FIRE SALES IN A MODEL OF COMPLEXITY

Ricardo J. Caballero
Alp Simsek

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ABSTRACT

Financial assets provide return and liquidity services to their holders. However, during severe financial crises many asset prices plummet, destroying their liquidity provision function at the worst possible time. In this paper we present a model of fire sales and market breakdowns, and of the financial amplification mechanism that follows from them. The distinctive feature of our model is the central role played by endogenous complexity: As asset prices implode, more "banks" within the financial network become distressed, which increases each (non-distressed) bank's likelihood of being hit by an indirect shock. As this happens, banks face an increasingly complex environment since they need to understand more and more interlinkages in making their financial decisions. This complexity brings about confusion and uncertainty, which makes relatively healthy banks, and hence potential asset buyers, reluctant to buy since they now fear becoming embroiled in a cascade they do not control or understand. The liquidity of the market quickly vanishes and a financial crisis ensues. The model exhibits a powerful "complexity-externality." As a potential asset buyer chooses to pull back, the size of the cascade grows, which increases the degree of complexity of the environment. This rise in perceived complexity induces other healthy banks to pull back, which exacerbates the fire sale and the cascade.

Ricardo J. Caballero
MIT
Department of Economics
Room E52-373a
Cambridge, MA 02142-1347
and NBER
caball@mit.edu

Alp Simsek
MIT
alpstein@MIT.EDU

1 Introduction

Financial assets provide return and liquidity services to their holders. However, during severe financial crises many asset prices plummet, destroying their liquidity provision function at the worst possible time. These fire sales are at the core of the amplification mechanism observed in severe financial crises: Large amounts of distressed asset sales depress asset prices, which exacerbates financial distress, leading to further asset sales, and the downward spiral goes on.

There are many instances of these dramatic fire sales and the chaos they trigger in recent financial history. As explained by Treasury Secretary Paulson and Fed Chairman Bernanke to Congress in an emergency meeting soon after Lehman's collapse, the main goal of the TARP during the subprime crisis as initially proposed was, precisely, to put a floor on the price of the assets held by financial firms in order to contain the sharp contractionary feedback loop triggered by the confusion and panic caused by Lehman's demise. And a few years back, after the LTCM intervention, then Fed Chairman Greenspan wrote in his congressional testimony of October 1, 1998:

> "Quickly unwinding a complicated portfolio that contains exposure to all manner of risks, such as that of LTCM, in such market conditions amounts to conducting a fire sale. The prices received in a time of stress do not reflect longer-run potential, adding to the losses incurred... ...a fire sale may be sufficiently intense and widespread that it seriously distorts markets and elevates uncertainty enough to impair the overall functioning of the economy. Sophisticated economic systems cannot thrive in such an atmosphere."

In this paper we propose a model of fire sales that builds on the idea that complexity, a feature strongly disliked by investors during downturns for the uncertainty it generates, rises endogenously during crises: As asset prices implode, more financial institutions (banks, for short) within the financial network may go under, which increases each bank's likelihood of being hit by an indirect shock from counterparty risk. This means that banks face an increasingly complex environment since they need to understand more and more interlinkages in making their financial decisions. Thus, perceived uncertainty rises with complexity and makes relatively healthy banks reluctant to buy since they now fear becoming embroiled in the cascade themselves, and no reasonable amount of research can rule out this option in the time available. The liquidity of the market quickly vanishes and a financial crisis ensues.

In Caballero and Simsek (2009) we show the basic interaction between the size of the equilibrium cascade in a financial network and the degree of complexity of the portfolio problem facing banks. In that model banks have bilateral linkages in order to insure each other against local liquidity shocks. The whole financial system is a complex network of linkages which functions smoothly in the environments for which it is designed, even though no bank knows with certainty all the many possible connections within the network (that is, each bank knows the identities of the other banks but not their exposures). However, these linkages may also be the source of contagion when an unexpected event of financial distress arises somewhere in the network. During normal times, banks only need to understand the financial health of their neighbors, which they can learn at low cost. In contrast, when a significant problem arises in parts of the network and the possibility of cascades arises, the number of nodes to be audited by each bank rises since it is possible that the shock may spread to the bank's counterparties. Eventually, the problem becomes too complex for them to fully figure out, which means that banks now face significant uncertainty and they react to it by retrenching into a liquidity-conservation mode.

This structure is the starting point of this paper. Our focus here is on secondary markets and on the feedback between that environment and the fire sales that arise in these markets. In particular, this paper introduces a secondary market in which banks in distress can sell their legacy assets to meet the surprise liquidity shock. The natural buyers of the legacy assets are other banks in the financial network, which may also receive an indirect hit. When the surprise shock is small, cascades are short and buyers can inspect their neighbors to rule out an indirect hit. In this case, buyers purchase the distressed banks' legacy assets at their "fair" prices (which reflect the fundamental value of the assets). In contrast, when the surprise shock is large, longer cascades become possible which increases the complexity of the environment, and buyers cannot rule out an indirect hit. As a precautionary measure, they hoard liquidity and disengage from trades in (now illiquid) legacy assets and may even turn into sellers. The price of legacy assets plummets to "fire-sale" levels.

A central aspect of our model is the dependence of the cascade size on the price of legacy assets, which leads to multiple equilibria for intermediate levels of the surprise shock. When legacy assets fetch a fair price in the secondary market, the banks in distress have access to more liquidity and thus the surprise shock is contained after fewer banks are bankrupt, leading to a relatively simple environment. When the environment is simple, the natural buyers rule out an indirect hit and demand legacy assets, which ensures that these assets trade at their fair prices. Set against this benign scenario is the possibility of a *fire-sale equilibrium* where the price of legacy assets collapses to fire-sale levels, which

leads to a longer cascade and a greater level of complexity. As the level of complexity increases, natural buyers become worried about an indirect hit and they hoard liquidity and/or sell their own legacy assets, which reinforces the collapse of asset prices.

This amplification mechanism is exacerbated by a *complexity-externality*. As a potential asset buyer chooses to pull back, the size of the potential cascade grows and with it the degree of complexity of the environment (each bank needs to explore larger segments of the network to understand the risk it is exposed to). This rise in complexity induces other healthy banks to pull back as a precautionary measure, which further exacerbates the fire sale and cascade.[1]

Our framework has two additional (and more standard) externalities: A network-liquidity externality that stems from the interlinkages of the financial system; when a bank chooses to raise external liquidity rather than to generate it from its own resources, it spreads the distress to other banks. And a fire-sale externality that arises from the negative effects that a bank's distressed asset sales have on other banks' balance sheets. These two externalities are present in many network and liquidity models. They interact but are distinct from the complexity externality that we highlight, which stems from the feature that any decision that lengthens the potential cascade, increases the complexity of the environments that other banks need to consider.

This paper is related to several strands of literature. In the canonical model of fire sales, these happen because during financial crises the natural buyers of the assets (other banks) also experience financial distress (cf. Shleifer and Vishny (1992,1997)). More recently, Brunnermeier and Pedersen (2008) show that even if some potential buyers are not distressed or constrained, these may choose not to arbitrage the fire sale in the short run because they anticipate a better deal in the future. Our model lies somewhere in between these two views: Most potential buyers are unconstrained, as in Brunnermeier and Pedersen (2008), but they are confused and hence fearful of going about their normal arbitrage role (and in this sense they are distressed as in Shleifer and Vishny (1992)). It is the complexity of the environment that sidelines potential buyers.

There is an extensive literature that highlights the possibility of network failures and contagion in financial markets. An incomplete list includes Allen and Gale (2000), Lagunoff and Schreft (2000), Rochet and Tirole (1996), Freixas, Parigi and Rochet (2000), Leitner (2005), Eisenberg and Noe (2001), and Cifuentes, Ferucci and Shin (2005) (see

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[1]Although we do not pursue this avenue in the paper, complexity probably plays a role in creating the incompleteness needed for a pecuniary externality to arise. If agents could sign contracts contingent on the events that follow a cascade, then the externality would be greatly reduced. However, as potential cascades lengthen, the number of contingencies that need to be written into contracts grow exponentially.

Allen and Babus (2008) for a recent survey). These papers focus mainly on the mechanisms by which solvency and liquidity shocks may cascade through the financial network. In contrast, we take these phenomena as the reason for the rise in the *complexity* of the environment in which banks make their decisions, and focus on the effect of this complexity on banks' prudential actions. It is also worth pointing out that the complexity mechanism we emphasize in this paper is operational *even for relatively small amounts of contagion.* The contagion literature is sometimes criticized because it is hard to believe that many financial institutions would be caught up in a cascade of bankruptcies.[2] That is, even if there is a cascade, it is reasonable to expect that it would eventually be contained (especially since banks take precautionary actions to fight the cascade). But as this paper illustrates, even *partial cascades* can have large aggregate effects, since they greatly increase the complexity of the environment.[3]

Our paper is also related to the literature on flight-to-quality and Knightian uncertainty in financial markets, as in Caballero and Krishnamurthy (2008), Routledge and Zin (2004), and Easley and O'Hara (2005); and to the related literature that investigates the effect of new events and innovations in financial markets, e.g. Liu, Pan, and Wang (2005), Brock and Manski (2008), and Simsek (2009). Our contribution relative to these literatures is in endogenizing the rise in uncertainty from the behavior of the financial network itself. More broadly, this paper belongs to an extensive literature on flight-to-quality and financial crises that highlights the connection between panics and a decline in the financial system's ability to channel resources to the real economy (see, e.g., Caballero and Kurlat (2008), for a survey).

The organization of this paper is as follows. In Section 2 we describe the financial network and the secondary market for assets, and we introduce a surprise shock (a rare event) in the network. We also discuss a benchmark case without complexity effects (because banks can understand the network at no cost). Section 3 contains our main results. There, banks have only local knowledge about the financial network, and a sufficiently large surprise shock increases the complexity of the environment and leads to

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[2]See, e.g.,Brunnermeier, Crockett, Goodhart, Persaud, an Shin (2009) which argue that the *domino model of financial contagion* is not useful for understanding financial contagion in a modern financial system since ".... It is only with implausibly large shocks that the simulations (of their model) generate any meaningful contagion. The reason is that the domino model paints a picture of passive financial institutions who stand by and do nothing as the sequence of defaults unfolds. In practice, however, they will take actions in reaction to unfolding events, and in anticipation of impending defaults..."

