|B) - 2c. \tag{9}$$

When rollover occurs at date 1, the bank's expected profit can be expressed as in the case of long term debt by the expected return of its portfolio $E(X_i)$ minus the investors' opportunity cost r_F^2, the expected bankruptcy costs $(1-q(g))(1-\alpha)E(X_i < \rho_{12}^B(g)|B)$, and the total due diligence costs $2c$.

When, after the realization of the bad signal, rollover does not occur and the bank is early liquidated at date 1, its expected profit is given by

$$\pi_i(g) = q(g)\left[E(X_i \geq r_f^2|G) - r_f^2\right] - 2c. \tag{10}$$

The bank now has positive expected profit only when with probability $q(g)$ the good signal is received. When with probability $1-q(g)$ the bad signal occurs, the bank is early liquidated and receives 0. Note that (9) and (10) imply that, in a given network g, the bank's expected profit is higher when debt is rolled over at date 1 than when it is not.

4.1 Investors' rollover decisions at date 1

The crucial difference between long and short term financing is that in the latter case the network structure matters for the equilibrium interest rates, bank profits and ultimately total welfare whereas it does not in the former case. The reason is that the probability distribution of the signal and the associated conditional probabilities of bank default at date 2 differ in the two networks.

To see this, we start by considering the distribution of the signal S. We focus on the case where $\ell^* = 2$ and bankruptcy only occurs when all projects in a bank's portfolio return R_L. Thus, the good signal arrives when all banks' portfolios return at least $(2R_L + R_H)/3$ and investors are able to obtain the opportunity cost r_f^2 at date 2. In contrast, the bad signal arrives when at least one of the banks has all three projects in its portfolio return

R_L at date 2. This means that the probability of $S = G$

$$q(g) = \Pr(\bigcap_i (X_i \geq r_f^2),$$

where $\Pr(\bigcap_i (X_i \geq r_f^2) = \Pr(X_1 \geq r_f^2, X_2 \geq r_f^2, ..., X_6 \geq r_f^2)$ represents the probability that none of the six banks defaults. The probability of $S = B$ is then $1 - q(g)$.

Tables 1 and 2 show all banks' portfolio return realizations and the number of banks defaulting for the clustered and unclustered networks, respectively. For simplicity, we assume that the probability of a project i returning R_H is $p = \frac{1}{2}$ and R_L is $1 - p = \frac{1}{2}$. This implies that all states are equally likely. Since there are 6 projects and each of them can have two possible returns, there are $2^6 = 64$ states numbered in the first column of both tables describing the possible project return realizations at date 2.

Table 1 is for the clustered network. The first set of columns shows the return realizations of the six projects. The second set of columns shows each bank's portfolio returns in the two clusters. The last column shows the total number of bank defaults. The good signal occurs when all banks have a portfolio return of at least $(2R_L + R_H)/3$ and no banks have a portfolio return R_L so there are no defaults. These are the unshaded states in the table. It can be seen that there are 49 of them. This means that the good signal arrives in the clustered network with probability

$$q(C) = \frac{49}{64}.$$

The remaining 15 states are the default states and are shaded in gray in the table. In 14 of these there are 3 banks defaulting and in 1 of them all 6 banks default. This is because banks hold identical portfolios within a cluster. There are 48 bank defaults across all states.

Table 2 is for the unclustered network. The first set of columns shows the return realizations of the six projects, while the second set shows each bank's portfolio returns. The last column shows the total number of defaults. It can be seen that there are now

39 unshaded states where all banks are solvent. This means that the good signal in the unclustered network occurs with probability

$$q(U) = \frac{39}{64}.$$

The remaining 25 shaded states are where at least one default occurs. In 12 of these 1 bank defaults, in 6 states 2 banks default, in 6 other states 3 banks default and in 1 state all 6 banks default. Again, there are 48 total bank defaults across all states, but they are now more spread out across the states. There are more default states but with less banks defaulting on average in each. The reason is that in the unclustered network banks are all connected but none holds identical portfolios. Thus risk is less concentrated in the unclustered than in the clustered network.

It can be seen that the probability of receiving the good signal $S = G$ is higher in the clustered network than in the unclustered network, that is

$$q(C) > q(U). \tag{11}$$

What matters for investors' rollover decisions are the conditional probability distributions of banks' portfolio returns. Tables 3 and 4 show these for the clustered and unclustered networks, respectively. In the clustered network there are 49 states with the good signal. Since none of them has any default, the probability of $X_i = R_L$ conditional on the good signal is 0 in Table 3. Counting the number of states among those unshaded in Table 1 where bank i has portfolio return $X_i = \frac{2R_L+R_H}{3}$ gives 21. Since this is the same for all 6 banks, the probability that bank i has $X_i = \frac{2R_L+R_H}{3}$ is $\frac{21}{49}$. Similarly for the other returns X_i given the good signal. There are 15 states where bankruptcy occurs and the bad signal is realized. Among those states each bank i has portfolio return $X_i = R_L$ in 8 states. Thus, the probability that any bank i has $X_i = R_L$ is $\frac{8}{15}$. Similarly for the other entries conditional on the bad signal.

The difference in the unclustered network is that there are 39 rather than 49 states where all banks are solvent and the good signal is realized. Again, since no banks default in these states, the probability of $X_i = R_L$ conditional on $S = G$ in Table 4 is 0. Among the 39 states, it can be seen from Table 2 that each bank i has $X_i = \frac{2R_L+R_H}{3}$ in 13 states. Thus, the probability for any bank to have $X_i = \frac{2R_L+R_H}{3}$ is $\frac{13}{39}$. Similarly for the other entries conditional on $S = G$. Among the shaded 25 states in Table 2 where bankruptcy occurs, it can be easily seen that each bank i has a portfolio return $X_i = R_L$ with probability $\frac{8}{25}$ as shown in Table 4. Similarly for the other entries conditional on $S = B$ in Table 4.

Comparing Tables 3 and 4, it can be seen that the conditional distributions of banks' portfolio returns are quite different in the two networks. In particular, the probability of $X_i = R_L$ conditional on $S = B$ in the clustered network, which is equal to $\frac{8}{15}$, is much higher than in the unclustered network, where it is $\frac{8}{25}$. This also implies that the conditional probability $\Pr(X_i \geq \rho_{12}^B(g)|B)$ that the bank is solvent and repays $\rho_{12}^B(g)$ to the investors at date 2 conditional on $S = B$ is higher in the unclustered than in the clustered network. That is,

$$\Pr(X_i \geq \rho_{12}^B(U)|B) > \Pr(X_i \geq \rho_{12}^B(C)|B) \tag{12}$$

for $\rho_{12}^B(g) \in [R_L, \frac{2R_L+R_H}{3}]$. This difference means that rollover decisions can also differ between the two networks. We study the clustered network first.

Proposition 3 *When the bad signal ($S = B$) is realized in the clustered network and $R_H > \frac{13}{12}R_L$,*

A. For $\alpha \geq \alpha_{LOW}(C)$, investors roll over the debt for a promised repayment $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$, where $\alpha_{LOW}(C) = \frac{45r_f^2 - 7(2R_L+R_H)}{24R_L}$.

B. For $\alpha_{MID}(C) \leq \alpha < \alpha_{LOW}(C)$, investors roll over the debt for a promised repayment $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$, where $\alpha_{MID}(C) = \frac{45r_f^2 - 4R_L - 8R_H}{3(10R_L+R_H)}$.

