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ABSTRACT

This paper studies optimal fiscal policy in an economy where heterogeneous agents with uncertain lifetimes coexist. We show that some plausible social welfare functions lead to time-inconsistent optimal plans, and we suggest restrictions on social preferences that avoid the problem. The normative prescriptions of a time-consistent utilitarian planner generalize the "two-part Golden Rule" suggested by Samuelson, and imply aggregate dynamics similar to those arising in the Cass-Koopmans-Ramsey optimal growth framework. We characterize lump-sum transfer schemes that allow the optimal allocation to be decentralized as the competitive equilibrium of an economy with actuarially fair annuities. The lump-sum transfers that accomplish this decentralization are age dependent in general.

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OPTIMAL TIME-CONSISTENT FISCAL POLICY WITH UNCERTAIN LIFETIMES

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Introduction

This paper studies the idea of optimal fiscal policy in an economy where heterogenous generations with uncertain lifetimes coexist. A growing literature studies the effects of social security and government debt issue in economies of this type, but it stops short of describing any intertemporal social welfare function that might justify the use of fiscal tools. Our primary concern is therefore the dynamic resource allocation chosen by a utilitarian planner who weighs the welfare of both existing and future generations. The basic model of the individual comes from Yaari (1965).[2]

Specification of an intertemporal social welfare function is complicated by the possibility that optimal plans are dynamically inconsistent in the sense of Strotz (1956). This possibility arises in any model with overlapping generations. Below we show that some plausible social welfare functions lead to time-inconsistency, and suggest restrictions on social preferences that avoid the problem.

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1 We are grateful for helpful discussions with Andrew Abel. Financial support was provided by the National Science Foundation and the Alfred P. Sloan Foundation.

2 For recent applications of uncertain-lifetime models to fiscal-policy questions, see Abel (1985), Blanchard (1985), and Eckstein, Eichenbaum, and Peled (1983), among others. Open-economy aspects of Blanchard's (1985) model are studied by Buiter (1984), Frenkel and Razin (1984), and Giovannini (1984). Early applications of Yaari's (1965) setup include Merton (1971) and Tobin (1967).

An intertemporal welfare analysis along the lines of that presented here has been developed by Samuelson (1967, 1968) in the context of Diamond's (1965) deterministic overlapping-generations model with capital.[3] However, special assumptions made by Samuelson obscure the potential for time inconsistency. It is therefore noteworthy that the normative prescriptions of our time-consistent utilitarian planner generalize those suggested by Samuelson. Moreover, the aggregate dynamics implied by an optimal plan are qualitatively similar to those derived by Cass (1965), Koopmans (1965), and Ramsey (1928) in models with identical, non-overlapping generations. We characterize lump-sum transfer schemes that allow the optimal allocation to be decentralized through a competitive economy with actuarially fair annuities.

An important finding of the paper is that the above-mentioned transfers are in general age dependent. It follows that the aggregative fiscal policies studied in the recent literature will usually fail to achieve the optimal allocation. If first-best fiscal policy tools are unavailable, however, the door is opened to the type of general-equilibrium time inconsistency studied by Kydland and Prescott (1977), Calvo (1978), and others.

The paper is organized as follows. Section I reviews the individual's problem and the competitive equilibrium of an economy in which there is no uncertainty at the aggregate level. Section II takes up utilitarian planning and the time-consistency problem. In section III the allocation chosen by a time-consistent utilitarian planner is characterized. Section IV discusses lump-sum redistribution schemes that

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[3] Phelps and Riley (1978) study the Rawlsian "maximin" case. Abel (1984) uses Samuelson's criterion to evaluate steady-state welfare in overlapping-generations models with money.

replicate the planner's preferred allocation as a competitive equilibrium. Section V is a brief summary of the results.

I. Competitive Equilibrium with Annuities

To make the paper self-contained, this section briefly reviews the intertemporal allocation problem of an individual with an uncertain lifetime who purchases actuarially fair annuities. While the conclusions merely repeat those of Yaari (1965), the presentation of the problem is slightly different and hopefully more transparent. The section concludes by describing the economy's aggregate dynamics in perfect-foresight equilibrium.

An individual born at time v (his "vintage") is uncertain about the length N of his life. Let F(.) denote the cumulative distribution function of the random variable N, so that $F(n) = \text{Prob}\{N \leq n\}$. Of course, $F(0) = 0$ and $\lim_{n\to\infty} F(n) = 1$. Implicit in our notation is the assumption that the distibution of N does not depend on v; in addition F(.) is assumed to be continuous and piecewise differentiable with an associated probability density function f(.) satisfying $F(n) = \int_0^n f(s)ds$. Individuals maximize the expected value over possible lifespans of a discounted integral of future instantaneous utilities. The time-t utility of a vintage-v individual is a function u(.) of consumption c(v,t).[4] If δ denotes the constant subjective discount rate, expected lifetime utility for an agent born on date v is

$$(1)\quad U(v) = \int_0^\infty f(n)\{\int_v^{v+n} u[c(v,t)]\exp[-\delta(t-v)]dt\}dn.$$

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[4] The function u(.) is assumed to be bounded, strictly concave, and twice continuously differentiable. Notice that u(.) is assumed to be independent of v. Only consumption paths c(v,.) that are piecewise continuous and right-hand differentiable are considered. To ensure interior solutions, the usual Inada conditions are imposed.

After integrating by parts, (1) can be written in the form

$$(2)\quad U(v) = \int_v^\infty u[c(v,t)][1 - F(t-v)]\exp[-\delta(t-v)]dt,$$

where $1 - F(t-v)$ is just the probability that an individual born on date v is alive on date t.

Define p(n) to be the instantaneous death probability faced by an individual of age n:

$$(3)\quad p(n) = f(n)/[1 - F(n)].$$

Because $f(n) = F'(n)$ and $F(0) = 0$, (3) implies

$$(4)\quad 1 - F(n) = \exp[-\int_0^n p(s)ds].$$

The objective function (2) therefore takes the form

$$(5)\quad U(v) = \int_v^\infty u[c(v,t)]\exp\{-\int_v^t [\delta + p(s-v)]ds\}dt.$$

As in Yaari (1965), the possibility of death leads to a higher subjective discount rate on future utility.

