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MACROECONOMIC POLICY UNDER CURRENCY INCONVERTIBILITY

Jorge Braga de Macedo

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ABSTRACT

This paper analyzes the macroeconomics of currency inconvertibility, building on the role of relative prices in a portfolio balance model. The relationship between black markets for foreign exchange and smuggling is first analyzed from the perspective of an individual importer. According to the portfolio view, the black market rate behaves like the financial rate in a dual market. The premium of the black market rate over the official rate is thus related to the probability of success in smuggling and the tariff.

Then the black market is analyzed using a simple three-good, two-asset general equilibrium model. Under the assumption of regressive exchange rate expectations, the portfolio view is contrasted with a monetary approach to the black market. The short-run and long-run effects of monetary and exchange rate policies on relative prices are assessed. Different assumptions about expected returns are contrasted, but emphasis is placed on the perfect foresight case. Unless expectations are static, official exchange rate policy has to adjust to the private valuation of foreign exchange, as stressed in the conclusion.

Jorge Braga de Macedo
Woodrow Wilson School
Princeton University
Princeton, NJ 08544
Ph. 609-452-6474

Introduction

Restricting the ability to convert the domestic currency into foreign exchange may involve restrictions on international trade or capital movements: currency inconvertibility is to be understood relative to particular items in the balance of recorded international transactions.[1] While inconvertibility promotes smuggling and black markets for foreign exchange, these may exist even when a currency is legally "convertible". In effect, the definition in the Articles of Agreement of the International Monetary Fund only requires the absence of restrictions on current account transactions.[2]

Some basic features of the black market for foreign exchange can be derived in partial equilibrium models. The relationship between recorded trade, smuggling and the demand for and supply of black market foreign exchange is evident in the choice between importing at the official exchange rate plus the tariff or at the black market rate. Under inconvertibility, therefore, domestic and foreign assets are not perfect substitutes. Applying the conventional monetary approach - where assets (and goods) are assumed to be perfect substitutes - to the inconvertible exchange rate would be misleading. On the contrary, the portfolio approach assumes a stock demand for the foreign, as well as for the domestic, currency and does not require perfect goods arbitrage. It can thus explain the observed diversification of currency portfolios and provide a useful framework in which to analyze macro-economic policy under inconvertibility.

The paper is divided into two parts. Part I contains a partial equilibrium model of the black market for foreign exchange which brings together the smuggling literature and the portfolio view. Different mechanisms of expectations formation are contrasted, but emphasis is placed on the perfect foresight case. It is shown under what condition the premium of the black market rate over the official rate is likely to be positively related to the probability of success in smuggling, along the lines of the classic "Beccaria formula".

In Part II, the black market for foreign exchange is analyzed using a simple three-good, two-asset general equilibrium model in the spirit of Kouri (1975). Under the assumption of "stabilizing", or regressive, expectations about returns to domestic and foreign currencies, the portfolio view is contrasted in section 2 with the monetary model of Blejer (1978a). Then, assuming static expectations, the black market premium (which is also the relative price of traded goods) and the relative price of non-traded goods are endogenously determined in temporary equilibrium (section 3), while the asset mix is determined in steady- state equilibrium (section 4). The effects of monetary and exchange rate policies on relative prices and asset stocks are analyzed in this set-up. The implications of perfect foresight are explored in Section 5. The conclusion summarizes the results.

I. Partial equilibrium

1. The Beccaria formula

The formal analysis on black markets can be traced back to the mid-eighteenth century, when - as Hume was writing on the specie-flow mechanism - Beccaria (1764-65) developed a break-even condition for smuggling activity, involving the probability of success and the tariff rate on imports:

(1) $zt = 1$

where z is the probability of success in smuggling

and t is one plus the tariff rate

A higher degree of enforcement of tariff regulations - to the extent that is implies a lower z - is associated with negative profits from smuggling unless a higher rate raises the domestic price and thus restores profitability. The objective of the analysis was precisely directing the attention of policy-makers to the trade-off between the revenue from higher tariffs and the associated higher enforcement costs.[3]

After some centuries of neglect, the literature picked up about ten years ago. Bhagwati and Hansen (1973) - who refer to Beccaria's work - showed that tariffs do induce smuggling but, together with Sheikh (1974), they claimed that a black market for foreign exchange would not be induced by trade restrictions alone. Sheikh (1976) adapted the traditional analysis of black markets by Michaely (1954) and others to the foreign exchange market.[4]

By introducing real costs to smuggling in the form of a "smuggling function", Pitt (1981) and (1982) showed that legal and illegal trade in goods and money coexist even in the absence of exchange restrictions. If some legal trade is necessary in order to engage in smuggling, the domestic price may be less than the cost of legal imports, a phenomenon

which he calls "price disparity". Along the same lines, Martin and Panagaryia (1983) propose a crime-theoretic approach to smuggling, which provides microeconomic foundations to Pitt's smuggling function but they neglect the black market for foreign exchange.
Noting that this revival of interest in the supervision aspect would have pleased Beccaria, a criminologist by training, we will show how the Pitt-Martin-Panagaryia (PMP) analysis is consistent with his formula, modified to allow for a black market for foreign exchange. In fact, the black market premium is a measure of the cost of smuggling. We begin by describing how this premium is determined in a portfolio model.

2. A portfolio model

Consider a small country whose residents are not authorized to hold foreign assets, but who nevertheless allocate their financial wealth between domestic and foreign currency.[5] The terms of trade between legal ("official") and illegal ("smuggled") imports and exports are given in the world market. Alternatively, as is done below, the same good is traded through both channels and there is a non-traded good.