C+D. For $\alpha < \alpha_{MID}(C)$, investors do not roll over the debt and the bank is early *liquidated at date 1.*

Proof. See the Appendix. ■

The proposition is illustrated in Figure 3, which plots investors' rollover decisions as a function of the exogenous parameters α and r_f^2. The result follows immediately from the investors' participation constraint at date 1. When the bad signal is realized, the bank continues at date 1 whenever investors can be promised a repayment that satisfies (7). Whether this is possible depends on the fraction α of the bank's portfolio return that the investors receive at date 2 when the bank defaults and on the opportunity cost r_f^2 they require over the two periods. When α is high or r_f^2 is low as in Region A in Figure 3, there exists a repayment $\rho_{12}^B(C)$ that satisfies (7). Investors roll over the debt and the bank continues. The promised repayment compensates the investors for the possibility that they obtain only αX_i in case of default. Given α is high, $\rho_{12}^B(C)$ does not need to be high for (7) to be satisfied. Thus, the equilibrium $\rho_{12}^B(C)$ lies in the lowest interval of the bank's portfolio return, $[r_f^2, \frac{2R_L+R_H}{3}]$. As α decreases or r_f^2 increases so that Region B is reached, investors still roll over the debt but require a higher promised repayment to compensate them for the greater losses in the case of bank default. Thus, $\rho_{12}^B(C)$ is higher and lies in the interval $[\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$. This also implies that, conditional on the realization of the bad signal, bankruptcy does not occur at date 2 only when all projects in a bank's portfolio pay off R_L but also when they pay $\frac{2R_L+R_H}{3}$. As α decreases or r_f^2 increases further so that Regions C and D below $\alpha_{MID}(C)$ are reached, it is no longer possible to satisfy (7) for any $\rho_{12}^B(g) \leq R_H$. Then, investors do not roll over the debt and the bank is early liquidated at date 1.

A similar result holds for the unclustered network.

Proposition 4 *When the bad signal ($S = B$) is realized in the unclustered network,*

A+B+C. For $\alpha \geq \alpha_{LOW}(U)$, investors roll over the debt for a promised repayment $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$, where $\alpha_{LOW}(U) = \frac{75r_f^2-17(2R_L+R_H)}{24R_L}$.

D. For $\alpha < \alpha_{LOW}(U)$, investors do not roll over the debt and the bank is liquidated at date 1.

Proof. See the Appendix. ■

Proposition 4 is also illustrated in Figure 3. As in the clustered network, debt is rolled over when investors can be promised a repayment ρ_{12}^B enough to satisfy their participation constraint (7) with equality. Whether such a repayment exists depends again on the parameters α and r_f^2. When they lie in the Regions A, B and C above $\alpha_{LOW}(U)$, it is possible to satisfy investors' participation constraint and the debt is rolled over. In contrast, when α and r_f^2 lie in Region D this is no longer possible and the debt is not rolled over. Note that differently from the clustered network, when rollover occurs the bank always offers investors a promised repayment $\rho_{12}^B(U)$ in the interval $[r_f^2, \frac{2R_L+R_H}{3}]$. The reason is that the probability $\Pr(X_i \geq \rho_{12}^B(U)|B)$ is sufficiently high to ensure that (7) can be satisfied for a low $\rho_{12}^B(U)$.

A comparison of propositions 3 and 4 shows that rollover occurs for a larger and early liquidation for a smaller parameter space in the unclustered network than in the clustered. The promised repayment is also the same or lower in the former.

4.2 Welfare with short term finance

We next consider welfare in the two networks with short term finance. As with long term finance, in both networks we can focus on the total welfare per bank as defined by the sum of a representative bank i's expected profit and its investors' expected returns. Welfare now depends on the investors' rollover decisions, since these affect the bank's expected profit. Using (9) and (10), when the bank is continued till date 2 welfare is given by

$$W(g) = E(X_i) - (1-q(g))(1-\alpha)E(X_i < \rho_{12}^B(g)|B) - 2c, \tag{13}$$

and when the bank is liquidated at date 1 after the bad signal

$$W(g) = q(g)\left[E(X_i \geq r_f^2|G)\right] + (1-q(g))r_f^2 - 2c. \tag{14}$$

In (13), welfare is given by the expected return of bank portfolio $E(X_i)$ minus the expected bankruptcy costs $(1-q(g))(1-\alpha)E(X_i < \rho_{12}^B(g)|B)$ and the due diligence costs $2c$. In contrast, in (14) welfare is given by the expected return of bank portfolio $q(g)\left[E(X_i \geq r_f^2|G)\right]$ when the good signal is realized and the bank is solvent plus the date 2 value of the liquidation proceeds $(1-q(g))r_f^2$ minus the due diligence costs $2c$.

Deriving $W(C)$ and $W(U)$ from (13) and (14) for the two networks gives the following result.

Proposition 5 *The comparison of total welfare in the two networks is as follows:*

A. For $\alpha \geq \alpha_{LOW}(C)$, total welfare is the same in the clustered and unclustered network: $W(C) = W(U)$.

B+C1. For $\alpha_W < \alpha < \alpha_{LOW}(C)$, total welfare is higher in the unclustered network than in the clustered network: $W(U) > W(C)$, where $\alpha_W = \frac{15r_f^2 - 3R_L - 4R_H}{8R_L}$.

C2+D. For $\alpha < \alpha_W$, total welfare is higher in the clustered network than in the unclustered network: $W(C) > W(U)$.

Proof. See the Appendix. ■

Figure 4 illustrates the proposition by showing the welfare in the clustered and unclustered network. It can be seen that with short term finance total welfare depends on the network structure. Which structure is better depends crucially on the parameters α and r_f^2. As (13) shows, the parameter α affects welfare when debt is rolled over as it determines the size of the expected bankruptcy costs that are lost when the bank defaults at date 2. As (14) shows, the parameter r_f^2 is important for welfare because when the debt is not rolled over it is equal to the date 2 value of the proceeds from early liquidation.

In Region A, where $\alpha \geq \alpha_{LOW}(C)$, investors roll over the debt for a promised total repayment $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$ in both networks. Each bank defaults when its portfolio pays off R_L and makes positive profits in all the other states in either network. As with long term finance, total welfare is then the same in both networks.

In Region B, where α lies in between $\alpha_{MID}(C)$ and $\alpha_{LOW}(C)$, investors still roll over the debt in both networks, but in the clustered network they now require a higher total promised repayment $\rho_{12}^{B}(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$. This implies that bank i defaults not only when $X_i = R_L$ but also when $X_i = \frac{2R_L+R_H}{3}$. As a result total welfare is lower in the clustered network relative to the unclustered network because expected bankruptcy costs are higher.

In Regions C1 and C2 in Figure 4 the debt is rolled over when the bad signal is realized in the unclustered network but not in the clustered one so that banks now make positive profits only when the good signal is realized in the latter network. Total welfare is then given by (13) and (14) in the unclustered and clustered networks, respectively. In the former, welfare is decreasing in the bankruptcy costs, $1-\alpha$. Thus, it decreases as α falls. In the latter, welfare is increasing with r_f^2 as this increases the proceeds from early liquidation and there are no bankruptcy costs. As α falls and r_f^2 increases, total welfare in the unclustered network becomes equal to that in the clustered network, and it then drops below.

Finally, in Region D, where $\alpha \leq \alpha_{LOW}(U)$, banks are early liquidated in both networks when the bad signal is realized so that total welfare is always given by (14). The clustered network attains higher welfare in this region as from (11) the good signal occurs more often. This leads to a higher expected return $q(g)\left[E(X_i \geq r_f^2|G)\right]$ in the clustered network and to a higher date 2 value of the early liquidation proceeds $(1-q(g))r_f^2$ in the unclustered network. The first term dominates, thus leading to higher total welfare in the clustered network.

