COMPETING ENGINES OF GROWTH:
INNOVATION AND STANDARDIZATION

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ABSTRACT

We study a dynamic general equilibrium model where innovation takes the form of the introduction new goods, whose production requires skilled workers. Innovation is followed by a costly process of standardization, whereby these new goods are adapted to be produced using unskilled labor. Our framework highlights a number of novel results. First, standardization is both an engine of growth and a potential barrier to it. As a result, growth in an inverse U-shaped function of the standardization rate (and of competition). Second, we characterize the growth and welfare maximizing speed of standardization. We show how optimal IPR policies affecting the cost of standardization vary with the skill-endowment, the elasticity of substitution between goods and other parameters. Third, we show that the interplay between innovation and standardization may lead to multiple equilibria. Finally, we study the implications of our model for the skill-premium and we illustrate novel reasons for linking North-South trade to intellectual property rights protection.

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1 Introduction

The diffusion of new technologies is often coupled with *standardization* of product and process innovations. New technologies, when first conceived and implemented, are often complex and require skilled personnel to operate. At this stage, their use in the economy is limited both by the patents of the innovator and the skills that these technologies require. Their widespread adoption and use first necessitates the tasks involved in these new technologies to become more routine and standardized, ultimately enabling their cheaper production using lower-cost unskilled labor. However, such standardization not only expands output but also implies that the rents accruing to innovators will come to an end. Therefore, the process of standardization is both an engine of economic growth and a potential discouragement to innovation. In this paper, we study this interplay between innovation and standardization.

The history of computing illustrates the salient patterns of this interplay. The use of silicon chips combined with binary operations were the big breakthroughs, starting the ICT revolution. During the first 30 years of their existence, computers could only be used and produced by highly skilled workers. Only a lengthy process of standardization made computers and silicon chips more widely available and more systematically integrated into the production processes, to such a degree that today computers and computer-assisted technologies are used at every stage of production with workers of very different skill levels. At the same time that the simplification of manufacturing processes allowed mass production of electronic devices and low prices, competition among ICT firms intensified enormously, first among few industry leaders and then more broadly at a global scale.

In our model, new products are invented via costly R&D and can first be produced only by skilled workers. This innovation process is followed by a costly process of standardization, whereby the previously new goods are adapted to be produced using unskilled labor.[1] Free entry into standardization makes it a *competing process*; standardization will be undertaken by newcomers, which may then displace incumbent producers. By shifting some technologies to low-skill workers, standardization alleviates the pressure on scarce high-skill workers, thereby raising aggregate demand

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[1] This view has a clear antecedent in Nelson and Phelps (1966), which we discuss further below. See also Autor, Levy and Murnane (2003) on the comparative advantage of unskilled workers in routine, or in our language "standardized," tasks. We can also interpret innovation as product innovation and standardization as process innovation. Evidence that firms engaging in product innovation (e.g., Cohen and Klepper, 1996) are smaller and more skill intensive than firms engaging in process innovation is consistent with our assumptions.

and fostering incentives for further innovation. Yet, the anticipation of standardization also reduces the potential profits from new products, discouraging innovation. This implies that while standardization—and the technology adoption that it brings—is an engine of economic growth, it can also act as a barrier to growth by potentially slowing down innovation.

Our baseline framework provides a simple model for the analysis of this interplay. Under some relatively mild assumptions, we establish the existence of a unique balance growth path that is saddle-path stable. We show that equilibrium growth is an inverse U-shaped function of the "extent of competition" captured by the cost of standardization. When standardization is very costly, growth is relatively slow because new products use skilled workers for a long while and this reduces their scale of production and profitability. On the other hand, when standardization is very cheap, growth is again relatively slow, this time because innovators enjoy ex post profits only for a short while. This inverse U-shaped relationship between competition and growth is consistent with the empirical findings in Aghion et al. (2005), and complements the theoretical channel highlighted in Aghion et al. (2001, 2005), which is driven by the interplay of their "escape competition" mechanism and the standard effects of monopoly profits on innovation.

In our model, the laissez-faire equilibrium is inefficient for two reasons. First, as in many models of endogenous technology, there is an appropriability problem: both innovating and standardizing firms are able to appropriate only a fraction of the gain in consumer surplus created by their investment and this makes the growth rate too low. Second, there is a new form of "business stealing" effect, whereby the costly standardization decisions reduce the rents of innovators.[2] The possibility that the laissez-faire equilibrium is inefficient and that growth is maximized by intermediate levels of competition implies that welfare and growth maximizing policies are not necessarily those that provide maximal intellectual property rights (IPR) protection to innovators. Under the assumption that a government can affect markups and the cost of standardization by regulating IPR protection, we characterize growth and welfare maximizing combinations of IPR and competition policies. Contrary to most of the literature, the optimal policy is not the result of a trade-off between the static cost of monopoly power and dynamic gains. Rather, in our model an excess

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[2]Another form of business stealing, studied extensively in Schumpeterian models of vertical innovation (e.g., Aghion and Howitt 1992), is when a monopoly is destroyed by new firms introducing a "better" version of an existing products. We suggest that standardization is also an important source of business stealing.

of property right protection may harm growth by increasing the overload on skilled workers, which are in short supply.

When the discount rate is small, we find that growth and welfare maximizing IPR policy involves lower protection when R&D costs (for new products) are lower, when markups for new products are higher and when the ratio of skilled to unskilled labor supply is greater. The latter comparative static result is a consequence of the fact that when there is a large supply of unskilled labor, standardization becomes more profitable and thus innovators require greater protection against standardization. We also show that when competition policy as well as IPR policy can be used, the optimal combination of policies involves no limits on monopoly pricing for new products, increased competition for standardized products and lower IPR protection than otherwise. Intuitively, lower IPR protection minimizes wasteful entry costs, but this may lead to excessive standardization and weak incentives to innovate. To maximize growth or welfare, this latter effect needs to be counteracted by lower markups for standardized products. We also show that trade liberalization in less-developed countries may create negative effects on growth by changing the relative incentives to innovate and standardize. However, if increased trade openness is coupled by optimal IPR policy, it always increases welfare and growth.

Finally, we show that under different parameter configurations or different assumptions on competition between innovators and standardizers, a new type of multiplicity of equilibria (of balanced growth paths) arises. When too much of the resources of the economy are devoted to standardization, expected returns from innovation are lower and this limits innovative activity. Expectation of lower innovation reduces interest rates and encourages further standardization. Consequently, there exist equilibria with different levels (paths) of innovation and standardization. It is noteworthy that this multiplicity does not rely on technological complementarities (previously studied and emphasized in the literature), and has much more of the flavor of "self-fulfilling equilibria," whereby the relative prices change in order to support equilibria consistent with initial expectations.

Our paper is related to several different literatures. In addition to the endogenous growth and innovation literatures (e.g., Aghion and Howitt, 1992, Grossman and Helpman, 1991, Romer, 1990, Segerstrom, Anant and Dinopoulos, 1990, Stokey, 1991), there are now several complementary frameworks for the analysis of technology adoption. These can be classified into three groups. The first includes models based on Nelson and Phelps's (1966) important approach, with slow diffusion of technologies across countries (and across firms), often related to the human capital of

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the workers employed by the technology adopting firms. This framework is incorporated into different types of endogenous growth models, for example, in Howitt (2000), Acemoglu, Aghion and Zilibotti (2006), and Acemoglu (2009, Chapter 18). Several papers provide more microeconomic foundations for slow diffusion. These include, among others, Jovanovic and Lach (1989), Jovanovic and Nyarko (1996), Jovanovic (2009) and Galor and Tsiddon (1997), which model either the role of learning or human capital in the diffusion of technologies. The second group includes papers emphasizing barriers to technology adoption. Parente and Prescott (1994) is a well-known example. Acemoglu (2005) discusses the political economy foundations of why some societies may choose to erect entry barriers against technology adoption. The final group includes models in which diffusion of technology is slowed down or prevented because of the inappropriateness of technologies invented in one part of the world to other countries (see, e.g., Acemoglu and Zilibotti, 2001, Atkinson and Stiglitz, 1969, Basu and Weil, 1998 and David, 1975). Gancia and Zilibotti (2009) propose a unified framework for studying technology diffusion in models of endogenous technical change. Our approach emphasizing standardization is different from, though complementary to, all three groups of papers.

Our paper is also related to Krugman's (1979) model of North-South trade and technology diffusion, whereby the South adopts new products with a delay. Krugman, in turn, was inspired by Vernon's (1966) model of the product cycle and his approach has been further extended by Grossman and Helpman (1991) and Helpman (1993).[3] Our approach differs from all these models because innovation and standardization make different use of skilled and unskilled workers and because we focus on a closed economy general equilibrium setup rather than the interactions between technologically advanced and backward countries as in these papers. A new implication of our alternative set of assumptions is that, differently from previous models, growth is an inverse-U function of standardization. More importantly, none of the above paper characterizes the optimal IPR policy and how it varies with skill abundance. Grossman and Lai (2004) and Boldrin and Levine (2005) study the incentives that governments have to protect intellectual property in a trading economy. Their framework, however, abstracts from the technology adoption choice and from the role of skill which are central to our analysis.

Finally, our emphasis on the role of skilled workers in the production of new goods and unskilled workers in the production of standardized goods makes our paper also

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[3]Similar themes are also explored in Bonfiglioli and Gancia (2008), Antras (2005), Dinopoulos and Segerstrom (2007, 2009), Lai (1998), Yang and Maskus (2001).

related to the literature on technological change and wage inequality; see, among others, Acemoglu (1998, 2003), Aghion, Howitt and Violante (2002), Caselli (1999), Galor and Moav (2000), Greenwood and Yorukoglu (1997), and Krusell, Ohanian, Rios-Rull and Violante (2000). The approach in Galor and Moav (2000) is particularly related, since their notion of ability-biased technological change also generates predictions for wage inequality similar to ours, though the economic mechanism and other implications are very different.

The rest of the paper is organized as follows. Section 2 builds a dynamic model of endogenous growth through innovation and standardization. It provides conditions for the existence, uniqueness and stability of a dynamic equilibrium with balanced growth and derives an inverse-U relationship between the competition from standardized products and growth. Section 3 presents the welfare analysis. After studying the first best allocation, it characterizes growth and welfare maximizing IPR and competition policies as function of parameters. As an application of these results, we discuss how trade liberalization in less developed countries affects innovation, standardization and the optimal policies. Section 4 shows how a modified version of the model may generate multiple equilibria and poverty traps. Section 5 concludes.

2   A Model of Growth through Innovation and Standardization

2.1   Preferences

The economy is populated by infinitely-lived households who derive utility from consumption C_t and supply labor inelastically. Households are composed by two types of agents: high-skill workers, with aggregate supply H, and low-skill workers, with aggregate supply L. The utility function of the representative household is:

$$U = \int_0^\infty e^{-\rho t} \log C_t dt,$$

where $\rho > 0$ is the discount rate. The representative household sets a consumption plan to maximize utility, subject to an intertemporal budget constraint and a No-Ponzi game condition. The consumption plan satisfies the standard Euler equation:

$$\frac{\dot{C}_t}{C_t} = r_t - \rho, \tag{1}$$

where r_t is the interest rate. Time-indexes are henceforth omitted when this causes no confusion.

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2.2 Technology and Market Structure

Aggregate output, Y, is a CES function defined over a measure A of goods available in the economy. As in Romer (1990), the measure of goods A captures the level of technological knowledge that grows endogenously through innovation. However, we assume that, upon introduction, new goods involve complex technologies that can only be operated by skilled workers. After a costly process of standardization, the production process is simplified and the good can then be produced by unskilled workers too. Despite this change in the production process, good characteristics remain unaltered so that all varieties contribute to final output symmetrically. Thus, Y is defined as:

$$Y = Z \left(\int_0^A x_i^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}} = Z \left(\int_0^{A_L} x_{L,i}^{\frac{\epsilon-1}{\epsilon}} \, di + \int_0^{A_H} x_{H,i}^{\frac{\epsilon-1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}, \tag{2}$$

where A_H is the measure of hi-tech goods, A_L is the measure of low-tech (standardized) goods and $A = A_H + A_L$. $\epsilon > 1$ is the elasticity of substitution between goods. The term $Z \equiv A^{\frac{\epsilon-2}{\epsilon-1}}$ is a normalizing factor that ensures that output is linear in technology and thus makes the final good production function consistent with balanced growth (without introducing additional externalities in R&D technology as we will see below). To see why, note that with this formulation when $x_i = x$, aggregate productivity, $Y/(Ax)$, is equal to A—as in AK models.[4]

From (2), the relative demand for any two goods $i, j \in A$ is:

$$\frac{p_i}{p_j} = \left(\frac{x_i}{x_j} \right)^{-1/\epsilon}. \tag{3}$$

We choose Y to be the numeraire, implying that the minimum cost of purchasing one unit of Y must be equal to one:

$$1 = A^{-1} \left(\frac{1}{A} \int_0^A (p_i)^{1-\epsilon} \, di \right)^{1/(1-\epsilon)}. \tag{4}$$

Each hi-tech good is produced by a monopolist with a technology that requires one unit of skilled labor per unit of output. Each low-tech good is produced by a

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[4]The canonical endogenous growth models that do not feature the Z term and allow for $\sigma \neq 2$ (e.g., Grossman and Helpman, 1991) ensure balanced growth by imposing an externality in the innovation possibilities frontier (R&D technology). Having the externality in the production good function instead of the R&D technology is no less general and simplifies our analysis.

monopolist with a technology that requires one unit of labor per unit of output. Thus, the marginal cost is equal to the wage of skilled workers, w_H, for hi-tech firms and the wage of unskilled workers, w_L, for low-tech firms. Since high-skill worker can be employed by both high- and low-tech firms, then $w_H \geq w_L$.

When standardization occurs, there are two potential producers (a high- and a low-tech one) for the same variety. The competition between these producers is described by a sequential entry-and-exit game. In stage (i) a low-tech firm can enter and produce a standardized version of the intermediate variety. Then, in stage (ii), the incumbent decides whether to exit or fight the entrant. Exit is assumed to be irreversible, i.e., when a hi-tech firm leaves the market it cannot go back to it and the low-tech firm becomes a monopolist. If the incumbent does not exit, the two firms compete *à la* Bertand (stage (iii)). We assume that all firms entering stage (iii) must produce (and thus pay the cost of producing) at least $\xi > 0$ units of output (without this assumption, stage (ii) would be vacuous, as incumbents would have a "weakly dominant" strategy of staying in and producing $x = 0$ in stage (iii)).

