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DISASTERS RISK AND BUSINESS CYCLES

François Gourio

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ABSTRACT

To construct a business cycle model consistent with the observed behavior of asset prices, and study the effect of shocks to aggregate uncertainty, I introduce a small, time-varying risk of economic disaster in a standard real business cycle model. The paper establishes two simple theoretical results: first, when the probability of disaster is constant, the risk of disaster does not affect the path of macroeconomic aggregates - a "separation theorem" between macroeconomic quantities and asset prices in the spirit of Tallarini (2000). Second, shocks to the probability of disaster, which generate variation in risk premia over time, are observationally equivalent to preference shocks. An increase in the perceived probability of disaster leads to a collapse of investment and a recession, an increase in risk spreads, and a decrease in the yield on safe assets. To assess the empirical validity of the model, I infer the probability of disaster from observed asset prices and feed it into the model. The variation over time in this probability appears to account for a significant fraction of business cycle dynamics, especially sharp downturns in investment and output such as 2008-IV.

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1 Introduction

The empirical finance literature has provided substantial evidence that risk premia are time-varying (see for instance Campbell and Shiller (1988), Fama and French (1989), Ferson and Harvey (1991), Cochrane (2005)). Yet, standard business cycle models such as the real business cycle model, or the DSGE models used for monetary policy analysis, largely fail to replicate the level, the volatility, and the cyclicality of risk premia. This seems an important neglect, since empirical work suggests a tight connection between risk premia and economic activity. For instance, Philippon (2008) and Gilchrist and Zakrajsek (2007) show that corporate bonds spreads are highly correlated with real physical investment, both in the time series and in the cross-section. A large research, summarized in Backus, Routledge and Zin (2008), shows that the stock market, the term premium, and (negatively) the short rate all lead the business cycle.[1]

I introduce time-varying risk premia in a standard real business cycle model, through a small, stochastically time-varying risk of economic "disaster", following the work of Rietz (1988), Barro (2006), and Gabaix (2007). Existing work has so far been confined to endowment economies, and hence does not consider the feedback from time-varying risk premia to macroeconomic activity. I prove two theoretical results, which hold under the assumption that a disaster reduces total factor productivity (TFP) and the capital stock by the same amount. First, when the risk of disaster is constant, the path for macroeconomic quantities implied by the model is the same as that implied by a model with no disasters, but a different discount factor β. This "observational equivalence" (in a sample without disasters) is similar to Tallarini (2000): macroeconomic dynamics are essentially unaffected by the amount of risk or the degree of risk aversion. Second, when the risk of disaster is time-varying, an increase in probability of disaster is observationally equivalent to a preference shock. This is interesting since these shocks appear to be important in accounting for the data, according to estimation of DSGE models with multiple shocks such as Smets and Wouters (2003). An increase in the perceived probability of disaster can create a collapse of investment and a recession, as risk premia rise, increasing the cost of capital. Demand for precautionary savings increase, leading the yield on less risky assets to fall, while spreads on risky securities increase. These business cycle dynamics occur with no change in current or future total factor productivity.

Quantitatively, I find that this parsimonious model can match many asset pricing facts - the mean, volatility, and predictability of returns - while maintaining the basic success of the RBC model in accounting for quantities. This is important since many asset pricing models which are successful in endowment economies do not generalize well to production economies (as explained in Jermann (1998), Lettau and Uhlig (2001), Kaltenbrunner and Lochstoer (2008)). This second shock also substantially increases the correlation between asset prices (or risk premia) and economic activity, making it closer to the data.

One obvious limitation of the paper is that the probability of disaster is hard to observe. As an empirical exercise, I infer the probability of disaster from asset prices. I then feed into the model this estimated probability of disaster. The variation over time in this probability appears to account for a

[1] Schwert (1989) and Bloom (2008) also show that stock market volatility negatively leads economic activity.

significant fraction of business cycle dynamics, and it especially helps matching sharp downturns.

This risk of an economic disaster could be a strictly rational expectation, or more generally it could reflect a time-varying belief, which may differ from the objective probability - i.e., waves of optimism or pessimism (see e.g. Jouini and Napp (2008)). For instance, during the recent financial crisis, many commentators, including well-known macroeconomists[2], have highlighted the possibility that the U.S. economy could fall into another Great Depression. My model studies the macroeconomic effect of such time-varying beliefs.[3] This simple modeling device captures the idea that aggregate uncertainty is sometimes high, i.e. people sometimes worry about the possibility of a deep recession. It also captures the idea that there are some asset price changes which are not obviously related to current or future TFP, i.e. "bubbles", "animal spirits", and which in turn affect the macroeconomy.

Introducing time-varying risk premia requires solving a model using nonlinear methods, i.e. going beyond the first-order approximation and considering "higher order terms". Researchers disagree on the importance of these higher order terms, and a fairly common view is that they are irrelevant for macroeconomic quantities. Lucas (2003) summarizes: "*Tallarini uses preferences of the Epstein-Zin type, with an intertemporal substitution elasticity of one, to construct a real business cycle model of the U.S. economy. He finds an astonishing separation of quantity and asset price determination: The behavior of aggregate quantities depends hardly at all on attitudes toward risk, so the coefficient of risk aversion is left free to account for the equity premium perfectly.*"[4] My results show, however, that these higher-order terms can have a significant effect on macroeconomic dynamics, when we consider shocks to the probability of disaster.[5]

The paper is organized as follows: the rest of the introduction reviews the literature. Section 2 studies a simple analytical example in an AK model which can be solved in closed form and yields the central intuition for the results. Section 3 gives the setup of the full model and presents some analytical results. Section 4 studies the quantitative implications of the model numerically. Section 5 considers

[2] Greg Mankiw (NYT, Oct 25, 2008): "*Looking back at [the great Depression], it's hard to avoid seeing parallels to the current situation. (...) Like Mr. Blanchard at the I.M.F., I am not predicting another Great Depression. But you should take that economic forecast, like all others, with more than a single grain of salt.*"

Robert Barro (WSJ, March 4, 2009): "*... there is ample reason to worry about slipping into a depression. There is a roughly one-in-five chance that U.S. GDP and consumption will fall by 10% or more, something not seen since the early 1930s.*"

Paul Krugman (NYT, Jan 4, 2009): "*This looks an awful lot like the beginning of a second Great Depression.*"

[3] Of course in reality this change in probability of disaster may be an endogenous variable and not an exogenous shock. But it is useful to understand the effect of an increase in aggregate risk (premia) on the macroeconomy.

[4] Note that Tallarini (2000) actually picks the risk aversion coefficient to match the Sharpe ratio of equity. Since return volatility is very low in his model - there are no capital adjustment costs - he misses the equity premium by several order of magnitudes.

[5] Cochrane (2005, p. 296-297) also discusses in detail the Tallarini (2000) result: "Tallarini explores a different possibility, one that I think we should keep in mind; that maybe the divorce between real business cycle macroeconomics and finance isn't that short-sighted after all (at least leaving out welfare questions, in which case models with identical dynamics can make wildly different predictions). (...) The Epstein–Zin preferences allow him to raise risk aversion while keeping intertemporal substitution constant. As he does so, he is better able to account for the market price of risk (...) but the quantity dynamics remain almost unchanged. In Tallarini's world, macroeconomists might well not have noticed the need for large risk aversion."

some extensions of the baseline model. Section 6 presents the empirical evaluation of the model, backing out the probability of disaster from asset prices.

Related Literature

Gabaix (2009) independently obtained some related results in contemporaneous work. His study has more analytical results, including some interesting examples where fluctuations in risk premia have no macroeconomic consequences. My paper has a more quantitative focus, using Epstein-Zin utility, and focuses on the traditional RBC setup, where a shock to the probability of disaster is equivalent to a preference shock.

This paper is mostly related to three strands of literature. First, a large literature in finance builds and estimates models which attempt to match not only the equity premium and the risk-free rate, but also the predictability of returns and potentially the term structure. Two prominent examples are Bansal and Yaron (2004) and Campbell and Cochrane (1999). However, this literature is limited to endowment economies, and hence is of limited use to analyze the business cycle or to study policy questions.

Second, a smaller literature studies business cycle models (i.e. they endogenize consumption, investment and output), and attempts to match not only business cycle statistics but also asset returns first and second moments. My project is closely related to these papers (A non-exhaustive list would include Jermann (1998), Tallarini (2000), Boldrin, Christiano and Fisher (2001), Lettau and Uhlig (2001), Kaltenbrunner and Lochstoer (2008), Campanele et al. (2008), Croce (2005), Gourio (2008c), Papanikolaou (2008), Kuehn (2008), Uhlig (2006), Jaccard (2008), Fernandez-Villaverde et al. (2008)). Most of these papers consider only the implications of productivity shocks, and generally study only the mean and standard deviations of return. In contrast, my model generates a large variation of economic risk premia, consistent with the empirical finance findings. Many of these papers abstract from hours variation. Several of these papers note that quantities dynamics are unaffected by risk aversion,[6] hence it is sometimes said that asset prices can be discarded. The recent studies of Swanson and Rudebusch (2008a and 2008b) are exceptions on all these counts. The long-run target is to build a medium-scale DSGE model (as in Smets and Wouters (2003) or Christiano, Eichenbaum and Evans (2005)) that is roughly consistent with asset prices.

Finally, the paper draws from the recent literature on "disasters" or rare events (Rietz (1988), Barro (2006), Barro and Ursua (2008), Farhi and Gabaix (2008), Gabaix (2007), Gourio (2008a,2008b), Julliard and Ghosh (2008), Martin (2007), Santa Clara and Yan (2008), Wachter (2008), Weitzmann (2007)). Disasters are a powerful way to generate large risk premia. Moreover, as we will see, disasters are relatively easy to embed into a standard macroeconomic model.

There has been much interest lately in the evidence that the stock market leads TFP and GDP, which has motivated introducing "news shocks" (e.g., Beaudry and Portier (2006), Jaimovich and Rebelo (2008)), but my model suggests that this same evidence could also be rationalized by variations in risk premia due to changes in the probability of disasters.

Last, the paper has the same flavor as Bloom (2008) in that an increase in aggregate uncertainty creates a recession, but the mechanism and the focus (asset prices here) is different. Fernandez-Villaverde

[6] Fernandez-Villaverde et al. (2008) use perturbation methods and report that the first three terms, which are calculated symbolically by the computer, are independent of risk aversion (there is, of course, a steady-state adjustment).

et al. (2009) also study the effect of shocks to risk in macro models, but they focus on open economy issues.

2 A simple analytical example in an AK economy

To highlight the key mechanism of the paper, consider a simple economy with a representative consumer who has power utility:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_t^{1-\gamma}}{1-\gamma},$$

where C_t is consumption and γ is the risk aversion coefficient (and also the inverse of the the intertemporal elasticity of substitution of consumption). This consumer operates an AK technology:

$$Y_t = AK_t,$$

where Y_t is output, K_t is capital, and A is productivity, which is assumed to be constant.[7] The resource constraint is:

$$C_t + I_t \leq AK_t.$$

The economy is randomly hit by disasters. A disaster destroys a share b_k of the capital stock. This could be due to a war which physically destroys capital, but there are alternative interpretations. For instance, b_k could reflect expropriation of capital holders (if the capital is taken away and then not used as effectively), or it could be a "technological revolution" that makes a large share of the capital worthless. It could also be that even though physical capital is not literally destroyed, some intangible capital (such as matches between firms, employees, and customers) is lost. Finally, one can imagine a situation where the demand for some goods falls sharply, rendering worthless the factories which produce them.[8]

Finally, the probability of a disaster varies over time. To maintain tractability I assume in this section that it is *i.i.d.*: p_t, the probability of a disaster at time $t+1$, is drawn at the beginning of time t from a constant cumulative distribution function F. The law of accumulation for capital is thus:

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + I_t, \text{ if } x_{t+1} = 0, \text{ (no disaster)} \\ &= ((1-\delta)K_t + I_t)(1-b_k), \text{ if } x_{t+1} = 1, \text{ (disaster)} \end{aligned}$$

where x_{t+1} is a binomial variable which is 1 with probability p_t and 0 with probability $1-p_t$. A disaster does not affect productivity A. I will relax this assumption in section 3.[9] Finally, I assume that the two random variables p_{t+1}, and x_{t+1} are independent. I also discuss this assumption in more detail in section 3.

[7] It is easy to extend this example to the case where A is stochastic; this does not affect the results, so in the interest of simplicity I omit this extension in the example and consider it in the full model of section 3.

[8] In a large downturn, the demand for some luxury goods such as boats, private airplanes, etc. would likely fall sharply. If this situation were to last, the boats-producing factories would never operate at capacity.

[9] In an AK model, a permanent reduction in productivity would lead to a permanent reduction in the growth rate of the economy, since permanent shocks to A affect the *growth rate* of output permanently.

This model has one endogenous state K and one exogenous state p, and there is one control variable C. There are two shocks: the realization of disaster $x' \in \{0, 1\}$, and the draw of a new probability of disaster p'. The Bellman equation for the representative consumer is:

$$\begin{aligned} V(K,p) &= \max_{C,I}\left\{\frac{C^{1-\gamma}}{1-\gamma} + \beta E_{p',x'}\left(V(K',p')\right)\right\} \\ s.t. &: \\ C + I &\leq AK, \\ K' &= \left((1-\delta)K + I\right)\left(1 - x'b_k\right). \end{aligned}$$

The assumptions made ensure that V is homogeneous, i.e. we can guess and verify that V is of the form $V(K,p) = \frac{K^{1-\gamma}}{1-\gamma}g(p)$, where g is defined through the Bellman equation:

$$g(p) = \max_i \left\{\frac{(A-i)^{1-\gamma}}{1-\gamma} + \beta\frac{(1-\delta+i)^{1-\gamma}\left(1-p+p(1-b_k)^{1-\gamma}\right)}{1-\gamma}\left(E_{p'}g(p')\right)\right\}, \tag{1}$$

where $i = \frac{I}{K}$ is the investment rate. This implies that consumption and investment are both proportional to the current stock of capital, but they typically depend on the probability of disaster as well:

$$\begin{aligned} C_t &= f(p_t)K_t, \\ I_t &= h(p_t)K_t. \end{aligned}$$

As a result, when a disaster occurs and the capital stock falls by a factor b_k, both consumption and investment also fall by a factor b_k. Given that there are no adjustment costs, the value of capital is equal to the quantity of capital, and hence it falls also by a factor b_k in a downturn. Finally, the return on an all-equity financed firm is:

$$R^e_{t,t+1} = (1-\delta+A)\left(1 - x_{t+1}b_k\right),$$

i.e. it is $1-\delta+A$ if there is no disaster, and $(1-\delta+A)(1-b_k)$ if there is a disaster. Clearly, the equity premium will be high, since the equity return and consumption are correlated and are affected by large shocks. Moreover, the equity premium is larger when p_t is higher, since risk is higher.[10]

Let us finally turn to the effect of p on the consumption-savings decision. Using equation (1), the first-order condition with respect to i yields, after rearranging:

$$\left(\frac{A-i}{1-\delta+i}\right)^{-\gamma} = \beta\left(1-p+p(1-b_k)^{1-\gamma}\right)\left(E_{p'}g(p')\right).$$

Given that p is *i.i.d.*, the expectation of g on the right-hand side is independent of the current p. The left-hand side is an increasing function of i. The term $(1-b_k)^{1-\gamma}$ is greater than unity if and only if $\gamma > 1$. Hence, i is increasing in p if $\gamma > 1$, it is decreasing in p if $\gamma < 1$, and it is independent of p if $\gamma = 1$.

