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THE AREA AND POPULATION OF CITIES:
NEW INSIGHTS FROM A DIFFERENT PERSPECTIVE ON CITIES

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The Area and Population of Cities: New Insights from a Different Perspective on Cities
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ABSTRACT

The distribution of the population of cities has attracted a great deal of attention, in part because it sharply constrains models of local growth. However, to this day, there is no consensus on the distribution below the very upper tail, because available data need to rely on the "legal" rather than "economic" definition of cities for medium and small cities. To remedy this difficulty, in this work we construct cities "from the bottom up" by clustering populated areas obtained from high-resolution data. This method allows us to investigate the population and area of cities for urban agglomerations of all sizes. We find that Zipf's law (a power law with exponent close to 1) for population holds for cities as small as 12,000 inhabitants in the USA and 5,000 inhabitants in Great Britain. In addition the distribution of city areas is also close to a Zipf's law. We provide a parsimonious model with endogenous city area that is consistent with those findings.

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1 Introduction

This paper builds on a recently-proposed algorithm to construct cities based on geographical features of high-quality micro data (Rozenfeld et al. 2008), rather than informative but somewhat arbitrary legal or administrative definitions. It allows us to take a fresh look at key quantities in urban economics, namely the population and the area of cities. We find that Zipf's law for population holds quite well, and well below the very upper tail of the city size distribution, where it had been shown to hold to a good degree of approximation (Gabaix and Ioannides 2004). We also find that the distribution of city areas follows a power law, with an exponent close to 1, the Zipf value. These findings help constrain further theories of cities and theories of geography. We present a baseline parsimonious model of cities, which features endogenous city area, and is consistent with these two key stylized facts, as well as others.

A key difficulty in studying cities is finding a practical way to define them (Zipf 1949; Krugman 1996; Eaton and Eckstein 1997; Dobkins and Ioannides 2001; Eeckhout 2004; Soo 2005; Batty 2006). A canonical method involves defining Metropolitan Statistical Areas (MSAs) obtained in the USA from the US Census Bureau (U.S. Census Bureau 2009). MSAs are defined for each major agglomeration, and attempt to capture their extent by merging administratively defined entities, counties in the USA, based on their social or economic ties. For instance, the MSA of Boston includes not only the administrative unit of Boston, but also adjacent Cambridge, MA. MSAs derive their appeal from a strong economic logic, but their construction requires qualitative analysis and is very time-consuming. Therefore, MSAs have been constructed only for the 276 most populated cities in the USA, and the corresponding Zipf's law has been documented only for the upper tail of the distribution (Gabaix and Ioannides 2004; Soo 2005).

Two main alternatives to the MSAs have been proposed in the literature. One method is to use administrative or legal borders of cities to define the so-called "places" as done by Eeckhout (2004) and Levy (2009). The analysis of 25,359 places in the USA has suggested that Zipf's law holds in the upper tail (Levy 2009) but fails in the bulk of the distribution, as legally defined cities follow a log-normal distribution rather than a power-law (Eeckhout 2004, 2009). The advantage of this definition is that it allows the study of the distribution of cities of all sizes. Still, it is problematic to define cities through their fairly arbitrary legal boundaries (the places method treats Cambridge and Boston as two separate units), and indeed, this is why researchers prefer agglomerations such as MSAs whenever such constructs are available. A second approach is to construct cities from micro data (Holmes and Lee 2009; Duranton and Overman 2005; Mori, Nishikimi and Smith 2008; Michaels, Rauch and Redding 2008). In particular, Holmes and Lee (2009) consider cities to be individual cells of six-by-six miles, for which the tail of the city size distribution is much less fat-tailed than

Zipf's law. However, this is probably because constraining cities to areas of six-by-six miles makes it nearly impossible to find a very large city. Hence, because of these methodological difficulties, the shape of distribution of agglomerations beneath the few hundred largest cities is still an open problem.

Here we build on an algorithm, the City Clustering Algorithm (CCA), that was recently introduced in (Rozenfeld et al. 2008) and based on previous studies done by Makse, Havlin and Stanley (1995) to build cities "from the bottom-up". The algorithm defines a "city" as a maximally connected cluster of populated sites defined at high resolution. Namely, a population cluster is made of contiguous populated sites within a prescribed distance ℓ that cannot be expanded: all sites immediately outside the cluster have a population density below a cutoff threshold. Rather than defining a city as one cell, as done by Holmes and Lee (2009), our method defines an agglomeration as a maximally connected cluster of potentially many cells.

We find that Zipf's law holds, to a good approximation, in the USA and GB, for both populations and areas. We also find that density has only a weak correlation with population and area. We propose that the two facts of Zipf's law for populations and areas could serve as tight constraints on models of cities. As we can measure area, we wish to model it. Hence, we provide a parsimonious urban model that incorporates areas, and generates Zipf's law for areas and populations.

In Section 2 we present the analyzed data and explain the CCA. In Section 3 we present our results for the population distribution of CCA clusters in the USA and GB. We also compare the CCA clusters with US Census MSAs and places and present a formal test of robustness of our clustering method. In Section 4 we show the results of the area distribution of CCA clusters in the USA and GB and present a study of the correlations between densities, areas, and populations for CCA clusters. In Section 5 we propose a model that can integrate the findings, and we summarize our conclusions in Section 6.

2 Data and Methods

2.1 Raw data

The data for the USA consists of the location and population of 61,224 points located throughout the area of the USA (U.S. Census Bureau 2001). Each point corresponds to a Federal Information Processing Standard (FIPS) census tract code (National Institute of Standards and Technology 2008) generated by the US Census Bureau ranging in population from 1,500 to 8,000 people, with a typical size of about 4,000 people. FIPS codes are uniquely specified by 11 digits. The first 2 digits correspond to the state code, the next 3 to the county within the state, and the next 6 correspond to the census tract code. For example FIPS 36061016500 corresponds to New York State (36), New York County (Manhattan, 061), census tract 016500 which is an area ranging from 58th Street to 60th Street and from 8th Ave. to 9th Ave. Figure 1 shows all FIPS for Manhattan Island in New York City and its surroundings. The location of the FIPS is not always equidistant. For instance the shortest distance between two FIPS is about 100 m as in appears in Manhattan, while in less populated areas like Wyoming, FIPS can be separated by about 100 km.

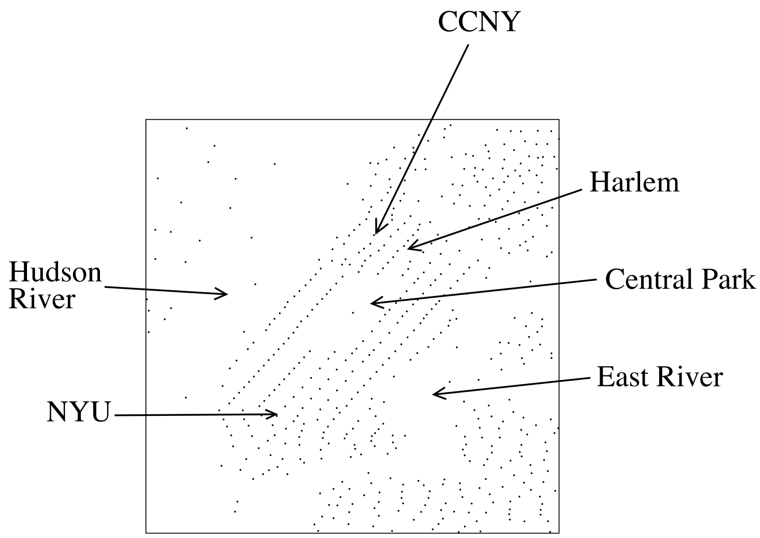
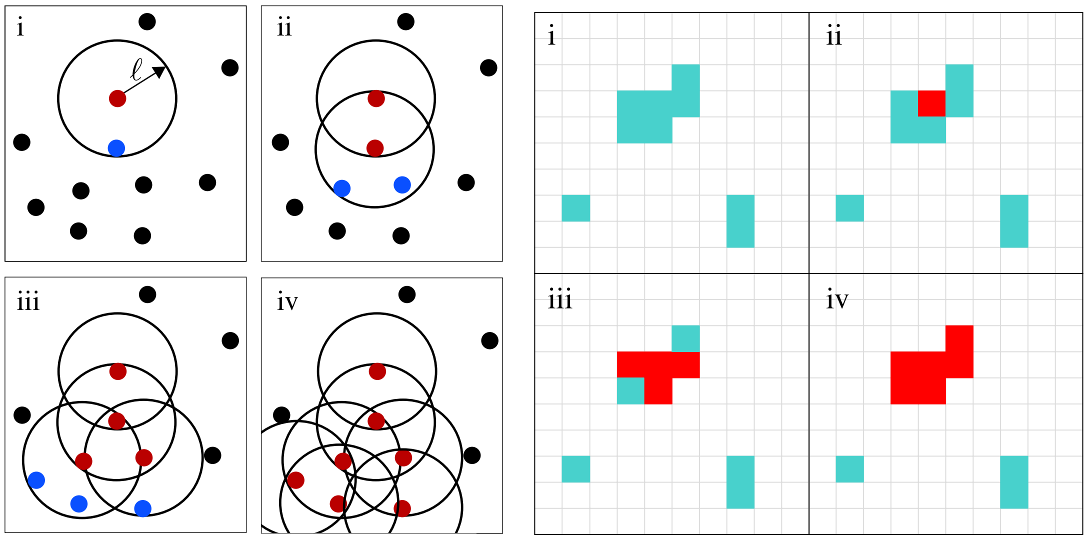


