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ABSTRACT

The long-run risks model of asset prices explains stock price variation as a response to persistent fluctuations in the mean and volatility of aggregate consumption growth, by a representative agent with a high elasticity of intertemporal substitution. This paper documents several empirical difficulties for the model as calibrated by Bansal and Yaron (BY, 2004) and Bansal, Kiku, and Yaron (BKY, 2007a). BY's calibration counterfactually implies that long-run consumption and dividend growth should be highly persistent and predictable from stock prices. BKY's calibration does better in this respect by greatly increasing the persistence of volatility fluctuations and their impact on stock prices. This calibration fits the predictive power of stock prices for future consumption volatility, but implies much greater predictive power of stock prices for future stock return volatility than is found in the data. Neither calibration can explain why movements in real interest rates do not generate strong predictable movements in consumption growth. Finally, the long-run risks model implies extremely low yields and negative term premia on inflation-indexed bonds.

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1 Introduction

The prices of long-term assets reflect both the cash flows that investors expect the assets to generate, and the discount rates that investors apply to those cash flows. One of the most basic questions in asset pricing is whether asset price movements are driven by changing expectations of cash flows, as in the traditional random walk model of stock prices, or by changing discount rates, as in the case of government bonds which make credible promises of fixed future cash flows. During the 1980's and 1990's, a number of papers argued that changing discount rates are the dominant influence on aggregate stock prices (e.g. Campbell and Shiller 1988, Campbell 1991), and proposed asset pricing models that have this property (e.g. Campbell and Cochrane 1999).

Bansal and Yaron (henceforth BY, 2004) have recently reemphasized the importance of cash-flow news for stock prices. Their "long-run risks" model has persistent variations in both the growth rate and the volatility of aggregate consumption, but consumption growth–which drives up stock prices by increasing investors' expectations of future cash flows—is the most important influence on stock prices in the calibration of the model proposed by BY. The long-run risks model has attracted a great deal of attention, with important subsequent work by Bansal, Khatchatrian, and Yaron (2005), Bansal, Kiku, and Yaron (2007a,b), Bansal, Dittmar, and Kiku (2008), Hansen, Heaton, and Li (2008), and Pakoš (2008) among others.

Of course, in general equilibrium the same underlying shocks may affect both expected cash flows and discount rates. In a consumption-based asset pricing model where the aggregate stock market is treated as a claim to aggregate consumption, expected future consumption growth increases both future dividends and real interest rates, and moves stock prices through both channels. The first effect raises stock prices while the second lowers them. If the intertemporal elasticity of substitution in consumption is one, then these two effects cancel out and expected future consumption growth has no effect on stock prices, which move one for one with current consumption.

The long-run risks model works within the consumption-based paradigm, but avoids exact cancellation of cash-flow and discount-rate effects. The model has four key features. First, there is a persistent predictable component of consumption growth. This component is hard to measure using univariate time-series methods, but investors perceive it directly and so it can move stock prices. Second, there is persistent variation in the volatility of consumption growth, which also moves stock prices and is roughly equal in importance for stock market variation. Third, consumption and dividends are not the same; the stock market is a claim to dividends, which are more volatile than consumption although correlated with consumption and sharing the same persistent predictable component and the same movements in volatility.

Finally, assets are priced by a representative investor who has Epstein-Zin-Weil preferences (Epstein and Zin 1989, Weil 1989). These preferences generalize power utility by treating the elasticity of intertemporal substitution (EIS) and the coefficient of relative risk aversion (RRA) as separate free parameters. In the long-run risks model, EIS is greater than one and RRA is many times greater than one. The level of EIS ensures that stock prices rise with expected future consumption growth and fall with volatility of consumption growth, while the level of RRA delivers high risk premia. Because EIS is greater than the reciprocal of RRA, asset risk premia are driven not only by covariances of asset returns with current consumption, as in the classic power-utility models of Hansen and Singleton (1983) and Mehra and Prescott (1985), but also by the covariances of asset returns with expected future consumption growth (Restoy and Weil 1998).[2]

The purpose of this paper is to evaluate the long-run risks model along a number of dimensions, most importantly challenging BY's claim that stock prices respond strongly to variation in expected future consumption growth. Some of our points have been made in recent papers by Bui (2007) and Garcia, Meddahi, and Tédongap (2008), but our examination of the long-run risks model is more comprehensive.

Alongside the original calibration of the long-run risks model by BY, we also consider a recent calibration proposed in an unpublished paper by Bansal, Kiku, and Yaron (henceforth BKY, 2007a). This new calibration responds to the concern that stock prices do not predict consumption growth by reducing the importance of persistent shocks to consumption growth and greatly increasing the importance of persistent shocks to volatility. Since volatility shocks move stock prices by changing risk premia, the BKY calibration increases the importance of discount-rate news and

[2]The Epstein-Zin-Weil model can alternatively be used to derive an augmented version of the classic Capital Asset Pricing Model, in which asset risk premia are driven not only by covariances of asset returns with the current return on the aggregate wealth portfolio, but also by covariances with news about future returns on wealth (Campbell 1993, 1996, Campbell and Vuolteenaho 2004). We do not explore this approach to the model here. Campbell (2003) gives a textbook treatment of the Epstein-Zin-Weil model under homoskedasticity.

reduces the importance of cash-flow news for stock prices. In this sense the BKY calibration is closer to the work of Calvet and Fisher (2007, 2008) than to the original BY model. We show that the BKY calibration is more successful than the original BY calibration of the long-run risks model, but still faces a number of empirical difficulties that should be addressed in future research.

The paper is organized as follows. Section 2 lays out the basic model, discusses the alternative calibrations of BY and BKY, and explains how the model is solved and simulated. BY use both numerical solutions and analytical solutions to a loglinear approximate model of the sort proposed by Campbell and Shiller (1988) and Campbell (1993). It is also possible to derive analytical solutions to a discrete-state approximation of the model (Garcia, Meddahi, and Tédongap 2008). Here we rely primarily on the loglinear approximation method, which appears to be highly accurate for reasonable values of the intertemporal elasticity of substitution, provided that one solves numerically for the parameter of loglinearization (Campbell 1993, Campbell and Koo 1997). We have also solved the model numerically but do not report the results here.

Section 3 explains the data that we use to evaluate the performance of the model, and presents basic moments from the data and the model as calibrated by BY and BKY. Section 4 examines the ability of stock prices to predict consumption growth, dividend growth, and stock returns, while Section 5 looks at stock price predictions of the volatility of these time series. Section 6 examines instrumental variables estimates of the elasticity of intertemporal substitution, which are usually less than one in aggregate data, and asks whether the long-run risks model can explain this fact. Section 7 studies the implications of the long-run risks model for the term structure of real interest rates, and Section 8 concludes.

2 The Long-Run Risks Model

Bansal and Yaron (2004), henceforth "BY", propose the following process for consumption and dividends:

$$
\begin{aligned}
g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\
\sigma_{t+1}^2 &= \sigma^2 + \nu_1(\sigma_t^2 - \sigma^2) + \sigma_w w_{t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} \\
& \quad w_{t+1}, e_{t+1}, u_{t+1}, \eta_{t+1} \sim i.i.d.\ \mathcal{N}(0,1).
\end{aligned}
\tag{1}
$$

Here g_{t+1} is the growth rate of consumption, x_t is a persistently varying component of the expected consumption growth rate, and σ_t^2 is the conditional variance of consumption with unconditional mean σ^2. Dividends are imperfectly correlated with consumption, but their growth rate $g_{d,t+1}$ shares the same persistent and predictable component x_t scaled by a parameter ϕ, and the conditional volatility of dividend growth is proportional to the conditional volatility of consumption growth.[3]

Both the expected consumption growth rate and the conditional variance of consumption follow first-order autoregressive processes. The variance process can take negative values, but this will happen only with small probability if the mean is high enough relative to the volatility of variance. In simulations, BY replace negative realizations of the conditional variance with a very small positive number, and we follow the same procedure here.

Table I reports parameter values from the calibrations of BY and Bansal, Kiku, and Yaron (2007a), henceforth "BKY". All parameters are given in monthly terms; thus mean consumption growth of 0.0015, or 15 basis points per month, corresponds to annualized growth of 1.8%. The persistence of the predictable component of consumption growth is 0.979 in BY and 0.975 in BKY, implying half-lives of between two and three years: $-\ln(2)/\ln(.979) = 33$ months for BY, and 27 months for BKY.

Dividends are more variable than consumption, and this is captured by the parameters ϕ and φ_d. The first measures the sensitivity of predictable dividend growth

[3] This process does not impose cointegration between consumption and dividends. Some more recent research, notably Bansal, Dittmar, and Kiku (2008) and Hansen, Heaton, and Li (2008), emphasizes such cointegration.

to predictable consumption growth, while the second measures the ratio of the standard deviations of dividend shocks and consumption shocks. The first parameter is 3 in BK and 2.5 in BKY, while the second is 4.5 in BK and 5.96 in BKY. Both calibrations imply that dividend growth is less predictable than consumption growth, but the difference between the two processes is accentuated in BKY. In addition, BKY introduces a contemporaneous correlation of consumption shocks and dividend shocks that is absent in BY.

The persistence of volatility is 0.987 in BY, implying a half-life slightly over four years, and 0.999 in BKY, implying an essentially infinite (58-year) half-life. Volatility shocks have similar standard deviations in the two calibrations (0.0000023 in BY and 0.0000028 in BKY), but the greater persistence of volatility in BKY implies that volatility shocks are very much more important for asset prices in that calibration. The original BY calibration is driven by long-run consumption growth risk, whereas in the BKY calibration long-run volatility risk is more important.

Because volatility is so persistent in the BKY calibration, it is much more likely to go negative than in the BY calibration. In simulations, we find negative realizations of volatility 1.3% of the time for the BKY process, but less than 0.001% of the time for the BY process. When we simulate 77-year paths of volatility using the BKY calibration, over half of them go negative at some point, whereas this happens less than 0.2% of the time for the BY process. Thus the censoring of negative volatility realizations has a nontrivial effect on the BKY calibration.

2.1 Solving and simulating the model

BY solve the long-run risks model using analytical approximations. They assume that the log price-consumption ratio for a consumption claim, z_t, is linear in the conditional mean and variance of consumption growth, the two state variables of the model:

$$z_t = A_0 + A_1 x_t + A_2 \sigma_t^2, \tag{2}$$

and that the log price-dividend ratio for a dividend claim, $z_{m,t}$, is similarly linear:

$$z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2. \tag{3}$$

Under the assumption that a representative agent has Epstein-Zin utility with coefficient of relative risk aversion γ and elasticity of intertemporal substitution ψ, the log stochastic discount factor for the economy is given by

$$\ln M_{t+1} = \theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}, \tag{4}$$

where $\theta = (1-\gamma)/(1-1/\psi)$ and $r_{a,t+1}$ is the return on a consumption claim, or equivalently the return on aggregate wealth.

BY use the Campbell-Shiller (1988) approximation for the return on the consumption claim in relation to consumption growth and the log price-consumption ratio:

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{5}$$

where κ_0 and κ_1 are parameters of linearization. Substituting equations (1) and (5) into equation (4), the innovation in the log SDF can be written as

$$m_{t+1} - \mathrm{E}_t\left(m_{t+1}\right) = -\lambda_\eta \sigma_t \eta_{t+1} - \lambda_e \sigma_t e_{t+1} - \lambda_w \sigma_w w_{t+1}, \tag{6}$$

where $\lambda_\eta = \gamma$, $\lambda_e = (1-\theta)\kappa_1 A_1 \varphi_e$, and $\lambda_w = (1-\theta)\, A_2 \kappa_1$. The $\lambda' s$ represent the market prices of risk for consumption shocks η_{t+1}, expected consumption growth shocks e_{t+1}, and volatility shocks w_{t+1} respectively.

In order to solve the model, one must find the unknown parameters A_0, A_1, A_2, A_{0m}, A_{1m}, A_{2m}, κ_0, and κ_1. Conditional on the linearization parameters κ_0 and κ_1, the A parameters can be found analytically. The parameters A_0 and A_2 determine the mean of the price-consumption ratio, $\overline{z}$, and the parameters κ_0 and κ_1 are simple nonlinear functions of $\overline{z}$. It is straightforward to iterate numerically until a fixed point for $\overline{z}$ is found. Campbell (1993) and Campbell and Koo (1997) study a somewhat simpler model, without volatility shocks, and find that the loglinear approximation method is highly accurate provided that numerical iteration is used to find a fixed point for $\overline{z}$, but approximation accuracy deteriorates noticeably if $\overline{z}$ is prespecified. Details of the approximate solution method for the long-run risks model are given in the appendix.

