THE DETERMINANTS OF STOCK AND BOND RETURN COMOVEMENTS

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ABSTRACT

We study the economic sources of stock-bond return comovements and its time variation using a dynamic factor model. We identify the economic factors employing a semi-structural regime-switching model for state variables such as interest rates, inflation, the output gap, and cash flow growth. We also view risk aversion, uncertainty about inflation and output, and liquidity proxies as additional potential factors. We find that macro-economic fundamentals contribute little to explaining stock and bond return correlations, but that other factors, especially liquidity proxies, play a more important role. The macro factors are still important in fitting bond return volatility; whereas the "variance premium" is critical in explaining stock return volatility. However, the factor model primarily fails in fitting covariances.

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1 Introduction

Stock and bond returns in the US display an average correlation of about 19 percent during the post–1968 period. A number of models have had some success in generating realistic average correlations using economic state variables. Yet, Shiller and Beltratti (1992) under-estimate the empirical correlation using a present value with constant discount rates, whereas Bekaert, Engstrom, and Grenadier (2005) over-estimate it in a consumption-based asset pricing model with stochastic risk aversion.

Figure 1 documents a more puzzling empirical phenomenon: the stock-bond return correlation displays very substantial time variation. The figure graphs realized quarterly correlations measured using daily returns, and the data-implied conditional correlation based on the bivariate DCC-MIDAS model of Colacito, Engle, and Ghysels (2009). We defer technical details about this statistical model to the Appendix, but come back to it below as it will serve as an empirical benchmark for our study. Note that during the mid-1990s, the stock-bond correlation was as high as 60 percent, to drop to levels as low as minus 60 percent by the early 2000s. There is a growing literature documenting this time variation using sophisticated statistical models (see also Guidolin and Timmermann (2006)), but much less work trying to disentangle its economic sources. In particular, the negative stock–bond return correlations observed since 1998 are mostly ascribed to a "flight-to-safety" phenomenon (see e.g. Connolly, Stivers, and Sun (2005)), where increased stock market uncertainty induces investors to flee stocks in favor of bonds. Campbell, Sunderam, and Viceira (2009) recently propose a pricing model for stock and bond returns, and assign a latent variable to capture the covariance between nominal variables and the real economy, which, in turn, helps to produce negative comovements between bond and stock returns.

This article asks whether a dynamic factor model in which stock and bond returns depend on a number of economic state variables can explain the average stock-bond return correlation and its variation over time. Our approach has a number of distinct features. First, we cast a wide net in terms of state variables. Our economic state variables do not only include interest rates, inflation, the output gap and cash flow growth, but also a "fundamental" risk aversion measure derived from consumption growth data based on Campbell and Cochrane (1999) and macro-economic uncertainty measures derived from survey data on inflation and GDP growth expectations. The latter variables may reflect true economic uncertainty as in David and Veronesi (2008), or heteroskedasticity as in Bansal and Yaron (2004) and Bekaert, Engstrom, and Xing (2009). In addition, we consider liquidity proxies, and the variance premium, a risk premium proxy representing the difference between the (square of the) VIX (the option-based "risk-neutral" expected conditional variance) and the conditional variance of future stock prices (see e.g. Carr and Wu (2009)).

Second, while we have estimated simple linear state variable models with various forms of heteroskedasticity, we focus our discussion on a state variable model that imposes structural

restrictions inspired by recent standard New–Keynesian models, and that incorporates regime–switching behavior. Both features improve the fit considerably. We use a regime-switching model to accommodate changes in monetary policy and to model heteroskedasticity in the shocks. As we will demonstrate, heteroskedasticity is a key driver of the time-variation in stock-bond return correlations. Moreover, macro-variables have witnessed important volatility changes over the sample period. For example, the lower variability of inflation and output growth observed since the mid-1980s, the so-called Great Moderation, could conceivably lead to lower correlations between stock and bond returns. Whether its timing actually helps matching the time variation in the stock-bond return correlations, including negative correlations at the end of the nineties, remains to be seen.

Third, given our model structure, we decompose the performance of the factor model in contributions of the various factors. This should provide useful input to future theoretical modelling of stock-bond return comovements. Moreover, we examine how well the model does with respect to each of the correlation components: covariances, and stock and bond return volatilities. In so doing, we add to the literature that examines the economic sources of stock and bond return volatility (see e.g. Schwert (1989), Campbell and Ammer (1993), and Engle, Ghysels, and Sohn (2008)).

One final point is of note. Our analysis is mostly at the quarterly frequency. This is the frequency at which data on the economic state variables used in the dynamic factor models are available. It may also be the highest frequency at which a fundamentals based model is expected to have explanatory power. While we do characterize the variation in stock-bond return correlations using daily return data to calculate ex-post quarterly correlations[1], our main benchmark is the long-run component of the Colacito, Engle, and Ghysels (2009) model. This flexible empirical model exploits the richness of daily data to estimate quarterly conditional correlations, directly comparable to the implied correlations from our factor model.

The remainder of this paper is organized as follows. Section 2 describes the factor model and develops the state variable model used to identify the economic factors. Section 3 details the estimation procedure and the model selection. Section 4 analyzes the fit of the factor model for correlation dynamics, whereas Section 5 decomposes the correlations into covariances, and stock and bond return volatilities. We find that macro-economic fundamentals contribute little to explaining stock and bond return correlations, but that other factors, especially liquidity proxies, play a more important role. The macro factors are still important in fitting bond return volatility; whereas the "variance premium" is critical in explaining stock return volatility. However, the factor model primarily fails in fitting covariances. A final section concludes.

2 Dynamic Stock and Bond Return Factor Model

In this section we present the general factor model linking stock and bond returns to structural factors. Section 2.1 considers the general dynamic factor model. Section 2.2 discusses the selec-

tion of the state variables and the modelling of their dynamics, which leads to the identification of the factors. Finally, section 2.3 discusses the modelling of the factor exposures.

2.1 The Dynamic Factor Model

Let $r_{e,t}$ denote the excess equity return and $r_{b,t}$ the excess bond return. We assume the following dynamics for $r_t = (r_{e,t}, r_{b,t})'$:

$$r_t = E_{t-1}(r_t) + \beta_t' F_t + \varepsilon_t \tag{1}$$

where $E_{t-1}(r_t)$ represents the expected excess return vector, $\beta_t = (\beta_{e,t}, \beta_{b,t})$ is a $n \times 2$ matrix of respectively stock and bond return factor loadings, and F_t is a $n \times 1$ vector containing the structural factors. The vector $\varepsilon_t = (\varepsilon_{e,t}, \varepsilon_{b,t})'$ represents return shocks not explained by the economic factors. For now, we model expected returns as constants but we investigate the robustness of our results to this assumption in Section 4.3.

The time variation in the betas $\beta_{e,t}$ and $\beta_{b,t}$ is generally modelled as:

$$\beta_t = \beta\left(I_{t-1}, S_t\right) \tag{2}$$

where I_t is the information set generated by a set of information variables at time t, and S_t is a discrete variable following a Markov chain, which we use to model sudden regime changes, as discussed below. The factors F_t represent innovations to a set of state variables X_t, and are distributed as:

$$F_t \sim N\left(\mathbf{O}, \Sigma_t\right) \tag{3}$$

where Σ_t is a $n \times n$ diagonal matrix containing the conditional variances of the structural factors, which are potentially time-varying. In particular, Σ_t generally may also depend on S_t. The off-diagonal elements are zero as we enforce structural factors to be orthogonal. Under the null of the model, the covariance matrix of the stock and bond return residuals ε_t is homoskedastic and diagonal. We denote the residual variances by h_e and h_b, respectively.

The factor model implies that the comovement between stock and bond returns follows directly from their joint exposure to the same economic factors. Let R be the set of values S_t can take on. Then, the conditional covariance can be written as:

$$cov_{t-1}\left(r_{e,t}, r_{b,t}\right) = \sum_{s_t \in R} \beta_e'\left(I_{t-1}, s_t\right) \Sigma\left(\left. s_t \right| I_{t-1}\right) \beta_b\left(I_{t-1}, s_t\right) P\left[\left. s_t \right| I_{t-1}\right]. \tag{4}$$

When the betas do not depend on the regime, it simplifies to:

$$cov_{t-1}\left(r_{e,t}, r_{b,t}\right) = \beta_{e,t-1}' \Sigma_{t-1} \beta_{b,t-1} \tag{5}$$

where Σ_{t-1} is conditioned on I_{t-1}. By dividing the covariance by the product of the stock and bond return volatilities, i.e. $\sqrt{\beta_{e,t-1}' \Sigma_{t-1} \beta_{e,t-1} + h_e}$ and $\sqrt{\beta_{b,t-1}' \Sigma_{t-1} \beta_{b,t-1} + h_b}$, we can decompose the model-implied conditional correlation between stock and bond returns, $\rho_{t-1}\left(r_{e,t}, r_{b,t}\right)$,

as follows:

$$\rho_{t-1}\left(r_{e,t}, r_{b,t}\right) = \frac{\beta_{e,t-1}^1 \beta_{b,t-1}^1 var_{t-1}(F_t^1)}{\sqrt{\beta_{e,t-1}' \Sigma_{t-1} \beta_{e,t-1} + h_e} \sqrt{\beta_{b,t-1}' \Sigma_{t-1} \beta_{b,t-1} + h_b}} + ... \quad (6)$$

$$... + \frac{\beta_{e,t-1}^n \beta_{b,t-1}^n var_{t-1}(F_t^n)}{\sqrt{\beta_{e,t-1}' \Sigma_{t-1} \beta_{e,t-1} + h_e} \sqrt{\beta_{b,t-1}' \Sigma_{t-1} \beta_{b,t-1} + h_b}}.$$

This decomposition clearly shows the standard effects of a linear factor model. First, factors with higher variances have the largest effect on comovement. Second, when the variance of a factor increases, its contribution to the comovement can become arbitrarily large. Third, if bond and stock betas have the same sign, increased factor variances lead to increased comovement, and vice versa. Consequently, to generate the substantial variation in comovements documented in Figure 1 in the context of this model, the volatility of the fundamentals must display substantial time variation. Moreover, to generate negative covariances, it must be true that there is at least one factor to which bonds and stocks have opposite exposures, and this factor must at times have substantial relative variance. When β's vary through time, they can also generate sign changes over time. When they are constant, however, the sole driver of time variation in the covariance between stock and bond returns is the heteroskedasticity in the structural factors. The betas determine the sign of the covariance. We now motivate which factors should be included in the factor model from the perspective of rational pricing models.

2.2   State Variable Model

2.2.1   Selection of State Variables

In standard rational pricing models, the fundamental factors driving stock and bond returns either affect cash flows or discount rates. We start out with describing a standard set of macro-factors, then describe how we measure potential risk premium variation. Finally, the literatures on bond (see Amihud and Mendelson (1991) and Kamara (1994)) and equity pricing (see Amihud (2002)) have increasingly stressed the importance of liquidity effects; so we consider two liquidity related state variables as well.

Standard Macro Factors   Our macro factors include the standard variables featured in macroeconomic models: the output gap[2], inflation and the short rate. All these variables may have both cash flow and discount rate effects, so their effect on bond and stock returns is not always easy to predict. Because bonds have fixed nominal cash flows, inflation is an obvious state variable that may generate different exposures between bond and stock returns. Analogously, if the output gap is highly correlated with the evolution of real dividends, it should affect stock but not bond returns.

However, both inflation and the output gap can also affect the real term structure of interest rates and therefore affect both bonds and equities. Because equities represent a claim on real

assets, the discount rate on stocks should not depend on nominal factors such as expected inflation. Yet, a recurring finding is that stocks seem to be very poor hedges against inflation and their returns correlate negatively with inflation shocks and expected inflation (see e.g. Fama and Schwert (1977)). One interpretation of this finding is that it represents money illusion (see Campbell and Vuolteenaho (2004)), another that it represents the correlation of inflation with rational risk premiums (see Bekaert and Engstrom (2009)). In this article, we leave the sign of the exposures unconstrained, giving the model maximal power to explain the data. In standard models, the (expected) output gap may reflect information about real rates as well and hence may induce positive correlation between stock and bond returns.

As is well known, the level of interest rates drives most of the variation in bond returns, and we include a short-term interest rate as a factor in our model. For long-term bonds, the relevant state variable is the long-term interest rate, which can in turn be decomposed into a short-term real rate, a term premium, expected inflation and an inflation risk premium. Increases in all these 4 components unambiguously decrease bond returns. To span the term and inflation premium components, we likely need more information than is present in our macro factors, and we therefore also use a number of direct "economic" risk premium proxies.

Risk Premium Factors   We use measures of economic uncertainty and risk aversion to capture stock and bond risk premia. For instance, Bekaert, Engstrom, and Grenadier (2005) show that stochastic risk aversion plays an important role in explaining positive stock-bond return correlations. The effects of risk aversion are, however, quite complex. In the models of Bekaert, Engstrom, and Grenadier (2005) and Wachter (2006), increases in risk aversion unambiguously increase equity and bond premiums, but their effect on interest rates is actually ambiguous. A rise in risk aversion may increase the real interest rate through a consumption smoothing effect or decrease it through a precautionary savings effect. Bansal and Yaron (2004) and Bekaert, Engstrom, and Xing (2009) stress economic uncertainty as a channel that may affect risk premiums and equity valuation. The effect of increases in uncertainty on equity valuation, while often thought to be negative, is actually ambiguous as increased uncertainty may lower real interest rates through precautionary savings effects. Hence, an increase in uncertainty may cause bonds and stocks to move in opposite directions depending on the relative strenghts of the term structure and risk premium effects.

An alternative motivation for the use of uncertainty measures follows from the learning models of Veronesi (1999) and David and Veronesi (2008). They show that higher uncertainty about future economic state variables makes investors' expectations react more swiftly to news, affecting both variances and covariances of asset returns.

To capture economic uncertainty, we use the Survey of Professional Forecasters (SPF) to create measures of inflation and output gap uncertainty. We use two measures of risk aversion. Our first measure is from Bekaert and Engstrom (2009): They create an empirical proxy for risk aversion, based on the external habit specification of Campbell and Cochrane (1999). This "fundamental"

risk aversion measure is generated solely by past consumption growth data, and behaves counter-cyclically. It is, however, unlikely that this measure fully captures equity risk premium variation. Recent work by Bollerslev, Tauchen, and Zhou (2009) shows that the variance premium, defined earlier, predicts equity returns. Drechsler and Yaron (2009) extend the model by Bansal and Yaron (2004) to allow for additional non-linearities in the consumption growth technology, and show that this premium depends on risk aversion and the non-Gaussian components of the model. We use an estimate of this risk premium as a factor, which helps us to interpret an often cited non-fundamental explanation for the occasionally observed negative stock-bond return correlations. Connolly, Stivers, and Sun (2005) use the VIX implied volatility measure as a proxy for stock market uncertainty and show that stock and bond return comovements are negatively and significantly related to stock market uncertainty. They interpret this finding as reflecting "flight-to-safety", where investors switch from the risky asset, stocks, to a safe haven, bonds, in times of increased stock market uncertainty, inducing corresponding price changes, and thus implying a negative correlation between stock and bond returns. Because the variance premium depends positively on VIX movements but depends negatively on the actual expected stock market volatility, we can determine whether this "flight-to-safety" effect is due to the "risk premium component" of the VIX, or rather reflects general stock market uncertainty.

Liquidity factors   We focus on transaction cost based measures of illiquidity. Liquidity can then affect the pricing of bonds and stocks in two main ways. First, liquidity may affect the betas, as economic shocks may not be transmitted quickly to observed returns in illiquid markets. This is a factor exposure effect. Second, liquidity may be a priced factor, and shocks that improve liquidity should increase returns. The impact of liquidity on stock and bond return comovements then obviously depends on how liquidity shocks comove across markets. For example, the monetary policy stance can affect liquidity in both markets by altering the terms of margin borrowing and by alleviating the borrowing constraints of dealers, or by simply encouraging trading activity. Liquidity effects may also correlate with the "flight-to-safety" phenomenon. Crisis periods may drive investors and traders from less liquid stocks into highly liquid Treasury bonds, and the resulting price-pressure effects may induce negative stock-bond return correlations. Some of these effects may persist at the quarterly frequency. Existing studies of the commonality in stock and bond liquidity (Chordia, Sarkar, and Subrahmanyam (2005) and Goyenko (2006)) are somewhat inconclusive as to which effect dominates. It is therefore important to let illiquidity shocks enter the model relatively unconstrained.

Our measure of bond market illiquidity is a monthly average of quoted bid-ask spreads of off-the-run bonds across all maturities, taken from Goyenko (2006). Our measure of equity market illiquidity uses the "zero return" concept developed in Lesmond, Ogden, and Trzcinka (1999), and is taken from Bekaert, Harvey, and Lundblad (2007). It is the capitalization-based proportion of zero daily returns across all firms, aggregated over the month. This measure has a positive and high correlation with more standard measures, such as Hasbrouck (2006)'s effective costs and Amihud (2002)'s price impact measures. Because liquidity has improved over time,

both illiquidity measures show near non-stationary behavior. For stocks, we first correct for the structural breaks induced by changes in tick size[3] in 1997 and 2001 by using the residuals from a regression on dummies for these break dates. Then, for both stock and bond illiquidity, we subtract a moving average of the levels of the previous 4 months from the current measures.

Eventually, we retain the following economic state variables: the output gap (y_t), inflation (π_t), risk aversion (q_t), nominal interest rate (i_t), cash flow growth (cg_t), output uncertainty (yd_t), inflation uncertainty (πd_t), stock market illiquidity ($sliq_t$), bond market illiquidity ($bliq_t$), and the variance premium (vp_t) for a total of 10 state variables, which we collect in a vector X_t. The data appendix provides full details about the measurement and construction of those variables.

