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ABSTRACT

Is there any need to “clean” up a banking system in the midst of a crisis, by closing or recapitalizing weak banks and taking bad assets off bank balance sheets, or can one wait till the crisis is over? We argue that an “overhang” of impaired banks that may be forced to sell assets soon can reduce the current price of illiquid assets sufficiently that weak banks have no interest in selling them. Anticipating a potential future fire sale, cash rich buyers have high expected returns to holding cash, which also reduces their incentive to lock up money in term loans. The potential for a worse fire sale than necessary, as well as the associated decline in credit origination, could make the crisis worse, which is one reason it may make sense to clean up the system even in the midst of the crisis. We discuss alternative ways of cleaning up the system, and the associated costs and benefits.

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In early 2009, the supply of credit in industrial countries appeared to be tightening substantially. Clearly credit quality deteriorates in a recession, which might suggest why banks were reluctant to lend. But lending across the quality spectrum seemed to fall in the last quarter of 2008, with new loans to investment grade borrowers down as much as new loans to below investment grade borrowers (Ivashina and Scharfstein (2009)). And, of course, the recessionary conditions themselves may not have been independent of bank lending.[1]

In what might seem a separate development, banks at that time also held on to large quantities of "Level 2" and "Level 3" assets – assets that were not frequently traded and for which the price was either based on models or largely hypothetical. In many cases, these were assets such as mortgage backed securities, for which a liquid market had existed, but where trading had dried up. The popular view was that there was a "buyers strike", as investors who traditionally had bought these assets had been seared by past negative returns and return volatility, and had abandoned these markets. Of course, an immediate question is why new "vulture" investors would not be attracted to these markets – the notion of a "buyers strike", at least over more than a few days, seems difficult to countenance. After all, there is always a price at which anything with positive value can be sold.

One explanation is that the banks themselves expected the value of these assets to rise and were reluctant to sell. This too seems implausible, at least prima facie. Why should banks have different expectations from the market, especially since the underlying factors driving the fundamental value of these securities, such as the state of housing markets, were common knowledge among those with expertise in trading mortgage backed securities? Another explanation is that banks were reluctant to recognize losses on these illiquid securities, since that would require writing down capital. Yet many of these banks had substantially more book capital than required by regulators. Moreover, the market could

[1] For overviews of the financial crisis, see Adrian and Shin (2008), Brunnermeier (2008), Caballero and Krishnamurthy (2009), Diamond and Rajan (2009), and Gorton (2009).

see the quantity of impaired assets held on bank balance sheets, and must have incorporated estimates of their value into equity prices. It is implausible to argue that bank management were eager to create uncertainty about the true value of their balance sheets, with the additional discount in equity valuation that would be implied, than to sell assets and remove uncertainty.[2]

We argue in this paper that the seizing up of term credit and the overhang of illiquid assets in distressed institutions may not be coincidental; they may have common roots, and have potential explanatory power beyond the Crisis of 2007-2010. The intuition is simple. Let a set of banks have a significant quantity of assets that have a limited set of potential buyers. One example of such an asset is a mortgage backed security which, in an environment where some mortgages have defaulted, can be valued accurately only by some specialized firms. Furthermore, let us assume that with some probability, the banks will need to realize cash quickly in the future. Such a need for liquidity may stem from unusual demands of the banks' customers, who draw on committed lines of credit or on their demandable deposits. It may also stem from panic, as depositors and customers, fearing a bank could fail, pull their deposits and accounts from the bank. Regardless of where the demand for liquidity comes from, it would force banks to sell assets or, equivalently, raise money, quickly. Given that the limited set of potential buyers or lenders for the bank's assets have limited resources, the asset would have to be sold at fire sale prices (as in Shleifer and Vishny (1992) and Allen and Gale (1994)).

One consequence of the fire sale is that it may depress asset values so much that the bank is insolvent. This may precipitate a run on the bank, which may cause more assets to be unloaded on the market, further depressing the price. Equally important, the returns to those who have liquid cash at such times can be extraordinarily high.

[2] Of course, one possibility is that what was going on was a form of forbearance -- if banks recognized all the losses, they would go below the regulatory minimum. Therefore, regulators were willing to suspend disbelief about asset quality so long as banks did not sell assets and make it impossible for regulators to continue the charade. Given that the authorities were attempting to get banks to clean up their balance sheets, and were willing to recapitalize banks up to a point, this explanation does not seem to us the most obvious one.

Folding back to today, the prospect of a future fire sale of the bank's asset can depress its current value – investors need to be enticed through a discount to buy the asset today, otherwise they have an incentive to hold back because of the prospect of buying the asset cheaper in the future. More generally, the high returns potentially available in the future to those who hold cash today can cause them to demand a high return today for parting with that cash today.

This is similar to standard dynamic asset pricing in financial markets and in markets with imperfect arbitrage such as Vayanos-Gromb (2002) or Kondor (2009), where future returns influence current required returns. But the elevated required rate of return now extends to the entire segment of the financial market that has the expertise to trade the security. If this segment also accounts for a significant fraction of the funding for potential new loans, the elevated required rate of return will be contagious and will spread to lending. Illiquidity can depress lending – a feature that may be absent in models where future asset values are uncertain for other reasons. Moreover, the institutional overhang will affect lending not only by distressed banks but also by healthy potential lenders, a feature that distinguishes this explanation from those where the reluctance to lend is based on the poor health of either the bank or its potential borrowers.

More surprising though, the bank's management, knowing that the bank could fail in some states in the future, do not have strong incentives to sell the illiquid asset today, *even though such sales could save the bank.* The reason is simple. By selling the asset today, the bank will raise cash that will bolster the value of its outstanding debt by making it safer. But in doing so, the bank will sacrifice the returns that it would get if the currently depressed value of the asset recovers. Indeed, because the states in which the depressed asset value recovers are precisely the states in which the bank survives, bank management would much rather prefer holding on to the illiquid assets and risking a fire sale and insolvency than selling the asset and ensuring its own stability in the future. This idea is clearly analogous to the risk shifting motive in Jensen and Meckling (1976) and the underinvestment motive in Myers (1977), though the bank "shifts" risk or under invests in our model by refusing to sell an illiquid asset than by taking on, or not taking on, a project.

The ingredients of our model are illiquidity and its effects on pricing combined with the unwillingness of the potentially insolvent owners of equity to take actions equivalent to buying costly insurance. These are well understood issues. What is new in our analysis is how they interact to produce the phenomenon we try to explain. Illiquidity can influence future and thus current returns, but when it leads to the potential insolvency of sellers it can cause the sellers to be unwilling to sell assets today at the price that buyers will pay, thus exacerbating the potential insolvency in the future and the associated price decline.

Our simple model predicts that illiquid banks will be on a "seller's strike." Sellers hold on, not because the available market price is irrationally low compared to its value in the near future, but because the alternative of holding on is more beneficial. The "ask" price it would take to get the bank to sell assets is too high given the price potential buyers will bid. Sales of the illiquid asset therefore dry up.

Our point is very general, and applies beyond this specific crisis. It holds whenever a set of privately owned distressed levered institutions hold any asset (loans, land, securities) that could potentially have a thin future market. These impaired institutions not only act as an overhang over the market, elevating required rates of return and further increasing financial fragility, but will also risk future insolvency by holding on to the assets, further depressing the market. Thus, there is an inherent source of adverse feedback in any financial crisis, which is why cleaning up the financial system may be an important contributor to recovery.

To unfreeze asset and credit markets, the authorities will have to move the holdings of potentially illiquid assets by distressed institutions into safer hands – potentially by forcing timely sales which effectively force the distressed institutions to purchase insurance, or by increasing the capacity of potential buyers to buy those assets. They can also remove the overhang of distressed institutions by recapitalizing them. By stabilizing the financial system and eliminating the possibility of fire sales, the authorities can eliminate the potential for high returns to be made in the market in the future, and increase the relative profitability of lending today, thus increasing its magnitude.

In section I, we present the model. In section II, we examine the sources of illiquidity; in section III we explore extensions to the model to show how the possibility of a market freeze is general. In section IV we explore the effects of several government interventions, especially forced sales. In section V, we relate the paper to the literature, including papers by Acharya, Gale, and Yorulmazer (2009), Allen, Carletti and Gale (2009), Allen and Gale (1994, 1998, 2000, 2003), Allen and Carletti (2008), Bhattacharya and Gale (1988), Bolton, Santos and Scheinkman (2008), Diamond and Rajan (2005), Heider, Hoerova and Holthausen (2009), Holmstrom and Tirole (1998), Shleifer and Vishny (1992, 2009), and Stein (2009). We will describe the relationships once we have described our model. We conclude in section V.