The intuition for this result is as follows: if p goes up, investment in physical capital becomes more risky and hence less attractive, i.e. the risk-adjusted return goes down.[11] The effect of a change in the

[10] Note, however, that the return on capital will not be volatile - in this example, it is constant in a sample without disasters. Adding leverage can create substantial volatility.

[11] By risk-adjusted return I mean $E(R^{1-\gamma})^{\frac{1}{1-\gamma}}$, where R is the physical return on capital. See Weil (1989).

return on the consumption-savings choice depends on the value of the IES, because of offsetting wealth and substitution effects. If the IES is unity (i.e. utility is log), savings are unchanged and thus the savings or investment rate does not respond to a change in the probability of disaster. But if the IES is larger than unity, i.e. $\gamma < 1$, the substitution effect dominates, and i is decreasing in p. Hence, an increase in the probability of disaster leads initially, in this model, to a decrease in investment, and an increase in consumption, since output is unchanged on impact. Next period, the decrease in investment leads to a decrease in the capital stock and hence in output. This simple analytical example thus shows that a change in the perceived probability of disaster can lead to a decline in investment and output. The key mechanism is the effect of *rate-of-return uncertainty* on the optimal savings decision.

Extension to Epstein-Zin preferences

To illuminate the respective role of risk aversion and the intertemporal elasticity of substitution, it is useful to extend the preceding example to the case of Epstein-Zin utility. Assume, then, that the utility V_t satisfies the recursion:

$$V_t = \left((1-\beta) C_t^{1-\gamma} + \beta E_t \left(V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}, \tag{2}$$

where θ measures risk aversion towards static gambles, γ is the inverse of the intertemporal elasticity of substitution (IES) and β reflects time preference. It is straightforward to extend the results above; the first-order condition now reads

$$\left(\frac{A-i}{1-\delta+i} \right)^{-\gamma} = \frac{\beta}{1-\beta} \left(1-p+p(1-b_k)^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \left(E_{p'} g(p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}},$$

and we can apply the same argument as above, in the realistic case where $\theta \geq 1$: the now *risk-adjusted* return on capital is $\left(1-p+p(1-b_k)^{1-\theta}\right)^{\frac{1}{1-\theta}}$; it falls as p rises; an increase in the probability of disaster will hence reduce investment if and only if the IES is larger than unity.[12] Hence, the parameter which determines the sign of the response is the IES, and the risk aversion coefficient (as long as it is greater than unity) determines the magnitude of the response only. While this example is revealing,[13] it has a number of simplifying features, which lead us to turn now to a quantitative model.

3 A Real Business Cycle model with Time-Varying Risk of Disasters

This section introduces a real business cycle model with time-varying risk of disaster and study its implications, first analytically, and then numerically. This model extends the simple example of the previous section in the following dimensions: (a) the probability of disaster is persistent instead of *i.i.d.*; (b) the production function is neoclassical and affected by standard TFP shocks; (c) labor is elastically

[12] The disaster reduces the mean return itself, but this is actually not important. We could assume that there is a small probability of a "capital windfall" so that a change in p does not affect the mean return on capital. Crucially, what matters here is the risk-adjusted return on capital, $E(R^{1-\theta})^{\frac{1}{1-\theta}}$, and a higher risk reduces this return.

[13] This example is related to work by Epaulard and Pommeret (2003), Jones, Manuelli and Siu (2005a, 2005b), and to the earlier work of Obstfeld (1994). It is easy to extend the example to the case of stochatic productivity A, which does not affect the results at all.

supplied; (d) disasters may affect total factor productivity as well as capital; (e) there can be capital adjustment costs.

3.1 Model Setup

The representative consumer has preferences of the Epstein-Zin form, and the utility index incorporates hours worked N_t as well as consumption C_t:

$$V_t = \left(u(C_t, N_t)^{1-\gamma} + \beta E_t \left(V_{t+1}^{1-\theta} \right)^{\frac{1-\gamma}{1-\theta}} \right)^{\frac{1}{1-\gamma}}, \tag{3}$$

where[14] the per period felicity function $u(C, N)$ is assumed to have the following form:

$$u(C, N) = C^{\upsilon}(1 - N)^{1-\upsilon}.$$

Note that γ is the inverse of the intertemporal elasticity of substitution (IES), and θ measures risk aversion towards static gambles, because u is homogeneous of degree one. But this is risk aversion over the bundle of consumption and leisure.

There is a representative firm, which produces output using a standard Cobb-Douglas production function:

$$Y_t = K_t^{\alpha} \left(z_t N_t \right)^{1-\alpha},$$

where z_t is total factor productivity (TFP), to be described below. The firm accumulates capital subject to adjustment costs:

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right) K_t, \text{ if } x_{t+1} = 0, \\ &= \left((1-\delta)K_t + \phi\left(\frac{I_t}{K_t}\right) K_t \right)(1 - b_k), \text{ if } x_{t+1} = 1, \end{aligned}$$

where ϕ is an increasing and concave function, which curvature captures adjustment costs, and x_{t+1} is 1 if there is a disaster at time $t+1$ (with probability p_t) and 0 otherwise (probability $1 - p_t$). At this stage b_k is a parameter, which could be zero - i.e., a disaster only affects TFP. We explore quantitatively the role of b_k in section 5.

The resource constraint is

$$C_t + I_t \leq Y_t.$$

Aggregate investment cannot be negative:

$$I_t \geq 0.$$

Finally, we describe the shock processes. Total factor productivity is affected by the "normal shocks" ε_t as well as the disasters. A disaster reduces TFP by a permanent amount b_{tfp} :

$$\begin{aligned} \log z_{t+1} &= \log z_t + \mu + \sigma\varepsilon_{t+1}, \text{ if } x_{t+1} = 0, \\ &= \log z_t + \mu + \sigma\varepsilon_{t+1} + \log(1 - b_{tfp}), \text{ if } x_{t+1} = 1, \end{aligned}$$

[14]Note that it is commonplace to have a $(1 - \beta)$ factor in front of $u(C, N)$ in equation (3), but this is merely a normalization, which it is useful to forgo in this case.

where μ is the drift of TFP, and σ is the standard deviation of "normal shocks". Here too, we will consider various values for b_{tfp}, including possibly zero - i.e., a disaster only destroys capital. Last, p_t follows a stationary Markov process with transition function Q. In the quantitative application, I will simply assume that the log of p_t follows an AR(1) process.

I assume that $p_{t+1}, \varepsilon_{t+1}$, and x_{t+1} are independent conditional on p_t. This assumption requires that the occurrence of a disaster today does not affect the probability of a disaster tomorrow. This assumption could be wrong either way: a disaster today may indicate that the economy is entering a phase of low growth or is less resilient than thought, leading agents to revise upward the probability of disaster, following the occurrence of a disaster; but on the other hand, if a disaster occurred today, and capital or TFP fell by a large amount, it is unlikely that they will fall again by a large amount next year. Rather, historical evidence suggests that the economy is likely to grow above trend for a while (Gourio (2008a), Barro et al. (2009)). In section 5, I extend the model to consider these different scenarios.

3.2 Bellman Equation

This model has three states: capital K, technology z and probability of disaster p; two independent controls: consumption C and hours worked N; and three shocks: the realization of disaster $x' \in \{0,1\}$, the draw of the new probability of disaster p', and the "normal shock" ε'. The first welfare theorem holds, hence the competitive equilibrium is equivalent to a social planner problem, which is easier to solve. Denote $V(K,z,p)$ the value function, and define $W(K,z,p) = V(K,z,p)^{1-\gamma}$. The social planning problem can be formulated as:[15]

$$\begin{aligned} W(K,z,p) &= \max_{C,I,N} \left\{ \left(C^{\upsilon}(1-N)^{1-\upsilon}\right)^{1-\gamma} + \beta \left(E_{p',z',x'} W(K',z',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\}, \qquad (4) \\ s.t. &: \\ C + I &\leq z^{1-\alpha} K^{\alpha} N^{1-\alpha}, \\ K' &= \left((1-\delta)K + \phi\left(\frac{I}{K}\right) K \right) (1 - x' b_k), \\ \log z' &= \log z + \mu + \sigma\varepsilon' + x' \log(1-b_{tfp}). \end{aligned}$$

A standard homogeneity argument implies that we can write $W(K,z,p) = z^{\upsilon(1-\gamma)} g(k,p)$, where $k = K/z$, and g satisfies the associated Bellman equation:

$$\begin{aligned} g(k,p) &= \max_{c,i,N} \left\{ \begin{array}{c} c^{\upsilon(1-\gamma)} (1-N)^{(1-\upsilon)(1-\gamma)} \\ +\beta e^{\mu\upsilon(1-\gamma)} \left(E_{p',\varepsilon',x'} e^{\sigma\varepsilon'\upsilon(1-\theta)} \left(1 - x' + x'(1-b_{tfp})^{\upsilon(1-\theta)}\right) g(k',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\} \qquad (5) \\ s.t. &: \\ c &= k^{\alpha} N^{1-\alpha} - i, \\ k' &= \frac{(1-x'b_k)\left((1-\delta)k + \phi\left(\frac{i}{k}\right)k\right)}{e^{\mu+\sigma\varepsilon'}(1-x'b_{tfp})}. \end{aligned}$$

Here $c = C/z$ and $i = I/z$ are consumption and investment detrended by the stochastic technology trend z. This simplification will lead to some analytical results, and can further be studied using standard numerical methods since k is stationary.

[15] Because we take a power $1-\gamma$ of the value function, if $\gamma > 1$, the max must be transformed into a min.

3.3 Asset Prices

It is straightforward to compute asset prices in this economy. The stochastic discount factor is given by the formula

$$M_{t,t+1}=\beta\left(\frac{C_{t+1}}{C_t}\right)^{\upsilon(1-\gamma)-1}\left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\upsilon)(1-\gamma)}\left(\frac{V_{t+1}}{E_t\left(V_{t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta}. \tag{6}$$

The price of a one-period risk-free bond is

$$P_{rf,t}=E_t\left(M_{t,t+1}\right)\overset{def}{=}P_{rf}(k,p).$$

This risk-free asset may not have an observable counterpart. Following Barro (2006), I will assume that government bonds are not risk-free but are subject to default risk during disasters.[16] More precisely, if there is a disaster, then with probability q the bonds will default and the recovery rate will be r. The T-Bill price can then be easily computed as

$$P_{1,t}=E_t\left(M_{t,t+1}(1-x_{t+1}q(1-r))\right)\overset{def}{=}P_1(k,p).$$

Computing the yield curve is conceptually easy using the standard recursion for zero-coupon bonds:

$$P_{n,t}=E_t\left(M_{t,t+1}P_{n-1,t+1}(1-x_{t+1}q(1-r)\right)\overset{def}{=}P_n(k,p).$$

Here I assume that a disaster simply reduces the face value of the bond (and does not affect its maturity). The ex-dividend value of the firm assets F_t is defined through the value recursion:

$$F_t=E_t\left(M_{t,t+1}\left(D_{t+1}+F_{t+1}\right)\right),$$

where $D_t=F(K_t,z_tN_t)-w_tN_t-I_t$ is the payout of the representative firm, and w_t is the wage rate, given by the marginal rate of substitution of the representative consumer between consumption and leisure. The equity return is then

$$R_{t,t+1}=\frac{D_{t+1}+F_{t+1}}{F_t}.$$

There is an alternative derivation of firm value and returns. Using the Q-theory, we see that

$$F_t=\frac{(1-\delta)K_t+\phi(I_t/K_t)K_t}{\phi'\left(\frac{I_t}{K_t}\right)},$$

where $(1-\delta)K_t+\phi(I_t/K_t)K_t=\frac{K_{t+1}}{1-x_{t+1}b_k}$ is the capital if no disaster occurs. (In the standard model, $p_t=0$, but here the amount of capital available tomorrow is unknown, since some capital is destroyed in the event of a disaster.) As a result, we can find an equivalent expression for the equity return, often known as the investment return, which holds as long as investment is positive:

$$\begin{aligned}R_{t,t+1} &= \frac{F_{t+1}+D_{t+1}}{F_t}=\frac{\frac{(1-\delta)K_{t+1}+\phi(I_{t+1}/K_{t+1})K_{t+1}}{\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)}+D_{t+1}}{\frac{(1-\delta)K_t+\phi(I_t/K_t)K_t}{\phi'\left(\frac{I_t}{K_t}\right)}}\\ &= (1-x_{t+1}b_k)\,\phi'\left(\frac{I_t}{K_t}\right)\left[\frac{1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)}{\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)}+\frac{\alpha K_{t+1}^{\alpha}z_{t+1}^{1-\alpha}N_{t+1}^{1-\alpha}-I_{t+1}}{K_{t+1}}\right].\end{aligned}$$

[16] Empirically, default often takes the form of high rates of inflation which reduces the real value of nominal government debt.

This expression is similar to that in Jermann (1998) or Kaltenbrunner and Lochstoer (2008), but for the presence of the term $(1 - x_{t+1}b_k)$, which reflects the capital destruction following a disaster. Finally, I will also compute the price of a leveraged claim on consumption, defined by its payoff C_t^{λ}, where λ is a leverage parameter. The motivation is that the dividend process implied by the model does not match well the dividend process in the data. In the real world, firms have financial leverage and operating leverage (e.g. fixed costs and labor contracts). This is a substantial source of profit volatility, which is not present in the model. Under some conditions, the only effect of this leverage is to modify the payout process. In the quantitative section I will use the model-implied price of a leveraged claim to consumption as the counterpart to the real-world equity.[17]

3.4 Analytical results

In this section, and before turning to the numerical analysis, we establish two simple, yet important, analytical results which follow directly from equation (5).

Proposition 1 *Assume that the probability of disaster p is constant, and that $b_k = b_{tfp}$ i.e. productivity and capital fall by the same amount if there is a disaster. Then, in a sample without disasters, the quantities implied by the model (consumption, investment, hours, output and capital) are the same as those implied by a model with no disasters ($p = 0$), but a different time discount factor $\beta^* = \beta(1 - p + p(1 - b_k)^{\upsilon(1-\theta)})^{\frac{1-\gamma}{1-\theta}}$. Assuming $\theta \geq 1$, we have $\beta^* \leq \beta$ if and only if $\gamma < 1$. Asset prices, however, will be different under the two models; in particular, let $\overline{R}$ be the gross return on equity in normal times, and let $\overline{d}$ be the dividend-capital ratio, then in a disaster, the return is $\overline{R}(1 - b_k) + b_k\overline{d}$, which is low, leading to a large equity premium.*

Proof. Notice that if $b_k = b_{tfp}$, then $k' = \frac{\left((1-\delta)k+\phi\left(\frac{i}{k}\right)k\right)}{e^{\mu+\sigma\varepsilon'}}$ is independent of the realization of disaster x'. Hence, we can rewrite the Bellman equation by decomposing the expectation of future values,

$$g(k) = \max_{c,i,N} \left\{ \begin{array}{c} c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} \\ +\beta e^{\mu\upsilon(1-\gamma)} \left(E_{x'} \left(1 - x' + x'(1-b_{tfp})^{\upsilon(1-\theta)}\right) E_{\varepsilon'} e^{\sigma\varepsilon'\upsilon(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

i.e.:

$$g(k) = \max_{c,i,N} \left\{ c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} + \beta^* e^{\mu\upsilon(1-\gamma)} \left(E_{\varepsilon'} e^{\sigma\varepsilon'\upsilon(1-\theta)} g(k')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\}.$$

We see that this is the same Bellman equation as the one in a standard neoclassical model, with discount rate β^*. As a result, the policy functions $c = C/z$, $i = I/z$, etc. are the same, so the implied quantities are the same, as long as no disaster occurs.[18] Asset prices, on the other hand, are driven by the stochastic

[17] The results are very similar if one defines dividends as a levered claim on output rather than consumption. The results are also similar, at least qualitatively, if one computes the levered return using the equity and long-term (five years) bond return, assuming that firms maintain a constant market or a constant bok leverage.

