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RULING OUT NONSTATIONARY SPECULATIVE BUBBLES

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ABSTRACT

There is a large and growing empirical literature that tests for the existence of asset-price bubbles or "sunspot" equilibria--equilibria unrelated to market fundamentals. Our view is that even tests for nonstationary asset-price bubbles should not be interpreted as such. In the present paper we extend earlier work of ours which provided a strong case for ruling out nonstationary speculative price bubbles in models based on individual maximizing behavior. In the first part of the paper we study the possibility of stochastic exploding price-level bubbles of the kind proposed by Blanchard (1979). As in our previous work, a scheme of fractionally backing the currency with real output is sufficient to preclude such bubbles. In the second part of the paper we examine conditions for ruling out implosive price-level bubbles, equilibrium paths along which the price level asymptotes to zero even though the monetary growth rate is constant. A condition on preferences implied by any reasonable monetary transactions technology is sufficient to prevent such bubbles from emerging. Given that anticipated future disturbances can lead to price paths which are qualitatively indistinguishable from bubble paths, and given the strong theoretical basis for ruling out nonstationary bubbles, our conclusion is that any "positive" evidence of bubbles should be regarded only as evidence of omitted variables.

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I. Introduction

There is now a large and growing empirical literature directed at testing for the existence of asset-price bubbles or "sunspot" equilibria--equilibria unrelated to market fundamentals. This empirical work already covers a broad range of markets and historical episodes, including the gold market, the stock market, the foreign exchange market, and the domestic currency market (during episodes of hyperinflation).[1] Although theorists have classified several different types of bubbles,[2] almost all empirical studies to date are concerned with nonstationary explosive or implosive (stochastic) bubbles of the type analyzed here. Perhaps the main reason why this particular class of bubbles is so important is that there is as yet no well-defined methodology for positively identifying other types of bubbles.

Our view is that even tests for nonstationary asset-price bubbles should not be interpreted as such. In Obstfeld and Rogoff (1983), and in the present paper, we provide a strong case for ruling out nonstationary speculative price bubbles as equilibria in models based on individual maximizing behavior. Given that anticipated future disturbances can lead to price paths which are qualitatively indistinguishable from bubble paths, and given the strong theoretical basis for ruling out nonstationary bubbles, our conclusion is that "positive" evidence of bubbles should be regarded only as evidence of a particular class of omitted variables. [Hamilton and Whiteman (1984) point out that bubbles and omitted variables are observationally equivalent, and present a particular formalization of that idea. Flood and Garber (1980) have noted the equivalence of bubbles and anticipated future disturbances.]

In our 1983 paper we demonstrated that explosive price bubbles are theoretically possible in models of pure fiat money. But such bubbles are not equilibria if there is some (possibly very small) probability that the government will provide some (possibly very small) real backing for the currency. Here we extend our previous work in two, literally opposite, directions (one explosive and one implosive). First, we study the possibility of having stochastic explosive bubbles of the type proposed by Blanchard (1979). In Blanchard's aggregative model, there exist price bubble paths which are nonstationary in mean even though the bubble bursts with probability one. This type of bubble might be appealing to those who believe that all real-world bubbles eventually burst. We demonstrate here that bubbles very similar to those described by Blanchard can indeed arise in a micro-based model if the monetary regime is one with pure fiat money. (The only difference is that the stochastic bubbles derived from our optimizing model do not burst with probability one. Instead, there is a positive probability that the real value of money will go to zero in finite time.) However, Blanchard-type bubbles only arise in the pure fiat money case. A fractional backing scheme, such as the one discussed in our previous paper, is sufficient to rule out such bubbles.

In the second part of the paper we study conditions for ruling out implosive price-level bubbles, equilibrium bubble paths along which the price level asymptotes to zero even though the growth rate of the money supply is constant. We derive a condition on preferences sufficient to preclude implosive bubbles whenever the money-supply growth rate μ is nonnegative. Our condition is implied by the weak requirement that the derived utility from holding real balances to reduce transaction costs be bounded from above. Almost any reasonable transactions technology will obey this requirement [see, for example, Feenstra (1984)].[3]

II. Explosive Stochastic Bubbles

Blanchard (1979) has suggested a class of stochastic bubbles driven by extrinsic uncertainty. [See also Shiller (1978), Flood and Garber (1980).] Along the explosive price paths analyzed by Blanchard, there is a non-zero probability each period that the price level will return to its stationary saddle-path value. In this section, we show how explosive stochastic price-level bubbles can arise in Brock's (1974, 1975) maximizing model of pure fiat money. Like nonstochastic bubbles, these too can be ruled out through the fractional backing scheme discussed in our earlier paper.