I. The Model

A set of identical banks at date 0 each owns financial assets (for example, mortgage backed securities) that will be worth Z at date 2. The bank is financed with demand deposits (or overnight paper) of face value D, with $Z > D$.[3] For now assume each bank has a local monopoly on financing and can raise a fixed quantity of deposits if it pays an interest rate of 0 (We discuss other motivations for the bank's fixed cost of raising deposits, including possible anticipated government bailouts of depositors or some long-term debt in section II). Depositors can demand repayment at date 1 or date 2. Everyone is risk neutral. We assume until section III that Z is a constant, which means that the assets are not risky when held to maturity. The model's primary implications are clear in this simple setting.

[3] For a model of why this might be the optimal form of financing for a bank in a world where aggregate liquidity shocks are low probability, see Diamond and Rajan (2001).

At date 1, banks face a liquidity shock with probability q, where a fraction f of their depositors withdraw. We will be more explicit about the sources of this shock later. Depositors demand cash (they cannot trade in the financial asset market and will not accept the asset in lieu of cash[4]). The bank will have to sell some of its asset for cash to meet this liquidity demand. The bank can raise money in anticipation of the shock by selling assets at date 0 for P_0 per unit of date-2 face value, or it can sell assets, after the shock has been realized at date 1, for P_1 per unit of date-2 face value. Note that if the liquidity shock does not hit at date 1, the bank will not part with the asset at that date for a per unit price less than 1.

We will describe how the price and trading are determined at date 0, given the date-1 price, first when the bank remains solvent when the liquidity shock hits (or if it has unlimited liability) and then when it becomes insolvent given the shock and has limited liability. We will show that trading can dry up at date 0 in the latter case.

Prices and Trading with Banks that are Solvent Given the Shock

Let us assume there are buyers who are not subject to liquidity shocks (such as banks with more liquid assets or longer term liabilities, private equity, or Warren Buffet) who can buy at date 1 after the shock hits, paying cash. The buyer is indifferent between buying at either date if the price gives him the same expected date 2 payoff per dollar spent, so long as the return is greater than the return on cash (so $P_0 \leq 1$ and $P_1 \leq 1$). The highest date-0 bid price of the buyer solves $\frac{1}{P_0} = q\frac{1}{P_1} + (1-q)$, or

$$P_0^{bid} = \frac{1}{q\frac{1}{P_1} + (1-q)} \qquad (1.1)$$

[4] Depositors could be thought of as unsophisticated and hence unable to accept or trade mortgage backed securities or bank loans.

Now consider the bank's decision on when to sell. If the bank postpones any sale until after the shock has hit at date 1, it will have to sell a fraction η_1 of the asset such that $\eta_1 Z P_1 = fD$, or $\eta_1 = \frac{fD}{ZP_1}$. If $\eta_1 > 1$, then the bank would be insolvent and unable to raise fD. For now, we assume that it is either solvent or it has unlimited liability, so it can raise the necessary amounts (potentially from its other assets) to pay withdrawing depositors and the depositors who stay till date 2. The payoff from selling at date 1 with probability *q* is $q\left[(1-\eta_1)Z-(1-f)D\right]+(1-q)\left[Z-D\right]$, which on substituting for η_1 simplifies to $Z-D-qfD(\frac{1}{P_1}-1)$.

In words, the bank pays an expected "illiquidity" cost of $qfD(\frac{1}{P_1}-1)$ whenever it has to sell at date 1 for a price $P_1 < 1$, which happens with probability *q*. Alternatively, the bank can sell at date 0 for P_0 and hold cash from date 0 to 1, to cover the case where it needs liquidity. If it sells early on date 0, it must sell a fraction of the asset given by $\eta_0 = \frac{fD}{ZP_0}$. The bank's payoff from selling just enough to meet the liquidity need is (note that with probability q the proceeds of sale of the fraction η_0 of the asset exactly pay off the fD of deposits):

$$q\left[(1-\eta_0)Z-(1-f)D\right]+(1-q)\left[(1-\eta_0)Z+\eta_0 P_0 Z-D\right]$$

$$=q\left[(1-\frac{fD}{ZP_0})Z-(1-f)D\right]+(1-q)\left[(1-\frac{fD}{ZP_0})Z+\frac{fD}{ZP_0}P_0 Z-D\right]$$

$$=Z-D-(\frac{1}{P_0}-1)fD. \qquad (1.2)$$

That is, by selling at date 0, the bank will pay the "illiquidity" cost of $(\frac{1}{P_0}-1)\,fD$ up front with certainty, which includes the cost of raising cash even though there might be no actual need. The bank is indifferent between selling at date 0 and date 1 when $(1-\frac{1}{P_0})\,fD = qfD(1-\frac{1}{P_1})$ or

$$P_0^{Ask} = \frac{1}{q\frac{1}{P_1}+(1-q)} \qquad (1.3)$$

This is also the bid price (see (1.1)), so both buyers and sellers are willing to trade on both dates so long as the date 0 price bears this relationship to the (yet-to-be-determined) date 1 price.

We do not model any intrinsic benefit from trade at date 0 as opposed to date 1 so the equilibrium price at date 0 reflects the future prices that can prevail. If we added some small and offsetting reasons for trade across banks and buyers, there would be active trading on both dates.

Limited Liability, Fire sales, and No Trade

We assumed above that the bank was solvent when it had to sell to meet the liquidity shock (or it had unlimited liability). What if the bank becomes insolvent conditional on the liquidity shock at date 1, and has limited liability? Clearly, the banker will never sell at date 0 if he fails even after doing so. Intuitively, the banker, in maximizing the value of equity, will want to maximize the value of the bank's assets conditional on survival. Since the bank survives only in the state with no liquidity shock, and because the asset pays off most when the banker holds it to maturity rather than if he sells it prematurely for a possibly discounted price $P_0 \le 1$, the banker prefers to hold the asset rather than sell it.

Now consider the case where the bank survives if it sells assets at date 0 for P_0 but it fails at date 1 if the liquidity shock occurs because assets are sold at fire sale prices.[5] From our previous analysis, the bank is willing to sell at P_0^{ask} at date 0 if the price allows it to avoid failure and

if $Z-D-(\frac{1}{P^{ask}{}_0}-1)fD \geq (1-q)(Z-D)$, where the right hand is the bank's expected payoff with no asset sales, given it fails conditional on the liquidity shock hitting. This requires $\frac{1}{P^{ask}{}_0} \leq 1+\frac{q(Z-D)}{fD}$,

which simplifies to $P_0^{Ask} \geq \frac{1}{1+q(\frac{Z-D}{fD})}$. We also know that given the price P_1, buyers are willing to pay $P_0^{bid} = \frac{1}{1+q(\frac{1}{P_1}-1)}$. The bid price is less than the ask, that is, no asset is offered for sale at prevailing prices at date 0 if $\frac{1}{P_1}-1 > \frac{Z-D}{fD}$. Simplifying, this condition is $fD > P_1\left[Z-(1-f)D\right]$, which is satisfied if the bank is insolvent conditional on the liquidity shock at date 1 and not having sold assets at date 0. We have

Proposition 1: If the bank is insolvent at date 1 conditional on the liquidity shock, it will never sell the asset at the bid price at date 0, even if by doing so it could remain solvent. No trade will take place for the asset at date 0.

In sum, so long as the "fire sale" price of the asset is so low at date 1 so as to drive the bank into insolvency, and the date 0 price reflects that future fire sale price, there will be no trade at date 0 – the market will freeze up. Intuitively, there is no point selling at date 0 for cash if the banker will not avoid

[5] We can allow the bank debt to be bailed out by the deposit insurance corporation, so long as the banker/equity is wiped out (see later).

failure at date 1 by doing so – the sale simply causes him to accept a discounted value for the asset in all states, including those in which he could hold it to maturity. Even if the banker could avoid failure doing so, he does so by making a transfer to the depositors in the state of the liquidity shock (from value he would have enjoyed if he held the asset to maturity in the state with no liquidity shock). Limited liability allows him to avoid having to make this transfer. Since the date-0 ask price when the bank is always solvent is exactly equal to the bid price, the date-0 ask price, if the bank is insolvent conditional on the shock and enjoys limited liability, has to be higher for the selling bank to be indifferent between selling and not. Hence no trade will occur.