5 Discussion

In this section we consider a number of extensions of the basic model. In particular, we discuss long term versus short term finance, different types of signal arriving at the interim date, a more general specification of the early liquidation proceeds, and finally different types of coordination mechanisms in the formation of linkages among banks.

5.1 Long term versus short term finance

In Section 3 we assumed that the maturity of the financing matches the maturity of the assets, while in Section 4 we considered short term finance. In practice, banks and other financial institutions have a choice of long and short term finance. There are a number of theories as to why different maturities are used. For example, Flannery (1986) and Diamond (1991) suggest that short term finance of long term assets can help overcome asymmetric information problems in credit markets. Calomiris and Kahn (1991) and Diamond and Rajan (2001) argue that short term debt in a bank's capital structure can play a role as a discipline device to ensure managers behave optimally. Brunnermeier and Oehmke (2009) suggest that creditors shorten the maturity of their claims to obtain priority, leading to an excessive use of short term debt. Another important rationale for the use of short term debt is the upward sloping yield curve. Borrowing short term at low rates to finance high yielding long term assets allows significant profits to be made.

We have not specifically modelled the choice of maturity structure. However, a simple way to do this is to assume that the short term rate r_f is sufficiently below the long term rate r_F so that the use of short term debt is optimal. This raises the issue of what determines the yield curve. One approach is that the rates for different horizons are determined by the access of investors to risk free technologies that last for different maturities.

5.2 Different types of signal

The core of our analysis is the interaction between the signal arriving at date 1, the network structure, and the funding maturity. So far the signal has been modelled as indicating whether at least one bank will default at date 2 without any information about the identity of potentially failing banks. Investors know the network structure but do not know any bank's position in it. Upon observing the signal, they update the conditional probability that their own bank will default at date 2. The important feature for our result is that the conditional probability of default in the clustered network is different from that in the unclustered network. The reason is that the signal generates a different information partition of the states in the two network structures. This leads to different rollover and early liquidation decisions with short term debt in the two networks.

Any signal that generates different information partitions and leads to different conditional probabilities across network structures will have the same qualitative effect as in our basic model. For example, a signal indicating that a particular bank, say bank 1, has gone bankrupt would lead to the same kind of results. Similarly for a signal indicating that a particular real sector is more likely to fail. This would correspond in our model to a signal indicating that a particular project or set of projects has a higher default probability than originally believed. This signal would generate different information partitions on banks' future defaults depending on the different compositions of banks' portfolios and would thus still lead to different conditional probabilities across the two networks.

A signal that does not lead to different conditional probabilities is one bringing general information about the fundamentals of the economy. For example, a signal indicating simply how many projects have a payoff of R_L at date 2 without specifying the identity of these projects or the banks owning them would rule out a number of states but would not generate different information partitions across the two networks. Another example would be a signal indicating a reduction of the same size in the success probability of all projects.

5.3 Early liquidation proceeds

In our basic model early liquidation gives proceeds r_f. This simplifies the analysis because it ensures that the date 1 repayment $r_{01}(g)$ promised to investors at date 0 is always equal to r_f. A more general formulation would be to assume that the early liquidation proceeds are βr_f with $\beta \leq 1$. A value of β less than 1 would mean that $r_{01}(g)$ would have to be greater than r_f in the case where there is early liquidation to allow the investors to recover their opportunity cost. There would be higher deadweight costs and thus lower welfare with early liquidation. This would affect the welfare analysis, but qualitatively the results would be similar.

5.4 What is the market failure?

An important feature of the network literature and of the equilibrium concept of Jackson and Wolinsky (1996) that we have used is that banks are not able to determine the network structure. Each bank individually chooses the links it wishes to have taking as given the choices of the other banks. Since banks form links simultaneously, this implies that with $\ell^* = 2$ either a clustered or an unclustered network can emerge. With long term finance the multiplicity of network structures does not matter since banks and investors are indifferent between them. However, with short term finance it does matter since systemic risk and welfare are different as described in Proposition 5. Investors are still indifferent as they always obtain their opportunity cost, whereas banks clearly prefer the network structure that gives them higher expected profits. The market failure in our analysis is the lack of a coordination mechanism that allows them to choose the preferred network.

One type of mechanism that may allow a degree of coordination would be to have banks condition their linkages on the connections between all other banks in the system. With this conditionality, it would be possible to ensure that only efficient networks are implemented. However, this kind of conditionality would be hard to implement particularly as the number of banks grows large and it is not observed in practice.

Government regulation could also potentially be used to ensure only the efficient network is chosen. This would require the gathering of a significant amount of information from banks and a determination of the optimal network structure. Such regulation may be difficult to implement.

6 Concluding remarks

Understanding connections among financial institutions is important for understanding systemic risk. In this paper we have developed a model where the number and shape of financial connections interact with the funding structure of financial institutions in determining systemic risk.

We have shown that the structure of financial networks matters for systemic risk and total welfare when banks use short term finance, but not when they use long term finance. The reason is that short term finance entails rollover risk, which is absent with a longer maturity of debt. Investors base the decision to roll over the debt on interim information about banks' future solvency. When negative information arrives, investors may infer that they will not to be able to recover the opportunity cost associated with the renewal of the debt. When this occurs, they do not roll over the debt thus forcing all banks into early liquidation. The rollover risk entailed by short term finance differs depending on the structure of connections among banks.

The key trade off between the clustered and the unclustered structure in our framework derives from the different overlap and risk concentration among banks' portfolios in the two networks. Banks have identical portfolios in each of the two groups when they are clustered, while they have diverse portfolios when they are unclustered. This implies different conditional probabilities in the two networks. The consequence is that there is more often early liquidation and hence systemic risk in the clustered than in the unclustered network, but the former can lead to higher welfare when the bankruptcy costs and the proceeds from early liquidation are high.

In our model banks swap projects. This allows us to use a standard approach based on network formation games. The analysis is simplified because swapping projects leads to symmetry. Allowing banks to buy and sell shares of projects would be an interesting extension. In addition to the symmetric equilibria that we have analyzed, there would also be asymmetric equilibria.

We have derived our results assuming that bankruptcy costs are constant irrespective of the number of banks defaulting. If, as in several other papers, such as Wagner (2010) and Ibragimov, Jaffee and Walden (2010), we were to assume that they were increasing in the number of defaults, the clustered network would be less attractive but our qualitative results would be similar. The case where the bankruptcy costs are independent of the number of bank defaults is an interesting benchmark.

Our results provide some insights on the desirability of risk concentration depending on the magnitude of the bankruptcy costs and the proceeds from early liquidation. The main insight is that when bankruptcy is inefficient but early liquidation is not, it is optimal to have fewer instances with more banks defaulting as in the clustered network rather than more frequent instances with less banks defaulting as in the unclustered network. In other cases it is better to spread out default across states as in the unclustered network.

The crucial market failure in our analysis is that banks choose their individual degree of diversification but do not determine the network structure. Hence there can be multiple network structures with different properties in terms of systemic risk for a given level of individual diversification. An important topic for future research concerns the implication of this result for financial regulation. One possibility is that governments and central banks are directly able to regulate the network of linkages. However, this would require a great deal of information. One measure to ensure clustered networks rather than unclustered networks if this was optimal might be to limit financial institutions to their home countries rather than allowing them to pursue opportunities in other countries. Much work clearly remains to be done on such policy issues.