The economy is one in which individuals receive each period y units of the perishable consumption good. Let c_t denote the representative individual's consumption at time t, M_t her nominal money holdings, and $\beta = 1/1+\delta$ her subjective discount rate. If the operator $E_t\{\cdot\}$ yields mathematical expectations conditional on time-t information, then the infinitely-lived individual agent's problem is to maximize

$$(1) \quad E_0\{\sum_{t=0}^{\infty} \beta^t [u(c_t) + v(M_t/P_t)]\}$$

subject to

$$(2) \quad M_t - M_{t-1} = P_t(y - c_t) + H_t ;$$

the individual agent treats the path of the price level $\{P_t\}_{t=0}^{\infty}$ and nominal transfers $\{H_t\}_{t=0}^{\infty}$ as exogenous. In (1), $u(\cdot)$ and $v(\cdot)$ are increasing, strictly concave, and have the usual smoothness properties. Further, $v(\cdot)$ satisfies the Inada conditions

$$(3) \quad \lim_{m\to 0} v'(m) = \infty, \ \lim_{m\to\infty} v'(m) = 0.$$

Initial individual money holdings M_{-1} are given.

The Euler equation characterizing an optimal path for the individual is

$$(4)\quad \frac{u'(c_t) - v'(M_t/P_t)}{P_t} = \beta E_t \{\frac{u'(c_{t+1})}{P_{t+1}}\}.$$

In equilibrium, desired consumption c_t must equal aggregate output y in each period, and nominal money demand must equal the money supply. The latter is assumed to be nonstochastic and (for simplicity) constant at level M. Accordingly, $H_t \equiv 0$. Let m_t stand for real balances M_t/P_t. Multiplying (4) by M and substituting y for c_t, we obtain

$$(5)\quad m_t[u'(y) - v'(m_t)] = \beta u'(y)E_t\{m_{t+1}\}.$$

In a rational-expectations equilibrium, the stochastic process $\{m_t\}_{t=0}^{\infty}$ must satisfy (5), where the conditional expections $E_t\{\cdot\}$ is taken with respect to the actual probability distribution of the price level.

The model as stated contains no instrinsic uncertainty. Under certainty there is a unique positive stationary level of real balances $\bar{m}$ satisfying (5) [so $v'(\bar{m}) = (1 - \beta)u'(y)$]. When real balances are constant at $\bar{m}$, the economy is on its saddle path. Following Blanchard (1979), we introduce extrinsic uncertainty as follows: Assume that if real balances are $\bar{m}$, then agents expect next period's price level to be $\bar{P} = M/\bar{m}$ with probability one. If the economy is on an explosive bubble path with $m_t < \bar{m}$, then agents expect that the price level will return ("crash") to its saddle-path level with probability π, but will continue on the bubble path with probability $1 - \pi$.

By (5), this means that given m_t, the value of m_{t+1} that prevails if a crash does not occur satisfies

(6) $m_t[u'(y) - v'(m_t)] = \beta u'(y)[\pi\bar{m} + (1 - \pi)m_{t+1}]$,

or

$$m_{t+1} = \frac{m_t[u'(y) - v'(m_t)] - \pi\beta u'(y)\bar{m}}{\beta(1 - \pi)u'(y)}.$$

Before the foregoing price process can define an equilibrium path for the pure fiat money economy, a final assumption is needed: If $m_t = 0$, then $m_{t+1} = 0$ with probability one. This assumption turns out to be necessary for the existence of stochastic price bubble paths, but it also implies that such bubble paths do not burst with probability one as in Blanchard's model.

Equation (6) leads to a diagram that shows the positive (feasible) realizations of m_t consistent with intertemporal rational expectations equilibrium. Define $A(m) \equiv m[u'(y) - v'(m)]$ and $B(m) \equiv \beta u'(y)[\pi\bar{m} + (1 - \pi)m]$. The difference equation (6) can then be written as

(7) $A(m_t) = B(m_{t+1})$.