Assume now, as in Blanchard's (1985) model, that a new cohort of individuals is born each instant and that there is no aggregate uncertainty even though each individual's lifespan is stochastic. If the size of each newly-born cohort is normalized to unity, there are exactly $1 - F(t-v)$ individuals of vintage v alive at any time $t \geq v$ and this cohort's size declines at rate $p(t-v)$. Those within a given cohort are assumed to be identical in all respects.

Individuals are prohibited from dying in debt, and can borrow only if they simultaneously buy insurance against that contingency. There exist insurance companies that buy and issue annuities which pay holders the age-dependent yield $r(t) + p(t-v)$ at time t but expire in the event of the owner's death. Because there are no bequests, those with positive nonhuman wealth will choose to hold it exclusively in the form of annuities, which pay a rate exceeding the market real interest rate while the owner lives. Borrowers effectively insure themselves against accidental death by issuing annuities to the insurance company. In short, insurance companies intermediate between all borrowers and lenders and also hold the private sector's net marketable assets, which in the present context will coincide with the capital stock. Under the assumptions of the preceding paragraph, and with costless free entry into the insurance industry, the insurance premium $p(t-v)$ is actuarially fair and the insurance company makes zero profits. The instantaneous effective borrowing rate faced at time t by an individual born at time v is thus $r(t) + p(t-v)$, where $r(t)$ is the real interest rate and $p(t-v)$ is the actuarially fair insurance premium.

The lifetime problem of an individual born on date v may now be stated as follows: choose a consumption path $\{c(v,t)\}_{t=v}^{\infty}$ so as to maximize (5) subject to

$$(6)\quad \int_v^{\infty} c(v,t)\exp\{-\int_v^t [r(s)+p(s-v)]ds\}dt \leq a(v,v).$$

In (6), $a(v,t)$ is the overall time-t wealth of an agent of vintage v. Wealth is the sum of of the present discounted value of wages (human wealth), the present discounted value of expected future transfer payments from the government, and capital.[5]

The Lagrangian for the problem can be written

$$L = \int_v^{\infty}\{u[c(v,t)]\exp[-\delta(t-v)] - \mu c(v,t)\exp[-\int_v^t r(s)ds\}\exp[-\int_v^t p(s-v)ds]dt,$$

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[5] It is easily verified that the solution to the individual's problem is time consistent.

where μ is a Lagrange multiplier. (Terms that do not involve $c(.,.)$ have been disregarded.) Maximization of L with respect $c(v,t)$ yields the necessary condition

$$(7)\quad u'[c(v,t)]\exp[-\delta(t-v)] - \mu\exp[-\int_v^t r(s)ds] = 0,$$

for all t. Let D be the time-derivative operator. Differentiating (7) with respect to time, we obtain

$$(8)\quad Dc(v,t) = -\{u'[c(v,t)]/u''[c(v,t)]\}[r(t) - \delta].$$

An optimal individual consumption plan obeys (8) while forcing the budget constraint (6) to bind. Note that the death probabilities do not affect the time derivative of consumption along an optimal path (although they do affect the level of consumption through (6)).

Turn next to the implied dynamic behavior of the economy's aggregates in perfect-foresight equilibrium. If each agent is endowed with one unit of labor, the labor force at any time t is a constant,

$$\int_{-\infty}^{t}\exp[-\int_v^t p(s-v)ds]dv.$$

Aggregate output Y may then be written as a function Y(K) of the economy's capital stock K.[6] The usual one-sector assumption fixes the consumption price of capital at unity. If C(t) denotes aggregate consumption,

$$(9)\quad C(t) = \int_{-\infty}^{t} c(v,t)\exp[-\int_v^t p(s-v)ds]dv,$$

and if the government consumes no goods itself, the economy's capital

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[6] The underlying production function is assumed to be homogeneous of degree one in capital and labor, to exhibit diminishing returns to each factor, and to obey the Inada conditions.

stock evolves according to

(10) $DK(t) = Y[K(t)] - C(t)$.

In perfect-foresight equilibrium, the expected real interest rate $r(t)$ must equal the marginal product of capital $Y'[K(t)]$ for all t. Combination of (8) and (9) shows that the time derivative of aggregate consumption is

$$DC(t) = c(t,t) - \int_{-\infty}^{t} p(t-v)c(v,t)\exp[-\int_{v}^{t} p(s-v)ds]dv - \{Y'[K(t)]-\delta\}\int_{-\infty}^{t}\{u'[c(v,t)]/u''[c(v,t)]\}\exp[-\int_{v}^{t} p(s-v)ds]dv \quad (11)$$

in equilibrium. The first two terms on the right-hand side of (11) sum to the difference between the consumption of the newly born and the overall consumption of those who die at time t. The third term is just the sum of the individual consumption changes dictated by the Euler equation (8).

The steady state is defined by $DC = DK = 0$. Without simplifying assumptions, it is quite difficult to characterize the steady-state allocation for this competitive economy. For example, if $p(n)$ is a constant p as in Merton (1971) and Blanchard (1985), and if $u(c) = c^{1-R}/(1-R)$ $(R > 0)$, then it is easy to see that the steady-state real interest rate $r^c = Y'(K^c)$ must satisfy $r^c < \delta + pR$. (K^c is the steady-state capital stock in the competitive model when there is no government intervention.)

The system becomes quite simple under Blanchard's (1985) additional assumptions that $R = 1$ (so that $u(c) = \log(c)$), that cohorts are born without nonhuman wealth, and that all individuals of any age have the same human wealth

(12) $h(t) = \int_{t}^{\infty} w(s)\exp\{-\int_{t}^{s}[r(z)+p]dz\}ds$,

where $w(t)$ is the real wage at t. When $R = 1$, the consumption function takes the form $c(v,t) = (\delta+p)[a(v,t)]$. The (constant) population is just $1/p$; and because aggregate human wealth $H(t)$ therefore equals $h(t)/p$ and aggregate nonhuman wealth equals the capital stock $K(t)$, (11) becomes

$$(13)\quad DC(t) = (\delta+p)h(t) + \{Y'[K(t)]-\delta-p\}(\delta+p)[H(t)+K(t)]$$
$$= \{Y'[K(t)]-\delta\}C(t) - p(\delta+p)K(t).$$

This simple characterization of the aggregate dynamics is generally inapplicable if the government taxes differentially according to age. As we shall see later, <u>differential taxation of this type is needed to support optimal growth paths as competitive equilibria</u>. We next discuss the meaning of "optimality" and the implied intertemporal resource allocation for a centrally-planned economy.

