NBER WORKING PAPER SERIES

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Behzad T. Diba

Herschel I. Grossman

Working Paper No. 1615

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 1985

The National Science Foundation Grant SES-8210129 has supported this research. The research reported here is part of the NBER's research programs in Financial Markets and Monetary Economics and Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

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ABSTRACT

A rational bubble would involve a self-confirming belief that an asset price depends on information that includes variables or parameters that are not part of market fundamentals. The existing literature shows that, if market fundamentals are economically interesting, i.e., forward looking, any rational bubbles would be either explosive or implosive. Further arguments based on the existing literature show that utility maximizing behavior implies finite bounds on asset prices and, accordingly, precludes both explosive and implosive rational price expectations, except for the possible case of an implosion in the value of fiat money. These arguments rule out both positive and negative rational bubbles, except for the possibility of rational inflationary bubbles.

This paper extends the theoretical analysis of rational bubbles in two ways. First, it shows that, although a supply response of the current asset stock to the current asset price dampens fluctuations in market fundamentals, such a response would cause a rational bubble to explode or to implode even faster. Thus, the explosiveness or implosiveness of rational bubbles is not an artifact of assuming that the asset stock evolves autonomously. Second, and more importantly, the present analysis considers the inception of rational bubbles and shows that, for a negative rational bubble--such as a rational inflationary bubble--to get started, a positive rational bubble also would have to have positive probability. Specifically, the expected initial absolute value of a potential negative rational bubble cannot exceed the expected initial value of a potential positive rational bubble. This result dramatically expands the theoretical basis for precluding rational bubbles. Specifically, because utility maximization directly rules out rational deflationary bubbles, the inception of a rational inflationary bubbles is also precluded.

Behzad Diba
Department of Economics
Tulane University
New Orleans, Louisiana 70118

Herschel I. Grossman
Department of Economics
Brown University
Providence, Rhode Island 02912

The recent literature includes various examples that apparently illustrate the theoretical possibility of rational bubbles in asset prices. In synthesizing these contributions, Burmeister, Flood, and Garber (1983)--henceforth BF&G--focus on a model in which the logarithm of asset demand, measured in units of a basket of consumption goods, depends linearly on the rational expectation of the rate of change in the asset price. Accordingly, the logarithm of the market-clearing asset price satisfies a first-order linear partial difference equation with a stochastic forcing term that consists of the variables that shift demand and supply. To motivate this setup, BF&G refer explicitly to the standard Cagan demand function for fiat money used in studies of inflation, interpreting the asset price as the inverse of the price level and the expected rate of decrease in the asset price as the expected inflation rate. This reference reflects the earlier interest of Flood & Garber (1980) in modelling hyper-inflations as negative rational bubbles in the value of fiat money. The same basic setup, however, also could represent an approximation to a portfolio balance equation that equates the demand and supply for a real asset, interpreting the asset price as the exchange ratio between the asset and a basket of consumption goods.

Within this framework, BF&G define the market-fundamentals component of the asset price to be the particular solution to the partial difference equation for the asset price that we obtain by setting the solution to the homogeneous equation equal to zero, and they define other solutions to the homogeneous equation to be the rational-bubbles component. Defined in this way, the market-fundamentals component relates the current price uniquely to the parameters of the demand and supply functions and, except in extreme cases of the forcing processes, to the current and <u>expected future</u> values of the stochastic forcing variables.[1] More interestingly, given their definitions, nothing in the specification of the BF&G model precludes the additional presence of a rational-bubbles component. As the discussion below explains, such a rational bubble would involve a self-confirming belief that the current relative asset price also depends on a variable (or a combination of variables) that is intrinsically irrelevant--that is, not part of market fundamentals--or on truly relevant variables in a way that involves parameters that are not part of market fundamentals.[2]

