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ABSTRACT

We estimate the effect of shifts in monetary policy using the term structure of interest rates. In our no-arbitrage model, the short rate follows a version of the Taylor (1993) rule where the coefficients on the output gap and inflation vary over time. The monetary policy loading on the output gap has averaged around 0.4 and has not changed very much over time. The overall response of the yield curve to output gap components is relatively small. In contrast, the inflation loading has changed substantially over the last 50 years and ranges from close to zero in 2003 to a high of 2.4 in 1983. Long-term bonds are sensitive to inflation policy shifts with increases in inflation loadings leading to higher short rates and widening yield spreads.

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1 Introduction

Over the last fifty years there have been dramatic changes in the intellectual foundation guiding monetary policy. The understanding of the nature of the trade-off between inflation on the one hand, and unemployment and real activity on the other, played a central role in this evolution. According to typical textbook accounts of this evolution, monetary policy in the 1960s was believed to have a lasting influence on unemployment and real activity, which could be achieved by trading off higher inflation with lower unemployment. In the late 1960s and 1970s, the work of Friedman and Phelps leading to the natural-rate hypothesis, the new understanding of the role of expectations, and the bruising macroeconomic experience of high inflation, all led to a fundamental shift in the paradigm. Since then, monetary policy's ability to influence unemployment and real activity is believed to be short lived, and has only a lasting effect on inflation.

Given this evolution of the understanding of the economic environment, it is difficult to believe that the conduct of monetary policy – the way policy authorities respond to the state of the economy – did not change in substantial ways. Sargent (1999) explains the rise of inflation in the 1970s and its subsequent fall in the 1980s on the basis of evolving beliefs about the trade-off between inflation and real activity. The Volcker disinflation in the early 1980s is an important and well-known example of the influence of the shifting paradigm on the actual conduct of monetary policy. The increased independence of central banks and the adoption of inflation targeting by many countries since the early 1990s are other clear manifestations of how the evolution of monetary policy theory has influenced monetary authorities.[1] Such shifts would have translated into time-varying responses of the Fed to inflation and real activity.

Not surprisingly, there has been a voluminous empirical literature attempting to document and quantify the importance of the changes in monetary policy.[2] One question that has received little attention so far is the implications of the shifting response of monetary policy to inflation and real activity on the term structure of interest rates. A growing number of studies that have employed macro factors in term structure models have found that macroeconomic fluctuations

[1] The implied evolution of monetary policy is complex and is not necessarily confined to a unidirectional evolution from worse to better. Romer and Romer (2002) discuss how the evolution of economic theory maps into changes in the way stabilization policies have been conducted. They argue that monetary policy in the 1980s is much more similar to monetary policy during the 1950s and 1960s than monetary policy during the 1970s.

[2] See, among many others, Clarida, Galí and Gertler (2000), Orphanides (2001), Cogley and Sargent (2001, 2005), Sims and Zha (2006), and Boivin (2006).

are an important source of uncertainty affecting bond risk premia.[3] Shifts in monetary policy could produce similar effects and be priced risk factors. In fact, monetary policy changes should affect the entire term structure because the actions of the Fed at the short end of the yield curve influence the dynamics of the long end of the yield curve through no-arbitrage restrictions.[4]

However, it is not clear how changing monetary policy affects long-term yields. Suppose the post-Volcker period can indeed be characterized as a much stronger desire to control inflation. On the one hand, for a given expected inflation rate, the higher sensitivity of short term interest rate to inflation might build up into higher long-term interest rates. On the other hand, the stronger stance on inflation, if credible, might lead to lower and less variable expected inflation, and thus, lower risk premia. Another interesting episode is the recent flattening yield curve between 2002 and 2005: an open question question is how much of this behavior is due to changes in monetary policy stances as opposed to other macro forces, such as the risk of deflation and low economic growth.

One central goal of this paper is to investigate the implications of the changes in the conduct of monetary policy on the shape and dynamics of the term structure of interest rates. To the extent that monetary policy has implications for the whole term structure, this also means that the entire yield curve, not just the short rate, contains potentially valuable information about monetary policy shifts. Exploiting additional information to identify policy shifts is useful because the literature has not come to a consensus in characterizing the nature of monetary policy shifts and their quantitative importance. On one side, Clarida, Galí and Gertler (2000) and Cogley and Sargent (2003, 2005) conclude that there have been important changes in the conduct of monetary policy that overall line up with a shift pre- and post-Volcker. On the opposite side of the debate, Orphanides (2001, 2003) and Sims and Zha (2005) find that either the conduct of monetary policy has not changed, or that if it did, the changes are not quantitatively important.

[3] A now large literature incorporating Taylor (1993) policy rules into term structure models following Ang and Piazzesi (2003) documents that the yield curve prices inflation and economic growth risk. Recently, Duffee (2006) disputes how much macro risk matters in bond prices, but most recent work including Buraschi and Jiltsov (2007), Ang, Bekaert and Wei (2008), Rudebusch and Wu (2008), and Joslin, Priebsch and Singleton (2009) find that inflation or economic growth, or both, play important roles in determining bond risk premia. Other studies that use other measures of macro factors like Pennachi (1991), who uses inflation surveys, and Buraschi and Jiltsov (2005), who use monetary aggregates, also find macro factors are priced.

[4] Two papers allowing for discrete regime shifts in monetary policy affecting the yield curve are Fuhrer (1996) and Bikbov (2006). As we explain below, our monetary policy shifts can approximate discrete regime shifts but better account for more a gradual evolution of monetary policy. Neither Fuhrer (1996) nor Bikbov (2006) estimate the price of risk of monetary policy changes.

Following this literature, we estimate monetary policy shifts by estimating changes in the parameters of a reaction function, where the Fed implements monetary policy through the setting of a short-term interest rate. Our study is the first to use a no-arbitrage model to identify policy shifts, which carry their own prices of risk, that allows information from the whole yield curve to be used.

We estimate a quadratic term structure model, where the dynamics of the short rate follow a version of Taylor's (1993) policy rule. Our no-arbitrage model allows for the Fed responses to inflation and output to potentially vary over time. Their evolution is assumed to obey a VAR that includes inflation and real activity. This allows changes in the policy parameters to be arbitrarily persistent and entertains the possibility that their current value may be influenced by the past behavior of the economy. For instance, an aggressive inflation response today might be due to inflation being high in the past. This is in contrast to most existing studies which assume that the time variation in the policy parameters is exogenous. Most importantly, modeling the policy shifts as stationary processes allows us to let agents form expectations with the knowledge that monetary policy is shifting. That is, agents are not oblivious to the fact that monetary policy changes over time and take into account future changes in forming prices. Since one objective is estimating the price of risk of policy shifts, this modeling approach is particularly desirable.[5]

We perform a series of exercises with the estimated model. We document the importance of the historical changes in monetary policy and discuss these changes in the context of evolving economic views. We investigate the effect of these policy changes on the term structure of interest rates by computing impulse responses and expected holding period returns. Finally, we directly estimate the price of risk of monetary policy shifts.

Our key findings can be summarized as follows. First, our estimates suggest that monetary policy changed substantially over the last 50 years. In particular, the Fed's sensitivity to inflation has changed markedly over time and our estimates are consistent with the broad contours of shifts in the intellectual framework behind monetary policy practice. In this respect, these estimates are largely consistent with the evidence reported in Clarida, Galí and Gertler (2000), and Cogley and Sargent (2005). One important feature of our results is that the evolution of monetary policy cannot be simply summarized by a once and for all shift of monetary policy under Volcker. For instance, we find that the response to inflation under Greenspan has been subject to large fluctuations and that in the early 1990s and in 2003-2004, it was as low or lower

[5] Note this modeling approach is in the spirit of Christopher Sims' perspective on policy intervention in a rational expectations context. See Sargent (1984) and Sims (1987).

than in the 1970s. The use of term structure information in the estimation of the policy rule leads to sharp parameter estimates, which statistically allow us to reject the hypothesis that the nominal short rate increased by more than inflation – the so-called Taylor principle – throughout the 1970s.

Second, we find shifts of monetary policy stances with regards to the output gap exhibit small variation. Our model estimates imply that most of the discretion in monetary policy has resulted from changing the response of the Fed to inflation rather than to output. The finding of very small variation in the output loadings is the opposite conclusion to estimates from models using random walks to capture the time variation of monetary policy coefficients. In these models estimated without yield curve information the policy loadings on output shocks exhibit much larger time variation.

Third, changes in monetary policy have a quantitatively important influence on the shape of the term structure. A surprise increase in the Fed response to inflation fluctuations, ceteris paribus, raises short term rates and increases the term spread. This suggests that investors perceive a higher response to inflation at the short-end of the yield curve as giving bonds of all maturities greater exposure to inflation and other macro risk. A stronger inflation policy response does not reduce inflation and other risk premia. Surprise increases in the inflation response induce a relatively large increase in yield spreads. In contrast, the effect of a surprise increase in the output gap stance increases the short rate and shrinks the term spread, which is also qualitatively similar to the effect of positive surprise to inflation or real activity. However, output gap components account for a relatively small proportion of yield movements.

We find that recently the stance of the Fed to inflation has decreased dramatically during the post-2001 period. The Fed response to inflation decreased to below one in 2001 and reached a low close to zero in 2003. This is consistent with an aggressive response of the Fed to a deflation threat during that period.[6] This would imply that the relationship between the Fed's forecast for inflation and current and past values of inflation and real activity changed over this period. Another interpretation is that short-term interest rates were held too low for too long a period of time in the face of deflationary threats from the aftermath of the 2001 recession and the September 2001 terrorist attacks.[7] Short rates reached 0.90% in 2003:Q2. If the Fed had held

[6] The Fed was concerned about the possibility of deflation at the time. Since contemporaneous inflation remained positive and we estimate the policy response to contemporaneous inflation, an aggressive reduction in the policy rate justified by expected deflation would be estimated in our framework as a weaker response to current inflation.

[7] This is a popular view taken by some media commentators including John B. Taylor in "How Government

its responses on the output gap and inflation constant at their values during 2000, the short rate during this period would have been 2.74%. Thus, according to this counter-factual experiment, interest rates would have been substantially higher if the Fed had not eased its stance to inflation as much as it did during the early 2000s.

The rest of the paper is organized as follows. Section 2 describes the modeling framework. It first describes the short rate equation, specified as a time-varying policy reaction function, and then derives bond prices based on a quadratic, arbitrage-free, term structure model. Section 3 describes the data. In Section 4 we discuss the parameter estimates, describe the estimated time series of the policy coefficients, show how policy changes affect the yield curve, and quantify how policy shift risk is priced. Section 5 concludes. The details of the bond pricing derivations and the Bayesian estimation technique can be found in the Appendix.

2 Model

We begin by describing a standard Taylor (1993) policy rule without policy shifts and then introduce shifts in inflation and output gap responses in Section 2.1. We model factor dynamics in Section 2.2 and compute bond prices in Section 2.3.

2.1 Policy Rules

2.1.1 Rule with No Policy Shifts

In a standard Taylor (1993) policy rule, the monetary authority sets the short rate as a linear function of inflation and the output gap:

$$r_t = \delta_0 + \bar{a} g_t + \bar{b} \pi_t + f_t^{\text{std}}, \tag{1}$$

where r_t is the short rate which we take to be the three-month T-bill yield, g_t is the output gap, and π_t is inflation.[8] In this specification, the Fed response to output and inflation in the systematic component of monetary policy, $\delta_0 + \bar{a} g_t + \bar{b} \pi_t$, is held fixed. The mean-zero residual in the

Created the Financial Crisis," *Wall Street Journal*, February 9, 2009.

[8] The original Taylor (1993) rule was applied to the federal funds rate (FFR). We follow Cogley and Sargent (2001), Ang and Piazzesi (2003), and many others by using the three-month T-bill, which has a correlation of 96.7% with the FFR over 1954:Q3 to 2007:Q4 (the FFR is unavailable in the beginning nine quarters of our full sample). Several reasons for the difference between T-bill yields and FFRs are greater liquidity for T-bills, the ability of T-bills to be used as collateral, and default risk (the FFR embeds default risk while T-bills do not). We use the T-bill yield as the basic building block because it has the same maturity as the quarterly frequency of the

standard policy rule, f_t^{std}, can be interpreted as a monetary policy shock where the superscript "std" refers to a standard Taylor rule. If f_t^{std} is correlated with the macro variables g_t and π_t then OLS does not yield consistent estimates of the Fed responses $\bar{a}$ and $\bar{b}$ to output gap and inflation shocks, respectively. However, Ang, Dong and Piazzesi (2006), Bikbov and Chernov (2006), and others show that f_t can be identified by the movements of long-term bond prices in a no-arbitrage model. Ang and Piazzesi (2003) show that estimating equation (1) yields a process of f_t that is very persistent and is highly correlated with short rate movements. We refer to the policy rule in equation (1) as the constant Taylor rule.

Our Taylor rule specification assumes that the policy instrument used by the Fed throughout this period is the short term interest rate. This might seem at odds with some anecdotal evidence, including some Fed official statements, suggesting that the Fed has used different instruments at different points in time. For instance, between 1979 and 1982, the Fed was officially targeting non-borrowed reserves. In practice, however, existing evidence suggests that using the short term interest is a good approximation to the operating procedure followed by Fed throughout that period, at least outside of the 1979-1982 period.[9] This is why most of the empirical literature modeling the Fed's behavior specifies a policy reaction function in terms of the short term interest rate.[10] In fact, all the existing evidence on changes in the conduct of monetary policy of which we are aware, including the evidence cited in this paper, is based on this assumption. Still, the fact remains that this assumption might be more problematic between 1979 and 1982, and we should keep in mind that this could contaminate our results for this period. However, this would not explain why the changes have persisted outside 1979-1982. In particular, this could not explain the fact that we observe a dramatically different conduct of monetary in the 1970s versus the 1980s, or during the early 2000s.

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macro variables, the model is specified at the quarterly frequency, and, like the other Treasury bonds used in the estimation, it is a risk-free rate.

[9] Bernanke and Mihov (1998) estimate parameters describing the operating procedure of the Fed over time. They find that the Fed followed something very close to an interest rate target since the 1950s, except perhaps between the Volcker non-borrowed reserved targeting experiment between 1979 and 1982. Cook (1989) argues that the Fed funds rate is a good indicator of monetary policy even during 1979-1982.

[10] Notable exceptions are Bakshi and Chen (1996) and Burashi and Jiltsov (2005) who specify a policy reaction function in terms of a money aggregate.

2.1.2 Rules with Policy Shifts

In order to capture the changing responses of the Federal Reserve to the macro environment, we let the policy responses on output and inflation vary over time. In our benchmark specification we set the systematic component of monetary policy to be

$$r_t = \delta_0 + a_t g_t + b_t \pi_t, \tag{2}$$

where the policy responses to g_t and π_t are stochastic processes. If there has been no change to the Fed's policy reaction function, then $a_t = \bar{a}$ and $b_t = \bar{b}$ for all t, otherwise time variation in a_t and b_t represent policy shifts in the relative importance of output gap or inflation shocks in the Fed's policy rule.

The monetary policy rule with policy shifts in equation (2) can be written in a similar form of the standard time-invariant Taylor rule in (1) by redefining the policy shock to explicitly depend on the level of the output gap and inflation combined with a time-varying policy stance:

$$\begin{aligned} r_t &= \delta_0 + (\bar{a} + a_t - \bar{a}) g_t + (\bar{b} + b_t - \bar{b}) \pi_t \\ &= \delta_0 + \bar{a} g_t + \bar{b} \pi_t + [(a_t - \bar{a}) g_t + (b_t - \bar{b}) \pi_t] \\ &= \delta_0 + \bar{a} g_t + \bar{b} \pi_t + f_t^{\text{bmk}} \end{aligned} \tag{3}$$

where the redefined discretionary policy shock is $f_t^{\text{bmk}} = (a_t - \bar{a}) g_t + (b_t - \bar{b}) \pi_t$, where the superscript "bmk" refers to the implied policy shock from our benchmark specification. Our estimations show that $(a_t - \bar{a}) g_t + (b_t - \bar{b}) \pi_t$ is highly correlated with the linear factor f_t^{std} from the constant Taylor rule (2). Thus, the constant Taylor rule may potentially confuse changes in systematic policy reaction components with true discretionary shocks. Previous research like Ang, Dong and Piazzesi (2006) has also shown that latent linear factors like f_t are correlated with output and inflation. In our setup, we decompose this traditional f_t term into policy shifts by the Fed (the $(a_t - \bar{a})$ and $(b_t - \bar{b})$ terms) and separate shocks to the output gap and inflation components.

In a final specification, we consider the possibility that there is also a linear policy shock in addition to the time-varying policy shifts in the benchmark specification in (2):

$$r_t = \delta_0 + a_t g_t + b_t \pi_t + f_t^{\text{ext}}, \tag{4}$$

where we specify f_t^{ext} to be orthogonal to a_t and g_t. We refer to equation (4) as the extended model. We find that by allowing the policy shifts a_t and b_t, the effect of the monetary policy shock factor f_t^{ext} becomes small. Thus, in presenting our results we concentrate on the benchmark model, but also compare our results across the models.