[3]The role of cascades in elevating complexity was also highlighted in Haldane's (2009) speech, who nicely captures the mechanism when he wrote that at times of stress "knowing your ultimate counter-party's risk becomes like solving a high-dimension Sudoku puzzle."

a breakdown in secondary markets. This section also highlights the dependence of the level of complexity on asset prices and demonstrates the possibility of multiple equilibria. In Section 4 we describe the three externalities in our setup and analyze their role in our main results. The paper concludes with a final remarks section and several appendices.

2 Equilibrium without Complexity

In this section, we describe the basic environment and characterize the equilibrium for a benchmark case in which financial institutions (banks, for short) have full knowledge of the interlinkages between all the banks in the financial network. In this benchmark we show that if the network is deep (i.e., there is a large number of banks) secondary markets do not break down and the financial network is resilient to a perturbation. That is, the size of financial cascades is contained and aggregate loan contraction is limited. These relatively benign results contrast with those we obtain in the next section once we introduce complexity.

2.1 The Environment

We consider an economy with three dates $\{0, 1, 2\}$ in which a single good (one dollar) can be kept in liquid reserves or it can be loaned to production firms. If kept in liquid reserves, a unit of the good yields one unit in the next date. Instead, if a unit is loaned to firms, it then yields $R > 1$ units at date 2. These new loans are completely illiquid.

The economy has n continuums of banks denoted by $\{b^j\}_{j=1}^n$. Each of these continuums is composed of identical banks and, for simplicity, we refer to each continuum b^j as *bank* b^j, which is our unit of analysis.[4] Each bank has initial assets which consist of $1 - y$ units of legacy loans, z units of demand deposits in one other bank, and y units of *flexible reserves* set aside for making new loans at date 0. The bank's liabilities consist of a unit measure of demand deposits held by retail depositors and z units of demand deposits held by one other bank. The demand deposit contracts in this economy pay 1 unit at date 1 (resp. R units at date 2) if the depositor arrives early (resp. late). There are no liquidity driven depositors. I.e. as long as the bank is solvent, all retail depositors would rather arrive at date 2. Note that a bank that loans all of its flexible reserves has just enough assets to pay its depositors at date 2. At date 0 (only) there is a secondary market for legacy loans and the (equilibrium) price for these loans is $r < 1$ (see below). The central trade-off in this economy will be whether the bank uses its flexible reserves to make new

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[4]The only reason for the continuum is for banks to take other banks' decisions as given.

loans and to purchase legacy loans in the secondary market, or whether it hoards some of this liquidity in response to a rare event (that we describe below).

The banks' cross demand deposit holdings form a *financial network* denoted by:

$$\mathbf{b}\left(\rho\right) = \left(b^{\rho(1)} \rightarrow b^{\rho(2)} \rightarrow b^{\rho(3)} \rightarrow \dots \rightarrow b^{\rho(n)} \rightarrow b^{\rho(1)}\right) \tag{1}$$

where $\rho : \{1, .., n\} \rightarrow \{1, .., n\}$ is a permutation that assigns bank $b^{\rho(i)}$ to *slot i* in the financial network. The arc $\rightarrow$ denotes that the bank in slot i (i.e., bank $b^{\rho(i)}$) has z units of demand deposits in the bank in the subsequent slot $i+1$, and slot n has slot 1 as a forward neighbor.[5] A central feature of the model is the banks' uncertainty about the financial network in (1), but we will shut down this ingredient until the next section. For now, all banks know all the financial linkages (cross-deposits) in the network.

At date 0, the banks learn that a *rare event* has happened and one bank, $b^{\bar{j}}$, will become distressed. Similar to Allen and Gale (2000), in order to remain solvent *this bank needs to make θ dollars of payment (to an outsider) at date 1.*

These losses might spill over to other banks via the financial network and may bring them into financial distress at date 1. To prepare for date 1, at date 0 the banks take one of the following actions $A_0^j = \{HS, H, B\}$, which are restricted to a discrete choice set for simplicity (see Caballero and Simsek (2009) for a related model with unrestricted action space). As the most extreme precautionary measure, the bank may choose $A_0^j = HS$, to *hoard* all of its flexible reserves y as liquidity and *sell* all of its legacy loans $1 - y$ in the secondary market, keeping a completely liquid balance sheet. As a less extreme measure, the bank may choose $A_0^j = H$, to hoard its flexible reserves as liquidity and to keep its legacy loans on its balance sheet. Alternatively, the bank may choose $A_0^j = B$, to keep its own legacy loans on its balance sheet and to be a potential *buyer* of loans. That is, this bank uses its flexible reserves either to make new loans or to buy legacy loans in the secondary market (whichever is more profitable). At date 1, the bank chooses whether to *keep* or *withdraw* its deposits on the forward neighbor bank, $A_1^j \in \{K, W\}$.

Given the rare event, a bank may not be able to pay its depositors the contracted amount $(1, R)$ (despite the precautionary measures it takes), but instead it ends up paying some $\left(q_1^j, q_2^j\right)$ to all depositors. Banks know that there are no liquidity driven retail depositors, thus a bank which is able to pay its depositors at least q_1^j at date 2 can refuse to pay the retail depositors if they arrive early. With this assumption, the continuation equilibrium for bank b^j at date 1 takes one of two forms. Either the bank is *solvent,* pays

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[5]In Caballero and Simsek (2009), we motivate the formation of the financial network for its role in facilitating bilateral liquidity insurance, as in Allen and Gale (2000).

$q_1^j = 1, q_2^j \geq 1$, and the retail depositors withdraw at date 2; or the bank is *insolvent,* pays $q_1^j < 1, q_2^j = 0$, and all depositors draw their deposits at date 1.

The bank chooses actions $\left(A_0^j, A_1^j\right)$ to maximize q_1^j until it can meet its liquidity obligations to depositors, that is, until $q_1^j = 1$. Increasing q_1^j beyond 1 has no benefit for the bank, thus once it satisfies its liquidity obligations, it then tries to maximize the return to late depositors q_2^j. Note that the banks make their decisions while facing uncertainty about the network structure. For expositional simplicity we assume that banks are infinitely risk averse (rather than just risk averse) with respect to the financial network, i.e. they evaluate their decisions according to the worst possible network realization which they find plausible.

Secondary Market for Legacy Loans

Legacy loans are traded in a centralized exchange that opens at date 0. Given the loan price r, the banks that choose $A^j = HS$ sell all of their legacy loans ($1 - y$ units for each bank) while the banks that choose $A^j = B$ are potential buyers of legacy loans and may spend up to y (their flexible reserves). There is no demand or supply for legacy loans outside of the network.

A legacy loan traded in the secondary market is held by a bank different than its originator, and it yields returns $R - \delta < R$ at date 2. We assume $\delta > 0$ which simplifies the subsequent analysis, but the economic results generalize also to the case $\delta = 0$.[6] Note that there is an upper bound on endogenous loan prices. In particular, potential buyers of legacy loans give up flexible reserves which they could also use to make new loans, thus each loan has an opportunity cost of R units at date 2. Since secondary loans return $R - \delta$ units at date 2, no potential buyer would demand legacy loans at a price greater than

$$r_{fair} \equiv (R - \delta)/R.$$

Legacy and new loans can be freely disposed of at the scrap value $r_{fire} \in (0, r_{fair})$, so r_{fire} is a lower bound on prices.

If $r < r_{fair}$, potential buyers spend all of their flexible reserves y on legacy loans, while if $r = r_{fair}$, they are indifferent between buying legacy loans and making new loans.

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[6]In other words, even though banks get returns slightly lower than R from legacy loans purchased in the secondary market, they are the natural buyers of these loans in the sense of Shleifer and Vishny (1992).
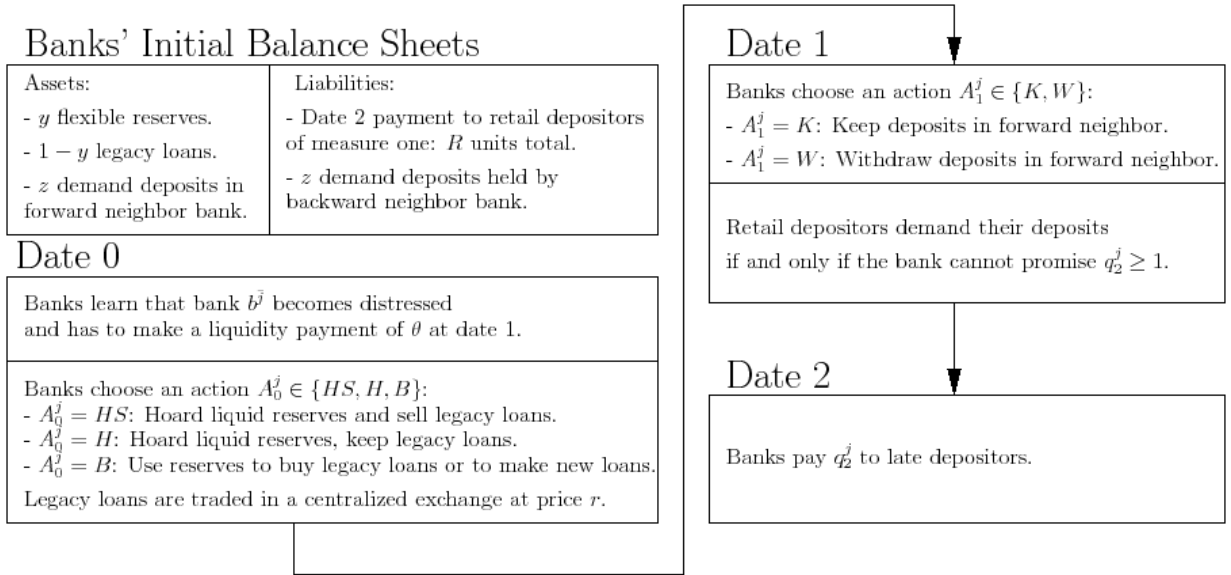
Banks' Initial Balance Sheets		Date 1

Banks' Initial Balance Sheets

Assets:	Liabilities:
- y flexible reserves.	- Date 2 payment to retail depositors of measure one: R units total.
- $1 - y$ legacy loans.	
- z demand deposits in forward neighbor bank.	- z demand deposits held by backward neighbor bank.