[18] Even after a disaster, the policy functions are the same, i.e. given the new levels of k and p (or K, z, and p), the two models predict the same quantities. However, the destruction of capital in a disaster is not possible in the model with $p = 0$ - the capital accumulation equation must hold without shocks (the large TFP decline is highly unlikely if shocks are normally distributed, but it is possible). If the capital stock is not observed, the observational equivalence result extends to any sample, including disasters or not.

discount factor, which has the following expression (see the computational appendix):

$$M(k,k',\varepsilon',x') = \beta\left(\frac{z'}{z}\right)^{(\gamma-\theta)\upsilon+\upsilon(1-\gamma)-1}\left(\frac{c(k')}{c(k)}\right)^{\upsilon(1-\gamma)-1}\left(\frac{1-N(k')}{1-N(k)}\right)^{(1-\upsilon)(1-\gamma)}\times ...$$

$$\left(\frac{g(k')^{\frac{1}{1-\gamma}}}{E_{z',x'}\left(\left(\frac{z'}{z}\right)^{\upsilon(1-\theta)}g(k')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta},$$

and of course the term z'/z depends on the realization of a disaster x'. In a disaster, the return on capital is

$$R_{t,t+1} = \phi'\left(\frac{I_t}{K_t}\right)\left[\frac{1-\delta+\phi\left(\frac{I_{t+1}}{K_{t+1}}\right)}{\phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)}(1-b_k)+\frac{\alpha K_{t+1}^{\alpha}z_{t+1}^{1-\alpha}N_{t+1}^{1-\alpha}-I_{t+1}}{K_{t+1}}\right]$$

and since the investment and the capital stock fall by the same amount, the investment rate is unaffected by the disaster; as a result the return is the same as if no disaster occurs, except for the term $1-b_k$; the dividend rate (profit less investment over capital) is itself unchanged. ■

Discussion of Result 1: This result is in the spirit of Tallarini (2000): fixing the asset pricing properties of a RBC model need not change the quantity dynamics. An economy with a high equity risk premium due to disasters ($p > 0$) is observationally equivalent to the standard stochastic growth model ($p = 0$), with a different β. One possible calibration of the model without disasters (e.g., Cooley and Prescott (1995)) is to pick β to match the observed return on stocks. This calibration would pick β^* and hence yield exactly the same implications as the model with disasters. Without the adjustment of β, the quantity implications are very slightly different. This is illustrated in the top panel of Figure 2 which depicts the impulse response of quantities to a TFP shock in three models: (a) the model with $p = 0$, (b) the model with constant positive p, and (c) the benchmark calibration with time-varying p. The differences can be seen in the scale (y-axis), but they are tiny. For this calibration, we have $\beta = .993$, and $\beta^* \approx .9924$. Of course, asset prices will be different, and in particular the equity premium will be higher, as seen in the bottom panel of Figure 1 - the average returns are very different across the three models. The observational equivalence is broken in a long enough sample since disasters must occur. (The observational equivalence would also be broken if one observes assets contingent on disasters, since the prices would be different under the two models.)

The assumption $b_k = b_{tfp}$, simplifies the analysis substantially: the steady-state of the economy shifts due to a change in z, but the ratio of capital to productivity is unaffected by the disaster, i.e. the economy is in the same position relative to its steady-state after the disaster and before the disaster. As a result, a disaster will simply reduce investment, output, and consumption by a factor $b_k = b_{tfp}$, and hours will be unaffected. The economy jumps from one steady-state to another steady-state, and there are no further transitional dynamics. Obviously, the possibility of disaster affects the choice of how much to save, and hence it changes β, but the response to a standard TFP shock is not affected. As emphasized by Cochrane (2005), in a RBC model there is little that agents can do to increase or decrease the amount of uncertainty that they face.[19]

[19] An interesting extension of the model is to have technologies with different levels of riskiness, i.e. different exposures to disasters. Then, an economy with high p would be different than an economy with low p in a more complex way, since agents would like to pick safer, lower-mean technologies.

This same result implies that the "steady-state" level of capital stock will be affected.[20] If risk aversion θ is greater than unity, and the IES is above unity, then $\beta^* < \beta$, leading people to save less and the steady-state capital stock is lower than in a model without disasters. While higher risk to productivity leads to higher precautionary savings, it is well known since Sandmo (1970) that rate-of-return risk can reduce savings (see Angeletos (2007), and Weil (1989) for related analysis).

While this first result is interesting, it is not fully satisfactory however, since the constant probability of disaster implies (nearly) constant risk premia, and hence P-D ratios are too smooth, and returns not volatile enough.[21] This motivates extending the result for a time-varying p.

Proposition 2 *Assume still that $b_k = b_{tfp}$, but that p follows a stationary Markov process. Then, in a sample without disaster, the quantities implied by the model are the same as those implied by a model with no disasters, but with stochastic discounting (i.e. β follows a stationary Markov process).*

Proof. This follows from a similar argument: rewrite the Bellman equation as:

$$g(k,p) = \max_{c,i,N} \left\{ \begin{array}{c} c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} \\ +\beta e^{\mu\upsilon(1-\gamma)} \left(E_{x'} \left(1 - x' + x'(1-b_{tfp})^{\upsilon(1-\theta)}\right) E_{\varepsilon',p'} e^{\sigma\varepsilon'\upsilon(1-\theta)} g(k',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

then define $\beta(p) = \beta E_{x'}\left(1 - x' + x'(1-b_{tfp})^{\upsilon(1-\theta)}\right)^{\frac{1-\gamma}{1-\theta}}$. By definition, $\beta(p) = \beta\left(1 - p + p(1-b_{tfp})^{\upsilon(1-\theta)}\right)^{\frac{1-\gamma}{1-\theta}}$, and, assuming $\theta \geq 1$, β is increasing in p if and only if $\gamma < 1$. We have:

$$g(k,p) = \max_{c,i,N} \left\{ (1-\beta)c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} + \beta(p) e^{\mu\upsilon(1-\gamma)} \left(E_{\varepsilon',p'} e^{\sigma\varepsilon'\upsilon(1-\theta)} g(k',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \right\},$$

i.e. the Bellman equation of a model with time-varying β, but no disasters. ■

Discussion of result 2: Result 2 shows that the time-varying risk of disaster has the same implications for quantities as a preference shock. It is well known that these shocks have significant effect on macroeconomic quantities (a point that we will quantify later). In a sense, this version of the model breaks the "separation theorem" of Tallarini (2000): the source of time-varying risk premia in the model will affect quantity dynamics.

This result is interesting in light of the empirical literature which suggests that "preference shocks" or "equity premium shocks" may be important (Smets and Wouters (2003) and the many papers that follow). Chari, Kehoe and McGrattan (2009) complain that these shocks lack microfoundations. My model provides a simple microfoundation, which allows to tie these shocks to asset prices precisely. Of course, my model is much "smaller" than the medium-scale models of Smets and Wouters (2003), or Christiano, Eichenbaum and Evans (2005), but I conjecture that this equivalence should hold in larger versions.

[20] By steady-state we mean the level to which the capital stock converges in the absence of small shocks ε and if no disasters are realized. Intuitively, the same result should hold for the average (ergodic) capital stock, with the shocks ε being realized.

[21] In an endowment economy where consumption and dividends follow random walk processes, these statements are exact. In our case, the processes are not exactly random walk, because of the TFP shock. However, the general intuition carries over, as we will see in the quantitative section.

Interestingly, this suggests that it is technically feasible to make DSGE models consistent with risk premia. A full non-linear solution of a medium-scale DSGE model is daunting. But under this result, we can solve the quantities of the model for $p = 0$ and a shock process for β, which we know is well approximated with a log-linear approximation. Appendix 1 details this solution method.

Note that Propositions 1 and 2 require that $b_k = b_{tfp}$; analytical results are impossible without this assumption. Numerical results suggest that tesult 1 is robust to this assumption, in that the dynamic response to a TFP shock is largely unaffected by the presence of disasters. Result 2, however, relies on this assumption more heavily.. If disasters affect only TFP, then an increase in p will lead people to want to hold more capital, for standard precautionary savings reasons. This is true regardless of the IES. We discuss further and relax the assumption $b_k = b_{tfp}$ in Section 5.

4 Quantitative Results

In general, the model cannot be solved analytically, so I resort to a numerical approximation. Of course, a nonlinear method is crucial to analyze time-varying risk premia. I use a standard policy function iteration algorithm, which is described in detail in appendix 2.

This section first presents the calibration. Next, I study the implications of the model for business cycle quantities and for the first and second moments of asset returns, as well as for the predictability of stock returns. Finally, I discuss the cyclicality of asset prices.

4.1 Calibration

Parameters are listed in Table 1. The period is one quarter. Many parameters follow the business cycle literature (Cooley and Prescott (1995)). Risk aversion is 6, but note that this is the risk aversion over the consumption-hours bundle. Since the share of consumption in the utility index is .3, the effective risk aversion to a consumption gamble is 1.8. (For the baseline calibration, hours worked do not change when there is a disaster, hence this utility index is three times less volatile than consumption in disasters.) Adjustment costs are zero in the baseline model, as in the standard RBC model.

The intertemporal elasticity of substitution of consumption (IES) is set equal to 2. There is a large debate regarding the value of the IES. Most direct estimates using aggregate data find low numbers (e.g. Hall (1988)), but this view has been challenged by several authors (see among others Bansal and Yaron (2004), Guvenen (2006), Mulligan (2004), Vissing-Jorgensen (2002)). As emphasized by Bansal and Yaron (2004), a low IES has the counterintuitive effects that higher expected growth lowers asset prices, and higher uncertainty increases asset prices. The IES plays a key role for only one part of my results, namely the response of macroeconomic quantities to an increase in the probability of disaster.

One crucial element of the calibration is the probability and size of disaster. I assume that $b_k = b_{tfp} = .43$ and the probability is .017 per year on average. These numbers are motivated by the evidence in Barro (2006) who reports this unconditional probability, and the risk-adjusted size of disaster is on average 43%. In my model, with $b_k = b_{tfp} = .43$, both consumption and output fall by 43% if there is a disaster. (Barro actually uses the historical distribution of sizes of disaster. In his model, this distribution is equivalent to a single disaster with size 43%.) Note that since TFP is $z^{1-\alpha}$, the actual drop in TFP is 30.2%.

Whether one should model a disaster as a capital destruction or a reduction in TFP is an important question. Clearly some disasters, e.g. in South America since 1945, or Russia 1917, affected TFP, perhaps by introducing an inefficient government and poor policies. On the other hand, World War II led in many countries to massive destructions and losses of human capital. It would be interesting to gather further evidence on disasters, and measure b_k and b_{tfp} directly. This is beyond the scope of this paper, and I will hence consider different values for these parameters in section 5 as a robustness test. As I discuss there, the benchmark case $b_k = b_{tfp}$ is parsimonious and makes the model consistent with several aspects of the data.

The second crucial element is the persistence and volatility of movements in this probability of disaster. I assume that the log of the probability follows an AR(1) process:

$$\log p_{t+1} = \rho_p \log p_t + (1 - \rho_p) \log \overline{p} + \sigma_p \varepsilon_{p,t+1},$$

where $\varepsilon_{p,t+1}$ is *i.i.d.* $N(0,1)$. The parameter $\overline{p}$ is picked so that the average probability is .017 per year, and I set $\rho_p = .96$ and the unconditional standard deviation $\frac{\sigma_p}{\sqrt{1-\rho_p^2}} = 1.5$, which allows the model to fit reasonably well the volatility and predictability of risk premia.[22] Regarding the default of government bonds during disasters, I follow the work of Barro (2006): conditional on a disaster, government bonds default with probability .6, and the default rate is the size of the disaster. For simplicity, I assume that all bonds (no matter their maturity) default by the same amount if there is a disaster.[23] Finally, the leverage parameters λ is set to 3, a standard value in the literature.

On top of this benchmark calibration, I will also present results from different calibrations (no disasters, constant probability of disasters, and in section 5 more extensions) to illustrate the sensitivity of the results.

Some may argue that this calibration of disasters is extreme. A few remarks are in order. First, a long historical view makes this calibration sound more reasonable, as shown by Barro (2006) and Barro and Ursua (2008). An example is the U.K., which sounded very safe in 1900, but experienced a variety of very large negative shocks during the century. As shown by Martin (2008), the key ingredient is that there is a tiny probability of a very large shock.

The key assumption is that beliefs regarding the possibility of large drops in GDP vary over time. The recent financial crisis offers an illustration, with many investors and economists worrying about a new Great Depression. Over the past year, some very prominent macroeconomists suggested that a new Great Depression is a distinct possibility, as illustrated by the quotes in the introduction. Clearly, some investors decided to pull out of the stock market in Fall 2008, partly out of the fear that the economy might continue to go down for a few more years, as it did in 1929. The recent crisis also illustrates some large declines in consumption or GDP: for instance, real consumption in Iceland is expected to drop

[22] See Gabaix (2007), Gourio (2008b), and Wachter (2008) for a similar calibration of the endowment economy model. Also, this equation allows the probability to be greater than one, however I will approximate this process with a finite Markov chain, which ensures that $0 < p_t < 1$ for all $t \geq 0$.

[23] Gabaix (2007) shows that if a disaster leads to a jump in inflation, then long-term bonds are more risky than short-term bonds, which can explain a variety of facts regarding the yield curve.

by 7.1% in 2008 and 24.1% in 2009, according to the official government forecast (as of January 2009). According to the IMF World economic outlook (April 2009), output in Germany, Ireland, Ukraine, Japan, Latvia, Singapore, Taiwan, are expected to contract by respectively 5.6%, 8.0%, 8.0%, 6.2%, 12.0%, 10.0%, 7.5% in 2009 *alone*.

It is also possible to change the calibration - e.g., increase risk aversion, which is only 2, and reduce the size of disasters. One can also employ fairly standard devices to boost the equity premium, and reduce the probability of disaster - e.g., the disasters may be concentrated on a limited set of agents, or some agents may have background risk (private businesses); or idiosyncratic risk might be countercyclical (becoming unemployed during the Great Depression was no fun). These features could all be added to the model, at a cost in terms of complexity, and would likely reduce the magnitude of disasters required to make the model fit the data.

4.2 Response to shocks

4.2.1 The dynamic effect of a disaster

Figures 1 presents the dynamics of quantities following a disaster, for each of the three possible types of disasters: the benchmark model ($b_k = b_{tfp} = .43$ as in the baseline calibration); a capital disaster ($b_k = .43$ and $b_{tfp} = 0$) whereby capital is destroyed but TFP is unaffected; and a TFP disaster ($b_{tfp} = .43$ and $b_k = 0$). Of course, in post WWII U.S., no disasters have occurred, so these pictures are not to be matched to any data. Yet, they matter, because *the properties of asset prices and quantities are driven by what would happen if there was a disaster*. For instance, to generate a large equity premium, a model must endogenously generate that consumption and stock returns are extremely low during disasters.[24] In the benchmark model, as implied by proposition 2, there are no transitional dynamics following the disaster: consumption drops by a factor $1 - b_k$, just like in the endowment economy of Rietz (1988) and Barro (2006); and the return on capital is approximately $1 - b_k$ due to the capital destruction, hence the model is successful in generating an equity premium.