The underlying intuition is a combination of an aggregate liquidity shortage leading to fire sale prices (Allen and Gale (2004), Diamond and Rajan (2005)), and risk shifting (Jensen and Meckling (1976)) or underinvestment (Myers (1977)). The banker focuses on the value he will get conditional on the bank surviving. Rather than selling at the date-0 illiquid value, which transfers some of the value of the proceeds to the depositors (akin to the Myers debt overhang problem), he would rather focus on preserving value in the survival states by holding on to the illiquid asset to maturity (akin to the Jensen and Meckling risk shifting problem). The risk shifting incentives of the banks make them unwilling to sell the assets because they will be giving up their option to put the assets to the debt holders at a low price conditional on the liquidity shock – or put differently, they will pay for insurance against default in some states without benefiting enormously from surviving in those states.

Note that from the banker's perspective, a sale of assets is equivalent to a sale of stock for cash. For the same reason that the bank will not sell assets for cash, it will not sell stock for cash given the prevailing prices in the market place. This is a form of underinvestment (Myers (1977), Ivashina and Scharfstein (2008)) whereby the bank will not issue stock because of the value transfer that goes to debt in states of insolvency, but it stems not from uncertain fundamental values but from the potentially low future (and low current) fire sale prices at which illiquid assets will have to be sold.[6]

In sum then, as expectations of date-1 liquidity fall so that the bank is insolvent conditional on the future shock, date-0 trading spontaneously dries up. Our model suggests then that distressed banks hold on to illiquid assets instead of selling them because they believe the price of the asset will be much higher in the future, conditional on their own survival.

II. The Sources of Illiquidity

Thus far, we have not described how the price P_1 is determined Clearly, this is critical to our analysis, for without a low P_1 there would be no illiquidity or potential insolvency at date 1, and no market freeze at date 0. Let the weight of the potentially "illiquid" banks we have described so far be normalized to 1. We will now distinguish between securities and loans on the bank's portfolio, which will add richness to our analysis, and will not qualitatively affect our previous analysis. Let fraction β of each bank's assets be composed of the financial security we have described so far. Let fraction $(1-\beta)$ of its assets be loans with face value Z maturing at date 2. We will assume these loans can be recalled by the bank at date 1 – borrowers will then have to liquidate projects to repay the bank, so as a short hand, we will say that banks liquidate loans. The bank's loan portfolio has differing liquidation values, with the range uniformly distributed between 0 and Z, that is, loans can be liquidated for values ranging from nothing to full face value. The possibility of liquidating loans gives the bank an alternative source of liquidity, in addition to selling financial assets. We assume loans cannot be sold at date 0 (they have little

[6] An oft mentioned rationale for why banks hold on to illiquid assets rather than selling them is the notion that their prices will go up in expectation. Indeed, it is easily shown that the price of the asset does rise in expectation so that $P_0 < qP_1 + (1-q)$. However, this is merely an artifact of Jensen's inequality and the need for returns to equalize over different horizons.

value in another lender's hands) nor can they be recalled immediately (the borrower has no cash after investing at date 0).

Liquid buyers (private equity, hedge funds, and liquid banks) can purchase the financial asset at either date, and start with θ in cash at date 0. Assume for simplicity that they are equity financed. Also, let these buyers also have the possibility of making term loans to industrial firms. If R is the date-2 return on a dollar lent at date 0, let the available volume of loans returning greater than or equal to R be $I(R)$, with $I(1)=\overline{I}$, $I'(R)<0$, $I''(R)>0$ when $R>1$. Loans made by liquid banks return nothing at date 1, though at the cost of additional unneeded complexity, we could assume a somewhat higher date 1 return. Liquid buyers can store any excess funds at date 0 at a rate of 1. The timeline is

|---|--|

<u>Date 0</u>	<u>Date 1</u>	<u>Date 2</u>
Illiquid bank sells securities (or not). Liquid buyers buy securities, make loans, and hold cash.	Liquidity shock hits (or not) and depositors withdraw from banks. Banks decide loans they want to liquidate. Banks sell securities and buyers buy with cash.	Loans and securities pay off. Banker consumes proceeds after paying deposits. Buyers consume.

Fire Sales and Lending

Let us now derive prices. At date 0, the implied interest rate on term loans has to match the return from buying the financial asset, that is, $\frac{1}{P_0}$. This means the amount lent by potential buyers at date 0 is $I(\frac{1}{P_0})$. Intuitively, the long term effective interest rate, and thus the extent of long term lending, is determined by the price of financial assets in the market.

Similarly, conditional on the liquidity shock at date 1, the illiquid bank will liquidate any loan at date 1 with liquidation value greater than $P_1 Z$ before it sells any securities. This means the total value of cash generated this way is $\frac{1}{Z}\int_{P_1 Z}^{Z} x dx = \frac{Z}{2}\left(1-\left(P_1\right)^2\right)$. Again, the implied interest rate used to judge whether to continue loans or not at date 1 is $\frac{1}{P_1}$, which depends on the price of financial assets, and hence available liquidity, on that date.

If $\theta - \overline{I} \geq fD$, then $P_0 = P_1 = 1$, there is no illiquidity, all industrial projects are funded, and no loans are liquidated. The asset will trade for full face value Z at all times. But if $\theta - \overline{I} < fD$, the asset will trade at a discount to face value. For the banks' date-1 needs for cash to be met, it must be that

$$(1-\beta)\frac{Z}{2}\left(1-\left(P_1\right)^2\right)+\left[\theta - I(\frac{1}{P_0})\right] = fD \qquad (1.4)$$

Equation (1.4) incorporates the first order condition for optimal loan liquidation by banks at date 1 and for optimal lending by buyers at date 0, given that the securities are sold for all of the date 1 cash possessed by buyers, and that this, together with loan liquidation, provides sufficient liquidity to meet the liquidity demand, fD. Also, we know that in equilibrium, $P_0 = \frac{1}{q\frac{1}{P_1}+(1-q)}$. Substituting in (1.4), we can solve for the single unknown, P_1. Note that P_1 is determined by the bank's need to recall enough loans to meet the aggregate demand for liquidity. Two conditions have to hold for (1.4) to be the equation determining P_1 : (1) the bank is solvent (2) the bank can actually raise fD in total at those prices. When the bank does not sell any securities at date 0, the necessary condition for solvency is

$$\left(1-\beta\right)P_1 Z P_1 + \left(1-\beta\right)\frac{Z}{2}\left(1-\left(P_1\right)^2\right) + \beta P_1 Z \geq (1-f)DP_1 + fD \qquad (1.5)$$

The first term on the left hand side of (1.5) is the value of the loans that it has not liquidated, the second term is the amount collected from liquidated loans, and the third term is the value of securities held. The first term on the right hand side is the value of deposits to be paid out while the second term is the value of deposits withdrawn. So long as (1.5) is met, the bank will be solvent even if it sells more securities at date 0 (because $P_0 > P_1$).

As P_1 falls, it becomes harder to meet the solvency constraint – asset illiquidity leads to insolvency as in Diamond and Rajan (2005). If the potential liquidity demand f and bank debt D are very high or the available cash liquidity net of industrial demand, $\theta - I$, low, so that (1.5) is not met, then the bank will be insolvent when the illiquidity shock hits and trading will cease at date 0.

The bank also has to have enough securities at date 1 to raise the needed cash in the hands of buyers, $\theta - I(\frac{1}{P_0})$. Equation (1.4) assumes that the bank can meet the liquidity demand by selling financial assets and liquidating loans with rate of return equal to that of selling a loan. This requires:

$$\beta Z P_1 \geq fD - (1-\beta)\frac{Z}{2}\left(1-\left(P_1\right)^2\right) \qquad (1.6)$$

For now, assume this condition is met. If (1.6) holds as a strict inequality, then only a fraction of securities would be sold at date 1. We have

Lemma 2: *(i) An increase in potential liquidity demand, f, or bank debt, D, as well as a decrease in the relative size of liquid entities, θ, will lead to a lower current and future expected price of the long dated asset Z. (ii) An increase in the probability of the liquidity shock, q, will lead to a decrease in the date 0 price P_0 and an increase in the date 1 price P_1. (iii) If there is a $f = f^R$ at which all sales of the long dated asset Z cease at date 0, then ceteris paribus, there will be no sales for any $f > f^R$. If there is a*

$D = D^R$ at which all sales of the long dated asset Z cease at date 0, then ceteris paribus, there will be no sales for any $D > D^R$. If there is a $\theta = \theta^R$ at which all sales of the long dated asset Z cease at date 0, then ceteris paribus, there will be no sales for any $\theta < \theta^R$.

Proof*:* See Appendix.