Date 0

Banks learn that bank $b^{\bar{j}}$ becomes distressed and has to make a liquidity payment of θ at date 1.

Banks choose an action $A_0^j \in \{HS, H, B\}$:
- $A_0^j = HS$: Hoard liquid reserves and sell legacy loans.
- $A_0^j = H$: Hoard liquid reserves, keep legacy loans.
- $A_0^j = B$: Use reserves to buy legacy loans or to make new loans.

Legacy loans are traded in a centralized exchange at price r.

Date 1

Banks choose an action $A_1^j \in \{K, W\}$:
- $A_1^j = K$: Keep deposits in forward neighbor.
- $A_1^j = W$: Withdraw deposits in forward neighbor.

Retail depositors demand their deposits if and only if the bank cannot promise $q_2^j \geq 1$.

Date 2

Banks pay q_2^j to late depositors.

Figure 1: Timeline of events.

Thus, the market clearing condition for legacy loans can be written as

$$(1 - y) \sum_j \mathbf{1}\left\{A^j = HS\right\} - \frac{y}{r} \sum_j \mathbf{1}\left\{A^j = B\right\} \quad \begin{cases} \geq 0 \text{ if } r = r_{fire} \\ = 0 \text{ if } r \in \left(r_{fire}, r_{fair}\right) \\ \leq 0 \text{ if } r = r_{fair} \end{cases} \quad . \quad (2)$$

The first term on the left hand side denotes the total supply of loans while the second term denotes the maximum potential demand. If the left hand side of Eq. (2) is negative for each $r \in [r_{fire}, r_{fair}]$, then legacy loans trade at their fair value r_{fair}, potential buyers are indifferent between buying legacy loans and selling new loans, and they buy just enough legacy loans to clear the market. If the left hand side of Eq. (2) is 0 for some $r \in [r_{fire}, r_{fair}]$, then r is the equilibrium price. If the left hand side is positive for each $r \in [r_{fire}, r_{fair}]$, then there is excess supply of loans and the price is given by the scrap value r_{fire} (note that this happens only if there are no potential buyers). We refer to this situation as a *breakdown* in the legacy loan market.

Equilibrium

Figure 1 recaps the timeline of events in this economy. We can now define equilibrium:

Definition 1. *An equilibrium is a collection of bank actions and payments* $\left[\left\{A_0^j\left(\rho\right), A_1^j\left(\rho\right)\right\}_j, \left\{q_1^j\left(\rho\right), q_2^j\left(\rho\right)\right\}_j\right]_{\mathbf{b}(\rho)}$ *and a price level* $r \in [r_{fire}, r_{fair}]$ *for legacy loans*

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such that, given the realization of the financial network $\mathbf{b}(\rho)$ *and the rare event, each bank* b^j *chooses its actions according to the worst case financial network that it finds plausible, the legacy loan market clears (cf. Eq. (2)), and the retail depositors withdraw deposits early if and only if* $q_2^j(\rho) < 1$.

We next turn to the characterization of this equilibrium. Let $\bar{\imath} \in \{1, .., n\}$ denote the slot of the distressed bank $b^{\bar{\jmath}=\rho(\bar{\imath})}$. Note that, for each financial network $\mathbf{b}(\rho)$ and for each bank b^j, there exists a unique $k \in \{0, .., n-1\}$ such that

$$j = \rho(\bar{\imath} - k),$$

which we define as the *distance* of bank b^j from the distressed bank.[7] The distance k is the payoff relevant information for a bank b^j and will play a key role in the analysis.[8]

2.2 The Benchmark without Complexity

We characterize the equilibrium in two steps: We start by describing the banks' actions and payoffs for a given price level of legacy loans, r; and then solve for the equilibrium price using the legacy loan market clearing condition (2).

Suppose the loan prices are fixed at some $r \in [r_{fire}, r_{fair}]$ and consider the banks' optimal actions and payments in this setting. Consider a distressed bank that needs to obtain liquidity at date 1 (e.g. the original distressed bank $b^{\rho(\bar{\imath})}$). This bank could try to obtain the required liquidity either by withdrawing its cross deposits at date 1 (i.e. by choosing $A_1^j = W$) and/or by taking a precautionary action at date 0 (i.e. by choosing $A_0^j \in \{H, HS\}$). It can be checked that the bank always weakly prefers ex-post liquidity withdrawal to the ex-ante precautionary actions, thus we fix banks' liquidity pecking order as follows:

Assumption (LPO). The liquidity pecking order is such that a bank that will need at least z units of liquidity at date 1 first chooses $A_1^j = W$, and then (if there is need) resorts to ex-ante precautionary measures.

Under assumption (LPO) and the parametric condition $\theta \geq z$, the original distressed bank $b^{\rho(\bar{\imath})}$, which has to make a liquidity payment of θ at date 1, withdraws its deposits from the forward neighbor bank $b^{\rho(\bar{\imath}+1)}$. This puts bank $b^{\rho(\bar{\imath}+1)}$ also in need of z units of liquidity, which also withdraws its deposits on the forward neighbor bank. As in Allen

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[7]We use modulo n arithmetic for the slot index i, e.g. $\bar{\imath} - k = -1$ represents the slot $n-1$.

[8]Note that the definition of equilibrium has built in a conservative feature which characterizes financial markets in distress: Banks make their precautionary decisions based on the worst outcome they find plausible. This feature plays no role in the benchmark since banks know the actual financial network.

and Gale (2000), this triggers further withdrawals until, in equilibrium, all cross deposits are withdrawn, i.e. $A_1^j = W$ for all j. In particular, a bank in need of liquidity (including the original distressed bank $b^{\rho(\bar{\iota})}$) tries, but cannot, obtain any net liquidity through cross withdrawals. Anticipating that it will not be able to obtain additional liquidity at date 1, the distressed banks try to obtain some liquidity by taking precautionary actions at date 0.

To simplify the characterization of the banks' date 0 actions, we assume

$$y + (1 - y) \, r_{fair} < 1 < (1 - y) \, R. \tag{3}$$

The left hand side of this condition is not new, as it follows from $\delta > 0$. It ensures that a bank that sells its legacy loans (i.e. that chooses the most precautionary action $A_0^j = HS$) does not have enough resources to promise at least 1 unit to its depositors at date 2, and thus it will be insolvent at date 1. The right hand side of this condition ensures that a bank that keeps its legacy loans on its book (i.e. that chooses $A_0^j \in \{H, B\}$) can always promise its depositors at least 1 unit at date 2. Under condition (3), a bank that expects to make a net liquidity payment at date 1 first considers hoarding liquidity, i.e. the action $A_0^j = H$. If the bank's expected loss at date 1 is less than its flexible reserves y, then it chooses $A_0^j = H$ and averts insolvency. If the bank's expected loss is greater than y, then the bank expects to be insolvent at date 1 even if it chooses $A_0^j = H$. Hence, this bank believes that it cannot avoid insolvency and thus it chooses $A^j = HS$ to improve its liquidation outcome. We refer to y as the bank's *buffer*, since this is the maximum level of losses the bank can sustain while remaining solvent.

We next conjecture that, under appropriate parametric conditions (including condition (3)), there exists a threshold $K(r) \in \{1, .., n - 2\}$, which depends on loan prices, such that all banks with distance $k \leq K(r) - 1$ are insolvent (there are $K(r)$ such banks) while the banks with distance $k \geq K(r)$ remain solvent. That is, the crisis will partially cascade through the network but will be contained after $K(r) \leq n - 2$ banks have failed. We refer to $K(r)$ as the *cascade size*.

The original distressed bank, $b^{\rho(\bar{\iota})}$, is insolvent as long as its required payment is greater than its buffer y (and greater than z so that assumption (LPO) applies), i.e. if and only if

$$\theta > \max(y, z). \tag{4}$$

Suppose this is the case so bank $b^{\rho(\bar{\iota})}$ is *insolvent*. Anticipating insolvency, this bank

chooses $A_0^{\rho(\bar{\imath})} = HS$, all retail depositors will arrive early at date 1, and the bank will pay

$$q_1^{\rho(\bar{\imath})} = \frac{y + r(1 - y) - \theta + z}{1 + z} < 1$$

to each depositor (where the inequality holds in view of condition (3)). Note that the bank will have $y + r(1 - y)$ units of liquidity at date 1, it will make a payment of θ, and it will receive z units in cross-deposits from its forward neighbor bank $b^{\rho(\bar{\imath}+1)}$(which is solvent in our conjectured equilibrium and thus will pay 1 unit for each unit of deposit).

Consider next the bank $b^{\rho(\bar{\imath}-1)}$ with distance 1 from the distressed bank. To remain solvent, this bank needs to pay 1 on its deposits to bank $b^{\rho(\bar{\imath}-2)}$ but it receives only $q_1^{\rho(\bar{\imath})} < 1$ on its deposits from the distressed bank $b^{\rho(\bar{\imath})}$, so it loses $z\left(1 - q_1^{\rho(\bar{\imath})}\right)$ in cross-deposits. Hence, bank $b^{\rho(\bar{\imath}-1)}$ will also become insolvent if (and only if) its losses from cross-deposits are greater than its buffer, $z\left(1 - q_1^{\rho(\bar{\imath})}\right) > y$, which can be rewritten as

$$q_1^{\rho(\bar{\imath})} < 1 - \frac{y}{z}.$$

If this condition fails, the only insolvent bank is the original distressed bank and the cascade size is $K(r) = 1$. If this condition holds, then bank $b^{\rho(\bar{\imath}-1)}$ anticipates insolvency and chooses $A_0^{\rho(\bar{\imath}-1)} = HS$, and it will pay its depositors

$$q_1^{\rho(\bar{\imath}-1)} = f\left(q_1^{\rho(\bar{\imath})}\right) \equiv \frac{y + r(1 - y) + zq^{\rho(\bar{\imath})}}{1 + z}. \tag{5}$$

