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ABSTRACT

This paper identifies a new source of business-cycle fluctuations. Namely, a common stochastic trend in neutral and investment-specific productivity. We document that in U.S. postwar quarterly data total factor productivity (TFP) and the relative price of investment are cointegrated. We show theoretically that TFP and the relative price of investment are cointegrated if and only if neutral and investment-specific productivity share a common stochastic trend. We econometrically estimate an RBC model augmented with a number of real rigidities and driven by a multitude of shocks. We find that in the context of our estimated model, innovations in the common stochastic trend explain a sizable fraction of the unconditional variances of output, consumption, investment, and hours.

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1 Introduction

Since the seminal work of Kydland and Prescott (1982) much attention has been devoted to the role of technology shocks as a source of business cycles. The early contributions, including that of Kydland and Prescott, limited attention to technology shocks that take the form of stationary disturbances to neutral productivity. Subsequent contributions, such as those by King, Plosser, and Rebelo (1988), Cogley and Nason (1995), and Rotemberg and Woodford (1996), study models in which technology shocks take the form of permanent disturbances to neutral productivity. More recently, a number of studies, starting with the work of Greenwood, Hercowitz, and Krusell (2000), have emphasized the role of stationary investment-specific productivity shocks as an engine of business cycles.

Following the introduction of the investment-specific technology shock, business-cycle researchers turned to studying the importance of this shock relative to that of the neutral productivity shock as a source of aggregate fluctuations. Fisher (2006), using VAR methods and long-run identification techniques, finds that the majority of business-cycle fluctuations are driven by investment-specific productivity shocks. Justiniano, Primiceri, and Tambalotti (2008) arrive at a similar conclusion in the context of an estimated structural model. By contrast, Smets and Wouters (2007) find that investment-specific productivity shocks play a negligible role in explaining output movements at business-cycle frequencies and assign instead a significant role to shifts in neutral technology.

The motivation for our analysis originates in two observations regarding the assumed stochastic properties of technology shocks in the literature referred to above. One observation concerns the fact that all existing studies assume that neutral and investment-specific productivity shocks follow independent stochastic processes. This assumption, however, is not based on empirical evidence, but appears to be made in a purely ad-hoc fashion. The second observation is that existing studies vary widely regarding the assumptions made about the long-run univariate properties of neutral and investment-specific productivity. Fisher (2006), for example, assumes that both the neutral and the investment-specific technology shocks follow independent nonstationary processes. Smets and Wouters (2007) assume instead that both, neutral and investment-specific technology shocks, follow independent but trend-stationary processes. Justiniano, Primiceri, and Tambalotti (2008) model neutral productivity shocks as having a nonstationary stochastic component and the investment-specific technology shock as having a trend-stationary component.

The first step in our analysis is to provide an empirical foundation for modeling the stochastic properties of the underlying productivity shocks. To this end, we begin by conducting an empirical analysis of the univariate and joint long-run properties of total factor productiv-

ity and the relative price of investment goods. We find that both of these time series contain a stochastic nonstationary component. This finding, together with the implications of a wide class of dynamic stochastic general equilibrium models linking the long-run properties of TFP and the price of investment to those of neutral and investment-specific technology, imply that both neutral and investment-specific productivity shocks should be modeled as containing a stochastic trend. In turn, the requirement that both types of productivity shock contain a stochastic trend can be fulfilled either by assuming that each productivity process contains an independent stochastic trend or by assuming that the two series share a single stochastic trend. Here again, we let the data inform us about which of these two modeling strategies is empirically more compelling. We therefore perform cointegration tests on TFP and the price of investment. We find that these two series appear to share a common stochastic trend. This finding calls for a change in the way productivity shocks should be modeled in business cycle studies. Specifically, the central implication of our empirical result is the emergence of a new source of business cycles, namely, a common stochastic trend in neutral and investment-specific productivity.

Accordingly, the second step in our investigation is to gauge the importance of our newly identified shock as a source of business cycles. To this end, we estimate a dynamic stochastic general equilibrium model driven by a multitude of shocks, including shocks to a common stochastic trend in neutral and investment-specific productivity. The skeleton of our theoretical model is the standard RBC structure. We augment this structure with four real frictions: habit formation in consumption, investment adjustment costs, variable capacity utilization, and imperfect competition in labor markets. These frictions have been shown to improve the RBC model's ability to match U.S. postwar data at business-cycle frequencies. We assume that business cycles are driven by seven shocks: two shocks to the common stochastic trend in neutral and investment-specific productivity, a stationary neutral productivity shock, a stationary investment-specific productivity shock, a preference shock, a wage-markup shock, and a government spending shock. We estimate the model by maximum likelihood using postwar U.S. quarterly data on output, consumption, investment, and hours. We find that in the context of our estimated model, the common stochastic trend in neutral and investment-specific productivity plays a sizable role in driving business cycles.

An important byproduct of our investigation is to provide an econometric justification for the common practice of associating the relative price of investment with an investment-specific productivity shock. Such association is valid only if the production technology transforming consumption goods into investment goods is linear. If instead this technology is not linear, then the relative price of investment is an endogenous variable that depends not only on the (exogenous) investment-specific technology shock, but also on the (endogenous)

amount of resources devoted to the production of investment goods. We estimate the curvature of the investment-good production function and find that this production technology is indeed linear. This finding validates the customary practice of treating the relative price of investment as an exogenous variable embodying investment-specific technological change.

The remainder of this paper is organized in eight sections. Section 2 presents a statistical analysis of the univariate and joint long-run properties of TFP and the relative price of investment. Section 3 presents the theoretical framework through which we evaluate the importance of the common productivity shock. Section 4 demonstrates theoretically that TFP and the relative price of investment are cointegrated if and only if neutral and investment-specific productivity are cointegrated. Section 5 presents a vector-error-correction model of the joint law of motion of the permanent components of neutral and investment-specific productivity. Section 6 presents the estimation of the model and discusses its fit. Section 7 analyzes the predicted role of the common productivity shock in generating business cycles. Section 8 provides sensitivity analysis. Section 9 concludes.

2 A Common Stochastic Trend in TFP and the Price of Investment

In this section, we empirically investigate two issues. One is whether TFP and the relative price of investment posses a stochastic trend. The second is whether these two series are cointegrated, that is, whether they are driven by a common stochastic trend.

Our unit-root and cointegration tests are conducted using quarterly U.S. data ranging from 1948:Q1 to 2006:Q4. The two time series are total factor productivity and the relative price of investment. Total factor productivity is taken from Beaudry and Lucke (2009). This time series covers the nonfarm business sector and is adjusted for variations in capital capacity utilization. The time series for the relative price of investment is based on our own calculations following the methodology proposed in Fisher (2006).[1]

2.1 Unit Root Tests

We begin by conducting tests of the null hypothesis that the logarithms of TFP and the relative price of investment have a unit root.[2] Table 1 presents the results. The table shows

[1]These data as well as an appendix detailing the procedure used in its construction are available from the authors upon request.