The case of a capital disaster is interesting because it leads endogenously to a recovery. The transitional dynamics here are exactly that of the standard Ramsey-Cass-Koopmans model (e.g., King and Rebelo (1993)). The return on capital is low on impact because of the destruction, but consumption does not fall as much as in the first case, given the anticipated recovery. Adding adjustment costs slows down the recovery, but makes the return on capital not as bad since Tobin's Q increases after the disaster.

Finally, a TFP disaster without a capital destruction leads to an excess of capital relative to its productivity. Investment falls to zero: the aggregate irreversibility constraint binds. Consumption and output then decline over time. In that case, the initial low return on capital is solely due to the binding irreversibility constraint - there is no capital destruction.

[24] Since leisure enters the utility function, low hours worked could also potentially help. Moreover, Epstein-Zin utility implies that state prices are also determined by continuation utility, (expected future consumption and hours worked), i.e. the full path of transitional dynamics following a disaster.

4.2.2 The dynamic effect of a TFP shock

As illustrated in figure 2, and discussed in the previous section, the dynamics of quantities in response to a TFP shock are similar to those of a standard model without disasters. Consumption, investment and employment are procyclical, and investment is the most volatile series. The model hence reproduces the basic success of the RBC model, and has the same deficiencies, e.g. employment is less volatile than in the data. The T-bill rate and the levered equity return are procyclical, but the return on physical capital is very smooth since there are no adjustment costs to capital adjustment.[25] These dynamics are very similar for all the possible calibrations of the model, except when adjustment costs are large. The fact that the response to a TFP shock is unaltered is an attractive property of the model, since most business cycle models which try to match asset prices have undesirable quantity implications.[26]

4.2.3 An increase in the probability of a disaster

We can now perform the key experiment of an increase in the probability of disaster, which leads to an increase in risk premia. Figure 3 plots the impulse response function to such a shock. For this experiment, I assume that the probability of disaster is initially at its long-run average (.017% per year or 0.00425% per quarter) and doubles at time $t = 6$.[27] Investment decreases, and consumption increases, as in the analytical example of section 2, since the elasticity of substitution is assumed to be greater than unity. Employment decreases too, through an intertemporal substitution effect: the risk-adjusted return to savings is low and thus working today is less attractive. (This is in spite of a negative wealth effect which tends to push employment up; given the large IES the substitution effect overwhelms the wealth effect both for consumption and for leisure.) Hence, output decreases because both employment and the capital stock decrease, even though there is no change in current or future total factor productivity. This is one of the main result of the paper: this shock to the "perceived risk" leads to a recession. After impact, consumption starts falling. These results are robust to changes in parameter values, except of course for the IES which crucially determines the sign of the responses, and the assumption that $b_k = b_{tfp}$ (as we discuss in section 5.1 below). The size of adjustment costs, and the level of risk aversion, affect only the magnitude of the response of investment and hours. These figures are consistent with proposition 2: the shock is equivalent, for quantities, to a preference shock to β.

The model predicts some negative comovement between consumption and investment, which may

[25] This model generate some positive autocorrelation of consumption growth, due to capital accumulation, hence the dynamics of consumption are qualitatively similar to those in Bansal and Yaron (2004). This could in principle generate larger risk premia, however, as argued by Kaltenbrunner and Lochstoer (2008), this effect is not quantitatively very important if shocks are permanent and the IES is not small.

[26] For instance, Jermann (1998) assumes fixed labor, since labor would be countercyclical if it was allowed to vary. Boldrin et al. (2001) resolve this by making assumptions regarding the timing of decisions.

[27] For clarity, to compute this figure, I assume that there are no realized disaster. The simulation is started of after the economy has been at rest for a long time (i.e. no realized disasters, no normal shocks, and no change in the probability of disaster). I obtain this figure by averaging out over 100,000 simulations which start at $t = 6$ in the same position, but then have further shocks to ε or p. Alternatively, one could compute a full non-linear impulse response function, which would take into account that the response of the economy depends also on the initial position, but this picture is meant as a mere illustration of the model mechanism.

seem undesirable. I discuss this further in Section 5.3, but note that the perfect correlation implied by the RBC model is at odds with the data (Table 3).

Regarding asset prices, figure 4 reveals that following the shock, the risk premium on equity increases (the spread between the red–crosses line and the black full line becomes larger), and the short rate decreases, as investors try to shift their portfolio towards safer assets - a "flight to quality".. Hence, in the model, an increase in risk premia coincides with an economic expansion. On impact (at $t = 6$), the increase in the risk premium lowers equity prices substantially, since the discount rate increases. Here too, the return on physical capital is very smooth, since there are no adjustment costs.[28]

To conclude this section, figures 5 give a snapshot of a simulation for quantities. This figure illustrates the key simplification of the model when $b_k = b_{tfp}$ - the time series (in log) of quantities, except hours, all shift down by the factor $1 - b_k$ when there is a disaster.

4.3 First and second moments of quantities and asset returns

Table 2 reports the standard business cycle moments obtained from model simulations for a sample without disasters. Table 3 presents the same statistics in a full sample, i.e. a sample with disasters. Row 2 shows the model when $b_k = b_{tfp} = 0$, i.e. a standard RBC model with an elasticity of substitution of 2. The success of the basic RBC model is clear: consumption is less volatile than output, and investment is more volatile than output. The volatility of hours is on the low side, a standard defect of the basic RBC model given the utility function.

Introducing a constant probability of disaster, in row 3, does not change the moments significantly, consistent with the impulse responses shown in the previous section, and with proposition 1. However, the presence of the new shock - the change in the probability of disaster - leads to additional dynamics, which are visible in row 4. Specifically, the correlation of consumption with output is reduced. Total volatility increases, since there is an additional shock, but this is especially true for investment and employment. Overall, the model overshoots the volatility of investment, but gets closer to the data for the other moments.

Turning to returns, tables 4 and 5 show that the benchmark model (row 4) can generate a large equity premium: 1.7% (=4*(.95-.52)) per year for unlevered equity and about 5% per year for a levered equity. Note that these risk premium are obtained with a risk aversion over consumption equal to 1.8. Moreover, these risk premia are computed over short-term government bonds, which are not riskless in the model. Whether these risk premia are calculated in a sample with disasters or without disasters does not matter much quantitatively - the risk premia are reduced by 10–30 basis point per quarter (see table 5). Note that the risk aversion is lower than in Barro (2006), because changes in the probability of disaster are an additional new source of risk.

Table 6 shows that the model also does not generate enough volatility in unlevered equity returns (only 8 basis points in a sample without disasters). The intuition from the consumption-based model is that shocks to the probability of disaster affect the risk-free rate and the equity premium by the

[28] A different calibration with substantial adjustment costs can generate some return volatility. However, large adjustment costs have undesirable implications for business cycle dynamics.

same amount, so that equity prices and average returns are not much affected by it. The intuition from the production-based point of view is that the return on capital is simply $1 - \delta + MPK_{t+1}$, since there are no adjustment costs in the benchmark calibration. The volatility is significantly higher in a sample with disasters (2.05% per quarter, see Table 7). Adding some financial or operating leverage, and possibly some wage rigidities may increase the return volatility. Rather than incorporating in detail all these mechanisms, I consider the implications of the model for a claim to levered consumption. In this case, we find that the volatility is of the right order of magnitude: 6.23% per quarter. Importantly, the model matches the low volatility of short-term interest rates, an improvement over Jermann (1998) and Boldrin, Christiano and Fisher (2001). Overall, I conclude that the model does a good job at fitting the first two moments of asset returns, if one allows for leverage.

Term Premium

The model generates a negative term premium, consistent with the evidence for indexed bonds in the US and UK. This negative term premium is not due to what happens during disasters, since short-term bonds and long-term bonds are assumed to default by the same amount. As usual, TFP shocks generate very small risk premia. The term premium is thus driven by the third shock, i.e. the shock to the probability of disaster. An increase in the probability of disaster reduces interest rates, as the demand for precautionary savings rises. As a result, long-term bond prices rise. Hence, long term bonds hedge against the shock to the probability of disaster, they have lower average return than the short-term bonds, and the yield curve is on average downward sloping.[29] However, the model does not generate enough volatility in the term premium, compared to that observed in the (nominal) Treasuries market. These results for the yield curve are similar to those in Gabaix (2007) and are highly dependent on the assumption that all bonds default by the same amount in disasters.

4.4 Time series predictability of returns

This section studies the ability of the model to match the evidence that equity returns are predictable, but dividends are not. Table 8 presents the evidence by showing the slope coefficients, t-stat and R^2 of the basic regression

$$R^e_{t \to t+k} - R^f_{t \to t+k} = \alpha + \beta \frac{D_t}{P_t} + \varepsilon_{t+k},$$

for different values of the horizon k, ranging from one to five years. As is well known, the slope coefficient β is positive, significant, and it increases with horizon, as does the regression R^2.[30] The same table reports the results from running these regressions in simulated data from the model. As can be seen, the model also generates a positive slope, which increases with the horizon, as does the R^2. When

[29] Note that states of nature with high probability of disaster are "bad states", even though consumption goes up. This is because the stochastic discount factor also includes current hours and future utility, and the higher uncertainty reduces the value due to risk aversion.

[30] Of course, these regression results are somewhat fragile, as shown by Stambaugh (1999), Ang and Bekaert (2007), Boudoukh, Richardson and Whitelaw (2008), and Goyal and Welch (2008) among others. Yet, this regression has motivated the development of models with time-varying risk premia such as Campbell and Cochrane (1999) or Bansal and Yaron (2004). Finally and most importantly, these results show that the volatility of the price-dividend ratio is largely due to the volatility of expected returns rather than dividends.

the probability of disaster is high, the dividend yield is high (prices are low), and expected returns are high. The model regressions, however, yield higher R^2 and coefficients than the data regressions.

Table 9 studies dividend predictability by showing the slope coefficients, t-stat and R^2 of the regression:

$$\frac{D_{t+k}}{D_t} = \alpha + \beta \frac{D_t}{P_t} + \varepsilon_{t+k},$$

again for horizons ranging from 1 to 5 years. In the data, there is essentially no predictability in dividends - the t-stat are less all than 1.2 and the R^2 less than 2%. In the model, the coefficient β is also insignificantly different from zero, and the R^2 is no higher than 12%. The model actually predicts negative dividend growth, consistent with the impulse response of quantities (figure 3). Overall, even though the quantitative match is not perfect, I conclude that both in the model and in the data, return predictability is much larger than dividend predictability.[31]

4.5 The relation between the stock market and output

While most of the research has focused on the equity premium and the stock market volatility puzzle, the cyclicality of asset prices is also intriguing. Here I concentrate on one measure, the covariogram of HP-filtered output, denoted y_t, and the stock market, measured as the price-dividend ratio. Figure 6 shows the quantity

$$\gamma_k = Cov(y_{t+k}, \log(P_t/D_t)),$$

for $k = -12, -11, ..., 0, ..., 12$ quarters. The black (full) line shows the data, reflecting the well-known pattern that output and the stock market are positively correlated. More precisely, the stock market leads GDP, so this covariance is highest for $k = 2$ quarters.

The blue line (circles) presents the covariogram for the model with only TFP shock, i.e. the basic RBC model. The model generates a positive covariance between the price-dividend ratio and output, because TFP shocks increase future cash flows, hence leading to an increase in the P-D ratio since the discount rate does not increase too much.[32] The volatility of the P-D ratio is quite small however, but the correlation between changes in the P-D ratio and changes in output is high, since there is only one shock. Finally, note that the model cannot replicate the fact that the stock market leads GDP - the largest covariance is on impact.

The red line (crosses) shows the covariogram for the benchmark model, i.e. including both TFP shocks and shocks to the probability of disaster. The key result is that the covariance is now about three times larger for low values of k. Hence, the model becomes closer to the data. The P-D ratio is substantially more volatile, but the covariance is only three times larger, because the effect of a shock to p on output is not very large, and the correlation of the two variables is now imperfect since there are two shocks. Very similar results are obtained if one uses investment instead of GDP in these figures.

[31] This generalizes the results of Gabaix (2007), Gourio (2008b), and Wachter (2008) in endowment economies to a full-fledged production economy. In these models, there is exactly zero dividend predictability in a sample without disasters. In my model, due to quantity dynamics following a change in p_t, there is some predictability in dividends.

[32] Note that in some versions of the model, this covariance is actually negative since procyclical discount rates make the P-D ratio countercyclical.

Finally, the model is still unable to replicate the fact that the stock market leads GDP, because the transitional dynamics are small compared to the response on impact.[33]

5 Robustness

In this section, I discuss several extensions of the baseline model.

5.1 TFP disasters vs. capital disasters

An interesting issue is whether one should model disasters as reductions in TFP or destructions of the existing capital stock. Decreases in TFP arise for instance because of poor government policies or extreme misallocation, while destructions of the capital stock can be due to wars, expropriations, or other effects (see the discussion in section 2). Tables 10 through 12 study the sensitivity of the key results to this assumption.

When disasters affect solely TFP, the model generates a sizeable equity premium, but it underpredicts somewhat the volatility of returns. This is to some extent a calibration issue, since capital is now less risky - it does not vanish during disasters. The key difference is that an increase in the probability of disaster now leads to an increase in the capital stock for precautionary savings reasons. As a result, in this case, and regardless of the IES, an increase in the probability of disaster leads to a boom in investment and output, i.e. the sign of the impulse response of 3 is reversed. The assumption $b_k = b_{tfp}$ ensures that the return on capital will be low if there is a disaster, and hence it makes agents reluctant to hold physical capital.

When disasters solely affect the capital stock, the model does not generate a significant equity premium. There are two main reasons: first, consumption on impact does not fall by a very large amount, as consumers anticipate the recovery; second, with Epstein-Zin utility future utility values matter, and in this instance they indicate that a recovery will soon erase the effects of the disaster. This is all in spite of a rise in hours on impact. Hence, it appears that capital disasters alone do not allow to match asset pricing facts.

5.2 Disasters Dynamics

As pointed out by Constantinides (2008), it is inadequate to model disasters as one-time jumps, since in reality they last more than a year. Barro et al. (2009) estimate a time-series process for disasters which takes into account the fact that after disasters start, they may continue for several years, and a recovery might then follow. In the spirit of this exercise, I consider the following variation on the model: a disaster leads only to a 20% drop in both productivity and the capital stock. However, a disaster also makes the probability of a disaster next period increase to 50%. Next period, either a disaster occurs, in which case the probability of a further disaster remains at 50%, or it doesn't, in which case this

[33] One could also try and replicate a similar pattern with the short rate and the term spread, emphasized by Backus, Routledge and Zin (2008). However these implications are sensitive to the (extraneous) assumptions made regarding inflation during disasters.

probability shifts back to a standard value. Tables 10 through 12 show the impact of this modification. While the disaster is substantially smaller (and the other parameters are kept constant), the model still generates a significant ex-ante equity premium - the fear of further disasters leads to an increase in the risk premium, and a further decline in asset prices, during the disaster. The model also gets reasonable return volatility. Figure 7 presents the response of macro quantities to a disaster (which is in this case not followed by a further disaster). Interestingly, this variation on the model makes investment drop by more than consumption, generating dynamics similar to the U.S. Great Depression. The key lesson from this illustrative computation is that adding some fear of further disasters is a very powerful ingredient.