From this point onwards, a pattern emerges. The payment by an insolvent bank $b^{\rho(\bar{\imath}-k)}$ (with $k \geq 1$) is given by

$$q_1^{\rho(\bar{\imath}-k)} = f\left(q_1^{\rho(\bar{\imath}-(k-1))}\right)$$

and this bank's backward neighbor $b^{\rho(\bar{\imath}-(k+1))}$ is also insolvent if and only if $q_1^{\rho(\bar{\imath}-k)} < 1 - \frac{y}{z}$. Hence, the payments by insolvent banks converge to the fixed point of the function $f(.)$, and if

$$y + r(1 - y) > 1 - \frac{y}{z}, \tag{6}$$

then there exists a unique $K(r) \geq 2$ such that

$$q_1^{\rho(\bar{\imath}-k)} < 1 - \frac{y}{z} \text{ for each } k \in \{0, .., K(r) - 2\} \tag{7}$$

$$\text{and } q_1^{\rho(\bar{\imath}-(K(r)-1))} \geq 1 - \frac{y}{z}.$$

If $n - 2$ is greater than the solution, $K(r)$, to this equation, i.e. if

$$n \geq K(r) + 2, \tag{8}$$

then, Eq. (7) shows that (in addition to the trigger-distressed bank $b^{\rho(\bar{\imath})}$) all banks $b^{\rho(\bar{\imath}-k)}$ with distance $k \in \{1,..,K(r)-1\}$ are insolvent since their losses from cross deposits are greater than their corresponding buffers. These banks choose $A_0^j = HS$ to improve their liquidation outcome. In contrast, bank $b^{\rho(\bar{\imath}-K(r))}$ is solvent, since it can meet its losses by hoarding liquidity, i.e. this bank chooses $A_0^{\rho(\bar{\imath}-K(r))} = H$. Since bank $b^{\rho(\bar{\imath}-K(r))}$ is solvent, all banks $b^{\rho(\bar{\imath}-k)}$ with distance $k \in \{K(r)+1,..,n-1\}$ are also solvent since they do not incur losses in cross-deposits. These banks are potential buyers of legacy loans, i.e. they choose $A_0^j = B$. This verifies our conjecture for a partial cascade of size $K(r)$ under conditions (3), (4), (6), and (8).

Note also that Eqs. (5) and (7) imply that $K(r)$ is decreasing in r: with a higher loan price, the liquidation value of each bank is greater, thus the crisis is contained after a smaller number of insolvencies. For future reference, we strengthen condition (6) to

$$y + r_{fire}(1-y) > 1 - \frac{y}{z}, \tag{9}$$

so that there exists a partial cascade for any $r \in [r_{fire}, r_{fair}]$. The next lemma summarizes the above discussion.

Lemma 1. *Suppose information is free so that banks know the financial network $\mathbf{b}(\rho)$, that the loan prices are exogenously fixed at $r \in [r_{fire}, r_{fair}]$, that assumption (LPO) holds, and that conditions (3), (4), (8), and (9) are satisfied. Then, there exists a partial cascade of size $K(r)$, where $K(r)$ is defined by Eq. (7). Banks with distance from the distressed bank $k \leq K(r) - 1$ are insolvent and they choose $A_0^j = HS$. The transition bank with distance $K(r)$ averts insolvency by choosing $A_0^j = H$. The remaining banks with distance $k \geq K(r) + 1$ are solvent and they choose $A_0^j = B$.*

Figure 2 displays the equilibrium cascade size $K(r)$ when the loan prices are exogenously fixed at some $r \in [r_{fire}, r_{fair}]$ for a particular parameterization of the model, illustrating that $K(r)$ is decreasing in r. The negative dependence of the cascade size on the price of loans will play an important role in the next section in which we consider the equilibrium with endogenous complexity

We next consider the legacy loan market clearing condition and solve for the equilibrium level of prices in the free-information benchmark. We claim that if n is sufficiently large, the endogenous loan price in the free-information benchmark is given by $r = r_{fair}$.

To see this, note that the insolvent banks (there are $K(r)$ of them) choose $A_0^j = HS$
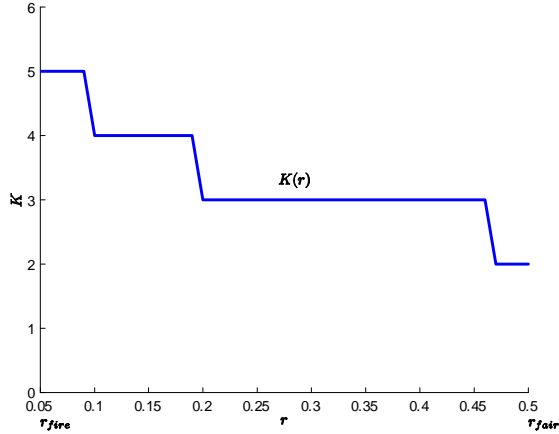
Figure 2: **Cascade size in the free-information benchmark.** The plot displays the cascade size $K(r)$ when the loan prices are exogenously fixed at $r \in [0, r_{fair}]$ (for a given level of θ).

and sell all of their existing loans, hence the aggregate supply of loans is given by $K(r)(1-y)$. The transition bank $b^{\rho(\bar{\imath}-K(r))}$ chooses to hoard liquidity, while the remaining solvent banks $\left\{ b^{\rho(\bar{\imath}-k)} \right\}_{k=K(r)+1}^{n-1}$ (there are $n - K(r) - 1$ of them) choose $A_0^j = B$, i.e. they are potential buyers of loans. Suppose n is sufficiently large so that

$$(n - K(r) - 1)\, y > r(1-y)K(r) \ \text{ for all } r \in [r_{fire}, r_{fair}]. \tag{10}$$

Under this condition, the demand from the potential buyers exceed the supply of loans for any price level $r < r_{fair}$, thus the loan market clearing condition (2) implies that $r = r_{fair}$.

Intuitively, if the cascade is only partial and banks know the financial network, then there exist banks which will remain solvent and know that much. These banks do not hoard liquidity and are ready to use their flexible reserves to purchase loans from distressed banks. When there are sufficiently many banks, the demand from these solvent banks is enough to absorb the supply from the distressed banks, ensuring that the secondary loans are traded at their fair price r_{fair}. We refer to condition (10) as the *deep secondary market assumption*. The next proposition summarizes the above discussion and characterizes the symmetric equilibrium for the free-information benchmark.

Proposition 1. *Suppose information is free so that banks know the financial network* $\mathbf{b}(\rho)$. *Suppose assumption (LPO) holds, conditions* (3), (4), (8), *and* (9) *are satisfied, and the deep secondary market assumption* (10) *holds. Then:*
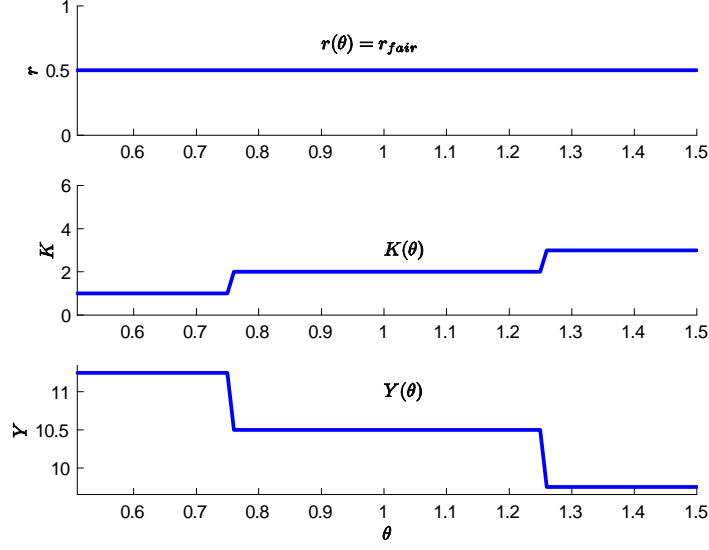
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Figure 3: **Equilibrium in the free-information benchmark.** The top, the middle, and the bottom panels respectively plot the loan prices, the cascade size, and the aggregate level of new loans as a function of the losses in the originating bank.

(i) The unique equilibrium price is given by $r = r_{fair}$.

(ii) For the continuation equilibrium (at date 1): All banks choose $A_1^j = W$. Banks with distance $k \leq K(r_{fair}) - 1$ are insolvent while the banks with distance $k \geq K(r_{fair})$ are solvent.

(iii) For the ex-ante equilibrium (at date 0): The insolvent banks choose $A_0^j = HS$, the transition bank with distance $K(r_{fair})$ chooses $A_0^j = H$, and all other solvent banks (i.e. banks with distance $k \geq K(r_{fair}) + 1$) choose $A_0^j = B$.

(iv) For the loan market: The insolvent banks sell all of their loans in the secondary market, while the solvent banks with distance $k \geq K(r_{fair}) + 1$ are indifferent between buying legacy loans and making new loans. These banks spend a portion of their flexible reserves to buy the legacy loans sold by the insolvent banks and they make new loans with the rest of their reserves. The aggregate level of new loans made in this economy is given by:

$$\mathcal{Y} = (n - K(r_{fair}) - 1)\, y - K(r_{fair})\,(1 - y)\, r_{fair}. \tag{11}$$

Discussion. Figure 3 displays the equilibrium in the benchmark economy for particular parameterization of the model. The figure demonstrates that the loan prices are fixed at r_{fair}, the cascade size is increasing in θ, and the aggregate level of new loans is decreasing

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in θ. Intuitively, as θ increases, there are more losses to be contained, which increases the spread of the insolvency. As the insolvency spreads, more banks hoard their flexible reserves as liquidity instead of making new loans, which lowers $\mathcal{Y}$. Note, however, that $\mathcal{Y}$ decreases "smoothly" with θ. These results offer a benchmark for the next section. There we show that once auditing becomes costly, both K and $\mathcal{Y}$ may experience large changes with small increases in θ.

3 Endogenous Complexity and Fire Sales

Let us repeat the steps of the previous section but now with our key assumption: *Banks only have local knowledge about the financial network* $\mathbf{b}\left(\rho\right)$.

Each bank observes its two forward neighbors but is otherwise uncertain about how the remaining banks are allocated to the remaining financial slots (cf. Eq. (1)). In particular, we assume this uncertainty takes a simple form: Banks with distance $k \leq 2$ know their distances from the distressed bank, and banks with distance $k \geq 3$ are uncertain about their distances and assign a positive probability to all distances $\tilde{k} \in \{3, 4, .., n-1\}$.

In this context, when the shock is small, the system behaves exactly as in the benchmark. But when the shock is large, banks need to understand distant (complex) linkages in order to assess the amount of counterparty risk they are facing. Their inability to figure out these complex linkages triggers a set of precautionary actions which overturns the relatively benign implications of the benchmark environment.