Figure 1: **Raw data for Manhattan.** In this plot we show all FIPS codes corresponding to Manhattan Island obtained from the raw data for the USA. Each point corresponds to a FIPS code specified by the US Census Bureau.

The data for Great Britain (GB) is uniformly gridded at high resolution. It consists of a grid with cell size 200 m overlaid on the area of GB for which the population in each cell is given. The source of the GB data is the ESRC (The 1981 and 1991 population census, Crown Copyright, ESRC purchase 2009) and is composed of 5.75 million square cells comprising a total population of about 55 million inhabitants in 1991. Given that the GB data is more fine-grained that of the US, it is arguably higher-quality. All datasets and results used and presented in this work may be downloaded from our web page.

2.2 The City Clustering Algorithm (CCA)

We start this section by providing a detailed explanation of the CCA (Rozenfeld et al. 2008). In Fig. 2a we show four steps of the CCA when it is applied to the USA. To define a CCA cluster, we first locate a populated site. Then, we recursively grow the cluster by adding all nearest-neighbor sites (populated sites within a distance smaller than the coarse-graining level, ℓ, from any site within the cluster) with a population density, D, larger than a threshold D_*. The cluster stops growing when no site outside the cluster with population density $D > D_*$ is at a distance smaller than ℓ from the cluster boundary. In this work, to minimize the number of free parameters, we set the threshold $D_* = 0$, and therefore clusters are recursively grown by merging all populated sites within a distance smaller than ℓ from any site within the cluster.



a **b**

Figure 2: **a,** CCA applied to the USA (continuum CCA). The points in this figure denote populated sites or FIPS. For our studies (and in this diagram) we use a density threshold $D_* = 0$. **(i)** We start a cluster selecting a populated site, red point, among all available populated sites. We draw a circle of radius ℓ and add all populated sites, blue point, that fall within the circle. **(ii)** We draw a circle from the new member of this cluster and add all populated sites (denoted by the two blue points) within the circle. **(iii)** Recursively, we keep drawing circles from all new cluster sites. The populated sites inside the circles (three blue points in this case) are merged into the cluster. **(iv)** The red points are the members of the cluster. Since no black point is at a distance smaller than ℓ from any red point, the cluster does not grow anymore. We start the process again selecting another initial point that has not been already assigned to any cluster. This process is repeated until all populated sites are assigned to a cluster. Notice that the choice of the initial condition, the first selected point, does not influence the outcome. **b,** CCA applied to GB (discrete CCA). **(i)** Cells are colored in blue if they are populated, otherwise they are in white. **(ii)** We initialize the CCA by selecting a random populated cell (red cell). Then, we merge all populated neighbors of the red cell as shown in **(iii)**. We keep growing the cluster by iteratively merging neighbors of the red cells until all neighboring cells are unpopulated, as shown in **(iv)**. Next, we pick another unburned populated cell and repeat the algorithm until all populated cells are assigned to a cluster.

Once the clusters are built, we calculate the population of a cluster as the sum of the populations of all sites within the cluster. Figure 3a shows a map of all identified clusters in the continental USA where colors correspond to the cluster population, and Fig. 3b shows a detail of the clusters in the northeastern USA for different ℓ.

Since the data of GB is already gridded, the CCA algorithm adopts a simpler form that in the USA. To apply the CCA to the GB data we start from a populated cell and at each step we grow the cluster by adding all populated cells neighboring the boundary of the cluster (see Fig 2b). The cluster stops growing when all cells neighboring the cluster have a density no greater than D_*.

As mentioned before, the data for GB differs from that of the USA, consisting of a grid with cell size 200 m overlaid on the map of GB. For this reason, since the data is already gridded at a very high resolution, we simply merge cells from the original data to obtain larger levels of coarse-graining at different grid sizes ℓ. We call this version of the CCA, the discrete CCA, while the version applied to the USA is the continuum CCA (see Fig. 2).

3 Population Distribution

3.1 Basic Results

We analyze the population data in the USA and GB to obtain the distribution, $P(S)$, measuring the probability density that a cluster has a population between S and $S + \mathrm{d}S$. Figure 4 shows the results of $P(S)$ for the USA for $\ell = 2$ km, $\ell = 3$ km, and $\ell = 4$ km for which we obtain 30,201, 23,499, and 19,912 clusters, respectively. We find that the population distribution follows a power-law of the form:

$$P\left(S\right) \sim S^{-\zeta-1}, \tag{1}$$

with an exponent of $\zeta \approx 1$, in approximate accordance with the value of Zipf's law. For example, when we estimate the exponent for $\ell = 3$ km and for clusters with $S > S_* = 12,000$ inhabitants (comprising 63% of the country's population) we find $\zeta = 0.97 \pm 0.03$ using an OLS estimator (the notation $\pm$ means that the standard deviation is 0.03). Figure 5 shows the Zipf exponent ζ for the USA for several value of ℓ. We observe that the exponent ζ remains approximately within 5% of the Zipf value in the range $\ell \in [2.5, 3.5]$ km.

Figure 6 displays the population distribution of the CCA clusters in GB for $\ell = 0.2$ km, $\ell = 0.6$ km, $\ell = 1$ km, $\ell = 1.8$ km, $\ell = 2$ km, and $\ell = 2.6$ km. For clusters with a population above a cutoff $S_* = 5,000$ inhabitants, the GB population follows a power-law to a good degree of approximation. Using an OLS regression, we estimate for $\ell = 1$ km (1,008 clusters with 83% of the country's population) a Zipf exponent $\zeta = 1.07 \pm 0.03$. As in the case of the USA, the exponent is similar for different choices of grid size ℓ.

To formally study the validity of our power-law fits, we employ the test proposed by Gabaix and Ibragimov (2010) and Gabaix (2009), offering a simple quantification of possible deviations from a pure power-law. The test for quadratic deviations is used to determine if a power-law is adequate to describe the city size distribution. The method is as follows. Sort