Another possible extension is to try and match the evidence that output rebounds in the aftermath of a disaster (Gourio (2008b), Barro et al. (2009)). When the disaster is modeled as a pure destruction of capital ($b_{tfp} = 0$) this rebound is an outcome of the model - the usual neoclassical transitional dynamics bring back the economy to its initial balanced growth path. However, when the disaster is modeled as a permanent reduction in total factor productivity ($b_{tfp} > 0$), there is no tendency for the economy to recover. To generate a recovery, one must assume that TFP itself will - perhaps randomly- recover. This is an interesting extension. The key issue is the behavior of real interest rates during the disaster. If they rise substantially, stock prices fall further, making the disaster even more risky ex-ante. Inversely, with Epstein-Zin utility and a high elasticity of substitution, interest rates will not rise much, and consumers understand that they are likely to enjoy a recovery, which will diminish somewhat the riskiness of disasters. From a calibration point of view, this would require to increase risk aversion somewhat. However, the other predictions of the model regarding the volatility and predictability of returns, and the macroeconomic effects of a change in the probability of disaster, would be largely unaffected.

5.3 Comovement of consumption and investment

An implication of the model that may seem odd is that, when the probability of disaster rises, consumption initially increases, while output, employment and investment fall. The wealth effect of the increase in the probability of disaster is for hours to go up, and for consumption to go down; but the substitution effect overwhelms this wealth effect, hence hours go down and consumption goes up. Given that productivity does not change, and the capital stock adjusts gradually, the labor demand schedule (marginal product of labor) is unchanged initially, and, as explained by Barro and King (1984), this makes it impossible to generate positive comovement.

It is not clear that this lack of comovement is necessarily a deficiency of the model. Consumption, investment, and hours are far from perfectly correlated in the data (see Table 3), which means that any model needs a shock which pushes consumption and investment in opposite directions sometimes. Indeed, we see in Table 3 that adding the shock to the probability of disaster brings the model closer to the data regarding the correlations. Moreover, while the impact response displays negative comovement, consumption eventually falls, although after a long delay.

Alternatively, there are some extensions of the model which may overturn this result. Here I discuss intuitively some possibilities.

Disasters are modeled for convenience as jumps, but in reality the contraction is not instantaneous. As a result, it is sometimes difficult for consumers and firms to determine if a decrease in productivity and output is a standard recession (a shock ε in our setup) or is the start of a large depression. A possible model is to assume that disasters last two periods; they are recognized as disasters only after they have ended. In real time, agents only observe productivity, which is driven by both shocks. As a result, a large decrease in productivity, due either to a disaster or to a large negative shock ε, will lead agents to anticipate a further decrease next period. Intuitively, the effect will be to make ε and p negatively correlated. Hence, consumption and output would be more correlated. This learning mechanism could hence attenuate the comovement puzzle.

Alternatively, it may be possible to employ non-standard preferences or adjustment cost formulations, as in Jaimovich and Rebelo (2009) for instance.

Finally and probably more interestingly, adding sticky prices might alter this result. Suppose that we embed the model in a standard New Keynesian setup. A perfect monetary policy could replicate the flexible price allocation, i.e. the results of this paper. In this case, an increase in the probability of disaster would require the central bank to decrease short-term interest rates. If, for some reason, monetary policy is not accomodative enough, or it is impossible to decrease interest rates because of the zero lower bound, then consumption would have to adjust. Since the real interest rate is too high, consumption would fall. This intuition suggests that this (very substantial) extension of the model may resolve the comovement puzzle.[34]

6 An Empirical Test

Disasters are difficult to measure, and shocks to the probability of disasters are almost impossible to observe. I have shown above that a simple, parsimonious framework can account for a variety of business cycle and asset pricing facts. However one may remain skeptical since these shocks are not directly measurable.

One indirect piece of evidence is that estimated DSGE models give a significant role to shocks to β in accounting for business cycle fluctuations.[35] However these shocks are picked without restrictions. My model suggests a natural restriction - shocks to the probability of disaster have a powerful effect on asset prices. This section provides a more precise test of the model by "backing out" the implied probability of disaster from observed asset prices, and feeding this implied probability of disaster in the model. The aim is to compare the quantity dynamics implied by the model with and without this new shock. Does the fit improves compared to a model with only the standard TFP shock? The asset-price that I use is the most natural one in the model, i.e. the price-dividend ratio.

The methodology is as follows. In the model, given a vector of parameters Θ, the price-dividend ratio $\frac{P_t}{D_t}$ is a function of $k_t = \frac{K_t}{z_t}$ (where K_t is the capital stock and z_t is TFP) and of the probability of disaster p_t :

$$\frac{P_t}{D_t} = \psi\left(k_t, p_t; \Theta\right).$$

[34] I thank Emmanuel Farhi for this suggestion.

[35] Or to shocks to the relative price of investment goods, which have similar dynamic properties.

Using data from the BEA, I can measure K_t and z_t, hence k_t. I then calculate, for each date, the value of the probability of disaster $\widehat{p}_t$ which allows to match exactly the observed price-dividend ratio in the data.[36]

Next, I feed this probability of disaster in the model, together with the measured TFP. For instance, the policy functions imply that aggregate non-durable consumption is $C_t = z_t c(k_t, p_t)$ which I can now compute. Finally, the implied series for consumption, investment, output and employment are HP-filtered and compared to the data and to the baseline RBC model (i.e. $p_t = 0$ for all t). The data is from 1948q1 to 2008q4.

Figure 8 presents the underlying probability of disaster. Given the methodology, this time series is a nonlinear function of the price-dividend ratio $\frac{P_t}{D_t}$ and the detrended capital stock k_t. The short-run fluctuations hence mostly reflect changes in the stock market value. Interestingly, the highest probability of disaster is estimated to occur at the very end of the sample, in the last quarter of 2008.

Figure 9 presents the quantity implications (this figure is best viewed with colors). Note that the RBC model does a reasonable job at matching macroeconomic aggregates, given TFP, until 1985, even if it does not generate quite enough volatility. The model with disaster risk is able to improve in some dimensions, e.g. investment and employment, by creating sharp downturns which help account for some big recessions, such as the mid-1970s and early 1980s episodes. A particularly interesting example is the current recession. Figure 10 "zooms in" on the most recent data. Little happens to TFP in 2008, hence the RBC model does not predict anything like the current recession. My model, however, generates a large drop in output, investment, and employment through the shock to p_t, as inferred from the stock market value. The comovement problem comes back with a vengeance, however, since the model suggests that consumption in 2008q4 should have gone up, while it fell markedly. As discussed in section 5.3, a richer version of the model may be able to account for this comovement.

Table 13 summarizes these results by computing a measure of fit, $\frac{Cov(\text{model, data})}{Var(\text{data})}$,for each of the two models. A close look at figure9 suggests that the model with the two shocks does much better if one introduces a lag between the model and the data. Hence, I also report this measure of fit when the model series are lagged by 2 quarters. There may well be delays to decisions and various adjustment costs which create such a delay, not captured in the simple version of the model. Finally, I report the same statistic for the subsample of recessions - which I defined as GDP 1.5% or more below the HP-filter trend. The table reveals, that, especially with the 2-quarter lag, the model provides a substantial improvement over the RBC model in terms of matching investment, and to some extent employment and output. The fit is especially good when one looks at the recessions. The model is, however, a step back for non-durable consumption, where the negative comovement implied by the model is too strong compared to the data. Overall I conclude from this exercise that shocks to the probability of disaster, as *restricted* by asset prices data, appear to help the RBC model fit the data, especially during severe

[36]The price-dividend ratio is from CRSP. (Similar results are obtained using the Shiller data.) As is standard in the literature, I divided the price by the sum of the last four quarters of dividends, given the substantial seasonality of dividends. Because of changes over the sample period in the distribution of equity payout between dividends and share repurchases, there are lower-frequency movements in the price-dividend ratio. Hence, I first removed these low-frequency movements using a HP filter (the smoothing parameter does not affect the results significantly, provided it is above 1600).

recessions, arguably the most interesting episodes.

This result should not be too surprising in light of the empirical findings that the stock market is correlated with GDP and investment. Intuitively, section 4 shows that the model generates a reasonable relation between asset prices and investment or output. Hence, feeding in asset prices from the data (through p_t) allows the model to improve quantities by using the empirical explanatory power of the stock market for investment or GDP.

7 Conclusion

This work shows how introducing disasters into a standard RBC model improves its fit of asset return data, preserves its relative success for quantity dynamics in response to TFP shocks, and finally creates some interesting new macroeconomic dynamics linked to the variation of risk premia. This parsimonious setup has the advantage of being tractable, which allows to derive some analytical results and makes it easy to embed it into richer models: researchers studying the connection between business cycle dynamics and risk premia may find this a useful modeling tool. For instance, much of the research on financial frictions has taken place in models which largely abstract from macroeconomic risk premia - a potentially significant limitation which been driven largely by technical considerations, and by the lack of setups with large and volatile risk premia.

As a result, there are many possible interesting extensions. First, it would be interesting to consider the effect of a time-varying risk of disaster in a richer business cycle model, e.g. one with collateral constraints or choice of financial leverage, or a standard New Keynesian model. Disaster risk seems also important for many emerging markets. Second, it would be interesting to consider richer dynamics for disasters - such as these sketched in section 5 - in more detail. Third, a change in the aggregate risk affects macroeconomic aggregates also by affecting the willingness to take on risk. This seems an interesting mechanism to explore: faced with an increase in the probability of an economic disaster, investors shift resources to technologies and projects which are less exposed to disasters. In doing so, they move the economy alongside a risk-return frontier, and pick projects which are less risky but also have lower expected returns. As a result, the expected output of the economy falls, and so does productivity.

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8 Appendix

8.1 A simple method to solve models with time-varying risk premia

The model in this paper is simple enough that a full nonlinear solution is easy and relatively fast to implement. However, many interesting extensions would require to add more state variables, making a full non-linear solution difficult. Results 1 and 2 of this paper, however, suggest a simple two-step solution method: under the assumption that $b_k = b_{tfp}$, the model has the same implications for quantities than a model without disaster, but with a different, and time-varying discount factor β. As we know this model can be solved precisely using a standard first-order linearization. Hence, the proposed methodology is as follows:

(1) Change the average value of β, using $\beta^* = \int \beta(p)d\mu(p) = \int \beta(1-p+p(1-b_k)^{\upsilon(1-\theta)})^{\frac{1-\gamma}{1-\theta}} d\mu(p)$ where μ is the invariant distribution of p.

(2) Solve for the policy functions $c(k,p)$, $N(k,p)$ using the equivalent model: $c(k,\beta), N(k,\beta)$ and a standard log-linearization.

(3) Solve for asset prices using the standard recursions. For simplicity I present them in the expected utility case:

(a) Equity paying out a process $D_t = z_t^\lambda d(k_t,p_t)$; the price-dividend ratio $f(p_t,k_t) = P_t/D_t$ satisfies:

$$f(p,k) = (1-p+p(1-b)^{\lambda+\upsilon(1-\gamma)-1}) \int_{\varepsilon'} \int_{p'} \frac{\beta u_1(c(k',p'),N(k',p'))}{u_1(c(k,p),N(k,p))} \frac{D(k',p')}{D(k,p)} \left(f(p',k')+1\right) dQ(p'|p)dH(\varepsilon'),$$

where H is the c.d.f. of the normal shock ε, and Q is the transition function of the process $\{p_t\}$.

(b) Yield curve: the price of a zero-coupon bond maturing in n periods satisfies:

$$q^{(n)}(k,p) = \left(p(1-b)^{\upsilon(1-\gamma)-1}(1-q(1-r))+1-p\right) \int_{\varepsilon'} \int_{p'} \frac{\beta u_1(C(k',p'),N(k',p'))}{u_1(C(k,p),N(k,p))} q^{(n-1)}(k',p')dH(\varepsilon')dQ(p'|p),$$

with $q^{(0)}(k,p) = 1$ and $u(C,N) = \frac{(C^\upsilon N^{1-\upsilon})^{1-\gamma}}{1-\gamma}$. This simplification relies on this form of preferences, which are consistent with balanced growth. These two recursions can be solved given the policy functions $C(k,p)$, $N(k,p)$, and $D(k,p)$ which are now known.

With recursive utility, the same procedure works, but one also needs to compute the value function, since it appears in the stochastic discount factor and hence in the asset price recursions. The value function can be computed easily once the policy functions are known, or it can be computed inexpensively using second-order perturbation methods as in Fernandez-Villaverde et al. (2008).

In general, for any given model, one should check that the equivalence results 1 and 2 hold, but I conjecture that they would in many models, and hence this opens the door to solving medium-scale macro model with time-varying risk premia.

8.2 Computational Method

This method used in the paper is presented for the case of a Cobb-Douglas production function, and a Cobb-Douglas utility function, but it can be used for arbitrary homogeneous of degree one production function and utility function. Also, this presentation does not allow for correlation between p and ε, and for recoveries, but it is straightforward to modify the method to allow for this.

The Bellman equation for the "rescaled" problem is:

$$g(k,p) = \max_{c,i,N} \left\{ \begin{array}{c} c^{\upsilon(1-\gamma)}(1-N)^{(1-\upsilon)(1-\gamma)} \\ +\beta e^{\mu\upsilon(1-\gamma)} \left(E_{p',\varepsilon',x'} e^{\sigma\varepsilon'\upsilon(1-\theta)} \left(1-x'+x'(1-b_{tfp})^{\upsilon(1-\theta)}\right) g(k',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

$$\begin{aligned} s.t. &\quad : \\ c+i &= k^{\alpha}N^{1-\alpha}, \\ k' &= \frac{(1-x'b_k)\left((1-\delta)k+\phi\left(\frac{i}{k}\right)k\right)}{e^{\mu+\sigma\varepsilon'}(1-x'b_{tfp})}. \end{aligned}$$

Because we take a power $\frac{1}{1-\gamma}$ of the value function, the max needs to be transformed in a min if $\gamma > 1$. To approximate numerically the solution of this problem, I proceed as follows:

(1) Pick a grid for k, and a grid for i, and approximate the process for p with a Markov chain with transition matrix Q. Discretize the normal shock ε, with probabilities $\pi(\varepsilon)$. I used 120 points for the grid for k, 1200 points for the grid for i, and 5 points for the grid for ε. Finally Q is picked as in Rouwenhorst (1995). That method is a better approximation for highly persistent processes. I used 9 points for the grid for p.

(2) Compute for any k, i in the grid the value $N(k,i)$ which solves

$$R(k,i) = \max_{N} \left(k^{\alpha}N^{1-\alpha} - i\right)^{\upsilon} (1-N)^{(1-\upsilon)}.$$

(3) The state space and action space are now discrete, so this is a standard discrete dynamic programing problem, which can be rewritten as follows, with one endogenous state, one exogenous state, and two additional shocks: a binomial variable x equal to one if a disaster occurs (with probability p) and the normal shock ε:

$$g(k,p) = \max_{i} \left\{ \begin{array}{c} R(k,i) + \beta e^{\mu\upsilon(1-\gamma)} \times ... \\ \left(\sum_{p',\varepsilon',x'\in\{0,1\}} \pi(\varepsilon')Q(p,p')e^{\sigma\varepsilon'\upsilon(1-\theta)} pr(x',p) \left(1-x'+x'(1-b_{tfp})^{\upsilon(1-\theta)}\right) g(k',p')^{\frac{1-\theta}{1-\gamma}} \right)^{\frac{1-\gamma}{1-\theta}} \end{array} \right\},$$

$$\begin{aligned} s.t. &\quad : \\ k' &= \frac{(1-x'b_k)\left((1-\delta)k+\phi\left(\frac{i}{k}\right)k\right)}{e^{\mu+\sigma\varepsilon'}(1-x'b_{tfp})}, \end{aligned}$$

where $pr(x',p) = p1_{x'=1} + (1-p)\,1_{x'=0} = px' + (1-p)(1-x')$. I solve this Bellman equation using modified policy iteration[37] (Judd (1998), p. 416), starting with a guess value close to zero. Recursive utility implies that the Blackwell sufficient conditions do not hold here, hence it is not obvious that the Bellman operator is a contraction. However, convergence occurs in practice as long as β and the probability of disasters are not too large. Note that to compute the expectation, we need the value function outside the grid points. I use linear interpolation in the early steps of the iteration, then switch to spline interpolation. The motivation is that linear interpolation is more robust, hence it is easier to make the iterations converge; but spline interpolation is more precise.