As in the benchmark, the original distressed bank $b^{\rho(\bar{\imath})}$ withdraws its deposits from the forward neighbor bank, which triggers further withdrawals until, in equilibrium, all cross deposits are withdrawn, i.e. $A_1^j = W$ for all j. Since the distressed bank knows that it cannot obtain any net liquidity at date 1, it tries to obtain liquidity at date 0. Under condition (4), it also knows that it cannot avoid insolvency, thus it chooses $A_0^{\rho(\bar{\imath})} = HS$.

Next consider a bank b^j with distance $k > 0$, and note that a *sufficient statistic* for this bank to choose action $A_0^j \in \{HS, H, B\}$ is *the amount it expects to receive in equilibrium from its forward neighbor.* In other words, to decide on the level of its precautionary measure, this bank only needs to know whether (and how much) it will lose in cross-deposits. Formally, if the bank chooses A_0^j at date 0 and its forward neighbor pays x at date 1, then this bank's payment can be written as a function $\left(q_1\left[A_0^j, x\right], q_2\left[A_0^j, x\right]\right)$. However, the bank chooses A_0^j while facing uncertainty about the financial network, and consequently about x. Note also that $q_1\left[A_0^j, x\right]$ and $q_2\left[A_0^j, x\right]$ are increasing in x for any choice of action. That is, the bank's payment is increasing in the amount it receives from

its forward neighbor regardless of the ex-ante precautionary measure it takes. Thus, since the bank is infinitely risk averse, it will choose its precautionary action *as if it will receive from its forward neighbor the lowest possible payment x*.

To characterize the bank's optimal action further, we define a useful notion of equilibrium. We say that the equilibrium allocation is *distance based and monotonic* if the banks' equilibrium payments can be written as an increasing function of their distance from the distressed bank. That is, there exists weakly increasing payment functions $Q_1, Q_2 : \{0, .., n-1\} \to \mathbb{R}$ such that

$$\left(q_1^{\rho(i-k)}(\rho), q_2^{\rho(i-k)}(\rho) \right) = (Q_1[k], Q_2[k])$$

for all $\mathbf{b}(\rho)$ and k. We conjecture (and verify in Appendix A.1) that the equilibrium is distance based and monotonic.

Under this conjecture, consider again a bank b^j with distance $k > 0$, and note that the payment of the bank's forward neighbor can be written as $x = Q_1[k-1]$. Then, the bank's uncertainty about the forward neighbor's payment $x = Q_1[k-1]$ reduces to its uncertainty about the forward neighbor's distance $k-1$, which is equal to one less than its own distance k. If the bank knows its distance to the distressed bank, i.e. if $k \in \{1, 2\}$, then it chooses its optimal action $A_0^j \in \{HS, H, B\}$ knowing that it will receive $x = Q_1[k-1]$ from its forward neighbor. On the other hand, if the bank is uncertain about its distance, i.e. if $k \geq 3$, then it assigns a positive probability to all distances $\tilde{k} \in \{3, ..., n-1\}$. Moreover, since $Q_1[.]$ is an increasing function, the payment $Q_1\left[\tilde{k}-1\right]$ is minimal for the distance $\tilde{k} = 3$. Hence a bank b^j with distance $k \geq 3$ chooses $A_0^j \in \{HS, H, B\}$ as if it will receive $x = Q_1[2]$ from its forward neighbor. In words, *the banks that are uncertain about their distances to the distressed bank choose their precautionary action as if they are closer to the distressed bank than they actually are*.

Formally, our next lemma establishes that all banks with distance $k \geq 3$ choose the action that the bank with distance $\tilde{k} = 3$ would choose if the information was freely available (characterized in Lemma 1). If the free-information cascade size is given by $K(r) = 3$, so that the bank with distance $\tilde{k} = 3$ would take the precautionary action $A_0^3 = H$ to avert insolvency, then all banks with $k \geq 3$ also take the precautionary action $A_0^j = H$ even though ex-post they end up not needing liquidity. If the free-information cascade size is even greater (i.e. if $K(r) \geq 4$) so that the bank with distance $\tilde{k} = 3$ would be insolvent and choose the most precautionary action $A_0^3 = HS$, then all banks with $k \geq 3$ fold by taking the most precautionary action $A_0^j = HS$, which ensures that their

insolvency (even though some of these banks would be solvent in the free-information economy). The proof of the following lemma is relegated to Appendix A.1 since most of the intuition is provided by the above discussion.

Lemma 2. *Consider the setup of Lemma 1, but now information is costly rather than free, so that banks know only their two forward neighbors and they are otherwise uncertain about the financial network $\mathbf{b}(\rho)$. Let $K(r)$ denote the cascade size that would obtain if information was freely available (characterized in Lemma 1).*

Then the equilibrium is distance based and monotonic. Each bank b^j with distance $k \in \{0, 1, 2\}$ chooses the same action that it would choose in the free-information economy (characterized in Lemma 1). Each bank b^j with distance $k \in \{3, .., n-1\}$ chooses the same action that the bank with distance 3 would choose in the free-information economy. In particular, there are three cases to consider, depending on the cascade size:

If $K(r) \leq 2$, then the crisis in the free-information economy would not cascade to bank with distance 3, which would choose $A_0^j = B$. Thus, each bank b^j with distance $k \in \{3, .., n-1\}$ chooses $A_0^j = B$, and the equilibrium actions and payments are identical to the free-information economy described in Lemma 1. In particular, the cascade size is given by $\hat{K}(r) = K(r)$.

If $K(r) = 3$, then the crisis in the free-information economy would cascade to and stop at bank with distance 3, which would choose $A_0^j = H$ to avert insolvency. Thus each bank b^j with distance $k \in \{3, .., n-1\}$ chooses $A_0^j = H$. The equilibrium cascade size is still the same as the free-information economy, i.e. $\hat{K}(r) = K(r)$, but the banks' actions and payments are different.

If $K(r) \geq 4$, then bank with distance 3 would be insolvent in the free-information economy and would choose $A_0^j = HS$. Thus, all banks choose $A_0^j = HS$ and they are all insolvent, i.e. the cascade size is given by $\hat{K}(r) = n$.

Note that the loan trade decisions of the banks with distance $k \geq 3$ depend on the free-information cascade size $K(r)$. In particular, if $K(r) \leq 2$, these banks are potential buyers in the loan market, while if $K(r) \geq 3$, they are either neutral in the loan market (if $K(r) = 3$) or they are potential sellers (if $K(r) \geq 4$). This dependence of the banks' loan trades on the free-information cascade size $K(r)$ (which itself depends on the loan price) plays a key role in subsequent analysis, where we endogenize the loan prices and complete the characterization of the equilibrium.

We next solve the equilibrium level of loan prices in the costly information setting and present our main results. There are three cases to consider depending on the cascade size $K(r)$ over the price range $r \in [r_{fire}, r_{fair}]$.

Case (i). If $K(r_{fire}) \leq 2$, then we conjecture that there is a unique symmetric equilibrium, that we refer to as the *fair-price equilibrium,* in which loan markets endogenously clear and loans trade at their fair price $r = r_{fair}$. To verify this conjecture, recall that $K(r)$ is a decreasing function so that $K(r) \leq K(r_{fire}) \leq 2$ for all $r \in [r_{fire}, r_{fair}]$. Hence, regardless of the endogenous price r, Lemma 2 implies that all banks with $k \geq 3$ choose $A_0^j = B$, and as such, these banks are potential buyers of loans. Thus, under the deep secondary market assumption (10), the unique equilibrium price is given by $r = r_{fair}$. Once the equilibrium price is determined, the remaining equilibrium allocations are uniquely determined as described in Lemma 2, proving our conjecture for this case.

Case (ii). If $K(r_{fair}) \geq 3$, we conjecture that there is a unique symmetric equilibrium, which we refer to as the *fire-sale equilibrium,* in which there is a breakdown in the loan market and $r = r_{fire}$. To see this, note that $3 \leq K(r_{fair}) \leq K(r)$ for all $r \in [r_{fire}, r_{fair}]$, thus Lemma 2 implies that all banks with $k \geq 3$ choose either $A_0^j = H$ (if $K(r) = 3$), or $A_0^j = HS$ (if $K(r) \geq 4$). In words, these banks are either neutral in the legacy loan market or they are sellers, in particular, they are *not* potential buyers. Using Lemma 2 once more, note that the banks with distance $k \leq 2$ are necessarily insolvent, so they choose $A_0^j = HS$ and become sellers in the legacy loan market. Thus, the legacy loan market features at least 3 sellers but no buyers. The market clearing condition (2) implies that $r = r_{fire}$, i.e. there is a breakdown in the secondary loan market.

Case (iii). If $K(r_{fair}) \leq 2 < 3 \leq K(r_{fire})$, we conjecture that there are two stable equilibria: one *fair-price equilibrium* and one *fire-sale equilibrium.* To see this, first suppose that the price of loans is given by $r = r_{fair}$ so that the cascade size satisfies $K(r) \leq 2$. Then, the analysis for case (i) applies unchanged, in particular, all banks with distance $k \geq 3$ are potential buyers and the price r_{fair} clears the market, verifying that there is a fair-price equilibrium. Next suppose that the price of loans is given by $r = r_{fire}$ so that the cascade size satisfies $K(r_{fire}) \geq 3$. Then, the analysis for case (ii) applies unchanged, in particular, all banks with distance $k \geq 3$ are either neutral in the loan market or they sell their loans, and loan prices collapse to $r = r_{fire}$, verifying that there is a fire-sale equilibrium.

We summarize these results in the following, and main, proposition.

Proposition 2. *Consider the setup of Proposition 1 with the difference here being that banks only have local understanding of the network, so that banks know only their two forward neighbors and they are otherwise uncertain about the financial network $\mathbf{b}(\rho)$.*

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Let $K(r)$ denote the cascade size in the free-information economy with price level $r \in [r_{fire}, r_{fair}]$ (characterized in Lemma 1).

(i) Fair-price equilibrium: If $K(r_{fire}) \leq 2$, there is a unique equilibrium in which the loan market clearing condition (2) is satisfied with equality and loans trade at their fair price $r = r_{fair}$.

