REAL ASPECTS OF EXCHANGE RATE REGIME
CHOICE WITH COLLAPSING FIXED RATES

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ABSTRACT

Typical evaluations of the choice of exchange rate regime employ a
criterion function that depends on the real performance of the economy, and
they focus on regimes that are expected to last indefinitely. This latter
feature is strongly contradicted by the transitory nature of actual regimes.
This paper extends the recent literature on collapses of fixed exchange rate
regimes with exogenous real sectors to examine how the predictions of two
popular models for the determination of some real economic variables must be
modified when agents rationally perceive that the fixed rate regime will be
transitory. The models studied are simple stochastic versions of the models
in Dornbusch (1976) and Flood and Marion (1982).

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Typical evaluations of the performance of an economy under alternative
exchange rate regimes proceed under the assumption that exchange-rate regimes
last forever.[1] This assumption is a gross contradiction of the fact that
exchange rate regimes are actually quite transitory. Countries never really
followed the "rules of the game" under the Gold Standard, and it evolved into
the Gold Exchange Standard. During the Great Depression currencies became
inconvertible. The Bretton Woods system was planned as a system of fixed
exchange rates, but it was recognized that countries would devalue and revalue
their currencies when in "fundamental disequilbrium". Although this term was
never formally defined, countries did devalue and revalue their currencies,
often by large amounts, during the two decades preceding the breakdown of the
system in 1971.

The purpose of this paper is to reexamine the determination of real
output in two popular models of the open economy accounting explicitly for the
transitory nature of fixed exchange rate regimes. Rational agents understand
that an exchange-rate regime may be temporary, and they incorporate
expectations of the collapse of the regime and its associated capital gains or
losses into their behavioral functions. The models we examine are simple
stochastic versions of the Dornbush (1976) model of exchange rate dynamics
with flexible output and the Flood and Marion (1982) model of wage indexation
in an open economy. We work with simplified versions of these models since
our point is that some of the implications of the models are drastically
altered when we allow agents to act on their understanding of regime
impermanence. These alterations are robust to more complex versions of the
models, but since we are presenting some counterexamples, model complexity
only obscures our points. Our work builds on previous research on the
temporary nature of government policies including Salant and Henderson (1978),
Krugman (1979), Flood and Garber (1983, 1984) and Flood and Marion (1983).

In Section 2 of the paper we develop our version of the Dornbusch model. For some configurations of the model, we demonstrate that the regime of permanently floating exchange rates always produces a higher unconditional variance of output than does a regime of permanently fixed exchange rates. This result is confirmed when we consider a temporary fixed-rate regime, although we find the disadvantage of a floating-rate regime diminishes as the possibility of an attack on a fixed rate regime rises. This result is an unsurprising extension of the original comparison between fixed and flexible exchange rates. What is surprising to us, however, is that output variance conditional on maintaining a fixed-rate regime which may collapse can be higher than output variance under floating rates. Intuitively, this measure of the conditional variance of real output corresponds to the sample or measured output variance during a fixed exchange rate regime that has not collapsed.

Our examination of the wage indexing model is presented in Section 3. It is well known that optimal wage indexation depends on the stochastic structure of the underlying economy, and a contribution of Flood and Marion (1982) was to demonstrate that the degree of wage indexation would not be invariant to the country's choice of exchange-rate regime. In their analysis, different permanent exchange rate regimes lead to different optimal wage indexing polices, and each policy is a fixed function of the time invariant stochastic structure of the economy. In this paper we discuss fixed exchange rate regimes which may collapse, and we find that the optimal degree of wage indexation is state dependent and thus time varying even though the stochastic structure of the economy is time invariant.

Our results are presented in the next two sections. For each type of model we first examine permanent exchange rate regimes turning then to a characterization of the economy when a fixed exchange rate regime may possibly collapse. In each model we take the domestic credit component of the money supply to be an exogenous process. This creates an inherent tension between the two government policies, the domestic credit process and the fixed exchange rate regime, which is resolved by the collapse of the regime. This appears to capture the actual priorities of governments without modeling the objective function of the monetary and fiscal authorities. Presumably, one reason why the domestic credit process has priority is because of the seignorage gained by the government from printing money.[2]

Because algebraic complexity quickly renders complex versions of each type of model analytically intractable, we worked with simple stripped down versions of each type of model while attempting to retain the essential economic aspects of the problems. Some of the derivations of various results are presented in a techical appendix to facilitate presentation of the results.

2A. A DORNBUSCH-TYPE MODEL

The original Dornbusch (1976) model depicted a medium-size open economy that was large in the market for goods produced at home and small in world asset markets. The crucial feature of the model was that domestic currency prices of domestic goods were predetermined while the country's exchange rate and other asset prices were currently determined and free to respond to all current shocks. This feature gave rise to the famous overshooting result. Since home goods prices were predetermined, the immediate response of the exchange rate to a money-market disturbance had to be larger than that response would have been had all prices been free to adjust. We use a version of the model that makes output demand determined. It consists of the following relations:

Glossary of Variables for Model I

m_t = logarithm of the money supply

h_t = logarithm of the domestic currency price of the domestic good

i_t = level of the domestic interest rate

i_t^* = level of the foreign interest rate

s_t = logarithm of the spot exchange rate

d_t = logarithm of the demand for domestic output

h_t^* = logarithm of the foreign currency price of the foreign good

b_t = logarithm of domestic credit

r_t = logarithm of international reserves

$\bar{y}$ = logarithm of the natural level of domestic output

u_t = white noise goods market disturbance with variance σ_u^2

v_t = white noise money supply disturbance with variance σ_v^2

Equations of Model I

Money Market Equilibrium

$$m_t - h_t = -\alpha i_t + \gamma d_t, \quad \alpha > 0, \gamma \geqslant 0 \qquad (1)$$

Capital Market Equilbrium

$$i_t = i_t^* + E_t s_{t+1} - s_t \qquad (2)$$

Goods Market Demand

$$d_t = \beta(h_t^* + s_t - h_t) + u_t \qquad (3)$$

Domestic Price Determination

$$\beta(h_t^* + E_{t-1}s_t - h_t) + E_{t-1}u_t = \bar{y} \tag{4}$$

Money Supply Definition

$$m_t = \omega b_t + (1-\omega)r_t, \quad 0 < \omega < 1 \tag{5}$$

Domestic Credit Process

$$b_t = \mu + b_{t-1} + v_t \tag{6}$$

Equation (1) is money market equilibrium. The logarithm of the real money

supply, $m_t - h_t$, equals the demand for real money balances, $- \alpha i_t + \gamma d_t$.[3]

The demand for money depends negatively on the opportunity cost of holding

money, the nominal interest rate, and it depends positively on the real demand

for home goods, d_t. The equilibrium condition for the world capital market is

given in equation (2) by the uncovered interest rate parity condition. The

demand for domestic output is specified in equation (3). Demand for home

goods depends negatively on the logarithm of the relative price of home goods

in terms of foreign goods, $h_t - (h_t^* + s_t)$, and u_t is a stochastic demand

disturbance. Equation (4) states that h_t is set at time t-1 at the level that

is expected to clear the market for domestic goods by equating expected demand

to full employment output, $\bar{y}$. Actual output in period t is determined by the

demand for it given the predetermined variables h_t and h_t^*. The composition of

the money supply is given by equation (5). The nominal money supply is

composed of a domestic credit component and an international reserves

component. Equation (6) states that the rate of growth of domestic credit,

$b_t - b_{t-1}$, is a constant μ plus a stochastic term, v_t.