[2]The econometric tests conducted in this section are carried out using the JMULTI software, which is freely available at `www.jmulti.de`. The detrending necessary to perform the DFGLS test was conducted by the authors.

Table 1: Testing the Null Hypothesis of Nonstationarity

Test	Variable	Lags (AIC)	Test Statistic	Critical Value (5%)	Reject Null of Nonstationarity
ADF	Log TFP	0	-1.76	-2.86	No
DFGLS	Log TFP	0	-1.07	-2.93	No
ADF	Log Price Investment	5	1.79	-2.86	No
DFGLS	Log Price Investment	5	-0.50	-2.93	No

Note: ADF stands for augmented Dickey-Fuller, and DFGLS stands for Dickey-Fuller Generalized Least Squares. In all cases the model includes a constant and no time trend.

Table 2: Testing the Null Hypothesis of Stationarity (the KPSS Unit Root Test)

Variable	Trend	Lags	Test Statistic	Critical Value (5%)	Reject Null of Stationarity
Log TFP	Yes	1	2.10	0.15	Yes
Log TFP	Yes	3	1.08	0.15	Yes
Log TFP	Yes	5	0.73	0.15	Yes
Log TFP	Yes	7	0.56	0.15	Yes
Log Price Investment	Yes	1	2.37	0.15	Yes
Log Price Investment	Yes	3	1.20	0.15	Yes
Log Price Investment	Yes	5	0.81	0.15	Yes
Log Price Investment	Yes	7	0.62	0.15	Yes

that under both the augmented Dickey-Fuller (ADF) and the Dickey-Fuller Generalized Least Squares (DFGLS) tests the null hypothesis of a unit root cannot be rejected at the standard 5 percent confidence level for both TFP and the relative price of investment.[3]

An alternative test of nonstationarity is to pose the null hypothesis that the time series is stationary in levels. The KPSS test is designed for this purpose. Table 2 displays the results of applying this test to TFP and the relative price of investment. We perform the test for lags ranging from 1 to 7 quarters, including always the possibility of a linear time trend. For both variables and for all lag specifications considered, the KPSS test rejects the hypothesis of stationarity of the individual time series at the 5 percent significance level.

We also perform the ADF and KPSS unit root tests on the growth rates of total factor productivity and the relative price of investment. The results of these tests suggests that both growth rates are stationary.

[3]In performing the ADF test, a constant but no time trend is included. The results are robust to including a time trend.

Table 3: Johansen Trace Test For Cointegration

Null Hypothesis	Alternative Hypothesis	Deterministic Trend	Lags (AIC)	p-value
$r = 0$	$r > 0$	No	6	0.00
$r = 0$	$r > 0$	Yes	7	0.01
$r = 0$	$r > 0$	Orthogonal	7	0.07

Note: The cointegration tests are performed on the logarithms of total factor productivity (corrected for variations in capacity utilization) and the relative price of investment. The sample period is 1948:Q1 to 2006:Q4. The variable r denotes the number of cointegrating vectors. AIC stands for Akaike Information Criterion.

Based on the unit-root tests performed above, we conclude that the logarithms of total factor productivity and the relative price of investment appear to be integrated processes of order one. That is, these two time series contain a stochastic trend. The implication of this empirical finding for modeling business cycles is to rule out model specifications in which either TFP or the price of investment or both are trend stationary. Such specifications are the most frequently used ones in the existing literature on the sources of business cycles. Our empirical findings thus far suggest that in modeling business cycles, the stochastic processes for neutral and investment-specific technology shocks should incorporate stochastic trends.

A further question regarding the appropriate modeling of neutral and investment-specific productivity shocks that has not been contemplated in the existing literature but that emerges naturally once the problem is analyzed from the perspective we are proposing is whether neutral and investment-specific productivity shocks are driven by two independent stochastic trends or by a single common one. We turn to this issue next.

2.2 Cointegration of TFP and the Relative Price of Investment

Thus far, we have established that both TFP and the relative price of investment posses a nonstationary stochastic component. The question we wish to investigate here is whether these two stochastic components are cointegrated or independent from each other. In other words, we wish to test the hypothesis that there is no cointegrating relationship between TFP and the price of investment. Rejection of this hypothesis would imply that the two time series are cointegrated. That is, that they are driven by a single stochastic trend.

To identify the number of independent stochastic trends that define the nonstationary components of TFP and the relative price of investment, we perform Johansen's trace test for cointegration. The results are shown in table 3. We set the lag length following the Akaike

information criterion. The Johansen test rejects the null hypothesis of zero cointegrating vectors at high confidence levels when no deterministic trend is included in the system (p-value of 0.00) and when a deterministic trend is included (p-value of 0.01). The hypothesis of no cointegrating relationships is rejected at a confidence level of 10 percent when an orthogonal trend is included (p-value of 0.07).

We conclude from the present empirical analysis that the hypothesis that TFP and the relative price of investment are driven by a single stochastic trend cannot be dismissed off hand. We do not conclude that this is the only empirically viable characterization of the long-run behavior of the TFP and investment price series. But our analysis speaks clearly for covering a gap created by the exclusive attention that has been paid thus far in the business-cycle literature to either models featuring independent stochastic trends in TFP and the price of investment (e.g., Fisher, 2006; and Altig, et al., 2005) or to models maintaining the assumption that the relative price of investment, or TFP or both are trend stationary (e.g., Smets and Wouters, 2007; and Justiniano et al., 2008). Accordingly, the remainder of this paper studies the consequences for business-cycle analysis of our novel assumption that TFP and the relative price of investment are cointegrated I(1) processes. Such analysis necessarily involves the use of a theoretical model of the business cycle, which we develop in the next section.

3 The Model

We develop a model of the business cycle for two purposes: First, we wish to establish what assumptions about the underlying shocks would give rise to the cointegration pattern between TFP and the price of investment documented above. Second, we wish to estimate, in the context of a structural DSGE model the fraction of the variance of output and other macroeconomic indicators explained by our newly identify source of aggregate fluctuations, namely shocks to the common stochastic trend in TFP and the Price of investment.

Our model economy is a real-business-cycle structure augmented with four real rigidities, habit formation in consumption, variable capacity utilization, investment adjustment costs, and imperfect competition in labor markets. The driving forces include stationary and nonstationary neutral and investment-specific productivity shocks, preference shocks, government spending shocks, and wage-markup shocks. This battery of shocks has been shown to be important for explaining business-cycle fluctuations in developed economies (see, for example, Smets and Wouters, 2007; and Justiniano, et al., 2009).

