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MODELING THE IMPACT OF WARMING IN CLIMATE CHANGE ECONOMICS

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ABSTRACT

Any economic analysis of climate change policy requires some model that describes the impact of warming on future GDP and consumption. Most integrated assessment models (IAMs) relate temperature to the level of real GDP and consumption, but there are theoretical and empirical reasons to expect temperature to affect the growth rate rather than level of GDP. Does this distinction matter in terms of implications for policy? And how does the answer depend on the nature and extent of uncertainty over future temperature change and its impact? I address these questions by estimating the fraction of consumption society would be willing to sacrifice to limit future increases in temperature, using probability distributions for temperature and impact inferred from studies assembled by the IPCC, and comparing estimates based on a direct versus growth rate impact of temperature on GDP.

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1   Introduction.

Any economic analyses of climate change policy must include a model of damages, i.e., a relationship that translates changes in temperature (and possibly changes in precipitation and other climate-related variables) to economic losses. Economic losses will of course include losses of GDP and consumption that might result from reduced agricultural productivity or from dislocations resulting from higher sea levels, but also the dollar-equivalent costs of possible climate-related increases in morbidity, mortality, and social disruption. Because of the lack of data and the considerable uncertainties involved, modeling damages is probably the most difficult aspect of analyzing climate change policy. There are uncertainties in other aspects of climate change policy — for example, how rapidly greenhouse gases (GHGs) will accumulate in the atmosphere absent an abatement policy, to what extent and how rapidly temperature will increase, and the current and future costs of abatement — but damages from climate change is the area we understand the least. It must be modeled, but it is important to understand the uncertainties involved and their policy implications.

Most quantitative economic studies of climate change policy utilize a "damage function" that relates temperature change directly to the levels of real GDP and consumption. Future consumption, for example, is taken to be the product of a loss function and "but-for consumption," i.e., consumption in the absence of any warming. With no warming, the loss function is equal to 1, but as the temperature increases the value of the loss function decreases. This approach is reasonably simple in that any projected path for temperature can be directly translated into an equivalent path for consumption. In other words, consumption at time t depends on temperature at time t. Given a social utility function that "values" consumption, one can then evaluate a particular policy (assuming we know its costs and can project its effects on GHG concentrations and temperature).

In a recent paper (Pindyck (2009)), I have argued that on both theoretical and empirical grounds, the economic impact of warming should be modeled as a relationship between temperature change and the *growth rate* of GDP as opposed to the level of GDP. This means that warming can have a permanent impact on future GDP and consumption. It makes

the analysis somewhat more complicated, however, because consumption at some future date depends not simply on the temperature at that date, but instead on the entire path of temperature, and thus the path of the growth rate of consumption, up to that date. The issue I address in this paper is the extent to which these two different approaches to modeling damages — temperature affecting consumption directly versus temperature affecting the growth rate of consumption — differ in terms of their policy implications.

This issue must be addressed in the context of uncertainty, which is at the heart of climate change policy. It is difficult to justify the immediate adoption of a stringent abatement policy based on an economic analysis that focuses on "most likely" scenarios for increases in temperature and economic impacts and uses consensus estimates of discount rates and other relevant parameters.[1] But one could ask whether a stringent policy might be justified by a cost-benefit analysis that accounts for a full distribution of possible outcomes.

In Pindyck (2009), I showed how probability distributions for temperature change and economic impact could be inferred from climate science and economic impact studies, and incorporated in the analysis of climate change policy. The framework I used, which I use again here, is based on a simple measure of "willingness to pay" (WTP): the fraction of consumption $w^*(\tau)$ that society would be willing to sacrifice, now and throughout the future, to ensure that any increase in temperature at a specific horizon H, ΔT_H, is limited to τ. Whether the reduction in consumption corresponding to some $w^*(\tau)$ is *sufficient* to limit warming to τ is a separate question that is not addressed; in effect WTP applies to the "demand" side of policy analysis. The advantage of this approach, however, is that there is no need to project GHG emissions and atmospheric concentrations, or estimate abatement costs. Instead the focus is on uncertainties over temperature change and its economic impact.

My earlier paper was based on what I called the current "state of knowledge" regarding global warming and its impact. I used information on the distributions for temperature change from scientific studies assembled by the IPCC (2007) and information about eco-

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[1]The Stern Review (2007) argues for a stringent abatement policy, but as Nordhaus (2007), Weitzman (2007), Mendelsohn (2008) and others point out, it makes assumptions about temperature change, economic impact, abatement costs, and discount rates that are outside the consensus range.

nomic impacts from recent "integrated assessment models" (IAMs) to fit displaced gamma distributions for these variables. But unlike existing IAMs, I modeled economic impact as a relationship between temperature change and the growth rate of consumption as opposed its level. I examined whether "reasonable" values for the remaining parameters (e.g., the starting growth rate and the index of risk aversion) can yield values of $w^*(\tau)$ well above 3% for small values of τ, which might support stringent abatement. I also used a counterfactual — and pessimistic — scenario for temperature change: Under "business as usual" (BAU), the atmospheric GHG concentration immediately increases to twice its pre-industrial level, which leads to an (uncertain) increase in temperature at the horizon H, and then (from feedback effects or further emissions) a gradual further doubling of that temperature increase.

In this paper, I use the same displaced gamma distributions for temperature change and economic impact, but I compare two alternative damage models — a direct impact of temperature change on consumption versus a growth rate impact. I calibrate and "match" the two models by matching estimates of GDP/temperature change pairs from the group of IAMs at a specific horizon. I then calculate and compare WTPs for both models based on expected discounted utility, using a constant relative risk aversion (CRRA) utility function.

I find that for either damage model, the resulting estimates of $w^*(\tau)$ are generally below 2% or 3%, even for τ around 2 or 3°C. This is because there is limited weight in the tails of the calibrated distributions for ΔT and its impact. Larger estimates of WTP result for particular combinations of parameter values (e.g., an index of risk aversion close to 1 and a low initial GDP growth rate), but overall, the results are consistent with moderate abatement. A direct impact generally yields a larger WTP than a growth rate impact, and the sign and extent of the difference varies with changes in parameter values. Overall, there are no substantial differences between the two models in terms of policy implications.[2]

The next section discusses the probability distribution and dynamic trajectory for tem-

[2]As with my earlier study, I ignore the implications of the opposing irreversibilities inherent in climate change policy and the value of waiting for more information. Immediate action reduces the largely irreversible build-up of GHGs in the atmosphere, but waiting avoids an irreversible investment in abatement capital that might turn out to be at least partly unnecessary, and the net effect of these irreversibilities is unclear. For a discussion of the interaction of uncertainty and irreversibility, see Pindyck (2007).

perature change. Section 3 discusses the two alternative ways of modeling economic impact, and the corresponding probability distributions that capture uncertainty over that impact. Section 4 explains the calculation of willingness to pay. Numerical results are presented and discussed in Sections 5 and 6, and Section 7 concludes.

2   Temperature.

According to the IPCC (2007), under "business as usual" (BAU), i.e., no abatement policy, growing GHG emissions would likely lead to a doubling of the atmospheric CO_2e concentration relative to the pre-industrial level by the end of this century. That, in turn, would cause an increase in global mean temperature that would "most likely" range between 1.0°C to 4.5°C, with an expected value of 2.5°C to 3.0°C. The IPCC report indicates that this range, derived from a "summary" of the results of 22 scientific studies the IPCC surveyed, represents a roughly 66- to 90-percent confidence interval, i.e., there is a 5 to 17-percent probability of a temperature increase above 4.5°C.

The 22 studies also provide rough estimates of increases in temperature at the outer tail of the distribution. In summarizing them, the IPCC translated the implied outcome distributions into a standardized form that makes the studies comparable, and created graphs showing multiple outcome distributions implied by groups of studies. Those distributions suggest that there is a 5% probability that a doubling of the CO_2e concentration relative to the pre-industrial level would lead to a global mean temperature increase of 7°C or more, and a 1% probability that it would lead to a temperature increase of 10°C or more. I fit a three-parameter displaced gamma distribution for ΔT to these 5% and 1% points and to a mean temperature change of 3.0°C. This distribution conforms with the distributions summarized by the IPCC. Finally, I assume (consistent with the IPCC's focus on the end of this century) that the fitted distribution for ΔT applies to a 100-year horizon H.

2.1 Displaced Gamma Distribution.

The displaced gamma distribution is given by:

$$f(x; r, \lambda, \theta) = \frac{\lambda^r}{\Gamma(r)} (x - \theta)^{r-1} e^{-\lambda(x-\theta)} \quad , \quad x \geq \theta \quad , \tag{1}$$

where $\Gamma(r) = \int_0^\infty s^{r-1} e^{-s} ds$ is the Gamma function. The moment generating function for (1) is

$$M_x(t) = \mathcal{E}(e^{tx}) = \left(\frac{\lambda}{\lambda - t} \right)^r e^{t\theta} \, .$$

Thus the mean, variance and skewness (around the mean) are $\mathcal{E}(x) = r/\lambda + \theta$, $\mathcal{V}(x) = r/\lambda^2$, and $\mathcal{S}(x) = 2r/\lambda^3$ respectively.

Fitting $f(x; r, \lambda, \theta)$ to a mean of 3°C, and the 5% and 1% points at 7°C and 10°C respectively yields $r = 3.8$, $\lambda = 0.92$, and $\theta = -1.13$. The distribution is shown in Figure 1. It has a variance and skewness around the mean of 4.49 and 9.76 respectively. Note that this distribution implies that there is a small (2.9 percent) probability that a doubling of the CO_2e concentration will lead to a *reduction* in mean temperature, consistent with several of the scientific studies. The distribution also implies that the probability of a temperature increase of 4.5°C or greater is 21%.

2.2 Trajectory for ΔT_t.

Recall that this fitted distribution for ΔT pertains to the 100-year horizon H. To allow for possible feedback effects and/or further emissions, I assume that $\Delta T_t \to 2\Delta T_H$ as t gets large. As summarized in Weitzman (2009), the simplest dynamic model relating ΔT_t to the GHG concentration G_t is the differential equation

$$d\Delta T/dt = m_1 [\ln(G_t/G_0)/\ln 2 - m_2 \Delta T_t] \, . \tag{2}$$

Assuming G_t initially doubles to $2G_0$, $\Delta T_t = \Delta T_H$ at $t = H$, and $\Delta T_t \to 2\Delta T_H$ as $t \to \infty$, implies that ΔT_t follows the trajectory:

$$\Delta T_t = 2\Delta T_H [1 - (1/2)^{t/H}] \, , \tag{3}$$

Thus if $\Delta T_H = 5°C$, ΔT_t reaches $2.93°C$ after 50 years, $5°C$ after 100 years, $7.5°C$ after 200 years, and then gradually approaches $10°C$.