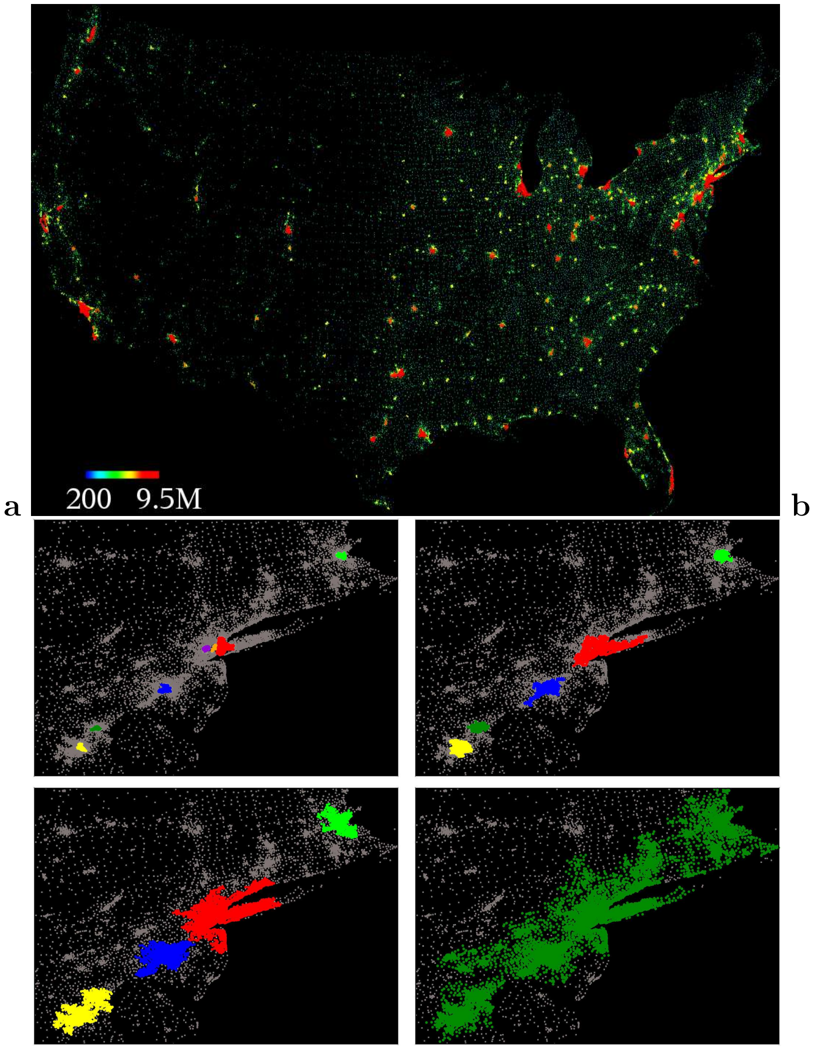


Figure 3: **CCA clusters in the USA. a,** CCA clusters applied to the entire USA. The map shows the different clusters obtained by the algorithm. The color indicates the population of each urban cluster (in logarithmic scale). **b,** Results of the CCA applied to the major clusters of the northeastern USA at different length scales. The top left panel shows the CCA clusters for $\ell = 1$ km separating the cities of Washington D.C., Baltimore, Philadelphia, Newark, Jersey City, New York, and Boston. The top right panel shows the results of the algorithm when the data is coarse-grained to $\ell = 2$ km. Here, for example, the cities of New York, Newark and Jersey City become part of the same cluster. The lower left panel shows the results for $\ell = 4$ km, where the main clusters are Washington D.C.-Baltimore; Philadelphia; New York-Newark-Jersey City-Long Island; and Boston-Cambridge. The lower right panel for $\ell = 8$ km shows a giant cluster comprising all major cities in the northeastern USA. The gray points are also identified as part of other clusters but for clarity we do not specify them with individual colors in this figure.

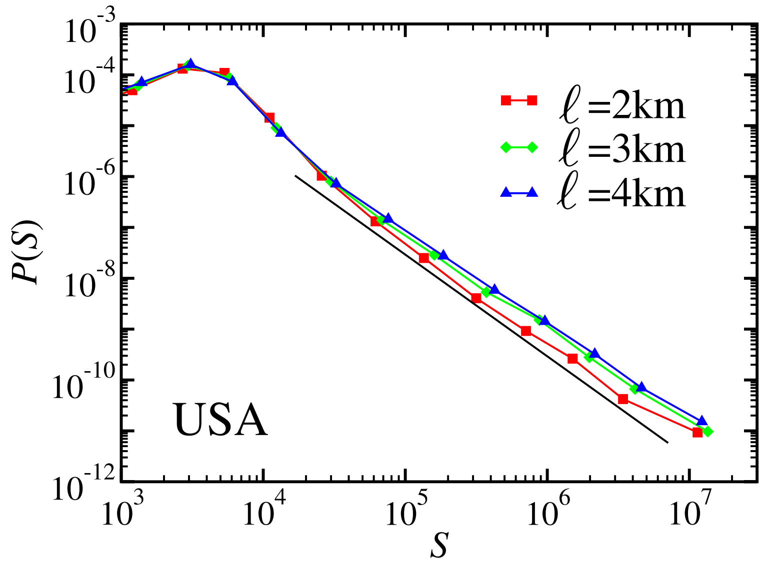


Figure 4: Probability distribution of cluster populations $P(S)$ for the USA at different coarse-graining scales ℓ. The black solid line denotes a power-law function with exponent -2, i.e. Zipf's Law.

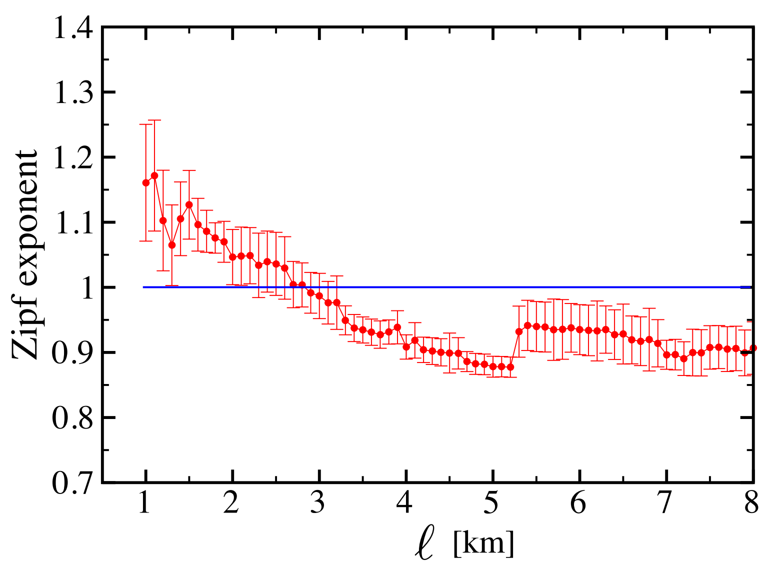


Figure 5: Zipf exponent ζ obtained for the USA clusters at different ℓ. The error bars correspond to ± 1 standard deviation.

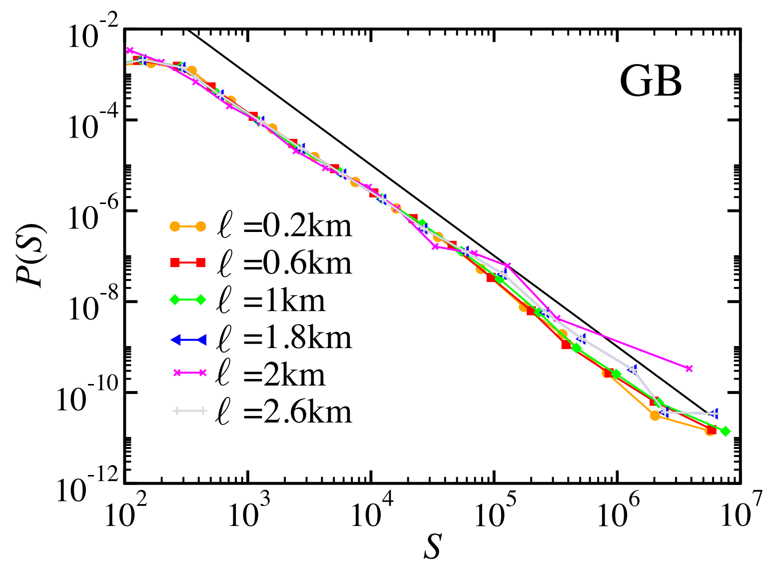


Figure 6: Probability distribution $P(S)$ for GB clusters at different coarse-graining scales ℓ.

the cities according to their rank i ($i = 1$ being the largest city) and run the OLS regression

$$\ln(i - 1/2) = \text{constant} - \zeta \ \ln S_i + q \ (\ln S_i - \gamma)^2 \quad (2)$$

where ζ (the power law exponent) and q (the quadratic deviation from a power law) are the parameters to estimate and $\gamma \equiv (\text{cov}((\ln S_j)^2, \ln S_j)) \, / \, (2\text{var}(\ln S_j))$. The recentering term γ ensures that the exponent ζ is the same whether the quadratic term is included or not, and therefore ζ may be estimated beforehand using a simple linear OLS. The quadratic test formalizes the intuition that a pure power law has $q = 0$ in the asymptotic limit, so a high value of $|q|$ indicates deviations from power-law behavior. Under the null of a power law, for large samples $\sqrt{2N} q_N / \zeta^2$ converges to a standard normal distribution (where N is the number of data-points). With probability 0.99, a standard normal is less than 2.57 in absolute value. Hence, let $q_c \equiv 2.57\zeta^2/\sqrt{2N}$, be the critical value for the absolute value of the quadratic term q at the 1% confidence level. If $|q| > q_c$ we reject the hypothesis that the data is well described by a power-law since the quadratic term becomes significant. Otherwise, if $|q| < q_c$, the quadratic term is insignificant and we do not reject the power-law hypothesis.