3.1 Households

Consider an economy populated by a large number of identical agents with preferences described over consumption, C_t, and hours worked, h_t,

$$E_0 \sum_{t=0}^{\infty} \beta^t b_t U(C_t - \theta C_{t-1}, h_t), \tag{1}$$

where β denotes the subjective discount factor, b_t is an exogenous stochastic preference shock, and $\theta \in [0, 1)$ measures the degree of internal habit formation. The period utility function U is assumed to be of the form

$$U(x, y) = [(x(1-y)^{\gamma})^{1-\sigma} - 1]/(1-\sigma).$$

Households are assumed to own physical capital. The capital stock, denoted K_t, is assumed to evolve over time according to the following law of motion

$$K_{t+1} = (1 - \delta(u_t))K_t + I_t^g \left[1 - S\left(\frac{I_t^g}{I_{t-1}^g}\right)\right], \tag{2}$$

where I_t^g denotes physical units of investment goods. Investment goods are produced using consumption goods via a technology of the form

$$I_t^g = a_t X_t^a H(I_t), \tag{3}$$

where I_t denotes gross investment measured in terms of physical units of consumption goods, and a_t and X_t^a are, respectively, stationary and nonstationary investment-specific technology shocks. The production function H is assumed to be of the form

$$H(I) = I^{\xi},$$

with $\xi \in (0, 1]$. In a decentralized version of this economy, the relative price of investment goods in terms of consumption goods, which we denote by p_t^I, is given by

$$p_t^I = \frac{1}{a_t X_t^a H'(I_t)}.$$

In the special case in which the production function of investment goods takes the linear form ($\xi = 1$), we have that the relative price of investment is simply given by the inverse of $a_t X_t^a$. This is the case most commonly assumed in the related literature. Rather than

imposing this assumption, we will let the data inform us about the curvature of the function H.

Owners of physical capital can control the intensity with which the capital stock is utilized. Formally, we let u_t measure capacity utilization in period t. The effective amount of capital services households supply to firms in period t is given by $u_t K_t$. We assume that increasing the intensity of capital utilization entails a cost in the form of a faster rate of depreciation. That is, we assume that the depreciation rate is an increasing and convex function $\delta(u_t)$ of the rate of capacity utilization. We adopt the functional form

$$\delta(u) = \delta_0 + \delta_1(u-1) + \delta_2/2(u-1)^2$$

for the function mapping the rate of capacity utilization to the depreciation rate.

The function S introduces investment adjustment costs. It is assumed that in the deterministic steady state the function S satisfies $S = S' = 0$ and $S'' > 0$. These assumptions imply the absence of adjustment costs up to first order in the vicinity of the steady state. They also imply that at the steady state the relative price of installed capital in terms of new capital goods, or Tobin's q, equals unity. We assume that the function S takes the form

$$S(x) = (\kappa/2)(x - \mu^{I^g})^2,$$

where $\kappa > 0$ is a parameter and μ^{I^g} denotes the growth rate of I_t^g along the deterministic growth path.

The budget constraint of the household is given by

$$C_t + I_t = \frac{W_t}{\mu_t} h_t + R_t u_t K_t + \Phi_t, \tag{4}$$

where R_t denotes the rental rate of capital and Φ_t denotes lump-sum profits net of lump-sum taxes. The variable $\mu_t \geq 1$ denotes an exogenous wage-markup shock. This markup represents a wedge between the wage rate paid by firms, W_t, and the marginal wage rate received by households, $W_t/\mu_t < W_t$. This wedge reflects the monopoly power of labor unions. The union rebates all profits to households in a lump-sum fashion. For more details of the underlying labor-market structure, see the appendix.

The first-order conditions associated with the household's optimization problem are (2), (3), (8), and

$$b_t U_1(C_t - \theta C_{t-1}, h_t) - \beta\theta E_t b_{t+1} U_1(C_{t+1} - \theta C_t, h_{t+1}) = \Lambda_t$$

$$-b_t U_2(C_t - \theta C_{t-1}, h_t) = \Lambda_t \frac{W_t}{\mu_t}$$

$$Q_t\Lambda_t = \beta E_t\Lambda_{t+1}\left[R_{t+1}u_{t+1} + Q_{t+1}(1-\delta(u_{t+1}))\right]$$

$$R_t = Q_t\delta'(u_t)$$

$$\frac{\Lambda_t}{a_t X_t^a H'(I_t)} = Q_t\Lambda_t\left[1 - S\left(\frac{I_t^g}{I_{t-1}^g}\right) - \frac{I_t^g}{I_{t-1}^g}S'\left(\frac{I_t^g}{I_{t-1}^g}\right)\right] + \beta E_t Q_{t+1}\Lambda_{t+1}\left(\frac{I_{t+1}^g}{I_t^g}\right)^2 S'\left(\frac{I_{t+1}^g}{I_t^g}\right),$$

where Λ_t and $\Lambda_t Q_t$ denote the Lagrange multipliers on the budget constraint, equation (8), and on the law of motion of capital, equation (2), respectively.

3.2 Firms

Output, denoted Y_t, is produced with a homogeneous-of-degree-one production function that takes as inputs effective units of capital, $u_t K_t$, and labor services, h_t. The technology is buffeted by a transitory neutral productivity shock denoted z_t and by a permanent neutral productivity shock denoted X_t^z. Formally, the production function is given by

$$Y_t = z_t F(u_t K_t, X_t^z h_t). \tag{5}$$

We assume that the production function F takes the familiar Cobb-Douglas form

$$F(x,y) = x^\alpha y^{1-\alpha}.$$

The demand for capital and labor services are given, respectively, by

$$z_t F_1(u_t K_t, X_t^z h_t) = R_t,$$

and

$$z_t X_t^z F_2(u_t K_t, X_t^z h_t) = W_t.$$

3.3 Equilibrium

The resource constraint of the economy is given by

$$C_t + I_t + G_t = Y_t \tag{6}$$

where G_t denotes government spending financed with lump-sum taxes.