II. Time-Consistent Utilitarian Planning

In this section we describe the objective function of a utilitarian planner whose preferences admit time-consistent optimal plans. The planner is utilitarian in the sense that his welfare objective is a weighted sum of the utilities of all generations, including those not yet born.

Samuelson (1967, 1968), building on a suggestion of Lerner (1959), was the first to study utilitarian planning in an economy with finitely-lived overlapping generations. In the model of Samuelson (1968), a generation lives for two periods and a new generation is born each period until a known date T in the future. Thus, the last generation is born on date T-1. If an individual born on date t enjoys lifetime utility $u[c(t,t),c(t,t+1)]$, Samuelson's planner maximizes

$$S(0) = \Sigma_0^{T-1}\beta^t u[c(t,t),c(t,t+1)],$$

subject to constraints discussed below, on date 0. The parameter $\beta \in (0,1]$ gives the rate at which the planner discounts future generations' utility (with $\beta = 1$ the Ramsey case). Although Samuelson assumes a finite horizon, he characterizes a hypothetical steady-state allocation satisfying first-order conditions for maximization of S(0).

The welfare criterion S(0) generally yields time-inconsistent programs if the planner is constrained only by the initial capital stock K(0) and the saving-investment relation $K(t+1) - K(t) = Y[K(t)] - c(t,t) - c(t-1,t)$. In other words, for $s > 0$, the consumption plan $\{c(t,t),c(t,t+1)\}_{t=0}^{\infty}$ maximizing S(0) subject to resource constraints does not maximize S(s) if followed from time s onward. The reason for this is that the Samuelson criterion does not attach an appropriate weight to the welfare of the old generation alive at the start of the plan. In each period s, maximization of S(s) requires that consumption by the old be zero, even though this was not planned when S(s-1) was maximized.

Samuelson (1968) in effect avoids time inconsistency by placing an additional constraint on the planner: the requirement that the consumption of the old in period s, c(s-1,s), be taken as given. If this predetermined consumption level for the old is interpreted as the level envisioned in the previous period's optimal plan, the planner is forced to pursue a time-consistent program. However, Samuelson does not offer this interpretation, and no suggestion is made regarding social institutions that might impose a time-consistency constraint.

The welfare criterion introduced here avoids time inconsistency by explicitly and appropriately accounting for the welfare of cohorts already alive at the planning period's start. No dynamic-consistency constraint is placed on the planner's actions. The social welfare function proposed in Samuelson (1967) also avoids time-inconsistency without side constraints, but only under restrictive assumptions on individual preferences that mask some of the issues discussed below. (Samuelson (1967) assumes that $u[c(t,t),c(t,t+1)] = v[c(t,t)] + v[c(t,t+1)]$, so that individuals do not discount future utility.) It is noteworthy (but not surprising) that the optimal plans explored below entail intertemporal and intergenerational allocation rules which generalize those derived by Samuelson (1967, 1968).

Our planner's objective is the sum of two components. First, there is the lifetime expected utility of the generations to be born, as measured from the moment of birth. Second, there is the expected utility, over the remainder of their lifetimes, of those cohorts currently alive. The remaining expected utility of a cohort currently alive is, like that of a cohort to be born, measured from the perspective of its birthdate. If it is assumed in addition that the planner discounts generations at a rate $\rho > 0$, the social welfare function is

$$(14)\quad W(0) = \int_0^\infty (\int_v^\infty u[c(v,t)]\exp\{-\int_v^t[\delta+p(s-v)]ds\}dt)\exp(-\rho v)dv + \int_{-\infty}^0 (\int_0^\infty u[c(v,t)]\exp\{-\int_v^t[\delta+p(s-v)]ds\}dt)\exp(-\rho v)dv$$

at time $t = 0$. $W(0)$ has an alternative interpretation. Its first component may be viewed as a weighted integral of instantaneous utilities actually enjoyed by members of future generations, discounted to the date of birth at the "risk-free" rate δ. (Recall that there is no <u>aggregate</u> uncertainty.) The second component is the weighted integral of utilities to be enjoyed by living members of the current generations, also discounted to their birthdates at rate δ. The planner's maximization must be carried out subject to an initial capital endowment $K(0)$ and the constraint (10), which is repeated here (after substitution from

(9)) for convenience:

$$(15)\quad DK(t) = Y[K(t)] - \int_{-\infty}^{t} c(v,t)\exp[-\int_{v}^{t} p(s-v)ds]dv.$$

It may appear unnatural to discount the utility of those already alive back to their birthdates, rather than to the present. After all, the planner is concerned with their welfare from the present (time t = 0) onward. However, this discounting scheme is necessary for the time consistency of optimal intertemporal allocations. Unless those alive and those to be born are treated symmetrically, the planner has an incentive to change the consumption previously planned for unborn generations once they come into existence.[7]

To appreciate the time inconsistency problem consider the "natural" social welfare function

$$(14')\quad V(0) = \int_{0}^{\infty}(\int_{v}^{\infty} u[c(v,t)]\exp\{-\int_{v}^{t}[\delta+p(s-v)]ds\}dt)\exp(-\rho v)dv + \int_{-\infty}^{0}(\int_{0}^{\infty} u[c(v,t)]\exp\{-\int_{0}^{t}[\delta+p(s-v)]ds\}dt)\exp[-\int_{v}^{0}p(s-v)ds]\exp(-\rho v)dv.$$

In (14′), the expected utility of the surviving members of current generations is measured from the perspective of time 0.[8] If it is assumed for simplicity that p(n) is a constant p, then for T > 0,

$$V(0) = V(T) + J_1(T) + J_2(T) + Q_1 + Q_2$$

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[7] An apparent alternative to (14) would treat current and future generations symmetrically by discounting all utility back to time 0. But this is equivalent to raising ρ to $\rho + \delta$ in (14).