$E_{t-j}x_t$ is the mathematical expectation of x_t conditional on information from date $t - j$, $j = 0, 1$. It is assumed that u_t and v_t are mutually and serially uncorrelated with $E_{t-1}u_t = E_{t-1}v_t = 0$ and with constant variances σ_u^2 and σ_v^2. The foreign price and foreign interest rate are also exogenous stochastic processes. We assume that $h_t^* = h_{t-1}^* + z_{t-1}^*$ where z_{t-1}^* is serially uncorrelated and uncorrelated with the model's other disturbances. Its variance is σ_{z*}^2, and it is an element of the time $t-1$ information set. This dating convention recognizes that foreign price setters are also setting prices for period t at time $t-1$. The foreign nominal interest rate, $i_t^* = i^* + x_t^*$ is assumed to be serially uncorrelated with mean i^*. The innovation in the foreign interest rate, x_t^*, is assumed to be orthogonal to the other stochastic processes and to have a constant variance σ_{x*}^2.

Our comparison of alternative exchange rate regimes focuses on the variability of real output. For the permanent regimes we measure variability with the conditional variance $V_{t-1}(y_t) = E_{t-1}(y_t - E_{t-1}y_t)^2$. For the collapsing fixed rate regime we use this measure as well as an alternative. In all regimes $\bar{y} = E_{t-1}d_t$, and in order to simplify the presentation, we normalized $\bar{y} = E_{t-1}d_t = 0$. Consequently, the conditional variance of output is

$$E_{t-1}(d_t^2) = \beta^2 V_{t-1}(s_t) + \sigma_u^2 + 2\beta C_{t-1}(s_t;u_t) \qquad (7)$$

where $C_{t-1}(\cdot;\cdot)$ denotes the conditional covariance operator.

2B. SOLUTIONS FOR PERMANENT EXCHANGE RATE REGIMES

When exchange rates are perfectly flexible, there is no intervention in the foreign exchange market implying that r_t is the constant $\bar{r}$. A solution for the exchange rate can be found by recognizing that the variables on the right-hand side of (8) completely describe the current state of the system. Consequently, the linearity of the model allows us to apply the method of undetermined coefficients, and we find

$$s_t = \lambda_0 + \lambda_1 h_t^* + \lambda_2 b_{t-1} + \lambda_3 x_t^* + \lambda_4 z_t^* + \lambda_5 v_t + \lambda_6 u_t \tag{8}$$

where

$$\lambda_0 = (1 + \alpha)\omega\mu + (1 - \omega)\bar{r} + \alpha i^*$$

$$\lambda_1 = -1$$

$$\lambda_2 = \omega$$

$$\lambda_3 = \alpha/(\alpha + \gamma\beta)$$

$$\lambda_4 = -\alpha/(\alpha + \gamma\beta)$$

$$\lambda_5 = (1+\alpha)\omega/(\alpha + \gamma\beta)$$

$$\lambda_6 = -\gamma/(\alpha + \gamma\beta).$$

From (7) and (8), the conditional variance of real output under a pure flexible exchange rate regime is

$$V_{t-1}(y_t)|_{Flex} = \beta^2(\lambda_3^2\sigma_{x*}^2 + \lambda_4^2\sigma_{z*}^2 + \lambda_5^2\sigma_v^2 + \lambda_6^2\sigma_u^2) + \sigma_u^2 + 2\beta\lambda_6\sigma_u^2. \quad (9)$$

If the exchange rate is permanently fixed at a level $\bar{s}$ and the government runs an exogenous domestic credit process, then r_t becomes endogenous. International reserves adjust during the period to equate the demand for money and the supply of money. Under a permanently fixed exchange rate regime, $s_t = \bar{s}$ for all t, and the conditional variance of output is simply

$$V_{t-1}(y_t)|_{Fix} = \sigma_u^2. \quad (10)$$

By comparing (9) and (10), one sees that the conditional variance of real output under a flexible exchange rate regime exceeds the conditional variance of real output under a fixed rate regime whenever

$$\beta\left[\alpha^2(\sigma_{x*}^2 + \sigma_{z*}^2) + (1+\alpha)^2\omega^2\sigma_v^2\right] > (2\alpha + \gamma\beta)\gamma\sigma_u^2. \quad (11)$$

Because the flexible exchange rate regime allows foreign price and interest rate disturbances and the domestic money supply disturbance to affect the exchange rate thereby altering the terms of trade, these disturbances add additional variance to the demand for and supply of real output under such a regime. A counterbalancing effect that tends to reduce the volatility of real output under flexible rates when compared to fixed rates is that positive real demand shocks appreciate the domestic currency thereby improving the terms of trade, and this change in relative prices reduces the demand for real output. The appreciation of the currency arises because the increase in real

demand increases the demand for money. The smaller is the elasticity of the demand for money with respect to real output, γ, the more likely is the volatility of real output to be greater under flexible exchange rates than under fixed rates. In order to provide a striking comparison in what follows, we assume that $\gamma = 0$. With this change, permanently flexible rates produce an unambiguously larger volatility of real output than permanently fixed exchange rates.

2C. SOLUTION FOR A TEMPORARY FIXED EXCHANGE RATE REGIME

Our fixed exchange rate solution from the previous section ignored the possibility that the economy may evolve into a state where it is worthwhile for agents to attack the government's foreign exchange reserves and end the fixed rate regime.[4] We assume that the monetary authority will defend the fixed rate regime only until its reserves hit some known lower bound, $\tilde{r}$. After reserves hit this bound, the monetary authority is assumed to abandon the fixed-rate regime forever. If agents successfully attack the fixed-rate regime, they produce the floating rate whose level is given by

$$\tilde{s}_t = \tilde{\lambda}_0 + \lambda_1 h_t^* + \lambda_2 b_{t-1} + \lambda_3 x_t^* + \lambda_4 z_t^* + \lambda_5 v_t \qquad (12)$$

where $\tilde{\lambda}_0 \equiv [(1-\omega)\tilde{r} + \alpha i^* + (1+\alpha)\omega\mu]$, and λ_1 through λ_5 are as above except with $\gamma = 0$. If $\tilde{s}_t > \bar{s}$, risk-free profits can be had at the expense of the monetary authority. If $\tilde{s}_t < \bar{s}$, attackers would make losses and consequently would not attack. If an attack took place when $\tilde{s}_t = \bar{s}$, no profits would be realized, and the post-attack exchange rate would be $\bar{s}$. Thus, we treat this case as part of the no attack situation without loss of generality, and an attack takes place if and only if $\tilde{s}_t > \bar{s}$.[5]

At time t-1, when setting nominal prices for period t, agents realize that the fixed exchange rate regime could collapse next period. Therefore, the conditional expectation of the exchange rate in period t is not simply $\bar{s}$, but it is the probability weighted average of the conditional expectations under the two possible regimes:

$$E_{t-1}s_t = (1 - \pi_{t-1})\bar{s} + \pi_{t-1}E_{t-1}(\tilde{s}_t|C_t), \tag{13}$$

where π_{t-1} is the probability at time t-1 of an attack at time t and $E_{t-1}(\tilde{s}_t|C_t)$ is the expectation at t-1 of $\tilde{s}_t$ conditional on a collapse at time t.

The probability of an attack next period, is $\pi_{t-1} = pr_{t-1}\{\tilde{s}_t - \bar{s} > 0\}$ where $pr_{t-1}\{x\}$ is the probability of event x given the t-1 information set. Examination of the solution for the flexible exchange rate in (12) indicates that the probability of an attack can be stated as $\pi_{t-1} = pr_{t-1}\{\varepsilon_t > \delta_{t-1}\}$ where $\varepsilon_t = \lambda_3 x^*_t + \lambda_4 z^*_t + \lambda_5 v_t$ is the innovation in the shadow flexible exchange rate that becomes the actual flexible rate if there is a collapse of the fixed rate regime at time t, and $\delta_{t-1} = \bar{s} - \tilde{\lambda}_0 - \lambda_1 h^*_t - \lambda_2 b_{t-1}$ is the amount by which the fixed exchange rate exceeds the conditional expectation of $\tilde{s}_t$ based on time t-1 information. As δ_{t-1} falls, the probability of collapse increases monotonically since a larger part of the distribution of ε_t can induce a collapse.

