RISK ALLOCATION, DEBT FUELED EXPANSION AND FINANCIAL CRISIS

Paul Beaudry
Amartya Lahiri

Risk Allocation, Debt Fueled Expansion and Financial Crisis
Paul Beaudry and Amartya Lahiri
NBER Working Paper No. 15110
June 2009
JEL No. E3,E4,G01

ABSTRACT

In this paper we discuss how several macroeconomic features of the 2001-2009 period may have resulted from a process in which financial markets were trying to allocate risk between heterogeneous agents when productive investment opportunities are scarce. We begin by showing how heterogeneity in terms of risk tolerance can cause financial markets to propagate transitory shocks and induce higher output volatility, albeit with a higher mean. We then show how this simple heterogeneous agent framework can explain an expansion driven by the growth in consumer debt, and why the equilibrium path of such an economy is likely fragile. In particular, we demonstrate that the emergence of a small amount of asymmetric information can make the economy susceptible to changes in expectations that can induce large reversals of financial flows, the freezing of assets and a recession that can persist despite high productivity.

Paul Beaudry
Department of Economics
University of British Columbia
Room 997-1873 East Mall
Vancouver, B.C. Canada V6T 1Z1
and NBER
paulbe@interchange.ubc.ca

Amartya Lahiri
Department of Economics
University of British Columbia
Vancouver, BC V6T 1Z1
alahiri@interchange.ubc.ca

1   Introduction

What caused the expansion of 2001-2007, and how is it related to the subsequent financial and economic contraction? There are currently many proposed explanations. For example, common explanations involve a combination of an overly expansive monetary policy, a saving glut driven by foreign investors, irrational exuberance involving house prices, opaque financial securities, fraudulent mortgage brokers, and excessive risk taking by poorly supervised bank executives. While all of these forces may have been present, they all suggest that the 2001-2007 expansion was built mainly on errors. Accordingly, the subsequent crisis is viewed by many as the natural and unavoidable outcome of a market economy gone mad.

In this paper we argue that to understand this episode it may be helpful to initially abstract from the many errors and abuses that arose during this period, and instead begin by identifying the nature of the allocation problem the economy was trying to solve. By adopting such a focus, it should become possible to examine whether the exploitation of objective gains from trade made possible through new financial relationships may explain the credit driven expansion that subsequently lead to a financial crisis. In our view, such an approach has the potential of isolating the more fundamental causes behind this episode and thereby may help identify solutions or remedies that can better prevent its recurrence and accelerate recovery. Since any explanation for developments occurring during this period will rely on capturing important macroeconomic interactions between real and financial factors, one of the aims of this paper is also to offer a new and simple framework in which these issues can be discussed.

There are several features which we believe are key to understanding this period. First, recall that the early years of the 21st century were particular from a historical standpoint on many dimensions. The recession of 2001 resulted in large part from a negative re-evaluation of the investment opportunities associated with information technology. In other words, this was a period where profitable investment opportunities in new technologies seemed much less abundant than in the late nineties. However, contrary to most recessions, profits in general did not decline and profits in the financial industry actually rose. Part of this rise in profits reflected the very strong productivity growth which was not matched by wage growth. Hence, the 2001-2007 period can be viewed as a period where profits where high but where productive investment opportunities were viewed as quite scarce. At the same time, one of the objectives of the financial industry during this period was to promote financial innovation that favored the allocation of risk toward those most willing and capable to bear it. This suggests that heterogeneity in terms of risk tolerance may be important for understanding the financial developments observed over the period. These observations lead us to ask the following set of questions: How does an economy with aggregate risk and heterogeneous agents (in terms of risk tolerance) adjust to a period of high profits/productivity with rather limited investment opportunities? Can the adjustment to such a situation explain features observed over the 2001-2007 period? Why may such an adjustment lead to a crisis, as seen post-2007?

Our approach to answering these questions is to begin by presenting a model of systematic risk allocation. In the model, agents differ in terms of risk tolerance. We use the model to

examine how an economy may adjust to good profit/productivity shocks in the absence of abundant investment opportunities. However before using the model to interpret recent observations, we use it to better understand what macro-risk allocation implies. For example, we show how and why the efficient allocation of systemic risk can give rise to macroeconomic outcomes that are more volatile in comparison to those arising in less developed financial markets. In fact, we show that efficient financial arrangements can cause simple transitory disturbances to be both amplified and propagated over time, thereby explaining how a developed financial system may be a contributor to macroeconomic fluctuations as opposed to being a stabilizing force.

The model we present has a set of key assumptions which allows it to capture real-financial interactions. In particular, we model employment decisions as inherently risky investment, as we assume that work must be executed before its production value is fully known. This simple modification implies that employment decisions are affected by the price of risk in the economy, and therefore is related to the allocation of risk. Hence, in the model employment decisions affect agents' willingness to hold financial assets, which in turn affects the price of risk which feeds back on employment and output. It is worth stressing that the interaction highlighted here is quite different from that found in much of the macro-financial literature which usually emphasizes how collateral constraints affect a firm's access to credit. Our approach, on the other hand, emphasizes the determinants of the supply of "risky credit" to firms (that is, the supply of credit for a firm with given risk characteristics). In fact, there is a key state variable that emerges in our setting which can be referred to as the amount of "risk capital" available in the economy. As more risk capital is offered on the market by risk tolerant individuals, risk premia decrease and employment expands. The opposite happens if this risk capital is depleted. The strong role of risk capital in determining macro-economic performance is one of the key feature of the model.

What happens in this economy when there is a good productivity outcome but no new investment possibilities? In the model, a good productivity outcome tends to disproportionately benefit risk tolerant individuals since they hold leveraged positions in risky assets and hence are the residual claimants to the income flows generated by these assets. This gives rise to a situation where high profitability in the financial sector induces an abundant supply of risky capital since risk tolerant individuals use their new wealth to take greater risky asset positions and thereby implicitly insure others. The situation is akin to a savings glut, but instead of being driven by an abundant supply of risk free credit from a foreign source, as is commonly discussed in the literature, it corresponds to a strong domestic supply of risk taking capital. We show how this supply of risk taking capital can simultaneously lead to an increase in consumer lending, increased employment, decrease in risk premia and increased leverage. All of these were observed over the period 2001-2007.

So if our narrative is relevant and the credit expansion of the early 2000s reflected in part the economy's response to a series of high profit outcomes, why did it end in a financial crisis and a severe recession? Here we argue that while these financial developments may have had many benefits, they did have an important drawback: they greatly increased the vulnerability of the economy to adverse selection and expectation driven swings in asset flows. We illustrate this within the model by showing how a small amount of asymmetric

information regarding the quality of consumer debt can lead to a drastic increase in the risk premium due to a freezing up of a certain asset class. This precipitates a large contraction in economic activity which can persist for a while despite productivity staying high.

While we do not claim that the adverse selection problem formalized in the model is the only source behind the recession that started in 2008, we believe it illustrates how the recycling of financial profits can lead to a financial system that is very vulnerable. For this reason, we view our narrative as telling a cautionary tale. While the allocation of risk to those most willing to bear it has the potential of favoring economic activity, it can, at the same time, also render the system sensitive to small perturbations, especially if those events cause risk tolerant individuals to withdraw their funds from the financing of risky endeavors. In fact, the collapse in our model leads to employment outcomes that are worse than those that would be observed under financial autarky.

The model also offers a new perspective on the expansion of securitization over this period. Instead of viewing this expansion as an exogenous event, the forces we highlight suggest that the economy as a whole was in need of financial innovation in order to help risk averse agents borrow from risk tolerant individuals who were flush with funds and searching to take on risky positions. Moreover, these risk tolerant individuals wanted these borrowing arrangements to be marketable and hopefully secure, so they could use them to support highly leveraged positions. Hence, in our view, the securitization process was not a driving force behind this period but was instead a natural outcome of the underlying economic problem the economy was trying to solve.[1]

Our work is related to two strands of the literature on finance and macroeconomics. The first is the work on credit market imperfections and the financial accelerator.[2] This literature formalizes environments in which the effects of bad shocks are amplified through financial frictions. Our model also generates a propagation mechanism for real shocks but the mechanism works differently as it does not rely on affecting the borrowing constraints of firms through the pricing of their assets or the value of their internal wealth. Instead our model emphasizes variation in the aggregate supply side of risk capital available on the market.

Our work is also related to the literature on behavioral finance which explores the allocations that arise in financial markets that are influenced by investor sentiment. Particularly relevant is a recent paper by Shleifer and Vishny (2009) who show how such markets can lead to procycical investment behavior as well as provide a propagation mechanism for shocks. Also related is the work of Acharya et al (2009) and Diamond and Rajan (2009) who investigate environments in which credit markets are prone to freezing up. It is worth reiterating that, aside from the differences in details of the mechanism at work, our work differs from most of the related work on this topic in that we try to provide an explanation for the entire

---

[1]In our model, house and land price appreciations do not play any role. However, the model can be easily extended to include such a mechanism. The main result of adding price appreciation is to amplify the effects we discuss.

[2]For work along this line, see Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (2000), Carlstrom and Fuerst (1997), Kiyotaki and Moore (1997) and Williamson (1987). Korinek (2009) explores the (sub) optimality and policy implications of the typical investment allocations that arise in these models.

boom-bust cycle of 2002-08 rather than just for the financial crisis episode in 2008.

The paper is structured as follows. In the next section we present some evidence which motivates our modeling choices and our narrative for the 2001-2007 period. Section 3 provides a short summary of the mechanisms at play in the model that we formalize. Section 4 presents a simple one-period model with heterogeneous agents and aggregate risk to highlight the role of insurance and the interaction of the real and financial sectors in the model. Section 5 extends the one-period model to a dynamic (overlapping generations) setting and derives the time paths for debt, risky assets, risk premia and production. In Section 6 we introduce default risk and asymmetric information into the model and show how the resulting adverse selection problem can cause multiple equilibria and expectation driven swings from optimistic equilibria with full risk pooling and low risk premia to pessimistic equilibria with little/no asset trade, low employment and high risk premia. Section 7 discusses the sensitivity of our main results to the availability of productive investment opportunities. The last section concludes. Proofs of all the Propositions are contained in the Appendix.

2 Empirical Patterns

Our narrative of the evolution of the US economy between 2001-2007 relies on a number of facts which we outline in this section. First, as is well known, this period was marked by a rapid build up of debt and leverage ratios. This can be seen in Figure 1 which shows that the Debt to GDP ratios of households, the private sector as well as the domestic financial sector all went up during this period.

Next, Figure 2 shows that the corporate profit rate rose across both the financial and non-financial sectors to historic highs during this period. Moreover, the high profit rate went hand-in-hand with high labor productivity, as Figure 3 makes clear.

With labor productivity being high and profits also rising, did firms start investing more during this period? Figure 4 shows the rate of non-residential investment during this period. Contrary to what one would expect to see during a high productivity phase, investment rates stayed relatively low. In fact, non-residential investment during this phase was lower than during the recession of 1981. This low investment rate is particularly surprising given the low real rates of interest during this period, see Figure 5, and the low spread on corporate debt, see Figure 6. To further place this low investment rate into perspective, Figure 7 reports the ratio of profits to non-residential investment. As can be seen in the figure, the size of profits have historically been about equal to 80% of non-residential investment. However, over the 2002-2007 period they represented more that 100%. These observations lead us to view the 2002-2007 period as one where productive investments appeared rather scarce even though there were substantial profits available for reinvestment.[3]

---

[3]A much more comprehensive and detailed survey of developments and events during the crisis at the end of this period can be found in Brunnermeier (2009).
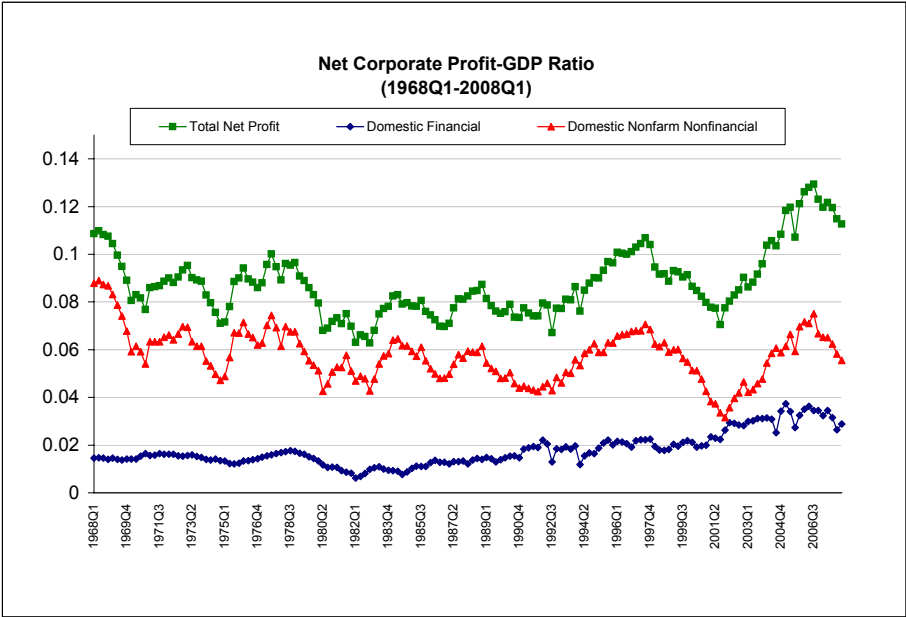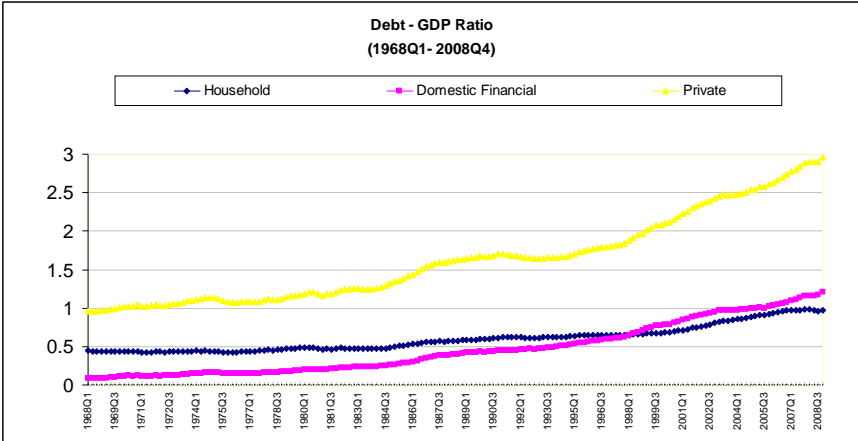
Figure 1: Debt to GDP Ratio

