NBER WORKING PAPER SERIES

EFFICIENT RECAPITALIZATION

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Working Paper 14929 http://www.nber.org/papers/w14929

NATIONAL BUREAU OF ECONOMIC RESEARCH 1050 Massachusetts Avenue Cambridge, MA 02138 April 2009

We thank seminar participants at the 2010 American Finance Association Meeting, Baruch College, the CEPR Bank Crisis Prevention and Resolution Conference, the CEPR Gerzensee Corporate Finance Meeting, the CEPR-HSG Corporate Finance and Economic Performance Conference, the Federal Reserve Chicago Bank Structure and Competition Conference, Harvard University, the NBER Asset Pricing and Corporate Finance Meeting, the NBER Monetary Economics Program Meeting, New York University, the New York Federal Reserve Liquidity Working Group, Princeton University, the OeNB Bank Resolution Workshop, the UBC Summer Finance Conference, the University of Binghamton, the University of Chicago, the University of Rochester and our discussants Marco Becht, Arnoud W.A. Boot, Max Bruche, Itay Goldstein, Ron Giammarino, Adriano Rampini, Gustav Sigurdsson, Juan Sole, and Luigi Zingales, for helpful comments and suggestions. An earlier version of this paper was circulated under "Efficient Bank Recapitalization." The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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Efficient Recapitalization Thomas Philippon and Philipp Schnabl NBER Working Paper No. 14929 April 2009, Revised September 2011 JEL No. G01,G2,G28,G33,G38,H0,H2,H81

ABSTRACT

We analyze government interventions to recapitalize a banking sector that restricts lending to firms because of debt overhang. We find that the efficient recapitalization program injects capital against preferred stock plus warrants and conditions implementation on sufficient bank participation. Preferred stock plus warrants reduces opportunistic participation by banks that do not require recapitalization, while conditional implementation limits free riding by banks that benefit from lower credit risk because of other banks' participation. Efficient recapitalization is profitable if the benefits of lower aggregate credit risk exceed the cost of implicit transfers to bank debt holders.

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Philipp Schnabl Leonard N. Stern School of Business Kaufman Management Center 44 West 4th Street, Room 9-76 New York, NY 10012 schnabl@stern.nyu.edu Firms invest too little if they are financed with too much debt. The reason is that the cash flow generated by new investments accrues to existing debt holders if the firm goes bankrupt. As a result, new investments can increase a firm's debt value while reducing its equity value. A firm that maximizes equity value may therefore forgo new investment opportunities and the extent of such under-investment increases as the firm gets close to bankruptcy. This is the well-know debt overhang problem first described in a seminal paper by Myers (1977).

This paper asks whether and how a government should intervene in a financial sector that suffers from debt overhang. We focus on debt overhang in the financial sector because interactions among financial institutions can amplify debt overhang at the aggregate level. Specifically, we analyze a general equilibrium model in which lending to firms is restricted when banks suffer from debt overhang. We assume debt overhang is caused by a negative aggregate shock to bank balance sheets and analyze whether and how the government can improve social welfare in this setting. The objective of the government is to increase socially valuable bank lending while minimizing the deadweight losses from raising new taxes.

We first show that one bank's decision to forgo profitable lending because of debt overhang reduces payments to households, which increases household defaults and thus worsen other banks' debt overhang. As a result, some banks do not lend because they expect other banks not to lend. If an economy suffers from such negative externalities, the social costs of debt overhang exceed the private costs, and the resulting equilibrium is inefficient.

Next, we analyze government interventions in which the government directly provides capital to banks and banks can decide whether to participate in the program. We assume that the government's options are limited. It cannot simply renegotiate with bank debt holders because debt claims are structured to avoid renegotiation and because bank debt holders are highly dispersed. We further assume that the government prefers to avoid regular bankruptcy procedures, possibly because a large-scale restructuring of the financial sector would trigger runs on other financial institutions and would impose large costs on the non-financial sector.

We allow banks to differ along two dimensions: the quality of their existing assets and the quality of their investment opportunities. Asset quality determines the severity of debt overhang, and welfare losses occur when high-quality investment opportunities are not undertaken. We assume that the government cannot observe the banks' investment opportunities and the banks' asset values but the banks can.

We find that government interventions generate two sources or rents for banks: 'macroeconomic' and 'informational' rents. Macroeconomic rents occur because of general equilibrium effects. These rents accrue to banks that do not participate in an intervention but benefit from the reduction in aggregate credit risk because of other banks' participation. As a result, there is a free-rider problem among banks. Informational rents occur because of private information. These rents accrue to banks that participate opportunistically. In general, macroeconomic rents imply that there is insufficient participation in the program, while informational rents mean that there is excessive participation.

We analyze the optimal design of interventions to eliminate both free-riding and opportunistic participation. To address free-riding, the efficient recapitalization policy conditions the implementation of an intervention on sufficient participation by banks. The intuition for this result is that banks have an incentive to coordinate participation because each bank's participation increases asset values in the economy. By conditioning on sufficient participation, the government makes each bank pivotal in whether the intervention is implemented and therefore reduces banks' outside options. In the limit, the government can completely solve the free-rider problem and extract the entire value of macroeconomic rents from banks.

To address opportunistic participation, the efficient recapitalization policy request equity in exchange for cash injections. We find that equity dominates other common forms of intervention, such as asset purchases and debt guarantees, because equity requires banks to share some of their upside with the government, which reduces participation by banks that can invest on their own. We show that the government can further reduce informational rents by asking for warrants at a strike price of bank asset values conditional on survival. Using warrants improves the self-selection of banks into the program: namely, banks that lend only because of the program. In the limit, the government uses preferred stock with warrants to completely eliminate opportunistic participation and extracts the entire value of informational rents from banks.

Finally, the government's cost of the efficient intervention depends on the severity of the debt overhang relative to the macroeconomic rents. Severe debt overhang increases the cost because the efficient intervention provides an implicit subsidy to bank debt holders. Larger macroeconomic rents reduce the cost because they allow the government to extract the value of lending externalities from banks. If the macroeconomic rents are small, then the intervention is costly and the government trades off the benefit of new lending with the deadweight loss of additional taxation. If the macroeconomic rents are large, then the government can recapitalize banks at a profit.

We discuss several extensions of the model. First, our benchmark model assumes a binary asset distribution and we show that all our results go through with a continuous asset distribution if we allow the government to use non-standard warrants that condition the strike price on the realization of bank asset values. However, such non-standard warrants may be difficult to implement in practice and, we therefore conduct a calibration to assess the efficiency loss of using more common interventions. We use data on U.S. financial institutions during the financial crisis of 2007-09 and compare pure equity injections with preferred stock plus standard warrants. We find that preferred stock plus standard warrants significantly outperforms pure equity injections with an efficiency loss that is about two thirds smaller. Second, we argue that the efficient intervention is more likely to succeed if a government starts the implementation with a small number of large banks. The reason is that large banks are more likely to internalize the positive impact of their participation decision on asset values and that a small number facilitates coordination among banks. Third, we show that constraints on cash outlays at the time of the bailout do not affect our results if the government can provide guarantees to private investors. Fourth, we find that heterogeneity among assets within banks generates additional informational rents and, as a result, using equity becomes even more attractive relative to asset purchases. Fifth, we show that deposit insurance decreases the cost of the intervention because the government is partly reducing its own expected insurance payments but it does not change the optimal form of the intervention.

We emphasize three contributions of our analysis. First, the conditional participation requirement can be interpreted as a mandatory intervention. Our paper thus provides a novel explanation for why governments may require participation in recapitalization efforts and why there seems to be insufficient take-up in the absence of such a requirement.¹ Second,

¹Mitchell (2001) reviews the empirical evidence and suggests that there is often too little take-up of government interventions

the preferred stock-warrants combination also limit risk-shifting and therefore emerges as the optimal solution in other studies of optimal security design (Green (1984)). In our model, banks cannot risk shift with their new investments since they are riskless as in Myers (1977), but risk shifting occurs through the reluctance to sell risky assets.² Our paper thus provides a novel mechanism for the optimality of preferred stock with warrants under asymmetric information. Third, other work on bank recapitalization mostly focuses on bank run externalities on the liabilities side of bank balance sheets. In contrast, our model focuses on lending externalities on the asset side of bank balance sheets. Our model therefore provides a novel motivation for government intervention even in the absence of bank runs.

Our results can shed light on the form of bank bailouts during the financial crisis of 2007-2009. In October 2008, the US government decided to inject cash into banks under the Troubled Asset Relief Program. Initial attempts to set up an asset purchase program failed and, after various iterations, the government met with the nine largest US banks and strongly urged *all* of them to participate in equity injections. Even though some banks were reluctant, all nine banks agreed to participate and the intervention was eventually implemented using a combination of preferred stock and warrants.

Our model relates to the discussion on the optimal regulation of financial institutions, and our analysis remains relevant even if one takes into account ex-ante moral hazard. Debt overhang creates negative externalities and, as in other models, it is therefore optimal to impose ex-ante restrictions on debt financing. Moreover, government interventions generate ex-ante moral hazard that may increase the ex-post cost of government interventions. In our model, we take debt overhang as given and rely on other research that links the financial crisis to securitization (Mian and Sufi (2009), Keys, Mukherjee, Seru, and Vig (2010)) and the tendency of banks to become highly levered (Adrian and Shin (2008), Acharya, Schnabl, and Suarez (2010)). We do not model the cost of imposing ex-ante restrictions and therefore cannot solve for their optimal level. However, we note that ex-ante moral hazard is caused by the ex-post provision of rents to banks. Our efficient recapitalization minimizes expost rents to banks, and therefore also minimizes ex-ante moral hazard conditional on any

 $^{^{2}}$ Selling risky assets for cash is formally equivalent to reverse-risk-shfiting. Our result that purchasing risky assets from banks is expensive is based on this insight.

given likelihood of government intervention. Hence, our solution would be part of optimal ex-ante regulations as long as there is a positively probability of bailout, because of time inconsistency issues (Chari and Kehoe (2009)), or because some ex-post bailouts are ex-ante optimal (Keister (2010)).

We note that our model focuses on banks with profitable lending opportunities that have risky debt. If banks have no such lending opportunities ("zombie banks"), there is no reason for the government to recapitalize these banks. However, because of the asymmetric information between the government and banks, the government is never quite sure which bank is a zombie bank and which bank simply suffers from debt overhang. We therefore think of our model as the optimal policy after the government has closed down obvious zombie banks. If the government has sufficient time, it can conduct bank stress tests to identify zombie banks prior to recapitalizing the financial sector.

Our model extends the existing literature on debt overhang. Debt overhang arises because renegotiations are hampered by free-rider problems among dispersed creditors and by contract incompleteness (Bulow and Shoven (1978), Gertner and Scharfstein (1991), and Bhattacharya and Faure-Grimaud (2001)). A large body of empirical research has shown the economic importance of renegotiation costs for firms in financial distress (Gilson, John, and Lang (1990), Asquith, Gertner, and Scharfstein (1994), Hennessy (2004)). Moreover, from a theoretical perspective, one *should* expect renegotiation to be costly for at least two reasons. First, the covenants that protect debt holders from risk shifting (Jensen and Meckling (1976)) are precisely the ones that can create debt overhang. Second, debt contracts are able to discipline managers only because they are difficult to renegotiate (Hart and Moore (1995)). Our model takes renegotiation costs as given and analyzes whether and how the government should intervene in this situation.