Figure 1 depicts the dynamics dictated by (7). As in Obstfeld and Rogoff (1983), figure 1 applies to the case where $\lim_{m\to 0} mv'(m) = 0$, which admits explosive deterministic bubbles under a pure fiat money regime.[4] One path that satisfies (7) is the saddle path, $m_t = \bar{m}$ for all t.

There are stochastic bubble paths that also satisfy (7). For example, consider the path beginning at m_0 in the figure. The initial price level is $P_0 = M/m_0$. At time zero agents expect a t = 1 price level of $\bar{P}$ (with

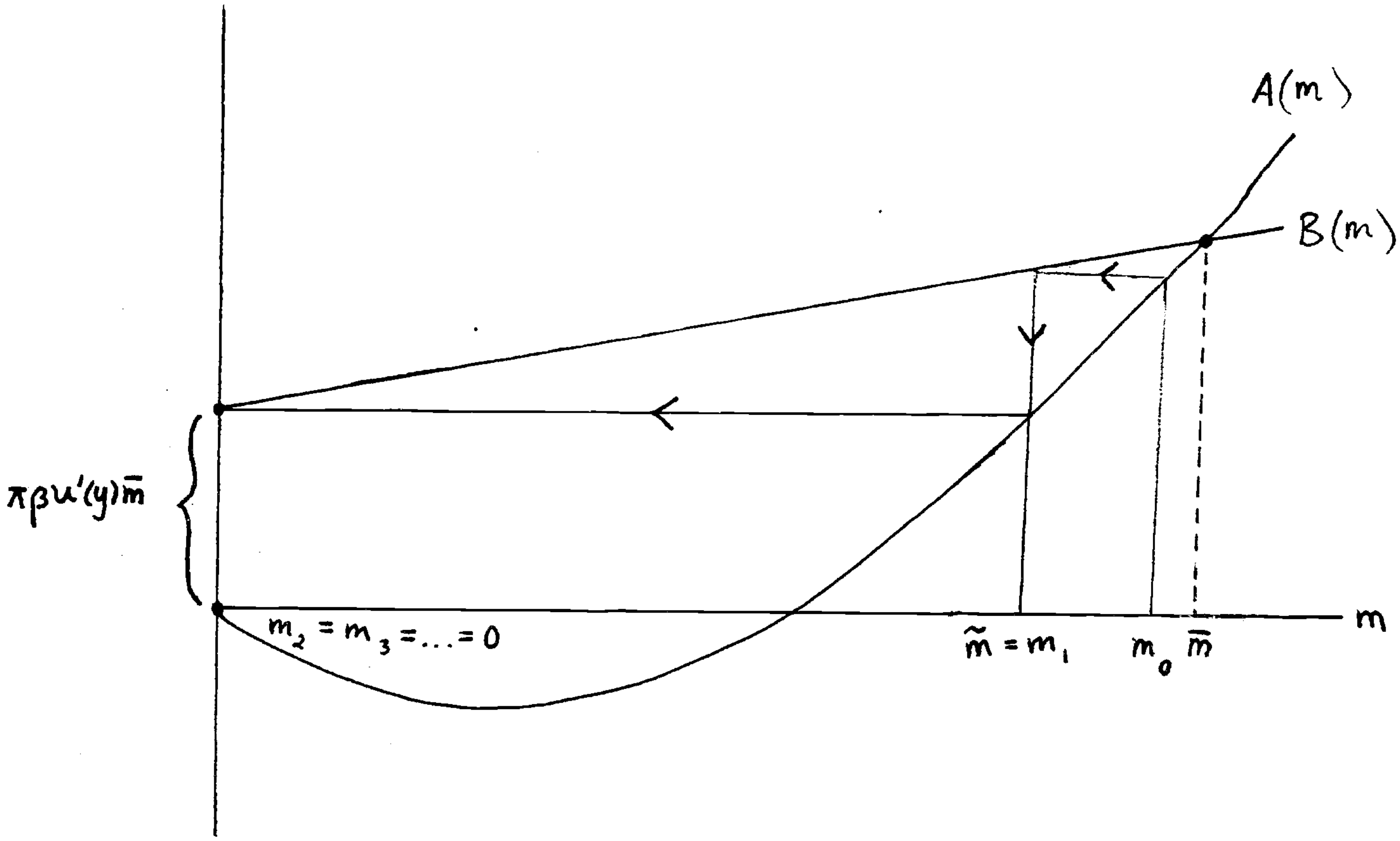

Figure 1

probability π) or $P_1 = M/m_1$ (with probability $1 - \pi$).[5] If no crash occurs at $t = 1$, the price level P_1 is equal to $\tilde{P}$, defined by