We compare the model with quarterly and annual data by simulating the model at a monthly frequency and then time-aggregating the data to a quarterly or annual frequency. First, we generate four sets of i.i.d. standard normal random variables and use these to construct the monthly series for consumption, dividends, and state variables using equation (1). Following BY, we replace negative realizations of conditional variance with a small positive number. Next, we construct quarterly (annual) consumption and dividend growth by averaging three (twelve) consumption and dividend levels, and then taking the growth rate of the average.[4] Low-frequency log market returns and risk free rates are the sums of monthly values, while the log price-consumption and price-dividend ratios use prices measured from the last month of the quarter or year.

The solution of the model depends on the preference parameters of the representative agent. Table I reports the parameters used in BY and BKY. BY consider relative risk aversion coefficients of 7.5 and 10, and assume an elasticity of intertemporal substitution of 1.5 and a time discount factor of 0.998 per month, equivalent to a pure rate of time preference of 2.4% per year. BKY set risk aversion at 10 and the EIS at 1.5, and use a higher time discount factor of 0.9989 per month, implying a pure rate of time preference of 1.3% per year.

3 Consumption and Financial Market Data

In order to evaluate the performance of the long-run risks model, we follow BY and use data on US nondurables and services consumption from the Bureau of Economic Analysis. We consider both a long-run annual series over the period 1930-2006, and a postwar quarterly US series over the period 1947:2-2007:3.

Basic data moments

Table II reports basic annual moments for US data over the period 1930-2006, and the annual population moments implied by the BY and BKY calibrations of the long-run risks model with relative risk aversion $\gamma = 10$. The population moments are calculated from a single simulation run over 1.2 million months or 100,000 years. Table III repeats this exercise for quarterly postwar US data and the quarterly population moments from the model. We look at five variables: the changes in log consumption and dividends, log stock return, log riskfree interest rate, and log price-dividend ratio. All variables are measured in real terms. For each variable, we report the mean, standard deviation, and first-order autocorrelation.

[4] We have verified that when we generate underlying iid consumption growth series using this procedure, the growth rate of time-aggregated consumption has a first-order autocorrelation of 0.25 as in the classic result of Working (1960).

It is apparent from Table II that the long-run risks model does a decent job of matching many basic properties of the long-run annual data, including the means, standard deviations, and first-order autocorrelations of consumption growth, dividend growth, and stock returns. The model understates the volatility of the riskless interest rate at about 1.3% in both calibrations, compared to 4.28% in the data. However, this is actually a strength of the model, since the data record the ex post real return on a short-term nominally riskless asset, not the ex ante (equal to ex post) real return on a real riskless asset. Volatile inflation surprises increase the volatility of the series in the data, but not in the model.

A much more serious discrepancy is that the long-run risks model greatly understates the volatility of the log price-dividend ratio. In the model, the standard deviation of z_m is 0.16 in the BY calibration and 0.26 in the BKY calibration, as compared with 0.46 in the annual data. Historical stock prices display low-frequency variation relative to cash flows, which is not captured by the model.[5]

The same issues arise in postwar quarterly data in Table III. At first glance, the behavior of quarterly dividend growth is an additional problem. The model implies a modest positive autocorrelation of dividend growth, but in quarterly data dividend growth has a first-order autocorrelation of -0.56. However this results merely from seasonality in dividend payments, a phenomenon that is commonly ignored in stylized asset pricing models. Dividend seasonality should not be regarded as an important omission of the long-run risks model.

A more serious difficulty is that postwar quarterly consumption growth has a much lower first-order autocorrelation than implied by the model. The autocorrelation in the data is only 0.20, less than the 0.25 autocorrelation that is obtained by time-averaging a continuous-time random walk (Working 1960). The BY and BKY calibrations imply autocorrelations of 0.38 and 0.34 respectively. It seems that the evidence for a persistent component of consumption growth is critically dependent on the use of consumption data from the period of the Great Depression.

Higher-order autocorrelations

Table IV sheds more light on the dynamic behavior of consumption and dividend growth by reporting the first five autocorrelations of these growth rates in the annual

[5] The historical standard deviation of the log price-dividend ratio is this high in part because stock prices were persistently high at the end of our sample period. If we end the sample in 1998, as in BY, we obtain a lower standard deviation of 0.36, still considerably higher than in the model.

1930-2006 data. The first two autocorrelations of consumption growth are positive, but the next three autocorrelations are negative, suggesting some longer-run mean reversion in the level of consumption. The first autocorrelation of dividend growth is positive, but the next four are negative, again suggestive of mean reversion.

Panel A of the table compares these data moments with those implied by the BY calibration of the long-run risks model, while Panel B compares the data with the BKY calibration of the model. In each panel, we report the model's population moment; the model's median moment in 100,000 finite-sample simulations of 924 months or 77 years; and the percentile of the cross-sectional distribution of the model's finite-sample moments that corresponds to the moment in the data. This last number can be interpreted as a p-value for a one-sided test of the model based on the data moment.

Not surprisingly, the long-run risks model implies that all autocorrelations of consumption and dividend growth should be positive. Table IV shows that the higher autocorrelations of annual consumption growth, particularly the third and fourth, are strikingly low relative to the predictions of the long-run risk model. All the autocorrelations of annual dividend growth are somewhat low, but the discrepancy is particularly severe in the first two autocorrelations. The BKY calibration performs somewhat better than the BY calibration, but there is some evidence against both calibrations in these test statistics.

A similar analysis can be performed with quarterly data, but more autocorrelations must be included to capture mean reversion that manifests itself over several years. Results are similar and to save space we do not report them here.

Sensitivity to preference parameters

Table V shows how the asset pricing properties of the long-run risks model vary with the preference parameters assumed for the representative agent. The table reports the mean and standard deviation of the log riskfree interest rate, equity risk premium, and log price-dividend ratio for the BY calibration (Panel A) and the BKY calibration (Panel B) with nine sets of parameters: relative risk aversion coefficients of 5, 10, and 15, and elasticities of intertemporal substitution of 0.5, 1.5, and 2.0. The base case is the central one in each three by three matrix, with values almost equal to those in Tables II and III (small differences result from randomness across simulations). The mean equity premium is calculated by adding one-half the variance of excess stock returns to the mean log stock return (a Jensen's Inequality correction), and subtracting the mean log interest rate.

It is immediately apparent from this table that the elasticity of intertemporal substitution ψ has an important effect on the behavior of asset prices in the long-run risks model. When $\psi < 1$, the riskless interest rate is high and volatile because the representative agent dislikes increasing consumption over time and would like to borrow from the future to flatten the upward-sloping and time-varying expected consumption growth path. Also, the equity premium is trivially small and the volatility of the price-dividend ratio is low, implying that the volatility of stock returns is close to the volatility of dividend growth.

The low volatility of the price-dividend ratio results from offsetting effects of expected consumption growth on stock prices. Rapid consumption growth raises stock prices by increasing expected future dividends, but lowers them by increasing real interest rates. The leverage parameter ϕ measures the strength of the effect on future dividends, while the reciprocal of the EIS, $1/\psi$, measures the strength of the effect on interest rates. If ϕ is close to $1/\psi$, then the two effects roughly cancel and stock prices respond only weakly to long-run growth shocks. The BY calibration assumes $\phi = 3$ and the BKY calibration assumes $\phi = 2.5$, so the case of $\psi = 0.5$ in Table V produces a relatively stable price-dividend ratio and a volatility of stock returns close to the volatility of dividend growth.[6]

The low equity premium with $\psi < 1$ is closely related. In the BY calibration, there is no contemporaneous correlation between consumption growth and dividend growth, so in a power utility model with $\psi = 1/\gamma$, the equity premium would be zero.[7] With Epstein-Zin utility, the equity premium depends not only on the covariance of the stock return with contemporaneous consumption growth, but also on its covariances with shocks to expected future consumption growth and consumption volatility. If $\psi > 1/\gamma$ as assumed in the long-run risks literature, an asset that pays off when there is an upward revision in expected consumption growth is risky and commands a premium. However, this premium is small when the response of stock prices to expected future consumption growth is weak. In addition, the sign of the premium for consumption volatility risk is ambiguous. When $\psi < 1$, stock prices increase when volatility increases and the risk premium for volatility shocks is negative (Lettau,

[6] Campbell (2003) gives a detailed account of these offsetting effects for a leveraged consumption claim in a homoskedastic model.

[7] The BKY calibration does allow for a positive contemporaneous correlation of consumption and dividend shocks, producing a small positive equity premium even in the power utility case.

Ludvigson, and Wachter 2008).

In the BKY calibration shown in Panel B, an extreme problem can arise in the case where $\psi < 1$. Because the rate of time preference is relatively low, and long-run uncertainty about consumption is high as a result of persistently time-varying volatility, consumers have a strong desire to save in the BKY calibration. If they are sufficiently risk averse, precautionary savings can make the equilibrium real interest rate negative and an infinite-lived consumption or dividend claim can have an infinite price. This happens when $\psi = 0.5$ and $\gamma = 10$ or 15. Since the consumption claim appears in the stochastic discount factor for Epstein-Zin utility, this problem prevents us from even calculating the equilibrium riskless interest rate for these cases.

The numbers in Table V make it clear that the long-run risks model can only match the level of the equity premium, and the volatility of stock prices in relation to dividends, if the elasticity of intertemporal substitution is greater than one. This observation is not new; it has been emphasized by BY and other papers in the long-run risks literature.

The relative importance of consumption and volatility shocks

The BY and BKY calibrations of the long-run risk model assign very different roles to movements in consumption growth and volatility. Movements in volatility have very little effect in the BY calibration, whereas they are primary in the BKY calibration. To establish this, we have calculated the moments shown in the central cases of Table V for two simpler models, one with constant volatility and time-varying expected consumption growth, and one with iid consumption growth.

In the BY calibration, the equity premium is zero with iid consumption growth, 5.31% with constant volatility and time-varying expected consumption growth, and 5.61% with time-varying volatility. The standard deviation of the log price-dividend ratio is zero with iid consumption growth and 0.16 with constant or time-varying volatility. It is apparent that time-variation in volatility is of little consequence for the results reported by BY.

In the BKY calibration, the results are very different. The equity premium is 1.61% with iid consumption growth, 3.82% with constant volatility and time-varying expected consumption growth, and 7.85% with time-varying volatility. (The positive equity premium with iid consumption growth results from the positive correlation of dividends and consumption growth assumed in the BKY calibration.) The standard deviation of the log price-dividend ratio is zero with iid consumption growth, 0.08 with constant volatility and time-varying expected consumption growth, and 0.27 with time-varying volatility. Most of the equity premium, and most of the variability in stock prices relative to dividends, result from time-varying volatility in the new calibration of the long-run risks model proposed by BKY.

4 Predictability of Stock Returns, Consumption, and Dividends

In the long-run risks model, the main cause of stock price variability relative to dividends is predictable and persistent variation in consumption growth, which creates similar variation in dividend growth. Thus, it is natural to test the model by evaluating the ability of the log price-dividend ratio to predict long-run consumption and dividend growth. At the same time, a large empirical literature has argued that the log price-dividend ratio predicts excess stock returns and not dividend growth or real interest rates (Campbell and Shiller 1988, Fama and French 1988, Hodrick 1992). This suggests that one should compare the predictability of excess returns with the predictability of consumption and dividend growth, in the data and in simulations from the long-run risks model.

We undertake this exercise in Table VI. Panel A of each table compares the data with the BY calibration of the long-run risks model, while Panel B uses the BKY calibration. In each panel of each table we regress excess stock returns, consumption growth, or dividend growth, measured over horizons of 1, 3, or 5 years, onto the log price-dividend ratio at the start of the measurement period. We report results both for annual data over the period 1930-2006 and for quarterly data over the period 1947.2-2007.3.

In the data, one must adopt a convention about the timing of measured consumption. Measured consumption is a flow that takes place over a discrete time interval, but in a discrete-time asset pricing model, consumption takes place at a point of time and consumption growth is measured over a discrete interval from one point of time to the next. To match the data to the model, one must decide whether measured consumption should be thought of as taking place at the beginning of each period, or the end. The former assumption gives a higher contemporaneous correlation of consumption growth and asset returns, and is advocated by Campbell (2003). The latter assumption generates a higher correlation between consumption growth and lagged financial market data, and is used by BY and Parker and Julliard (2005) among others. Here we report results using both assumptions to clarify how this affects the results. In each case we time-aggregate the model from monthly data using the same timing assumption so that the comparison of data and model is legitimate.