Our paper relates to the theoretical literature on bank bailouts. Gorton and Huang (2004) argue that the government can bail out banks in distress because it can provide liquidity more effectively than private investors. Diamond and Rajan (2005) show that bank bailouts can backfire by increasing the demand for liquidity and causing further insolvency. Diamond (2001) emphasizes that governments should only bail out the banks that have specialized knowledge about their borrowers. Aghion, Bolton, and Fries (1999) show that bailouts can be designed so as not to distort ex-ante lending incentives. Farhi

and Tirole (2009) examine bailouts in a setting in which private leverage choices exhibit strategic complementarities due to the monetary policy reaction. Corbett and Mitchell (2000) discuss the importance of reputation in a setting where a bank's decision to participate in a government intervention is a signal about asset values, and Philippon and Skreta (2009) formally analyze optimal interventions when outside options are endogenous and information-sensitive. Mitchell (2001) analyzes interventions when there is both hidden actions and hidden information. Landier and Ueda (2009) provide an overview of policy options for bank restructuring. Bhattacharya and Nyborg (2010) examine bank bailouts in a model where the government wants to eliminate bank credit risk. In contrast, our paper focuses on the form of efficient recapitalization under debt overhang.

Two other theoretical papers share our focus on debt overhang in the financial sector. Kocherlakota (2009) analyzes a model where it is the insurance provided by the government that generates debt overhang. He analyzes the optimal form of government intervention and finds an equivalence result similar to our symmetric information equivalence theorem. Our papers differ because we focus on debt overhang generated within the private sector and we consider the problem of endogenous selection into the government's programs. In Diamond and Rajan (2009) as in our model, debt overhang makes banks unwilling to sell their toxic assets. In effect, refusing to sell risky assets for safe cash is a form of risk shifting. But while we use this initial insight to characterize the general form of government interventions, Diamond and Rajan (2009) study its interactions with trading and liquidity. In their model, the reluctance to sell leads to a collapse in trading which increases the risks of a liquidity crisis.

The paper also relates to the empirical literature on bank bailouts. Allen, Chakraborty, and Watanabe (2009) provide empirical evidence consistent with the main predictions of our model: they find that interventions work best when they target equity injections into the banks that have material risks of insolvency. Giannetti and Simonov (2009) find that bank recapitalizations result in positive abnormal returns for the clients of recapitalized banks as predicted by our debt overhang model. Glasserman and Wang (2009) develop a contingent claims framework to estimate market values of securities issued during bank recapitalizations such as preferred stock and warrants.

Finally, our paper also relates to the literature on macroeconomic externalities across

firms. Lamont (1995) analyzes the importance of macroeconomic expectations in an economy where firms may suffer from debt overhang. In his model the feedback mechanism works through imperfect competition in the goods market and can generate multiple equilibria. In contrast, we focus on optimal government policy in a setup where banks differ in asset quality and investment opportunities. Moreover, we analyze the feedback mechanism through the repayment of household debt to the financial sector. Bebchuk and Goldstein (2009) consider bank bailouts in a global games framework with exogenous strategic complementarities. In contrast, our model endogenizes complementarities across banks and allows for heterogeneity within the financial sector.

The paper proceeds as follows. Section 1 sets up the formal model. Section 2 solves for the decentralized equilibrium with and without debt overhang. Section 3 analyzes macroeconomic rents. Section 4 analyzes informational rents. Section 5 describes two extensions to our baseline model. Section 6 discusses the relation of our results to the financial crisis of 2007-2009. Section 7 concludes.

1 Model

We present a general equilibrium model with a financial sector and a household sector. We refer to all financial firms as banks and we assume that banks own industrial projects. The model has a continuum of households, a continuum of banks, and three dates, t = 0, 1, 2.

1.1 Banks

All banks are identical at t = 0, with existing assets financed by equity and long term debt with face value D due at time 2. At time 1, banks become heterogenous along two dimensions: they learn about the quality of their existing assets and they receive investment opportunities. Figure 1 summarizes the timing, technology, and information structure of the model.

The assets deliver a random payoff a = A or a = 0 at time 2.³ The probability of a high payoff depends on the idiosyncratic quality of the bank's portfolio and on the aggregate

³The assumption that assets pay off 0 in the low state is primarily for notational convenience. As we show in an the working paper version of this paper, we can also allow banks to hold safe assets that pay off S for sure. The only additional assumption is that senior debt holders have covenants in place to prevent the sale of S for the benefit of equity holders. In this case, the model is effectively unchanged where D is replaced by $D \square S$.

performance of the economy. We capture macroeconomic outcomes by the aggregate payoff \bar{a} , and idiosyncratic differences across banks by the random variable ε . At time 1, all private investors learn the realization of ε for each bank. We define the probability of a good outcome conditional on the information at time 1 as:

$$p(\bar{a},\varepsilon) \equiv \Pr(a=A|\varepsilon,\bar{a}).$$

The variables are defined so that the probability $p(\varepsilon, \bar{a})$ is increasing in ε and in \bar{a} . Note that p is also the expected payoff per unit of face value for existing assets of quality ε in the aggregate state \bar{a} . The average payoff in the economy is simply

$$\bar{p}\left(\bar{a}\right) \equiv \int_{\varepsilon} p\left(\bar{a},\varepsilon\right) dF_{\varepsilon}\left(\varepsilon\right),$$

where F_{ε} is the cumulative distribution of asset quality across banks. The variable \bar{a} is a measure of common performance for all banks' existing assets and satisfies the accounting constraint:

$$\bar{p}\left(\bar{a}\right)A = \bar{a}.\tag{1}$$

Banks receive new investment opportunities at time 1. All new investments cost the same fixed amount x at time 1 and deliver income $v \in [0, V]$ at time 2. The payoff v is heterogenous across banks and is known to the financial sector at time 1.⁴ A bank's type is therefore defined by ε , the bank-specific deviation of asset quality from average bank asset quality, and v, the quality of its investment opportunities.

Let *i* be an indicator for the bank's investment decision: i = 1 if the bank invests at time 1, and zero otherwise. The decision to invest depends on the banks's type and on the aggregate state, so we have $i(\varepsilon, v, \bar{a})$. Banks must borrow an amount *l* in order to invest. We normalize the banks' cash balances to zero, so that the funding constraint is $l = x \cdot i$. We will later allow the government to inject cash in the banks to alleviate this funding constraint. At time 2 total bank income *y* is:

$$y = a + v \cdot i,$$

There are no direct deadweight losses from bankruptcy. Let r be the gross interest rate between t = 1 and t = 2. Under the usual seniority rules at time 2, we have the following

⁴As in the original Myers (1977) model.

payoffs for long term debt holders, new lenders, and equity holders:

$$y^D = \min(y, D); \quad y^l = \min(y \Box y^D, rl); \quad y^e = y \Box y^D \Box y^l.$$

We assume that banks suffer from debt overhang, or equivalently, that long term debt is risky.

Assumption A1 (Risky Debt): V < D < A.

Under assumption A1, in the high payoff state (a = A) all liabilities are fully repaid $(y^D = D)$ and $y^l = rl$ and equity holders receive the residual $(y^e = y \Box D \Box rl)$. In the low payoff state (a = 0) long term debt holders receive all income $(y^D = y)$ and other investors receive nothing: $(y^l = y^e = 0)$. Figure 2 summarizes the payoffs to investors by payoff state.

1.2 Households

At time 0 all consumers are identical. Each consumer owns the same portfolio of long term debt and equity of banks.⁵ They also have various types of loans due to the banks at time 2 with face value A. These loans could be mortgages, auto loans, student loans, credit card debt, or other consumer loans.

At time 1, each consumer receives an identical endowment \bar{w}_1 and they have access to a storage technology which pays off one unit of time-2 consumption for an investment of one unit of time-1 endowment. Consumers can also lend to banks. Consumers are still identical at time 1 and we consider a symmetric equilibrium where they make the same investment decisions. They lend \bar{l} to banks and they store $\bar{w}_1 \Box \bar{l}$. At time 2 they receive income w_2 which is heterogenous and random across households. Let \bar{y}^e , \bar{y}^D , and \bar{y}^l be the aggregate payments to holders of equity, long term debt, and short term debt. The total income of the household is therefore:

$$n_2 = \underbrace{\bar{w}_1 \Box \bar{l}}_{\text{safe storage risky labor income financial income}}^{\bar{w}_1 \Box \bar{l}} + \underbrace{w_2}_{\text{risky labor income financial income}}^{\bar{w}_1 \oplus \bar{l}} + \underbrace{\bar{y}^e + \bar{y}^D + \bar{y}^l}_{\text{financial income}}$$
(2)

The household defaults if and only if $n_2 < A$. There are no direct deadweight losses of default so the bank recovers n_2 in case of default. The aggregate payments (or average

⁵The assumption that all households hold the same portfolio is a simplifying assumption that facilitates the exposition of the model.

payment) from households to banks are therefore:

$$\bar{a} = \int \min\left(n_2, A\right) dF_w\left(w_2\right). \tag{3}$$

Note that the mapping from household debt to bank assets endogenizes the aggregate payoff \bar{a} but leaves room for heterogeneity of banks' assets quality captured by the parameter ε . This heterogeneity is needed to analyze the consequences of varying quality of assets across banks. Finally, we need to impose the market clearing conditions. Let \mathcal{I} be the set of banks that invest at time 1: $\mathcal{I} \equiv \{(\varepsilon, v) \mid i = 1\}$. Aggregate investment at time 1 must satisfy $\bar{l} = \bar{x} (\mathcal{I}) \equiv x \iint_{\mathcal{I}} dF(\varepsilon, v)$ and consumption (or GDP) at time 2 is:

$$\bar{c} = \bar{w}_1 + \bar{w}_2 + \iint_{\mathcal{I}} \left(v \Box x \right) dF\left(\varepsilon, v\right).$$
(4)

2 Equilibrium

2.1 First best equilibrium

We assume that households have sufficient endowment to finance all positive NPV projects.

Assumption A2 (Excess Savings): $\bar{w}_1 > \bar{x}(1_{v>x})$

Under assumption A2, the time-1 interest rate is pinned down by the storage technology, which is normalized to 1.

In the first best equilibrium, banks choose investments at time 1 to maximize firm value $V_1 = E_1[a] + v \cdot i \Box E_1[y^l]$ subject to the time 1 budget constraint $l = x \cdot i$, and the break even constraint for new lenders $E_1[y^l] = l$. This implies that firm value $V_1 = E_1[a] + (v \Box x) \cdot i$. Therefore, investment takes place when a banks has a positive NPV project, or equivalently, when v > x.

The unique first best solution is for investment to take place if and only if v > x, irrespective of the value of ε and $E_1[\bar{a}]$. The first best equilibrium is unique and first-best consumption is $\bar{c}^{FB} = \bar{w}_1 + \bar{w}_2 + \int_{v>x} (v \Box x) dF_v(v)$. We can think of the first best as a world in which banks can pledge the PV of new projects to households (no debt overhang). Hence, positive NPV projects can always be financed. Figure 3 illustrates investment under the first best.⁶

⁶Notice the equivalence between maximizing firm value and maximizing equity value with efficient bar-

2.2 Debt overhang equilibrium without intervention

Under debt overhang, we assume that banks maximize equity value $E_1 [y^e | \varepsilon] = E_1 [y \Box y^D \Box y^l | \varepsilon]$ taking as given the priority of senior debt $y^D = \min(y, D)$. Recall that the idiosyncratic shock ε is known at time 1. With probability $p(\bar{a}, \varepsilon)$ the bank is solvent and repays its creditors, and shareholders receive $A \Box D + (v \Box rl) \cdot i$. With probability $1 \Box p(\bar{a}, \varepsilon)$ the bank is insolvent, and shareholders get nothing. Using the break even constraint for new lenders, $r = 1/p(\bar{a}, \varepsilon)$, equity holders solve:

$$\max_{i} p(\bar{a},\varepsilon) \left(A \Box D + \left(v \Box \frac{x}{p(\bar{a},\varepsilon)} \right) \cdot i \right).$$

The condition for investment is $p(\bar{a}, \varepsilon) v > x$, which is more restrictive than under the first best because of debt overhang. The investment domain without government intervention is therefore:

$$\mathcal{I} = I(\bar{a}, 0) \equiv \left\{ (\varepsilon, v) \mid p(\bar{a}, \varepsilon) > \frac{x}{v} \right\}.$$
(5)

The index 0 in the investment set indicates that there is no intervention by the government. At time 2, we aggregate across all banks and we have the accounting identity:

$$\underbrace{\bar{a} + \iint_{\mathcal{I}} v dF(\varepsilon, v)}_{\text{gargate bank income}} = \underbrace{\bar{y}^e + \bar{y}^D + \bar{y}^l}_{\text{payments to households}} \tag{6}$$

Using (2) and (6), we can write household income n_2 as:

$$n_{2} = w_{2} + \bar{w}_{1} + \bar{a} + \iint_{\mathcal{I}} \left(v \Box x \right) dF\left(\varepsilon, v\right)$$
(7)

With the exception of risky time 2 income w_2 , all terms in household income are identical across households. The three unknowns in our model are the repayments from households to banks \bar{a} , the investment set I, and the income of households n_2 . The three equilibrium conditions are therefore (3), (5), (7). We solve the model backwards. First, we examine the equilibrium at time 2, when the investment set is given. We then solve for the equilibrium at time 1, when investment is endogenous.

gaining. We can always write $V_1 = E_1 \left[y \Box y^l \right] = E_1 \left[y^e + y^D \right]$. The maximization program for firm value is equivalent to the maximization of equity value $E_1 \left[y^e \right]$ as long as we allow renegotiation and transfer payments between equity holders and debt holders at time 1.