Figure 2: Profits to GDP Ratio
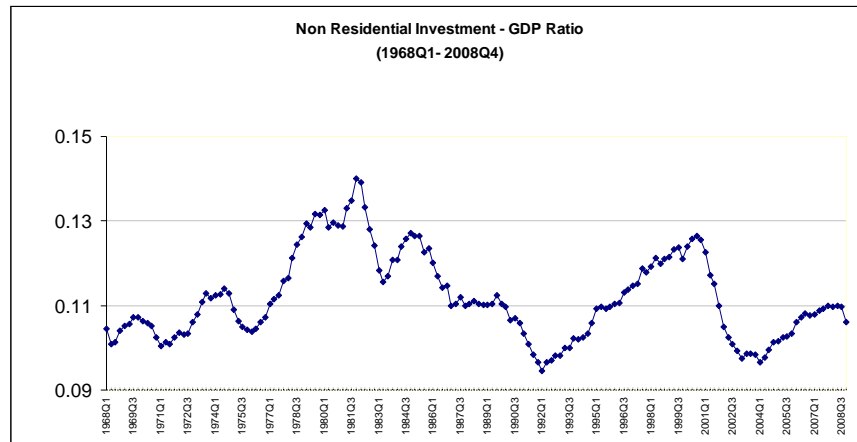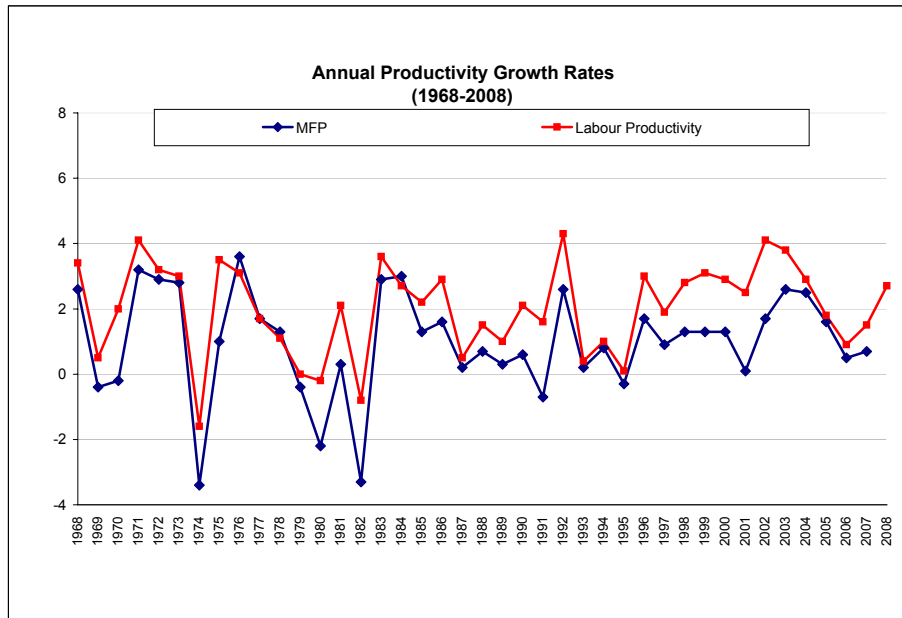
Figure 3: Productivity

Figure 4: Non-Residential Investment

7

Figure 5: Real Interest rate

Figure 6: Risk Premium
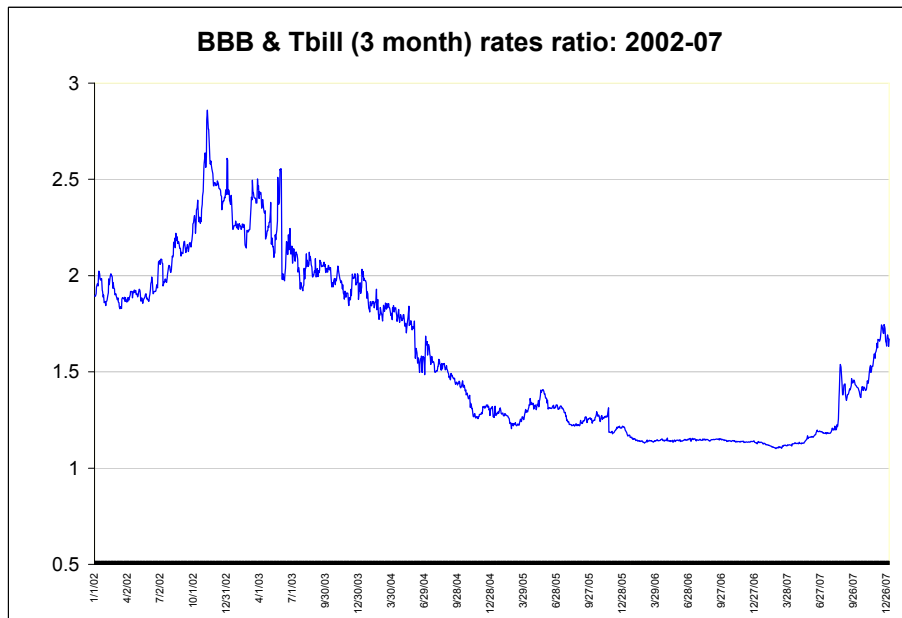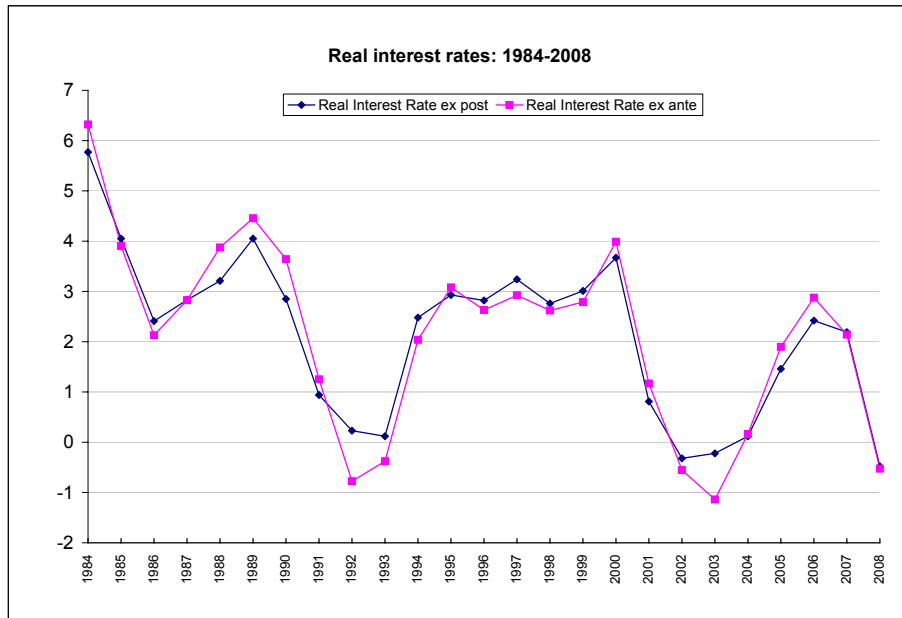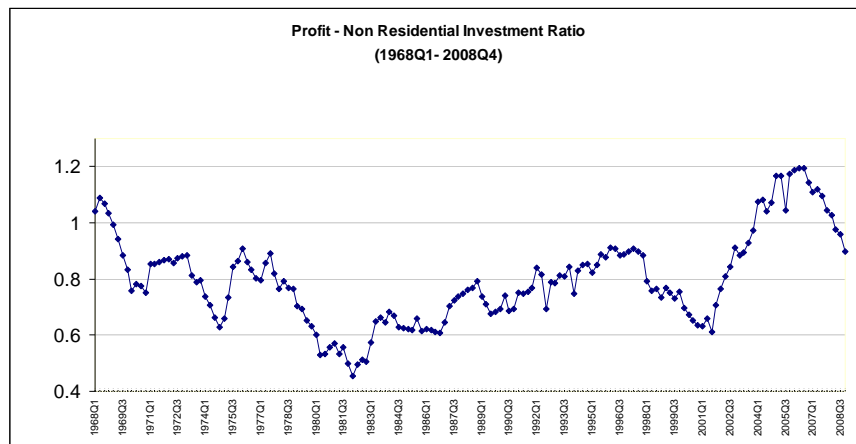
Figure 7: Profits to Non-Residential Investment

Profit - Non Residential Investment Ratio
(1968Q1- 2008Q4)

3   Summary of mechanisms

Before formally presenting the model, it is helpful to first outline the main assumptions and mechanism at play. In particular, since the model is very stylized, it is important to emphasize what elements and behavior the model is attempting to capture. Recall that the objective of the model is to illustrate how and when an expansion driven by increased consumer debt and financial leverage can arise, and why such an expansion is likely to be vulnerable to sudden reversals. The two main features of the model are: (a) agents with different degrees of risk tolerance interact to determine asset prices and, most importantly, the risk premium; (b) the risk premium influences economic activity. The risk premium directly influences economic activity in the model due to the assumption that the actual value of output is unknown at the time of hiring. Heterogeneity in risk preference is introduced by allowing for two types of individuals – risk neutral and risk averse.[4] Risk neutral individuals, who we call financiers, will be endowed with preferences that induce them to act like a dynasty which maximizes end of period wealth and consumes a fix proportion of its resources every period. Risk averse households, who we refer to as workers, act optimally to determine their borrowing, labor supply and asset holdings.

The key market in the model is the market for risk capital as it jointly determines the risk premium and employment. The demand for risk capital comes from firms and depends on technological characteristics. The supply of risk capital comes from the different agents (households and financiers) with different degrees of risk tolerance. Financiers use their holdings of household debt to finance their supply of risk capital to the market. The dynamics in the model are driven by variations in the supply of risk taking capital. Risk neutral financiers are willing to take on risk in search of high returns and are only constrained in their risk taking by a solvency requirement. Since these financiers will hold very risky portfolios, their capacity to bear risk will be sensitive to firms' profit outcomes. When profits are high, financiers gain disproportionately. This increases the resources available to them for lending to households today which, in turn, allows financiers to acquire more risky assets tomorrow. The additional investment in risky assets induces a decrease in the risk premium and raises employment. In contrast, when a bad profit outcome arises, these financiers lose substantially. This reverses the previous mechanism and leads to a rise in the risk premium as well as a fall in employment.

The responses highlighted above are all efficient and there is no sense of any financial "crisis" in what we described. To get at that aspect we then introduce default risk and private information. In particular we show that once households can default on their debt and financiers know the default rate on their own portfolios, the market for risky credit is subject to adverse selection due to the familiar "lemons" problem. In particular, financiers with better quality household debt on their books may have an incentive to withhold these from the risk capital market. Such an environment is prone to multiple equilibria which are

---

[4]There is a growing literature which analyzes implications of heterogeneity in terms of risk tolerance. Most of this literature focuses on asset price. See for example Chan and Kogan (2002), or Garleanu and Panageas (2008). Our paper differs from the literature by focusing on interactions between real and financial factors when agents differ in terms of risk tolerance.

sensitive to expectations. Hence, such an economy could easily move from an "optimistic" equilibrium in which agents behave as if there were no adverse selection to a "pessimistic" equilibrium in which the market for risk capital completely freezes up as all agents expect only the worst quality assets to be sold on this market. From the perspective of the model then, a financial crisis is a switch from an "optimistic" equilibrium to a "pessimistic" equilibrium.

4   A one period model

In order to flesh out the key mechanisms at play, we start by presenting a simple one-period model of a closed economy with two types of agents – workers and financiers – who differ in their degree of risk aversion. The framework is designed to illustrate the macroeconomic properties of an economy which aims to efficiently allocate aggregate risk between agents with different degrees of risk tolerance. While the one period structure precludes any discussion of dynamics, it highlights the nature of the interaction between the real and financial sides of the model. We should note that the one-period model takes as given the inherited asset positions of workers and financiers. These positions will be rendered endogenous in the next section.

Consider a one-period economy that produces output using the technology

$$y = Al \tag{1}$$

where A denotes labor productivity. Productivity is stochastic and follows an i.i.d. binomial process

$$A = \begin{cases} 1 & \text{with probablity } q \\ \theta & \text{with probability } 1-q \end{cases}$$

where $\theta < 1$. Productivity is the sole source of uncertainty in this economy.

Events unfold in this economy as follows: at the beginning of the period asset markets as well as labor markets open. Stocks and bonds are traded in asset markets while employment and wage contracts are struck in labor markets. Thereafter the productivity shock is revealed whereupon output is produced, claims are settled and agents consume.

4.1   Firms

Firms hire labor to produce output using the technology given by equation (1). Labor hiring decisions and wage payments need to be made before observing the productivity shock for the period. Without loss of generality, we can assume that firms finance this by issuing shares (risky claims) to output produced by labor. Each share pays one unit of output in the good state (when $A = 1$) and θ units of output in the bad state. Thus, firms maximize

$$p^s S - wl$$

subject to the solvency constraint $S \leq l$, where p^s denotes the price of stock, S denotes the number of shares sold, w is the wage rate and l is the level of employment. If $w > p^s$, then

firms will not hire, if $w < p^s$, firms will want to hire an infinite amount. Hence, market equilibrium will require[5]:

$$p^s = w.$$

4.2 Workers

The economy is inhabited by a continuum of identical worker-households of measure one. Workers have one unit of labor time that they can allocate to work or leisure. Utility of workers is given by

$$V^w = u\left(c^w + g\left(1-l\right)\right), \quad u' >, u'' < 0, g' > 0, g'' \leq 0,$$

where c^w denotes consumption of workers and l is labor supplied to the market.

We assume that each worker inherits a stock of debt d which needs to be paid off before the period ends. Thus, workers face the state-contingent budget constraints

$$p^s s^w + p^b b^w = wl$$

$$c_g^w = s^w + b^w - d$$

$$c_b^w = \theta s^w + b^w - d$$

where c_g^w denotes consumption by workers in the good state when $A = 1$ while c_b^w denotes consumption in the bad state when $A = \theta$. s^w is stock holdings of workers while b^w denote their bond holdings. The first equation gives the budget constraint that workers face in their asset market transactions while the last two equations are the budget constraints that dictate their state contingent consumption allocations.

It is useful at this stage to note that the (percentage) expected excess return to holding stock versus bonds is given by $\frac{p^b E[A]}{p^s} - 1$. In the rest of the paper we will refer to $\frac{p^b}{p^s}$ as the risk premium as it is an affine transformation of the excess return.

The workers utility maximization problem leads to two first order conditions:

$$\frac{p^s}{p^b} = g'\left(1-l\right),$$

$$\frac{p^s}{p^b} = \frac{qu'\left(c_g^w + g\left(1-l\right)\right) + \theta\left(1-q\right)u'\left(c_b^w + g\left(1-l\right)\right)}{qu'\left(c_g^w + g\left(1-l\right)\right) + \left(1-q\right)u'\left(c_b^w + g\left(1-l\right)\right)}.$$

In this economy hiring labor is a risky activity since labor productivity is unknown prior to the employment decision. The market price of this risk over the risk-free alternative is $\frac{p^s}{p^b}$. The first condition shows that at an optimum workers equate the marginal utility from safe,

---

[5]This extremely simple relationship between the price of stock and the wage results from the assumption that there are constant returns to labor in the model. If we allowed for decreasing returns to labor, which is an easy generalization, the price of stock would vary more than wages which would be more empiricaly reasonable.

non-market work with $\frac{p^s}{p^b}$ which is the cost of withholding this labor from the risky market activity. The second condition dictates the optimal mix of stocks and bonds in the mature worker's portfolio. Note that worker behavior only depends on the ratio $\frac{p^s}{p^b}$, not on each price individually.