Regardless of the behavior of other producers or other prices in this economy, a subgame-perfect equilibrium of this game must have the following features: standardization in sector j will be followed by the exit of the high-skill incumbent whenever $w_H > w_L$. If the incumbent did not exit, competition in stage (iii) would result in all of the market being captured by the low-tech firm due to its cost advantage and the incumbent would make a loss on the $\xi > 0$ units that it is forced to produce. Thus, as long as the skill premium is positive, firms contemplating standardization can ignore any competition from incumbents. However, if $w_H < w_L$ incumbents would fight entrants and can dominate the market. Anticipating this, standardization is not profitable in this case and will not take place. Finally, in the case where $w_H = w_L$, there is a potential multiplicity of equilibria, where the incumbent is indifferent between fighting and exiting. In what follows, we will ignore this multiplicity and adopt the *tie-breaking rule* that in this case the incumbent fights (we modify this assumption in Section 4). We summarize the main results of this discussion in the following Proposition.

Proposition 1 *In any subgame-perfect equilibrium of the entry-and-exit game described above, there is only one active producer in equilibrium. Whenever $w_H > w_L$ all hi-tech firms facing the entry of a low-tech competitor exit the market. Whenever $w_H \leq w_L$ hi-tech incumbents would fight entry, and no standardization occurs.*

In the rest of the paper, we focus on the limit economy as $\xi \to 0$.[5] Since in equilibrium there is only one active producer, the price of each good will always be a markup over the marginal cost:

$$p_H = \left(1 - \frac{1}{\epsilon}\right)^{-1} w_H \quad \text{and} \quad p_L = \left(1 - \frac{1}{\epsilon}\right)^{-1} w_L. \tag{5}$$

Symmetry and labor market clearing pin down the scale of production of each firm:

$$x_L = \frac{L}{A_L} \quad \text{and} \quad x_H = \frac{H}{A_H}, \tag{6}$$

where recall that H is the number of skilled workers employed by hi-tech firms and L is the remaining labor force. Markup pricing implies that profits are a constant fraction of revenues:

$$\pi_H = \frac{p_H H}{\epsilon A_H} \quad \text{and} \quad \pi_L = \frac{p_L L}{\epsilon A_L}. \tag{7}$$

At this point, it is useful to define the following variables: $n \equiv A_H/A$ and $h \equiv H/L$. That is, n is the fraction of hi-tech goods over the total and h is the relative endowment of skilled workers. Then, using demand (3) and (6), we can solve for relative prices as:

$$\frac{p_H}{p_L} = \left(\frac{x_H}{x_L}\right)^{-1/\epsilon} = \left(h \frac{1-n}{n}\right)^{-1/\epsilon} \tag{8}$$

and

$$\frac{w_H}{w_L} = \frac{p_H}{p_L} = \left(h \frac{1-n}{n}\right)^{-1/\epsilon}. \tag{9}$$

Intuitively, the skill premium w_H/w_L depends negatively on the relative supply of skill ($h = H/L$) and positively on the relative number of hi-tech firms demanding skilled workers. Note that $w_H = w_L$ at:

$$n^{\min} \equiv \frac{h}{h+1}.$$

For simplicity, we restrict attention to initial states of technology such that $n > n^{\min}$. As an implication of Proposition 1, if we start from $n > n^{\min}$, the equilibrium will always remain in the interval $n \in \left[n^{\min}, 1\right]$. We can therefore restrict attention to this interval, over which skilled workers never seek employment in low-tech firms.

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[5] The focus on the limit economy is for simplicity. We could alternative model the game differently, and assume away stage (ii). Although conceptually similar, this case is less tractable.

Using (7) and (8) yields relative profits:

$$\frac{\pi_H}{\pi_L} = \left(h \frac{1-n}{n} \right)^{1-1/\epsilon}. \tag{10}$$

This equation shows that the relative profitability of hi-tech firms, π_H/π_L, is increasing in the relative supply of skill, H/L, because of a standard market size effect and decreasing in the relative number of hi-tech firm, A_H/A_L. The reason for the latter effect is that a larger number of firms of a given type implies stiffer competition for labor and a lower equilibrium firm scale.

Next, to solve for the level of profits, we first use symmetry into (4) to obtain:

$$\frac{p_H}{A} = \left[(1-n) \left(\frac{p_L}{p_H} \right)^{1-\epsilon} + n \right]^{1/(\epsilon-1)}, \text{ and} \tag{11}$$

$$\frac{p_L}{A} = \left[1 - n + n \left(\frac{p_H}{p_L} \right)^{1-\epsilon} \right]^{1/(\epsilon-1)}.$$

Using these together with (8) into (7) yields:

$$\pi_H = \frac{H}{\epsilon} \left[1 + \left(\frac{1}{n} - 1 \right)^{\frac{1}{\epsilon}} h^{\frac{1-\epsilon}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} n^{\frac{2-\epsilon}{\epsilon-1}}, \text{ and} \tag{12}$$

$$\pi_L = \frac{L}{\epsilon} \left[1 + \left(\frac{1}{n} - 1 \right)^{-\frac{1}{\epsilon}} h^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} (1-n)^{\frac{2-\epsilon}{\epsilon-1}}.$$

Note that, for a given n, profits per firm remain constant. Moreover, the following lemma formalizes some important properties of the profit functions:

Lemma 1 *Assume $\epsilon \geq 2$. Then, for $n \in \left[n^{\min}, 1 \right]$:*

$$\frac{\partial \pi_H}{\partial n} < 0 \quad and \quad \frac{\partial \pi_L}{\partial n} > 0. \tag{13}$$

Moreover, π_L is a convex function of n.

Proof. See the Appendix. ∎

The condition $\epsilon \geq 2$ is sufficient—though not necessary—for the effect of competition for labor to be strong enough to guarantee that an increase in the number of hi-tech (low-tech) firms reduces the absolute profit of hi-tech (low-tech) firms. In the

rest of the paper, we assume that the restriction on ϵ in Lemma 1 is satisfied.[6]

2.3 Standardized Goods, Production and Profits

Substituting (6) into (2), the equilibrium level of aggregate output can be expressed as:

$$Y = A \left[(1-n)^{\frac{1}{\epsilon}} L^{\frac{\epsilon-1}{\epsilon}} + n^{\frac{1}{\epsilon}} H^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \tag{14}$$

showing that output is linear in the overall level of technology, A, and is a constant-elasticity function of H and L. From (14), we have that

$$\frac{\partial Y}{\partial n} = \frac{A^{1-\frac{1}{\epsilon}} Y^{\frac{1}{\epsilon}}}{\epsilon - 1} \left[\left(\frac{H}{n} \right)^{\frac{\epsilon-1}{\epsilon}} - \left(\frac{L}{1-n} \right)^{\frac{\epsilon-1}{\epsilon}} \right], \tag{15}$$

which implies that aggregate output is maximized when $n/(1-n) = h$. Intuitively, production is maximized when the fraction of hi-tech products is equal to the fraction of skilled workers in the population, so that $x_L = x_H$ and prices are equalized across goods. Equation (14) is important in that it highlights the value of technology diffusion: by shifting some technologies to low-skill workers, standardization "alleviates" the pressure on scarce high-skill workers, thereby raising aggregate demand. It also shows that the effect of standardization on production, for given A, disappears as goods become more substitutable (high ϵ). In the limit as $\epsilon \to \infty$, there is no gain to smoothing consumption across goods ($x_L = x_H$) so that Y only depends on aggregate productivity A.

Finally, to better understand the effect of technology diffusion on innovation, it is also useful to express profits as a function of Y. Using (2)-(4) to substitute $p_H = A^{\frac{\epsilon-2}{\epsilon}} (Y/x_H)^{1/\epsilon}$ into (7), profits of a hi-tech firm can be written as:

$$\pi_H = \frac{(Y/A)^{1/\epsilon}}{\epsilon} \left(\frac{H}{n} \right)^{\frac{\epsilon-1}{\epsilon}}. \tag{16}$$

A similar expression holds for π_L. Notice that profits are proportional to aggregate demand, Y. Thus, as long as faster technology diffusion (lower n) through standardization raises Y, it also tends to increase profits. On the contrary, an increase in

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[6]An elasticity of substitution between products greater than 2 is consistent with most empirical evidence in this area. See, for example, Broda and Weinstain (2006).

$n \geq n^{\min}$ reduces the instantaneous profit rate of hi-tech firms:

$$\frac{\partial \pi_H}{\partial n} \frac{n}{\pi_H} = \frac{1}{\epsilon} \frac{\partial Y}{\partial n} \frac{n}{Y} - \frac{\epsilon - 1}{\epsilon} < 0. \tag{17}$$

2.4 Innovation and Standardization

We model both innovation, i.e., the introduction of a new hi-tech good, and standardization, i.e., the process that turns an existing hi-tech product into a low-tech variety, as costly activities. We follow the "lab-equipment" approach and define the costs of these activities in terms of output, Y. In particular, we assume that introducing a new hi-tech good requires μ_H units of the numeraire, while standardizing an existing hi-tech good costs μ_L units of Y. We may think of μ_L as capturing the technical cost of simplifying the production process plus any policy induced costs due to IPR regulations restricting the access to new technologies.

Next, we define V_H and V_L as the net present discounted value of a firm producing a hi-tech and a low-tech good, respectively. These are given by the discounted value of the expected profit stream earned by each type of firm and must satisfy the following Hamilton-Jacobi-Bellman equations:

$$\begin{aligned} rV_L &= \pi_L + \dot{V}_L \\ rV_H &= \pi_H + \dot{V}_H - mV_H, \end{aligned} \tag{18}$$

where m is the arrival rate of standardization, which is endogenous and depends on the intensity of investment in standardization. These equations say that the instantaneous profit from running a firm plus any capital gain or losses must be equal to the return from lending the market value of the firm at the risk-free rate, r. Note that, at a flow rate m, a hi-tech firm is replaced by a low-tech producer and the value V_H is lost.

Free-entry in turn implies that the value of innovation and standardization can be no greater than their respective costs:

$$V_H \leq \mu_H \quad \text{and} \quad V_L \leq \mu_L.$$

If $V_H < \mu_H$ ($V_L < \mu_L$), then the value of innovation (standardization) is lower than its cost and there will be no investment in that activity.

A *dynamic equilibrium* is a time path for $(C, \ x_i, \ A, \ n, \ r, \ p_i)$ such that monopolists maximize the discounted value of profits, the evolution of technology is determined by free entry in innovation and standardization, the time path for prices is consistent with market clearing and the time path for consumption is consistent with household maximization. We will now show that a dynamic equilibrium can be represented as a solution to two differential equations. Let us first define:

$$\chi \equiv \frac{C}{A}; \quad y \equiv \frac{Y}{A}; \quad g \equiv \frac{\dot{A}}{A}.$$

The first differential equation is the law of motion of the fraction of hi-tech goods, n. This is the state variable of the system. Given that hi-tech goods are replaced by a low-tech goods at the endogenous rate m, the flow of newly standardized products is $\dot{A}_L = mA_H$. From this and the definition $n = (A - A_L)/A$ we obtain:

$$\dot{n} = (1 - n)\, g - mn. \tag{19}$$

The second differential equation is the law of motion of χ. Differentiating χ and using the consumption Euler equation (1) yields:

$$\frac{\dot{\chi}}{\chi} = r - \rho - g \tag{20}$$

Next, to solve for g, we use the aggregate resource constraint. In particular, consumption is equal to production minus investment in innovation, $\mu_H \dot{A}$, and in standardization, $\mu_L \dot{A}_L$. Noting that $\dot{A}/A = g$ and $\dot{A}_L/A = mn$, we can thus write:

$$\chi = y - \mu_H g - \mu_L mn$$

Substituting for g from this equation (i.e., $g = (y - \mu_L mn - \chi)/\mu_H$) into (19) and (20) gives the following two equation dynamical system in the (n, χ) space:

$$\frac{\dot{\chi}}{\chi} = r - \rho - \frac{y - \mu_L mn - \chi}{\mu_H} \tag{21}$$

$$\frac{\dot{n}}{n} = \left(\frac{1 - n}{n}\right) \frac{y - \chi}{\mu_H} - m \left(1 + (1 - n)\frac{\mu_L}{\mu_H}\right), \tag{22}$$

Note that y is a function of n (see equation (14)). Finally, r and m can be found

as functions of n from the Hamilton-Jacobi-Bellman equations. First, note that, if there is positive imitation ($m > 0$), then free-entry implies $V_L = \mu_L$. Given that μ_L is constant, V_L must be constant too, $\dot{V}_L = 0$. Likewise, if there is positive innovation ($g > 0$), then $\dot{V}_H = 0$. Next, equations (18) can be solved for the interest rate in the two cases:

$$r = \frac{\pi_L}{\mu_L} \quad \text{if} \quad m > 0 \tag{23}$$

$$r = \frac{\pi_H}{\mu_H} - m \quad \text{if} \quad g > 0. \tag{24}$$

We summarize these findings in the following proposition.

Proposition 2 *A dynamic equilibrium is characterized by (i) the autonomous system of differential equations (21)-(22) in the (n, χ) space where*

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\begin{aligned}
y &= y(n) = \left[(1-n)^{\frac{1}{\epsilon}} L^{\frac{\epsilon-1}{\epsilon}} + n^{\frac{1}{\epsilon}} H^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}, \\
r &= r(n) = \max \left\{ \frac{\pi_L(n)}{\mu_L}, \frac{\pi_H(n)}{\mu_H} - m \right\}, \\
m &= m(n) = \begin{cases} 0 & \text{if} \quad r > \frac{\pi_L(n)}{\mu_L} \\[2mm] \frac{y(n)-\chi}{n\mu_L} & \text{if} \quad r > \frac{\pi_H(n)}{\mu_H} - m \end{cases},
\end{aligned}
$$

and $\pi_H(n)$ and $\pi_L(n)$ are given by (12), (ii) a pair of initial conditions, n_0 and A_0, and (iii) the transversality condition $\lim_{t\to\infty} \left[\exp \left(-\int_0^t r_s ds \right) \int_0^{A_t} V_i di \right] = 0$.