[37] This turns out to be significantly faster than value iteration for this application.

(4) Given g, we have $V(K,z,p) = z^{\upsilon} g(k,p)^{\frac{1}{1-\gamma}}$. We also obtain the policy functions $C = zc(k,p)$, $I = zi(k,p)$, $N = N(k,p)$, and the output policy function $Y = zk^{\alpha}N(k,p)^{1-\alpha}$. Because these policy functions are defined on a discrete grid, I use interpolation in the simulations and impulse responses to obtain more accurate results. (Linear or spline interpolations yield nearly the same results.)

(5) To compute asset prices, we need the stochastic discount factor, which is given by the standard formula:

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\upsilon(1-\gamma)-1} \left(\frac{1-N_{t+1}}{1-N_t}\right)^{(1-\upsilon)(1-\gamma)} \left(\frac{V_{t+1}}{E_t\left(V_{t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta}.$$

Using homogeneity, the SDF between two states $s = (k,p)$ and $s' = (k',p')$ is:

$$\begin{aligned} M(s,s',\varepsilon',x') &= \beta\left(\frac{z'c(k',p')}{zc(k,p)}\right)^{\upsilon(1-\gamma)-1}\left(\frac{1-N(k',p')}{1-N(k,p)}\right)^{(1-\upsilon)(1-\gamma)}\left(\frac{z'^{\upsilon}g(k',p')^{\frac{1}{1-\gamma}}}{E_{z',p',x'}\left(z'^{\upsilon(1-\theta)}g(k',p')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta} \\ &= \beta\left(\frac{z'}{z}\right)^{(\gamma-\theta)\upsilon+\upsilon(1-\gamma)-1}\left(\frac{c(k',p')}{c(k,p)}\right)^{\upsilon(1-\gamma)-1} \times ... \\ & \quad ...\left(\frac{1-N(k',p')}{1-N(k,p)}\right)^{(1-\upsilon)(1-\gamma)}\left(\frac{g(k',p')^{\frac{1}{1-\gamma}}}{E_{z',p',x'}\left(\left(\frac{z'}{z}\right)^{\upsilon(1-\theta)}g(k',p')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta}. \end{aligned}$$

Note that we first need to compute the conditional expectation which appears on the denominator of the last term. Denote $k' = j(k,p,\varepsilon',x')$ the detrended capital next period, wich depends on the detrended investment $i(k,p)$ and on the realization of the shocks next period ε' and x' (but not p'). The conditional expectation is obtained as:

$$E_{z',p',x'}\left(\left(\frac{z'}{z}\right)^{\upsilon(1-\theta)} g(k',p')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}} =$$

$$= \sum_{p',\varepsilon'} Q(p,p')\Pr(\varepsilon')e^{\upsilon(1-\theta)\mu+\upsilon(1-\theta)\sigma\varepsilon'}\left(p(1-b_{tfp})^{\upsilon(1-\theta)}g(j(k,p,\varepsilon',1),p')^{\frac{1-\theta}{1-\gamma}} + (1-p)g(j(k,p,\varepsilon',0),p')^{\frac{1-\theta}{1-\gamma}}\right).$$

(6) We can now obtain the price of a one-period asset, with payoff $d(k',z',p',x',\varepsilon')$. e.g. a pure risk-free asset $d=1$, or a short-term government bond: $d = 1-q(1-r)x'$, as

$$P(k,p) = E_{x',p',\varepsilon'}M(s,s',x',\varepsilon')d(k',z',p',x',\varepsilon').$$

For instance, for a pure risk-free asset, the formula is:

$$E_tM_{t,t+1} = \frac{\begin{array}{c} \beta\sum_{p'}\sum_{\varepsilon'}Q(p,p')\Pr(\varepsilon')e^{((\gamma-\theta)\upsilon+\upsilon(1-\gamma)-1)\left(\mu+\sigma\varepsilon'\right)} \times ... \\ ...\left(\begin{array}{c} p(1-b_{tfp})^{(\gamma-\theta)\upsilon+\upsilon(1-\gamma)-1}c(j(k,p,\varepsilon',1),p')^{\upsilon(1-\gamma)-1}\times \\ (1-N(j(k,p,\varepsilon',1),p'))^{(1-\upsilon)(1-\gamma)}\,g(j(k,p,\varepsilon',1),p')^{\frac{\gamma-\theta}{1-\gamma}} \\ +(1-p)c(j(k,p,\varepsilon',0),p')^{\upsilon(1-\gamma)-1}\times \\ (1-N(j(k,p,\varepsilon',0),p'))^{(1-\upsilon)(1-\gamma)}\,g(j(k,p,\varepsilon',0),p')^{\frac{\gamma-\theta}{1-\gamma}} \end{array}\right)\end{array}}{c(k,p)^{\upsilon(1-\gamma)-1}\left(1-N(k,p)\right)^{(1-\upsilon)(1-\gamma)}E_{z',p',x'}\left(\left(\frac{z'}{z}\right)^{\upsilon(1-\theta)}g(k',p')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{\gamma-\theta}{1-\theta}}}.$$

(7) Next, we can obtain the term structure of interest rates on government bonds, using the recursion:

$$P_n(k,p) = E_{x',p',\varepsilon'}\left(M(s,s',x',\varepsilon')\left((1-x'q(1-r))\,P_{n-1}(k',p')\right)\right),$$

where $q(1-r)$ is the expected loss rate of government bonds during disasters. It is assumed that government bonds are risk-free if there is no disaster.

(8), Finally, we can compute the price of equity claims. I consider three types of "equities". Define the representative firm's dividend as earnings less investment:

$$D_t = Y_t - w_t N_t - I_t.$$

The first equity is simply a claim to the stream $\{D_t\}$. Let F_t denote its price, which satisfies the standard recursion:

$$F_t = E_t\left(M_{t,t+1}\left(F_{t+1} + D_{t+1}\right)\right).$$

Note that D_t can be written as $D_t = z_t d(k_t, p_t)$, where $d(k,p) = \alpha k^{\alpha} N^{1-\alpha} - i(k,p)$. Hence, we can rewrite the firm value recursion as:

$$z f(k,p) = E_{s'|s}\left(M(s,s') \times \left(z' d(k',p') + z' f(k',p')\right)\right), \tag{7}$$

where $F_t = z_t f(k_t, p_t)$. The equity return is then

$$\begin{aligned} R^e_{t+1} &= \frac{F_{t+1} + D_{t+1}}{F_t} \\ &= \frac{z_{t+1}}{z_t} \frac{f(k_{t+1}, p_{t+1}) + d(k_{t+1}, p_{t+1})}{f(k_t, p_t)}. \end{aligned} \tag{8}$$

To solve the recursion 7 in practice, I iterate starting with an initial guess $f(k,p) = 0$. The recursion can be rewritten as:

$$\begin{aligned} f(k,p) &= E_{s'|s}\left(M(s,s') \frac{z'}{z}\left(d(k',p') + f(k',p')\right)\right) \\ &= \beta E_{p',\varepsilon',x'}\left[\begin{array}{c} \left(\frac{z'}{z}\right)^{(\gamma-\theta)\upsilon+\upsilon(1-\gamma)} \left(\frac{c(k',p')}{c(k,p)}\right)^{\upsilon(1-\gamma)-1} \left(\frac{1-N(k',p')}{1-N(k,p)}\right)^{(1-\upsilon)(1-\gamma)} \times \ldots \\ \left(\frac{g(k',p')^{\frac{1}{1-\gamma}}}{E_{z',p',x'}\left(\left(\frac{z'}{z}\right)^{\upsilon(1-\theta)} g(k',p')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{1}{1-\theta}}}\right)^{\gamma-\theta} \left(d(k',p') + f(k',p')\right) \end{array}\right]. \end{aligned}$$

This conditional expectation can be written down, as

$$f(k,p) = \frac{\begin{array}{c} \beta \sum_{p'} \sum_{\varepsilon'} Q(p,p') \Pr(\varepsilon') e^{((\gamma-\theta)\upsilon+\upsilon(1-\gamma))(\mu+\sigma\varepsilon')} \times \ldots \\ \left(\begin{array}{c} p(1-b_{tfp})^{(\gamma-\theta)\upsilon+\upsilon(1-\gamma)} c(j(k,p,\varepsilon',1),p')^{\upsilon(1-\gamma)-1} \left(1 - N(j(k,p,\varepsilon',1),p')\right)^{(1-\upsilon)(1-\gamma)} \times \\ g(j(k,p,\varepsilon',1),p')^{\frac{\gamma-\theta}{1-\gamma}} \left(d(j(k,p,\varepsilon',1),p') + f(j(k,p,\varepsilon',1),p')\right) \\ +(1-p) c(j(k,p,\varepsilon',0),p')^{\upsilon(1-\gamma)-1} \left(1 - N(j(k,p,\varepsilon',0),p')\right)^{(1-\upsilon)(1-\gamma)} \times \\ g(j(k,p,\varepsilon',0),p')^{\frac{\gamma-\theta}{1-\gamma}} \left(d(j(k,p,\varepsilon',0),p') + f(j(k,p,\varepsilon',0),p')\right) \end{array}\right) \end{array}}{c(k,p)^{-\gamma} E_{z',p',x'}\left(\left(\frac{z'}{z}\right)^{\upsilon(1-\theta)} g(k',p')^{\frac{1-\theta}{1-\gamma}}\right)^{\frac{\gamma-\theta}{1-\theta}}}.$$

Note that the return computed by solving this recursion for firm value can be compared to the expression obtained by the investment return. The two must give the same result. In practice it is a useful numerical check to perform.

Finally, The levered equity assumes that the payoff streams is $\{C_t^\lambda\}$. It is easy to adapt the same method to price the claims to these assets. Finally, I obtain the model statistics by simulating 1000 samples of length 400, started at the nonstochastic steady-state, and cutting off the first 200 periods. The Matlab(c) programs will be made available on my web page.

8.3 Data Sources

Business cycle and return moments of Tables 2-7: consumption is nondurable + services consumption, investment is fixed investment, and output is GDP, from the NIPA Table 1.1.3, quarterly data 1947q1-2008q4. Hours is nonfarm business hours from the BLS productivity program (through FRED: HOABNS). The return data is from Ken French's webpage, (benchmark factors, aggregated to quarterly frequency, and deflated by the CPI (CPIAUCSL through FRED)).

Table 8: the data is taken from John Cochrane's webpage (Cochrane, 2008) and the regression results replicate exactly his specification (for the simple regression of return on dividend yield last year). This is also the CRSP value weighted return, and the risk-free rate is from Ibottson.

Section 6: the price-dividend ratio is from CRSP. The capital stock is obtained from the annual fixed assets tables, and is interpolated. TFP is computed as output divided by labor to the power 2/3 and capital to the power 1/3.

Parameter	Greek Letter	Value
Capital share	α	.34
Depreciation rate	δ	.02
Adjustment cost curvature	η	0
Trend growth of TFP	μ	.0025
Discount factor	β	.993
IES	$1/\gamma$	2
Share of consumption in utility	υ	.3
Risk aversion over the cons.–leis. bundle	θ	6
Standard deviation of ordinary TFP shock	σ	.01
Size of disaster in TFP	b_{tfp}	.43
Size of disaster for capital	b_k	.43
Persistence of $\log(p)$	ρ_p	.96
Unconditional std. dev. of $\log(p)$	$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	1.5
Leverage	λ	3
Recovery rate for bonds during disasters	$1-r(1-b)$	.828

Table 1: Parameter values for the benchmark model. The time period is one quarter.

Sample without disasters	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\sigma(\Delta \log Y)$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data	0.57	2.68	0.92	0.98	0.45	0.68
No disaster	0.44	2.63	0.42	0.90	0.98	0.99
Constant p	0.44	2.67	0.42	0.91	0.98	0.99
Benchmark	0.56	3.84	0.65	0.98	0.52	0.92

Table 2: Business cycle quantities: second moments implied by the model, for different calibrations. Quarterly data. This is based on a sample without disasters. rho(C,Y) and rho(I,Y) are the correlation of the growth rate of C and of Y, and I of Y, respectively. Data sources in appendix.

Sample with disasters	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\sigma(\Delta \log Y)$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data*	0.57	2.68	0.92	0.98	0.45	0.68
No disaster	0.44	2.63	0.42	0.90	0.98	0.99
Constant p	0.91	1.31	0.12	3.10	0.99	0.97
Benchmark	0.92	1.61	0.21	3.11	0.83	0.89

Table 3: Business cycle quantities: second moments implied by the model, for different calibrations. Quarterly data. This is based on a full sample (i.e., including disasters).

Sample without disasters	$E(R_f)$	$E(R_b)$	$E(R_e)$	$E(R_{c,lev})$
Data	—	0.21	1.91	1.91
No disaster	0.91	0.91	0.91	0.96
Constant p	0.23	0.52	0.97	1.30
Benchmark	0.24	0.52	0.95	1.85

Table 4: Mean returns implied by the model for (a) pure risk-free asset, (b) a one-quarter government bond, (c) a claim to dividends (d) a claim on levered consumption. Quarterly data. Statistics computed in a sample without disasters. Data sources in appendix.

Sample with disasters	$E(R_f)$	$E(R_b)$	$E(R_e)$	$E(R_{c,lev})$
Data*	—	0.21	1.91	1.91
No disaster	0.91	0.91	0.91	0.96
Constant p	0.23	0.45	0.79	0.95
Benchmark	0.23	0.43	0.75	1.47

Table 5: Mean returns implied by the model for (a) pure risk-free asset, (b) a one-quarter government bond, (c) a claim to dividends (d) a claim on levered consumption. Quarterly data. Statistics computed in a full sample (i.e. including disasters). Data sources in appendix.

Sample without disasters	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e)$	$\sigma(R_{c,lev})$
Data	—	0.81	8.14	8.14
No disaster	0.07	0.07	0.07	2.51
Constant p	0.07	0.07	0.07	2.44
Benchmark	0.96	0.60	0.08	6.23

Table 6: Standard deviations of returns implied by the model for (a) pure risk-free asset, (b) a one-quarter government bond, (c) a claim to dividends (d) a claim on levered consumption. Quarterly data. Statistics computed in a sample without disasters. Data sources in appendix.

Sample with disasters	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e)$	$\sigma(R_{c,lev})$
Data*	—	0.81	8.14	8.14
No disaster	0.07	0.07	0.07	2.51
Constant p	0.07	0.84	2.07	5.19
Benchmark	0.99	1.16	2.05	7.93

Table 7: Standard deviation of returns implied by the model for (a) pure risk-free asset, (b) a one-quarter government bond, (c) a claim to dividends (d) a claim on levered consumption. Quarterly data. Statistics computed in a full sample (i.e. including disasters). Data sources in appendix.