$$(8) \quad \frac{u'(y) - v'(M/\tilde{P})}{\tilde{P}} = \frac{\pi\beta u'(y)}{\bar{P}} .$$

At $\tilde{P}$, agents expect real balances to jump to their saddle-path value with probability π, and to be zero with probability $1 - \pi$. Equation (8) implies that individuals have no incentive to reduce their real balances at $m_1 = \tilde{m}$, despite the fact that their money holdings may lose their real value forever. (The possible capital loss is compensated by the current utility of money in reducing transaction costs and the possible capital gain in the event of a crash.)[6]

If P subsequently jumps to $\bar{P}$, the economy remains on the saddle-path forever. If P subsequently becomes infinite, the Euler equation becomes (and remains) $[u'(y) - v'(m_t)]/P_t = \beta u'(y)/P_{t+1}$, which is satisfied (given that $\lim_{m\to\infty} mv'(m) = 0$) if $P = \infty$ and $m = 0$ in all future periods.

It is easy to see that the stochastic bubble equilibria just described cannot occur if the government provides a trivial amount of real backing for the currency. Suppose that the government owns a portion of the aggregate endowment y and rebates its share to the public in the form of lump-sum transfers. As in Obstfeld and Rogoff (1983), the government can guarantee a minimal real redemption value for money by promising to back each currency unit by a claim to a small fraction of its endowment. This suffices to rule out as equilibria the bubble paths considered above, since $m = 0$ (and $\pi = 0$) is no longer an equilibrium.[7]

We have not discussed implosive stochastic bubbles in this section becuase their analysis rests on the same principles as the analysis of implosive nonstochastic bubbles. That subject is taken up next.

III. Ruling Out Implosive Price-Level Paths

This section studies implosive price bubbles in a fully deterministic version of the previous section's model. Because the precise rate of money growth is now central to the discussion, we no longer restrict it to be zero as in section II. A continuous-time framework is more convenient for the analysis, and in this case the Euler equation for real balances is

$$(9) \quad \dot{m}_t = [\mu + \delta - v'(m_t)]\, m_t \quad ,$$

where, again, μ is the instantaneous growth rate of the nominal money supply and δ is the instantaneous subjective discount rate [see Brock (1974)]. To simplify notation, the utility function has been normalized so that $u'(y) = 1$. It is assumed for now that $\mu \geq 0$.

The positive steady-state level of real balances $\bar{m}$ is given by $v'(\bar{m}) = \mu + \delta$. To rule out as a possible equilibrium a solution to (9) with $m_0 > \bar{m}$, it is sufficient to show that this implosive path violates the transversality condition

$$(10) \quad \lim_{t\to\infty} e^{-\delta t}/P_t = 0.$$

Since (10) is necessary but not generally sufficient for an equilibrium when $\mu \geq 0$, violation of (10) is sufficient, but not always necessary, to rule out an implosive path. The appendix shows why Euler equation (9) and condition (10) are jointly necessary for an equilibrium under the present assumptions.[8]

The following result provides the basis of our analysis:

<u>Theorem 1</u>. A necessary and sufficient condition for an Euler path starting with $m_0 > \bar{m}$ to violate the transversality condition (10) is that

the integral

$$(11) \quad \int_{m_0}^{\infty} \frac{v'(m)\, dm}{[\mu + \delta - v'(m)]m}$$

be convergent.[9]

<u>Proof</u>. Along an Euler path with $m_0 > \bar{m}$, (9) implies that

$$(12) \quad m_t = m_0 e^{(\mu + \delta)t - \int_0^t v'(m_s)\, ds} .$$

Because $m_t = e^{\mu t} M_0/P_t$,

condition (10) is equivalent to

$$(13) \quad \lim_{t\to\infty} e^{-\int_0^t v'(m_s)\, ds} = 0,$$

which holds if and only if

$$(14) \quad \lim_{t\to\infty} \int_0^t v'(m_s)\, ds = \infty .$$

Now change variables from s to m in (14), using the fact that, by (9), $dm = [\mu + \delta - v'(m)]m\, ds$ along the path we are studying. Then (14) holds along this path if and only if the utility-of-money function $v(\cdot)$ satisfies

$$\int_{m_0}^{\infty} \frac{v'(m)\, dm}{[\mu + \delta - v'(m)]\, m} = \infty .$$

It follows that (10) is violated if and only if (11) converges. This completes the proof.