The result in the BF&G analysis that, except in extreme cases of the forcing processes, the only well-defined solution for the market-fundamentals component is forward looking accords with the usual economic intuition. This result obtains because BF&G assume, analogously to a downward sloping demand curve, that the demand for the asset depends positively on the expected rate of change in the asset price. This assumption means that the eigenvalue of the partial difference equation governing the asset price is greater than unity. Importantly, the eigenvalue greater than unity, as BF&G observe, also implies that a rational bubble would involve an explosion, either positive or negative, of the rational expectation of the logarithm of the asset price. Specifically, with the eigenvalue greater than unity, the existence of a rational bubble would imply that the expected values of the logarithm of the asset price conditional on current information either increase or decrease (at an increasing rate) into the infinite future.[3]

The main attraction of the simple first-order log-linear model of an asset price analyzed by BF&G is that it provides explicit mathematical representations of the market-fundamentals component and the rational-bubbles component for general specifications of the stochastic processes generating the variables that are part of or not part of market fundamentals. Unfortunately, however, the log-linear approximation to the asset demand function can have seriously misleading implications for the behavior of the value of asset demand at extremely high or low asset prices. Specifically, as various authors have observed, the implications of utility maximizing behavior can directly preclude the explosive behavior of the expected value of the logarithm of the asset price associated with the rational bubbles analyzed by BF&G. The general idea--see, for example, Tirole (1982), Blanchard and Watson (1982), or Gray (1984)--is that, under any reasonable specification of preferences, asset holders, even if their planning horizons are infinite, are not willing to adopt plans that involve permanent postponement of consumption.

The development of this idea differs in a subtle, but important, way between positive and negative rational bubbles. For positive rational bubbles, the relevant concept of consumption includes all sources of utility except the service flow from the asset stock itself. The essential argument is that, at a sufficiently high, but finite, asset price and associated value for the asset stock, asset holders would be unwilling to continue to hold on to the asset stock no matter how fast they expected the asset price to be increasing. For example, if the price of gold were already so high that (say) an ounce of gold could buy the world, even if gold itself yields a positive service flow, we should expect, contrary to the implication of the log-linear demand function, that holders would spend their gold now rather than plan to wait (forever) until the price of gold reached infinity. Given this prospective disappearance of the demand for gold, which involves reinforcing substitution effects and wealth effects, asset holders could not rationally expect the price of gold to increase explosively without bound, as would have to be the case under a positive rational bubble.

Taking into account a potentially unlimited succession of overlapping generations modifies this argument slightly. In this expanded framework, although each generation could plan to consume in the finite future by selling its asset holdings to the next generation, the endowments of each new generation would limit the prices at which each old generation could exchange its asset holdings for consumption. Consequently, given that current asset holders do not rationally expect consumption endowments to explode, again they also could not rationally expect the price of any asset to increase explosively without bound.

For negative rational bubbles, the relevant concept of consumption is the service flow from the asset itself. The analogous argument is that, at a sufficiently low, but positive, asset price, asset holders would want to acquire more than the existing asset stock no matter how fast they expected the asset price to be decreasing. For example, if the price of gold were to fall so low that the wages of a few moments of work effort could buy a ton of gold, keeping in mind that the asset price approaches zero as its logarithm becomes increasingly negative, the multitude of consumers who admire the beauty of gold would exercise their demand for gold now rather than, as the log-linear demand function implies, plan to wait (forever) until the price of gold reached zero. Given this prospective arbitrarily large demand for gold, which involves the eventual dominance of substitution effects over wealth effects, asset holders could not rationally expect the logarithm of the price of gold to decrease explosively, as would have to be the case under a negative rational bubble.

As noted above, the log-linear approximation to the demand functions used by BF&G could apply to fiat money as well as to a real asset like gold. Unlike the analogous argument against positive bubbles, however, the preceding argument against negative bubbles is relevant only for real assets. It does not apply to inflationary bubbles. Specifically, as various authors have shown--for recent contributions, see Kingston (1982), Obstfeld and Rogoff (1983), or Gray (1984)--an expectation of an implosive decrease in the value of fiat money to zero could be rational as long as fiat money is inessential in the sense that, at a finite expected rate of decrease of its price, asset holders would be willing to reduce their holdings increasingly close to zero in exchange for finite increases in consumption. As Obstfeld and Rogoff and Gray explain, this condition seems consistent with a reasonable view of the way in which fiat money enhances utility.