The volatility of real output under a fixed exchange rate regime that may collapse is

$$V_{t-1}(y_t)|_{C.Fix} = \beta^2 E_{t-1}(s_t - E_{t-1}s_t)^2 + \sigma^2_u \tag{14}$$

which is clearly larger than $V_{t-1}(y_t)|_{Fix}$. It is also true that $V_{t-1}(y_t)|_{C.Fix}$ is less than $V_{t-1}(y_t)|_{Flex}$. In order to determine this result, consider the following decomposition of the conditional variance of the exchange rate under the two regimes:

$$V_{t-1}(s_t)|_{C.Fix} = (1-\pi_{t-1})\bar{s}^2 + \pi_{t-1}E_{t-1}(\tilde{s}_t^2|C_t)$$

$$- [(1-\pi_{t-1})\bar{s} + \pi_{t-1}E_{t-1}(\tilde{s}_t|C_t)]^2 \tag{15a}$$

$$V_{t-1}(s_t)|_{Flex} = (1-\pi_{t-1})E_{t-1}(\tilde{s}_t^2|NC_t) + \pi_{t-1}E_{t-1}(\tilde{s}_t^2|C_t)$$

$$- [(1-\pi_{t-1})E_{t-1}(\tilde{s}_t|NC_t) + \pi_{t-1}E_{t-1}(\tilde{s}_t|C_t)]^2 \tag{15b}$$

Expressions (15a) and (15b) utilize the facts that the unconditional moment of a distribution is the probability weighted sum of the conditional moments and that the exchange rate does not collapse (NC_t) when $\varepsilon_t < \delta_{t-1}$. In the Appendix we demonstrate that subtracting (15a) from (15b) and rearranging terms gives

$$(1 - \pi_{t-1})V_{t-1}(s_t|NC_t) + \pi_{t-1}(1 - \pi_{t-1})\{[\bar{s} - E_{t-1}(\tilde{s}_t|NC_t)]^2 +$$

$$2[\bar{s} - E_{t-1}(\tilde{s}_t|NC_t)][E_{t-1}(\tilde{s}_t|C_t)-\bar{s}]\} \tag{16}$$

which is unambiguously positive since $E_{t-1}(\tilde{s}_t|C_t) > \bar{s} > E_{t-1}(\tilde{s}_t|NC_t)$. This demonstrates that a regime of collapsing fixed rates has a more volatile real

output than a regime of permanently fixed rates, but the volatility is not larger than under a pure flexible rate regime.

These results are not particularly surprising since as δ_{t-1} increases, the probability of an attack decreases, and the conditional variance of output approaches that under a fixed exchange rate regime. Similarly, as δ_{t-1} decreases, the probability of an attack increases, and the conditional variance of real output approaches that under a flexible exchange rate regime. For intermediate values of δ_{t-1}, the conditional variance is between the two extreme regimes.

The conditional variance used in the above comparisons takes into account the forecast errors that arise from both parts of the distributions of ε_t which we have denoted C_t ($\varepsilon_t > \delta_{t-1}$) and NC_t ($\varepsilon_t < \delta_{t-1}$). An alternative way to examine the volatility of output under a potentially collapsing fixed exchange rate regime is to estimate the variance of output by examining only the realizations of output under a regime that could collapse but had not collapsed during the sample. This corresponds to the sample variance that one would measure by using all the data during a regime of fixed rates up to the point of a collapse.

The volatility of real output in this case is denoted

$$V_{t-1}(y_t)\big|_{C.Fix.2} = \beta^2(\bar{s} - E_{t-1}s_t)^2 + \sigma_u^2. \tag{17}$$

In order to conduct an analytical comparison of (17) and (15b), it is ne essary to consider an actual probability distribution for ε_t. Analytical tractablity also rules out obvious candidates like the normal distribution, but the point of the analysis is to demonstrate that the squared exchange rate forecast error in (17) can be greater than the one in (15b) for some

distributions. Consequently, we examined the model under the hypotheses that ε_t had either a uniform distribution or a double exponential probability distribution. Only the results for the uniform distribution are reported in the text.

When ε_t is uniformly distributed, we have

$$f(\varepsilon_t) = 1/2\eta, \qquad -\eta \leqslant \varepsilon_t \leqslant \eta \qquad\qquad (18)$$

$$f(\varepsilon_t) = 0 , \qquad\qquad \text{elsewhere,}$$

where $f(\varepsilon_t)$ is the probability density function for the random variable ε_t. The unconditional expected value of ε is zero and its unconditional variance is $\eta^2/3$. It also follows that

$$\pi_{t-1} = (\eta - \delta_{t-1})/2\eta, \qquad -\eta \leqslant \delta_{t-1} \leqslant \eta. \qquad\qquad (19)$$

In the following discussion we will assume $-\eta \leqslant \delta_{t-1} \leqslant \eta$, since this is the region of interest. If $\delta_{t-1} > \eta$, $\pi_{t-1} = 0$, and δ_{t-1} cannot be less than $-\eta$, since this would make $\pi_{t-1} = 1$ and would cause the collapse in period t-1.

In the Appendix we use (7) and (18) to find the conditional variance of output during a fixed-rate regime which may collapse. This expression is

$$V_{t-1}(y_t)|_{\text{C.Fix}} = \beta^2[-(\eta - \delta_{t-1})^4/16\eta^2 + (\eta^3 - \delta_{t-1}^3)/6\eta] \qquad (20a)$$

$$- \beta^2[\delta_{t-1}(\eta - \delta_{t-1})/2] + \sigma_u^2$$

We also use (7) and (18) to find the conditional variance of output during a permanently floating exchange rate regime. This expression is

$$V_{t-1}(y_t)\big|_{Flex} = \beta^2\eta^2/3 + \sigma_u^2. \tag{20b}$$

In Figure 1, (20a) and (20b) are plotted against δ_{t-1}. Expression (20b) is independent of δ_{t-1}, and it is the upper limit of (20a) as $\delta_{t-1} \to -\eta$. The lower limit of (20b) as $\delta_{t-1} \to \eta$ is σ_u^2, which is output variance for a fixed rate regime without collapse. These results confirm what was found earlier for arbitrary probability distributions. When $\delta_{t-1} = \eta$, an attack next period is unprofitable with probability one, so output variance is σ_u^2. When $\delta_{t-1} = -\eta$, an attack next period happens with probability one, so output variance is $[\beta^2\eta^2/3] + \sigma_u^2$. For values of δ_{t-1} between η and $-\eta$, output variance is a convex combination of the two extremes with weights that vary nonlinearly with δ_{t-1}.

In our examination of these results, however, we asked what would happen to measured output variance during the fixed-rate regime as δ changed but the regime did not collapse. The Appendix demonstrates that output variance during the fixed-rate regime would be given by

$$V_{t-1}(y_t)\big|_{C.Fix.2} = \beta^2(\eta-\delta_{t-1})^4/16\eta^2 + \sigma_u^2. \tag{20c}$$

This quantity also depends on δ_{t-1} and is plotted in Figure 1. The interesting feature of this line is that the maximum variance for output is above output variance for a permanent float. Indeed, the volatilities in (20b) and (20c) are equal when $\delta_{t-1} = -[(16/3)^{1/4} - 1]\eta = -.52\eta$. The implication of this relationship is that while δ_{t-1} is in the range below

$-.52\eta$, measured output variance under the fixed exchange rate regime is

<u>larger</u> than output variance would be under a floating exchange rate regime.