The cascade size is the same as the free-information benchmark, i.e. $\hat{K} = K(r_{fair}) \leq 2$. The insolvent banks (with distance $k \leq \hat{K} - 1$) choose $A_0^j = HS$ and sell their loans, while the solvent banks with distance $k \geq \hat{K} + 1$ choose $A_0^j = B$ and are indifferent between using their reserves to make new loans or to buy legacy loans in the secondary market. The aggregate level of new loans is equal to the benchmark Eq. (11).

(ii) Fire sale equilibrium: If $K(r_{fair}) \geq 3$, then there is a unique equilibrium in which there is a breakdown in the secondary loan market, i.e. there is an excess supply of loans and $r = r_{fire}$.

(ii.1) If $K(r_{fire}) = 3$, all banks with distance $k \geq 3$ choose $A_0^j = H$. These banks remain solvent and the cascade size is $\hat{K} = K(r_{fire})$.

(ii.2) If $K(r_{fire}) \geq 4$, all banks with distance $k \geq 3$ choose $A_0^j = HS$. These banks go under and the cascade size in this case is $\hat{K} = n$.

In either sub-case, all flexible reserves are hoarded as liquidity and there are no new loans, $\mathcal{Y} = 0$.

(iii) Multiple equilibria: If $K(r_{fair}) \leq 2 < 3 \leq K(r_{fire})$, there are two stable equilibria.

In the fair-price equilibrium, loans trade at their fair price $r = r_{fair}$, the cascade size is $\hat{K} = K(r_{fair}) \leq 2$, the solvent banks are indifferent between making new loans or purchasing loans in the secondary market, and the aggregate level of new loans $\mathcal{Y}$ is given by the benchmark Eq. (11).

In the fire sale equilibrium, $r = r_{fire}$, the cascade size is given by either $\hat{K} = 3$ (if $K(r_{fire}) = 3$) or $\hat{K} = n$ (if $K(r_{fire}) \geq 4$), all banks hoard liquidity and (if $K(r_{fire}) \geq 4$) dump their loans in the secondary market, and there are no new loans, $\mathcal{Y} = 0$.

Discussion. Figure 4 displays the equilibria with network uncertainty for a particular parameterization of the model. The top panel reproduces the cascade size $K(r)$ in the free-information benchmark as a function of the losses in the originating bank θ, when loan prices are fixed at $r = r_{fire}$ and $r = r_{fair}$. The remaining three panels display the equilibria with network uncertainty, illustrating the characterization in Proposition 2. Note that there is a unique equilibrium for small and large levels of θ, however, there are multiple equilibria for intermediate levels of θ.
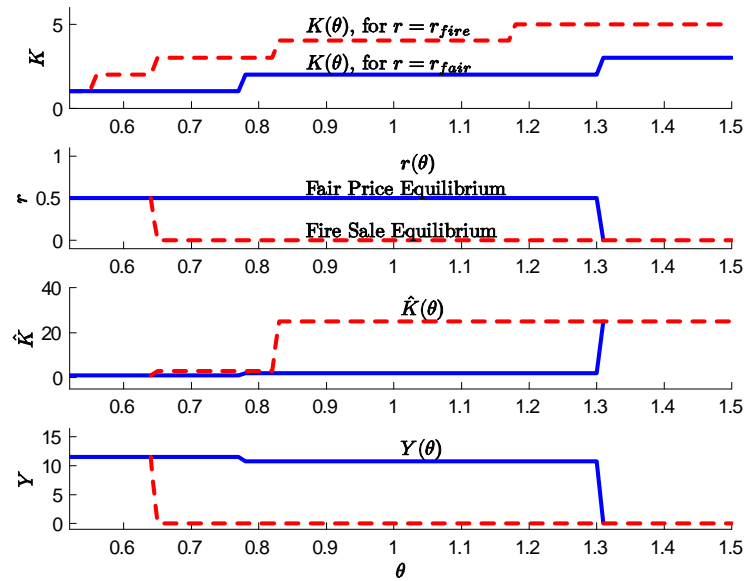
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Figure 4: **Equilibria with network uncertainty.** The top panel plots the cascade size $K(r)$ in the free-information benchmark as a function of the losses in the originating bank θ, when loan prices are fixed at $r \in \{0, r_{fair}\}$. The remaining three panels display the equilibria with network uncertainty: They respectively plot the loan prices, the cascade size and the aggregate level of new loans as a function of θ, for both the fair-price and the fire-sale equilibria.

When θ is sufficiently small so that $K(r_{fire}) \leq 2$, i.e. when the free-information benchmark features a short cascade even for the price level $r = r_{fire}$, there is a unique fair price equilibrium. This equilibrium features a low level of complexity thus the banks that are uncertain about their distance to the distressed bank can rule out an indirect hit. Consequently, these banks use their reserves to make new loans and to demand assets. The aggregate level of new loans is the same as in the free-information benchmark, and assets trade at their fair prices.

In contrast, when θ is sufficiently large so that $K(r_{fair}) \geq 3$, i.e. when the cascade size in the free-information benchmark is sufficiently large, there is a unique fire-sale equilibrium. In this equilibrium, the aggregate level of new loans makes a very large and discontinuous drop to zero. That is, when the losses (measuring the severity of the initial shock) are beyond a threshold, the cascade size becomes so large that banks are unable to tell whether they are connected to the distressed bank or not. All uncertain banks act as if they are closer to the distressed bank than they actually are, hoarding much more liquidity than in the free-information benchmark and leading to a severe credit crunch episode. Moreover, these banks, some of which would be potential buyers in the free-information benchmark, become sellers and this leads to a collapse in asset prices. This result provides a rationale for the collapse of asset prices in an environment in which complexity suddenly (and endogenously) rises. Furthermore, it is possible (if $K(r_{fire}) \geq 4$) that the uncertain banks panic so much that they take an extremely precautionary action to increase their liquidation outcome. However, ex-post, this action ensures that the uncertain banks become insolvent.

When θ is in an intermediate range, the cascade size is manageable if price of loans is high (i.e. $K(r_{fair}) \leq 2$), however it becomes unmanageable if loans trade at the fire-sale price (i.e. $K(r_{fire}) \geq 3$). In this case, the interaction between asset prices and the endogenous level of complexity generates multiple equilibria.

In the fair-price equilibrium, loans trade at a higher price and the cascade size is relatively small, which reduces the level of complexity. With the lower level of complexity, the banks that are uncertain about their distance to the distressed bank become potential buyers of loans, which ensures that loans trade at the higher price and that the cascade is shorter.

Set against this benign scenario is the possibility of a fire-sale equilibrium, in which the price of loans collapses and there is a longer cascade, which increases the level of complexity. As the level of complexity increases, banks that are uncertain about their distances panic and sell their loans, which reinforces the collapse of loan prices.

Note also that, whenever there are multiple equilibria, the fair-price equilibrium Pareto

dominates the fire-sale equilibrium for all banks and their depositors. Intuitively, the fire-sale equilibrium entails two distinct social costs: fewer new loans and more banks become insolvent. In the next section, we identify the negative externalities in our setup that account for these social costs.

4 Three Externalities

In this section, we discuss the various externalities present in our setup and we highlight the role they play in our main results. Our model features three distinct externalities, which we call the *network-liquidity* externality, the *fire-sale* externality and the *complexity* externality. The first of these emerges directly from the interlinkages between banks. The second externality stems from the interaction of loan prices and the endogenous determination of bank runs. These two externalities are relatively standard. Models of interconnected banks typically feature externalities akin to our network-liquidity externality (see, for example, Rotemberg (2009)), while models based on the Diamond and Dybvig (1983) setup typically lead to fire-sale externalities whenever prices affect the likelihood of bank runs (see, for example, Allen and Gale (2005)). The *complexity externality* is novel to our analysis, and it emerges from the interaction of loan prices and the complexity of the financial network.

4.1 Network-liquidity Externality

In our setup, a distressed bank can obtain liquidity through two distinct sources: It can either withdraw its deposits on the forward neighbor bank, or it can hoard its liquid reserves (or it can sell its legacy loans, as a last measure). Since aggregate liquidity is scarce, a bank that chooses the former option puts its forward neighbor bank also in distress, imposing a *liquidity externality* on it. In particular, if the bank decides to withdraw its cross deposits, the forward neighbor bank scrambles for liquidity and faces insolvency if it is unable to meet this liquidity demand. A distressed bank in our setup always (weakly) prefers to withdraw deposits (cf. assumption (LBO)).[9] In Appendix A.2, we study the polar opposite case, enforced by the government, in which a bank that needs liquidity first hoards its own liquid reserves, and thus avoids inflicting a liquidity externality whenever it can. Our analysis shows that internalizing the liquidity externalities in this fashion

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[9]The bank's preference for liquidity withdrawal is strict if the forward neighbor pays $q_2^j < R$. In the more general model analyzed in Caballero and Simsek (2009), the bank always strictly prefers to withdraw deposits and thus always inflicts a liquidity externality.

leads to shorter cascades in aggregate (see Proposition 3 in the appendix).

Intuitively, the scramble for liquidity is like a hot potato which the banks can either pass straight to their neighbors, or which they can cool down a bit using their resources before passing it on. When all banks pass it without cooling it, the hot potato eventually reaches a *vulnerable* bank, i.e. a bank which is sufficiently close to the original distressed bank. This bank cannot pass the potato to its neighbor, which is already bankrupt. Moreover, the resources of this bank alone are not sufficient to cool down the potato hence the bank burns (i.e. it cannot meet the liquidity demand and becomes insolvent), which lengthens the cascade. In contrast, when each bank cools down the potato before passing it on, then the potato is cold before it reaches the vulnerable bank, leading to a much shorter cascade.

4.2 Fire Sale Externality

Consider a bank that decides to sell some loans, leading to a small decline in loan prices. This action has a small positive effect on the net budgets of the banks that buy legacy loans, while it has a small negative effect on the net budgets of the banks that sell legacy loans. Absent further effects, the welfare impacts of these budget changes would typically "net out," which is the content of the first welfare theorem. However, a small decline also increases the likelihood of a bank run in our setup, leading to a further decline in the welfare of these banks and their depositors. Consequently, the effects of price changes do not necessarily "net out," and loan prices feature a negative pecuniary externality.

To formalize this point, let us consider the benchmark economy analyzed in Section 2.2. This setting shuts down the complexity externality (since the banks know the financial network) which we will analyze in the next subsection.