[8] As noted above, Samuelson (1967) studies the maximization of a social welfare function similar to (14) in an overlapping-generations model with deterministic two-period lifetimes. However, he assumes a finite planning horizon, no individual time preference, and no generational preference on the part of the planner. In Samuelson's framework, therefore, $\delta = 0$, which would imply no distinction between (14) and (14′).

where

$J_1(T) =$

$$\int_{-\infty}^{T}\{\int_{T}^{\infty}u[c(v,t)]\exp[-(\delta+p)(t-T)]dt\}\exp[p(v-T)]\exp(-\rho v)[\exp(-\delta T)-1]dv,$$

$J_2(T) =$

$$\int_{0}^{T}\{\int_{T}^{\infty}u[c(v,t)]\exp[-(\delta+p)(t-T)]dt\}\exp[p(v-T)]\exp(-\rho v)\{\exp[\delta(v-T)]-\exp(-\delta T)\}dv,$$

$$Q_1 = \int_{0}^{T}\{\int_{v}^{T}u[c(v,t)]\exp[-(\delta+p)(t-v)]dt\}\exp(-\rho v)dv,$$

$$Q_2 = \int_{-\infty}^{0}\{\int_{0}^{T}u[c(v,t)]\exp[-(\delta+p)t]dt\}\exp(pv)\exp(-\rho v)dv.$$

The quantities Q_1 and Q_2 are predetermined as of time T, but $J_1(T)$ and $J_2(T)$ are not. It follows that a plan maximizing V(0) given K(0) will not in general maximize V(T) given the resulting K(T). The reason for this is that any plan optimal at time 0 must maximize $V(T) + J_1(T) + J_2(T)$--not V(T)-- given K(T); otherwise it can be dominated by a plan that yields the same values for K(T), Q_1, and Q_2, but a higher value for $V(T) + J_1(T) + J_2(T)$. A planner with preferences described by (14′) will therefore be time inconsistent. Arriving at T with capital K(T), he will prefer to maximize V(T) and so will deviate from the plan that maximized V(0).

Yet another alternative to (14), particularly appealing when the instantaneous death probability is a constant p, is to treat all those alive at time zero as if they had just been born i.e., as if they were all of vintage zero. It is easily verified that this "egalitarian" scheme, like (14′), generally yields a time-inconsistent optimum. The exception occurs when $\rho = \delta$, so that the planner's objective is identical to a special case of (14), that in which consumption is optimally equal across cohorts at any time.

The time consistency of plans maximizing W(0) subject to the constraints can be seen by noting that for $T > 0$

$$W(0) = W(T) + R_1 + R_2,$$

where

$$R_1 = \int_0^T (\int_v^T u[c(v,t)]\exp\{-\int_v^t[\delta+p(s-v)]ds\}dt)\exp(-\rho v)dv,$$

$$R_2 = \int_{-\infty}^0 (\int_0^T u[c(v,t)]\exp\{-\int_v^t[\delta+p(s-v)]ds\}dt)\exp(-\rho v)dv.$$

R_1 and R_2 are predetermined as of time T. It follows that any plan maximizing W(0) given K(0) must maximize W(T) given the implied value of K(T). Otherwise there would exist a plan yielding the same values of K(T), R_1, and R_2, but a higher value of W(T). And this would contradict the assumed optimality of the initial plan.[9]

An implication of our analysis is that the credibility of a planner will depend on the way he weights generations' utilities. This complication is not found in the Cass-Koopmans framework with nonoverlapping generations. There, it is a matter of indifference whether the planner discounts instantaneous utility according to the time at which it is enjoyed or the generation that enjoys it. Here the distinction is crucial.[10]

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[9] It should now be clear that there is a time-consistent analogue of the Samuelson criterion S(0) which embodies the discounting convention incorporated in our welfare measure W(0). That criterion is $\{u[c(-1,-1),c(-1,0)]/\beta\} + S(0)$, which is maximized at time 0 given the (predetermined) consumption level $c(-1,-1)$ enjoyed in their youth by those currently old.

[10] If one were to insist that optimal plans should be time consistent, our findings would give some guidance regarding admissible social welfare functions. This is reminiscent of Koopmans' (1965) discussion, where its is shown that a nonnegative planner's discount rate is required to ensure the existence of an optimal plan when population is growing. Note that our analysis would be unchanged if the discount rate applied by the planner to generation v at time t were $\exp[-\rho(v-t)]$ (as in Cass and Koopmans) rather than $\exp[-\rho v]$.

III. The Optimal Allocation over Time

The intertemporal resource allocation chosen by a time-consistent utilitarian planner is studied in this section. That optimum is qualitatively similar to the one arising in the familiar Cass-Koopmans-Ramsey growth problem with identical nonoverlapping generations, but here the generational discount factor ρ determines the long-run marginal product of capital, as suggested by Samuelson (1968).

It is easiest to solve the planner's problem of maximizing (14) subject to (15) by the method of optimal control (Arrow and Kurz, 1970). After changing the order of integration, (14) may be written in the form

$$(16)\quad W(0) = \int_0^\infty \{\int_{-\infty}^t u[c(v,t)]\exp[-\int_v^t p(s-v)ds]\exp[(\delta-\rho)v]dv\}\exp(-\delta t)dt.$$

Equation (16) yields yet another interpretation of the planner's objective. W(0) is just the discounted integral, over all future dates, of a weighted sum of instantaneous utilities of those currently alive. The planner applies the individual subjective discount factor δ in weighting the aggregate utility enjoyed on different dates. In adding up utilities enjoyed on a given date by agents of different ages, vintage is discounted at the net rate $\delta - \rho$.[11]

Let $\lambda(t)$ denote the costate variable for the problem of maximizing (16) subject to (15). Then the associated Hamiltonian is written

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[11] The criterion (16) may be viewed as a discounted sum of static "Benthamite" social welfare functions; again, see Samuelson (1967). Observe that even if u(.) is bounded the summands in (16) can be unbounded if the generational discount factor ρ is sufficiently large. In general, therefore, an optimal plan need not exist: one needs to assume that the welfare weight attached to previous generations does not grow "too quickly" relative to the rate at which members of those generations die and δ. This problem can be avoided by postulating a finite age n such that F(n) = 1. Existence of an optimum is assumed in the discussion below.