2.6   BALANCED GROWTH PATH

A Balanced Growth Path (BGP) is a dynamic equilibrium such that $\dot{n} = \dot{m} = 0$ and, hence, the skill premium and the interest rate are at a steady-state level. An "interior" BGP is a BGP where, in addition, $m > 0$ and $g > 0$. Equation (19) implies that an interior BGP must feature $m^{ss} = g(1-n)/n = (r - \rho)(1-n)/n$. To find the associated BGP interest rate, we use the free-entry conditions for standardization and innovation. Using (12), the following equation determines the interest rate consistent with $m > 0$:

$$
\begin{aligned}
r_L(n) &\equiv \frac{\pi_L}{\mu_L} \tag{25} \\[2mm]
&= \frac{L}{\mu_L \epsilon} \left[1 + \left(\frac{1}{n} - 1 \right)^{-\frac{1}{\epsilon}} h^{\frac{\epsilon-1}{\epsilon}} \right]^{\frac{1}{\epsilon-1}} (1-n)^{\frac{2-\epsilon}{\epsilon-1}}.
\end{aligned}
$$

13

Next, the free-entry condition for hi-tech firms, conditional on the BGP standardization rate, determines the interest rate consistent with the BGP:

$$r_H^{ss}(n) \equiv \frac{\pi_H}{\mu_H} - m^{ss} = n\frac{\pi_H}{\mu_H} + (1-n)\rho \tag{26}$$

$$= \frac{H}{\mu_H \epsilon}\left[1 + \left(\frac{1}{n}-1\right)^{\frac{1}{\epsilon}} h^{\frac{1-\epsilon}{\epsilon}}\right]^{\frac{1}{\epsilon-1}} n^{\frac{1}{\epsilon-1}} + (1-n)\rho.$$

The curves $r_H^{ss}(n)$ and $r_L(n)$ can be interpreted as the (instantaneous) return from innovation (conditional on $\dot{n} = 0$) and standardization, respectively. In the space (n,r), the BGP value of n can be found as their crossing point: in other words, along a BGP, both innovation and standardization must be equally profitable.[7] We summarize the preceding discussion in the following proposition:

Proposition 3 *An interior BGP is a dynamic equilibrium such that $n = n^{ss}$ where n^{ss} satisfies*

$$r_L(n^{ss}) = r_H^{ss}(n^{ss}), \tag{27}$$

and $r_L(n^{ss})$ and $r_H^{ss}(n^{ss})$ are given by (25) and (26), respectively. Given n^{ss}, the BGP interest rate is $r^{ss} = \pi_L(n^{ss})/\mu_L$, and the standardization rate is $m^{ss} = (r^{ss} - \rho)(1 - n^{ss})/n^{ss}$, where π_L is as in (12) [evaluated at $n = n^{ss}$]. Finally, A_H, A_L, Y and C all grow at the same rate, $g^{ss} = r^{ss} - \rho$.

To characterize the set of BGP, we need to study the properties of $r_L(n)$ and $r_H^{ss}(n)$. Due to the shape of π_L, $r_L(n)$ is increasing and convex. Provided that ρ is not too high, $r_H^{ss}(n)$ is non-monotonic (first increasing and then decreasing) and concave in n.[8] The intuition for the non monotonicity is as follows. When n is high, competition for skilled workers among hi-tech firms brings π_H down and this lowers the return to innovation. Moreover, when n is higher than $h/(h+1)$ aggregate productivity and Y are low, because skilled workers have too many tasks to perform, while unskilled workers too little. This tends to reduce π_H even further. When n is low, π_H is high, but the flow rate of standardization is high as well (since, recall, $m^{ss} = g(1-n)/n$) and this brings down the return to innovation.

Figure 1 shows the BGP relationship between r and n. Note that, as long as $m > 0$, the equilibrium must lie on the (dashed) $r_L(n)$ curve. An interior BGP must

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[7]It can also be verified straightforwardly that the allocation corresponding to this crossing point satisfies the transversality condition.

[8]A formal argument can be found in the proof of Proposition 4.
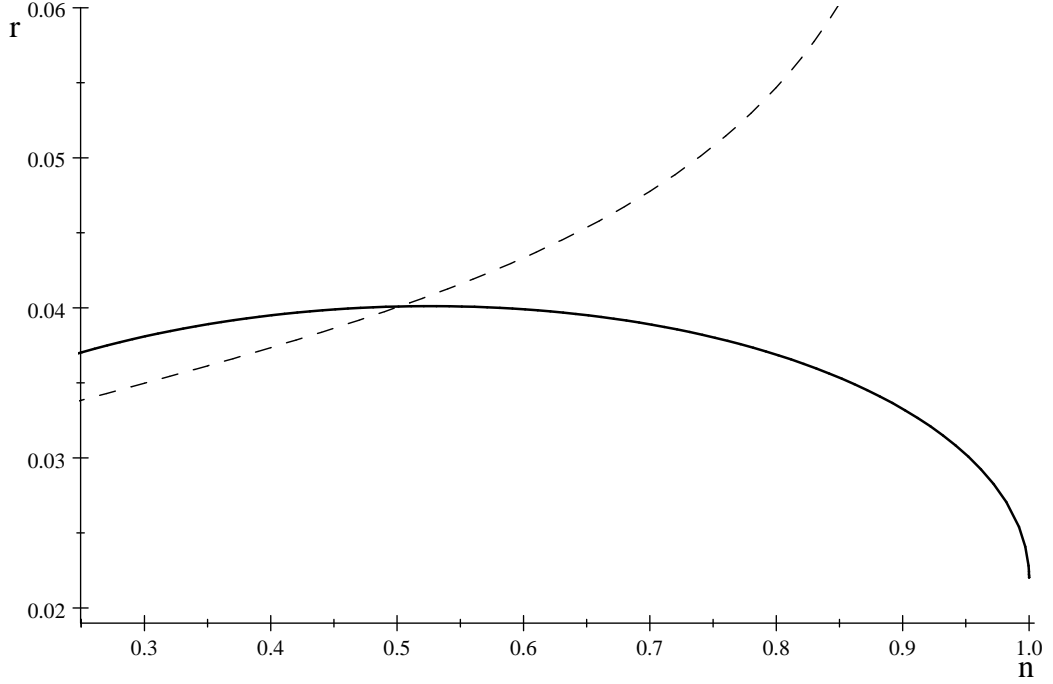
Figure 1: Solid $= r_H^{ss}(n)$, Dashed $= r_L(n)$

also lie on the (solid) $r_H^{ss}(n)$ curve. Thus, the interior BGP value of n is identified by the intersection. The following assumptions guarantee the existence and uniqueness of a BGP, and that such BGP is interior.

Assumption 1 $: 0 < \rho < H/(\mu_H \epsilon)$.

This condition is standard: it guarantees that innovation is sufficiently profitable to sustain endogenous growth and that the transversality condition is satisfied.

Assumption 2 $: \mu_H < \mu_L \frac{h}{1+h-\epsilon \rho \mu_L/(H+L)}$.

Assumption 2 ensures that $r_H^{ss}(n^{\min}) > r_L(n^{\min})$, ruling out the uninteresting case in which standardization is always more profitable than innovation when n is expected to stay constant, and guarantees that the BGP is interior and unique. We state the existence and uniqueness of the BGP in a formal proposition.

Proposition 4 *Suppose Assumptions 1-2 hold. Then there exist a unique BGP equilibrium.*

15

Proof. See the Appendix. ∎

Proposition 4 establishes the existence and uniqueness of a BGP equilibrium, denoted by (n^{ss}, χ^{ss}). The next goal is to prove the (local) existence and uniqueness of a dynamic equilibrium converging to this BGP. Unfortunately, the analysis of dynamics is complicated by several factors. First, the dynamic system (21)-(22) is highly nonlinear. Second, it may exhibit discontinuities in the standardization rate (and thus in the interest rate) along the equilibrium path. Intuitively, at (n^{ss}, χ^{ss}) there is both innovation and standardization (otherwise we could not have $\dot{n} = 0$). It is relatively easy to prove that, similar to models of directed change (e.g., Acemoglu 2002 and Acemoglu and Zilibotti 2001), there exists a dynamic equilibrium converging to the BGP featuring either only innovation (when $n < n^{ss}$) or only standardization (when $n > n^{ss}$). This implies that when the economy approaches the BGP from the left, the standardization rate and the interest rate both jump once the BGP is reached. In particular, when $n < n^{ss}$, there is no standardization, thus, $m = 0$, while in BGP we have $m > 0$. Since throughout there is innovation and thus the value of hi-tech firms must remain constant at $V_H = \mu_H$, there can be no jump in $r + m$. Consequently, there must be an exactly offsetting jump in interest rate r when the BGP is reached and the standardization rate, m, jumps.[9]

However, it turns out to be more difficult to prove that there exist no other dynamic equilibria. In particular, we must rule out the existence of equilibrium trajectories (solutions to (21)-(22)) converging to (n^{ss}, χ^{ss}) with *both* innovation and standardization out of BGP. Numerical analysis suggests that no such trajectory exists as long as Assumptions 1 and 2 are satisfied. In particular, the system (21)-(22) under the condition that $m = \pi_H(n)/\mu_H - \pi_L(n)/\mu_L$ (i.e., under the condition that there is both innovation and standardization) is globally unstable around (n^{ss}, χ^{ss}). However, we can only prove this analytically under additional conditions. In particular, we must impose the following parameter restriction:[10]

$$\frac{\epsilon - 1}{\epsilon}(h + 2) + \epsilon > \frac{2h + 1}{h(h + 1)}. \tag{28}$$

Proposition 5 *Suppose that Assumptions 1 and 2 and (28) hold. Then there ex-*

---

[9]Note that the discontinuous behavior of the standardization rate and interest does not imply any jump in the asset values, V_H and/or V_L. Rather, the rate of change of these asset values may jump locally.

[10]This restriction ensures that $n_{\min} = h/(1 + h)$ be not too small, which is key in the proof strategy (see Appendix). For example, when $\epsilon = 2$, it requires $n_{\min} = h/(1 + h) > 0.28$ and when $\epsilon = 3$, $n_{\min} = h/(1 + h) > 0.21$.

ists $\bar{\rho} > 0$ such that, for $\rho < \bar{\rho}$, the interior BGP is locally saddle-path stable. In particular, if n_{t_0} is in the neighborhood of its BGP value, n^{ss}, and $n_{t_0} > n^{ss}$ [resp., $n_{t_0} < n^{ss}$], then there exists a unique path converging to the BGP such that for some finite $\bar{t} > t_0$, we have $\tau \in [t_0, \bar{t}]$, $m_\tau > 0$, $g_\tau = 0$ and $\dot{n}_\tau < 0$ [resp., $m_\tau = 0$, $g_\tau > 0$ and $\dot{n}_\tau > 0$], and the economy attains the BGP at $\bar{t}$ (i.e., for all $\tau \geq \bar{t}$, we have $n_\tau = n^{ss}$, $m_\tau = m^{ss}$, and $g_\tau = g^{ss}$).

Proof. See the Appendix. ∎

2.7 GROWTH AND STANDARDIZATION: AN INVERSE-U RELATIONSHIP

How does the cost of standardization, μ_L, affect the BGP growth rate, g^{ss}? Answering this question is important from both a normative and a positive perspective. First, policies such as IPR protection are likely to have an impact on the profitability of standardization. Therefore, knowing the relationship between standardization and growth is a key step for policy evaluation. Second, the difficulty to standardize may vary across technologies and over time.

The cost of standardization affects $r_L(n)$, but not $r_H^{ss}(n)$. Thus, increasing the cost of standardization amounts to shifting the $r_L(n)$ curve in Figure 1 and therefore the intersection, form $n = n^{\min}$ (low μ_L) to $n \to 1$ (high μ_L). The effect on the growth rate depends in turn on the relationship between g^{ss} and n:

$$g^{ss}(n) = r_H^{ss}(n) - \rho = n\left(\frac{\pi_H(n)}{\mu_H} - \rho\right).$$

This expression highlights the trade-off between innovation and standardization: a high standardization rate (and thus a low n) increases the instantaneous profit rate $\pi_H(n)$, but lowers the expected profit duration. Taking the derivative and using (17) yields:

$$
\begin{aligned}
\frac{\partial g^{ss}(n)}{\partial n} &= \frac{\pi_H(n)}{\mu_H} - \rho + \frac{n}{\mu_H}\frac{\partial \pi_H(n)}{\partial n} \\
&= \frac{\pi_H(n)}{\epsilon \mu_H}\left(1 + \frac{\partial Y(n)}{\partial n}\frac{n}{Y(n)}\right) - \rho.
\end{aligned}
$$

From (15), $\partial Y(n)/\partial n = 0$ at $n = n^{\min}$. For $n > n^{\min}$, we have $\partial Y(n)/\partial n < 0$ with $\lim_{n\to 1}\partial Y(n)/\partial n = -\infty$. Thus:

$$\left.\frac{\partial g^{ss}(n)}{\partial n}\right|_{n=n^{\min}} = \frac{H+L}{\mu_H \epsilon^2} - \rho \quad \text{and} \quad \lim_{n\to 1}\frac{\partial g^{ss}(n)}{\partial n} = -\infty.$$

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Provided that $\rho < \frac{H+L}{\mu_H \epsilon^2}$, $g^{ss}(n)$ is an inverse-U function of n. Intuitively, at $n = 1$ the wage of unskilled workers is zero and hence the marginal value of transferring technologies to them (in terms of higher aggregate demand and thus also profits) is infinite. Instead, at $n = n^{\min}$, aggregate output Y is maximized and marginal changes in n have second order effects on aggregate production. Moreover, given that future profits are discounted, the impact of prolonging the expected profit stream (high n) on innovation vanishes if ρ is high. When $\rho < \frac{H+L}{\mu_H \epsilon^2}$, growth is maximized at $n^* \in (n^{\min}, 1)$ that solves:

$$1 - \rho \frac{\epsilon \mu_H}{\pi_H(n^*)} = -\frac{\partial Y(n^*)}{\partial n^*} \frac{n^*}{Y(n^*)}. \tag{29}$$

The condition $\rho < \frac{H+L}{\mu_H \epsilon^2}$ is satisfied whenever Assumption 1 (which we imposed above and which guarantees $g > 0$) and $\epsilon < 1 + 1/h$, i.e., if skilled workers are sufficiently scarce. It is also satisfied when ρ and μ_H are sufficiently low. Now recalling that in BGP n is an increasing function of μ_L, we have the following result (proof in the text):

Proposition 6 *Let g^{ss} be the BGP growth rate and assume $\rho < \frac{H+L}{\mu_H \epsilon^2}$. Then, g^{ss} is an inverse U-shaped function of the cost of standardization.*

Figure 1 provides a geometric intuition. Starting from a very high μ_L such that $r_H^{ss}(n)$ is in its decreasing portion, a decrease in μ_L moves the equilibrium to the left along the schedule $r_H^{ss}(n)$. This yields a lower n^{ss} and thus higher growth. Therefore, in this region, a decrease in μ_L increases growth. However, after the maximum of the $r_H^{ss}(n)$ schedule is passed, further decreases in μ_L reduce n and growth.