4.3 Financiers

Financiers are also identical and of unit mass. The representative financier maximizes utility V^F where

$$V^F = c^F$$

where c^F denotes financier consumption. Hence, the linearity of financier preferences implies that they are risk neutral which is a key source of difference relative to workers.

Financiers are born with an initial endowment of assets d (which is the inherited debt of workers) which becomes due back to them at the end of the period. Thus, the state contingent budget constraints of the financiers are given by

$$p^s s^F + p^b b^F = 0$$

$$c_g^F = s^F + b^F + d$$

$$c_b^F = \theta s^F + b^F + d$$

where the first equation is the constraint on asset market transactions of financiers while the last two are the state-contingent constraints on their consumption flows. Note that s^F denotes stock holdings of financiers while b^F denotes their bond holdings. Combining these constraints yields the following restriction on all admissible state-contingent consumption allocations:

$$\left(\frac{p^s}{p^b} - \theta\right) c_g^F + \left(1 - \frac{p^s}{p^b}\right) c_b^F = (1 - \theta) d. \tag{2}$$

Equation (2) makes clear that the effective price of wealth in the good state is $\frac{p^s}{p^b} - \theta$ while the corresponding price in the bad state is $1 - \frac{p^s}{p^b}$. Lastly, admissible consumption allocations must satisfy financier solvency which requires that $c_g^F \geq 0$ and $c_b^F \geq 0$.

Financiers have a portfolio problem to solve in order to maximize their expected end of period consumption. The solution for this optimization problem dictates that at an optimum we must have

$$\frac{p^s}{p^b} \leq q + (1 - q)\theta = E[A].$$

If this condition is not satisfied then the marginal utility from c_g^F would be negative which would lead to a violation of the solvency restrictions $c_g^F \geq 0$. Intuitively, if the condition is violated then the risk premium is so low that the relative cost of taking equity positions is greater than the expected returns from stocks which would induce financiers to take short positions in stocks.

Given the linearity of financier preferences, there are two potential cases to be analyzed.

Case 1: $\frac{p^s}{p^b} < E[A]$. In this case the financier will choose an asset portfolio which results in $c_b^F = 0$ and $c_g^F = \frac{(1-\theta)d}{\frac{p^s}{p^b} - \theta}$ where the latter expression follows directly from equation (2). Note that from the financier budget constraint in the bad state (see above), $c_b^F = 0$ implies that $b^F = -\theta s^F - d < 0$, i.e., financiers issue bonds to finance the purchase of a leveraged portfolio of stock.

Case 2. If $\frac{p^s}{p^b} = E[A]$, then the financier is indifferent between all consumptions pairs that satisfy equation (2).

4.4   Equilibrium

The equilibrium in this one period economy arises when goods and asset markets clear. The state-contingent goods market clearing conditions are

$$c_g^w + c_g^F = l$$

$$c_b^w + c_b^F = \theta l$$

The corresponding asset market clearing conditions are

$$b^w + b^F = 0$$

$$s^w + s^F = S$$

Definition: *A Walrasian Equilibrium for this economy consists of prices* $\left\{ w, \frac{p^b}{p^s} \right\}$ *and allocations* $\left\{ c_g^w, c_b^w, c_g^F, c_b^F, l \right\}$ *such that the allocations are optimal given prices and all markets clear at those prices.*

Throughout the paper we shall maintain the following two assumptions:

Assumption 1: $E[A] = q + (1 - q)\theta > g'(1)$

Assumption 2: $g'(0) = \infty$.

In order for the equilibrium of this economy to potentially involve positive levels of employment, it is necessary to assume that the technology is sufficiently productive relative to workers' value of time. This is captured by Assumption 1. Assumption 2 will ensure that equilibrium levels of employment are interior $(0 < l < 1)$. This will eliminate the need to cover potential corner solutions where $l = 1$.

4.5   Autarky

Before examining equilibrium outcomes for the economy with both workers and financiers, it is useful to first examine how this economy would behave if there were no financiers. In this case, the equilibrium characterization of worker behavior is very simple. The Walrasian

14

equilibrium is characterized by a time-invariant level of employment denoted by l^a, and a time invariant risk premium $\frac{p_a^b}{p_a^s}$. These two variables are determined by the equations:

$$\frac{p_a^s}{p_a^b} = g'\left(1 - l^a\right)$$

$$\frac{p_a^s}{p_a^b} = \frac{qu'\left(l^a + g\left(1 - l^a\right)\right) + \theta\left(1 - q\right)u'\left(\theta l^a + g\left(1 - l^a\right)\right)}{qu'\left(l^a + g\left(1 - l^a\right)\right) + \left(1 - q\right)u'\left(\theta l^a + g\left(1 - l^a\right)\right)}$$

where we have substituted out the state contingent consumption levels of workers by using the market clearing conditions. Output for this economy is given by Al^a.

The autarkic case makes clear the role that financiers play in this one period economy. Financiers provide insurance to workers and firms by acquiring claims to risky production. This margin is missing in the autarkic case with no financiers. This then leads to the question: How does the presence of individuals with different levels of risk tolerance affect the behavior of an economy when financial markets can act to allocate aggregate/systemic risk between them?

4.6 Comparative Statics

The market clearing conditions along with the household optimality conditions define a system of two equations that describe the effect of the state variable debt d on equilibrium employment l and the risk premium p^b/p^s. We summarize these effects in two propositions. Proposition 1 gives a description of the relations between employment and debt, while Proposition 2 expresses the relationship for the risk premium.

Proposition 1: The level of employment is a continuous and weakly increasing function of the debt level d. This function, which we denote by $l = \phi^l(d)$, is strictly increasing in d when $d \in \left(0, \tilde{d}\right)$, $\tilde{d} > 0$, and is constant for all $d \geq \tilde{d}$. Moreover, $\phi^l(d) > l^a$ for all $d > 0$. The cutoff level of debt $\tilde{d}$ is defined by the solution to the following equations, where $\tilde{l}$ is implicitly defined by $q + \theta\left(1 - q\right) = g'(1 - \tilde{l})$.

$$g'(1 - \tilde{l}) = \frac{qu'\left(\tilde{l} - \left(\frac{(1-\theta)\tilde{d}}{g'(1-\tilde{l})-\theta}\right) + g(1 - \tilde{l})\right) + (1 - q)\theta u'(\theta\tilde{l} + g(1 - \tilde{l}))}{qu'\left(\tilde{l} - \left(\frac{(1-\theta)\tilde{d}}{g'(1-\tilde{l})-\theta}\right) + g(1 - \tilde{l})\right) + (1 - q)u'\left(\theta\tilde{l} + g(1 - \tilde{l})\right)}.$$

Proposition 2: The risk premium is a continuous and weakly decreasing function of the debt level d. This function, which we denoted by $\frac{p^b}{p^s} = \phi^p(d)$, is strictly decreasing in d when $d \in \left(0, \tilde{d}\right)$, $\tilde{d} > 0$, and is constant for all $d \geq \tilde{d}$. Moreover, $\phi^p(d) < \frac{p_a^b}{p_a^s}$ for all $d > 0$.

Propositions 1 and 2 indicate that the equilibrium of our model has the property that when households are more indebted, employment is higher and the risk premium is lower. The

easiest way to understand these results is by first noticing that debt reflects the initial wealth position of financiers. Since financiers are risk neutral, their initial wealth can be thought of as the amount of funds available for taking on risk. For this reason, it is useful to think of d as the amount of risk capital available to financiers for providing insurance to firms. When financiers have more wealth, they are able to take a more leveraged position in risky claims (stock). This increases the price of risky claims and, consequently, decreases the risk premium. The decline in the risk premium pushes firms to increase employment. As long as the expected value of investing in risky assets is greater than the value of investing in safe assets, financiers will choose a portfolio mix between bonds and stock which leads to positive net pay-outs only in the good state. This has the effect of maximizing the downward pressure on the risk premium and favoring employment. It is only when the risk premium disappears, i.e., when $d > \tilde{d}$ (and hence $\frac{p^b}{p^s} = q + (1-q)\theta$), that debt no longer has a positive effect on employment. Any increase in debt beyond this threshold level is used solely to build a portfolio that keeps the risk premium at zero.

5   The model with dynamics

We now extend the model to a multi-period setting in order to illustrate the dynamics that would likely arise in the type of economy we explored in the previous section. While previously we had just started off financiers and worker-households with some inherited asset positions (specifically, their debt positions), we now endogeneize those asset positions and track out the implied equilibrium dynamics.

We start by focusing on a pure consumption-loan version of the model; that is, a version where there are no available productive investment opportunities. We choose this version since it allows us to capture in an extreme form the desired property that profitable investment opportunities in physical capital are scarce. Moreover, it allows for a clear description of the macroeconomic implications of aggregate risk sharing, separate from any role played by collateral and physical capital accumulation. We believe that this is a good way to highlight a new mechanism that may have been underlying the 2002-2008 episode. As we will show at the end of this section, the results from our consumption-loan model extend almost trivially to a situation where consumers can invest in durable goods (such as housing). For this reason, the debt taken on by consumers in the model can be interpreted as being akin to mortgages.[6]

Our dynamic model economy builds on an overlapping generations setup inhabited by two kinds of agents – workers and financiers. At every date t new generations of workers and financiers are born who live for two periods. Thus, at each date mature workers overlap with young workers and mature financiers overlap with young financiers. We consider a closed economy with the same production structure as in the one-period model and the

---

[6]In the following section, we will examine how the results change when we allow for productive but risky investment opportunities. We will show how certain implications of our model are diluted when there is an abundance of risky investment opportunities, and this explains why we believe the model is particularly relevant for the 2002-2008 period where such opportunities were not abundant.

same stochastic description for the productivity parameter A. We have chosen to adopt a two period overlapping generations setting to discuss dynamics since we believe it is the simplest way to extend our one period model and illustrate its multi-period implications. In particular, our goal is to highlight why the dynamics of employment and risk premium are likely to exhibit important asymmetries.

The timing of events is as follows: at the beginning of every period both asset markets and labor markets open. Mature agents buy and sell risky claims and risk free bonds in asset markets while employment and wage decisions are made in the labor market. We will refer to the risky claim as a stock and the risk free claim as a bond, although the risky claim can alternatively be thought of as a risky bond. After these markets close, a new cohort of agents is born, the productivity shock is revealed and output for the period is produced. After the productivity realization, all outstanding claims in asset markets are settled including the stock and bond claims contracted at the beginning of the period. At this point young agents may receive transfer from old agents and they can use these proceeds to borrow or lend in the debt market.[7]

5.1 Workers

At every date a continuum of workers of measure one is born. Workers now live for two periods. In the first period of life they receive y units of the good as an endowment while they have one unit of labor time in the second period of life which they can use for either work or leisure. The lifetime welfare of a representative worker born at date t is given by

$$V_t^y = E_t \left[u \left(c_t^y \right) + u \left(c_{t+1}^o + g \left(1 - l_{t+1} \right) \right) \right],$$

where c^y denotes consumption when young, c^o is consumption when mature and l denotes labor supply. We assume throughout that $u' > 0$, $u'' < 0$, $g' > 0$, $g'' \leq 0$. We deliberately adopt a utility structure where there are no wealth effects on labor supply as we want to illustrate mechanisms that do not rely on this force. The structure we choose forces worker-households to face non-trivial decisions in terms of saving, labor supply and risk taking.

Workers can access a debt market at the end of the first period of life in order to borrow or lend. In the second period of life, workers supply labor and receive wages that they can invest in stocks and bonds. Stocks and bonds payout at the end of period after the realization of A_t.[8] The state contingent flow budget constraints facing the worker in each period of life are

$$c_t^y = y + p_t^d d_t \tag{3}$$

[7]In principle, we could allow for the young agents to use the proceeds from debt market transactions to buy a consumer durable (which could be interpreted as housing) which renders consumption services in both the first and second period of life. However, to clarify the mechanisms at work, we choose to focus for now on the case where goods are non-durables and we discuss the extension to the durable goods case later.

[8]We should note that the structure of the asset market is sufficiently rich to mimic a full state contingent claims market. In particular, since the payout on stocks is state-contingent we could equivalently formalize this environment using state-contingent claims markets.

$$p_{t+1}^s s_{t+1}^o + p_{t+1}^b b_{t+1}^o = w_{t+1} l_{t+1} \qquad (4)$$

$$c_{gt+1}^o = s_{t+1}^o + b_{t+1}^o - d_t \qquad (5)$$

$$c_{bt+1}^o = \theta s_{t+1}^o + b_{t+1}^o - d_t \qquad (6)$$

where p^d denotes the price of debt, d denotes debt incurred by a young worker, p^s and p^b denote the price of stock and bonds respectively, w denotes wages, while s and b denote stocks and bonds respectively. c_{gt+1}^o denotes consumption of the mature worker at date $t+1$ in the good state when $A = 1$. Analogously, c_{bt+1}^o is consumption of the mature worker at date $t+1$ in the bad state when $A = \theta$.

The workers maximization problem leads to three optimality conditions:

$$\frac{p_{t+1}^s}{p_{t+1}^b} = g'(1 - l_{t+1}) \qquad (7)$$

$$\frac{p_{t+1}^s}{p_{t+1}^b} = \frac{qu'\left(c_{gt+1}^o + g(1 - l_{t+1})\right) + \theta(1-q)u'\left(c_{bt+1}^o + g(1 - l_{t+1})\right)}{qu'\left(c_{gt+1}^o + g(1 - l_{t+1})\right) + (1-q)u'\left(c_{bt+1}^o + g(1 - l_{t+1})\right)} \qquad (8)$$

$$p_t^d u'\left(c_t^y\right) = qu'\left(c_{gt+1}^o + g(1 - l_{t+1})\right) + (1-q)u'\left(c_{bt+1}^o + g(1 - l_{t+1})\right) \qquad (9)$$

The first two conditions are identical to the optimality conditions in the one-period model analyzed above. The third condition, which is the new feature of the dynamic model, determines optimal borrowing by young agents in the first period of life. It is the standard Euler equation which equates the marginal rate of substitution between current and expected future income with the relative price of current consumption $1/p^d$ (which is the rate of return to saving).