The top part of Table VI shows regression coefficients, t statistics, and R^2 statistics for predicting excess stock returns, first in the annual data and then in the quarterly postwar data. At the left, we report the estimated regression coefficients in the data, then the population coefficients implied by the long-run risks model, the median coefficients generated by finite-sample regressions on simulated data from the long-run risks model, and finally the percentiles of the simulated coefficients that correspond to the estimated coefficients in the data. In the center, we report the t statistics of the estimated coefficients, and at the right we report R^2 statistics. Just as with the regression coefficients, we first report the R^2 statistics in the data, then the population values implied by the long-run risks model, median values from simulations, and the percentiles of the simulated statistics corresponding to the statistics in the data.

The remaining parts of Table VI repeat these exercises for consumption growth, first with the end-of-period timing convention and then with the beginning-of-period timing convention, and finally for dividend growth.

Panel A of Table VI shows a striking contrast between the patterns in the data and in the BY calibration of the long-run risks model. In the data, the log price-dividend ratio predicts excess stock returns negatively, with a coefficient whose absolute value increases strongly with the horizon. At a 5-year horizon, the R^2 statistic is 28% in long-run annual data and 30% in postwar quarterly data. However there is relatively little predictability of consumption growth in the data. At a one-year horizon there is some predictability in annual data if the end-of-period timing assumption is used, but this predictability dies out rapidly and disappears entirely under the beginning-of-period timing assumption. There is no predictability of consumption growth under either timing convention in postwar quarterly data. Dividend growth predictability also appears to be short-term, dying out rapidly as one lengthens the horizon, and is absent in postwar quarterly data.

These empirical patterns are the reverse of the population predictions of the long-run risks model. According to the model, regressions of excess returns on log price-dividend ratios have true R^2 statistics that are never greater than 0.1%, while regressions of consumption growth on stock returns have true R^2 statistics that increase rapidly with the horizon. At five years, the true explanatory power of the log price-dividend ratio is 37% for consumption growth with the end-of-period consumption timing assumption, 24% for consumption growth with the beginning-of-period timing assumption, and 27% for dividend growth.

The contrast between the data and the predictions of the model is almost as strong when we use the finite-sample distributions of the model statistics rather than their population values. For excess returns, median finite-sample R^2 statistics remain very small under the null that the long–run risks model describes the data. However, the finite-sample distribution of these statistics has a fat right tail because of the well-known Stambaugh (1999) bias in predictive regressions with persistent regressors whose innovations are correlated with innovations in the dependent variable. The bias affects not only the coefficients, but also the t statistics and R^2 statistics of predictive regressions (Cavanagh, Elliott, and Stock 1995). Because of this problem, the predictability of excess returns can only be used to reject the model statistically at longer horizons. For consumption and dividend growth, Stambaugh bias is a much less serious concern, and both the regression coefficients and R^2 statistics deliver strong statistical rejections of the long-run risk model at almost all horizons.

BY conduct a similar exercise to this with qualitatively similar results. However, they report only median finite-sample regression coefficients and R^2 statistics, and the contrast between the model and the data is much less extreme in their results. We cannot explain the discrepancy, but we do note that Bui (2007) and Garcia, Meddahi, and Tédongap (2008) report results similar to ours. More work is needed to understand this issue.

Panel B of Table VI repeats this analysis for the BKY calibration of the long-run risks model. Recall that this calibration greatly increases the persistence of volatility; it therefore increases the effect of volatility on asset prices and the predictive power of the log price-dividend ratio for excess stock returns, and reduces the predictive power for consumption and dividend growth. At a five year horizon, the true explanatory power of the log price-dividend ratio is 4% for excess stock returns, 4% for consumption growth with the end-of-period timing assumption, 2% for consumption growth with the beginning-of-period timing assumption, and 1% for dividend growth. In finite samples there are enough simulations in which stock prices have spuriously increased predictive power for stock returns, and decreased predictive power for consumption and dividend growth, that statistical rejections of the model are less extreme for this calibration. In annual data only consumption predictability seems anomalous with respect to this calibration, and only at certain horizons (longer for the end-of-period timing convention, and shorter for the beginning-of-period timing convention). In postwar quarterly data, longer-term dividend predictability also provides evidence against the model.

The contrast between the long-run risks model and the data can be better understood by studying the timing of the relationship between stock prices and consumption growth. Consider a regression of K-period time-aggregated consumption growth onto the log price-dividend ratio, with a lead of j periods:

$$\Delta c_{t+j} + ... + \Delta c_{t+j+K} = \alpha_{jK} + \beta_{jK}(p_t - d_t) + \varepsilon_{jKt}.$$

When $j \geq 1$, this is a predictive regression of the sort we have reported in Table VI. The long-run risks model implies that both the regression coefficient and the R^2 statistic of the regression should be highest when j is around $-K/2$, for then the K-period consumption growth rate is centered at time t and best approximates the unobserved state variable x_t that drives stock prices. The regression coefficient and R^2 statistic decline slowly as we move j away from $-K/2$, and remain high even when $j \geq 0$.

Figures 1-4 plot the regression coefficient β_{jK} and R^2 statistic R^2_{jK} against j, for several alternative horizons K. Figures 1 and 2 are based on annual data, while Figures 3 and 4 are based on quarterly data. Figures 1 and 3 show regression coefficients, and Figures 2 and 4 show R^2 statistics. The top panel of each figure shows a 1-year horizon, the middle panel shows a 3-year horizon, and the bottom panel shows a 5-year horizon. Each panel contains three curves, one for the BY calibration, one for the BKY calibration, and one for the historical data. The BKY curves are much lower than the BY curves, but the historical curves are typically lower again.

It is also noteworthy that in the postwar quarterly data shown in Figures 3 and 4, the historical curves are particularly shifted down in the predictive region $j \geq 1$ at the right of the figures. In this sense the empirical curves appear shifted to the left relative to the theoretical curves. To the extent that stock prices are related to consumption growth, they appear relatively more responsive to lagged consumption growth, and relatively less predictive of future consumption growth, than the long-run risks model implies. Responsiveness of stock prices to lagged consumption growth is a phenomenon that is captured by habit formation models such as Campbell and Cochrane (1999).

In summary, the long-run risks model, as its name suggests, tends to generate stock prices that reveal the long-run prospects for consumption and dividend growth. This does not seem to be the case in the data, so extreme movements in volatility, as assumed by BKY, are needed to bring the model into even rough concordance with the data.

5 Stock Prices and Volatility

Movements of consumption volatility are also important drivers of stock prices in the long-run risks model, and particularly so in the BKY calibration of that model. Thus it is appropriate to evaluate the model by asking whether it matches the ability of stock prices to predict future realized volatility of consumption, dividends, or excess stock returns. In Table VII, we do this using a measure of realized volatility suggested by Bansal, Khatchatrian and Yaron (2005). As before, Panel A refers to the BY calibration and Panel B refers to the BKY calibration of the long-run risks model.

We begin by fitting an AR(1) process for each variable y_t that we are interested in:

$$y_{t+1} = b_0 + b_1 y_t + u_{t+1}. \tag{7}$$

Then we calculate K-period realized volatility as the sum of the absolute values of the residuals over K periods:

$$Vol_{t,t+K-1} = \sum_{k=0}^{K-1} |u_{t+k}|. \tag{8}$$

Finally, we regress the log of K-period realized volatility onto the log price-dividend ratio:

$$\ln\left[Vol_{t+1,t+K}\right] = \alpha_c + \beta_c(p_t - d_t) + \xi_t. \tag{9}$$

For consumption and dividends, the data interval is annual or quarterly. It is possible to calculate realized stock return volatility in the same manner, treating the raw data as annual or quarterly, and we do this at the top of each panel of Table VII. However, since stock returns are measured more frequently, a better measure of volatility can be obtained by starting with monthly data, a procedure we follow immediately below.

Table VII shows that the log price-dividend ratio predicts consumption volatility, with a negative sign, at horizons from 1 to 5 years. The effect is highly statistically significant, and the explanatory power of the regressions at a 5-year horizon is 24% in long-run annual data and 34% in postwar quarterly data. The evidence that the log price-dividend ratio predicts dividend or return volatility is considerably weaker. The only statistically significant prediction for return volatility is in annual data, at a 1-year horizon, measuring volatility from monthly data.

The BY calibration of the long-run risks model, in Panel A, generates a relation between stock prices and consumption or dividend volatility with the same negative sign that we observe in the data. However, the effect in the model is far weaker than in the data; the theoretical regression coefficients are much smaller than the empirically estimated coefficients, and the explanatory power of the regressions is trivially small both in population and in finite samples. The BY calibration of the long-run risks model is strongly rejected statistically on the basis of its lack of explanatory power for consumption and dividend volatility. It does, however, fit the observed weak relation between stock prices and the future volatility of stock returns.

The BKY calibration of the model, in Panel B of Table VII, has a much more persistent volatility process. This increases the predictive power of stock prices for consumption and dividend volatility to levels that match the data quite well. Unfortunately, the model also predicts that stock prices should be extraordinarily good predictors of the future volatility of stock returns. If volatility is measured from monthly return data, the population R^2 at a five-year horizon is over 70% in Panel B of Table VII, as compared with 8% in long-run annual data and 1% in postwar quarterly data. Thus the BKY calibration creates a new puzzle: if stock prices are driven by persistent changes in the volatility of consumption, which in turn moves the stock market, why don't they forecast the future volatility of the stock market itself?

To ensure the robustness of these results, we have also considered a measure of realized volatility used by Campbell (2003). We start by regressing each variable of interest y_{t+1} onto the log price-dividend ratio:

$$y_{t+1} = b_0 + b_1(p_t - d_t) + u_{t+1}. \tag{10}$$

In the second stage, we regress the K-period average of squared residuals onto the log price-dividend ratio:

$$\frac{\sum_{k=0}^{K-1} u_{t+1+k}^2}{K} = \alpha_c + \beta_c(p_t - d_t) + \xi_t. \quad (11)$$

The general pattern of results using this method is very similar to those using the Bansal, Khatchatrian and Yaron (2005) method.

The message of this section is that the BY calibration of the long-run risks model greatly understates the effect of consumption volatility on stock prices. The BKY calibration does much better in this respect, in effect changing the driving force of the model from consumption growth to consumption volatility. However, this leads to a new difficulty, which is that there has only been a weak historical relation between stock prices and the volatility of stock returns themselves. An interesting challenge for future research will be to build a model that matches the strong relation between stock prices and consumption volatility without generating a counterfactually strong relation between stock prices and stock return volatility.

6 Estimating the Elasticity of Intertemporal Substitution

We have noted that the long-run risks model critically depends on the assumption that the elasticity of intertemporal substitution is greater than one. In a model with constant variance, this implies that the real interest rate should be perfectly correlated with, but less volatile than, predictable consumption growth.

Hansen and Singleton (1983), followed by Hall (1988), Campbell and Mankiw (1989), and others, have used an instrumental variables (IV) regression approach to estimate the elasticity of intertemporal substitution from the homoskedastic Euler equation. One way to run the regression is as

$$r_{i,t+1} = \mu_i + \left(\frac{1}{\psi}\right) \Delta c_{t+1} + \eta_{i,t+1}. \quad (12)$$

In general the error term $\eta_{i,t+1}$ will be correlated with realized consumption growth so OLS is not an appropriate estimation method. However $\eta_{i,t+1}$ is uncorrelated with any variables in the information set at time t. Hence any lagged variables correlated with asset returns can be used as instruments in an IV regression to estimate $1/\psi$.

Alternatively, one can reverse the regression and estimate

$$\Delta c_{t+1} = \tau_i + \psi r_{i,t+1} + \zeta_{i,t+1}. \tag{13}$$

If the orthogonality conditions hold, then the estimate of ψ in (13) will asymptotically be the reciprocal of the estimate of $1/\psi$ in (12). In a finite sample, however, Staiger and Stock (1997) have shown that the IV estimator is poorly behaved if the right hand side variable is difficult to predict. This means that if the Euler equation holds and ψ is small, it is better to estimate (13); however, if ψ is large, it is better to estimate (12).

Hall (1988) estimated an extremely small value of ψ using this approach. Campbell and Mankiw (1989) found some predictability of consumption growth associated with predictable income growth, but little predictable variation associated with interest rates, again implying a low ψ. Campbell (2003) summarizes these results and finds similar patterns in international data.