The intuition behind Lemma 2 (i) is straightforward. Turning to lemma 2 (ii), an increase in the probability of the liquidity shock will make the returns to holding cash to buy assets at date 1 higher, so the date 0 price of the asset has to fall. In turn, however, this implies less lending, so more cash will be available to meet the liquidity demand, and the date 1 price of the asset will rise. Therefore the higher the probability of illiquidity, the greater the precautions the system takes against it, and less the chance of individual failure conditional on the shock. Lemma 2 (iii) suggests that as expected liquidity conditions deteriorate, there will eventually be a "sudden stop" in date 0 trading as banks become insolvent conditional on the liquidity shock.

Proposition 2: An increase in potential liquidity demand, f, the face value of bank debt, D, or the probability of the liquidity shock, q, as well as a decrease in the relative size of liquid entities, θ, will lead to a reduction in date-0 lending.

Proof: Lending increases in P_0. P_0 decreases in $f, D,$ and q and increases in θ from lemma 2. Hence the proposition. Q.E.D.

As the returns to buying illiquid assets increases, date-0 lending slows. Indeed, if date-0 trading in the long dated asset halts, liquid buyers may have plenty of cash on their balance sheet which is not being lent, in anticipation of buying assets cheaply at a date-1 fire sale. To the outsider politician, this may seem excessive caution (after all, the liquid buyers have no fear of liquidity shocks), and they may want to mandate more date-0 lending for the liquid buyer. However, as we have argued, this could well be a rational equilibrium phenomenon where potential lenders in effect demand a premium for locking up their money.

Illiquid Banks

Now let us turn to the bank's liquidity condition (1.6) – that by selling all of its liquid securities, it can raise enough to provide the additional cash it needs at date 1 to meet the liquidity demand by depositors (after it liquidates all loans with returns less than or equal to the return from selling securities). If this condition is not satisfied, then the bank will have to sell all the securities it has, βZ , for all the cash buyers have. In this case, P_1 solves

$$P_1 \beta Z = \left[\theta - I(\frac{1}{P_0}) \right] \tag{1.7}$$

Now the bank will have recall more loans than specified in (1.4) to meet the remaining cash short fall, and these will be loans that it would not otherwise recall at the prevailing interest rate of $\frac{1}{P_1}$. This will reduce the bank's solvency. If $\beta Z P_1 < fD - (1-\beta)\frac{Z}{2}$, the bank cannot meet the demands of its depositors even after selling all securities and recalling all loans. It is then illiquid as well as insolvent.

Bank runs and Inefficiency

Our focus thus far has been on explaining why the market for specialized assets with limited liquidity possessed by buyers can freeze and how this effects other lending. We have ignored any efficiency consequences of bank insolvencies, fire sales, or credit contractions. Provided there are no externalities from lending (that is, surplus generated by a borrower that the lender does not internalize), fire sales when banks are solvent do not produce inefficient outcomes.[7] They are simply a way liquidity is obtained from buyers and transferred to those claimants on the bank who need liquidity. The expected payment from buyer to seller is independent of whether the trade is at date 0 or 1. However, insolvency will precipitate a run on the bank. This will cause the bank sell all its securities for the available cash in the market, and to recall all its loans regardless of their liquidation value, a source of significant inefficiency.

An Example

Let the base case be *Z*=2, θ=0.3, *f*=0.58, $q=0.2$, β=0.5, $I(1/P_0)=0.3*1.2^{1-\left(\frac{1}{P_0}\right)}$. Given these parameters, P_0=0.354, P_1=0.0988 and the amount of date 0 lending is 0.215. The bank could sell just 24% of its securities holdings at date 0, not sell any securities at date 1, and be solvent. However, if it does not sell any at date 0, it will find it has to sell 86 percent of its securities portfolio at the depressed price conditional on the liquidity shock, and will become insolvent. Yet it prefers not to sell, because the value of equity is higher conditional on no sale than conditional on the date-0 sale, for reasons we have explained.

In Figure 1, we plot date 0 and date 1 security prices as we vary the amount of available cash with liquid buyers, θ. At levels of θ below 0.32, the banks will fail conditional on the liquidity shock, and will be run. Securities prices are low and lending even lower. Interestingly, an increase in liquidity from 0.29 to 0.3 makes little difference in prices or lending because it does not alter the fact that the bank will be run, and that the date-1 fire sale prices conditional on the shock will be very low, so date-0 prices will be low.

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[7] Our model has no such externalities, but they are present in Diamond-Rajan (2005).