5.2   Financiers

In every period a new cohort of financiers is born which lives for two periods. Every cohort has a continuum of financiers of measure one. In the first period of life young financiers born at date t receive an endowment f of the good as well as transfers T_t from old financiers. They can either consume these resources or use these to lend to other young agents. In the second period of life they begin by transacting in stock and bond markets, which allows them to leverage their inherited asset position and create a risky portfolio. The only constraint on financiers is solvency in that their consumption cannot be negative. At the end of the period, they receive payments on all their claims. Financiers, as before, are assumed to be risk neutral.

We will assume that financiers get no utility from consuming in the first period of their life. This simplifies the analysis as it will make financiers decision when young trivial, since they will simply lend all there resources to young worker-households. Allowing for financiers to

consume when young can be easily accommodated, but does not provide additional insight. The second important assumption we make about financiers is that they get utility from transferring bequests to the next generation of financiers. In particular, we will assume that their objective is to maximize

$$V_t^F = E_t \left[\min \left[\gamma c_{t+1}^F, (1 - \gamma) T_{t+1} \right] \right],$$

where c^F denotes consumption by financiers and γ controls the utility weight on bequests relative to own consumption.[9] Denoting the end-of-period resources of mature financiers by F, it is clear that $T_{t+1} = \gamma F_{t+1}$.

The objective of financiers born at date t reduces then to just maximizing the value of their end-of-life resources. It is worth emphasizing that the assumptions imposed on the financiers make them act similarly to a dynasty which maximizes expected wealth every period and consumes a fix fraction of its resources. The bequest motive is a very simple way of linking outcomes across time. Although we have assumed that workers to not behave in such a dynastic fashion, this is again for simplicity as the analysis can be easily extended to include this possibility.

Since young financiers can access debt markets at the end of the first period to lend, their budget constraint when young is

$$0 = f + T_t + p_t^d d_t^F$$

where d^F denotes debt issued by young financiers. Market clearing for debt requires that the total debt of young workers and financiers add to zero, i.e., $d + d^F = 0$. Hence, $d^F = -d$, i.e., the debt of young workers must be owed to the young financiers. Using this relationship, the flow budget constraints facing financiers in each state and period of life are given by

$$p_t^d d_t = f + T_t, \tag{10}$$

$$p_{t+1}^s s_{t+1}^F + p_{t+1}^b b_{t+1}^F = 0, \tag{11}$$

$$F_{gt+1} = s_{t+1}^F + b_{t+1}^F + d_t, \tag{12}$$

$$F_{bt+1} = \theta s_{t+1}^F + b_{t+1}^F + d_t, \tag{13}$$

where F_{gt+1} denotes financier resources at the end of period $t + 1$ in the good state when $A = 1$ while F_{bt+1} denotes resources in the bad state when $A = \theta$. The only limit on asset positions is that they must satisfy solvency in that $F_{gt+1} \geq 0$ and $F_{bt+1} \geq 0$. As we showed in the one period model, all admissible choices by financiers must satisfy the consolidated resource constraint for the mature financier in the second period of life given by

$$\left(\frac{p_{t+1}^s}{p_{t+1}^b} - \theta \right) F_{gt+1} + \left(1 - \frac{p_{t+1}^s}{p_{t+1}^b} \right) F_{bt+1} = (1 - \theta) d_t. \tag{14}$$

---

[9]Financiers thereby act partially in a dynastic fashion.

The portfolio problem facing mature financiers is identical to the one we solved in the one-period model earlier. Hence, those solutions apply here. The new aspect in this dynamic version of the model is the lending by financiers when young. Since they do not consume in the first period of life, they simply lend their resources in the debt market. Using equation (10), the debt bought by financiers is given by

$$d_t = \frac{f + \gamma F_t}{p_t^d}$$

5.3 Equilibrium

We now describe the equilibrium for this economy. In equilibrium transfers received by young financiers must satisfy

$$T_t = \gamma F_t.$$

Moreover, goods markets must clear in both the good and bad states. Thus, we must have

$$c_{gt}^y + c_{gt}^o + (1 - \gamma) F_{gt} = y + f + l_t, \tag{15}$$

$$c_{bt}^y + c_{bt}^o + (1 - \gamma) F_{bt} = y + f + \theta l_t. \tag{16}$$

The first condition describes market clearing in the good state: total consumption of young and mature workers, and financiers must exhaust total output in that state. The second equation is the analogous condition for the bad state.

Lastly, asset market clearing requires that both bond markets and equity market must clear

$$b_t^F + b_t^o = 0,$$

$$s_t^F + s_t^o = S_t,$$

Definition: *A Walrasian Equilibrium for this economy consists of a sequence of prices $\{w_t, \frac{p_t^b}{p_t^s}, p_t^d\}$ and a sequence of allocations $\{c_{gt}^y, c_{bt}^y, c_{gt}^o, c_{bt}^o, F_{gt}, F_{bt}, l_t\}$, such that all agents find their allocations to be optimal given prices, and all markets clear.*[10]

In principle, these sequences could depend on the whole history of realizations of A_t. However, as we shall show, the equilibrium can be represented in recursive form where the level of debt plays the role of a state variable. In particular, we will show the existence of an equilibrium transition equation for debt of the form $d_t = \phi^d (d_{t-1}, A_t)$, whereby current debt depends only on past debt and the current realization of the state of nature. Other variables can then be expressed as functions of debt.

In the rest of the paper we shall impose the following restriction on preferences:

---

[10]Note that the equilibrium only determines $\frac{p_t^b}{p_t^s}$, not each individually. This is because the second period asset market is really only a risk market.

Assumption 3: $u(x) = \log x$.

Assumption 3 is more restrictive than needed, but is sufficient for demonstrating the mechanisms that we are interested in.

5.4 Equilibrium Characterization

We now discuss the equilibrium properties of our model economy. We start by describing the equilibrium allocations. Combining equations (3) and (10) gives consumption of young workers in each state:

$$c_{gt}^y = y + f + \gamma F_{gt},$$
$$c_{bt}^y = y + f + \gamma F_{bt}.$$

The market clearing conditions then directly yield the state contingent consumptions of mature workers:

$$c_{gt}^o = l_t - F_{gt},$$
$$c_{bt}^o = \theta l_t - F_{bt}.$$

The three key variables of interest in the model are: the level of debt taken on by workers d_t, the employment rate l_t and the risk premium captured by $\frac{p_t^b}{p_t^s}$. Both employment and the risk premium at date t are determined within the period as outcomes of the solution to equations (7) and (8). This solution is identical to the solutions for employment and the risk premium in the one-period model that we derived previously. Hence, Propositions 1 and 2 continue to apply here.

The solution for debt, d_t, however, depend on both the current state of nature as well as the initial level of debt, d_{t-1}. From equation (10) we have $p_t^d d_t = f + \gamma F_t$. From the young workers' first order condition for optimal borrowing we also have

$$p_t^d = \frac{q u' \left(c_{gt+1}^o + g\left(1 - l_{t+1}\right)\right) + (1 - q) u' \left(c_{bt+1}^o + g\left(1 - l_{t+1}\right)\right)}{u'\left(c_t^y\right)}.$$

The equilibrium evolution of debt in this economy can be derived by combining these two expressions.

In light of Propositions 1 and 2, there are two potential regions of d to be considered: $d \leq \tilde{d}$ and $d > \tilde{d}$. In each of these regions the dynamic evolution of debt is dependent on the current state of nature (the realization of A) aside from the inherited level of debt. This is easy to see from the fact that bequests to young financiers, γF_t, as well consumption of young workers, c_t^y, depend directly on the current state of nature. Hence, both the price of debt and the level of current debt will depend on the current state of nature as well as the inherited level of debt.

In view of this dependence, the equilibrium dynamic evolution of debt can be summarized by a pair of transition equations:

$$d_t = \begin{cases} \phi^d\left(d_{t-1}, A_t\right) & \text{if } d_{t-1} \leq \tilde{d} \\ \tilde{\phi}^d\left(d_{t-1}, A_t\right) & \text{if } d_{t-1} > \tilde{d} \end{cases}.$$

We summarize the key dynamic properties of this economy in the following Proposition:

Proposition 3: The equilibrium evolution of debt is characterized by three key features:

1) For a given state of nature ($A_t = 1$ or $A_t = \theta$), debt at time t is a continuous and weakly increasing function of d_{t-1}. For $A_t = 1$, debt is strictly increasing in previous debt. For $A_t = \theta$, $d_t = \underline{d}$ for $d_{t-1} \leq \tilde{d}$ while d_t is strictly increasing in d_{t-1} for $d_{t-1} \geq \tilde{d}$.

2) There exists a fixed point for $\phi^d(d, 1)$, denoted by $\bar{d} = \phi^d(\bar{d}, 1)$, and another one for $\phi^d(d, \theta)$, denoted by $\underline{d}$. The debt in this economy will fluctuate within the range $\bar{d}$ and $\underline{d}$.

3) For a given level of d_{t-1}, d_t is always greater after a good realization of the state of nature than after a bad realization, i.e, $\phi^d(d_{t-1}, 1) > \phi^d(d_{t-1}, \theta)$ and $\tilde{\phi}^d(d_{t-1}, 1) > \tilde{\phi}^d(d_{t-1}, \theta)$.

The debt dynamics take the following form: suppose the initial level of debt is some $d \in [\underline{d}, \bar{d})$. If the state of nature is good, then debt will grow. Debt will continue to grow as long as the state is good, and it will gradually approach $\bar{d}$. However, if at any date the state of nature is bad, then debt falls. If $\bar{d} < \tilde{d}$ then it falls immediately to $\underline{d}$. If $\bar{d} > \tilde{d}$ then it can take several periods of bad outcomes for debt to converge to $\underline{d}$. Figure 8 depicts the dynamics for the case $\bar{d} < \tilde{d}$ while Figure 9 shows the dynamics when $\bar{d} > \tilde{d}$. The solid arrows in the two figures depict the dynamic behavior of debt in response to good productivity shocks while the dashed arrows show the response of the economy to a low productivity shock.

Given the results expressed in Propositions 1 and 2, the dynamics for employment and the risk premium follow easily from Proposition 3. Thus, employment will continuously rise following a set of good outcomes, but it will drop when a bad state arises. In particular, if $\bar{d} < \tilde{d}$, it will drop immediately to it lowest level $\phi^l(\underline{d})$ following the realization of the bad state. This implies that the fall in employment will be greater the longer an expansion period has been. The behavior of the risk premium is a mirror image of employment.

This highlights a key feature of our model. The presence of debt in financier portfolios links periods and thereby acts as a transmission mechanism for shocks. In good states financiers accumulate more claims on households (higher d) which raises the level of resources that they have in the next period to provide insurance cover for risky employment (we call this risk capital). The presence of risk capital and its dependence on the current state of nature links adjoining periods and thereby provides a propagation mechanism. A good productivity shock raises resources of financiers which then translates into more insurance, a lower risk premium and greater employment tomorrow. Similarly, a transitory negative productivity shock in any period (say a low θ) translates into low employment in the next period due to its effect on financier balance sheets today.

5.5 An Illustrative Example

In order to illustrate the dynamic evolution of the economy we now present a simple example where the dis-utility from labor (or equivalently, the technology for home production) is linear, that is,

$$g(1-l) = (1-l)g^*, \quad \theta < g^* < 1.$$

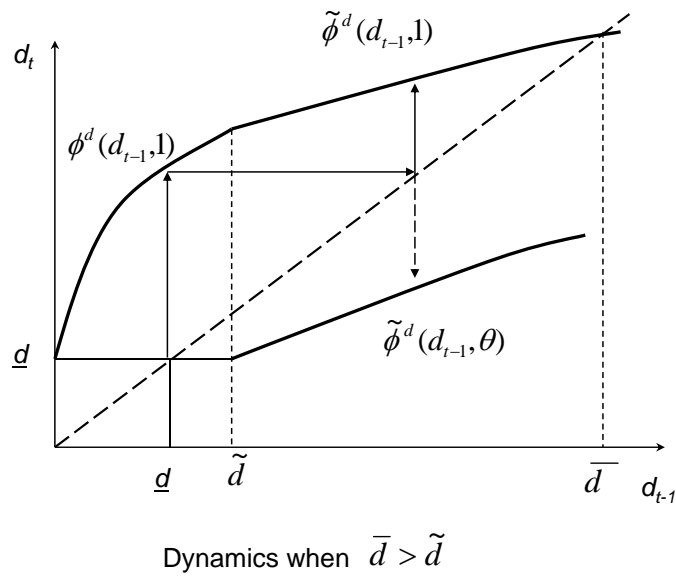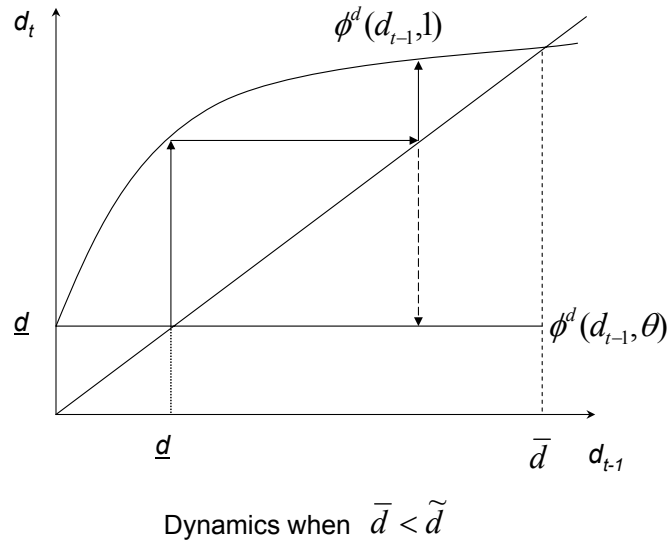22

Figure 8: Debt dynamics when $\bar{d} < \tilde{d}$

Dynamics when $\bar{d} < \tilde{d}$

Dynamics when $\bar{d} > \tilde{d}$

Figure 9: Debt dynamics when $\bar{d} > \tilde{d}$

23

Under this specification equilibrium employment[11] is given by

$$l_t = \frac{(1-q)(1-\theta)}{(1-g^*)(g^*-\theta)}d_{t-1} + \frac{[q+(1-q)\theta - g^*]}{(1-g^*)(g^*-\theta)}.$$

Using this solution for employment in the equilibrium difference equation for debt gives the following state contingent solutions for d_t:

1. If $A = 1$ then

$$d_t = \left(\frac{f + \psi d_{t-1}}{y + 2f + 2\psi d_{t-1}}\right)g^*$$

where $\psi \equiv \frac{\gamma(1-\theta)}{g^*-\theta}$. It is easy to check that d_t is increasing in d_{t-1} in this case. Moreover, this mapping between current and past debt has a unique positive steady state which is given by

$$\bar{d} = \left(\frac{1}{4\psi}\right)\left[-(y + 2f - \psi g^*) + \sqrt{(y + 2f - \psi g^*)^2 + 8\psi g^*}\right].$$

Clearly, $d_t = \left(\frac{f}{y+2f}\right)g^* > 0$ when $d_{t-1} = 0$ and $d_t = \frac{g^*}{2} < \infty$ when $d_{t-1} \to \infty$. Thus, for very low levels of d_{t-1} we must have $d_t > d_{t-1}$ while $d_t < d_{t-1}$ for arbitrarily high levels d_{t-1}. This, along with the fact that d_t is increasing in d_{t-1}, is sufficient to establish that the steady state is stable.

