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ABSTRACT

Pollution taxes are believed to burden low-income households that spend a greater than average share of income on pollution-intensive goods. Some propose to offset that effect by returning revenue to low-income workers via reduced labor tax. We build analytical general equilibrium models with both skilled and unskilled labor, and we solve for expressions that show the change in the real net wage of each group. A decomposition shows the effect of the tax rebate, the effect on the uses side of income (higher product prices), and the effect on the sources side of income (relative wage rates). We also include numerical examples. Even though the pollution tax injures both types of labor, we find that returning all of the revenue to the low-skilled workers is still not enough to offset the effect of higher product prices. Moreover, changing wage rates may further hurt low-skilled labor. In almost all of our examples, the rebate of all revenue to low-skilled labor still does not prevent a reduction in their overall real net wage.

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Market-based pollution policies, such as taxes or tradable emissions permits, can efficiently address environmental problems by forcing polluters to take into account the full cost of pollution. In order to evaluate policies and determine the tradeoffs among competing goals of economic efficiency and distributional objectives, it is important to know not only the overall costs and benefits of environmental policies but also the effects they have on different people. Additionally, the distribution of the costs depends on what is done with any revenue. Emissions permits that are distributed freely to polluting firms do not generate any government revenue, but pollution taxes or auctioned permits do. The way in which the government spends this revenue or redistributes it to taxpayers helps determine the final incidence of the policy.

One of the common arguments against pollution taxes is that households with lower income tend to spend a larger fraction of their income on polluting or pollution-intensive goods, so a tax that raises the prices of polluting goods may disproportionately hurt those with low income. However, a pollution tax may affect not only output prices, but relative wage rates as well. To consider both the uses side and the sources side of income, this paper employs a new analytical general equilibrium model in the spirit of Harberger (1962) to solve for the incidence of a pollution tax when the government has a revenue neutrality requirement. Unlike the standard Harberger model with labor and capital, however, our simple model assumes two types of labor: skilled and unskilled labor. Any exogenous shock such as a change in the pollution tax can change the demand for skilled labor relative to unskilled labor, and thus can affect relative wage rates. We assume that low-income unskilled workers spend a higher fraction of income on the polluting good, and that government tries to offset this effect by using pollution tax revenue to reduce pre-existing labor taxes on low-income unskilled workers. Then we can solve for effects on each wage rate to determine the distribution of net burdens from this overall tax reform. We show that this rebate is not enough to offset higher prices for pollution-intensive goods such as gasoline, electricity, and home heating oil. Moreover, changes in factor prices can further burden low-income families.

Related to this topic is a literature on the “double dividend”, the notion that an environmental tax reform can clean up the environment and generate revenue to cut pre-existing distortionary taxes. Several studies using analytical and computational general equilibrium models have studied this question about revenue-neutral environmental policy reform, but the question is about efficiency rather than distributional effects.[1] Although some of these studies solve for the change in wage rate, they do not discuss equity issues and do not report whether the real low-skilled wage rises or falls.

A few papers do consider the distributional side of an environmental tax swap, using computational models. Using data on the carbon content of different products and the purchases of different income groups, Metcalf (1999, 2009) shows how an environmental tax reform could be made less regressive or perhaps not regressive at all. Using both annual income and lifetime income as measures of well-being, he finds that the use of pollution tax revenue to reduce taxes for low-income households may be distributionally neutral or even slightly progressive. Burtraw, Sweeney and Walls (2009) also calculate incidence based on product price increases (without factor price changes). Dinan and Rogers (2002) use a general equilibrium model with labor and capital, but they do not show distributional effects on the sources side. They study the distributional effects of a permit system that seeks a 15% decrease in carbon emissions, where the permits are either auctioned off or given away.[2] West and Williams (2004) empirically estimate the uses-side incidence of a gasoline tax under several scenarios, one of which is using tax revenue to decrease the tax on labor. They find a gain in efficiency and a decrease in the regressivity, but the gas tax is still not progressive.[3]

To our knowledge, no paper in this literature employs analytical general equilibrium models to analyze the distributional effects on both the uses side and sources side from a revenue-neutral pollution tax, where the tax revenue is used to lower a pre-existing labor tax (such as the U.S. payroll tax). A limitation of computational models is that the specific result depends on assumptions about parameter values. In contrast, this paper finds closed-form solutions for the change in the real unskilled wage and skilled wage. Since these solutions hold for any parameter values, they show general conditions under which the relative real wage ratio rises or falls. We also use plausible parameter values to calculate burdens on each type of labor.

The model here builds upon the model of Fullerton and Heutel (2007), where

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[1] See Bovenberg and de Mooij (1994a, 1994b, 1998), Bovenberg and Van der Ploeg (1994), Parry (1995), Goulder (1995), Bovenberg and Goulder (1997), Parry et al (1999), and Fullerton and Metcalf (2001).

[2] They assume that government can use permit revenue to cut corporate taxes, to cut payroll taxes, or to give households lump-sum rebates. The last scenario is the only one that might not be regressive.

[3] Hassett et al (2009) emphasize redistribution on a lifetime basis and by region. Metcalf et al (2008) use a computational model with tax shifting to labor and capital.

labor and capital are used in both sectors, and pollution is used in only one of them. They assume one representative household, so they do not distinguish different expenditure patterns, and they employ no revenue-neutral rebate.[4] As with their model, we assume two inputs into production along with pollution, and no assumptions are made about the functional form for production. Here, however, we distinguish two groups of households. Instead of using labor and capital as the production inputs, this paper uses unskilled and skilled labor, abstracting from the use of capital. Low-income unskilled workers spend a higher fraction of income on pollution-intensive goods. Thus the focus remains on real relative burdens borne by high and low wage earners, with a swap of a higher pollution tax for a lower unskilled wage tax. All of the prior general equilibrium incidence studies mentioned above consider labor to be one homogenous input.[5]

Here, we allow for two different types of labor, where the wage for skilled workers may rise or fall relative to the wage for unskilled labor. Other studies have shown the effects on rising wage inequality from various exogenous shocks. Higher demand for skilled labor relative to unskilled labor may occur with changes over time in technology, trade patterns, globalization, labor force composition, and other exogenous changes.[6] Similarly, a pollution tax could increase relative demand for skilled labor and raise the equilibrium skilled wage rate, if firms substitute out of pollution and into skilled labor. Conversely, it could increase the unskilled wage rate if firms substitute more into unskilled labor. Which is more likely? These cross-price substitution parameters have never been estimated, but we suggest that any major environmental policy reform is likely to spur sophisticated abatement technologies that favor high-skilled labor.

Given the generality of our model, the pollution tax is not guaranteed to raise the price of the dirty good, or even to reduce pollution.[7] Yet such possibilities do not merely confirm the notion that "in general equilibrium, anything can happen." To avoid that

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[4] Instead, they assume that the government uses pollution tax revenue to purchase the two commodities in the economy in the same proportion as do households.

[5] In these prior models, some individuals may have a larger endowment of effective labor units, but every labor unit earns the same wage rate. Therefore, individuals at all income levels with different endowments are affected the same way when the single wage rate changes in response to an environmental tax.

[6] Several papers focus on the role of skill-biased-technical-change and the effect on relative skilled and unskilled wages, including Bound and Johnson (1992), Katz and Murphy (1991), and Berman et al. (1994). Hanson and Harrison (1999) consider trade liberalization as an explanation for rising wage inequality, and Autor et al. (2005) evaluate the effect of shifts in labor force composition.

[7] Such possibilities were shown in Bovenberg and de Mooij (1998) and in Fullerton and Heutel (2007).

trap, we carefully categorize those perverse cases and show that they require extreme parameter values that are highly unlikely. Our paper is not about perverse cases. Even without those perverse cases, however, we demonstrate multiple ways in which low-income workers might suffer despite receiving all of the tax rebate. We say nothing about whether low-income families "ought" to be protected, only whether they *can* be protected from a fall in their real net wage rate.

We decompose the change in their real net wage into the effect on their gross wage, the effect of the tax rebate, and the effect of product prices. Both groups face higher product prices, and yet we find that the return of *all* additional pollution tax revenue just to the low-income group is still not enough to offset the effect of higher product prices on their real net wage. One reason is that burdens on the two groups exceed the pollution tax revenue (because of deadweight loss or "excess burden"). This effect is not captured in papers mentioned above that focus just on product prices and return of revenue. Moreover, we find that additional changes in relative wage rates may offset or exacerbate that burden on low-skilled workers. Based on our data, the dirty sector is low-skilled intensive, so the pollution tax reduces demand for low-skilled labor and suppresses their wage. In addition, the low-skilled wage may fall if the pollution tax induces substitution into high-skilled labor and out of low-skilled labor. Their relative wage can only rise if low-skilled labor is a good enough substitute for pollution to offset the fact that the polluting sector is low-skill intensive *and* the fact that the tax rebate is not enough to offset their disproportionate spending on the dirty good. In almost all of our numerical examples, the real net wage of low-income workers falls. Only in rare cases would the return of revenue protect low-income workers.