BY have criticized this literature on the grounds that time-varying volatility causes time-variation in the intercept of the Euler equation and biases the estimate of the elasticity of intertemporal substitution. While this criticism is correct in principle, it is an empirical question whether there is a large downward bias. In Table VIII we simulate the long-run risks model to see whether IV estimates of ψ are importantly downward biased.

Table VIII reports two-stage least squares estimates of equations (13) and (12), in both annual and quarterly data. As before, Panel A refers to the BY calibration of the long-run risks model, and Panel B to the BKY calibration. The first two rows of each part of Table VIII report results using the log short-term real interest rate as the asset return, while the second two rows use the realized log stock return. We use the beginning-of-period timing convention for consumption. The instruments are the asset return, the consumption growth rate, and the log price-dividend ratio, lagged twice to avoid difficulties caused by time-aggregation of the consumption data (Wheatley 1988, Campbell and Mankiw 1989). The columns report empirical estimates, population estimates implied by the long-run risks model, median finite-sample estimates implied by the model, and finally the percentiles of the finite-sample distribution of model estimates corresponding to the empirical estimates.

Panel A of Table VIII shows that the original BY calibration implies no downward bias in IV estimates of ψ. The population values of the regression coefficients are always above the true elasticity of intertemporal substitution of 1.5, and when the riskless interest rate is used as the asset return the discrepancy between the model estimates and the data estimates provides strong statistical evidence against the model. There is however a serious finite-sample problem with IV estimates of ψ using the stock return as the asset return. If the long-run risks model is true, these estimates are strongly downward-biased and extremely noisy, so they cannot be used to reject the model. Presumably the poor finite-sample performance of IV regressions with stock returns reflects a weak instrument problem.

In the BKY calibration, in Panel B of Table VIII, we do find that IV estimates of the elasticity of intertemporal substitution are generally biased downwards. This reflects the greater importance of time-varying volatility in this calibration of the model. In fact, when the stock return is used as the asset return we get a negative population estimate of ψ. Even in the BKY calibration, however, the low IV estimates of ψ in postwar quarterly data provide some statistical evidence against the model.

These results highlight an empirical difficulty for the long-run risks model, that (particularly in postwar quarterly data) the real interest rate is so volatile relative to predictable variation in consumption growth. It is hard to reconcile this with the assumption of the model that assets are priced by a representative agent with an elasticity of intertemporal substitution greater than one.

Some authors have looked at disaggregated data and have found greater predictable variation in consumption growth than appears in aggregate data. Attanasio and Weber (1993) and Beaudry and van Wincoop (1996) have found higher values for ψ using disaggregated cohort-level and state-level consumption data. Vissing-Jorgensen (2002) points out that many consumers do not participate actively in asset markets; using household data she finds a higher value for ψ among asset market participants. But these results do not confirm the long-run risks model because that model is calibrated to aggregate consumption data. A general equilibrium model with limited asset market participation and long-run risk in the consumption of stock market participants is a different model that remains to be explored.

7 The Term Structure of Real Interest Rates

The long-run risks literature has focused primarily on stock prices, but the model has important implications for the term structure of real interest rates as well. In a consumption-based model with power utility, the risk premia on long-term real bonds relative to short-term real bonds (real term premia) depend on the covariance between innovations to consumption and innovations to real interest rates. If consumption follows a mean-reverting process such that positive shocks to consumption are expected to reverse themselves through slower subsequent consumption growth, then a positive consumption shock causes real interest rates to fall, and bond prices to rise. In this case real bonds are risky and there is a positive real term premium. On the other hand, if consumption growth follows a persistent process such that positive shocks cause upward revisions in expected future consumption growth, then a positive consumption shock causes real interest rates to increase and bond prices to fall. In this case real bonds hedge consumption risk and have a negative real term premium (Campbell 1986).

With Epstein-Zin utility as assumed by the long-run risks model, revisions in expected future consumption growth command a risk premium even if they are uncorrelated with shocks to contemporaneous consumption growth. Since increases in expected consumption growth drive up real interest rates and drive down bond prices, the long-run risks model with a positive risk premium for consumption growth implies a negative real term premium. This fact has been pointed out by Piazzesi and Schneider (2006).

In Table IX we report the moments of yields and returns on real perpetuities for the BY calibration (Panel A) and BKY calibration (Panel B) of the long-run risks model. As in Table V, we consider a matrix of preference parameters with relative risk aversion γ of 5, 10, and 15, and elasticity of intertemporal substitution ψ of 0.5, 1.5, and 2.0. The BY and BKY calibrations are the central cases of Panel A and Panel B respectively. In each panel we report the mean real perpetuity yield, mean yield spread relative to the short-term riskless interest rate, the real term premium, and the standard deviation of excess real bond returns.

In Panel A, we find that with a coefficient of relative risk aversion of 15, the price of a real perpetuity becomes infinite so the moments of returns are undefined. Even when we do have defined moments, real perpetuity prices are extremely high, implying negative yield spreads and real term premia of about -2% in Panel A for the BY base case. While there is relatively little data available on inflation-indexed bond yields, what data we do have suggest higher average yields and real term premia that are close to zero. The behavior of the real term structure is another troubling difficulty for the long-run risks model of asset prices.

In Panel B, the problem is even more serious. The persistence of volatility in the BKY calibration makes long-run consumption extremely uncertain, lowering the safe long-term discount rate in the manner described by Martin (2008a,b) and Weitzman (2007, 2009).[8] Thus the price of a real consol is infinite for every parameter combination considered except $\psi = 0.5$ and $\gamma = 5$.

8 Conclusion

The long-run risks model of asset prices (Bansal and Yaron 2004) is an important advance in that it allows economists to understand asset price variation in an economy with persistent shocks to both consumption growth and volatility, while making a realistic distinction between aggregate consumption and dividends.

However, the model has several important difficulties as a quantitative description of US financial history. First and most salient, in US data there is little evidence either for long-run persistent fluctuations in consumption and dividend growth rates, or for the ability of stock market participants to predict these growth rates. This implies that the long-run risks model cannot use persistent variations in consumption growth to explain stock market variation.

Bansal, Kiku, and Yaron (2007a) appear to recognize this problem and recalibrate the model to emphasize persistent variations in consumption volatility. However this creates a second difficulty, which is that although stock prices strongly predict future consumption volatility, they have little predictive power for the future volatility of stock returns. The discrepancy between these two types of volatility movements is an interesting issue for future research.

[8] Barro (2006, 2009) also studies the effects of fat-tailed consumption shocks on asset prices in models with power utility and Epstein-Zin utility, respectively. However, he considers the short-term real interest rate rather than the long-term real bond yield.

A third difficulty for the long-run risk model is that aggregate consumption growth does not respond to variations in the short-term real interest rate in the manner required by the model's assumption that the elasticity of intertemporal substitution is greater than one. Although Bansal and Yaron (2004) correctly point out that time-varying consumption volatility can bias traditional estimates of the elasticity of intertemporal substitution that assume homoskedastic consumption growth, this bias is not large. Another challenge for future research is to resolve the apparent contradiction between the smoothness of consumption in the face of real interest variation, which suggests a low elasticity of intertemporal substitution, and the negative response of stock prices to consumption volatility, which suggests a high elasticity.

Finally, the long-run risks model generates extremely low yields and negative term premia on long-term inflation-indexed bonds. In the calibration of Bansal, Kiku, and Yaron (2007a), this even implies an infinite price for an inflation-indexed perpetuity. While inflation-indexed bond yields were extremely low in the early stages of the financial crisis of 2007-08, they have been high enough in most other periods to suggest that this too is a challenge for the long-run risks model of asset prices.

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Appendix: Solving the Long-Run Risks Model

This appendix provides solutions for all assets used in the text for the generalized BKY endowment process

$$\begin{aligned}
g_{t+1} &= \mu + x_t + \sigma_t \eta_{t+1} \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1} \\
\sigma^2_{t+1} &= \sigma^2 + \nu_1(\sigma^2_t - \sigma^2) + \sigma_w w_{t+1} \\
g_{d,t+1} &= \mu_d + \phi x_t + \varphi_d \sigma_t u_{t+1} + \pi_d \sigma_t \eta_{t+1} \\
& \quad w_{t+1}, e_{t+1}, u_{t+1}, \eta_{t+1} \sim i.i.d.\ \mathcal{N}(0,1).
\end{aligned} \tag{14}$$

In the BY model $\pi_d = 0$ so BY solutions are a special case. The Euler equation for the economy is

$$\mathrm{E}_t \left[\exp \left(\theta \ln \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} + r_{i,t+1} \right) \right] = 1, \tag{15}$$

where $r_{a,t+1}$ is the log return on the consumption claim and $r_{i,t+1}$ is the log return on any asset. All returns are given by the approximation of Campbell and Shiller (1988) $r_{i,t+1} = \kappa_{0,i} + \kappa_{1,i} z_{t+1,i} - z_{t,i} + g_{i,t+1}$.

Define a vector of state variables $Y_t' = [1 \;\; x_t \;\; \sigma_t^2]$ and the coefficients on the log price consumption ratio $z_t = A' Y_t$ where $A' = [A_o \;\; A_1 \;\; A_2]$. For any other asset i define coefficients in the same manner $A_i' = [A_{o,i} \;\; A_{1,i} \;\; A_{2,i}]$. This appendix prices the consumption claim, the dividend claim $z_{t,m} = A_m' Y_t$ and a levered consumption claim $z_{t,l} = A_l' Y_t$. The consumption claim has leverage $l = 1$, while a real consol bond, that pays one unit of the consumption good in all future periods, has leverage $l = 0$, We find z_t, $z_{t,m}$ and $z_{t,l}$ by the method of undetermined coefficients, using the fact that the Euler equation must hold for all values of Y_t'.

The risk premium on any asset is

$$\begin{aligned}
\mathrm{E}_t\left(r_{i,t+1} - r_{f,t}\right) + \frac{1}{2}\mathrm{Var}_t(r_{i,t+1}) &= -\mathrm{Cov}_t\left(m_{t+1} - E_t m_{t+1}, r_{i,t+1} - E_t r_{i,t+1}\right) \\
&= \sum_{j=n,e,w} \beta_{i,j} \lambda_j \sigma^2_{j,t},
\end{aligned} \tag{16}$$

where $\beta_{i,j}$ is the beta and $\sigma^2_{j,t}$ the volatility of the j^{th} risk source, and the λ_j represent the price of each risk source as defined in the text.

Consumption claim

The risk premium for the consumption claim is

$$\mathrm{E}_t\left[r_{a,t+1} - r_{f,t}\right] + \frac{1}{2}\mathrm{Var}_t\left(r_{a,t+1}\right) = \lambda_n \beta_{a,n}\sigma_t^2 + \lambda_e \beta_{a,e}\sigma_t^2 + \lambda_w \beta_{a,w}\sigma_w^2, \qquad (17)$$

where $\beta_{a,n} = 1$, $\beta_{a,e} = \kappa_1 A_1 \varphi_e$ and $\beta_{a,w} = \kappa_1 A_2$. The conditional variance of the consumption claim is equal to

$$\mathrm{Var}_t\left(r_{a,t+1}\right) = \left[\beta^2_{a,n} + \beta^2_{a,e}\right]\sigma_t^2 + \beta^2_{a,w}\sigma^2_w \qquad (18)$$

The coefficients A' for the log price-consumption ratio z_t are

$$\begin{aligned} A_0 &= \frac{\ln\delta + \mu(1-\frac{1}{\psi}) + \kappa_0 + \beta_{a,w}\sigma^2\left(1-\nu_1\right) + \frac{1}{2}\theta\beta^2_{a,w}\sigma^2_w}{(1-\kappa_1)} \\ A_1 &= \frac{1-\frac{1}{\psi}}{1-\kappa_1\rho}. \\ A_2 &= \frac{\frac{1}{2}\left[\left(\theta - \frac{\theta}{\psi}\right)^2 + \left(\theta\beta_{a,e}\right)^2\right]}{\theta\left(1-\kappa_1\nu_1\right)} \end{aligned} \qquad (19)$$