2.2 Factor Dynamics

We collect the macro and policy variables in the state vector $X_t = [g_t \; \pi_t \; a_t \; b_t \; f_t]^\top$, where f_t is either f_t^{std} in the constant Taylor rule or f_t^{ext} in the extended Taylor rule, which follows the stationary VAR:

$$X_t = \mu + \Phi X_{t-1} + \Sigma \varepsilon_t, \tag{5}$$

where $\varepsilon_t \sim$ IID $N(0, I)$. We order the macro variables first in the VAR. The constant Taylor rule and benchmark specifications omit the dynamics of (a_t, b_t) and f_t, respectively, and are special cases of the extended model.

We parameterize Φ as

$$\Phi = \begin{pmatrix} \Phi_{gg} & \Phi_{g\pi} & \Phi_{ga} & 0 & \Phi_{gf} \\ \Phi_{\pi g} & \Phi_{\pi\pi} & 0 & \Phi_{\pi b} & \Phi_{\pi f} \\ \Phi_{ag} & 0 & \Phi_{aa} & 0 & 0 \\ 0 & \Phi_{b\pi} & 0 & \Phi_{bb} & 0 \\ 0 & 0 & 0 & 0 & \Phi_{ff} \end{pmatrix}. \tag{6}$$

The upper 2×2 matrix of Φ represents a regular VAR of output and inflation. The coefficients Φ_{ga} and $\Phi_{\pi b}$ allow changes in the policy coefficients to influence the future path of output and inflation.[11] Similarly, non-zero Φ_{gf} and $\Phi_{\pi f}$ imply that discretionary linear policy shocks affect output and inflation next period. Previous research by Clarida, Galí and Gertler (2000) and many others find negative estimates of $\Phi_{\pi f}$ so tighter monetary policy reduces future inflation. We capture an endogenous response of the Fed policy to changing output and inflation in the coefficients Φ_{ag} and $\Phi_{b\pi}$. Specifically, we allow the response of inflation and output to depend on whether past inflation or output is high or low. If $\Phi_{b\pi}$ is positive, then the Fed becomes more aggressive in responding to inflation shocks when past inflation is high.[12]

[11] We do not allow the Fed's response to inflation to influence the future output gap or the Fed's output gap sensitivity to influence future inflation ($\Phi_{gb} = \Phi_{\pi a} = 0$). In systems with latent factors, the same reduced-form model may often be produced by arbitrarily scaling or shifting the coefficients governing the dynamics of a_t and b_t in Φ or Σ. To identify a_t and b_t, we allow their shocks to be correlated in Σ, but do not allow any feedback between a_t and b_t in Φ.

[12] A version of the Lucas critique would suggest that the time variation in the policy rule could imply time variation in the upper left 2×2 block of the matrix Φ. Unfortunately, modeling this time variation takes us outside the tractable class of quadratic term structure models (see below) and we can no longer derive long-term bond prices. In our current setup, the first two equations have thus to be interpreted as a first-order approximation to the true dynamics of these two variables.

We set Σ to take the following form:

$$\Sigma = \begin{pmatrix} \Sigma_{gg} & 0 & 0 & 0 & 0 \\ \Sigma_{\pi g} & \Sigma_{\pi\pi} & 0 & 0 & 0 \\ \Sigma_{aa} & \Sigma_{a\pi} & \Sigma_{aa} & 0 & 0 \\ \Sigma_{bg} & \Sigma_{b\pi} & \Sigma_{ba} & \Sigma_{bb} & 0 \\ 0 & 0 & 0 & 0 & \Sigma_{ff} \end{pmatrix}. \tag{7}$$

Our specification of Σ allows inflation, output and policy shifts to be contemporaneously correlated. We also specify that conditional shocks to f_t are orthogonal to all other factors as an identifying assumption.

We treat the policy variables, a_t and b_t, and the policy shock factor, f_t as latent factors. We are especially interested in the variation of a_t and b_t through the sample. We assume that the time variation in the policy coefficients is a covariance stationary process, that is all the eigenvalues of Φ lie inside the unit circle. Under our formulation, agents form expectations taking into account the probability that monetary policy will shift in the future according to a known stationary law of motion. That is, agents know that monetary policy has changed and will change again. Our specification thus accounts for a version of the Lucas critique, in the spirit of the Sims' perspective on policy intervention in a rational expectations context (see Sargent, 1984; Sims, 1987). Furthermore, the time variation of a_t and b_t is also allowed to endogenously depend on past macro variables, as is potentially the future path of g_t and π_t allowed to depend on the current monetary policy stance. Since the persistence of the process could be estimated to be arbitrarily high, our setup can approximate the random walk specification that have been used in previous studies (see, among others, Cooley and Prescott, 1976; Cogley and Sargent, 2001, 2005; Cogley, 2005; Boivin, 2006; Justiniano and Primiceri, 2006).

The benchmark Taylor rule with changing policy stances (equation (2)) and the extended version which also incorporates additional policy shocks (equation (4)) are examples of regression models with stochastically varying coefficients. Using only macro data and short rates, these systems may be asymptotically identified (see Pagan, 1980). However, it is hard to use only one observable variable, short rates, to identify two or more latent processes in small samples. Fortunately, it is not only the short rate that responds to policy shifts – we identify the variation in a_t, b_t, and f_t by using information from the entire yield curve. A further advantage of using the entire term structure is that we can identify the prices of risk that agents assign to the policy authority's time-varying policy rules. Thus, we can infer the effect on long-term yields of a policy shift by the Fed on its inflation stance, as well as the traditional analysis of tracing through the effect of an inflation shock on the term structure. We now show how bond prices embed the dynamics of all factors through no arbitrage.

2.3 Bond Prices

To derive bond prices from the policy shift model of equation (2), we write the short rate as a quadratic function of the factors $X_t = [g_t \ \pi_t \ a_t \ b_t \ f_t]^\top$:

$$r_t = \delta_0 + \delta_1^\top X_t + X_t^\top \Omega X_t \tag{8}$$

where δ_0 is a scalar and $\delta_1 = [0\ 0\ 0\ 0\ 1]^\top$, which picks up the linear factor f_t. In the quadratic term $X_t^\top \Omega X_t$ in equation (8), Ω is specified as

$$\Omega = \begin{pmatrix} 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 \\ \frac{1}{2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{9}$$

The short rate is linear in the observable macro variables and the quadratic form results from the interaction of the stochastic policy coefficients a_t and b_t with the macro factors g_t and π_t. The constant Taylor rule model is a standard affine term structure model where a_t and b_t are constant at $a_t = \bar{a}$ and $b_t = \bar{b}$ and the vector of loadings in the short rate takes the form $\delta_1 = [\bar{a}\ \bar{b}\ 1]^\top$ for the factors $[g\ \pi\ f^{\text{std}}]^\top$.

To price long-term bonds, we specify the pricing kernel to take the form:

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t^\top \lambda_t - \lambda_t^\top \varepsilon_{t+1}\right), \tag{10}$$

with the time-varying prices of risk depending on the state variables X_t following Duffee (2002) and others:

$$\lambda_t = \lambda_0 + \lambda_1 X_t, \tag{11}$$

for the 4×1 vector λ_0 and the 4×4 matrix λ_1. The prices of risk control the response of long-term yields to macro and policy shocks and cause the expected holding period returns of long-term bonds to vary over time (see Dai and Singleton, 2002). We can rewrite equation (10) to emphasize how each shock is separately priced:

$$m_{t+1} = \exp\left(-r_t - \frac{1}{2}\lambda_t^\top \lambda_t - \lambda_t^g \varepsilon_t^g - \lambda_t^\pi \varepsilon_t^\pi - \lambda_t^a \varepsilon_t^a - \lambda_t^b \varepsilon_t^b - \lambda_t^f \varepsilon_t^f\right),$$

for $\varepsilon_t = [\varepsilon_t^g \ \varepsilon_t^\pi \ \varepsilon_t^a \ \varepsilon_t^b \ \varepsilon_t^f]^\top$ and $\lambda_t = [\lambda_t^g \ \lambda_t^\pi \ \lambda_t^a \ \lambda_t^b \ \lambda_t^f]^\top$. If a risk is not priced, then the corresponding row of λ_t is equal to zero and payoffs of an asset correlated with those factor innovations receive no risk premium. Of particular interest are the risk premia parameters λ_t^a and λ_t^b on the policy shift variables a_t and b_t. These have not been examined before because the prices of risk in equation (11) have almost exclusively been employed in traditional affine macro-term structure models where the policy coefficients are constant (see, for example, Ang and Piazzesi, 2003).

The pricing kernel prices zero-coupon bonds from the recursive relation

$$P_t^n = \mathrm{E}_t[m_{t+1} P_{t+1}^{n-1}],$$

where P_t^n is the price of a zero-coupon bond of maturity n quarters at time t. Equivalently we can solve the price of a zero-coupon bond as

$$P_t^n = \mathrm{E}_t^{\mathbb{Q}} \left[\exp \left(- \sum_{i=0}^{n-1} r_{t+i} \right) \right], \tag{12}$$

where $\mathrm{E}_t^{\mathbb{Q}}$ denotes the expectation under the risk-neutral probability measure $\mathbb{Q}$, under which the dynamics of the state vector X_t are characterized by the risk-neutral constant and companion form matrix:

$$\begin{aligned} \mu^Q &= \mu - \Sigma\lambda_0 \\ \Phi^Q &= \Phi - \Sigma\lambda_1, \end{aligned} \tag{13}$$

where X_t follows the process

$$X_t = \mu^Q + \Phi^Q X_{t-1} + \Sigma\varepsilon_t$$

under $\mathbb{Q}$. In our estimation, we impose Φ^Q to take the same restrictions as the companion form under the real measure, Φ, given in equation (6). The relevant dynamics for bond prices are given by the risk-neutral parameters μ^Q and Φ^Q.

The quadratic short rate (2) or (8), combined with the linear VAR in equation (5), and the pricing kernel (10) gives rise to a quadratic term structure model. We can write the bond price for maturity n implied by the model as:

$$P_t^n = \exp(A_n + B_n^\top X_t + X_t^\top C_n X_t), \tag{14}$$

where the terms A_n, B_n, and C_n are given in Appendix A. Hence, if we denote the yield on a zero-coupon bond with maturity n quarters as $y_t^n = -1/n \log P_t^n$, yields are quadratic functions of X_t:

$$y_t^n = a_n + b_n^\top X_t + X_t^\top c_n X_t, \tag{15}$$

where $a_n = -A_n/n$, $b_n = -B_n/n$, and $c_n = -C_n/n$. This analytical form enables the estimation of the model and allows us to investigate how the entire term structure responds to policy changes and macro shocks.

We define an excess holding period return as the return on holding a long-term bond in excess of the short rate:

$$xhpr_{t+1}^n = \log \frac{\hat{P}_{t+1}^{n-1}}{\hat{P}_t^n} - r_t,$$

where the notation $xhpr_{t+1}^n$ denotes that the excess holding period return applies to a zero coupon bond of n periods today at time t. The conditional expected excess holding period return implied by the model is also given by a quadratic function:

$$\mathrm{E}_t[xhpr_{t+1}^n] = \bar{A}_n + \bar{B}_n^\top X_t + X_t^\top \bar{C}_n X_t, \tag{16}$$

where the coefficients $\bar{A}_n$, $\bar{B}_n$ and $\bar{C}_n$ are given in Appendix A.

Since the yields are quadratic functions of the state variables, the model belongs to the class of quadratic term structure models developed by Longstaff (1989), Beaglehold and Tenney (1992), Constantinides (1992), Leippold and Wu (2002, 2003), and Ahn, Dittmar and Gallant (2002).[13] None of these authors incorporate observable macro factors or investigate policy shifts. Ahn, Dittmar and Gallant (2002) and Brandt and Chapman (2003) demonstrate that quadratic models have several advantages over the Duffie and Kan (1996) affine class in adding more flexibility to better match conditional moments of yields and matching correlations across yields. The non-linearity of yields also aids in estimating prices of risk because there is an additional source of identification, through the non-linear mapping of state variables to yields, that is absent in an affine setting. Our model naturally shares these advantages. However, while we share the main technical methodology of the general class of quadratic term structure models, in our setting the quadratic structure arises naturally by allowing policy shifts in a Taylor policy rule, rather than immediately assuming the use of a quadratic term structure model. Thus, we provide some economic interpretation behind a general quadratic term structure model and interpret the factors and prices of risk in an interesting and important policy application.

[13] These quadratic models are related to the broader class of Wishart term structure models as they have linear representations of yields involving factors X_t and second moments of factors, $vech(X_t X_t')$. In these models, the quadratic term itself follows an affine process, as shown by Filipovic and Teichmann (2002) and Gourieroux and Sufana (2003). Buraschi, Cieslak and Trojani (2007) show that the quadratic short rate process can be supported in a Cox, Ingersoll and Ross (1985) production economy with a representative agent.

To estimate the model, we assume that all yields, including the short rate, are measured with error. Specifically, we assume:

$$\tilde{y}_t^n = y_t^n + u_t^n, \tag{17}$$

where y_t^n is the model-implied yield in equation (15), $\tilde{y}_t^n$ is the yield observed in data, and u_t^n IID $\sim N(0, \sigma_n^2)$, are additive measurement errors for all yields n. The quadratic form of the yields implies that there is not a one-to-one correspondence between certain yields assumed to be observed without error and latent state variables. Thus, standard Kalman filtering techniques for estimating affine models cannot be used to estimate our quadratic term structure model. We employ a Bayesian filtering algorithm that requires no approximation to estimate the model, which we detail in Appendix B.

3 Data

All our data is at a quarterly frequency and the sample period is from June 1952 to December 2007. The output gap is constructed following Rudebusch and Svensson (2002) and is given by

$$g_t = \frac{1}{4}\frac{Q_t - Q_t^*}{Q_t^*}, \tag{18}$$

where Q_t is real GDP and Q_t^* is potential GDP. We obtain real GDP from the Bureau of Economic Analysis (BEA), which is produced using chained 2000 dollars. We use the measure of potential output published by the Congressional Budget Office (CBO) in the Budget and Economic Outlook using chained 1996 dollars. To make the BEA series comparable to the CBO series, we translate real GDP to 1996 dollars. Finally, we demean the output gap and divide the output gap by four to correspond to quarterly units. Since we will be using per quarter short rates, this allows us to read the coefficient on the output gap as an annualized number. Our series for inflation is the year-on-year GDP deflator expressed as a continuously compounded growth rate. This is also divided by four to be in per quarter units. In addition to the one-quarter short rate, our term structure of interest rates comprises take zero-coupon bond yields from CRSP of maturities 4, 8, 12, 16, and 20 quarters. These are all expressed as continuously compounded yields per quarter.

Figure 1 plots the output gap, inflation, and the short rate over our sample in annualized terms. The output gap decreases during all the NBER recessions and reaches a low of -7.1% during the 1981:Q3 to 1983:Q4 recession. The output gap strongly trends upwards during the expansions of the 1960's, the mid-1980's, and the 1990's. Inflation is slightly negatively correlated with the output gap at -0.245. Inflation rises to near 10% during the mid-1970's and early 1980's, but otherwise remains below 5%. In the data, the correlation between the output gap and the short rate is -0.147 and the correlation between inflation and the short rate is 0.698. These correlations are matched closely by the model, with implied correlations of g_t and π_t with the short rate of -0.135 and 0.788, respectively.

As a benchmark, we report OLS estimates of simple Taylor (1993) rules where the short rate is a linear combination of macro factors and lagged inflation:

$$\begin{array}{ccccccccc} r_t & = & \underset{(0.001)}{0.005} & + & \underset{(0.059)}{0.025}\, g_t & + & \underset{(0.063)}{0.906}\, \pi_t & + & \varepsilon_t, \end{array} \tag{19}$$

where standard errors are reported in parentheses. Adding lagged short rates we obtain

$$\begin{array}{ccccccccccc} r_t & = & \underset{(0.000)}{0.000} & + & \underset{(0.028)}{0.072}\, g_t & + & \underset{(0.040)}{0.143}\, \pi_t & + & \underset{(0.031)}{0.872}\, r_{t-1} & + & \varepsilon_t, \end{array} \tag{20}$$

which can be written in partial adjustment format as:

$$r_t = 0.001 + 0.872\, r_{t-1} + (1 - 0.872)(0.562\, g_t + 1.117\, \pi_t) + \varepsilon_t.$$

These estimates are very similar to those reported in the literature. We report these OLS coefficients for comparison. In our model, the latent factor f_t^{std} in the constant Taylor rule or extended model and the redefined residual term $f_t^{\text{bmk}} = (a_t - \bar{a})g_t + (b_t - \bar{b})\pi_t$ in the benchmark model are correlated with the regressors. This implies that our estimated (time-varying) loadings may be potentially different from the OLS estimates.

In Table 1 we report summary statistics of the factors in data and implied by the benchmark model. The factors and yields are expressed in percentage terms at a quarterly frequency. The model provides a good match to the data, with model-implied unconditional means and standard deviations very close to the moments in data. In Panel A, the unconditional moments of the output gap and inflation implied by the model are well within 95% confidence bounds of the data estimates. Panel B of Table 1 compares the yields in data with the model-implied yields. We construct the posterior moments of the model-implied yields by using the generated latent factors in each iteration from the Gibbs sampler estimation. The tight posterior standard deviations indicate that the draws of the latent a_t and b_t factors in the estimation result in yields that very closely lie around the data yields. All of the model-implied estimates are very similar to the data. Note that we match the mean of the short rate exactly in the estimation.