2.2.2   General State Variable Dynamics

To identify the structural factors F_t, we must specify the dynamics of the state variables X_t. The general model has the following form:

$$X_t = \mu\left(S_t\right) + \Omega_1\left(S_t\right) E_t\left(X_{t+1}\right) + \Omega_2\left(S_t\right) X_{t-1} + \Gamma\left(S_t\right) F_t \tag{7}$$

where μ is a 10×1 vector of drifts, and Ω_2 represents the usual feedback matrix. We also allow the dynamics of the factors to depend on expectations of future values as is true in many standard macro–models through Ω_1. Γ is a 10×10 matrix of structural parameters, capturing the contemporaneous correlation between the fundamental state variables. Finally, all coefficient matrices generally depend on S_t, which represents a set of latent regime variables, modelled in the Hamilton (1989) tradition. The regime variables can capture structural changes in the macro-economic relations, as induced, for example, by changes in monetary policy. Monetary economists debate the effects of heteroskedasticity in the fundamental shocks versus shifts in monetary policy on the identification of economic and monetary policy shocks. By letting the conditional variance of F_t also depend on regime variables, we accommodate both interpretations of the data (see also below).

Without further restrictions, the model in Equation (7) is surely overparametrized, but it nests a large number of reasonable dynamic models. For example, assume $\Omega_2 = 0$, and eliminate the regime dependence in μ, Ω_1 and Γ, and we obtain a simple first-order VAR with heteroskedastic shocks. We estimated several variants of such models, using a non-structural identification of the shocks in (7) by imposing a Choleski decomposition. Such models were easily out-performed by a semi-structural model, nested in Equation (7). For this model, we split the state variables into two sets: "pure macro variables", $X_{t,ma} = [y_t, \pi_t, q_t, i_t]'$ (recall that q_t is computed from only consumption data), and the other variables $X_{t,nma} = [cg_t, yd_t, \pi d_t, sliq_t, bliq_t, vp_t]'$. To model $X_{t,ma}$, we make use of a New Keynesian model, which we describe in the next sub-section. To identify the shocks to $X_{t,nma}$, we use a simple empirical model that recognizes that these factors may depend on the macro variables themselves. For example, Goyenko, Subrahmanyam, and Ukhov (2008) show that inflation and monetary policy affect bond liquidity, and that illiquidity

increases in recessions. Specifically, the model is:

$$X_{t,nma} = \mu_{nma}(S_t) + \phi_{nma}X_{t-1,nma} + \Gamma_{nma}^{ma}X_{t,ma} + F_{t,nma} \qquad (8)$$

with ϕ_{nma} a diagonal matrix; Γ_{nma}^{ma} a 6×4 matrix capturing the contemporaneous covariance with the macro-economic state variables and $F_{t,nma}$ the vector of (uncorrelated) 'structural' shocks. With this model structure, the non-macro factors may partially inherit autoregressive dynamics from the pure macro-economic variables, and the $F_{t,nma}$ shocks must be interpreted as being purged from the pure macro-shocks. Finally, note that we allow the drifts to depend on the regime variable S_t. Because inflation and output uncertainty are likely highly correlated with macro-economic heteroskedasticity (see Giordani and Soderlind (2003); Evans and Wachtel (1993)), the assumption of heteroskedastic macro shocks makes this dependence logically necessary. Regime-dependent drifts may also help model the structural changes (see Hasbrouck (2006); Goyenko (2006)) affecting the liquidity in both stock and bond markets.

2.2.3 A Structural Model for $X_{t,ma}$

The structural model for $X_{t,ma}$ extends a standard New-Keynesian three-equation model (see e.g. Bekaert, Cho, and Moreno (2008)) comprising an IS or demand equation, an aggregate supply (AS) equation, and a forward looking monetary policy rule, to accommodate time-varying risk aversion:

$$y_t = a_{IS} + \mu E_t(y_{t+1}) + (1 - \mu) y_{t-1} + \eta q_t - \phi (i_t - E_t(\pi_{t+1})) + F_t^y \qquad (9)$$

$$\pi_t = a_{AS} + \delta E_t(\pi_{t+1}) + (1 - \delta) \pi_{t-1} + \lambda y_t + F_t^\pi \qquad (10)$$

$$q_t = \alpha_q + \rho_q q_{t-1} + F_t^q \qquad (11)$$

$$i_t = a_{MP} + \rho i_{t-1} + (1 - \rho) [\beta(S_t^{mp}) E_t(\pi_{t+1}) + \gamma(S_t^{mp}) y_t] + F_t^i. \qquad (12)$$

The μ and δ parameters represent the degree of forward-looking behavior in the IS and AS equations. If they are not equal to one, the model features endogenous persistence. The ϕ parameter measures the impact of changes in real interest rates on output and λ the effect of output on inflation. They are critical parameters in the monetary transmission mechanism, and high and positive values imply that monetary policy has significant effects on the real economy and inflation. Because all these parameters arise from micro-founded models, for example representing preference parameters, we assume them to be time invariant.

The variable q_t reflects time-varying risk aversion. We build on the model in Bekaert, Engstrom, and Grenadier (2005) to append stochastic risk aversion to the New-Keynesian IS-curve (see Appendix C for details). The model nests a variant of the Campbell and Cochrane (1999) external habit model, and q_t represents the negative of their surplus ratio and thus the local curvature of the utility function. While output shocks and risk aversion are negatively correlated in the model, η in Equation (9) can nevertheless not be definitively signed. It reflects counteracting consumption-smoothing and precautionary savings effects of risk aversion on interest rates.

The monetary policy rule is the typical forward-looking Taylor rule with smoothing parameter ρ. However, as in Bikbov and Chernov (2008), we allow systematic monetary policy to vary with a regime variable. There is substantive evidence that monetary policy has gone through activist and more accommodating spells (see e.g. Cho and Moreno (2006), Boivin (2005)). We let S_t^{mp} take on two values and it follows a standard Markov-chain process with constant transition probabilities.

While it is theoretically possible to obtain the rational expectations solution of the model in Equations (9)-(12), the model implies highly non-linear restrictions on the parameters, further complicated by the presence of regime-switching in the structural shocks. Bikbov and Chernov (2008) estimate a slightly simpler version of this model incorporating term structure data and note that without these additional data the identification of the regimes is rather poor. Our strategy is different. We replace the forward-looking rational expectations with survey forecast measures for expectations of the output gap and inflation. Let $X_{t,sur}$ be a vector containing the median of the individual survey forecasts for the output gap and expected inflation plus two zeros. Using these forecasts, we write the model in compact matrix notation as:

$$B_{11}(S_t^{mp})X_{t,ma} = \alpha + A_{11}(S_t^{mp})X_{t,sur} + B_{12}X_{t-1,ma} + F_{t,ma}, \qquad F_{t,ma} \sim N(0,\Sigma_t) \qquad (13)$$

where A_{11}, B_{11}, and B_{12} follow straightforwardly from Equations (7) to (12), leading to the following reduced form:

$$X_{t,ma} = c(S_t^{mp}) + \Omega_1(S_t^{mp})X_{t,sur} + \Omega_2(S_t^{mp})X_{t-1,ma} + \Gamma(S_t^{mp})F_{t,ma} \qquad (14)$$

with $\Gamma(S_t^{mp}) = inv(B_{11}(S_t^{mp}))$; $c(S_t^{mp}) = \Gamma(S_t^{mp})\,\alpha$; $\Omega_1(S_t^{mp}) = \Gamma(S_t^{mp})A_{11}(S_t^{mp})$; $\Omega_2(S_t^{mp}) = \Gamma(S_t^{mp})B_{12}$. The variable S_t^{mp} switches certain structural parameters in the A_{11} and B_{11} matrices. This structural model provides an economic interpretation to the contemporaneous relations between the state variables and a natural identification of the shocks F_t^y, F_t^π, F_t^q, and F_t^i.

The model can be estimated using limited maximum likelihood (we do not specify the dynamics of the survey forecasts). The use of the survey forecasts therefore both adds additional information and permits to identify the structural parameters with a relatively easy and straightforward estimation procedure. The quality of the model identification depends to a large extent on the quality of the survey forecasts. A recent paper by Ang, Bekaert, and Wei (2007) suggests that the median survey forecast of inflation is the best inflation forecast out of sample, beating time series, Philips curve and term structure models. Elliott, Komunjer, and Timmermann (2008) document biases in output and inflation forecasts, but argue that this may be due to the common assumption that forecasters use a symmetric loss function. In an empirical exercise, they find that only a modest degree of asymmetry is required to overturn rejections of rationality and symmetric loss. The value of surveys is also increasingly recognized in state-of-the-art term structure models (see e.g. D'Amico, Kim, and Wei (2009) and Chernov and Mueller (2009)) and monetary policy research.

2.2.4 Identification of F_t

In a simple VAR model, the F_t shocks can be computed from observations on the X_t variables. However, in our model, the F_t shocks also depend on S_t. We follow the typical Hamilton (1989) model, where agents in the economy observe the regime, but the econometrician does not. In estimating the state variable model, we hence make inferences about the realizations of the regime variable S_t as well. Once these are identified, we can simply retrieve F_t from the data using Equations (8) and (14). The identification scheme works well, because our estimation typically yields well-behaved smoothed regime probabilities close to one or zero.

2.2.5 Variance Dynamics of Factors

The factors F_t are assumed to be heteroskedastic with variance-covariance matrix Σ_t. Our modelling of Σ_t is inspired by direct empirical evidence of changing fundamental variances. Macroeconomists have noted a downward trend in the volatility of output growth and inflation from 1985 onwards (see e.g. Stock, Watson, Gali, and Hall (2003) and Blanchard and Simon (2001)), a phenomenon known as the Great Moderation. While some macroeconomists have attributed the Great Moderation to improved monetary policy (see Cogley and Sargent (2005)), a possibility already accommodated in our structural model, Sims and Zha (2005) and Ang, Bekaert, and Wei (2008) have identified important cyclical changes in the variance of fundamental shocks. We therefore model Σ_t as a function of a latent regime variable S_t. We allow each shock to have its own regime variable: $S_t = \{S_t^y, S_t^\pi, S_t^q, S_t^i, S_t^{cg}, S_t^{sliq}, S_t^{bliq}, S_t^{vp}\}$. Shocks to output and inflation uncertainty share the same regime variable as output and inflation. To retain tractability, we assume the regime variables to be independent Markov chain processes. Each regime variable can take on two values with the transition probabilities between states assumed constant. In particular, for each variable j, we have

$$var(F_t^j | S_t) = \exp\left(\alpha_j(S_t^j)\right).$$

A regime-switching model does not accommodate permanent structural breaks: at each point in time there is a probability that the variance may revert to a higher variability regime. We view such potential re-occurrence of similar regimes, which may not have been observed for a long time, as plausible. For example, the 2008-2009 crises will likely prove the Great Moderation to be a temporary, rather than permanent phenomenon. The computational and estimation complexities of state-of-the-art models accommodating both structural changes and regime-switching behavior (see Pesaran, Pettenuzzo, and Timmermann (2006)) make them hard to apply in our setting. We did entertain alternative models in which the factor variances depend on the own lagged state variables X_{t-1}^j. However, these models under-perform the regime switching models.

2.3 Time Variation in Betas

The benchmark model forces the betas to be constant, i.e. $\beta_{e,t} = \beta_e$, $\beta_{b,t} = \beta_b$. Simple affine pricing models imply that stock and bond return innovations are constant beta functions of the innovations in the state variables. Linearized versions of many present value models for equity pricing (see e.g. Campbell and Ammer (1993) and Bekaert, Engstrom, and Grenadier (2005)) imply a similar constraint on the betas. While there are economic reasons for time variation in betas, we deliberately limit the complexity of the models we consider, for two reasons. First, time variation in the betas could spuriously pick up non-fundamental sources of comovement. Second, with 10 state variables, allowing time variation in all betas very quickly leads to parameter proliferation that the amount of data we have cannot bear. We therefore consider a limited number of parsimonious models investigating the most likely economic sources of time variation in the betas. In a first set of models, the betas depend on instruments measured at time $t - 1$ (state dependent betas); in a second set of models, the betas depend on a subset of the regime variables S_t identified in the state variable model (regime-switching betas).

2.3.1 State Dependent Betas

We consider two different state dependent models. In a first model, we select a different economic source for beta variation for each factor. That is for state variable i, the beta is modelled as:

$$\beta_{j,t-1}^i = \beta_{j,0}^i + \beta_{j,1}^i z_{t-1}^{ij} \tag{15}$$

where $j = e, b$ and z_{t-1}^{ij} is a particular instrument, which we now describe for each factor. First, in the model of David and Veronesi (2008), widening the dispersion in beliefs increases the effect of economic shocks on returns. Our measures of output and inflation uncertainty can be viewed as proxies to belief dispersion ("uncertainty") regarding economic growth and inflation expectations. Hence, we let the sensitivity to output gap, inflation, and cash flow growth shocks be a function of respectively output, inflation, and cash flow uncertainty. The latter is proxied by the dispersion in cash flow predictions obtained from the Survey of Professional Forecasters. Second, because we use a constant maturity bond portfolio, interest rate changes affect the duration of the portfolio and consequently its interest rate sensitivity. As interest rates increase, the bond portfolio's lower duration should decrease its sensitivity to interest rate shocks. This line of thought applies to stocks as well, as stocks are long-duration assets with stochastic cash flows. The duration of stock returns actually depends on its dividend yield. We therefore allow the betas of stocks with respect to interest rate shocks to be a function of the level of the (log) dividend yield (corrected for repurchases), denoted by dy_t. Unfortunately, it is conceivable that behavioral factors may indirectly account for the resulting time variation in betas, if they are correlated with valuation effects reflected in dividend yields. Third, we let the exposure to risk aversion and variance premium shocks be a function of (lagged) risk aversion and the lagged variance premium themselves. This permits non-linear effects in the relationship between risk aversion changes and stock and bond returns. The effects of shocks to risk aversion may be mitigated at very high risk aversion levels, or they may be amplified if the economy is in or near a crisis. Finally, we also allow for level effects in the liquidity betas: a liquidity shock may

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affect returns less rapidly in illiquid markets. Hence, we allow the stock (bond) liquidity beta be a function of the lagged stock (bond) liquidity level.

In two alternative state dependent models, all betas depend on either the fundamental risk aversion measure or the variance risk premium. Here, the variance risk premium is viewed as a market-based indicator of risk aversion. In any pricing model, the price effects of economic shocks may depend on the discount rate. For example, high discount rates may decrease the magnitude of cash flow effects. Alternatively, Veronesi (1999) demonstrates how the degree of risk aversion exacerbates the nonlinear effects of uncertainty on asset prices in the context of a learning model. If risk aversion varies substantially through time, such effects may be important.

2.3.2 Regime-Switching Betas

An alternative dynamic model for betas is a regime-switching beta specification. While there are many reasons to expect betas to show regime-switching behavior, we preserve the structural interpretation of the implied stock-bond return correlation dynamics by using regime variables exogenously extracted from the state variables, without using stock and bond returns. First, with the Lucas critique in mind, one model considers betas to be a function of S_t^{mp}, the monetary policy regime. Second, both theoretical work (David and Veronesi (2008)) and empirical work (Boyd, Hu, and Jagannathan (2005); Andersen, Bollerslev, Diebold, and Vega (2007)) suggest betas may depend on the business cycle. Because many of our state variables have strong correlations with the business cycle, we let each factor's beta depend on its own regime variable to allow this possibility[4]. Finally, as discussed before, when a market is very illiquid, shocks may take time to filter through into prices. Consequently, we consider a model where all betas depend on the two liquidity regime variables. While such liquidity effects may indeed reflect trading costs, it is equally plausible that they reflect risk premium variation, given our quarterly data frequency.

3 Estimation and Model Selection

3.1 Model Estimation

We follow a two-stage procedure to estimate the bivariate model presented in Equation (1). In a first stage, we estimate the state variable model, then we estimate the factor model conditional on the economic factor shocks identified in the first step. From an econometric point of view, it would be more efficient to jointly estimate the factor and state variable models. However, an important risk of a one-step estimation procedure is that the parameters of the state variable model are estimated to help accommodate the conditional stock-bond return correlation, which would make the economic interpretation of the factors problematic.

We estimate the structural model using limited-information maximum likelihood because we replace unobservable conditional expected values by observable measures based on survey fore-

casts. We use quarterly data from 1968 till 2007, which we describe in detail in Appendix A. The state variable model features 10 state variables and 9 regime variables (which must be integrated out of the likelihood function), leading to a model with 82 parameters. Therefore, we consider "pared-down" systems in addition to the full model. We retain all parameters with t-statistics over 1, computed with White (1980) heteroskedasticity-consistent standard errors, and re-estimate the constrained system[5]. Fortunately, the retained parameters from the constrained estimation are remarkably similar to their unconstrained estimation counterparts. For the regime-dependent intercepts in $X_{t,nma}$, we use a Wald test for equality across regimes instead. If we fail to reject at the 24 percent level (corresponding roughly to a t-stat = 1 criterion), we continue to use a regime invariant drift. With the state variable model estimated, we identify the regimes and thus the factors at each point in time; finally, we estimate the factor model in Equation (1) using simple regression analysis.

3.2 State Variable Model Selection and Fit

An adequate state variable model must satisfy a number of requirements. First, the model should accurately describe the dynamics of the state variables themselves. To this end, we perform a battery of specification tests on the residuals of the state variable model. Appendix D describes these tests in detail. For each equation, we test the hypotheses of a zero mean and zero serial correlation (two lags) of the residuals; unit mean and zero serial correlation (two lags) for the squared residuals; appropriate skewness and kurtosis. In performing these tests, we recognize that the test statistics may be biased in small samples, especially if the data generating process is as non-linear as we believe it to be. Therefore, we use critical values from a small Monte Carlo analysis also described in Appendix D. For both the mean and the variance, we do the tests on the expected value and the serial correlation both separately and jointly. Hence, there are a total of 80 ($[3 + 3 + 2] \times 10$) such tests. The model described above features only 5 rejections, two at the five percent and three at the one percent level. Table 1, Panel A reports the Monte Carlo p-values of the joint test across means, variances, and higher moments for each variable. We reject the model only for inflation uncertainty, probably because of some remaining serial correlation, at the 5% level. The second line reports Monte Carlo p-values for a covariance test. We use the state variable model to uncover uncorrelated factor shocks, and here we test for each shock whether its joint covariances with all other factor shocks are indeed zero. We reject the hypothesis of zero covariances at the 5 percent level only for the output gap, and quite marginally so. All in all, given the complexity of the model at hand, we view its performance as a success.