The first section below presents the theoretical model. Section II shows solution results and analyzes special cases in order to interpret them. Section III discusses these results and the implications. Section IV undertakes a numerical calculation based on those theoretical results, and Section V concludes.

I. The Model

The model presented here is similar to one in Fullerton and Heutel (2007), with the addition of three features: we consider two household and labor types instead of one, we model the government's revenue neutrality condition, and we solve for the labor tax reduction that exactly rebates the extra pollution tax revenue.

One household supplies high-wage skilled labor, while the other household supplies low-wage unskilled labor. The low-income families spend a disproportionately high fraction of income on pollution-intensive products like gasoline, electricity, and heating oil (Metcalf 1999, 2009). We assume that government tries to offset their added burden on the uses side by using the extra pollution tax revenue to lessen the payroll tax, a pre-existing tax on the labor of low-wage families. The incidence of the new tax system on the sources side is characterized by the proportional changes in the two wage rates, in response to a small exogenous increase in the pollution tax.[8]

The setting for the model is a perfectly competitive two-sector economy, with production of a "clean" good, denoted by X, and a "dirty" good, denoted by Y. Skilled and unskilled labor are used to produce both goods, and pollution is also an input in the dirty sector. The outputs are taxed at rates τ_X and τ_Y, so the prices faced by consumers are $p_X(1+\tau_X)$ and $p_Y(1+\tau_Y)$, respectively. The constant returns to scale production functions are $X=X(L_X, H_X)$ and $Y=X(L_Y, H_Y, Z)$, where L is low-skilled labor with wage w, H is high-skilled labor with wage h, and Z is pollution. The use of pollution is also taxed, so the price of pollution is simply its tax rate, τ_Z.[9] Both types of labor are mobile between the two sectors, with fixed total amounts of each ($\overline{L}$ and $\overline{H}$). Resource constraints for these two inputs are $L_X + L_Y = \overline{L}$ and $H_X + H_Y = \overline{H}$. Totally differentiating the resources constraints yields

$$\hat{H}_X \lambda_{HX} + \hat{H}_Y \lambda_{HY} = 0 \qquad (1)$$

$$\hat{L}_X \lambda_{LX} + \hat{L}_Y \lambda_{LY} = 0 \qquad (2)$$

where a hat indicates a proportional change ($\hat{H}_X = dH_X/H_X$), and λ_{ij} is sector j's share of input i (e.g., $\lambda_{HX} = H_X/\overline{H}$).

Producers of X can substitute between skilled and unskilled labor according to an elasticity of substitution, σ_X. Rearranging the definition of σ_X provides a behavioral equation describing how producers of X respond to any change in input prices, p_L and

[8] We started this model with one type of labor and one type of capital, just as in Fullerton and Heutel (2007) and many other papers back to Harberger (1962), but we soon realized that those two factors do not clearly delineate who has high income or low income. The ratio of capital income to labor income plotted against total income reveals a U-shaped pattern, primarily because low-income retirees may have no labor income. A lifecycle model is way beyond the scope of this paper. Instead, the two types of labor in this paper clearly identify who has high income or low income. Skilled labor differs from unskilled labor, the ratio of their use may differ by industry, and either one might be a better substitute for pollution.

[9] The pollution tax rate, τ_Z, is a per unit tax, whereas τ_X and τ_Y are *ad valorem* tax rates.

p_H, namely $\hat{H}_X - \hat{L}_X = \sigma_X(\hat{p}_L - \hat{p}_H)$. These two input prices are the costs that producers face and are gross-of-tax, so $p_L = w(1+\tau_L)$ and $p_H = h(1+\tau_H)$. We assume that the increased pollution tax τ_Z generates revenue used to lower τ_L, the wage tax on low-skilled labor. For simplicity, all other tax rates are assumed to stay constant (τ_X, τ_Y, and τ_H). Thus, the proportional change in the price of low-skilled labor is $\hat{p}_L = \hat{w} + \hat{\tau}_L$ where $\hat{w} \equiv dw/w$, but where we define $\hat{\tau}_L$ as $d\tau_L/(1+\tau_L)$. Since τ_H does not change, $\hat{p}_H = \hat{h}$. Thus we have:

$$\hat{H}_X - \hat{L}_X = \sigma_X(\hat{w} + \hat{\tau}_L - \hat{h}). \tag{3}$$

Because the dirty sector uses three factors of production, it has slightly more complicated responses. The firms' input demands are functions of input prices and output. Following Mieszkowski (1972), differentiate these demand functions to get:

$$\hat{H}_Y = a_{HH}\hat{p}_H + a_{HL}\hat{p}_L + a_{HZ}\hat{p}_Z + \hat{Y}$$
$$\hat{L}_Y = a_{LH}\hat{p}_H + a_{LL}\hat{p}_L + a_{LZ}\hat{p}_Z + \hat{Y}$$
$$\hat{Z} = a_{ZH}\hat{p}_H + a_{ZL}\hat{p}_L + a_{ZZ}\hat{p}_Z + \hat{Y}$$

where a_{ij} is the elasticity of demand for factor i with respect to the price of factor j. Then $a_{ij} = e_{ij}\theta_{Yj}$, where e_{ij} is the Allen elasticity of substitution between inputs i and j (Allen, 1938), which is negative when the two inputs are complements and positive for substitutes. Also, θ_{Yj} is the share of sales revenue from Y that is used to purchase factor j (e.g., $\theta_{YH} = h(1+\tau_H)H_Y/p_Y Y$). In the case where a tax per unit of pollution is the only price of pollution, then $p_Z = \tau_Z$ and $\hat{p}_Z = \hat{\tau}_Z$ (where $\hat{p}_Z \equiv dp_Z/p_Z$ and $\hat{\tau}_Z \equiv d\tau_Z/\tau_Z$). Substitute the Allen elasticities and the proportional price changes into the differentiated factor demands above, and subtract the third equation from the first two:

$$\hat{H}_Y - \hat{Z} = \theta_{YH}(e_{HH} - e_{ZH})\hat{h} + \theta_{YL}(e_{HL} - e_{ZL})(\hat{w} + \hat{\tau}_L) + \theta_{YZ}(e_{HZ} - e_{ZZ})\hat{\tau}_Z \tag{4}$$
$$\hat{L}_Y - \hat{Z} = \theta_{YH}(e_{LH} - e_{ZH})\hat{h} + \theta_{YL}(e_{LL} - e_{ZL})(\hat{w} + \hat{\tau}_L) + \theta_{YZ}(e_{LZ} - e_{ZZ})\hat{\tau}_Z \tag{5}$$

These two equations represent how producers of Y react to changes in prices and tax rates that are attributable to an exogenous increase in the pollution tax, τ_Z.

The perfect competition and constant returns to scale assumptions imply that sales revenue in each sector equals the sum of payments to factors of production. Differentiating these conditions, we have:

$$\hat{X} + \hat{p}_X = \theta_{XH}(\hat{h} + \hat{H}_X) + \theta_{XL}(\hat{w} + \hat{\tau}_L + \hat{L}_X) \quad (6)$$

$$\hat{Y} + \hat{p}_Y = \theta_{YH}(\hat{h} + \hat{H}_Y) + \theta_{YL}(\hat{w} + \hat{\tau}_L + \hat{L}_Y) + \theta_{YZ}(\hat{\tau}_Z + \hat{Z}). \quad (7)$$

Then, using the perfect competition assumption, differentiate each sector's production function to describe how output quantities change with inputs:

$$\hat{X} = \theta_{XH}\hat{H}_X + \theta_{XL}\hat{L}_X \quad (8)$$

$$\hat{Y} = \theta_{YH}\hat{H}_Y + \theta_{YL}\hat{L}_Y + \theta_{YZ}\hat{Z}. \quad (9)$$