Equilibrium at time 2

Let us define the sum of time 1 endowment and investment as

$$K(\mathcal{I}) = \bar{w}_1 + \iint_{\mathcal{I}} \left(v \Box x \right) \cdot dF\left(\varepsilon, v\right).$$
(8)

Note that K is fixed at date 2 because investment decisions are taken at time 1. Using equation (8), we can write equation (7) as $n_2 = w_2 + \bar{a} + K$. Using (3) we obtain the equilibrium condition for \bar{a} :

$$\bar{a} = \int \min(w_2 + \bar{a} + K, A) \, dF^w(w_2) \,. \tag{9}$$

We now make a technical assumption:

Assumption 3: $\int \min(w_2 + \bar{w}_1, A) dF^w(w_2) > 0.$

Assumption A3 rules out a multiple equilibria at time 1 Allowing for multiple equilibria complicates the analysis but does not affect our main results.

The following Lemma gives the properties of the aggregate performance of existing assets at time 2:

Lemma 1 There exists a unique equilibrium $\bar{a}(K)$ at time 2. Moreover, \bar{a} is increasing and concave in K.

Proof. The slope on the left hand side of equation (9) is 1. The slope on the right hand side is $F^w(\hat{w}_2) \in [0,1]$ where $\hat{w}_2 = A \Box \bar{a} \Box K$ is the income of the marginal household (the differential of the boundary term is zero since the integrated function is continuous). There is therefore at most one solution. Moreover, under assumption A3 the RHS is strictly positive when $\bar{a} = 0$. When $\bar{a} \to \infty$ the RHS goes to A, which is finite. Therefore the equilibrium exists and is unique. At the equilibrium, the slope of the RHS must be strictly less than one, so the solution must satisfy $F(\hat{w}_2) < 1$. The comparative statics with respect to K is:

$$\frac{\partial \bar{a}}{\partial K} = \frac{F\left(\hat{w}_{2}\right)}{1 \Box F\left(\hat{w}_{2}\right)} > 0$$

So the function \bar{a} is increasing in K. Moreover we have

$$\frac{\partial \hat{w}_2}{\partial K} = \Box \mathbf{1} \Box \frac{\partial \bar{a}}{\partial K} < 0$$

Since \hat{w}_2 is decreasing in K, the slope of \bar{a} is decreasing and the function is concave.

The shape of the function \bar{a} is intuitive because the impact of additional income only increases payment of households in default. Hence, if the share of households in default decreases with income K, the impact of additional income K decreases.

Equilibrium at time 1

We can now turn to the equilibrium at time 1. We have just seen in equation (9) that \bar{a} increases with K at time 2. At time 1, K depends on the anticipation of \bar{a} because investment depends on the expected value of existing assets through the debt overhang effect. To see this, let us rewrite equation (8) as:

$$K(\bar{a}) = \bar{w}_1 + \iint_{\varepsilon > \hat{\varepsilon}(\bar{a}, v)} (v \Box x) dF(\varepsilon, v), \qquad (10)$$

The cutoff $\hat{\varepsilon}$ is defined implicitly by $p(\bar{a}, \hat{\varepsilon}) v = x$, which implies $\frac{\partial \hat{\varepsilon}}{\partial \bar{a}} = \Box \frac{\partial p/\partial \bar{a}}{\partial p/\partial \varepsilon}$ and therefore:⁷

$$\frac{\partial K}{\partial \bar{a}} = \int_{v > x} \left(v \Box x \right) \frac{\partial p}{\partial \bar{a}} \frac{f\left(v, \hat{\varepsilon}(v)\right)}{\partial p / \partial \varepsilon} dv$$

This last equation shows that K is increasing in \bar{a} since all the terms on the right-handside are positive. The economic intuition is straightforward. When banks anticipate good performance on their assets, they are less concerned with debt overhang and are more likely to invest. The sensitivity of K to \bar{a} depends on the extent of the NPV gap $v \Box x$, the elasticity of p to \bar{a} , and the density evaluated at the boundary of marginal banks (the term $\partial p/\partial \varepsilon$ is simply a normalization given the definition of ε). Figure 4 illustrates investment under the debt overhang equilibrium.

The important question here is whether the equilibrium is efficient. The simplest way to answer this question is to see if a pure transfer program can lead to a Pareto improvement. This is what we do in the next section.

2.3 Debt overhang equilibrium with cash transfers

We study here a simple cash transfer program. The government announces at time 0 that it gives $m \ge 0$ to each bank. The government raises the cash by imposing a tax m on

⁷We can restrict our analysis to the space where v > x since from (5) we know that there is no investment outside this range.

households' endowments \bar{w}_1 . The deadweight loss from taxation at time 1 is χm . Non distorting transfers correspond to the special case where $\chi = 0$.

Consider the investment decision for banks. Banks receive cash injection m. It is straightforward to show that if a bank is going to invest, it will first use its cash m, and borrow only $x \square m$. The break even constraint for new lenders remains $r = 1/p(\bar{a}, \varepsilon)$. If the bank does not invest it can simply keep m on its balance sheet. Equity holders therefore maximize:

$$\max_{i} p(\bar{a},\varepsilon) \left[A \Box D + i \cdot \left(v \Box \frac{x \Box m}{p(\bar{a},\varepsilon)} \right) + (1 \Box i) \cdot m \right].$$

This yields the investment condition $p(v \square m) > x \square m$ which defines the investment domain:

$$\mathcal{I} = I(\bar{a}, m) \equiv \left\{ (\varepsilon, v) \mid p(\bar{a}, \varepsilon) > \frac{x \Box m}{v \Box m} \right\}.$$
(11)

Households do not care about transfers because they are residual claimants: what they pay as taxpayers, they receive as bond and equity holders. We therefore only need to modify the definition of K to include the deadweight losses at time by replacing \bar{w}_1 by $\bar{w}_1 \Box \chi m$ in equation (8). Conditional on K, the equilibrium at time 2 is unchanged and equation (9) gives the same solution $\bar{a}(K)$. At time 1 we now have:

$$K(\bar{a},m) = \bar{w}_1 + \iint_{I(\bar{a};m)} (v \Box x) f(v,\varepsilon) \, d\varepsilon dv \Box \chi m.$$
(12)

The cutoff $\hat{\varepsilon}$ is defined implicitly by $p(\bar{a},\hat{\varepsilon})(v \Box m) = (x \Box m)$. The system is therefore described by the increasing and concave function $\bar{a}(K)$ in (9) which implies $d\bar{a} = \bar{a}_K dK$ and the function $K(\bar{a}, m)$ in (12) which implies $dK = K_{\bar{a}}d\bar{a} + K_m dm.^8$

At this point, we need to discuss briefly the issue of multiple equilibria. Without debt overhang, K would not depend on \bar{a} and there would be only one equilibrium. With debt overhang, however, there is a positive feedback between investment, the net worth of households, and the performance of outstanding assets. We can rule out multiple equilibria when $\bar{a}_K K_{\bar{a}} < 1$. A simple way to ensure unicity is to have enough heterogeneity in the economy (either in labor income, or in asset quality). When the density f is small, the slope of K is also small, and the condition $\bar{a}_K K_{\bar{a}} < 1$ is satisfied.⁹ Since multiple equilibria are

⁸We are using the standard notations $\bar{a}_K = \frac{\partial \bar{a}}{\partial K}$ and $K_{\bar{a}} = \frac{\partial K}{\partial \bar{a}}$. ⁹In any case, multiple equilibria simply correspond to the limiting case when $\bar{a}_K K_{\bar{a}}$ goes to one, and, as will be seen shortly, they only reinforce the efficiency of government interventions.

not crucial for the insights of this paper, we proceed under the assumption that the debt overhang equilibrium is unique.

The impact of cash injection m on average repayment \bar{a} is

$$\frac{d\bar{a}}{dm} = \frac{\bar{a}_K K_m}{1 \ \Box \ \bar{a}_K K_{\bar{a}}}$$

and from (4), we see that consumption at time 2 satisfies

$$d\bar{c} = dK(\bar{a}, m) = \frac{K_m}{1 \Box \bar{a}_K K_{\bar{a}}} dm.$$

From the definition of the cutoff we get $\frac{\partial \hat{\varepsilon}}{\partial m} \frac{\partial p}{\partial \varepsilon} = \Box \frac{(v \Box x)}{(v \Box m)^2}$. Differentiating (12) we therefore have:

$$\frac{\partial K}{\partial m} = \int_{v>x} \frac{(v \Box x)^2}{(v \Box m)^2} \frac{f(v,\hat{\varepsilon})}{\partial p/\partial \varepsilon} dv \Box \chi.$$
(13)

The sensitivity of K to m increases in the NPV gap $v \Box x$ and the density evaluated at the boundary of marginal banks and decreases in deadweight loss of taxation χ . Importantly, the equilibrium always improves when $\chi = 0$, which shows that the decentralized equilibrium is not efficient.

Proposition 1 The decentralized equilibrium under debt overhang is inefficient. Non distorting transfers from households to banks at time 1 lead to a Pareto superior outcome.

Figure 5 illustrates investment in the debt overhang equilibrium with cash transfers. If tax revenues can be raised without costs – i.e., if taxes do not create distortions and if tax collection does not require any labor or capital – then these revenues should be used to provide cash to the banks until debt overhang is eliminated. In such a world the issue of efficient recapitalization does not arise, since the government has in effect access to infinite resources.

If government interventions are costly, however, we see from (13) that the benefits of cash transfers are reduced. The overall impact of the cash transfers can even be negative if deadweight losses are large. In such a world, it become critical for the government to minimize the costs of its interventions. This is the issue we address now.

3 Macroeconomic rents

We consider first interventions at time 0 when the government and firms have the same information about uncertain asset values and investment opportunities. This allow us to focus on macroeconomic rents and abstract from informational rents. For interventions at time 0, we show that the critical feature is to allow the government to design programs conditional on aggregate participation. However, the form of the intervention does not matter.

3.1 Government and shareholders

The objective of the government is to maximize the expected utility of the representative agent. All consumers are risk neutral and identical as of t = 0 and t = 1. Hence, the government simply maximizes

$$\max_{\Box} E\left[\bar{c}\left(\Box\right)\right] \tag{14}$$

where \Box describes the specific intervention. Let $\Psi(\Box)$ be the expected net transfer from the government to financial firms. We assume that raising taxes is inefficient and leads to a deadweight loss at time 1 equal to $\chi\Psi(\Box)$. The government takes into account this deadweight loss in its maximization program.