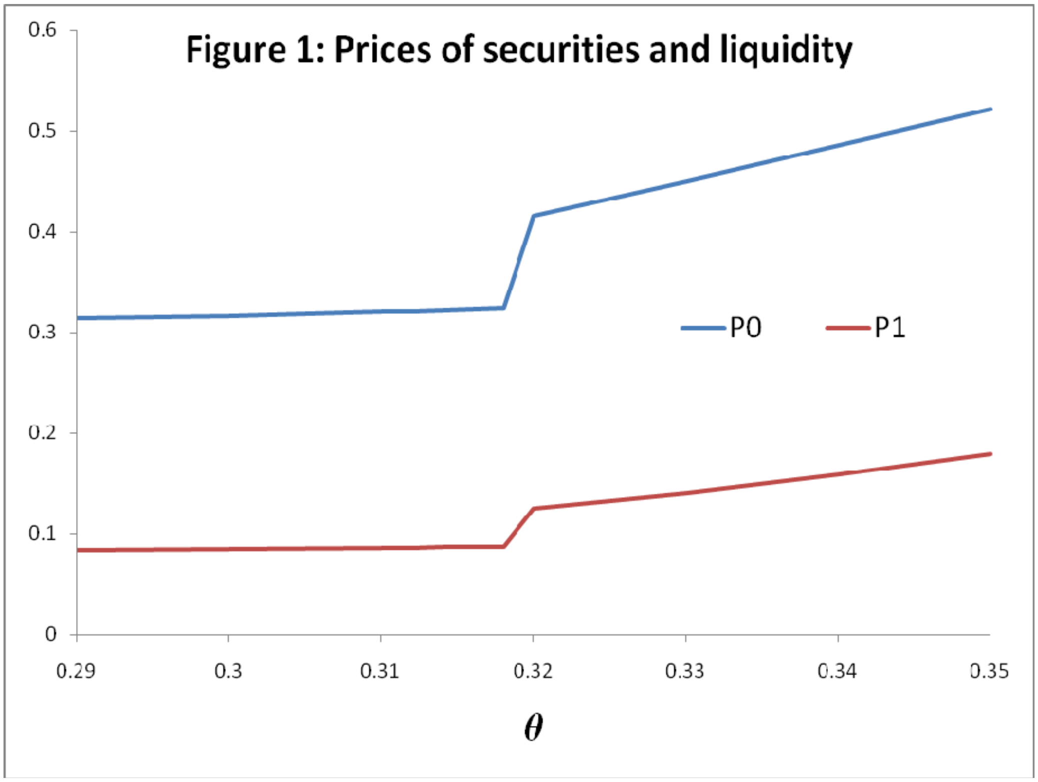

Figure 1: Prices of securities and liquidity

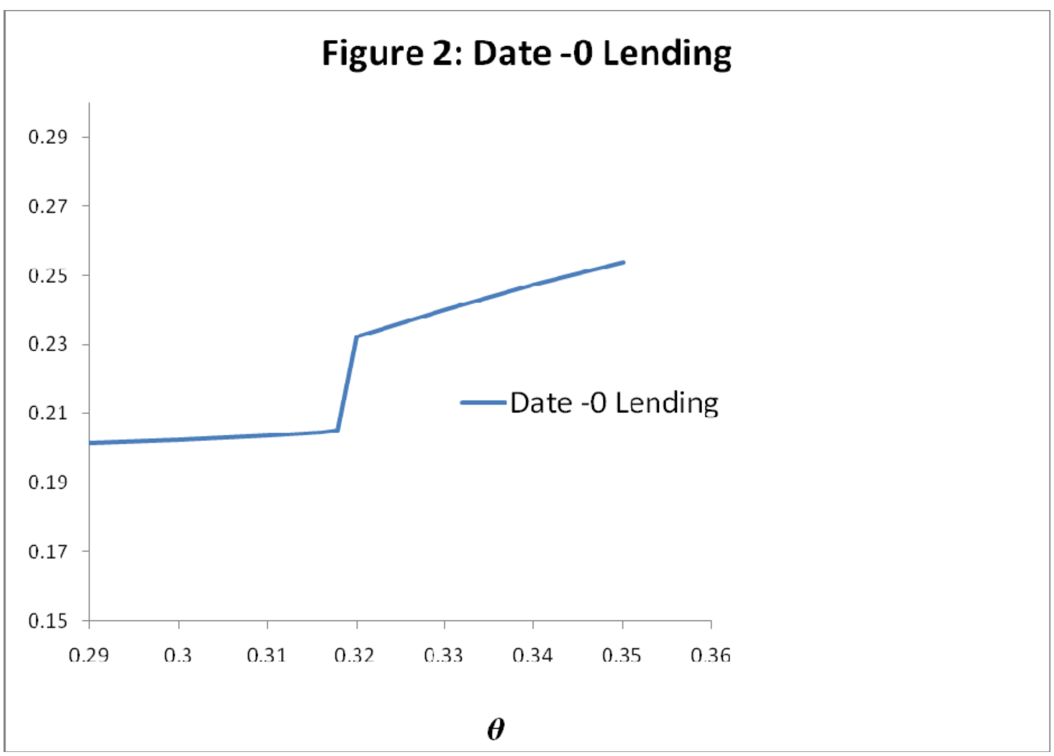

Figure 2: Date -0 Lending

However, if enough liquidity is infused into the system so that the bank is not insolvent at date1, securities prices are considerably higher at date 1 and date 0, and consequently, date-0 lending (see Figure 2) jumps.

In sum then, the private sector bank does not internalize the consequences of its own illiquidity and failure on future economy-wide liquidity conditions, on future available returns, and thus on current required returns and lending today. While the bank could sell today and avoid future failure (and the spillovers from its failure to the price of securities as well as to interest rates), it prefers not to, focusing instead on maximizing value in the future states in which it expects to survive.

Key Assumptions of the Basic Model

It may be useful to discuss our key assumptions. We have referred to the bank's required need at date 1 of *fD* of cash as a liquidity shock, without specifying the source. It could be a need by depositors for working capital for their own businesses during a period of limited liquidity, or represent a fear-based withdrawal by some (uninsured) depositors or other short-term creditors who come to doubt the bank's viability. One example is the loss of access to interbank loan markets where other banks anticipate future problems with the bank and prefer to collect before the bank fails or is closed (as in Smith (1991)). Diamond-Rajan (2005) provides a general equilibrium model of this type of run-based withdrawals with a shortage of liquidity, stressing the two-way causality between illiquidity and insolvency.

The key contractual element that can lead to a fire sale is that short-term debt can be withdrawn (or committed lines of credit can be drawn down) before the assets mature. If there was no possibility of needing to sell assets before date 2, then the market values of the assets would not be depressed by forced fire sales and as a result there would be no risk of insolvency. With no insolvency, the market would not freeze.

In our model, banks refuse to sell at date 0 as soon as there is any possibility they might default. If there were some frictional reasons for trade (for example, some buyers are willing to buy at a slightly higher price than the bid price we calculate), then there would be some trade at date 0 if banks expect to be solvent at date 1 or if the probability of bank insolvency were very small (the benefits would exceed the small value of the put foregone), but not if the probability becomes higher. If the chance of receiving a liquidity shock that causes insolvency varies across otherwise identical banks, we would see the extent of asset trade varying with the probability of the liquidity shock in a more continuous way.

The assets are illiquid in our model because the set of (equally) informed buyers is limited and they have finite borrowing capacity (as in Shleifer and Vishny (1992)).[8] Hence buying capacity, rather than asymmetry of information between buyers and sellers of assets, drives our results. Of course, over time we would expect that if there were substantial quantities of illiquid assets, more potential buyers would acquire the necessary skills. So illiquidity of this kind would, at best, be a medium term phenomenon.

Term lending (as opposed to overnight lending) would be curtailed so long as such lending is done by *either* the illiquid banks or the potential buyers of their assets. In other words, it is because of the future high expected cost of capital/rate of return for entities that suffer the liquidity shock, or can buy the illiquid assets, that today's required rate of return is high and lending is depressed. For firms that can borrow from institutions outside this group (such as small firms borrowing from local financiers, who have no capacity to understand, and hence buy, the illiquid assets), lending would be less constrained. There are, of course, many channels through which illiquidity can spill into the rest of the economy – the inter-bank market would be one channel.

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[8] The notion is that any buyers outside the set of the skilled would find it hard to tell the few bad securities from the majority of good ones, and could well face a substantial lemons problem if they tried to buy.

Finally, we have assumed banks' cost of funding does not vary if they do or do not sell assets. Implicit in this is the assumption that bank depositors do not have the expertise to trade the illiquid asset, and as a result changes in the expected return offered in secondary markets available to banks do not lead them to change the interest rates offered to depositors. Also, we assume that when a bank subject to liquidity shocks sells assets early and reduces the default risk to depositors, the deposit rate does not fall -- This is based on our assumption that banks have a local monopoly over deposits and depositors in-elastically supply a fixed quantity of deposits at date 0 at the offered interest rate (normalized to 0) and do not demand additional compensation for greater default risk. This assumption allows us to focus on the essential driving force of the model.

More generally, the lack of full adjustment of the bank's cost of funding to asset sales arises when one portion of bank debt (such as insured deposits, or deposits which might, ex-post, end up being insured, or existing long-term debt as in Myers (1977)) is relatively insensitive to bank actions, while another portion of debt (such as overnight borrowing, uninsured demand deposits, and cash in brokerage accounts) is sensitive to the bank's health and susceptible to run. The unwillingness to sell which causes the freeze occurs even if all deposits are insured, as long as the bank's equity value is wiped out when the liquidity shock makes it insolvent. The implications would be qualitatively similar.

III. Two Extensions

Before we move to discussing interventions, let us discuss two possible extensions. The first is to allow heterogeneity in the exposure to the liquidity shock.

3.1. Heterogeneity in the Shock

Suppose the probability of the aggregate liquidity shock is q as before, but conditional on a liquidity shock, each bank i has a probability $\delta^i \leq 1$ of getting the shock. Assume further that the fraction of banks that get a shock, conditional on the shock occurring, is constant – there is no aggregate uncertainty conditional on the shock.

One might expect that banks with a lower anticipated exposure to the shock might require a higher price to sell at date 0 – after all, they are less likely to need liquidity. Simple algebra suggests, however, that if banks remain solvent given the shock, the probability of the shock does not affect the price they ask for the asset. As before, therefore, the bid and ask prices are identical when there is no insolvency. The intuition is interesting. The ask price at date 0 that we computed earlier in (1.3) fully compensated banks for the cost of selling early and having spare liquidity if the aggregate shock does not hit. In addition, the ability to reinvest spare date-0 cash at the low security price P_1, conditional on the aggregate shock, fully compensates them for having spare liquidity at date 0 and having the aggregate but not the bank-specific shock hit. This is why the size of the idiosyncratic shock does not matter if banks are solvent.

But if banks are insolvent conditional on the shock hitting (that is, if $\beta Z P_1 + (1-\beta)\left(\frac{Z}{2}\right)(1-P_1^2) - fD + \left[(1-\beta)P_1 Z - (1-f)D\right]P_1 < 0$), δ^i does determine the size of the put option. Noting that if a bank survives at the ask price that gives it the incentive to forego its put option, the bank will sell all its securities at date 0, and not just enough to meet the liquidity shock, we have after some simple algebra

$$P_0^{Ask,\delta^i} = \frac{q\delta^i\left[\dfrac{fD-(1-\beta)\left(\frac{Z}{2}\right)(1-P_1^2)}{P_1} - (1-\beta)P_1 Z + (1-f)D\right] + \left(q(1-\delta^i)+(1-q)\right)\beta Z}{\left[\dfrac{q}{P_1}+(1-q)\right]\beta Z} \qquad (1.8)$$

If the bank is insolvent conditional on the liquidity shock in the absence of any sales at date 0, the numerator increases in δ^i. Hence we have the ask price now increasing with the size of the idiosyncratic shock δ^i.

Proposition 3: If banks have differing exposure δ^i *to the aggregate liquidity shock, the ask price does not vary with* δ^i *if banks are solvent conditional on the liquidity shock hitting them, but increases in* δ^i *if they are insolvent conditional on the shock hitting them.*

If there are reasons to trade at date 0, as mentioned at the end of the previous section, then the banks with the lowest probability of receiving a liquidity shock will be the most willing to trade at date 0. This has implications for government actions to eliminate a market freeze. We discuss this in section IV.