Second, the model should also identify factor shocks that help explain the stock and bond return covariance and volatility dynamics. We report two sets of results for the constant beta model. In Panel B, we report a series of specification tests; in Panel C, we report a number of model diagnostics. The specification tests are applied to the estimated cross-product of the stock and bond residuals from Equation (1), $\hat{z}_t = \hat{\varepsilon}_{e,t}\hat{\varepsilon}_{b,t}$, and to the standardized stock and bond residuals

individually, $\hat{z}_t = \hat{\varepsilon}_{e(b),t}/\sqrt{h_{e(b)}}$, where $h_{e(b)}$ is the constant variance of the residuals of the factor model for respectively stocks and bonds. Under the null hypothesis that the model is correctly specified and captures stock-bond return dynamics, we have

$$E\left[\hat{z}_t\right] \;=\; 0 \tag{16}$$

$$E\left[\hat{z}_t\hat{z}_{t-k}\right] \;=\; 0, \text{ for } k \geq 1. \tag{17}$$

We conduct the zero mean test and the serial correlation (4 lags) test both individually and jointly within a GMM framework. The test statistics follow χ^2 distributions with respectively 1, 4, and 5 degrees of freedom. For the cross-product of the residuals, the zero mean test only rejects at the 10% level, and the serial correlation test does not reject at any level. A joint test of (16) and (17) fails to reject as well. Hence, the model appears to fit both the average covariance level between bond and stock returns and some first order covariance dynamics reasonably well. The analogous tests for standardized bond and stock residuals do not reject at any significance level. For the stock and bond residuals we perform two additional tests, a test for serial correlation on squared standardized residuals (the variance test), using 4 lags again, and Engle and Ng (1993) "sign bias" tests. The latter test whether the model captures volatility asymmetry: negative shocks raise volatility more than positive shocks (see Campbell and Hentschel (1992) and Bekaert and Wu (2000)). The actual tests check the validity of the orthogonality conditions $(a), (b),$ and (c) :

$$(a) \quad E\left[(\hat{z}_t^2 - 1)\,\mathbf{1}\left\{\hat{z}_{t-1} < 0\right\}\right] = 0$$
$$(b) \quad E\left[(\hat{z}_t^2 - 1)\,\mathbf{1}\left\{\hat{z}_{t-1} < 0\right\}\hat{z}_{t-1}\right] = 0$$
$$(c) \quad E\left[(\hat{z}_t^2 - 1)\,\mathbf{1}\left\{\hat{z}_{t-1} \geq 0\right\}\hat{z}_{t-1}\right] = 0$$

where $\mathbf{1}$ represents an indicator function. These conditions correspond to respectively the Sign Bias, the Negative Sign Bias, and the Positive Sign Bias tests of Engle and Ng (1993). The sign bias test examines whether or not the model captures the differential effect of negative and positive return shocks on volatility. The negative (positive) sign bias test focuses on the differential effect that large and small negative (positive) returns shocks have on volatility. The joint test, denoted by "Asym", is distributed as χ^2 with 3 degrees of freedom. Columns 4 and 5 in Panel B of Table 1 report the test results. The model easily passes both the "Variance" and "Asym" tests with the p-values being at least 20%.

The model selection statistics, reported in Panel C of Table 1, compare our model-implied conditional correlations, calculated as in Equation (6), with two benchmarks, namely realized correlations measured using daily returns of the following quarter, and the data-implied conditional correlations based on the MIDAS model referred to before. We expect the latter to give us a good picture of how the actual conditional correlations at the quarterly frequency vary through time, whereas the former essentially tests the predictive power of the models for future correlations. We consider several measures of "closeness". First, we compute the mean absolute deviation between the model-implied correlation and our proxy for the actual conditional correlation and the next quarter's realized correlation. For completeness, we also compute the

distance measure with respect to the unconditional correlation, which can help us interpret poor fits. Second, we compute the correlation between the two benchmark correlations and the model-implied correlations. It may be that the model cannot fit the correlation level, but performs well with respect to the time variation in correlations. Third, we also run a regression of our two benchmark correlations onto our model-implied correlations, expecting a zero constant and unitary slope. This measure provides information on, inter alia, the relative variability of the various correlation measures.

To better compare different models, we also create a statistical yardstick for the magnitude of the different diagnostics. We start with the benchmark factor model (see below), using constant betas in Equation (1). We then collect the estimated betas for both stock and bond returns and their corresponding covariance matrix, and draw 1000 different beta vectors from a normal distribution with expected beta and variance-covariance matrix equal to the estimated beta and variance-covariance matrix. With these simulated beta vectors, we create distributions for all the model diagnostics, and we show the 90 percent confidence intervals in the table. The first line of Panel C reports the diagnostic for our main model with constant betas. The model produces correlations that are 18.6% different from the correlations produced by the MIDAS model, 24.3% different from the realized correlations next quarter, and it deviates from the unconditional correlation by 6.6%, on average being mostly lower. For the latter two, we can compare the factor model's performance with that of the MIDAS model (see last line). The factor model fits the unconditional correlation actually better than the MIDAS model does (which has a 19.5% average distance), but it does not perform as well with respect to fitting the realized correlations. The MIDAS model's fit is also outside the 90% confidence interval. The correlation of the factor model-implied correlations with both MIDAS correlations and realized correlations is relatively low at 0.394 and 0.389 respectively. The MIDAS model has a 0.688 correlation with realized correlations. Finally, the regressions show no mean bias, but the regression slope coefficients are larger than one, significantly so for the realized correlation regression. However, the confidence intervals from the simulation experiment reveal that the regression constants and slopes likely will constitute weak tests of model fit.

Overall, the fit is satisfactory, albeit not great. In an earlier version of the article, we considered a large number of alternative models, in terms of numbers of factors (considering models with fewer factors), the identification of the factors (considering non-structural VARs) and the heteroskedasticity structure of the factors (state dependent). The current semi-structural model with regime-switching factors outperforms these models using the measures of fit just described. Panel C illustrates this outperformance for a number of alternative models: three non-structural VARS (a homoskedastic model, a model with state dependent heteroskedasticity, and a model with regime switching volatility), and two structural models (a homoskedastic model, and a model where the heteroskedasticity is state dependent rather than regime-switching). In the state dependent models, the conditional variance of the shocks depends on the lagged level of the variable itself; the output, inflation, and cash flow growth variances depend additionally on the lagged level of their uncertainty measures. In terms of the distance measures, these models

all perform worse on all three diagnostics, significantly so for the non-structural homoskedastic and state dependent models. Imposing structural restrictions and a regime-switching structure clearly improves the fit. The correlation measures reveal a similar story. The alternative models all generate lower regression slopes, which are in some cases very close to 1. However, the alphas are always significantly different from zero. We discuss the fit of the benchmark model in more detail in Section 4.

3.3   Estimation State Variable Model

To conserve space, we report parameter estimates for the state variable model in Appendix E. We focus the discussion on the identification of regimes and the volatility dynamics of the models as they determine the fundamental stock and bond return correlations. In the New-Keynesian model, the structural parameters are of independent interest but a detailed discussion is beyond the scope of this article[6]. Let us only comment on the regime variable for systematic monetary policy in the interest rate equation. Our β estimates reveal an activist monetary policy regime (with $\beta = 2.09$) and an accommodating monetary policy regime (with β smaller and insignificantly different from 1). The coefficient on the output gap, γ, is significantly positive in both regimes, but larger in the second regime.

Figure 2 plots the smoothed regime probabilities for all regime variables. All models show significant regime-switching volatility both in statistical and economic terms Figure 3 then plots the conditional volatilities of the various factors. We discuss the two figures in tandem. We first focus on the regime variable affecting the volatility of the output gap and inflation shocks. For both inflation and output volatility, there is a near permanent switch to the low volatility regime, which is consistent with the idea of a Great Moderation. For output, this switch occurs in 1988; for inflation only in 1992. Of course, in our regime-switching model, there is a positive probability that the high volatility regime will re-occur. In terms of volatility levels (Figure 3), inflation and output volatilities are often but not always higher in NBER recessions.

A stronger counter-cyclicality is observed for interest rate shocks, which are also less variable than inflation but more variable than output shocks. In Figure 2, Panel B, we see that the high interest rate volatility regime mostly occurs during recessions, including the 1980-1982 Volcker period. This is consistent with the results in Bikbov and Chernov (2008) who also categorize the Volcker period as a period of discretionary monetary policy, with interest rate shocks becoming more variable. Unlike Bikbov and Chernov (2008), our structural model identifies systematic monetary policy to be activist during this period. The model also shows that the 1990 and 2001 recessions were accompanied by an accommodating monetary policy regime, but that activist monetary policy spells became more frequent from 1980 onwards.

Panel C of Figure 2 shows the regimes for the fundamental risk aversion and variance premium factors. Fundamental risk aversion is notably higher in the 1974-75, 1980-82 and early nineties

recessions, but not in 2001. The high volatility regime typically seems to outlast the recession itself and is also relatively high around 1997. The variance premium regime is more clearly counter-cyclical and also shows a peak towards the end of the sample. As Panel B of Figure 3 shows, the variance premium shock is multiple times more volatile than the fundamental risk aversion factor.

The cash flow volatility regime (Panel D, Figure 2) shows frequent shifts, but also appears to shift mostly into the lower volatility regime post 1990, with the exception of another switch into the high volatility regime during the recession of 2001. Note that these are cash flow shocks cleansed of the influence of the macro-economic state variables. The last Panel in Figure 2 shows the stock and bond market illiquidity regimes. For stock illiquidity, the regime is mostly in the high variance state, only shifting occasionally to lower variances but most notably doing so for a lengthy time period post 2001. For bond illiquidity, the picture is similar, but the regime variable goes to the lower volatility state already around 1992, with only one spike up afterwards. Figure 3 shows that bond illiquidity shocks were mostly more variable than stock illiquidity shocks until the shift to the low variability regime in 1992, and mostly less variable thereafter.

4 Correlation Dynamics

4.1 Model Fit

Table 2 repeats the fit of the constant beta benchmark model, including the 90% confidence intervals. All the other lines reflect the performance of the candidate models for time variation in betas we discussed before. Because these models have at least 20 parameters (10 factors with at least one interaction term), we consider two versions of these models, an unrestricted version and a version where sources of time variation leading to coefficients with t-stats less than one in absolute value are removed, and the model re-estimated.

The most striking feature of the table is that the constant beta model proves a very difficult to beat benchmark. Let's first focus on the distance measures. No other model matches the unconditional correlation as well as the constant beta model. With respect to the distance relative to future realized correlations, seven of the 16 estimated models with time-varying betas perform significantly better than the constant beta model, but barely so. In terms of distance to the MIDAS model, the majority of the models perform worse than the constant beta model and no model performs significantly better. The best models in terms of distance measures are the models with either risk aversion or the variance premium as an instrument and the regime-switching models with either the own regime variable or the illiquidity regime variables as sources of beta variation. In terms of correlation with either the MIDAS benchmark or future realized correlations, these same models also perform reasonably well, with the exception of the illiquidity regime switching beta model. Instead, the model with the monetary policy regime betas performs better on this score. However, the best model here clearly is the model

where the variance premium is an instrument. Both its restricted and unrestricted versions lead to correlations that are significantly higher than the constant beta model. Another nice feature of this model is that the regression slopes are very close to one (although the alphas are significantly different from zero using MIDAS correlations). We therefore select this model as an alternative benchmark, and note that the model with restrictions performs slightly better.

Figure 4, Panel A graphs the MIDAS conditional correlations together with the correlations implied by the two benchmark models (constant betas and betas that vary with the variance premium). Clearly, the benchmark models produce less variable correlation dynamics than the MIDAS model does. The constant beta model never produces negative correlations, and while correlations decrease around 2000, the model cannot match the steep decrease in correlations the MIDAS model generates. The time-varying beta model does better in this regard, which explains why it correlates more with the MIDAS model than the constant beta model does. We now explore the economics behind the correlations in more detail, starting with examining the beta exposures.

4.2   Factor Exposures

Table 3 presents the beta estimates for the three retained models. Let's first focus on the constant beta specification. For equity returns, many factors are either highly (fundamental risk aversion, bond illiquidity and the variance risk premium) or borderline (the output gap, inflation, output and inflation uncertainty and stock market illiquidity) statistically significant. Only the interest rate and bond illiquidity shocks significantly affect bond returns however. Nevertheless, more often than not the signs of the betas are consistent with expectations. The risk aversion variables (fundamental and the variance risk premium) and the liquidity factors produce negative coefficients with only one exception (the totally insignificant variance premium shock for bond returns), consistent with them being discount rate factors. The interest shock is negative for both bond and stock returns, but is much more important for bond returns both in economic magnitude and statistical significance. However, inflation and the output gap matter more for equity returns (with a positive sign for the output gap and a negative one for inflation), whereas they are insignificant for bond returns. Output and inflation uncertainty only show some explanatory power for stock returns, with a negative sign for output and a positive sign for inflation uncertainty. The former could simply reflect a discount rate effect, whereas the latter is potentially consistent with the learning models of Veronesi (1999), in which uncertainty decreases the equity risk premium.

To better gauge the economic importance of the factors in driving variation in bond and stock returns, Table 3 also reports the standard deviation of the factors. For the statistically important factors in the stock return equation, the effect of a one standard deviation change is mostly between 1 and 2 percent. The economically most important factor, by far, is the variance premium, for which a one standard deviation move changes stock returns by over 3.5 percent.

For bond returns, a one standard deviation positive shock to interest rates (bond illiquidity) decreases bond returns by 1.8 (1.4) percent.

The next 4 columns in the table report the betas for two time-varying beta specifications, one unrestricted, one zero-ing out the coefficients with t-stats less than 1. For equity returns, about half of the betas are significantly related to the variance risk premium. With one exception, the coefficient on the variance risk premium is positive. Recall that the variance premium's variance and level have a strong cyclical pattern, being higher in recessions. Hence, the exposure of stocks to a cash flow shock such as the output gap is increased in absolute magnitude in recessions, but the exposure to discount rate shocks (including the variance premium itself) is mitigated. The exception is the bond illiquidity beta, which is even more negative when the variance risk premium is high. For bond return betas, we fail to find significant exposures to the variance premium.

Note that the bond and stock exposures are mostly of the same sign, explaining why the constant beta model cannot generate negative correlations. However, with time-varying betas, the equity return exposures to inflation, the interest rate, and output uncertainty switch signs at the end of the 1990s, contributing to negative model-implied correlations. Hence, the model likely uses the variance premium dependence to partially capture a "flight-to-safety" effect. When risk aversion is high in a recession or crisis, interest rates may be low, bond prices increase but stocks are now positively correlated with interest rate shocks and stock prices fall.

4.3 Time-Varying Expected Returns

Our model produces unconditional correlations that are too low. In the data, this correlation amounts to 19 percent, but the benchmark model produces an average correlation of 14 percent. One potential channel to increase unconditional correlations not present in our current model is time variation in expected returns. For instance, in the model of Bekaert, Engstrom, and Xing (2009) risk premiums on stocks and bonds are highly correlated, thus increasing the unconditional correlation between stock and bond returns. In addition, mis-measurement of expected returns may affect the estimation of conditional covariance dynamics. An assumption of constant risk premiums seems particularly strong in light of the important structural shifts that we uncovered in the variances of fundamental variables such as inflation and the output gap. Such important changes may lead to abrupt changes in risk premiums, which are unaccounted for in our present model. In fact, Lettau, Ludvigson, and Wachter (2008) claim that the decline in macroeconomic volatility may have led to a decline in the equity risk premium.

We consider two extensions to our models to accommodate time variation in expected stock and bond excess returns. First, we model expected excess returns as a linear function of the lagged level of the default spread, short rate, term spread, dividend yield, the consumption-wealth ratio (CAY from Lettau and Ludvigson (2001)), and the VIX premium. Second, we use the regime probabilities identified in the structural factor model estimation as instruments for expected

returns in univariate regressions. The results are disappointing. In the instrumental variables regression, the short rate, CAY, and the default spread are significant predictors with the same sign in the bond and stocks return regressions. Thus, they can possibly help generate positive covariation between stock and bond premiums. However, the increase in the unconditional correlation is small, and amounts to only 1.74 percent. Structural changes, as identified by the regime variables, do not seem to affect expected stock and bond returns in a meaningful way.

4.4 Economic Factor Contributions

Our model counts 10 state variables, prompting the question to what extent different factors contribute to the model's fit. To determine this, we re-estimate the factor model, leaving out various factors, and then report the deterioration in fit. The confidence intervals reported in Table 2 can again tell us something about the statistical significance of the worsening in fit, at least for the constant beta model. Table 4 reports the results. We first split our variables into pure macro variables (the output gap, inflation, the interest rate and the risk aversion measure computed from consumption data), and the rest of the variables (uncertainty, cash flow growth, illiquidity, and the variance premium).

The main message is clear and consistent across the constant and time-varying beta models: the fit worsens considerably more when dropping the non-macro variables as opposed to the macro variables. The deterioration in fit by dropping the macro variables is only economically (albeit not statistically) important for the unconditional correlation in the constant beta model. Within the set of macro variables, fundamental risk aversion is relatively important for the distance measures, but does not help in fitting the correlation with actual bond-stock return correlations. Among the non-macro variables, the cash flow variable overall contributes the least, followed closely by the variance premium. Of course, the variance premium also contributes indirectly through its effect on betas.