A competitive equilibrium is a set of processes C_t, h_t, I_t^g, K_{t+1}, u_t, I_t, Y_t, Λ_t, R_t, W_t, and Q_t satisfying

$$K_{t+1} = (1-\delta(u_t))K_t + I_t^g\left[1 - S\left(\frac{I_t^g}{I_{t-1}^g}\right)\right]$$

$$C_t + I_t + G_t = Y_t$$

$$Y_t = z_t F(u_t K_t, X_t^z h_t)$$

$$b_t U_1(C_t - \theta C_{t-1}, h_t) - \beta\theta E_t b_{t+1} U_1(C_{t+1} - \theta C_t, h_{t+1}) = \Lambda_t$$

$$-b_t U_2(C_t - \theta C_{t-1}, h_t) = \Lambda_t \frac{W_t}{\mu_t}$$

$$Q_t \Lambda_t = \beta E_t \Lambda_{t+1} \left[R_{t+1} u_{t+1} + Q_{t+1}(1 - \delta(u_{t+1})) \right]$$

$$R_t = Q_t \delta'(u_t)$$

$$\frac{\Lambda_t}{a_t X_t^a H'(I_t)} = Q_t \Lambda_t \left[1 - S\left(\frac{I_t^g}{I_{t-1}^g}\right) - \frac{I_t^g}{I_{t-1}^g} S'\left(\frac{I_t^g}{I_{t-1}^g}\right) \right] + \beta E_t Q_{t+1} \Lambda_{t+1} \left(\frac{I_{t+1}^g}{I_t^g}\right)^2 S'\left(\frac{I_{t+1}^g}{I_t^g}\right)$$

$$z_t F_1(u_t K_t, X_t^z h_t) = R_t$$

$$z_t X_t^z F_2(u_t K_t, X_t^z h_t) = W_t$$

$$I_t^g = a_t X_t^a H(I_t)$$

given exogenous stochastic processes a_t, X_t^a, X_t^z, z_t, μ_t, b_t, and G_t and initial conditions I_{-1}^g, K_0, and C_{-1}.

4 Theoretical Implications for Cointegration

In section 2, we presented evidence suggesting that total factor productivity and the relative price of investment are cointegrated processes. In our model economy, TFP and the price of investment are given, respectively, by

$$TFP_t = z_t (X_t^z)^{1-\alpha},$$

and

$$p_t^I = \frac{1}{a_t X_t^a \xi I_t^{\xi - 1}}.$$

Recall that z_t and a_t are stationary exogenous random variables, whereas X_t^z and X_t^a are nonstationary exogenous random variables. Along a balanced growth path with a stationary investment share in output, the trend of investment must equal the trend in output, which we denote by X_t^Y. In turn, the equilibrium trend of output along the balanced growth path can be shown to be related to the productivity trends X_t^z and X_t^a as follows:

$$X_t^Y = (X_t^z)^{\frac{1-\alpha}{1-\alpha\xi}} (X_t^a)^{\frac{\alpha}{1-\alpha\xi}}.$$

Our empirical finding that the logarithms of TFP and the price of investment are cointegrated means that there exists a scalar ζ such that

$$TFP_t^{\zeta} p_t^I$$

is a stationary process. Using the definitions of TFP and p_t^I given above and the expression for the equilibrium trend in output, it follows that this cointegration restriction implies that

$$\frac{(X_t^z)^{\zeta(1-\alpha\xi)+(1-\xi)}}{X_t^a}$$

must be stationary. This last expression states that if TFP and p_t^I are cointegrated, then X_t^z and X_t^a must themselves be cointegrated. By a similar argument, one can readily establish that the converse is also true. That is, if X_t^z and X_t^a are cointegrated, then so are TFP and the price of investment. We summarize this result in the following proposition:

Proposition 1 *In the model with equilibrium conditions given in section 3.3, TFP and the price of investment are cointegrated if and only if X_t^z and X_t^a are cointegrated.*

An alternative way to arrive at the conclusion that neutral and investment-specific productivity are driven by a common stochastic trend is to study the cointegration properties of stock prices and TFP in U.S. postwar data through the lens of a theoretical model.

There are two standard theoretical measures of the value of the stock market. One is given by the value of the stock of physical capital calculated as the product of the capital stock, K_t, and marginal Tobin's Q, Q_t. Under this measure, the value of the stock market is given by $Q_t K_t$. It can be shown that in our model, the trend in $Q_t K_t$ is the same as the trend in output, X_t^Y, which, as established earlier in this paper, is a linear combination of the trend in neutral technology, X_t^z, and the trend in investment-specific technology, X_t^a. The second standard theoretical measure of the value of the stock market is given by the value of the firm, V_t, which can be written recursively as $V_t = Y_t - W_t h_t - I_t + \beta b_t E_t \Lambda_{t+1}/\Lambda_t V_{t+1}$. That is, the value of the firm is given by the present discounted value of output net of wage payments and investment spending. It can be shown that the equilibrium trend of V_t is the same as that of output, X_t^Y, which as mentioned before is a geometric combination of the trends in neutral and investment specific productivity, X_t^z and X_t^a.

In sum, regardless of which of the two definitions of the value of the stock market one uses, its trend is given by a combination of the trends in neutral and investment specific productivity, X_t^z and X_t^a. At the same time, the trend in TFP is, as we already deduced, given by $(X_t^z)^{1-\alpha}$, and therefore depends only upon the trend in neutral productivity. It

follows from these theoretical arguments that stock prices and total factor productivity are cointegrated if and only if X_t^z and X_t^a share a common stochastic trend. We summarize this result in the following proposition:

Proposition 2 *In the model with equilibrium conditions given in section 3.3, TFP and stock prices are cointegrated if and only if X_t^z and X_t^a are cointegrated.*

The key empirical question is therefore whether stock prices and TFP appear to be I(1) cointegrated processes in postwar U.S. data. To this end, we measure stock prices by the Standard and Poor 500 index deflated by the GDP deflator and expressed in per capita terms by dividing by the population between 15 and 65 years of age. The Dickey Fuller test fails to reject the null hypothesis of a unit root at standard confidence levels in specifications with and without a linear trend. Similarly, the KPSS test rejects the hypothesis of stationarity at standard confidence levels with or without the assumption of a linear trend. Finally, the Johansen cointegration test rejects the hypothesis of no cointegration relationship between stock prices and TFP when no linear trend is included, when a linear trend is included, and when an orthogonal linear trend is included. Beaudry and Portier (2006) also find that TFP and stock prices are cointegrated time series. This evidence provides further support to the strategy of modeling productivity in the neutral and investment-specific sectors as sharing a common stochastic trend.