In order to discuss the distribution of net burdens in the simplest possible way, we assume that the economy has two types of consumers, low-skilled and high-skilled laborers, where subscripts L and H denote these groups (e.g. X_L is the amount of good X consumed by low-skilled workers). These two groups may spend on X and Y in different proportions, but they have the same elasticity of substitution in utility, σ_U, between X and Y. This simplifying assumption makes the model easier to solve, and yet does not detract from the purpose of the paper to find effects on these two groups. The elasticity of substitution has major impacts on economic efficiency, but it has only second-order effects on burdens. The first-order impact of $\hat{p}_X$ and $\hat{p}_Y$ on the relative burden of each group is the major pre-existing difference in the fraction of income that each group spends on Y. Rearranging the definition of σ_U yields behavioral equations that show how the two consumers' demands respond to changes in output prices:

$$\hat{X}_L - \hat{Y}_L = \sigma_U(\hat{p}_Y - \hat{p}_X) \quad (10)$$

$$\hat{X}_H - \hat{Y}_H = \sigma_U(\hat{p}_Y - \hat{p}_X) \quad (11)$$

The two goods are purchased by both types of workers. The total quantity of good X can then be expressed as $X = X_L + X_H$, and similarly $Y = Y_L + Y_H$. Differentiating these equations yields:

$$\hat{X} = \hat{X}_L \frac{X_L}{X} + \hat{X}_H \frac{X_H}{X} \quad (12)$$

$$\hat{Y} = \hat{Y}_L \frac{Y_L}{Y} + \hat{Y}_H \frac{Y_H}{Y}. \quad (13)$$

The government budget constraint requires a fixed amount, G, matched by tax revenue:

$$G = \tau_Z Z + h\tau_H H + w\tau_L L + p_Y \tau_Y Y + p_X \tau_X X.$$

Rather than specify direct government spending on X and Y, we say that the fixed revenue G is returned to the two groups [δG to the high-income group, and $(1-\delta)G$ to the low-income group]. Since G is fixed, these lump-sum transfers are fixed and do not affect our results. Only the *increase* in pollution tax revenue is used to cut τ_L (leaving G fixed). With the revenue neutrality condition ($dG = 0$) and the assumption that only tax rates τ_Z and τ_L can change, we differentiate the government budget constraint and rearrange to solve for the proportional change in the labor tax:[10]

$$\hat{\tau}_L = \frac{1}{\overline{L}p_L}[-Z\tau_Z(\hat{\tau}_Z + \hat{Z}) - \overline{H}h\tau_H\hat{h} - \tau_L\hat{w} - p_Y\tau_Y Y(\hat{p}_Y + \hat{Y}) - p_X\tau_X X(\hat{X} + \hat{p}_X)]. \quad (14)$$

The two consumer groups allocate spending on the two goods according to their preferences and budget constraints. For example, the budget constraint for skilled labor is $p_X(1+\tau_X)X_H + p_Y(1+\tau_Y)Y_H = h(H_X + H_Y) + \delta G$. Of the two spending equations, one is independent. The second can be derived from the first, using Eqs. (6) and (7). Therefore, only one consumer group budget constraint is necessary, and differentiation of the skilled labor budget constraint yields:

$$p_X(1+\tau_X)X_H(\hat{p}_X + \hat{X}_H) + p_Y(1+\tau_Y)Y_H(\hat{p}_Y + \hat{Y}_H) = h(H_X + H_Y)\hat{h}. \quad (15)$$

Equations (1)-(15) are fifteen linear equations in sixteen unknowns: $\hat{H}_X, \hat{H}_Y, \hat{L}_X, \hat{L}_Y, \hat{w}, \hat{h}, \hat{p}_X, \hat{X}, \hat{X}_L, \hat{X}_H, \hat{p}_Y, \hat{Y}, \hat{Y}_L, \hat{Y}_H, \hat{Z}$, and $\hat{\tau}_L$. Good X is chosen as numeraire, so that $\hat{p}_X = 0$. The model can be solved through successive substitution to obtain equations for each of the remaining fifteen endogenous variables in terms of exogenous parameters and the exogenous change in the pollution tax, $\hat{\tau}_Z$.

II. Results

The variables of most interest are the price changes, to determine the relative incidence on skilled and unskilled labor. Table 1 shows solutions for these selected variables and the change in pollution, $\hat{Z}$.

The expressions for $\hat{p}_H$ and $\hat{p}_L$ are "closed form" solutions, because they contain only parameters and the exogenous policy shock ($\hat{\tau}_Z$), but the expression for $\hat{w}$ contains $\hat{\tau}_L$ (which is endogenous). All of these solutions *can* be expressed just in terms of

[10] Tax rates τ_X and τ_Y are fixed, but changes in X and Y will affect revenue.

exogenous parameters and $\hat{\tau}_Z$, but that would make the expressions much longer. For example, a closed-form solution for $\hat{Z}$ can be obtained by substituting expressions for $\hat{h}$ and $\hat{w} + \hat{\tau}_L$ into the $\hat{Z}$ expression shown in Table 1, and that $\hat{Z}$ can be used to obtain closed-form expressions for $\hat{\tau}_L$ and $\hat{w}$.

Table 1: Proportional changes in prices, wage rates, and amount of pollution

$$\hat{h} = \hat{p}_H = \frac{\theta_{XL}\theta_{YZ}}{D}[A(e_{HZ}-e_{ZZ}) - B(e_{LZ}-e_{ZZ}) - (\gamma_H-\gamma_L)(\sigma_U N + J)]\hat{\tau}_Z \qquad \text{(16a)}$$

$$\hat{w} = \hat{p}_L - \hat{\tau}_L = \frac{\theta_{XH}\theta_{YZ}}{D}[A(e_{ZZ}-e_{HZ}) - B(e_{ZZ}-e_{LZ}) + (\gamma_H-\gamma_L)(\sigma_U N + J)]\hat{\tau}_Z - \hat{\tau}_L \qquad \text{(16b)}$$

$$\hat{p}_Y = \frac{(\theta_{YL}\theta_{XH}-\theta_{YH}\theta_{XL})}{D}\theta_{YZ}[A(e_{ZZ}-e_{HZ}) - B(e_{ZZ}-e_{LZ}) + (\gamma_H-\gamma_L)(\sigma_U N + J)]\hat{\tau}_Z + \theta_{YZ}\hat{\tau}_Z \qquad \text{(16c)}$$

$$\begin{aligned}\hat{Z} = -\frac{1}{C}[[&\theta_{YH}[\beta_H(e_{HH}-e_{ZH}) + \beta_L(e_{LH}-e_{ZH}) + \sigma_U N + J] - \frac{1}{C_H}h\bar{H}(\frac{Y_H}{Y_L} - \frac{X_H}{X_L})]\hat{h} \\ &+ \theta_{YL}[\beta_H(e_{HL}-e_{ZL}) + \beta_L(e_{LL}-e_{ZL}) + \sigma_U N + J](\hat{w} + \hat{\tau}_L) \qquad \text{(16d)} \\ &+ \theta_{YZ}[\beta_H(e_{HZ}-e_{ZZ}) + \beta_L(e_{LZ}-e_{ZZ}) + \sigma_U N + J]\hat{\tau}_Z]\end{aligned}$$

$$\begin{aligned}\hat{\tau}_L = [\frac{1+\tau_L}{\bar{L}p_L}][&(-Z\tau_Z + T_X(\theta_{XH}\gamma_H + \theta_{XL}\gamma_L) - T_Y)\hat{Z} + [-Z\tau_Z + \qquad \text{(16e)} \\ &T_X\theta_{YZ}(\theta_{XH}\gamma_H(e_{HZ}-e_{ZZ}) + \theta_{XL}\gamma_L(e_{LZ}-e_{ZZ})) - T_Y\theta_{YZ}(1+\theta_{YH}(e_{HZ}-e_{ZZ}) + \theta_{YL}(e_{LZ}-e_{ZZ}))]\hat{\tau}_Z \\ &+ [-\bar{H}\tau_H h + M\bar{L}p_L + T_Y(F-\theta_{YH}) + (T_X\theta_{XH}\gamma_H - T_Y\theta_{YH})E_H + (T_X\theta_{XL}\gamma_L - T_Y\theta_{YL})E_L]\hat{h}]\end{aligned}$$

$$\textit{where}\;\; \gamma_H \equiv \frac{\lambda_{HY}}{\lambda_{HX}} = \frac{H_Y}{H_X}, \quad \gamma_L \equiv \frac{\lambda_{LY}}{\lambda_{LX}} = \frac{L_Y}{L_X}, \quad \beta_H \equiv \tfrac{X}{X_L}\theta_{XH}\gamma_H + \tfrac{Y}{Y_L}\theta_{YH}, \quad \beta_L \equiv \tfrac{X}{X_L}\theta_{XL}\gamma_L + \tfrac{Y}{Y_L}\theta_{YL},$$

$$A \equiv \gamma_L\beta_H + \gamma_H(\beta_L + \tfrac{Y}{Y_L}\theta_{YZ}), \quad B \equiv \gamma_H\beta_L + \gamma_L(\beta_H + \tfrac{Y}{Y_L}\theta_{YZ}), \quad C \equiv \beta_H + \beta_L + \tfrac{Y}{Y_L}\theta_{YZ},$$