A Appendix

Derivation of sufficiency of condition (4). To ensure that bankruptcy only occurs when all projects in a bank's portfolio return R_L for any $\ell_i = 0, .., 5$, we need to show that there exists a value of r in the interval $[r_F^2, \frac{\ell_i R_L + R_H}{1+\ell_i}]$ that satisfies the investors' participation constraint (1). Substituting $\Pr(X_i < r) = (1-p)^{1+\ell_i}$ and $\Pr(X_i \geq r) = 1-(1-p)^{1+\ell_i}$ into (1), this requires

$$(1-(1-p)^{1+\ell_i})\frac{\ell_i R_L + R_H}{1+\ell_i} + (1-p)^{1+\ell_i}\alpha R_L \geq r_F^2 \tag{15}$$

for any $\ell_i = 0, .., 5$. To show that (4) is sufficient for (15) to hold, we show that the left hand side of (15) is decreasing in ℓ_i for $\ell_i = 0, .., 5$. To see this, we differentiate the left hand side of (15) with respect to ℓ_i and obtain

$$\begin{aligned}
&\frac{\left(1-(1-p)^{1+\ell_i}\right)}{1+\ell_i}\left[R_L - \frac{(\ell_i R_L + R_H)}{1+\ell_i}\right] + (1-p)^{1+\ell_i} Log(1-p)\left[\alpha R_L - \frac{(\ell_i R_L + R_H)}{1+\ell_i}\right] \\
&\leq \left[\frac{(1-(1-p)^{1+\ell_i})}{1+\ell_i} + (1-p)^{1+\ell_i} Log(1-p)\right]\left[R_L - \frac{(\ell_i R_L + R_H)}{1+\ell_i}\right]. \qquad (16)
\end{aligned}$$

It is sufficient that the last expression is negative for any $\ell_i = 0, ..., 5$. To see this is the case, initially consider the first term $\left[\frac{(1-(1-p)^{1+\ell_i})}{1+\ell_i} + (1-p)^{1+\ell_i} Log(1-p)\right]$. Its value is 0 when it is evaluated at $p = 0$. Differentiating it with respect to p gives

$$-(1+\ell_i)(1-p)^{\ell_i} Log(1-p) > 0$$

for any $p \in (0,1)$. This guarantees that the first term is positive for any $\ell_i = 0, ..., 5$. The second term is $R_L - \frac{(\ell_i R_L + R_H)}{1+\ell_i} < 0$ since $R_H > R_L$. Together, these imply that the right hand side of (16) is negative and hence also that the left hand side of (15) is decreasing in ℓ_i as required. It is then sufficient to assume that (15) holds for $\ell_i = 5$ to ensure that it holds for any other ℓ_i. $\square$

Proof of Proposition 1. Given that condition (4) implies that bankruptcy only occurs when all projects in a bank's portfolio return R_L, a bank's expected profit (3) with $\ell = 2$ simplifies to

$$\pi_i(g) = E(X_i) - r_F^2 - (1-p)^3(1-\alpha)R_L - 2c.$$

To show pairwise stability, we first consider severing a link. Suppose that bank 1 severs the link with bank 3 so that its portfolio is now $\frac{2}{3}\theta_1 + \frac{1}{3}\theta_2$ and its profit is

$$\pi_1(g - \ell_{13}) = E(X_i) - r_F^2 - (1-p)^2(1-\alpha)R_L - c.$$

Bank 1 does not deviate if $\pi_i(g) \geq \pi_1(g - \ell_{13})$, which is satisfied for $c \leq p(1-p)^2 R_L$.

Suppose now that bank 1 adds a link with bank 4 so that its portfolio is now $\frac{1}{6}\theta_1 + \frac{1}{3}\theta_2 + \frac{1}{3}\theta_3 + \frac{1}{6}\theta_4$ and its profit is

$$\pi_1(g + \ell_{14}) = E(X_i) - r_F^2 - (1-p)^4(1-\alpha)R_L - 3c$$

when bankruptcy occurs when all projects pay off R_L. If bankruptcy occurs more often than this, the expected profit from the deviation will be lower. Thus, it is sufficient for the deviation not to be profitable that $\pi_i(g) \geq \pi_1(g+\ell_{14})$ which requires $c \geq p(1-p)^3(1-\alpha)R_L$. Since all banks are symmetric, this shows that $\ell^* = 2$ is a pairwise stable equilibrium for the range of c given in the proposition.

To see that $\ell^* = 2$ is the Pareto dominant equilibrium it is sufficient to show that bank's expected profit is highest in this case since the investors always obtain their opportunity cost. First note that (5) is concave in ℓ. Combining this with the condition that c lies in the range given in the proposition, it follows that a bank's expected profit in the equilibrium with $\ell^* = 2$ is greater than in either the equilibrium with $\ell^* = 1$ or $\ell^* = 3$ or any other equilibrium. $\square$

Proof of Proposition 3. We proceed in two steps. First, we find the minimum value of α as a function of the short term risk free rate r_f^2 in each interval of the bank's portfolio return X_i such that investors' participation constraint (7) is satisfied for a feasible promised repayment $\rho_{12}^B(C)$. Second, we compare the functions representing the minimum values of α found in the first step to find the equilibrium value of $\rho_{12}^B(g)$.

Step 1. We start by determining the minimum value of α such that (7) is satisfied for $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$. Substituting $\rho_{12}^B(C) = \frac{2R_L+R_H}{3}$ in (7) and using the distribution probability $\Pr(X_i = x|B)$ as in Table 3, we obtain

$$\frac{7}{15}\frac{2R_L + R_H}{3} + \alpha\frac{8}{15}R_L = r_f^2,$$

from which

$$\alpha_{LOW}(C) = \frac{45r_f^2 - 7(2R_L + R_H)}{24R_L}.$$

This implies that for any $\alpha \geq \alpha_{LOW}(C)$, there exists a value of $\rho_{12}^B(C) \in [r_f^2, \frac{2R_L+R_H}{3}]$ such that investors roll over their debt. Analogously, for $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$, we obtain

$$\frac{4}{15}\frac{R_L + 2R_H}{3} + \alpha(\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3}) = r_f^2$$

from which

$$\alpha_{MID}(C) = \frac{45r_f^2 - 4R_L - 8R_H}{3(10R_L + R_H)}.$$

Finally, for $\rho_{12}^B(C) \in [\frac{R_L+2R_H}{3}, R_H]$ we obtain

$$\frac{1}{15}R_H + \alpha(\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3} + \frac{3}{15}\frac{R_L + 2R_H}{3}) = r_f^2$$

from which

$$\alpha_{HIGH}(C) = \frac{15r_f^2 - R_H}{11R_L + 3R_H}.$$

The interpretation of $\alpha_{MID}(C)$ and $\alpha_{HIGH}(C)$ is the same as the one for $\alpha_{LOW}(C)$.

Step 2. To find the equilibrium value of $\rho_{12}^B(C)$ defined as the minimum promised repayment that satisfies (7), we now compare the functions $\alpha_{LOW}(C)$, $\alpha_{MID}(C)$ and $\alpha_{HIGH}(C)$. We then obtain:

$$\alpha_{MID}(C) - \alpha_{LOW}(C) = \frac{7R_H^2 + 20R_HR_L + 108R_L^2 - 45r_f^2(2R_L + R_H)}{24R_L(10R_L + R_H)}.$$

We note that $\alpha_{MID}(C) - \alpha_{LOW}(C)$ is positive for $r_f^2 < \overline{r}_f^2 = \frac{7R_H^2+20R_HR_L+108R_L^2}{45(2R_L+R_H)} < \frac{5R_L+R_H}{6}$, and negative otherwise. Similarly, it can be shown that $\alpha_{HIGH}(C)-\alpha_{MID}(C) > 0$ for any $r_f^2 \in [\overline{r}_f^2, \frac{5R_L+R_H}{6}]$ and $R_H > \frac{13}{12}R_L$, while $\alpha_{HIGH}(C) - \alpha_{LOW}(C) > 0$ for any $r_f^2 \in [R_L, \overline{r}_f^2]$. Given that in equilibrium the bank offers the minimum level of $\rho_{12}^B(C)$ that satisfies (7), the proposition follows. □

Proof of Proposition 4. We proceed in two steps as in the proof of Proposition 3.