This result is due entirely to the fact that exchange rate prediction

errors can be larger, for some ranges of δ, for a fixed-rate regime which

might collapse but does not than for a floating-rate regime. In our example

with exchange rate innovations drawn from a uniform distribution, this occurs

whenever $\delta_{t-1} < -.52\eta$. The lower is δ_{t-1}, the more likely is a collapse of

the regime, but if actual draws of ε are not greater than δ_{t-1}, the regime

remains viable and output variance is larger than under flexible rates.

3A. A FLOOD-MARION TYPE MODEL

One version of the Flood-Marion (1982) model depicted an open economy

that was small both in the market for goods and in the market for assets.

Wage contracts were set one period in advance of the realizations of

disturbances. Wages, however, were indexed to the price level in a manner to

minimize the deadweight loss from wages being different than their level in a

frictionless (no contract) economy.[6] The major result of Flood and Marion was

that the degree of wage indexation depended on the choice of exchange-rate

regime. They found that a small open economy would index wages fully under a

regime of permanently fixed exchange rates, but wages would only be indexed

partially under a regime of floating exchange rates. Their results indicated

that the degree of wage indexing would remain constant under any single regime

as long as the underlying stochastic structure of the economy was constant.

In our version of the Flood-Marion model, the fixed exchange rate regime

may collapse. Hence, wages are not fully indexed for such a regime, and the

degree of indexation is time varying even though the underlying exogenous

stochastic structure is time invariant.

We demonstrate this result in a stripped-down version of the Flood-Marion model consisting of the following relations:

Glossary of Variables for Model II

m_t = logarithm of the money supply

p_t = logarithm of the price level

b_t = logarithm of domestic credit (assumed constant)

r_t = logarithm of international reserves

p_t^* = logarithm of the foreign price level

s_t = logarithm of the exchange rate

y_t = logarithm of domestic output

$\hat{y}_t$ = logarithm of the benchmark level of domestic output

u_t = domestic real disturbance with variance σ_u^2

v_t^* = foreign price disturbance with variance σ_v^2*

Equations of Model II

Money Market Equilibrium

$$m_t - p_t = \alpha + u_t \qquad (21)$$

Law of One Price

$$p_t = p_t^* + s_t \qquad (22)$$

Aggregate Supply

$$y_t = \bar{y} + z(p_t - E_{t-1}p_t) + u_t \qquad (23)$$

Benchmark Output

$$\hat{y}_t = \bar{y} + \rho u_t, \quad 0 < \rho < 1 \qquad (24)$$

Money Supply Definition

$$m_t = \omega \bar{b} + (1 - \omega)r_t \qquad (25)$$

Foreign Price Process

$$p_t^* = p_{t-1}^* + v_t^* \qquad (26)$$

Equation (21) states that the supply of real money balances, $m_t - p_t$, is equal to the demand for them, $\alpha + u_t$. To make the model analytically tractable, we simplified the money demand function, removing from the Flood-Marion model the opportunity cost of holding money as well as the portion of output that is responsive to prices. Money demand thus depends only on a constant, α, and the productivity disturbance, u_t. The qualitative properties of our results do not depend on these simplifications, but they enormously simplify some later calculations. Equation (22) gives the goods arbitrage condition, which is equivalent to purchasing power parity in this one-good model. Equation (23), which is derived in the Appendix of Flood and Marion, is the aggregate supply function. The variable z is proportional to one minus the optimal degree to which the nominal wage is indexed to the price level. Since z is a deterministic linear function of the degree of indexing, the optimal wage contract may be found by minimizing the loss function (given below) with respect to z. Equation (24), which is also derived in the Appendix of Flood and Marion, gives the frictionless benchmark (no indexing) level of output, $\hat{y}_t$. Equation (25) specifies that the money supply is composed of domestic credit, which we hold constant, and international reserves. As specified in equation (26), the foreign price follows a random walk. The disturbance terms, u_t and v_t^* have zero unconditional means and are mutually and serially uncorrelated.

As in Flood and Marion, it is assumed that wage contracts are set according to

$$\min_{z} E_{t-1}(y_t - \hat{y}_t)^2 \quad , \qquad (27)$$

where $E_{t-1}x_t$ is the mathematical expectation of x_t conditional on information from time t-1, which includes all variables dated t-1 and earlier as well as the structure of the model. The problem posed in (27) is to minimize a quadratic measure of deadweight loss by an appropriate choice of z.

By substituting from (23) and (24), the optimal indexing problem (27) may be rewritten as

$$\min_{z} E_{t-1}[z(p_t - E_{t-1}p_t) + \lambda u_t]^2 \qquad (28)$$

where $0 < \lambda = 1 - \rho < 1$. The solution to (28) denoted z_{t-1} to reflect the possibility that it may depend on time t-1 information is

$$z_{t-1} = - \lambda C_{t-1}(p_t;u_t)/V_{t-1}(p_t) \qquad (29)$$

where $C_{t-1}(\cdot;\cdot)$ denotes the conditional covariance and $V_{t-1}(\cdot)$ denotes the conditional variance.

3B. SOLUTIONS FOR PERMANENT EXCHANGE RATE REGIMES

When the exchange rate is flexible, there is no intervention in the foreign exchange market. Thus, international reserves, r_t, are constant at the level $\bar{r}$, and the exchange rate is $s_t = \bar{m} - \alpha - u_t - p_t^*$, where $\bar{m} \equiv \omega\bar{b} + (1 - \omega)\bar{r}$. The domestic price prediction error,

$p_t - E_{t-1}p_t$, equals $v_t^* + s_t - E_{t-1}s_t$, which is equal to $- u_t$.

Hence, the optimal value of z is $z_{|Flex} = \lambda$, and this value of z will set the loss function to zero. This special result is due to the assumed absence of independent disturbances to the money supply or to money demand in this simplified version of the model. In general, the optimal z will depend on the relative sizes of the variances of the various shocks (see Flood and Marion, p. 54). Presently, the important point is that $z_{|Flex}$ is positive and time invariant.

When the exchange rate is permanently fixed, r_t becomes demand determined, and the domestic price is given by $p_t = p_t^* + \bar{s}$, where $\bar{s}$ is the fixed exchange rate. In this circumstance the domestic price prediction error, $p_t - E_{t-1}p_t$, is simply the prediction error of foreign prices, v_t^*. Consequently, since v_t^* and u_t are uncorrelated, the solution to (29) is to set $z_{|Fix} = 0$ to screen foreign price disturances out of the domestic loss function. This is exactly the result in Flood and Marion.

3C. SOLUTION FOR A TEMPORARY FIXED EXCHANGE RATE REGIME

The fixed-rate solution in the previous section presumed that the fixed-rate regime would last forever. As before, however, for some configurations of the state variables of the economy it may be profitable for agents to attack the government's stock of foreign exchange reserves and end the fixed-rate regime. We continue to assume that the monetary authority will defend the fixed rate only until international reserves hit some known lower bound, $\tilde{r}$, and that after hitting this bound, the fixed-rate regime will be abandoned forever in favor of a flexible exchange rate regime. In this model, if agents attack the fixed rate, they will produce the floating rate given by

$$\tilde{s}_t = \tilde{m} - p_t^* - \alpha - u_t, \tag{30}$$

where $\tilde{m} \equiv \omega\bar{b} + (1 - \omega)\bar{r}$. Thus, when forming wage contracts based on information available at time $t-1$, the contracting parties must plan for the possibility that the fixed-rate regime will be attacked successfully at time t. The probability of an attack taking place at t, based on $t-1$ information, is $\pi_{t-1} = pr_{t-1}\{\tilde{s}_t - \bar{s} > 0\}$, or $\pi_{t-1} = pr_{t-1}\{(u_t + v_t^*) < \gamma_{t-1}\}$, where $\gamma_{t-1} \equiv (\tilde{m} - p_{t-1}^* - \alpha - \bar{s})$.