$$\begin{aligned}D \equiv C\sigma_X &+ A[\theta_{XH}\theta_{YL}(e_{HL}-e_{LZ}) - \theta_{XL}\theta_{XH}(e_{HH}-e_{HZ})] - B[\theta_{XH}\theta_{YL}(e_{LL}-e_{LZ}) - \theta_{XL}\theta_{YH}(e_{HL}-e_{HZ})] \\ &- (\gamma_H-\gamma_L)(\sigma_U N + J)(\theta_{XH}\theta_{YL} - \theta_{XL}\theta_{YH}) - (\gamma_H-\gamma_L)\frac{1}{C_H}h\bar{H}(\frac{Y_H}{Y_L} - \frac{X_H}{X_L})\end{aligned}$$

$$T_X \equiv p_X\tau_X X, \quad T_Y \equiv p_Y\tau_Y Y, \quad F \equiv \frac{\theta_{YL}\theta_{XH}}{\theta_{XL}}, \quad M \equiv \frac{\tau_L}{1+\tau_L}\frac{\theta_{XH}}{\theta_{XL}},$$

$$E_H \equiv \theta_{YH}(e_{HH}-e_{ZH}) - \frac{\theta_{YL}\theta_{XH}}{\theta_{XL}}(e_{HL}-e_{ZL}), \quad E_L \equiv \theta_{YH}(e_{LH}-e_{ZH}) - \frac{\theta_{YL}\theta_{XH}}{\theta_{XL}}(e_{LL}-e_{ZL})$$

$$N \equiv \frac{X}{X_L} + (1-\alpha_H)(\frac{Y_H}{Y_L} - \frac{X_H}{X_L}), \quad J \equiv \alpha_H(\frac{Y_H}{Y_L} - \frac{X_H}{X_L})$$

$$C_H \equiv p_X(1+\tau_X)X_H + p_Y(1+\tau_Y)Y_H, \quad \text{and} \quad \alpha_H \equiv \frac{p_Y(1+\tau_Y)Y_H}{p_X(1+\tau_X)X_H + p_Y(1+\tau_Y)Y_H}$$

For further simplification, parameters are combined into definitions. That is, γ_H and γ_L are relative factor intensities. Our data below indicate that $\gamma_L > \gamma_H$, so the dirty sector is low-skilled labor intensive (or, equivalently, the clean sector uses skilled labor intensively). The constants A and B are difficult to interpret, but they are weights on the terms involving the Allen elasticities (e_{ij}). In equations (16), these terms represent "substitution effects" and indicate how producers of Y substitute among inputs when pollution becomes more expensive. The constant C partly determines the change in pollution ($\hat{Z}$) in (16d). The constants A, B, and C are all positive, but the denominator D cannot be signed in general. Constants T_X and T_Y represent tax payments received by the government on the two goods prior to the pollution tax increase, and these help to determine the magnitude of the decrease in the unskilled labor tax rate. The E constants that appear in the solution for $\hat{\tau}_L$ help determine how the substitution effects impact the magnitude of $\hat{\tau}_L$. The terms N and J summarize the relative consumption of the two goods by the two consumer groups. While N can be shown to be positive, the sign of J is ambiguous without additional assumptions (discussed below).[11]

Two effects appear in (16a), (16b), and (16c) that are comparable to effects identified by Mieszkowski (1967) and discussed by Fullerton and Heutel (2007). First, the "substitution effect" is the term $[A(e_{HZ} - e_{ZZ}) - B(e_{LZ} - e_{ZZ})]$. Through this term, the higher pollution tax ($\hat{\tau}_Z > 0$) tends to help high-skilled labor ($\hat{h} > 0$) whenever e_{HZ} is larger than e_{LZ}, that is, when H is a better substitute for pollution than is L. Second, the "output effect" is represented here by the term $(\gamma_H - \gamma_L)(\sigma_U N + J)$. If low-skilled labor is used more intensively in the dirty sector, then $(\gamma_H - \gamma_L) < 0$. Because σ_U and N are both positive, then the first part of this term, $(\gamma_H - \gamma_L)\sigma_U N$ is negative. A higher tax on emissions, $\hat{\tau}_Z$, reduces the dirty output in a way that depends on consumer preferences represented by σ_U, and therefore reduces demand for L relative to H (reducing w and raising h). It places more burden on the factor intensively employed in the dirty sector.

The additional term J means that this effect on factor prices also depends on the relative consumption by the two groups. For concreteness, assume that low-skilled consumers spend a greater proportion of income on the dirty good than do high-skilled consumers. In other words, assume Y_L/X_L exceeds Y_H/X_H, which makes J negative.

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[11] By rearranging terms, $N = (x_L + \alpha_H x_H)/x_L + (1 - \alpha_H) y_L / y_H$ where both terms are positive.

The additional J term does not appear in results of Fullerton and Heutel (2007) because they assume government spends the tax revenue in the same way as the one representative consumer. Here, the extra tax revenue is rebated to unskilled labor, with disproportionate spending on the dirty good, so this particular use of the revenue increases purchases of *Y*. This J term helps the factor used intensively in *Y*, offsetting the usual output effect.[12]

Because the solutions for these variables are complicated, with offsetting effects, the determination of the overall sign of each is difficult. In this most general case, the tax on pollution might even reduce the price of the dirty good, or increase the amount of pollution. Therefore, we employ special cases that simplify the expressions and allow more definitive results. The rest of this section focuses on the sources side of income, and a later section focuses on the uses side.

Case 1: Equal Factor Intensities ($\gamma_H = \gamma_L = \gamma$)

For the first special case, suppose that the two sectors have the same ratio of high-skilled to low-skilled labor ($H_Y / H_X = L_Y / L_X$). This condition eliminates the output effect, $(\gamma_H - \gamma_L)(\sigma_U N + J)$ and leaves the sign of each price change to depend only on the substitution effects – the e_{ij} parameters. In this simple case, the solutions are:

$$\hat{p}_Y = \theta_{YZ}\hat{\tau}_Z \tag{17a}$$

$$\hat{h} = \hat{p}_H = \frac{\theta_{XL}\theta_{YZ}\gamma(e_{HZ} - e_{LZ})}{D_1}\hat{\tau}_Z \tag{17b}$$

$$\hat{w} = \hat{p}_L - \hat{\tau}_L = \frac{-\theta_{XH}\theta_{YZ}\gamma(e_{HZ} - e_{LZ})}{D_1}\hat{\tau}_Z - \hat{\tau}_L \tag{17c}$$

where $D_1 = (\sigma_X - \theta_{XL}\theta_{YH}\gamma e_{HH} - \theta_{XH}\theta_{YL}\gamma e_{LL}) + \gamma(\theta_{XL}\theta_{YH} + \theta_{XH}\theta_{YL})e_{HL}$. In this case, $\hat{p}_Y$ in (17a) is clearly positive, since θ_{YZ} is positive, and because the pollution tax is increased ($\hat{\tau}_Z > 0$). The sign of $\hat{h}$ depends on whether $e_{HZ} > e_{LZ}$ and $D_I > 0$. To put a sign on this denominator, define

Condition 1: $\quad e_{HL} > \dfrac{-\sigma_X + \theta_{XL}\theta_{YH}\gamma e_{HH} + \theta_{XH}\theta_{YL}\gamma e_{LL}}{\gamma(\theta_{XL}\theta_{YH} + \theta_{XH}\theta_{YL})}$.

The denominator D_I is positive if and only if Condition 1 holds. Since $e_{HH} < 0$, $e_{LL} < 0$, and all other terms are positive, the ratio on the right side of the inequality is strictly

[12] The N and J terms show how expenditure patterns affect demands for X and *Y*, and thus demands for L and *H*, with effects on the sources side. Later, we show how different expenditures affect the uses side.

negative. For this condition to hold, it is sufficient that $e_{HL} > 0$, which means that low-skilled labor and high-skilled labor are substitutes. More generally, Condition 1 may still hold if low-skilled and high-skilled labor are not too complementary.

If that condition holds, and high-skilled labor is a better substitute for pollution than unskilled labor ($e_{HZ} > e_{LZ}$), then the pollution tax raises the high-skilled wage ($\hat{h} > 0$). If Condition 1 fails, and the two types of labor are "too" complementary, then $\hat{h}$ is negative.[13] Less unlikely is that Condition 1 holds but unskilled labor is a better substitute for pollution, where again $\hat{h}$ is negative.[14]

The sign of $\hat{w}$ depends on $\hat{\tau}_L$ and on the $\hat{p}_L$ term (with sign opposite to that of $\hat{h} = \hat{p}_H$). The extra pollution tax revenue is used to reduce the low-skilled tax, so $\hat{\tau}_L$ is negative.[15] The reduction in this tax rate has a powerful, positive impact on $\hat{w}$, since the goal of the tax swap is to raise the net wage for low-income families. If unskilled labor is a better substitute for pollution than is high-skilled labor ($e_{LZ} > e_{HZ}$), then the $\hat{p}_L$ term in (17c) is positive, as long as D_I is positive. In that case $\hat{w}$ is unambiguously positive. The positive effect of $\hat{\tau}_L$ on $\hat{w}$ can be overwhelmed, however, if unskilled labor is enough less of a substitute for pollution ($e_{HZ} >> e_{LZ}$), so that a reduction in Z leads to a large reduction in demand for L. Then the net unskilled wage may fall.