The essential difference between fiat money and real assets in this regard is that utility does not derive from physical units of fiat money--that is, nominal balances--but rather from the value of these units--that is, real balances. A negative rational bubble in the price of (say) gold would mean that the expected cost of a unit of the services of gold in terms of units of other sources of utility would decrease towards zero. The above argument against negative rational bubbles is that this expectation would imply the contradictory expectation that the demand for physical units of gold eventually would become arbitrarily large. In contrast, with an inflationary rational bubble, although the expected price of a unit of fiat money would decrease towards zero, the expected cost of a unit of the services of fiat money in terms of units of other sources of utility would not decrease. Accordingly, there is no reason to expect the demand for either real or nominal balances to become arbitrarily large and thwart the inflationary bubble.

To summarize the arguments in the existing literature, the log-linear asset demand function analyzed by BF&G suggests that rational bubbles are theoretically possible. In general, however, this functional form, although a useful approximation for some purposes, is not globally consistent with utility maximizing behavior. Specifically, the implications of any reasonable specification of preferences enable us directly to rule out both positive and negative rational bubbles a priori, except for the possibility, stressed by BF&G, of rational inflationary bubbles.

The present paper extends the theoretical analysis of rational bubbles in two ways. First, it introduces into the BF&G model a supply response of the current asset stock to the current asset price. This analysis shows that a rational bubble would imply either explosive or implosive price expectations even if the behavior of the asset stock dampens fluctuations in market fundamentals. Second, and more importantly, the present paper considers explicitly the inception of rational bubbles--a problem from which the existing literature largely abstracts. This analysis shows that, for a negative rational bubble to get started, a positive rational bubble also would have to have positive probability. This result dramatically expands the theoretical basis for precluding rational bubbles, including the apparently exceptional case of rational inflationary bubbles.

1. Analytical Framework

The log-linear model analyzed by BF&G assumes that the logarithm of the value of the demand for the asset at date t, in terms of a basket of consumption goods, depends positively on the expected rate of change of the price of the asset from date t to date $t+1$ relative to the price of the consumption basket, and that the price of the asset adjusts to equate this demand to the value of the existing asset stock. Thus, the relative price of the asset satisfies

(1) $$Q_t + P_t = \beta(E_t P_{t+1} - P_t) + d_t,$$

where Q_t is the logarithm of the physical asset stock at date t,

P_t is the logarithm of the ratio of the asset price to a relevant index of prices of consumable goods and services,

E_t is an operator that denotes a rational expectation, i.e., an expectation consistent with this model, conditional on information available at date t,

d_t represents the effects on demand of all factors other than the expected rate of change of the relative price of the asset,

and β is a positive constant.In this setup, the variable (or combination of variables) represented by d_t is stochastic and its innovations are independent of past prices.

The preceding discussion pointed out that, although BF&G interpret Q_t as the logarithm of the nominal stock of fiat money and P_t as the logarithm of the inverse of the price level, equation (1) could also represent an approximation to a portfolio balance equation for a real asset. Nevertheless, as stressed above, this log-linear demand function, in general, is not a good approximation at extremely high or low asset prices. Specifically, utility maximization suggests that, for either fiat money or a real asset, demand would become arbitrarily small at a sufficiently high current price, no matter how high the expected future price. Also, for a real asset, but not necessarily for fiat money, demand would become arbitrarily large at a sufficiently low current price, no matter how low the expected future price. Given that the asset stock is always positive, but finite, we can incorporate these considerations into the log-linear model by imposing limits on the range of possible values of P_t, and on the relevance of equation (1), of the form

(2) $\underline{P} \leqslant P_t \leqslant \overline{P}$ for real assets and

$P_t \leqslant \overline{P}$ for fiat money,

where $\underline{P}$ and $\overline{P}$ are finite.

BF&G assume that the current asset stock evolves autonomously. The present analysis extends this framework to allow the current asset stock to depend on the current asset price. Specifically, the current asset stock satisfies

(3) $Q_t = \alpha P_t + s_t,$

where α is a non-negative constant

and s_t represents the effects on the current asset stock of all factors other than the asset price.

Like d_t, s_t is also stochastic with innovations independent of past prices.