We assume the government can make a take-it-or-leave-it offer to bank equity holders. Equity holders then decide whether they want to participate in the intervention. The government faces the same debt overhang problem as the private sector, that means the government cannot renegotiate the claims of long term debt holders. Moreover, we assume the government can restrict dividend payments to shareholders at time 1. This is necessary because under debt overhang the optimal action for equity holders is to return cash injections to equity holders.

At time 0, banks do not yet know their idiosyncratic asset value ε and investment opportunities v. Hence, all banks are identical and when participation is decided at time 0, we can without loss of generality consider programs where all banks participate. To be concrete, we first consider three empirically relevant interventions: equity injections, asset purchases, and debt guarantees.

In an asset purchase program, the government purchases an amount Z of risky assets

at a per unit price of q. If a bank decides to participate, its cash balance increases by m = qZ and the face value of its assets becomes $A \square Z$. In an equity injection program, the government offers cash m against a fraction α of equity returns. In a debt guarantee program the government insures an amount S of debt newly issued at time 0 for a per unit fee of ϕ . The rate on the insured debt is one and the cash balance of the banks becomes $m = S \square \phi S$.

To study efficient interventions it is critical to understand the participation decisions of equity holders. The following value function will prove useful throughout our analysis. Conditional on a cash injection m, the time 0 value of equity value is:

$$E_{0}\left[y^{e}|\bar{a},m\right] = \bar{p}\left(\bar{a}\right)\left(A \Box D + m\right) + \iint_{I\left(\bar{a},m\right)}\left(p\left(\bar{a},\varepsilon\right)v \Box x + \left(1 \Box p\left(\bar{a},\varepsilon\right)\right)m\right)dF(\varepsilon,v), \quad (15)$$

In this equation, one must of course also recognize that in equilibrium \bar{a} depends on m, as explained earlier. The first term is the expected equity value of long term assets plus the cash injection using the unconditional probability of solvency $\bar{p}(\bar{a})$. The second term is the time 0 expected value of new investment opportunities. This value is positive when the bank's type belongs to the investment set I defined in Equation (11). Note that cash adds an extra term to the expected value of investment opportunities because the cash spent on investment is not given to debt holders at time 2. For bank equity holders, the opportunity cost of using cash for investment is therefore less than the opportunity cost of raising funds from lenders at time 1.

3.2 Free participation

In this section we study interventions in which the implementation of an intervention is independent of a bank's decision. We refer to this setup as interventions with free participation:

Definition 1 An intervention satisfies free participation if the program offered to a bank only depends on that bank's participation decision.

We first study an asset purchase program. Banks sell assets with face value Z and receive cash m = qZ. It is easy to see that the government does not want to buy assets to the point that default occurs in both states. We can therefore restrict our attention to the case where $A \Box Z > D$. After the intervention, the equilibrium takes place as in the decentralized debt overhang equilibrium. We know that the investment domain in the equilibrium where all the banks participate is $I(\bar{a}(m), m)$ defined in (11). From the perspective of the government, we can define the equilibrium investment set as:

$$\hat{I}(m) \equiv I(\bar{a}(m), m),$$

which recognizes that the cash injection determines the macro state \bar{a} . Let $\mathcal{T} = [\varepsilon_{\min}, \varepsilon_{\max}] \times [0, V]$ be the state space. We then have the following Lemma:

Lemma 2 Consider an asset purchase program (Z,q) with free participation at time 0. Let m = qZ. This program implements the investment set $\hat{I}(m)$ at the strictly positive cost:

$$\Psi_0^{free}(m) \equiv m \iint_{\mathcal{T} \setminus \hat{I}(m)} (1 \Box p(\bar{a}(m), \varepsilon)) dF(\varepsilon, v) \Box \iint_{\hat{I}(m) \setminus I(\bar{a}(m), 0)} (p(\bar{a}(m), \varepsilon) v \Box x) dF(\varepsilon, v).$$
(16)

Proof. The cost to the government is $m \Box \bar{p}(\bar{a})Z$. The participation constraint of banks is $E_0[y^e|\bar{a},m] \Box \bar{p}(\bar{a})Z \ge E_0[y^e|\bar{a},0]$. Using (15), we can write a binding constraint as

$$\bar{p}(\bar{a})\left(Z \Box m\right) = m \iint_{\hat{I}(m)} (1 \Box p\left(\bar{a},\varepsilon\right)) dF(\varepsilon,v) + \iint_{\hat{I}(m) \setminus I(\bar{a},0)} (p\left(\bar{a},\varepsilon\right)v \Box x) dF\left(\varepsilon,v\right)$$

From the definition of $\bar{p}(\bar{a})$ we then get the cost function $\Psi_0^{free}(m)$. Finally, both terms on the RHS of (16) are positive. The first is obvious. The second is also positive because $p(\bar{a},\varepsilon) v \Box x$ is negative over the domain $\hat{I}(m) \setminus I(\bar{a},0)$.

We can interpret this result both from the perspective of time 0 and time 1.

At time 0, all banks are identical. The government pays per-asset price q for assets with market price $\bar{p}(\bar{a})$ such that each bank receives a per-asset subsidy of $(q \Box \bar{p}(\bar{a}))$. We note that the participation constraint for equity holders is binding such that they are indifferent between participating and not participating (assuming that asset payoffs \bar{a} remain unchanged). Put differently, the total subsidy of $Z * (q \Box \bar{p}(\bar{a}))$ is an implicit transfer to debt holders. At time 1, banks learn about their asset quality ε and investment opportunity v. The cost of the program can then be interpreted in terms of the right-hand side of equation (16). The first term reflects the transfer of wealth from the government to the debt holders of banks that do not invest: debt value simply increases by $(1 \Box \bar{p}) m$ over the set of banks that do not invest: debt value simply increases by $(1 \Box \bar{p}) m$ over the set of banks that do not invest $\mathcal{T} \setminus \hat{I}(m)$. The second term measures the subsidy needed to induce equity holders to investment over the expanded domain $\hat{I}(m)$ compared to the investment domain $I(\bar{a}, 0)$. This domain is the set of banks that invest only because of the program. The expression is positive because $p(\bar{a}(m), \varepsilon) v < x$ for all banks that only invest because of the program.

We note that the program is always implemented at positive costs. This result comes from the fact that the government provides a positive subsidy to every bank $(q \Box \bar{p}(\bar{a})) Z$ but does not capture the increase in bank asset values from $\bar{p}(\bar{a}(0), 0)$ to $\bar{p}(\bar{a}(m), m)$ under free participation.

We can now examine the optimal form of the intervention. We compare asset purchases with equity injections and debt guarantees.

Proposition 2 Under symmetric information, the type of financial security used in the intervention is irrelevant.

Proof. See Appendix.

Proposition 2 says that an asset purchase program (Z, q) is equivalent to a debt guarantee program with S = Z and $q = 1 \Box \phi$. It is also equivalent to an equity injection program (m, α) , where m = qZ and q and α are chosen such that at time 0 all banks are indifferent between participating and not participating in the program. All programs implement the same investment set $\hat{I}(m)$ and have the same expected cost $\Psi_0^{free}(m)$

The key to this irrelevance theorem is that banks decide whether to participate before they receive information about investment opportunities and asset values. The government thus optimally chooses the program parameters such that bank equity holders are indifferent between participating and not participating. The cost to the government is thus independent of whether banks are charged through assets sales, debt guarantee fees, or equity injections.

3.3 Conditional participation

We now focus on the participation decision. So far we assumed that banks can decide whether to participate independently of other banks' participation decisions. We now allow the government to condition the program offered to one bank on the participation of other banks. We call this a program with conditional participation. In effect, the offer by the government holds only if all banks participate in the program. The key is that if a bank that was supposed to participate decides to drop out, then the program is canceled for all banks. It is straightforward to see that the equivalence result of Proposition 2 holds for conditional programs, and we have the following proposition:

Proposition 3 A program with conditional participation implements the investment set $\hat{I}(m)$ at cost

$$\Psi_{0}^{cond}\left(m\right) = \Psi_{0}^{free}\left(m\right) \square \mathcal{M}\left(m\right)$$

where $\mathcal{M}(m) \equiv E_0[y^e | \bar{a}(m), 0] \Box E_0[y^e | \bar{a}(0), 0] \geq 0$ measures macroeconomic rents.

Proof. The government offers a program that is implemented only if all the banks opt in. If they do, the equilibrium is $\bar{a}(m)$. If anyone drops out, the equilibrium is $\bar{a}(0)$. Let $E[y^g]$ be the expected payments to the government. The participation constraint is $E_0[y^e|\bar{a}(m),m] \Box E[y^g] \ge E_0[y^e|\bar{a}(0),0]$. By definition, we have $E_0[y^e|\bar{a}(0),0] =$ $E_0[y^e|\bar{a}(m),0] \Box \mathcal{M}(m)$. The cost to the government is $mE[y^g]$. Using a binding participation constraint, we therefore obtain $\Psi_0^{cond}(m) = \Psi_0^{free}(m) \Box \mathcal{M}(m)$.

The key point is that free riding occurs because banks do not internalize the impact of their participation on the health of other banks. The program with conditional participation is less costly because the government makes each bank pivotal for the implementation for the program and therefore appropriates the macroeconomic rents created by its intervention

The comparison of a program with free participation relative to a program with conditional participation allows to precisely study the sources of macroeconomic rents $\mathcal{M}(m)$. Let $\Delta_p(\varepsilon) = p(\bar{a}(m), \varepsilon) \Box p(\bar{a}(0), \varepsilon)$. Using equation (15), we can rewrite macroeconomic rents such that

$$\mathcal{M}(m) = \bar{\Delta}_p \left(A \Box D \right) + \iint_{\hat{I}(0)} \Delta_p \left(\varepsilon \right) dF \left(\varepsilon, v \right) + \iint_{I(\bar{a}(m),0) \setminus \hat{I}(0)} \left(p\left(\bar{a}, \varepsilon \right) v \Box x \right) dF \left(\varepsilon, v \right).$$

This expression decomposes the macroeconomic rents to shareholders into three components. The first term is higher repayment rate on assets in place, the second term is the higher expected value of investments that would have been made even without intervention, and the third term is the expected benefit of expanding the equilibrium investment set. Finally, the costs of the conditional participation program can be negative when the macroeconomic rents are large. In this case, the government can recapitalize banks and end up with a profit. We can therefore summarize our results in the following theorem.

Theorem 1 The government must use a conditional participation program in order to capture the macroeconomic value of its intervention. Under symmetric information, the type of security used in the intervention is irrelevant.

We note that this mechanism may be difficult to implement in practice. In particular, when there is a large number of banks and if some bank equity holders decide against participation for reasons outside of our model. Also, there exists an equilibrium in which no bank participates because each bank expects other banks not to participate. We discuss the implementation of this mechanism in the extension section.

4 Informational rents

In this section we consider interventions at time 1, when banks know their types but the government does not. The macroeconomic rents that we have studied in the previous section still exist but we do not need to repeat our analysis. For brevity, we study only programs with free participation and we focus on the consequences of information asymmetry.

4.1 Complete information benchmark

We first discuss participation and investment under perfect information and derive the minimum cost of an intervention. We note that this setting is different from the time 0 setting where banks and the government have the same information but they still face uncertainty about asset values and investment opportunities. Instead we assume that the government is perfectly informed about each bank's asset values and investment opportunities. For example, this would be the case if banks can credibly reveal their information to the government. Under perfect information, the government simply decides which banks should participate and provides enough capital such that bank equity's participation constraint is binding. We can thus provide a general characterization of the minimum cost of any intervention with free participation:

Lemma 3 Consider a program with free participation that implements the investment set \mathcal{I} . Let $\min = \mathcal{I} \setminus I(\bar{a}, 0)$. The cost of the program cannot be lower than:

$$\Psi_{1}^{\min} = \Box \iint_{\Omega^{\min}} \left(p\left(\bar{a}, \varepsilon\right) v \ \Box \ x \right) dF\left(\varepsilon, v\right).$$

Proof. Note that $I(\bar{a}, 0)$ is the set of banks that can invest alone, and ^{min} is the set of types that invest only thanks to the program. The best the government can do with $I(\bar{a}, 0)$ is to make sure they do not participate. Voluntary participation means that equity holders in ^{min} must get at least $p(A \Box D)$. The government and old equity holders must share the residual surplus whose value is

$$p(A \Box D) + p(\bar{a}, \varepsilon) v \Box x$$

Hence the expected net payments to the government must be at least $\iint_{\Omega^{\min}} (p(\bar{a}, \varepsilon) v \Box x) dF(\varepsilon, v)$. These payments are negative by definition of $^{\min}$, and therefore the lower bound Ψ_1^{\min} is strictly positive.