Table 2 summarizes these results and conditions.[16] In the first row of Table 2, $e_{HZ} < e_{LZ}$, so low-skilled labor is a better substitute for pollution, and the increased pollution tax definitely raises the net low-skilled labor wage (even before accounting for $\hat{\tau}_L < 0$).[17] In the next two rows of Table 2, high-skilled labor is a better substitute for pollution than low-skilled labor, a situation that tends to help high-skilled labor and reduce the low-

[13] Intuition is difficult for the perverse case where Condition 1 fails and $D_I < 0$. How could the pollution tax hurt high-skilled labor, even though H is a better substitute for pollution ($e_{HZ} > e_{LZ}$)? The higher price of pollution induces substitution into H, but if L and H are sufficiently complementary, then the firm wants to employ more L, which raises w relative to h.

[14] If *both* conditions fail, so that $D_I < 0$ *and* $e_{HZ} < e_{LZ}$, then h would rise.

[15] We assume that the pollution tax rate is on the normal side of the Laffer curve, so that increasing the rate yields additional revenue that can be used to cut τ_L.

[16] The $\hat{\tau}_L$ term is endogenous. The three conditions in Table 2 can be expressed in terms of exogenous parameters only, but these conditions would then be extremely long and complicated.

[17] But even this case does not guarantee that L is held harmless, because this analysis does not yet account for the effect on L from spending disproportionately on Y when $\hat{p}_Y > 0$. So far, we discuss only the sources side ($\hat{w}$ and $\hat{h}$), but later we discuss effects on the two groups from the uses side ($\hat{p}_Y > 0$).

skilled wage. In the second row, the decrease in τ_L is *not* enough to overcome this injury, so the net-of-tax low-skilled wage falls. In the third row, however, the decrease in τ_L is large enough to offset this burden, so that the low-skilled wage rises. The intuition for the sign of $\hat{h}$ is similar and depends on whether low-skilled labor or high-skilled labor is a better substitute for pollution. Since the high-skilled labor tax rate does not change, though, any burden on the high-skilled wage cannot be offset by tax changes.

Table 2: Changes in *w* and *h* for the case of equal factor intensities

If high-skilled labor and unskilled labor are not too complementary (D_1>0):	$\hat{h}$	$\hat{w}$
(1) $e_{HZ} < e_{LZ}$ *(unskilled labor is a better substitute for pollution)*	< 0	> 0
(2) $e_{HZ} > e_{LZ}$ and $\hat{\tau}_L > \frac{-\theta_{XH}\theta_{YZ}\gamma(e_{HZ} - e_{LZ})}{D_1}\hat{\tau}_Z$	> 0	< 0
(3) $e_{HZ} > e_{LZ}$ and $\hat{\tau}_L < \frac{-\theta_{XH}\theta_{YZ}\gamma(e_{HZ} - e_{LZ})}{D_1}\hat{\tau}_Z$	> 0	> 0

Case 2: Equal Factor Intensities and $e_{HZ} = e_{LZ}$

This case is a special version of Case 1 with equal factor intensities ($\gamma_H = \gamma_L = \gamma$), with the additional requirement that low-skilled labor and high-skilled labor are equally substitutable for pollution. In this most simple case, the solutions are now just:

$$\hat{p}_Y = \theta_{YZ}\hat{\tau}_Z > 0 \tag{18a}$$

$$\hat{w} = -\hat{\tau}_L > 0 \tag{18b}$$

$$\hat{h} = 0 \tag{18c}$$

The change in the price of Y is the same as in Case 1 and is positive. Whereas equal factor intensities removes the output effect, setting $e_{HZ} = e_{LZ}$ removes the substitution effect. Now relative wage rates do not change at all ($\hat{p}_L = \hat{p}_H = 0$), but pollution tax revenue is used to cut the low-skilled labor tax ($\hat{\tau}_L < 0$, so $\hat{w} > 0$). These simplifying assumptions remove factor price effects and leave only the product price effects analyzed by Metcalf (1999, 2009) and Burtraw et al (2009). The remaining question, analyzed below, is whether the rebate is enough to overcome the burden from $\hat{p}_Y > 0$.

Case 3: Fixed Input Proportions (e_{ij} = 0)

In this case, all the elasticities are set to zero, so that the substitution effects disappear, and only the output effects remain. Now results are driven by whether sector Y is high-skilled labor intensive or low-skilled labor intensive. The solutions are:

$$\hat{p}_Y = \frac{[C\sigma_X - (\gamma_H - \gamma_L)\frac{1}{C_H} h\overline{H}(\frac{Y_H}{Y_L} - \frac{X_H}{X_L})]\theta_{YZ}}{D_3}\hat{\tau}_Z \tag{19a}$$

$$\hat{h} = \hat{p}_H = -\frac{\theta_{XL}\theta_{YZ}(\gamma_H - \gamma_L)(\sigma_U N + J)}{D_3}\hat{\tau}_Z \tag{19b}$$

$$\hat{w} = \hat{p}_L - \hat{\tau}_L = \frac{\theta_{XH}\theta_{YZ}(\gamma_H - \gamma_L)(\sigma_U N + J)}{D_3}\hat{\tau}_Z - \hat{\tau}_L \tag{19c}$$

where $D_3 = C\sigma_X - (\gamma_H - \gamma_L)(\sigma_U N + J)(\theta_{XH}\theta_{YL} - \theta_{XL}\theta_{YH}) - (\gamma_H - \gamma_L)\frac{1}{C_H} h\overline{H}(\frac{Y_H}{Y_L} - \frac{X_H}{X_L})$.

The denominator D_3 can be positive or negative, depending on whether the dirty sector is low-skilled or high-skilled labor intensive, as well as on other parameters. Even with the removal of the substitution effects, the results are still complicated and difficult to sign. An additional simplification is that the elasticity of substitution in consumption between X and Y, σ_U, is equal to unity (the value used in the numerical section of Fullerton and Heutel, 2007). This simplifying assumption means that the term $(\sigma_U N + J)$ is now unambiguously positive, and it allows us to sign the results based on factor intensities in the dirty industry and several other conditions. The Appendix contains a diagram of these sub-cases and shows the signs of $\hat{p}_Y$, $\hat{h}$, and $\hat{w}$.

When the dirty sector is high-skilled intensive ($\gamma_H > \gamma_L$), the results are definitive: p_Y increases, h decreases, and w increases. These results are consistent with the intuition stated earlier that the output effect places more of the burden on the factor used intensively in the dirty sector. When the dirty industry is low-skilled intensive ($\gamma_H < \gamma_L$), the situation is more complicated. In this case, unskilled labor might be hurt despite their tax cut. Also, the dirty good's price could actually decrease. When the industries have very different factor intensities, so that γ_L is much larger than γ_H, then w likely decreases. The output effect hurts intensively-used unskilled labor, which can overtake the opposing decrease in the tax on unskilled labor. A full categorization of results for case 3 is provided in Figure 1 of the Appendix.

III. Effects on the Uses Side

Thus far, we have only discussed the effect that a pollution tax swap has on the sources side of income for both types of workers. We next consider the uses side and changes in the real net unskilled wage. Define $\omega \equiv p_L(1-\tau_L)/p_Q^L = w/p_Q^L$, where p_Q^L is a price index, $p_Q^L \equiv \alpha_L p_Y(1+\tau_Y)+(1-\alpha_L)p_X(1+\tau_X)$.[18] Differentiation yields $\hat{p}_Q^L = \alpha_L \hat{p}_Y$. We then calculate the change in the real net unskilled wage:

$$\hat{\omega} = \hat{p}_L - \hat{\tau}_L - \hat{p}_Q^L. \qquad (20)$$

This equation nicely decomposes the effect on the real net wage into three components: the change in the gross wage, the change in the tax rate, and the change from product prices. Similarly, define the real net wage for skilled labor as $\psi \equiv p_H(1-\tau_H)/p_Q^H$ $= h/p_Q^H$, with analogous definitions of p_Q^H and α_H. Since τ_H is fixed, we have:

$$\hat{\psi} = \hat{p}_H - \hat{p}_Q^H. \qquad (21)$$

In Case 1 with equal factor intensities, we show above that $\hat{p}_Y > 0$, so $\hat{w}$ may be positive or negative. Thus $\hat{\omega}$ is definitely negative when $\hat{w} < 0$; in particular, when high-skilled and unskilled labor are not too complementary, and low-skilled labor is highly complementary with pollution, then the real unskilled wage falls. For $\hat{w} > 0$, it is more likely that the real net wage increases if α_L is small, that is, if unskilled laborers do not spend too disproportionately on good Y. Case 2 simplifies the analysis even more, with the additional assumption that high-skilled labor and unskilled labor are equally substitutable for pollution. In this case $\hat{p}_Y > 0$, and the real wage increases if $-\hat{\tau}_L > \alpha_L \hat{p}_Y$. Thus, if the wage tax cut is very large, or if unskilled workers do not spend too much on Y, then their real wage is likely to rise.