This is illustrated in Figure 2, which shows a trajectory for ΔT when it is unconstrained (and ΔT_H happens to equal $5°C$), and when it is constrained so that $\Delta T_H \leq \tau = 3°C$. Note that even when constrained, ΔT_H is a random variable and (unless $\tau = 0$) will be less than τ with probability 1; in Figure 2 it happens to be $2.5°C$. If $\tau = 0$, then $\Delta T = 0$ for all t.

3   Impact of Warming.

Most economic studies of climate change assume that ΔT has a *direct impact* on GDP (and/or consumption), modeled via a "loss function" $L(\Delta T)$, with $L(0) = 1$ and $L' < 0$. Thus GDP at some horizon H is $L(\Delta T_H)\text{GDP}_H$, where GDP_H is but-for GDP in the absence of warming. This "direct impact" approach has been used in all of the integrated assessment models that I am aware of. However, there are reasons to expect warming to affect the *growth rate* of GDP as opposed to the level. At issue is how these two alternative approaches to modeling the impact of warming — direct versus growth rate — differ in their implications for estimates of willingness to pay to limit warming.

3.1   Direct Impact.

The most widely used loss function has been the inverse-quadratic. For example, the recent version of the Nordhaus (2008) DICE model uses the following loss function:

$$L = 1/[1 + \pi_1 \Delta T + \pi_2 (\Delta T)^2] .$$

Weitzman (2008) introduced the exponential loss function which is very similar to the inverse quadratic for small values of ΔT, but allows for greater losses when ΔT is large:

$$L(\Delta T) = \exp[-\beta(\Delta T)^2] . \tag{4}$$

I will use this loss function of eqn. (4) when calculating WTP under the direct impact assumption.

To introduce uncertainty over the impact of warming, I will treat the parameter β as a random variable that can be described by a 3-parameter displaced gamma distribution. Although the IPCC does not provide standardized distributions for lost GDP corresponding to any particular ΔT as it does for climate sensitivity, it does survey the results of several IAMS. As discussed below, I use the information from the IPCC along with other studies to infer means and confidence intervals for β.

3.2 Growth Rate Impact.

There are three reasons to expect warming to affect the growth rate of GDP as opposed to the level. First, some effects of warming are likely to be permanent: for example, destruction of ecosystems from erosion and flooding, extinction of species, and deaths from health effects and weather extremes. If warming affected the level of GDP directly, e.g., as per eqn. (4), it would imply that if temperatures rise but later fall, e.g., because of stringent abatement or geo-engineering, GDP could return to its but-for path with no permanent loss. This is not the case, however, if ΔT affects the growth rate of GDP.

Suppose, for example, that temperature increases by 0.1°C per year for 50 years and then decreases by 0.1°C per year for the next 50 years. Figure 3 compares two consumption trajectories: C_t^A, which corresponds to the loss function of eqn. (4), and C_t^B, which corresponds to the following growth rate loss function:

$$g_t = g_0 - \gamma \Delta T_t \tag{5}$$

The example assumes that without warming, consumption would grow at 0.5 percent per year — trajectory C_t^0 — and both loss functions are calibrated so that at the maximum ΔT of 5°C, $C^A = C^B = .95C^0$. Note that as ΔT falls to zero, C_t^A reverts to C_t^0, but C_t^B remains permanently below C_t^0.

Second, resources needed to counter the impact of higher temperatures would reduce those available for R&D and capital investment, reducing growth. Adaptation to rising temperatures is equivalent to the cost of increasingly strict emission standards, which, as Stokey (1998) has shown with an endogenous growth model, reduces the rate of return

on capital and lowers the growth rate. As a simple example, suppose total capital $K = K_p + K_a(T)$, with $K_a'(T) > 0$, where K_p is directly productive capital and $K_a(T)$ is capital needed for adaptation to the temperature T (e.g., stronger retaining walls and pumps to counter flooding, new infrastructure and housing to support migration, more air conditioning and insulation, etc.). If all capital depreciates at rate δ_K, $\dot{K}_p = \dot{K} - \dot{K}_a = I - \delta_K K - K_a'(T)\dot{T}$, so that the rate of growth of K_p is reduced.

Third, there is empirical support for a growth rate effect. Using historical data on temperatures and precipitation over the past 50 years for a panel of 136 countries, Dell, Jones, and Olken (2008) have shown that higher temperatures reduce GDP growth rates but not levels. The impact they estimate is large — a decrease of 1.1 percentage points of growth for each 1°C rise in temperature — but significant only for poorer countries.[3]

To calculate WTP when ΔT affects the growth rate of GDP, I assume that in the absence of warming, real GDP and consumption would grow at a constant rate g_0, but warming will reduce this rate according to eqn. (5). This simple linear relation was estimated by Dell, Jones, and Olken (2008), and can be viewed as at least a first approximation to a more complex loss function. I introduce uncertainty by making the parameter γ, like β, a random variable drawn from a displaced gamma distribution.

3.3   Distributions for β and γ.

To compare the effects of a direct versus growth rate impact on estimates of WTP, we need to fit and "match" the distributions for β and γ. This is done as follows.

Using information from a number of IAMs, I fit the three parameters in a displaced gamma distribution for β in the exponential-quadratic loss function of eqn. (4). I then translate this into an equivalent distribution for γ using the trajectory for GDP and consumption implied by eqn. (5) for a temperature change-impact combination projected to occur at hori-

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[3] "Poor" means below-median PPP-adjusted per-capita GDP. Using World Bank data for 209 countries, "poor" by this definition accounts for 26.9% of 2006 world GDP, which implies a roughly 0.3 percentage point reduction in world GDP growth for each 1°C rise in temperature. In a follow-on paper (2009), they estimate a model that allows for adaptation effects, so that the long-run impact of warming is smaller than the short-run impact. They find a long-run decrease of 0.51 percentage points of growth for each 1°C rise in temperature, but again only for poorer countries.

zon H. From eqns. (3) and (5), the growth rate is $g_t = g_0 - 2\gamma\Delta T_H[1-(1/2)^{t/H}]$. Normalizing initial consumption at 1, this implies:

$$C_t = e^{\int_0^t g(s)ds} = \exp\left\{-\frac{2\gamma H\Delta T_H}{\ln(1/2)} + (g_0 - 2\gamma\Delta T_H)t + \frac{2\gamma H\Delta T_H}{\ln(1/2)}(1/2)^{t/H}\right\} \ . \qquad (6)$$

Thus γ is obtained from β by equating the expressions for C_H implied by eqns. (4) and (6):

$$\exp\left\{-\frac{2\gamma H\Delta T_H}{\ln(1/2)} + (g_0 - 2\gamma\Delta T_H)H + \frac{\gamma H\Delta T_H}{\ln(1/2)}\right\} = \exp\{g_0 H - \beta(\Delta T_H)^2\} \ , \qquad (7)$$

so that β and γ have the simple linear relationship:

$$\gamma = 1.79\beta\Delta T_H/H \ . \qquad (8)$$

3.3.1 Distribution for β.

To fit a displaced gamma distribution for β, I utilitze the IPCC's survey of several IAMS. This information from the IPCC, along with other studies, allow me to infer means and confidence intervals for β. These IAMs yield a rough consensus regarding possible economic impacts: for temperature increases up to $4°$C, the "most likely" impact is from 1% to at most 5% of GDP. (Of course this consensus might arise from the use of similar ad hoc damage functions in various IAMs.) Of interest is the outer tail of the distribution for this impact. There is some chance that a temperature increase of $3°$C or $4°$C would have a much larger impact, and we want to know how that affects WTP.

Based on its survey of impact estimates from four IAMs, the IPCC (2007b) concludes that "global mean losses could be 1–5% of GDP for $4°$C of warming."[4] In addition, Dietz and Stern (2008) provide a graphical summary of damage estimates from several IAMs, which yield a range of 0.5% to 2% of lost GDP for $\Delta T = 3°$C, and 1% to 8% of lost GDP for $\Delta T = 5°$C. I treat these ranges as "most likely" outcomes, and use the IPCC's definition of "most likely" to mean a 66 to 90-percent confidence interval. Using the IPCC range and, to be conservative, assuming it applies to a 66-percent confidence interval, I take the mean loss for $\Delta T = 4°$C to be 3% of GDP, and the 17-percent and 83-percent confidence

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[4]The IAMs surveyed by the IPCC include Hope (2006), Mendelsohn et al (1998), Nordhaus and Boyer (2000), and Tol (2002). For a recent overview of economic impact studies, see Tol (2009).

points to be 1% of GDP and 5% of GDP respectively. We can then use eqn. (4) to get the mean, 17-percent, and 83-percent values for β, which I denote by $\bar{\beta}$, β_1, and β_2 respectively. For example, $.97 = e^{-\bar{\beta}(4)^2}$, so that $\bar{\beta} = .00190$. Likewise, $\beta_1 = .000628$ and $\beta_2 = .00321$. Fitting a displaced gamma distribution to these numbers yields $r = 4.5$, $\lambda = 1528$, and $\theta = \bar{\beta} - r/\lambda = -.00105$.

Figure 4 shows the fitted distribution for β. Also shown is the fitted distribution when "most likely" is taken to mean a 90-percent confidence interval, so that β_1 and β_2 instead apply to the 5- and 95-percent confidence points.