Dividend Claim

The innovation in the market return $r_{m,t+1} - E_t(r_{m,t+1})$ is

$$r_{m,t+1} - \mathrm{E}_t(r_{m,t+1}) = \varphi_d\sigma_t u_{t+1} + \beta_{m,\eta}\sigma_t\eta_{t+1} + \beta_{m,e}\sigma_t e_{t+1} + \beta_{m,w}\sigma_w w_{t+1}, \qquad (20)$$

where $\beta_{m,\eta} = \pi_d$, $\beta_{m,e} = \kappa_{1,m}A_{1,m}\varphi_e$ and $\beta_{m,w} = \kappa_{1,m}A_{2,m}$, which implies that the risk premium on the dividend claim is

$$\mathrm{E}_t\left[r_{m,t+1} - r_{f,t}\right] + \frac{1}{2}\mathrm{Var}_t\left(r_{m,t+1}\right) = \lambda_\eta \beta_{m,\eta}\sigma_t^2 + \lambda_e \beta_{m,e}\sigma_t^2 + \lambda_w \beta_{m,w}\sigma_w^2. \qquad (21)$$

In the BY calibration $\pi_d = 0$ so the premium from consumption shocks is zero. The coefficients A_m' for the log price-dividend ratio are as follows

$$
\begin{aligned}
A_{0,m} &= \frac{\left[\begin{array}{c} \theta\log(\delta) + \mu(\theta - \frac{\theta}{\psi}) - \lambda_w\sigma^2(1-\nu_1) + (\theta-1)\left[\kappa_0 + A_0\left(\kappa_1 - 1\right)\right] \\ +\kappa_{0,m} + \beta_{m,w}\sigma^2(1-\nu_1) + \mu_d + \frac{1}{2}\left[\beta_{m,w} - \lambda_w\right]^2\sigma_w^2 \end{array}\right]}{(1-\kappa_{1,m})} \\
A_{1,m} &= \frac{\phi - \frac{1}{\psi}}{1-\kappa_{1,m}\rho}. \\
A_{2,m} &= \frac{(1-\theta)\,A_2(1-\kappa_1 v_1) + \frac{1}{2}\left[(\pi_d - \lambda_n)^2 + \left(\beta_{m,e} - \lambda_e\right)^2 + \varphi_d^2\right]}{(1-\kappa_{1,m}\nu_1)}
\end{aligned} \tag{22}
$$

Levered consumption claim

The innovation to a levered consumption claim is

$$
r_{l,t+1} - \mathrm{E}_t r_{l,t+1} = \beta_{l,\eta}\sigma_t\eta_{t+1} + \beta_{l,e}\sigma_t e_{t+1} + \beta_{l,w}\sigma_w w_{t+1}, \tag{23}
$$

where $\beta_{l,\eta} = l$, $\beta_{l,e} = \kappa_{1,l}A_{1,l}\varphi_e$ and $\beta_{l,w} = \kappa_{1,l}A_{2,l}$ so that the risk premium on the claim is equal to

$$
\mathrm{E}_t\left[r_{l,t+1} - r_{f,t}\right] + \frac{1}{2}\mathrm{Var}_t\left(r_{l,t+1}\right) = \lambda_n\beta_{l,\eta}\sigma_t^2 + \lambda_e\beta_{l,e}\sigma_t^2 + \lambda_w\beta_{l,w}\sigma_w^2, \tag{24}
$$

and the coefficients in the log price-levered consumption ratio are

$$
\begin{aligned}
A_{0,l} &= \frac{\left[\begin{array}{c} \theta\ln(\delta) + \mu((\theta-1) + \frac{\theta}{\psi} + l) - \lambda_w\sigma^2(1-\nu_1) + (\theta-1)\left(\kappa_0 + A_0(\kappa_1 - 1)\right) \\ +\kappa_{0,l} + \beta_{l,w}\sigma^2(1-\nu_1) + \frac{1}{2}\sigma_w^2\left(\beta_{l,w} - \lambda_w\right)^2 \end{array}\right]}{1-\kappa_{1,l}} \\
A_{1,l} &= \frac{(l - 1/\psi)}{(1-\kappa_{1,l}\rho)}
\end{aligned} \tag{25}
$$

$$
A_{2,l} = \frac{(\theta-1)\,A_2(\kappa_1\nu_1 - 1) + \frac{1}{2}\left[(l-\lambda_\eta)^2 + \left(\beta_{l,e} - \lambda_e\right)^2\right]}{1-\kappa_{1,l}\nu_1}. \tag{26}
$$

The consumption claim is the special case $l = 1$, and a real consol bond is the special case $l = 0$.

Riskfree interest rate

To derive the riskfree rate, we use the Euler equation for a riskless asset:

$$\begin{aligned} r_{f,t} &= -\theta \log(\delta) + \frac{\theta}{\psi} \mathrm{E}_t \left[g_{t+1} \right] + (1-\theta) \mathrm{E}_t r_{a,t+1} \\ & -\frac{1}{2} \mathrm{Var}_t \left[\frac{\theta}{\psi} g_{t+1} + (1-\theta) r_{a,t+1} \right]. \end{aligned} \tag{27}$$

We subtract $(1-\theta) r_{f,t}$ from both sides and divide by θ, assuming $\theta \neq 0$. It follows that:

$$\begin{aligned} r_{f,t} &= -\log(\delta) + \frac{1}{\psi} \mathrm{E}_t \left[g_{t+1} \right] + \frac{(1-\theta)}{\theta} \mathrm{E}_t \left[r_{a,t+1} - r_{f,t} \right] \\ & -\frac{1}{2\theta} \mathrm{Var}_t \left(m_{t+1} \right), \end{aligned} \tag{28}$$

$\mathrm{Var}_t \left(m_{t+1} \right) = \left(\lambda_n^2 + \lambda_e^2 \right) \sigma_t^2 + \lambda_w^2 \sigma_w^2$ and $\mathrm{E}_t \left[r_{a,t+1} - r_{f,t} \right]$ is given above.

Linearization parameters

For any asset, the linearization parameters are determined endogenously by the following system of equations as discussed in Campbell and Koo (1997) and Bansal, Kiku and Yaron (2007):

$$\begin{aligned} \overline{z_i} &= A_{0,i}(\overline{z_i}) + A_{2,i}(\overline{z_i}) \sigma^2 \\ \kappa_{1,i} &= \frac{\exp(\overline{z_i})}{1 + \exp(\overline{z_i})} \\ \kappa_{0,i} &= \ln(1 + \exp(\overline{z_i})) - \kappa_{1,i} \overline{z_i} \end{aligned} \tag{29}$$

The solution is determined numerically by iteration until reaching a fixed point of $\overline{z_i}$. The dependence of $A_{0,i}$ and $A_{2,i}$ on the linearization parameters is discussed in the previous sections.

Table I

Long Run Risks Parameters

Preference Parameters

	Risk Aversion	EIS	Discount Factor
	γ	ψ	δ
BY	7.5-10	1.5	.998
BKY	10	1.5	.9989

Consumption Growth Dynamics

	Mean Growth	Persistence	LR Vol Multiple
	μ	ρ	φ_e
BY	.0015	.979	.044
BKY	.0015	.975	.038

Dividend Growth Dynamics

	Mean Growth	Leverage	Div Vol Multiple	Cons Exposure
	μ_d	ϕ	φ_d	π_d
BY	.0015	3	4.5	0
BKY	.0015	2.5	5.96	2.6

Volatility Parameters

	Base SD	Vol of Volatility	Vol Persistence
	σ	σ_w	ν_1
BY	.0078	.0000023	.987
BKY	.0072	.0000028	.999

Table I displays the model parameters for Bansal and Yaron (2004) (BY) and Bansal, Kiku and Yaron (2007) (BKY). All parameters are given in monthly terms. The standard deviation of the long run innovations is equal to the volatlity of consumption growth times the long run volatility multiple (LR Vol multiple) and the standard deviation of dividend growth innovations is equal to the volatility of consumption growth times the volatility multiple for dividend growth (Div Vol Multiple). Cons Exposure is the magnitude of the impact of the one period consumption shock on dividend growth. Leverage is the exposure of dividend growth to long run risks.

Table II

Long Run Risks Moments

Yearly Time Interval

Moment	**Model BY**	**Model BKY**	**Data 1930-2006**
$E(\Delta c)$	1.79	1.82	1.95
$\sigma(\Delta c)$	2.92	2.96	2.16
$AC1(\Delta c)$	0.51	0.44	0.44
$E(\Delta d)$	1.66	1.85	1.02
$\sigma(\Delta d)$	11.57	16.42	10.69
$AC1(\Delta d)$	0.40	0.29	0.14
$E(r_e)$	6.62	6.58	6.20
$\sigma(r_e)$	16.88	21.35	18.34
$AC1(r_e)$	0.03	0.02	0.04
$E(r_f)$	2.56	0.99	0.99
$\sigma(r_f)$	1.30	1.28	4.28
$AC1(r_f)$	0.85	0.86	0.59
$E(p-d)$	3.00	3.04	3.31
$\sigma(p-d)$	0.16	0.26	0.46
$AC1(p-d)$	0.77	0.95	0.88

Table II displays moments for the analytical solutions of the Long Run Risks model with the BY calibration in column 1 and the BKY calibration in column 2. The model moments are calculated from a simulation of 1.2 million months. The consumption growth rate and dividend growth rate are calculated by first aggregating monthly consumption to yearly levels, then computing the growth rate, then taking logs. The return on equity and the risk free rate are aggregated to a yearly level by adding log returns within a year. For the log price-dividend ratio the yearly value is taken from the last month of the year. The third column displays the moments from the 1930-2006 yearly dataset. All returns and growth rates are in logs.

Table III

Long Run Risks Moments

Quarterly Time Interval

Moment	**Model BY**	**Model BKY**	**Data 1947.2-2007.3**
$E(\Delta c)$	1.77	1.80	2.06
$\sigma(\Delta c)$	2.47	2.63	1.09
$AC1(\Delta c)$	0.38	0.34	0.20
$E(\Delta d)$	1.66	1.84	1.87
$\sigma(\Delta d)$	10.63	16.29	26.92
$AC1(\Delta d)$	0.27	0.22	-0.56
$E(r_e)$	6.62	6.58	7.38
$\sigma(r_e)$	16.62	21.18	15.45
$AC1(r_e)$	0.01	0.01	0.05
$E(r_f)$	2.56	0.99	0.83
$\sigma(r_f)$	1.34	1.32	1.73
$AC1(r_f)$	0.96	0.96	0.38
$E(p-d)$	3.00	3.04	3.40
$\sigma(p-d)$	0.16	0.26	0.38
$AC1(p-d)$	0.94	0.99	0.97

Table III displays moments for the analytical solutions of the Long Run Risks model with the BY calibration in column 1 and the BKY calibration in column 2 The moments are calculated from a simulation of 1.2 million months. The consumption growth rate and dividend growth rate are calculated by first aggregating monthly consumption to quarterly levels, then computing the growth rate, then taking logs. The return on equity and the risk free rate are aggregated to a quarterly level by adding log returns within a quarter. For the log price-dividend ratio the quarterly value is taken from the last month of the year. The fourth column displays the moments from the 1947.2-2007.3 quarterly dataset. All returns and growth rates are in logs.

Table IV: Panel A

Autocorrelations of Consumption and Dividends: BY Calibration

Consumption Autocorrelations				
Moment	$\widehat{\boldsymbol{\rho}}$	$\boldsymbol{\rho}(pop)$	$\boldsymbol{\rho}(50\%)$	$\%(\widehat{\boldsymbol{\rho}})$
$AC1$	0.444	0.509	0.468	0.416
$AC2$	0.156	0.282	0.225	0.317
$AC3$	-0.104	0.216	0.156	0.034
$AC4$	-0.237	0.170	0.103	0.006
$AC5$	-0.010	0.130	0.066	0.292

Dividend Autocorrelations				
Moment	$\widehat{\boldsymbol{\rho}}$	$\boldsymbol{\rho}(pop)$	$\boldsymbol{\rho}(50\%)$	$\%(\widehat{\boldsymbol{\rho}})$
$AC1$	0.139	0.395	0.366	0.028
$AC2$	-0.259	0.158	0.120	0.002
$AC3$	-0.064	0.124	0.083	0.132
$AC4$	-0.034	0.094	0.052	0.251
$AC5$	-0.035	0.071	0.030	0.302

Table IV.A displays consumption and dividend autocorrelations in yearly data and for the BY calibration. Population values are from a simulation of 1.2 million months.The medians are from 100,000 samples of equivalent length to the data (924 months) and the proportion of those samples with an estimate at or below that of the data is also displayed. The consumption growth rate and dividend growth rate are calculated by first aggregating monthly consumption to yearly levels, then computing the growth rate, then taking logs.