If foreigners do not hold the inconvertible domestic currency in their portfolios, the domestic currency price of foreign currency in the black market will be such that the existing stock is willingly held. Furthermore, the only way for domestic residents to acquire foreign currency is through the underinvoicing of exports and the overinvoicing of imports. In the absence of endogenous reported capital flows and errors and omissions, as well as interest payments on unrecorded capital flows, excess supply of the two traded goods equals the change in the stock of foreign currency of the private sector and the central bank respectively.

The black market rate is determined so that the private stock of black market foreign exchange is willingly held, and demand α is proportional to wealth for simplicity. Demand increases with expected depreciation of the black market rate or with a higher probability of success in smuggling:

$$(2) \qquad eF = \alpha(\psi, z)W \qquad \varepsilon = \alpha_{\psi}\psi/\alpha$$
$$\gamma = \alpha_{z}z/\alpha$$

where e is the black market exchange rate

F is the stock of black market foreign exchange (in foreign currency)

W is private financial wealth (in domestic currency)

and ψ is the expected rate of change of the black market rate.

The measure of relative real return is the expected change in the black market rate because domestic residents are assumed to be identical in their consumption preferences.[6]

Given an official exchange rate, $\tilde{e}$, the requirement of portfolio balance can be written in terms of the black market premium and of financial wealth valued at the official rate:

$$(3) \qquad pF = \alpha\tilde{W}$$

where $p = e/\tilde{e}$ is the black market premium and $\tilde{W} = W/\tilde{e}$ is financial wealth valued at the official exchange rate.

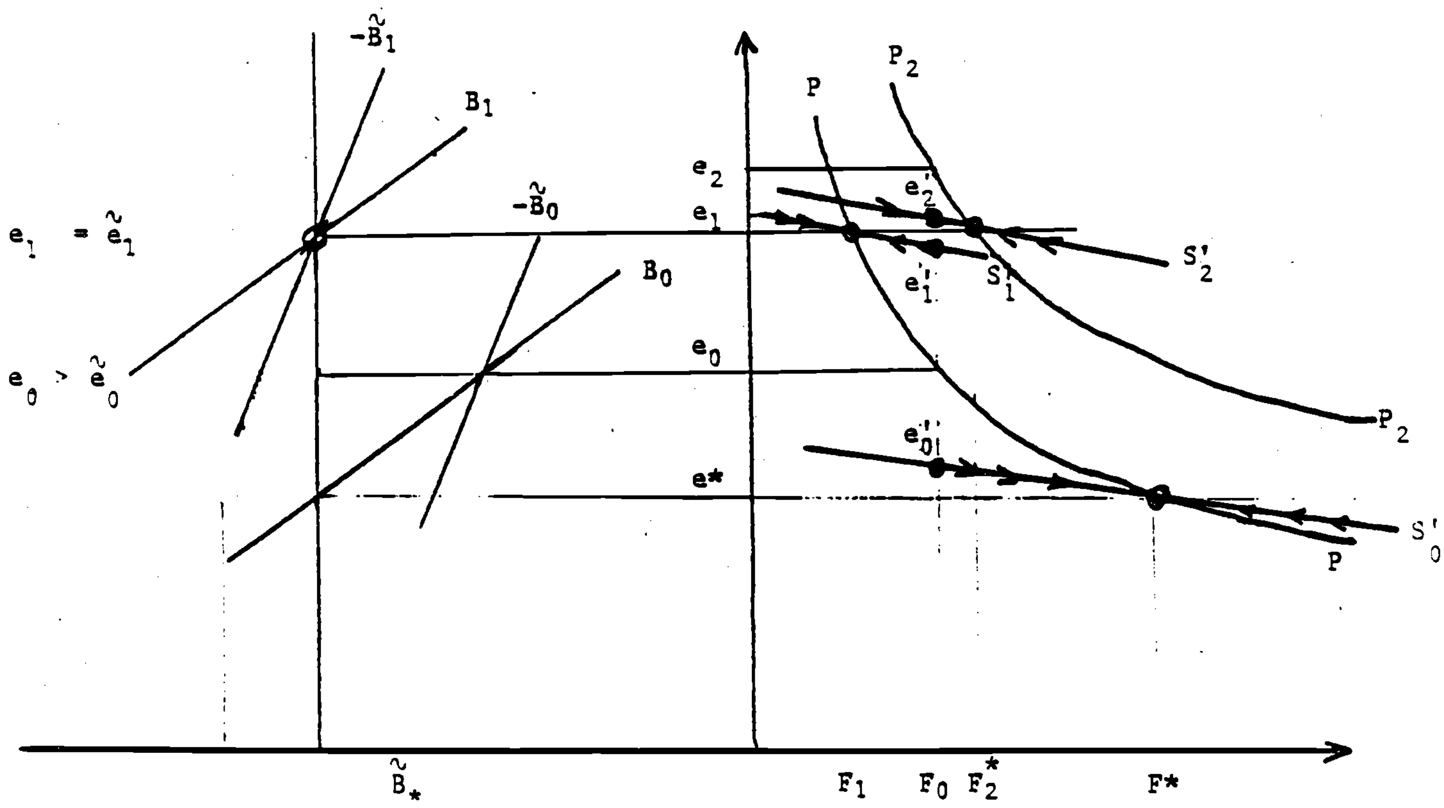
The change in F is given by smuggling and other unreported current account transactions. The unreported surplus, B, is zero for a particular value of the premium, taken to be one, and, if the elasticities condition holds, improves with an increase in the premium. Denoting rates of change by dots:

(4) $\dot{F} = B(p) \quad B(1) = 0 \quad B' > 0$

Assuming that the opposite holds for the reported trade surplus, $\tilde{B}$, we show temporary equilibrium in Figure 1, a variant of the Kouri (1983) diagram. The right panel shows the instantaneous determination of the black market exchange rate, given demand for foreign currency and no expected change in relative returns, along a rectangular hyperbola PP. Under perfect foresight, demand for foreign currency will increase when the black market rate depreciates, constraining its initial value for a given stock of foreign currency F_0 to e_0', on the saddle path S_0' (below the corresponding initial value under static expectations, e_0). For a given official exchange rate, $\tilde{e}_0$ ($< e_0$ so that $p_0 > 1$), the left panel of Figure 1 plots the unreported trade surplus in foreign currency, B (which equals the increase in F) and the exogenous reported trade deficit, $-\tilde{B}$. Assuming a stronger effect of the black market rate on B, the relative slopes will be as drawn. For simplicity, assume that the unreported trade balance equilibrium associated with the given official exchange rate is at $e^* < e_0$.