As before, when there is a possibility that the exchange rate regime will collapse, the agents take account of the possibility in determining their contractual relations. In this case, the optimal degree of indexing depends on the probability of the collapse because this affects the unconditional time $t-1$ covariance and variance in (29).

As is shown in the Appendix, the solution to (29) for this case is

$$z_{t-1} = \frac{-\pi_{t-1} \lambda E_{t-1}[u_t(\tilde{s}_t - \bar{s})|C_t]}{\sigma_{v*}^2 + \pi_{t-1}E_{t-1}[(\tilde{s}_t - \bar{s})^2|C_t] + \pi_{t-1}^2[\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]^2 + 2\pi_{t-1}E_{t-1}[(\tilde{s}_t - \bar{s})v_t^*|C_t]} \tag{31}$$

where σ_{v*}^2 is the unconditional variance of v_t^*, and $E_{t-1}(\cdot|C_t)$ is the expectation operator based on time $t-1$ information but conditional on a collapse taking place at t. The expression on the right hand side of (31), although complicated, is simply the covariance of λu_t and p_t conditional only on $t-1$ information divided by the variance of p_t conditional only on $t-1$ information.

In typical wage indexing models the optimal z is a time invariant function of the variances and covariances of the underlying disturbances. Our main point in this section is that it will not in general be time invariant

when stochastic process switching is possible. This point can be seen at an intuitive level by examining (31) and noticing that z_{t-1} depends on π_{t-1}, the state dependent probability at t-1 of an attack at t.

To make additional progress concerning z_{t-1}, we again made specific assumptions about the probability distribution functions for v_t^* and u_t. In the Appendix we assume that the orthogonal disturbances v_t^* and u_t are each uniformly distributed on the interval $[-\eta,\eta]$. This assumption leads to the closed form but complicated expression for z_{t-1} shown in the Appendix. Since the final expression for z_{t-1} is so complicated, we simulated it for a few values of γ_{t-1} in the interval $[-2\eta,0]$, and we set $\eta = 1$. In this situation we found:

$$\lim_{\gamma_{t-1} \to -2} z_{t-1} = 0 \tag{32a}$$

$$\gamma_{t-1} = -1 \implies z_{t-1} = (.107)\lambda \tag{32b}$$

$$\gamma_{t-1} = 0 \implies z_{t-1} = (3/8)\lambda. \tag{32c}$$

Example (32a) is the Flood-Marion result; when $\gamma_{t-1} \to -2$, it is impossible for $v_t^* + u_t$ to be less than γ_{t-1}. Consequently, $\pi_{t-1} \to 0$, and an attack on the fixed-rate regime at time t is impossible because it cannot be profitable. It is thus optimal to set $z_{t-1} = 0$ (index wages fully), thereby screening all foreign disturbances out of the loss function. For larger values of γ_{t-1}, such as -1 and 0 in examples (32b) and (32c), an attack is possible and contracts written at t-1 allow for this contingency by setting z_{t-1} at optimal values between 0, the optimal value for a fixed exchange rate with no attack possible, and λ, the optimal value for a permanently floating exchange rate.

The point of this simple example is to note that an economy's optimal choice of wage indexation to the price level may not depend simply on the policy currently being pursued by the government and the constant covariance structure of exogenous disturbances. It may also depend on agents' rational beliefs about the possibility of switches in government policies in the future. Consequently, the observation that indexing in wage contracts is time varying need not necessarily be associated with variation in the conditional covariance matrix of the underlying exogenous disturbances, it may instead be associated with time variation in agents' beliefs concerning the permanance of currently implemented policies.[7]

4. CONCLUSIONS

The purpose of this paper was to examine how some aspects of the real economy, such as the determination of output, relative prices and real wages, are influenced by the potential collapse of a fixed exchange rate regime. We examined the implications of the potential switch in government policies in simple versions of two popular international macroeconomic models. In both models, agents must take an action that predetermines some nominal variable. Consequently, monetary policy and the choice of exchange rate regime have real effects. In both models the monetary authority is only willing to defend its exchange rate until its reserves hit a known lower bound. In each case the monetary authority also conducts an exogenous domestic credit policy. If the authorities were willing to conduct an endogenous domestic credit policy, there would be no need for international reserves, and the probability of collapse of the fixed exchange rate regime would be zero.

Clearly, one interesting area for future work is the linkage of the domestic credit policy and the constraints imposed by the government budget

constraint with the choice of exchange rate regime. Eaton (1984) has investigated this problem in a neoclassical context with exogenous real output.

Another interesting area for future work is to focus on devaluations as opposed to the switch from fixed to flexible rates investigated here. What determines the timing and magnitude of a devaluation? Blanco and Garber (1984) have begun such an investigation.

APPENDIX

A1. RESULTS FOR SECTION 2

This Appendix provides the derivation of some results in Section 2. The first result involves a comparision of the conditional variance of the exchange rate under flexible rates in (15b) and under collapsing fixed rates in (15a). These equations are reproduced here as (A1) and (A2) respectively:

$$V_{t-1}(s_t)\big|_{Flex} = (1 - \pi_{t-1})E_{t-1}(\tilde{s}_t^2|NC_t) + \pi_{t-1}E_{t-1}(\tilde{s}_t^2|C_t)$$

$$- [(1 - \pi_{t-1})E_{t-1}(\tilde{s}_t|NC_t) + \pi_{t-1}E_{t-1}(\tilde{s}_t|C_t)]^2 \tag{A1}$$

$$V_{t-1}(s_t)\big|_{C.Fix} = (1 - \pi_{t-1})\bar{s}^2 + \pi_{t-1}E_{t-1}(\tilde{s}_t^2|C_t)$$

$$- [(1 - \pi_{t-1})\bar{s} + \pi_{t-1}E_{t-1}(\tilde{s}_t|C_t)]^2 . \tag{A2}$$

Subtracting (A2) from (A1) gives

$$(1 - \pi_{t-1})[E_{t-1}(\tilde{s}_t^2|NC_t) - \bar{s}^2] - (1 - \pi_{t-1})^2\{[E_{t-1}(\tilde{s}_t|NC_t)]^2 - \bar{s}^2\}$$

$$- 2(1 - \pi_{t-1})\pi_{t-1}[E_{t-1}(\tilde{s}_t|NC_t) - \bar{s}]E_{t-1}(\tilde{s}_t|C_t)$$

$$= (1 - \pi_{t-1})\{E_{t-1}(\tilde{s}_t^2|NC_t) - [E_{t-1}(\tilde{s}_t|NC_t)]^2\} +$$

$$\pi_{t-1}(1-\pi_{t-1})\{[E_{t-1}(\tilde{s}_t|NC_t)]^2 - \bar{s}^2 - 2[E_{t-1}(s_t|NC_t) - \bar{s}]E_{t-1}(\tilde{s}_t|C_t)\}$$

$$= (1 - \pi_{t-1}) V_{t-1}(\tilde{s}_t | NC_t) + \pi_{t-1}(1 - \pi_{t-1})\{[\bar{s} - E_{t-1}(\tilde{s}_t | NC_t)]^2 +$$

$$2 [\bar{s} - E_{t-1}(\tilde{s}_t | NC_t)][E_{t-1}(\tilde{s}_t | C_t) - s]\} \tag{A3}$$

where $V_{t-1}(\tilde{s}_t | NC_t)$ is the time $t-1$ conditional variance given no collapse at t. Expression (A3) is (16) in the text, and it is clearly positive since $E_{t-1}(\tilde{s}_t | C_t) > \bar{s} > E_{t-1}(\tilde{s}_t | NC_t)$.