As was noted above, convergence of (11) is sufficient to rule out implosive bubbles when $\mu \geq 0$. But when $\mu = 0$, transversality condition (10) is sufficient, as well necessary, for an equilibrium (see the appendix). In this case, therefore, divergence of (11) is necessary and sufficient for equilibrium.

The following is a case in which $\mu = 0$ and (11) diverges, so that implosive Euler paths are equilibria:

Example.[10] Suppose that for large m, $v'(m) = 1/\log(m)$. Then (11) becomes

$$(15) \quad \int_{m_0}^{\infty} \frac{dm}{m[\delta \log(m) - 1]} .$$

Because $\delta \log(m) > 1$ when $m_0 > \bar{m}$ and m is given by (9),

$$(16) \quad \int_{m_0}^{\infty} \frac{dm}{m[\delta \log(m) - 1]} > \int_{m_0}^{\infty} \frac{dm}{\delta m \log(m)} .$$

It is therefore enough to prove that the right-hand side of (16) is divergent. But

$$(17) \quad \int_{m_0}^{\infty} \frac{dm}{m \log(m)} = \log[\log(\infty)] - \log[\log(m_0)] = \infty.$$

Thus (10) is satisfied along this Euler path.

Fortunately, equilibrium implosive price paths are not a problem when $\mu \geq 0$. The following calls into question the economic relevance and interpretation of any implosive price path along which transversality condition (10) holds:

Theorem 2. If (10) holds along an implosive price path, then

(18) $$\lim_{m \to \infty} v(m) = \infty.$$

Proof. By the concavity of $v(\cdot)$, for $m > \bar{m}$,

(19) $$\frac{v(m) - v(\bar{m})}{m - \bar{m}} \geq v'(m).$$

Therefore

(20) $$\int_{m_0}^{\infty} \frac{[v(m) - v(\bar{m})]dm}{(m-\bar{m})m[\mu+\delta-v'(m)]} \geq \int_{m_0}^{\infty} \frac{v'(m)dm}{[\mu+\delta-v'(m)]m},$$

where the right-hand side of (20) equals the integral (11). If there exists a finite B such that for all m,

(21) $$v(m) \leq B,$$

then the left-hand side of (20) is bounded by

(22) $$\int_{m_0}^{\infty} \frac{[B - v(\bar{m})]dm}{(m-\bar{m})m[\mu+\delta - v'(m)]}.$$

The integral (22) is convergent becuase $\mu+\delta-v'(m)$ approaches $\mu+\delta$ monotonically as $m \to \infty$ and $\mu+\delta-v'(m_0) > 0$ (see footnote 8). Therefore (11) can diverge only if $v(m)$ is unbounded as $m \to \infty$. So if $v(m)$ is bounded, Theorem 1 tells us that (10) cannot hold.

No plausible transactions technology can yield the result that the derived utility of money increases without bound for a fixed consumption level, even if $v'(m) > 0$ for all finite m. We therefore conclude that implosive bubbles cannot reasonably occur in this model when $\mu \geq 0$.[11]

What about cases where $\mu < 0$? These are relevant to discussions of the "optimal quantity of money." It turns out that imploding bubbles can be equilibria in these cases even if (10) does not hold.[12] The reason why (10) is no longer necessary is explained by Brock (1974, p. 764, and 1975, p. 145), but it is worth repeating. Transversality condition (10) ensures that an agent cannot increase her utility by reducing her nominal balances permanently by a dollar, provided this is feasible (see the appendix). But this reduction is simply not feasible when nominal balances are declining toward zero along the initial Euler path. The dollar would eventually have to be repurchased to keep money holdings nonnegative, and the cost of this transaction would, by (9), nullify the initial gain in instantaneous utility. (This problem goes away if monetary growth, though currently negative, goes to zero at some finite time in the future.)