Equation (3) incorporates simplifying assumptions that serve to minimize the mathematical complexity of the analysis by preserving the property that the logarithm of the asset price satisfies a first-order partial difference equation even if α is positive. First, the asset price affects the asset stock with no lag. A more realistic model for a real asset might include production and consumption lags and, hence, have the current asset stock depend on past expectations of the current price. Second, only the current asset price affects the current asset stock. A more realistic model for a real asset might have the current price affect the flow of production or consumption of the asset and, hence, have the current asset stock depend on past prices. Either of these extensions in general would lead to a higher-order partial difference equation for the asset price. (See Evans and Honkapolija (1983) for a discussion of mathematical properties of solutions for models that imply such higher-order systems.) Equation (3) also does not address the peculiar issues concerning public finance and monetary institutions involved in motivating the issuance of fiat money.

2. Market Fundamentals

Combining equations (1) and (3) to eliminate Q_t yields the following relation involving P_t, E_tP_{t+1}, d_t, and s_t:

$$(4) \qquad (1+\alpha+\beta)P_t - \beta E_tP_{t+1} - d_t + s_t = 0.$$

The general solution of equation (4) for P_t, which is also subject to condition (2), is the sum of a particular solution and the general solution to the homogeneous equation. Following BF&G, denote the particular solution in terms of current and expected future values of the exogenous variables d and s and the parameters α and β to be the market-fundamentals component of price, F_t. Thus, F_t satisfies

$$(5) \qquad F_t = (1+\alpha+\beta)^{-1}(\beta E_tF_{t+1} + d_t - s_t).$$

Also following BF&G, denote the homogeneous solution, which can involve current and past realizations of other variables as well as other parameters, to be the potential rational-bubbles component of price, B_t. Thus, B_t satisfies

(6) $$E_t B_{t+1} - \beta^{-1}(1+\alpha+\beta)B_t = 0.$$

In general,

(7) $$P_t = F_t + B_t,$$

subject to condition (2).

The solution of equation (5) for F_t requires the derivation of an expression for $E_t F_{t+1}$ in terms of expectations of the forcing variables. Assuming that condition (2) does not impose a binding constraint on the evolution of F_t, the assumption of rational expectations implies that in forming $E_t F_{t+1}$ market participants behave as if they know that market fundamentals will conform to equation (5) in all future periods. Leading equation (5) j periods, $j \geq 1$, and applying the operator E_t gives the partial difference equation,

(8) $$\begin{aligned} E_t F_{t+j} &= (1+\alpha+\beta)^{-1} E_t(\beta E_{t+j} F_{t+j+1} + d_{t+j} - s_{t+j}) \\ &= (1+\alpha+\beta)^{-1}[\beta E_t F_{t+j+1} + E_t(d_{t+j} - s_{t+j})]. \end{aligned}$$

Equation (8) is a partial, rather than an ordinary, difference equation because $E_t F_{t+j}$ depends on both t and j.

To solve for $E_t F_{t+1}$, fix t and treat equation (8) as an ordinary difference equation in j. Because the eigenvalue, $\beta^{-1}(1+\alpha+\beta)$, is greater than unity, the forward-looking solution to this equation involves a convergent sum, as long as the expected difference, $E_t(d_{t+i} - s_{t+i})$, for any t does not grow with i at a geometric rate equal to or greater than

$\beta^{-1}(1+\alpha+\beta)$. (An important advantage of the log-linearity of equation (4) is that it permits the derivation of explicit expressions for the market-fundamental and rational-bubble components of price without imposing any additional restrictions on these processes. For example, to allow maximum generality, d_t and s_t can be moving average processes of infinite order and can accommodate any specific autoregressive representation of the demand and supply functions as a special case.)