Proposition 6 also has interesting implications for the skill premium. Recall that, in this model, the skill-premium is the market value of being able to operate new technologies and produce hi-tech goods. For this reason, it is increasing in the fraction of hi-tech firms (see equation (9)). Since growth is an inverse-U function of n, the model also predicts a inverse U-shaped relationship between growth and wage inequality, as shown in Figure 2. Intuitively, a very high skill-premium could be a sign that standardization is so costly to slow down growth. A very low skill premium, however, might be a sign of too fast standardization and thus weak incentives to innovate.
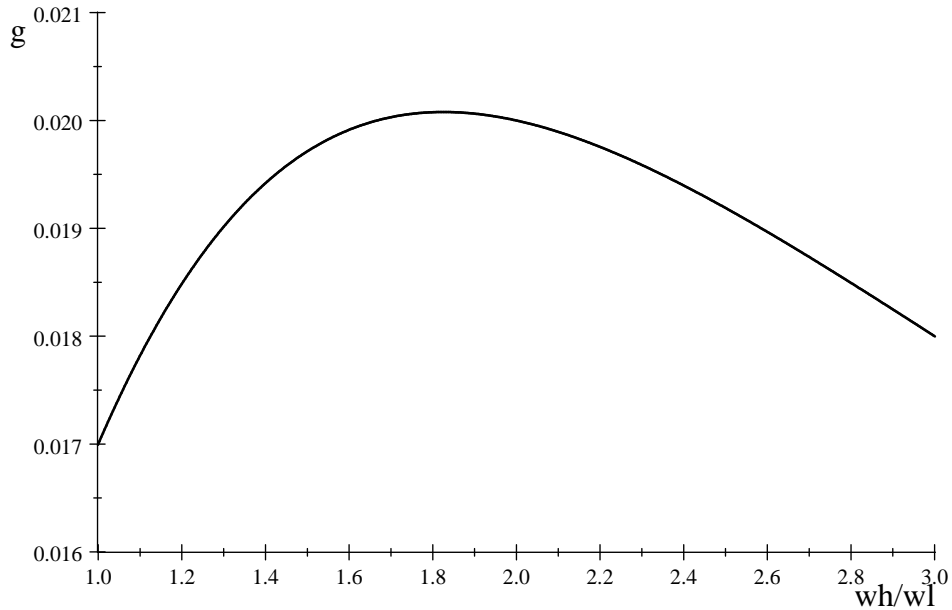
Figure 2: Growth and the Skill Premium

3 Welfare Analysis and Optimal Policies

We now turn to the normative analysis. We start by characterizing the Pareto optimal allocation for a given μ_L, representing the technical cost of standardization. This allows us to identify the inefficiencies that are present in the decentralized equilibrium. Next, we focus on the constrained efficient allocation that a government could achieve with a limited set of instruments. In particular, we allow the government to increase the cost of standardization above μ_L through IPR regulations and to influence the level of competition. Finally, we briefly discuss how North-South trade affects the optimal policies.

3.1 Pareto Optimal Allocation

The Pareto optimal allocation is the one chosen by a social planner seeking to maximize the utility of the representative agent, subject to the production function (14) and for given costs of innovation, μ_H, and standardization, μ_L. The current value Hamiltonian for the problem is:

$$\mathcal{H} = \ln\left(Y - I_H - I_L\right) + \xi_H \frac{I_H}{\mu_H} + \xi_L \frac{I_L}{\mu_L}$$

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where I_H and I_L are investment in innovation and standardization, respectively. The control variables are I_H and I_L, while the state variables are A and A_L, with co-state variables ξ_H and ξ_L, respectively. From the first order conditions, the Pareto optimal n solves:

$$\frac{\partial Y}{\partial A} \frac{1}{\mu_H} = \frac{\partial Y}{\partial A_L} \frac{1}{\mu_L} \tag{30}$$

That is, the planner equates the marginal rate of technical substitution between hi-tech and low-tech products to their relative development costs. The Euler equation for the planner is:

$$\frac{\dot{C}}{C} = \frac{\partial Y}{\partial A} \frac{1}{\mu_H} - \rho.$$

By comparing these results to those in the previous section, we can see that the decentralized equilibrium is inefficient for two reasons.

First, there is a standard appropriability problem whereby firms only appropriate a fraction of the value of innovation/standardization so that R&D investment is too low. To isolate this inefficiency, consider the simplest case $L = 0$, so that there is no standardization and $Y = AH$, $\pi_H = H/\epsilon$. In this case, the social return from innovation is H while the private return is only $r = H/\epsilon < H$. The same form of appropriability effect also applies when $L > 0$.

Second, there is too much standardization relative to innovation due to a business stealing externality: the social value of innovation is permanent while the private benefit is temporary. A particularly simple case to highlight this inefficiency is when $\epsilon = 2$ so that (30) simplifies to:

$$\frac{n}{1-n} = h \left(\frac{\mu_L + \mu_H}{\mu_H} \right)^2,$$

In the decentralized equilibrium, instead, the condition $r_L(n) = r_H^{ss}(n)$ yields:

$$\frac{n}{1-n} = h \left[\left(\frac{r}{m+r} \right) \frac{\mu_L}{\mu_H} \right]^2$$

Clearly, n is too low in the decentralized economy.

To correct the first inefficiency, subsidies to innovation (and standardization) are needed. On the other hand, the business stealing externality can be corrected by introducing a licensing policy requiring low-tech firms to compensate the losses they impose on hi-tech firms. In particular, suppose that firms that standardize must pay a one-time licensing fee μ_L^{lic} to the original inventor. In this case, the free-entry

conditions together with the Hamilton-Jacobi-Bellman equations (18) become:

$$V_L = \frac{\pi_L}{r} = \mu_L^{lic} + \mu_L$$

$$V_H = \frac{\pi_H - m\left(V_H - \mu_L^{lic}\right)}{r} = \mu_H.$$

Clearly, the business stealing effect is removed when $\mu_L^{lic} = \mu_H$, that is, when low-tech firms compensate the hi-tech produces for the entire capital loss μ_H. We summarize these results in the proposition (proof in the text).[11]

Proposition 7 *The Pareto optimal allocation can be decentralized using a subsidy to innovation and a license fee imposed on firms standardizing new products.*

3.2 CONSTRAINED EFFICIENCY: OPTIMAL μ_L

Proposition 7 shows how the Pareto optimal allocation can be decentralized. However, the subsidies to innovation require lump-sum taxes and in addition, the government would need to set up and operate a system of licensing fees. In practice, both of these might be difficult.[12] Motivated by this reasoning, we now analyze a constrained efficient policy, where we limit the instruments of the government. In particular, we assume that the government can only affect the standardization cost through IPR regulations restricting the access to new technologies, and ask what would the optimal policy be in this case.[13] More precisely, we find the (constant) μ_L^* that maximizes

---

[11]Note that there is no static distortion due to monopoly pricing. This is because in our model all firms only use inelastically supplied factors (skilled and unskilled labor). Thus, markups do not distort the allocation. In a more general model, subsidies to production would also be needed correct for the static inefficiency.

[12]Some reasons emphasized in the literature why licensing may fail include asymmetric information and bilateral monopoly (e.g., Bessen and Maskin, 2006). See also Chari et al. (2009) on the difficulties of using market signals to determine the value of existing innovations.

[13]We do not consider patent policies explicitly for a number of reasons. Patents are often perceived as offering relatively weak protection of IPR, less than lead time and learning-curve advantage in preventing duplication. Patents disclose information, the application process is often lengthy and cannot prevent competitors from "inventing around" patents. Overall, Levin et al. (1987) found that patents increase imitation costs by 7-15%. This support our approach of modeling IPR protection as an additional cost.

BGP utility:

$$\max_{\mu_L} \rho W = \rho \int_0^\infty \left[\ln\left(\frac{C}{A}\right) + \ln\left(A_0 e^{gt}\right) \right] e^{-\rho t} dt \tag{31}$$

$$= \ln\left(\chi^{ss}\right) + \frac{g^{ss}}{\rho}.$$

The optimal policy maximizes a weighted sum of the consumption level and its growth rate. In turn, $g^{ss}(\mu_L) = n\left[\pi_H(n)/\mu_H - \rho\right]$ evaluated at the $n(\mu_L)$ that solves (27) and $\chi^{ss} = y(n) - g(\mu_L)\left[\mu_H + \mu_L(1-n)\right]$, evaluated at the same n. In general, problem (31) does not have a closed-form solution. Nonetheless, we can make progress by considering two polar cases.

3.2.1 Optimal/Growth Maximizing Policy: $\rho \to 0$

As $\rho \to 0$, the optimal policy is to maximize g^{ss}. For this case, we have simple analytic results. Manipulating the first order condition (29), the optimal n^* is implicitly defined by:

$$\left(\frac{1-n^*}{n^*} h\right)^{\frac{\epsilon-1}{\epsilon}} = 1 - \frac{1}{n^*}\left(1 - \frac{1}{\epsilon}\right) \tag{32}$$

Note that the LHS is decreasing in n, from infinity to zero, while the RHS is increasing in n, ranging from minus infinity to $1/\epsilon$. Thus, the solution n^* is always interior and unique. Using the implicit function theorem yields:

$$\frac{\partial n^*}{\partial \epsilon} \frac{\epsilon}{n^*} = \frac{\epsilon(1-n^*)}{\epsilon-1} > 0 \quad \text{and} \quad \frac{\partial n^*}{\partial h} \frac{h}{n^*} = (1-n^*)(1-\epsilon+n^*\epsilon) > 0 \tag{33}$$

because, from (32), $\epsilon n^* - \epsilon + 1 = \epsilon (n^*)^{\frac{1}{\epsilon}} (1-n^*)^{\frac{\epsilon-1}{\epsilon}} h^{\frac{\epsilon-1}{\epsilon}} > 0$. That is, the optimal fraction of hi-tech goods is increasing in the relative skill-endowment and in the elasticity of substitution across products.

Once we have n^*, we can use the indifference condition between innovation and standardization, $\frac{V_H}{V_L} = \frac{\mu_H}{\mu_L}$, to solve for the μ_L^* that implements n^*. When $\rho \to 0$ and $m = g(1-n)/n$ we obtain:

$$\frac{\pi_H}{\pi_L} = \frac{\mu_H}{\mu_L}\frac{1}{n} \to \left(\frac{1-n}{n}h\right)^{\frac{\epsilon-1}{\epsilon}} = \frac{\mu_H}{\mu_L}\frac{1}{n} \tag{34}$$

The equilibrium fraction of hi-tech goods, n, depends on relative profits (π_H/π_L),

relative R&D costs (μ_H/μ_L), and the standardization risk faced by hi-tech firms (captured by the factor $1/n$). Note that a decline in the relative skill supply, h, leads to a more than proportional fall in n because π_H/π_L falls *and* m rises. Substituting $\left(\frac{1-n^*}{n^*}h\right)^{\frac{\epsilon-1}{\epsilon}}$ from (32) we can find the optimal standardization cost as:

$$\mu_L^* = \mu_H \left(n^* - 1 + \frac{1}{\epsilon}\right)^{-1} = \frac{\mu_H \epsilon}{1 - \epsilon + n^* \epsilon}. \tag{35}$$

This expression has the advantage that it only depends on h through n^* and can be used to study the determinants of the optimal policy. Differentiating (35) and using (33), we obtain the following proposition (proof in the text).

Proposition 8 *Consider the case $\rho \to 0$. BGP welfare and growth are maximized when the cost of standardization μ_L satisfies (35) and n^* is the solution to (32). The following comparative static results hold:*

$$\frac{\partial \mu_L^*}{\partial \mu_H} \frac{\mu_H}{\mu_L^*} = 1$$

$$\frac{\partial \mu_L^*}{\partial h} \frac{h}{\mu_L^*} = -\epsilon n^* (1 - n^*) < 0$$

$$\frac{\partial \mu_L^*}{\partial \epsilon} \frac{\epsilon}{\mu_L^*} = \frac{n^* \epsilon - 1}{\epsilon - 1} > 0.$$

The results summarized in this proposition are intuitive. Changes in the cost of innovation should be followed by equal changes in the cost of standardization, so as to keep the optimal n constant. A decline in the relative supply of skilled workers makes technology diffusion relatively more important. However, the incentive to standardize increases so much (both because of the change in instantaneous profits *and* because the equilibrium m increases too) that the optimal policy is to make standardization more costly. Thus, somewhat surprisingly, a higher abundance of unskilled worker calls for stronger IPR protection. Finally, given that $\epsilon \geq 2$, a lower elasticity of substitution makes the diffusion of technologies to low-skill workers more important for aggregate productivity and reduces the optimal IPR, μ_L^*.[14]

3.2.2 Optimal Policy: high ρ

To understand how the optimal policy changes with the discount rate, we consider the other polar case. In particular, assume that $\rho \to \frac{H+L}{\mu_H \epsilon}$. As we will see, this is

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[14] To see this, recall that $\epsilon n^* - \epsilon + 1 > 0$. Then, $\epsilon \geq 2$ implies $(\epsilon n^* - 1)/(\epsilon - 1) > 0$.

the highest ρ compatible with positive growth. In this case, Section 2.7 shows that g is maximized at the corner $n^{\min} = h/(h + 1)$. Moreover, for $n \to n^{\min}$ we have $\pi_H = \frac{L+H}{\epsilon}$ and $g = \frac{H+L}{\mu_H \epsilon} - \rho \to 0$ (since $\rho \to \frac{H+L}{\mu_H \epsilon}$). Next, the result that g must be close to zero yields $\chi^{ss} = y$, which is also maximized at $n^{\min}$. Thus, with high discounting the optimal policy is the same as the one that maximizes static output (and consumption) only. Reaching this point requires setting $\mu_L^* = \mu_H$. Note that, in this extreme scenario, the optimal policy becomes independent of h and other parameters. Comparing the policy $\mu_L^* = \mu_H$ to (35) shows, not surprisingly, that high discounting implies a lower optimal protection of IPR.