A simple way to understand this result is to imagine what would happen if the government could write contracts contingent on investment. For the shareholders of type (ε, v) , the value of investment is $p(\bar{a}, \varepsilon) v \Box x$, which is negative outside the private investment region $I(\bar{a}, 0)$. If the government has perfect information, it can offer a contract with a typespecific payment contingent on investment. The minimum the government would have to offer type (ε, v) would be $\Box (p(\bar{a}, \varepsilon) v \Box x)$. We define an intervention's informational rents as the subsidy provided to bank equity holders in excess of this amount.

We note that the government cannot simply use observed assets prices to implement the intervention because the expectation of an intervention may in turn affect prices (see Bond, Goldstein, and Prescott (2010) and Bond and Goldstein (2010)). Credit default swap prices of US banks during the financial crisis of 2007 to 2009 provide clear evidence of this issue. Most market participants expected some form of intervention if a crisis became sufficiently severe and indeed the government intervened several times after credit default swaps reached critical levels. Hence, it is unlikely that credit default swaps reflected the probability of default in the absence of government interventions.

4.2 Participation and investment under asymmetric information

We now examine participation and investment under asymmetric information at time 1. We first compare asset purchases, debt guarantees, and equity injections. The objective function of the government is the same as in the previous section. The participation decisions are based on equity value which is now conditional on each bank's type (ε, v) . The structure of the programs is the same as at time 0, but the government must now take into account the endogenous participation decisions of banks. Under free participation, banks opt in if and only if $E_1[y^e|\bar{a}, \varepsilon, v, \Box]$ is greater than $E_1[y^e|\bar{a}, \varepsilon, v, 0]$.

There are several cases to consider: opportunistic participation, inefficient participation, and efficient participation. Consider opportunistic participation first. It happens when a bank takes advantage of a program even though it would have invested without it. We define the net value of opportunistic participation as:

$$U(\bar{a},\varepsilon,v;\Box) \equiv E_1[y^e|\bar{a},\varepsilon,v,\Box,i=1] \Box E_1[y^e|\bar{a},\varepsilon,v,0,i=1].$$
(17)

Consider now inefficient participation. It happens when a bank participates but fails to invest. We define the net value of inefficient participation as:

$$NIP(\bar{a},\varepsilon,v;\Box) \equiv E_1[y^e|\bar{a},\varepsilon,v,\Box,i=0] \Box E_1[y^e|\bar{a},\varepsilon,v,0,i=0]$$
(18)

It is straightforward to show that the government should always prevent inefficient participation, and that it can do so by charging a small fee. We always make sure that out program satisfy NIP < 0 for all types. Finally, efficient participation occurs when a bank that would not invest alone opts in the program. We define the net value of opportunistic participation as:

$$L(\bar{a},\varepsilon,v;\Box) \equiv E_1[y^e|\bar{a},\varepsilon,v,\Box,i=1] \Box E_1[y^e|\bar{a},\varepsilon,v,0,i=0].$$
⁽¹⁹⁾

We will see that U = 0 defines an *upper* participation schedule and that L = 0 defines a *lower* participation schedule (hence our choice of notations).

The participation set of any program \Box is therefore

$$(\bar{a}, \Box) = \{(\varepsilon, v) \mid L(\bar{a}, \varepsilon, v; \Box) > 0 \land U(\bar{a}, \varepsilon, v; \Box) > 0\}.$$
(20)

Note that L > 0 and NIP < 0 implies that there is always investment conditional on participation. The investment domain under the program is the combination of the investment set $I(\bar{a}, 0)$ (banks that would invest without government intervention) and the participation set (\bar{a}, \Box) . With a slight abuse of notation, we define:

$$I(\bar{a},\Box) = I(\bar{a},0) \cup (\bar{a},\Box).$$
(21)

Note that the overlap between the two sets, $I(\bar{a}, 0) \cap {}_1(\bar{a}, \Box)$, represents opportunistic participation. Opportunistic participation is participation by banks that would invest even without the program.

4.3 Comparison of standard interventions

We now compare the relative efficiency of the three standard interventions (described earlier) under asymmetric information. We study first the asset purchase program. The upper participation curve (17) is defined by $U^a(\bar{a},\varepsilon,v;Z,q) = (q \Box p(\bar{a},\varepsilon))Z$. Banks participate only if the price q offered by the government exceeds the true asset value $p(\bar{a},\varepsilon)$. This is the adverse selection problem between the government and the financial sector. The NIP-constraint (18) only requires q < 1, which is always satisfied by efficient interventions. The lower bound schedule (19) is given by $L^a(\bar{a},\varepsilon,v;Z,q) = p(\bar{a},\varepsilon) v \Box x + (q \Box p(\bar{a},\varepsilon)) Z$. The lower- and the upper-schedules define the participation set ${}^a_1(\bar{a},Z,q)$ from (20). The expected cost of the asset purchase program is:

$$\Psi_{1}^{a}(\bar{a},q,Z) = Z \iint_{\Omega^{a}(\bar{a},Z,q)} (q \Box p(\bar{a},\varepsilon)) dF(\varepsilon,v).$$
(22)

Figure 6 shows the investment and participation sets for asset purchases under asymmetric information. The figure distinguishes three regions of interest: efficient participation, opportunistic participation, and independent investment. The efficient participation region comprises the banks that participate in the intervention and that invest because of the intervention. The opportunistic region comprises the banks that participate in the intervention but would have invested even in the absence of the intervention. The independent investment region comprises the banks that invest without government intervention. As is clear from the figure, the government's trade-off is between expanding the efficient participation region and reducing the opportunistic participation region.

From cost equation (22) we see that an asset purchase qZ is less costly than an equivalent a cash transfer qZ for three reasons. First, the independent investment region reduces opportunistic participation without reducing investment. Second, the pricing q < 1 excludes banks that would not invest. Third, the government receives Z in the high-payoff state which lowers the government's cost without affecting investment. Let us now compare asset purchases to debt guarantees:

Proposition 4 Equivalence of asset purchases and debt guarantees. An asset purchase program (Z,q) with participation at time 1 is equivalent to a debt guarantee program with S = Z and $q = 1 \Box \phi$.

Proof. See Appendix.

The equivalence of asset purchases and debt guarantees comes from the fact that both programs make participation contingent on asset quality $p(\bar{a}, \varepsilon)$ but not investment opportunity v. To see this result, consider the upper-bound schedule. If $q = 1 \Box \phi$, banks with asset quality $p \in [1 \Box \phi, 1]$ choose not to participate. Hence, asset purchase program and debt guarantees have the same upper-bound schedule. Next, note that the net benefit of asset purchases is $(q \Box p)$, whereas the net benefit of debt guarantees is $(1 \Box \phi \Box p)$. Hence, asset purchases and debt guarantees have the same lower bound schedule. The NIP constraint for asset purchases is p < 1, which is equivalent to $\phi > 0$. The last step is to show that both asset purchases and debt guarantees have the same cost to the government, which is true since they yield the same net benefit to participants. We can finally compare debt guarantees and asset purchases to equity injections: **Proposition 5** Dominance of equity injection. For any asset purchase program (Z,q) with participation at time 1, there is an equity program that achieves the same allocation at a lower cost for the government.

Proof. See Appendix.

The dominance of equity injection over debt guarantees and asset purchases comes from the fact the equity injections are dependent both on asset quality ε and investment opportunity v. To understand this result, it is helpful to define the function $X(\bar{a}, \varepsilon; m, \alpha)$ as the part of the net benefit from participation that is tied to existing assets:

$$X(\bar{a},\varepsilon;m,\alpha) \equiv (1 \Box \alpha) m \Box \alpha p(\bar{a},\varepsilon) (A \Box D).$$
⁽²³⁾

In words, a participating bank receives net cash injection $(1 \Box \alpha) m$ and gives up share α of the bank's expected equity value $p(\bar{a}, \varepsilon) (A \Box D)$. To compare equity injections with other programs, start by choosing an arbitrary asset purchase program. Then choose $X(\bar{a}, \varepsilon; m, \alpha)$ such that the lower-bound schedule of the asset purchase program coincides with the lowerbound schedule of the equity injection program. Under both programs, equity holders at the lower-bound schedule receive no surplus and are indifferent between participating and not participating. For given level of asset quality ε , the cost of participation for banks with a good investment opportunity v is higher under the equity injection program than under the asset purchase program because the government receives a share in both existing assets and new investments. As a result, there is less opportunistic participation with equity injections than with asset purchases.

Figure 7 shows the participation and investment regions under the equity injection program. The increase in cost of participation relative to the asset purchase program has two effects. First, conditional on participation, the cost to the government is smaller because the government receives a share in the investment opportunity v. Second, there is less opportunistic participation because participation is more costly. As a result, equity injections and asset purchases implement the same level of investment but equity injections are less costly to the government relative to asset purchases.¹⁰ The macroeconomic feedback

¹⁰The final step in the proof is to show that the NIP constraint is the same under both programs. This is true because the equity injection provides lower rents to participating banks than the asset purchase

from equation (12) only reinforces the dominance of equity injection. Finally, we note that participating banks receive informational rents in an equity injection program. It is therefore straightforward to show that equity injections do not achieve the minimum cost under perfect information.

4.4 Efficient interventions

We now analyze the efficient intervention in our setting. In particular, we examine whether an intervention with warrants and preferred stock can eliminate informational rents and achieve the minimum cost under perfect information Under the efficient intervention, the government injects cash m at time 1 in exchange for state contingent payoffs at time 2. New lenders at time 1 must break even and we can without loss of generality restrict our attention to the case where the government payoffs depend on the residual payoffs $y \Box y^D \Box y^l$. As in previous sections, we analyze cost minimization for a given investment set.¹¹

It is far from obvious whether the government can reach the minimum cost under perfect information. The surprising result is that it can do so with warrants and preferred stock.

Theorem 2 Consider the family of programs $\Box = \{m, h, \eta\}$ where the government provides cash m at time 1 in exchange for preferred stock with face value (1 + h)m and a portfolio of $(1 \Box \eta)/\eta$ warrants at the strike price $A \Box D$. These programs implement the same set of investment domains as equity injections, but at a lower cost. In the limit $\eta \to 0$, opportunistic participation disappears and the program achieves the minimum cost:

$$\lim_{\eta \to 0} \Psi_1\left(\bar{a}, \Box\right) = \Psi_1^{\min}.$$

Proof. See Appendix.

Figure 8 shows the investment and participation region under the optimal intervention. The efficient intervention completely eliminates informational rents. The intuition for this result

programs. Hence, if the no-efficient participation holds under the asset purchase program, it also holds under the equity injection program. We can also show that equity programs at time 1 cannot be improved by mixing them with a debt guarantee or asset purchase program. Pure equity programs always dominate. The proof is available upon request.