3.2. Risky vs Illiquid Assets

Thus far, we have assumed that the illiquid assets have a value Z with certainty. This simplifying assumption creates some expositional difficulty, for it is hard to imagine then why the asset is illiquid, since anyone with cash could buy the asset and hold it to maturity. In particular, there would be no cost to the government of liquidity intervention, as long as the market price per unit of face value is less than one.

One reason why the government could be reluctant to intervene is that the asset might require management (as with a bank loan), and the government may not have management capability. But this is not a plausible explanation for arm's length mortgage backed securities. Another reason, and one we have relied on so far, is that only the specialist knows the value of the security or, equivalently, a fraction of the illiquid assets could be very poor in quality, so any casual buyer will end up with an adversely selected lot of securities (unless he buys the whole portfolio, which even the government may not have the funds to do). But a third reason, is there could be fundamental uncertainty about the asset's value, which overlays (and is partly responsible) for its illiquidity. To analyze this, let us start by examining an asset whose value is uncertain, ignoring illiquidity for the moment, then bring back illiquidity.

Risky assets

Suppose the date 2 value is not Z with certainty, but is Z with probability 1-q and $P_1 Z < Z$ with probably q, where the date 1 value, $P_1 Z$, is now low because date 2 payments are expected to be low. In this case, the asset is liquid and the gross rate of return from buying it at the low value at date 1 is the normal expected rate of return (one each period). Of course, because the asset is risky, a bank that would fail if the lower asset value were realized at date 1 might still ask for a price at date 0 that exceeds the fundamental value of the asset, because it values the option to shift risk to lenders, as in Allen and Gale (2000b).

Note that the ask price required by the risky bank would be the same as derived for the base case earlier, because the bank's payoff from not selling is (1-q)(Z-D) as before. The bid price would be somewhat higher than the bid price we found earlier (because buyers require a gross rate of return of only 1 in the future, implying that their highest bid would be the fundamental value of the asset, that is $qP_1 Z + (1-q)Z > P_o^{Bid}$). Because the hurdle rate for new date-0 lending would not be elevated by expectations of future fire sales, lending by potential buyers is normal.[9] Therefore, to explain both a general credit freeze, including few new loans by healthy unlevered buyers, and a freeze in securities markets, the fear of fire sales is, we believe, a more relevant model.

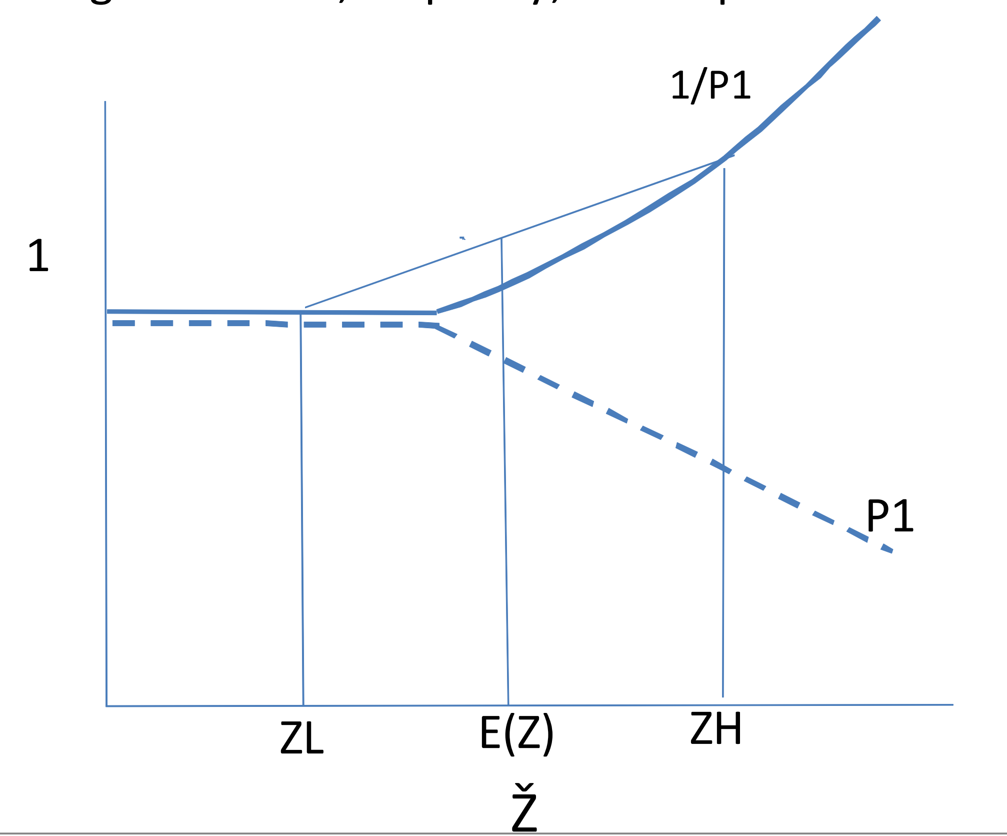
Illiquid and Risky Assets

Adding the possibility that assets are illiquid, in addition to being risky, is straightforward, so we will not pursue it. There is though a final but subtle consequence of considering both illiquidity and risk at

[9] This also means injecting future liquidity would have no effect on current or future asset prices. The market would still be frozen.

the same time. Suppose the realization of the fundamental value of $\tilde{Z}$ conditional on the liquidity shock is either low, Z_L , or high, Z_H , with expected value Z. At date 1 the available pool of liquidity will tend to bind more when asset prices are high – the departure from fundamental value will be greater because P_1 will be smaller, and the prospective returns, $\frac{1}{P_1}$, for investors from purchasing at that time will be higher. As a result, using Jensen's inequality, a mean preserving spread in the fundamental date-2 value of the asset, conditional on the liquidity shock, will result in an increase in the expected return to having cash in those states. Thus risk overlaid on illiquidity will further depress lending at date 0.

Figure 3: Risk, illiquidity, and Expected Returns



The point of introducing an overlay of risk on our basic model of illiquidity was to show how illiquidity is fundamental to the point we have made. There are interesting interactions between illiquidity and risk, however, which deserve further exploration.

IV. Interventions

There are several reasons that government regulators or central banks might consider intervening to eliminate the market freeze and even influence the market price.

(i) Interventions that force some banks to sell securities early can keep banks from being run and their assets being dumped on the market. Interventions that reduce the fire sale pricing can have a similar effect.

(ii) The authorities may need a market price to help determine if the banks are solvent. When Z is random, banks and expert buyers may know the long-run value of assets, Z, but the regulators may not know. In addition, regulators may not know the amount of available liquidity, θ. Therefore, useful valuation information can be obtained about both fundamental values and expected liquidity conditions by unfreezing the market. Note that illiquidity would depress the bid price below fundamentals, while the put option, stemming from both illiquidity and risky fundamentals, could elevate the ask price, even above fundamental value.

(iii) An intervention that reduces the fire sale pricing can increase the flow of credit from potential buyers and sellers, both at date 0 (new loans) and at date 1 (reduce unnecessary liquidation of old loans).

Possible interventions include forcing distressed institutions' to sell, paying enough to induce voluntary sales to the authorities, increasing the capacity and incentive of potential buyers to buy distressed assets, providing liquidity directly to banks, or removing the overhang of distressed institutions by recapitalizing or closing them. In any intervention, the authorities may be handicapped by not knowing the fundamental value of assets themselves. In what follows, we examine various interventions and the insights offered by the model on their likely costs and benefits.

Forced Date-0 Asset Sales

We argued earlier that because the security is illiquid, the bank may be insolvent conditional on the shock if it holds to sell at date 1. This, of course, makes it even more determined to hold the security till date 1. If, however, it sold the security at date 0, it would get a higher price and perhaps avoid default. To the extent that this reduces the number of banks that are run at date 1 and that therefore sell all their securities into the market, it could also push up bid prices for the securities, and thus increase lending at both dates. Of course, banks that expect to fail conditional on the liquidity shock do not have an incentive to sell assets at date 0 at the price buyers are willing to pay, so sales will have to be forced by the government. We state the result for the standard case where the bank is liquid but insolvent.

Theorem 4: Let P_1 be determined by (1.4), let the liquidity condition (1.6) hold, but let the bank be insolvent if it retains all its securities till date 1 so that (1.5) is violated.

(i) *Then if there is an η_0 with $\eta_0 \in (0,1]$ such that*

$$\left(1-\beta\right)P_1 Z.P_1 + \left(1-\beta\right)\frac{Z}{2}\left(1-\left(P_1\right)^2\right) + \beta Z(\eta_0 P_0 + (1-\eta_0)P_1) \geq (1-f)DP_1 + fD \qquad (1.9)$$

$$\beta\eta_0 P_0 \leq \left[\theta - I(\frac{1}{P_0})\right] \qquad (1.10)$$

then all the banks can be made fail-safe by requiring them to sell η_0 of their securities at date 0.