Case 3 is unique because $\hat{p}_Y$ may be positive or negative. Therefore, $\hat{\omega}$ is definitely positive when $\hat{w} > 0$ and $\hat{p}_Y < 0$. Otherwise, as before, $\hat{\omega}$ may still be positive as long as unskilled labor's expenditure share on Y is not too large. It is also possible that $\hat{\omega} > 0$ in the perverse case where both the net wage and p_Y fall.

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[18] The weight α_L is unskilled labor's share of expenditures on Y (using initial values, so α_L is a fixed parameter). Thus, $\alpha_L \equiv p_Y(1+\tau_Y)Y_L/[p_X(1+\tau_X)X_L + p_Y(1+\tau_Y)Y_L]$. Below we set initial prices to 1.0 and output tax rates to zero, so we have $\alpha_L \equiv Y_L/(Y_L + X_L)$.

We now summarize all our analytical results. Ignoring unlikely perverse cases where p_Y may fall or Z may rise, we have identified several reasons that the real net wage of low-income workers (ω) may fall, even when pollution tax revenue is used to cut their labor tax rate. First, ω may fall if H is better than L as a substitute for pollution. Second, ω may fall if the dirty sector is low-skill intensive. Third, ω may fall if low-skill labor spends disproportionately on the dirty good.

IV. Numerical Analysis

Because the general model's results are complicated and difficult to sign, we now assign plausible parameter values and solve numerically for changes in wage rates and other variables. We then can change certain parameter values, in a sensitivity analysis, to see if and when extreme parameter values generate perverse results. This section provides only an illustration of plausible magnitudes in the analytical model, however, not a fully detailed numerical simulation of the pollution tax reform (particularly since we omit capital and changes in returns to capital).

We use several data sources. First, we define the "dirty" sector as the fourteen most polluting industries using data from the EPA's Toxic Release Inventory (TRI) for 2006. The "clean" sector then includes all other industries.[19] The TRI contains information on nearly 650 chemical releases for various kinds of manufacturing, electric utilities, and metal and coal mining. We use this information along with data from the Occupational Employment Statistics (OES) survey from 2007 in order to identify factor income shares for each sector.[20] The OES has data on employment and average wages for each of over 800 occupations, grouped by NAICS industry code. We classify skilled labor as occupations with mean annual wage of at least $50,000. Adding total compensation to workers in the clean sector implies that the clean sector represents about

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[19] No industry is perfectly "clean", especially when considering intermediate inputs that are ignored here. We simply separate the dirtiest 14 from other industries. These initial steps follow Fullerton and Heutel (2007), but here we use more recent data. TRI industry data use 3 and 4-digit NAICS codes. Aggregating to the 3-digit level, the industries we put in the "dirty" sector are those with the most on- and off-site disposal of monitored chemicals: mining, utilities, chemical manufacturing, primary metals, paper manufacturing, waste management, food manufacturing, beverage and tobacco manufacturing, petroleum and coal product manufacturing, fabricated metals, transport equipment, plastics and rubber manufacturing, nonmetallic mineral products manufacturing, and wood products manufacturing. The "clean" sector is comprised of the remaining 75 industries. This data set is available at http://www.epa.gov/triexplorer/industry.htm.

[20] The Occupational Employment Statistics survey is available from the Bureau of Labor Statistics' website: http://www.bls.gov/data.

93% of income.[21] The share of the clean sector's compensation to low-skilled workers is about 54%, and the share of the dirty sector's compensation to low-skilled workers is about 64%, indicating that the dirty sector is indeed low-skilled labor-intensive.

This information allows us to specify that $\theta_{XL} = 0.54$ and $\theta_{XH} = 0.46$ in the clean sector. In the dirty sector, however, we do not have data on the fraction of sales revenue attributable to pollution, so we cannot yet determine all factor shares. Following Fullerton and Heutel (2007), we choose $\theta_{YZ} = 0.25$. The remaining 75% of sales revenue is paid to labor, of which 64% is to low-skilled labor (so $\theta_{YL} = 0.48$), and the remainder to high-skilled labor (so $\theta_{YH} = 0.27$). We define a unit of each input or output so that all initial prices are one ($w = h = p_Z = p_X = p_Y = 1$). Also, perfect competition implies zero profits, so $X = (1 + \tau_H)H_X + (1 + \tau_L)L_X$ and $Y = (1 + \tau_H)H_Y + (1 + \tau_L)L_Y + Z$. Defining total factor income to equal one as well means that $(1 + \tau_X)X + (1 + \tau_Y)Y = 1$. Using all these relationships, we can then solve for all the initial factor allocations and parameters shown in the top portion of Table 3.

Table 3: Summary of parameters

H_Y	= 0.017	L_Y	= 0.029
H_X	= 0.301	L_X	= 0.459
θ_{YH}	= 0.317	θ_{YL}	= 0.433
θ_{XH}	= 0.455	θ_{XL}	= 0.545
λ_{HY}	= 0.052	λ_{LY}	= 0.059
λ_{HX}	= 0.948	λ_{LX}	= 0.941
γ_H	= 0.055	γ_L	= 0.063

$\tau_H = 0.4$, $\tau_L = 0.1$

$\tau_X = 0$, $\tau_Y = 0$, $\tau_Z = 1$

$\alpha_H = 0.05$, $\alpha_L = 0.12$, $\delta = .39$

We also need numerical values for the tax rates (τ_X, τ_Y, τ_H, and τ_L), expenditure shares (α_H and α_L), the fraction of initial tax revenue returned to high-skilled labor (δ), and elasticities. We choose zero for the tax rates on X and Y, 10% as the tax rate on unskilled labor, and 40% as the tax rate on skilled labor.[22]

[21] For each industry, we multiply employment in each skilled occupation by the mean annual wage for that occupation, and sum over all skilled occupations to calculate total compensation to skilled labor. Similarly, for unskilled labor, we take employment in each occupation times the mean wage for each, and sum.

[22] Excise and other output taxes in the two sectors are similar to each and do not add much to the model, so we choose the simplest case where the tax rates on X and Y are both zero. Labor tax rates of 10% and 40% for unskilled and skilled labor, respectively, approximate payroll and income taxes for the two groups.

We approximate the expenditure shares for each group of workers using data on income quintiles from the 2007 Consumer Expenditure Survey (CEX). The bottom three quintiles correspond to our low-skilled workers, with average pre-tax incomes of \$10,531, \$27,674, and \$46,213 respectively. The top two quintiles, representing high-skilled workers, have mean incomes of \$72,460 and \$158,388. We add up expenditure per quintile on natural gas, electricity, fuel oil, and gasoline in order to obtain an approximation of the fraction of income spent on the "dirty" good.[23] This yields α_L = 12% (so 0.88 is the fraction spent on the clean good by low-skilled workers) and α_H = 5% (so 0.95 is the fraction spent on the clean good by high-skilled workers). We assume that initial government revenue is simply distributed in proportion to each group's initial income, so δ = 39%.

We use an elasticity of substitution in production for the clean sector of one, and an elasticity of substitution in utility of one (following Fullerton and Heutel, 2007). No study has estimated which factors of production are better substitutes for pollution, especially for our two factors (high-skilled and low-skilled labor). The point here is to see how much these Allen cross-price elasticities matter, so we use values ranging from –1 to 1. These then determine the own-price elasticities. We are now able to solve for the relevant magnitudes in changes from the initial equilibrium, as shown in Table 4.

Table 4: A 10% Increase in the Pollution Tax (% changes)

Row	(1) e_{HZ}	(2) e_{LZ}	(3) Pollution $\hat{Z}$	(4) Low-Skill Wage $\hat{w}$	(5) High-Skill Wage $\hat{h}$	(6) Price of Dirty Good $\hat{p}_Y$
1	1	-0.5	-3.932	0.173	0.124	2.494
2	0	0	-2.932	0.285	0.012	2.499
3	0.5	0	-4.567	0.195	0.048	2.498
4	1	0	-6.190	0.105	0.083	2.496
5	-0.5	0.5	-3.488	0.311	-0.065	2.503
6	0	0.5	-5.148	0.219	-0.029	2.501
7	0.5	0.5	-6.797	0.128	0.007	2.500
8	1	0.5	-8.433	0.038	0.043	2.498
9	-0.5	1	-5.676	0.246	-0.106	2.505
10	0	1	-7.350	0.154	-0.069	2.503
11	0.5	1	-9.012	0.062	-0.033	2.501

[23] Our dirty industries include mining, chemicals, and primary metals, but these outputs are not purchased directly by consumers and so do not appear in the CEX. The most pollution-intensive consumer goods categories are natural gas, electricity, fuel oil, and gasoline. Consumer expenditure for these four categories is about 6.8%, which is close to our definition of the dirty sector (representing 7% of income).