¹¹In general, the government can offer a menu of contracts to the banks in order to obtain various investment sets. The actual choice depends on the distribution of types F(p, v) and the welfare function W but we do not need to characterize it. We simply show how to minimize the cost of implementing any particular set.

is that the initial shareholders receive the following payment in the high-payoff state:

$$f(y^e) = \min(y^e, A \Box D) + \eta \max(y^e \Box A \Box D, 0)$$
(24)

Shareholders are full residual claimants up to the face value of old assets $A \square D$ and η residual claimants beyond. When η goes to zero, the entire increase in equity value due to investment is extracted by the government via warrants. As a result, the opportunistic participation region disappears and only the banks that really need the capital injection to invest participate in the program.¹²

Four properties of this optimal program are worth mentioning. First, we use preferred stock because it is junior to new lenders at time 1 and senior to common equity, but the program could also be implemented with a subordinated loan. Second, it is important that the government also takes a position that is *junior* to equity holders. The warrants give the upside to the government, which limits opportunistic participation. Third, the use of warrants limits risk shifting incentives since the government owns the upside, not the old equity holders (see, for instance, Green (1984)). Fourth, the use of warrants may allow the government to credibly commit to protecting new equity holders. This may be important for reasons outside the model if investors worry about outright nationalization of the banks.

We also note that our results describe the optimal intervention for a given investment set. The optimal investment set is the solution to the government objection function (14) and depends on the distribution of asset values and investment opportunities $F(\varepsilon, v)$ and the deadweight losses of taxation χ . We note that implementing the optimal investment set may require a menu of programs.

Our results on the efficient intervention can be extended to alternative payoff structures. It is straightforward to allow for uncertainty in asset values in the low payoff state. We think this type of uncertainty is the most relevant for banks because banks usually hold loans and debt securities with a fixed face value A and thus primarily face downside risk. It is clear from the discussion above that such downside risk in the low payoff state does not affect banks' participation and investment choices in the high-payoff state and therefore does not

¹²In practice, there might be a lower bound on ε because the government might not want to own the banks. An approximate optimal program could then be implemented at this lower bound ε . Similarly, the rate h is chosen to rule out inefficient participation (the NIP constraint). In theory, any h > 0 would work, but in practice, parameter uncertainty could prevent h from being too close to zero.

affect our results. We also examine more general asset distributions in the extensions of our model.

5 Extensions

In this section we present five extensions to our baseline model. We consider the consequences of heterogenous assets within banks, continuous assets distributions, deposit insurance, constrains on cash outlays, and the implementation of conditional participation.¹³

5.1 Continuous asset distribution

Our benchmark model assumes a binary payoff structure for assets in place. We can generalize our model to a continuous asset distribution $F(a|\varepsilon)$ over $[0, \infty)$. As before, we assume that ε parameterizes the quality of assets in place. We now discuss how our main results change in this more general setup.

It is clear that our results on conditional participation still hold (Theorem 1). It is relatively straightforward to show that equity injections continue to dominate debt guarantees and asset purchases (Proposition 5). The main challenge is to solve for the efficient mechanism. As we show in the Internet Appendix, a modified version of the preferred stock-warrant combination continues to eliminate all informational rents. The modification is that the strike price of warrants must be set after asset prices are realized. The key point of the implementation is that banks whose assets perform well ex-post are rewarded by a lower dilution of their equity.¹⁴

We note that such warrants are non-standard and may be difficult to implement in practice. Hence, this raises the question about the efficiency loss of using more common interventions such as pure equity injections or preferred stock with standard warrants.

To address this question, we calibrate the model using data from the financial crisis of 2007 to 2009. We model asset payoffs using a beta distribution because this distribution

¹³Other extension on the sale of safe assets is available upon request. This exentions does not generate new insights that justify its inclusion in this section.

¹⁴Technically, by adjusting the strike price based on the realized asset value a, the government can provide the same incentives as in the model with binary asset payoffs. We note that this security design it is perfectly consistent with the assumptions we have made regarding information and contracts. If assets trade, the government only need to use ex-post market prices. Even if assets do not trade, the government can implement the optimal intervention by buying a small random sample of assets, observe the ex-post performance, and set the strike price accordingly.

is well-suited to match recovery rates on asset with a fixed upper bound payoffs (such as bank loans or fixed income securities). We choose the distribution parameters to match five-year credit default swap prices of the six largest financial institutions as of December 2008. We choose these banks because they are representative of the U.S. financial system at that time. We report the CDS prices and the implied price discount in Table 1. The CDS prices vary from 160 to 660 basispoints and the implied price discounts vary from 8% to 28%.

We normalize asset size to one and assume that senior debt represents 50 percent of assets (A = 1, D = 0.5). We assume that the average cost of investment represent 30 percent of assets (x = 0.3). We choose the distribution of investment opportunities v to match the empirical distribution of market to book. Using data prior to the 2008 financial crisis, we find that the median net investment opportunities are 11 percent of assets $(v \Box x = 0.11)$.¹⁵

We consider three levels of interventions: none, intermediate, and complete. The no intervention and complete intervention scenario yield total investment relative to efficient investment of 73% and 100%, respectively. We choose the size of the intermediate intervention to achieve an investment level of 87%. We consider the excess cost of two recapitalization policies: equity injections and preferred stock with standard warrants. The excess cost is computed as a share of total cost under the symmetric information benchmark.¹⁶

We report the result in Table 2. We find that the excess cost of equity injections are about three times larger than the excess cost of preferred stock with warrants. In the intermediate intervention scenario, the excess costs of preferred stock plus warrants are 16% and the excess costs of pure equity injections are 40%. In the complete intervention scenario, the excess costs are 40% and 110%, respectively. These results suggest that the use of warrants significantly reduces the excess cost of government interventions.

5.2 Implementation of conditional participation

The government can capture macroeconomic rents and reduce the cost its intervention by making participation conditional on participation. The implementation of conditional participation may be difficult in practice because with a large number of banks, a single bank

¹⁵All other details of the calibration are available in the Internet Appendix.

¹⁶This benchmark is equivalent to the asymmetric information case and the implementation with nonstandard warrants.

could halt the program's implementation for reasons outside of the model. This raises the question of whether there is a robust mechanism to implement a conditional participation requirement.

We argue that the government can increase the likelihood that a conditional participation requirement be successful by targeting a small number of large banks.

First, a small number of banks facilitates coordination among participants and reduces the likelihood that any bank might deviate for idiosyncratic reasons. It is important to understand that, from the perspective of the government, the free-riding problem is the opposite of the anti-trust problem. The government wants to facilitate communication and coordination among banks.

Second, the government should target the largest banks, for two reasons. First, large banks internalize some of the positive impact of their participation on the macro state \bar{a} and therefore on their own credit risk and funding costs. All else equal, this increases their willingness to participate. Second, the largest banks have (by definition) the greatest impact on the macro state \bar{a} for a given number of participating banks. If the banking sector is relatively concentrated, this allows the government to capture a large share of the macroeconomic rents. After the government captures most of the macroeconomic rents, it can offer the program to smaller banks without conditional participation.

5.3 Constraint on cash outlays

The government objective function trades off the efficiency gains from recapitalization with the deadweight losses from additional taxation. We have assumed that the government uses the net present value of gains and losses in its calculations. However, political economy considerations may impose additional constraints. Specifically, the government may be constrained in terms of cash outlay m at time 0 because of the budgetary approval process.

Such constraints on cash outlays make guarantees (which do not require outlays at time 0) more appealing than fully funded programs. This does not change our fundamental analysis, however, because it is always possible to transform a funded program into a guarantee program. For instance, in the context of the asset purchase program, the government can offer to insure private investor against losses on their assets holdings, instead of directly purchasing assets. Under this guarantee, private investors should be willing to purchase bank assets at face value. The expected cost of this asset purchase insurance is the same as the expected cost of the debt guarantee.

More generally, if cash outlays are a constraint, the government can lever up private money. For instance the government can provide capital (or guarantees) to a funding vehicle that borrows from private investors. The money raised can then be used to purchase equity and other securities. The security design problem is then separated from the timing of cash flows.

5.4 Heterogeneous assets within banks

We consider an extension of our model to allow for asset heterogeneity within banks. Suppose that the face value of assets at time 0 is A + A'. All these assets are ex-ante identical. At time 1, the bank learns which assets are A' and which assets are A. The A assets are just like before, with probability $p(\bar{a}, \varepsilon)$ of A and $1 \Box p(\bar{a}, \varepsilon)$ of 0. The A' assets are worth zero with certainty. The ex-ante problems are unchanged, so all programs are still equivalent at time 0.

The equity and debt guarantee programs are unchanged at time 1. So equity still dominates debt guarantee. But the asset purchase program at time 1 is changed. For any price q > 0 the banks will always want to sell their A' assets. This will be true in particular of the banks without profitable lending opportunities.

Proposition 6 With heterogeneous assets inside banks, there is a strict ranking of programs: equity injection is best, debt guarantee is intermediate, asset purchase program is worse.

The main insight from this extension is that adverse selection across banks is different from adverse selection across assets within banks. Adverse selection within banks increases the cost of the asset purchase program but does not affect the other programs.

5.5 Deposit insurance

Suppose long term debt consists of two types of debt: deposits Δ and unsecured long term debt B such that

$$D = \Delta + B.$$

Suppose that the government provides insurance for deposit holders and that deposit holders have priority over unsecured debt holders. Then the payoffs are are:

$$y^{\Delta} = \min(y, \Delta); y^{B} = \min^{\Box} y \Box y^{\Delta}, B$$

Proposition 7 The costs of time 0 and time 1 programs decrease. The equivalence results and ranking of both time 0 and time 1 programs remain unchanged.

Proof. See Internet Appendix.

The intuition is that the government has to pay out deposit insurance in the low-payoff state. Hence, every cash injection lowers the expected cost of deposit insurance in the lowpayoff state one-for-one. As a result, the government recoups the cash injection both in the high- and low-payoff state. Put differently, a cash injection represents a wealth transfer to depositors and, because of deposit insurance, a wealth transfer to the government. Hence, the equivalence results and the ranking of interventions remain unchanged.

6 Discussion of financial crisis of 2007/09

The financial crisis of 2007-2009 has underlined the importance of debt overhang. Recent empirical work on the financial crisis documents the decline in bank lending (Ivashina and Scharfstein (2008)) and the reduction in investment by financially constrained firms (Campello, Graham, and Harvey (2010)). There is broad agreement among many observers that debt overhang is an important reason for this development (see Allen, Bhattacharya, Rajan, and Schoar (2008), and Fama (2009), among others). There is also a broad agreement that macroeconomic externalities are potentially large and can justify the direct provision of capital by the government (see Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) and Bernanke (2009)).

The crisis has also shown the difficulty of finding effective solutions to the debt overhang problem. Several experts have expressed concerns that existing bankruptcy procedures for financial institutions are insufficient for reorganizing the capital structure. As an alternative, Zingales (2008) argues for a law change that allows for forced debt-for-equity swaps. Coates and Scharfstein (2009) suggest to restructure bank holding companies instead of bank subsidiaries. Ayotte and Skeel (2009) argue that Chapter 11 proceedings are adequate if managed properly by the government. Assuming that restructuring can be carried effectively, these approaches reduce debt overhang at low cost to the government. However, Swagel (2009) argues that the government lacks the legal authority to force restructuring and that changing bankruptcy procedures is politically infeasible once banks are in financial distress. Moreover, concerns for systemic risk and contagion make it difficult to restructure financial balance sheets in the midst of a financial crisis. Aside from the costs of its own failure, the bankruptcy of a large financial institution may trigger further bankruptcies because of runs by creditors and counterparty risks (Heider, Hoerova, and Holthausen (2008)).