To summarize, a very weak preference restriction prevents imploding bubbles when $\mu \geq 0$, but this restriction is inapplicable when $\mu < 0$. Note, however, that a government promise to place a floor on the money-price of output by unlimited purchases of goods with money at a sufficiently low P prevents the emergence of imploding bubbles. And the government will never have to exercise its guarantee. Placing a ceiling on the real value of money is the mirror image of the threatened intervention that precludes exploding bubbles in Obstfeld and Rogoff (1983). Wallace (1981) studies a similar scheme in an overlapping-generations economy.

IV. Conclusion

In this paper we have provided further microeconomic justification for ruling out nonstationary price bubbles, stochastic and otherwise. Given the strong a priori grounds for ruling out such bubbles, it would appear that it is not meaningful to test empirically for their presence. How can one then interpret the existing empirical literature that tests for nonstationary bubbles? Consider once again the model of section II, and assume there are no bubbles. Assume instead that in period t, agents know that there will be an election at the end of period t + 3. Furthermore, agents know that if the current government stays in power, the money supply will remain forever at its current level $\bar{M}$; but the money supply will be $2\bar{M}$, from period t + 4 onward, if the opposition is elected at the end of t + 3. The opposition party is expected to win with an exogenous probability π. What does the path of the price level look like if, ex post, the incumbent party wins the election and the money supply never changes? It is easy to show that the price level will follow a path such as the one depicted in figure 2.

An econometrician unaware of the election's signifcance for the stochastic properties of the money supply process might mistake the price path in figure 2 for evidence of an extrinsic price bubble, one that bursts in period t + 4. However, the analysis of this paper suggests that one should not adopt that interpretation. In the absence of any observed change in the money supply, a price path like the one in figure 2 should be interpreted as evidence of a market fundamental that cannot be observed by the econometrician--an omitted variable.[13]