4 Empirical Results

Section 4.1 discusses the parameter estimates starting from the constant Taylor rule model and working up to the extended model. Section 4.2 documents how the Fed reaction to output gap and inflation shocks have changed over time. In Section 4.3, we discuss how the yield curve reacts to changes in the Fed's policy parameters. Section 4.4 characterizes risk premia of long-term bonds. In Section 4.5, we present a counter-factual experiment of how the yield curve in the early 2000s might have been had the Fed not lowered its output gap and inflation stance as much as it did during this time.

4.1 Parameter Estimates

We report parameter estimates of the constant Taylor rule model in Table 2, the benchmark model in Table 3, and the extended model in Table 4. Each table reports posterior means of the model parameters, with posterior standard deviations in parentheses.

Across all specifications we find that high inflation Granger-causes lower economic growth and higher economic activity Granger-causes higher inflation consistent with a Phillips curve. For example, in the benchmark specification $\Phi_{g\pi} = -0.083$ with a posterior standard deviation of 0.036 and $\Phi_{\pi g} = 0.064$ with a posterior standard deviation of 0.011. In the conditional covariance matrix, Σ, conditional shocks to the output gap and inflation have almost zero correlation. These effects have been noted before in standard VAR macro models like Christiano, Eichenbaum and Evans (1996, 1999). Consistent with previous macro-affine models estimated in the literature, there are several significant price of risk parameters for g_t and π_t indicating that macro risk plays an important role in bond pricing.

4.1.1 Constant Taylor Rule Model

Table 2 reports that for the constant Taylor rule model the policy rule is given by

$$\begin{array}{ccccccccc} r_t & = & 0.008 & + & 0.364\, g_t & + & 0.609\, \pi_t & + & f_t^{\text{std}}, \\ & & (0.002) & & (0.063) & & (0.237) & & \end{array}$$

which are different to the OLS estimates in equation (19) as the policy shock factor f_t^{std} has an unconditional correlation with g_t and π_t of 7.2% and -16.4%, respectively. In particular, the long-term output gap response $\bar{a} = 0.364$ compared to the OLS estimate of 0.025 and the long-term inflation response is $\bar{b} = 0.609$ compared to the OLS estimate of 0.906.

Not surprisingly, Table 2 shows that the f^{std} factor is highly persistent with $\Phi_{ff} = 0.940$ as f^{std} inherits the high autocorrelation of the short rate. The correlation of f^{std} with the short rate is 0.849. Thus, in common with other affine estimations like Ang and Piazzesi (2003), f is a "level" factor in the sense of Knez, Litterman and Scheinkman (1994) and affects all yields across the term structure in a parallel fashion. Table 2 shows there is some evidence that high f values Granger-cause lower economic activity and lower inflation with coefficients of $\Phi_{gf} = -0.023$ and $\Phi_{\pi f} = -0.015$. While statistically weak, these coefficients are economically consistent with previous estimates in the literature that monetary policy shocks influence future inflation and output.

4.1.2 Benchmark Model

In the benchmark model the response of the Fed to the output gap and inflation vary through time. In Table 3 we report two estimates for the long-run Fed responses, which we refer to as $\bar{a}$ and $\bar{b}$. The first "sample" estimate is the average posterior values of a_t and b_t over the sample. The second long-run estimate is the population long-run mean implied by the VAR. Both estimates are similar to each other. The sample long-run response to the output gap is $\bar{a} = 0.356$ with a posterior standard deviation of 0.047. The long-run inflation response in the sample is $\bar{b} = 1.117$ with a posterior standard deviation of 0.138. The corresponding population VAR-implied long-run values are very similar at $\bar{a} = 0.372$ and $\bar{b} = 1.154$, respectively.[14]

According to the benchmark model, the long-run inflation response of $\bar{b} = 1.154$ is higher than the OLS estimate of 0.906 in equation (19) and is also higher than the constant Taylor rule estimate of 0.609 in Table 2. This suggests that the time variation of a_t and b_t plays an important role in determining the short rate. We further explore the policy shift dynamics of the output gap and inflation responses below.

The benchmark model's implied policy factor, f_t^{bmk}, in equation (3) is dependent on the time-varying a_t and b_t coefficients and should be highly correlated with the latent f_t^{std} factor from the model with the constant Taylor rule. This is indeed the case with a correlation of 0.864 between f_t^{std} and f_t^{bmk}. Thus, most of the variation of a standard linear policy factor is attributed to time-varying policy stances in the benchmark model.

Table 3 shows there is little evidence that changes in a_t and b_t affect the future path of output

[14] The posterior standard deviations for the VAR-implied values are larger the sample estimates because the population-implied value from the VAR can takes a wider range of values, especially when a simulated value for Φ is close to the unit circle.

and inflation, with estimates of $\Phi_{ga} = \Phi_{\pi b} = 0$. In contrast, we find large endogenous responses of the Fed to the macro environment, but these estimates have posterior standard deviations that are quite wide. A 1% increase in the output gap lowers a_t by 1.114, with a posterior standard deviation of 0.763, and a 1% increase in inflation increases b_t by 2.682, but this coefficient has a fairly large posterior standard deviation of 2.595. Thus, overall we find only weak statistical evidence that monetary policy stances endogenously respond to past output gap and inflation realizations.

In the conditional volatility matrix, the conditional shocks of a_t and b_t have a correlation of 85.4% with each other indicating that the Fed is likely to raise or lower the responses on inflation and output simultaneously. Conditional shocks to the macro factors and a_t and b_t have relatively low correlations, but some of these are significant. The conditional correlation of shocks to π_t and shocks to a_t and b_t are -0.332 and -0.203, respectively, with posterior standard deviations of 0.077 and 0.075. This implies that the Fed has a slight tendency to lower its stances to macro shocks at times when larger macro shocks are expected. The conditional volatility matrix also reveals that the conditional volatility of b_t is 0.048 and is approximately 10 times the conditional volatility of a_t, so the Fed's stance to output gap shocks has been much more stable than the Fed's response to inflation. Below, we further investigate the time-series variation of a_t and b_t.

4.1.3 Extended Model

Table 4 reports the estimates of the extended model, which are largely similar to the benchmark model for the common parameters. The estimates of the long-run Fed responses to the output gap and inflation are also very similar across the benchmark and extended models. For example, the sample long-run inflation response is 1.075 in the full model and 1.117 in the benchmark model. Similar to the benchmark model, we find weak evidence of Granger-causality of past inflation to next-period b_t values. In the extended model, $\Phi_{b\pi} = 2.952$, with a posterior standard deviation of 2.334, compared to $\Phi_{b\pi} = 2.682$ with a posterior standard deviation of 2.595 in the benchmark model.

The extended model has a linear latent f_t^{ext} model in addition to time-varying policy loadings. Table 4 shows that the conditional volatility of f_t^{ext} is 0.007×10^{-3} which is several orders of magnitude smaller than the conditional volatilities of the policy shift parameters a_t and b_t, which are 0.003 and 0.036, respectively. The correlation between f_t^{ext} and the short rate is low at 0.228 and thus must of the movements in the short rate come from changing g_t and π_t interacted with monetary policy shifts. Put another way, time-varying a_t and b_t in the benchmark monetary policy shock, f_t^{bmk} accounts for most of the movements of the short rate, and the extended model's remainder monetary policy effect, f_t^{ext}, plays a relatively small role in explaining short rate movements.

This is also seen in a formal variance decomposition of the short rate, where f^{ext} accounts for 4.6% of the total short rate variance, which is computed as

$$1 - \frac{\text{var}_f(r)}{\text{var}(r)},$$

where $\text{var}_f(r)$ is the variance of the short rate computed through the sample where f^{ext} is set to be constant at its sample mean and $\text{var}(r)$ is the variance of the short rate in data.[15] In contrast, the corresponding variance decomposition for f^{bmk} in the benchmark model is 0.257. This is as expected. The benchmark model already allows the output gap and inflation response to vary over time and further allowing an independent f^{ext} factor in addition to the a_t and b_t variation indicates that the role of f^{ext} is small. Furthermore, since the estimated time-series paths of a_t and b_t are very similar across the benchmark and the extended models, we concentrate on the benchmark model for looking at how Fed policy shifts have changed over time, which we turn to next.

4.2 Policy Shifts in Output and Inflation Responses

4.2.1 Short Rate Components

In the benchmark model, short rates move due to movements in the output gap component, $a_t g_t$, or movements in the inflation component, $b_t \pi_t$. A formal variance decomposition is given by (See Appendix C for details):

$$\begin{array}{ccccccc} \text{var}(r_t) & = & \text{var}(a_t g_t) & + & \text{var}(b_t \pi_t) & + & 2\text{cov}(a_t g_t,\, b_t \pi_t) \\ 100\% & = & 7.74\% & & 98.67\% & & \text{-}6.41\% \end{array}$$

Thus, almost all movements in the short rate are attributable to inflation and the Fed response to inflation. The variation of the output component of the short rate is relatively very small. Although the unconditional standard deviation (in annual terms) in the output gap and inflation are similar at 2.35% and 2.20% for g_t and π_t, respectively (see Table 1), the smaller policy responses on output shocks and the relatively larger responses on inflation cause the inflation component to dominate.

[15] The variance decompositions of long-term yields in the extended model in terms of f^{ext} are also very small. For example, the variance decomposition for the 20-quarter yield for f^{ext} is 0.008.

Figure 2 reports the decomposition of the short rate into these two components in the benchmark model. The policy factors are evaluated at the best estimates of a_t and b_t through the sample, together with the short rate. The correlation between the actual short rate and the fitted components $\delta_0 + a_t g_t + b_t \pi_t$ is 0.976, indicating that movements in the macro variables and policy rule account for almost all of the variation in the short rate and the observation error component is small. The bottom panel of Figure 2 visually confirms the very high attribution to $b_t \pi_t$ in the variance decomposition of the short rate by clearly demonstrating that the inflation components shadow the level of the short rate while the output gap terms are relatively stable.

Figure 3 displays the policy parameters a_t and b_t over the sample from the benchmark model. We plot the mean posterior estimates at each point in time of the Fed's response to output and inflation produced by the Gibbs sampler, along with two posterior standard deviations. There are two main differences between the Fed's output gap and inflation responses. First, the overall variation of the output gap loading is small compared to the inflation loading variation. The sample standard deviation of the posterior mean of a_t is 0.167 compared to 0.552 for the b_t loadings. Thus, the Fed has exhibited relatively little change in its responsiveness to economic growth and comparatively large changes in its inflation response.[16]

Second, the Fed places relatively more importance on responding to inflation than it does to the output gap. The inflation loading in the second panel of Figure 3 has a long-run mean of $\bar{b} = 1.117$ compared to $\bar{a} = 0.356$ and ranges from a low of 0.08 in 2003:Q3 to a high of 2.43 in 1983:Q4. These estimates lend support to the conjecture that the changes in monetary policy, at least to inflation, during this period were substantial.

4.2.2 Shifts in Inflation Stance

We now comment in detail on changes in Fed sensitivities to inflation plotted in the bottom panel of Figure 3. The response to inflation during the 1950s starts well below one at around 0.2 and sharply increases to above 2 during the late 1950s. In the last quarter of 1959 b_t reaches a temporary high of 2.23. From this high, the Fed's inflation coefficient starts to decrease during the 1960s, dips below one in the mid-1960s and stays low through the 1970s until 1979-1980. For instance, by the end of the 1970 recession, the response to inflation is less than 0.5.

In the late 1970s the Fed's inflation response starts to increase. Interestingly, and as in

[16] A previous version of the paper shows that there is significantly more variation in the output gap loading when the additional term structure information that this model brings to bear on the identification of policy stances is ignored. These results are available upon request.

Boivin (2006), the sharpest increase in the inflation response is not in late 1979, as is often assumed because of the appointment of Volcker in July 1979, but after 1981. At the beginning of January 1979 b_t starts out at 1.17 and reaches a high of 2.42 in June 1984. From this high the inflation loading starts to decrease over the 1980s and early 1990s. In 1992:Q3 b_t decreases to 0.68 before increasing to 2.05 in December 1994. Interestingly, this increase is completely consistent with anecdotal accounts of the Fed's "preemptive strike on inflation," which is the name given to this unprecedented episode as the Fed started to tighten monetary policy well before any concrete signs of inflation started to materialize (see Beckner, 1996, for further details on these events).

Recently the response to inflation falls well below one during the 2001 recession and the aftermath of the September 2001 terrorism acts. Specifically, the short rate declines from 4.25% in 2001:Q1 to 0.90% in 2003:Q2. During this time the Fed's response to inflation shocks also sinks below one in 2001:Q1 to 0.98 reaching a low of 0.08 in 2003:Q3. Thus, we find that the last few years of monetary policy under Greenspan was similar to monetary policy in the 1950s and 1970s with policy coefficients of inflation below one. From the low in 2003:Q3 to the 2007:Q2, the Fed response to inflation increases sharply, rising above one in 20055:Q4 to a value of 1.45 in 2007:Q2. Our estimates indicate a slight decrease in the inflation loading during the second part of 2007 associated with the beginning of the current financial and economic crisis.

The time-series pattern of the inflation coefficient is in broad agreement with the evidence reported in Clarida, Galí and Gertler (2000), Cogley and Sargent (2005), and Boivin (2006). In general, we find a low response to inflation in the 1970s and much higher response in the early 1980s. The response in most of the 1960s and 1970s is such that a unit increase in inflation translates into a less than a unit increase in the nominal policy rate, thus a decline in the real rate, and hence implies an easing of monetary policy. If agents had been expecting the response to inflation to remain permanently below one, it might have been possible for inflation expectations, and thus economic fluctuations, to be driven by non-fundamental sunspot shocks. Some commentators argue that the failure to rule out the presence of such shocks is responsible for the greater economic volatility of the 1970s (see the discussion by Taylor, 1999; Clarida, Galí and Gertler, 2000). It is important to note that in the context of our model, however, agents understand that the response to inflation shift in the future. As long as they believe that the long-run mean of the response to inflation is above one, sunspot fluctuations should be ruled out, even if the response is temporarily below one.

It is also reassuring to observe that the secular evolution of our estimates of inflation stances is also consistent with the evolution of the intellectual debate and development of monetary policy theory. In particular, Romer and Romer (2002) argue that this evolution has not been a linear improvement in the understanding of the economy, but rather "it is a more interesting evolution from a crude but fundamentally sensible model of how the economy worked in the 1950s, to more formal but faulty models in the 1960s and 1970s, and finally to a model that was both sensible and sophisticated in the 1980s and 1990s." This lines up well with our estimated response to inflation, where monetary policy in the late 1950s is quite similar to the one observed in the early 1980s under Volcker and in the mid-1990s under Greenspan.

Interestingly, the more recent evidence suggests that monetary policy in the 2003-2004 period is closer to monetary policy observed in the 1970s with both periods having inflation responses below one. However, there is an important difference with the 1970s: during the 2003-2004 episode, the Fed was concerned about the possibility of deflation. Since we are modeling the Fed's response to contemporaneous inflation, the estimated decline in the response to inflation could be explained by expected deflation at the time that was not reflected in current inflation. This explanation requires that the historical relationship between the Fed's forecast for inflation at that point in time and other macro variables broke down during that period. Whether it is due to markedly different forecasts for inflation or an actual change in the Fed's response to inflation, this period certainly stands out as unusual relative to our historical estimates.

A final comment is that the timing of monetary policy shifts in Figure 3 is broadly consistent with the general increase in volatility in the 1960s and 1970s and the general decline of macro volatility in the mid-1980s, suggesting monetary policy could have played a significant role in this pattern of change in inflation and economic growth.[17] This view might be reinforced by the fact that the increased volatility in the last few years of our sample is preceded, according to our estimates, by a substantial reduction in the response to inflation between 2003-2004. However, this remains highly speculative at this stage.

4.3 Policy Shifts and the Yield Curve

To gauge how important each factor is in determining the dynamics of yields, we compute unconditional variance decompositions in Table 5. Movements in GDP growth account for only a small proportion of yield curve movements whereas inflation accounts for 60.3% of short

[17] This issue is explored by, among others, Stock and Watson (2003), Boivin and Giannoni (2006), Sims and Zha (2006), and Justiniano and Primiceri (2006).

rate movements and 19.1% of long rate movements. Time-varying output gap loadings have their greatest impact on yield variation at the middle part of the yield curve with a variance decomposition reaching 11.6% at at a one-year maturity. Similarly, the attribution of yield variance to movements in b_t is the highest at a two-year maturity with a value of 27.5%. The majority of yield variation is given by the inflation term $b_t\pi_t$ for all maturities, which echoes the high comovement between $b_t\pi_t$ and the short rate in Figure 8. These variance decompositions suggest that the main determinants of yield curve variation are inflation and inflation loadings.

In Figure 4 we plot the impulse response of the yield curve to macro shocks and inflation policy shifts. Since the yields are non-linear functions of macro and policy variables, we compute the impulse responses numerically, which we detail in Appendix D. We graph in columns the response of an unconditional one-standard deviation shock to each factor and trace the effect on the short rate r_t, the 20-quarter long rate, y_t^{20}, and the yield spread, $y_t^{20} - r_t$, which are presented in rows. The units on the x-axis are in quarters whereas the impulse responses are expressed in annualized percentage terms.