Return Predictability	Data			Model		
horizon	coeff.	t-stat	R2	coeff.	t-stat	R2
1	3.83	2.47	0.07	8.65	3.40	0.19
2	7.42	3.13	0.11	15.93	4.73	0.31
3	11.57	4.04	0.18	22.32	5.64	0.38
4	15.81	4.35	0.20	28.16	6.23	0.43
5	17.30	4.01	0.18	33.71	6.75	0.46

Table 8: Regression of cumulated sock market excess return on lagged dividend yield, for different horizons, in the model and in the data. The table reports the slope coefficient, the t-stat (based on the OLS standard error), and R2, for each horizon from 1 year to 5 years. In the model, these regressions are computed using the levered equity, for the benchmark calibration. Data from Cochrane (2008).

Dividend Predictability	Data			Model		
horizon	coeff.	t-stat	R2	coeff.	t-stat	R2
1	.07	.05	.00	-.34	0.63	0.03
2	-.42	-.26	.00	-.72	1.02	0.06
3	.16	.09	.00	-.88	1.12	0.08
4	1.37	.73	.01	-.94	1.4	0.10
5	2.42	1.20	.02	-.91	1.12	0.11

Table 9: Regression of cumulated dividend growth on lagged dividend yield, for different horizons, in the model and in the data. The table reports the slope coefficient, the t-stat (based on the OLS standard error), and R2, for each horizon from 1 year to 5 years. In the model, these regressions are computed using the levered equity, for the benchmark calibration. Data from Cochrane (2008).

Robustness	$\frac{\sigma(\Delta \log C)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log I)}{\sigma(\Delta \log Y)}$	$\frac{\sigma(\Delta \log N)}{\sigma(\Delta \log Y)}$	$\sigma(\Delta \log Y)$	$\rho_{C,Y}$	$\rho_{I,Y}$
Data	0.57	2.68	0.92	0.98	0.45	0.68
Benchmark	0.56	3.84	0.65	0.98	0.52	0.92
Capital disasters ($b_{tfp} = 0$)	0.52	3.95	0.60	0.96	0.62	0.92
TFP disasters ($b_k = 0$)	0.61	7.40	0.80	1.12	0.34	0.90
Multiperiod disasters	0.63	5.24	0.78	1.04	0.36	0.91

Table 10: Extensions of the model: Business cycle quantities. Statistics computed in a sample without disasters.

Robustness	$E(R_f)$	$E(R_b)$	$E(R_e)$	$E(R_{c,lev})$
Data	—	0.21	1.91	1.91
Benchmark	0.24	0.52	0.95	1.85
Capital disasters ($b_{tfp} = 0$)	1.09	1.19	1.33	1.30
TFP disasters ($b_k = 0$)	0.85	1.12	0.87	1.81
Multiperiod disasters	0.76	0.96	1.27	2.18

Table 11: Extensions of the model: Mean returns. Statistics computed in a sample without disasters.

Robustness	$\sigma(R_f)$	$\sigma(R_b)$	$\sigma(R_e)$	$\sigma(R_{c,lev})$
Data	—	0.81	8.14	8.14
Benchmark	0.96	0.60	0.08	6.23
Capital disasters ($b_{tfp} = 0$)	0.16	0.11	0.10	2.45
TFP disasters ($b_k = 0$)	0.31	0.20	1.67	3.54
Multiperiod disasters	1.00	0.65	0.09	5.03

Table 12: Extensions of the model: standard deviation of returns. Statistics computed in a sample without disasters.

Measure of Fit (in %)	A: Full sample		B: With 2q lag		C: Recessions + 2q lag	
	RBC	RBC+p	RBC	RBC+p	RBC	RBC+p
C	17.4	7.4	10.1	0.1	-11.4	-21.5
I	37.1	46.6	41.7	63.2	47.6	96.6
N	5.1	5.7	15.3	23.6	14.5	32.2
Y	47.6	50.8	41.7	49.5	32.8	50.3

Table 13: Covariance between model and data, for the RBC model and the benchmark model of this paper. The time series for each model are computed by feeding in TFP and, for the benchmark model, feeding in the probability of disaster extracted from the P-D ratio as explained in section 6. Panel A: full sample fit. Panel B: fit computed by lagging both models by 2 quarters. Panel C: fit using only the recession data points.

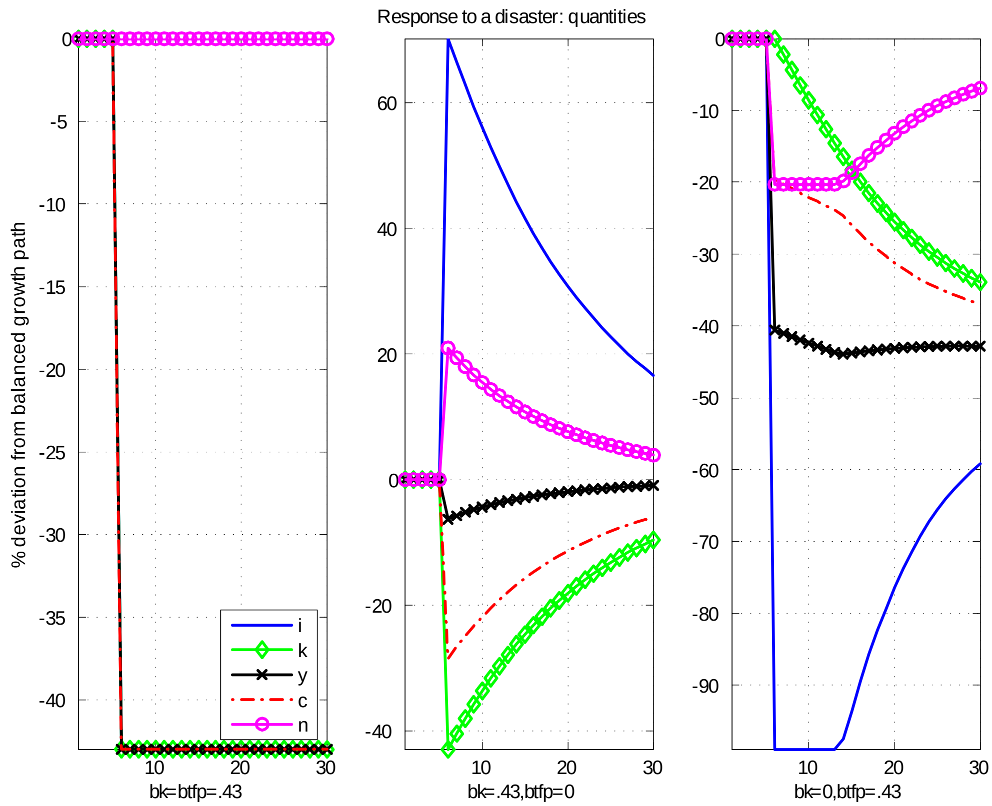

Figure 1: **The different types of disasters.** Response of quantities (Consumption C, Investment I, Capital K, Employment N, Output Y) to a disaster at $t = 6$, in % deviation from balanced growth path. Left panel: $b_k = b_{tfp}$; Middle panel: $b_k = .43, b_{tfp} = 0$. Right panel: $b_{tfp} = .43, b_k = 0$.

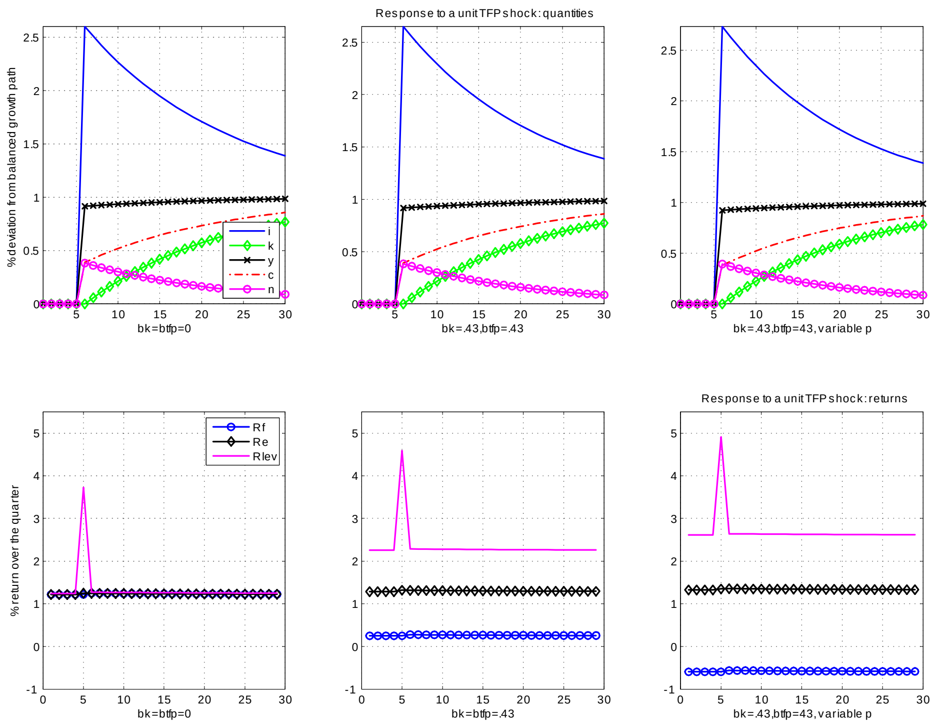

Figure 2: **(Near) Observational equivalence of quantity dynamics to a TFP shock.** The figure plots the impulse response function of quantities (C,I,N,K,Y) and returns (risk-free rate, equity return, levered equity) to a permanent TFP shock at $t = 6$. Left panel: model without disasters. Middle panel: model with constant probability of disasters. Right panel: model with time-varying probability of disaster (benchmark). All other parameters (including β) are kept constant across the three panels.

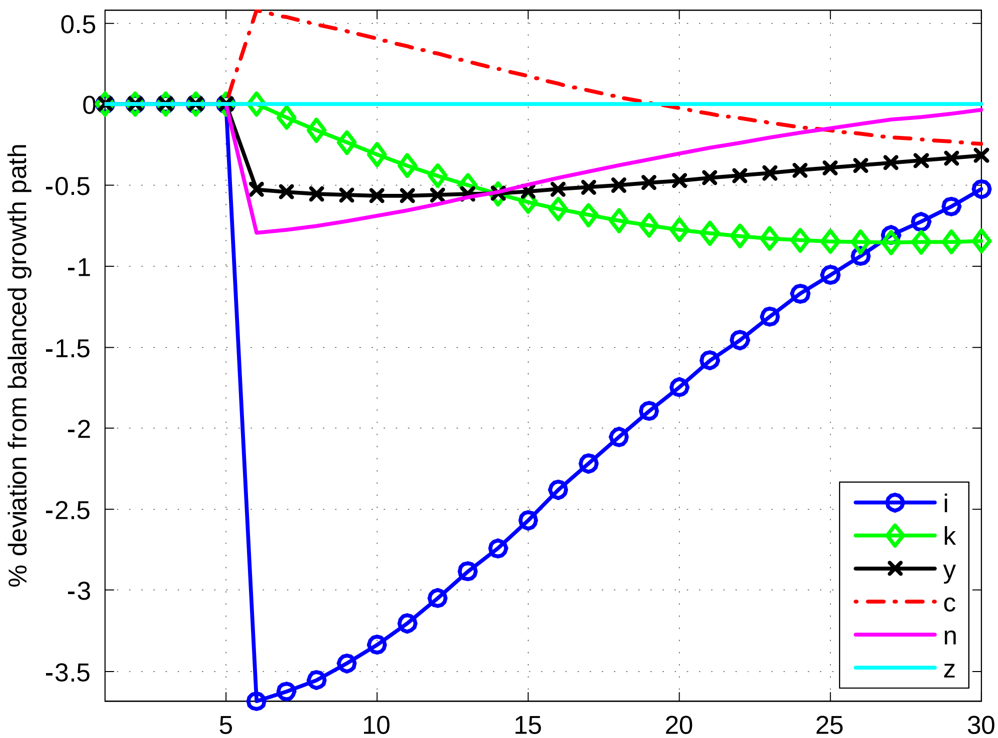

Figure 3: **The effect of an increase in the probability of economic disaster on macroeconomic quantities**. Impulse response of (C,I,K,N,Y,Z) to a shock to the probability of disaster at $t = 6$. The probability of disaster goes from its long-run average (0.425% per quarter) to twice its long-run average then mean-reverts according to its AR(1) law of motion. For clarity, this figure assumes that there is no shock to TFP, and no disaster realized.

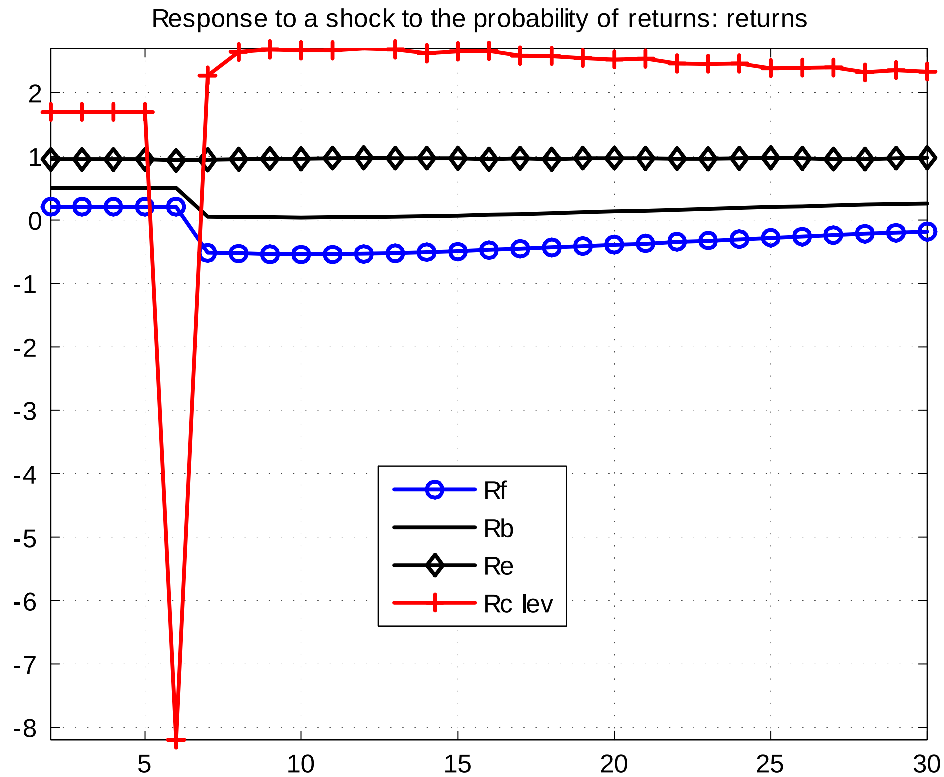

Figure 4: **The effect of an increase in the probability of economic disaster on asset returns and spreads**. Impulse response of asset returns to a shock to the probability of disaster at $t = 6$. The probability of disaster doubles at $t = 6$, starting from its long-run average. The figure plots the risk-free rate, the short-term government bond return, the equity return, and the levered equity return.