Figure 1

A Portfolio Model
of the black market rate



If the associated reported deficit $-\tilde{B}^*$ is corrected by devaluation of the official exchange rate to its long-run equilibrium level, $\tilde{e}_1$, the two schedules will shift up and intersect on the vertical axis at a point where, by choice of units, $e_1 = \tilde{e}_1$ so that $p_1 = 1$. The implications of the official devaluation differ depending on the expectations formation mechanism. If expectations are static, the black market exchange rate will gradually depreciate from e_0 to e_1 as the stock of foreign assets decreases from F_0 to F_1. Under perfect foresight, the black market rate jumps from e_0' to e_1' and then gradually depreciates to the same long-run value. If the official devaluation generates an increase in the demand for foreign assets from PP to P_2P_2, furthermore, the black market rate will overshoot to e_2 (e_2' under perfect foresight) and gradually appreciate back to e_1 as the stock of foreign assets increases to F_2^*. The same would be true of an exogenous increase in z: in the short-run the premium would rise, in the long-run F would rise.

The analysis remains applicable under a crawling peg regime, where the rate of depreciation equals the reported trade deficit as a proportion of central bank reserves. It does, however, assume that these are large enough, so as to be able to neglect the "collapse problem" of Krugman (1979).[7]

3. Special risks

Consider the profit ρ in domestic currency of a price-taking importer. It varies depending on whether smuggling is successful (ρ_z) or not (ρ_{1-z}). Setting the foreign currency price at unity, we get profits as a linear function of the amounts imported through legal and illegal channels:

(5) $\rho_z = P(L + S) - \tilde{e}tL - eS$

(6) $\rho_{1-z} = PL - \tilde{e}tL - eS$

where P is the domestic price

L is the amount legally imported

S is the amount smuggled

Under perfect competition, profits will be zero at the optimum, which is independent of demand conditions. If the probability of success is exogenous to the firm, expected profits will also be linear in the decision variables, S and L, since they are a z-weighted average of (5) and (6):

(7) $\rho = z\ \rho z + (1 - z)\ \rho_{1-z}$

The basic idea behind the so-called "crime-theoretic" approach is, however, to make the special risks, measured by the probability of success, depend on the decision variables of the firm. Assume it is a decreasing function of the ratio of smuggled to legal imports.

(8) $z = z(s)\ ;\ \zeta = -sz'/z$

where $s = \frac{S}{L}$

Using (3) and (8) in (7), first-order conditions for profit maximization can be solved for the optimal combination of price and import pattern, given the tariff and the premium:

(9) $\tilde{P}(1 + z\zeta s) = t + \gamma\zeta sp$ LL locus

(10) $z\tilde{P}(1 - \zeta) = p(1 - \gamma\zeta)$ SS locus

where $\tilde{P} = P/\tilde{e}$

To preserve the zero-profit condition for legal imports in (9), an increase in the smuggling ratio requires a lower price in terms of the official exchange rate if both z and p are elastic or inelastic with respect to s and z respectively. This implies a downward sloping LL locus. If one of the functions is elastic and the other is inelastic, though, the LL locus will be upward sloping. The slope of the LL locus is given by:

(11) $$\left.\frac{d\tilde{P}}{ds}\right|_{LL} = -\left(\frac{1-\gamma}{1-\zeta}\right) \frac{\zeta p}{1+\zeta zs}$$

To preserve the zero-profit condition for smuggling imports in (10), an increase in the smuggling ratio requires a higher price unless p is elastic with respect to the probability of success. The slope of the SS locus is given by:

(12) $$\left.\frac{d\tilde{P}}{ds}\right|_{SS} = \frac{1-\gamma}{(1-\zeta)^2}\ \frac{zp}{s}$$

Adapting a diagram from Martin and Panagaryia (1983), we present the inelastic case (γ, $\zeta < 1$) in Figure 2. An increase in the tariff from t_0 to t_1 raises $\tilde{P}$ and s. A decrease in z (or F) from z_0 (or F0) to z_1 (or F_1) raise $\tilde{P}$ but lowers s.

Now eliminating $\tilde{P}$ between (9) and (10) we obtain a more general version of the Beccaria formula:

(13) $$zt = pA$$

where $$A = \frac{1-\zeta[\gamma-zs(1-\gamma)]}{1-\zeta}$$

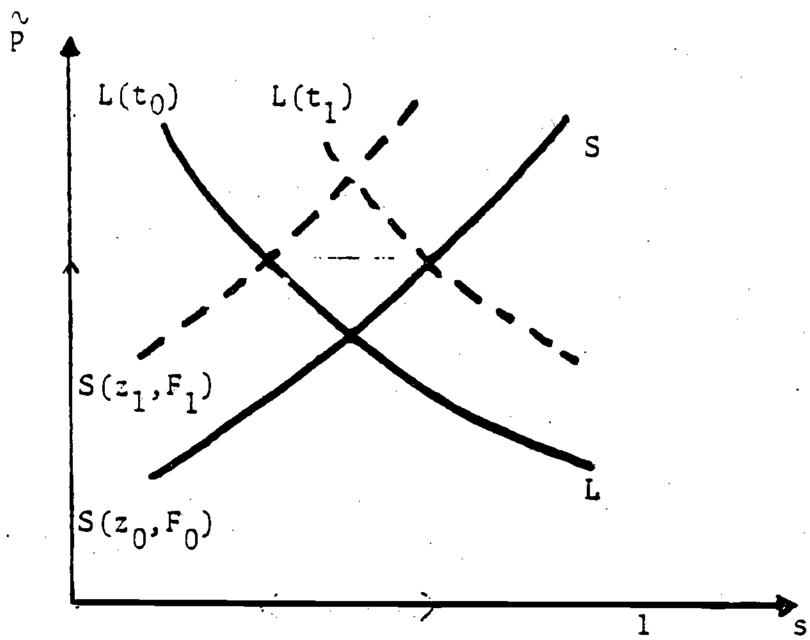
Note first that, if demand for black market foreign exchange increases in proportion with the probability of success ($\gamma = 1$) or if z is exogenously given ($\zeta = 0$), then A = 1 in (13). This is a sufficient condition for the premium to be less than the tariff ($t > p$). Put another way, given the probability of success in smuggling, an increase in the premium lowers profits unless there is an increase in the tariff. The Beccaria is now between more supervision (lowering z)and a higher tariff or an official devaluation (lowering p/t).

On the other hand, a new trade-off arose between profits from the smuggling activity and profits in the black market for foreign exchange. Higher supervision of smuggling is now associated with a lower black market. In Figure 2, or in the elastic case, we have A > 1 so that again $t > p$.[8]

Figure 2

The PMP model of smuggling

(inelastic case)



While some basic features of the black market for foreign exchange can be derived in partial equilibrium models, the interaction between the private and official valuation of foreign exchange, on the one hand, and the role of the black market premium as a relative price, on the other, call for a general equilibrium approach. In the model presented next, we determine not only the black market premium but also the relative price of non-traded goods, their ratio being the real exchange rate. To sharpen the analysis, we will neglect the role of the black market premium emphasized in the previous section, namely determining whether a transaction will go through the official or the black foreign exchange market.[9]

II General equilibrium

1. The Model

Exports and imports are either traded through the official market or smuggled. Their prices in foreign currency are determined in the world market. It would, of course, be possible to reinterpret the results in terms of the Beccaria analysis by allowing for a tariff on the official import.

Using the wealth definition, the portfolio balance condition is (3) becomes:

$$(14) \qquad pF = \frac{\alpha}{1 - \alpha} \tilde{M} = h\tilde{M}$$

where $\tilde{M} = M/\tilde{e}$ is the domestic money stock, M, valued at the official rate

$h = eF/M$ is the currency ratio

and $\tilde{W} = \tilde{M}\,(1 + h)$

The assumption of regressive expectations implies, in turn, that:

(15) $\psi = \psi(p)\ ;\ \tilde{\psi} = -\psi_p p/\psi$

Excess-supply functions for the official traded good, the smuggled good and the non-traded good depend on prices (e, $\tilde{e}$ and P_N) and wealth. An income effect could easily be added but, as noted by Kouri (1975), it would be irrelevant. We take the official exchange rate as the numeraire.

The market for non-traded goods is always in equilibrium. Neglecting the cross-price effect (and indicating the sign of the partial derivatives over the respective argument), excess supply for non-traded goods is given by:

(16) $N(\overset{+}{q}, \overset{-}{\tilde{W}}) = 0$

where $q = P_N/\tilde{e}$ is the relative price of the non-traded good.

As mentioned, excess supply for traded goods equals the accumulation of foreign currency by the central bank and the private sector through the reported and unreported trade balances respectively. The domestic money stock is made up of domestic credit (C) and foreign exchange reserves, valued at the official exchange rate. The balance of payments is the only endogenous source of money creation: capital gains on the foreign currency value of reserves go into central bank net-worth forever. Neglecting reported capital flows, (and denoting proportional rates of change by hats), we get:

$$(17) \qquad \dot{F} = B(\overset{+}{p}, \overset{-}{q}, \overset{-}{\tilde{W}})$$

$$(18) \qquad \hat{\dot{M}} = \tilde{B}(\overset{-}{p}, \overset{-}{q}, \overset{-}{\tilde{W}}) + \delta\tilde{M}$$

where $\delta = \frac{\dot{C}}{M} - \hat{\tilde{e}}$

2. A Comparison of stock and flow specifications

If $\delta \neq 0$ but the excess supply of traded goods is zero, we obtain from log differention of (14) and (16) the solution for the change in the relative prices as a function of δ:

$$(19) \qquad \hat{q} = \frac{\omega_N}{\nu} \delta$$

$$(20) \qquad \hat{p} = \frac{1}{1+\underline{\varepsilon}} \delta$$

where $\omega_N = -\tilde{W}\partial N/\partial\tilde{W}$ is the positive semi-elasticity of the excess supply of non-traded goods with respect of real wealth;

$\nu = -q\partial N/\partial q$ is the positive semi-elasticity of the excess supply of non-traded goods with respect to the relative price of non-traded goods.

and $\underset{=}{\varepsilon} = \frac{\varepsilon\tilde{\psi}}{1-\alpha}$ is the positive elasticity of h with respect to the premium.

We contrast (19) and (20) with the flow specification of the black market rate due to Blejer (1978a). His model collapses into the usual monetarist model of exchange rate determination when the real official exchange rate is fixed by a purchasing-power-parity reaction function.[10] To make the models compatible, this last aspect and many inessential complications are ignored.