3.3 OTHER COMPETITION POLICIES

In practice, several other competition policies, besides licensing fees and intellectual property rights, are used in order to affect the profitability of standardization. We now briefly discuss the implications of such policies. Suppose that the government can directly affect markups in the hi-tech and the low-tech sectors. In particular, it can set $\epsilon_H \geq \epsilon$ and $\epsilon_L \geq \epsilon$ in the pricing equations (5).

When markups vary across firms, profits (16) become:

$$\pi_L = \frac{y^{1/\epsilon}}{\epsilon_L} \left(\frac{L}{1-n} \right)^{\frac{\epsilon-1}{\epsilon}} \quad \text{and} \quad \pi_H = \frac{y^{1/\epsilon}}{\epsilon_H} \left(\frac{H}{n} \right)^{\frac{\epsilon-1}{\epsilon}} \tag{36}$$

From $r_L(n) = \pi_L / \mu_L$ and the above expressions, it is immediate to see that the BGP n only depends on the product $\mu_L \epsilon_L$. This result highlights that competition policy (ϵ_L) and IPR protection (μ_L) are substitutes. Intuitively, with lower mark-ups (high ϵ_L) for low-tech firms, there is less entry in the L-sector. Yet, the government can offset this effect by reducing μ_L, so that it becomes easier to standardize. On the contrary, $g^{ss}(n)$ does not depend on ϵ_L, so that n^* is as before. Given that intervening on μ_L or ϵ_L is equally effective to implement a desired n^*, the optimal mix depends on the relative costs of the two policies.

Now when we also have $\rho \to 0$, equation (35) becomes:

$$\mu_L = \frac{\epsilon_H}{\epsilon_L} \cdot \frac{\mu_H \epsilon}{1 - \epsilon + n^* \epsilon}. \tag{37}$$

Then, under the assumptions that ϵ_L can be changed at no cost, it is easy to see that

the optimal policy is:

$$\epsilon_H = \epsilon$$
$$\mu_L^* = \mu_L^{\min}$$
$$\epsilon_L = \frac{\epsilon_H}{\mu_L^{\min}} \cdot \frac{\mu_H \epsilon}{1 - \epsilon + n^* \epsilon}$$

where $\mu_L^{\min} \geq 0$ is the minimum "technical" cost of standardization (i.e., with no IPR protection). Intuitively, full monopoly among hi-tech firms ensures high innovation; $\mu_L^* = \mu_{L\,\min}$ minimizes the resources spent on standardization; high competition among low-tech firms yields the optimal n^*. If the desired level of competition ϵ_L cannot be achieved, then μ_L^* should be adjusted upward accordingly.[15]

3.4 North-South Trade and IPR Policy

We now ask how trade opening in countries with a large supply of unskilled workers affects the optimal IPR policy. This question is interesting because there is an unsettled debate on whether trade liberalization in less developed countries should be accompanied by tighter IPR protection, as implied by the TRIPs Agreement, or by less strict IPR policies, which serve to encourage technology diffusion to less advanced economies. We can investigate this question using our model.

Consider an integrated world economy (the North), described by the model in Section 2. For simplicity, let us also assume that there is a single large developing country endowed with unskilled workers only (the South). Without trade, we assume that Northern technologies are copied at no cost by competitive firms in the South. However, this form of technology transfer is imperfect: when a low-tech good is introduced in the South, labor productivity there is only a fraction $\varphi \in (0, 1]$. There is no innovation in the South.

Now imagine that the South opens its economy to trade. We assume that economic integration allows Northern firms to produce in the South. In the new integrated equilibrium factor prices are equalized (or else firms would relocate to the country where

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[15] Another way to highlight the same result is that policy does not affect markups, but rather patent duration in the low-tech sector. In the model considered so far, patent length is infinite in the low-tech sector. Suppose, however, that patent duration is finite and, once the patent expires, the good is produced by unskilled workers under competitive conditions. Here, the key trade-off is between the cost of standardization and the duration of the subsequent monopoly position in the low-tech sector. The gist of the argument is that the best combination is, in a sense, low IPR everywhere (low μ_L and short patents). However, it has to be carefully tailored, since the cost-relative-to-duration must be pinned down so as to get the right n.

labor is cheaper) and Southern firms are replaced by their Northern counterpart. This result stems from the fact that Northern firms are more productive and can capture the entire market by charging a price equal to or lower than the marginal cost of the Southern imitators, $p_L \leq w_L/\varphi$. However, if $\varphi > (1 - 1/\epsilon)$, Northern firms must compress their markup to keep Southern imitators out.

In sum, the effect of trade opening in the South is isomorphic to an increase in the world endowment of L and possibly a reduction in the markup and profit margins of low-tech firms (higher ϵ_L). What are the implications for the BGP growth rate and the optimal IPR policy? The change in L and ϵ_L have opposite effects on π_L (see equation (36)) and hence on the return from standardization, so the $r_L(n)$ curve in Figure 1 may either shift up or down. The $r_H^{ss}(n)$ curve, instead, always shifts up because the greater supply of low-tech goods increases the price and thus the profitability of hi-tech products. As a result, in the new BGP, n^{ss} and g^{ss} may be higher or lower. Despite this ambiguity, it is easy to see that trade opening is necessarily growth (and welfare) enhancing if IPR policy, μ_L, is correctly adjusted. This follows immediately from the upward shift of the $r_H^{ss}(n)$ curve, implying that the maximum attainable r must be higher.

The crucial question, then, is how μ_L should be changed. As already seen, a higher L/H increases the optimal level of IPR protection, μ_L^*. On the other hand, higher competition among low-tech firms, ϵ_L, calls for a reduction in μ_L, to compensate for the fall in profit margins (see equation (37)). The net effect depends on which force dominates. If the liberalizing country is large and inefficient (low φ), the competitive pressure posed by imitators on low-tech firms is weak, while the threat to hi-tech firms, due to the increased incentives to standardize, is high. In this case, integration should be followed by a tightening of IPR policies.[16]

4   EXTENSION: MULTIPLE EQUILIBRIA AND POVERTY TRAPS

We have so far assumed that, at $w_H = w_L$, standardization stops implying that in a BGP n stays in the range $(n^{\min}, 1)$. This is an immediate consequence of the assumption that, at $w_H = w_L$, incumbent hi-tech firms fight (see Proposition 1) so that low-tech firms do not find entry profitable. Under this assumption and Assumption 2, Proposition 4 established the uniqueness of a BGP equilibrium. However, either

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[16]These are the policies that a world planner would choose starting from the optimum. Yet, governments of individual countries face different incentives, because an increase in μ_L leads to a higher skill premium and redistributes income towards skill-abundant countries. This conflict of interests between the North and the South is studied, among others, by Grossman and Lai (2004).

when we adopt the alternative tie-breaking rule—whereby at $w_H = w_L$ hi-tech firms facing the entry of a low-tech competitor exit—or relax Assumption 2, the model may generate multiple equilibria and potential poverty traps. In this section, we briefly discuss this possibility.

For brevity, we focus on the case where Assumption 2 still holds but the competition between hi-tech firms and entrants at $w_H = w_L$ is resolved according to the polar opposite tie-breaking rule. Even under this alternative tie-breaking rule, we still have that $w_H \geq w_L$ for any $n \in [0, 1]$ since skilled worker can always take unskilled jobs. But in contrast to Proposition 1 now standardization may continue even at $n < n^{\min}$.

As a first step in the analysis of this case, we characterize the static equilibrium for low levels of n. Recall that $w_H = w_L$ at $n = n^{\min}$. For $n < n^{\min}$, the skill premium is constant at $w_H = w_L$ and some high skill workers are employed in low-tech firms. In this case, the allocation of labor between the two type of firms, h, is determined endogenously by equation (9) after setting $w_H = w_L$. This yields $h = n/(1 - n)$ and a profit rate of $\pi_H = \pi_L = \frac{L+H}{\epsilon}$. In other words, for sufficiently low n, it is as if workers were perfect substitutes, prices are equalized $p_H = p_L$, and so are profits.

To find the steady states, we draw the $r_L(n)$ and $r_H^{ss}(n)$ schedules over the entire domain $n \in [0, 1]$. Figure 3 shows the determination of n^{ss} for two possible $r_L(n)$ schedules, corresponding to different values of μ_L. Compared to Figure 1, the first part of both schedules is a straight line, as there the skill premium is constant and equal to one. The interior BGPs are again the intersections between the $r_H^{ss}(n)$ (solid line) and $r_L(n)$ (dashed line) schedules.

In addition to balanced growth equilibria, now there might exist "corner steady-states" such that $n = g = m = 0$ and $r = \rho$. A corner steady state can arise in two different circumstances: (i) at $n = 0$, there is no incentive to innovate nor to standardize, i.e., $\rho > \frac{\pi_L}{\mu_L} = \frac{H+L}{\mu_L \epsilon}$ and $\rho > \frac{\pi_H}{\mu_H} = \frac{H+L}{\mu_H \epsilon}$; (ii) at $n = 0$, firms have an incentive to standardize, i.e., $\rho < \frac{H+L}{\mu_L \epsilon}$, but there are no goods to standardize, since $n = 0$. Moreover, innovation is discouraged by the expectation that new hi-tech goods would trigger a high standardization rate. Formally, innovating firms expect that $m > \frac{H+L}{\mu_H \epsilon} - \rho$ whenever $n > 0$. This conjecture does not violate the resource constraint since the absolute investment in standardization would be infinitesimal when $n = 0$ even though the standardization *rate* were high. The uninteresting case in which $r_L(n)$ lays above $r_H^{ss}(n)$ for all n is still ruled out by Assumption 2.

As shown in Figure 3, depending on the standardization cost, there are two regimes:

High μ_L : For $\mu_L > (H + L) / (\rho \epsilon)$ (lower $r_L(n)$ schedule in Figure 3), there is a
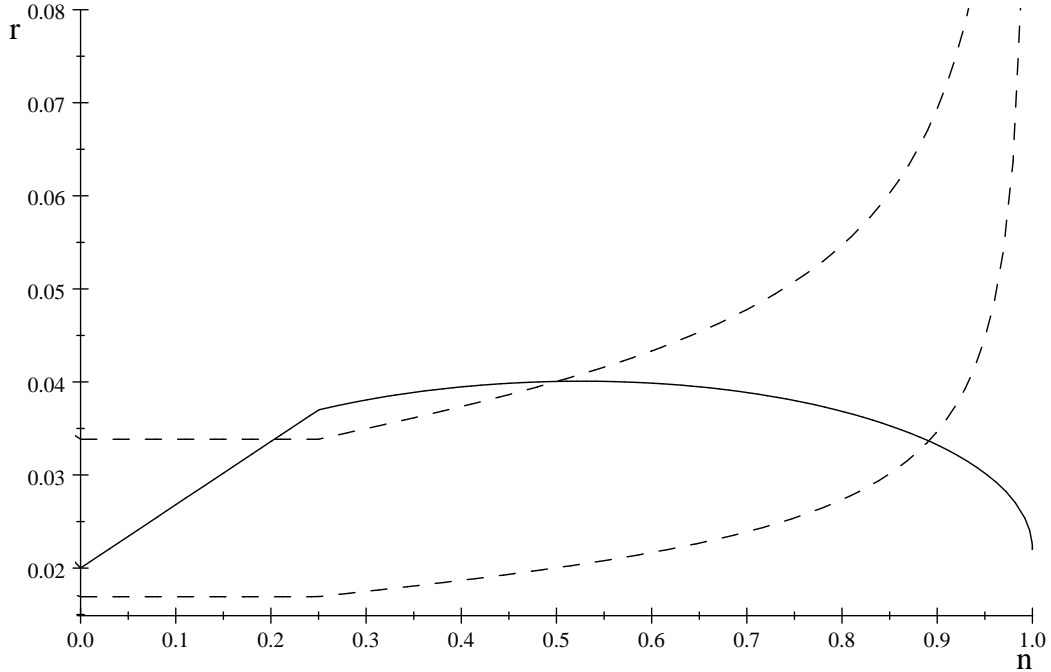
Figure 3: Solid $= r_H^{ss}(n)$, Dashed $= r_L(n)$

unique steady state (BGP) corresponding to the unique crossing point of the $r_L(n)$ and $r_H^{ss}(n)$ schedules.

Low μ_L : For $\mu_L < (H + L) / (\rho\epsilon)$ (upper $r_L(n)$ schedule) there are two interior and a corner steady state. The two interior steady states can be seen in Figure 3. In this case, a corner steady state also exists, since $r_H^{ss}(0) = \rho < r_L(0) = (H + L) / (\mu_L\epsilon)$. Hence, standardization is profitable at $n = 0$.

The reason for the potential multiplicity is a complementarity between investment decisions by firms. If firms expect n to be high in the BGP, they also anticipate a low standardization rate, m, and this encourages further innovation. Greater innovation in turn increases the demand for resources (i.e., the demand for "investment" rather than consumption) and raises the interest rate. A greater interest rate reduces the value of standardization more than the value of innovation), confirming the expectation of a low m. In contrast, when a large fraction of the resources of the economy are devoted to standardization, expected returns from innovation decline and this limits innovative. Expectation of lower innovation reduces the interest rates, leading to reverse reasoning— i.e., encouraging standardization (more than innovation) and

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confirming the expectation of a high m.[17] Note that this complementarity was also present in the model analyzed in the previous sections. Yet, it did not give rise to multiplicity because Assumption 2 guarantees that the other candidate steady states correspond to levels of n below $n^{\min}$, which was ruled out by Proposition 1 under our baseline tie-breaking rule.[18] Thus, the fact that standardization is profitable only if unskilled labor is strictly cheaper than skilled labor prevents the economy from falling to low-growth traps where innovation is discouraged by the expectation of a very fast standardization rate.

We summarize the characterization of the set of steady-state equilibria in the following proposition (proof in the text).

Proposition 9 *Suppose that Assumptions 1 and 2 hold. Then:*

1. *If $\mu_L > (H+L)/(\rho\epsilon)$ there exists a unique BGP which is interior.*

2. *If $\mu_L < (H+L)/(\rho\epsilon)$ there exist two interior BGP equilibria and a corner steady state.*

It is also noteworthy that the non-monotonic relationship between the g^{ss} and μ_L and the policy analysis derived in the previous sections now apply to the higher interior BGP. The main novelty, however, is that too low a cost of standardization may lead to multiple steady states, with the equilibrium determined by self-fulfilling expectations, and stagnation.