2. If $A = \theta$ then

$$d_t = \underline{d} = \left(\frac{f}{y + 2f}\right)g^*$$

Lastly,

$$d_t|_{A_t=1} > d_t|_{A_t=\theta}$$

which follows directly from the expressions for equilibrium d_t in the two cases, along with the fact that d_t is increasing in d_{t-1} for $A_t = 1$. Hence, debt in a good state must be greater than debt in a bad state. Hence, as long as the economy gets good productivity draws ($A_t = 1$), debt will keep growing along with employment. As soon as a bad productivity shocks hits the economy, debt will jump down to $\underline{d}$ as will employment in the following period. The process will then start up again.

5.6 Discussion of portfolio positions

It is instructive to clarify the evolution of the balance sheet of financiers. When a financier enters the second period of his life, he initially holds only debt and therefore his balance sheet looks as follows:

---

[11]Since this example does not satisfy the Inada condition that $g'(0) = \infty$, it becomes necessary to now check that given the equilibrium solution for d_{t-1}, employment is bounded between 0 and 1. For the purposes of the example, this can be accomplished by appropriate restrictions on g^*, q and θ (as well as the other parameters of the system).

Assets	Liabilities
d_{t-1}	Own Equity

In general, a risk neutral financiers will not be satisfied with such a balance sheet since his return on own equity is rather low. To increase his expected return on equity, the financier transacts on the second period assets market to buy stocks in the amount:

$$s_t^F = \frac{d_{t-1}}{\frac{p_t^s}{p_t^b} - \theta},$$

and issues bounds in the amount:

$$b_t^F = \frac{\frac{p^s}{p^b} d_{t-1}}{\frac{p^s}{p^b} - \theta}.$$

After these transactions, a financier's balance sheet is given by

Assets	Liabilities
$p_t^s s_t^F$	b_t
d_{t-1}	Own Equity

(Note that in the above the price of bonds are normalized to 1). These transactions in the asset market allow the financier to construct a leveraged portfolio which gives him an expected return on equity which is greater than that associated with his initial position. Furthermore, it can be verified that the resulting return on equity is also higher than simply holding a pure equity position, which is directly due to the leveraging. Although this new asset position is more risky, it is preferred by the financier.[12]

In describing the evolution of the financier asset position, we believe it is informative to keep track of gross positions rather than immediately netting out d_{t-1} and b_t. In fact, in our model d_{t-1} and b_t are slightly different as d_{t-1} represents long term debt, while b_t is more akin to short term debt. Obviously, as they have the same maturity date, they eventually become perfect substitutes. One advantage of keeping track of these gross position is that it may better capture actual positions observed in financial markets due to the fact that many individuals have both short and long positions in debt. If we instead net out the debt position, the financiers balance sheet will look as below, where we see that the financier has created a leveraged position in stocks.

---

[12]There are many ways to describe the type of transaction undertaken by financiers. For example, financiers can be seen as performing a swap between their holdings of household debt from the previous period for risky equity claims on firms – a type of "debt-equity" swap.

Assets	Liabilities
$p_t^s s_t^F$	$b_t - d_{t-1} = \frac{\theta d_{t-1}}{\frac{p^s}{p^b} - \theta}$
	Own Equity

The preceding discussion fleshes out what we believe to be a key role that is played by financial markets. Financial market do not generally insure workers directly by issuing state contingent claims, but instead may insure them indirectly. In our set up this is done by allowing firms to pay workers non-contingent wages, with the financiers picking up most of the residual risk by buying equity. As opposed to transactions in pure state contingent assets, actual transactions in the real world are probably better understood as reflecting such combinations of asset trades. For, example, it is easy with these gross flows to understand how debt can be viewed as collateral for taking higher leveraged positions in other risky assets.

5.7   Allowing for consumer durables

Our consumption loan model illustrates how a series of good productivity outcomes can generate a debt fueled expansion. The debt in the model is pure consumption debt, as it is used to buy consumption goods and is therefore not backed by any real assets. In contrast, over the 2002-2008 period, much of the growth in consumer debt was associated with mortgages and the financing of consumer durables. For this reason it is important to stress that the insights from the previous propositions also apply to a situation which allows for consumer durables and where accordingly consumer debt can be interpreted as asset backed securities. To see this, suppose now that workers when young can use both own resources and any borrowed resources to buy a consumer durable. Suppose that this consumer durable has the following properties. It provides one unit of consumption services in its first period and provides s units of consumption services in its second period (after a second period, it is assumed to depreciate fully). For simplicity, let us assume that the production of one unit of consumer durable requires one unit of output, and that the services of consumer durables are perfectly substitutable for the consumption services of output. In this case, our previous analysis extends with only minor changes.

The main difference in this case is that the equation for the equilibrium determination of employment has to be modified. Previously, employment at time t was determined by the condition

$$g'(1 - l_t) = \frac{qu'\left(l_t - \left(\frac{(1-\theta)d_{t-1}}{g'(1-l_t)-\theta}\right) + g(1 - l_t)\right) + (1 - q)\theta u'\left(\theta l_t + g(1 - l_t)\right)}{qu'\left(l_t - \left(\frac{(1-\theta)d_{t-1}}{g'(1-l_t)-\theta}\right) + g(1 - l_t)\right) + (1 - q)u'\left(\theta l_t + g(1 - l_t)\right)}.$$

In the presence of durable consumer good , the condition becomes

$$g'(1-l_t) = \frac{qu'\left(l_t + sh_{t-1} - \left(\frac{(1-\theta)d_{t-1}}{g'(1-l_t)-\theta}\right) + g(1-l_t)\right) + (1-q)\theta u'\left(\theta l_t + sh_{t-1} + g(1-l_t)\right)}{qu'\left(l_t + sh_{t-1} - \left(\frac{(1-\theta)d_{t-1}}{g'(1-l_t)-\theta}\right) + g(1-l_t)\right) + (1-q)u'\left(\theta l_t + sh_{t-1} + g(1-l_t)\right)}.$$

where sh_{t-1} represents the services from a stock h_{t-1} of consumer durable bought last period. The purchases of consumer durable h_{t-1} are given by $h_{t-1} = p^d_{t-1}d_{t-1} + y$, where the price of debt (p^d_{t-1}) is determined by the inter-temporal condition

$$\frac{1}{\frac{1}{p^d_{t-1}} - s} = \frac{qu'\left(l_t - \left(\frac{(1-\theta)d_{t-1}}{g'(1-l_t)-\theta}\right) + g(1-l_t)\right) + (1-q)\theta u'(\theta l_t + g(1-l_t))}{u'(h_{t-1})}.$$

It can be easily verified that our previous characterization for the mapping between inherited debt and employment (and risk premium) continues to hold in this case. For example, it is still the case that higher levels of debt d_{t-1} lead to greater employment and a lower risk premium. Furthermore, the equilibrium determination of debt dynamics remains essentially unchanged in this case as debt at t is still given by $p^d_t d_t = f + \gamma\left(\frac{(1-\theta)d_{t-1}}{g'(1-l_t)-\theta}\right)$ if $A_t = 1$ and $p^d_t d_t = f$ if $A_t = \theta$. Hence, our previous analysis can be seen as applying to a situation where financiers fund the purchase of consumer durables by young workers, and then use the resulting debt to build a leveraged position in stocks which favors the expansion of employment needed to repay the debt. Whether debt is used to purchase durables or non-durables is not central to the mechanism.

6   Default Risk and Adverse Selection

Thus far we have focused on an environment where all information is available to all agents. While it was a useful assumption that allowed us to highlight a set of mechanisms, it did not permit us to study environments in which asset markets may freeze up due to adverse selection. We turn to this issue now by introducing information heterogeneity into the model. While the idea that asymmetric information may cause markets to freeze up is well established, the value added of this section is to illustrate how adverse selection in an asset market can cause major disruptions in economic activity.

In order to simplify the analysis, in the remainder of the paper we shall maintain the following assumption which eliminates the need to consider corner situations where the risk premium is completely eliminated:

Assumption 4: f and y are such that $\bar{d} < \tilde{d}$.

Let us now suppose that workers who undertake debt when young may default on this debt when old. Specifically, let ψ^i be the probability that a worker i will have productive market labor at the beginning of the second period life. Hence, with probability $1 - \psi^i$ worker i's labor productivity in market work is zero. Each worker draws a $\psi \in [0,1]$ from an

i.i.d. distribution with density $f(\psi)$. Since there is a continuum of workers of unit mass, the mean of the distribution, $E[\psi]$, is also the expected fraction of workers that will have a productive market labor. Lastly, assume that the actual productivity of market labor of a prospective worker is revealed to both the worker and firms just before the labor market opens. For this reason, there will not be any information problem in the labor market. The source of asymmetric information will revolve only around asset markets, in particular the market for existing debt.

In the second period of life, if the worker has productive market labor, then he will behave as before since this is optimal. If not, he does not work and simply defaults on debt and gets $u(g(1))$ as utility.[13] Hence, $E[\psi]$ is the expected probability of a young worker not defaulting on his debt in the second period of life and $1 - \psi^i$ is the probability of default of worker i.

We assume that each financier's portfolio of debt brought into a period is characterized by a specific ψ. This corresponds to an environment where each financier that has debt has bought it from a continuum of workers with the same ψ, i.e., each financier buys debt from young workers from a single point in the distribution of ψ.[14]

Since the quality of outstanding debt is heterogeneous, it is helpful to assume that in every period asset markets begin operations with the opening of a market for existing debt. In particular, let us assume that intermediaries can setup and offer to buy up risky debt from individual financiers in order to pool the different default risks. The assets held by one of these intermediaries will be equivalent to a synthetic package of debt whose payoff has the expected repayment rate of the whole distribution of debt sold to it. Financiers can sell their debt to these intermediaries, while the intermediaries finance themselves by issuing bonds to households. Note that the overall quality of the portfolio held by the intermediaries will, in equilibrium, be common knowledge, as all agents will be able to infer who supplied their debt to the market. Hence, these intermediaries will be able to finance themselves by issuing risk free bonds. The potential for an adverse selection problem will arise between financiers and the intermediaries, as we assume that intermediaries cannot directly asses the quality of debt being sold to it by an individual financier. We denote the average payoff per unit of debt held by the intermediary by $\bar{\psi}$.

A financier who sells his debt to an intermediary can use the proceeds to build, as before, a new portfolio position. In the following we shall denote the price paid by an intermediary for a unit of debt previously held by a financier to be p^k. The total quantity of debt bought by intermediaries will be denoted by k, and total value of bonds issued to buy the debt is $p^k k$, as we assume free entry in the intermediary sector.

---

[13] We are assuming that labor continues to be productive in home production even when its market productivity is zero.

[14] This assumption considerably simplifies the analysis, especially the asymmetric information case we study later. There are alternative ways of setting up the debt portfolio of financiers but they come at the cost of tractability and algebraic complications.

6.1 Symmetric Information

We present here the case where there is heterogeneity across debt quality but no asymmetric information. In particular, let us assume for now that no one in the asset market knows the individual ψ of the debt held by financiers (including the financiers themselves). In this case, we will show that our preceding equilibrium analysis can be carried through with almost no changes. This section is nevertheless useful for understanding the case with asymmetric information.

In the first period of life, young workers will recognize the probability of not having a productive market labor when mature and will take this into account when borrowing. The constraint faced by workers in the first period of life is still given by equation (3). The constraints faced by mature workers with productive market labor are also unchanged. Thus, the optimality conditions for the labor-leisure choice as well as the portfolio choice between stocks and bonds, equations (7) and (8) respectively, remain unaltered. The first order condition for optimal borrowing of young workers does change though due to the default probability. The new condition is given by:

$$p_t^d u'\left(c_t^y\right) = E[\psi]\left[qu'\left(c_{gt+1}^o + g\left(1 - l_{t+1}\right)\right) + (1 - q)\, u'\left(c_{bt+1}^o + g\left(1 - l_{t+1}\right)\right)\right]. \qquad (17)$$

The important new element introduced by the presence of default risk is an additional equilibrium condition for the price of debt bought by financiers. This arbitrage condition associated with free entry into the intermediary sector is given by:

$$\bar{\psi} = \frac{p_{t+1}^k}{p_{t+1}^b} \qquad (18)$$

The budget constraint facing young financiers is unaffected by the introduction of default risk. For a mature financier who sells his debt to an intermediary, the constraints in the second period of life are now given by:

$$p_{t+1}^s s_{t+1}^F + p_{t+1}^b b_{t+1}^F = p_{t+1}^k d_t,$$
$$F_{gt+1} = s_{t+1}^F + b_{t+1}^F \geq 0,$$
$$F_{bt+1} = \theta s_{t+1}^F + b_{t+1}^F \geq 0.$$

If the financier does not sell his debt on the market, he has no liquidity to build a new portfolio and therefore must simply hold his existing debt to maturity. Under the current assumption that financiers do not know the ψ of their own portfolio, no financier has any incentive to not sell their debt to the intermediaries. Since all debt gets sold to intermediaries, the average payoff on the debt held by intermediaries is

$$\bar{\psi} = \int_0^1 \psi f\left(\psi\right) d\psi = E[\psi].$$

Moreover, in equilibrium, we must also have the aggregate relationship

$$k_t = d_{t-1}.$$

The portfolio re-balancing by mature financiers through debt sales implies that their end-of-period resources are now given by

$$F_t^g = \frac{(1-\theta)\,E[\psi]d_{t-1}}{\frac{p_t^s}{p_t^b} - \theta},$$

$$F_t^b = 0,$$

while the stock position of financiers is given by

$$s_t^F = \frac{E[\psi]d_{t-1}}{\frac{p_t^s}{p_t^b} - \theta},$$

Lastly, the equilibrium state-contingent consumption levels of mature workers are given by

$$\hat{c}_{gt}^o = l_t - \frac{(1-\theta)E[\psi]d_{t-1}}{g'(1-l_t) - \theta},$$

$$\hat{c}_{bt}^o = \theta l_t.$$

with the equilibrium level of employment being determined by the condition:

$$g'(1-l_t) = \frac{qu'\left(l_t - \frac{(1-\theta)E[\psi]d_{t-1}}{g'(1-l_t)-\theta} + g(1-l_t)\right) + \theta(1-q)u'(\theta l_t + g(1-l_t))}{qu'\left(l_t - \frac{(1-\theta)E[\psi]d_{t-1}}{g'(1-l_t)-\theta} + g(1-l_t)\right) + (1-q)u'(\theta l_t + g(1-l_t))} \tag{19}$$

The transition equation for debt remains unchanged as $d_t = \frac{f+T_t}{p_t^d}$, although the actual transfer does change as F_{gt} changes.