Arguing that excess demand for non-traded goods is a given fraction, τ, of excess demand in the goods market, and therefore of the ex-ante excess supply of money, Blejer posits that changes in the relative price of non-traded goods are proportional to δ:

$$(21) \qquad \hat{q} = \tau\delta$$

By interpreting τ as ω_N/ν, (21) is of course the same as (19). The difference comes from the specification of the black market for foreign exchange in flow terms. Supply per unit of time, S^B, is a positive function of the black market premium:

$$(22) \qquad \log S^B = C_{11} + C_{12} \log p$$

Demand per unit of time, D^B, is a positive function of the expected rate of change of the black market rate and a negative function of expected inflation, as proxies for the real return to holding black market foreign exchange and holding the non-traded good. Since own and cross effects are the same, and expectations are assumed to be regressive, we get:[13]

(23) $\log D^B = C_{21} - C_{22} \log \xi$

where $\xi = e/P_N$

Note that (23) is not consistent with domestic residents having the same consumption preferences, as assumed in (12), where ψ, rather than the expected change in ξ, measures the real return differential.

Log differentiating (22) and (23) and equating yields a link between the two relative prices p and q, represented in Figure 3 by point E:

(24) $\hat{p} = c\hat{q}$

where $c = C_{22}/(C_{12} + C_{22})$

Substituting (24) into (21) we obtain, instead of (20):

(25) $\hat{p} = c\tau\delta$

According to (25), the effect on the premium is dampened by $c\tau < 1$. Clearly, if q is not responsive to δ and $\tau = 0$, the black market premium is constant. The same happens when the flow supply of foreign exchange does not respond to the premium - say because expectations are static - and c = 0.

However in (20), if $\tilde{\psi} = 0$ we get $\hat{p} = \delta = \hat{q}/\tau$, so that the premium moves by more than the relative price of non-traded goods, as seems plausible. The equilibrium is now at E' in Figure 3, and the flow model understates the change in the premium by E'E. Since $\tilde{\psi} > 0$, under regressive expectations the premium will be lower. It will nevertheless be larger than at E if the elasticity of expectations and of the currency ratio are sufficiently larger than the ratio of the flow supply and demand elasticities, as at E". The condition is:

$$(25') \qquad \tilde{\varepsilon}\,\tilde{\psi} > C_{12}/C_{22}$$

where $\tilde{\varepsilon} = \dfrac{\varepsilon}{1-\alpha}$.

We now analyze, in the case $\tilde{\psi} = \gamma = 0$, the temporary equilibrium of the system, where excess supply for traded goods is non-zero, and steady-state equilibrium, where asset stocks are constant.[12]

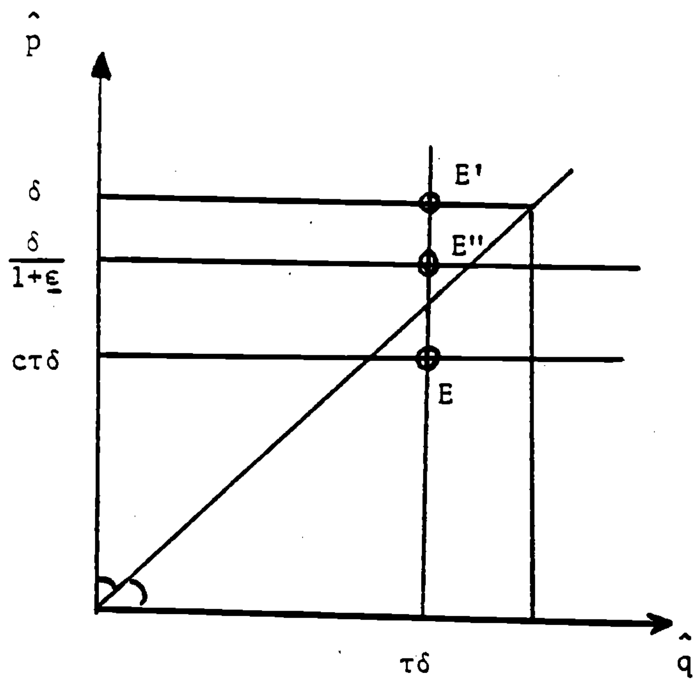
3. Temporary equilibrium

Equations (14) and (16) determine the relative prices of the smuggled and non-traded good in terms of the numeraire, given asset stocks. An increase in the return to foreign assets (or in the probability of success in smuggling) would increase h, that is the black market rate. For a given h, (14) states that there is a single value of p consistent with portfolio balance. Similarly, from (16), an increase in q must be

Figure 3

Stock and Flow

Equilibria Compared



matched by an increase in wealth, that is in p, in order to eliminate the associated excess-supply of non-traded goods.

$$(26) \qquad \hat{p} = \hat{\tilde{M}} - \hat{F}$$

$$(27) \qquad \hat{q} = \tau\hat{\tilde{M}}$$

According to (26) and (27), an increase in foreign money decrease p and leaves q unchanged, while domestic monetary expansion increases p one-to-one and q by τ. If the relative price effect dominates the wealth effect ($\tau < 1$), then, domestic monetary expansion will imply a real depreciation of the black market rate expressed in terms of non-traded goods. The same will be true for a once-and-for-all devaluation of the official exchange rate, which leaves the demand for foreign currency unchanged, so p falls, and generates an excess demand for non-traded goods, so q falls:

$$(28) \qquad \hat{\xi} = (1 - \tau)\hat{\tilde{M}} - \hat{F}$$

It can similarly be shown that an increase in h will increase p more than q (i.e. lead to a real depreciation) unless $\tau > 1 + 1/h$.