Step 1. We determine first the minimum value of α such that (7) is satisfied for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$. Substituting $\rho_{12}^B(U) = \frac{2R_L+R_H}{3}$ in (7) and using the distribution probability $\Pr(X_i = x|B)$ as in Table 4, we obtain

$$\frac{17}{25}\frac{2R_L + R_H}{3} + \alpha\frac{8}{25}R_L = r_f^2,$$

from which

$$\alpha_{LOW}(U) = \frac{75r_f^2 - 17(2R_L + R_H)}{24R_L}.$$

As before, this implies that for any $\alpha \geq \alpha_{LOW}(U)$, there exists a value of $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ such that investors roll over their debt. Analogously, for $\rho_{12}^B(U) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$ and $\rho_{12}^B(U) \in [\frac{R_L+2R_H}{3}, R_H]$, respectively, we obtain

$$\frac{6}{25}\frac{R_L + 2R_H}{3} + \alpha(\frac{8}{25}R_L + \frac{11}{25}\frac{2R_L + R_H}{3}) = r_f^2$$

from which

$$\alpha_{MID}(U) = \frac{75r_f^2 - 6(R_L + 2R_H)}{46R_L + 11R_H};$$

and

$$\frac{1}{25}R_H + \alpha(\frac{8}{25}R_L + \frac{11}{25}\frac{2R_L + R_H}{3} + \frac{5}{25}\frac{R_L + 2R_H}{3}) = r_f^2$$

from which

$$\alpha_{HIGH}(U) = \frac{25r_f^2 - R_H}{17R_L + 7R_H}.$$

Step 2. We now compare the functions $\alpha_{LOW}(U)$, $\alpha_{MID}(U)$ and $\alpha_{HIGH}(U)$ to find equilibrium value of $\rho_{12}^B(C)$. After some algebraic manipulation it is easy to see that $\alpha_{LOW}(U) < \alpha_{MID}(U) < \alpha_{HIGH}(U)$ for any $r_f^2 \in [R_L, \frac{5R_L+R_H}{6}]$. Thus, the proposition follows given that the bank always offers investors the minimum total repayment that satisfies (7). $\square$

Proof of Proposition 5. The proposition follows immediately from the comparison of total welfare in the two networks in the different regions. We analyze each region in turn.

Region A. For $\alpha \geq \alpha_{LOW}(C) > \alpha_{LOW}(U)$, (7) is satisfied for $\rho_{12}^B(g) \in [r_f^2, \frac{2R_L+R_H}{3}]$ and investors roll over the debt in both networks. Given this, from (13) total welfare is given by

$$W(g) = \frac{R_L + R_H}{2} - \frac{8}{64}(1-\alpha)R_L - 2c \tag{17}$$

for $g = U, C$ as a bank's expected probability of default at date 2 is the same in two structures.

Region B. For $\alpha_{LOW}(C) > \alpha \geq \alpha_{MID}(C) > \alpha_{LOW}(U)$, (7) is satisfied for $\rho_{12}^B(C) \in [\frac{2R_L+R_H}{3}, \frac{R_L+2R_H}{3}]$ in the clustered network and for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ in the unclustered network. Investors roll over the debt in both networks but the bank default probabilities now differ in the two structures. From (13) and Table 3, total welfare in the clustered network is given by

$$W(C) = \frac{R_L + R_H}{2} - \frac{15}{64}(1-\alpha)[\frac{8}{15}R_L + \frac{3}{15}\frac{2R_L + R_H}{3}] - 2c, \tag{18}$$

and by (17) in the unclustered network. It follows immediately that $W(U) > W(C)$.

Regions C1 and C2. For $\alpha_{MID}(C) > \alpha \geq \alpha_{LOW}(U)$, (7) cannot be satisfied for any $\rho_{12}^B(C) \leq X_i$ in the clustered network, whereas it is still satisfied for $\rho_{12}^B(U) \in [r_f^2, \frac{2R_L+R_H}{3}]$ in the unclustered network. Thus, the bank is liquidated and, from (14), total welfare in the clustered network is now equal to

$$W(C) = \frac{49}{64}\left[\frac{21}{49}\frac{2R_L + R_H}{3} + \frac{21}{49}\frac{R_L + 2R_H}{3} + \frac{7}{49}R_H\right] + \frac{15}{64}r_f^2 - 2c,$$

whereas $W(U)$ is still given by (17) in the unclustered network.

Comparing $W(C)$ and $W(U)$ gives

$$W(U) - W(C) = \frac{1}{64}[4R_H + (3 + 8\alpha)R_L - 15r_f^2].$$

Equating this to zero and solving for α as a function of r_f^2 gives the boundary between Regions C1 and C2:

$$\alpha_W = \frac{15r_f^2 - 3R_L - 4R_H}{8R_L}.$$

It can be easily seen that $W(U) > W(C)$ for $\alpha > \alpha_W$ and $W(U) < W(C)$ for $\alpha < \alpha_W$.

Region D. For $\alpha < \alpha_{LOW}(U)$, (7) cannot be satisfied for any $\rho_{12}^B(g) \leq X_i$ so that banks are early liquidated in both networks. Total welfare is still as in (18) in the clustered network, while, from (14), it equals

$$W(U) = \frac{39}{64}\left[\frac{13}{39}\frac{2R_L + R_H}{3} + \frac{19}{39}\frac{R_L + 2R_H}{3} + \frac{7}{39}R_H\right] + \frac{25}{64}r_f^2 - 2c$$

in the unclustered network. The difference between the two expressions is given by

$$W(C) - W(U) = \frac{1}{32}(2R_H + 3R_L + -5r_f^2),$$

which is positive for any $r_f^2 \in [R_L, \frac{5R_L+R_H}{6}]$. $\square$

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Table 1: States of the world, banks' portfolio returns and defaults in the clustered network