In the first column, a positive output shock initially increases short rates and decreases spreads. A unit unconditional standard deviation shock to g_t of 2.35% first increases the short rate by 0.79% and reaches a peak of 1.31% at 12 quarters. The effect on the long yield is smaller, which initially increases by 0.42%. Consequently the term spread initially shrinks by -0.37% before shrinking to its maximum absolute value of -0.50% at 13 quarters. A similar pattern is observed for a shock to inflation on the yield curve in the second column. A 2.20% shock to inflation causes the short rate to jump 0.93% and shrinks the term spread by 0.66%. These effects die out faster than the output gap shock, with the effect of an inflation shock on the short rate dying out by 30 quarters. These results are similar to those reported by Ang and Piazzesi (2003), among many others, who show that the macro shocks have a greater influence on the short end of the yield curve compared to the long end of the yield curve.

The third and fourth columns of Figure 4 show the response of the yield curve to monetary policy shifts. Note that these responses would be the same across the yield curve if monetary policy shifts were not priced or the price of a_t and b_t risk were constant. The third column traces the response of an unconditional one standard deviation change in a_t, which is 0.170. This causes the short rate to increase to 1.70% and the 20-quarter yield to increase to 0.64%. Whereas the shock to the long bond dies out quickly after 20 quarters, the shock to a_t on the short rate persists up to 40 quarters. Thus, changing the output gap response mostly affects the short end of the yield curve and causes the term spread to shrink.

In contrast, the last column shows inflation policy shifts affect the long end of the yield curve more than the short end. In the last column we shock the short rate, long yield, and term spread by an unconditional one standard deviation shock in b_t, which is 0.560. This increases the short rate by 1.81%, which dies out after 30 quarters. The 20-quarter yield moves almost twice as much as the short rate to 3.36%. The shock to b_t also has a more persistent effect on the long end of the yield curve than the short rate, which does not reach close to zero until around 40 quarters.

The stronger effect on the long end of the yield curve of changing inflation stances compared to changing economic growth stances is also observed in Figure 5. We plot the yield curve as a function of maturity in quarters on the x-axis and trace the response of the yield curve after an initial shock at $t = 0$ for various t in quarters in rows. The left-hand column, which plots the effect of a unit unconditional standard deviation shock to a_t has overall smaller responses, which die out more quickly, than the term structure responses in the right-hand column, which traces the effect of an initial unconditional one standard deviation shock to b_t. Figure 5 shows the effect of the shocks to a_t and b_t are monotonic across the yield curve with the a_t shocks affecting short maturities more whereas the long end of the yield curve is more sensitive to changes in b_t. After 20 quarters the whole yield curve is still around 50 basis points higher for the b_t shock, with the long yield slightly higher than the short rate, but only the short end of the yield curve remains around 50 basis points for the a_t shock. In summary, long-term yields are especially sensitive to policy changes in inflation loadings.

What can explain the stronger effect of b_t on long-term yields compared to short-term yields? In principle, if investors perceived the stronger inflation response as implying lower and less variable inflation, it would suggest b_t might carry a negative price of risk and long-term yields may decrease to reflect lower risk premia when b_t is higher. Instead, according to our estimates, when b_t changes the short rate, and by no arbitrage the entire yield curve, becomes more exposed to inflation risk. This suggests b_t carries a positive risk premium. Moreover, since b_t does not affect (or barely affects) the path of future inflation, raising b_t then causes all bond prices to be exposed to greater inflation and other macro risk – and since b_t also responds to inflation but not vice versa, this effect is magnified. Long-maturity bonds pick up this feedback sensitivity and hence are relatively more sensitive to changes in b_t. To investigate this further we now examine bond risk premia and factor prices of risk.

4.4 Risk Premia

In this section we characterize risk premia of long-term bonds by computing expected excess holding period returns. We first examine impulse responses of risk premia and then directly examine the price of risk of monetary policy shifts.

4.4.1 Risk Premia Responses

Figure 6 graphs the response of expected excess holding period returns, $\mathrm{E}_t[xhpr^n_{t+1}]$, defined in equation (16) of the $n = 4$-quarter bond plotted in the solid line and the $n = 20$-quarter bond plotted in the dashed line. The shocks are unit unconditional standard deviation shocks to each factor and the conditional risk premia are evaluated at the model's implied unconditional mean. The top row shows that bond risk premia are strongly counter-cyclical, as documented by many studies. Positive output gap shocks decrease risk premia with a 2.35% shock to g_t producing a 0.32% decrease in the intermediate-term bond risk premium and a 1.96% decrease in the long-term bond risk premium. These shocks are very persistent and for the long bond do not die out until after 40 quarters. Similarly, during economic expansions when inflation is high, risk premia also decrease. These responses are relatively larger than the risk premia responses to the output gap. In particular, a 2.20% inflation shock decreases the long-bond risk premium by 2.71%.

The bottom row of Figure 6 shows the risk premia responses to shocks in monetary policy stances. The 0.170 shock to a_t increases the risk premium of the four-quarter bond by 0.19% while the same shock produces a maximum decrease in the 20-quarter bond risk premium of 0.36% after 15 quarters. The non-linear effect comes from the quadratic form and implies that an unexpected more responsive stance to output gap shocks is felt non-monotonically across the yield curve with higher risk premia on intermediate-bonds and lower risk premia on long-term bonds. However, the absolute size of the the risk premia responses to changes in a_t shocks is relatively small, which is consistent with the small role that output gap components play in determining the short rate and term structure.

The lower right-hand panel of Figure 6 shows that risk premia are very responsive to b_t shocks. A 0.560 shock to b_t increases the risk premium on the four-quarter bond by 1.79% and the 20-quarter bond by 6.86%. Both responses die out monotonically and disappear after 40 quarters for the long bond. These responses are large and reflect a risk premium for agents being subject to changing inflation policy stances, which we now directly examine.

4.4.2 Interpreting Price of Risk Parameters

To directly interpret the λ_0 and λ_1 price of risk coefficients consider first a standard CRRA representative agent economy with the pricing kernel

$$m_{t+1} = \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} = \exp(-\gamma(\mu_c + \sigma_c \varepsilon^c_{t+1})),$$

where C_t is aggregate consumption, γ is the representative agent's risk aversion, μ_c and σ_c are the mean and volatility of log consumption growth, respectively, and $\varepsilon^c_{t+1} \sim N(0,1)$ is the shock to consumption growth. In this economy consider a security paying off ε^c_{t+1}, which is a unit consumption shock. This has price

$$\begin{aligned} P_t &= \mathrm{E}_t[m_{t+1}\varepsilon^c_{t+1}] = \mathrm{E}_t[e^{-\gamma(\mu_c+\sigma_c\varepsilon^c_{t+1})}\varepsilon^c_{t+1}] \\ &= -\gamma e^{-r_t}, \end{aligned} \tag{21}$$

where the risk-free rate $r_t = \gamma\mu_c - \frac{1}{2}\gamma^2\sigma_c^2$. Equation (21) shows that the price of this security is equal to a bond multiplied by minus the degree of risk aversion. The security has a mean zero payoff since $\mathrm{E}[\varepsilon^c_{t+1}] = 0$. If agents are risk neutral, then the price of the security has the same zero value as its mean payoff. If agents are risk averse, $\gamma > 0$, then the price of the security is negative. In this case, agents bid down the price of the security below its risk neutral price and must be paid to bear consumption risk. Consequently, consumption risk carries a positive risk premium.

In the term structure model there is no direct correspondence to representative risk aversion because there are multiple shocks, the prices of risk vary over time, and the prices of risk of a_t and b_t also depend on the correlated movements of g_t and π_t as well as each other. Nevertheless, we can use the difference between the actual price and risk-neutral price of claims to the factor shocks to provide economic intuition for the policy shift risk priced by the yield curve. The prices of unit shock payoffs are given by

$$\begin{aligned} \mathrm{E}_t[m_{t+1}\varepsilon_{t+1}] &= \mathrm{E}_t\left[\exp\left(-r_t - \frac{1}{2}\lambda_t^\top\lambda_t - \lambda_t^\top\varepsilon_{t+1}\right)\varepsilon_{t+1}\right] \\ &= -\lambda_t e^{-r_t} = -(\lambda_0 + \lambda_1 X_t)e^{-r_t}, \end{aligned} \tag{22}$$

where we use the definition of the pricing kernel in equation (10) and the short rate $r_t = \delta_0 + \delta_1^\top X_t + X_t^\top \Omega X_t$ is also a function of X_t.

Equation (22) carries the same intuition as the simple CRRA economy in equation (21). In this case the effect of risk aversion is specified over multiple factors and if the price of a factor shock is negative, the risk premium attached to that factor shock is positive, and vice versa. In the model the prices of risk also depend on the level of the state variables. Note that because there is no structural representative agent utility in the model, we cannot directly translate the price of a factor shock to an overall aggregate measure of risk aversion.

Figure 7 plots the price of a unit shock to a_t and b_t as a function of the policy loadings. We denote with vertical lines the steady-state values of $\bar{a} = 0.372$ and $\bar{b} = 1.154$. Figure 7 shows that both the price of risk of a_t and the price of risk of b_t are positive indicating that agents demand a risk premium for bearing a_t and b_t factor shocks. Note that the pure factor risk prices do not translate directly into a one-to-one relation into risk premia, as Figure 6 shows. This happens even in an affine model because multiple factors are correlated, but the effects are exacerbated in our non-linear setting. Nevertheless, for changing inflation stances, which play the most important role in explaining term structure movements, the higher risk premia on b_t shocks when b_t is high is consistent with agents demanding higher risk premia on bonds when b_t is high (see Figure 6). Intuitively, when b_t is high the entire yield curve becomes more exposed to inflation risk and the risk of b_t itself. Agents dislike this risk and bid down the prices of bonds and increase long-term yields.

It is also possible to price the implied monetary policy shock, f^{bmk}, in equation (3) implied by the benchmark model:

$$\mathrm{E}_t[m_{t+1} f^{\text{bmk}}] = \mathrm{E}_t[m_{t+1}[(a_t - \bar{a})g_t + (b_t - \bar{b})\pi_t],$$

which can be computed in closed form as a function of various quadratic terms. Evaluated at the posterior mean of all factors, this price is 0.001 and is close to zero for all sample values of the parameters. Hence, the benchmark model implies that agents apply a price of risk to monetary policy shifts but not to a linear monetary policy shock.

4.5 The Post-2001 Episode

It is an interesting question to see what the yield curve would have looked like had the Fed not changed its inflation loading over the post-2001 period. Some commentators have raised the possibility that short-term interest rates were held too low for too long after the Fed lowered interest rates to respond to the September 2001 terrorist attacks and the 2001 recession. However, during the early 2000s, inflation was low, possibly even below an implicit target (see Figure 1) and the output gap was negative, so interest rates may have declined over this period even with unchanged policy coefficients. Our model provides a way to precisely quantify at what level interest rates would have been in the counter-factual experiment where the Fed did not change its stance to output or inflation in the years following 2001.[18]

Figure 8 reports the results of a counter-factual experiment where we hold the Fed weight on the output gap and inflation at the average weight of a_t and b_t over 2000 and trace the effects on the yields post-2001. We allow the other macro factors, g_t and π_t, to take their sample values. Figure 8 plots the path of the short rate and term spread if the Fed had maintained the same output and inflation stance as in 2000 in the dashed lines and overlays the actual short rate and term spread in the solid lines.

The top panel of Figure 8 shows that had the Fed not changed its output and inflation stances since 2000, short rates would indeed have been higher post-2001 than in data. During 2002 the average difference between the actual short rate and the theoretical short rate had the Fed not changed its policy stance is 1.19%. Thus, even with no additional policy response to the terrorist attacks and the recession, short term interest rates would have fallen. However, the gap between the short rate in data and the theoretical short rate with no policy changes widens in 2003 and 2004 to 2.12% and 2.96%, respectively. Short rates reach a minimum level of 0.90% in 2003:Q2, whereas at this time the short rate without any policy shifts would have been 2.74%. Even in 2005, short rates in data remain considerably below the short rates predicted by the Fed's 2000 policy stance with an average difference of 1.37%.

The bottom panel graphs the five-year term spread. Figure 8 shows that there is qualitatively little difference between the slope of the yield curve over 2001-2004 comparing actual data and the counter-factual exercise where the Fed did not take a more dovish stance. The overall pattern of both the data and the predicted term spread with no policy changes is similar with both exhibiting an overall decrease post-2001 over the next five years. In summary, if the Fed had maintained its output gap and inflation stance during 2000 over the early 2000s, the overall level of the yield curve would have been much higher than observed in data but there would have been little effect on the slope of the yield curve compared to its data realization. Thus, the reduction in the response to inflation between post-2001 does not explain any part in the flattening of the yield curve during this period (the so-called Greenspan conundrum).

[18] Naturally, like any counter-factual experiment, the usual caveats on the Lucas critique apply since agents would have reacted differently if they would have known the Fed's monetary policy stance would be fixed under the counter-factual.

5 Conclusion

Despite how convincing the anecdotal evidence and historical accounts may be, diverging conclusions have been reached in the literature concerning the importance, or even the existence, of changes in the conduct of monetary policy over the last 50 years. The literature has also concentrated on using only short rate information to estimate changes in policy stances, but potential shifts in monetary policy should affect the entire term structure; the actions of the Fed at the short end of the yield curve influence the dynamics of the long end of the yield curve through no-arbitrage restrictions. These shifts in monetary policy are in principle another source of uncertainty affecting bond risk premia. Thus, long-term bonds provide valuable information on identifying monetary policy shifts.

In this paper we propose to model monetary policy and the term structure of interest rates jointly using a quadratic term structure model, where the coefficients of the short rate equation – which describe the monetary policy response to the state of the economy – can change over time. The model allows the entire yield curve to be used to estimate potential monetary policy shifts. Importantly, long-term bonds are priced by agents who care about shifting monetary policy risk. These agents are not oblivious to the fact that monetary policy changes over time and take into account future changes in forming prices.

We find that monetary policy has changed in quantitatively important ways which are almost entirely summarized by the evolution of the the Fed's response to inflation. The response of the Fed to the output gap has remained relatively constant. Our estimates of the time-varying inflation response line up largely with narrative accounts of monetary policy and with some existing empirical estimates. The Fed's response to inflation lies below one during the 1970s, increases during the 1980s, and again decreases below one during the early 2000s. The changing response to inflation carries a positive price of risk with an unexpected increase in the Fed's response to inflation increasing the short rate and increasing the term spread. Intuitively investors perceive a higher policy loading to inflation at the short-end of the yield curve as giving bonds of all maturities greater exposure to inflation and other risk.

Appendix

A Bond Pricing

The price of a one-period zero-coupon bond is given by:

$$\begin{aligned} P_t^1 &= \exp(-r_t) = \exp(-\delta_0 - \delta_1^\top X_t - X_t^\top \Omega X_t) \\ &= \exp(A_1 + B_1^\top X_t + X_t^\top C_1 X_t), \end{aligned} \tag{A-1}$$

where $A_1 = -\delta_0$, $B_1 = -\delta_1 = -[0\ 0\ 0\ 0\ 1]^\top$, and $C_1 = -\Omega$, with Ω given in equation (9).

Under measure $\mathbb{Q}$, the price of a n-period zero-coupon bond, P_t^n, is:

$$\begin{aligned} P_t^n &= \mathrm{E}_t^{\mathbb{Q}}(\exp(-r_t)P_{t+1}^{n-1}) \\ &= \mathrm{E}_t^{\mathbb{Q}}(\exp\left(-r_t + A_{n-1} + B_{n-1}^\top X_{t+1} + X_{t+1}^\top C_{n-1} X_{t+1}\right)) \\ &= \exp\left(-r_t + A_{n-1} + B_{n-1}^\top(\mu^Q + \Phi^Q X_t) + (\mu^Q + \Phi^Q X_t)^\top C_{n-1}(\mu^Q + \Phi^Q X_t)\right) \\ &\quad \times \mathrm{E}_t^{\mathbb{Q}}(\exp\left((B_{n-1}^\top \Sigma + 2(\mu^Q + \Phi^Q X_t)^\top C_{n-1}\Sigma)\varepsilon_{t+1} + \varepsilon_{t+1}^\top \Sigma^\top C_{n-1}\Sigma\varepsilon_{t+1}\right)). \end{aligned} \tag{A-2}$$

To take the expectation, note that the expectation of the exponential of a quadratic Gaussian variable is given by:

$$\mathrm{E}[\exp(A^\top \epsilon + \epsilon^\top \Gamma \epsilon)] = \exp\left(-\frac{1}{2}\ln\det(I - 2\Psi\Gamma) + \frac{1}{2}A^\top(\Psi^{-1} - 2\Gamma)^{-1}A\right)$$

for $\epsilon \sim N(0, \Psi)$. This can be derived by general properties of Gaussian quadratic forms (see Mathai and Provost, 1992; Searle, 1997).