The main result undoubtedly is that the illiquidity variables are the most important contributors to the correlation dynamics. Recall that we actually find that bonds and stocks both load negatively on stock and bond illiquidity, so that liquidity variation induces positive correlation between stock and bond returns. This common exposure was particularly helpful in the early part of the sample (where liquidity was still poor), although no super-clear subsample patterns emerge. While more work in this area is surely needed, some recent research helps give credence to our results. Li, Wang, Wu, and He (2009) show that systematic liquidity risk is priced in the bond market, but they do not consider stock returns. Bansal, Connolly, and Stivers (2009) show that the usual stock liquidity measures help predict stock and bond return correlations, and have effects independent of the usual VIX - "flight-to-safety" effect. They focus on a relatively short 1997-2005 period however. Finally, Goyenko and Sarkissian (2008) also document a strong link between bond illiquidity and stock returns in 46 different markets.

5 Covariance and Volatility Dynamics

5.1 Correlation decomposition

The correlation is a scaled statistic involving covariances, and both stock and bond return volatility. Using the MIDAS model as a "well-fitting" empirical benchmark allows us to decompose the performance of our main model. Table 5 repeats our standard diagnostic fit measures[7], but we now replace the three components of the correlation, one-by-one, with their MIDAS counterparts. For example, the line "MIDAS covariance" replaces the covariance produced by the model by that of the MIDAS model. Clearly, the model's fit improves dramatically, nearly matching the fit of the MIDAS model, which is repeated on the last line of the Table. The exception is the constant in the regression which is now significantly positive, owing to the MIDAS model producing too high covariances on average. Not surprisingly, this must mean that the use of MIDAS volatilities is correspondingly less important. While using MIDAS's stock volatility model leads to improvements in most cases, using MIDAS's bond volatility worsens the fit relative to using the model's. What is particularly striking is that the slope coefficients in the regressions become quite small once the MIDAS bond volatility replaces that of the factor model. As we will see below, this owes to the MIDAS model producing quite volatile bond return volatility while the factor model really only captures a small part of the variation in bond return volatility. Consequently, the results here need not necessarily mean that the factor model fits bond return volatility better than the MIDAS model. In fact, the poor distance measures here result from the MIDAS model producing relatively low bond return volatility in the 70s at the time that the benchmark model already overshoots actual correlations.

The main conclusion to be drawn is that our factor model primarily suffers from a poor fit of bond-stock return covariances. It appears that it does not fit stock return volatility significantly worse than the MIDAS model does. Of course, it remains possible that the MIDAS model also represents a poorly fitting benchmark model for stock return volatility. We now investigate the performance of the model with respect to the three components of correlation in more detail.

5.2 Covariances

Table 6 reports our diagnostic measures but applied to covariances, rather than correlations. We leave out the regression evidence because it proved not powerful in distinguishing models. The overall fit of the covariances matches the patterns we observed for correlations. Relative to the unconditional covariance, the model performs reasonably well, outperforming the MIDAS model. The time-varying beta model is worse than the constant beta model in terms of distance measures, but produces higher correlations with realized and MIDAS covariances.

In terms of the contributions of the various factors, the non-macro variables, particularly the liquidity variables, are most important. However, no single factor contributes in a meaningful way to lower the distance measure with respect to the realized covariances. Yet, the model as a whole does perform better than the unconditional mean.

5.3   Stock Volatility

Table 7 reports the same diagnostics for the conditional stock market volatility. The model produces a stock market volatility that is on average 6.3% different from the unconditional volatility. The MIDAS benchmark does better with an average distance measure of 2.8%. However, the model does about as well as the MIDAS model predicting future realized stock return volatilities. Not surprisingly the distance between the MIDAS and factor models is rather small at less than 3%. While time-varying betas lower the distance with respect to the unconditional standard deviation of stock returns, they increase it with respect to predicting realized volatilities. The correlation between MIDAS conditional volatilities and the ones produced by the economic factor model are relatively high; yet the MIDAS model produces stock return volatilities that are much more highly correlated with realized volatilities.

As a final diagnostic, we compute the R^2 of the factor model for stock returns. This is not only important to understand the fit with stock market volatility: If the factors fit only a small fraction of the return variance then it is unrealistic to hope for a satisfactory fit for the covariance of stock and bond returns. The literature on stock returns in particular has a long but controversial exponent arguing that stock returns are excessively volatile (see for instance the old debate between Shiller (1981) and Kleidon (1986)).

We report the adjusted regression R^2, adjusted for 10 regressors in the constant beta model and 20 in the time-varying beta model. The adjusted R^2's are respectively 27% for the constant and 35% for the time-varying beta model. This may appear low, especially when compared to the MIDAS model, which explains 65% of total stock market variance. However, the comparison is somewhat unfair. First, for the MIDAS model, our R^2 counterpart takes the ratio of the average conditional variance produced by the MIDAS model divided by the unconditional stock market variance. We could compute a similar "model-based" R^2 for the factor model, which would be close to the regression R^2 if the conditional variances of the factors are relatively close to their empirical counterparts. Fortunately, they are, with the biggest deviation (the model variance being 10% larger than the empirical variance) occurring for the variance premium. Moreover, we cannot easily adjust the MIDAS model for "the number of regressors", so a fairer comparison would be with an "unadjusted model R^2". For the constant beta model, this R^2 is 38.4% and for the time-varying beta model it is even over 70%, better than the MIDAS model.

The contribution analysis generally shows again that the non-macro variables are relatively more important than the macro variables. However, the macro variables do contribute something to the fit, especially with respect to the distance measures. Among the non-macro variables, the illiquidity factors remain very important but the variance premium is now the dominant factor. There is a flurry of recent research on the properties of the variance premium, mostly on its predictive properties with respect to stock returns (Bollerslev, Tauchen, and Zhou (2009)). Our article shows that it is one of the main drivers of stock market volatility. Its important role also explains the somewhat strange decomposition analysis for the distance measures relative

to realized variances in the time-varying beta model. There, leaving out the non-macro variables worsens the fit considerably, but seemingly no single economic variable is responsible for the good fit, as leaving them out univariately or in pairs always leads to improvements in fit. Because the variance premium enters the betas, and the factor model is re-estimated when factors are dropped, the model retains the flexibility to match the data reasonably well through the beta exposures.

5.4   Bond Return Volatility

Table 8 reports diagnostics on the fit of bond return volatility. The MIDAS benchmark performs slightly better with respect to the distance measures relative to realized volatilities. Because the average bond volatility is only 8.98% (compared to 17.38% for stocks), the average distances are relatively worse than for stock returns. In terms of correlations, the factor model's bond volatility correlates only 18.3% with realized correlations, and only 20.8% with the MIDAS model's bond volatility. The MIDAS model's bond volatility also correlates much more highly with realized volatilities. We conclude that the economic model has actually a harder time fitting bond volatility than it does stock market volatility. This is an interesting finding, as the academic literature has primarily looked for links between macro-economic fundamentals and stock market volatility. While some of the seminal articles have been skeptical (see especially Schwert (1989)), some more recent articles did find a link: Flannery and Protopapadakis (2002) show links between stock market volatility and inflation and money growth; Hamilton and Lin (1996) establish a business cycle link for stock market volatility, and Diebold and Yilmaz (2009) and Engle and Rangel (2008) find a link between volatile fundamentals and volatile stock markets in a cross-section of countries. On the contrary, the work on bond returns appears surprisingly limited.

We find that the factor model explains about the same amount of variation in bond returns as in stock returns (the adjusted R^2 is 30%). However, the factors contributing to this fit are very different than the factors explaining stock returns. Here, the fundamental variables contribute the most to the fit; the non-macro variables contribute relatively little. Of the fundamental variables, the short rate, not surprisingly, is the dominant factor. All in all, this "discrepancy" between the factors explaining bond and stock returns is intriguing and deserves further scrutiny. Baker and Wurgler (2008) recently find interesting links between bonds and "bond-like" stocks, which they attribute to changes in investor sentiment. Their main focus is on co-predictability rather than comovement, but it would be interesting to test whether fundamentals have a different effect on bonds than on "bond-like" stocks. Also, their sentiment index may capture value-relevant behavioral factors not captured by our risk aversion indicators. In fact, none of our variables capturing discount rate factors (fundamental risk aversion, the variance premium, bond and equity market liquidity) correlates more than 20% with the Baker-Wurgler index.

6 Conclusions

The substantial time variation in stock-bond return correlations has long been viewed as puzzling. Without assessing what time variation in correlations a formal model of fundamentals can generate, this may be a premature judgment. For instance, much has been made of the negative correlation between bond and stock returns in recent times. However, the real economy and the inflation process have undergone some remarkable changes recently. In particular, it is well known that output and inflation volatility have decreased substantially since 1985. If bonds and stocks have similar exposures to these economic factors, their correlation should have decreased. It is also conceivable that these fundamental changes have affected risk aversion, a factor on which bonds and stocks may load with a different sign. While it remains difficult to think of economic factors that would cause a sudden and steep decrease of stock-bond return correlations into negative territory, it remains useful to quantify how much of the correlation dynamics can be attributed to fundamentals. This is what this paper sets out to do using a dynamic factor model with fundamental factors.

Importantly, we consider a large number of factors, both pure economic factors, and factors measuring risk aversion and illiquidity. We also consider a large number of model specifications, some with scant structural restrictions. Interestingly, the performance of our fundamental models improves when factor shocks are partially 'structurally' identified by means of a New-Keynesian model, and we focus the article mostly on this model. Overall, we fail to find a fully satisfactory fit with stock-bond return correlations. Our model fails to forecast realized correlations as well as a benchmark empirical model, using the MIDAS framework of Colacito, Engle, and Ghysels (2009). While this model is a backward-looking empirical model, it uses daily return data efficiently and generally fits the data very well. Our model does fit the unconditional correlations between stocks and bond returns better than the MIDAS model does. Not unlike the pattern observed in the data, our fundamental-based model generates positive correlation until the end of the 1980s, and decreasing correlations afterwards. However, a model with constant factor exposures fails to generate negative correlations. We do obtain negative correlations when we allow the factor exposures to depend on a risk premium proxy extracted from options data, the so-called variance premium. This model also produces time-varying correlations that correlate substantially with correlation movements in the data, although the timing and magnitude is far from perfect.

We then analyze our results along several dimensions, producing a variety of useful directions for future research. First, given the long tradition of viewing stocks as excessively volatile, the poor fit of our factor model may simply be due to its failure to account for actual stock return volatility. However, we show that the model primarily fails in fitting covariances and that its fit with respect to stock volatility is actually relatively better than its fit with respect to bond volatility. Second, the factors most useful in explaining correlations and covariances are the non-macro variables, especially stock and bond market illiquidity factors. Much more research must be directed towards analyzing the dynamic effects of liquidity. Third, there is an

interesting dichotomy between the fit of stock and bond market volatility dynamics. For bonds, fundamental factors do play a relatively large role, but for stocks, the liquidity factors, and the variance premium are hugely important factors. These empirical results for sure complicate the creation of general equilibrium models that can jointly rationalize bond and stock pricing (see Bekaert, Engstrom, and Grenadier (2005) for a recent effort).

Of course, we have focused our analysis on standard macro factors, and more intricate models would likely yield variables that we have not considered. In models with incomplete markets for example, the distribution of income and wealth may matter. Yet, our economic factors include uncertainty measures (for both inflation and output forecasts) which may be motivated using models involving heterogeneity and learning. However, we find that these variables are not nearly as important in driving second moment return dynamics as are liquidity variables. These liquidity factors may be correlated with the "flight-to-safety" effects that have been documented in the literature (see especially Connolly, Stivers, and Sun (2005)). In the end our fundamental model does not seem to produce an entirely satisfactory fit of the "flight-to-safety" effects that are likely at the heart of the negative correlations observed post 2000. To test this more formally, we re-estimated the factor model over the 1986-2007 sample period, including the VIX as an additional factor. We know that VIX increases are associated with lower correlations between bond and stock returns, but it is conceivable that this effect is a pure discount rate effect, captured by our two risk premium measures, in particular, by the variance premium. However, we find that the VIX still comes in significantly with a negative sign. While it is always possible that our variance premium estimate is an imperfect proxy to the risk premium variation hidden in VIX movements, we believe research into "flight-to-safety" effects and their interaction with liquidity factors remain of first-order importance to fully understand stock-bond return comovements.

Notes

[1]Autocorrelation in daily stock and bond returns potentially biases our estimates of quarterly stock and bond return volatilities and correlations. While we do find a moderate degree of autocorrelation in both stock and bond returns, correcting for this bias (using 4 Newey and West (1987) lags) does not meaningfully alter stock-bond return volatilities and correlations.

[2]The output gap uses a quadratic trend to measure potential output (see the data appendix for details).

[3]In 1997, quoted tick sizes decreased from eights to sixteens, and in 2001 a system of decimalization was introduced.

[4]For completeness, we estimated a model with betas a function of NBER dummies, but it performs less well than the regime-switching beta model described here.

[5]We then also impose the economic restriction that ϕ must be positive, setting its value at 0.1.

[6]We find mostly parameters in line with the extant literature, including a rather weak monetary transmission mechanism (see Bekaert, Cho, and Moreno (2008)).

[7]We left out the unconditional correlation as the MIDAS model fits the unconditional correlation worse than our model.

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Appendix

A   Data Appendix

Our dataset consists of stock and bond returns and a number of economic (fundamental) state variables for the US. Our sample period is from the fourth quarter of 1968 to the fourth quarter of 2007 for a total of 157 observations. The economic state variables are seasonally adjusted. We now describe the exact data sources and how the variables are constructed:

1. **Stock Excess Returns (r_e):** End-of-quarter NYSE/AMEX/NASDAQ value-weighted returns including dividends, from the Center for Research in Security Prices (CRSP) Stock File Indices. The returns are in excess of the US 3-month T-bill rate.

2. **Bond Excess Returns (r_b):** End-of-quarter returns on 10-year Treasury bonds, from the CRSP US Treasury and Inflation Module, in excess of the US 3-month T-bill rate.

3. **Output Gap (y):** The output measure is real Gross Domestic Product (GDP), from the Bureau of Economic Analysis. The gap is computed as the percentage difference between output and its quadratic trend.

4. **Expected Output Gap (ye):** The expected output gap is constructed as follows:

$$
\begin{aligned}
E_t\left[y_{t+1}\right] &= E_t\left[\frac{g_t}{g_t}\left(\frac{g_{t+1}}{tr_{t+1}}-1\right)\right] \\
&= g_t\frac{E_t\left[\frac{g_{t+1}}{g_t}\right]}{tr_{t+1}}-1
\end{aligned}
$$

where g_t is the level of real GDP at time t, and tr_t the (quadratic) trend value of real GDP at time t. We use survey-based expectations of the level of real GDP for the current and next quarter to measure $E_t\left[\frac{g_{t+1}}{g_t}\right]$. The source is the Survey of Professional Forecasters (SPF).

5. **Output Uncertainty (yd):** Average SPF respondents' assessment of real output uncertainty, taken from Bekaert and Engstrom (2009).

6. **Inflation (π):** Log difference in the Consumer Price Index for All Urban Consumers (All Items), from the Bureau of Labor Statistics.

7. **Expected Inflation (πe):** Median survey reponse of expected growth in the GDP deflator over the next quarter, from the Survey of Professional Forecasters (SPF).

8. **Inflation Uncertainty (πd):** Average SPF respondents' assessment of inflation uncertainty, taken from Bekaert and Engstrom (2009).

9. **Fundamental Risk Aversion (fra):** Our measure of fundamental risk aversion is based on the external habit specification of Campbell and Cochrane (1999), and taken from Bekaert and Engstrom (2009).

10. **Nominal Risk-free Rate** (i): 3-Month Treasury Bill (secondary market rate) from the Federal Reserve.

11. **Cash Flow Growth** (cg)**:** Dividend growth including repurchases, taken from Bekaert and Engstrom (2009). The source for the dividends is CRSP. The source for the repurchases is Securities Data Corporation. Dividend growth is transformed into cash flow growth using the ratio of repurchases to (seasonally adjusted) dividends.

12. **Stock Market Illiquidity** ($sliq$): Capitalization-based proportion of zero daily returns across all firms, aggegated over the quarter, obtained from Bekaert, Harvey, and Lundblad (2007).

13. **Bond Market Illiquidity** ($bliq$): Monthly average of quoted bid-ask spreads across all maturities, taken from Goyenko (2006). He uses securities of 1 month, 3 months, 1, 2, 3, 5, 7, 10, 20, and 30 years to maturity, and deletes the first month of trading, when the security is 'on-the-run', as well as the last month of trading. Consequently, he calculates a monthly equally-weighted average of quoted spreads from daily observations for each security. Finally, the market-wide illiquidity measure is calculated as an equally-weighted average across all securities for each month. We take the end of quarter value as a quarterly measure.

14. **Variance premium** (vp)**:** We calculate the Variance Premium (on a quarterly basis) by subtracting the fitted MIDAS variance from the VIX squared. The VIX is the implied volatility of highly liquid S&P500 index options, and is taken at the end of the quarter from the Chicago Board of Options Exchange (CBOE). Unfortunately, the VIX is only available from January 1986 onwards. For the 1968-1985 period, we use a projection on the following explanatory variables: the Michigan Consumer Confidence Index (CC), the Realized Variance (RV), the consumption-wealth ratio (CAY), the Term Spread (TS), and the Default Spread (DS). This specification resulted after insignificant variables were removed from a richer specification, estimated on 1986-2007 data, also involving liquidity and sentiment variables.

	Const	CC	RV	CAY	TS	DS	R^2
Param	-0.0826	0.0006	0.2752	0.2961	0.0031	0.0095	50.19
pval	0.003	0.0230	0.0004	0.001	0.082	0.049	

The variables stock return, bond return, (expected) output gap, (expected) inflation, interest rate and cash flow growth are expressed in percentages on a quarterly basis. The stock illiquidity, bond illiquidity and variance premium are multiplied by 100 to make them similar in order of magnitude. The uncertainty measures and the risk aversion measure are the original series as taken from Bekaert and Engstrom (2009).