4. Steady-state equilibrium

Steady-state asset stock are such that, aside from portfolio balance and equilibrium in the non-traded goods market, there is zero excess-supply of the smuggled good, so that B = 0 and, therefore, zero excess-supply of the official traded good as well, $\tilde{B} = 0$. We thus need $\delta = 0$ in steady-state.

Substituting for p and q from the temporary equilibrium equations, the asset accumulation equations in (17) and (18) can be used to show that the reported (unreported) trade balances deteriorate sufficiently with increases in domestic (foreign) currency for the system to be stable.

Consider now the steady-state effects of monetary and exchange rate policies when the currency ratio is fixed. The total differential of the system can be expressed as a function of disturbances which affect the currency ratio and official exchange rate policies:

$$(29)\qquad \begin{pmatrix} 1 & 0 & -1 & 1 \\ 0 & \nu & -\omega_N & 0 \\ -\tilde{\pi} & -\tilde{\mu} & -\tilde{\omega} & 0 \\ \pi & -\mu & -\omega & 0 \end{pmatrix} \begin{pmatrix} \hat{p} \\ \hat{q} \\ \hat{\tilde{M}} \\ \hat{F} \end{pmatrix} = \begin{pmatrix} \hat{h} \\ \alpha\omega_N dh + (\nu - \omega_N)\hat{\tilde{e}} \\ \alpha\tilde{\omega}\, dh - (\tilde{\pi}+\tilde{\mu}+\tilde{\omega})\hat{\tilde{e}} - \tilde{M} d\delta \\ \alpha\omega\, dh + (\pi-\mu-\omega)\hat{\tilde{e}} \end{pmatrix}$$

where $\tilde{\pi}(\pi)$ is the positive semi-elasticity of $\tilde{B}(B)$ with respect to the premium

$\tilde{\mu}(\mu)$ is the positive semi-elasticity of $\tilde{B}(B)$ with respect to the relative price of non-traded goods;

and $\tilde{\omega}(\omega)$ is the positive semi-elasticity of $\tilde{B}(B)$ with respect to real wealth

A once-and-for-all devaluation leaves the stock of foreign assets unchanged and raises e, P_N and M in proportion, so that p, q and $\tilde{M}$ are also unchanged. An increase in domestic credit creation - or a decline in the rate of crawl - increases p, q and $\tilde{M}$, whereas the effect on F is ambiguous:

(30) $$\hat{p} = \frac{\Omega}{\Delta} \tilde{M}d\delta$$

(31) $$\hat{q} = \frac{\tau\pi}{\Delta} \tilde{M}d\delta$$

(32) $$\hat{\tilde{M}} = \frac{\pi}{\Delta} \tilde{M}d\delta$$

(33) $$\hat{F} = \frac{\pi-\Omega}{\Delta} \tilde{M}d\delta$$

where $\Omega = \mu\tau + \omega$;

$\tilde{\Omega} = \tilde{\mu}\tau + \tilde{\omega}$;

and $\Delta = \pi\tilde{\Omega} + \tilde{\pi}\Omega$.

When own-price effects are stronger than wealth and cross-price effects, $\pi > \Omega$. Then, when δ rises, p rises by less than $\tilde{M}$, so that F rises. Defining ratios of semi-elasticities, as in (21) above, we can rewrite the condition in (33) as:

(33') $$1 - \phi\tau > \tau'$$

where $\phi = \mu/\pi$

and $\tau' = \omega/\pi$.

The condition for a real depreciation when the rate of crawl increases (or, equivalently, for a fall in ξ after a rise in δ) is stronger than (33'). It is given by:[13]

$$(34) \qquad (1 - \phi)\tau > \tau'$$

An increase in h has no steady-state effects on relative prices. It increases F by $1 + \alpha$ and decreases $\tilde{M}$ by α. We now allow for endogenous changes in h. Rather than making the currency ratio a function of the premium, though, we focus on the case of perfect foresight.

4. The implication of perfect foresight

The well-known portfolio model of flexible exchange rates with non-traded goods of Calvo and Rodriguez (1977) asssumes that expectations about relative returns are continuously realized, so that $\psi = \hat{e}$. Using this in (14) and inverting, we obtain a differential equation for the black market rate. Since the rate of change of P_N is pegged to the rate of crawl by the assumption that the market for non-traded goods clears, we solve out for q. We also assume that the official rate is constant. Then we express the system in terms of p, $\tilde{M}$ and F by taking a linear approximation around steady-state equilibrium, where p = 1 by choice of units.

$$
(35) \qquad
\begin{array}{ccccc}
\dot{p} & & 1/\tilde{\varepsilon} & -1/\tilde{\varepsilon} & 1/\tilde{\varepsilon} & p-1 \\
\hat{\tilde{M}} & = & -(\pi+\alpha\tilde{\Omega}) & -(1-\alpha)\tilde{\Omega} & -\alpha\tilde{\Omega} & \frac{\tilde{M}-\tilde{M}^*}{\tilde{M}^*} \\
F & & \pi-\alpha\Omega & -(1-\alpha)\Omega & -\alpha\Omega & \frac{F-F^*}{F^*}
\end{array}
$$

To solve the characteristic equation of the system in (35) we assume that $\Omega = \tilde{\Omega}$ and obtain:[14]

$$(36) \qquad \lambda^2[1 - \tilde{\varepsilon}(\lambda + \Omega)] + \lambda(\Omega + \Pi) + \Omega\Pi = 0$$

where $\quad \Pi = \pi + \tilde{\pi}$

The trace and determinant are given by:

$$(37) \qquad \lambda_1\lambda_2\lambda_3 = \Pi\Omega/\tilde{\varepsilon}$$

$$(38) \qquad \lambda_1 + \lambda_2 + \lambda_3 = \frac{1}{\tilde{\varepsilon}} - \Omega$$

Noting that $\lambda_3 = -\Omega$ satisfies (36), we get two roots of opposite signs:

$$(39) \qquad \lambda = -\frac{1}{2\tilde{\varepsilon}} \pm \quad \frac{1}{4\tilde{\varepsilon}^2} + \frac{\Pi}{\tilde{\varepsilon}}$$

The solution is therefore saddle-point-stable with one direction of instability associated with the jump variable p and two directions of stability associated with the assets stocks.