$$(17)\quad \int_{-\infty}^{t} u[c(v,t)]\exp[-\int_{v}^{t} p(s-v)ds]\exp[(\delta-\rho)v]dv$$
$$+\ \lambda(t)\{Y[K(t)] - \int_{-\infty}^{t} c(v,t)\exp[-\int_{v}^{t} p(s-v)ds]dv\}.$$

Necessary conditions for an optimal plan are

$$(18)\quad u'[c(v,t)]\exp[(\delta-\rho)v] = \lambda(t)$$

plus the equation of motion for the costate,

$$(19)\quad D\lambda(t) = \lambda(t)\{\delta - Y'[K(t)]\}.$$

Equation (18) implies that at any time t, consumption evolves across cohorts according to the equation

$$(20)\quad \partial c(v,t)/\partial v = -\{u'[c(v,t)]/u''[c(v,t)]\}(\delta - \rho).$$

By (18) and (19), the consumption of a given cohort evolves over time according to

$$(21)\quad \partial c(v,t)/\partial t = -\{u'[c(v,t)/u''[c(v,t)]\}\{Y'[K(t)] - \delta\}.$$

What is the meaning of equation (20)? As was noted earlier, the difference $\delta - \rho$ can be viewed as the <u>net</u> rate at which the planner discounts a given cohort's welfare according to its age. If an allocation is optimal, there must be no incentive to shift consumption between cohorts at any time t. But this implies that the rate at which the marginal utility of consumption changes as age rises (i.e., as v falls) must equal $\delta - \rho$. This is what equation (20) states. The case $\rho = \delta$ yields the "egalitarian" plan mentioned in the previous section, under which all individuals have the same consumption level at any point in time.

The intertemporal allocation condition (21) is identical to the condition (8) achieved by the competitive economy. This, incidentally, can be used to show the Pareto efficiency of the market allocation (since population growth is zero and the marginal product of capital is always positive). What is the rationale for (21) in a planning context? For an agent of vintage v, the sum $\delta + p(t-v)$ is the instantaneous expected-utility cost, at time t, of postponing consumption. If the government shifts one unit of a given cohort's consumption into the future, the instantaneous return on the investment to those left alive is $Y'[K(t)] + p(t-v)$ because the proceeds are divided among a smaller group. Since the net return available to the government equals that offered by the insurance company, the rate at which a cohort's marginal utility changes over time must equal $\delta + p(t-v) - Y'[K(t)] - p(t-v) = \delta - Y'[K(t)]$, as in the competitive allocation.

The solution to the planning problem is quite general, in that no special assumptions about the form of the instantaneous utility function u(.) are required. To make the nature of the solution more transparent, it is useful to analyze first a special case. We then show that the main implications of this special case also hold in general.

Assume temporarily that u(.) belongs to the constant relative risk aversion class, so that $-cu''/u' = R$. Then conditions (20) and (21) yield a simple characterization of the behavior of aggregate consumption C(t) along an optimal path. Differential equation (20) now has the solution

(22) $c(v,t) = c(t,t)\exp[(\delta-\rho)(v-t)/R]$.

Aggregate consumption at time t may therefore be expressed as

(23) $C(t) = c(t,t)\int_{-\infty}^{t}\exp\{-(1/R)\int_{v}^{t}[\delta-\rho+Rp(s-v)]ds\}dv$

(cf. (9)). The convergence condition $\lim_{t\to\infty}\exp\{\int_{0}^{t}[\rho-\delta-Rp(s)]ds\} = 0$ is imposed to ensure that (23) is well defined. Without this convergence

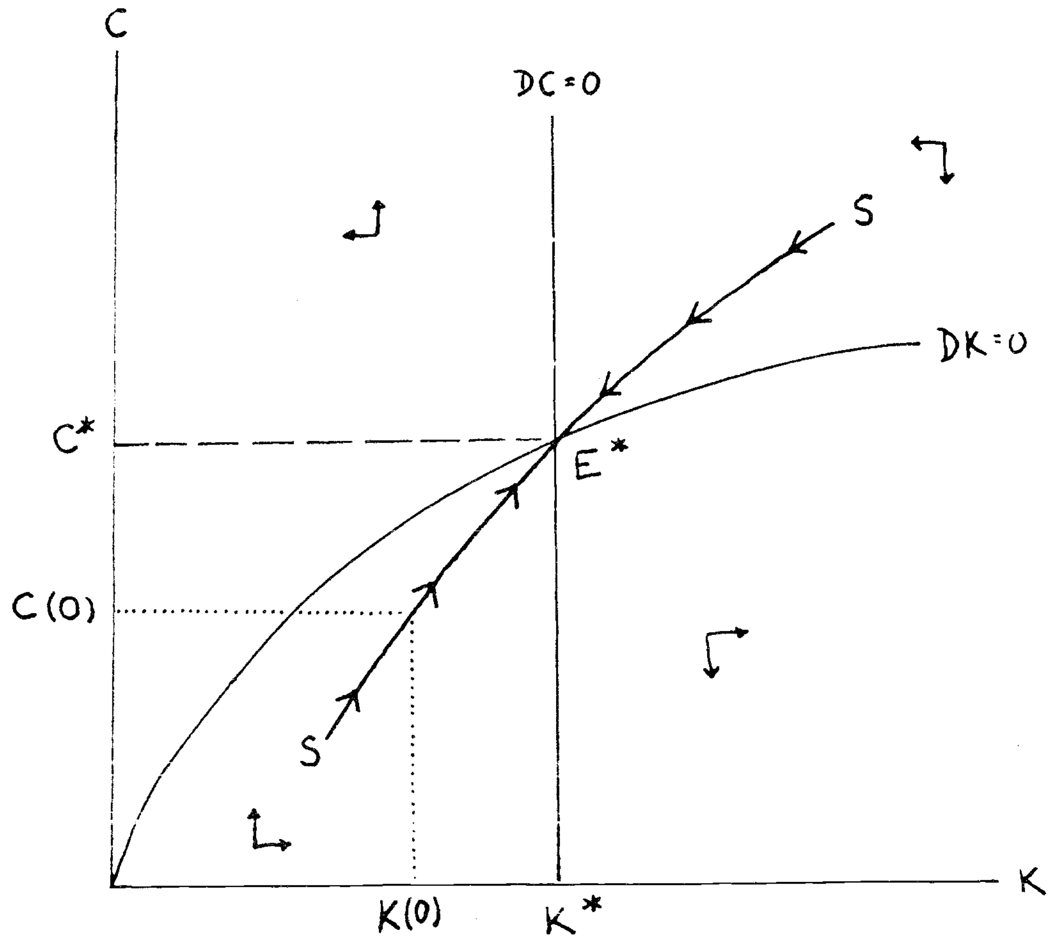

Figure 1

condition an optimal plan would not exist (see footnote 11).