B A Component Model for Dynamic Stock-Bond Correlations

In a recent paper, Colacito, Engle, and Ghysels (2008) introduce a component model for dynamic correlations. Their model combines the DCC model of Engle (2002) and the GARCH-MIDAS model of Engle, Ghysels, and Sohn (2006). Consider the vector of stock and bond returns $r_t = [r_{e,t}, r_{b,t}]'$, and assume it follows the following process:

$$r_t \sim {}_{iid}N(\mu, H_t) \tag{B-1}$$

$$H_t = D_t R_t D_t \tag{B-2}$$

where μ is the vector of unconditional means, H_t the bivariate conditional variance-covariance matrix, and D_t a two-by-two diagonal matrix with the conditional stock and bond return volatilities on the diagonal, and

$$R_t = E_{t-1}\left[\xi_t \xi_t'\right] \tag{B-3}$$

$$\xi_t = D_t^{-1}(r_t - \mu) \tag{B-4}$$

This model is conveniently estimated in two steps. First, the conditional volatilities in D_t are estimated, to be followed by the conditional correlation matrix R_t.

A GARCH-MIDAS component model for conditional stock and bond return variances.

Assume that the univariate return follows the following GARCH-MIDAS process:

$$r_{i,t} = \mu_i + \sqrt{m_{i,\tau} \times g_{i,t}}\,\xi_{i,t}, \qquad i = \{e, b\}, t = (\tau - 1)N_v^i + 1, ..., \tau N_v^i \tag{B-5}$$

where $g_{i,t}$ and $m_{i,\tau}$ are the short and long run variance components of the daily returns of asset i. The short run component $g_{i,t}$ varies at the daily frequency t, while the long-term component $m_{i,\tau}$ - with a time subscript τ - only changes every N_v^i days. The short run variance of stock and bond returns follows a simple GARCH(1,1) process:

$$g_{i,t} = (1 - \alpha - \beta) + \alpha_i \frac{(r_{i,t} - \mu)^2}{m_{i,\tau}} + \beta_i g_{i,t-1} \tag{B-6}$$

The low-frequency component $m_{i,\tau}$ is a weighted sum of L_v^i lags of realized variances (RV) over a long horizon:

$$m_{i,\tau} = \bar{m}_i + \theta_i \sum_{l=1}^{L_v^i} \varphi_l(\omega_v^i)\, RV_{i,\tau-l} \tag{B-7}$$

where the realized variances involve N_v^i daily non-overlapping squared returns:

$$RV_{i,\tau} = \sum_{t=(\tau-1)N_v^i+1}^{\tau N_v^i} (r_{i,t})^2 \tag{B-8}$$

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Because we compare the long-term MIDAS components with our quarterly model counterparts, we set N_v^i equal to the number of trading days within one quarter. As a weighting function, we use a beta function with decay parameter ω_v^i :

$$\varphi_l\left(\omega_v^i\right) = \frac{\left(1 - \frac{l}{L_v^i}\right)^{\omega_v^i - 1}}{\sum_{j=1}^{L_v^i}\left(1 - \frac{j}{L_v^i}\right)^{\omega_v^i - 1}} \tag{B-9}$$

The weight attached to past realized variances will depend on the two parameters ω_v^i and L_v^i. The latter determines the number of lagged realized variances taken into account. The decay parameter ω_v^i determines the weight attached to those past realized variances. In case of $\omega_v^i = 1$, the past L_v^i will receive an equal weight of $1/L_v^i$. In the likely case of $\omega_v^i > 1$, past realized variances will gradually get less and less weight. The larger ω_v^i, the larger the decay. In our empirical analysis, we allow both the decay parameter ω_v^i and the L_v^i lags of realized variances to differ between stock and bond returns. We will select optimal values for both parameters using the likelihood profiling procedure discussed in Engle, Ghysels, and Sohn (2006).

A component model for conditional stock and bond return correlations

In the second step, we calculate correlations based on the standardized residuals ξ_t. More specifically, we calculate a conditional matrix Q_t, whose elements are given by:

$$q_{i,j,t} = \bar{\rho}_{i,j,\tau}\left(1 - a - b\right) + a\xi_{i,t-1}\xi_{j,t-1} + bq_{i,j,t-1} \tag{B-10}$$

$$\bar{\rho}_{i,j,\tau} = \sum_{l=1}^{L_c^{ij}}\varphi_l\left(\omega_c\right)c_{i,j,\tau-l} \tag{B-11}$$

$$c_{i,j,\tau} = \frac{\sum_{t=(\tau-1)N_c^{ij}+1}^{\tau N_c^{ij}}\xi_{i,k}\xi_{j,k}}{\sqrt{\sum_{t=(\tau-1)N_c^{ij}+1}^{\tau N_c^{ij}}\xi_{i,k}^2\sum_{t=(\tau-1)N_c^{ij}+1}^{\tau N_c^{ij}}\xi_{j,k}^2}} \tag{B-12}$$

where the weighting scheme is similar to the one used in (B-9). The long-run correlation $\bar{\rho}_{i,j,\tau}$ is a weighted sum of L_c^{ij} lags of realized correlations, calculated on N_c^{ij} daily non-overlapping returns. As for the long run variance, we consider one quarter of such daily returns. The conditional correlations between stock and bond returns at the daily frequency can then be easily calculated as:

$$\rho_{e,b,t} = \frac{q_{e,b,t}}{\sqrt{q_{e,e,t}}\sqrt{q_{b,b,t}}}$$

Estimation Results

Panel A of the table below reports the estimation results for the conditional bond and equity return variances. All parameters are significant at the 1 percent level. The sums of α and β are 0.946 and 0.965, for bonds and stocks respectively. The likelihood profiling procedure selects an optimal number of lags of realized variances of respectively 8 quarters for bonds and 3 quarters

for stocks. The decay parameter ω_v^i is substantially larger than 1 for bonds, indicating that the weight attached to past realized bond return variances decreases rapidly with the number of lags. For equity volatility, in contrast, the parameter estimate is close to one, implying a relatively flat weighting function. The long-term components of stock and bond return volatility are plotted in Panels C and D of Table 4, respectively.

Panel B shows the estimation results for the MIDAS conditional stock and bond return correlations. The likelihood function peaks at 12 lags of quarterly realized correlations. The estimate of the decay parameter ω_c implies a rather rapidly decreasing weighting function. The a and b parameters are both highly significant, and their sum of 0.949 is significantly below 1. The long-term component of stock and bond return correlations is plotted in Panel A of Figure 4. The long-term correlation is around its unconditional value of 20 percent during most of the 1980s, to increase to levels up to 60 percent in the late 1990s. The stock-bond return correlations drops to slightly negative levels at the end of the 1990s, to become extremely negative (up to minus 60 percent) around 2003.

Panel A: Estimates for MIDAS Variance Model							
	α	β	μ	$\bar{m}$	θ	ω_v	L_v^i
Bond Volatilty							
Estim	0.847	0.099	0.0002	2.24E-06	0.016	3.314	8
St. Error	(0.022)	(0.015)	(0.0001)	(9.56E-06)	(0.004)	(0.830)	
Equity Volatility							
Estim	0.877	0.088	0.0006	3.55E-05	0.0084	1.044	3
St. Error	(0.013)	(0.014)	(0.0001)	(6.83E-06)	(0.004)	(0.034)	
Panel B: Estimates for MIDAS Correlation Model							
	a	b				ω_c	L_c
Estim	0.050	0.899				4.531	12
St. Error	(0.011)	(0.028)				(2.013)	

C   An IS Equation with Stochastic Risk Aversion

In a typical model with external habit à la Campbell and Cochrane (1999), the log of the real pricing kernel m_{t+1} is given by:

$$m_{t+1} = -\gamma \Delta c_{t+1} + \gamma \Delta q_{t+1} \tag{C-13}$$

where Δc_{t+1} is (logarithmic) consumption growth, and γ is the curvature parameter in the utility function (we ignore the discount factor β, without any consequence for the derivations to follow):

$$U(C_t) = \left(\frac{C_t - H_t}{1 - \gamma} \right)^{1-\gamma}. \tag{C-14}$$

We define q_t as $\log(Q_t)$, where $Q_t = \frac{C_t}{C_t - H_t}$ is the inverse of the surplus ratio. The dynamics of q_t are specified as follows:

$$q_t = \mu_q + \phi_{qq} q_{t-1} + \sigma_{qq} \sqrt{q_{t-1}} \varepsilon_t^q \tag{C-15}$$

where μ_q, ϕ_{qq}, and σ_{qq} are parameters, and ε_t^q a standard normal innovation process. Notice that ε_t^q is the sole source of conditional uncertainty in the stochastic risk aversion process. Note that the fact that we model q_t as a square root process makes the conditional variance of the pricing kernel depend positively on the level of Q_t.

The consumption process is defined as:

$$\Delta c_t = E_{t-1}[\Delta c_t] + \sigma_{cc} \sqrt{q_{t-1}} \left[\left(1 - \lambda^2 \right)^{\frac{1}{2}} \varepsilon_t^c + \lambda \varepsilon_t^q \right] \tag{C-16}$$

where σ_{cc} and λ are parameters, and ε_t^c a standard normal innovation process specific to the consumption growth process. We do not need to specify the conditional mean dynamics as we will solve for the interest rate as a function of expected consumption. Furhermore, we assume that ε_t^c and ε_t^q are jointly $N(0, I)$, so that:

$$Cov_{t-1}[\Delta c_t, q_t] = \lambda \sigma_{qq} \sigma_{cc} q_t. \tag{C-17}$$

It is easy to see that λ is the conditional correlation between 'risk aversion' and consumption growth. This correlation would be -1 in the Campbell-Cochrane setup, and is generally expected to be negative to induce counter-cyclical risk aversion.

In any log-normal model, we have:

$$r_t = -E_t[m_{t+1}] - \frac{1}{2} Var_t[m_{t+1}] \tag{C-18}$$

where r_t is the real interest rate. From (C-13), (C-15) and (C-16), it follows:

$$Var_t[m_{t+1}] = \gamma^2 q_t \left[\sigma_{cc}^2 + \sigma_{qq}^2 - 2\lambda \sigma_{qq} \sigma_{cc} \right]. \tag{C-19}$$

Substituting in (C-18), and using (C-13), we obtain:

$$
\begin{aligned}
r_t &= \gamma E_t[\Delta c_{t+1}] - \gamma \left[\mu_q + \left(\phi_{qq} - 1 \right) q_t \right] - \frac{\gamma^2}{2} q_t \left[\sigma_{cc}^2 + \sigma_{qq}^2 - 2\lambda \sigma_{qq} \sigma_{cc} \right] \\
&= -\gamma \mu_q + \gamma E_t[c_{t+1}] - \gamma c_t + \tilde{\eta} q_t
\end{aligned} \tag{C-20}
$$

with $\tilde{\eta} = -\gamma \left(\phi_{qq} - 1 \right) - \frac{\gamma^2}{2} \left[\sigma_{cc}^2 + \sigma_{qq}^2 - 2\lambda\sigma_{qq}\sigma_{cc} \right]$.

Solving for c_t, we find:

$$c_t = -\mu_q + E_t \left[c_{t+1} \right] - \frac{1}{\gamma} r_t + \eta q_t \qquad \text{(C-21)}$$

where $\eta = \frac{\tilde{\eta}}{\gamma} = \left(1 - \phi_{qq} \right) - \frac{\gamma}{2} \left[\sigma_{cc}^2 + \sigma_{qq}^2 - 2\lambda\sigma_{qq}\sigma_{cc} \right]$. If q_t is persistent, and for sufficiently high γ, and sufficiently negative λ, we likely have $\eta < 0$, but its sign is generally indeterminate.

To turn (C-21) into an IS equation, two more steps are required. First, we define

$$r_t = i_t - E_t \left[\pi_{t+1} \right] + \pi_r \qquad \text{(C-22)}$$

where π_r is a term arising from a Jensen's inequality. Consequently, we implicitly assume a constant inflation risk premium. Second, we must get from c_t to detrended output. One common way to do this is to assume:

$$y_t = c_t + g_t \qquad \text{(C-23)}$$

where g_t is a demand shock assumed *iid*, reflecting all the 'gaps' between output and consumption (such as government spending). If we linearly detrend, i.e. $\tilde{y}_t = y_t - \delta t$, we get:

$$\tilde{y}_t = \tilde{c}_t + g_t. \qquad \text{(C-24)}$$

Substituting (C-22) and (C-24) into (C-21), we obtain:

$$\tilde{y}_t = \alpha_{IS} + E_{t+1} \left[\tilde{y}_{t+1} \right] - \frac{1}{\gamma} \left(i_t - E_t \left[\pi_{t+1} \right] \right) + \eta q_t + F_t^y \qquad \text{(C-25)}$$

where α_{IS} reflects a collection of all the constant terms and $F_t^y = -g_t$.

D Specification Tests for the State Variable Model

D.1 Univariate Tests

Consider the following model, which encompasses our reduced-form state variable model:

$$y_t = \mu(S_t) + X_{t-1}\beta + \sigma(S_t)\varepsilon_t \tag{D-26}$$

For our purposes, it suffices to assume that S_t can take on two values. 1 or 2. Let π_{t-1} be the conditional probability of $S_t = 1$, and π_{t-1} the corresponding probability of $S_t = 2$. We can define the residuals for the model as:

$$q_t = y_t - X_{t-1}\beta - (\pi_{t-1}\mu_1 + (1 - \pi_{t-1})\mu_2) \tag{D-27}$$

where μ_1 and μ_2 are the means in respectively regime 1 and 2. The conditional variance V_{t-1} of q_t is:

$$V_{t-1} = \pi_{t-1}\sigma_1^2 + (1 - \pi_{t-1})\sigma_2^2 + \pi_{t-1}(1 - \pi_{t-1})(\mu_2 - \mu_1)^2 \tag{D-28}$$

where σ_1^2 and σ_2^2 are the variances in respectively regime 1 and 2. As calculated in Timmermann (2000), the conditional skewness Sk_{t-1} is given by:

$$Sk_{t-1} = \frac{\pi_{t-1}(1 - \pi_{t-1})(\mu_1 - \mu_2)\left(3(\sigma_1^2 - \sigma_2^2) + (1 - 2\pi_{t-1})(\mu_2 - \mu_1)^2\right)}{\left[\pi_{t-1}\sigma_1^2 + (1 - \pi_{t-1})\sigma_2^2 + \pi_{t-1}(1 - \pi_{t-1})(\mu_2 - \mu_1)^2\right]^{\frac{3}{2}}} \tag{D-29}$$

while the conditional Kurtosis K is equal to:

$$K_{t-1} = \frac{\pi_{t-1}\left\{3\sigma_1^2 + (\mu_1 - \mu)^4 + 6\sigma_1^2(\mu_1 - \mu)^2\right\} + (1 - \pi_{t-1})\left\{3\sigma_2^2 + (\mu_2 - \mu)^4 + 6\sigma_2^2(\mu_2 - \mu)^2\right\}}{\left[\pi_{t-1}\sigma_1^2 + (1 - \pi_{t-1})\sigma_2^2 + \pi_{t-1}(1 - \pi_{t-1})(\mu_2 - \mu_1)^2\right]^2} \tag{D-30}$$

To perform tests using GMM, we actually use the unconditional probabilities π and compute unconditional moments V, Sk, and K (We also performed tests using ex-ante probabilities, but then must use unscaled versions of Sk_{t-1} and K_{t-1}. Results were very similar).

We test for a zero mean and no second order correlation by testing whether or not b_1, b_2, and b_3 are zero in:

$$E[q_t] - b_1 = 0$$
$$E[(q_t - b_1)(q_{t-1} - b_1)] - b_2 = 0$$
$$E[(q_t - b_1)(q_{t-2} - b_1)] - b_3 = 0$$

Define $\tilde{q}_t$ as $(q_t - b_1)^2/V - 1$. We test for a well specified variance by testing whether or not b_4, b_5, and b_6 are equal to zero:

$$E[\tilde{q}_t] - b_4 = 0$$
$$E[\tilde{q}_t\tilde{q}_{t-1}] - b_5 = 0$$
$$E[\tilde{q}_t\tilde{q}_{t-2}] - b_6 = 0$$

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We test for excess skewness by testing whether or not b_7 is equal to zero in:

$$\frac{E\left[(q_t - b_1)^3\right]}{E\left[(q_t - b_1)^2\right]^{\frac{3}{2}}} - Sk - b_7 = 0$$

and for excess kurtosis by testing whether or not b_8 is equal to zero in:

$$\frac{E\left[(q_t - b_1)^4\right]}{E\left[(q_t - b_1)^2\right]^2} - K - b_8 = 0$$

We estimate b_1 to b_8 using GMM with a Newey-West (1987) weighting matrix with number of lags equal to 5. The tests for zero mean, unit variance, zero skewness, and zero excess kurtosis follow a $\chi^2(1)$ distribution, the tests for second order autocorrelation a $\chi^2(2)$ distribution. We also perform a small sample analysis of the test statistics. For each series, we use the estimated parameters from the state variable model to simulate a time-series of similar length as our sample. For 500 of such simulated time-series, we calculate the test statistics, and use the resulting distribution to derive empirical probability values.

D.2 Covariance Test

To investigate whether our state variable model adequately captures the covariance between the factor shocks, we test whether the following conditions hold:

$$E\left[q_{i,t}q_{j,t}\right] = 0, \text{ for } i = 1, .., N \ ; \ j = 1, .., N; i \neq j$$

where N represents the number of state variables. A joint test for the covariances between all factor shocks follows a χ^2 distribution with $N(N-1)/2$ degrees of freedom. We also test for each of the N variables whether its shocks have a zero covariance with all other factor shocks. This test follows a χ^2 distribution with 9 degrees of freedom. As for the univariate tests, we also perform a small sample analysis of the test statistics.

E Estimation Results State Variable Model

The table below reports the estimation results of the semi-structural state variable model discussed in Section 2.2.