State	*States of the world*						*Banks' portfolio returns*						Total defaults
							Cluster 1			*Cluster* 2			
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	X_1	X_2	X_3	X_4	X_5	X_6	
1	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	0
2	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
3	R_H	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
4	R_H	R_H	R_H	R_H	R_H	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
5	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
6	R_H	R_H	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
7	R_L	R_H	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
8	R_H	R_H	R_H	R_L	R_L	R_H	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
9	R_H	R_H	R_H	R_H	R_L	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
10	R_H	R_H	R_H	R_L	R_H	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
11	R_H	R_L	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
12	R_H	R_L	R_H	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
13	R_H	R_L	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
14	R_H	R_L	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_H	R_H	R_H	0
15	R_H	R_H	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
16	R_H	R_H	R_L	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
17	R_H	R_H	R_L	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
18	R_L	R_L	R_H	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_H	R_H	R_H	0
19	R_L	R_H	R_H	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
20	R_L	R_H	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
21	R_L	R_H	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
22	R_L	R_H	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_H	R_H	R_H	0
23	R_H	R_H	R_H	R_L	R_L	R_L	R_H	R_H	R_H	R_L	R_L	R_L	3
24	R_H	R_L	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
25	R_H	R_L	R_H	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
26	R_H	R_L	R_H	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
27	R_H	R_H	R_L	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
28	R_H	R_L	R_L	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
29	R_H	R_H	R_L	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
30	R_H	R_H	R_L	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
31	R_H	R_L	R_L	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
32	R_H	R_L	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
33	R_L	R_H	R_H	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
34	R_L	R_L	R_H	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
35	R_L	R_H	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
36	R_L	R_H	R_H	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
37	R_L	R_L	R_H	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
38	R_L	R_L	R_H	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
39	R_L	R_L	R_L	R_H	R_H	R_H	R_L	R_L	R_L	R_H	R_H	R_H	3
40	R_L	R_H	R_L	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
41	R_L	R_H	R_L	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
42	R_L	R_H	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
43	R_H	R_L	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_L	R_L	R_L	3
44	R_H	R_H	R_L	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_L	R_L	R_L	3
45	R_H	R_L	R_L	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
46	R_H	R_L	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
47	R_H	R_L	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
48	R_L	R_H	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_L	R_L	R_L	3
49	R_L	R_L	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
50	R_L	R_L	R_H	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
51	R_L	R_L	R_H	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
52	R_L	R_L	R_L	R_L	R_H	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	3
53	R_L	R_L	R_L	R_H	R_H	R_L	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	3
54	R_L	R_L	R_L	R_H	R_L	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	3
55	R_L	R_H	R_L	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
56	R_L	R_H	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
57	R_L	R_H	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
58	R_H	R_L	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
59	R_L	R_L	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
60	R_L	R_L	R_L	R_L	R_L	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
61	R_L	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
62	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
63	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
64	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	6

Table 2: States of the world, banks' portfolio returns and defaults in the unclustered network

State	*States of the world*						*Banks' portfolio returns*						Total defaults
	θ_1	θ_2	θ_3	θ_4	θ_5	θ_6	X_1	X_2	X_3	X_4	X_5	X_6	
1	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	R_H	0
2	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	0
3	R_H	R_H	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
4	R_H	R_H	R_H	R_H	R_H	R_L	$(R_L+2R_H)/3$	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
5	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	0
6	R_H	R_H	R_L	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	0
7	R_L	R_H	R_H	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	R_H	$(R_L+2R_H)/3$	0
8	R_H	R_H	R_H	R_L	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
9	R_H	R_H	R_H	R_H	R_L	R_L	$(R_L+2R_H)/3$	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
10	R_H	R_H	R_H	R_L	R_H	R_L	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
11	R_H	R_L	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	0
12	R_H	R_L	R_H	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
13	R_H	R_L	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
14	R_H	R_L	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	0
15	R_H	R_H	R_L	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	0
16	R_H	R_H	R_L	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
17	R_H	R_H	R_L	R_H	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
18	R_L	R_L	R_H	R_H	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	$(R_L+2R_H)/3$	0
19	R_L	R_H	R_H	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
20	R_L	R_H	R_H	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
21	R_L	R_H	R_H	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
22	R_L	R_H	R_L	R_H	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	0
23	R_H	R_H	R_H	R_L	R_L	R_L	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	1
24	R_H	R_L	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
25	R_H	R_L	R_H	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
26	R_H	R_L	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
27	R_H	R_H	R_L	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	1
28	R_H	R_L	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	1
29	R_H	R_H	R_L	R_L	R_H	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	0
30	R_H	R_H	R_L	R_H	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
31	R_H	R_L	R_L	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
32	R_H	R_L	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
33	R_L	R_H	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	1
34	R_L	R_L	R_H	R_H	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	1
35	R_L	R_H	R_H	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
36	R_L	R_H	R_H	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
37	R_L	R_L	R_H	R_L	R_H	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
38	R_L	R_L	R_H	R_H	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
39	R_L	R_L	R_L	R_H	R_H	R_H	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	R_H	$(R_L+2R_H)/3$	1
40	R_L	R_H	R_L	R_L	R_H	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	0
41	R_L	R_H	R_L	R_H	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
42	R_L	R_H	R_L	R_H	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	0
43	R_H	R_L	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	1
44	R_H	R_H	R_L	R_L	R_L	R_L	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	$(2R_L+R_H)/3$	2
45	R_H	R_L	R_L	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	2
46	R_H	R_L	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	1
47	R_H	R_L	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
48	R_L	R_H	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	2
49	R_L	R_L	R_H	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	R_L	2
50	R_L	R_L	R_H	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	1
51	R_L	R_L	R_H	R_L	R_L	R_H	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
52	R_L	R_L	R_L	R_L	R_H	R_H	$(2R_L+R_H)/3$	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	2
53	R_L	R_L	R_L	R_H	R_H	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	2
54	R_L	R_L	R_L	R_H	R_L	R_H	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	1
55	R_L	R_H	R_L	R_L	R_L	R_H	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	1
56	R_L	R_H	R_L	R_H	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(R_L+2R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	1
57	R_L	R_H	R_L	R_L	R_H	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	0
58	R_H	R_L	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	$(2R_L+R_H)/3$	3
59	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	3
60	R_L	R_L	R_L	R_L	R_L	R_H	$(2R_L+R_H)/3$	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
61	R_L	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	3
62	R_L	R_L	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	3
63	R_L	R_H	R_L	R_L	R_L	R_L	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	$(2R_L+R_H)/3$	R_L	R_L	R_L	3
64	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	R_L	6

	$X_i = R_L$	$X_i = \frac{2R_L+R_H}{3}$	$X_i = \frac{R_L+2R_H}{3}$	$X_i = R_H$
$\Pr(X_i = x \mid G)$	0	$\frac{21}{49}$	$\frac{21}{49}$	$\frac{7}{49}$
$\Pr(X_i = x \mid B)$	$\frac{8}{15}$	$\frac{3}{15}$	$\frac{3}{15}$	$\frac{1}{15}$

Table 3: Conditional distribution of bank i's portfolio returns in the *clustered* network

	$X_i = R_L$	$X_i = \frac{2R_L+R_H}{3}$	$X_i = \frac{R_L+2R_H}{3}$	$X_i = R_H$
$\Pr(X_i = x \mid G)$	0	$\frac{13}{39}$	$\frac{19}{39}$	$\frac{7}{39}$
$\Pr(X_i = x \mid B)$	$\frac{8}{25}$	$\frac{11}{25}$	$\frac{5}{25}$	$\frac{1}{25}$

Table 4: Conditional distribution of bank i's portfolio returns in the *unclustered* network