Recall that the cascade size $K(r)$ is decreasing in r: with a lower loan price, the liquidation payment of each bank is greater (cf. Eq. (5)), thus the crisis is contained after a smaller number of insolvencies (cf. Eq. (7)). In particular, there exists some $\bar{r} \in [r_{fire}, r_{fair}]$ such that, for any $\varepsilon > 0$ we have

$$K(\bar{r} - \varepsilon/2) = K(\bar{r} + \varepsilon/2) + 1.$$

That is, the cascade size increases by one in response to an arbitrarily small decrease in loan prices. The small price drop from $\bar{r} + \varepsilon/2$ to $\bar{r} - \varepsilon/2$ leads to the insolvency of one more bank, inflicting a discrete negative effect on the welfare of this bank and its backward neighbor, while it has a continuous effect on the welfare of other banks. Hence,

for sufficiently small ε, the net welfare effect of the price change from $\bar{r} + \varepsilon/2$ to $\bar{r} - \varepsilon/2$ is negative, demonstrating the fire sale externality.[10]

The general intuition behind this externality is that a drop in asset prices lowers the liquidation value of an insolvent bank, which increases the probability of a bank run in the counterparties of this bank.

4.3 Complexity Externality

Consider next the model analyzed in Section 3 in which banks only have local information about the financial network. Similar to the above analysis for the fire sale externality, a bank that sells a loan has a small negative impact on loan prices, which in turn increases the length of the cascade. However, in this case, longer cascades also increase the *complexity* of the financial network. Since banks are averse to complexity (which we model as infinite risk aversion with respect to the financial network), an increase in complexity leads to a welfare reduction for banks that are uncertain about the financial network.

To formalize this point, consider the set of parameters that lead to multiple equilibria in Proposition 2, $K(r_{fair}) \leq 2 < 3 \leq K(r_{fire})$. Then, there exists $\bar{r} \in [r_{fire}, r_{fair}]$ such that, for any $\varepsilon > 0$ we have

$$K(\bar{r} - \varepsilon/2) = 3 > K(\bar{r} + \varepsilon/2) = 2.$$

That is, small decline in loan prices makes the cascade size in the free-information benchmark exceed the critical threshold of 2. Recall that the banks with distance $k \in \{3, .., n-1\}$ mimic the bank with distance 3 in the free-information benchmark. Then, when the loans trade at price $\bar{r} + \varepsilon/2$, these banks know that they will not lose anything in cross deposits, thus they do not take any precaution and they pay out at least $\left(q_1^j = 1, q_2^j = R\right)$ to their depositors (and perhaps more, if they can acquire legacy loans at a discount).

In contrast, when the loans trade at price $\bar{r} - \varepsilon/2$, the banks with distance $k \in \{3, .., n-1\}$ are worried that they might suffer an indirect hit from the cascade. Since these banks are infinitely risk averse, their welfare is greatly reduced. Moreover, these banks hoard liquidity to precaution against the worst case scenario, and they end up paying $\left(q_1^j = 1, q_2^j < R\right)$ to their depositors. Hence, a small drop in prices inflicts a discrete

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[10]The discrete nature of the fire sale externality is due to our modeling assumptions, in particular, our assumption that the good equilibrium is selected whenever there are multiple equilibria for bank runs. However, the externality would also be present in other variants of the model, as long as the probability of a bank run is increasing in the date 1 losses (i.e. the severity of the liquidity shock) of the bank.

negative effect on all of the banks with distance $k \in \{3, .., n-1\}$ and their depositors, demonstrating the complexity externality.

The complexity externality may also lead to multiple Pareto-ranked equilibria in our setup, as we have already seen in Proposition 2. In particular, an increase in the level of complexity due to a reduction in the price of assets not only lowers the welfare of many banks and their depositors, but also induces these banks to take extreme precautionary measures, which, in some instances, includes further asset sales. The sale of assets by banks in panic mode reduces asset prices further, which leads to a vicious cycle culminating in the fire-sale equilibrium. In contrast, an increase in asset prices reduces the complexity externality, which may mitigate the precautionary measures and turn more sellers into buyers, leading to a virtuous spiral towards the fair price equilibrium.

Finally, note that the complexity externality is potentially much more potent than the fire sale externality analyzed in the previous subsection and highlighted in the literature. The reason is that the fire-sale externality affects banks that are on the verge of insolvency. In contrast, the complexity externality affects all banks that are uncertain about the financial network, which, in practice, includes virtually all financial institutions. The greater scope of the complexity externality also leads to widespread (precautionary) actions, which has the potential to create aggregate effects, price changes, and multiple equilibria.

5   Conclusion

In this paper we provide a model that illustrates how fire sales and complexity can trigger a very powerful negative feedback loop within a financial network. More severe fire sales lengthen the potential cascades, and raise the complexity of the environment exponentially. This triggers confusion among potential asset buyers, which pull back and exacerbate the fire sale. In extreme scenarios these potential buyers can turn into sellers, leading to a complete collapse in secondary markets.

In our model, the distressed institutions' motive for fire sales is to improve their outcome in the worst case scenario of insolvency. This is an extreme assumption made only to simplify the exposition. In reality, the distressed institutions may sell assets for a multitude of reasons that do not imply anticipating insolvency. E.g. they may be forced to do so by regulatory requirements, or they may hope to avoid a run by obtaining more liquidity. These reasons can be incorporated into our framework while preserving our main conclusions.

Having said this, the particular insolvency motive we consider also raises the question of what would happen if the distressed institutions chose to gamble for resurrection by not selling their assets, which would improve their outcome in good states at the cost of a greater bankruptcy risk. Our model suggests that gambling for resurrection may be a mixed blessing for the aggregate. Gambling by potential buyers, that is, institutions that are far from the cascade but that do not know this, would limit the fire sales and the downward spiral of prices. On the other hand, gambling by institutions near the cascade would increase the cascade size and trigger the complexity mechanism.

We did not explore policy questions, but it is apparent that our environment creates many policy opportunities during crises. Supporting secondary markets, insulating financial institutions from counterparty risk, and stress testing (increasing transparency and supporting if needed) systemically important financial institutions, are all policies practiced during the current financial crisis and supported by our framework.

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A   Appendix

A.1   Proofs Omitted in the Main Text

Proof of Lemma 2

Case (i): $K(r) \leq 2$.   To prove that the conjectured actions and payments constitute an equilibrium, first note that the original distressed bank optimally chooses $A_0^{\rho(\bar{\imath})} = HS$ and pays out the same level $Q_1[0]$ that it would pay in the free-information economy (since it receives the full amount z from its forward neighbor, $b^{\rho(\bar{\imath}+1)}$, which is solvent in the conjectured equilibrium). Next consider a bank with distance $k \in \{1, 2\}$. Under the conjectured equilibrium, this bank knows that it will receive $x = Q_1[k-1]$ from its forward neighbor, which is equal to what it would receive in the free-information economy. Hence, it optimally chooses the same action it would choose when information is free.

Consider next a bank with distance $k \geq 3$. This bank is uncertain about its distance, thus it chooses its date 0 action as if it will receive $x = Q_1[2]$ from its forward neighbor. Under the conjectured equilibrium, the cascade size is $K(r) \leq 2$, thus we have $Q_1[2] = 1$. Since the bank makes no losses in cross deposits (even in the worst case scenario of $\tilde{k} = 3$), it optimally chooses $A_0^j = B$, verifying the optimality of the conjectured action.

Since the banks' actions are the same as the free-information economy, their payments are also identical to that case, which verifies (by Lemma 1) that the equilibrium is distance

based and monotonic.

Case (ii): $K(r) = 3$. Similar to the previous case, the original distressed bank and the banks with distance $k \in \{1, 2\}$ optimally choose the same action and make the same payments as in the free-information economy.

Consider next a bank with distance $k \geq 3$. This bank is uncertain about its distance, thus it chooses its date 0 action as if it will receive $x = Q_1[2]$ from its forward neighbor. Since $K(r) = 3$, under the conjectured equilibrium, we have $Q_1[2]$ (which is equal to its level in the free-information benchmark) lies in the interval $(1 - y/z, 1)$. Hence, the bank with distance $k \geq 3$ expects (with some probability) to make losses less than its buffer, thus it chooses $A_0^j = H$ to counter these losses, verifying the optimality of its conjectured action.

These banks with distance $k \geq 3$ are solvent, but they pay potentially less than the free-information economy. In particular, the bank with distance $k = 3$ pays the same amount $(Q_1[3], Q_2[3])$ that it would pay in the free-information case, while the banks with distance $k \geq 4$ end up paying

$$Q_1[k] = 1 \text{ and } Q_2[k] = y + (1 - y) R \in (1, R),$$

where $Q_2[k] \geq 1$ follows from Eq. (3). It follows also that the equilibrium is distance based and monotonic.

Case (iii): $K(r) \geq 4$. In this case, note that the banks' payments, given the conjectured actions, are the solutions to the following system of equations:

$$Q_1[0] = \frac{y + (1 - y) r - \theta + z Q_1[n - 1]}{1 + z} \tag{12}$$

$$Q_1[k] = \frac{y + (1 - y) r + z Q_1[k - 1]}{1 + z}, \text{ for } k \in \{1, .., n - 1\}.$$

Note that, by condition (3), the solution to this system satisfies $Q_1[k] < 1$ for all k, verifying that the banks are indeed insolvent given their actions. The system in Eq. (12) also verifies that the equilibrium allocation is distance based and monotonic.

To verify the optimality of banks' actions, first note that Eq. (12) implies the payments $\{Q_1[k]\}_{k=0}^{K(r)-1}$ are lower than what they would be in the free-information economy, because the forward neighbor of the original distressed bank pays $Q_1[n - 1] < 1$ in this case (while it would be solvent and would pay 1 unit per deposit in the free-information economy). Consider a bank with distance $k \in \{1, 2\}$, and recall that this bank chooses its date 0

action knowing that it will receive $Q_1 [k-1]$ from its forward neighbor. Since $K(r) \geq 4$, this bank would be insolvent in the free-information economy. In the present case, $Q_1 [k-1]$ is even lower than what it would be in the free-information economy. Thus the bank with distance $k \in \{1, 2\}$ is necessarily insolvent, and it optimally chooses $A_0^j = HS$.