In brief, the equilibrium conditions with default are identical to those without default up to the following two transformations: take the new equilibrium system, and define $\frac{p_t^d}{E[\psi]}$ as the price of debt, and $E[\psi]d_t$ as debt. The resultant system is identical to the one without default. Hence, Propositions 1 to 3 continue to hold.

6.2   Asymmetric Information

Now we introduce private information into the model. In particular, let us suppose that mature financiers learn the repayment rate ψ in their portfolios at the beginning of the period when they enter the asset market. The first thing to note is that when the ψ characterizing the portfolio of a financier is only known to that financier, there will potentially be an adverse selection problem in the asset market. Financiers with relatively high quality debt (low default probability) may have an incentive not to offer their debt on the secondary

market since the market will value their debt at the average default rate, which is higher than the default rate of their portfolio. This implies that only low quality debt may be offered for sale to intermediaries.

Let us denote by $\hat{\psi}$ the conjectured quality of debt offered to intermediaries, i.e., the conjectured average repayment rate of the debt that is offered on the market As we saw in the previous subsection, the zero profit condition for intermediaries implies that the market price of the debt offered on the market is given by

$$\frac{p_t^k}{p_t^b} = \hat{\psi}$$

The problem facing mature workers remains unaffected by this new environment relative to the symmetric information case analyzed above. For financiers however, the problem changes. Every financier now has to make a choice about whether or not he wants to offer his debt holdings to intermediaries. How will financiers decide whether to sell their debt or to keep it? For a given conjecture $\hat{\psi}$, the budget constraint facing a financier i with debt type ψ^i who decides to place his debt on the market is

$$p_{t+1}^s s_{t+1}^{Fi} + p_{t+1}^b b_{t+1}^{Fi} = p_t^k d_t$$

with the solvency constraints

$$F_{gt+1}^i = s_{t+1}^{Fi} + b_{t+1}^{Fi} \geq 0$$
$$F_{bt+1}^i = \theta s_{t+1}^{Fi} + b_{t+1}^{Fi} \geq 0$$

With $\frac{p^s}{p^b} \leq E[A]$, and under our maintained assumption that $\bar{d} < \tilde{d}$, a financier who offers his debt on the market at date t will use the proceeds to build a risky position which results in the payoffs:

$$F_{gt}^i = \frac{(1-\theta)\hat{\psi}_t d_t^i}{\frac{p_t^s}{p_t^b} - \theta},$$
$$F_{bt}^i = 0,$$

Since the probability of the good state is q, the expected payoff from selling his debt on the market is $\frac{q(1-\theta)p_t^b \hat{\psi}_t d_t^i}{\frac{p_t^s}{p_t^b} - \theta}$.[15]

Alternatively, financier i can choose not to supply his debt to the market and, instead, hold on to it. In this event his expected payoff is $p_t^b \psi^i d^i$. Hence, financier i will choose to build a risky portfolio if and only if the expected payoff from doing so exceeds the payoff from holding on to it:

---

[15]Recall that if $\frac{p^s}{p^b} = E[A]$ then financiers are indifferent between all combinations that satisfy their budget constraint. In that special case we shall continue to focus on the equilibrium where $F_g > 0$ and $F_b = 0$.

31

$$\frac{q(1-\theta)\hat{\psi}_t}{\frac{p_t^s}{p_t^b} - \theta} > \psi_t^i.$$

Now consider the marginal type who is just indifferent between using his debt holding to build a risky portfolio or simply holding on to his debt. For this marginal type, which we denote by ψ^m, we must have

$$\frac{q(1-\theta)\hat{\psi}_t}{\frac{p_t^b}{p_t^s} - \theta} = \psi_t^m.$$

Clearly, all financiers with $\psi^i > \psi^m$ will choose to hold on to their debt while types with $\psi^i \leq \psi^m$ will offer their debt holdings on the market.

The preceding implies that the average quality of debt offered for sale at time t is given by

$$\hat{\psi}_t = \frac{\int_0^{\psi_t^m} \psi f(\psi)d\psi}{F(\psi_t^m)}$$

where ψ^m denotes the highest financier type who sells his debt on the market and $F(\psi)$ is the cumulative density function of $f(\psi)$. Hence, the consistency condition for this behavior to be optimal is that ψ^m must satisfy

$$\psi_t^m = \frac{q(1-\theta)\int_0^{\psi_t^m} \psi f(\psi)d\psi}{F(\psi_t^m)\left(\frac{p_t^s}{p_t^b} - \theta\right)} \tag{20}$$

where $\frac{p_t^s}{p_t^b} = g'(1-l_t)$ and l_t is the solution to:

$$g'(1-l_t) = \frac{qu'\left(\tilde{c}_{gt}^o + g(1-l_t)\right) + \theta(1-q)u'\left(\tilde{c}_{bt}^o + g(1-l_t)\right)}{qu'\left(\tilde{c}_{gt}^o + g(1-l_t)\right) + (1-q)u'\left(\tilde{c}_{bt}^o + g(1-l_t)\right)} \tag{21}$$

where

$$\tilde{c}_{gt}^o = l_t - d_{t-1}\int_{\psi^m}^1 \psi f(\psi)d\psi - \frac{(1-\theta)\hat{\psi}_t d_{t-1}}{g'(1-l_t) - \theta},$$

$$\tilde{c}_{bt}^o = \theta l_t - d_{t-1}\int_{\psi^m}^1 \psi f(\psi)d\psi.$$

The last two expressions are the optimal consumption levels of mature workers in the good and bad states respectively. Note that the consumption levels of mature workers in this case differ from their corresponding levels under symmetric information by the term $\left(\int_{\psi^m}^1 \psi f(\psi)d\psi\right)d_{t-1}$. This term simply reflects the fraction of aggregate debt that is not placed on the market by financiers and on which they get repaid by mature workers with productive market labor. As ψ^m goes to one this term becomes vanishingly small and the consumption levels $\tilde{c}_{gt}^o$ and $\tilde{c}_{bt}^o$ tend to approach those under symmetric information.

Equations (20) and (21) jointly define the equilibrium for this economy at every date t in terms of ψ_t^m and l_t as a function of the inherited stock of debt d_{t-1}. The equilibrium values of all the other endogenous variables can then be recovered recursively using these two solutions for ψ_t^m, l_t and the initial level of debt d_{t-1}. For future reference, we denote the solutions for equilibrium employment and the risk premium p^s/p^b as

$$\tilde{l}_t = \tilde{\phi}^l \left(d_{t-1}, \psi_t^m \right)$$

$$\frac{\tilde{p}_t^s}{\tilde{p}_t^b} = \tilde{\phi}^p \left(d_{t-1}, \psi_t^m \right)$$

In general, there can be multiple solutions to the equilibrium system of equations (20) and (21). Below we illustrate the potential multiple equilibria by describing alternative equilibria.

6.2.1 Pessimistic Equilibrium: $\psi^m = 0$

In this model economy there always exists an equilibrium in which $\psi^m = 0$. We call this the "pessimistic" equilibrium, as it corresponds to the case where the market interprets any offer of debt as reflecting the worse type of debt, that is debt with a zero expected repayment rate. Since this equilibrium always exists, we will begin by characterizing it. In the pessimistic equilibrium, the asset market available to mature financiers breaks down in the sense that financiers do not trade their debt on this market. Thus, they have no liquidity to build a new portfolio. Hence it must be workers that buy all the risky claims supplied by firms looking to finance working capital.

Proposition 4: There always exists a pessimistic equilibrium where $\psi^m = 0$ and the asset holdings S and b of financiers are zero.

We next summarize the response of equilibrium employment to changes in initial debt and contrast this equilibrium with that in the autarkic case with no financiers. The main element to notice in Proposition 5 is that the debt-employment relationship in the pessimistic equilibrium is almost the exact opposite of the one derived in Proposition 1.

Proposition 5: The pessimistic equilibrium is characterized by employment being a decreasing function of d_{t-1}. Moreover, for all $d_{t-1} > 0$, employment is lower than in autarky.

The reason why employment is low and is a decreasing function of debt in the pessimistic equilibrium, is that now the presence of debt increases the risk premium. This occurs because workers, who are the agents that determine the risk premium at the margin, dislike risk even more when they are more indebted and no one is willing to insure them against bad aggregate outcomes. Effectively, workers have no insurance against fluctuating labor income but have to repay a larger debt when d_{t-1} is higher. Hence, their net risk exposure becomes higher with higher d_{t-1}. The market response is for employment to decrease as labor time is shifted to the safe home production technology $g\left(1-l\right)$. The important new

aspect is that the equilibrium mapping between debt and employment is decreasing, that is more debt leads to lower employment, as stated in Proposition 6.

Proposition 6: In the pessimistic equilibrium current debt is an increasing function of past debt and is independent of the current state. Moreover, there is a unique steady state level of debt in this equilibrium, d^p, and $d^p < \bar{d}$.

A key feature of Proposition 6 is that the evolution of debt is not affected by the current state. This has an important implication for the dependence of current employment on past states of nature. Since mature financiers do not acquire any risky claims, their end-of-period resources are $E[\psi]d_{t-1}$ in both states (their portfolio is concentrated entirely in safe claims). This implies that their bequests to young financiers are also not dependent on the current state. Hence, resources available to young financiers to lend to young workers every period are $f + \gamma E[\psi]d_{t-1}$ which too is independent of the current state. Thus, in the pessimistic equilibrium, current employment becomes independent of the state in the previous period. High productivity shocks today do not translate into high employment tomorrow since transitory productivity shocks are no longer propagated across periods. Clearly, if the economy gets stuck in the pessimistic equilibrium with low employment, a sequence of good productivity shocks will not necessarily lift the economy out of it.

To summarize, in the presence of asymmetric information, there always exists a pessimistic equilibrium in which the market for debt breaks down due to adverse selection induced "market for lemons" problem. Relative to the symmetric information case, in the pessimistic equilibrium employment is lower and the risk premium is higher. Moreover, both the evolution of debt and employment become non-state contingent in this equilibrium.

6.2.2 Optimistic equilibrium: $\psi^m = 1$

Next we turn to the potential existence of other equilibria. Conditional on existence, one potential equilibrium is particularly interesting. This is the one where the market becomes optimistic and all mature financiers choose to offer their debt on the market, i.e., $\psi^m = 1$. In this subsection we ask whether this equilibrium can exist and, if so, under what conditions.

Recall that a financier with type ψ^i debt would choose to sell his debt on the market if and only if

$$\frac{q(1-\theta)\hat{\psi}_t}{\frac{p_t^s}{p_t^b} - \theta} \geq \psi^i,$$

where $\hat{\psi}$ is the conjectured average quality of debt sold on the market. Combining this with the consistency condition (equation 20) shows that for the holder of the highest quality debt ($\psi^i = 1$) to place his debt on the market it must be that

$$\frac{q(1-\theta)\bar{\psi}}{\frac{p_t^s}{p_t^b} - \theta} \geq 1,$$

where we have used the fact that when all debt is sold on the market the average quality of debt is given by

$$\hat{\psi}_t\Big|_{\psi_t^m=1} = \int_0^1 \psi f(\psi) d\psi = \bar{\psi}.$$

Recall that in the asymmetric information case $\frac{\tilde{p}_t^s}{\tilde{p}_t^b} = \tilde{\phi}^p(d_{t-1}, \psi_t^m)$.

Condition 1: $\frac{q(1-\theta)\bar{\psi}}{\tilde{\phi}^p(d_{t-1},1)-\theta} > 1$.

Proposition 7: As long as Condition 1 holds there will always exist an optimistic equilibrium with $\psi^m = 1$.

Clearly, for the optimistic equilibrium with $\psi^m = 1$ to exist the density function of debt types $f(\cdot)$ must have sufficient mass concentrated near 1, so that Condition 1 is satisfied. The optimistic equilibrium is characterized by full pooling of default risk by all financiers and the market behaves as if there was no asymmetric information. Hence, the endogenous variables of the model behave exactly as in the previously analyzed case of a given exogenous default probability and symmetric information. We call this the optimistic equilibrium as it resembles the equilibrium without adverse selection.[16]

6.2.3   An Example

We now provide a simple example to illustrate the mechanics of the model under asymmetric information. Assume that

$$g(1 - l_t) = (1 - l_t) g^*,$$

where $g^* > 0$ is a positive constant. This makes the home production technology linear in its labor input (or equivalently, it makes the utility from leisure linear in time). The direct implication of this assumption is that $p^s/p^b = g^*$ is now a constant as is the risk premium. Hence, the consistency condition (equation 20) is sufficient to pin down the equilibrium ψ^m. All other endogenous variables at date t can then be determined recursively as functions of ψ_t^m, d_{t-1} and A_t.

It is easy to see that the logic of the multiple equilibria described above continues to apply. $\psi^m = 0$ is always a solution to equation (20) which implies that the pessimistic equilibrium always exists. The relevant condition for the optimistic equilibrium to exist can now be written as

$$\bar{\psi} \geq \frac{g^* - \theta}{q(1 - \theta)}.$$

This amounts to a restriction on the parameters and the density function since $\int_0^1 \psi f(\psi) d\psi = \bar{\psi}$.

---

[16]More generally, under adverse selection there are many possible equilibrium configurations. Whether or not these arise depend on the parameter configuration and the properties of the density function $f(.)$.

As a specific case, assume that $f(\psi)$ follows a uniform distribution in the interval $[0,1]$. Define $\frac{q(1-\theta)\int_0^{\psi^m}\psi f(\psi)d\psi}{F(\psi^m)(g^*-\theta)} \equiv G(\psi^m)$. Under our distributional assumption on f we have

$$G(\psi^m) = \frac{q(1-\theta)}{(g^*-\theta)}\frac{\psi^m}{2}.$$

Note that G is rising in ψ^m but becomes a constant $\frac{q(1-\theta)}{g^*-\theta}$ for all $\psi^m \geq 1$. Figure 10 plots ψ^m and $G(\psi^m)$ against ψ^m. There are three possibilities:

Case 1: $\frac{q(1-\theta)}{g^*-\theta} < 2$: In this case $G(1) = \frac{q(1-\theta)}{2(g^*-\theta)} < 1$. Hence, the pessimistic equilibrium is the only possible equilibrium. This case is depicted by the G^p schedule in the figure.

Case 2: $\frac{q(1-\theta)}{g^*-\theta} > 2$: In this case, $G(1) = \frac{q(1-\theta)}{2(g^*-\theta)} > 1$. There are two equilibria in this case – the pessimistic equilibrium with $\psi^m = 0$ and the optimistic equilibrium with $\psi^m = 1$. This case is depicted by the G^o schedule in the figure.

Case 3: $\frac{q(1-\theta)}{g^*-\theta} = 2$: In this case $G(\psi^m) = \psi^m$. Hence the G function coincides with the 45 degree line. There are thus a continuum of equilibria in the interval $[0,1]$.