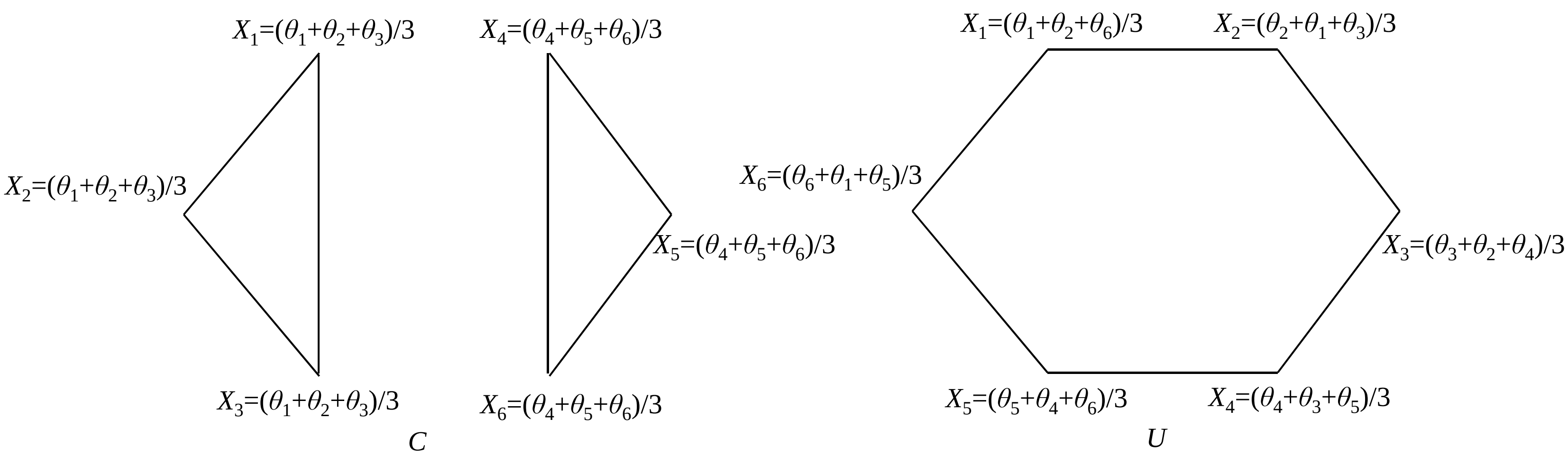

Figure 1: Clustered (C) and unclustered (U) network

The figure depicts the clustered (C) and the unclustered (U) networks. In the former, banks are connected in two clusters of three banks each. Within each cluster, banks hold identical portfolios X_i but the two clusters are independent of each other. In the latter, banks are all connected in a circle. Each of them exchanges projects only with the two neighboring banks and none of the banks holds identical portfolios.

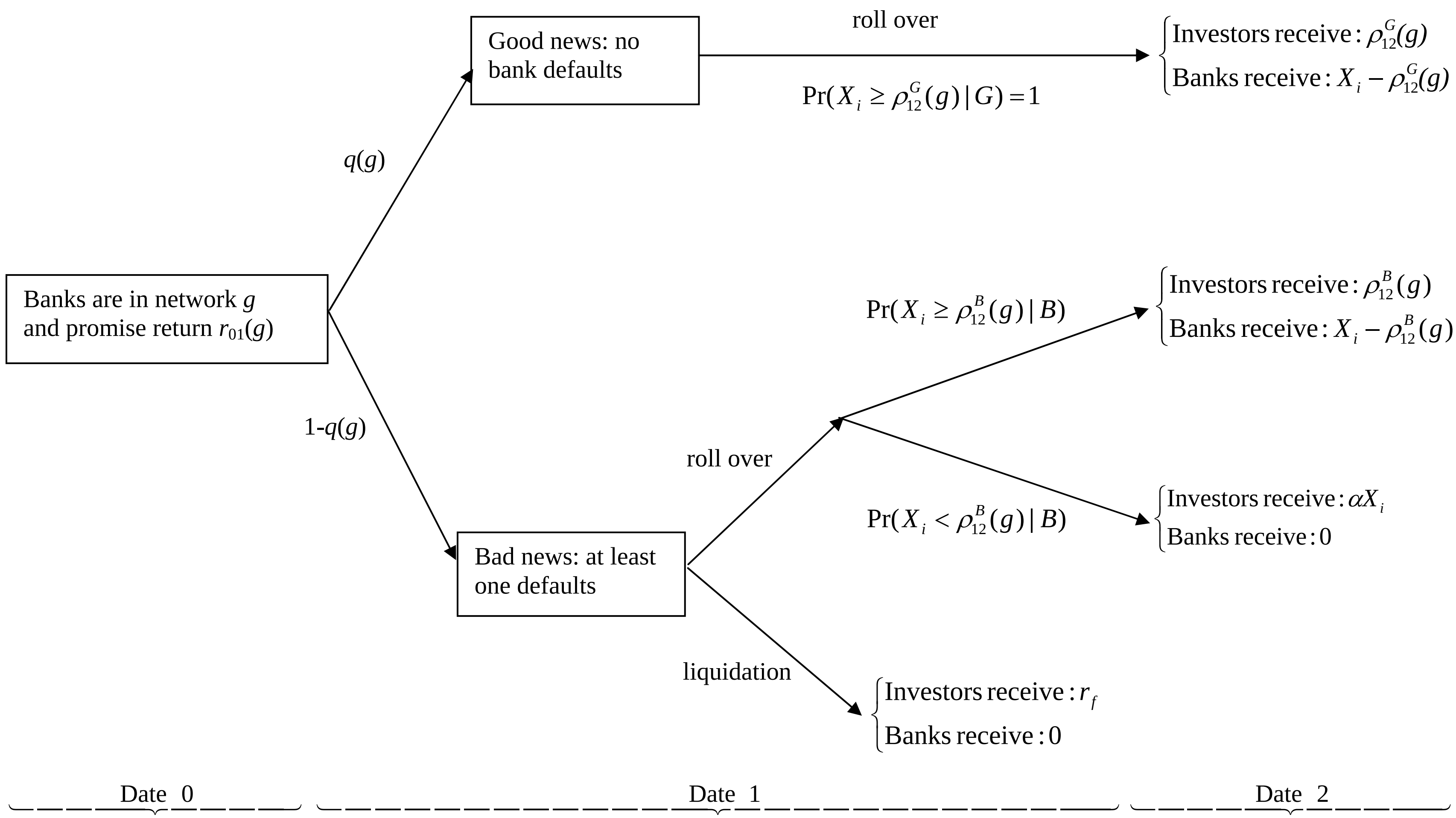

Figure 2: Sequence of events

The figure shows the timing of the model with short term finance. At date 0 each bank in network g=C,U raises one unit of funds in exchange for a promised return $r_{01}(g)$ at date 1. At the beginning of date 1, before investors are repaid, a signal $S=G,B$ is realized. With probability $q(g)$, it brings the good news that all banks will be solvent at date 2. With probability $1\text{-}q(g)$ it brings the bad news that at least one bank will default at date 2. Investors decide whether to retain $r_{01}(g)$ or roll it over for a total promised repayment of $\rho_{12}{}^S(g)$ at date 2. When the debt is rolled over, the bank continues till date 2. If it remains solvent, which occurs with probability $\Pr(X_i \geq \rho_{12}{}^S(g)/\mathrm{B})$, investors obtain $\rho_{12}{}^S(g)$ and the bank X_i - $\rho_{12}{}^S(g)$. If the bank defaults at date 2, , which occurs with probability $\Pr(X_i < \rho_{12}{}^S(g)/\mathrm{B})$, investors obtain αX_i and the bank zero. When the debt is not rolled over, the bank is forced into early liquidation at date 1. Investors obtain r_f and the bank zero.