Table 4 shows results for the relevant changes of interest, assuming that the pollution tax rate increases by 10%. The different rows show results when the elasticities e_{HZ} and e_{LZ} are varied, while e_{HL} is held constant at a value of one (so skilled and unskilled labor are substitutes).[24] For all of the combinations of parameters shown in the table, the increase in the pollution tax always reduces the amount of pollution ($\hat{Z} < 0$) and increases the price of the dirty good ($\hat{p}_Y > 0$).

In all rows, the output effect dampens the unskilled wage, since the dirty sector is L-intensive. The net wage for unskilled workers (*w*) always rises, however, because it includes the rebate of pollution tax revenues through the reduction in τ_L. The overall changes in w are quite small, ranging from about 0.04% to 0.31%. This net wage rises most when the substitution effect favors low-skilled labor, that is, when *L* is a better substitute for pollution (when $e_{LZ} > e_{HZ}$, as in rows 5 and 9). The price of the dirty good rises, however, so the effect on the *real* net wage is not yet clear.

In Table 4, the changes for the high-skilled wage ($\hat{h}$) are positive or negative, but small in magnitude. Through the substitution effect, the higher pollution tax tends to raise the high-skilled wage ($\hat{h}$>0) whenever $e_{HZ} > e_{LZ}$ (high-skilled labor is a better substitute for pollution than is unskilled labor, as in rows 1, 3, 4, and 8). When $e_{LZ}>e_{HZ}$, however, the high-skilled wage falls (in rows 5, 6, 9, 10, and 11).

While changes in both real net wage rates may be small, the taxed sector is only 7% of national income. The nationwide wage rates are driven primarily by the clean sector, which employs 93% of labor. Thus, even a 10% higher pollution tax has only small effect on w relative to h. As shown in Case 2 above, the tax has *no* effect on *relative* wage rates when factor intensities are the same in the two sectors and substitution parameters are equal ($e_{HZ} = e_{LZ}$). The extra pollution tax also collects only a small amount of revenue. Indeed, the change must be small for our linear approximations to be valid. The fact that it's small, however, does not detract from our ability to address the question in this paper, namely whether the rebate of all extra revenue is enough to make up for higher product prices and for changes in factor prices. The answer to that question is based on our decomposition of all effects on each real net wage rate.

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[24] We also tried $e_{HL} = e_{HZ} = e_{LZ}$ =1, where the implied own-price elasticities are: e_{HH} = –2.2, e_{LL} = –1.2, and e_{ZZ} = –3. These elasticity value are not "perverse", but they do not result in positive additional revenue, violating our assumption that the pollution tax rate is on the normal side of the Laffer curve.

We undertake that decomposition in Table 5, where the rows correspond exactly to the rows of Table 4, as indicated by e_{LZ} and e_{HZ} in the first two columns. Then column (3) shows just the effect on the gross nominal wage p_L, which already includes both the output effect and the substitution effect. The output effect always acts to reduce the low-skilled wage, because the dirty sector is low-skilled labor intensive. The substitution effect exacerbates that output effect when $e_{LZ} < e_{HZ}$ (as in the first row), and it offsets the output effect when $e_{LZ} > e_{HZ}$ (as in rows 5, 6, 9, 10, and 11).

Table 5: A Decomposition of Effects from Factor Prices, Rebates, and Product Prices

Row	(1) e_{HZ}	(2) e_{LZ}	(3) Factor Price $\hat{p}_L$	(4) Tax Rebate $-\hat{\tau}_L$	(5) Product Prices $-\hat{p}_Q^L$	(6) Real Net Wage $=\hat{\omega}$	(7) Factor Price $\hat{p}_H$	(8) Product Prices $-\hat{p}_Q^H$	(9) Real Net Wage $=\hat{\psi}$
1	1	-0.5	-0.103	0.277	-0.299	-0.126	0.124	-0.125	-0.001
2	0	0	-0.010	0.295	-0.300	-0.015	0.012	-0.125	-0.113
3	0.5	0	-0.040	0.235	-0.300	-0.105	0.048	-0.125	-0.077
4	1	0	-0.069	0.175	-0.300	-0.194	0.083	-0.125	-0.042
5	-0.5	0.5	0.054	0.257	-0.300	0.011	-0.065	-0.125	-0.190
6	0	0.5	0.024	0.195	-0.300	-0.081	-0.029	-0.125	-0.154
7	0.5	0.5	-0.006	0.134	-0.300	-0.172	0.007	-0.125	-0.118
8	1	0.5	-0.036	0.074	-0.300	-0.262	0.043	-0.125	-0.082
9	-0.5	1	0.088	0.158	-0.301	-0.054	-0.106	-0.125	-0.231
10	0	1	0.058	0.096	-0.300	-0.147	-0.069	-0.125	-0.194
11	0.5	1	0.028	0.034	-0.300	-0.238	-0.033	-0.125	-0.158

The real net wage change is $\hat{\omega} = \hat{p}_L - \hat{\tau}_L - \hat{p}_Q^L$ for low-skill labor, $\hat{\psi} = \hat{p}_H - \hat{p}_Q^H$ for high-skill labor.

Column (4) shows the effect of the tax rebate, τ_L, which always helps low-skilled labor. Interestingly, the amount of that rebate varies, because it depends on the amount of revenue from the pollution tax. That pollution tax revenue is small when pollution abatement is relatively easy, so the rebate is small when *both* types of labor are good substitutes for pollution (rows 8 and 11). The final component is the effect of product prices, p_Q^L, in column (5), which always reduces the *real* net wage. These three components add to the total effect on the real net wage ω in column (6).

The real net wage falls in almost every case ($\hat{\omega} < 0$, where $\hat{\omega} = \hat{p}_L - \hat{\tau}_L - \hat{p}_Q^L$). It rises slightly in row 5 only, where low-skilled labor is much better than high-skilled labor as a substitute for pollution ($e_{LZ} > e_{HZ}$). This result answers the key question of our paper: only under special conditions can the rebate of revenue to the low-income group necessarily protect them against a loss in real income.

An estimated distribution of elasticities in Table 5 could be used in a Monte Carlo simulation to calculate the expected change in the real net wage ω. Without such estimates, we cannot state the probability of each row in Table 5. Yet the table makes clear that Monte Carlo simulations are not necessary. Any feasible set of probability weights for the rows of Table 5 could only yield an expected fall in the real net wage.

Importantly, the decomposition allows us to address *why* the real net wage falls. Notice first that the effect of the rebate is relatively large. In this model, however, the tax rebate in column (4) is *never* enough to offset the effect of product prices in column (5). This result seems somewhat remarkable, since the pollution tax places some burden on high-skilled labor as well as on low-skilled labor, and yet all of the pollution tax revenue is given as a rebate just to low-skilled labor! Why? In this general equilibrium model, because of "excess burden", consumers lose more than the revenue from the tax, especially in the cases where abatement is easy (where e_{LZ} and e_{HZ} are both positive, and revenue is small, as in rows 8 and 11).[25]

Thus, the rebate of the entire tax is not enough to protect low-income families just from the effects of higher prices for electricity, heating fuel, and gasoline. This problem is even worse when factor prices also change to hurt low-skilled labor, especially where H is better than L as a substitute for pollution (rows 1,3, 4, 8).

In the last three columns of Table 5, the effect of the tax on the gross wage for high-skilled labor ($\hat{p}_H$) plus the effect from higher produce prices, ($-\hat{p}_Q^H$) add to the effect on their real net wage ($\hat{\psi}$). The gross nominal wage may rise or fall, depending on elasticities, but high-skilled labor gets no rebate in this tax swap. The effect of higher prices always swamps any positive effect on the wage rate, however, so the real net wage always falls. In Table 5, both real net wage rates always fall (except for a tiny gain to low-skilled labor in row 5 discussed above).

So far, in all scenarios, our illustrative parameters have yielded positive values for the change in the nominal net low-skilled wage ($\hat{w} > 0$ in every row of Table 4). We next perform sensitivity analysis to see the effect on w with more extreme parameter values. As described in Case 1 of the previous section, whenever H and L are not

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[25] Our general equilibrium model could be used to derive and calculate excess burden, but we choose here to focus instead on distributional effects. Suffice it to say that marginal excess burden per additional dollar of tax revenue is small when e_{LZ} and e_{HZ} are small. It rises with the value of those parameters and becomes infinite at the peak of the Laffer curve where the extra revenue is zero.