Government may therefore decide to recapitalize banks as, for example, the U.S. government in October 2008. Surprisingly, while there was at least some agreement regarding the diagnostic (debt overhang), there was considerable disagreement about the optimal form of government intervention outside restructuring. The original bailout plan proposed by former Treasury Secretary Paulson favors asset purchases over other forms of interventions. Stiglitz (2008) argues that equity injections are preferable to asset purchases because the government can participate in the upside if financial institutions recover. Soros (2009a) also favors equity injections over asset purchases because otherwise banks sell their least valuable assets to the government. Diamond, Kaplan, Kashyap, Rajan, and Thaler (2008) argue that the optimal government policy should be a combination of both asset purchases and equity injections because asset purchases establish prices in illiquid markets and equity injections encourage new lending. Bernanke (2009) suggests that in addition to equity injections and debt guarantees the government should purchase hard-to-value assets to alleviate uncertainty about bank solvency. Geithner (2009) argues that asset purchases are necessary because they support price discovery of risky assets.¹⁷

¹⁷Other observers have pointed out common elements among the different interventions without necessarily endorsing a specific one. Ausubel and Cramton (2009) argue that both asset purchases and equity injections require to put a price on hard-to-value assets. Bebchuk (2008) argues that both asset purchases and equity injections have to be conducted at market values to avoid overpaying for bad assets. Soros (2009b) argues that bank recapitalization has to be compulsory rather than voluntary. Kashyap and Hoshi (2008) compare the financial crisis of 2007-2009 with the Japanese banking crisis and argue that in Japan both asset purchases and capital injections failed because the programs were too small. Scharfstein and Stein (2008) argue that government interventions should restrict banks from paying dividends because, if there is debt overhang, equity holders favor immediate payouts over new investment. Acharya and Backus (2009) suggest that public lender of last resort interventions would be less costly if they borrowed some of the standard tools used in

Our paper make threes contributions to this debate. First, we believe an analytical approach to this question is helpful because it allows the government to implement an interventions in which financial institutions are treated equally and government actions are predictable. This approach is preferable to tailor-made interventions that are more likely to be influenced and distorted by powerful incumbents (see Hart and Zingales (2008), Johnson (2009)). Second, we distinguish the economic forces that matter by providing a benchmark at which the government can recapitalize at a profit and under which the form of government interventions is irrelevant. Under symmetric information, all interventions implement the same level of lending at the same expected costs. Under asymmetric information, our analysis shows how the government can use equity and warrants to minimize the expected cost to taxpayers. Third, our analysis clarifies why government interventions are costly. Under symmetric information, debt holders receive an implicit transfer. Under asymmetric information, participating banks receive informational rents because otherwise they would choose not to participate.

We believe our analysis captures some important considerations made in practice. Regarding macroeconomic externalities, the International Monetary Fund and the European Bank for Reconstruction and Development coordinated an agreement ("Vienna Initiative") among 15 banks to overcome the free-rider problem with regard to their lending in Eastern Europe. Specifically, the banks jointly agreed to roll over credit lines to their Eastern European subsidiaries and the initiative is widely believed to have reduced the impact of the financial crisis on Eastern Europe. This mechanism is strongly suggestive of lending externalities among banks. Regarding the optimal mechanism, Swagel (2009) notes that the terms of the Capital Purchase Program, the first round of U.S. recapitalization efforts in October 2008, consisted of providing cash injections in exchange for preferred stock and warrants. Similarly, the investor Warren Buffet provided \$5 billion to Goldman Sachs in September 2008 in exchange for preferred stock and warrants. This structure is qualitatively consistent with the optimal intervention.

private contracts for lines of credit. , Kacperczyk and Schnabl (2009) describe a government intervention, the Commercial Paper Funding Facility, to support non-financial firms directly.

7 Conclusion

In this paper, we study the efficiency and welfare implications of different government interventions in a standard model with debt overhang. We find that government interventions generate informational and macroeconomic rents for banks. Informational rents accrue to banks that participate in an intervention but do not change their level of investment as a result. Macroeconomic rents accrue to banks that do not participate but benefit from the rise in asset values because of other banks' participation. We show that the efficient intervention minimizes informational rents by using preferred stock with warrants and minimizes macroeconomic rents by conditioning implementation on sufficient bank participation. The first feature allows the government to extract the upside of new investments and the second feature reduces banks' outside options. If macroeconomic rents are large, then the efficient intervention recapitalizes banks at a profit.

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Proof of Proposition 2

Cash injection

The government offers cash m against fraction α of equity capital. The government recognizes that the equilibrium is $\bar{a}(m)$ which yields the investment domain $I^{i}(m)$. At time 0, equity holders participate in the voluntary intervention if

$$(1 \Box \alpha) E_0[y^e | \bar{a}, m] \ge E_0[y^e | \bar{a}, 0].$$
(25)

The cost of the program to the government is

$$\Psi_{0}^{e}(m,\alpha) = m \Box \alpha E_{0}\left[y^{e}|\bar{a},m\right].$$

Because the investment domain does not depend on α , the government chooses equity share α such that the participation constraint (25) binds. Using the participation constraint (25) to eliminate α from the cost function yields

$$\Psi_{0,free}^{e}(m,\alpha) = m \Box \left(E_0 \left[y^e | \bar{a}, m \right] \Box E_0 \left[y^e | \bar{a}, 0 \right] \right).$$

Using expected shareholder value at time 0

$$E_{0}\left[y^{e}|\bar{a},m\right] \Box E_{0}\left[y^{e}|\bar{a},0\right] = \bar{p}\left(\bar{a}\right)m + m \int_{I^{i}(m)} \int_{I^{i}(m)} \left(1 \Box p\left(\bar{a},\varepsilon\right)\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) dF\left(\varepsilon,v\right) dF\left(\varepsilon,v\right) + \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right)v \Box x\right) dF\left(\varepsilon,v\right) dF\left(\varepsilon,v\right)$$

Therefore the cost of the government is

$$\begin{split} \Psi_{0,free}^{e}\left(m,\alpha\right) &= m \Box \alpha E_{0}\left[y^{e}|\bar{a},m\right] \\ &= \left(1 \Box \bar{p}\left(\bar{a}\right)\right) m \Box m \int_{I^{i}(m)} \int_{I^{i}(m)} \left(1 \Box p\left(\bar{a},\varepsilon\right)\right) dF\left(\varepsilon,v\right) \Box \int_{I^{i}(m) \setminus I(\bar{a},0)} \int_{I^{i}(m) \setminus I(\bar{a},0)} \left(p\left(\bar{a},\varepsilon\right) v \Box x\right) dF\left(\varepsilon,v\right) \\ &= \Psi_{0}^{free}\left(m\right) \end{split}$$

Debt guarantee

The government recognizes that the equilibrium conditional on intervention is given by the function $\bar{a}((1 \Box \phi) S)$. Using equation (15), we see that conditional on participation, the equity value at time 0 is $E_0[y^e|\bar{a}, (1 \Box \phi) S] \Box \bar{p}(\bar{a})S$. If a bank opts out, equity value becomes $E_0[y^e|\bar{a}, 0]$. Since participation only depends on $m = (1 \Box \phi) S$, the government chooses the program such that the participation constraint binds:

$$\bar{p}\left(\bar{a}\right)S = \bar{p}\left(\bar{a}\right)m + (1 \Box \phi)S \iint_{I^{i}(m)} (1 \Box p\left(\bar{a},\varepsilon\right))dF(\varepsilon,v) + \iint_{I^{i}(m)\setminus I(\bar{a},0)} (p\left(\bar{a},\varepsilon\right)v \Box x)dF\left(\varepsilon,v\right)$$

The cost to the government is

$$\Psi_{0,free}^{e} = \left(1 \Box \bar{p}\left(\bar{a}\right)\right) S \Box \phi S$$

Plugging the participation constraint into the cost function yields the expected cost $\Psi_0^{free}(m)$ defined in equation (16) The programs is equivalent to an asset purchase program when $Zq = (1 \Box \phi) S$.

Proof of Proposition 4

We omit \bar{a} to shorten the notations but all the calculations are conditional on the equilibrium value of \bar{a} . We must show equivalence along four dimensions: (i) the NIP constraint, (ii) the upper schedule, (iii) the lower schedule, and (iv) the cost function. Upon participation and investment, equity value is

$$E_{1}[y^{e}|i=1; S, \phi] = p(\varepsilon) (A \Box D) + p(\varepsilon) v \Box x + (1 \Box \phi \Box p(\varepsilon)) S$$

Participation without investment yields

$$E_1[y^e|i=0; S, \phi] = p(\varepsilon) (A \Box D \Box \phi S).$$

Now consider the three constraints:

• NIP: $E_1[y^e|i=0; S, \phi] < E_1[y^e|i=0; 0, 0]$ or

 $\phi > 0.$

• Upper schedule: $E_1[y^e|i=1; S, \phi] > E_1[y^e|i=1; 0, 0]$ or

$$U(\varepsilon, v; S, \phi) = (1 \Box \phi \Box p(\varepsilon)) S.$$

• Lower schedule: $E_1[y^e|i=1; S, \phi] > E_1[y^e|i=0; 0, 0]$ or

$$L(\varepsilon, v; S, \phi) = p(\varepsilon) v \Box x + (1 \Box \phi \Box p(\varepsilon)) S$$

Using the notations of the asset purchase program, the participation set is ${}^{a}_{1}(S, 1 \Box \phi)$, the investment domain is $I_{1}^{a}(S, 1 \Box \phi)$ and the expected cost of the program is

$$\Psi_{1}\left(S,1 \Box \phi\right) = \phi S \Box S \int_{\Omega^{a}(S,1 \Box \phi)} \int_{\Omega^{a}(S,1 \Box \phi)} (1 \Box p(\varepsilon)) dF(\varepsilon,v)$$

Now if we set S = Z and $q = 1 \Box \phi$, we see that the *NIP* constraint, the upper and lower schedules, and the cost functions are the same as for the asset purchase program. The two programs are therefore equivalent.

Proof of Proposition 5

Equity value at time 1 with cash injection m is:

$$E_1[y^e|\varepsilon, v, \bar{a}, m] = p(\bar{a}, \varepsilon) \left(A \Box D + m\right) + \mathbb{1}_{(\varepsilon, v) \in I(\bar{a}, m)} \left(p(\bar{a}, \varepsilon) v \Box x + (1 \Box p(\bar{a}, \varepsilon))m\right)$$

We omit \bar{a} to shorten the notations but all the calculations are conditional on the equilibrium value of \bar{a} . We first analyze the equity injection program at time 1. Upon participation and investment, equity value (including the share going to the government) is

$$E_{1}[y^{e}|i=1;m] = p(\varepsilon) (A \Box D) + p(\varepsilon) v \Box x + m$$

Participation without investment yields

$$E_1[y^e|i=0;m] = p(\varepsilon) \left(A \Box D + m\right)$$

Now consider the three constraints

• NIP: $(1 \Box \alpha) E_1[y^e | i = 0; m] < E_1[y^e | i = 0, 0]$ or:

$$(1 \square \alpha) m < \alpha (A \square D)$$

• Upper schedule: $(1 \Box \alpha) E_1[y^e | i = 1; m] \Box E_1[y^e | i = 1; 0]$ or:

$$U^{e} = (1 \Box \alpha) m \Box \alpha (p(\varepsilon) (A \Box D) + p(\varepsilon) v \Box x).$$

• Lower schedule: $(1 \Box \alpha) E_1[y^e | i = 1; m] \Box E_1[y^e | i = 0; 0]$ or:

$$L^{e} = (1 \Box \alpha) (p(\varepsilon) v \Box x + m) \Box \alpha p(\varepsilon) (A \Box D).$$

If we define the function $X(\varepsilon; m, \alpha) \equiv (1 \Box \alpha) m \Box \alpha p(\varepsilon) (A \Box D)$ as in equation (23), we can rewrite the program as:

$$\begin{array}{rcl} L^{e} & = & (1 \Box \alpha) \left(p \left(\varepsilon \right) v \Box x \right) + X \left(\varepsilon ; m, \alpha \right) \\ U^{e} & = & \alpha \left(p \left(\varepsilon \right) v \Box x \right) \Box X \left(\varepsilon ; m, \alpha \right) \end{array}$$

The participation set is

$$^{e}(m,\alpha) = \left\{ (\varepsilon,v) \mid L^{e} > 0 \land U^{e} > 0 \right\}.$$

The cost function is therefore

$$\Psi_{1}^{e}(m,\alpha) = \int_{\Omega^{e}(m,\alpha)} \int_{\Omega^{e}(m,\alpha)} (m \Box \alpha E_{1}[y^{e}|i=1;m]) dF(\varepsilon,v).$$

We can rewrite the cost function such that

$$\Psi_{1}^{e}\left(m,\alpha\right)=\int_{\Omega^{e}\left(m,\alpha\right)}X\left(\bar{a},\varepsilon;m,\alpha\right)dF\left(\varepsilon,v\right)\,\Box\,\alpha\!\int_{\Omega^{e}\left(m,\alpha\right)}\left(p\left(\varepsilon\right)v\,\Box\,x\right)dF\left(\varepsilon,v\right).$$

The following table provides a comparison of the government interventions:

	Asset purchase	Equity injection		
Participation	$^{a}\left(\bar{a},Z,q ight)$	$^{e}\left(\bar{a},m,lpha ight)$		
Investment	$I(\bar{a}) \cup {}^{a}(\bar{a}, Z, q)$	$I\left(\bar{a}\right) \cup {}^{e}\left(\bar{a},m,\alpha\right)$		
NIP-constraint	q < 1	$(1 \square \alpha) m < \alpha (A \square D)$		
Cost function	$\Psi_{1}^{a}\left(\bar{a},Z,q\right)$	$\Psi_{1}^{e}\left(\bar{a},m,\alpha\right)$		

Now let us prove that the equity injection program dominates the other two programs. Take an asset purchase program (Z, q). We are going to construct an equity program that

has same investment at lower cost. To get equity with same lower bound graph we need to ensure that:

$$L^{e}(\varepsilon, v; m, \alpha) = L^{g}(\varepsilon, v; q, Z)$$
 for all ε, v .