(ii) *If there is no* η_0 *with* $\eta_0 \in (0,1]$ *such that (1.9) and (1.10) are satisfied, then not all the banks can be saved through forced date-0 sales. The maximum fraction of banks that can be saved is given by* γ, *where* γ, P_1, *and* η_0 *solve*

$$\gamma\beta\eta_0 P_0 = \left[\theta - I(\frac{1}{P_0})\right] \tag{1.11}$$

$$(1-\gamma)\beta Z P_1 + \gamma\left(fD - (1-\beta)\frac{Z}{2}\left(1-\left(P_1\right)^2\right)\right) = \left[\theta - I(\frac{1}{P_0})\right] \tag{1.12}$$

$$\left(1-\beta\right)P_1 Z.P_1 + \left(1-\beta\right)\frac{Z}{2}\left(1-\left(P_1\right)^2\right) + \beta Z(\eta_0 P_0 + (1-\eta_0)P_1) = (1-f)DP_1 + fD \tag{1.13}$$

(iii) *If no feasible solution exists to (ii), then the banking system cannot be prevented from failing through securities sales at date 0.*

(iv) *If some banks can be saved from failing through securities sales at date 0, both* P_0 *and* P_1 *are higher (relative to the situation where all banks fail conditional on the shock), as are date 0 and date 1 lending.*

Proof: Omitted

The reason a forced sale at date 0 could contribute to bank solvency is that the sales price pools the value of the security across future states (buyers essentially provide the bank insurance). So the bank is worth more conditional on the liquidity shock if it sold at date 0 than it would be worth if it did not sell and got hit by the liquidity shock. Indeed, the price of the security conditional on the shock is also now higher because failed banks do not now depress the price by dumping all their securities on the market. This implies that the sale at date 0 also fetches a higher price.

One problem, of course, may be that the value of securities that have to be sold at date 0 to ensure bank solvency may exceed the available cash with buyers -- (1.10) may not hold. This may occur even if the available cash, coupled with loans that are liquidated by the bank at date 1, is enough to meet the needs of depositors – i.e., even if there is enough liquidity to meet the demands of depositors. In this case, only a fraction γ of banks can then be saved through sales at date 0. Of course, these banks may now have excess liquidity even when hit with the liquidity shock, and can use the excess to purchase securities cheaply from other firms. Finally, forced asset sales will not work if there is little cash with buyers (so securities prices are low and solvency cannot be assured) or if the probability of the liquidity shock is high (so selling at date 0 does not add much to the value).

Another interesting case, different from the one we have examined above, is one where the liquidity shock is so high, and therefore the date 1 price is so low, that a bank would need to sell all its securities and would also need to liquidate loans with higher future returns than $\frac{1}{P_1}$ ((1.6) does not hold). Securities are priced to draw out all the cash in the market (and the price is determined by (1.7)). We know that the bank would be insolvent at date 1 at this price (otherwise, it would have been willing to sell at date 0). Interestingly, if the authorities force a sale at date 0, the date-0 price will be the cash-in-the-market price – it will be based on the cash available to purchase securities, and not on the potentially higher expected value of the security given by (1.3) (which incorporates the higher value in the no-shock state). Selling all of the security at date 0 will set a price that solves $P_0 \beta Z = \left[\theta - I(\frac{1}{P_0}) \right]$, which is the same as the date-1 price. Even so, forced sales may help bank solvency – at the low cash-in-the-market date-0 price, P_0, buyers will set aside more cash to purchase securities, thus boosting their date 0 price relative to the date 1 price that prevailed when date 0 sales were not mandated. Forced sales produce a lower P_0, and lower lending by potential buyers, $I(\frac{1}{P_0})$, who divert the proceeds to security purchases. Only if this margin of lower $I(\frac{1}{P_0})$ is large, will there be a benefit to forced sales.

By contrast, if $I(\frac{1}{P_0})$ were unchanged at the lower P_0, the equilibrium P_0 would equal the P_1 from (1.7) and the banks will be insolvent at date 0. This is the potential downside to forced sales when liquidity is in short supply. Because the forced date-0 sale makes sellers accept a cash-in-the market price, it will reduce their date-0 value beyond just the loss of the put option (and buyers will now get a windfall because the security will sell for lower than the value implied by date-1 returns without forced sales). If the authorities are mistaken about the extra available liquidity, forced sales would render banks insolvent at date 0, even if they were solvent at date 0 without the forced sale.

Voluntary Sales to the Authorities

Thus far, we have looked at forced sales. The authorities can offer to buy securities at date 0 at a price banks will voluntarily sell at, so as to establish securities prices, keep the securities off market in an entity like the Resolution Trust Corporation, and prevent some bank failures. The problem, however, is that a bank will sell only at or above the ask price given in (1.8), which depends on a given bank's likelihood of receiving the liquidity shock, and which is above the private buyer's bid price.[10]

So long as the magnitude of the purchase by the authorities is such as to not eliminate bank failure, the value of the put option of the marginal selling bank will determine the price that banks will set in a reverse auction (similar to that considered in the initial TARP program in the United States in 2008).

[10] Unlike the private market, though, the authorities may be willing to tolerate some losses because they internalize the economy-wide positive spillovers from having a healthy, functioning, financial sector. The losses could take the form of rents needed to ensure incentive-compatibility for buyers that can acquire the asset on their behalf, subsidies to induce banks to sell and partly reveal the value of assets, or losses made in buying the illiquid assets or claims on the illiquid assets that the authorities do not have great expertise in.

A reverse auction will provide a price, but if authorities are unsure of the motivation for the lack of selling (and do not know q, δ^i, or f), it will not reveal the value of Z to them. Three additional features of the reverse auction are clear.

(i) If there are banks with different exposures to the liquidity shock, the banks with the lowest value of the put option (the lowest exposure to the liquidity shock) would agree to sell at the lowest price (this is from expression (1.14)) .

(ii) Conditional on agreeing to sell, they would want to sell all their securities, since at any price over the bid price, selling and reinvesting conditional on the liquidity shock beats holding to maturity.[11] If these selling banks were banned from repurchasing the illiquid security if they did not receive a liquidity shock on date 1, they would demand a higher price for parting with it at date 0.

(iii) The price at which banks will sell will increase with the amount of money the government puts into purchasing assets – because the fewer the assets left with the banks after the reverse auction, the lower the date-1 price discount conditional on the liquidity shock (this is assuming that the cash the authorities put into purchasing is additional to that in the hands of private buyers). At the same time, the price will also approach the price private buyers will pay because a higher date-1 price will eliminate the possibility of failure and hence the put option. Interestingly, therefore, government purchases can restart an illiquid private market at a higher price. Note that if date-1 illiquidity is not the problem but the problem is low fundamentals, the price will not move regardless of the amount the government puts in.

[11] To see this, the value of selling at the date 0 bid price and reinvesting conditional on the liquidity shock at date 1 is $\frac{P_0}{P_1} q + P_0(1-q) = 1$ when $P_0 = P_0^{Bid} = \frac{1}{\frac{q}{P_1} + (1-q)}$

Liquidity infusion 1: Lend to liquid buyers at date 0 or date 1.

We have already examined one form of liquidity infusion, which is for the government to buy assets directly. An alternative is for it to lend to liquid buyers at date 0 or date 1, thus augmenting θ and boosting prices (see lemma 2). Buyers certainly have the expertise to value the assets, the real issue being whether they have the incentives to use government financing in a reasonable way. Some rents/subsidies may have to be offered to them to give them the incentive to use their expertise on behalf of the government. To the extent that buyers get these subsidies only if they purchase assets, they may have an incentive to pay for the put options banks hold. Note that if the government has insured depositors, and cannot avoid illiquidity induced failures, the subsidy to buyers, which is passed on to banks, reflects the (perhaps unwise) commitments it has taken through deposit insurance (though, as discussed above, the government can reduce the put by forcing banks to sell securities). The subsidy is more questionable if the bank is insolvent, or if the banks are significantly funded through uninsured deposits.

Liquidity infusion 2: Lend to the banks at date 1

An apparently straightforward solution to the problem of a fire sale at date 1 is to lend freely to the banks experiencing liquidity withdrawals. If the government could commit to do this on a sufficiently large scale, which would require it to value the banks' assets at date 1, the illiquid pricing would be eliminated as would be the date 0 market freeze. However, if as we argue in section 3.1, regulators are unsure about the asset values (Z), they would be unsure if they were providing liquidity or participating in a bail out. If the public believed, rightly or wrongly, that lending was a bail out, it might be unwilling to support it. As a result, it might be difficult for the authorities to commit to unconditional date-1 liquidity support at date 0, preventing them from ending the freeze.