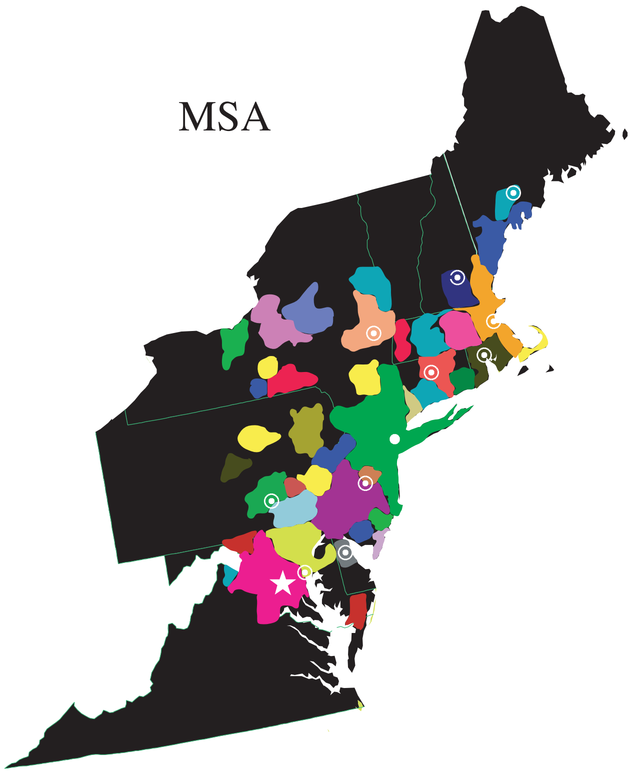
For the USA, when we consider the distribution of city sizes for cities larger than $S_* = 12,000$ for $\ell = 3$ km, we obtain $|q| = 0.0291$ and $q_c = 0.0413$. Since $|q| < q_c$, we conclude that we can disregard the quadratic correction to the OLS fit and consider that the power-law describes the empirical distribution of city sizes. In the case of GB, we consider $S_* = 5,000$ and $\ell = 1$ km, for which $|q| = 0.0521$ and $q_c = 0.0522$. Although $|q|$ and q_c are very close, the fact that $|q| < q_c$ indicates that we cannot reject the hypothesis that the power-law describes the city size distribution for GB. We conclude that Zipf's law is a good description of city sizes with population above $S_* = 12,000$ inhabitants in the USA and $S_* = 5,000$ inhabitants in GB. This comprises $1,947$ clusters (for $\ell = 3$ km) and a population of 171.3 million out of a total population of 271.1 million in the USA, and 1,007 clusters (for $\ell = 1$ km) and a population of 45.3 milllion out of a total population of 54.5 million in GB, in contrast to previous samples (Soo 2005) typically having a few hundred cities.

So far, we have only focused on the part of the distribution where a power law fit could not be statistically rejected. Now, somewhat more loosely, we turn to a visual inspection of Fig. 4 and Fig. 6. We see that the distribution is arguably well-approximated by a power law, in a region covering cities above 3,000 inhabitants in the USA, and cities above 300 inhabitants in GB. The deviations from the power law, while statistically significant, are not very large economically. Hence, we also submit that, for the modelling a cities, the domain of an approximate power law is quite large. This domain comprises 17609 clusters and a population of 259.3 million (96% of the total population) in the USA, and 9214 clusters and a population of 53.1 million (96% of the total population) in GB.

3.2 Comparison between CCA clusters, MSAs, and Places

Although the CCA allows one to choose the observation level of population clusters, ℓ, it may be desirable to have an objective way to choose ℓ. For this purpose, we perform a comparison with the MSAs in the USA which may be considered a benchmark for plausibly well-constructed cities. MSAs are defined starting from a highly populated central county with population larger than 50,000 and adding its surrounding counties if they have social or economic ties such as large commuting patterns between the regions. Figures 7a and 7b show a comparison between the MSAs of the northeastern USA and the clusters obtained using CCA.

In order to find the value of ℓ that best matches the MSAs we match each MSA with the most populated overlapping CCA cluster. For this purpose, from the US Census Bureau, we obtain the counties (and corresponding FIPS) that belong to each MSA. An overlap between an MSA and a CCA cluster exists if they share at least one FIPS code. This overlapping procedure leads to several CCA clusters corresponding to one particular MSA. To obtain a one-to-one correspondence, among all overlapping CCA clusters we select the one with the largest population. We compare the size of the obtained CCA cluster with the corresponding MSA by computing the correlation, $\rho(\ell)$, between the logarithm of the cluster population, $S_i^{\mathrm{CCA}}(\ell)$, and the logarithm of the population of the MSA, S_i^{MSA}. Figure 8a shows the cross-plot of $\log S_i^{\mathrm{MSA}}$ versus $\log S_i^{\mathrm{CCA}}(\ell)$ for $\ell = 3$ km displaying an approximately linear behavior. Figure 8b shows the correlation analysis between CCA clusters and MSAs by plotting $\rho(\ell)$ for other values of ℓ. We quantify the regression, $\log S_i^{\mathrm{CCA}}(\ell) = a(\ell) + b(\ell) \log S_i^{\mathrm{MSA}}(\ell)$, by measuring the value of the linear regression slope $b(\ell)$ as a function of ℓ. We find that $b(\ell) \approx 1.2$ for $\ell > 2$ km. Correlation in log sizes is very good for values of ℓ between 2 km



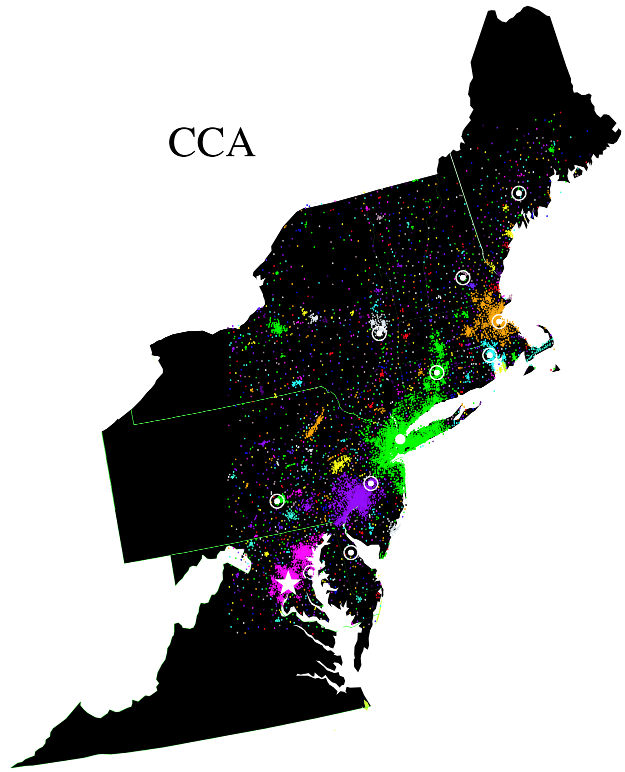


Figure 7: **Comparison between the MSAs and the CCA clusters. a,** MSAs for the northeastern USA. For example, New York county (Manhattan) with a population larger than 50,000 is a center of a MSA. Jersey City belongs to the same MSA since a large number of its population commute to Manhattan, setting economic and social ties between the two regions. **b,** CCA clusters for the northeastern USA for $\ell = 5$ km. Each cluster or MSA is plotted with a different color. For instance, the MSA centered in New York City (in green in **a**) is composed of several clusters. The largest overlapping cluster found with the CCA is in green in **b**. The white concentric circles correspond to the location of the state capitals in the considered region. The star denotes Washington D.C. and the white full circle corresponds to New York City.