After taking the expectation and equating the terms with

$$P_t^n = \exp(A_n + B_n^\top X_t + X_t^\top C_n X_t),$$

the coefficients A_n, B_n, and C_n are given by the recursions:

$$\begin{aligned} A_n &= -\delta_0 + A_{n-1} + B_{n-1}^\top \mu^Q + \mu^{Q^\top} C_{n-1}\mu^Q - \frac{1}{2}\ln\det(I - 2\Sigma^\top C_{n-1}\Sigma) \\ &\quad + \frac{1}{2}(\Sigma^\top B_{n-1} + 2\Sigma^\top C_{n-1}\mu^Q)^\top (I - 2\Sigma^\top C_{n-1}\Sigma)^{-1}(\Sigma^\top B_{n-1} + 2\Sigma^\top C_{n-1}\mu^Q) \\ B_n^\top &= -\delta_1^\top + B_{n-1}^\top \Phi^Q + 2\mu^{Q^\top} C_{n-1}\Phi^Q + 2(\Sigma^\top B_{n-1} + 2\Sigma^\top C_{n-1}\mu^Q)^\top (I - 2\Sigma^\top C_{n-1}\Sigma)^{-1}\Sigma^\top C_{n-1}\Phi^Q \\ C_n &= -\Omega + \Phi^{Q^\top} C_{n-1}\Phi^Q + 2(\Sigma^\top C_{n-1}\Phi^Q)^\top (I - 2\Sigma^\top C_{n-1}\Sigma)^{-1}(\Sigma^\top C_{n-1}\Phi^Q) \end{aligned} \tag{A-3}$$

If the model were specified in continuous time, then the recursions in equation (A-3) are versions of the ordinary differential equations derived by Ahn, Dittmar and Gallant (2002) on the bond pricing coefficients.

To compute conditional excess holding period returns, we use the exponential quadratic form for zero-coupon bond prices in equation (14) to write:

$$\begin{aligned} xhpr_{t+1}^n &= \log\frac{P_{t+1}^{n-1}}{P_t^n} - r_t \\ &= A_{n-1} + B_{n-1}^\top X_{t+1} + X_{t+1}^\top C_{n-1} X_{t+1} - (A_n + B_n^\top X_t + X_t^\top C_n X_t) \\ &\quad + (A_1 + B_1^\top X_t + X_t^\top C_1 X_t). \end{aligned} \tag{A-4}$$

Since $X_{t+1} \sim N(\mu + \Phi X_t, \Sigma\Sigma^\top)$, we can write the expectation of a quadratic form, $\mathrm{E}_t(X_{t+1}^\top C X_{t+1})$, as:

$$\mathrm{E}_t(X_{t+1}^\top C X_{t+1}) = tr(C\Sigma\Sigma^\top) + (\mu + \Phi X_t)^\top C(\mu + \Phi X_t).$$

This allows us to compute the expectation as:

$$\mathrm{E}_t[xhpr_{t+1}^n] = \bar{A}_n + \bar{B}_n^\top X_t + X_t^\top \bar{C}_n X_t, \tag{A-5}$$

where

$$\begin{aligned}\bar{A}_n &= A_{n-1} - A_n + A_1 + tr(C_{n-1}\Sigma\Sigma^\top) + \mu^\top C_{n-1}\mu + B_{n-1}^\top \mu \\ \bar{B}_n &= \Phi^\top B_{n-1} - B_n + B_1 + 2\Phi^\top C_{n-1}\mu \\ \bar{C}_n &= \Phi^\top C_{n-1}\Phi - C_n + C_1. \end{aligned} \qquad \text{(A-6)}$$

B Estimating the Model

The model is estimated using a Bayesian Gibbs sampling algorithm. While there are several examples of these types of estimations for affine models (see, among others, Lamoureux and Witte, 2002; Johannes and Polson, 2005; Ang, Dong and Piazzesi, 2006; Dong, 2006), these cannot be directly employed to estimate the quadratic model because in an affine setting, drawing the latent factors requires a Kalman filter. The Kalman filter assumes that yields are linear functions of state variables, whereas they are non-linear functions in the quadratic model. In this appendix, we develop an acceptance-rejection algorithm to draw the latent factors without approximation.

For ease of notation, we group the macro variables as $M_t = [g_t\; \pi_t]^\top$ and the latent factors as $L_t = [a_t\; b_t\; f_t]^\top$ and rewrite the dynamics of $X_t = \left[M_t^\top\; L_t^\top\right]^\top$ in equation (5) as:

$$\begin{pmatrix} M_t \\ L_t \end{pmatrix} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} + \begin{pmatrix} \Phi_{11} & \Phi_{12} \\ \Phi_{21} & \Phi_{22} \end{pmatrix} \begin{pmatrix} M_{t-1} \\ L_{t-1} \end{pmatrix} + \begin{pmatrix} \Sigma_{11} & 0 \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix} \begin{pmatrix} \varepsilon_{M,t} \\ \varepsilon_{L,t} \end{pmatrix}, \qquad \text{(B-1)}$$

where $\varepsilon_t = (\varepsilon_{M,t}^\top\, \varepsilon_{L,t}^\top)^\top \sim$ IID $N(0, I)$ and Σ_{11} and Σ_{22} are lower triangular.

The parameters of the model are $\Theta = (\mu,\, \Phi,\, \Sigma,\, \delta_0, \delta_1,\, \Omega,\, \mu^Q,\, \Phi^Q,\, \sigma_u)$, where μ^Q and Φ^Q are parameters governing the state variable process under the risk neutral probability measure, and σ_u denotes the vector of observation error volatilities $\{\sigma_n\}$. We draw μ^Q and Φ^Q, but invert the prices of risk λ_0 and λ_1 using the relations:

$$\begin{aligned}\lambda_0 &= \Sigma^{-1}(\mu - \mu^Q) \\ \lambda_1 &= \Sigma^{-1}(\Phi - \Phi^Q). \end{aligned} \qquad \text{(B-2)}$$

The latent factors $L_t = \{a_t\; b_t\; f_t\}$ are generated in each iteration of the Gibbs sampler. Note that Ω and δ_1 are not estimated, given that they are fixed from equation (9). We also do not draw δ_0, but set this parameter to match the sample mean of the short rate in each iteration.

We simulate 500,000 observations in addition to using a burn-in period of 50,000. We sample every fifth observation to lower the serial correlation of the parameter draws. To check the adequacy of the number of simulations, we use the tests of Geweke (1992) and Raftery and Lewis (1992). For all parameters the simulation length is more than adequate except for some companion form parameters where the stationarity constraint is binding. These parameters are estimated to be always close to the unit circle no matter how many iterations are used as they capture the high persistence of the factors.

We now detail the procedure for drawing each of these variables. We denote the factors $X = \{X_t\}$ and the set of yields for all maturities in data as $\tilde{Y} = \{\tilde{y}_t^n\}$. Note that the model-implied yields $Y = \{y_t^n\}$ differ from the yields in data, Y, by observation error. By definition, $\tilde{Y} = Y + u$, where $u = \{u_t^n\}$ is the set of all observation errors for all yields. This notation also implies that the short rate in data, $\tilde{r}_t$, is the same as $\tilde{y}_t^1$.

B.1 Drawing the Latent Factors

We use a single-move algorithm based on Jacquier, Polson and Rossi (1994, 2004) adapted to our model. We derive a draw from the distribution $P(L_t|\tilde{Y}, L_{-t}, M)$, where L_t is the t-th observation of the latent factors, L_{-t} denotes all the latent factors except the t-th observation, and $\tilde{Y}$ and M are the complete time-series of yields and macro variables, respectively. We use the notation $\tilde{Y}_t$ and M_t to denote the t-th observation of the set of yields and macro variables. We draw the latent factors L_t conditional on the macro factors, yields, and other parameters.

From the Markov structure of the model, we can write:

$$P(L_t|L_{-t}, \tilde{Y}, M, \Theta) \propto P(L_t|L_{t-1}, M, \Theta) P(\tilde{Y}_t|L_t, M, \Theta) P(L_{t+1}|L_t, M, \Theta). \qquad \text{(B-3)}$$

To keep the notation to a minimum, we write this as:

$$P(L_t|L_{-t}) \propto P(L_t|L_{t-1}) P(\tilde{Y}_t|L_t) P(L_{t+1}|L_t).$$

Since M and Θ are treated as known, we can write the dynamics for L_t in equation (B-1) as

$$\begin{aligned} L_t &= \mu_2 + \Sigma_{12}\varepsilon_{M,t} + \Phi_{21}M_{t-1} + \Phi_{22}L_{t-1} + \Sigma_{22}\varepsilon_{L,t} \\ &= \mu_{L,t} + \Phi_L L_{t-1} + \Sigma_L \varepsilon_{L,t}, \end{aligned} \tag{B-4}$$

where $\mu_{L,t} = \mu_2 + \Sigma_{12}\varepsilon_{M,t}$, $\Phi_L = \Phi_{22}$, and $\Sigma_L = \Sigma_{22}$. Since M is observable and we hold Θ as fixed, $\mu_{L,t}$ is known at time t.

Each conditional distribution of the RHS of equation (B-3) is known. From equation (B-4) we have

$$P(L_t|L_{t-1}) \propto \exp\left(-\frac{1}{2}(L_t - \mu_{L,t} - \Phi_L L_{t-1})^\top (\Sigma_L \Sigma_L^\top)^{-1}(L_t - \mu_{L,t} - \Phi_L L_{t-1})\right). \tag{B-5}$$

Similarly, from the VAR in equation (B-4) we can write:

$$P(L_{t+1}|L_t) \propto \exp\left(-\frac{1}{2}(L_{t+1} - \mu_{L,t} - \Phi_L L_t)^\top (\Sigma_L \Sigma_L^\top)^{-1}(L_{t+1} - \mu_{L,t} - \Phi_L L_t)\right). \tag{B-6}$$

Finally, the likelihood of bond yields, $P(\tilde{Y}_t|L_t)$, is given by:

$$P(Y_t|L_t) \propto \exp\left(-\frac{1}{2}\sum_n \left[\frac{(\tilde{y}_t^n - (a_n + b_n^\top X_t + X_t^\top c_n X_t))^2}{\sigma_n^2}\right]\right), \tag{B-7}$$

where $X_t = [L_t^\top \; M_t^\top]$ and the summation is taken over yield maturities n. In the likelihood, the model-implied yield, $y_t^n = a_n + b_n^\top X_t + X_t^\top c_n X_t$, is given in equation (15), and σ_n^2 is the observation error variance of the yield of maturity n.

We can combine equations (B-5)-(B-7) and complete the square to obtain:

$$\begin{aligned} P(L_t|L_{-t}) &\propto P(\tilde{Y}_t|L_t)\exp\Big(-\frac{1}{2}\Big[L_t^\top(\Phi_L'^\top(\Sigma_L\Sigma_L^\top)^{-1}\Phi_L + (\Sigma_L\Sigma_L^\top)^{-1})L_t \\ &\quad -2(L_{t+1}^\top(\Sigma_L\Sigma_L^\top)^{-1}\Phi_L + L_{t-1}^\top\Phi_L(\Sigma_L\Sigma_L^\top)^{-1} - \mu_L(\Sigma_L\Sigma_L^\top)^{-1}\Phi_L + \mu_L(\Sigma_L\Sigma_L^\top)^{-1})L_t\Big]\Big) \\ &\propto P(\tilde{Y}_t|L_t)\exp\left(-\frac{1}{2}(L_t - \mu_t^*)^\top(\Sigma_t^*)^{-1}(L_t - \mu_t^*)\right) \end{aligned} \tag{B-8}$$

where

$$\begin{aligned} \Sigma_t^* &= \left(\Phi_L'^\top(\Sigma_L\Sigma_L^\top)^{-1}\Phi_L + (\Sigma_L\Sigma_L^\top)^{-1}\right)^{-1} \\ \mu_t^* &= \Sigma_t^*(L_{t+1}^\top(\Sigma_L\Sigma_L^\top)^{-1}\Phi_L + L_{t-1}^\top\Phi_L(\Sigma_L\Sigma_L^\top)^{-1} - \mu_{L,t}(\Sigma_L\Sigma_L^\top)^{-1}\Phi_L + \mu_{L,t}(\Sigma_L\Sigma_L^\top)^{-1})^\top. \end{aligned}$$

Since this distribution is not recognizable, we use a Metropolis draw. We draw a proposal from the distribution $N(\mu_t^*, \Sigma_t^*)$ and then the acceptance probability is based on the likelihood of $P(\tilde{Y}_t|L_t)$.

In the three-factor constant Taylor rule model, yields are linear functions of the factors and there is no need for the single-move algorithm. In this case, we employ the more efficient Carter and Kohn (1994) forward-backward algorithm to first run a Kalman filter forward and then sample f_t backwards. When the single-move algorithm is employed, it produces parameter values and posterior sample paths of f_t that are almost identical to those produced by the forward-backward algorithm. Since we specify the mean of f_t to be zero for identification, we set each generated draw of this factor to have a mean of zero.

In the benchmark four-factor specification, we additionally require that at each point in time both a_t and b_t are non-negative for purposes of identification.

In the extended five-factor model, we impose a prior for the draw of a_t and b_t period by period. Specifically, the prior used is given by the uniform distribution on the interval $[k_t^p - \sigma_{k,t}^p \,;\, k_t^p + \sigma_{k,t}^p]$ for $k = a, b$; where k_t^p and $\sigma_{k,t}^p$ represent the posterior mean and standard deviation of factor k in period t from the estimated benchmark model. The motivation for imposing this prior is that we want the latent factor f_t^{ext} in the extended model to capture only for short rate and term structure movements not accounted for by the four factors $[g_t \; \pi_t \; a_t \; b_t]^\top$ since the model specifies f_t^{ext} as a factor orthogonal to $[g_t \; \pi_t \; a_t \; b_t]^\top$.

B.2 Drawing μ and Φ

We follow Johannes and Polson (2005) and explicitly differentiate between $\{\mu, \Phi\}$ under the real measure and $\{\mu^Q, \Phi^Q\}$ under the risk-neutral measure. As X_t follows a VAR in equation (5), we follow standard Gibbs sampling and use conjugate normal priors and posteriors for the draw of μ and Φ. We note that the posterior of μ and Φ conditional on X, $\tilde{Y}$ and the other parameters is:

$$\begin{aligned} P(\mu, \Phi|\Theta_-, X, \tilde{Y}) &\propto P(\tilde{Y}|\Theta, X)P(X|\mu, \Phi, \Sigma)P(\mu, \Phi) && \text{(B-9)} \\ &\propto P(\tilde{Y}|\Sigma, \delta_0, \delta_1, \mu^Q, \Phi^Q, \sigma_\eta, X)P(X|\mu, \Phi, \Sigma)P(\mu, \Phi) \\ &\propto P(X|\mu, \Phi, \Sigma)P(\mu, \Phi), && \text{(B-10)} \end{aligned}$$

where Θ_- denotes the set of all parameters except μ and Φ, and $P(X|\mu, \Phi, \Sigma)$ is the likelihood function of the VAR, which is normally distributed from the assumption of normality for the errors in the VAR. The validity of going from the first line to the second line is ensured by the bond recursion in equation (A-3): given μ^Q and Φ^Q, the bond price is independent of μ and Φ. We specify the prior $P(\mu, \Phi)$ to be $N(0, 1000)$, which effectively represents an uninformative prior. We draw μ and Φ separately for each equation in the VAR system (5). Given that we impose the restriction that f_t is mean zero for identification, we set μ_f to zero.

B.3 Drawing $\Sigma\Sigma^\top$

To draw $\Sigma\Sigma^\top$, we note that the posterior of $\Sigma\Sigma^\top$ conditional on X, $\tilde{Y}$ and the other parameters is:

$$P(\Sigma\Sigma^\top|\Theta_-, X, \tilde{Y}) \propto P(Y|\Theta, X)P(X|\mu, \Phi, \Sigma)P(\Sigma\Sigma^\top), \quad \text{(B-11)}$$

where Θ_- denotes the set of all parameters except Σ. This posterior suggests an Independence Metropolis draw. We draw $\Sigma\Sigma^\top$ from the proposal density

$$q(\Sigma\Sigma^\top) = P(X \mid \mu, \Phi, \Sigma)P(\Sigma\Sigma^\top),$$

which is an Inverse Wishart (IW) distribution if we specify the prior $P(\Sigma\Sigma^\top)$ to be IW, so that $q(\Sigma\Sigma^\top)$ is an IW natural conjugate. The proposal draw $(\Sigma\Sigma^\top)^{m+1}$ for the $(m+1)$th draw is then accepted with probability α, where

$$\begin{aligned} \alpha &= \min\left\{ \frac{P((\Sigma\Sigma^\top)^{m+1} \mid \Theta_-, X, \tilde{Y})}{P((\Sigma\Sigma^\top)^{m} \mid \Theta_-, X, \tilde{Y})} \frac{q((\Sigma\Sigma^\top)^{m})}{q((\Sigma\Sigma^\top)^{m+1})}, 1 \right\} \\ &= \min\left\{ \frac{P(\tilde{Y} \mid (\Sigma\Sigma^\top)^{m+1}, \Theta_-, X)}{P(\tilde{Y} \mid (\Sigma\Sigma^\top)^{m}, \Theta_-, X)}, 1 \right\}, && \text{(B-12)} \end{aligned}$$

where $P(\tilde{Y}|\mu, \Phi, \Theta_-, X)$ is the likelihood function of all yields, including the short rate, which is normally distributed from the assumption of normality for the observation errors. From equation (B-12), α is just the ratio of the likelihoods of the new draw of $\Sigma\Sigma^\top$ relative to the old draw.