Using the forward operator L^{-1}, this forward-looking solution is, for $j = 1$,

$$(9) \qquad E_t F_{t+1} = (1+\alpha+\beta)^{-1} E_t \{[1-\beta(1+\alpha+\beta)^{-1}L^{-1}]^{-1}(d_{t+1} - s_{t+1})\}$$

$$= \beta^{-1} \sum_{i=1}^{\infty} [\beta(1+\alpha+\beta)^{-1}]^{i} E_t(d_{t+i} - s_{t+i}).$$

Substituting equation (9) for $E_t F_{t+1}$ into equation (5) gives the market-fundamentals component of price,

$$(10) \qquad F_t = (1+\alpha+\beta)^{-1}\{d_t - s_t + \sum_{i=1}^{\infty} [\beta^{-1}(1+\alpha+\beta)]^{-i} E_t(d_{t+i} - s_{t+i})\}.$$

Equation (10) says that F_t is proportionate to a weighted sum of current and expected future realizations of the variables that shift demand and supply. The factor of proportion is inversely related to β, and the weights are powers of the eigenvalue such that the contribution of $E_t(d_{t+i} - s_{t+i})$ to F_t declines exponentially with i.

3. Rational Bubbles

The general solution for P_t also includes the rational-bubbles component, B_t, which satisfies the homogeneous equation (6). The assumption of rational expectations implies that in forming $E_t B_{t+j}$, for all $j > 0$, asset holders behave as if they know that any rational bubble component would conform to equation (6) in all future periods. Accordingly, any solution to equation (6) would have the property

(11) $$E_t B_{t+j} = [\beta^{-1}(1+\alpha+\beta)]^j \; B_t \quad \text{for all} \quad j > 0.$$

Equation (11) confirms the result stressed above that with an eigenvalue greater than unity, the existence of a rational bubble would imply that the expected values of the logarithm of the asset price conditional on current information either increase or decrease (at an increasing rate) into the infinite future. Thus, equation (11) shows that, except for the possibility of a negative value of B_t in the case of fiat money, any solution to equation (6), other than $B_t = 0$, would imply rational expectations inconsistent with condition (2), which specifies finite limits on the range of possible logarithms of asset prices. This result limits interest in nontrivial solutions to equation (6) to the potential case of rational inflationary bubbles.

Solutions to equation (6), satisfy the stochastic difference equation

(12) $$B_{t+1} - \beta^{-1}(1+\alpha+\beta)B_t = z_{t+1},$$

where z_τ, a random variable (or combination of variables), representing new information available at date τ, satisfies

(13) $$E_t z_\tau = 0 \quad \text{for} \quad \tau > t.$$

The key to the relevance of equation (12) for the general solution for P_t is that equation (6) relates B_t to $E_t B_{t+1}$, rather than to B_{t+1} itself as would the case in a perfect-foresight model.

In this formulation, the realizations of z_τ embody all sources of divergence between P_t and F_t. In other words, z_τ is the source of potential rational bubbles. The random variable z_τ can be intrinsically irrelevant--that is, unrelated to the forcing variables present in F_t--or it can depend on truly relevant variables, like d_t and s_t, through parameters that are not present in F_t. The only critical property of z_τ, given by equation (13), is that its expected future values are always zero. (In the model developed by Blanchard and Watson (1982) the analog to z_τ satisfies equation (13) even though it is not variance stationary.)

A solution to equation (12) is

$$(14) \qquad B_t = \sum_{\tau=1}^{t} [\beta^{-1}(1+\alpha+\beta)]^{t-\tau} z_t,$$

where date one, a given point in the finite past, marks the inception of the process generating z_τ, or, equivalently, the first nonzero realization from this process. (Note that for P_t to be finite, the inception of a rational bubble could not have been infinitely long ago.) Equation (14) says that the rational-bubbles component is a weighted sum of current and past realizations of z_τ. The weights are powers of the eigenvalue such that the contribution of z_τ to B_t increases exponentially with the difference between t and τ. For example, the initial realization z_1 contributes only the amount z_1 to B_1, but contributes $[\beta^{-1}(1+\alpha+\beta)]^{t-1} z_1$ to B_t.