Table IV: Panel B

Autocorrelations of Consumption and Dividends: BKY Calibration

Consumption Autocorrelations				
Moment	$\widehat{\boldsymbol{\rho}}$	$\boldsymbol{\rho}(pop)$	$\boldsymbol{\rho}(50\%)$	$\%(\widehat{\boldsymbol{\rho}})$
$AC1$	0.444	0.433	0.397	0.658
$AC2$	0.156	0.195	0.148	0.521
$AC3$	-0.104	0.144	0.094	0.080
$AC4$	-0.237	0.107	0.055	0.015
$AC5$	-0.010	0.082	0.029	0.385

Dividend Autocorrelations				
Moment	$\widehat{\boldsymbol{\rho}}$	$\boldsymbol{\rho}(pop)$	$\boldsymbol{\rho}(50\%)$	$\%(\widehat{\boldsymbol{\rho}})$
$AC1$	0.139	0.283	0.259	0.154
$AC2$	-0.259	0.036	0.012	0.017
$AC3$	-0.064	0.023	0.005	0.296
$AC4$	-0.034	0.024	-0.003	0.404
$AC5$	-0.035	0.012	-0.008	0.412

Table IV.B displays consumption and dividend autocorrelations in yearly data and for the BKY calibration. Population values are from a simulation of 1.2 million months. The medians are from 100,000 samples of equivalent length to the data (924 months) and the proportion of those samples with an estimate at or below that of the data is also displayed. The consumption growth rate and dividend growth rate are calculated by first aggregating monthly consumption to yearly levels, then computing the growth rate, then taking logs.

Table V: Panel A

Moments and Preference Parameters: BY Calibration

$\mathbf{E}\left(r_f\right)$

Data		0.99		
		ψ		
		0.5	1.5	2.0
γ	5	5.89	3.14	2.73
	10	6.06	2.58	2.11
	15	6.52	2.00	1.46

$\sigma\left(r_f\right)$

Data		4.28		
		ψ		
		0.5	1.5	2.0
γ	5	3.86	1.30	0.97
	10	3.87	1.31	1.00
	15	3.89	1.34	1.05

$\mathbf{E}\left(r_e - r_f\right) + \frac{1}{2}var(r_e - r_f)$

Data		6.89		
		ψ		
		0.5	1.5	2.0
γ	5	0.69	2.70	3.09
	10	1.95	5.58	6.05
	15	3.39	8.21	8.89

$\sigma\left(r_e - r_f\right)$

Data		18.37		
		ψ		
		0.5	1.5	2.0
γ	5	13.08	17.06	17.77
	10	13.06	16.56	17.12
	15	12.96	16.56	16.68

$\mathbf{E}\left(p - d\right)$

Data		3.31		
		ψ		
		0.5	1.5	2.0
γ	5	3.23	3.65	3.71
	10	2.92	3.00	3.01
	15	2.61	2.64	2.64

$\sigma\left(p - d\right)$

Data		0.463		
		ψ		
		0.5	1.5	2.0
γ	5	0.069	0.171	0.183
	10	0.067	0.161	0.172
	15	0.068	0.183	0.166

Table V.A displays moments for the BY calibration for different levels of the EIS and RRA. The moments for each combination of preference parameters are generated from an independent 1.2 million month simulation. The corresponding moments from the 1930-2006 yearly dataset are displayed above the moments from the model.

Table V Panel B

Moments and Preference Parameters BKY Calibration

$\mathbf{E}(r_f)$

Data 0.99				
		ψ		
		0.5	1.5	2.0
γ	5	4.65	4.62	4.63
	10	NA	0.97	0.29
	15	NA	-0.18	-0.99

$\sigma(r_f)$

Data 4.28				
		ψ		
		0.5	1.5	2.0
γ	5	3.32	3.40	3.31
	10	NA	1.32	1.12
	15	NA	1.51	1.37

$\mathbf{E}(r_e - r_f) + \frac{1}{2}var(r_e - r_f)$

Data 6.89				
		ψ		
		0.5	1.5	2.0
γ	5	1.33	1.36	1.39
	10	NA	7.85	8.60
	15	NA	13.22	13.88

$\sigma(r_e - r_f)$

Data 18.37				
		ψ		
		0.5	1.5	2.0
γ	5	19.50	19.93	19.42
	10	NA	21.45	22.13
	15	NA	21.45	22.27

$\mathbf{E}(p - d)$

Data 3.31				
		ψ		
		0.5	1.5	2.0
γ	5	3.79	3.79	3.79
	10	NA	3.03	3.04
	15	NA	2.42	2.43

$\sigma(p - d)$

Data 0.463				
		ψ		
		0.5	1.5	2.0
γ	5	0.15	0.14	0.14
	10	NA	0.27	0.30
	15	NA	0.30	0.33

Table V.B displays moments for the BKY calibration for different levels of the EIS and RRA. The moments for each combination of preference parameters are generated from an independent 1.2 million month simulation. The corresponding moments from the 1930-2006 yearly dataset are displayed above the moments from the model. NA represent cases where the price of the consumption claim is infinite. NA is listed for the risk free rate because it is a function of the expected return on the consumption claim. The price of the log-price dividend ratio is undefined in these cases because it depends on coefficients governing the relationship between the log price-consumption ratio and the state variables.

Table VI: Panel A
Predictability of Excess Returns, Consumption and Dividends: BY Calibration

$\sum_{j=1}^{J}(r_{e,t+j}-r_{f,t+j})$ 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	-0.059	-0.007	-0.049	0.471	-1.262	0.022	0.000	0.007	0.770
3 Y	-0.229	-0.026	-0.144	0.403	-2.678	0.143	0.000	0.018	0.956
5 Y	-0.421	-0.039	-0.230	0.358	-3.617	0.278	0.000	0.027	0.986
1947.2-2007.3 Quarterly Data									
4 Q	-0.116	-0.008	-0.063	0.362	-2.723	0.081	0.000	0.009	0.969
12 Q	-0.293	-0.020	-0.179	0.384	-2.953	0.204	0.000	0.023	0.973
20 Q	-0.478	-0.031	-0.288	0.370	-3.072	0.304	0.000	0.034	0.978

$\sum_{j=1}^{J}(\Delta c_{t+j})$ End of Period Timing 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	0.012	0.114	0.113	0.000	1.704	0.068	0.390	0.361	0.005
3 Y	0.010	0.286	0.271	0.000	0.659	0.013	0.435	0.394	0.001
5 Y	-0.001	0.388	0.350	0.001	-0.043	0.000	0.373	0.322	0.000
1947.2-2007.3 Quarterly Data									
4 Q	0.003	0.110	0.109	0.000	0.628	0.006	0.281	0.269	0.001
12 Q	-0.001	0.262	0.245	0.002	-0.102	0.000	0.348	0.307	0.001
20 Q	-0.002	0.353	0.304	0.012	-0.119	0.000	0.302	0.243	0.006

$\sum_{j=1}^{J}(\Delta c_{t+j})$ Beginning of Period Timing 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	0.001	0.097	0.092	0.001	0.175	0.000	0.280	0.242	0.000
3 Y	-0.010	0.230	0.207	0.004	-1.245	0.018	0.280	0.228	0.029
5 Y	-0.016	0.308	0.259	0.016	-1.466	0.040	0.235	0.176	0.134
1947.2-2007.3 Quarterly Data									
4 Q	0.000	0.103	0.102	0.001	0.063	0.000	0.248	0.225	0.000
12 Q	-0.004	0.246	0.226	0.005	-0.385	0.003	0.306	0.255	0.009
20 Q	-0.005	0.331	0.279	0.022	-0.319	0.003	0.266	0.200	0.028

$\sum_{j=1}^{J}(\Delta d_{t+j})$ 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	0.064	0.343	0.339	0.004	1.793	0.074	0.228	0.207	0.084
3 Y	0.076	0.860	0.816	0.006	0.978	0.034	0.288	0.257	0.035
5 Y	0.051	1.171	1.053	0.016	0.716	0.013	0.265	0.224	0.036
1947.2-2007.3 Quarterly Data									
4 Q	0.009	0.331	0.328	0.004	0.374	0.002	0.156	0.146	0.007
12 Q	0.002	0.790	0.733	0.016	0.028	0.000	0.229	0.196	0.002
20 Q	0.002	1.065	0.911	0.041	0.020	0.000	0.213	0.167	0.003

Table VI.A displays coefficients, t-statistics, and R-squared from predictive regressions of excess returns, consumption growth and dividend growth on log price-dividend ratios using the BY calibration. Standard errors are Newey-West with 2*(horizon-1) lags. Population values are from a simulation of 1.2 million months. The medians from 100,000 samples of equivalent length to the data (924 or 726 months) and the proportion of those samples with an estimate at or below that of the data is also displayed.

Table VI: Panel B
Predictability of Excess Returns, Consumption and Dividends: BKY Calibration

$\sum_{j=1}^{J}(r_{e,t+j}-r_{f,t+j})$ 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	-0.059	-0.078	-0.121	0.661	-1.262	0.022	0.009	0.012	0.642
3 Y	-0.229	-0.226	-0.344	0.607	-2.678	0.143	0.026	0.034	0.871
5 Y	-0.421	-0.368	-0.537	0.570	-3.617	0.278	0.041	0.053	0.930
1947.2-2007.3 Quarterly Data									
4 Q	-0.116	-0.078	-0.133	0.539	-2.723	0.081	0.009	0.014	0.919
12 Q	-0.293	-0.230	-0.371	0.565	-2.953	0.204	0.027	0.039	0.918
20 Q	-0.478	-0.374	-0.575	0.549	-3.072	0.304	0.042	0.060	0.923

$\sum_{j=1}^{J}(\Delta c_{t+j})$ End of Period Timing 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	0.012	0.022	0.046	0.142	1.704	0.068	0.037	0.078	0.456
3 Y	0.010	0.052	0.107	0.148	0.659	0.013	0.042	0.092	0.176
5 Y	-0.001	0.069	0.134	0.183	-0.043	0.000	0.036	0.084	0.011
1947.2-2007.3 Quarterly Data									
4 Q	0.003	0.021	0.048	0.105	0.628	0.006	0.028	0.061	0.132
12 Q	-0.001	0.048	0.102	0.160	-0.102	0.000	0.034	0.079	0.026
20 Q	-0.002	0.062	0.123	0.227	-0.119	0.000	0.030	0.077	0.040

$\sum_{j=1}^{J}(\Delta c_{t+j})$ Beginning of Period Timing 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	0.001	0.018	0.036	0.133	0.175	0.000	0.025	0.050	0.046
3 Y	-0.010	0.040	0.077	0.174	-1.245	0.018	0.025	0.056	0.289
5 Y	-0.016	0.053	0.092	0.235	-1.466	0.040	0.021	0.057	0.423
1947.2-2007.3 Quarterly Data									
4 Q	0.000	0.020	0.044	0.116	0.063	0.000	0.024	0.050	0.015
12 Q	-0.004	0.045	0.093	0.178	-0.385	0.003	0.030	0.067	0.110
20 Q	-0.005	0.059	0.111	0.246	-0.319	0.003	0.026	0.069	0.111

$\sum_{j=1}^{J}(\Delta d_{t+j})$ 1930-2006 Sample									
Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
1 Y	0.064	0.054	0.113	0.367	1.793	0.074	0.008	0.020	0.834
3 Y	0.076	0.133	0.265	0.326	0.978	0.034	0.011	0.034	0.504
5 Y	0.051	0.176	0.329	0.337	0.716	0.013	0.010	0.042	0.284
1947.2-2007.3 Quarterly Data									
4 Q	0.009	0.051	0.119	0.256	0.374	0.002	0.005	0.016	0.203
12 Q	0.002	0.116	0.255	0.297	0.028	0.000	0.008	0.034	0.014
20 Q	0.002	0.150	0.307	0.342	0.020	0.000	0.007	0.045	0.011

Table VI.B displays coefficients, t-statistics, and R-squared from predictive regressions of excess returns, consumption growth and dividend growth on log price-dividend ratios in the yearly datasets using the BKY calibration. Standard errors are Newey-West with 2*(horizon-1) lags. Population values are from a simulation of 1.2 million months. The medians from 100,000 samples of equivalent length to the data (924 or 726 months) and the proportion of those samples with an estimate at or below that of the data is also displayed.