5   Concluding Remarks

New technologies often diffuse as a result of costly adoption and standardization decisions. Such standardization also creates cheaper ways of producing new products, for example, substituting cheaper unskilled labor for the more expensive skilled labor necessary for the production of new complex products. This process endogenously

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[17]To see the role of the interest rate, consider a more general formulation of preferences where θ is the inverse of the intertemporal elasticity of substitution. In this case, the Euler equation takes the form $\dot{C}/C = (r - \rho)/\theta$. Using this, equation (26) becomes:

$$r_H^{ss}(n) = \frac{\theta n}{n\theta + 1 - n}\frac{\pi_H}{\mu_H} + \rho\left(\frac{1-n}{n\theta + 1 - n}\right).$$

Note that, as $\theta \to 0$ the $r_H^{ss}(n)$ curve becomes flat and the BGP is necessarily unique.

[18]When Assumption 2 is relaxed, it is possible that the $r_L(n)$ and $r_H^{ss}(n)$ schedules cross twice over the range $n \in \left[n^{\min}, 1\right]$.

generates competition to original innovators. In this paper, we studied the implications of this costly process of standardization, emphasizing both its role as an engine of growth and its potential negative effects on innovation (because of the "business stealing" effect that it creates).

Our analysis has delivered a number of new results. First, the tension between innovation and standardization generates an inverse U-shaped relationship between competition and growth. Second, while technology diffusion is potentially beneficial, it can also have destabilizing effects. Standardization can open the door to multiple equilibria (multiple growth paths). Finally, we characterized the optimal competition and IPR policy and how it depends on endowments and other parameters, such as the elasticity of substitution between products. We found that innovation rents should be protected *more* when skilled workers are perceived as *scarcer*, that is, when they are in short supply and when the elasticity of substitutions between goods is high. We also showed that these results provide new reasons for linking North-South trade to intellectual property rights protection.

It is also worth noting that a key feature of our analysis is the potential competition between standardized products and the original hi-tech products. We believe that this is a good approximation to a large number of cases in which standardization takes place by different firms (and often in the form of slightly different products). Nevertheless, the alternative, in which standardization is carried out by the original innovator, is another relevant benchmark. In our follow-up work, Acemoglu et al. (2010), we study a model of offshoring, where offshoring can be viewed as a costly process of standardization carried out by the original innovator to make goods producible in less developed countries with cheaper labor.

Our model yields a number of novel predictions that can be taken to the data. In particular, it suggests that competition and IPR policy should have an impact on skill premia. Furthermore, data on product and process innovation might be used to test the existence of a trade-off between innovation and standardization at the industry level. These seem interesting directions for future work.

6 APPENDIX

6.1 PROOF OF LEMMA 1

Recall $\pi_H = \frac{p_H H}{\epsilon_H A_H} = \frac{p_H H}{\epsilon_H n A}$. From (11) it is immediate to see that $\frac{\partial \pi_H}{\partial n} < 0$ if $p_H \geq p_L$, which is true in equilibrium. To establish the properties of π_L, note that:

$$\frac{\partial (\epsilon - 1) \ln \pi_L}{\partial n} = \frac{\frac{1}{\epsilon} \left(\frac{n}{1-n}\right)^{\frac{\epsilon+1}{\epsilon}} \left(\frac{1}{n^2}\right) h^{\frac{\epsilon-1}{\epsilon}}}{1 + \left(\frac{n}{1-n}\right)^{\frac{1}{\epsilon}} h^{\frac{\epsilon-1}{\epsilon}}} + \frac{\epsilon - 2}{1 - n} > 0 \ \text{ if } \ \epsilon > 2 - \frac{\frac{1}{\epsilon}\frac{1}{n} h^{\frac{\epsilon-1}{\epsilon}}}{\left(\frac{1-n}{n}\right)^{\frac{1}{\epsilon}} + h^{\frac{\epsilon-1}{\epsilon}}}.$$

For $\epsilon \geq 2$, $\lim_{n \to 1} \frac{\partial \pi_L}{\partial n} = \infty$. Convexity of π_L follows immediately because the function $\frac{\partial \pi_L}{\partial n}$ has no critical point.

6.2 NOTES ON FIGURES

The benchmark economy used to draw all figures has the following parameter values:

$$\rho = 0.02; \ \epsilon = 2; \ \mu_H = 22.7, \ \mu_L = 59.1; \ H = 1; \ L = 3$$

implying in steady state:

$$g = 0.02; \ r = 0.02; \ m = 0.02; \ n = 0.5; \ \frac{w_H}{w_L} = 1.5.$$

6.3 PROOF OF PROPOSITION 4

A BGP must be a rest point of the dynamical system (21)-(22). We first note that there cannot be a rest point at the boundaries $n = n^{\min}$ and $n = 1$ in view of Proposition 1. Thus any BGP must be interior as defined in Proposition 3, or equivalently, it must be a zero of the dynamical system (21)-(22). We denote such a zero by (n^{ss}, χ^{ss}), where n^{ss} satisfies $r_L(n^{ss}) = r_H^{ss}(n^{ss})$ (see again Proposition 3). We prove the existence of a unique interior BGP by showing that there is a unique value $n^{ss} \in (n^{\min}, 1)$ such that $r_L(n^{ss}) = r_H^{ss}(n^{ss})$, that there is a unique corresponding value of χ^{ss} and that at (n^{ss}, χ^{ss}) the transversality condition is satisfied.

We prove the first step by establishing that $r_H^{ss}(n^{ss})$ is a continuous inverse U-shaped function whereas $r_L(n^{ss})$ is a continuous, increasing and convex function. Moreover $r_H^{ss}(n^{\min}) > r_L(n^{\min})$ (which follows immediately from Assumption 2) and $\lim_{n \to 1}(r_L(n^{ss}) - r_H^{ss}(n^{ss})) = \infty$. Then, the intermediate value theorem establishes the existence of such a BGP, while the shape of the two func-

tions implies uniqueness. Let $\phi(x) \equiv \left[x + x \left(\frac{1}{x} - 1 \right)^{\frac{1}{\epsilon}} h^{\frac{1-\epsilon}{\epsilon}} \right]^{\frac{1}{\epsilon-1}}$ where $\epsilon \geq 2$ and $h \geq 0$. Standard algebra establishes that $\phi(x)$ is a continuous inverse U-shaped concave function, such that $\lim_{x \to 0} \phi'(x) = \infty$ and $\lim_{x \to 1^-} \phi'(x) = -\infty$. Thus, $\phi(x)$ has a unique interior maximum in the unit interval. Next, note that $r_H^{ss}(n^{ss}) = \phi(n^{ss}) \cdot H / (\mu_H \epsilon) + (1 - n^{ss}) \rho$. Since $r_H^{ss}(n^{ss})$ is a linear transformation of $\phi(n^{ss})$, it is also a continuous inverse U-shaped concave function, with a unique interior maximum in the unit interval. Consider now $r_L(n^{ss})$. Since $r_L(n^{ss}) = \pi_L(n^{ss}) / \mu_L$, Lemma 1 establishes that $r_L(n^{ss})$ is increasing and convex, with $\lim_{n^{ss} \to 1} r_L(n^{ss}) = \infty$ (implying that $\lim_{n^{ss} \to 1} (r_L(n^{ss}) - r_H^{ss}(n^{ss})) = \infty$).

Next, straightforward algebra immediately implies that, conditional on $n = n^{ss}$, $m = m(n^{ss})$ and $z = z(n^{ss})$ there exists a unique value $\chi = \chi^{ss}$ that yields a zero of the dynamical system (21)-(22). Finally, since in BGP $r = \rho + g > g$ (from (1) and Assumptions 1-2), the transversality condition is satisfied in the unique candidate BGP.

6.4 PROOF OF PROPOSITION 5

Recall that dynamic equilibria are given by solutions to dynamical system (21)-(22) with boundary conditions given by the initial condition $n = n_0$ and the transversality condition. By the same argument as in the proof of Proposition 4, there cannot be any dynamic equilibrium path where $n \to n^{\min}$ and $n \to 1$. Any dynamic equilibrium must thus either converge to the unique (interior) BGP (n^{ss}, χ^{ss}) or involves cycles. We will show in this proof that starting from any initial condition $n = n_0$ in the neighborhood of n^{ss} (the BGP), there exists a unique path converging to (n^{ss}, χ^{ss}) and that there cannot be cycles, thus establishing local saddle-path stability of the dynamic equilibrium.

Because there are two sources of technical change (innovation and standardization), we first distinguish between three possible types of potential dynamic equilibria (which may converge to the BGP, (n^{ss}, χ^{ss})).

CASE 1: $V_H = \mu_H$ and $V_L < \mu_L$ ($\Rightarrow m = 0$ and $g = (y(n) - \chi) / \mu_H$). In this case, from Proposition 3 the dynamics are governed by the following system of ordinary differential equations:

$$\frac{\dot{\chi}}{\chi} = \frac{\pi_H(n)}{\mu_H} - \rho - \frac{y(n) - \chi}{\mu_H} \tag{38}$$

$$\dot{n} = (1 - n) \frac{y(n) - \chi}{\mu_H}.$$

CASE 2: $V_H < \mu_H$ and $V_L = \mu_L$ ($\Rightarrow m = (y(n) - \chi)/(n\mu_L)$ and $g = 0$). In this case, again from Proposition 3 the dynamics are governed by the following system of ordinary differential equations:

$$\frac{\dot{\chi}}{\chi} = \frac{\pi_L(n)}{\mu_L} - \rho \tag{39}$$
$$\dot{n} = -\left(\frac{y(n) - \chi}{\mu_L}\right).$$

CASE 3: $V_H = \mu_H$ and $V_L = \mu_L$ ($\Rightarrow m = \pi_H(n)/\mu_H - \pi_L(n)/\mu_L$ and $g = (y(n) - \mu_L mn - \chi)/\mu_H$). In this case, the dynamics are governed by the following system of ordinary differential equations:

$$\frac{\dot{\chi}}{\chi} = \frac{\pi_L(n)}{\mu_L} - \rho - \frac{y(n) - \mu_L mn - \chi}{\mu_H} \tag{40}$$
$$\dot{n} = (1 - n)\frac{y(n) - \chi}{\mu_H} - mn\left(1 + (1 - n)\frac{\mu_L}{\mu_H}\right).$$

In all three cases, the differential equations are defined over the region $\chi \in [0, y(n)]$, $n \in [n_{\min}, 1]$.

Recall that in the BGP $m > 0$ and $g > 0$. This implies that $V_H = \mu_H$ and $V_L = \mu_L$. Therefore, (n^{ss}, χ^{ss}) is a zero of the dynamical system (40), but it is *not* a zero either of (38) or of (39). Nevertheless, we will show that CASE 3 cannot describe dynamic equilibrium behavior at any point with $(n, \chi) \neq (n^{ss}, \chi^{ss})$. Instead, the equilibrium will be given by either CASE 1 or CASE 2 (depending on whether n is above or below n^{ss}) and will be unique. Then under the equilibrium dynamics, the economy will converge in finite time to (n^{ss}, χ^{ss}), and then a jump in m and r will create a switch to CASE 3 at that point, and since (n^{ss}, χ^{ss}) is a zero of (40), the economy will have reached the BGP and will stay at (n^{ss}, χ^{ss}) thereafter.

We prove by first establishing several Lemmas. First, Lemma 2 establishes that, if $n < n^{ss}$, there exists a unique trajectory converging to (n^{ss}, χ^{ss}) (it is immediate that, if $n < n^{ss}$, there exists no trajectory converging to (n^{ss}, χ^{ss}) following the dynamics (39), since these would imply $\dot{n} \leq 0$). Second, Lemma 3 establishes that, if $n > n^{ss}$, there exists a unique trajectory converging to (n^{ss}, χ^{ss}) following the dynamics (39) (it is immediate that, if $n > n^{ss}$, there exists no trajectory converging to (n^{ss}, χ^{ss}) following the dynamics given by (38), since these imply $\dot{n} \geq 0$). Third, Lemma 4 provides a complete characterization of equilibrium dynamics when the transition involves either only innovation or only standardization, followed by a jump

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in either the innovation of standardization rate as the economy reaches (n^{ss}, χ^{ss}), but continuous changes in asset values. Fourth, Lemma 5 establishes that (under the sufficient conditions of the Proposition) there exists no trajectory converging to (n^{ss}, χ^{ss}) following the dynamics (40). Finally, Lemma 6 rules out transitional dynamics in the neighborhood of the BGP in which there is a jump from CASE 3 to either CASE 1 or CASE 2 or between CASE 1 and CASE 2. These lemmas together establish local saddle-path stability.

Lemma 2 *Suppose $n_0 < n^{ss}$. Then, there exists a unique trajectory attaining (n^{ss}, χ^{ss}) in finite time following the dynamics of CASE 1, (38). This trajectory features monotonic convergence in n ($\dot{n} > 0$).*

Proof. Consider the phase diagram depicting the system of differential equations (38) shown in Figure 4. This system has no zero over the feasible region $[n_{\min}, 1] \times [0, y(n)]$ (i.e., over the region where $n \in [n_{\min}, 1]$ and $\chi \in [0, y(n)]$). In particular, in the interior of the region $[n_{\min}, 1] \times [0, y(n)]$, and hence at (n^{ss}, χ^{ss}), $\dot{n} > 0$ and $\dot{\chi} \gtreqqless 0 \Leftrightarrow \chi \gtreqqless \hat{\chi}(n)$, where $\hat{\chi}(n) = \mu_H \rho + y(n) - \pi_H(n) < y(n)$. The last inequality follows from Assumption 1. Although whether $\hat{\chi}(n^{ss}) \gtreqqless \chi(n^{ss})$ is in general ambiguous, the phase diagram shows that there is a unique trajectory (and a unique initial level of the control variable, χ_0) converging in finite time to (n^{ss}, χ^{ss}). In particular, since (n^{ss}, χ^{ss}) is not a zero of the system (38), the determination of the converging trajectory can be expressed as an initial value problem with (n_T^{ss}, χ_T^{ss}) being the boundary (terminal) condition. From the standard result of existence and uniqueness of solutions for systems of ordinary differential equations, this initial value problem has a unique solution. Fixing the initial condition n_0 yields a unique solution for T (the length of the transition) and χ_0. The monotonicity of the dynamics of n ensures that this solution is unique, i.e., there does not exist two solutions (n_0, T, χ_0) and (n_0, T', χ_0') with $T \neq T'$ and $\chi_0 \neq \chi_0'$. This argument also proves that convergence is attained in finite time. ■

Lemma 3 *Suppose $n_0 > n^{ss}$. Then, there exists a unique trajectory attaining (n^{ss}, χ^{ss}) in finite time following the dynamics of CASE 2, (39). This trajectory features monotonic convergence ($\dot{n} < 0$ and $\dot{\chi} > 0$).*

Proof. The proof is similar to that of Lemma 2. The dynamical system again has no zero over the feasible region $[n_{\min}, 1] \times [0, y(n)]$. In particular, for $n_0 \geq n^{ss}$, $\dot{n} > 0$ and $\dot{\chi} > 0$. The latter follows from the observation that $\dot{\chi} \gtreqqless 0 \Leftrightarrow \pi_L(n)/\mu_L \gtreqqless \rho$,

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Figure 4: Saddle Path, $n_0 < n^{ss}$

where $\pi'_L(n) < 0$ and $\pi_L(n^{ss})/\mu_L > \rho$, implying that $\pi_L(n)/\mu_L > \rho$ for all $n \geq n^{ss}$. The phase diagram in Figure 5 shows that there is a unique trajectory converging in finite time to (n^{ss}, χ^{ss}). This trajectory features monotonic dynamics. ∎

The previous two Lemmas together imply our key characterization result.