5 Modeling Cointegration Between Neutral and Investment-Specific Productivity

Based on the results contained in propositions 1 and 2, we impose the following cointegration relationship between X_t^z and X_t^a:

$$x_t \equiv \psi \ln(X_t^z) - \ln(X_t^a) \text{ is stationary.} \tag{7}$$

Without loss of generality, we assume that the deterministic steady-state value of x_t is zero. Let

$$\mu_t^z \equiv \frac{X_t^z}{X_{t-1}^z}$$

and

$$\mu_t^a \equiv \frac{X_t^a}{X_{t-1}^a}$$

denote, respectively, the gross growth rates of X_t^z and X_t^a. Then, we postulate the following vector error correction model (VECM) for the joint law of motion of X_t^z and X_t^a:

$$\begin{bmatrix} \ln(\mu_t^z/\mu^z) \\ \ln(\mu_t^a/\mu^a) \end{bmatrix} = \begin{bmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{bmatrix} \begin{bmatrix} \ln(\mu_{t-1}^z/\mu^z) \\ \ln(\mu_{t-1}^a/\mu^a) \end{bmatrix} + \begin{bmatrix} \kappa_1 \\ \kappa_2 \end{bmatrix} x_{t-1} + \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & D_{22} \end{bmatrix} \begin{bmatrix} \epsilon_t^1 \\ \epsilon_t^2 \end{bmatrix},$$

where the innovations to the common trend in neutral and investment-specific productivity, ϵ_t^1 and ϵ_t^2 are iid normal with mean zero and variances $\sigma_{\epsilon^1}^2$ and $\sigma_{\epsilon^2}^2$, respectively. Without loss of generality, we set $D_{11} = D_{22} = 1$ and $D_{12} = 0$.

We refer to ϵ_t^1 and ϵ_t^2 as the common shocks to neutral and investment-specific productivity. And we refer to z_t as the neutral productivity shock and to a_t as the investment-specific productivity shock. The central goal of the remainder of the paper is to ascertain the joint contribution of the common technology shocks ϵ_t^1 and ϵ_t^2 to business-cycle fluctuations and to compare it to the contributions of the sector-specific productivity shocks z_t and a_t. Our VECM formulation encompasses, as polar cases, the two most common formulations of shock dynamics in the related literature. In one of these formulations, it is assumed that both TFP and the price of investment are stationary (e.g., Smets and Wouters, 2007). This setup arises when one assumes that $\sigma_{\epsilon^1} = \sigma_{\epsilon^2} = 0$. The second case is one in which TFP and the price of investment are assumed to possess independent stochastic trends (e.g., Fisher, 2006). This case arises when $\rho_{21} = \rho_{12} = \kappa_1 = \kappa_2 = D_{21} = 0$. As discussed earlier, the key implication of the empirical analysis of section 2 is that both of these formulations are strongly rejected by the data.

In addition to the shocks to the common trend in neutral and investment-specific productivity, our model features five stationary shocks: a neutral productivity shock, z_t, an investment-specific productivity shock, a_t, a preference shock, b_t, a government spending shock, g_t, and a wage-markup shock, μ_t.

These exogenous random variables are all assumed to follow univariate AR(1) processes:

$$\ln z_t = \rho_z \ln z_{t-1} + \sigma_z \epsilon_t^z,$$

$$\ln a_t = \rho_a \ln a_{t-1} + \sigma_a \epsilon_t^a,$$

$$\ln b_t = \rho_b \ln b_{t-1} + \sigma_b \epsilon_t^b$$

$$\ln\left(\frac{g_t}{\bar{g}}\right) = \rho_g \ln\left(\frac{g_{t-1}}{\bar{g}}\right) + \sigma_g \epsilon_t^g,$$

and

$$\ln\left(\frac{\mu_t}{\bar{\mu}}\right) = \rho_\mu \ln\left(\frac{\mu_{t-1}}{\bar{\mu}}\right) + \sigma_\mu \epsilon_t^\mu.$$

The innovations ϵ_t^z, ϵ_t^a, ϵ_t^b, ϵ_t^g, and ϵ_t^μ are assumed to be i.i.d. with mean zero and standard deviations equal to one. The parameters $\bar{g}$ and $\bar{\mu}$ denote the steady-state values of g_t and μ_t, respectively. We assume that the trend component of government spending is cointegrated with that of output. Specifically, we assume that

$$G_t = g_t X_t^G$$

and that

$$X_t^G = \left(X_{t-1}^G\right)^{\rho_{xg}} \left(X_{t-1}^Y\right)^{1-\rho_{xg}},$$

where X_t^G denotes the trend component of government spending, g_t is the cyclical component of government spending, assumed to be exogenous and stochastic, and ρ_{xg} is a smoothing parameter controlling the speed of transmission of shocks to the trend in output to the trend in government spending.

6 Model Estimation

Our main goal is to ascertain the contribution to business-cycle fluctuations of the novel shock we have introduced, namely, the common stochastic trend in neutral and investment-specific productivity. In our VECM formulation this common stochastic trend is driven by two innovations, ϵ_t^1 and ϵ_t^2. We do so using the model economy developed in section 3 as the data generating process. We calibrate a subset of the structural parameters of the model and estimate the remaining parameters using maximum likelihood.

We assign a value of 2 to the preference parameter σ, a value of 0.025 to the steady-state rate of depreciation δ_0, a value of 0.99 to the subjective discount factor β, a value of 10 percent to the wage markup, a value of 20 percent to the share of government spending in output, a value of 1.0049 to the steady-state gross quarterly growth rate of output, μ^y, and a value of 0.9957 to the steady state gross quarterly growth rate of the price of investment, μ^{p^I}. We normalize the steady-state rate of capacity utilization, u, to unity. This normalization pins down the parameter δ_1. Table 4 summarizes the calibration. We estimate the remaining structural parameters of the model by maximum likelihood.

The log-linearized version of our dynamic, stochastic, general equilibrium model is of the form

$$y_t = y_t^* + \eta^{me} \epsilon_t^{me}$$

$$y_t^* = g_x x_t$$

$$x_{t+1} = h_x x_t + \eta \epsilon_{t+1},$$

Table 4: Calibrated Parameter Values

Parameter	Value
σ	2
δ_0	0.025
β	0.99
μ^y	1.0049
μ	1.10
g/y	0.20
u	1
μ^{p^I}	0.9957

where y_t is a vector containing four observables, namely, the growth rates of output, consumption, and investment, and the logarithm of hours all expressed in deviations from their means. The variables in y_t are measured with error. The true values, contained in the vector y_t^*, are unobservable. The measurement errors are captured by the term $\eta^{me}\epsilon_t^{me}$, where ϵ_t^{me} is a normally distributed i.i.d. vector of order four with mean zero and variance/covariance matrix equal to the identity matrix. The matrix η^{me} is diagonal and of order 4 by 4. We estimate the elements of its diagonal. We constrain the measurement errors to capture at most 25 percent of the variance of the corresponding variables. The vector x_t is unobservable and contains the states of the system. The matrices g_x and h_x are functions of the calibrated and estimated structural parameters of the model. The structural disturbances are collected in the 7-by-1 vector ϵ_t, which distributes normally with mean zero and variance/covariance matrix equal to the identity matrix. The matrix η is a function of the standard deviations σ_i for $i = 1, 2, a, z, b, \mu, g$.