Table VII: Panel A

Predictability of Volatility: BY Calibration

Excess Returns, Consumption and Dividends

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Excess Return Volatility 1930-2006 Sample									
1 Y	-0.081	-0.123	-0.136	0.526	-0.316	0.001	0.000	0.007	0.205
3 Y	-0.059	-0.115	-0.114	0.537	-0.335	0.003	0.001	0.017	0.214
5 Y	-0.017	-0.113	-0.091	0.555	-0.114	0.000	0.002	0.024	0.073
1947.2-2007.3 Quarterly Data									
4 Q	-0.144	-0.138	-0.130	0.430	-0.785	0.011	0.003	0.007	0.487
12 Q	0.047	-0.118	-0.109	0.414	0.337	0.003	0.006	0.018	0.561
20 Q	0.086	-0.102	-0.086	0.364	0.986	0.015	0.007	0.026	0.665

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Excess Return Volatility Estimated from Monthly Data 1930-2006 Sample									
1 Y	-0.075	-0.026	-0.021	0.087	-2.298	0.127	0.011	0.014	0.966
3 Y	-0.051	-0.022	-0.018	0.180	-1.339	0.092	0.018	0.028	0.793
5 Y	-0.039	-0.020	-0.015	0.242	-1.164	0.081	0.019	0.035	0.707
1947.2-2007.3 Quarterly Data									
4 Q	-0.004	-0.024	-0.021	0.657	-0.290	0.001	0.009	0.015	0.144
12 Q	0.006	-0.021	-0.017	0.717	0.395	0.004	0.016	0.033	0.180
20 Q	0.009	-0.019	-0.014	0.723	0.671	0.014	0.017	0.041	0.303

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Consumption Volatility 1930-2006 Sample									
1 Y	-0.481	-0.128	-0.140	0.336	-1.835	0.035	0.000	0.006	0.906
3 Y	-0.491	-0.122	-0.124	0.258	-2.248	0.122	0.001	0.015	0.957
5 Y	-0.564	-0.113	-0.104	0.183	-3.200	0.235	0.002	0.022	0.983
1947.2-2007.3 Quarterly Data									
4 Q	-0.684	-0.134	-0.139	0.146	-3.693	0.191	0.003	0.006	0.983
12 Q	-0.657	-0.116	-0.120	0.096	-3.686	0.313	0.005	0.016	0.988
20 Q	-0.543	-0.100	-0.099	0.056	-3.928	0.335	0.006	0.025	0.990

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Dividend Volatility 1930-2006 Sample									
1 Y	-0.530	-0.146	-0.143	0.319	-1.576	0.035	0.000	0.006	0.899
3 Y	-0.478	-0.144	-0.126	0.269	-1.638	0.070	0.002	0.015	0.862
5 Y	-0.496	-0.123	-0.104	0.223	-1.498	0.084	0.003	0.023	0.815
1947.2-2007.3 Quarterly Data									
4 Q	-0.534	-0.138	-0.137	0.101	-1.858	0.082	0.003	0.007	0.997
12 Q	-0.441	-0.120	-0.118	0.059	-0.974	0.070	0.006	0.017	0.976
20 Q	-0.426	-0.103	-0.098	0.049	-0.877	0.076	0.006	0.025	0.932

Panels 1, 3 and 4 of table VII.A display coefficients, t-statistics, and R-squared from regressions of the log of the sum of absolute residuals from an AR(1) model of consumption growth, dividend growth or excess returns on the log price-dividend ratio using the BY calibration. In panel 2, the dependent variable is instead the standard deviation of monthly excess log returns over a 1, 3 or 5 year horizon. The monthly excess log return for this panel is calculated using CRSP data for stock returns and one month Tbills and the standard deviation is multiplied by the square root of 12 to express it in annualized terms. All standard errors are Newey-West with lags equal to 2*(horizon-1). Population values are from a simulation of 1.2 million months. Medians are from a series of 100,000 samples of equivalent length to the data (924 and 726 months). The percentile is the proportion of the 100,000 samples with an estimate at or below that of the data. The beginning of period timing convention is used for consumption.

Table VII: Panel B

Predictability of Volatility: BKY Calibration

Excess Returns, Consumption and Dividends

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Excess Return Volatility 1930-2006 Sample									
1 Y	-0.081	-1.315	-1.366	0.884	-0.316	0.001	0.085	0.035	0.078
3 Y	-0.059	-1.268	-1.219	0.914	-0.335	0.003	0.273	0.117	0.064
5 Y	-0.017	-1.336	-1.085	0.914	-0.114	0.000	0.364	0.153	0.022
1947.2-2007.3 Quarterly Data									
4 Q	-0.144	-1.301	-1.258	0.954	-0.785	0.011	0.342	0.163	0.075
12 Q	0.047	-1.252	-1.093	0.976	0.337	0.003	0.519	0.277	0.025
20 Q	0.086	-1.213	-0.943	0.973	0.986	0.015	0.564	0.291	0.067

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Excess Return Volatility Estimated from Monthly Data 1930-2006 Sample									
1 Y	-0.075	-0.234	-0.225	0.961	-2.298	0.127	0.572	0.358	0.143
3 Y	-0.051	-0.231	-0.205	0.972	-1.339	0.092	0.703	0.457	0.059
5 Y	-0.039	-0.226	-0.184	0.969	-1.164	0.081	0.721	0.443	0.062
1947.2-2007.3 Quarterly Data									
4 Q	-0.004	-0.230	-0.219	0.993	-0.290	0.001	0.583	0.321	0.007
12 Q	0.006	-0.228	-0.194	0.992	0.395	0.004	0.714	0.411	0.012
20 Q	0.009	-0.224	-0.169	0.987	0.671	0.014	0.734	0.391	0.033

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Consumption Volatility 1930-2006 Sample									
1 Y	-0.481	-1.420	-1.511	0.816	-1.835	0.035	0.095	0.042	0.451
3 Y	-0.491	-1.382	-1.402	0.829	-2.248	0.122	0.290	0.148	0.439
5 Y	-0.564	-1.336	-1.262	0.780	-3.200	0.235	0.372	0.193	0.576
1947.2-2007.3 Quarterly Data									
4 Q	-0.684	-1.473	-1.431	0.783	-3.693	0.191	0.371	0.200	0.484
12 Q	-0.657	-1.408	-1.252	0.753	-3.686	0.313	0.524	0.315	0.497
20 Q	-0.543	-1.360	-1.081	0.763	-3.928	0.335	0.561	0.321	0.522

Periods	$\widehat{\beta}$	$\beta(pop)$	$\beta(50\%)$	$\%(\widehat{\beta})$	t	$\widehat{R}^2$	$R^2(pop)$	$R^2(50\%)$	$\%(\widehat{R}^2)$
Dividend Volatility 1930-2006 Sample									
1 Y	-0.530	-1.483	-1.563	0.810	-1.576	0.035	0.102	0.045	0.436
3 Y	-0.478	-1.431	-1.421	0.833	-1.638	0.070	0.305	0.149	0.306
5 Y	-0.496	-1.384	-1.270	0.804	-1.498	0.084	0.393	0.191	0.288
1947.2-2007.3 Quarterly Data									
4 Q	-0.534	-1.301	-1.462	0.849	-1.858	0.082	0.342	0.203	0.263
12 Q	-0.441	-1.415	-1.268	0.854	-0.974	0.070	0.521	0.315	0.140
20 Q	-0.426	-1.364	-1.091	0.822	-0.877	0.076	0.558	0.318	0.154

Panels 1, 3 and 4 of table VII.B display coefficients, t-statistics, and R-squared from regressions of the log of the sum of absolute residuals from an AR(1) model of consumption growth, dividend growth or excess returns on the log price-dividend ratio using the BKY calibration. In panel 2, the dependent variable is instead the standard deviation of monthly excess log returns over a 1, 3 or 5 year horizon. The monthly excess log return for this panel is calculated using CRSP data for stock returns and one month Tbills, and the standard deviation is multiplied by the square root of 12 to express it in annualized terms. All standard errors are Newey-West with lags equal to 2*(horizon-1). Population values are from a simulation of 1.2 million months. Medians are from a series of 100,000 samples of equivalent length to the data (924 and 726 months). The percentile is the proportion of the 100,000 samples with an estimate at or below that of the data. The beginning of period timing convention is used for consumption.

Table VIII: Panel A

Long Run Risks and the EIS: BY Calibration

$$\Delta c_{t+1} = \tau_i + \psi r_{i,t+1} + \zeta_{i,t+1}$$

Asset	Sample	$\widehat{\psi}$	$\psi(pop)$	$\psi(50\%)$	$\%(\widehat{\psi})$
$r_{f,t+1}$	1930-2006	0.147	1.646	1.229	0.016
	1947.2-2007.3	0.230	1.462	1.379	0.002
$r_{e,t+1}$	1930-2006	0.048	2.081	0.067	0.459
	1947.2-2007.3	-0.025	1.551	0.052	0.411

$$r_{i,t+1} = \mu_i + \left(\frac{1}{\psi}\right) \Delta c_{t+1} + \eta_{i,t+1}.$$

Asset	Sample	$1/\left(\widehat{1/\psi}\right)$	$\left[1/\left(\widehat{1/\psi}\right)\right](pop)$	$\left[1/\left(\widehat{1/\psi}\right)\right](50\%)$	$\%\left[1/\left(\widehat{1/\psi}\right)\right]$
$r_{f,t+1}$	1930-2006	1.014	1.647	1.483	0.117
	1947.2-2007.3	0.728	1.463	1.503	0.010
$r_{e,t+1}$	1930-2006	0.059	2.136	0.387	0.375
	1947.2-2007.3	-0.192	1.599	0.465	0.432

Table VIII.A displays the EIS estimates using both the risk free rate and the market return as the asset for the BY calibration. Population values are from a simulation of 1.2 million months. Medians are from a series of 100,000 samples of equivalent length to the data (924 and 726 months). The percentile is the proportion of the 100,000 samples with an estimate at or below that of the data. The instruments are consumption growth, the log price-dividend ratio and returns for the asset, all lagged twice. The beginning of period timing convention is used for consumption. In the model, the EIS is 1.5.

Table VIII: Panel B

Long Run Risks and the EIS: BKY Calibration

		$\Delta c_{t+1} = \tau_i + \psi r_{i,t+1} + \zeta_{i,t+1}$			
Asset	Sample	$\widehat{\psi}$	$\psi(pop)$	$\psi(50\%)$	$\%(\widehat{\psi})$
$r_{f,t+1}$	1930-2006	0.147	0.933	0.916	0.121
	1947.2-2007.3	0.230	1.051	1.207	0.029
$r_{e,t+1}$	1930-2006	0.048	-0.158	0.036	0.542
	1947.2-2007.3	-0.025	-0.311	0.006	0.396

		$r_{i,t+1} = \mu_i + \left(\frac{1}{\psi}\right) \Delta c_{t+1} + \eta_{i,t+1}.$			
Asset	Sample	$1/\left(\widehat{1/\psi}\right)$	$\left[1/\left(\widehat{1/\psi}\right)\right]$ (pop)	$\left[1/\left(\widehat{1/\psi}\right)\right]$ (50%)	$\%\left[1/\left(\widehat{1/\psi}\right)\right]$
$r_{f,t+1}$	1930-2006	1.014	1.564	1.545	0.218
	1947.2-2007.3	0.728	1.454	1.653	0.030
$r_{e,t+1}$	1930-2006	0.059	-2.077	0.194	0.379
	1947.2-2007.3	-0.192	-1.170	0.097	0.394

Table VIII.B displays the EIS estimates using both the risk free rate and the market return as the asset for the BKY calibration. Population values are from a simulation of 1.2 million months. Medians are from a series of 100,000 samples of equivalent length to the data (924 and 726 months). The percentile is the proportion of the 100,000 samples with an estimate at or below that of the data. The instruments are consumption growth, the log price-dividend ratio and returns for the asset, all lagged twice. The beginning of period timing convention is used for consumption. In the model, the EIS is 1.5.