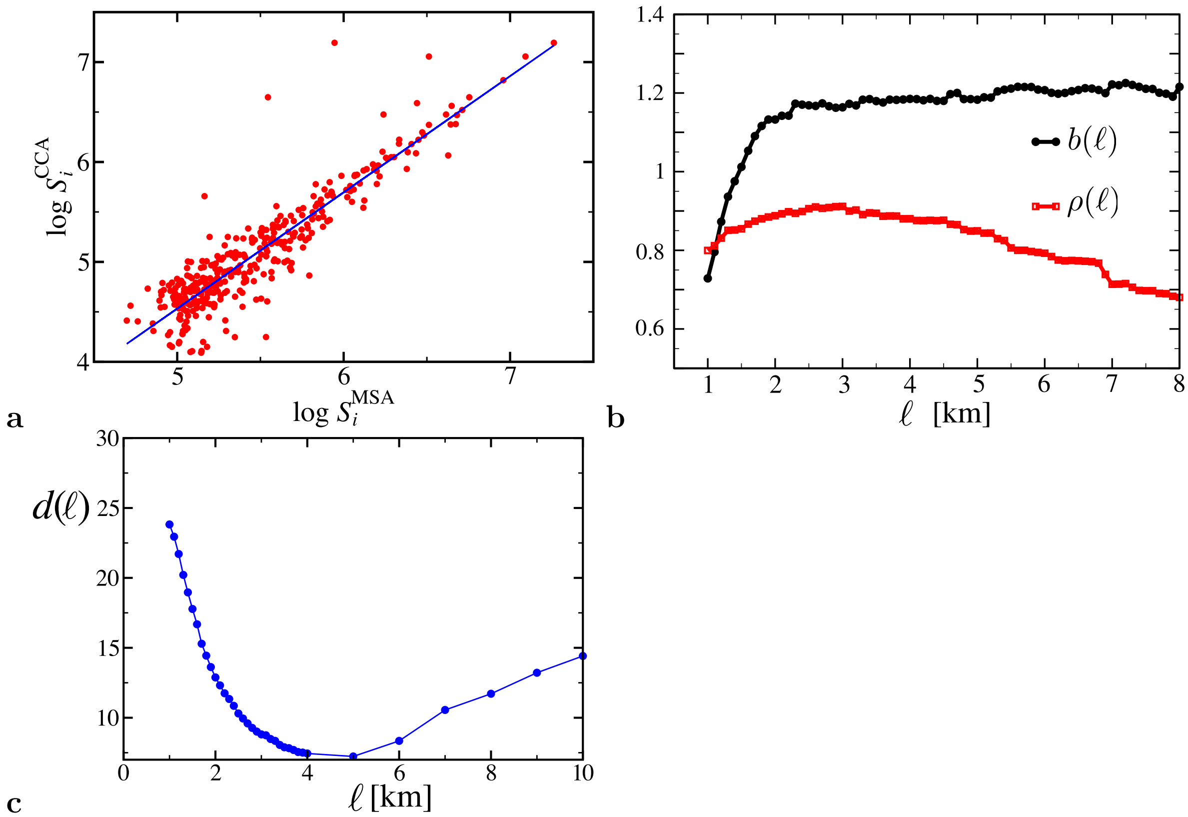


Figure 8: **a,** Population of the CCA cluster in the USA for $\ell = 3km$ vs its corresponding MSAs, using the one-to-one correspondence explained in the text. **b,** Correlation analysis between CCA clusters and MSAs by plotting $\rho(\ell)$ for different values of ℓ. We quantify the regression, $\ln S_i^{\rm CCA}(\ell) = a(\ell) + b(\ell) \ln S_i^{\rm MSA}(\ell)$, by measuring the value of the linear regression slope $b(\ell)$ as a function of ℓ. **c,** Euclidean distance between MSAs and CCA clusters.

and 6 km; the correlation, displayed in Fig. 8b, is very high for this range of ℓ. We find that $\rho(\ell)$ exhibits a maximum value of $\rho \approx 0.91$ for $\ell \in [2.5, 3.5]$ km, so that we consider $\ell = 3$ km as the optimal value.

We present another plausible measure of similarity between MSAs and CCA clusters, based on the Euclidean distance. We define the distance, $d(\ell)$, between MSAs and CCA as

$$d(\ell) \equiv \sqrt{\sum_i [\ln(S_i^{\rm MSA}) - \ln(S_i^{\rm CCA}(\ell))]^2}, \tag{3}$$

where the sum is over all the MSAs and their corresponding CCA clusters. In Fig. 8c we show the distance between overlapped MSAs and CCA clusters as a function of ℓ. We find that when $\ell = 5$ km the distance (in population) is minimized, and that it is very low between 2.5 km $\leq \ell \leq$ 6 km in approximate agreement with the log correlation analysis of Fig. 8a,b.

In addition to the MSAs, we compare the CCA clusters with US Census Bureau "places" previously analyzed in Eeckhout (2004) where a log-normal distribution of city sizes was found. We first find a one-to-one correspondence between CCA clusters and places, in analogy to the previous match between MSAs and CCA clusters. In contrast to MSAs, US Census places take into account all towns, villages, and cities and are based only on their administrative or political boundaries (Eeckhout 2004; Holmes and Lee 2009). The smallest and largest places are Lost Spring, Wyoming, with exactly one resident, and the political entity of New York City (Manhattan, Brooklyn, Queens, Bronx, and Staten Island) with population 8.0 million.

From the US Census Bureau we obtain the geographical location of each US Census place. Then, we identify each place with a unique FIPS code. Accordingly, each place is associated with a unique CCA cluster. This association leads to many places corresponding to a single CCA. To obtain the one-to-one correspondence, among all overlapping places we consider the one with the largest population.

In Fig. 9a we show that, the smallest cities found with the CCA do not correspond well to US Census places; however, for cities above population $S = 10,000$ CCA and Census places do exhibit a correlation coefficient of $\rho = 0.79$. A detailed comparison between CCA clusters and places shows that the number of small CCA clusters is smaller than that for places because the CCA tends to group small places that are geographically connected into a larger cluster. Therefore, the construction based on places overestimates the number of small cities and underestimates the number of large cities in comparison with CCA, resulting in the size distribution of places to being less fat tailed than the distribution for CCA clusters. This discrepancy, which may find its root in the fact that places are purely based on legal boundaries of locations (Holmes and Lee 2009), may explain the finding of a log-normal distribution of places (Eeckhout 2004), whose full elucidation is beyond the scope of this paper. Here, we show results for $\ell = 3$ km as representative, but other values of ℓ lead to the same conclusions.

We also perform a comparison between MSAs and places. In Fig. 9b we observe a good congruence in the whole range for which MSAs are defined. Notice that MSAs by definition have a minimum population of 50,000. Therefore, when looking for the one-to-one correspondence, only large places are considered, leading to a good congruence, as found between large CCA cluster and large places, with correlation $\rho = 0.87$.

3.3 Robustness Checks

In this section we test whether the results shown in Section 3.1 could be forced by the CCA, or in other words, whether they could be an artifact of the CCA. Starting with the actual location of the FIPS in the USA we randomize the data by placing all 61,224 FIPS at random positions in a rectangle of the same area as the USA. Then we apply the CCA to

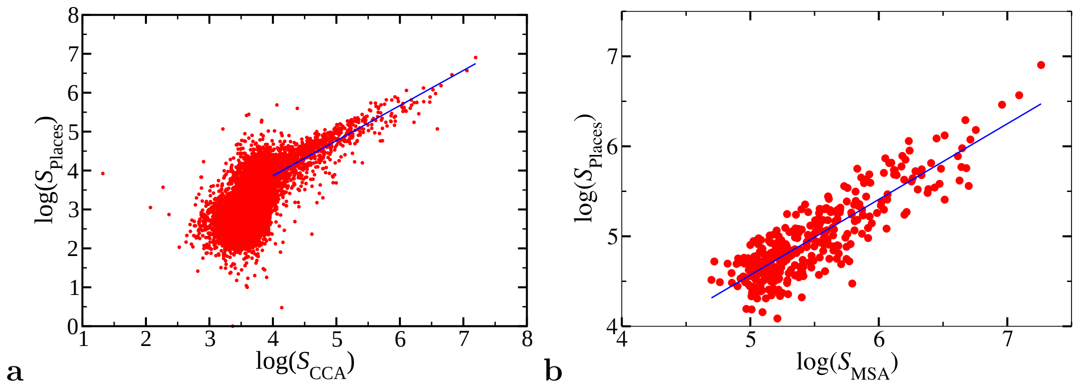


Figure 9: **a,** Log of population of US Census places vs. the log of population of their corresponding CCA clusters for $\ell = 3$km. The straight line corresponds to a least square fit with slope $b = 0.90 \pm 0.02$ and y-intercept $a = 0.25 \pm 0.09$, from where the correlation coefficient $\rho = 0.79$ is obtained for cities with population larger than 10,000. **b,** Log of population of US Census places vs. the log of population of their corresponding MSA. The straight line corresponds to a least square fit with slope $b = 0.84 \pm 0.03$ and y-intercept $a = 0.37 \pm 0.14$, from where the correlation coefficient $\rho = 0.87$ is obtained.

obtain the corresponding clusters. This randomization procedure preserves the population of each FIPS. In Fig. 10 we show the population distribution for the shuffled data and for the original data. These results show that the shuffled data does not exhibit Zipf's law. The largest cluster for the shuffled data contains 196,112 inhabitants: the reshuffling prevents the emergence of very large clusters. This suggests that the CCA is not forcing the data to present a power-law for the population distribution, and that Zipf's law arises purely from the data.