We estimate by maximum likelihood the parameters defining the exogenous processes driving business cycles in our model along with other structural parameters. Specifically, we estimate the parameters defining the VECM model for the evolution of the cointegrated trends X_t^z and X_t^a, which are ρ_{ij}, κ_i, and σ_{ϵ^i} for $i, j = 1, 2$ and D_{21}. We also estimate the standard deviations and serial correlations of the AR(1) processes governing the laws of motion of the remaining five exogenous shocks. We further estimate two preference parameters, θ and γ, four technology parameters, α, ξ, κ, and δ_2, and the smoothing parameter for government purchases, ρ_{xg}.

We estimate the model on U.S. quarterly data ranging from 1948:Q1 to 2006:Q4. Table 5 displays estimated parameter values. The table also displays standard errors of the estimated parameters. These standard errors are computed using the method proposed by Chernozhukov and Hong (2003).

To our knowledge, this paper presents the first attempt to estimate using full information

Table 5: Maximum-Likelihood Estimates of Model Parameters

Parameter	ML Estimate	Standard Error
ξ	1.00	0.06
θ	0.31	0.07
κ	2.46	0.78
δ_2	0.11	0.26
ϵ_{hw}	1.85	0.10
α	0.37	0.03
ρ_{xg}	0.81	0.14
ρ_z	0.99	0.25
ρ_a	0.93	0.28
ρ_g	0.98	0.06
ρ_b	0.00	0.25
ρ_μ	0.97	0.01
ρ_{11}	0.13	0.22
ρ_{12}	0.08	0.10
ρ_{21}	1.07	0.70
ρ_{22}	0.58	0.22
κ_1	0.03	0.08
κ_2	0.39	0.28
D_{21}	0.87	2.54
σ_z	0.00	0.10
σ_a	1.11	0.41
σ_g	1.29	0.22
σ_b	0.92	0.94
σ_μ	0.64	0.07
σ_{ϵ^1}	0.37	0.14
σ_{ϵ^2}	3.20	0.77
σ_{gy}^{me}	0.50	0.01
σ_{gc}^{me}	0.27	0.04
σ_{gi}^{me}	1.26	0.07
σ_{h}^{me}	0.00	0.03

Note: The sample period is 1948:Q1 to 2006:Q4.The observables are the growth rates of output, consumption, and investment, and the logarithm of hours. Each of the observables is assumed to be measured with error. Standard errors are computed using the method proposed by Chernozhukov and Hong (2003). The symbol ϵ_{hw} stands for the implied Frisch elasticity of labor supply in the absence of habit formation.

Table 6: Predicted and Empirical Moments

	g^Y	g^C	g^I	h
Standard Deviations				
Model	0.98	0.56	2.45	3.65
Data	1.00	0.55	2.52	3.97
Correlation with Output Growth				
Model		0.41	0.67	0.11
Data		0.48	0.66	0.04
Serial Correlation				
Model	0.40	0.19	0.41	0.97
Data	0.33	0.11	0.48	0.97

Note: g^i, for $i = Y, C, I$ denotes the growth rates of output, consumption, and investment, respectively, and h denotes the logarithmic deviation of hours worked from its mean. Standard deviations are expressed in percentage points.

methods and in the context of a DSGE model the curvature of the technology transforming consumption goods into investment goods, embodied in the parameter ξ. Our estimate shows that this production function appears to be linear ($\xi = 1$). This implies that the relative price of investment coincides with the inverse of investment-specific productivity, an assumption maintained in much of the existing literature on investment-specific technology shocks, and one that is often criticized for not being based on econometric evidence. The variances of the measurement errors for the growth rates of output, consumption, and investment attain their maximum allowed values of 25 percent of the variances of the corresponding time series, whereas the estimated measurement error in hours is nil.

6.1 Model Fit

Table 6 presents predicted and observed second moments of output growth, consumption growth, investment growth, and hours. The model fits the data quite well along all dimensions considered in the table. It correctly predicts the volatility ranking hours, investment, output, consumption. The model also captures the procyclicality of consumption and investment. Finally, as shown in the bottom panel of table 6, the model satisfactorily replicates the serial correlations of output growth, consumption growth, investment growth, and hours.

Table 7: Variance Decomposition

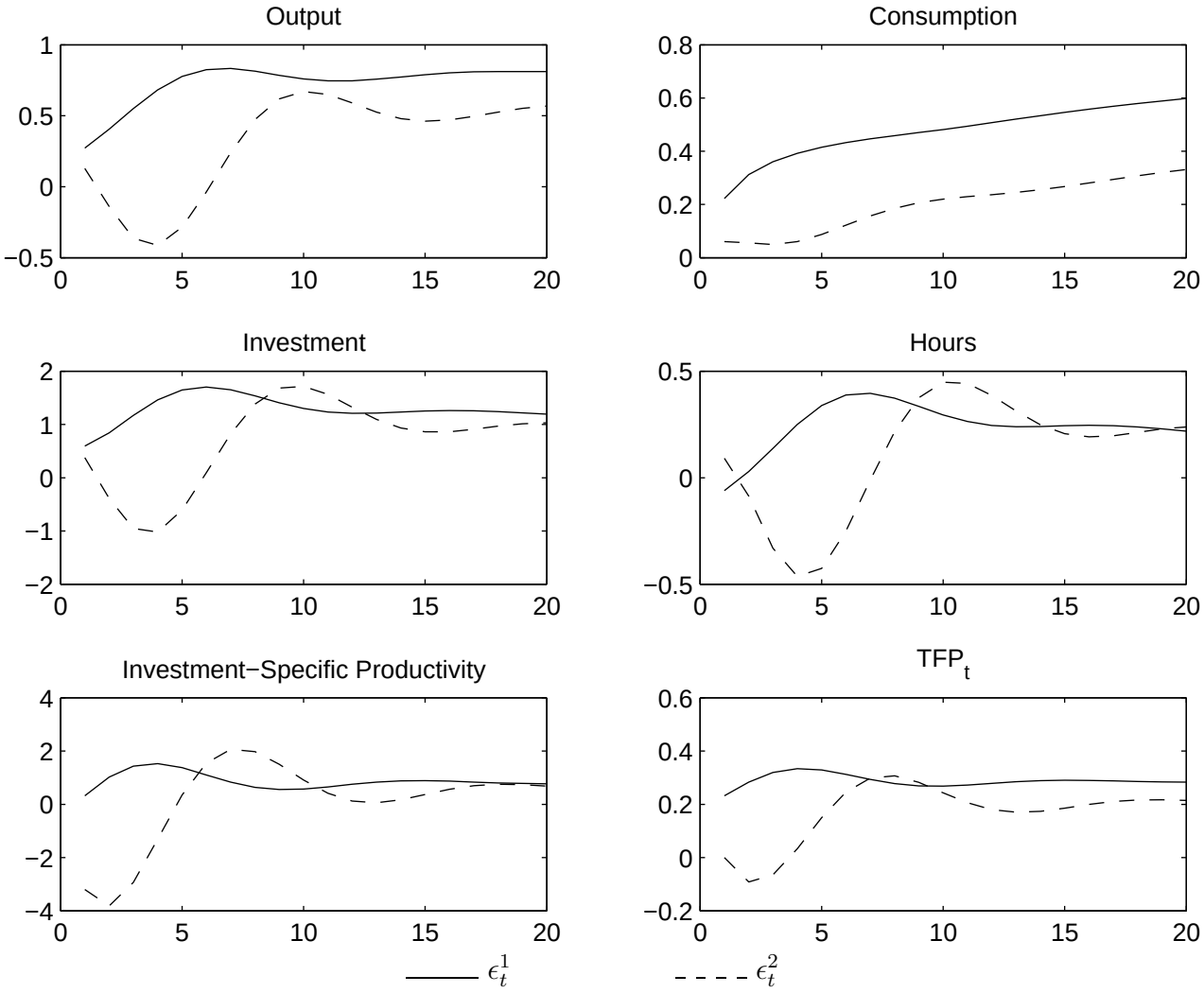
Type of Shock	g^Y	g^C	g^I	h
Common Stochastic Trend	0.75	0.33	0.80	0.35
Stationary TFP Shock	0.00	0.00	0.00	0.00
Stationary ISP Shock	0.07	0.01	0.16	0.07
Wage-Markup Shock	0.10	0.45	0.03	0.42
Preference Shock	0.01	0.19	0.01	0.00
Government Spending Shock	0.07	0.01	0.00	0.17