3.3.2 Distribution for γ.

The mean, 17-percent, and 83-percent values for β applied to a $\Delta T_H = 4°C$ at a horizon $H = 100$ years, so from eqn. (8), $\gamma = .0716\beta$. Thus the mean, 17-percent, and 83-percent values for γ are, respectively, $\bar{\gamma} = .0001360$, $\gamma_1 = .0000450$, and $\gamma_2 = .0002298$. Now, suppose $f(x; r, \lambda, \theta)$ is the displaced gamma distribution for x, and we want the distribution $f(y; r_1, \lambda_1, \theta_1)$ for $y = ax$. We can make use of the fact that the expectation, variance, and skewness of x and of y are related as follows: $\mathcal{E}(y) = a\mathcal{E}(x) = ar/\lambda + a\theta$, $\mathcal{V}(y) = a^2\mathcal{V}(x) = a^2 r/\lambda^2$, and $\mathcal{S}(y) = a^3\mathcal{S}(x) = 2a^3 r/\lambda^3$. This implies that $\theta_1 = a\theta$, $r_1 = r$, and $\lambda_1 = \lambda/a$. Thus the matched distribution for γ will be the same as that for β, except that $\lambda_1 = 1528/.0716 = 21{,}340$ and $\theta_1 = .0716(-.00105) = -.0000752$. The distribution for γ, shown graphically in Pindyck (2009), will have exactly the same shape as the distribution for β but a different scaling.

4 Willingness to Pay.

Given the distributions for ΔT and β or γ, I posit a CRRA social utility function:

$$U(C_t) = C_t^{1-\eta}/(1-\eta) \ , \tag{9}$$

where η is the index of relative risk aversion (and $1/\eta$ is the elasticity of intertemporal substitution). Social welfare is measured as the expected sum over time of discounted utility:

$$W = \mathcal{E} \int_0^\infty U(C_t)e^{-\delta t}dt \ , \tag{10}$$

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where δ is the rate of time preference, i.e., the rate at which utility is discounted. Note that this rate is different from the *consumption* discount rate, which in the Ramsey growth context would be $R_t = \delta + \eta g_t$. If ΔT affects consumption directly, then $R_t = \delta + \eta g_0$, and does not change over time. If ΔT affects the growth rate of consumption, then $R_t = \delta + \eta g_0 - 2\eta\gamma\Delta T_H[1 - (1/2)^{t/H}]$, so R_t falls over time as ΔT increases.[5]

For both the direct and growth rate impact models, I calculate the fraction of consumption — now and throughout the future — society would sacrifice to ensure that any increase in temperature at a specific horizon H is limited to an amount τ. That fraction, $w^*(\tau)$, is the measure of willingness to pay.[6]

4.1   WTP: Direct Impact.

Using eqn. (3), if ΔT_H and β were known, social welfare would be given by:

$$W = \int_0^\infty U(C_t)e^{-\delta t}dt = \frac{1}{1-\eta}\int_0^\infty e^{\rho_0 - \rho_1 t - 2\rho_0(1/2)^{t/H} + \rho_0(1/2)^{2t/H}}dt \ , \tag{11}$$

where

$$\rho_0 = 4(\eta - 1)\beta(\Delta T_H)^2 \ , \tag{12}$$

$$\rho_1 = (\eta - 1)g_0 + \delta \ . \tag{13}$$

Suppose society sacrifices a fraction $w(\tau)$ of present and future consumption to keep $\Delta T_H \leq \tau$. With uncertainty over ΔT_H and β, social welfare at $t = 0$ is:

$$W_1 = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta}\mathcal{E}_{0,\tau}\int_0^\infty e^{\tilde{\rho}_0 - \tilde{\rho}_1 t - 2\tilde{\rho}_0(1/2)^{t/H} + \tilde{\rho}_0(1/2)^{2t/H}}dt \ , \tag{14}$$

where $\mathcal{E}_{0,\tau}$ denotes the expectation at $t = 0$ over the distributions of ΔT_H and β conditional on $\Delta T_H \leq \tau$. (Tildes are used to denote that ρ_0 and ρ_1 are functions of two random variables.) If no action is taken to limit warming, social welfare would be:

$$W_2 = \frac{1}{1-\eta}\mathcal{E}_0\int_0^\infty e^{\tilde{\rho}_0 - \tilde{\rho}_1 t - 2\tilde{\rho}_0(1/2)^{t/H} + \tilde{\rho}_0(1/2)^{2t/H}}dt \ , \tag{15}$$

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[5]If $2\eta\gamma\Delta T_H > \delta + \eta g_0$, R_t becomes negative as ΔT grows. This is entirely consistent with the Ramsey growth model, as pointed out by Dasgupta et al (1999).

[6]The use of WTP as a welfare measure goes back at least to Debreu (1954), was used by Lucas (1987) to estimate the welfare cost of business cycles, and was used in the context of climate change (with $\tau = 0$) by Heal and Kriström (2002) and Weitzman (2008).

where $\mathcal{E}_0$ again denotes the expectation over ΔT_H and β, but now with ΔT_H unconstrained. Willingness to pay to ensure that $\Delta T_H \leq \tau$ is the value $w^*(\tau)$ that equates $W_1(\tau)$ and W_2.[7]

Given the distributions $f(\Delta T)$ and $g(\beta)$ for ΔT_H and β respectively, denote by $M_\tau(t)$ and $M_\infty(t)$ the time-t expectations

$$M_\tau(t) = \frac{1}{F(\tau)} \int_{\theta_T}^{\tau} \int_{\theta_\beta}^{\infty} e^{\tilde{\rho}_0 - \tilde{\rho}_1 t - 2\tilde{\rho}_0(1/2)^{t/H} + \tilde{\rho}_0(1/2)^{2t/H}} f(\Delta T)g(\beta)d\Delta T d\beta \quad (16)$$

and

$$M_\infty(t) = \int_{\theta_T}^{\infty} \int_{\theta_\beta}^{\infty} e^{\tilde{\rho}_0 - \tilde{\rho}_1 t - 2\tilde{\rho}_0(1/2)^{t/H} + \tilde{\rho}_0(1/2)^{2t/H}} f(\Delta T)g(\gamma)d\Delta T d\gamma \quad , \quad (17)$$

where $\tilde{\rho}_0$ and $\tilde{\rho}_1$ are given by eqns. (12) and (13), θ_T and θ_β are the lower limits on the distributions for ΔT and β, and $F(\tau) = \int_{\theta_T}^{\tau} f(\Delta T)d\Delta T$. Thus $W_1(\tau)$ and W_2 are:

$$W_1(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} \int_0^\infty M_\tau(t)dt \equiv \frac{[1 - w(\tau)]^{1-\eta}}{1-\eta} G_\tau \quad (18)$$

and

$$W_2 = \frac{1}{1-\eta} \int_0^\infty M_\infty(t)dt \equiv \frac{1}{1-\eta} G_\infty \quad . \quad (19)$$

Setting $W_1(\tau)$ equal to W_2, WTP is given by:

$$w^*(\tau) = 1 - [G_\infty/G_\tau]^{\frac{1}{1-\eta}} \quad . \quad (20)$$

The solution for $w^*(\tau)$ depends on the distributions for ΔT and γ, the horizon $H = 100$ years, and the parameters η, g_0, and δ (values for which are discussed below). We will examine how w^* varies with τ; the cost of abatement should be a decreasing function of τ, so given estimates of that cost, one could use these results to determine abatement targets.

4.2   WTP: Growth Rate Impact.

If ΔT_H instead affects the growth rate of consumption as in eqn. (5), and if ΔT_H and γ were known, social welfare would be:

$$W = \int_0^\infty U(C_t)e^{-\delta t}dt = \frac{1}{1-\eta} \int_0^\infty e^{\omega_0 - \omega_1 t - \omega_0(1/2)^{t/H}} dt \quad , \quad (21)$$

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[7]I calculate WTP using a finite horizon of 500 years. After some 200 years the world will likely exhaust the economically recoverable stocks of fossil fuels, so that GHG concentrations will fall. In addition, so many other economic and social changes are likely that the relevance of applying CRRA expected utility over more than a few hundred years is questionable.

where

$$\omega_0 = 2(\eta - 1)\gamma H \Delta T_H / \ln(1/2) \ . \tag{22}$$

$$\omega_1 = (\eta - 1)(g_0 - 2\gamma \Delta T_H) + \delta \ , \tag{23}$$

If society sacrifices a fraction $w(\tau)$ of present and future consumption to keep $\Delta T_H \leq \tau$ and there is uncertainty over ΔT_H and γ, social welfare at $t = 0$ is:

$$W_1(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1 - \eta} \mathcal{E}_{0,\tau} \int_0^\infty e^{\tilde{\omega}_0 - \tilde{\omega}_1 t - \tilde{\omega}_0 (1/2)^{t/H}} dt \ . \tag{24}$$

If no action is taken to limit warming, social welfare would be:

$$W_2 = \frac{1}{1 - \eta} \mathcal{E}_0 \int_0^\infty e^{\tilde{\omega}_0 - \tilde{\omega}_1 t - \tilde{\omega}_0 (1/2)^{t/H}} dt \ . \tag{25}$$

Once again, WTP is the value $w^*(\tau)$ that equates $W_1(\tau)$ and W_2. Defining $M_\tau(t)$ and $M_\infty(t)$ as before, but with $g(\gamma)$ instead of $g(\beta)$, eqns. (18), (19), and (20) again apply.

5   Results.

WTP is essentially a measure of the "demand" side of policy — the maximum amount society would be willing to sacrifice to obtain the benefits of limited warming. The case for an actual GHG abatement policy will depend on the cost of that policy as well as the benefits. The framework I use does not involve estimates of abatement costs — I only estimate WTP as a function of τ, the abatement-induced limit on any increase in temperature at the horizon H. Clearly the amount and cost of abatement needed will decrease as τ is made larger, so I consider a stringent abatement policy to be one for which τ is "low," which I take to be at or below the expected value of ΔT under a business-as-usual (BAU) scenario, i.e., about 3°C, and $w^*(\tau)$ is "high," i.e., at least 3 percent. At issue in this paper is the extent to which estimates of WTP depend on whether ΔT is assumed to affect the level of consumption directly versus the growth rate of consumption.

In addition to the distributions for ΔT and the impact parameters β or γ, WTP depends on the values for the index of relative risk aversion η, the rate of time discount δ, and the base level real growth rate g_0. To explore the case for a stringent abatement policy, I make

conservative assumptions about η, δ, and g_0, in the sense of choosing numbers that would lead to a higher WTP.