This table reports the estimation results of the state variable model as outlined in Section 2.2. Panel A reports the drift parameters of the fundamental state variables. The parameters are expressed in percentages. The drift parameters for the state variables yd_t, πd_t and vp_t are allowed to switch according to respectively the regime variables S_t^y, S_t^π, and S_t^{vp}. Panel B reports the structural parameters of the state variable model. Part 1 shows the structural parameters of the IS, AS and risk aversion equation. Part 2 shows the structural parameters of the monetary policy rule. Part 3 shows the feedback parameters of the non-macro variables. Panel C reports the parameters of the gamma matrix for the non-macro variables. Panel D shows the regime-switching volatilities of the structural factors. The volatilities are expressed in percentages on a quarterly basis. Panel E reports the transition probabilities for the nine regime-switching variables The regime-switching variables are respectively denoted as S_t^y, S_t^π, S_t^q, S_t^i, S_t^{cg}, S_t^q, S_t^{sliq}, S_t^{bliq}, S_t^{vp}, and S_t^{mp}. The parameter P is the probability to stay in the high volatility state (active monetary policy state for S_t^{mp}). The parameter Q is the probability to stay in the low volatility state (accomodating monetary policy state S_t^{mp}). Standard errors are reported between parentheses.

Panel A: Constants

	y_t	π_t	q_t	i_t	cg_t	yd_t	πd_t	$sliq_t$	$bliq_t$	vp_t
Regime 1	-0.064	0.014	0.075	0.030	-0.510	2.447	-0.129	-1.710	-0.028	0.024
	(0.099)	(0.032)	(0.044)	(0.043)	(0.911)	(0.738)	(0.280)	(0.818)	(0.032)	(0.013)
Regime 2						1.405	-0.264			0.006
						(0.605)	(0.108)			(0.003)

Panel B: Structural Parameters

1. Output, Inflation and Risk Aversion Parameters

	μ	η	ϕ	δ	λ	ρ_q
Estim	0.665	0.028	0.100	0.379	0.098	0.982
St.error	(0.046)	(0.023)		(0.101)	(0.060)	(0.011)

2. Monetary Policy Parameters

	ρ	$\beta(S_t^{mp}=1)$	$\beta(S_t^{mp}=2)$	$\gamma(S_t^{mp}=1)$	$\gamma(S_t^{mp}=2)$
Estim	0.832	2.093	0.829	0.878	1.415
St.error	(0.038)	(0.353)	(0.197)	(0.363)	(0.321)

3. Feedback Parameters of Non-Macro Variables

	ρ_{cg}	ρ_{yd}	$\rho_{\pi d}$	ρ_{sliq}	ρ_{bliq}	ρ_{vp}
Estim	-0.130	0.921	0.968	0.652	0.536	0.553
St.error	(0.117)	(0.019)	(0.012)	(0.067)	(0.068)	(0.081)

Panel C: Gamma Matrix for Non-Macro Variables

	y_t	π_t	q_t	i_t	cg_t	yd_t	πd_t	$sliq_t$	$bliq_t$	vp_t
cg_t	0.431	-	0.254	-0.210	1	0	0	0	0	0
	(0.208)		(0.228)	(0.113)						
yd_t	-0.223	-0.173	-0.445	0.260	0	1	0	0	0	0
	(0.147)	(0.127)	(0.147)	(0.086)						
πd_t	0.249	-	-	0.128	0	0	1	0	0	0
	(0.077)			(0.078)						
$sliq_t$	0.371	0.143	0.387	-	0	0	0	1	0	0
	(0.159)	(0.148)	(0.197)							
$bliq_t$	-	-	-	-	0	0	0	0	1	0
vp_t	-	-0.004	-	0.003	0	0	0	0	0	1
		(0.002)		(0.002)						

Panel D: Volatility Parameters

	y_t	π_t	q_t	i_t	cg_t	yd_t	πd_t	$sliq_t$	$bliq_t$	vp_t
Regime 1	0.129	0.367	0.226	0.290	1.584	0.531	0.889	1.288	2.061	0.027
	(0.039)	(0.050)	(0.049)	(0.044)	(0.334)	(0.154)	(0.212)	(0.103)	(0.138)	(0.005)
Regime 2	0.075	0.214	0.061	0.070	0.411	0.265	0.208	0.188	0.138	0.007
	(0.011)	(0.025)	(0.010)	(0.011)	(0.061)	(0.029)	(0.025)	(0.035)	(0.016)	(0.001)

Panel E: Transition Probabilities

	S_t^y	S_t^π	S_t^q	S_t^i	S_t^{cg}	S_t^{sliq}	S_t^{bliq}	S_t^{vp}	S_t^{mp}	
P	0.907	0.850	0.904	0.849	0.585	0.969	0.964	0.550	0.908	HIGH
	(0.112)	(0.117)	(0.073)	(0.116)	(0.263)	(0.032)	(0.034)	(0.227)	(0.085)	
Q	0.975	0.929	0.954	0.911	0.884	0.921	0.966	0.907	0.934	LOW
	(0.027)	(0.056)	(0.038)	(0.070)	(0.087)	(0.117)	(0.036)	(0.064)	(0.058)	

Table 1: Specification Tests for State-Variable and Factor Model

Panel A reports the specification tests for the state variable model. The first line shows the Monte-Carlo p-values from a joint test on the standardized residuals of a zero mean, unit variance, no second order autocorrelation in first and second moments, zero skewness, and zero excess kurtosis. The second line reports Monte-Carlo p-values for a test of zero covariances of the factor shocks of one state variable with the factor shocks of all other state variables. Panel B reports specification tests for the factor model with constant factor exposures. We test for a zero mean and serial correlation (4 lags) in the level, both individually and jointly. The variance column performs the serial correlation test on squared residuals. 'Asym' is a joint test for the sign bias and the negative and positive sign biases of Engle and Ng (1993). The first line applies tests to the standardized residuals of the stock and bond residuals from our benchmark factor model; the second and third line to the standardized residuals of the stock and bond residuals from our benchmark factor model in which factors are generated using three different non-structural and semi-structural models, respectively. Panel C presents the model selection tests for the constant factor model with homoskedastic, state-dependent, and regime-switching variances is our benchmark case. The distance measures compute the mean absolute deviation of the model-implied correlations from respectively the MIDAS conditional correlations (column 1), the realized correlations (column 2), and the unconditional correlation (column 3). The correlation measures compute the unconditional correlation between our model-implied conditional correlations and respectively the MIDAS conditional correlations (column 4) and the realized correlations (column 5). We also report the constant and slope coefficient of a regression of respectively the realized correlations and the MIDAS conditional correlations on our model-implied conditional correlations. Estimation results are shown respectively in columns 6 and 7 (for realized correlations), and columns 8 and 9 (for MIDAS correlations). Standard errors of the regression coefficients are reported between parentheses. Panel C also shows the results of a Monte Carlo simulation applied to the factor model with constant factor exposures, as explained in Section 3.2. Distance and correlation measures which fall outside the 90% confidence interval are denoted by a *. The bottom line of the table shows the model selection tests for the MIDAS correlations relative to the realized correlations as an additional benchmark.

Panel A: Specification Tests State Variable Model

	y_t	π_t	q_t	i_t	cg_t	gd_t	πd_t	$sliq_t$	$bliq_t$	vp_t
Univariate Test	0.100	0.499	0.762	0.392	0.094	0.088	0.023	0.984	0.794	0.123
Covariance Test	0.045	0.183	0.301	0.243	0.073	0.063	0.343	0.419	0.489	0.120

Panel B: Specification Tests Factor Model (Constant Beta)

	Mean Zero	Mean AC	Mean Joint	Variance	Asym
Covariance	0.076	0.236	0.148		
Stocks	0.676	0.446	0.936	0.271	0.373
Bonds	0.316	0.483	0.675	0.388	0.992

Panel C: Model Selection Statistics

Model	Distance Measures MD	Distance Measures Real	Distance Measures Unc	Corr Measures MD	Corr Measures Real	Regression Real Const	Regression Real Slope	Regression MD Const	Regression MD Slope
Benchmark	0.186	0.243	0.066	0.394	0.389	-0.044	1.659	0.018	1.322
						(0.051)	(0.317)	(0.040)	(0.249)
1) Results 1000 Simulations									
Percentile 5%	*0.176*	*0.237*	*0.056*	*0.275*	*0.248*	*-0.117*	*1.039*	*-0.065*	*0.864*
Percentile 95%	*0.239*	*0.276*	*0.153*	*0.439*	*0.449*	*0.118*	*2.517*	*0.152*	*2.052*
2) Non-Structural State Variable Models									
Homoskedastic	0.265*	0.298*	0.159*	-	-	-	-	-	-
State-Dependent	0.245*	0.287*	0.145	0.160*	0.099*	0.164	0.594	0.170	0.754
						(0.036)	(0.479)	(0.028)	(0.374)
Regime-Switching	0.228	0.271	0.141	0.225*	0.258	0.152	0.702	0.179	0.483
						(0.028)	(0.212)	(0.022)	(0.168)
3) Alternative Structural State Variable Models									
Homoskedastic	0.228	0.272	0.096	-	-	-	-	-	-
State-Dependent	0.209	0.256	0.107	0.339	0.296	0.082	1.194	0.107	1.077
						(0.038)	(0.311)	(0.030)	(0.241)
4) MIDAS versus Realized Correlation									
	-	0.174	0.195	-	0.688	0.012	0.874	-	-

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Table 2: Model Selection Tests for Model-Implied Conditional Correlations

This table presents the model selection tests for alternative beta specifications in the factor model. We differentiate between 9 specifications for the factor exposures as explained in Section 2.3. Panel 1 reports results for the constant beta specification. Next, we consider three specification with state-dependent factor exposures, respectively using factor-specific state variables as instruments (Panel 2), risk aversion as instrument (Panel 3), and the variance premium as instrument (Panel 4). Consequently, we report results for three specifications with regime-switching factor exposures, respectively using the monetary policy regime variable S_t^{mp} as instrument (Panel 5), the own regime variable as instrument (Panel 6), and the illiquidity regime variables S_t^{sliq} and S_t^{bliq} as instruments (Panel 7). Finally, we have two specifications for the factor exposures combining regime variables with state variables, respectively using the monetary policy regime variable S_t^{mp} and risk aversion as instruments (Panel 8), and the monetary policy regime variable S_t^{mp} and variance premium as instruments (Panel 9). For each beta specification, we consider two cases, one without restrictions and one in which coefficients are forced to be zero if | t-stat | <1 in the unrestricted case. To facilitate comparison, the first panel also shows the results of the Monte Carlo simulation applied to the factor model with constant factor exposures (see Section 3.2). The distance measures compute the mean absolute deviation of the model-implied correlation from respectively the MIDAS conditional correlation (column 1), the realized quarterly correlation (column 2), and the unconditional correlation (column 3). The correlation measures compute the unconditional correlation between our model-implied conditional correlation and respectively the MIDAS (column 4) and realized (column 5) correlation. To test the predictive power of the various models for future correlations, we regress respectively the realized and the MIDAS conditional correlation on our model-implied conditional correlation. Estimation results are shown respectively in columns 6 and 7 (for realized correlation), and columns 8 and 9 (for MIDAS correlation) Standard errors of the regression coefficients are reported between parentheses. Distance and correlation measures which fall outside the 95% (or 5%) percentile are denoted by a *. The bottom line of the table shows the model selection tests for the MIDAS correlation relative to the realized correlation as an additional benchmark.

Factor Model	Distance Measures			Corr Measures		Regression Real		Regression MD	
	MD	Real	Unc	MD	Real	Const	Slope	Const	Slope
1) Constant Betas									
No Restrictions	0.186	0.243	0.066	0.394	0.389	-0.044	1.659	0.018	1.322
						(0.051)	(0.317)	(0.040)	(0.249)
Results 1000 Simulations									
S.E.	0.021	0.013	0.030	0.052	0.064	0.074	0.479	0.066	0.365
Percentile 5%	0.176	0.237	0.056	0.275	0.248	-0.117	1.039	-0.065	0.864
Percentile 95%	0.239	0.276	0.153	0.439	0.449	0.118	2.517	0.152	2.052
2) State-Dependent Betas - Original State-Dependent Model									
No Restrictions	0.215	0.255		0.416	0.409	0.134	1.022	0.160	0.819
						(0.026)	(0.184)	(0.020)	(0.144)
Restrictions	0.206	0.249		0.410	0.417	0.095	1.235	0.131	0.956
						(0.029)	(0.217)	(0.023)	(0.171)
3) State-Dependent Betas - Risk Aversion as Instrument									
No Restrictions	0.188	0.236*		0.452*	0.421	0.039	1.106	0.077	0.935
						(0.036)	(0.192)	(0.028)	(0.149)
Restrictions	0.193	0.242		0.475*	0.466*	-0.002	1.809	0.051	1.453
						(0.038)	(0.277)	(0.030)	(0.217)

Factor Model	Distance Measures			Corr Measures		Regression Real		Regression MD	
	MD	Real	Unc	MD	Real	Const	Slope	Const	Slope
4) State-Dependent Betas - Variance Premium as Instrument									
No Restrictions	0.187	0.234*	0.104	0.444*	0.463*	0.037	1.092	0.089	0.826
						(0.033)	(0.169)	(0.027)	(0.134)
Restrictions	0.180	0.231*	0.094	0.477*	0.495*	0.004	1.324	0.064	1.004
						(0.035)	(0.187)	(0.028)	(0.149)
5) Regime-Switching Betas - MP Regime Variable as Instrument									
No Restrictions	0.198	0.250	0.099	0.476*	0.439	0.001	1.927	0.043	1.646
						(0.039)	(0.318)	(0.030)	(0.245)
Restrictions	0.193	0.249	0.085	0.489*	0.439	-0.035	2.023	0.008	1.774
						(0.044)	(0.334)	(0.034)	(0.255)
6) Regime-Switching Betas - Own Regime Variable as Instrument									
No Restrictions	0.180	0.234*	0.074	0.448*	0.470*	-0.080	1.803	0.002	1.351
						(0.047)	(0.273)	(0.038)	(0.217)
Restrictions	0.180	0.235*	0.066	0.429	0.452*	-0.117	1.922	-0.024	1.434
						(0.055)	(0.305)	(0.044)	(0.244)
7) Regime-Switching Betas - Illiquidity Regime Variables as Instruments									
No Restrictions	0.180	0.235*	0.135	0.415	0.429	0.028	0.920	0.082	0.700
						(0.037)	(0.156)	(0.029)	(0.124)
Restrictions	0.181	0.235*	0.132	0.384	0.414	0.032	0.917	0.090	0.669
						(0.037)	(0.162)	(0.030)	(0.130)
8) State-Dependent RS Betas - MP Regime Var and Risk Aversion as Instruments									
No Restrictions	0.235	0.266	0.176	0.372	0.382	0.148	0.741	0.173	0.568
						(0.025)	(0.144)	(0.020)	(0.114)
Restrictions	0.207	0.239	0.145	0.410	0.415	0.084	0.741	0.122	0.576
						(0.031)	(0.131)	(0.024)	(0.103)
9) State-Dependent RS Betas - MP Regime Var and Variance Premium as Instruments									
No Restrictions	0.229	0.267	0.147	0.291	0.295	0.141	0.564	0.167	0.437
						(0.028)	(0.147)	(0.022)	(0.116)
Restrictions	0.197	0.241	0.109	0.416	0.404	0.059	0.956	0.099	0.776
						(0.034)	(0.175)	(0.027)	(0.137)
MIDAS versus Realized Correlation									
No Restrictions	-	0.174	0.195	-	0.688	0.012	0.874	-	-
						(0.024)	(0.074)		

Table 3: Estimation Results for the Factor Models

This table reports the estimation results for the factor model with constant exposures and the best performing factor model with time-varying exposures, which is the model with the variance premium as instrument. For the latter, we further differentiate between the model without and with restrictions. The estimated betas are shown with White (1980) heteroskedasticity-consistent standard errors between parentheses. The first column further shows the standard deviation of the different factors to facilitate the interpretation of the beta estimates. The mean and the standard deviation of our instrument (i.e. the variance premium) is respectively 2.126 and 1.724. Stock and bond returns are in quarterly percentages.