4 Investigation of the Geography of Cities: Areas and Densities

4.1 Areas

The CCA presents a unique feature in that it allows the definition of the area of cities not based on administrative boundaries. Such a feature in not present in agglomerations defined by Places or MSAs. Thus, the spatial analysis of the CCA allows us to examine a possible feature of the origin of the Zipf's law: highly populated cities may have a large geographic area. Therefore, it is of interest to study the distribution of areas (Makse, Havlin and Stanley 1995), $P(A)$, defined by the CCA.

As explained above, the data of GB consists of a high resolution grid with cell size 200m. Therefore, after applying the CCA, we calculate the area of a cluster in GB as the number of cells in the cluster multiplied by the area of a cell, ℓ^2.

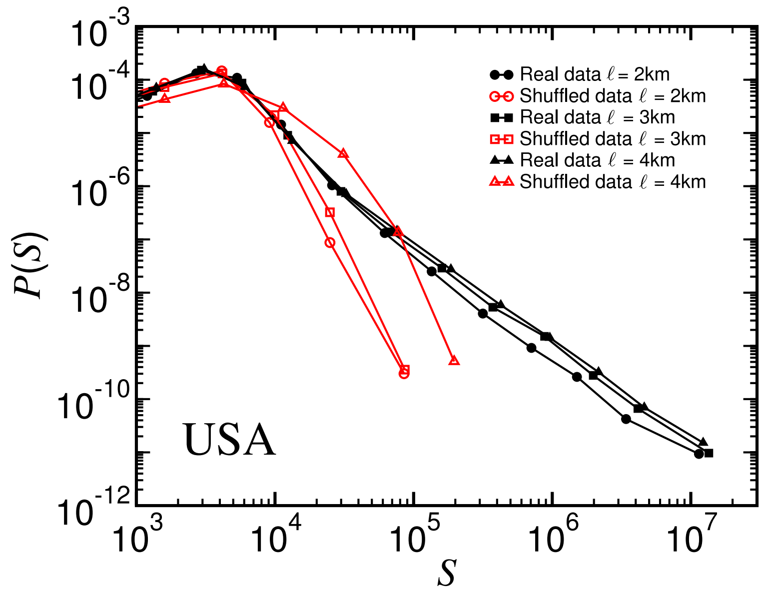


Figure 10: Population distribution for shuffled data. The black lines correspond to the real data studied in Section 3.1. The red lines correspond to the shuffled data, showing a change in the population distribution and suggesting that the results of Section 3.1 are not an artifact of the CCA.

The case of the USA is more complicated. The data consists of 61,224 populated points on the map. Each point corresponds to a different FIPS code, defined by the US Census Bureau. USA FIPS are simply a partition of the map of the USA, so that any point in the map belongs to one FIPS code, and each FIPS has an associated area which is given by the US Census Bureau in the dataset. In the USA, FIPS codes are not homogeneously distributed. In the New York City area, there is high resolution, which means that there are many FIPS covering a small area, but in the state of Wyoming or Utah the resolution is quite low, so that there are FIPS with a large area. For instance, FIPS in Manhattan typically cover an area of about 0.20 km^2 while in the state of Utah FIPS 49003960100 covers a large area of 15,962 km^2. Therefore, when ℓ is of the order of a few kilometers, a FIPS in the Wyoming area will remain isolated in its own cluster, but still its area will be extremely large, typically a couple of orders of magnitude larger than ℓ^2. Therefore, since the area of isolated points is very large, these points will appear at the tail and in the middle of the distribution $P(A)$, overestimating the outcome for middle and large areas. Accordingly, in order to compute the $P(A)$, we do not take into account clusters containing only 1 or 2 FIPS since they overestimate the amount of land they cover. Moreover, the population of those isolated points is typically small and rarely exceeds $S = 10,000$. In fact, we find that removing all clusters with only 1 or 2 FIPS is practically the same as removing all clusters with population smaller than 10,000: only 7% of clusters with 1 or 2 FIPS have a population larger than 10,000.

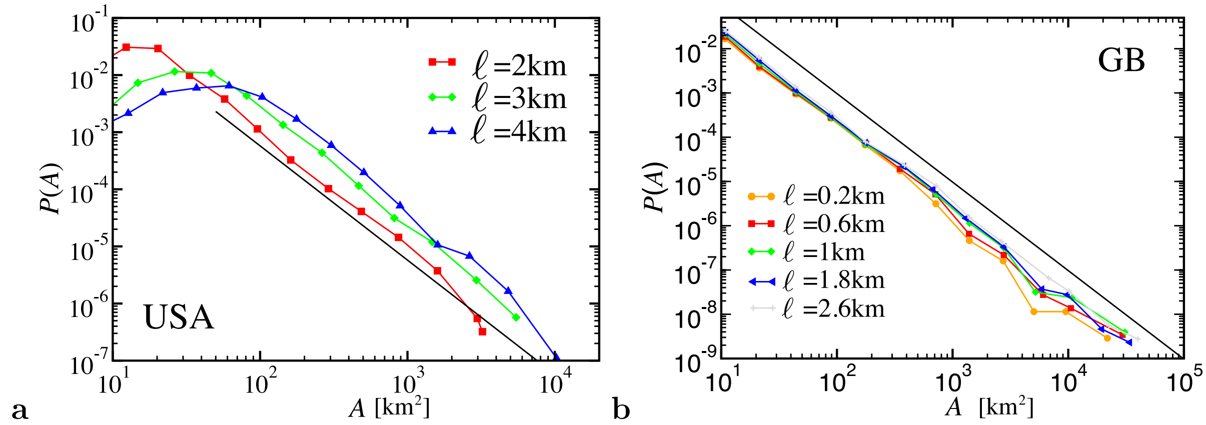


Figure 11: **a,** Probability distribution of the areas, $P(A)$, for the USA for different ℓ. **b,** Probability distribution $P(A)$ of the areas of the clusters in GB at different coarse-graining scales ℓ. The distribution of city areas for GB is also consistent with Zipf's law. We find $\zeta_A = 0.97 \pm 0.04$, for $\ell = 1$ km. The black solid lines denote Zipf's law, i.e. a power-law function with exponent -2.

In Fig. 11a we report the results of $P(A)$ for the USA. We find a power-law distribution of the form

$$P(A) \sim A^{-\zeta_A - 1}, \tag{4}$$

with a Zipf exponent $\zeta_A = 1.07 \pm 0.04$, for $\ell = 3$ km. In Fig. 11b we show the results of $P(A)$ for GB. As for the USA, we find that the area distribution for GB follows a power-law with exponent $\zeta_A = 0.97 \pm 0.04$, for $\ell = 1$ km. This extends the results obtained in (Makse et al. 1998) for areas distributions surrounding a city like London and Berlin (Makse, Havlin and Stanley 1995) and in UK (Makse et al. 1998). The result of the Zipf's law for areas in the US appears to be new.

This result may be an important update for calibrated models of cities where transport costs of goods or people play an important role (Brakman, Garretsen and van Marrewijk 2009; Fujita, Krugman and Venables 2001). The Zipf's law for areas implies that some cities have very large areas, and those cities' viability may mean that transport costs cannot be too large, or are mitigated in economically interesting ways. We come back to this topic in Section 5.

In Fig. 12a we study the correlations between areas and populations for the USA and GB. We find that the linear OLS regression $\ln A = a + b \ln S$ leads to the results shown in Table 1, indicating a strong correlation between areas and population in log sizes. Indeed, the finding of $b \simeq 1$ indicates that population is, to a good degree of approximation, simply proportional to area. This finding motivates us to study city density in more detail.