The finance and macroeconomics literature has estimates of η ranging from 1.5 to 6, and estimates of δ ranging from .01 to .04. The historical real growth rate g ranges from .02 to .025. It has been argued, however, that for intergenerational comparisons δ should be close to zero, on the grounds that society not should value the well-being of our great-grandchildren less than our own. Likewise, while values of η well above 2 may be consistent with the (relatively short-horizon) behavior of investors, we might use lower values for inter-generational welfare comparisons. Because I want to determine whether current assessments of uncertainty over temperature change and its impact generate a high enough WTP to justify stringent abatement, I will stack the deck in favor of our great-grandchildren and use relatively low values of η and δ: around 2 for η and 0 for δ. Also, WTP is a decreasing function of the base growth rate g_0, so I will set $g_0 = .02$, the low end of the historical range.

5.1 No Uncertainty.

It is useful to begin by considering a deterministic world in which the trajectory for ΔT and the impact of that trajectory are known with certainty. Then eqns. (18) and (19) for the direct impact case would simplify to:

$$W_1 = \frac{[1 - w(\tau)]^{1-\eta}}{1 - \eta} \int_0^\infty e^{\rho_0 - \rho_1 t - 2\rho_0(1/2)^{t/H} + \rho_0(1/2)^{2t/H}} dt \ , \tag{26}$$

$$W_2 = \frac{1}{1 - \eta} \int_0^\infty e^{\rho_0 - \rho_1 t - 2\rho_0(1/2)^{t/H} + \rho_0(1/2)^{2t/H}} dt \ , \tag{27}$$

where now $\bar{\beta}$, the mean of β, replaces β in eqn. (12) for ρ_0. (I will use the means of β and γ as their certainty-equivalent values.) Likewise, eqns. (24) and (25) for the case of a growth rate impact would simplify to:

$$W_1(\tau) = \frac{[1 - w(\tau)]^{1-\eta}}{1 - \eta} \int_0^\infty e^{\omega_0 - \omega_1 t - \omega_0(1/2)^{t/H}} dt \ , \tag{28}$$

$$W_2 = \frac{1}{1 - \eta} \int_0^\infty e^{\omega_0 - \omega_1 t - \omega_0(1/2)^{t/H}} dt \ , \tag{29}$$

where now the mean $\bar{\gamma}$ replaces γ in eqns. (22) and (23) for ω_0 and ω_1.

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For both impact models, I calculate the WTP to keep ΔT zero for all time, i.e., $w^*(0)$, over a range of values for ΔT at the horizon $H = 100$. For this exercise, I set $\eta = 2$, $\delta = 0$, and $g_0 = .020$. The results are shown in Figure 5, where $w_c^*(0)$ applies to the case where ΔT affects C directly, and $w_g^*(0)$ applies to the case where ΔT affects the growth rate of C.

The graph says that if, for example, $\Delta T_H = 6°$C, $w_c^*(0)$ is about .03, and $w_g^*(0)$ is about .022. Thus if ΔT affects consumption directly, society should be willing to give about 3% of current and future consumption to keep ΔT at zero instead of 6°C. But if ΔT affects the growth rate of consumption, the willingness to pay is only about 2.2%. (Remember that the "known ΔT" applies to time $t = H$. ΔT_t follows the trajectory given by eqn. (3).)

Note that both $w_c^*(0)$ and $w_g^*(0)$ become much larger as the *known* ΔT_H becomes larger than 8°C; such temperature outcomes, however, have low probability. In addition, these curves have different shapes: $w_c^*(0)$ is a convex function of ΔT_H, while $w_g^*(0)$ is a (nearly) linear function of ΔT_H.[8] This means that for small changes in temperature, a growth rate impact model will yield a slightly higher WTP, but for very large changes in temperature, the direct impact model yields much larger WTPs. Whether this difference matters for estimates of WTP under uncertainty depends on the probability distributions for ΔT_H and β and γ. With sufficient probability mass in the right-hand tails of the distributions, the two impact models should yield different numbers for WTP. We explore this below.

5.2   Uncertainty Over Temperature and Economic Impact.

I now allow for uncertainty over both ΔT and the relevant impact parameter (either β or γ), using the calibrated distributions for each. WTP is given by eqns. (16) to (20) for the direct impact model, and eqns. (24) and (25) for the growth rate impact. The calculated values of

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[8]To see that $w_c^*(0)$ is a convex function of ΔT_H, note that if $\tau = 0$, eqns. (26) and (27) imply that

$$w_c^*(0) = 1 - \left[\rho \int_0^\infty e^{-\rho t - 4(1-\eta)\bar{\beta}\Delta T_H^2 \psi(t)} dt \right]^{\frac{1}{1-\eta}}$$

where $\psi(t) = [1 - (1/2)^{t/H}]^2$. Just differentiate to see that $dw_c^*/d\Delta T_H > 0$ for all values of ΔT_H and $\bar{\beta}$, and and $d^2 w_c^*/d\Delta T_H^2 > 0$ for sufficiently small values of ΔT_H and $\bar{\beta}$ (in our case, as long as $\Delta T_H < 15.8\ °$C). Similarly, we can show that $dw_g^*/d\Delta T_H > 0$ and $d^2 w_c^*/d\Delta T_H^2$ is a small negative number (in our case -.000063), a curvature small enough so that in Figure 5, $w_g^*(0)$ appears linear.

WTP as are shown as functions of τ in Figure 6 for $\delta = 0$, $g_0 = .020$, and $\eta = 2$ and 1.5. Note that if $\eta = 2$, WTP is always less than 1.5%, even for $\tau = 0$. To obtain a WTP above 2% requires a lower value of η. As Figure 6 shows, if $\eta = 1.5$, $w^*(\tau)$ reaches about 3% for τ around 0 or 1°C, but only when the impact of warming occurs through the growth rate of consumption. When the impact is direct, w^* is always below 2.5%.

Because relatively large values of WTP can only be obtained for small values of η, the top two lines in Figure 6 have the greatest policy relevance. But note that when $\eta = 1.5$, the difference between $w_g^*(\tau)$ and $w_c^*(\tau)$ is only significant for τ below 2°C. It seems unlikely that a politically and economically feasible policy would be adopted that would prevent any warming, or limit it to 1 or even 2°C. If we belive that a "feasible" policy is one that limits ΔT to its expected value of around 3°C, then as Figure 6 shows, the direct and growth rate impact models give similar values for WTP.

On the other hand, what if we take the view that the "correct" value of η is less than 1.5? Figure 7 shows the dependence of WTP on the index of risk aversion, η. It plots $w^*(3)$, i.e., the WTP to ensure $\Delta T_H \leq 3$°C at H = 100 years, for $g_0 = .02$, as a function of η. Although $w^*(3)$ is below 2% for values of η above 1.5, it approaches 5% as η is reduced to 1 (the value used in Stern (2007)). The reason is that while future utility is not discounted (because $\delta = 0$), future consumption is implicitly discounted at the initial rate ηg_0. If η is made smaller, potential losses of future consumption have a larger impact on WTP. Also, as discussed further below, $w_c^*(3) > (<) \, w_g^*(3)$ when $\eta > (<) \, 1.3$.

These estimates of WTP are based on zero discounting of future uility. While there may be an ethical argument for zero discounting, $\delta = 0$ is outside the range of estimates of the rate of time preference obtained from consumer and investor behavior. However, estimates of WTP above 3% depend critically on this assumption of a $\delta = 0$. Figure 8 again plots $w_c^*(3)$ and $w_g^*(3)$, but this time with $\delta = .01$. Note that for either impact model, discounting future utility, even at a very low rate, will considerably reduce WTP. With $\delta = .01$, $w^*(3)$ is again below 2% for all values of η, and for either impact model.

The results so far indicate that for either impact model, large values of WTP require fairly extreme combinations of parameter values. However, these results are based on distributions

for ΔT_H, β and γ that were calibrated to studies in the IPCC's 2007 report and concurrent economic studies, and those studies were done several years prior to 2007. More recent studies suggest that "most likely" values for ΔT in 2100 might be higher than the 1.0°C to 4.5°C range given by the IPCC. For example, a recent report by Sokolov et al (2009) suggests an expected value for ΔT in 2100 of around 4 to 5°C, as opposed to the 3.0°C expected value that I used. Thus I recalculate WTP for both impact models, for both $\delta = 0$ and .01, but this time shifting the distribution for ΔT_H to the right, so that it has a mean of 5°C, corresponding to the upper end of the 4 to 5°C range in Sokolov et al (2009). (The other moments of the distribution remain unchanged, and H is again 100 years).

The results are shown in Figures 9 and 10. Now if $\delta = 0$ and η is below 1.5, $w^*(3)$ is above 3% when the impact of ΔT occurs through the growth rate, and above 4% when the impact is direct, and reaches around 10% if $\eta = 1$. Even if $\delta = .01$, $w_c^*(3)$ exceeds 4% when $\eta = 1$ (although $w_g^*(3)$ only reaches 2..5%). Thus there are parameter values and plausible distributions for ΔT that yield a large WTP. Those parameter values and distributions are outside the current consensus range, but that may change as new studies of warming and its impact become available.

As Figures 7 to 10 show, for either value of δ, $w_c^*(3)$ is usually higher than $w_g^*(3)$, and when $\delta = .01$ it is considerable higher. In the Ramsey growth context, the consumption discount rate is $\delta + \eta g_t$, so even if $\delta = 0$, future consumption (although not utility) is discounted (less so for small values of η). When ΔT affects consumption directly, the loss of consumption is greater at shorter horizons (but smaller at long horizons), making $w_c^*(3) > w_g^*(3)$. (In Figure 7, $w_c^*(3) < w_g^*(3)$ when $\eta < 1.3$ because with a low consumption discount rate, the larger long-run reduction in consumption from a growth rate impact overwhelms the smaller short-run impact, even in expected value terms.)

6 Modeling and Policy Implications.

The integrated assessment models that I am aware of all relate temperature change to the level of real GDP and consumption. As we have seen, this will often yield a higher WTP

— and thus yield higher estimates of optimal GHG abatement — than will a model that relates temperature change to the growth rate of GDP and consumption. How important is the difference, and what do these results tell us about modeling?

In Pindyck (2009), using a model that related temperature change to the growth rate of consumption, I found that for temperature and impact distributions based on the IPCC and "conservative" parameter values (e.g., $\delta = 0$, $\eta = 2$, and $g_0 = .02$), WTP to prevent even a small increase in temperature is around 2% or less, which is inconsistent with the immediate adoption of a stringent GHG abatement policy. To what extent do those results change when temperature change directly affects the level of consumption? And more broadly, what are the policy implications of the results in this paper?