Figure 10: Multiple equilibria example
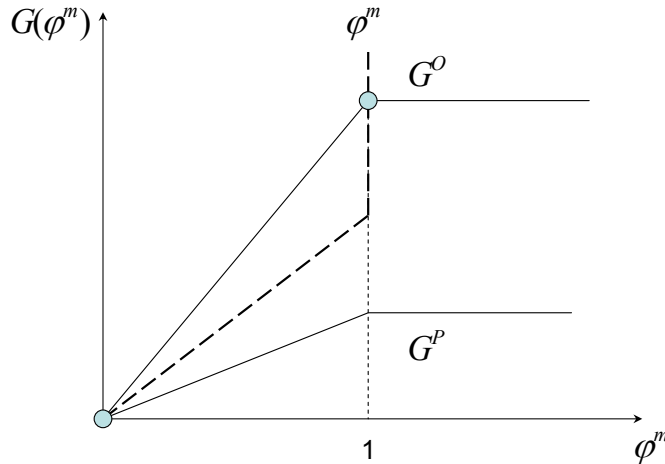
Example: *f* is uniform in [0,1]

6.2.4 A Scenario: Debt Fueled Expansion and Financial Crisis

We now use our previous results to sketch out one potential scenario that could emerge in the context of our model. Suppose we are in Case 2, i.e., $\frac{q(1-\theta)}{g^*-\theta} > 2$. Hence, there

are two equilibria. Recall that in the optimistic equilibrium with complete risk pooling by financiers, good productivity shocks ($A = 1$) will imply greater debt creation today and higher employment and debt tomorrow. A bad productivity shock would lead to debt falling to $\underline{d}$ and then start rising again in response to good states or staying at $\underline{d}$ in bad states.

Suppose our model economy starts off with some debt $d_0 < \bar{d}$. Assume that the economy is initially in the optimistic equilibrium. A sequence of good productivity shocks will thus induce rising debt and rising employment. Now suppose that at some date t there is an abrupt switch in expectations to pessimism: financiers conjecture that the debt being offered on the market is of the worst possible type, i.e., $\hat{\psi} = 0$. This will immediately lead to the economy switching to the pessimistic equilibrium in which the market for existing debt will freeze up. Consequently, financiers will simply hold their debt to maturity and there will be no insurance on offer for risky production. The increasing riskiness of market employment will induce a precipitous decline in employment below even the autarky level.

One way of thinking about a map between our model and financial crisis episodes is to view the switch in the equilibrium from the optimistic to the pessimistic as the financial crisis episode. Viewed through this lens, a financial crisis episode in our structure is particularly debilitating because, as long as the economy stays in the pessimistic equilibrium, good productivity shocks are not going to help raise employment and expected output. Moreover, in this equilibrium, creating more debt will now make the employment situation even worse as workers will be faced with bigger repayment obligations but no insurance against risky labor income. Thus, a financial crisis in our model can have very large negative effects on employment even if the economy is fortunate enough not to have bad productivity shocks.

6.3 Allowing for consumer durables

As we discussed in the previous section, extending our model to allow for consumer durables did not significantly change the results in the absence of worker heterogeneity. In the presence of worker heterogeneity and adverse selection, does this still remain the case? The answer depends on what a financier can recover from a worker who defaults on his debt. To see this, let us first consider the extreme case where the financier cannot recover anything from a defaulting worker (for example, this would be the case if the foreclosure process is very costly). In such a case, it can be verified that the extension of the model to allowing consumer durable has again no substantial impact on the analysis. In contrast, if the financier could recover the full value of his debt by appropriating the durable goods of a defaulting worker, then the adverse selection problem would not arise. Hence, the key element determining the relevance of the adverse selection problem is not whether debt is used to finance consumer durables, but is instead whether the consumer durables can be used to pay back all the debt or whether the real backing of the debt relies mainly on the workers income. Given that the foreclosure process is viewed as generally very costly, we believe that a substantial fraction of debt is primarily supported by the capacity of the borrower to repay using his income stream, in which case the adverse selection problem described previously can emerge.

7   Productive Investment Projects

In our presentation of motivating facts, we documented that non-residential investment was quite weak over the period 2002-2007 despite low interest rates, low risk premium and high profits. We interpreted this observation as indicating that productive risky investment opportunities were likely quite scarce over this period. In response to this observation, in our model we did not include any option for agents to invest in productive capital in order to illustrate how an economy may function when profits are high and productive investment opportunities are scarce. As we have shown, this leads to a type of risk capital glut whereby risk tolerant individuals try to both lend and insure risk averse households in order to generate a risky portfolio position which has a higher expected return.

In this section we briefly discuss how our results relate to our assumption that productive (but risky) investment opportunities are scarce. To this end, consider an extension of our baseline model where output now depends not just on employment but also on previous investment in physical capital, denoted by K_t. For simplicity, let us assume that production takes the following form: $y_t = A_t l_t + J_t K_t$, where in the good state (with probability q), $A_t = 1$ and $J_t = J$ while in the bad state $A_t = \theta$ and $J_t = 0$. The capital stock K_t arises from output invested at the end of period $t-1$, and it lasts for one period. The case where $J = 0$ corresponds to our previous analysis. What happens if $J > 0$? Obviously, the answer depends on the value of J. If J is sufficiently small, then this investment opportunity is not attractive and it can be verified that the previous analysis remains unchanged. However if J is sufficiently large, then the equilibrium behavior can be quite different. We illustrate this using the case without any default risk. Proposition 8 summarizes the main implications of allowing for highly productive real investment opportunities in that version of the model.

Proposition 8:   When output takes the form $y_t = A_t l_t + J_t K_t$, there exist a $\bar{J}$, such that if $J \geq \bar{J}$ then equilibrium behavior is characterized by: (1) young financiers investing all their resources into capital accumulation, and there is no debt accumulation (financiers do not lend to workers). Moreover, financiers simply hold this asset to maturity; and (2) Employment and the risk premium are constant over time.

Proposition 8 indicates that the behavior of an economy may depend heavily on the availability of risky physical investment opportunities. In the case where such opportunities are scarce (as we argued was the case over the 2002-2007 period), the economy may go through a debt fueled expansion that can crash either due to bad profit outcomes ($A = \theta$) or, even worse, due to the emergence of adverse selection which causes the asset market to freeze. In contrast, if risky investment opportunities are abundant and productive, then such a scenario is much less likely. The economy would still be subject to fluctuations, but such fluctuations have much less effect on employment since employment is not as reliant on the availability of risk capital to insure workers. For example, in the case of our model economy with $J \geq \bar{J}$, fluctuations would affect investment and output, but not employment.

While we believe this to be an extreme illustration, this comparison suggests that different cyclical episodes may have very different characteristics depending on the force driving the expansion. If the expansion is driven by good physical investment opportunities – as was

likely the case in the 1990s – the subsequent contraction may be quite different than in the case where the expansion is instead driven by a "glut" of risk taking capital.

8   Conclusion

In this paper we have provided a description of an economic environment in which (a) there is systemic aggregate risk; and (b) there is heterogeneity across agents in terms of their risk tolerance. Our model has focused on two key functions of financial markets: they facilitate lending to households for consumption smoothing purposes and they provide insurance to risk averse agents by shifting production risk to agents with greater risk tolerance. We have shown that in this environment, good productivity shocks can lead to more resources with financiers which, in turn, facilitates greater lending to households and more insurance provisions by these financial intermediaries for risky production as the risk premium declines. Crucially, the resources available to financier act as a form of risk capital which links current states to future economic activity. Hence, financial markets tend to propagate transitory productivity shocks over time through this risk capital.

We have also shown how the presence of default risk and private information with financial intermediaries regarding the default rates on their debt portfolios gives rise to an adverse selection problem in asset market. This can easily lead to multiple equilibria which are sensitive to expectations. Pessimism about the quality of assets on offer in asset markets can lead to a freezing of transactions – a financial crisis – and low employment while optimism about the average quality of assets in the market can give rise to an equilibrium with high employment which resembles one with no adverse selection at all. Financial crises in our model are thus associated with a switch in expectations from optimism to pessimism. These crises are associated with precipitous falls in employment and a shutting down of insurance markets. Perhaps, most strikingly, once an economy gets into the pessimistic equilibrium, productivity shocks are no longer transmitted across periods. Hence, the economy can be stuck in a low employment phase for long periods of time despite continual good productivity realizations.

We have argued that this model provides a candidate description of important underlying forces behind both the financial crisis of 2008 and the prior expansion during the 2001-2007 period which saw rising output and debt/GDP ratios, falling risk premia, and high productivity. While this model clearly omits many elements relevant during this episode, we believe the margins isolated in this paper offer an insight into the deeper or more fundamental causes.

A Appendix

A.1 Proof of Proposition 1

Combining equations ([7](#)) and ([8](#)) gives

$$g'(1-l) = \frac{qu'\left(l - \left(\frac{(1-\theta)d}{g'(1-l)-\theta}\right) + g(1-l)\right) + (1-q)\theta u'(\theta l + g(1-l))}{qu'\left(l - \left(\frac{(1-\theta)d}{g'(1-l)-\theta}\right) + g(1-l)\right) + (1-q)u'(\theta l + g(1-l))}.$$

This expression implicitly defines the equilibrium relationship $l = \phi^l(d)$, $\frac{\partial \phi^l}{\partial d} > 0$ where the sign of the derivative of the ϕ^l function follows from differentiating the key equation above and applying the implicit function theorem. To derive the cut-off level of debt $\tilde{d}$, note that the financier optimality conditions dictate that p^s/p^b is bounded above by $E(A) = q + \theta(1-q)$. From equation ([7](#)) $p^s/p^b = g'(1-l)$ which leads to the solution for $\tilde{l}$. The solution for $\tilde{d}$ then follows directly from the equilibrium relationship $l = \phi^l(d)$. The result that $\phi^l(d) > l^a$ follows trivially from the fact that ϕ^l is increasing in d and that $d = 0$ in the autarkic case.

A.2 Proof of Proposition 2

The proof follows by combining the household optimality condition for optimal labor-leisure $p^s/p^b = g'(1-l)$ with Proposition 1 which showed that ϕ^l is increasing in d and l is bounded above by $\tilde{l}$.

A.3 Proof of Proposition 3

We start by noting that the equilibrium for this economy is described by the solution to the following system of equations:

$$g'(1-l_{t+1}) = \frac{qu'\left(l_{t+1} - \left(\frac{(1-\theta)d_t}{g'(1-l_{t+1})-\theta}\right) + g(1-l_{t+1})\right) + \theta(1-q)u'(\theta l_{t+1} + g(1-l_{t+1}))}{qu'\left(l_{t+1} - \left(\frac{(1-\theta)d_t}{g'(1-l_{t+1})-\theta}\right) + g(1-l_{t+1})\right) + (1-q)u'(\theta l_{t+1} + g(1-l_{t+1}))}$$

$$p_t^d = \frac{qu'\left(c_{gt+1}^o + g(1-l_{t+1})\right) + (1-q)u'\left(c_{bt+1}^o + g(1-l_{t+1})\right)}{u'(c_t^y)}$$

$$p_t^d d_t = f + \gamma F_t$$

where F_t and c_t^y are dependent on the realization of A at date t. Combining the last two equations and substituting in the allocations for F_t and c_t^y in each of the two states yields four difference equations which describe the state-contingent equilibrium dynamics of this economy:

1. If $d_{t-1} \leq \tilde{d}$ and $A_t = 1$, then

$$d_t = \frac{\left(f + \frac{\gamma(1-\theta)d_{t-1}}{g'(1-\phi^l(d_{t-1}))-\theta}\right)u'\left(f + y + \frac{\gamma(1-\theta)d_{t-1}}{g'(1-\phi^l(d_{t-1}))-\theta}\right)}{qu'\left(\phi^l(d_t) - \frac{(1-\theta)d_t}{g'(1-\phi^l(d_t))-\theta} + g(1-\phi^l(d_t))\right) + (1-q)u'(\theta\phi^l(d_t) + g(1-\phi^l(d_t)))}.$$

2. If $d_{t-1} \le \tilde{d}$ and $A_t = \theta$, then

$$d_t = \frac{f u' (f + y)}{q u' \left(\phi^l (d_t) - \frac{(1-\theta)d_t}{g'(1-\phi^l(d_t)) - \theta} + g (1 - \phi^l (d_t)) \right) + (1 - q) u' (\theta \phi^l (d_t) + g (1 - \phi^l (d_t)))}.$$

This expression is independent of d_{t-1} and depends only on the time independent variables f and y and constant parameters. We denote the solution for d in this case by $\underline{d}$.

3. If $d_{t-1} > \tilde{d}$ and $A_t = 1$, then

$$d_t = \frac{\left(f + \gamma \left(d_{t-1} + \frac{(1-q)\tilde{d}}{q} \right) \right) u' \left(f + y + \gamma \left(d_{t-1} + \frac{(1-q)\tilde{d}}{q} \right) \right)}{q u' \left(\phi^l \left(\tilde{d} \right) - d_t - \frac{(1-q)\tilde{d}}{q} + g \left(1 - \phi^l \left(\tilde{d} \right) \right) \right) + (1 - q) u' \left(\theta \phi^l \left(\tilde{d} \right) - (d_t - \tilde{d}) + g \left(1 - \phi^l \left(\tilde{d} \right) \right) \right)}.$$

4. If $d_{t-1} > \tilde{d}$ and $A_t = \theta$, then

$$d_t = \frac{\left(f + \gamma(d_{t-1} - \tilde{d}) \right) u' \left(f + y + \gamma(d_{t-1} - \tilde{d}) \right)}{q u' \left(\phi^l \left(\tilde{d} \right) - d_t - \frac{(1-q)\tilde{d}}{q} + g \left(1 - \phi^l \left(\tilde{d} \right) \right) \right) + (1 - q) u' \left(\theta \phi^l \left(\tilde{d} \right) - (d_t - \tilde{d}) + g \left(1 - \phi^l \left(\tilde{d} \right) \right) \right)}.$$

The last two expressions follow from the fact that when $d_{t-1} > \tilde{d}$ the financiers build risky positions by buying stocks using only $\tilde{d}$ of their initial debt holdings. Beyond this there is no further gain from taking risky positions since the risk premium achieves its lowest feasible level at this point. Hence, financiers hold onto to all debt holdings in excess of $\tilde{d}$. Thus,

$$F_{bt} = d_{t-1} - \tilde{d},$$

$$F_{gt} = \frac{(1 - \theta) \tilde{d}}{\phi^p \left(\tilde{d} \right) - \theta} + d_{t-1} - \tilde{d} = d_{t-1} + \frac{(1 - q) \tilde{d}}{q},$$

where we have used the fact that $\phi^p \left(\tilde{d} \right) = q + \theta (1 - q)$ implies that $\frac{1 - \phi^p \left(\tilde{d} \right)}{\phi^p \left(\tilde{d} \right) - \theta} = \frac{1 - q}{q}$.