The second set of results involves the comparison of measures of the variance of real output under alternative regimes in (20a, b, c) under the hypothesis that the innovation in the exchange rate, ε_t, has a uniform probability distribution on the interval $[-\eta, \eta]$.

The expression of interest is $E_{t-1}(y_t - E_{t-1}y_t)^2$. From the model in the text $y_t - E_{t-1}y_t = \beta(s_t - E_{t-1}s_t) + u_t$, since h_t and h_t^* are known at time $t-1$. The disturbance u_t is uncorrelated with $\tilde{s}_t$, and it is trivially uncorrelated with $\bar{s}$. Therefore,

$$E_{t-1}(y_t - E_{t-1}y_t)^2 = \beta^2 E_{t-1}(s_t - E_{t-1}s_t)^2 + \sigma_u^2, \tag{A4}$$

where σ_u^2 is the unconditonal variance of u_t. Our problem then is to find

$$E_{t-1}(s_t - E_{t-1}s_t)^2 = (1 - \pi_{t-1})E_{t-1}[(s_t - E_{t-1}s_t)^2 | NC_t]$$

$$+ \pi_{t-1}E_{t-1}[(s_t - E_{t-1}s_t)^2 | C_t], \tag{A5}$$

where $\pi_{t-1} = pr(\varepsilon_t > \delta_{t-1}) = (\eta - \delta_{t-1})/2\eta$, and $E_{t-1}s_t = (1 - \pi_{t-1})\bar{s} + \pi_{t-1}E_{t-1}(\tilde{s}_t | C_t)$. Using this information and recognizing that NC_t implies $s_t = \bar{s}$ and C_t implies $s_t = \tilde{s}_t$ in (A5) we derive

$$E_{t-1}(s_t - E_{t-1}s_t)^2 = -\pi_{t-1}^2[\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]^2 \qquad (A6)$$

$$+ \pi_{t-1}E_{t-1}[(\tilde{s}_t - \bar{s})^2|C_t].$$

In the text $\tilde{s}_t$ is given by equation (12). Therefore

$$E_{t-1}[\tilde{s}_t|C_t] = \tilde{\lambda}_0 + \lambda_1 h_t^* + \lambda_2 b_{t-1} + E_{t-1}(\varepsilon_t|C_t), \qquad (A7)$$

where $E_{t-1}(\varepsilon_t|C_t) = \frac{1}{2}[\delta_{t-1} + \eta]$ since the collapse takes place if and only if $\varepsilon_t > \delta_{t-1}$, and ε_t is uniformly distributed conditional on collapse. Equation (A7) may thus be used to obtain $E_{t-1}(\tilde{s}_t|C_t) - \bar{s} = \frac{1}{2}(\eta - \delta_{t-1})$, since $\delta_{t-1} = \bar{s} - \tilde{\lambda}_0 - \lambda_1 h_t^* - \lambda_2 b_{t-1}$ which implies that

$$[E_{t-1}(\tilde{s}_t|C_t) - \bar{s}]^2 = (1/4)(\eta - \delta_{t-1})^2. \qquad (A8)$$

The second term in (A6) contains $E_{t-1}[(\tilde{s}_t - \bar{s})^2|C_t]$, which is $E_{t-1}[(\varepsilon_t - \delta_{t-1})^2|C_t] = E_{t-1}(\varepsilon_t^2|C_t) - 2\delta_{t-1}E_{t-1}(\varepsilon_t|C_t) + \delta_{t-1}^2$. Since $E_{t-1}(\varepsilon_t^2|C_t) = \int_{\delta_{t-1}}^{\eta} \varepsilon_t^2(\eta - \delta_{t-1})^{-1}d\varepsilon_t = (\eta^3 - \delta_{t-1}^3)/3(\eta - \delta_{t-1})$, we find

$$E_{t-1}[(\tilde{s}_t - \bar{s})^2|C_t] = [(\eta^3 - \delta_{t-1}^3)/3(\eta - \delta_{t-1})] - \delta_{t-1}\eta . \qquad (A9)$$

Use (A8) and (A9) in (A6) to obtain

$$E_{t-1}[(s_t - E_{t-1}s_t)^2] = -\pi_{t-1}^2(1/4)[\eta - \delta_{t-1}]^2$$

$$+ \pi_{t-1}\{[(\eta^3 - \delta_{t-1}^3)/3(\eta - \delta_{t-1})] - \delta_{t-1}\eta\}. \qquad (A10)$$

Then, use text equation (19) for π_{t-1} in (A10) and that result in (A4) to obtain text equation (20a).

Output variance during the fixed-rate regime which may collapse but has not is

$$E_{t-1}[(y_t - E_{t-1}y_t)^2|NC_t] = \beta^2[\bar{s} - E_{t-1}s_t]^2 + \sigma_u^2. \qquad (A11)$$

But, $\bar{s} - E_{t-1}s_t = \pi_{t-1}[\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]$. Therefore,

$E_{t-1}[(y_t - E_{t-1}y_t)^2|NC_t] = \beta^2\pi_{t-1}^2[\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]^2 + \sigma_u^2$, and we may use previous results of the Appendix to obtain

$$E_{t-1}[(y_t - E_{t-1}y_t)^2|NC_t] = \beta^2(\eta - \delta_{t-1})^4/16\eta^2 + \sigma_u^2, \qquad (A12)$$

which is equation (20c) in the text.

AII. PROPERTIES OF FIGURE 1

The curve labeled (20a) in Figure 1 plots $V_{t-1}(y_t)|_{C. \text{ Fix}}$ as a function of δ_{t-1}. It has maximum value at $\delta_{t-1} = -\eta$ of $\dfrac{\beta^2\eta^2}{3} + \sigma_u^2$ and a minimum at $\delta_{t-1} = \eta$ of σ_u^2. The slope of the function is

$$\frac{\beta}{4\eta^2}(\eta - \delta_{t-1})(\delta_{t-1}^2 - \eta^2) < 0$$

since $|\delta_{t-1}| < \eta$. The derivative of the slope is

$$\frac{\beta^2}{4\eta^2}[(\eta^2 - \delta_{t-1}^2) + 2(\eta - \delta_{t-1})\delta_{t-1}]$$

so the curve's inflection point lies at $\delta_{t-1} < 0$ since the second derivative

is positive at $\delta_{t-1} = 0$.

The curve labeled (20c) in Figure 1 plots $V_{t-1}(y_t)|_{C.Fix.2}$ as a

function of δ_{t-1}. It has a maximum value at $\delta_{t-1} = -\eta$ of $\beta^2\eta^2 + \sigma_u^2$. The

derivative of this function is $-\beta^2(\eta - \delta_{t-1})^3/4\eta^2 < 0$ and the second deriva-

tive is $3\beta^2(\eta - \delta_{t-1})^2/4\eta^2 > 0$. The two curves intersect when real solutions

for δ_{t-1} obtained by setting (20a) equal to (20c) lie in the range $-\eta < \delta_{t-1} < \eta$.