6.1   Implications for Modeling.

The difference in WTPs for a direct versus a growth rate impact is largest for large temperature changes and for higher consumption discount rates. As we saw in Figure 5 for the case of no uncertainty, $w_c^*(0)$ is a convex function of ΔT and thus becomes increasingly greater than $w_g^*(0)$ as ΔT gets larger. Likewise, when there is uncertainty but the *expected* temperature change is increased from 3°C to 5°C, the difference between $w_c^*(3)$ and $w_g^*(3)$ becomes larger. And note from Figures 9 and 10 that the difference between $w_c^*(3)$ and $w_g^*(3)$ is proportionally larger when the consumption discount rate $(\delta + \eta g_0)$ is larger, i.e., when η is larger and/or when δ is .01 rather than 0.

If the consumption discount rate is large (i.e., if η is large and/or $\delta > 0$), almost any model will yield estimates of WTP and optimal abatement levels that are small. This is simply the result of discounting over long horizons (greater than 50 years). That is why model-based analyses that call for stringent abatement policies assume $\delta = 0$ and relatively low values for η. (Stern (2007, 2008), for example, uses $\delta = 0$ and $\eta = 1$.) Thus if we limit our analyses to the low end of the consensus range for η (around 1.5), even with $\delta = 0$, the choice of impact model will matter if evolving climate science studies yield increasingly large estimates of expected temperature change.

Which impact model — direct versus growth rate — should one use for modeling? A

direct impact model is simpler, easier to understand, and perhaps easier to estimate or calibrate. But as I have argued at the beginning of this paper, there are strong theoretical and empirical arguments that favor the growth rate impact. Until new studies demonstrate otherwise, it seems to me that it is difficult to make the case for a direct impact.

6.2 Implications for Policy.

The results in this paper supplement those in Pindyck (2009) in terms of implications for policy. We can summarize those implications as follows.

First, although the direct impact model often yields higher estimates of WTP, it is still the case that using temperature and impact distributions based on the IPCC (2007) and concurrent economic studies, for most parameter values our WTP estimates are still too low to support a stringent GHG abatement policy. Of course these estimates do not suggest that no abatement is optimal. For example, a WTP of 2% of GDP is in the range of cost estimates for compliance with the Kyoto Protocol.[9]

In addition to the effects of discounting discussed above, our low estimates of WTP are due to the limited weight in the tails of the distributions for ΔT and the impact parameter β or γ. The probability of a realization in which $\Delta T \geq 4.5°C$ in 100 years *and* the impact parameter is one standard deviation above its mean is less than 5 percent. An even more extreme outcome in which $\Delta T = 7°C$ (and the impact parameter is one standard deviation above its mean) would imply about a 9 percent loss of GDP in 100 years for a growth rate impact, but the probability of an outcome this bad or worse is less than 1 percent. And this low-probability loss of GDP in 100 years would involve much smaller losses in earlier years.

Second, although these estimates of WTP are consistent with the current consensus regarding future warming and its impact as summarized in IPCC (2007), that consensus may be wrong, especially with respect to the tails of the distributions. Indeed, based on recent studies, that consensus may already be shifting towards more dire estimates of warming and its impact. As we saw from Figures 9 and 10, shifting the temperature distribution to the

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[9]See the survey of cost studies by the Energy Information Administration (1998), and the more recent country cost studies surveyed in IPCC (2007c).

right so that $\mathcal{E}(\Delta T_H)$ is 5°C instead of 3°C results in substantially higher estimates of WTP.

7   Conclusions.

If we are to use economic models to evaluate GHG abatement policies, how should we treat the impact of possible future increases in temperature? One could argue that we simply do not (and cannot) know much about that impact because we have had no experience with substantial amounts of warming, and there are no models or data that can tell us much about the impact of warming on production, migration, disease prevalence, and a variety of other relevant factors. Instead, I have taken existing IAMs and related models of economic impact at face value and treated them analogously to the climate science models that are used to predict temperature change or its probability distribution. In this way I obtained a (displaced gamma) distribution for an impact parameter that relates temperature change to consumption, or to the growth rate of consumption.

We have seen that in most cases, a direct impact yields a higher WTP than a growth rate impact. The reason is that when ΔT affects consumption directly, the loss of consumption is greater at short horizons (but smaller at long horizons). Consumption discounting can give these short-horizon effects more weight. Even if future utility is not discounted ($\delta = 0$), the consumption discount rate ($\delta + \eta g_0$) is still positive, and can be large if η is large.

Overall, I would argue that the choice of a direct versus growth rate impact should be based on the underlying economics, and the growth rate specification has both theoretical and empirical support. But even with a direct impact model, using temperature and impact distributions based on the IPCC (2007) and concurrent economic studies, for most parameter values our WTP estimates are still too low to support a stringent GHG abatement policy. Of course there are parameter values and plausible distributions for ΔT that yield a large WTP. Those distributions and parameter value are outside the current consensus range, but that range may change as new studies of warming and its impact are done.

References

Dasgupta, Partha, Karl-Göran Mäler, and Scott Barrett, "Intergenerational Equity, Social Discount Rates, and Global Warming," Chap. 7 in P. Portney and J. Weyant, Eds., *Discounting and Intergenerational Equity*, Resources for the Future, 1999.

Debreu, Gerard, "A Classical Tax-Subsidy Problem," *Econometrica*, Jan. 1954, **22**, 14–22.

Dell, Melissa, Benjamin F. Jones, and Benjamin A. Olken, "Climate Change and Economic Growth: Evidence from the Last Half Century," NBER Working Paper No. 14132, June 2008.

Dell, Melissa, Benjamin F. Jones, and Benjamin A. Olken, "Temperature and Income: Reconciling New Cross-Sectional and Panel Estimates," *American Economic Review*, May 2009, **99**, 198–204.

Dietz, Simon, and Nicholas Stern, "Why Economic Analysis Supports Strong Action on Climate Change: A Response to the *Stern Review*'s Critics," *Review of Environmental Economics and Policy*, Winter 2008, **2**, 94–113.

Energy Information Administration, "Comparing Cost Estimates for the Kyoto Protocol," Report No. SR/OIAF/98-03, Oct. 1998.

Heal, Geoffrey, and Bengt Kriström, "Uncertainty and Climate Change," *Environmental and Resource Economics*, June 2002, **22**, 3–39.

Hope, Chris W., "The Marginal Impact of CO_2 from PAGE2002: An Integrated Assessment Model Incorporating the IPCC's Five Reasons for Concern," *Integrated Assessment*, 2006, **6**, 1–16.

Intergovernmental Panel on Climate Change (IPCC), "Climate Change 2007: The Physical Science Basis," Cambridge University Press, 2007a; "Climate Change 2007: Impacts, Adaptation, and Vulnerability," 2007b; "Climate Change 2007: Mitigation of Climate Change," 2007c.

Lucas, Robert E., Jr., *Models of Business Cycles*, Oxford: Basil Blackwell, 1987.

Mendelsohn, Robert, "Is the *Stern Review* an Economic Analysis?" *Review of Environmental Economics and Policy*, Winter 2008, **2**, 45–60.

Mendelsohn, Robert, W.N. Morrison, M.E. Schlesinger, and N.G. Andronova, "Country-Specific Market Impacts of Climate Change," *Climatic Change*, 1998, **45**, 553–569.

Nordhaus, William D., "A Review of the *Stern Review on the Economics of Climate Change*," *Journal of Economic Literature*, Sept. 2007, 686–702.

Nordhaus, William D., *A Question of Balance: Weighing the Options on Global Warming Policies*, Yale University Press, 2008.

Nordhaus, William D., and J.G. Boyer, *Warming the World: Economic Models of Global Warming*, M.I.T Press, 2000.

Pindyck, Robert S., "Uncertainty in Environmental Economics," *Review of Environmental Economics and Policy*, Winter 2007.

Pindyck, Robert S., "Uncertain Outcomes and Climate Change Policy," NBER Working Paper, August 2009.

Sokolov, A.P., P.H. Stone, C.E. Forest, R. Prinn, M.C. Sarofim, M. Webster, S. Paltsev, C.A. Schlosser, D. Kicklighter, S. Dutkiewicz, J. Reilly, C. Wang, B. Felzer, and H.D. Jacoby, "Probabilistic Forecast for 21st Century Climate Based on Uncertainties in Emissions (without Policy) and Climate Parameters," MIT Joint Program on the Science and Policy of Global Change, Report No. 169, January 2009.

Stern, Nicholas, *The Economics of Climate Change: The Stern Review*, Cambridge University Press, 2007.

Stern, Nicholas, "The Economics of Climate Change," *American Economic Review*, May 2008, **98**, 1–37.

Stokey, Nancy, "Are There Limits to Growth?" *International Economic Review*, Feb. 1998, **39**, 1–31.

Tol, Richard S.J., "Estimates of the Damage Costs of Climate Change," *Environmental and Resource Economics*, 2002, **21**, 47–73 (Part 1: Benchmark Estimates), 135–160 (Part 2: Dynamic Estimates).

Tol, Richard S.J., "The Economic Effects of Climate Change," *Journal of Economic Perspectives*, Spring 2009, **23**, 29–51.

Weitzman, Martin L., "A Review of the *Stern Review on the Economics of Climate Change*," *Journal of Economic Literature*, Sept. 2007, 703–724.

Weitzman, Martin L., "Some Dynamic Economic Consequences of the Climate-Sensitivity Inference Dilemma," unpublished manuscript, Feb. 2008.