7 Common-Trend Shocks and Business Cycles

How important is the common stochastic trend in neutral and investment-specific productivity identified in this paper in generating business cycles? To address this question, we compute the variance decomposition of key macroeconomic variables implied by our estimated DSGE model. Table 7 displays the shares of the predicted variances of output growth, consumption growth, investment growth and hours attributable to the common stochastic trend, i.e., to ϵ_t^1 and ϵ_t^2 jointly, and to each of the remaining five driving forces included in the model. Shocks to the common trend emerge as the main drivers of business cycles in our estimated model. They explain about three fourth of the variances of output and investment growth and about one third of the predicted variances of consumption growth and the level of hours. This result is remarkable because in the existing literature, which has not allowed for the possibility of a common stochastic trend in neutral and investment-specific productivity, the lion's share of business-cycle fluctuations is typically allocated to shifts in either neutral or investment-specific productivity. In our model, by contrast, these two sources of uncertainty explain each a negligible fraction of the predicted movements in output growth, consumption growth, investment growth, and hours.

To gauge the pattern of comovement induced by the common-trend shocks, figure 1 displays the impulse response functions to one-standard-error innovations in the common-trend shocks ϵ_t^1 and ϵ_t^2. The common-trend shock ϵ_t^1, shown with a solid line, generates an expansion in output, consumption, and investment. Hours initially contract due to the wealth effect caused by the increase in productivity. After the initial contraction, hours display a hump-shaped response, expanding gradually at first, reaching a peak response six quarters after the innovation and then declining slowly toward their steady-state value. The initial contraction in hours is in line with a number of empirical studies that use Blanchard-Quah-type methods to identify the effect of permanent shifts in productivity. The negative response of hours to permanent shifts in productivity documented by empirical studies has

Figure 1: Impulse Responses to a One-Standard-Error Innovation in the Common-Trend Shocks ϵ_t^1 and ϵ_t^2



Note. Output, consumption, investment, neutral productivity, and investment-specific productivity are measured as percent deviations from their respective balanced-growth paths. Hours are measured as percentage deviations from steady state.

been interpreted by some as being at odds with the predictions of the RBC model. Figure 1 shows that such interpretation is misplaced. In fact, one can state more generally that a weak or negative response of hours to productivity shocks that are either stationary and highly persistent or nonstationary is a quite robust prediction of the RBC model.

The second common-trend shock, ϵ_t^2, whose impulse responses are shown with broken line in figure 1, possesses the characteristics of a technological diffusion. It generates an increase in both neutral and investment-specific productivity in the long run, but a decline in both types of productivity during the initial transition. The response of productivity resembles the adoption of new technologies that, by replacing old, well-established ones, induces a temporary slump in productivity. This diffused response of productivity translates into an initial contraction in output, investment and employment. All of these variables later recover and by period ten the economy is booming. Forward-looking consumers understand that the initial slump is merely a prelude to permanently higher output and as a consequence choose to increase spending from the outset. Our estimates show that the diffused, permanent productivity shock ϵ_t^2 is a key driver of business cycles; it alone explains more than half of the predicted variations in output and investment.

8 Sensitivity Analysis

In this section we analyze the robustness of our results to three modifications of the baseline setup. The first robustness check analyzes the role of measurement errors. Our rationale for including measurement error is twofold. First, our theoretical model is of a closed economy. The data we confront the model with, however, is taken from an open economy. As a result, domestic absorption necessarily equals output in the model but not in the data. The introduction of measurement error is meant to partially bridge this theoretical gap. Second, it is widely accepted that NIPA data is not free of measurement error, particularly in the time series for private spending. We begin by addressing the question of whether the importance of common-trend shocks is sensitive to the fact that the variances of measurement errors are capped at 25 percent of the variance of the associated observables. To this end, we reestimate a version of the model in which measurement errors are unrestricted. We find that in this case, measurement error explains 30 percent of the variance of output growth, 25 percent of the variance of consumption growth, 33 percent of the variance of investment growth, and virtually nothing of the variance of hours. These figures are not too different from those that emerge from our baseline estimation. In fact, a likelihood-ratio test of the null hypothesis that the variance of measurement errors are no greater than 25 percent of the variance of the associated observables cannot be rejected in favor of the alternative of

unrestricted measurement errors. Moreover, the unrestricted estimation assigns an even greater importance to shocks to the common productivity trend than does the restricted estimation that caps measurement error at 25 percent. Specifically, in the unrestricted case, ϵ_t^1 and ϵ_t^2 explain jointly 86 percent of the predicted variance of output growth, 36 percent of consumption growth, 93 percent of investment growth, and 48 percent of hours.

An alternative setup is one in which a theoretical model that does not allow for measurement error is forced to explain data that are strongly suspected of suffering from some amount of measurement error. We address this issue by testing the null hypothesis that measurement errors are nil in all variables against the alternative hypothesis that measurement errors are unrestricted. We find that the null hypothesis of no measurement error is strongly rejected by the data. The associated likelihood-ratio test has a p-value of about virtually 0. Even in this case, the common-trend shocks are estimated to be important drivers of the business cycle, explaining 42 percent of output growth, 32 percent of consumption growth, and 50 percent of investment growth.