Factors	Std. Factors	Stock						Bond					
		Constant	Time-Varying		Time-Varying Restr			Constant	Time-Varying		Time-Varying Restr		
		β_0	β_0	β_1	β_0	β_1		β_0	β_0	β_1	β_0	β_1	
y	0.090	10.723 (6.482)	-1.253 (11.469)	9.400 (5.866)	-1.865 (11.143)	9.675 (5.629)		-4.029 (3.326)	-4.695 (5.834)	-0.448 (2.544)	-4.029 (3.326)	-
π	0.276	-4.113 (2.264)	-10.055 (3.320)	3.478 (1.685)	-10.219 (3.331)	3.548 (1.681)		-0.735 (0.995)	-2.429 (1.738)	0.681 (0.777)	-0.735 (0.995)	-
q	0.140	-12.458 (4.686)	-13.142 (10.321)	0.429 (3.922)	-11.992 (4.747)	-		-2.757 (3.065)	0.887 (7.996)	-1.560 (3.211)	-2.757 (3.065)	-
i	0.178	-1.882 (3.434)	-18.374 (7.187)	7.365 (3.739)	-19.229 (7.594)	7.724 (3.919)		-10.020 (1.818)	-9.550 (3.924)	-0.179 (1.515)	-10.020 (1.818)	-
cg	0.823	-0.734 (0.723)	-0.886 (1.674)	0.089 (0.682)	-0.690 (0.624)	-		-0.516 (0.529)	-1.378 (0.968)	0.244 (0.279)	-0.516 (0.529)	-
yd	0.338	-3.396 (2.136)	-7.009 (3.458)	2.156 (2.108)	-7.674 (3.264)	2.571 (1.883)		-0.241 (0.955)	-0.545 (1.767)	0.227 (0.721)	-0.241 (0.955)	-
πd	0.537	2.355 (1.236)	1.540 (2.202)	0.544 (1.157)	2.527 (1.336)	-		0.616 (0.727)	1.079 (1.136)	-0.197 (0.521)	0.616 (0.727)	-
$sliq$	1.131	-1.170 (0.635)	-1.261 (1.141)	-0.070 (0.608)	-1.389 (0.593)	-		-0.435 (0.300)	-0.955 (0.580)	0.244 (0.301)	-0.435 (0.300)	-
$bliq$	1.567	-1.724 (0.340)	-0.578 (0.719)	-0.620 (0.368)	-0.607 (0.675)	-0.632 (0.343)		-0.872 (0.234)	-1.003 (0.393)	0.033 (0.168)	-0.872 (0.234)	-
vp	1.205	-2.997 (0.545)	-5.026 (0.706)	0.543 (0.278)	-5.052 (0.592)	0.554 (0.166)		0.057 (0.250)	-0.039 (0.381)	0.058 (0.108)	0.057 (0.250)	-

Table 4: Economic Contributions for Model-Implied Conditional Correlations

This table shows the economic contributions of the different factors for the model-implied conditional correlations. Panel A shows the results for the factor model with constant exposures. Panel B shows the results for the best performing factor model with time-varying exposures, which is the model with the variance premium as instrument, imposing zero restrictions on parameters with absolute t-statistics below one. For both panels, we repeat the results of the model selection tests of the model including all factors (line 'Full Model'). Next, we compute the model selection test measures leaving out certain factors. We differentiate between leaving out macro factors and leaving out non-macro factors. For each measure we compute the deterioration of the restricted model relative to the full model. For the distance measures, we compute the deterioration as 100 times (distance restricted model - distance full model)/distance full model. For the correlation measures, we report the difference between the correlation measure for the full model and the correlation measure for the restricted model, multiplied by 100. All numbers are expressed in percentages. In Panel A, a * means that the restricted model performs significantly worse than the full model with constant betas, based on the simulation results in Table 2. MD stands for MIDAS; Real for realized.

Factor Model	Distance Measures			Corr Measures	
	MD	Real	Unc	MD	Real
Panel A: Constant Betas					
Full Model	*0.186*	*0.243*	*0.066*	*0.394*	*0.389*
minus macro var	7.6	2.8	32.3	1.9	-0.3
minus y_t, π_t, i_t	6.4	2.6	25.2	2.6	0.5
minus q_t	8.8	3.3	35.4	-0.6	-2.2
minus non-macro var	33.9*	18.8*	113.7	30.7*	37.4*
minus cg_t	2.3	1.0	5.6	1.1	0.4
minus $sliq_t, bliq_t$	30.6*	16.8*	106.7	21.9*	26.7*
minus $yd_t, \pi d_t$	6.8	2.6	23.2	-1.5	-2.3
minus vp_t	2.9	1.6	3.7	5.8	5.3
Panel B: Time-Varying Betas					
Full Model	*0.180*	*0.231*	*0.094*	*0.477*	*0.495*
minus macro var	10.5	7.4	-4.6	6.3	6.8
minus y_t, π_t, i_t	8.1	5.7	-0.6	10.7	10.5
minus q_t	8.7	4.6	12.9	1.8	1.2
minus non-macro var	36.1	22.4	66.9	23.6	24.4
minus cg_t	2.1	1.0	2.3	1.0	0.5
minus $sliq_t, bliq_t$	31.9	19.6	58.6	20.0	21.7
minus $yd_t, \pi d_t$	2.5	1.4	-11.6	-3.7	-4.5
minus vp_t	1.0	0.7	-3.7	2.8	2.0

Table 5: Correlation Decomposition

This table shows results of the model selection tests for the model-implied conditional correlations imposing respectively the MIDAS data-implied covariance, stock volatility and bond volatility. Panel A shows the results for the factor model with constant exposures. Panel B shows the results for the best performing factor model with time-varying exposures, which is the model with the variance premium as instrument. The first line of each model repeats the results of the model selection tests for the benchmark model without imposing the MIDAS moments. The bottom line of the table shows the model selection tests for the MIDAS correlations relative to the realized correlations. MD stands for MIDAS; Real for realized.

Factor Model	Distance Measures		Corr Measures		Regression Real		Regression MD	
	MD	Real	MD	Real	Const	Slope	Const	Slope
Panel A: Constant Betas								
Benchmark	0.186	0.243	0.394	0.389	-0.044	1.659	0.018	1.322
					(0.051)	(0.317)	(0.040)	(0.249)
MIDAS Covariance	0.104*	0.200*	0.938*	0.647*	0.083	0.945	0.081	1.078
					(0.022)	(0.090)	(0.008)	(0.032)
MIDAS Stock Volatility	0.175*	0.235*	0.409	0.397	-0.031	1.131	0.025	0.917
					(0.048)	(0.210)	(0.038)	(0.165)
MIDAS Bond Volatility	0.214	0.267	0.204	0.214	0.097	0.489	0.136	0.366
					(0.044)	(0.180)	(0.034)	(0.142)
Panel B: Time-Varying Betas								
Benchmark	0.180	0.231	0.477	0.495	0.004	1.324	0.064	1.004
					(0.035)	(0.187)	(0.028)	(0.149)
MIDAS Covariance	0.111	0.205	0.943	0.642	0.084	1.003	0.081	1.161
					(0.022)	(0.097)	(0.008)	(0.033)
MIDAS Stock Volatility	0.192	0.232	0.439	0.466	0.008	0.831	0.070	0.617
					(0.037)	(0.127)	(0.029)	(0.102)
MIDAS Bond Volatility	0.221	0.263	0.312	0.332	0.084	0.535	0.127	0.396
					(0.035)	(0.123)	(0.028)	(0.097)
MIDAS All	-	0.174	-	0.688	0.012	0.874		
					(0.024)	(0.074)		

Table 6: Economic Contributions for Stock and Bond Return Covariance

This table shows the economic contributions of different factors for the model-implied conditional covariance. Panel A shows the results for the factor model with constant exposures. Panel B shows the results for the best performing factor model with time-varying exposures, which is the model with the variance premium as instrument. For both panels, we repeat the results of the model selection tests of the factor model including all factors (line 'Full Model'). Next, we compute the model selection tests leaving out certain factors. We differentiate between leaving out macro factors and leaving out non-macro factors. For each measure we compute the deterioration of the restricted model relative to the full model. For the distance measures, we compute the deterioration as 100 times (distance restricted model - distance full model)/distance full model. For the correlation measures, we report the difference between the correlation measure for the full model and the correlation measure for the restricted model, in percentages. The distance measures for the full model are multiplied by 100 to improve readability. In Panel A, * means that the restricted model performs significantly worse than the full model, based on the simulation results in Table 2. In Panel B, a † in the line for the full model with time-varying betas means that the model performs significantly better than the full model with constant betas. MD stands for MIDAS; Real for realized.

Factor Model	Distance Measures			Corr Measures	
	MD	Real	Unc	MD	Real
Panel A: Constant Betas					
Full Model	0.227	0.305	0.124	0.415	0.420
minus macro var	1.5	-4.5	3.4	1.7	-0.7
minus y_t, π_t, i_t	2.2	-3.3	5.6	2.1	0.2
minus q_t	0.0	-5.9	4.7	0.4	-0.1
minus non-macro var	12.1	-4.7	73.0	21.4*	22.8*
minus cg_t	0.2	-1.8	0.4	1.1	-0.6
minus $sliq_t, bliq_t$	9.3	-5.5	66.4	17.8*	15.4*
minus $yd_t, \pi d_t$	-1.8	-5.7	-2.7	-1.0	-0.8
minus vp_t	0.7	0.2	2.9	2.5	2.3
Panel B: Time-Varying Betas					
Full Model	0.258	0.302	0.180	0.416	0.517†
minus macro var	-10.6	-3.9	-26.0	0.8	6.4
minus y_t, π_t, i_t	-17.5	-2.0	-8.3	-5.0	10.8
minus q_t	2.3	-1.3	4.2	8.0	4.6
minus non-macro var	10.9	3.7	38.3	33.2	25.5
minus cg_t	-1.4	-2.4	-0.1	0.4	0.2
minus $sliq_t, bliq_t$	7.1	1.1	32.4	29.0	21.8
minus $yd_t, \pi d_t$	-15.8	-8.1	-18.3	-9.9	-3.0
minus vp_t	-5.6	-1.2	-3.6	-3.2	0.4
MIDAS Benchmark	-	0.251	0.252	-	0.615

Table 7: Economic Contributions for Stock Volatility

This table shows the economic contributions of different factors for stock volatility. Panel A shows the results for the factor model with constant exposures. Panel B shows the results for the best performing factor model with time-varying exposures, which is the model with the variance premium as instrument. For both panels, we repeat the results for the model selection tests of the factor model including all factors (line 'Full Model'). Next, we compute the model selection tests leaving out certain factors. We differentiate between leaving out macro factors and leaving out non-macro factors. For each measure we compute the deterioration of the restricted model relative to the full model. For the distance measures, we compute the deterioration as 100 times (distance restricted model - distance full model)/distance full model. For the correlation measures, we report the difference between the full model correlation measure for the full model and the correlation measure for the restricted model, in percent. The distance measures of the full model are expressed in standard deviation units. The last colum reports the adjusted R-squared from the factor model and the absolute deterioration in adjusted R-squared when factors are eliminated. In Panel A, a * means that the restricted model performs significantly worse than the full model with constant betas, based on the simulation results in Table 2. MD stands for MIDAS; Real for realized.

Factor Model	Distance Measures			Corr Measures		R^2 Measure
	MD	Real	Unc	MD	Real	
Panel A: Constant Betas						
Full Model	*0.028*	*0.038*	*0.063*	*0.451*	*0.248*	*0.270*
minus macro var	31.9	3.6	14.9	-14.4	-10.5	4.6%
minus y_t, π_t, i_t	14.3	1.0	7.1	-7.2	-4.8	2.0%
minus q_t	21.5	2.2	10.3	-8.3	-6.6	3.0%
minus non-macro var	227.1*	120.8*	100.6*	46.0*	31.0*	22.8%
minus cg_t	8.1	1.6	4.1	0.5	0.3	-0.1%
minus $sliq_t, bliq_t$	86.9*	25.0*	39.0*	-17.0	-9.3	9.6%
minus $yd_t, \pi d_t$	24.7	2.9	11.7	-8.7	-5.8	0.8%
minus vp_t	80.7*	36.2*	36.3*	47.5*	30.9*	14.3%
Panel B: Time-Varying Betas						
Full Model	*0.024*	*0.046*	*0.041*	*0.541*	*0.187*	*0.347*
minus macro var	-0.9	-11.3	23.8	16.6	7.4	8.2%
minus y_t, π_t, i_t	-17.5	-12.7	24.7	20.9	10.0	7.1%
minus q_t	-6.1	-7.0	10.9	-4.0	-4.0	2.7%
minus non-macro var	212.3	54.7	175.0	27.1	18.3	29.8%
minus cg_t	-2.1	-1.6	3.3	0.3	0.3	-0.1%
minus $sliq_t, bliq_t$	8.0	-15.7	48.7	0.2	-0.1	10.4%
minus $yd_t, \pi d_t$	-11.2	-5.5	16.3	-2.1	-0.2	2.1%
minus vp_t	62.9	-1.3	86.2	1.0	-0.7	20.7%
MIDAS Benchmark	-	0.034	0.042	-	0.481	0.662

Table 8: Economic Contributions for Bond Volatility

This table shows the economic contributions of different factors for bond volatility. Panel A shows the results for the factor model with constant exposures. We repeat the results of the model selection tests of the factor model including all factors (line 'Full Model'). Next, we compute the model selection tests leaving out certain factors. We differentiate between leaving out macro factors and leaving out non-macro factors. For each measure we compute the deterioration of the restricted model relative to the full model. For the distance measures, we compute the deterioration as 100 times (distance restricted model - distance full model)/distance full model. For the correlation measures, we report the difference between the correlation measure for the full model and the correlation measure for the restricted model, in percent. The distance measures of the full model are expressed in standard deviation units. The last colum reports the adjusted R-squared from the factor model and the absolute deterioration in adjusted R-squared when factors are eliminated. In Panel A, * means that the restricted model performs significantly worse than the full model with constant betas, based on the simulation results in Table 2.

| Factor Model | Distance Measures | | | Corr Measures | | R^2 Measure |
	MD	Real	Unc	MD	Real	
Panel A: Constant Betas						
Full Model	*0.030*	*0.030*	*0.042*	*0.208*	*0.183*	0.307
minus macro var	25.4*	23.4*	22.7*	14.3*	16.9*	12.7
minus y_t, π_t, i_t	25.7*	23.6*	23.0*	14.3*	16.9*	13.2
minus q_t	5.2	3.9	6.1	0.9	1.0	0.2
minus non-macro var	4.3	-0.4	7.4	-7.8	-9.8	7.5
minus cg_t	0.4	0.5	0.3	-0.6	-0.6	0.3
minus $sliq_t, bliq_t$	3.5	-1.2	6.9	-7.8	-9.9	8.5
minus $yd_t, \pi d_t$	2.6	1.6	3.6	-0.1	-0.1	-0.5
minus vp_t	-0.2	-0.2	-0.2	0.1	0.0	-0.5
MIDAS Benchmark	-	0.019	0.024	-	0.651	-0.722

Figure 1: Realized and Conditional MIDAS Stock-Bond Return Correlation

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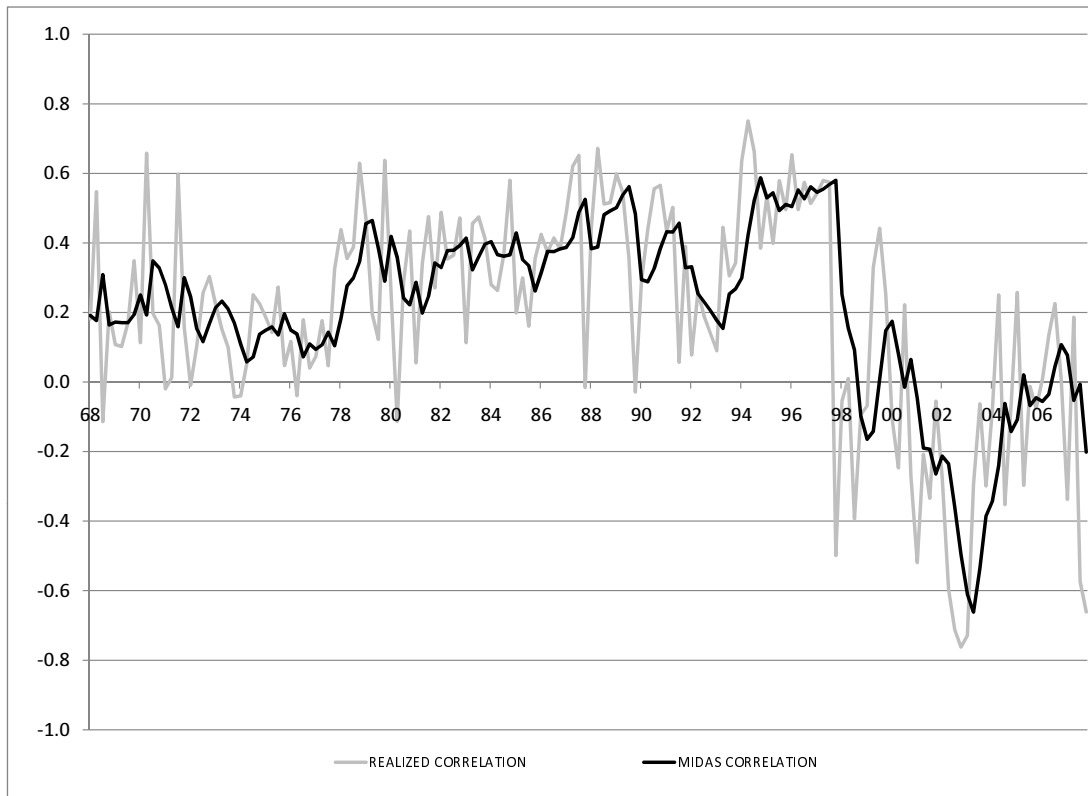
This figure graphs realized quarterly correlations measured using daily returns, and the data-implied conditional correlation based on the bivariate DCC-MIDAS model of Colacito, Engle, and Ghysels (2009). See Appendix B for the technical details about this model.
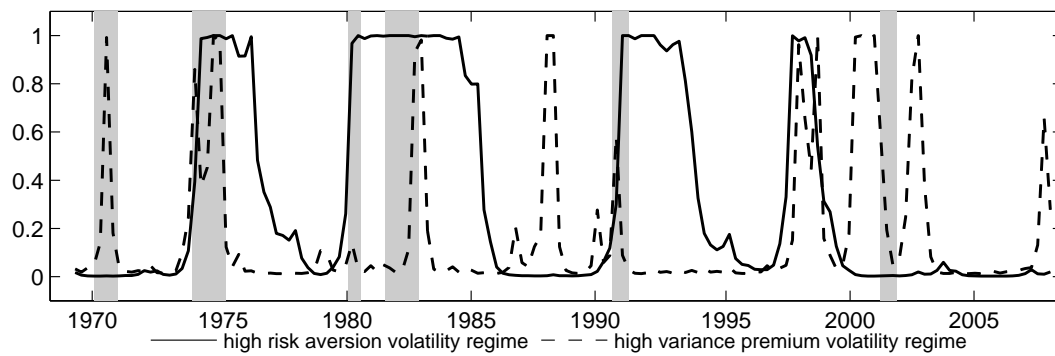
Figure 2: Smoothed Probabilities of Regimes
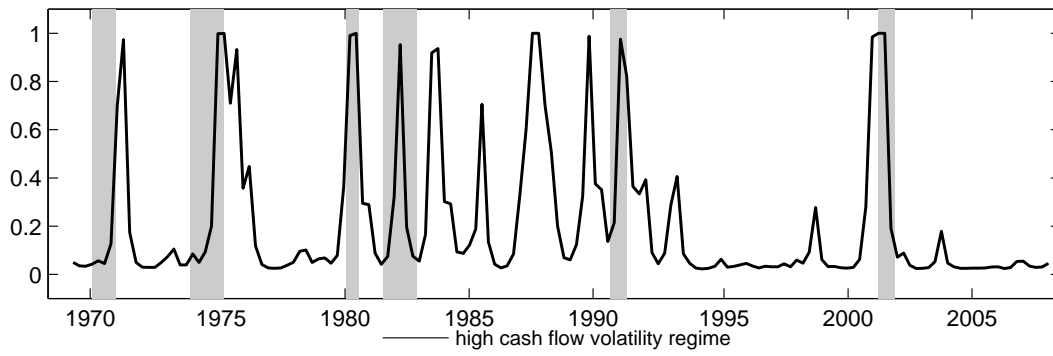
Panel A: Output and Inflation Volatility Regime