Differentiation shows that the coefficient of $c(t,t)$ in (23) is constant over time. Thus, aggregate consumption and the consumption of the newly born are proportional, so that

$$(24)\quad DC(t)/C(t) = Dc(t,t)/c(t,t).$$

Combining (21) and (22), we find that

$$(25)\quad Dc(t,t)/c(t,t) = (1/R)\{Y'[K(t)] - \rho\}.$$

Equations (24) and (25) now show that the central planner will cause aggregate consumption to evolve according to the rule

$$(26)\quad DC(t)/C(t) = (1/R)\{Y'[K(t)] - \rho\}.$$

This is identical to the condition that would govern the evolution of aggregate consumption in the standard Cass-Koopmans-Ramsey representative-agent planning model with discount rate ρ.

Equations (15) and (26) describe the aggregate dynamics implied by utilitarian planning. These dynamics may be pictured with the aid of figure 1. Aggregate consumption equals ouput along the $DK = 0$ locus, so the capital stock is stationary there. On the $DC = 0$ locus aggregate consumption is stationary (although individuals' consumption levels may change over time). The system is saddlepoint stable as usual, with a unique path SS converging to the steady state equilibrium E*. SS yields the optimal initial aggregate consumption level $C(0)$ associated with any given initial capital stock $K(0)$. The optimal steady-state capital stock K^* is determined by the condition

$$(27)\quad Y'(K^*) = \rho,$$

and thus depends only on the production function and the rate at which the planner discounts according to age. Optimal steady-state consumption is of course given by

(28) $C^* = f(K^*)$.

It is noteworthy that condition (27) is the same as that derived by Samuelson (1968) in a model where the planner is constrained to pursue a time-consistent plan. Indeed, conditions (20) and (21) also have analogues in Samuelson's framework.

The main results of the constant relative risk aversion case can be generalized. In particular, the steady state described by (27) and (28) is independent of the instantaneous utility function u(.), and the consumptions of all cohorts rise or fall monotonically along the transition path. While it is no longer possible in general to express the economy's dynamics in terms of aggregate consumption and the capital stock, an alternative two-variable representation can be developed.

These assertions are established as follows. Use necessary condition (18) to write c(v,t) as

(29) $c(v,t) = \phi\{\lambda(t)\exp[(\rho-\delta)v]\}$, $\phi' < 0$.

Recall that aggregate consumption is given by equation (9). After a change of variables from v to n = t - v (i.e., from "vintage" to "age"), (9) becomes

$$C(t) = \int_0^\infty c(t-n,t)\exp[-\int_0^n p(s)ds]dn,$$

so that upon substitution of (29) we have

(30) $C(t) = \int_0^\infty \phi\{\lambda(t)\exp[(\rho-\delta)t]\exp[-(\rho-\delta)n]\}\exp[-\int_0^n p(s)ds]dn.$

Define $\eta(t) \equiv \lambda(t)\exp[(\rho-\delta)t]$. Then (29) and (30) imply that aggregate consumption can be written as a declining function of $\eta(t)$, $C[\eta(t)]$.

Equation (15) may now be expressed in the form

(31) $DK(t) = Y[K(t)] - C[\eta(t)]$.

Differentiation of $\eta(t)$ and application of (19) show that

(32) $D\eta(t) = \eta(t)\{\delta - Y'[K(t)]\} + \eta(t)(\rho - \delta) = \eta(t)\{\rho - Y'[K(t)]\}$.

Equation (32) is identical to the dynamic equation for the costate variable in the standard representative-agent model with time-preference rate ρ.

The phase diagram for the system in $\eta(t)$ and $K(t)$ described by equations (31) and (32) is shown in figure 2. Because $C(\eta^*) = K^*$, where η^* is the steady-state value of η, aggregate consumption and the capital stock converge toward the stationary values C^* and K^* defined by (27) and (28). The qualitative behavior of consumption is the same as in the constant relative risk aversion case of figure 1, both in the aggregate and at the cohort level.

IV. Optimal Fiscal Policy

The goal of this section is to show how fiscal policy can be used to decentralize optimal utilitarian plans in the competitive economy of section I. Because the competitive equilibrium without government intervention is efficient, only lump sum taxes need be used to generate the optimal plan as a competitive equilibrium. However, the tax an individual pays will in general vary according to his age and calendar time. To keep the analysis simple we consider only balanced-budget fiscal policy. Equivalent policies could involve government debt issue,