Close Some Banks at Date 0

If there is insufficient liquidity for all banks to survive the liquidity shock, then all will be run and all will fail if each waits until date 1 to sell assets. There might be sufficient liquidity for some to survive if the others are closed and their assets taken “off market”.

If banks are insolvent, the authorities can close them (if the banks are not “too big to fail”). To use solvency as a basis to close banks, the authorities will have to determine the value of assets, for which they will either need to generate a market price or they will have to hire experts to value the assets.[12] They will also have to hold the illiquid assets in some holding entity (similar to the Resolution Trust Corporation) and sell them over time once the likelihood of the liquidity shock falls. Closure thus allows the authorities to remove the overhang of illiquid assets, and bring down required rates of return, but it does not absolve them of the need to value assets or pump in resources (to finance the holding entity).

The problem comes when the banks appear solvent today and thus cannot be failed, but could become insolvent in the future – as in our model. Closure may not be an option for the “walking wounded”. It may also not be an option for the banks that are difficult to fail for a variety of reasons.

Recapitalization

If the authorities are willing to infuse capital they could help when the problem is primarily one of solvency (if when date 1 interest rates go high enough to generate enough liquidity from calling loans to meet depositor withdrawals, the bank is insolvent). Indeed, this is the situation in our initial example. If, for instance, the authorities were willing to give the bank bonds of value 0.018 at date 1 that would pay

[12] A market price will be sufficient to value assets held on the books so long as there are easily observable and verifiable characteristics that put the traded assets in the same equivalence class as the book assets – e.g., mortgage backed securities based on sub-prime mortgages originated by A in new development B in city C. If, however, there are still intrinsic differences between assets that require expertise (mortgages originated in the south-side of the development have different default characteristics than those originated in the north side, and only south side mortgages are traded), then there is no alternative to hiring experts.

off at date 2, they could save the bank from being run. This would increase the date-1 security price above the "run" price, and increase lending at date 0.

A commitment to inject capital as needed to keep the bank from failing does create moral hazard because the bank will be bailed out no matter how poorly it does. Moreover, this intervention does not induce the banker to sell securities early (because the banker knows his stake will be severely diluted even if the bank is rescued, and thus his down side payoff is very low, much as the situation where he is run). It does keep some assets off the market by preventing a run, and hence boosts securities prices, and thus lending. Indeed, the stress tests conducted by the U.S. government on 19 large banks in May 2009 might have been interpreted as a signal that the government would stand behind the banks regardless of their eventual loss levels. This eliminated the possibility of a large bank failure, and began the process of raising the prices of illiquid assets

V. Related Literature

Some recent work has explained market freezes by appeals to asymmetric information, along the lines of the original insight of Ackerlof (1970). Bolton, Santos and Scheinkman (2008) and Heider, Hoerova and Holthausen (2009) present interesting models of securities sales based on private information of existing banks about the value of their assets. For example, in Bolton, Santos, and Scheinkman (2008), long horizon investors cannot tell whether short horizon investors sell because they need liquidity or because they have adverse information about asset quality. This leads to a price discount, which gets worse over time because the potential seller gets to know more about the asset. The seller thus has to decide whether to sell now in response to a liquidity need, or to attempt to ride out the crisis with the possibility of selling in the future at a much greater discount. There are both immediate trading equilibria and late trading equilibria, with the latter resembling our trading freeze. The clear difference in our model is the assumption of no information asymmetry within the set of buyers and sellers.

Acharya, Gale, and Yorulmazer (2009) show that borrowing freezes can arise when the information structure in the market shifts from one where the arrival of no news is good news (and the asset price goes up) to one where the arrival of no news is bad news (and the asset price goes down). In the latter situation, the borrowing capacity of a bank may be very low when it intends to roll over its borrowing repeatedly. The shift in information structure in the market can, therefore, cause lending to banks to dry up. Our paper explains, by contrast, why long term lending to industry, where there is no rollover risk, can also dry up.

Allen, Carletti and Gale (2009) present a model of freezes without asymmetric information, with limited liquidity as we assume, but without any risk of default. The market freezes there when there is ample liquidity but most of the liquidity risk is systematic rather than bank specific. The interbank market freezes because each bank wants to hold liquidity on its balance sheet rather than to borrow or lend it when nearly all banks will borrow or lend (rather than take offsetting positions).

There are closely related studies of aggregate liquidity shortages. Diamond-Rajan [2005] model contagious bank failures due to limited aggregate liquidity. In their model, there is both individual bank risk about the proportion of their loans that generate liquid repayments quickly and aggregate uncertainty about the supply of liquidity. The potential failure of enough banks forces banks to call in bank-specific loans. Banks choose to increase interest rates to attempt to attract deposits from other banks, and this can bring down all other banks when liquidity is too low. The model assumes that the deposit market is competitive and that all assets, including bank deposits and short-term debt, must offer the same return as the loans that banks make. The model we present in this paper has similar features, except we assume that deposit markets are local monopolies (or at least require a lower return than bank assets, and the return does not move one for one with returns in asset markets). The effect of limited liquidity is via the price of banks' tradable assets, which affects the rate of return expected in the market over time, and thus lending.

Our paper is also related to Shleifer and Vishny (2009), where banks expand and contract lending based on their ability to securitize loans in a sentiment driven market. In their model, parameters are assumed such that banks would not want to hoard cash in order to buy assets when market sentiment falls. This then drives the pro-cyclicality of lending. However, banks would hoard securities and not sell them at such times, in anticipation of a recovery in prices. Our rationale for why banks hoard securities is different, since there are buyers in our market who are not infected by negative sentiment. The reason in our model is that banks prefer the higher return they get conditional on survival by holding on to the asset, to the lower unconditional return they get from selling.

The model of illiquid asset markets where prices are set by the quantity of liquidity in the market is closely related to that used in Bhatacharya-Gale (1987) and Allen-Gale (1998, 2003). This is related, yet somewhat different from the model of liquidity in Holmstrom-Tirole (1998), which relies on collateral value as the limit to liquidity of an asset, rather than limited purchasing power.

Finally, our paper is related to Phillipon and Schnabl (2009). Unlike our paper, they treat the problem of credit contraction in a crisis as primarily due to a debt overhang faced by banks as in Myers (1977). Our focus on the institutional overhang of distressed institutions is clearly different.

Conclusion

While our model is simple, it offers a way to think about the problems that ail any financial market and lending in a crisis. The simple message is that in a crisis, credit is unlikely to flow freely unless the problem of institutional overhang is dealt with – unless the solvency of illiquid institutions is assured, or the illiquid assets they hold are moved to entities that will not unload them quickly. The task of the authorities is to facilitate such a clean-up at minimum cost to the taxpayer. We have suggested some possible interventions that could be effective. More work is clearly needed to understand the links between solvency, liquidity, and lending better.

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Proof of Lemma 2;

We sketch the proof when the change in parameters does not cause a change in whether the bank defaults or not. Incorporating such a change is straightforward. (i) Totally differentiating (1.4), we get

$\frac{dP_1}{df} = \frac{D}{-(1-\beta)ZP_1 + I'\left(\frac{1}{P_0{}^2}\right)\frac{\partial P_0}{\partial P_1}}$ which is negative since the denominator is negative. Similarly, we can show $\frac{dP_1}{dD} < 0$ and $\frac{dP_1}{d\theta} > 0$. Since $P_0 = \frac{1}{q\frac{1}{P_1} + (1-q)}$, lemma 2 (i) follows. (ii) Ceteris paribus, an increase in q leads to a decrease in P_0. From (1.4), this must imply that P_1 will increase in equilibrium, since liquid buyers will lend less and store more cash. (iii) The condition for all sales to cease is that the bank ceases to be solvent. The solvency condition is given by (1.5), which on simplifying is

$P_1\left[(1-\beta)\left(\frac{Z}{2}\right)P_1 + \beta Z - D\right] > fD(1-P_1) - (1-\beta)\left(\frac{Z}{2}\right)$. This is clearly satisfied if $P_1 = 1$, so long as $D < Z$. As f increases, P_1 falls, from lemma 2 (i). So the right hand side of the inequality increases, while the left hand side falls. Hence provided the bank fails for some f^R, it should fail for all $f > f^R$. The other conditions follow similarly. Q.E.D.