Suppose now that a crawling peg policy is implemented from a steady-state situation where the rate of crawl is zero. In steady-state the premium is constant, so that the rate of depreciation of the black market rate has to increase. This makes foreign currency more attractive and raises the currency ratio.

Suppose now that the expected rate of change of the black market is given by the rate of crawl, $\psi = \hat{\tilde{e}}$, the case of hyperopic expectation.[15] The currency ratio also increases.

We now return to the diagram in the right panel of Figure 1 above, in order to represent a simplified version of the general equilibrium system, given by a monetary policy such that $\dot{\tilde{M}} = 0$ at all times, so that we can solve out for $\tilde{M}$ and obtain a two-by-two dynamic system. Using the ratios of elasticities already introduced, we get:

$$(40) \qquad \begin{bmatrix} \dot{p} \\ \dot{F} \end{bmatrix} = \begin{bmatrix} \frac{1}{\varepsilon}\left(1 + \frac{1}{\tilde{\tau}' + \tilde{\phi}\tau}\right) & \frac{1}{\varepsilon} \\ \pi\left(1 + \frac{\tau' + \phi\tau}{\tilde{\tau}' + \tilde{\phi}\tau}\right) & 0 \end{bmatrix} \begin{bmatrix} p - 1 \\ \frac{F - F^*}{F} \end{bmatrix}$$

where $\tilde{\tau}' = \tilde{\omega}/\tilde{\pi}$;

and $\tilde{\phi} = \tilde{\mu}/\tilde{\pi}$.

The trace of the matrix in (40) is positive and the determinant is negative, so that once again the roots are of opposite sign and the system is saddle-point stable. Since the wealth effects cancel, the locus where $\dot{F} = 0$ is horizontal in p, F space, and defines the long-run value of the premium.[16]. The linear approximation to the portfolio balance locus, $\dot{p} = 0$, has negative slope less than one in absolute value, given by $(\tilde{\tau}' + \tilde{\phi}\tau)/(1 + \tilde{\tau}' + \tilde{\phi}\tau)$. Now reinterpret Figure 1 as if the premium were measured on the vertical axis, and the $\dot{F} = 0$ locus were a straight line. Then, as stated in Part I, under perfect foresight, demand for foreign currency increases when the black market rate depreciates, constraining its initial value for a given stock of foreign currency to a value below the corresponding initial value under static expectations. The effect of an increase in the demand for foreign assets is also smaller than under static expectations. It is followed by a continuous decline of the black market premium along a higher perfect foresight path. Conversely, the effect of an exogenous increase in the foreign demand for the smuggled good is a downward shift in the $\dot{F} = 0$ locus, which leads to a jump fall in p and a continuous decline along a lower perfect foresight path to F_2. Under static expectations, there would be no jump fall in p, but only a decline along the portfolio balance locus.

Conclusion

The paper has analyzed regimes of currency inconvertibility using the portfolio approach to exchange rate determination developed by Kouri (1983) and others. Reference was also made to a "crime-theoretic" approach pioneered by Beccaria (1764) and to the monetary approach of Blejer (1978a). In a simple three-good, two-asset general equilibrium model, the condition for a crawling peg to induce real depreciation of the black market rate in terms of non-traded goods was shown to be the dominance of the price effect over the wealth effect. Increases in the demand for foreign assets increase relative prices in the short-run and the currency ratio in the long-run.

Allowing the currency ratio to be changed by monetary and exchange rate policy, it was shown that, under perfect foresight, it would be raised by an increase in the rate of crawl. The assumption of perfect foresight may seem particularly strained in a black market, but the result also holds when expectations are hyperopic, so, in the long run that the private valuation of foreign currency adjusts to the official one.

The contrast of the various expectations formation mechanisms brings out an important dilemma of currency inconvertibility. If the monetary authorities believe that, in the long-run, the private valuation of foreign currency adjusts to the official one, they also have to accept that the wrong exchange rate policy will set the black market for foreign exchange off the stable path where expectations are continuously realized, making domestic money worthless. To rule out destabilizing expectations, the official valuation has to adjust to the private valuation, as in a dual market for foreign exchange.

Such a regime has the advantage of giving a warning signal which may prevent the collapse of a fixed exchange rate regime. The literature on this latter issue, beyond the scope of the present paper, has been surveyed by Garber (1983), who mentions in passing the potential role of a dual exchange rate in preventing the speculative attack. This, in turn, provides a justification for the weak standard of convertibility found in the IMF Articles of Agreement.

NOTES

1. Thus, restricting convertibility for current account transactions would tend to promote factor mobility and trade in assets. Conversely restricting convertibility for capital account transactions would tend to promote trade in goods.
2. In response to the greater mobility of capital, some governments have attempted to recapture foreign exchange by offering special advantages to the types of transactions, where evasion of exchange controls would be easier, like tourist services and migrant's remittances. According to the IMF's latest Annual Report on Exchange Arrangements and Exchange Restrictions, only 33 out of 148 member countries have fully convertible currencies, while 67 (Western Europe, Middle-East and Latin America) have accepted convertibility for current account transactions only.
3. As Cooper (1974) showed for the case of Indonesia, tariff collections as a percent of imports were less than schedule rates at higher duties because of the increased incentive to smuggling activity. While higher collections are presumably achievable with greater law enforcement, net revenues might still be less than proportional to rates.
4. A brief survey of this literature is in Macedo (1979).
5. See Macedo (1982a) and (1982b, Essay III) and Dornbusch et al. (1983) for the analysis of black markets for foreign exchange in Egypt, Portugal and Brazil, respectively, using the portfolio approach.