Weitzman, Martin L., "Additive Damages, Fat-Tailed Climate Dynamics, and Uncertain Discounting," unpublished manuscript, April 2009.
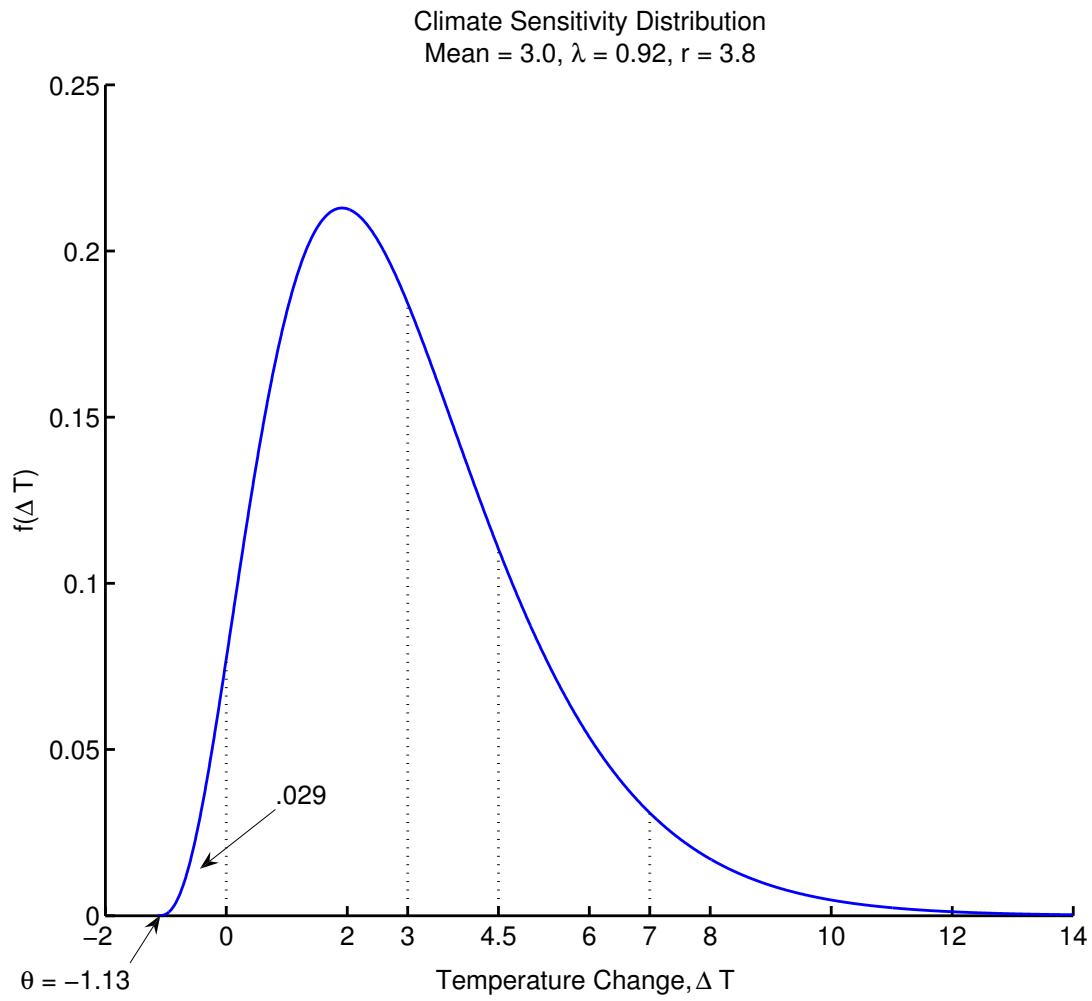
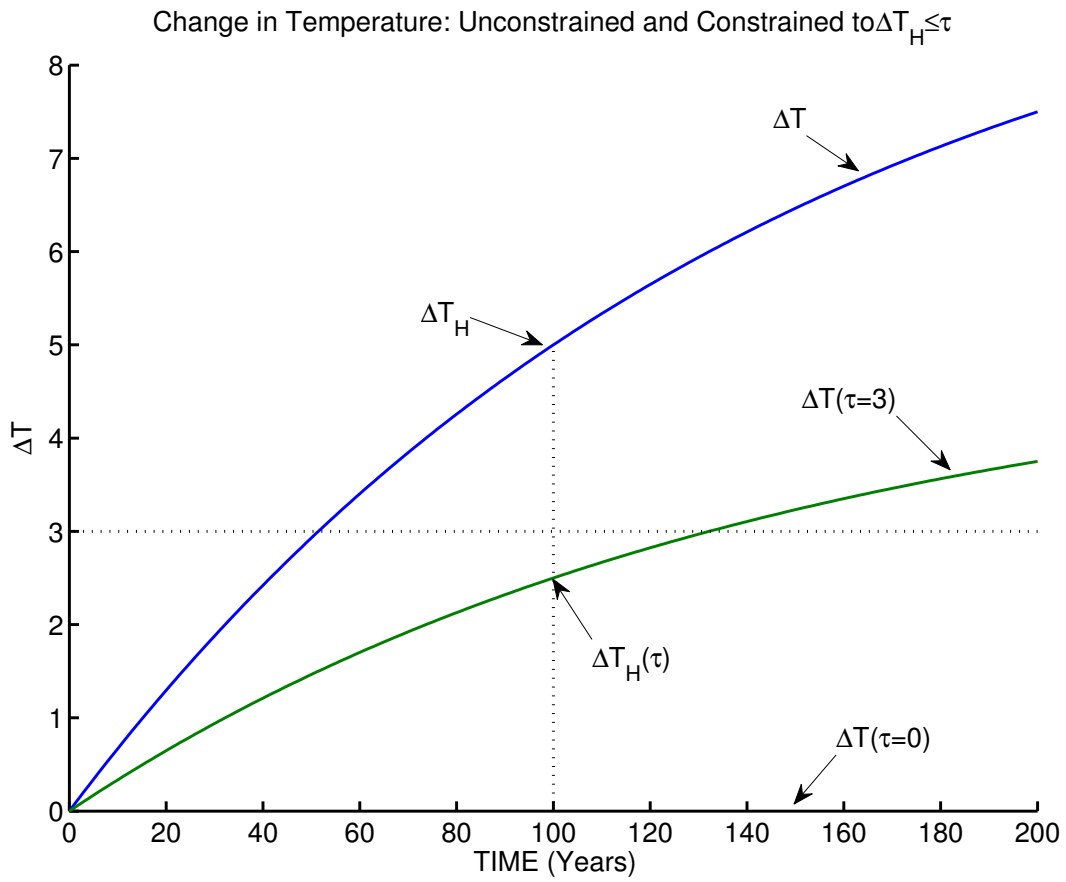Figure 1: Distribution for Temperature Change.