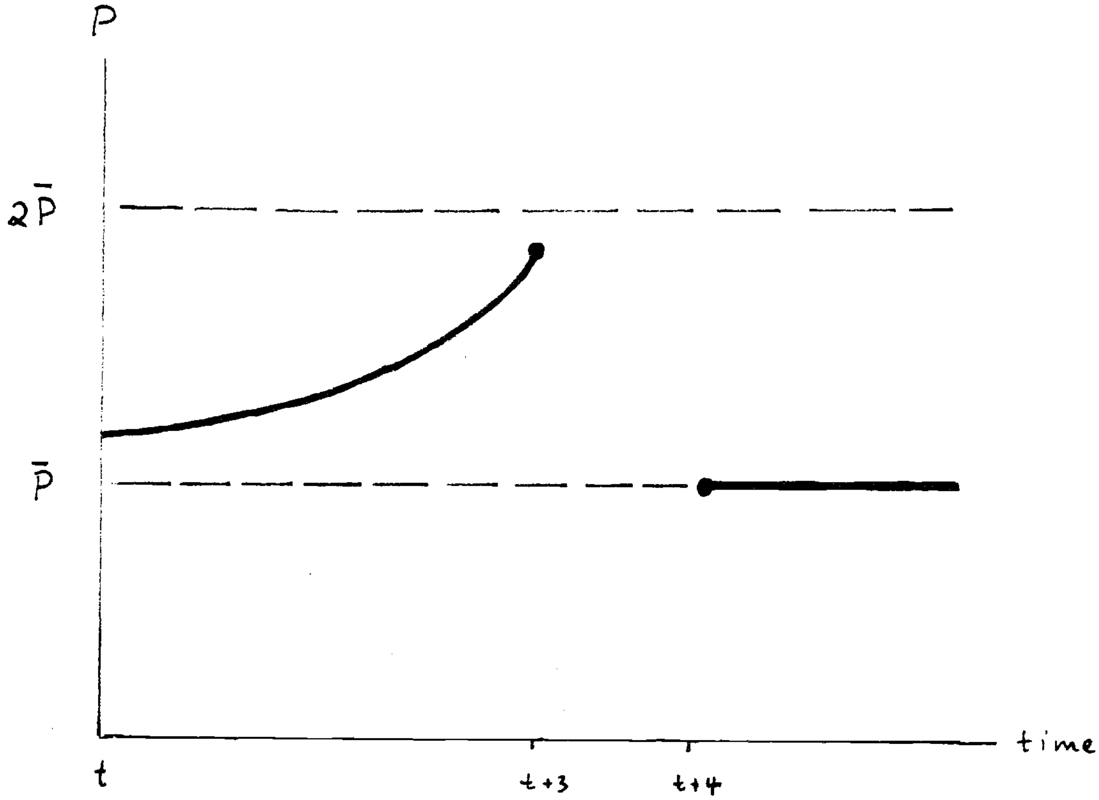

Figure 2

Appendix

This appendix establishes that when monetary growth is nonnegative, the condition

$$\text{(A1)}\quad \lim_{t\to\infty} \beta^t/P_t = 0$$

is necessary for a price-level path $\{P_t\}$ to be an equilibrium in this paper's model. (The discrete-time version of the model is used to aid intuition.) A somewhat less formal proof is given in Obstfeld and Rogoff (1983). Brock (1974) shows that the condition

$$\text{(A2)}\quad \lim_{t\to\infty} \beta^t m_t = 0$$

is sufficient for an equilibrium.[14] (A1) is equivalent to (A2), and thus necessary and sufficient, when monetary growth is zero, as assumed in our 1983 paper. Clearly (A2) implies (A1) if the money stock is growing at a nonnegative rate, but the converse is not true in general.

Consider the deterministic version of the first-order condition (4). Iterating this relation forward yields

$$\text{(A3)}\quad u'(c_t)/P_t = \sum_{i=0}^{\infty} \beta^{t+i} v'(M_{t+i}/P_{t+i})/P_{t+i}$$

$$+ \lim_{T\to\infty} \beta^T u'(c_{t+T})/P_{t+T} \; .$$

If markets clear, so that $c_{t+i} = y$ and $M_{t+i} = M^s_{t+i}$ (the money supply) for all i, it must be true that

$$\text{(A4)}\quad u'(y)/P_t = \sum_{i=0}^{\infty} \beta^i v'(M^s_{t+i}/P_{t+i})/P_{t+i}$$

$$+ \lim_{T\to\infty} \beta^T u'(y)/P_{t+T} \; .$$

Assume that

(A5) $\lim_{T\to\infty} \beta^T/P_{t+T} > 0.$

We will show that (A5) implies $\{P_t\}$ is not an equilibrium. In other words, an agent facing prices $\{P_t\}$ can raise her lifetime utility by deviating from the plan $c_{t+i} = y$, $M_{t+i} = M^s_{t+i}$.

Consider the following deviation: Consume $P_t e$ dollars at time t, reducing nominal balances by this amount in period t and all succeeding periods. Note that this is always a feasible deviation, for e sufficiently small, if M^s is not shrinking over time.

The deviation causes an immediate consumption gain that exceeds $u'(y+e)e$ (by concavity). It causes a lifetime loss of liquidity services less than

$$\sum_{i=0}^{\infty} \beta^i v'[(M^s_{t+i} - P_t e)/P_{t+i}]\ (P_t e/P_{t+i}).$$

It is therefore certainly true that if there is a positive e such that

(A6) $$\frac{u'(y+e)}{P_t} > \sum_{i=0}^{\infty} \beta^i v'[(M^s_{t+i} - P_t e)/P_{t+i}]/P_{t+i} ,$$

$\{P_t\}$ cannot be an equilibrium path. This conclusion is also reached by Brock (1974, p. 763).

But (A4) and (A5) imply that

(A7) $$u'(y)/P_t > \sum_{i=0}^{\infty} \beta^i v'(M^s_{t+i}/P_{t+i})/P_{t+i} .$$

At $e = 0$, $u'(y+e)$ is decreasing in e and $v'[(M^s_{t+i} - P_t e)/P_{t+i}]$ is increasing in e (for all i); but the continuity of these functions ensures that if (A7) holds, we can find a positive e small enough that (A6) holds as well. It follows by contradiction that (A1) is a necessary condition of equilibrium when money growth is nonnegative.

Footnotes

1. The econometric studies include papers by Blanchard and Watson (1982), Diba and Grossman (1982), Flood and Garber (1980), Meese (1984), West (1984a, 1984b), and Woo (1984).