B.4 Drawing μ^Q and Φ^Q

We draw μ^Q and Φ^Q with a Random Walk Metropolis algorithm assuming a flat prior. We draw each parameter separately in μ^Q, and each row in Φ^Q. The accept/reject probability for the draws of μ^Q and Φ^Q is the ratio of the likelihood of bond yields based on candidate and last draw of μ^Q and Φ^Q:

$$\begin{aligned} \alpha &= \min\left\{ \frac{P((\mu^Q, \Phi^Q)^{m+1} \mid \Theta_-, X, \tilde{Y})}{P((\mu^Q, \Phi^Q)^{m} \mid \Theta_-, X, \tilde{Y})} \frac{q((\mu^Q, \Phi^Q)^{m})}{q((\mu^Q, \Phi^Q)^{m+1})}, 1 \right\} \\ &= \min\left\{ \frac{P(\tilde{Y} \mid (\mu^Q, \Phi^Q)^{m+1}, \Theta_-, X)}{P(\tilde{Y} \mid (\mu^Q, \Phi^Q)^{m}, \Theta_-, X)}, 1 \right\}, && \text{(B-13)} \end{aligned}$$

In each iteration, we invert λ_0 and λ_1 and report the estimates of the prices of risk instead of μ^Q and Φ^Q. We discard non-stationary draws of Φ^Q.

B.5 Drawing σ_u

Drawing the variance of the observation errors, σ_u^2, is straightforward, because we can view the observation errors η as regression residuals from equation (17). We draw the observation variance $(\sigma_\eta^n)^2$ separately from each yield. We specify a conjugate prior $IG(0, 0.00001)$, so that the posterior distribution of σ_η^2 is a natural conjugate Inverse Gamma distribution. The prior information roughly translates into a 30bp bid-ask spread in Treasury securities, which is consistent with studies on the liquidity of spot Treasury market yields (see, for example, Fleming, 2000).

C Short Rate Variance Decomposition

For the short rate variance decomposition presented in Section 4.2.1, we write the short rate given by equation (8) as

$$r_t = \delta_0 + \delta_1^\top X_t + X_t^\top \Omega^1 X_t + X_t^\top \Omega^2 X_t, \tag{C-1}$$

where the matrices Ω^1 and Ω^2 have elements $\Omega^1_{ga} = \Omega^1_{ag} = \Omega^2_{\pi b} = \Omega^2_{b\pi} = 0.5$ and zeros elsewhere. Then, the unconditional variance of the short rate can be decomposed as:

$$\begin{aligned} \text{var}(r_t) &= \text{var}(a_t g_t) + \text{var}(b_t \pi_t) + 2\text{cov}(a_t g_t,\, b_t \pi_t) \\ &= \text{var}(X_t^\top \Omega^1 X_t) + \text{var}(X_t^\top \Omega^2 X_t) + 2\text{cov}(X_t^\top \Omega^1 X_t,\, X_t^\top \Omega^2 X_t), \end{aligned} \tag{C-2}$$

where

$$\begin{aligned} \text{var}(X_t^\top \Omega^1 X_t) &= 2tr(\Omega^1 \Sigma_X \Sigma_X^\top)^2 + 4\mu_X^\top \Omega^1 \Sigma_X \Sigma_X^\top \Omega^1 \mu_X \\ \text{var}(X_t^\top \Omega^2 X_t) &= 2tr(\Omega^2 \Sigma_X \Sigma_X^\top)^2 + 4\mu_X^\top \Omega^1 \Sigma_X \Sigma_X^\top \Omega^2 \mu_X \\ 2\text{cov}(X_t^\top \Omega^1 X_t,\, X_t^\top \Omega^2 X_t) &= 4tr(\Omega^1 \Sigma_X \Sigma_X^\top \Omega^2 \Sigma_X \Sigma_X^\top) + 8\mu_X^\top \Omega^1 \Sigma_X \Sigma_X^\top \Omega^2 \mu_X, \end{aligned}$$

and Σ_X is the unconditional covariance matrix of X_t implied by the VAR in equation (5).

D Impulse Responses

Since the yields are non-linear, we follow Gallant, Rossi and Tauchen (1993) and Potter (2000), among others, and compute the impulse response functions using simulation. We start with the sample series of data (g_t and π_t) and the posterior means of the latent factors (a_t and b_t) at each observation t. We term these points X_t^*. From the VAR in equation (5), we construct an orthogonalized error term ν_t by taking the Cholesky of $\Sigma\Sigma^\top$. To construct the impulse response for the jth variable of X_t, we first draw a shock v_t that represents a shock only to variable j from the error term distribution ν_t. From the points X_t^*, we construct a new series where each observation has been shocked by v_t, which we denote as $X_t^v = X_t^* + v_t$.

The impulse response functions are taken as the difference between the averaged response of the yields to the evolution of X_t^* without shocks to the evolution of the shocked X_t^v series:

$$\text{E}(y_{t+k}^n | X_t^v) - \text{E}(y_{t+k}^n | X_t^*).$$

Using the VAR in equation (5), we simulate out the value of X_{t+k}^v from X_t^v and the value of X_{t+k}^* from X_t^*. This is done at each observation t. Then, we construct the yields, y_{t+k}^n, from equation (15) corresponding to the state vectors X_{t+k}^v and X_{t+k}^*. We take values of $k = 1 \ldots 60$ quarters.

The impulse responses are computed at each observation by taking the average of the sample paths of y_{t+k}^n computed using X_{t+k}^v minus the average of the sample paths of y_{t+k}^n computed using X_{t+k}^*. We report the average of the impulse responses across all observations t. This procedure results in impulse responses that are identical to impulse responses computed for traditional VAR systems for large numbers of observations.

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Table 1: Summary Statistics

Panel A: Moments of Macro Factors

	Means (%)		Standard Deviations (%)		Autocorrelations	
	Data	Model	Data	Model	Data	Model
g	0.000	-0.094	2.349	2.372	0.930	0.927
	(0.323)	(0.528)	(0.056)	(0.116)	(0.034)	(0.021)
π	3.419	3.327	2.199	2.407	0.982	0.983
	(0.321)	(2.163)	(0.262)	(1.051)	(0.026)	(0.007)

Panel B: Moments of Yields

	$n = 1$	$n = 4$	$n = 8$	$n = 12$	$n = 16$	$n = 20$
Means (%)						
Data	5.083	5.429	5.616	5.775	5.895	5.973
	(0.403)	(0.400)	(0.397)	(0.388)	(0.385)	(0.380)
Model	5.083	5.437	5.660	5.766	5.862	6.005
	–	(0.051)	(0.025)	(0.007)	(0.008)	(0.010)
Standard Deviations (%)						
Data	2.820	2.784	2.749	2.683	2.656	2.615
	(0.347)	(0.308)	(0.304)	(0.297)	(0.296)	(0.284)
Model	2.796	2.782	2.754	2.701	2.644	2.597
	(0.166)	(0.060)	(0.013)	(0.006)	(0.004)	(0.008)

The table lists various moments of the factors in data and implied by the four-factor benchmark model. All the factors and yields are expressed in annualized percentage terms. All standard errors are reported in parentheses. Panel A lists moments of the output gap and inflation. for the benchmark model Panel B reports data and benchmark model unconditional moments of n-quarter maturity yields. The benchmark model has factors $X_t = [g_t \, \pi_t \, a_t \, b_t]^\top$ the short rate equation (2), factor dynamics in equation (5), prices of risk in equation (11), and observation error standard deviations in equation (17) for yields of maturity n quarters. For the model, we construct the posterior distribution of unconditional moments by computing the unconditional moments implied from the parameters in each iteration of the Gibbs sampler. We compute the posterior distribution of the model-implied yields using the generated latent factors in each iteration of the Gibbs sampler. In Panels A and B, the data standard errors are computed using GMM with robust standard errors. The sample period is June 1952 to December 2007 and the data frequency is quarterly.

Table 2: Constant Taylor Rule Model

Short Rate Parameters

δ_0	$\bar{a}$	$\bar{b}$
0.008	0.364	0.609
(0.002)	(0.063)	(0.237)

VAR Parameters

		Φ			Volatility $\times 1000$/Correlation		
	$\mu \times 1000$	g	π	f	g	π	f^{std}
g	0.658	0.904	-0.082	-0.023	0.004	0.018	0
	(0.267)	(0.025)	(0.027)	(0.017)	(0.000)	(0.067)	–
π	-0.022	0.062	1.004	-0.015	0.018	0.001	0
	(0.121)	(0.011)	(0.012)	(0.010)	(0.067)	(0.000)	–
f^{std}	0.016	0	0	0.940	0	0	0.008
	(0.185)	–	–	(0.026)	–	–	(0.001)

Risk Premia Parameters

		λ_1		
	λ_0	g	π	f^{std}
g	0.243	9.497	7.661	-3.475
	(0.797)	(23.28)	(18.96)	(29.28)
π	-3.056	100.89	18.76	-98.69
	(2.482)	(25.16)	(17.66)	(78.13)
f^{std}	0.133	0	0	-5.944
	(0.197)	–	–	(13.71)

Observation Error Standard Deviation

	$n = 1$	$n = 4$	$n = 8$	$n = 12$	$n = 16$	$n = 20$
σ_u^n	0.177	0.106	0.052	0.032	0.044	0.058
	(0.009)	(0.006)	(0.003)	(0.002)	(0.003)	(0.003)

The table lists parameter values for the constant Taylor rule model for the factors $X_t = [g_t \, \pi_t \, f_t^{\text{std}}]^\top$ with the short rate equation (1), factor dynamics in equation (5), prices of risk in equation (11), and observation error standard deviations in equation (17) for yields of maturity n quarters. Any parameters without standard errors are not estimated. We report the posterior mean and posterior standard deviation (in parentheses) of each parameter. In the Volatility/Correlation matrix, we report standard deviations of each factor along the diagonal multiplied by 1000 and correlations between the factors on the off-diagonal elements. The zero entries in the λ_1 matrix result from the companion form Φ taking the form of equation (6) under both the risk neutral and the real measure. The sample period is June 1952 to December 2007 and the data frequency is quarterly.

Table 3: Benchmark Model

Short Rate Parameters

	$\bar{a}$		$\bar{b}$	
δ_0	Sample	VAR	Sample	VAR
0.003	0.356	0.372	1.117	1.154
(0.001)	(0.063)	(0.186)	(0.115)	(0.503)

VAR Parameters

		Φ				Volatility $\times 1000$/Correlation			
	$\mu \times 1000$	g	π	a	b	g	π	a	b
g	0.563	0.911	-0.083	0.000	0	0.004	0.011	-0.038	-0.021
	(0.690)	(0.025)	(0.036)	(0.001)	–	(0.000)	(0.067)	(0.089)	(0.106)
π	0.142	0.064	0.990	0	0.000	0.011	0.001	-0.332	-0.203
	(0.164)	(0.011)	(0.007)	–	(0.000)	(0.067)	(0.000)	(0.077)	(0.075)
a	23.042	-1.114	0	0.937	0	-0.038	-0.332	4.072	0.854
	(11.50)	(0.763)	–	(0.027)	–	(0.089)	(0.077)	(0.994)	(0.056)
b	66.593	0	2.682	0	0.922	-0.021	-0.203	0.854	47.832
	(41.70)	–	(2.595)	–	(0.027)	(0.106)	(0.075)	(0.056)	(14.95)

Risk Premia Parameters

		λ_1			
	λ_0	g	π	a	b
g	0.905	18.545	62.700	2.376	0
	(0.360)	(11.83)	(17.63)	(1.152)	–
π	-3.001	31.444	169.53	-0.027	-0.345
	(0.486)	(12.88)	(12.32)	(0.178)	(0.218)
a	-0.632	-6.314	62.881	-0.577	-0.118
	(0.434)	(15.28)	(17.03)	(0.595)	(0.077)
b	0.082	23.948	-9.540	1.175	-0.663
	(0.574)	(23.08)	(29.95)	(0.832)	(0.274)

Observation Error Standard Deviation

	$n=1$	$n=4$	$n=8$	$n=12$	$n=16$	$n=20$
σ_u^n	0.168	0.069	0.030	0.021	0.024	0.026
	(0.043)	(0.026)	(0.010)	(0.003)	(0.003)	(0.002)

Note to Table 3

The table lists parameter values for the benchmark model for the factors $X_t = [g_t\, \pi_t\, a_t\, b_t]^\top$ with the short rate equation (2), factor dynamics in equation (5), prices of risk in equation (11), and observation error standard deviations in equation (17) for yields of maturity n quarters. Any parameters without standard errors are not estimated. We report the posterior mean and posterior standard deviation (in parentheses) of each parameter. For the short rate parameters, we report two estimated long-run means $\bar{a}$ and $\bar{b}$ for a_t and b_t, respectively. The "sample" mean is the posterior mean of the latent factors averaged across the sample. For the "population" mean we compute the population mean of the latent factors implied by the VAR parameters in each iteration and report the posterior average. In the Volatility/Correlation matrix, we report standard deviations of each factor along the diagonal multiplied by 1000 and correlations between the factors on the off-diagonal elements. The zero entries in the λ_1 matrix result from the companion form Φ taking the form of equation (6) under both the risk neutral and the real measure. The sample period is June 1952 to December 2007 and the data frequency is quarterly.

Table 4: Extended Model

Short Rate Parameters

	$\bar{a}$		$\bar{b}$	
δ_0	Sample	VAR	Sample	VAR
0.003	0.374	0.388	1.075	1.101
(0.000)	(0.018)	(0.184)	(0.031)	(0.516)

VAR Parameters

		Φ				
	$\mu \times 1000$	g	π	a	b	f^{ext}
g	0.355	0.907	-0.080	0.001	0	-0.058
	(0.740)	(0.025)	(0.037)	(0.001)	–	(0.048)
π	0.116	0.064	0.989	0	0.000	-0.014
	(0.165)	(0.011)	(0.007)	–	(0.000)	(0.015)
a	21.23	-1.267	0	0.945	0	0
	(11.07)	(0.693)	–	(0.027)	–	–
b	57.74	0	2.952	0	0.925	0
	(36.87)	–	(2.334)	–	(0.026)	–
f^{ext}	0	0	0	0	0	0.672
	–	–	–	–	–	(0.107)

	Volatility $\times 1000$/Correlation				
g	0.004	0.024	0.006	-0.009	0
	(0.000)	(0.068)	(0.082)	(0.078)	–
π	0.024	0.001	-0.359	-0.185	0
	(0.068)	(0.000)	(0.068)	(0.086)	–
a	0.006	-0.359	3.413	0.854	0
	(0.082)	(0.068)	(0.708)	(0.068)	–
b	-0.009	-0.185	0.854	35.818	0
	(0.078)	(0.086)	(0.068)	(6.437)	–
f^{ext}	0	0	0	0	0.007
	–	–	–	–	(0.001)

Table 4 Continued

Risk Premia Parameters

		λ_1				
	λ_0	g	π	a	b	f^{ext}
g	0.711	18.63	65.50	3.130	0	-28.27
	(0.925)	(11.97)	(18.12)	(0.707)	–	(22.72)
g	-3.154	30.15	164.15	-0.076	-0.390	-15.49
	(0.570)	(11.94)	(12.26)	(0.220)	(0.156)	(15.94)
g	-0.896	-8.92	63.33	-0.422	-0.152	-6.031
	(0.391)	(15.15)	(14.84)	(0.537)	(0.071)	(7.302)
g	0.360	27.04	-14.59	0.878	-0.712	3.899
	(0.609)	(24.48)	(30.25)	(0.997)	(0.335)	(6.053)
f^{ext}	0.063	0	0	0	0	79.33
	(0.072)	–	–	–	–	(52.67)

Observation Error Standard Deviation

	$n=1$	$n=4$	$n=8$	$n=12$	$n=16$	$n=20$
σ_u^n	0.081	0.038	0.024	0.018	0.022	0.024
	(0.024)	(0.004)	(0.002)	(0.001)	(0.002)	(0.002)

The table lists parameter values for the benchmark model for the factors $X_t = [g_t\, \pi_t\, a_t\, b_t\, f_t^{\text{ext}}]^\top$ with the short rate equation (4), factor dynamics in equation (5), prices of risk in equation (11), and observation error standard deviations in equation (17) for yields of maturity n quarters. Any parameters without standard errors are not estimated. We report the posterior mean and posterior standard deviation (in parentheses) of each parameter. For the short rate parameters, we report two estimated long-run means $\bar{a}$ and $\bar{b}$ for a_t and b_t, respectively. The "sample" mean is the posterior mean of the latent factors averaged across the sample. For the "population" mean we compute the population mean of the latent factors implied by the VAR parameters in each iteration and report the posterior average. In the Volatility/Correlation matrix, we report standard deviations of each factor along the diagonal multiplied by 1000 and correlations between the factors on the off-diagonal elements. The zero entries in the λ_1 matrix result from the companion form Φ taking the form of equation (6) under both the risk neutral and the real measure. The sample period is June 1952 to December 2007 and the data frequency is quarterly.