Table IX: Panel A

Consol Bond Moments: BY Calibration

$\mathbf{E}(y_b)$

		ψ 0.5	ψ 1.5	ψ 2.0
γ	5	3.98	2.25	2.05
	10	0.45	0.45	0.45
	15	NA	NA	NA

$\mathbf{E}(y_b - r_f)$

		ψ 0.5	ψ 1.5	ψ 2.0
γ	5	-1.95	-0.86	-0.69
	10	-5.66	-2.14	-1.63
	15	NA	NA	NA

$\mathbf{E}(r_b - r_f) + \frac{1}{2}var(r_b - r_f)$

		ψ 0.5	ψ 1.5	ψ 2.0
γ	5	-1.50	-0.80	-0.65
	10	-4.97	-2.05	-1.57
	15	NA	NA	NA

$\sigma(r_b - r_f)$

		ψ 0.5	ψ 1.5	ψ 2.0
γ	5	9.85	3.58	2.76
	10	11.82	4.37	3.45
	15	NA	NA	NA

Table IV.A displays moments for a claim to a consol bond that pays one unit of the consumption good in all future periods for the BY Calibration. The yield spread and bond yield are the average of monthly yield spreads and bond yields multiplied by 1200. The excess return of the bond and standard deviation are calculated by first aggregating monthly excess returns to annual levels and then computing the excess return or standard deviation. All moments are calculated from a simulation of 1.2 million months. NA refers to cases where the price of the consol bond is infinite. Bond prices are finite when the discount factor is reduced to .995, .996 or .996 for EIS of 0.5, 1.5 and 2.0. The discount factor in the model is .998.

Table IX: Panel B

Consol Bond Moments: BKY Calibration

$\mathbf{E}\left(y_b\right)$

		ψ		
		0.5	1.5	2.0
γ	5	3.05	NA	NA
	10	NA	NA	NA
	15	NA	NA	NA

$\mathbf{E}\left(y_b - r_f\right)$

		ψ		
		0.5	1.5	2.0
γ	5	-1.49	NA	NA
	10	NA	NA	NA
	15	NA	NA	NA

$\mathbf{E}\left(r_b - r_f\right) + \frac{1}{2}var(r_b - r_f)$

		ψ		
		0.5	1.5	2.0
γ	5	-1.16	NA	NA
	10	NA	NA	NA
	15	NA	NA	NA

$\sigma(r_b - r_f)$

		ψ		
		0.5	1.5	2.0
γ	5	9.47	NA	NA
	10	NA	NA	NA
	15	NA	NA	NA

Table IX.B displays moments for a claim to a consol bond that pays one unit of the consumption good in all future periods for the BKY Calibration. The yield spread and bond yield are the average of monthly yield spreads and bond yields multiplied by 1200. The excess return of the bond and standard deviation are calculated by first aggregating monthly excess returns to annual levels and then computing the excess return or standard deviation. All moments are calculated from a simulation of 1.2 million months. NA refers to cases where the price of the consumption claim or the consol bond is infinite. For an EIS of .5 and risk aversion of 10 and 15 the price of the consumption claim is infinite and the consol bond price is undefined as a result. All other NAs represent cases where the price of the consumption claim is finite but the price of the consol bond is not. Both bonds and the consumption claim have finite prices when the discount factor is reduced to .998 for EIS of 1.5 and 2.0 with RRA 5, .996 for EIS of 0.5, 1.5 and 2.0 with RRA 10 and .993 for EIS of 0.5, 1.5 and 2.0 with RRA 15 . The discount factor in the model is .9989.

Figure 1 Predictability of Consumption with Leads or Lags: Yearly Coefficients

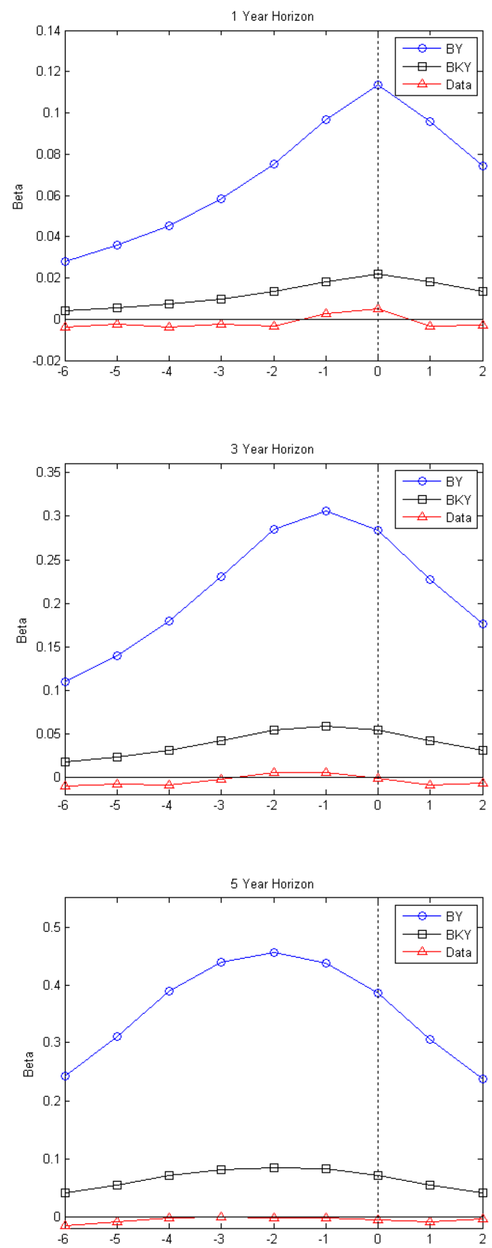

Figure 1 displays the coefficients from a regression of consumption growth over a 1, 3 or 5 year horizon on the log price-dividend ratio at different leads and lags. Each datapoint on the graph represents a different regression for that time horizon and lead or lag. The x-axis is the parameter j as outlined in the text. Model regressions are from a simulation of 1.2 million months. The data is the 1930-2006 yearly dataset.

Figure 2 Predictability of Consumption with Leads or Lags: Yearly R-Squared

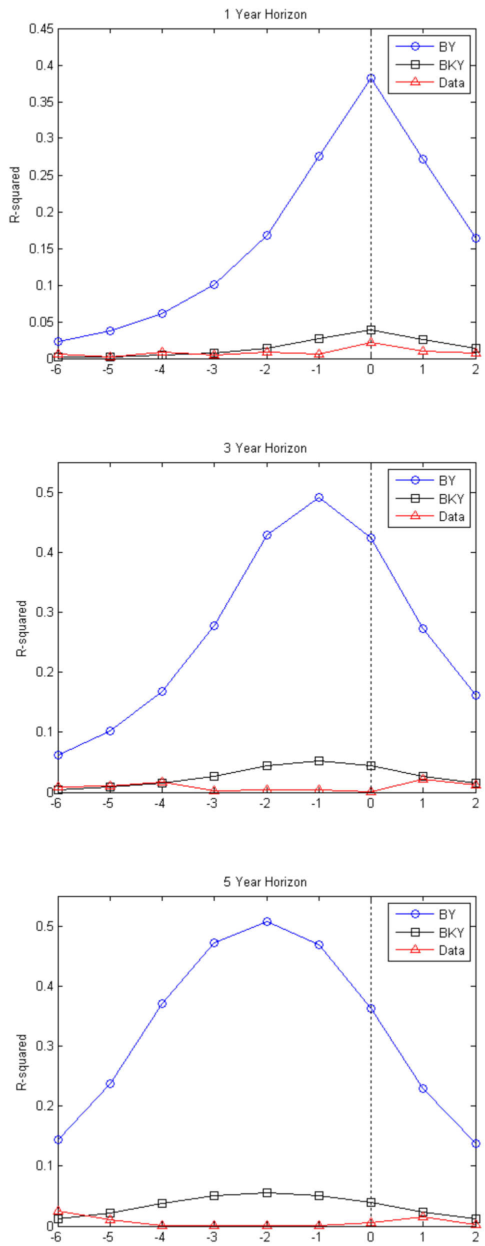

Figure 2 displays the R-squared from a regression of consumption growth over a 1, 3 or 5 year horizon on the log price-dividend ratio at different leads and lags. Each datapoint on the graph represents a different regression for that time horizon and lead or lag. The x-axis is the parameter j as outlined in the text. Model regressions are from a simulation of 1.2 million months. The data is the 1930-2006 yearly dataset.

Figure 3 Predictability of Consumption with Leads or Lags: Quarterly Coefficients

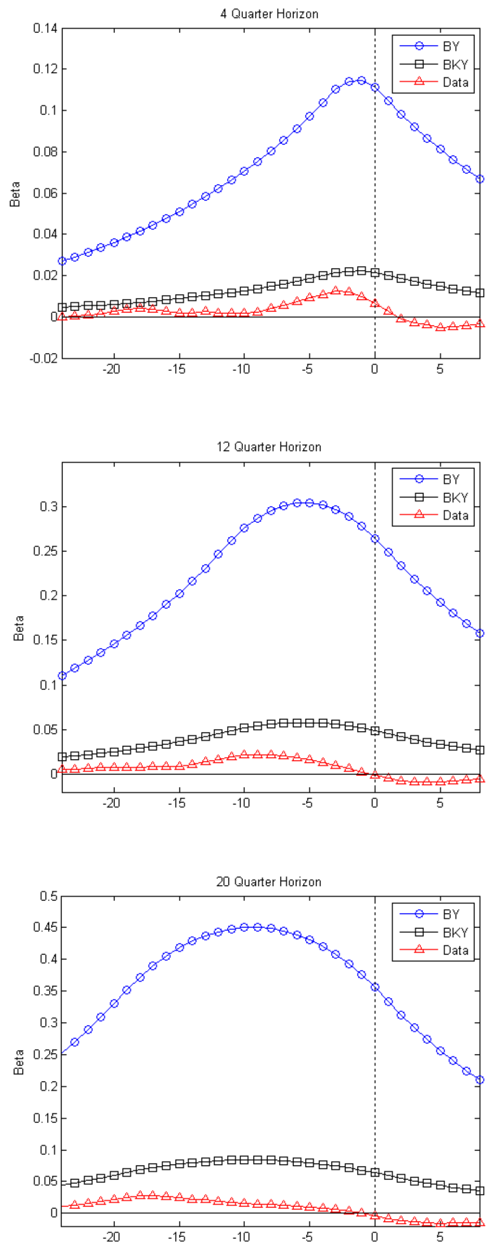

Figure 3 displays the coefficients from a regression of consumption growth over a 4, 12 or 20 quarter horizon on the log price-dividend ratio at different leads and lags. Each datapoint on the graph represents a different regression for that time horizon and lead or lag. The x-axis is the parameter j as outlined in the text. Model regressions are from a simulation of 1.2 million months. The data is the 1947.2-2007.3 quarterly dataset.

Figure 4 Predictability of Consumption with Leads or Lags: Quarterly R-Squared

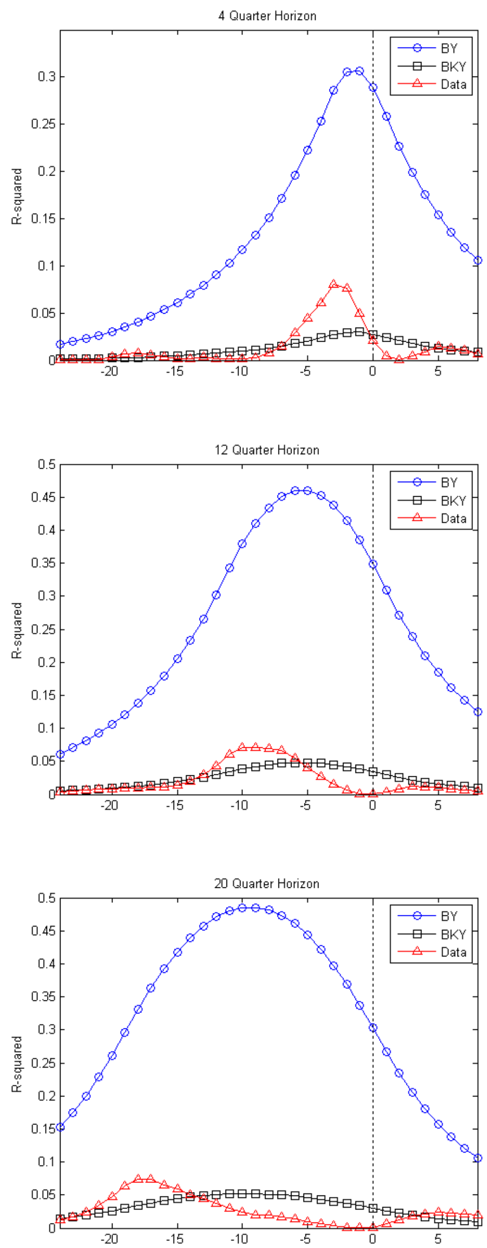

Figure 4 displays the coefficients from a regression of consumption growth over a 4, 12 or 20 quarter horizon on the log price-dividend ratio at different leads and lags. Each datapoint on the graph represents a different regression for that time horizon and lead or lag. The x-axis is the parameter j as outlined in the text. Model regressions are from a simulation of 1.2 million months. The data is the 1947.2-2007.3 quarterly dataset.