It is easy to see that this is indeed possible if we identify term by term: $\frac{\alpha}{1 \Box \alpha} = \frac{Z}{A \Box D}$ and m = qZ. In this case we also have $I^a(\bar{a}, S, \phi) = I^e(\bar{a}, m, \alpha)$. The NIP constraint are also equivalent since $(1 \Box \alpha) m < \alpha (A \Box D) \Leftrightarrow q < 1$.

Now consider the upper bound. Consider the lowest point on the upper schedule of the asset purchase program, i.e., the intersection of $U^e = 0$ with $p(\varepsilon) v \Box x = 0$. At that point $(\tilde{\varepsilon}, \tilde{v})$, we have $p(\tilde{\varepsilon}) = q$ and $\tilde{v} = x/q$. Using the fact that lower bounds are equal to zero, we can write $X(\varepsilon; m, \alpha) = (1 \Box \alpha) (1 \Box p(\varepsilon)) qZ$ for all ε, v . This implies that $X(\tilde{\varepsilon}; m, \alpha) = 0$, and therefore $U^e(\tilde{\varepsilon}, \tilde{v}; m, \alpha) = \alpha (p(\tilde{\varepsilon}) \tilde{v} \Box x) \Box X(\bar{a}, \tilde{\varepsilon}; m, \alpha) = 0$. Therefore the upper schedule $U^e(\varepsilon, v; m, \alpha) = 0$ also passes by this point. But the schedule $U^e(\varepsilon, v; m, \alpha) = 0$ is downward slopping in (ε, v) , so the domain of inefficient participation is smaller (see Figure 7) than in the asset purchase case. Formally, we have just shown that:

$$f(m,\alpha) \subset {}^{g}(S,\phi).$$

As an aside, it is also easy to see that the schedule $U^e(\varepsilon, v; m, \alpha) = 0$ is above the schedule $pv \square x = 0$ so it does not get rid completely of opportunistic participation, but it helps. The final step is to compare the cost functions $\Psi_1^a(q, Z)$ and $\Psi_1^e(m, \alpha)$. By definition of the participation domain, we know that lower bound $L^e(\bar{a}, \varepsilon, v; m, \alpha) > 0$. Therefore:

$$\Box \int_{\Omega^{e}(m,\alpha)} \int_{\Omega^{e}(m,\alpha)} (p(\varepsilon) v \Box x) dF(p,v) < \frac{X(\varepsilon;m,\alpha)}{1 \Box \alpha} \text{ for all } (\varepsilon,v) \in e(m,\alpha)$$

Therefore

$$\Psi_{1}^{e}\left(m,\alpha\right) < \frac{1}{1 \ \square \ \alpha} \int_{\Omega^{e}(m,\alpha)} \int_{X\left(\varepsilon;m,\alpha\right)} dF\left(\varepsilon,v\right) = Z \int_{\Omega^{e}(m,\alpha)} \int_{\Omega^{e}(m,\alpha)} \left(q \ \square \ p\left(\varepsilon\right)\right) dF\left(\varepsilon,v\right)$$

Since $q \Box p(\varepsilon) > 0$ for all $(\varepsilon, v) \in e(m, \alpha)$, and since $e(m, \alpha) \subset a(q, Z)$, we have

$$\Psi^{e}(m,\alpha) < \Psi^{a}(q,Z).$$

Finally, note that the comparison is conditional on equilibrium \bar{a} . However, the equity injection requires lower taxes and therefore lead to higher equilibrium level \bar{a} , only reinforcing our proposition.

Proof of Theorem 2

Consider the equity payoffs for a bank in the program:

$$y^e = \max\left(a \Box D + i \cdot \left(v \Box \frac{x \Box m}{p_{\varepsilon}} \Box m\right) \Box hm, 0\right).$$

In the good state, as soon as $y^e > A \square D$, the warrants are in the money and the number of shares jumps to $1 + \frac{1 \square \eta}{\eta} = \frac{1}{\eta}$. So the old shareholders get only a fraction η of the value beyond $A \square D$. The payoff function for old shareholders is therefore:

$$f(y^e) = \min(y^e, A \Box D) + \eta \max(y^e \Box A \Box D, 0).$$

Old shareholders are full residual claimants up to the face value of old assets $A \square D$ and η residual claimants beyond. Now let us think about their decisions at time 1.

The NIP-constraint is simply h > 0. The value for old shareholders conditional on participation and investment is

$$E_1\left[f\left(y^e\right)|\varepsilon, v, \Box, i=1\right] = p_{\varepsilon}\left(A \Box D + \eta\left(v \Box \frac{x \Box m}{p_{\varepsilon}} \Box (1+h)m\right)\right)$$

The lower schedule (efficient participation) is therefore

$$L(\varepsilon, v; \Box) = \eta \left(p_{\varepsilon} v \Box x + m \left(1 \Box \left(1 + h \right) p_{\varepsilon} \right) \right).$$

For any $\eta > 0$, we can see that the lower schedule is equivalent to that of an equity injection with $\frac{\alpha}{1\square\alpha} = \frac{m(1+h)}{A\square D}$, and that of an asset purchase with m = qZ and $q = \frac{1}{1\square h}$. If we take $h \to 0$ we get the lower bound of a simple cash injection program, with an investment set simply equal to $I(\bar{a}, m)$. In general, we have an investment set \mathcal{I} .

The upper schedule (opportunistic participation) is:

$$U(\bar{a},\varepsilon,v;\Box) = \eta m (1 \Box (1+h) p_{\varepsilon}) \Box (1 \Box \eta) (p_{\varepsilon}v \Box x)$$

When $\eta \to 0$, the upper bound schedule $\{U = 0\}$ converges to the schedule $\{p_{\varepsilon}v \Box x = 0\}$. In this limit, there is no opportunistic participation and

$$\lim_{\eta \to 0} \quad (\Box) = \mathcal{I} \setminus I(\bar{a}, 0) = {}^{\min}$$

Finally, the expected payments to the old shareholders converge to $p_{\varepsilon} (A \Box D)$. So the government receives expected value $p_{\varepsilon} v \Box x + m$ by paying m at time 1. The total cost therefore converges to:

$$\lim_{\eta \to 0} \Psi_1(\bar{a}, \Box) = \Box \iint_{\Omega^{\min}} (p_{\varepsilon} v \Box x) dF(\varepsilon, v) = \Psi_1^{\min}.$$

QED.

Table 1Empirical Distribution of Credit Default Swap Prices

This table provides information on the empirical distribution of credit default swap (CDS) prices. The CDS prices are used for the model calibration. "Spread" is the average five-year CDS on senior debt in basispoints (bp) as of December 2008. "Five-year discount" is the implied value of senior debt computed from CDS prices. "Citi" denotes Citibank, "BoA" denotes Bank of America, "JPM" denotes J.P. Morgan, "AIG" denotes American International Group, "GS" denotes Goldman Sachs, and "MS" denotes Morgan Stanley.

Financial Institution	Citibank	BoA	JPM	AIG	GS	MS
Spread (bp)	250	200	160	660	310	430
5-year discount	0.88	0.91	0.92	0.72	0.86	0.81

Table 2 Calibration Results

This table reports the results of the model calibration. "Intervention" denotes the level of intervention. "Actual/Efficient Investment" denotes actual investment as a share of efficient investement. "Equity Excess Cost" denotes the excess cost of equity injections as a share of total cost under symmetric information. "Warrant Excess Cost" denotes the excess cost of preferred stock with standard warrents (fixed strike price) as a share of total cost under symmetric information.

Intervention	Actual/Efficient Investment	Equity Excess Cost	Warrant Excess Cost
None	73%	0	0
Intermediate	87%	41%	16%
Complete	100%	110%	40%

Fig 1: Information & Technology

This figure plots the information structure and technology of our model. Existing banks assets pay off A with probability p or 0 with probability (1-p) at time 2. At time 1, banks receive a new investment opportunity that requires an investment of x at time 1 and yields a payoff of v at time 2. The blue-shaded circle indicates that banks know the distribution of future asset values and investment opportunities at time 1 but the government does not.



Fig 2: Payoffs

This figure shows the payoffs to bank equity and debt holders at time 2 as a function of the bank's investment decision and the realization of bank asset values. D denotes the face value of senior debt, l denotes the face value of junior debt, r denotes the interest rate on junior debt, and ε denotes asset quality. The other variables are defined in Figure 1.



Fig 3: First Best

This figure shows first-best investment as a function of asset quality and the quality of investment opportunities. The blue-shaded area indicates the set of banks that invest under the first best solution ("Efficient Investment").



Fig 4: Debt Overhang

This figure shows investment under debt overhang. The blue-shaded area indicates the set of banks that invest ("Efficient Investment"). The red-shaded area indicates the set of banks that have a profitable investment opportunity but do not invest because of debt overhang ("Debt Overhang").



Fig 5: Cash at time 0

This figure shows investment after a cash injection at time 0. The grey-shaded area indicates the set of banks that invest even without the cash injection ("Efficient Investment"). The blue-shaded area indicates the set of banks that invest only because of the cash injection ("Efficient Additional Investment"). The red-shaded area indicates the set of banks that have a profitable investment opportunity but do not invest ("Debt Overhang").



Fig 6: Asset Purchase at time 1

This figure shows investment and participation after a time 1 asset purchase program. The set of banks that invest is the same as in Figure 5. The grey-shaded area indicates the set of banks that do not participate but invest ("Efficient No-Participation"). The yellow-shaded area indicates the set of banks that participate but would invest even in the absence of the program ("Opportunistic Participation"). The blue-shaded area indicates the set of banks that participate set of banks that participate and invest only because of the program ("Efficient Participation").



Figure 7: Equity injection at time 1

This figure shows investment and participation after a time 1 equity injection. The investment region is the same as in Figure 5. The participation regions are defined in Figure 6. We note that the "Opportunistic Participation" region shrinks and the "Efficient No-Participation" region expands relative to Figure 6.



Figure 8: Efficient Mechanism

This figure shows investment and participation under the efficient mechanism. The investment region is the same as in Figure 5. The participation regions are defined in Figure 6. We note that the "Opportunistic Participation" region disappears and the "Efficient No-Participation" region expands relative to Figure 7.