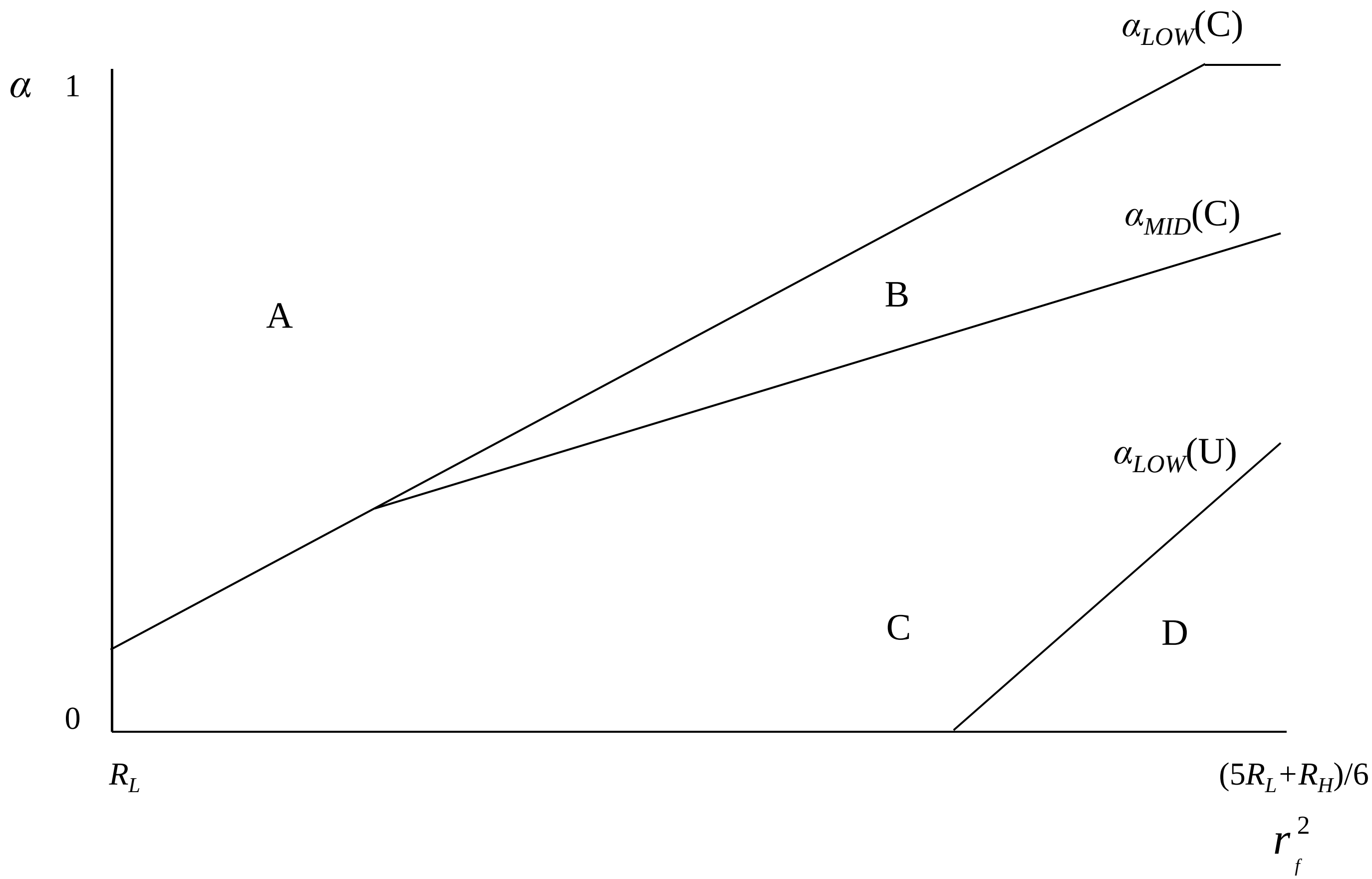

Figure 3: Investors' rollover decision in the clustered and unclustered networks

The figure depicts investors' rollover decision in both networks when the bad signal arrives as a function of the opportunity cost r_f^2 and the fraction α of the bank's portfolio return that investors receive in case of default. In Region A debt is rolled over for a repayment $\rho_{12}^B(g) \in (r_f^2, (2R_L + R_H)/3]$ in both networks. In Region B rollover occurs still in both networks but in the clustered network the repayment is now $\rho_{12}^B(g) \in ((2R_L + R_H)/3, (R_L + 2R_H)/3]$. In Region C debt is rolled over in the unclustered network but not in the clustered one. In Region D rollover does not occur in either networks.

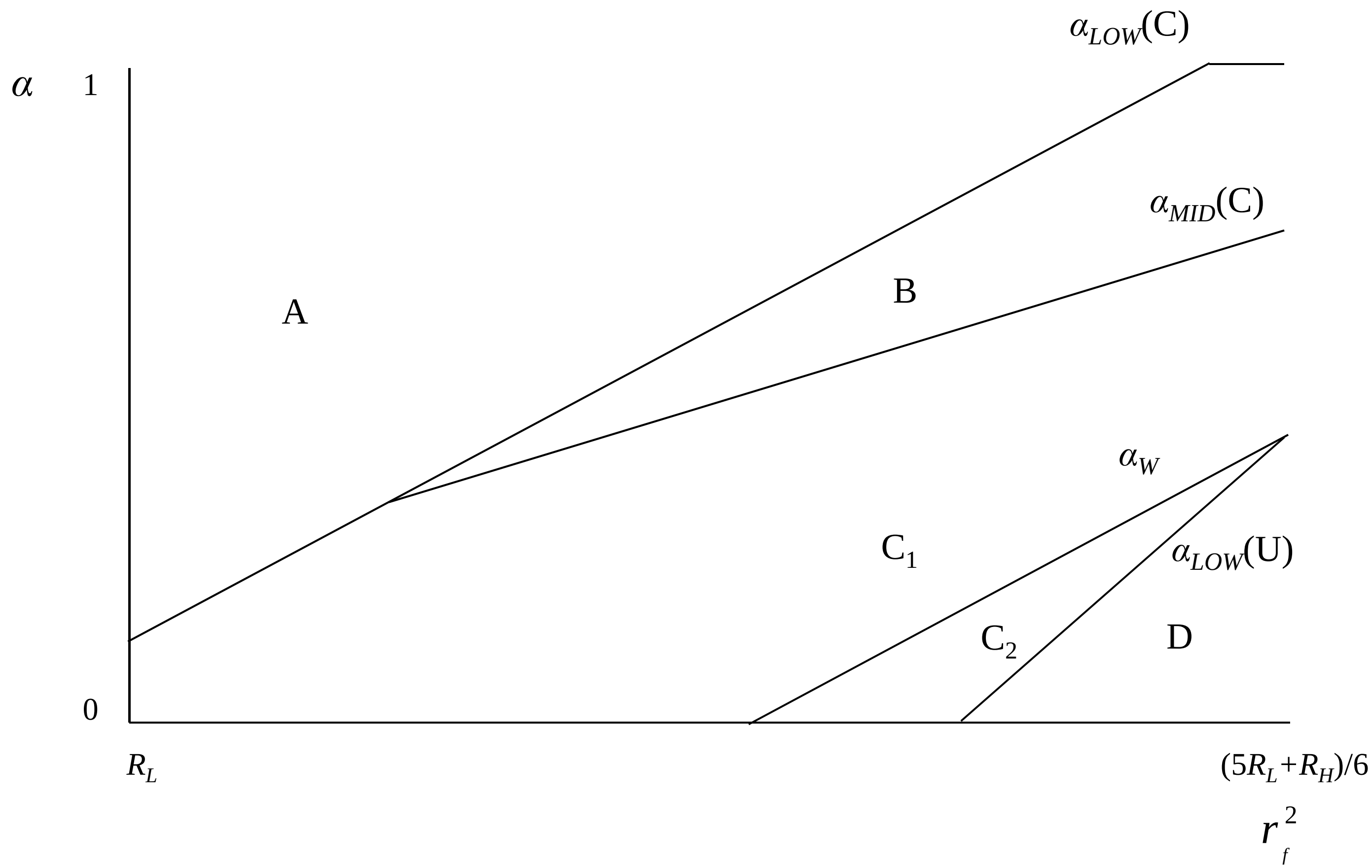

Figure 4: Total welfare in the clustered and unclustered networks

The figure depicts total welfare in the clustered and unclustered networks as a function of the investors' opportunity cost r_f^2 and the fraction α of the bank's portfolio return that they receive in case of default. In Region A, total welfare is the same in both networks. In Region B+C_1, total welfare is higher in the unclustered network. In Region C_2+D, total welfare is higher in the clustered network.