——— high output volatility regime − − − high inflation volatility regime
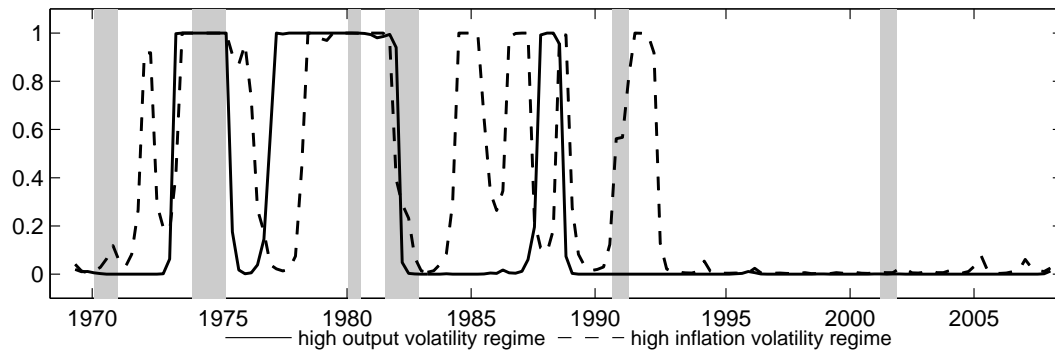
Panel B: Interest Rate Volatility and Monetary Policy Regime

——— high interest rate volatility regime − − − active monetary policy regime
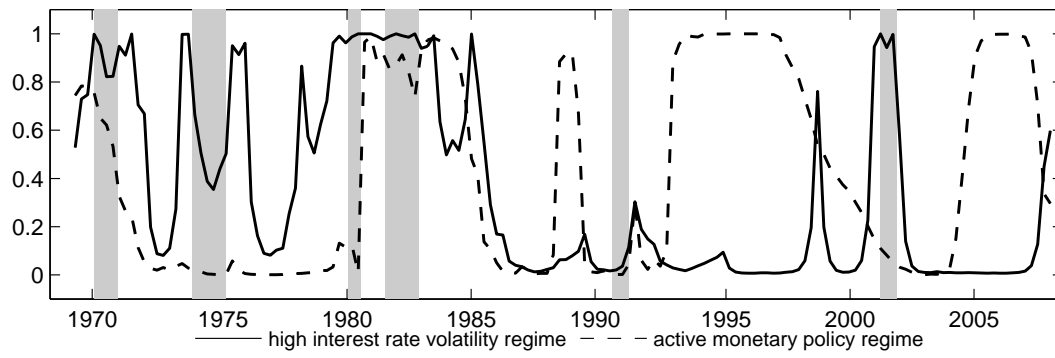
Panel C: Risk Aversion Volatility and Variance Premium Regime

——— high risk aversion volatility regime − − − high variance premium volatility regime

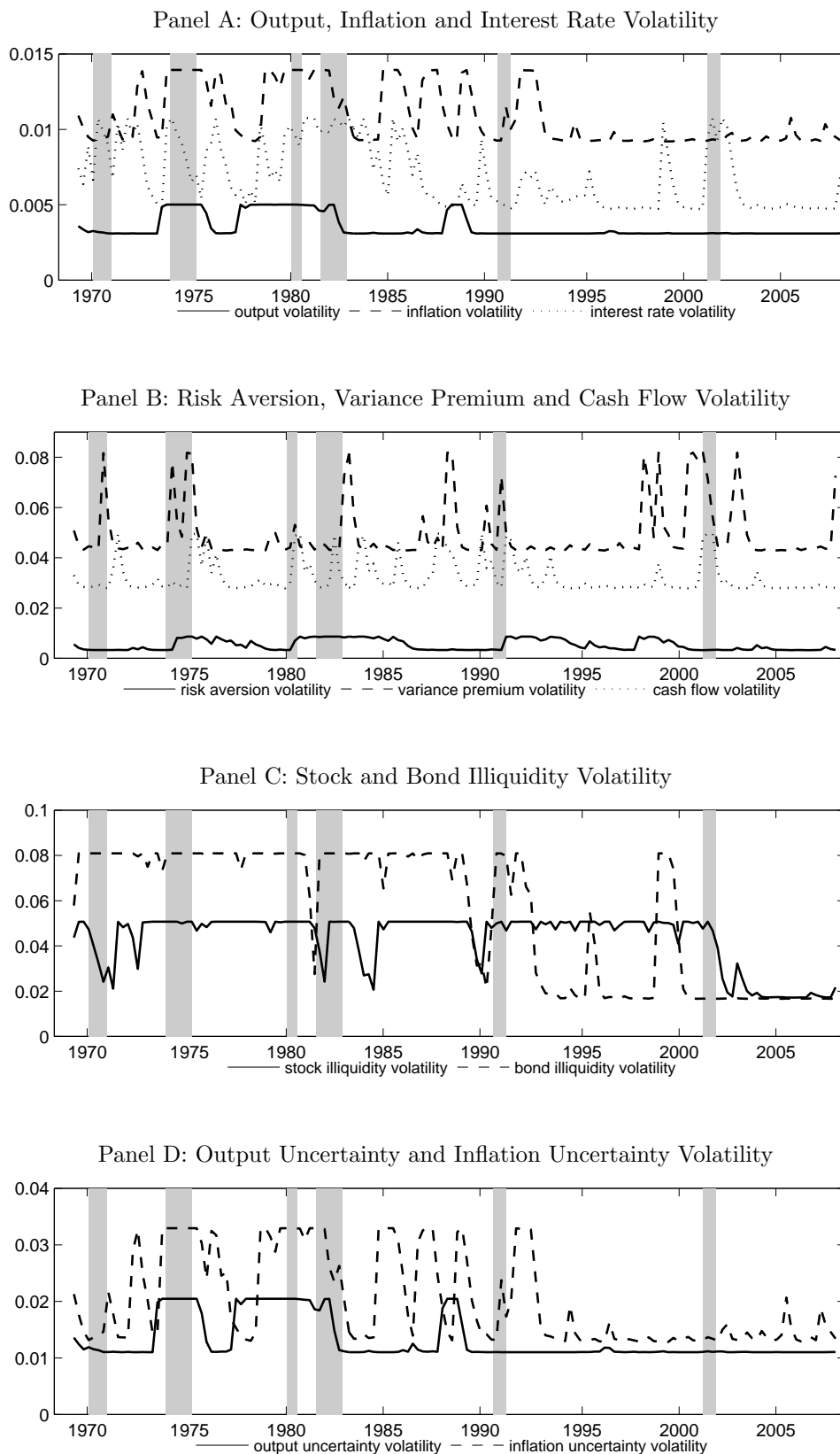Panel D: Cash Flow Volatility Regime

Panel E: Stock and Bond Illiquidity Regime
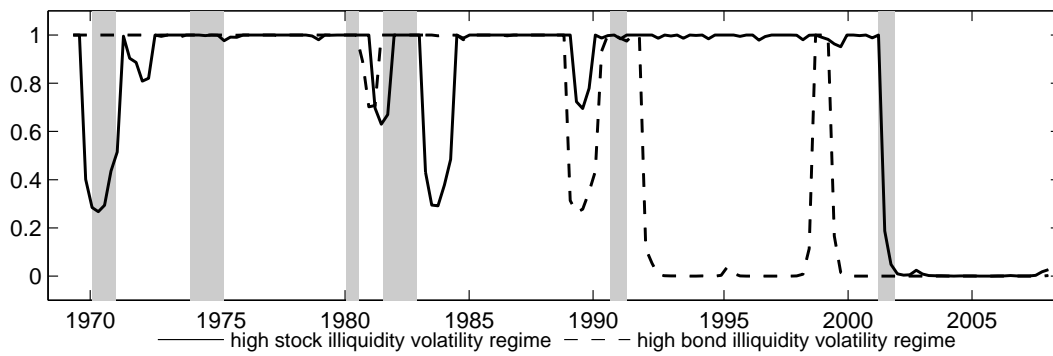
This figure shows the smoothed probabilities of the nine independent regimes in our state variable model. The different regimes are defined in Section 2.2.3 for the active monetary policy regime and Section 2.2.5 for the different volatility regimes. Panel A shows the smoothed probability of a high output gap and inflation volatility regime. Panel B shows the smoothed probability of a high interest rate volatility regime and the smoothed probability of an active monetary policy regime in which the FED aggressively stabilizes the price level. Panel C shows the smoothed probability of a high risk aversion volatility regime and of a high variance premium regime. Panel D shows the smoothed probability of a high cash flow growth shock volatility regime. Finally Panel E shows the smooted probability of a high stock and bond illiquidity regime. NBER recessions are shaded gray.

Figure 3: Volatility of the Factors

Panel A: Output, Inflation and Interest Rate Volatility

—— output volatility — — — inflation volatility ⋯⋯ interest rate volatility

Panel B: Risk Aversion, Variance Premium and Cash Flow Volatility

—— risk aversion volatility — — — variance premium volatility ⋯⋯ cash flow volatility

Panel C: Stock and Bond Illiquidity Volatility

—— stock illiquidity volatility — — — bond illiquidity volatility

Panel D: Output Uncertainty and Inflation Uncertainty Volatility

—— output uncertainty volatility — — — inflation uncertainty volatility

This figure shows the conditional volatilities (annualized) of the various factors identified in our state variable model. Panel A shows respectively output, inflation and interest rate volatility. Panel B shows respectively risk aversion, the variance premium and cash flow volatility. Panel C shows stock and bond illiquidity volatility. Finally Panel D shows output uncertainty and inflation uncertainty volatility. NBER recessions are shaded gray.
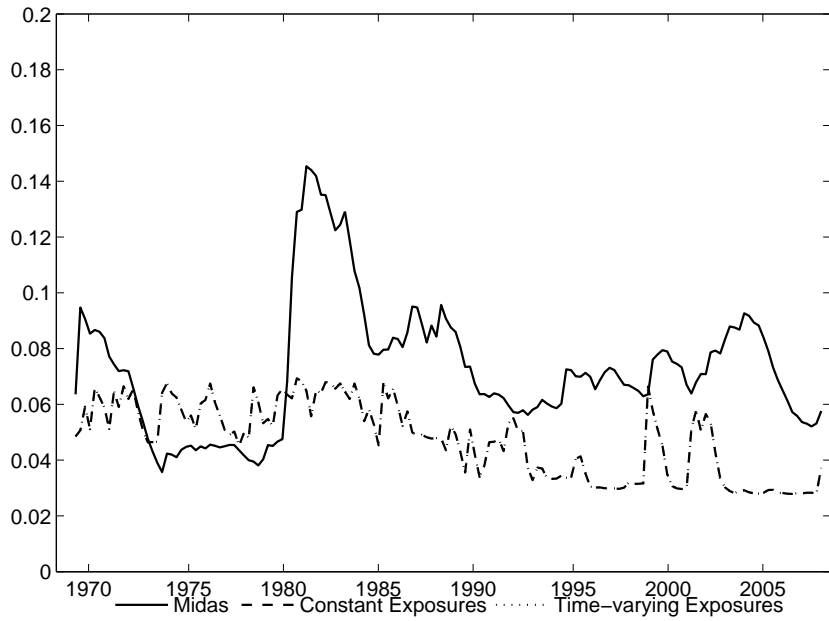
Figure 4: Data-Implied and Model-Implied Moments

Panel A: Correlation
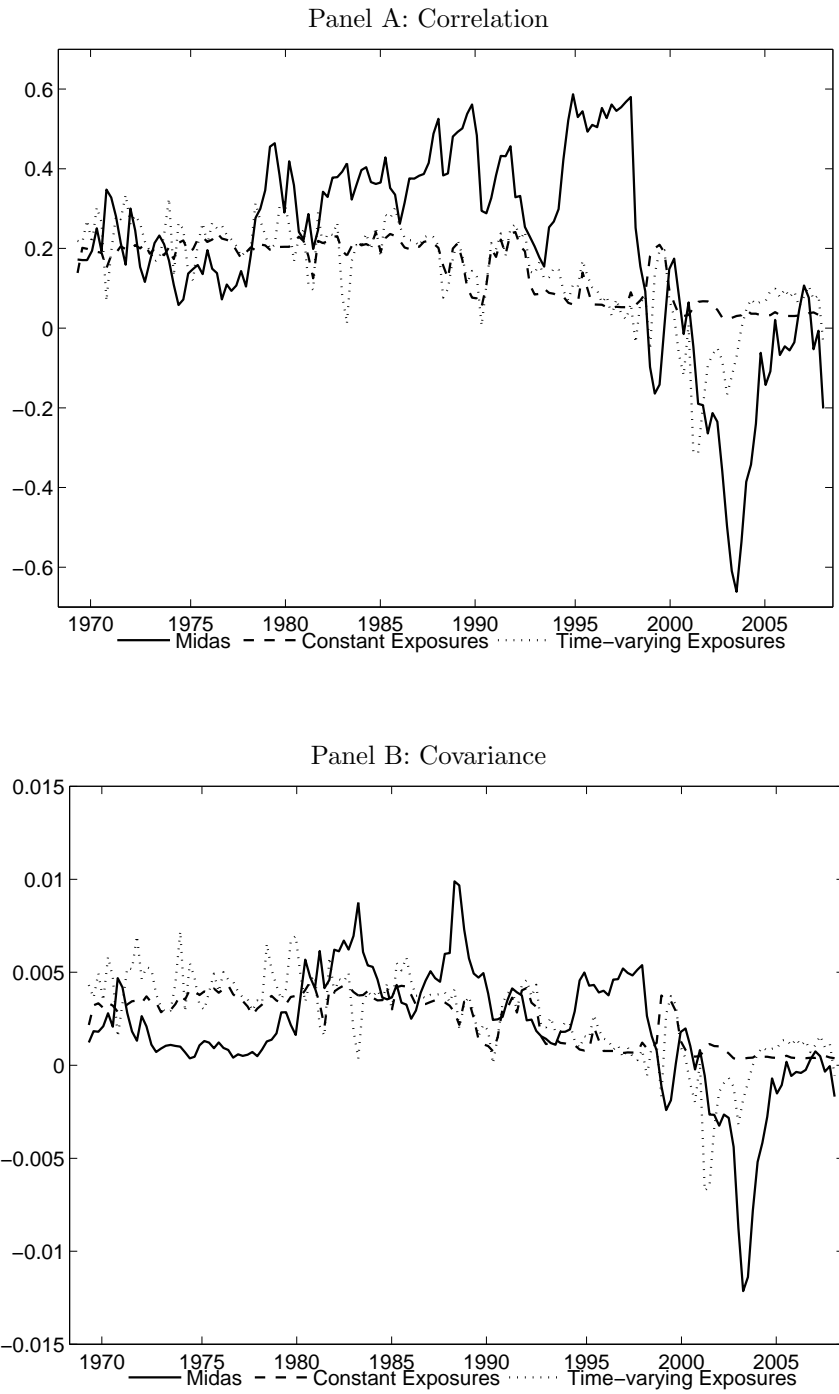
Panel B: Covariance

Panel C: Stock Volatility
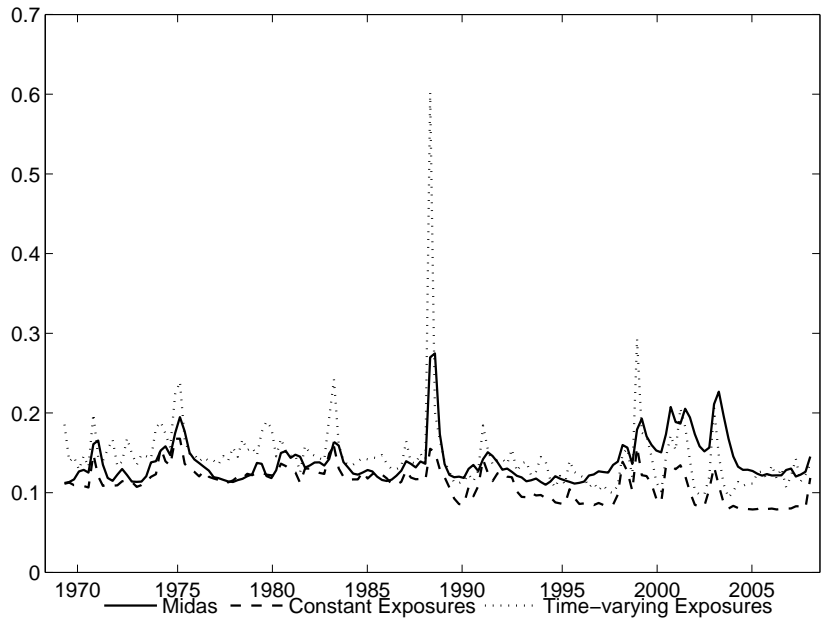
Panel D: Bond Volatility

This figure plots the model-implied correlations (Panel A), the model-implied covariances (Panel B), the model-implied stock volatility (Panel C) and the model-implied bond volatility (Panel D), for both the factor model with constant exposures and the best performing factor model with time-varying exposures, which is the model with the variance premium as instrument. In addition, we plot the data-implied conditional moments based on the MIDAS model described in Apppendix B.