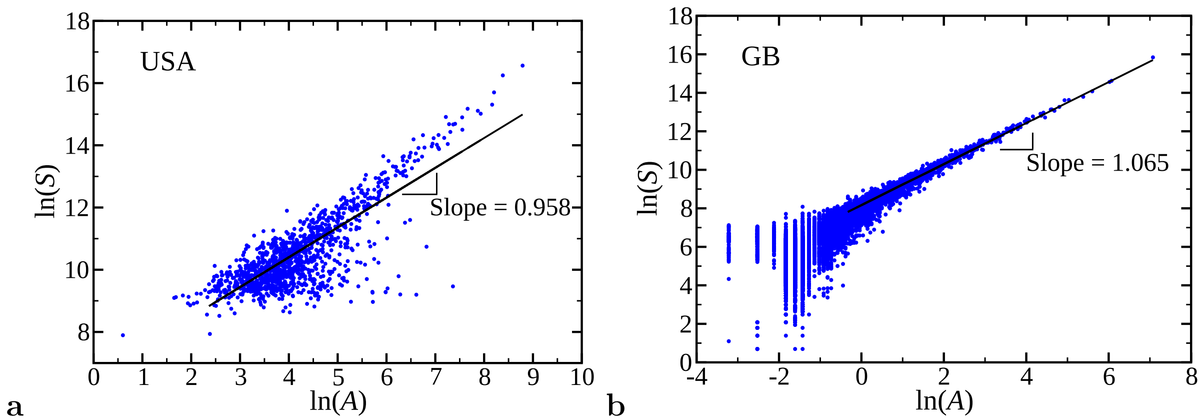


Figure 12: Logarithm of the population, S versus the logarithm of the area, A, for **a,** the USA with $\ell = 3$ km and **b,** GB for $\ell = 1$ km. The black lines denote the OLS regression (see Table 1.)

Table 1: Results of the OLS regression analysis of $\ln S = a + b \ln A$, where A is the area and S the population. We report results for $S_* = 12,000$ and $\ell = 3$ km for the USA, and $S_* = 5,000$ and $\ell = 1$ km for GB. Standard errors are reported in parentheses.

	USA	GB
$\ln A$	0.958	1.065
	(0.020)	(0.007)
Constant	6.567	8.166
	(0.085)	(0.010)
Observations	1064	1007
R^2	0.686	0.921

4.2 Densities

In this section we study the population density, $D = S/A$. [1] We study the behavior of D versus S and A by performing the linear regressions $\ln D = a + b \ln A$, and $\ln D = a + b \ln S$. Table 2 shows the results of the OLS regression estimates with $S_* = 12,000$ and $\ell = 3$ km for the USA, and $S_* = 5,000$ and $\ell = 1$ km for GB (other choices of ℓ lead to the same conclusions). We find that population density has very little relation to the area: the coefficients are very close to 0. It has a slightly higher link with population. Of course, measurement error in the variables may bias the measurement.

Still, the link between density and area is perhaps surprisingly weak. Some urban systems, like New York City, are quite dense, but even then, the effects are moderate: the density of New York City is only 3.7 times the national median even though its population is 485

[1] See Bryan, Minton and Sarte (2007) for an alternative analysis of density. They find that density has fallen in the US over the past seven decades.

Table 2: Results of the OLS regression analysis of $\ln D = a + b\,\ln A$ and $\ln D = a + b\,\ln S$, where $D = S/A$ is the density, A the area, and S the population. We report results for $S_* = 12,000$ and $\ell = 3$ km for the USA, and $S_* = 5,000$ and $\ell = 1$ km for GB. Standard errors are reported in parentheses.

	$\ln D = a + b\,\ln A$			$\ln D = a + b\,\ln S$	
	USA	GB		USA	GB
$\ln A$	-0.042	0.065	$\ln S$	0.284	0.099
	(0.020)	(0.007)		(0.015)	(0.006)
Constant	6.567	8.166	Constant	3.357	7.299
	(0.086)	(0.010)		(0.159)	(0.057)
Observations	1064	1007	Observations	1064	1007
R^2	0.004	0.007	R^2	0.256	0.050

times the national median. Of course, we obviate here a consideration of the interesting heterogeneity within cities; but for the purposes of this paper such a study may be deferred to later work. We find that density has a very small dispersion: the standard deviation of its natural logarithm is 0.28 for the USA and 0.09 for GB. In contrast, the corresponding quantity for areas and population is about 1. Hence, we conclude that city area covaries greatly with population, and little with density. We next propose a model that is consistent with this finding, as well as the power law scaling of city sizes.

5 Model

Recent economic theories that are compatible with Zipf's law generally rely on the existence of random growth (Champernowne 1953; Simon 1955; Krugman 1996; Levy and Solomon 1996; Gabaix 1999a; Dobkins and Ioannides 2001; Davis and Weinstein 2002; Gabaix and Ioannides 2004; Eeckhout 2004; Duranton 2006, 2007; Rossi-Hansberg and Wright 2007; Córdoba 2008): cities follow a proportional growth process where the distribution of the percentage growth rate is the same for small and large cities. Small cities, however, grow faster (Glaeser et al. 1992; Glaeser, Scheinkman and Shleifer 1995; Rozenfeld et al. 2008), which prevents the distribution from becoming degenerate. Some theories obtain Zipf's law only approximately, and do not obtain it over the range that we find in the present work. Accordingly, we present a parsimonious model that generates an approximate Zipf's law for population and area.

We first describe the model at a given point in time. Cities are indexed by $i \in [0, 1]$. City i employs S_i workers, and has a competitive sector producing good i, which it produces in quantity $y_i = b_i S_i$, where b_i is the productivity. The aggregate good is a Dixit-Stiglitz aggregator with elasticity of substitution $\eta > 1$:

$$Y = \left(\int y_i^{(\eta-1)/\eta} di \right)^{\eta/(\eta-1)} \tag{5}$$

There is a potentially unbounded quantity of land, but making usable an area a of land necessitates an investment pa, for some unit cost p. This reflects e.g. maintenance cost, roads and other infrastructure to occupy a land area a (the cost is in the consumption good, but it could equivalently be in units of labor). Hence, as in Rossi-Hansberg and Wright (2007), land use is endogenous. As a result, if A is total land use, and C total consumption, the resource constraint is $C + pA \leq Y$.

Consumers' utility is $u(c, a) = c^{1-\beta} a^{\beta}$ with $0 < \beta < 1$, where c is the consumption of the good, and a the consumption of land. Workers are free to choose their cities, so that utilities are the same across cities. Hence, the competitive equilibrium is also the solution to the planner's problem that equalizes utility across agents, and allocates population S_i in each city i (subject to $\int S_i di = S$, the total population), and allocates their per capita consumption of good c_i and land area a_i, to maximize total utility subject to the resource constraint:

$$\max_{S_i, c_i, a_i} \int u(c_i, a_i) S_i di \text{ subject to}$$

$$\begin{aligned} \int (c_i + pa_i) S_i di &\leq \left(\int (b_i S_i)^{(\eta-1)/\eta} di \right)^{\eta/(\eta-1)} \\ \forall i, j, u(c_i, a_i) &= u(c_j, a_j) \\ \int S_i di &= S \end{aligned}$$

The solution method is standard. The labor allocated to producing good i, i.e., the population living in city i is:

$$S_i = \frac{B_i}{\overline{B}} S \tag{6}$$

where $B_i \equiv b_i^{\eta-1}$ and $\overline{B} \equiv \int B_i di$. GDP is $Y = \overline{B}^{1/(\eta-1)} S$. A fraction $1 - \beta$ of income is devoted to consumption of the good, and a fraction β to land use. Each consumer purchases a quantity of land $\nu \equiv \beta \overline{B}^{1/(\eta-1)}/p$. So, the total quantity of land in city i, A_i, is ν times the number of inhabitants of city i:

$$A_i = \nu S_i \tag{7}$$

Hence city area and city population are proportional.