A3. RESULTS FOR SECTION 3

In this section we derive the optimal z for the collapsing fixed rate

regime. The general solution of (28) is

$$z_{t-1} = -\lambda C_{t-1}[p_t; u_t]/V_{t-1}(p_t). \tag{A13}$$

We first work on the term in the numerator of A(13). When a speculative

attack is possible, the expected price level reflects the probability of the

attack, π_{t-1}:

$$E_{t-1}(p_t) = (1-\pi_{t-1})E[(p_{t-1}^* + v_t^* + \bar{s})|NC_t] +$$

$$\pi_{t-1}E[(p_{t-1}^* + v_t^* + \tilde{s}_t)|C_t] \tag{A14}$$

$$= p_{t-1}^* + (1-\pi_{t-1})\bar{s} + \pi_{t-1}E(\tilde{s}_t|C_t).$$

The conditional covariance in (A13) is

$$E_{t-1}[u_t(p_t - E_{t-1}p_t)] = (1 - \pi_{t-1})E_{t-1}[u_t(p_t - E_{t-1}p_t)|NC_t]$$

$$+ \pi_{t-1}E_{t-1}[u_t(p_t - E_{t-1}p_t)|C_t]. \tag{A15}$$

Combining (A14) and (A15) gives

$$E_{t-1}[u_t(p_t - E_{t-1}p_t)] = (1 - \pi_{t-1})\{E_{t-1}(u_t v_t^*|NC_t)$$

$$+ \pi_{t-1}[\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]E_{t-1}(u_t|NC_t)\} +$$

$$+ \pi_{t-1}\{E_{t-1}(u_t v_t^*|C_t) + E_{t-1}[(\tilde{s}_t - \bar{s})u_t|C_t]$$

$$+ \pi_{t-1}[\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]E_{t-1}(u_t|C_t)\}. \tag{A16}$$

Since $E_{t-1}(u_t v_t^*) = (1 - \pi_{t-1})E_{t-1}(u_t v_t^*|NC_t) + \pi_{t-1}E_{t-1}(u_t v_t^*|C_t) = 0$, and $E_{t-1}(u_t) = (1 - \pi_{t-1})E_{t-1}(u_t|NC_t) + \pi_{t-1}E_{t-1}(u_t|C_t) = 0$, we may write (A16) as

$$E_{t-1}[u_t(p_t - E_{t-1}p_t)] = \pi_{t-1}E_{t-1}[u_t(\tilde{s}_t - \bar{s})|C_t], \tag{A17}$$

which is used in the numerator of text equation (31).

For later calculations it is useful to recall from the text that $\tilde{s}_t - \bar{s} = [\gamma_{t-1} - u_t - v_t^*]$, where $\gamma_{t-1} \equiv [\tilde{m} - p_{t-1}^* - \alpha - \bar{s}]$. Hence,

$$- \lambda C_{t-1}(p_t - E_{t-1}p_t; u_t) = - \lambda \pi_{t-1}\{\gamma_{t-1}E_{t-1}(u_t|C_t) - E(u_t^2|C_t) - E(v_t^* u_t|C_t)\}.$$

$$\tag{A18}$$

We turn now to the expression in the denominator of A(13). A derivation similar to that above gives

$$E_{t-1}(p_t - E_{t-1}p_t)^2] = \sigma_{v*}^2 + \pi_{t-1}E_{t-1}[(\tilde{s}_t - \bar{s})^2|C_t] - \pi_{t-1}^2[(\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]^2$$

$$+ 2\pi_{t-1}E_{t-1}[(\tilde{s}_t - \bar{s})v_t^*|C_t]. \qquad (A19)$$

It will prove useful to expand some of the terms on the right hand side of (A19). In particular, consider the following terms:

$$E_{t-1}[(\tilde{s}_t - \bar{s})^2|C_t] = \gamma_{t-1}^2 - 2\gamma_{t-1}[E_{t-1}(u_t|C_t) + E_{t-1}(v_t^*|C_t)]$$

$$+ E_{t-1}(u_t^2|C_t) + E_{t-1}(v_t^{*2}|C_t) + 2E_{t-1}(u_t v_t^*|C_t), \qquad (A20a)$$

$$[\bar{s} - E_{t-1}(\tilde{s}_t|C_t)]^2 = \gamma_{t-1}^2 - 2\gamma_{t-1}[E_{t-1}(u_t|C_t) + E_{t-1}(v_t^*|C_t)]$$

$$+ [E_{t-1}(u_t|C_t) + E_{t-1}(v_t^*|C_t)]^2, \qquad (A20b)$$

$$E_{t-1}[(\tilde{s}_t - \bar{s})v_t^*|C_t] = \gamma_{t-1}E_{t-1}(v_t^*|C_t) - E_{t-1}(u_t v_t^*|C_t) - E_{t-1}(v_t^{*2}|C_t). \qquad (A20c)$$

In order to make further progress on these expressions we assumed u_t and v_t^* to be uniformly and independently distributed on the interval $[-n, n]$. In the calculations reported here, we impose the additional assumption that $-2n < \gamma_{t-1} < 0$. We impose this additional restriction because the joint distribution of (u_t, v_t^*) conditional on $u_t + v_t^* < \gamma_{t-1}$ is quite complicated when $\gamma_{t-1} > 0$, and we need only examine a few different values of γ_{t-1} to establish the point that z_{t-1} will be time varying. Also, when $\gamma_{t-1} < -2n$, the probability of a collapse is zero.

The following results are a few basic facts concerning the joint
conditional distribution of u_t and v_t:

$$E_{t-1}(u_t|C_t) = E_{t-1}(v_t^*|C_t) = (\gamma_{t-1} - n)/3 \qquad (A21a)$$

$$E_{t-1}[u_t^2|C_t] = E_{t-1}[v_t^{*2}|C_t] = \frac{(\psi_{t-1}/3)(\psi_{t-1}^3 + n^3) - (\psi_{t-1}^4 - n^4)/4}{(1/2)(2n + \gamma_{t-1})^2} \qquad (A21b)$$

$$E_{t-1}[u_t v_t^*|C_t] = \frac{[(\gamma_{t-1}^2 - n^2)/2](\psi_{t-1}^2 - n^2) - (2\gamma_{t-1}/3)(\psi_{t-1}^3 + n^3)}{(2n + \gamma_{t-1})^2} +$$

$$+ \frac{(\psi_{t-1}^4 - n^4)/4}{(2n + \gamma_{t-1})^2} \qquad (A21c)$$

$$\sigma_{v*}^2 = E_{t-1}(v_t^{*2}) = n^2/3 \qquad (A21d)$$

$$\pi_{t-1} = \frac{(\gamma_{t-1} + 2n)^2}{8n^2} \qquad (A21e)$$

where $\psi_{t-1} \equiv (\gamma_{t-1} + n)$.

We now rewrite (A13) using all of the above to obtain

$$z_{t-1} = \frac{-\pi_{t-1}\lambda\, x_{1t-1}}{\sigma_{v*}^2 + \pi_{t-1}x_{2t-1} - \pi_{t-1}^2 x_{3t-1} + 2\pi_{t-1}x_{1t-1}} \qquad (A22)$$

where, for $-2n < \delta_{t-1} < 0$, π_{t-1} is given in (A21e), σ_{v*}^2 is given in (A21d),
and

$$x_{1t-1} = [\frac{\gamma_{t-1}(\gamma_{t-1} - n)}{3} - \{\frac{(\psi_{t-1}/3)(\psi_{t-1}^3 + n^3) - (1/4)(\psi_{t-1}^4 - n^4)}{(1/2)(2n + \gamma_{t-1})^2}\}$$

$$- \{\frac{[(\gamma_{t-1}^2 - n^2)/2](\psi_{t-1}^2 - n^2) - (2\gamma_{t-1}/3)(\psi_{t-1}^3 + n^3) + (\psi_{t-1}^4 - n^4)/4}{(2n + \gamma_{t-1})^2}\}]$$

(A23a)

$$x_{2t-1} = \gamma_{t-1}^2 - [4\gamma_{t-1}(\gamma_{t-1} - n)/3] +$$

$$+ \frac{(\gamma_{t-1}^2 - n^2)(\psi_{t-1}^2 - n^2) + (4n/3)(\psi_{t-1}^3 + n^3) - (1/2)[(\psi_{t-1}^4 - n^4]}{(2n + \gamma_{t-1})^2}$$

(A23b)

$$x_{3t-1} = [(1/3)\gamma_{t-1} + (2/3)n]^2.$$

(A23c)

The simulations reported in the text used $n = 1$ and $\gamma_{t-1} = -2, -1, 0.$

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FOOTNOTES

[1]Early contributions to the debate on fixed versus flexible exchange rates are
surveyed by Ishiyama (1975) and Tower and Willett (1976). Some of the
more recent analytical contributions to this area that are based on
rational expectations include Flood (1979), Lapan and Enders (1980),
Helpman (1981), Weber (1981), Eaton (1982), Kimbrough (1983), Turnovsky
(1983), and Aizenman and Frenkel (1984).