Table 5: Yield Curve Variance Decompositions

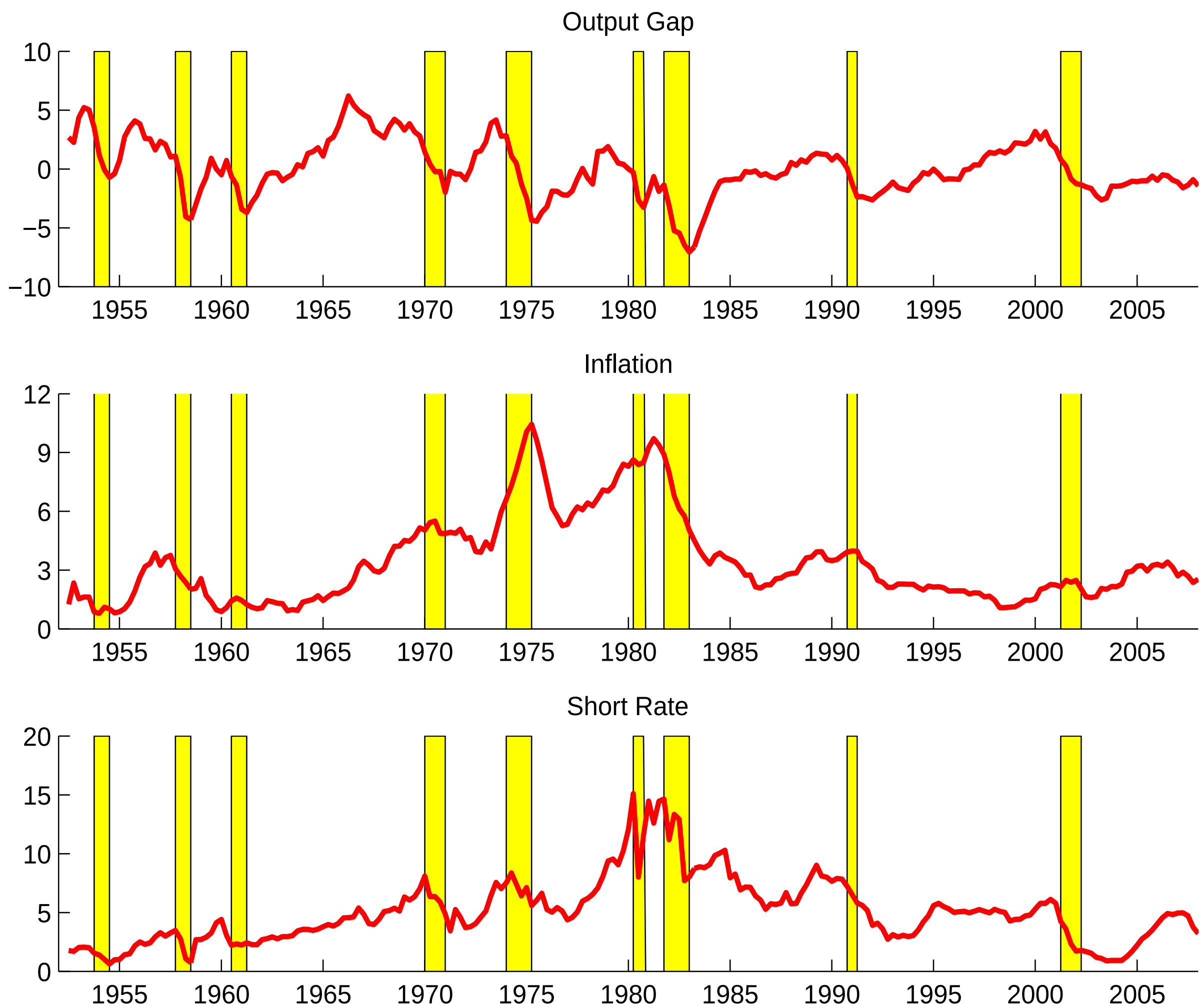
	$n=1$	$n=4$	$n=8$	$n=12$	$n=16$	$n=20$
g	-0.083	-0.132	-0.152	-0.150	-0.140	-0.127
π	0.603	0.493	0.388	0.307	0.241	0.191
a	0.059	0.116	0.026	-0.089	-0.182	-0.249
b	0.170	0.254	0.275	0.247	0.213	0.191
g and a	-0.083	-0.046	-0.137	-0.241	-0.320	-0.373
π and b	0.913	0.865	0.734	0.590	0.471	0.389

The table reports variance decompositions of yields implied by the benchmark model. The variance decompositions are produced by computing

$$1 - \frac{\text{var}_\theta(y_t^n)}{\text{var}(y_t^n)}$$

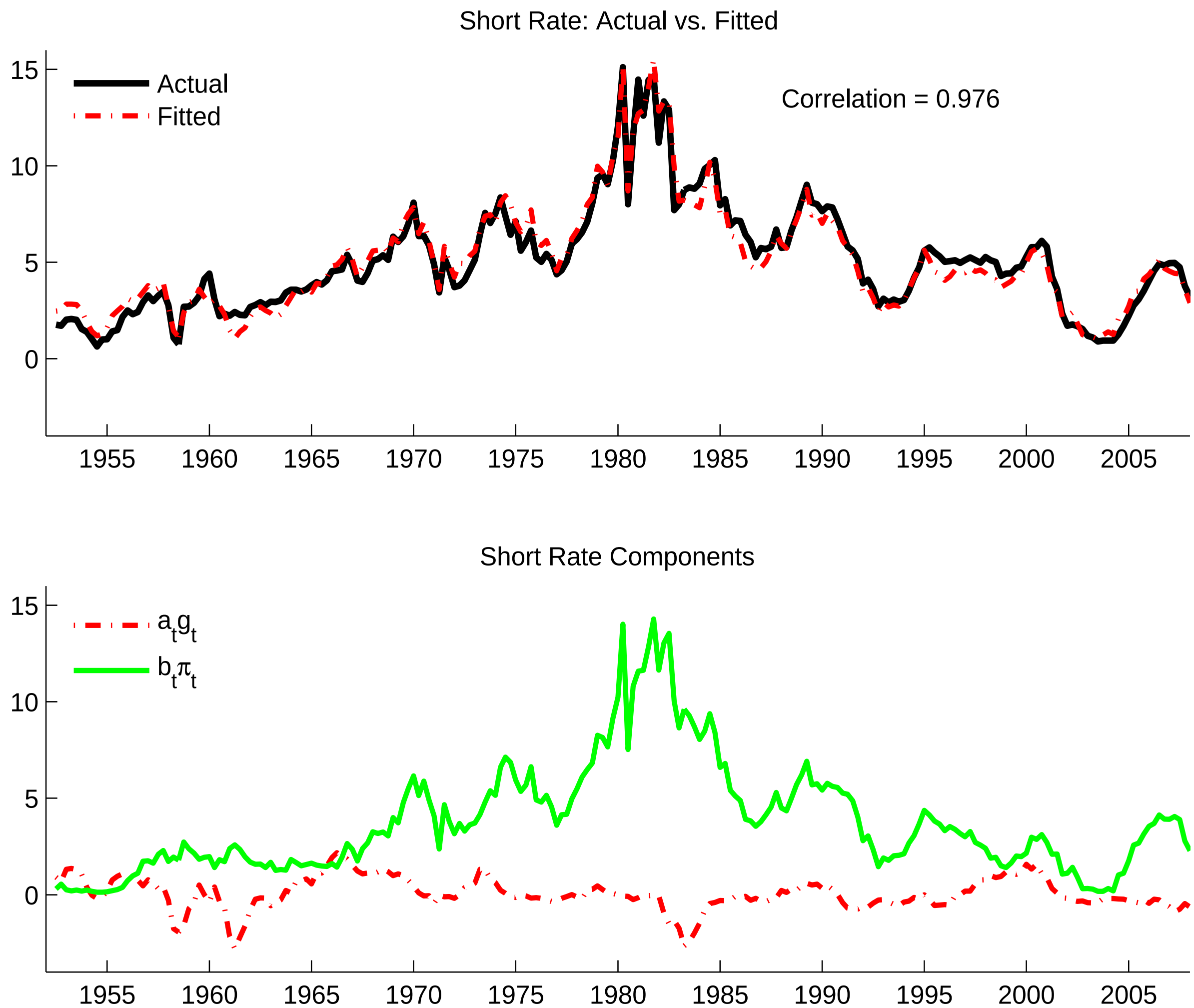
where $\text{var}_\theta(y_t^n)$ is the variance of the n-quarter yield implied by the model through the sample computed by setting the time-varying factor θ equal to its posterior mean and $\text{var}(y_t^n)$ is the full model-implied variance of the n-quarter yield through the sample.

Figure 1: Output Gap, Inflation, and the Short Rate



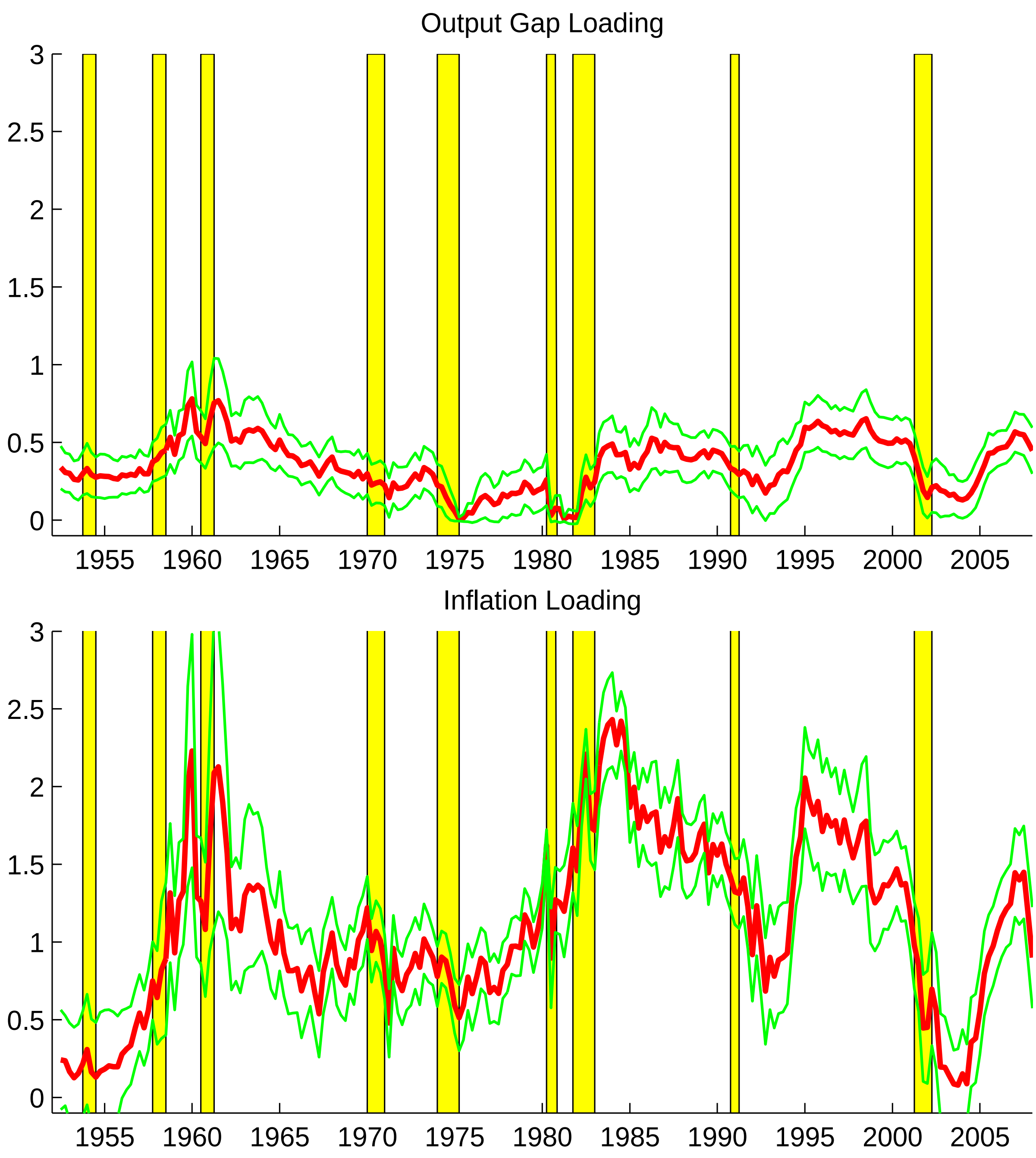
We plot the output gap, inflation, and the short rate. The output gap is defined as the proportional difference between actual and potential real GDP. Inflation is the year-on-year GDP deflator. The short rate is the three-month T-bill yield. We overlay the NBER recession periods in shaded bars. The sample period is from June 1952 to December 2007 and the data frequency is quarterly. All data is in annualized percentage terms.

Figure 2: Components of the Short Rate



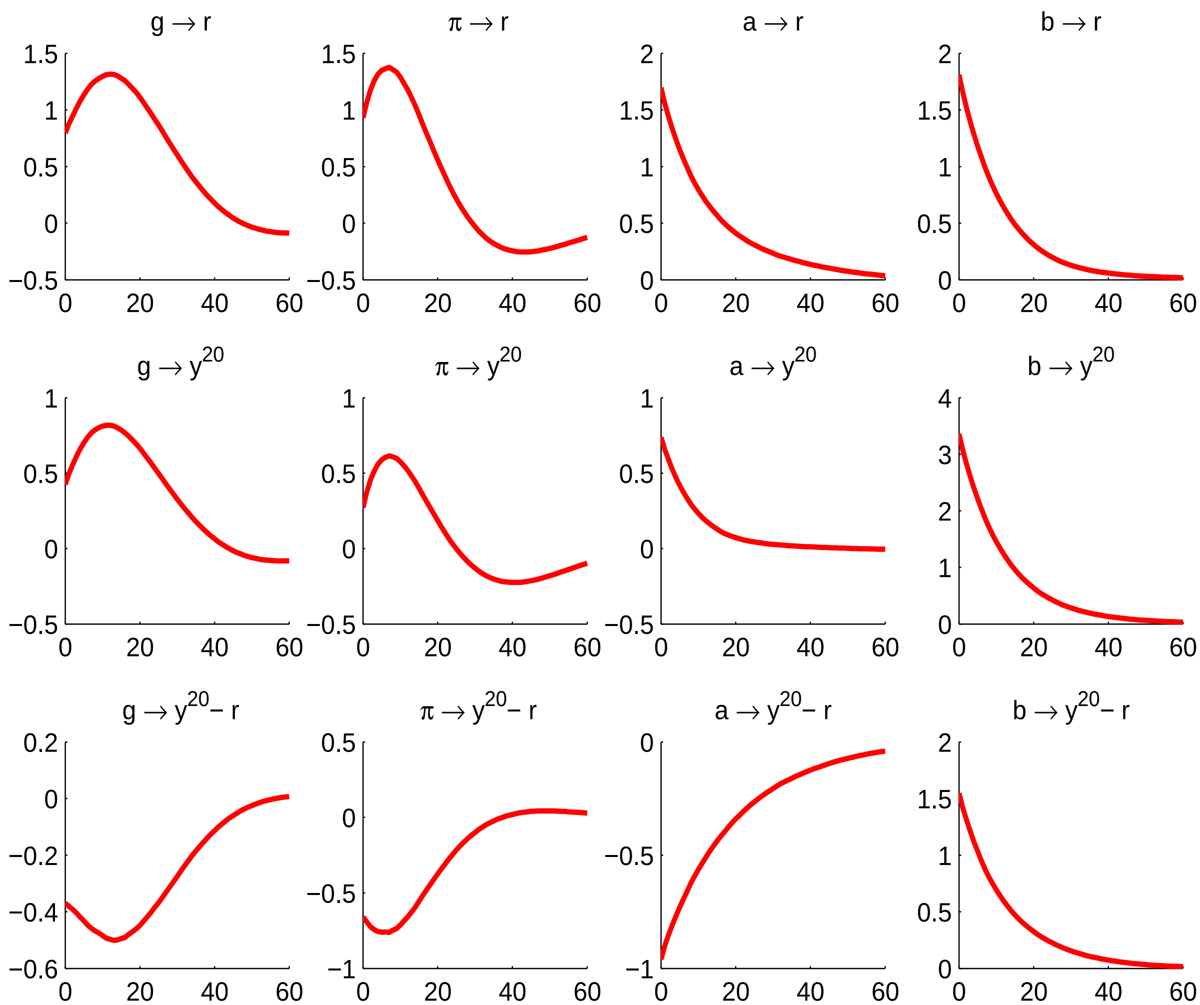
The top panel plots the short rate together with the fitted components from the benchmark model, $\delta_0 + a_t g_t + b_t \pi_t$, where the policy factors a_t and b_t are evaluated at their posterior means at each observation from the Gibbs sampler. All variables are in annualized units. The bottom panel plots each short rate component separately. The sample period is from June 1952 to December 2007 and the data frequency is quarterly.