A further sensitivity exercise consists in extending the sample to include the great recession of 2008. Estimating the model over the period 1948:1 to 2010:1 delivers results that are consistent with our baseline estimates. Specifically, over the longer sample the common-trend shocks explain 83 percent of the variance of output growth, 33 percent of the variance of consumption growth, 97 percent of the variance of investment growth, and 39 percent of the variance of hours. The most salient feature of the great-recession years is the unprecedented collapse in hours worked. Specifically, between 2006:4 and 2010:1, actual hours worked per capita fell by almost 15 percent, reaching the lowest level in the entire sample. There is an ongoing literature aimed at identifying the sources of the great contraction of 2008. This literature has not yet arrived at a definite conclusion regarding the type of shock that is responsible for it. There is relative consensus, however, in that problems originated in the financial sector had much to do with the downturn. Because our theoretical model abstracts from both financial frictions and financial disturbances, we hesitate to include the last three years in our baseline sample.

We have pointed out elsewhere (Schmitt-Grohé and Uribe, 2008) that DSGE models that include an investment-specific productivity shock but do not include the price of investment in the set of observables tend to deliver business cycles in which the predicted ratio of the standard deviation of the growth rate of the price of investment to the standard deviation of the growth rate of TFP is significantly higher than its empirical counterpart. This problem also arises in the present study. As a partial remedy, we perform a constrained maximum-likelihood estimation in which we restrict the predicted ratio of the standard deviations of the price of investment and TFP to be at most 0.65. In the data, this ratio has a mean of

0.55 and a standard deviation of 0.05. So our upper bound of 0.65 is the sum of the mean ratio of standard deviations and two standard deviations of the ratio. Under the constrained estimate, the common-trend shocks continue to explain the bulk of movements in output, consumption, and investment. However, the fit of the model deteriorates when the constraint is imposed.

9 Conclusion

In this paper we identify a new source of business cycles. It takes the form of a common stochastic trend in neutral and investment-specific productivity. We identify this novel source of aggregate fluctuations by means of three empirical facts whose joint theoretical implications have been overlooked in the large related literature. Namely, the facts that TFP contains a unit root, that the relative price of investment contains a unit root, and that TFP and the price of investment appear to be cointegrated processes. A key insight of this paper is that these three facts can be theoretically reconciled if and only if neutral and investment-specific productivity are assumed to be cointegrated processes.

The second contribution of our investigation is to quantify the importance of shocks to the common stochastic trend in driving aggregate fluctuations in postwar U.S. data. To this end, we perform a maximum-likelihood estimation of a mainstream dynamic stochastic general equilibrium model featuring a number of real rigidities and driving forces. A novel element of our model is that the laws of motion of the permanent components of neutral and investment-specific productivity are modeled as a bivariate vector error correction model, thereby allowing for cointegration. Our estimated DSGE models implies that the common stochastic trend in neutral and investment-specific productivity is a non negligible driver of short-run fluctuations. We regard these estimates as a first pass at evaluating the role of a new source of business cycles. It remains for future research to establish the sensitivity of this result to model specification and to the set of observables used in estimation.

We conclude by stressing that the central insight of this paper, namely that neutral and investment-specific productivity share a common stochastic trend has potentially an important implication for the ongoing debate on the relative importance of neutral and investment-specific productivity as drivers of business cycles. Viewed through the lens of our analysis, the question of whether business cycles are driven by neutral or investment-specific productivity shocks is ill posed. For it fails to contemplate the possibility that business cycles are driven in part by a common productivity component.

Appendix

Modeling An Exogenous Wage Markup

We introduce a variable wage markup by assuming the existence of a union that sells differentiated labor services monopolistically. The household receives total labor compensation in the amount $W_t h_t$, where W_t denotes the real wage rate paid by firms to the union. However, in deciding the number of hours worked the household faces a marginal wage rate equal to $\widetilde{W}_t$, which the household takes as exogenously given. The difference between the average and the marginal labor income, given by $\Phi_t^U \equiv (W_t - \widetilde{W}_t)h_t$, is a monopoly rent, which the labor union rebates to households in a lump-sum fashion. Households take Φ_t^U as exogenously given. In practice, workers pertaining to a labor union are offered a contract specifying a wage rate and a fixed number of hours of work. This scheme is equivalent to the theoretical contract we propose, which specifies a wage rate, $\widetilde{W}_t$, a lump-sum profit, and the ability to choose the number of hours of work. The budget constraint of the household is then given by

$$C_t + I_t = \widetilde{W}_t h_t + R_t u_t K_t + \Phi_t^U + \Phi_t^F, \tag{8}$$

where R_t denotes the rental rate of capital and Φ_t^F denotes profits received from the ownership of firms.

Labor services hired by firms, denoted h_t^d, are a composite of differentiated types of labor inputs, aggregated by means of the following function

$$h_t^d = \left[\int_0^1 h_{jt}^{\frac{1}{\mu_t}} dj\right]^{\mu_t},$$

where h_{jt} denotes labor services of type j used in period t, and μ_t is an exogenous and stochastic variable. The intratemporal elasticity of substitution between differentiated types of labor input is given by $\mu_t/(\mu_t - 1)$. As will be clear shortly, μ_t can be interpreted as the equilibrium gross markup charged by a labor union. Given a desired level of the composite labor input, h_t^d, the firm chooses h_{jt} for all $j \in [0, 1]$ by minimizing the total cost of labor, given by $\int_0^1 W_{jt} h_{jt} dj$, subject to the constraint imposed by the above aggregator technology, where W_{jt} denotes the wage paid to labor input of type j. The implied demand for labor of type j is given by

$$h_{jt} = \left(\frac{W_{jt}}{W_t}\right)^{-\frac{\mu_t}{\mu_t - 1}} h_t^d, \tag{9}$$

where $W_t \equiv \left[\int_0^1 W_{jt}^{-\frac{1}{\mu_t - 1}} dj\right]^{-(\mu_t - 1)}$ denotes the cost of one unit of the labor composite. That

is, at the optimum, $\int_0^1 W_{jt} h_{jt} dj = W_t h_t^d$.

The wage rate for labor services of type j, W_{jt}, is set by a labor union that is the monopolistic supplier of this type of labor. The union maximizes profits, which are given by $(W_{jt} - \widetilde{W}_t)h_{jt}$, subject to the demand for labor, equation (9). The optimality condition associated with this problem is

$$W_{jt} = \mu_t \widetilde{W}_t.$$

It is clear from this expression that μ_t represents a wage markup.

In a symmetric equilibrium, we have that $h_{jt} = h_t^d = h_t$ and $W_{jt} = W_t$ for all j. Therefore, we can drop the indices j and d.

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