Figure 2: Temperature Change: Unconstrained and Constrained So $\Delta T_H \leq \tau$
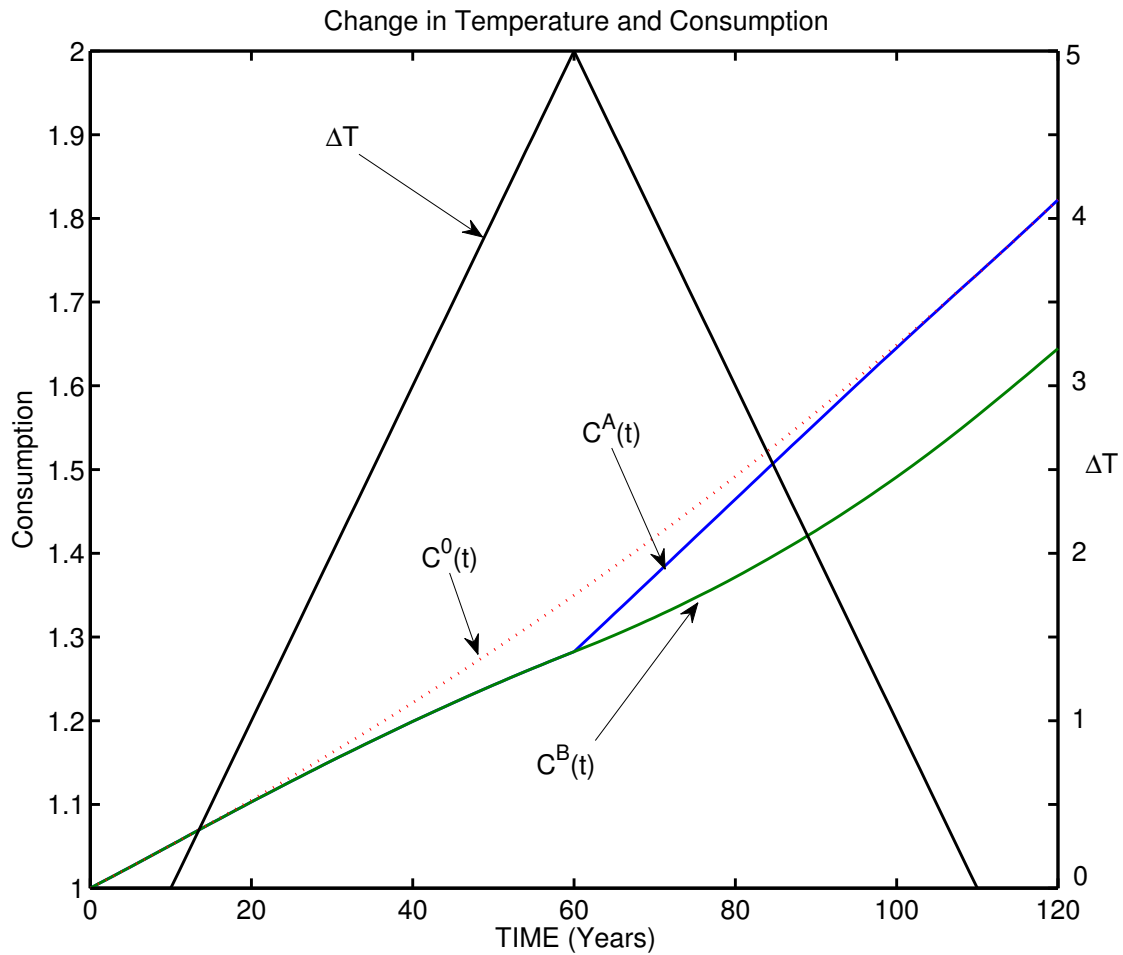
Figure 3: Example of Economic Impact of Temperature Change. (Note temperature increases by 5°C over 50 years and then falls to original level over next 50 years. C^A is consumption when ΔT reduces level, C^B is consumption when ΔT reduces growth rate, and C^0 is consumption with no temperature change.)
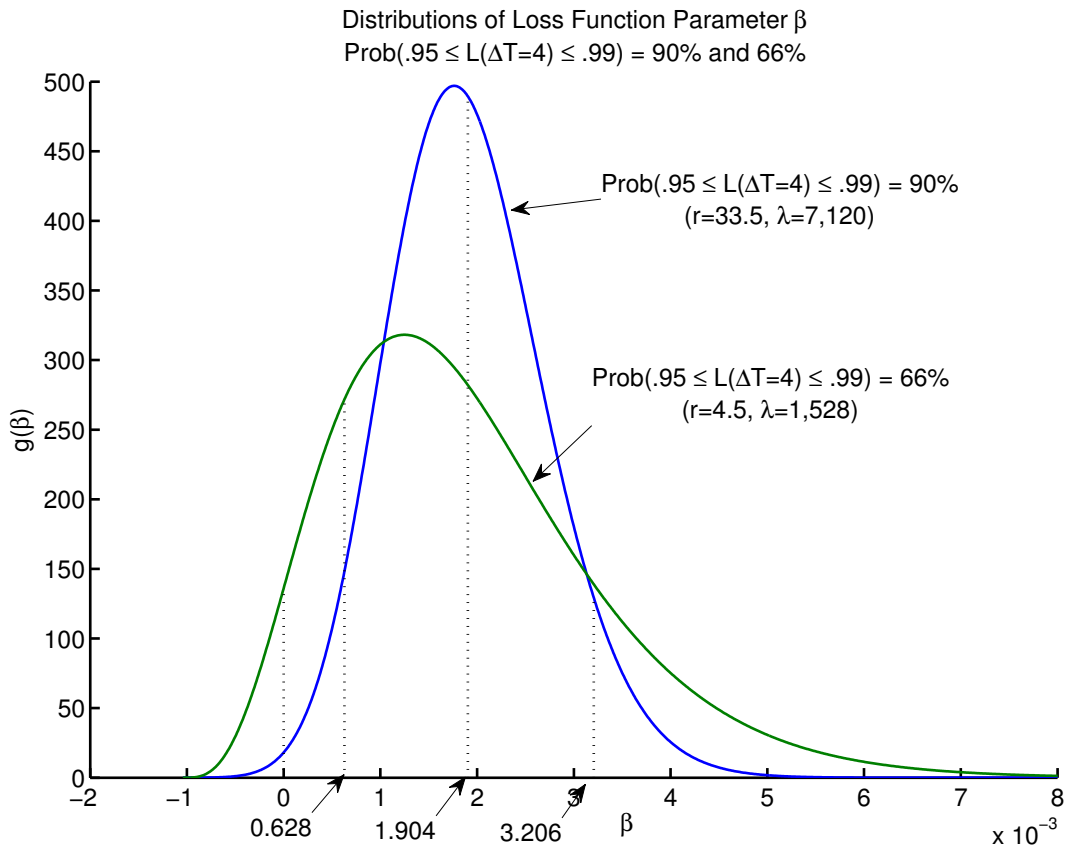
Figure 4: Distributions for Loss Function Parameter β
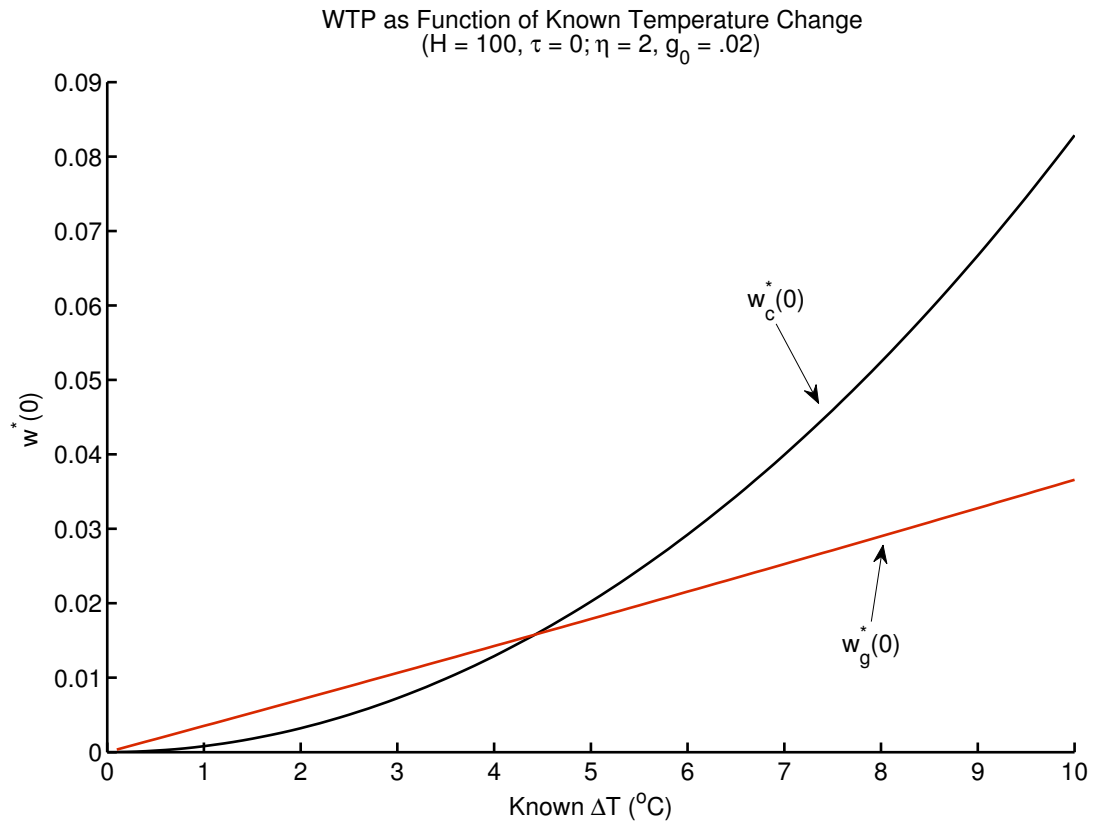
Figure 5: WTP, Known Temperature Change, $\eta = 2$, $g_0 = .020$, and $\delta = 0$
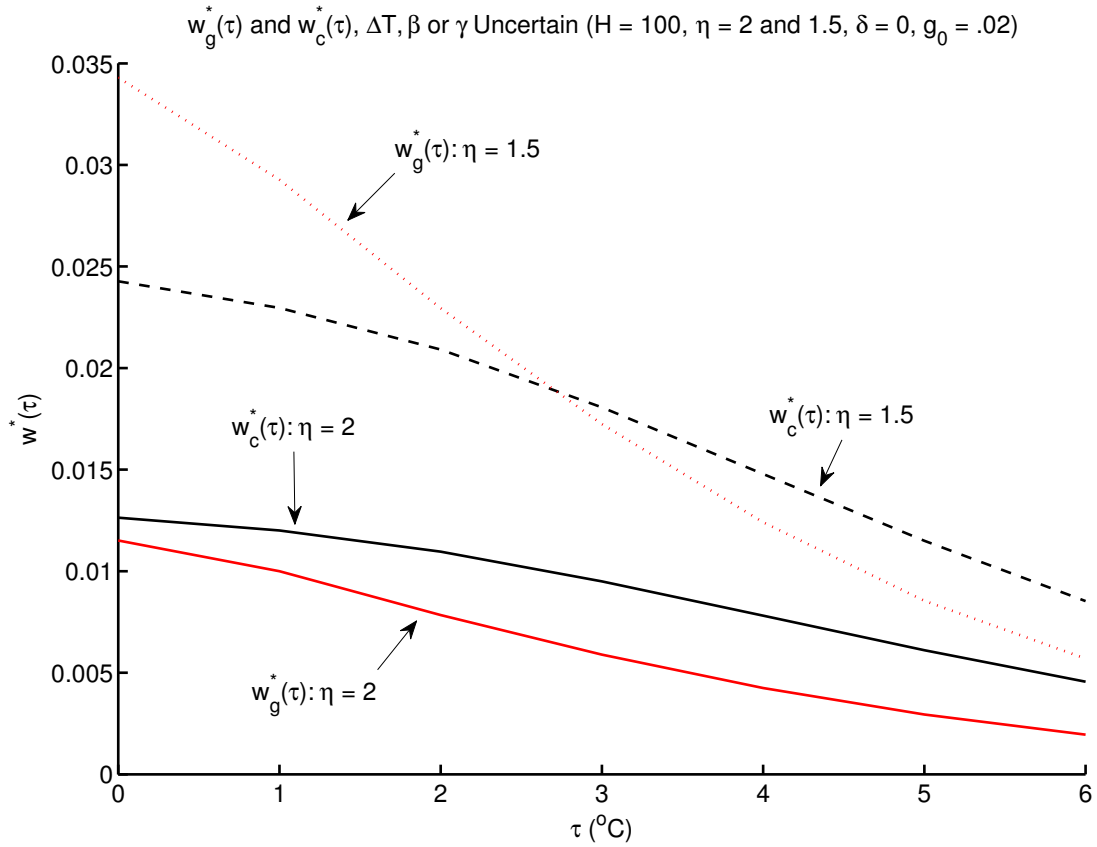
Figure 6: WTP, Both ΔT and γ Uncertain. $\eta = 2$ and 1.5, $g_0 = .020$ and $\delta = 0$
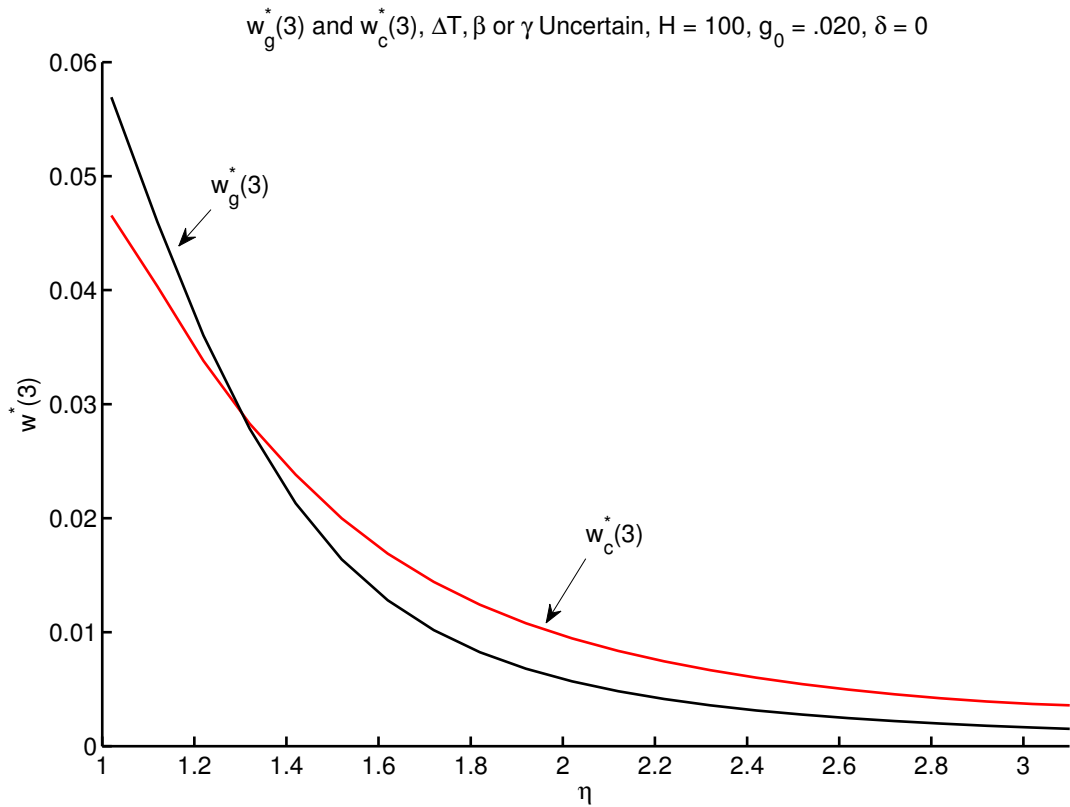
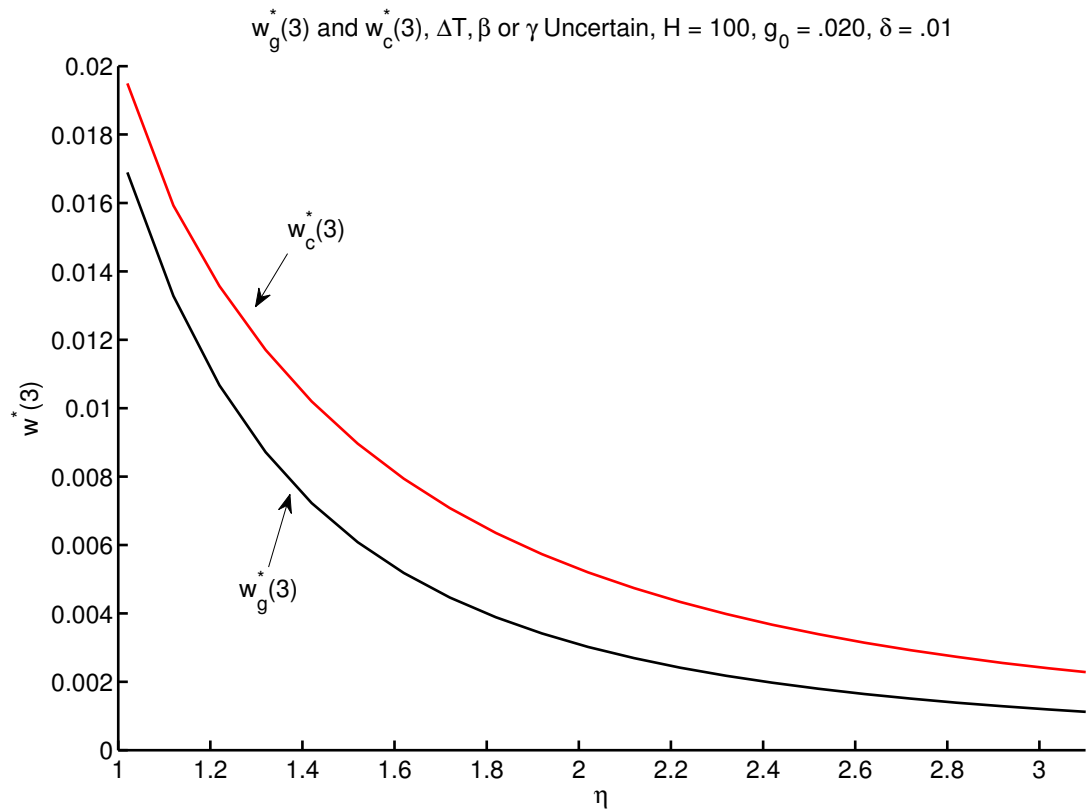Figure 7: WTP Versus η for $\tau = 3$. $g_0 = .020$ and $\delta = 0$