In order to proceed further it is convenient to use the definition

$$W_{t+1} \equiv \frac{(1 - \theta) d_t}{g' (1 - l_{t+1}) - \theta}$$

and analyze the equilibrium dynamics of the economy in terms of W. Note that since l_t is a function of d_{t-1}, which is the state variable of the original system, W_t will be the state variable of the new system. We should also note that

$$\frac{dW_{t+1}}{dd_t} = \left[\frac{1 - \theta}{g' (1 - l_{t+1}) - \theta} \right] \left[1 + \left(\frac{dg'' (1 - l_{t+1})}{g' (1 - l_{t+1}) - \theta} \right) \frac{\partial l_{t+1}}{\partial d_t} \right] > 0,$$

which implies that the mapping between W_t and d_{t-1} is monotone. To see the latter inequality, one can totally differentiate the first equation of this section to get

$$\frac{\partial l_{t+1}}{\partial d_t} = \frac{A (1 - \theta) / [g' (1 - l_{t+1}) - \theta]}{-p_t^d u' (c_t^y) g'' (1 - l_{t+1}) + A \left[1 - g' (1 - l_{t+1}) - \frac{(1-\theta)dg''(1-l_{t+1})}{g'(1-l_{t+1})-\theta} \right] + B} > 0$$

41

where

$$A = -q\left[1 - g'\left(1 - l_{t+1}\right)\right] u''\left(g, t+1\right) > 0$$

$$B = -\left(1 - q\right)\left[g'\left(1 - l_{t+1}\right) - \theta\right]^2 u''\left(b, t+1\right) > 0$$

where the inequalities follow from the fact that $\theta < g' < 1$. Note that in the above we are using the notation $u\left(g, t\right)$ to denote utility in the good state in period t and $u\left(b, t\right)$ to denote period-t utility in the bad state.

Using this expression for $\frac{\partial l_{t+1}}{\partial d_t}$ it is easy to check that

$$\left(1 - g'\left(1 - l_{t+1}\right) - \frac{\left(1 - \theta\right) d g''\left(1 - l_{t+1}\right)}{\left[g'\left(1 - l_{t+1}\right) - \theta\right]^2}\right) \frac{\partial l_{t+1}}{\partial d_t} < \frac{1 - \theta}{g'\left(1 - l_{t+1}\right) - \theta}$$

This can be rearranged to give

$$\frac{d g''\left(1 - l_{t+1}\right)}{\left[g'\left(1 - l_{t+1}\right) - \theta\right]} \frac{\partial l_{t+1}}{\partial d_t} + 1 > \frac{\left[1 - g'\left(1 - l_{t+1}\right)\right]\left[g'\left(1 - l_{t+1}\right) - \theta\right]}{1 - \theta} \frac{\partial l_{t+1}}{\partial d_t} > 0.$$

Hence, $\frac{d g''\left(1 - l_{t+1}\right)}{\left[g'\left(1 - l_{t+1}\right) - \theta\right]} \frac{\partial l_{t+1}}{\partial d_t} + 1 > 0$ which implies that $\frac{dW_{t+1}}{dd_t} > 0$.

Using the change of variable from d to W, we can rewrite the equilibrium conditions describing the economy as

$$g'(1 - l_{t+1}) = \frac{qu'\left(l_{t+1} - W_{t+1} + g(1 - l_{t+1})\right) + \theta(1 - q)u'(\theta l_{t+1} + g(1 - l_{t+1}))}{qu'\left(l_{t+1} - W_{t+1} + g(1 - l_{t+1})\right) + (1 - q)u'\left(\theta l_{t+1} + g(1 - l_{t+1})\right)}$$

$$p_t^d = \frac{qu'\left(l_{t+1} - W_{t+1} + g(1 - l_{t+1})\right) + (1 - q)u'\left(\theta l_{t+1} + g(1 - l_{t+1})\right)}{u'\left(c_t^y\right)}$$

$$p_t^d d_t = f + \gamma F_t$$

Totally differentiating the first condition gives

$$\frac{\partial l_{t+1}}{\partial W_{t+1}} = \frac{-q(1 - g')u''(g, t+1)}{-p_t^d u'\left(c_t^y\right) g''\left(1 - l_{t+1}\right) - q\left(1 - g'\right)^2 u''\left(g, t+1\right) - (1 - q)\left(\theta - g'\right)^2 u''\left(b, t+1\right)} > 0.$$

For future reference it useful to note that the above also implies that

$$0 < \left(1 - g'\right) \frac{\partial l_{t+1}}{\partial W_{t+1}} < 1.$$

We shall denote this implicit solution for l_t by $\tilde{\phi}^l\left(W_t\right)$.

Throughout the following analysis we shall maintain Assumption 3 (see Section 5.3 above) so that preferences are given by

$$u(x) = \log x.$$

First, consider the case where $A_t = 1$. In this event one can combine the three equilibrium conditions above along with the equilibrium allocations for F_t and c_t^y in the good state to get

$$W_{t+1} = \left(f + \gamma W_t\right)\left[\frac{u'\left(y + f + \gamma W_t\right)}{qu'\left(l_{t+1} - W_{t+1} + g\left(1 - l_{t+1}\right)\right)}\right].$$

42

In deriving this expression we have used the fact that

$$\frac{(1-q)\, u'\,(b,t+1)}{q u'\,(g,t+1)} = \frac{1 - g'\,(1 - l_{t+1})}{g'\,(1 - l_{t+1}) - \theta}.$$

Under our assumption on preferences the equilibrium difference equation reduces to

$$q W_{t+1} = (f + \gamma W_t)\left[\frac{l_{t+1} - W_{t+1} + g\,(1 - l_{t+1})}{y + f + \gamma W_t}\right]. \tag{22}$$

Totally differentiating this expression gives

$$\frac{dW_{t+1}}{dW_t} = \frac{\gamma \left[\frac{y}{y + f + \gamma W_t}\right] [l_{t+1} - W_{t+1} + g(1 - l_{t+1})]}{\left[1 - \left\{(1 - g'\,(1 - l_{t+1}))\frac{\partial l_{t+1}}{\partial W_{t+1}} - 1\right\}\right][f + \gamma W_t]}$$

As we showed above, $(1 - g')\frac{\partial l_{t+1}}{\partial W_{t+1}} < 1$. Hence the denominator in the expression for $\frac{dW_{t+1}}{dW_t}$ is clearly positive as is the numerator. Thus, $\frac{dW_{t+1}}{dW_t} > 0$. Noting that $\frac{dW_{t+1}}{dd_t} > 0$ then implies that d_t must be an increasing function of d_{t-1} when $A_t = 1$.

Second, recall that from expression 2 (the equilibrium difference equation when $A = \theta$) above that d_t is independent of d_{t-1} when $A_t = \theta$.

Next, straight forward differentiation of the equilibrium difference equations governing the system when $d_{t-1} > \tilde{d}$ (the third and fourth equilibrium difference equations described at the beginning of this section above) shows that $\frac{dd_t}{dd_{t-1}} > 0$ in both cases. This completes the proof of part 1 of Proposition 3. Part 3 follows trivially from part 2.

For part 2 of the Proposition we start by noting that the fixed point for the equilibrium map under $A = 1$ and $d_{t-1} < \tilde{d}$ is derived by first solving for $\bar{W}$ from the expression

$$q\bar{W} = \left(f + \gamma\bar{W}\right)\left[\frac{\tilde{\phi}^l\,(\bar{W}) - \bar{W} + g\left(1 - \tilde{\phi}^l\,(\bar{W})\right)}{y + f + \gamma\bar{W}}\right]$$

and using the definition of W to get

$$\bar{d} = \frac{\bar{W}\left[g'\left(1 - \tilde{\phi}^l\,(\bar{W})\right) - \theta\right]}{1 - \theta}.$$

To see the existence of such a fixed point, first note that from equation (22) $W_{t+1} > 0$ when $W_t = 0$. Next, divide both sides of equation (22) by W_t and to get

$$q\omega_{t+1} = \left(\frac{f}{W_t} + \gamma\right)\left[\frac{\frac{l_{t+1} + g(1 - l_{t+1})}{W_t} - \omega_{t+1}}{\frac{y + f}{W_t} + \gamma}\right].$$

Taking the limit of both sides as $W_t \to \infty$ gives

$$(1 + q)\lim_{W_t \to \infty}\omega_{t+1} = 0.$$

Hence, $\lim_{W_t \to \infty}\omega_{t+1} = 0$. Since $\omega_{t+1} > 0$ around $W_t = 0$ and $\omega_{t+1} = 0$ as W_t goes to infinity, ω_{t+1} must equal one somewhere in the interior of this range for W_t as W_{t+1} is continuous. Hence, there always exists such a fixed point.

43

The equilibrium transition equation for d_t when $A = \theta$ is independent of d_{t-1} (see the second difference equation at the beginning of this appendix). The solution to this expression is also the fixed point for this mapping $\underline{d}$. The fixed points for the equilibrium mappings from d_{t-1} to d_t when $d_{t-1} > \tilde{d}$ are determined analogously from the corresponding difference equations described above.

A.4 Proof of Proposition 4

The consistency condition given by equation (20) is trivially satisfied when $\psi^m = 0$. Employment, for a given debt level d_{t-1}, is now given implicitly by

$$g'(1 - l_t) = \frac{qu'(l_t - E[\psi]d_{t-1} + g(1 - l_t)) + (1 - q)\theta u'(\theta l_t - E[\psi]d_{t-1} + g(1 - l_t))}{qu'(l_t - E[\psi]d_{t-1} + g(1 - l_t)) + (1 - q)u'(\theta l_t - E[\psi]d_{t-1} + g(1 - l_t))}. \quad (23)$$

This has a well defined solution l^p given by

$$l_t^p = \tilde{\phi}^l(d_{t-1}, 0).$$

A.5 Proof of Proposition 5

The proof is straightforward as it follows from differentiating equation (23) with respect to l_t and d_{t-1} and using the implicit function theorem to get $\frac{\partial l_t^p}{\partial d_{t-1}} < 0$. The second statement implies that $l^a \geq l_t^p$ for all t. This follows directly from the fact that $\frac{\partial l_t^p}{\partial d_{t-1}} < 0$ and that $d_{t-1} > 0$ when both financiers and workers coexist while $d_{t-1} = 0$ corresponds to the autarkic case.

A.6 Proof of Proposition 6

The dynamics of debt in this case are given by

$$d_t = \frac{f + \gamma E[\psi]d_{t-1}}{p_t^d},$$

where p_t^d is determined by

$$p_t^d u'(y + f + \gamma E[\psi]d_{t-1}) = E[\psi]\left[qu'(l_{t+1} - d_t + g(1 - l_{t+1})) + (1 - q)u'(\theta l_{t+1} - d_t + g(1 - l_{t+1}))\right].$$

This gives an implicit mapping between $d_t = \phi_p^d(d_{t-1})$. From the previous two equations it is clear that this transition equation is not state contingent. Straightforward differentiation shows d_t is increasing in d_{t-1}.

A.7 Proof of Proposition 7

The proof follows directly by combining Condition 1 with the condition under which the financier with the highest quality debt sells his debt on the market $-q(1 - \theta)\bar{\psi} \geq \frac{p_t^s}{p_t^b} - \theta$.

A.8 Proof of Proposition 8

Begin by defining a $\bar{J}$ using the following condition:

$$\bar{J} = \frac{u'(y)}{qu'(l^a + g(1 - l^a))}$$

where l^a is the autarky level of employment. Furthermore, let K_t^F represent the amount of resources invested into capital accumulation by young financiers at time $t - 1$, and let K_t^W represent the amount of resources invested into capital accumulation by young workers at time $t - 1$. For the case where $J \geq \bar{J}$, the equilibrium determination of l_t and K_t^W is given by the following two time invariant conditions:

$$J = \frac{u'(y - K^w)}{qu'\left(l + \bar{J}K^w + g(1 - l)\right)}$$

and

$$g'(1 - l) = \frac{qu'(l + JK^w + g(1 - l)) + (1 - q)\theta u'(\theta l + g(1 - l))}{qu'(l + JK^W + g(1 - l)) + (1 - q)u'(\theta l + g(1 - l))}.$$

Furthermore, the equilibrium behavior is characterized by no trade in asset markets, as financiers invest all their resources in capital accumulation. This lead to a simple dynamic equation for K_t^F, where by $K_t^F = f + \gamma J K_{t-1}^F$ when the good state arise at time $t - 1$, and $K_t^F = f$ when the bad state arises at $t - 1$. To check that this constitutes an equilibrium, one can verify that these allocations satisfy the financiers and the workers' optimality conditions when asset prices are given by the marginal conditions of the workers. Hence, when $J \geq \bar{J}$, employment and risk premium are constant over time and financiers do not lend to workers.

References

[1] Acharya, V., D. Gale, and T. Yorulmazer, 2009, "Rollover Risk and Market Freezes," mimeo.

[2] Bernanke, Ben, and Mark Gertler, 1989, "Agency Costs, Net Worth and Business Fluctuations," *American Economic Review* 79, pp. 14-31.

[3] Bernanke, Ben, Gertler, Mark and Simon Gilchrist, 2000, "The Financial Accelerator in a Quantitative Business Cycle Framework", in *The Handbook of Macroeconomics*, North-Holland.

[4] Brunnermeier, M., 2009, "Deciphering the 2007-08 Liquidity and Credit Crunch," *Journal of Economic Perspectives* 23, pp. 77-100.

[5] Chan, Yeung L., and Leonid Kogan, 2002, "Catching Up with the Joneses: Hetrogenous Preferences and the Dynamics of Asset Prices", *Journal of Political Economy*, vol. 110, no. 6, pp. 1255-1285.

[6] Carlstrom, Charles T., and Timothy S. Fuerst, 1997, "Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis", *American Economic Review*, vol. 87, no. 5, pp. 893-910.

[7] Diamond, D., and R. Rajan, 2009, "The Credit Crisis: Conjectures About Causes and Remedies," forthcoming in *AER Papers and Proceedings*.

[8] Garleanu, Nicolae and Stavros Panageas, 2008, "Old, Young, Conservative and Bold: The implications of Heterogeneity and Finite Lives for Asset Pricing," mimeo, University of C. Berkeley.

[9] Kiyotaki, Nobuhiro, and John Moore, 1997, "Credit Cycles," *Journal of Political Economy* 105, pp. 211-248.

[10] Korinek, Anton, 2009, "Systemic Risk-Taking: Amplification Effects, Externalities and Regulatory Responses," mimeo, University of Maryland.

[11] Shleifer, Andrei, and Robert W. Vishny, 2009, "Unstable Banking," NBER Working Paper 14943.

[12] Williamson, Stephen, 1987, "Financial Intermediation, Business Failures, and Real Business Cycles," *Journal of Political Economy*, vol. 95, no. 6.