Lemma 4 *There exists equilibrium dynamics with the following characteristics.*
If $n_0 < n^{ss}$, the economy converges in finite time to (n^{ss}, χ^{ss}) following the system of differential equations (38), with monotonic convergence in n. Throughout this convergence, $V_H = \mu_H$, $V_L < \mu_L$, $m = 0$ and $g = (y(n) - \chi)/\mu_H$. When the economy reaches (n^{ss}, χ^{ss}), there is a discrete increase in standardization offset by a fall in the interest rate such that $r(n^{ss}) = r'(n^{ss}) + m(n^{ss})$. Thereafter, $V_H = \mu_H$ and $V_L = \mu_L$. If $n_0 > n^{ss}$ the economy converges in finite time to (n^{ss}, χ^{ss}) following the system of differential equations (39), with monotonic convergence in n and χ. Throughout this convergence, $V_H < \mu_H$, $V_L = \mu_L$, $m = (y(n) - \chi)/(n\mu_L) > 0$ and $g = 0$. When the economy reaches (n^{ss}, χ^{ss}), there is a discrete fall in standardization offset by an increase in innovation such that $y(n^{ss}) - \chi$ remains constant. Thereafter, $V_H = \mu_H$ and $V_L = \mu_L$.

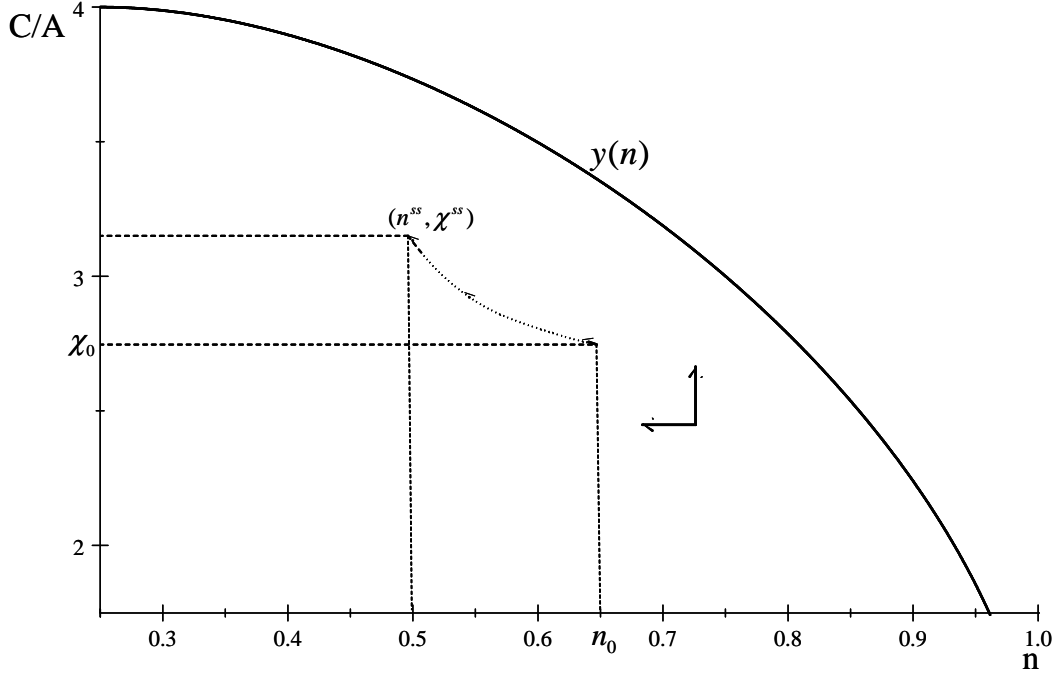**Proof.** The proof follows from Lemmas 2 and 3 combined with the following obser-

Figure 5: Saddle Path, $n_0 > n^{ss}$

vations. Suppose we start with $n_0 < n^{ss}$, then the dynamic equilibrium is given by the system of differential equations (38), so from Lemma 2 until T, we have $m = 0$. At T, we reach (n^{ss}, χ^{ss}) and m jumps from zero to its steady state, m^{ss}. This is offset by an equal jump down in r implying that V_H does not change (i.e., it remains at $V_H = \mu_H$). Moreover, at T, V_L attains its steady state (BGP) value, $V_L = \mu_L$. Note that there is no discontinuity in the asset value V_L, since the change in r and m are perfectly anticipated, causing a continuous change in the value V_L before the actual change occurs to reach $V_L = \mu_L$ exactly at T (the continuity of V_L ensures that there is no arbitrage opportunity in buying and selling shares of L-sector firms). Thus at this point, the dynamics switch to those given by the system of differential equations (40) with both innovation and standardization. Since (n^{ss}, χ^{ss}) is a zero of (40), the economy stays at (n^{ss}, χ^{ss}) thereafter. The fact that this path satisfies the transversality condition follows by the same argument as in the proof of Proposition 4.

Next, suppose that we start with $n_0 > n^{ss}$. Then the dynamic equilibrium is given by the system of differential equations (39) and from Lemma 3, until T, $g = 0$. At T, investment in standardization and m_T fall discretely (the latter declining to m^{ss}), and investment in innovation and g jumps up (the latter increasing to g^{ss}). There is

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no change in overall investment and thus neither r, nor consumption nor V_L change at T. As m jumps down, V_H attains its stead state value, $V_H = \mu_H$ exactly at T (note that the path of V_H is continuous at T, as the change in m is perfectly anticipated by investors). As a result, again at T, the dynamics switch to those given by the system of differential equations (40) with both innovation and standardization. Since (n^{ss}, χ^{ss}) is a zero of (40), the economy stays at (n^{ss}, χ^{ss}) thereafter. The fact that this path satisfies the transversality condition again follows by the same argument.. ■

We next show that transitional dynamics converging to (n^{ss}, χ^{ss}) *cannot* feature *both* $V_H = \mu_H$ and $V_L = \mu_L$ since the system (40) is unstable in the neighborhood of (n^{ss}, χ^{ss}).

Lemma 5 *Suppose $n_0 \neq n^{ss}$. Then, under the (sufficient) conditions of the Proposition, there exists no trajectory converging to (n^{ss}, χ^{ss}) following the dynamical system (40).*

Proof. The proof, which is long, is presented in the next subsection. ■

Lemmas 2-5 establish that in the neighborhood of (n^{ss}, χ^{ss}), we must have either $V_H < \mu_H$ or $V_L < \mu_L$, implying that there is either only innovation or only standardization. However, the results established so far do not rule out "switches" between different regimes while $n_t \neq n^{ss}$, and thus cycles. Moreover, with such switches, the equilibrium might also be indeterminate, with multiple paths starting from some initial n_0 converging to the BGP. Lemma 6 rules out all of these possibilities by showing that in the neighborhood of the BGP, there cannot be a switch from the dynamics given by any one of (38), (39) and (40) to one of the other two. (For notational convenience, in this lemma, we write $V_{H,t} = \mu_H$ to mean that $V_{H,t'} = \mu_H$ for t' in a neighborhood of t).

Lemma 6 *Consider an equilibrium trajectory in the neighborhood of the BGP, (n^{ss}, χ^{ss}). Then, there cannot be a switch from any one of (38), (39) and (40) to one of the other two, i.e., if at t_0 in the neighborhood of (n^{ss}, χ^{ss}), we have $V_{H,t_0} = \mu_H$ and $V_{L,t_0} = \mu_L$, then an equilibrium cannot involve $V_{H,t} < \mu_H$ and/or $V_{L,t} < \mu_L$ for $t > t_0$; if we have $V_{H,t_0} = \mu_H$ and $V_{L,t_0} < \mu_L$, then an equilibrium cannot involve $V_{H,t} < \mu_H$ and/or $V_{L,t} = \mu_L$ for $t > t_0$; and if we have $V_{H,t_0} < \mu_H$ and $V_{L,t_0} = \mu_L$, then an equilibrium cannot involve $V_{H,t} = \mu_H$ and/or $V_{L,t} < \mu_L$ for $t > t_0$.*

Proof. We will prove that if in the neighborhood of (n^{ss}, χ^{ss}), we have $V_{H,t_0} = \mu_H$ and $V_{L,t_0} = \mu_L$, then an equilibrium cannot involve $V_{H,t} = \mu_H$ and $V_{L,t} < \mu_L$ for $t > t_0$. The other cases are analogous.

Suppose to obtain a contradiction that this is the case and denote the last instance where $V_L = \mu_L$ by T (i.e., $V_{L,T+\varepsilon} < \mu_L$ for $\varepsilon > 0$. We need to distinguish two cases. First, $V_{L,t'} < \mu_L$ for all $t' > t$, and second, there exists $T' > t$, such that we again have $V_{L,T'} = \mu_L$.

Case 1: the fact that $V_{L,t'} < \mu_L$ for all $t' > t$ contradicts the hypothesis that the equilibrium path will converge to the BGP.

Case 2: we write $V_{L,T}$ as follows:

$$
\begin{aligned}
V_{L,T} &= \int_T^\infty \exp\left(-\int_T^\tau r(n_\nu)d\nu\right) \pi_L\left(n_\tau\right) d\tau \\
&= \int_T^{T'} \exp\left(-\int_T^\tau r(n_\nu)d\nu\right) \pi_L\left(n_\tau\right) d\tau + \exp\left(-\int_T^{T'} r\left(n_\tau\right) d\tau\right) \mu_L,
\end{aligned}
$$

where the equality exploits the fact that by hypothesis $V_{L,T'} = \mu_L$. Moreover, we also have, again by hypothesis, that $V_{L,T} = \mu_L$, which implies

$$
\int_T^{T'} \exp\left(-\int_T^\tau r(n_\nu)d\nu\right) \pi_L\left(n_\tau\right) d\tau = \left(1 - \exp\left(-\int_T^{T'} r\left(n_\tau\right) d\tau\right)\right) \mu_L. \quad (41)
$$

Suppose next that $n_T > n^{ss}$. By the instability result in Lemma 5, this implies $\dot{n}_T > 0$ and thus $n_\tau > n^{ss}$ for all $\tau \in [T, T']$. But then from Lemma 1, $\pi_L(n_\tau) > \pi_L(n^{ss})$ for all $\tau \in [T, T']$. Moreover, since $V_{H,\tau} = \mu_H$ and $V_{L,\tau} < \mu_L$, we also have that for all $\tau \in [T, T']$,

$$
r\left(n_\tau\right) = \frac{\pi_H\left(n_\tau\right)}{\mu_H} > \frac{\pi_L\left(n_\tau\right)}{\mu_L} > \frac{\pi_L\left(n^{ss}\right)}{\mu_L} = r\left(n^{ss}\right),
$$

where the second inequality again follows from Lemma 1 in view of the fact that $n_\tau > n^{ss}$ for all $\tau \in [T, T']$. But then,

$$
\begin{aligned}
\int_T^{T'} \exp\left(-\int_T^\tau r(n_\nu)d\nu\right) \pi_L\left(n_\tau\right) d\tau &< \left(\int_T^{T'} \exp\left(-\int_T^\tau r(n_\nu)d\nu\right) d\tau\right) \pi_L\left(n_{ss}\right) \\
&< \left(1 - \exp\left(-\int_T^{T'} r(n_\tau)d\tau\right)\right) \mu_L,
\end{aligned}
$$

where the second inequality follows from the fact that $r\left(n_\tau\right) < r\left(n^{ss}\right)$ for all $\tau \in [T, T']$. This inequality contradicts (41).

Suppose instead that $n_T < n^{ss}$. By the instability result in Lemma 5, $\dot{n}_T < 0$ and thus $n_\tau < n^{ss}$ for all $\tau \in [T, T']$. Moreover, by the same reasoning for all $\tau \in [T, T']$,

$\pi_H(n_\tau) > \pi_H(n^{ss})$ and since $V_{L,\tau} < \mu_L$, $m(n_\tau) = 0$. Therefore,

$$
\begin{aligned}
V_{H,T} &= \int_T^\infty \exp\left(-\int_T^\tau (r(n_\nu) + m(n_\nu))\, d\nu\right) \pi_H(n_\tau)\, d\tau \\
&= \int_T^{T'} \exp\left(-\int_T^s r(n_\nu) d\nu\right) \pi_H(n_\tau)\, d\tau + \exp\left(-\int_T^{T'} r(n_\tau) d\tau\right) \mu_H. \quad (42)
\end{aligned}
$$

But since for all $\tau \in [T, T']$

$$
r(n_\tau) = \frac{\pi_H(n_\tau)}{\mu_H} > \frac{\pi_H(n^{ss})}{\mu_H}
$$

the first term in (42) is strictly greater than $\left(1 - \exp\left(-\int_t^T r(n_\tau)\, d\tau\right)\right) \mu_H$ and thus contradicts $V_{H,T} = \mu_H$. ∎

Lemmas 2-6 establish the results of the Proposition. In particular, Lemmas 5 and 6 imply that starting at $n_t \neq n^{ss}$ in the neighborhood of the BGP we must have $V_H < \mu_H$ or $V_L < \mu_L$. If $V_H = \mu_H$ and $V_L = \mu_L$, either we diverge from the BGP in view of 5, or we have to switch to a regime where $V_H < \mu_H$ or $V_L < \mu_L$, which is ruled out by Lemma 6. If we have $V_H < \mu_H$ or $V_L < \mu_L$ in the neighborhood of the BGP, then Lemmas 2-4 imply that there exists a unique path converging to the BGP. This completes the proof of the Proposition.