"too" complementary, the denominator is positive ($D_1 > 0$). Then if high-skilled labor is a *much* better substitute for pollution, the low-skilled nominal net wage may fall.[26] Here we find that if $e_{LZ} = -1$, then $\hat{w}$ is negative and $\hat{h}$ is positive whenever e_{HZ} is bigger than about 2.4 (still keeping $e_{HL} = 1$). These new parameter values are a bit different from those in Tables 4 and 5, but they generate a very different outcome.

Conversely, we also consider the case where high-skilled and low-skilled labor are highly complementary with each other. We take $e_{HL} = -5$ and then check for values of e_{HZ} and e_{LZ} where $\hat{w}$ is negative. For example, $e_{LZ} = 10$ and $e_{HZ} = 9$ is a case where $\hat{w}$ is negative, and $\hat{h}$ is positive, even though low-skilled labor is a better substitute for pollution. Other combinations of e_{HZ} and e_{LZ} also can make w fall, but their values must still be quite large.

Other extreme parameters can yield other perverse outcomes such as a decrease in the price of the dirty good or an increase in pollution. Without a comprehensive search, we were able to find one example where pollution rises ($e_{HL} = 5$, $e_{HZ} = -8$, and $e_{LZ} = 6$, which imply that $e_{HH} = -0.52$, $e_{LL} = -7.12$, and $e_{ZZ} = -0.25$). The point is certainly not that a pollution tax is likely to raise pollution. Rather, finding this one case confirms that only extreme elasticity values can generate perverse results.

The intended effect of a tax rebate for low-income workers is for their net wage not to fall relative to the high-skilled wage. However, we have identified several examples where a very different result is possible. For policy reasons, it is important to understand that any intended protection of low-skilled labor may not be realized. We have shown that even when the nominal low-skilled wage increases, low-skilled labor may still bear a disproportionate share of the tax burden. Still, however, low-skilled labor in these cases could be hurt even more without the rebate.

V. Conclusion

In order to evaluate different policies, it is important to understand not only the efficiency costs of environmental taxes but distributional effects as well. Using pollution tax revenue to reduce pre-existing labor taxes can lessen the regressivity of the tax, and that might make a pollution tax more politically viable. While the double dividend literature has focused on the efficiency side of this kind of tax swap, this paper considers equity issues and the circumstances under which real net wages may rise or fall. The

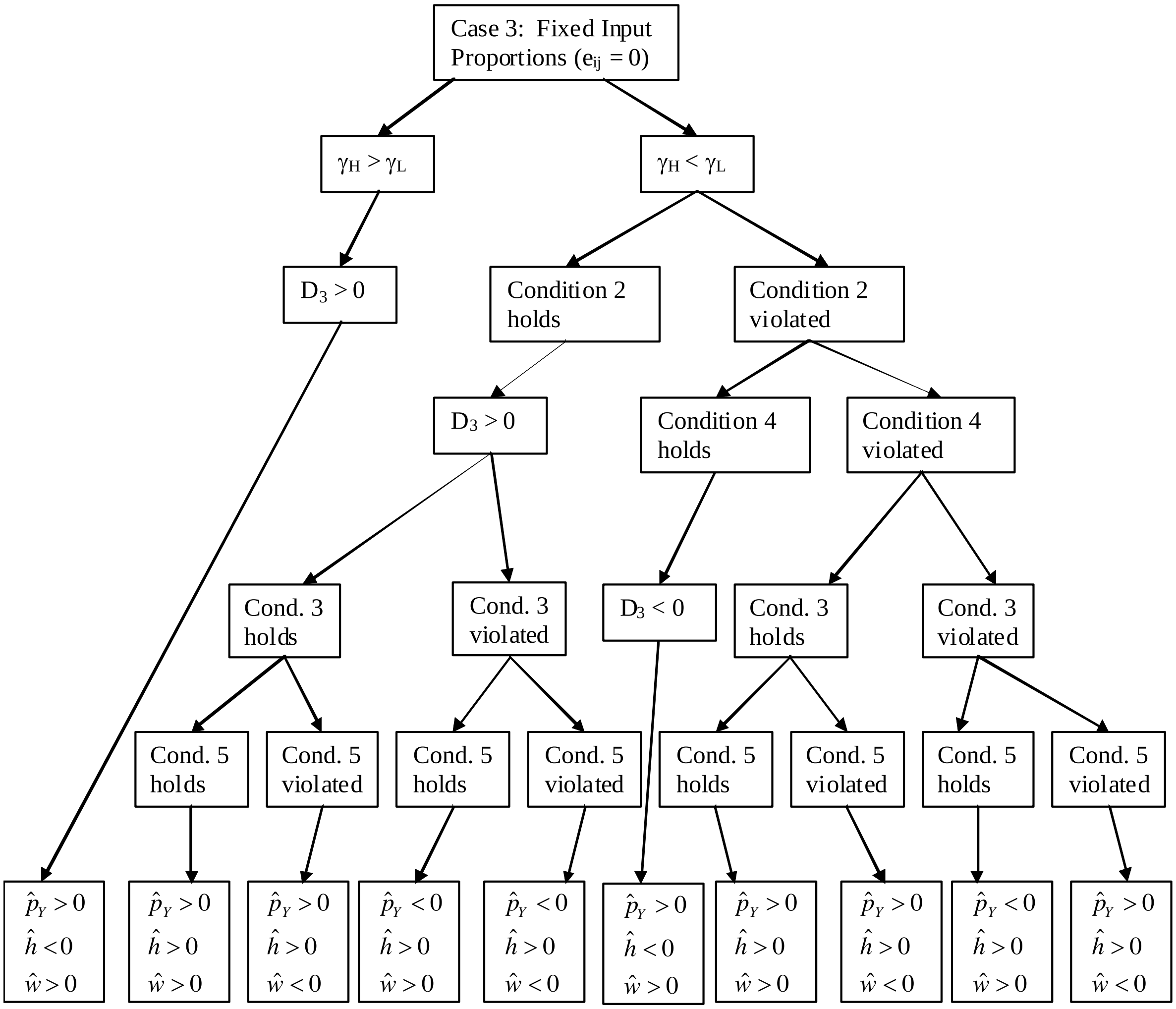
[26] Our numerical example does not exactly match Case 1, however, because factor intensities are not equal.

model developed here is simple but can provide key insights into the effects of a pollution tax increase and labor tax decrease on low-skilled and high-skilled workers.

Using a general equilibrium model, we derive closed-form solutions for changes in wage rates of both types of workers, the price of the dirty good, and the amount of pollution. We find perverse cases where the price of the dirty good may fall, or where pollution may rise, but these cases are unlikely and are not the point of this paper. More to the point is that even without extreme parameter values, the use of the pollution tax revenue to cut low-income workers' labor tax may not raise their real net wage. The tax cut for low-skilled workers certainly has a positive impact on the net low-skilled wage, but for three reasons this effect may not be enough to overcome the burden imposed by an increased pollution tax. First, their real net wage may fall if high-skilled labor is better as a substitute for pollution than is low-skilled labor. Second, it may fall if the dirty sector makes intensive use of low-skilled labor. And third, of course, it may fall if low-income families spend disproportionately on the dirty good.

In our numerical analysis, using reasonable parameter values, we find that the nominal net low-skilled wage increases in all cases, but low-skilled workers can still bear a disproportionate burden of the tax, due to the increase in the price of the dirty good. This numerical simulation does not provide definitive results for the incidence of this tax swap, but it does indicate what parameters need better estimation, and what values of those parameters make policy unable to offset the adverse effects of pollution taxes on low-income families.

Appendix Figure 1: Results for Case 3



Condition 2 (unskilled labor does not spend too disproportionately on Y):

$$(\sigma_U N + J)(\theta_{XH}\theta_{YL} - \theta_{XL}\theta_{YH}) + \frac{1}{C_H} h\overline{H}\left(\frac{Y_H}{Y_L} - \frac{X_H}{X_L}\right) > 0$$

Condition 3 (similar enough factor intensities): $\gamma_L - \gamma_H < \dfrac{C\sigma_X C_H}{h\overline{H}\left(\frac{X_H}{X_L} - \frac{Y_H}{Y_L}\right)}$

Condition 4 (different enough factor inensities):

$$\gamma_L - \gamma_H > \frac{-C\sigma_X}{(\sigma_U N + J)(\theta_{XH}\theta_{YL} - \theta_{XL}\theta_{YH}) + \frac{1}{C_H} h\overline{H}\left(\frac{Y_H}{Y_L} - \frac{X_H}{X_L}\right)}$$

Condition 5 (similar enough factor intensities): $\dfrac{\theta_{XH}\theta_{YZ}}{D_3}(\gamma_L - \gamma_H)(\sigma_U N + J)\hat{\tau}_Z < -\hat{\tau}_L$

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