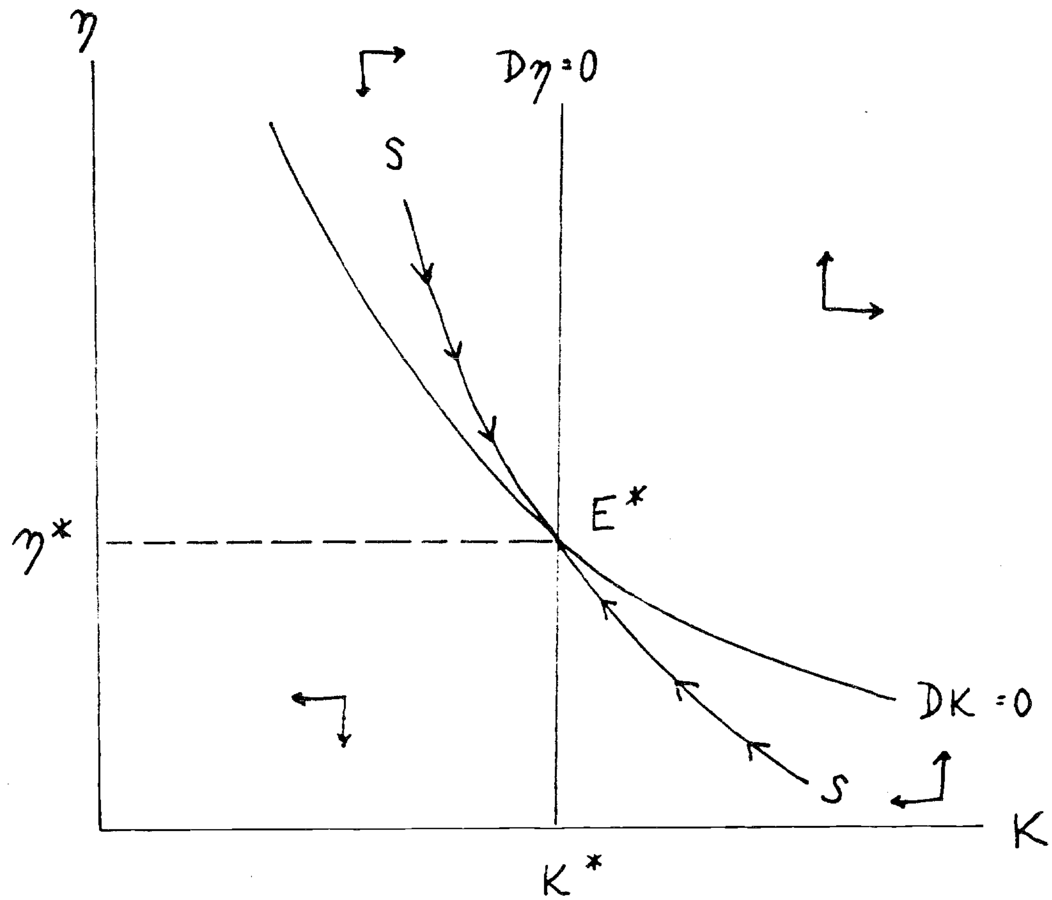

Figure 2

but this is not discussed explicitly below.

To make the main points it is sufficient to work with a special case of the model, that studied by Blanchard (1985). It is therefore assumed once again that the instantaneous death probability is a constant p, that the instantaneous utility function is logarithmic, and that individuals are born with human wealth only. The last paragraphs of section I discuss individual and aggregate behavior in this case when there is no government intervention, and the reader may wish to review them before proceeding.

Given an initial capital stock K(0), the optimal plan generates paths $\{K(t)\}_{t=0}^{\infty}$ for capital and $\{C(t)\}_{t=0}^{\infty}$ for aggregate consumption. These in turn yield paths for the shadow real interest rate $r(t) = Y'[K(t)]$, the shadow wage $w(t) = p\{Y[K(t)] - K(t)Y'[K(t)]\}$, and the shadow present value of per capita wage income h(t) (given by (12), and the same for all agents alive at time t). To decentralize the optimal intertemporal allocation, the government must endow each agent at birth with a transfer stream inducing the desired consumption levels when the shadow price paths $\{r(t)\}_{t=0}^{\infty}$ and $\{w(t)\}_{t=0}^{\infty}$ are expected. Let $\tau(v,t)$ denote the transfer payment received by an agent of vintage v at time t, and let b(v,t) denote the present value of these payments at time t given the expected real-interest rate path. We will consider fiscal policies having the property that the government's budget is balanced on each date t, so that net transfers to the public are zero:

(33) $\int_{-\infty}^{t} \tau(v,t)\exp[-p(t-v)]dv = 0.$

The present value of transfers faced by an agent born at time t = 0 is b(0,0). That individual's consumption is therefore

(34) $c(0,0) = (\delta+p)[h(0) + b(0,0)]$

when $u(c) = \log(c)$; and, if optimality condition (20) holds at $t = 0$,

$$(35)\quad c(v,0) = (\delta+p)[h(0) + k(v,0) + b(v,0)]$$
$$= (\delta+p)[h(0) + b(0,0)]\exp[(\delta-\rho)v].$$

Since aggregate consumption at $t = 0$ must equal the optimal level $C(0)$, integration of (35) over the population leads to

$$(35)\quad b(0,0) = [(\delta+p-\rho)/(\delta+p)]C(0) - h(0).$$

Equation (35) implies that the transfer streams of those born before $t = 0$ have present values given by

$$(37)\quad b(v,0) = [h(0) + b(0,0)]\exp[(\delta-\rho)v] - h(0) - k(v,0).$$

After the transfer streams for those alive at $t = 0$ are announced by the government, its only remaining choice variables are the transfer payments to be made to those born on subsequent dates. Suppose these are chosen in such a way that the budget is balanced on each date (i.e., (33) holds) and

$$(38)\quad b(t,t) = [h(0) + b(0,0)]\exp\{\textstyle\int_0^t[r(s)-\rho]ds\} - h(t).$$

The resulting competitive equilibrium will then replicate the optimal plan.

To verify this result, it must be shown that the implied cohort consumption paths satisfy (20) and (21). (We have already seen that the correct initial consumption levels will prevail.) By the individual's Euler equation (8), it will automatically be the case that for $t \geq v \geq 0$,

$$(39)\quad c(v,t) = c(v,v)\exp\{\textstyle\int_v^t[r(s)-\delta]ds,$$

where $c(v,v) = (\delta+p)[h(v) + b(v,v)]$. It therefore follows from (38) that

(40) $$c(v,t) = (\delta+p)[h(0) + b(0,0)]\exp\{\int_0^t [r(s)-\delta]ds\}\exp[(\delta-\rho)v].$$

In the case $v < 0$, (37) implies that (40) holds for all $t \geq 0$. Equations (20) and (21) follow immediately upon differentiation of (40).

For the sake of intuition, it is useful to analyze the balanced-budget fiscal policy that supports the steady state pictured in figure 1 as a competitive equilibrium. Assume, therefore, that the capital stock is initially at the level $K*$ defined by equation (27).