6. Strictly speaking, this also requires constant expenditure shares. See Kouri (1975, p. 77, note 1).
7. The "collapse" literature is surveyed by Garber (1983).
8. In the case where LL is upward sloping, the existence of a solution ($A > 0$) requires that, when $\zeta > 1$ and $1 > \gamma\, zs/(1 + zs)$, we also have $\zeta[\gamma - zs\,(1 - \gamma)] > 1$, so that $A < 1$ and if $A < z$, then $t < p$.
9. In Macedo (1982b, Essay II), S, L and their ratios depend on p. An alternative, suggested by Al Fishlow, would be to make the ratio a function of ψ.
10. Other references in the monetarist spirit are Culbertson (1975) who used annual data on black market rates in India, the Philippines and Turkey for purchasing power parity calculations; Giddy (1978) who attempted to show the "efficiency" of black markets for foreign exchange in Columbia, Brazil and Israel by testing the randomness of the black market premium and changes therein using weekly data; Blejer (1978b) who used annual data on a black-market-based real exchange rate for Brazil, Chile and Colombia to estimate money demand functions in these countries; and Canto (1983), who performed time-series analysis on data from the Dominican Republic.
11. Under rational expectations the "real rate" ξ would be a double exponential function of time.
12. See the analysis in Macedo (1983) for the cases where $\tilde{\psi} \neq 0$, so that the currency ratio changes, where there are terms-of-trade changes and where alternative assumptions about capital gain on reserves are explored.
13. This real appreciation is used to characterize the "Egyptian disease" in Macedo (1982a).

14. This allows $\pi \neq \tilde{\pi}$ because the elasticity relative to the numeraire can also differ.

15. See Allen and Kenen (1980, p. 242).

16. If, instead, we assumed that $\tilde{M} = \tilde{M}^*$ at all times, the $\dot{F} = 0$ locus would be upward sloping because of the wealth effect. None of the conclusions would be changed.

References

Allen, P.R. and P. Kenen (1980). Asset Markets, Exchange Rates and Economic Integration, Cambridge.

Beccaria, C. (1764-65), Tentativo Analitico Sui Contrabbandi, reprinted in Scrittori Classici Italiana di Economia Politica, Vol. XII, Milan, 1804, p. 235-241.

Bhagwati, J. and B. Hansen (1973), A Theoretical Analysis of Smuggling, Quarterly Journal of Economics, May.

Blejer, M. (1978a), Exchange Restrictions and the Monetary Approach to the Exchange Rate, in J. Frenkel and H. Johnson eds., The Economics of Flexible Exchange Rates, Addison-Wesley.

_________ (1978b), Black Market Exchange Rate Expectations and the Domestic Demand for Money: Some Empirical Results, Journal of Monetary Economics.

Calvo, G. and C. Rodriguez (1977), A Model of Exchange Rate Determination Under Currency Substitution and Rational Expectations, Journal of Political Economy, June.

Canto, V. (1983), Monetary Policy, "Dollarization" and Parallel Market Exchange Rates: The Case of Dominican Republic, draft, University of Southern California, February.

Cooper, R. (1974), Tariffs and Smuggling in Indonesia, in Bhagwati ed., Illegal Transactions in International Trade, North-Holland.

Culbertson, W. (1975), Purchasing Power Parity and Black Market Exchange Rates, Economic Inquiry, May.

Dornbusch, R. et al. (1983), The Black Market for Dollars in Brazil, Quarterly Journal of Economics, February.

Garber, P. ((1983), The Collapse of Asset Price Fixing Regimes, draft, University of Rochester, November.

Giddy, I. (1978), The Efficiency of Black Markets for Foreign Exchange, draft, Columbia Business School.

Kouri, P. (1975), Essays in the Theory of Flexible Exchange Rates, Ph.D Thesis, Massachusetts Institute of Technology.

__________ (1983), Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model, in J. Bhandari and B. Putnam (eds.), The International Transmission of Disturbances under Flexible Exchange Rates, MIT Press.

Krugman, P. (1979), A Model of Balance of Payments Crises, Journal of Money Credit and Banking.

Macedo, J. (1979), Exchange Rate Behavior Under Currency Inconvertibility, Economic Growth Center Discussion Paper no. 319, Yale University, September.

__________ (1982a), Currency Diversification and Export Competitiveness: A Model of the "Dutch Disease" in Egypt, Journal of Development Economics, December.

__________ (1982b), Portfolio Diversification and Currency Inconvertibility: Three Essays in International Monetary Economics, Lisbon (Portugal); Servicos Graficos da Universidade Nova de Lisboa.

__________ (1983), Currency Inconvertibility, Portfolio Balance and Relative Prices, in T. Basar and L.F. Pau (eds.), Dynamic

Modelling

and Control of National Economics 1983, Pergamon Press, 1984.

Martin, L. and A. Panagariya (1983), Smuggling, Trade and Price Disparity: A Crime Theoretic Approach, forthcoming Journal of International Economics .

Michaely, M. (1954), A Geometrical Analysis of Black Market Behavior, American Economic Review, September.

Pitt, M. (1981), Smuggling and Price Disparity, Journal of International Economics, December.

__________ (1982), Smuggling and the Black Market for Foreign Exchange, draft, University of Minnesota, December.

Sheikh, M. (1974), Smuggling, Production and Welfare, Journal of International Economics.

__________ (1976), Black Markets for Foreign Exchange, Capital Flows and Smuggling, Journal of Development Economics, 3.