Figure 3: Time-Varying Policy Coefficients



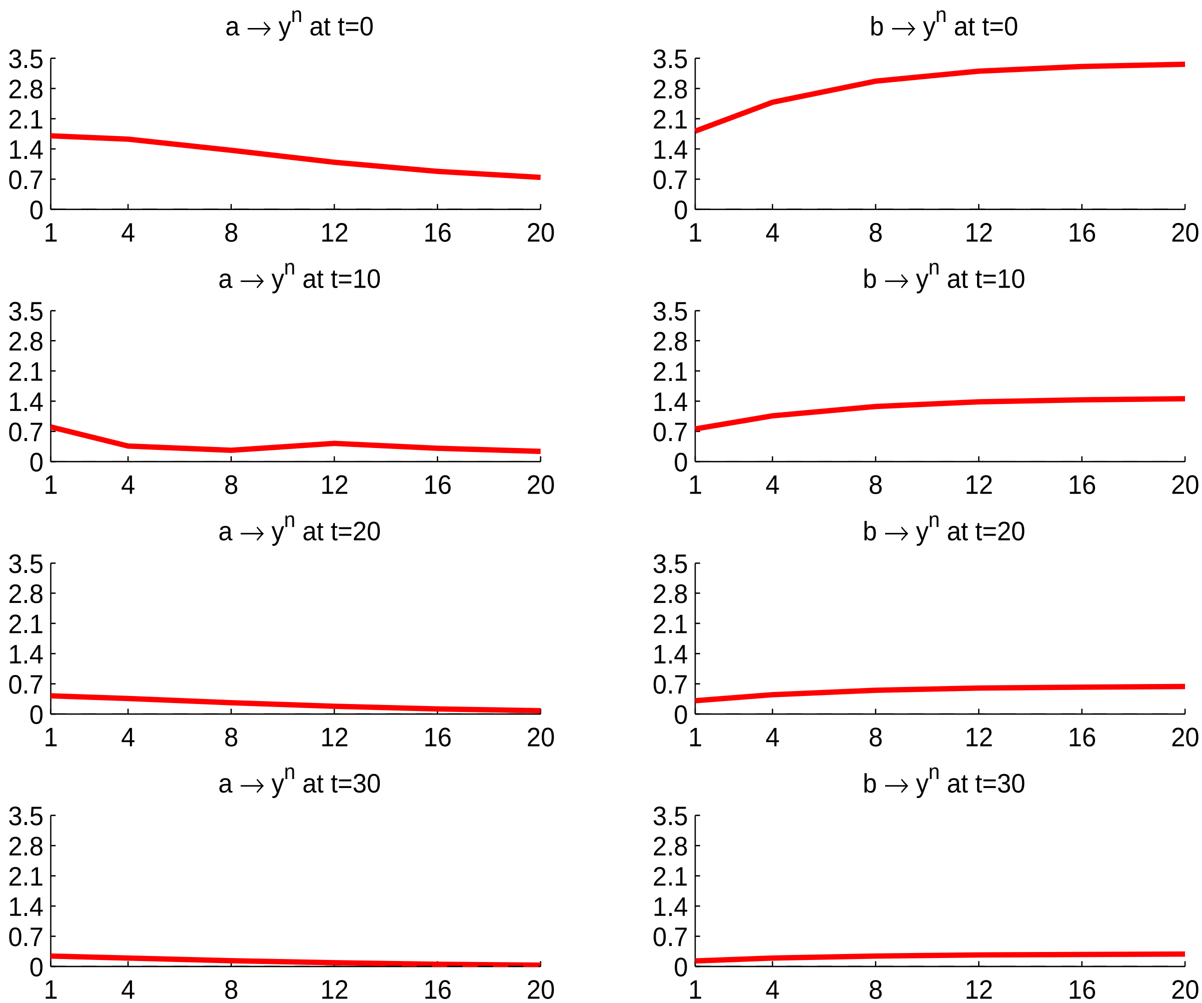
We plot the posterior mean of the time-varying loadings a_t and b_t in the thick lines together with two posterior standard deviation bands in thin lines from the benchmark model. We overlay the NBER recession periods in shaded bars. The sample period is from June 1952 to December 2007 and the data frequency is quarterly.

Figure 4: Yield Curve Impulse Responses to Factor Shocks



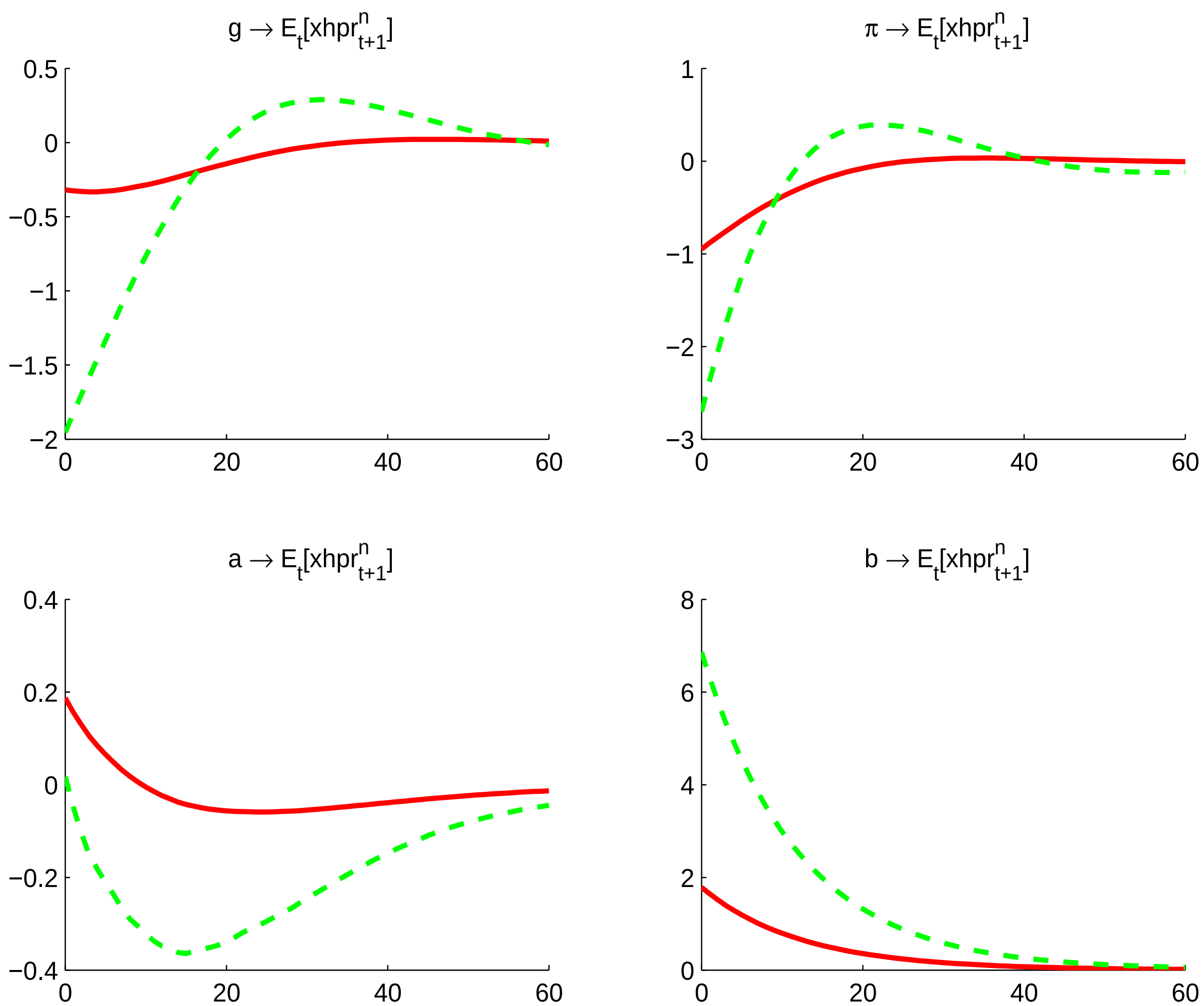
We plot the impulse responses of the short rate, r_t, the 20-quarter yield, y_t^{20}, and the yield spread, $y_t^{20} - r_t$, to an unconditional one-standard deviation shock in the output gap and inflation (g_t and π_t respectively) in the first two columns and an unconditional one-standard deviation shock to a_t and b_t in the last two columns. We compute impulse responses following the method in Appendix D. Units on the x-axis are in quarters and the responses of yields on the y-axis are annualized and in percentages.

Figure 5: Term Structure Responses to Factor Shocks



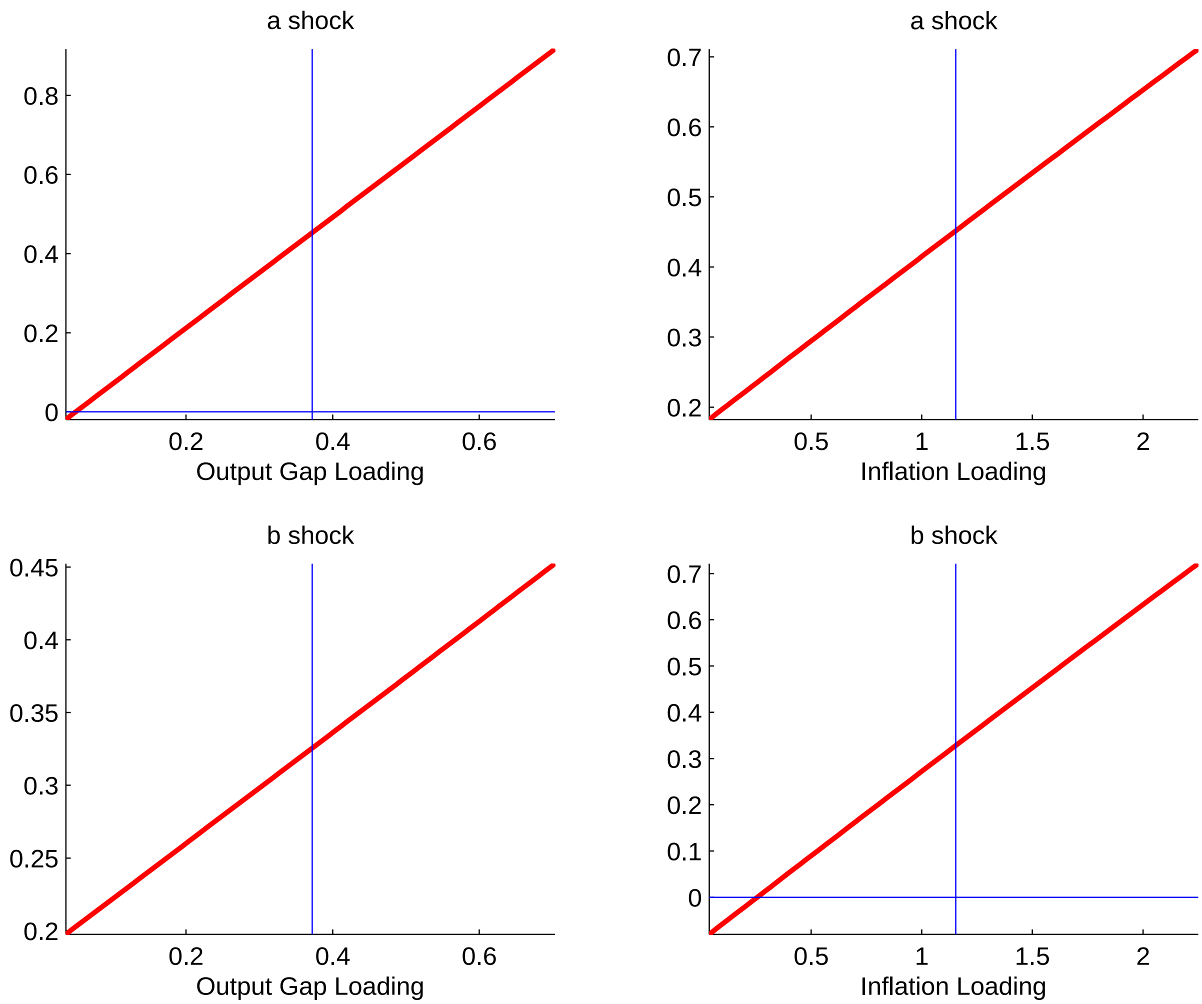
We plot the impulse responses of an unconditional one-standard deviation shock in the output gap (left column) and inflation (right column) on the term structure of yields, y_t^n. We compute impulse responses following the method in Appendix D. Yield maturities in quarters, n, are on the x-axis and the responses of yields on the y-axis are annualized and in percentages. The initial shock occurs at $t = 0$ and the expected yield curve is plotted at various t in quarters after the initial shock in rows. The x-axis is marked as a horizontal dotted line.

Figure 6: Risk Premia Responses to Factor Shocks



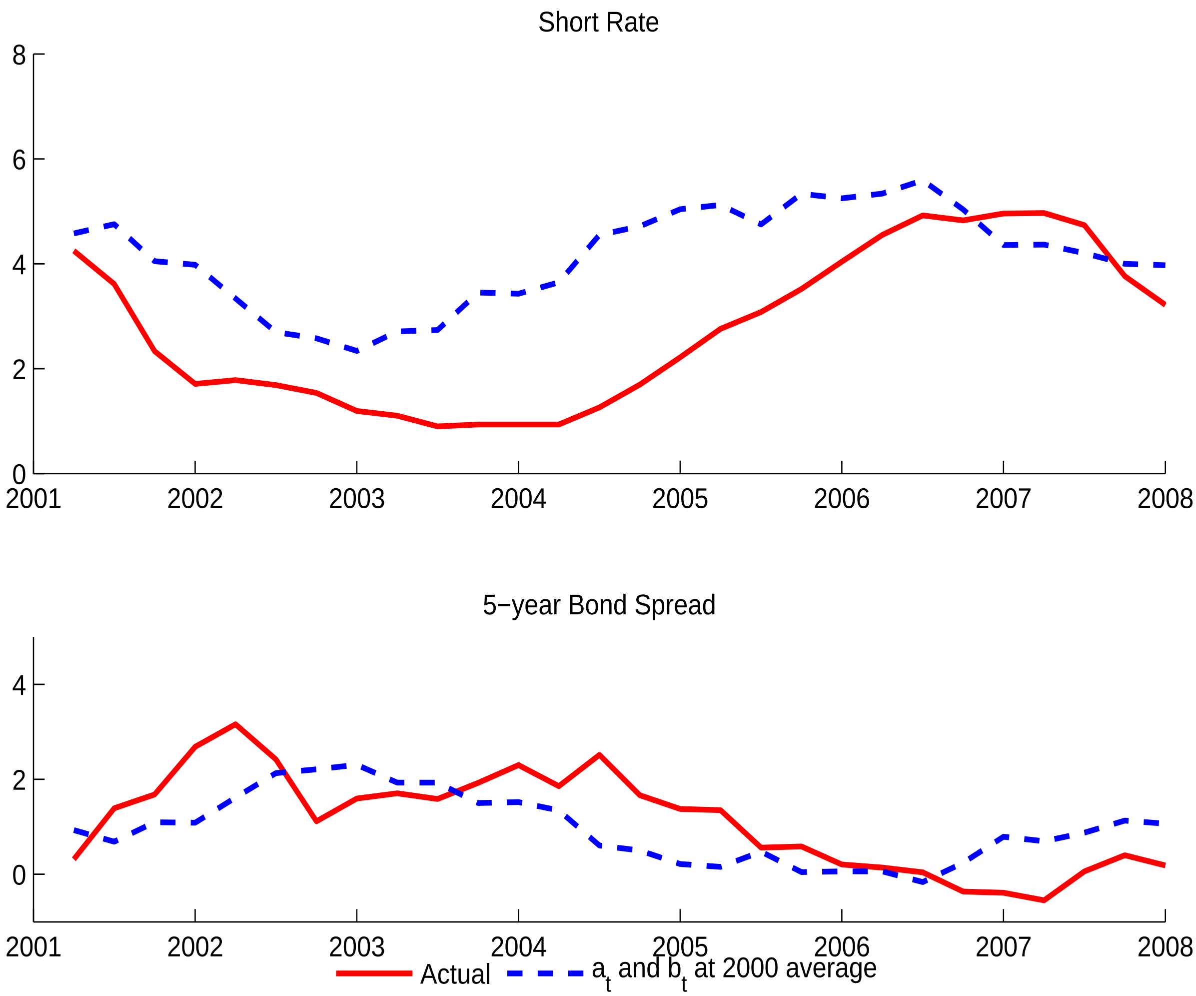
We plot the impulse responses of an unconditional one-standard deviation shock to the output gap, inflation, the inflation loading, and the output gap loading (clockwise from the top-left panel) to the expected excess holding period return, $\mathrm{E}_t[xhpr^n_{t+1}]$, of the $n = 4$-quarter bond in the solid line and the $n = 20$-quarter bond in the dashed line. The conditional risk premia are evaluated at the model's implied unconditional mean. Units on the x-axis are in quarters and the responses of risk premia on the y-axis are annualized and in percentages.

Figure 7: The Price of Risk of Monetary Policy Shifts



The figure plots the price of a unit shock to a_t (first row) and a unit shock to b_t (second row) as a function of the output gap loading a_t or inflation loading b_t. We denote with vertical lines the steady-state value of $\bar{a} = 0.372$ and $\bar{b} = 1.154$. When altering a_t and b_t, we set all other factors equal to their expected conditional values given the value of a_t or b_t. We plot plus and minus two unconditional standard deviations of a_t and b_t on the x-axis.

Figure 8: The Post-2001 Episode



The figure plots the short rate (top panel) and the 5-year term spread (bottom panel), which is the 5-year yield minus the 3-month T-bill, from the results of a counter-factual experiment. We hold the Fed weight on the output gap and inflation constant at its average level over 2000 and allow all other factors to take their sample values. The figure plots the effect on the yield curve post-2001 in the dashed lines along with the actual paths of the yield curve in the solid lines. Units on the y-axis are annualized and in percentages.