2. See, for example, Azariadis (1981), Cass and Shell (1983), Diamond and Dybvig (1983), and Farmer and Woodford (1984).

3. Brock (1974, pp. 760-764) gives a condition on $v(\cdot)$ sufficient to rule out implosive equilibrium price paths when money growth is nonnegative. Our 1983 paper gives the impression that when the money stock is constant, no condition on preferences beyond a standard Inada condition is needed to preclude imploding bubbles, but this is not the case. The condition we derive here is somewhat more general than Brock's (see footnote 9, below), and is necessary as well as sufficient in the case $\mu = 0$.

4. It is easy to see that stochastic as well as deterministic explosive price-level bubbles are not equilibria when $\lim_{m \to 0} mv'(m) > 0$. Obstfeld and Rogoff (1983) show that this case is economically unreasonable, since it implies that $v(0) = -\infty$.

5. Note that the inflation rate conditional on no crash occurring must rise with π to offset the possibility of a sharp capital gain on real balances.

6. If π did not become zero once real balances were zero at $t = 2$, there would have to be some probability that real balances would become negative in period $t = 3$. Otherwise the equilibrium condition (7) could not be satisfied. But the price level cannot be negative with free disposal. Thus, the stochastic bubble equilibria described here do not crash with

probability one as in Blanchard's (1979) aggregative model.

7. William Brock and Mark Gertler, in unpublished notes, have reached conclusions similar to those reached here. In particular, they show how extrinsic bubbles can be ruled out in a stochastic growth model.

8. See Brock (1974), Gray (1984), or Obstfeld and Rogoff (1983) for intuitive interpretations of the transversality condition.

9. Brock (1974, p. 760), gives a sufficient condition for the violation of (10) when $\mu \geq 0$, that $v'(m) \leq m^{\lambda}$ $(\lambda < 0)$ for m sufficiently large. It is easy to confirm that for any function which meets Brock's condition (e.g., any member of the constant relative risk aversion family), (11) converges.

To see that $\int_{m_0}^{\infty} m^{-\gamma} dm/m(\mu + \delta - m^{-\gamma}) = \int_{m_0}^{\infty} c\ dm/m(m^{\gamma} - c)$ converges for $\gamma > 0$ [where $c \equiv 1/(\mu + \delta)$], expand by partial fractions and write c^{-1} times this integral (for $c \neq 0$) as

$$\int_{m_0}^{\infty} \left[\frac{-1}{cm} + \frac{m^{\gamma-1}}{c(m^{\gamma}-c)}\right] dm$$

$$= \lim_{m_1 \to \infty} \left[-\frac{1}{c}\log(m) + \frac{1}{c\gamma}\log(m^{\gamma}-c)\right] \Big|_{m_0}^{m_1}$$

$$= \lim_{m_1 \to \infty} \log\left[\frac{(m^{\gamma} - c)^{1/\gamma}}{m}\right]^{1/c} \Big|_{m_0}^{m_1}$$

$$= -\log\left[\frac{(m_0^{\gamma} - c)^{1/\gamma}}{m_0}\right]^{1/c} < \infty .$$

10. Guillermo Calvo and Roque Fernandez suggested this example.

11. The condition given in Theorem 2 is necessary, but not sufficient, for an implosive bubble path to be an equilibrium path when $\mu \geq 0$. The transversality condition (10) is violated for all $v(\cdot)$ in the constant relative risk aversion class (see footnote 8), and some of these are unbounded as $m \to \infty$. Of course if $\lim_{m \to \infty} v(m) = \infty$, the individual's objective function (1) may be unbounded for price paths $\{P_t\}$ such that $P_t \to 0$. This would require reformulation of the individual's problem in terms of an "overtaking" principle.

12. Given our assumption that $v'(\cdot)>0$, $\mu+\delta>0$ is required for the existence of equilibrium. Brock (1974, 1975) discusses the possibility that $v'(\cdot)\leq 0$ for sufficiently large m. Note that if $v'(m) = 0$ for some finite m, (10) cannot hold along an implosive Euler path for $\mu \geq 0$.

13. Again, see Hamilton and Whiteman (1984).

14. It is easy to see that this sufficient condition never holds for imploding Euler paths with $\mu > 0$, and always holds when $\mu < 0$.

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