[2]Fischer (1982) demonstrates that seignorage accounts for as much as ten
percent of total government revenue for countries such as Argentina and
Brazil. The literature on the monetary approach to the balance of
payments, e.g. Frenkel and Johnson (1975), was quite clear that a policy
of fixing the exchange rate made the total money supply endogenous.

[3]The Dornbusch model deflates nominal money balances by the domestic price of
the domestic good instead of deflating them by a price index composed of
the domestic price of domestic goods and the domestic price of foreign
goods. For our purposes this is a harmless simplification which we also
adopt.

[4]Although we examine only fixed and freely floating exchange rates, our
methods could easily encompass "crawling pegs" or any rule that makes the
fixed exchange rate a deterministic function of the previous state. Our
methods could also be extended to analyze a "dirty float" of the form
$m_t - m_{t-1} = \psi s_t + v_t$, such as that studied by Aizenman and Frenkel
(1984). In such a circumstance, while the private sector could not attack
the money printing rule, the fiscal authority might require a change in
the rule if it were to call for less money printing than were required for
deficit finance. The inability of Southern Cone countries to stay on
preannounced crawling peg schedules is perhaps an example of the type of
fiscal authority attack we have in mind. In such countries an exchange
rate policy seems not to be permanently viable unless it accomodates the
fiscal authority's revenue requirements.

[5]The conditions we use for an attack are those in Flood and Garber (1984). An alternative view, where the agents who would attack the price fixing scheme are small and disorganized is proposed by Obstfeld (1984). According to the Obstfeld view, $\tilde{s}_t > \bar{s}$ is necessary for an attack but not always sufficient. The condition is necessary and sufficient in the present model though, as Obstfeld (1984) has shown.

[6]Indexing wages to prices is motivated only by the large number of such contracts actually implemented. The value of the loss function can generally be made smaller if contracts include more complex indexing to the state as in Karni (1983) or Aizenman and Frenkel (1984). Our intent, however, is not to specify an ideal contract, but rather to indicate how actual contracts affect the economy in a second best environment.
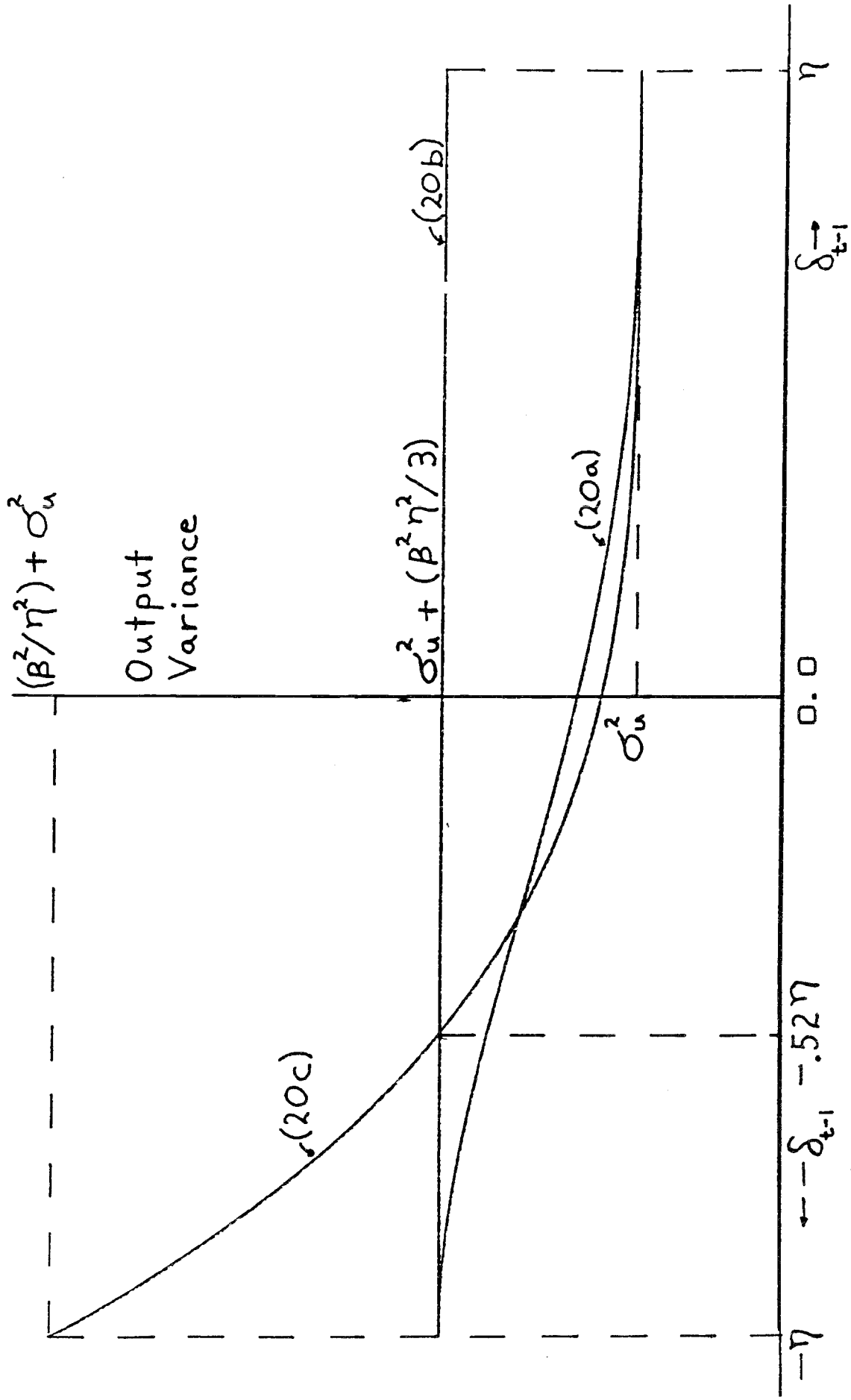
[7]Cecchetti (1984) derives and documents changes in an implied indexing parameter for union wage changes from 1964-1978 for U. S. unionized manufacturing. The movement in the parameter may be due to movement in the conditional variance of the exogenous monetary and real processes of the economy, but the analysis in this paper suggests that such movement might also arise from agents' perceptions of the likelihood of switches in government policies. Since the contracting paradigm also leads to the aggregate supply curve studied by Lucas (1973), the perceived impermanence of existing exchange rate regimes is an additional reason why the slope parameters would differ across countries.

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FIGURE 1

$(\beta^2/\eta^2) + \sigma_u^2$

Output Variance

$\sigma_u^2 + (\beta^2\eta^2/3)$

(20a)

(20b)

(20c)

σ_u^2

$\delta_{t-1} \longrightarrow$

0.0

$\longleftarrow -\delta_{t-1}$ $-.52\eta$

η

$-\eta$

Scale: $\eta=1$, $\beta=2$, $\sigma_u^2=1$