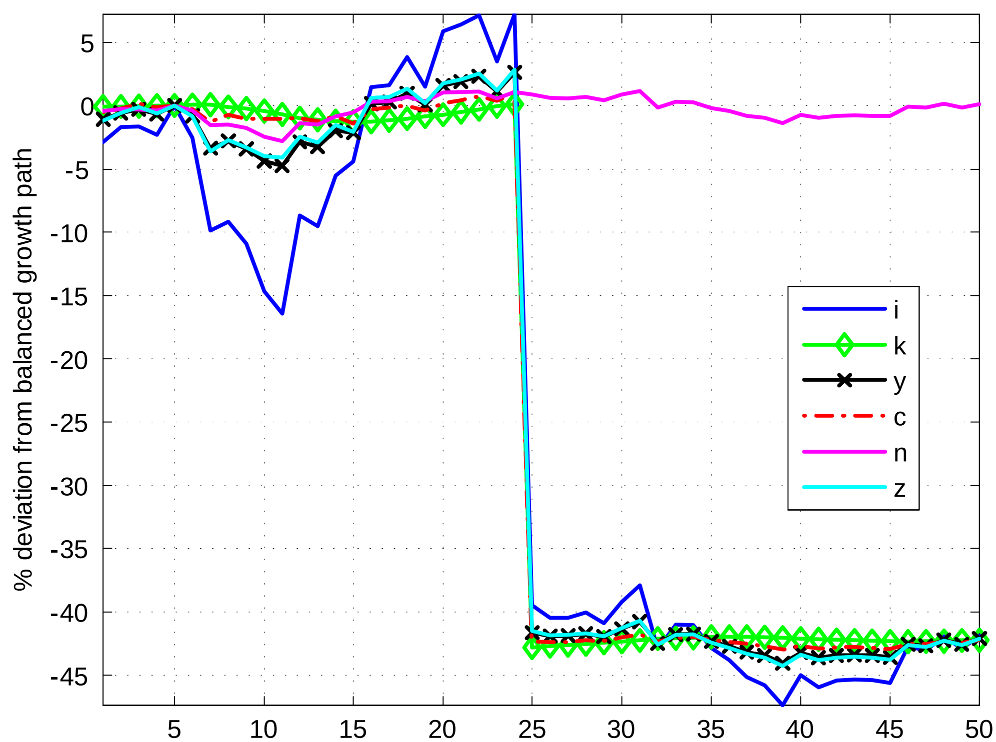

Figure 5: **A sample path for macroeconomic quantities.** Macroeconomic variables (I,K,N,Y,C) and TFP (Z) are driven by standard normally distributed shocks to TFP, shocks to the probability of disaster, and realization of disasters (in this sample, there is one disaster at $t = 25$).

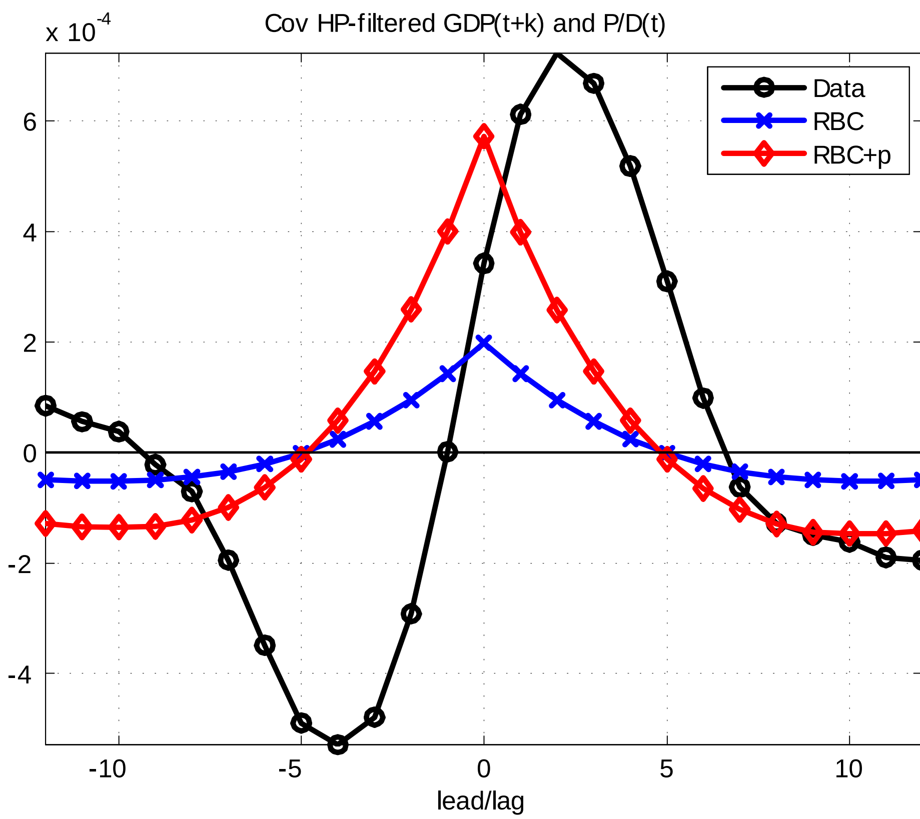

Figure 6: **Relation between the stock market and GDP in the model and in the data.** Cross-covariogram of the (HP-filtered) log P-D ratio and GDP, in the data (black circles), the RBC model, i.e. the model with only TFP shocks, (blue crosses) and the benchmark model with both p-shocks and TFP shocks (red diamond). The model covariograms are obtained by running 1000 simulations of length 200 each, and averaging.

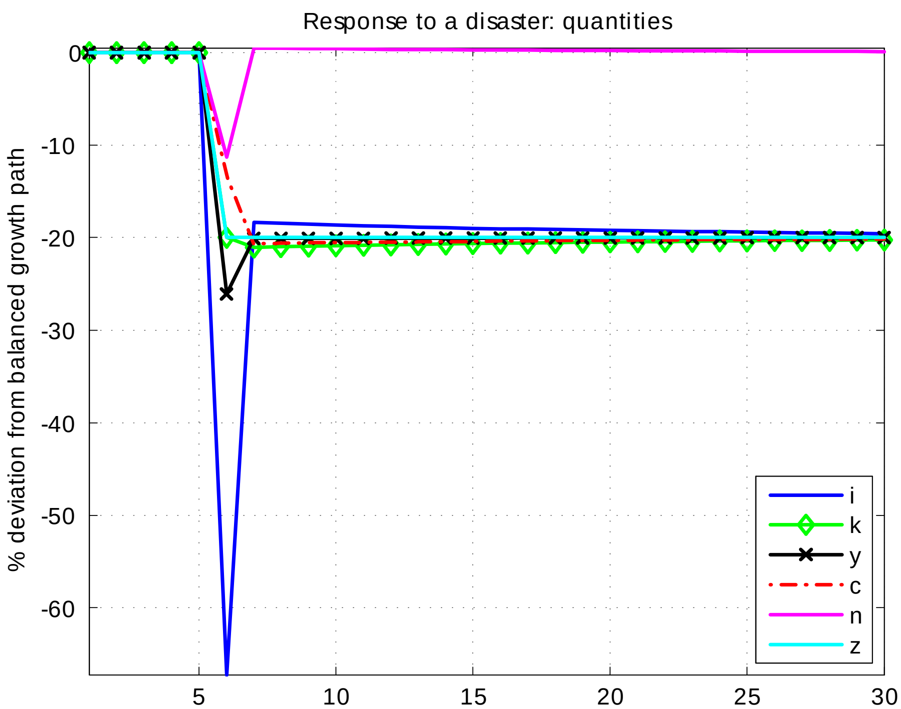

Figure 7: **Impact of a disaster when disasters may last several periods. (Section 5.2)** This picture plots a sample path for macroeconomic quantities where a 20% disaster occurs at $t = 6$, and hence the probability of a further disaster rises to .5, but no further disaster occurs.

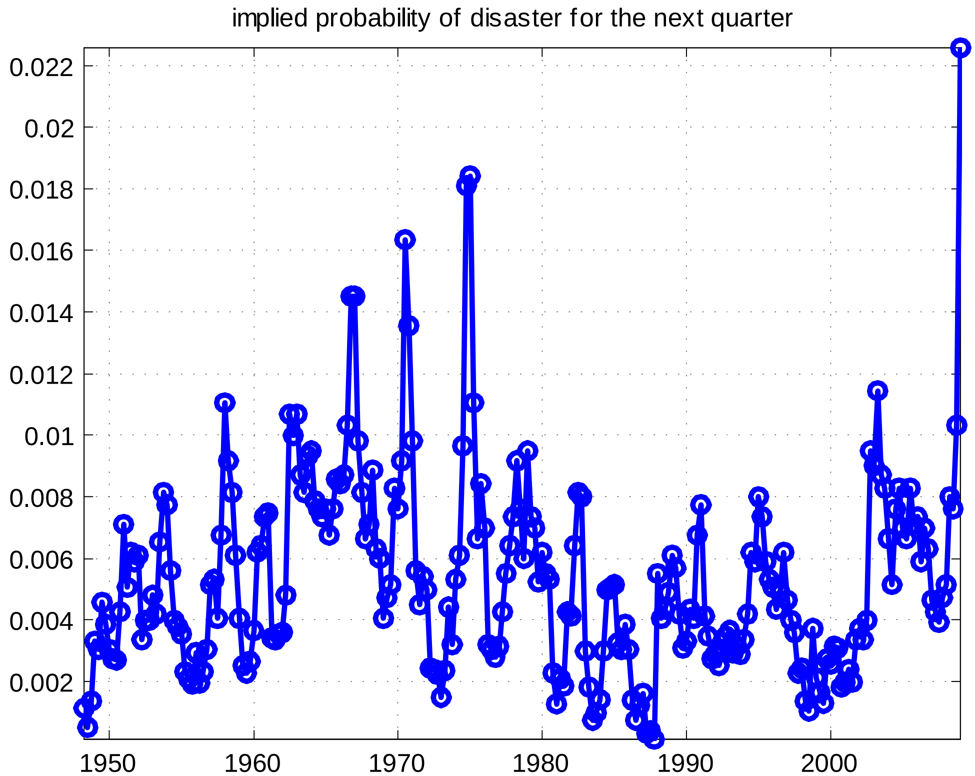

Figure 8: **Time-series for the quarterly probability of disaster (1948q1 to 2008q4).** This picture plots the path for p_t, as implied by the model given the observed price-dividend ratio from CRSP and the measured capital stock and TFP (see section 6).

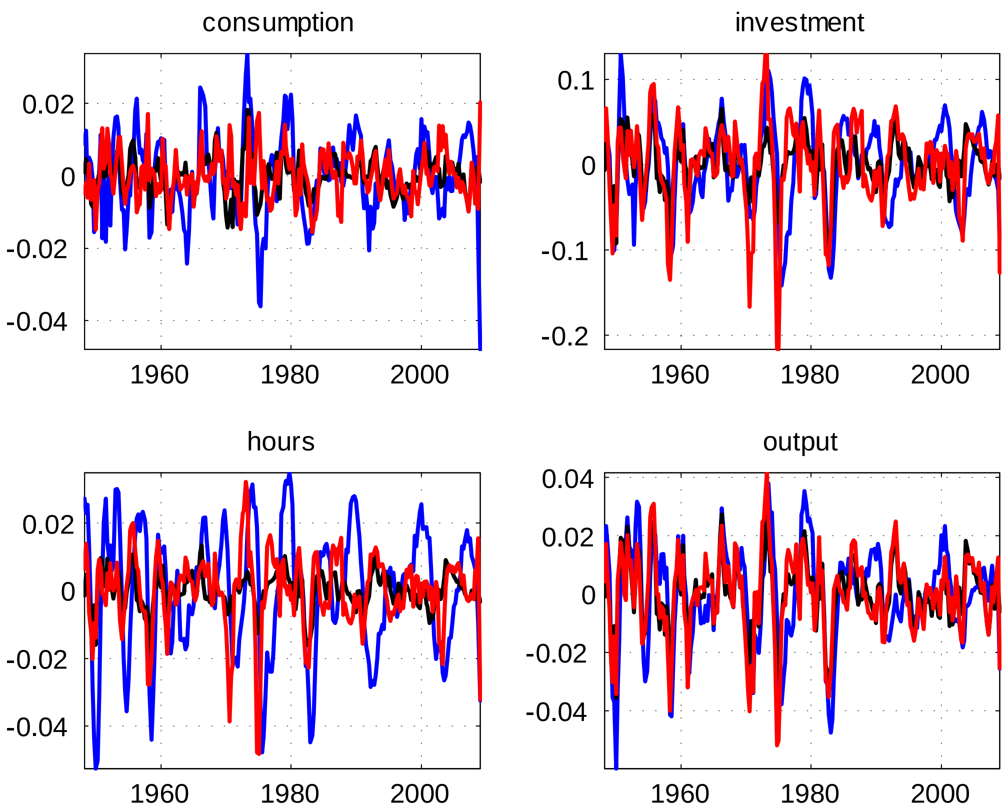

Figure 9: **Time-series of Consumption, Investment, Employment and Output, in the data, in the RBC model, and in the benchmark model.** This picture plots the data, together with the model-implied time series for macro aggregates for the RBC model (when TFP is fed into the model) and for the benchmark model (when both TFP and the probability of disaster are fed into the model). See section 6 for details. All series are logged and HP-filtered, 1947q1-2008q4.

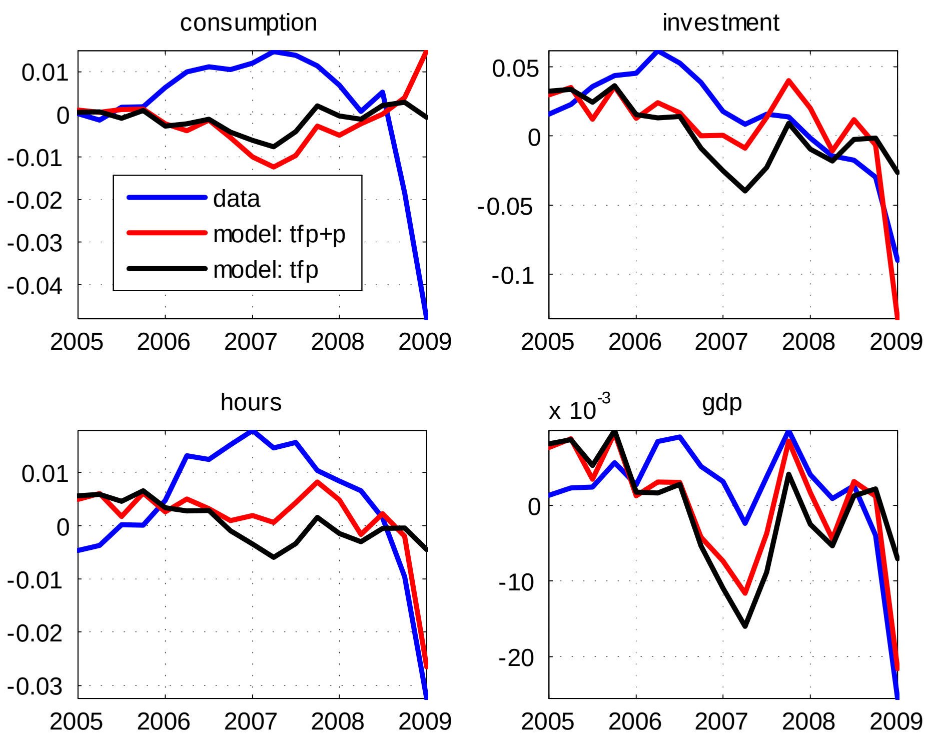

Figure 10: **Time-series of Consumption, Investment, Employment and Output, in the data, in the RBC model, and in the benchmark model, from 2005q1 to 2008q4.** This picture is a zoomed-in version of the previous figure, and displays the data, the model-implied time series for macro aggregates for the RBC model (when TFP is fed into the model) and for the benchmark model (when both TFP and the probability of disaster are fed into the model). See section 6 for details. All series are logged and HP-filtered, over 1947q1-2008q4, then cut from 2005q1 onwards.