6.5   PROOF OF LEMMA 5

We take a linear approximation of the dynamical system (40) around (n^{ss}, χ^{ss}) :

$$
\begin{aligned}
\frac{\dot{\chi}}{\chi} &\simeq F_\chi(\chi^{ss}, n^{ss}) \cdot (\chi - \chi^{ss}) + F_n(\chi^{ss}, n^{ss}) \cdot (n - n^{ss}) \\
\frac{\dot{n}}{n} &\simeq G_\chi(\chi^{ss}, n^{ss}) \cdot (\chi - \chi^{ss}) + G_n(\chi^{ss}, n^{ss}) \cdot (n - n^{ss})
\end{aligned}
$$

where subscripts denote partial derivatives and

$$
\begin{aligned}
F(\chi, n) &\equiv r(n) - \rho - \frac{y(n) - \mu_L m(n)n - \chi}{\mu_H}, \\
G(\chi, n) &\equiv \left(\frac{1-n}{n}\right) \frac{y(n) - \chi}{\mu_H} - m(n)\left(1 + (1-n)\frac{\mu_L}{\mu_H}\right).
\end{aligned}
$$

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implying that:

$$F_\chi(\chi, n) = \frac{1}{\mu_H} > 0, \; G_\chi(\chi, n) = -\left(\frac{1 - n^{ss}}{n^{ss}}\right)\frac{1}{\mu_H} < 0.$$

Solving for the schedules such that, respectively, $\dot{\chi} = 0$ and $\dot{n} = 0$ yields:

$$\begin{aligned}
\chi(n)\big|_{\dot{\chi}=0} &= y(n) - \mu_L m(n) n - \mu_H(r(n) - \rho) \\
\chi(n)\big|_{\dot{n}=0} &= y(n) - \mu_L m(n) n - \mu_H m(n)\left(\frac{n}{1-n}\right),
\end{aligned}$$

with slopes:

$$\chi'(n)\big|_{\dot{\chi}=0} = -\frac{F_n(\chi^{ss}, n^{ss})}{F_\chi(\chi^{ss}, n^{ss})} \quad \text{and} \quad \chi'(n)\big|_{\dot{n}=0} = -\frac{G_n(\chi^{ss}, n^{ss})}{G_\chi(\chi^{ss}, n^{ss})}.$$

Suppose there were trajectories featuring both innovation and standardization converging to (n^{ss}, χ^{ss}). Then, either one or both eigenvalues of the linearized system would be negative. We show that this is impossible and that under the sufficient conditions of the Proposition both eigenvalues must be positive. Let the two eigenvalues of the linearized system be denoted by λ_1 and λ_2. We know that

$$\begin{aligned}
\lambda_1 + \lambda_2 &= F_\chi(\chi^{ss}, n^{ss}) + G_n(\chi^{ss}, n^{ss}) \\
\lambda_1 \cdot \lambda_2 &= F_\chi(\chi^{ss}, n^{ss}) \cdot G_n(\chi^{ss}, n^{ss}) - F_n(\chi^{ss}, n^{ss}) \cdot G_\chi(\chi^{ss}, n^{ss}).
\end{aligned}$$

Claim 1 *The following inequality holds*

$$-\frac{F_n(\chi^{ss}, n^{ss})}{F_\chi(\chi^{ss}, n^{ss})} < -\frac{G_n(\chi^{ss}, n^{ss})}{G_\chi(\chi^{ss}, n^{ss})}.$$

Hence, $\lambda_1 \cdot \lambda_2 > 0$

Proof. We need to show that $\chi'(n^{ss})\big|_{\dot{\chi}=0} < \chi'(n^{ss})\big|_{\dot{n}=0}$. Define $\Delta(n) = \chi(n)\big|_{\dot{\chi}=0} - \chi(n)\big|_{\dot{n}=0} = \mu_H m(n)\left(\frac{n}{1-n}\right) - \mu_H(r(n) - \rho)$. We know that $m(n) = 0$ for $n \geq n^{\max} > n^{ss}$. Thus, at $n = n^{\max}$ we have:

$$\Delta(n^{\max}) = -\mu_H(r(n) - \rho) = -\mu_H\left(\frac{\pi_H(n)}{\mu_H} - \rho\right) < 0$$

by Assumption 1. Next, recall that at $n^{\min} = \frac{H}{H+L}$ we have $y(n) = H + L$, $m(n) =$

$\frac{H+L}{\epsilon}\left(\frac{1}{\mu_H}-\frac{1}{\mu_L}\right)$ and $r\left(n\right)=\frac{H+L}{\epsilon\mu_L}$. Thus:

$$\Delta\left(n^{\min}\right)=\mu_H\frac{H+L}{\epsilon}\left(\frac{1}{\mu_H}-\frac{1}{\mu_L}\right)h-\mu_H\left(\frac{H+L}{\epsilon\mu_L}-\rho\right)>0$$

by Assumption 2. Moreover, we know that n^{ss} is unique. Thus, $\chi\left(n^{ss}\right)\big|_{\dot{\chi}=0}=\chi\left(n^{ss}\right)\big|_{\dot{n}=0}$ for a unique value of n^{ss}. Then, by the intermediate value theorem, $\chi\left(n\right)\big|_{\dot{\chi}=0}>\chi\left(n\right)\big|_{\dot{n}=0}$ for all $n<n^{ss}$ and $\chi\left(n\right)\big|_{\dot{\chi}=0}<\chi\left(n\right)\big|_{\dot{n}=0}$ for all $n>n^{ss}$. ∎

Since we know that $\lambda_1\cdot\lambda_2>0$, showing that $\lambda_1+\lambda_2>0$ establishes that the system has two positive eigenvalues and is therefore unstable. The condition $\lambda_1+\lambda_2>0$ can be written as:

$$\chi'\left(n\right)\big|_{\dot{n}=0}=-\frac{G_n\left(\chi^{ss},n^{ss}\right)}{G_\chi\left(\chi^{ss},n^{ss}\right)}>\frac{F_\chi\left(\chi^{ss},n^{ss}\right)}{G_\chi\left(\chi^{ss},n^{ss}\right)}=-\left(\frac{n}{1-n}\right)\frac{1}{\mu_H}. \tag{43}$$

A sufficient condition for (43) to be satisfied is that the locus $\dot{n}=0$ be upward sloping in a neighborhood of the BGP. Let us first consider the case where $\rho\to 0$. Using $y\left(n\right)=\epsilon\left[n\pi_H\left(n\right)+\left(1-n\right)\pi_L\left(n\right)\right]$ and $m\left(n\right)=\frac{\pi_H(n)}{\mu_H}-\frac{\pi_L(n)}{\mu_L}$ into $\chi\left(n\right)_{|\dot{n}=0}$:

$$\chi\left(n\right)\big|_{\dot{n}=0}=y\left(n\right)-m\left(n\right)\left(\frac{n}{1-n}\mu_H+n\mu_L\right)=\left[\pi_H\left(n\right)n\right]\cdot A\left(n\right)$$

where $A\left(n\right)\equiv\left[\frac{\mu_L}{\mu_H}+\frac{\pi_L(n)}{\pi_H(n)}\left(1+\frac{\epsilon}{n}\right)+\frac{1}{1-n}\left(\frac{\pi_L(n)}{\pi_H(n)}\frac{\mu_H}{\mu_L}-1\right)\right]$. We know that the factor $\pi_H\left(n\right)n$ is inverted U-shaped, with a maximum at $n^*>n_{\min}$ (it corresponds to the maximum g characterized in Section 2.7). Thus, for $n\in\left[n_{\min},n^*\right]$ a sufficient condition for $\chi'\left(n\right)_{|\dot{n}=0}>0$ is $\frac{\partial A(n)}{\partial n}>0$:

$$\frac{\partial A\left(n\right)}{\partial n}=\frac{\partial}{\partial n}\left(\frac{\pi_L\left(n\right)}{\pi_H\left(n\right)}\right)\left(1+\frac{\epsilon}{n}\right)-\frac{\pi_L\left(n\right)}{\pi_H\left(n\right)}\frac{\epsilon}{n^2}+\frac{1}{\left(1-n\right)^2}\left(\frac{\pi_L\left(n\right)}{\pi_H\left(n\right)}\frac{\mu_H}{\mu_L}-1\right)+\frac{\partial}{\partial n}\left(\frac{\pi_L\left(n\right)}{\pi_H\left(n\right)}\right)\frac{1}{1-n}$$

For $\rho\to 0$, in the BGP we have $\frac{\pi_L(n)}{\pi_H(n)}=\frac{\mu_L}{\mu_H}n$ and $\frac{\partial}{\partial n}\left(\frac{\pi_L(n)}{\pi_H(n)}\right)=\frac{\epsilon-1}{\epsilon}\frac{\pi_L(n)}{\pi_H(n)}\frac{1}{(1-n)n}=\frac{\epsilon-1}{\epsilon(1-n)}\frac{\mu_L}{\mu_H}$. Thus:

$$\frac{\partial A\left(n\right)}{\partial n}=\frac{\mu_L}{\left(1-n\right)\mu_H}\left[\frac{\epsilon-1}{\epsilon}+\epsilon-\frac{1}{n}+\frac{\epsilon-1}{\epsilon\left(1-n\right)}+\frac{\mu_H}{\mu_L}\right].$$

A sufficient condition for $\chi'\left(n\right)\big|_{\dot{n}=0}>0$ is then:

$$\frac{\epsilon-1}{\epsilon}+\epsilon-\frac{1}{n}+\frac{\epsilon-1}{\epsilon\left(1-n\right)}+\frac{\mu_H}{\mu_L}>0$$

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Finally, note that in the BGP $\frac{\pi_L(n)}{\pi_H(n)} = \frac{\mu_L}{\mu_H}n \rightarrow \frac{\mu_H}{\mu_L} = \frac{\pi_H(n)}{\pi_L(n)}n = \left(\frac{1-n}{n}h\right)^{\frac{\epsilon-1}{\epsilon}}n$. Thus, the sufficient condition become:

$$\frac{\epsilon-1}{\epsilon} + \epsilon - \frac{1}{n} + \frac{\epsilon-1}{\epsilon(1-n)} + \left(\frac{1-n}{n}h\right)^{\frac{\epsilon-1}{\epsilon}}n > 0$$

Since this expression is increasing in n, we only need to verify that it is positive at $n_{\min}$. At $n_{\min}$ we have $\left(\frac{1-n}{n}h\right)^{\frac{\epsilon-1}{\epsilon}} = 1$ and the condition becomes:

$$\frac{\epsilon-1}{\epsilon} + \epsilon - \frac{1}{n} + \frac{\epsilon-1}{\epsilon(1-n)} + n > 0. \tag{44}$$

Substituting $n_{\min} = h/(1+h)$ yields (28) in the text.

Finally, for $n > n^*$, rewrite the necessary condition as:

$$\frac{\chi'(n)|_{\dot{n}=0}}{\chi(n)|_{\dot{n}=0}} = \frac{\pi'_H(n)}{\pi_H(n)} + \frac{1}{n} + \frac{\partial A}{\partial n}\frac{1}{A} > -\left(\frac{n}{1-n}\right)\frac{1}{\mu_H\chi(n)|_{\dot{n}=0}}$$

From equations (16) and (17):

$$\frac{\pi'_H(n)}{\pi_H(n)} + \frac{1}{n} = \frac{1}{\epsilon}\frac{[y(n)]^{\frac{1-\epsilon}{\epsilon}}}{\epsilon-1}\left(\frac{H}{n}\right)^{\frac{\epsilon-1}{\epsilon}} - \frac{1}{\epsilon}\frac{[y(n)]^{\frac{1-\epsilon}{\epsilon}}}{\epsilon-1}\left(\frac{L}{1-n}\right)^{\frac{\epsilon-1}{\epsilon}} + \frac{1}{n\epsilon}.$$

Substituting $[y(n)]^{\frac{\epsilon-1}{\epsilon}} = (1-n)^{\frac{1}{\epsilon}}L^{\frac{\epsilon-1}{\epsilon}} + n^{\frac{1}{\epsilon}}H^{\frac{\epsilon-1}{\epsilon}}$ into this expression, we have

$$-\frac{1}{\epsilon(\epsilon-1)}\frac{(1-n)^{-1}}{1+\left(\frac{n}{1-n}\right)^{\frac{1}{\epsilon}}h^{\frac{\epsilon-1}{\epsilon}}} < \frac{\pi'_H(n)}{\pi_H(n)} + \frac{1}{n}$$

This implies that $\frac{\partial A(n)}{\partial n}\frac{1}{A(n)} > \frac{1}{\epsilon(\epsilon-1)}\frac{\frac{1}{1-n}}{1+\left(\frac{n}{1-n}\right)^{\frac{1}{\epsilon}}h^{\frac{\epsilon-1}{\epsilon}}}$ is sufficient for $\chi'(n)|_{\dot{n}=0} > 0$. Now using the fact that in BGP $\frac{\pi_L(n)}{\pi_H(n)} = \frac{\mu_L}{\mu_H}n$, we obtain that $A(n) = 1 - \frac{\mu_H}{\mu_L} + n + \epsilon$. Thus, a sufficient condition for $\chi'(n)|_{\dot{n}=0} > 0$ is:

$$\frac{\frac{\epsilon-1}{\epsilon} + \epsilon - \frac{1}{n} + \frac{\epsilon-1}{\epsilon(1-n)} + \frac{\mu_H}{\mu_L}}{1 - \frac{\mu_H}{\mu_L} + n + \epsilon} > \frac{1}{\epsilon(\epsilon-1)}\frac{1}{1+\left(\frac{n}{1-n}\right)^{\frac{1}{\epsilon}}h^{\frac{\epsilon-1}{\epsilon}}} \tag{45}$$

The LHS is increasing in n, the RHS is decreasing. Thus, if this condition is satisfied at n^*, it will also be satisfied for higher values of n. Since $n^* \geq 1/2$ (see (32)), it can be verified that condition (45) is always satisfied when (44) holds. In sum, (44) is sufficient to prove that the dynamical system with both innovation and standardization

is locally unstable in the limit where $\rho \to 0$. By continuity, the same result applies for $\rho < \bar{\rho}$ for some $\bar{\rho} > 0$ sufficiently small.

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