BF&G distinguish between deterministic bubbles and stochastic bubbles. The significance of this distinction would seem to be to separate factors that have no effect on the conditional variance of P_t from factors that make P_t more variable than F_t. The expression for B_t in equation (14) shows that this separation depends on the point in time at which it is made. Specifically, at date $t-j$, $j > 0$, only the realizations of z_τ from date $t-j+1$ to date t are both unknown and relevant for B_t. Thus, if z_τ has constant variance σ_τ^2, equation (14) implies that the variance of B_t conditional on information available at date $t-j$ is

$$\sigma_\tau^2 \sum_{\tau=t-j+1}^{t} [\beta^{-1}(1+\alpha+\beta)]^{2(t-\tau)}.$$

Accordingly, from the perspective of date $t-j$, the stochastic part of B_t is

$$\sum_{\tau=t-j+1}^{t} [\beta^{-1}(1+\alpha+\beta)]^{t-\tau} z_{\tau},$$

and the deterministic part, which is the rest of B_t and is known at date $t-j$, is

$$\sum_{\tau=0}^{t-j} [\beta^{-1}(1+\alpha+\beta)]^{t-\tau} z_{\tau}.$$

Specifically, from the perspective of date zero, B_t, for all $t > 0$, is entirely stochastic.[4]

4. The Effect of Endogeneity of the Asset Stock

Conventional demand and supply analysis suggests that, ceteris paribus, a larger positive response of supply to price should dampen price fluctuations. In the present model, this conclusion is clearly applicable to the market-fundamentals component of price. Specifically, equation (10) implies that, given the process generating $\{d_t - s_t\}$, the variance of F_t conditional on information available at any date $t-j$, $j > 0$, is negatively related to α.

Interestingly, however, this dampening effect on the variance of the relative price would not apply to a potential rational-bubbles component. Instead, equation (14) implies that, although the variance of B_t conditional on information available at any date $t-1$ equals the conditional variance of z_t and is independent of α, the variance of B_t conditional on information available at any earlier date $t-j$, $j > 1$, is positively related to α. This effect on the variance of the asset price results from the fact that a positive value of α would transmit a rational bubble in the asset price to the asset stock. The effect on the stock in turn would amplify the implications of the rational bubble for expected future asset prices.

This perverse effect of α on the conditional variance of a hypothetical rational-bubbles component of the asset price reflects the self-confirming nature of the expectations generated by a nonzero realization of z_τ. Note that in the market-fundamentals component of the asset price, a positive innovation in $d_t - s_t$, with $E_t(d_{t+i} - s_{t+i})$ for all $i > 0$ unchanged, leaves $E_t F_{t+1}$ unchanged. Hence, market clearing requires a positive innovation in F_t, but this required innovation is smaller the larger is α. In contrast, given a positive realization of z_t, implying a positive innovation in B_t, the existence of a rational bubble would require a larger positive innovation in $E_t B_{t+1}$ to satisfy market clearing. The larger α, the larger this required innovation in $E_t B_{t+1}$.

For example, a current positive realization of z_t, which according to equation (14) would raise B_t by that amount, also would raise Q_t by an amount proportionate to α. But, given B_t, the larger is α and, hence, the larger is Q_t, the larger has to be $E_t B_{t+1}$ in order to keep demand equal to the current asset stock. Moreover, given $E_t B_{t+1}$, the larger is α, the larger is the rational expectation of Q_{t+1}, and, hence, the larger has to be $E_t B_{t+2}$, and so on.

5. The Inception of Rational Inflationary Bubbles

The preceding analysis considered solutions to equation (12) given the presumption that nontrivial solutions exist that do not violate condition (2). The arguments developed above have already ruled out this presumption for all cases except the possibility of rational inflationary bubbles. This section considers the possible inception of a rational inflationary bubble.

As discussed above, condition (2), together with the assumption of rational expectations, implies for the case of fiat money that any solution to equation (6) must satisfy $B_t \leq 0$ for all values of t. In other words, the rational-bubble component of the value of fiat money can never become positive. Whether or not this implication of condition (2) turns out to be a binding constraint on the actual time path of B_t given by equation (14) depends on the realizations of z_τ, for all $\tau = 1 \ldots t$. Specifically, for any solution to equation (12), such as equation (14), to be a nontrivial solution to equation (6) for fiat money, the new information contained in z_τ must enter the price of fiat money in such a way that

$$(15) \qquad z_\tau \leq - \beta^{-1}(1+\alpha+\beta)B_{\tau-1} \geq 0 \quad \text{for all} \quad \tau = 1 \ldots t.$$

Taking account of condition (2), the process generating z_τ must satisfy condition (15) in addition to equation (13).