Consider next a bank with distance $k \geq 3$. This bank is uncertain about its distance and thus it chooses its date 0 action as if it will receive $x = Q_1 [2]$ from its forward neighbor. Since $K(r) \geq 4$, and since the payments $\{Q_1 [k]\}_{k=0}^{K(r)-1}$ are even lower than the free-information economy, we necessarily have $Q_1 [2] < 1 - y/z$. In other words, the bank with distance $k \geq 3$ believes that it might experience losses beyond its buffer and go bankrupt. Consequently, it chooses the most extreme precautionary action $A_0^j = HS$ to improve their liquidation outcome, verifying the optimality of the conjectured action.

A.2 Equilibrium without Liquidity Externalities

In the main text, the bank always considers deposit withdrawal as the first source of liquidity (cf. assumption (LBO)), thus imposing a liquidity externality on its forward neighbor bank. To clarify the role of liquidity externalities, in this appendix we analyze the equilibrium under a government policy that prevents banks from withdrawing cross deposits unless they have used all of their liquid reserves.

To facilitate the analysis, it helps to consider a slightly greater action space for banks:

$$\left(A_0^j, A_1^j \right) \in \{H (\tilde{y}), HS, B\} \times \{W (\tilde{z}), K\}, \tag{13}$$

where $A_0^j = H (\tilde{y})$ denotes that the bank hoards $\tilde{y} \in (0, y]$ units of its flexible reserves, and $A_1^j = W (\tilde{z})$ denotes that the bank withdraws $\tilde{z} \in (0, z]$ units of its deposits on the forward neighbor bank. In other words, we allow the banks to *partially* hoard their flexible reserves and to *partially* withdraw their cross deposits. Next, we introduce the main assumption of this appendix: the government enforces the *reverse* of the liquidity pecking order in assumption (LPO) so that banks *do not* impose a liquidity externality on their forward neighbors, whenever they can avoid doing so.

Assumption (LPO-R). Consider a bank that needs liquidity at date 1 and whose forward neighbor bank is solvent. The government imposed liquidity pecking order for this bank is such that the bank first considers hoarding its flexible reserves, i.e. it considers the action $A_0^j = H (\tilde{y})$. The bank withdraws some of its cross deposits (chooses $A_1^j = W (\tilde{z})$) only if $A_0^j = H (y)$ is not sufficient to meet its liquidity demand.

Finally, we strengthen the right hand side of condition (3) to

$$1 + z < (1 - y) R, \tag{14}$$

so that a bank's own legacy loans are enough to promise all depositors (including the backward neighbor bank) at least 1 unit at date 2.

Under these assumptions and when n is sufficiently large, we conjecture that there is an equilibrium in which all banks with distance $k \geq 1$ (i.e. all banks except potentially the original distressed bank) are solvent. In other words, the cascade size is at most 1, in stark contrast with the equilibria characterized in Propositions 1 and 2.

In the conjectured equilibrium, the original bank $b^{\rho(\bar{\imath})}$ hoards all of its liquid reserves and also withdraws its deposits on bank $b^{\rho(\bar{\imath}+1)}$, i.e. it chooses $A_0^{\rho(\bar{\imath})} = HS$ and $A_1^{\rho(\bar{\imath})} = W(z)$, which puts bank $b^{\rho(\bar{\imath}+1)}$ in need of z units of liquidity. Under the new assumption for the liquidity pecking order, bank $b^{\rho(\bar{\imath}+1)}$ (with distance $n - 1$) first resorts to hoarding its flexible reserves. If $z \leq y$, then this bank chooses $\left(A_0^{\rho(\bar{\imath}+1)} = H(z), A_1^{\rho(\bar{\imath}+1)} = K \right)$, that is, the bank meets its liquidity payments purely by hoarding its flexible reserves, and it does not withdraw any cross deposits. Otherwise $z > y$ and this bank chooses $\left(A_0^{\rho(\bar{\imath}+1)} = H(z), A_1^{\rho(\bar{\imath}+1)} = W(z - y) \right)$. In this case, consider bank $b^{\rho(\bar{\imath}+2)}$ with distance $n - 2$, which needs to find $z - y$ units of liquidity. This bank's response is similar to bank $b^{\rho(\bar{\imath})}$: if $z - y \leq y$, then the bank meets its liquidity payment purely by hoarding its flexible reserves, and otherwise it withdraws an even smaller amount from its forward neighbor.

It follows that a pattern emerges for the banks' cross withdrawal decisions. In particular, let $\bar{n} \geq 1$ denote the unique integer such that

$$y\bar{n} \geq z > y(\bar{n} - 1)$$

and suppose $n > \bar{n}$. Then, for each $j \in \{1, .., \bar{n} - 1\}$, the bank $b^{\rho(\bar{\imath}+j)}$ with distance $n - j$ chooses $\left(A_0^{\rho(\bar{\imath}+j)} = H(y), A_1^{\rho(\bar{\imath}+j)} = W(z - jy) \right)$, while the bank $b^{\rho(\bar{\imath}+\bar{n})}$ with distance $n - \bar{n}$ chooses $\left(A_0^{\rho(\bar{\imath}+\bar{n})} = H(z - (\bar{n} - 1)y), A_1^{\rho(\bar{\imath}+\bar{n})} = K \right)$. Since the bank with distance $n - \bar{n}$ keeps its cross deposits, the remaining banks with distance $k \in \{1, ..., n - \bar{n} - 1\}$ do not need any liquidity, and these banks withdraw their deposits if and only if their forward neighbors are insolvent.

Going back to the original distressed bank $b^{\rho(\bar{\imath})}$, note that, unlike in the previous sections, this bank obtains a total of $y + z$ units of liquidity: y units from its buffer and z units from cross deposits (if it remains solvent). Hence, this bank is insolvent if and only

if

$$\theta > y + z, \tag{15}$$

which is a stronger condition than (4). Suppose this condition is satisfied so that bank $b^{\rho(\bar{\imath})}$ is insolvent and consider the backward neighbor bank $b^{\rho(\bar{\imath}-1)}$. This bank chooses $A_1^{\rho(\bar{\imath}-1)} = W(z)$ and receives some $q_1^{\rho(\bar{\imath})} z < z$ from the original distressed bank. Despite incurring some losses, in view of condition (14), bank $b^{\rho(\bar{\imath}-1)}$ is solvent and it can promise its late depositors (including the backward neighbor bank) at least 1 unit at date 2, i.e. it pays $\left(q_1^{\rho(\bar{\imath}-1)} = 1, q_2^{\rho(\bar{\imath}-1)} > 1 \right)$. Since the backward neighbor bank $b^{\rho(\bar{\imath}-2)}$ does not need liquidity, it chooses to keep its deposits in bank $b^{\rho(\bar{\imath}-1)}$. Repeating this reasoning, all banks with distances $k \in \{1, .., n - \bar{n} - 2\}$ are solvent, and thus all banks with distances $k \in \{2, .., n - \bar{n} - 1\}$ choose $\left(A_0^{\rho(\bar{\imath}-k)} = B, A_1^{\rho(\bar{\imath}-k)} = K \right)$, in particular, keeping their deposits in their forward neighbor banks.

Note also that there is at most one seller in the conjectured equilibrium (the original distressed bank $b^{\rho(\bar{\imath})}$) while the banks with distance $k \in \{2, .., n - \bar{n} - 1\}$ are potential buyers of loans. Hence, under the deep secondary market assumption (i.e. the analogue of condition (10) for this setup),

$$(n - \bar{n} - 2)\, y > 1 - y, \tag{16}$$

the unique equilibrium price is given by $r = r_{fair}$. Note also that this analysis applies regardless of whether the banks know the financial network $\mathbf{b}(\rho)$ or whether they only have local knowledge (i.e. they know only their two forward neighbors). We summarize this result in the following proposition.

Proposition 3. *Suppose the banks' action space is extended to* (13), *assumption (LPO) for the liquidity pecking order is replaced by assumption (LPO-R) (which is imposed by the government), conditions* (15) *and* (16) *hold. Then, regardless of whether the banks know the financial network $\mathbf{b}(\rho)$ or just their two forward neighbors, there is a symmetric equilibrium in which loan prices are given by $r = r_{fair}$ and the cascade size is equal to 1. In particular, the original distressed bank is insolvent while all other banks are solvent. At date 1, the banks with distance $k \in \{1, 0, n - 1, .., n - \bar{n} - 1\}$ withdraw all or some of their cross deposits, while the banks with distance $k \in \{2, 3, .., n - \bar{n}\}$ keep their deposits in the forward neighbor bank.*

Proposition 3 establishes our main result in this appendix: If the banks avoid inflicting a liquidity externality, then the cascade size is much shorter relative to the cases analyzed in Propositions 1 and 2. To see the intuition, consider the equilibrium in Proposition 3

and consider what the banks would do if we removed the government imposed liquidity pecking order in assumption (LPO-R). Consider bank $b^{\rho(\bar{\imath}+\bar{n})}$, which meets its liquidity demand from its backward neighbor purely by hoarding its flexible reserves. For this bank, hoarding reserves delivers 1 unit of liquidity at an opportunity cost of R (in period 2), while withdrawing cross deposits would deliver 1 unit of liquidity at an opportunity cost of $q_2^{\rho(\bar{\imath}+n+1)}$. Given the characterization in 3, it can be checked that $q_2^{\rho(\bar{\imath}+n+1)} < R$ (as some of the losses in the original bank will spillover to bank $b^{\rho(\bar{\imath}+n+1)}$). Hence, if not restricted by government policy, bank $b^{\rho(\bar{\imath}+n)}$ would strictly prefer to withdraw its cross deposits to hoarding liquidity, which would put bank $b^{\rho(\bar{\imath}+n+1)}$ in distress. Similarly, absent government policy, bank $b^{\rho(\bar{\imath}+n+1)}$ would prefer to withdraw its cross deposits and the scramble for liquidity would continue to cascade in similar fashion. Eventually, a *vulnerable* bank, $b^{\rho(\bar{\imath}-1)}$, which is sufficiently close to the original distressed bank (and thus has incurred some losses) would become distressed. This bank might be unable to find the required liquidity and might become insolvent. Hence, the liquidity externalities have the potential to make vulnerable banks go insolvent. The government imposed liquidity order in assumption (LPO-R) internalizes these liquidity externalities, which in turn ensures that the cascade size is shorter.

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