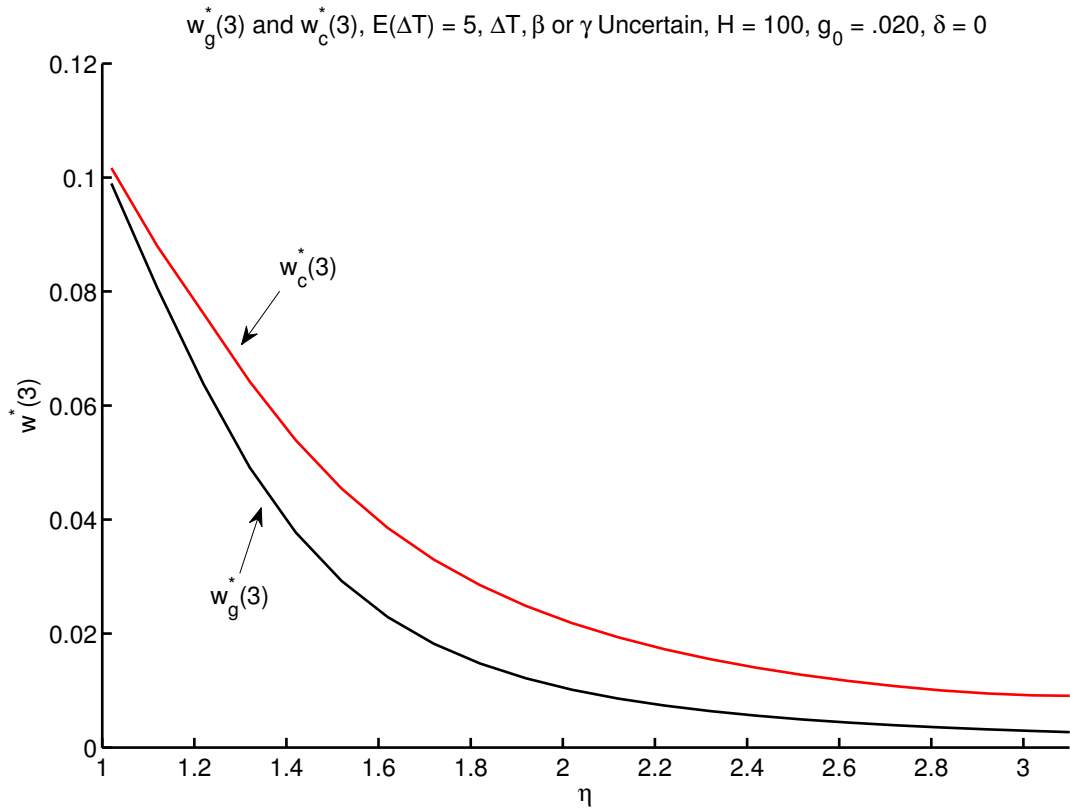Figure 8: WTP Versus η for $\tau = 3$. $g_0 = .020$ and $\delta = .01$

Figure 9: WTP Versus η for $\tau = 3$. $\mathcal{E}(\Delta T_H) = 5°$C, $g_0 = .020$, $\delta = 0$
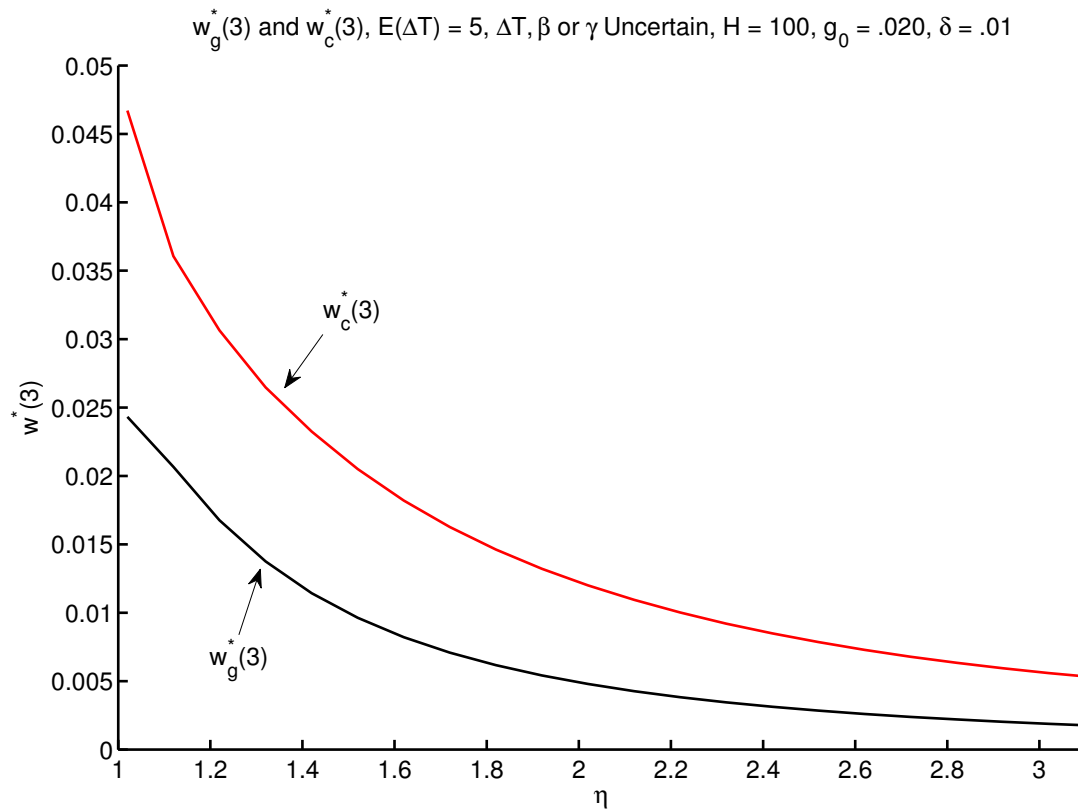
Figure 10: WTP Versus η for $\tau = 3$. $\mathcal{E}(\Delta T_H) = 5°\text{C}$, $g_0 = .020$, $\delta = .01$