For $\tau = 1$, condition (15) implies $z_1 \leq 0$. But, given equation (13), $z_1 \leq 0$ implies that z_1 equals zero with probability one. Thus, a rational inflationary bubble cannot get started at date one, nor, by extension, at any subsequent date. In other words, the inception of a rational inflationary bubble would involve a contradiction, because, for a rational inflationary bubble to get started at date τ, expectations would have to fail to be rational at date $\tau-1$. Specifically, asset holders would have to fail to recognize at date $\tau-1$ that z_τ would have to satisfy both equations (13) and (15) in order to have a self-confirming effect on P_τ.

The essential idea underlying this line of argument is that, because the inception of a rational bubble involves an innovation in the asset price, the expected initial values of a positive rational bubble and a negative rational bubble would have to be equal. Accordingly, given that rational expectations hold at all times, the fact that condition (2) directly rules out rational deflationary bubbles means that rational inflationary bubbles also cannot get started.

6. Summary

A rational bubble would involve a self-confirming belief that an asset price depends on information that includes variables or parameters that are not part of market fundamentals. The existing literature shows that, if market fundamentals are economically interesting, i.e., forward looking, any rational bubbles would be either explosive or implosive. Further arguments based on the existing literature show that utility maximizing behavior implies finite bounds on asset prices and, accordingly, precludes both explosive and implosive rational price expectations, except for the possible case of an implosion in the value of fiat money. These arguments rule out both positive and negative rational bubbles, except for the possibility or rational inflationary bubbles.

The preceding discussion extended the theoretical analysis of rational bubbles in two ways. First, it showed that, although a supply response of the current asset stock to the current asset price dampens fluctuations in market fundamentals, such a response, given the self-confirming nature of expectations inherent in a rational bubble, would cause a rational bubble to explode to to implode even faster. Thus, the explosiveness or implosiveness of rational bubbles is not an artifact of assuming that the asset stock evolves autonomously. Second, and more importantly, it considered the inception of rational bubbles and showed that, for a negative rational bubble--such as a rational inflationary bubble--to get started, a positive rational bubble also would have to have positive probability. Specifically, the expected initial absolute value of a potential negative rational bubble cannot exceed the expected initial value of a potential positive rational bubble. This result implies that, because utility maximization directly rules out rational deflationary bubbles, theoretical analysis also precludes the inception of a rational inflationary bubble.

FOOTNOTES

[1] The BF&G definition of market fundamentals seems natural and straightforward if the forcing variables reflect aspects of technology and resource endowments--for example, the flow of services from the stock of a real asset or the economizing on transactions services made possible by the stock of fiat money. The interpretation of market fundamentals would be more subtle for a hypothetical asset that did nothing except permit the shifting of consumption over time on attractive terms. Although not of any direct empirical relevance, the analysis of such a pure store of value has stimulated theoretical interest. For example, Weil (1984) and Tirole (1985) derive conditions under which a pure store of value could have a positive price. They also, unfortunately, create some semantic confusion by defining such a positive price to be a "bubble". It would seem more consistent with the definitions of BF&G to denote the equilibria derived by Weil and Tirole, if consistent with rational expectations, to be fundamental.

[2] The concept of a rational bubble as defined by BF&G is not a peculiarity of linear models. For example, Azariadis (1981) illustrates the possible effect on price of intrinsically irrelevant variables within a general preference structure that does not necessarily imply a linear demand function. To make his analysis tractable, however, Azariadis restricts the price process to be a two-state Markov chain.

[3] The results of Mussa (1984) underscore the association of economically interesting market fundamentals with explosive or implosive rational bubbles. He shows that various examples of attempts to construct alternative models in which potential rational bubbles are convergent all preclude a forward-looking market-fundamentals solution for some relevant price variable.

[4]The seminal study of rational inflationary bubbles by Flood and Garber (1980) focuses on a term that is the product of the eigenvalue raised to the power t and a constant. According to the present analysis, such a constant would represent a single nonzero realization of z_τ at date one. The bubble term that Flood and Garber consider would represent the effect of this single realization on the time path of price.

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