Next, we wish to see why Zipf's law might arise. We consider a dynamic version of the above static description. Consumers have utility $E\left[\int e^{-\delta t} u(c_t, a_t) dt\right]$ with some discount rate δ, which is assumed to be sufficiently large for utility to be finite. As there are no adjustment costs, for given productivities, the dynamic model yields the same allocation of workers and land across cities as in the static model. We take a model that merges the random growth models of cities and the model of random growth of firms developed by Luttmer (2007). We postulate that (elasticity-adjusted) productivity B_i of city i evolves as a geometric Brownian motion:

$$\frac{dB_{it}}{B_{it}} = gdt + \sigma dz_{it} \tag{8}$$

where g is the mean of the growth rate of productivity of an existing city, σ the volatility of that growth rate, and z_{it} are independent Brownian motions. However, if a city is too unproductive, it can "refresh" its productivity as a fraction of the average productivity:

$$B_{it} \geq \pi \overline{B}_t \tag{9}$$

where $\pi \in (0, 1)$ is a constant. Here, we simply postulate that it can reset its productivity for free, by simply imitating the average productivity, but only imperfectly: its reset productivity is only a fraction π of the average productivity. Luttmer (2007) presents a much more elaborate microfoundation for this idea, including the π, but the above model is useful for its simplicity. All in all, B_{it} follows a geometric Brownian motion, reflected at $\pi \overline{B}_t$.

The following Proposition characterizes the behavior of this economy.

Proposition *(i) The steady state distribution of city population and city area is a power-law with exponent ζ:*

$$\zeta = \frac{1}{1-\pi} \tag{10}$$

Indeed, $S_{it}/\overline{S}_t$, $A_{it}/\overline{A}_t$ and $B_{it}/\overline{B}_t$ are all equal and follow the Pareto distribution $P(X \geq x) = (x/\pi)^{-\zeta}$ for $x \geq \pi$. The exponent ζ tends to 1 (the Zipf's law value) when the friction π coming from the reflecting barrier tends to 0.

(ii) City population S is proportional to city area A, and density $D = S/A$ is independent of city size.

(iii) The fraction of income spent on housing is independent of city size.

Proof The proof method is as in Gabaix (1999a) (see also Gabaix (2009) and the references therein). Denote by $\overline{g}$ the growth rate $\overline{B}_t$ on the balanced growth path. The relative share of city i, $s_{it} \equiv B_{it}/\overline{B}_t$ follows a geometric Brownian motion, with $ds_{it}/s_{it} = (g - \overline{g})dt + \sigma dz_{it}$, with a reflecting barrier, $s_{it} \geq \pi$. Calling $\mu = g - \overline{g}$, the steady state density $p(s)$ follows the Forward Kolmogorov equation:

$$0 = -(\mu s p(s))' + \frac{1}{2}\left(\sigma^2 s^2 p(s)\right)''$$

Integration of this equation yields $p(s) = ks^{-\zeta-1}$ for $\zeta = 1 - 2\mu/\sigma^2$ and some constant k. By construction $E[s] = 1$. Given $\int_\pi^\infty p(s)\,ds = 1$, we have

$$1 = \frac{\int_\pi^\infty p(s)\,sds}{\int_\pi^\infty p(s)\,ds} = \frac{\int_\pi^\infty ks^{-\zeta-1}sds}{\int_\pi^\infty ks^{-\zeta-1}sds} = \pi\frac{\zeta}{\zeta-1}$$

which yields $\zeta = 1/(1-\pi)$. The steady state distribution can be written $P(s_{it} \geq x) = (x/\pi)^{-\zeta}$. Finally, by (6) and (7), the distribution of populations and areas is a Pareto with the same exponent ζ.

We also note that $\zeta = 1 - 2(g - \overline{g})/\sigma^2$. This yields the value of the growth rate of productivity: $\overline{g} = g + \sigma^2 (\zeta - 1)/2$. The (endogenous) growth rate of average productivity is higher than the (exogenous) growth rate of a city above the reflecting barrier, because this reflecting barrier makes small cities grow faster. In the Zipf limit where $\pi \to 0$, hence $\zeta \to 1$, the difference between the two growth rates, $\overline{g} - g$, goes to 0. ■

This economy reflects our main empirical findings, (i) and (ii). Point (iii) reflects the findings of Davis and Ortalo-Magné (2008), who find that the fraction of income spent on housing is roughly constant over time and across city sizes.

We note that here, following Rossi-Hansberg and Wright (2007) and Van Nieuwerburgh and Weill (2009), land is not exogenous but instead it is acquired. This is a legitimate modelling idealization in our view. Take a city such as Dallas, which starts with vast quantities of unoccupied land around it. It can grow in a fairly unlimited way, but it needs to pay for the land use, e.g. building infrastructure such as road, electricity and running water. It makes sense to model this activity as a constant-return to scale activity, at least in the first approximation. At the other end of the spectrum, we may have New York. But even it has grown considerably by geographical expansion, which lends credence to our model. It would be interesting, and surely desirable, to extend the model with some sort of increasing cost of land use (given some limit). We conjecture that, if the random growth effects are large enough, this will modify the power law distribution, but will not eliminate it. A calibration of the deviation from the constant return to scale model, and the deviation of the power law, would be useful, but we will not attempt it here.

Here cities are basically constant-return-to-scales economies, except for one large Marshallian force that makes a given good only producible in one city (as "secrets of the trade" may be exclusive that city). Of course, this is a stark model, but it is parsimonious, and is consistent with our scaling facts. In addition, external effects linked to cities may not be huge. For instance, Glaeser (1998) reports quantitatively moderate deviations from the hypothesis that cities are constant-return-to scale (see also Bettencourt et al. (2007)). For instance, Glaeser (1998) reports that the average commute time in cities of less than 100,000 is 20.5 minutes each way, while in cities of more than 1,000,000 it is 31.9 minutes each way[2]. This difference may be small compared to the huge differences in size and area that our model focuses on.

Our model postulates that Gibrat's law holds. However, deviations from Gibrat's law

[2]In a related vein, Ciccone and Hall (1996) estimate that a doubling of density increases productivity by 5.5%, while Davis, Fisher and Whited (2009) finds an increase of 2%. This is a arguably small deviation from the constant-return to scale benchmark we use in our model.

have been found in the literature and for the CCA clusters (e.g. Glaeser, Scheinkman and Shleifer (1995), and Rozenfeld et al. (2008)). A simple theoretical solution to this apparent tension between the data and the idealization used in models based on Gibrat's law is discussed in Gabaix and Ioannides (2004), Section 3.2.2. Urban growth may accommodate a wide range of growth processes exhibiting a Pareto distribution, but also deviations from Gibrat's law, as long as they contain a unit root (which satisfies Gibrat) with respect to the logarithm of city size: in particular, growth processes can have some mean-reverting component that violates Gibrat's law. Under that hypothesis, the deviations from Gibrat's law would come from the mean-reverting component of the growth process, but Gibrat's law in the unit root part of the process would ensure the Pareto law. More research is needed to empirically assess this possibility. It would likely require empirical studies of Gibrat's law over long time intervals.

We think that the model could be extended to add positive and negative agglomeration externalities, which also can generate random growth, as in Gabaix (1999b), Eeckhout (2004), and Rossi-Hansberg and Wright (2007). We also eschew a detailed modelling of the heterogeneity within a city, such as the one in Lucas and Rossi-Hansberg (2002). Such developments would be very welcome, but we propose to defer them to future research.

6 Conclusion

We have used a "bottom-up" approach which allows us to construct cities independently of their "legal" definition, instead using a more geographical and economic basis. The resulting data extend the domain of validity of Zipf's law to a considerable range: we show that when cities are constructed independently of their administrative boundaries, Zipf's law appears to be a genuine regularity for the bulk of the city size distribution. Second, we are able to analyze city areas, which allows for the estimation of a potentially very important quantity in urban economics, and anchors the definition of cities much more in geography. We find evidence for a power-law distribution of areas, with an exponent close to 1. Third, we presented a model incorporating both population and area, that matches our "macro" facts. Fourth, we provide a public good by putting on our web page the correspondence between ZIP code and our Clusters, so that other researchers can use the agglomerations constructed with the CCA, and study dimensions of local economics other than areas and populations.

In the present work we have investigated only two countries. It is natural to extend this study to more countries, an investigation that might offer confirmation of the scaling laws for areas, population and density that we have found, and also perhaps find economically interesting deviations from them. The minimalist model presented here could be extended to incorporate richer specification of the internal structure of cities. We think that this "bottom-up" approach could be useful for a host of urban questions. Combining our geographical approach with land price data could lead to a much more constrained and geography-based theory of the macro and internal structure of cities.

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