Let us first ask why $K*$ would generally not be a steady-state equilibrium without government intervention. To be concrete, take the case $Y'(K*) = \rho = \delta$. If the interest rate and the wage were expected to remain constant at δ and $w*$ forever, each agent in a new cohort would be born with total (human) wealth

(41) $$h(t) = w*/(\delta+p) = p[Y(K*) - Y'(K*)K*]/(\delta+p)$$

(by (12)) and his lifetime consumption path would be flat at the level $p[Y(K*) - Y'(K*)K*]$. Asymptotically aggregate consumption would clearly approach labor's share in national output; and because labor's share is less than total output, this is inconsistent with goods-market equilibrium in a steady state. In the competitive steady state, $Y'(K^c) > \delta$. Each cohort's consumption thus rises over time (by (8)), so that aggregate consumption equals the lower level of national output.

Return now to the decentralization problem. Since we are assuming a steady state, the payment received by an individual depends on age $t-v$ only, and so may be written as $\tau(t-v)$. The present value of these payments given the expected path of the real interest rate, $b(v,t)$, may be written as $b(t-v)$ in the present context. It is assumed as before that the government's budget is balanced. This condition is now written

(42) $\int_{-\infty}^{t}\tau(t-v)\exp[-p(t-v)]dv = 0.$

To support the optimal steady state associated with ρ as an equilibrium, fiscal policy must confront each agent with a transfer path inducing aggregate consumption equal to $Y(K*)$ when the interest rate is expected to remain at $Y'(K*) = \rho$ forever. Let $k(t-v)$ denote the capital held by an agent of age $t-v$ (of course $k(0) = 0$). Since all agents have the same wage income, the consumption of an individual aged $t-v$ is

(43) $c(t-v) = p(\delta+p)[Y(K*) - Y'(K*)K*]/(\rho+p) + (\delta+p)[k(t-v) + b(t-v)],$

where

(44) $b(t-v) = \int_{t}^{\infty}\tau(s-v)\exp[-(\rho+p)(s-t)]ds.$

Integrating (43) over the entire population, we find that aggregate consumption is

(45) $C(t) = (\delta+p)[Y(K*) - Y'(K*)K*]/(\rho+p)$
$+ (\delta+p)\{K* + \int_{-\infty}^{t}b(t-v)\exp[-p(t-v)]dv\}.$

Equate the value of $C(t)$ given by (45) to $Y(K*)$ and recall (44). This yields

(46) $\int_{-\infty}^{t}\{\int_{t}^{\infty}\tau(s-v)\exp[-(\rho+p)(s-v)]ds\}\exp[-p(t-v)]dv$
$= [(\rho-\delta)/(\rho+p)(\delta+p)]Y(K*) - [p/(\rho+p)]K*.$

To make sense of the implied fiscal policy we need to interpret the left-hand side of (46). After changing the order of integration, this may be written in the form

(47) $\int_{0}^{\infty}\tau(n)\{\int_{0}^{n}\exp(-ps)\exp[-(\rho+p)(n-s)]ds\}dn$

$$= (1/\rho)\int_0^\infty \tau(n)\exp[-(\rho+p)n][\exp(\rho n) - 1]dn.$$

The left-hand side of (47) is just a weighted sum of the transfer payments made at each age n. The weight given to $\tau(n)$ is in turn a sum, each term of which equals the number of agents in a cohort of age n-s (s $\geq$ 0) times the discount factor each applies to $\tau(n)$. By the government budget constraint (42), (46) and (47) can be combined to yield a formula giving the optimal present value of government transfers at birth

(48) $\int_0^\infty \tau(n)\exp[-(\rho+p)n]dn = [\rho p/(\rho+p)]K* - [\rho(\rho-\delta)/(\rho+p)(\delta+p)]Y(K*).$

Any transfer path $\{\tau(n)\}_{n=0}^{\infty}$ that simultaneously satisfies (42) and (48) will induce a level of aggregate consumption that is constant at Y(K*). In general, many such paths exist. Before concluding that the problem of decentralizing the steady state has been solved, it is necessary to check the optimality condition (20) governing the allocation of consumption among contemporary cohorts. But by the individual Euler condition (8),

(49) $\partial c(t-v)/\partial t = c(t-v)[Y'(K*) - \delta].$

Condition (20) follows immediately from the observations that $\partial c/\partial t = -\partial c/\partial v$ in a steady state and that $Y'(K*) = \rho$.

Equation (48) allows us to determine whether a newborn agent's discounted lifetime transfers b(0) will be positive or negative under optimal fiscal policy. Direct calculation shows that

(50) $b(0) \lessgtr 0$ as $\rho \gtrless \delta + p(\delta+p)[K*/Y(K*)].$

By (13), however, the steady-state real interest rate in the absence of fiscal intervention is $Y'(K^c) = \delta + p(\delta+p)[K^c/Y(K^c)]$. It follows from (50) that the government must set b(0) positive if it wishes to maintain a stationary capital stock K* greater than the laissez-faire level K^c, and must set b(0) negative in the opposite case. In other words, additional capital accumulation requires negative (unfunded) social security, an unsurprising result in view of those obtained by Diamond (1965). Faced at birth with a declining path of transfer payments, each agent accumulates capital so as to smooth his consumption. By setting the path of transfers according to (48), the government can equate aggregate saving to zero.

The foregoing results are underlined by considering again the special case $\rho = \delta$. Under this temporary assumption, (48) reduces to

$$(51)\quad b(0) = \delta pK^*/(\delta+p) = pY'(K^*)K^*/(\delta+p).$$

Equation (51) states that the transfer system endows each agent at birth with the per capita present discounted value of capital's share in national income so that, by (41), $a(0) = h(0) + b(0) = pY(K^*)/(\delta+p)$. Individual consumption is flat at $pY(K^*)$, and a declining path of transfers induces a flow demand for capital just equal to flow supply "bequeathed" to the economy by those who die.

V. Conclusion

This paper has studied the problem of time-consistent utilitarian planning in an economy where individual lifetimes are stochastic. A dynamically consistent optimal allocation is characterized by a generalized version of Samuelson's (1968) "two-part golden rule". However, the economy's aggregate dynamics are quite similar to those arising in the planning models of Cass (1965), Koopmans (1965), and Ramsey (1928), which postulate homogeneous nonoverlapping generations. Through appropriate lump-sum transfers, the optimal allocation can be realized as the equilibrium of a competitive economy with actuarially fair annuities.

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