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TRADING FRICTIONS AND HOUSE PRICE DYNAMICS

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<u>ABSTRACT</u>

We construct a model of trade with matching frictions. The model provides a simple characterization for the joint proces of prices, sales and inventory. We compare the implications of the model to certain properties of housing markets. The model can generate the large price changes and the positive correlation between prices and sales that we see in the data. Unlike the data, prices are negatively autocorrelated and high inventory predicts price appreciation. We investigate several amendments to the model.

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1. Introduction

We construct a model of trade with matching frictions and use it to analyze several puzzling features of the housing markets. Our baseline model is closely related to Shimer's (2007) model of mismatch between workers and firms. Whereas Shimer's focus was on the distribution of unemployment and vacancies, our focus is on the dynamics of equilibrium prices. The economy is made up of a large number of markets each with a collection of homes. Potential buyers sample markets searching for homes to purchase. Owners experience relocation shocks and become sellers.

Markets in this economy swing between periods of tight supply and excess supply. Sometimes there are more buyers than sellers, and the market is tight. Sometimes there are more sellers than buyers and there is excess supply. During periods of excess supply buyers extract surplus from sellers. Their ability to do so is limited by sellers option to wait and sell in the future. Sellers who make a sale must be at least as well off as sellers who do not. This links prices across periods. During periods of tight supply, sellers extract surplus from buyers. Their ability to do so is limited by buyers option to search elsewhere. This links prices across markets. In this sense, sellers–being tied to a location–are responsible for intertemporal arbitrage, whereas buyers–being free to search across markets–are responsible for intratemporal arbitrage.

The model generates a very simple relationship characterizing the evolution of prices over time: whenever there is excess demand, sellers extract the maximal price; whenever there is excess supply, sellers must be indifferent between sales today and sales tomorrow. This simple characterization places strong restrictions on the dynamics of prices and allows for easy calibration.

We investigate the ability of this model to match three sets of facts. The first two are standard in the literature. The first is the positive autocorrelation in house price appreciation as documented by Case and Shiller (1989). The puzzle is that these swings do not appear to

be explained by fundamentals. Case and Shiller (1989) argue that serial correlation in rents does not explain inertia in price changes. In order to control for difficulties in measuring rents, Glaeser and Gyourko (2006) use wages to proxy for fundamentals. They also are unable to generate significant positive serial correlation in price changes. The second set of facts is that prices are volatile and price changes are correlated with the volume of transactions (Stein [1985], Ortalo-Magne and Rady [2006]). The third fact is the negative correlation between the inventory of unsold homes and future price appreciation. Several authors have included lagged measures of the inventory of unsold homes in aggregate pricing equations (eg. Peach [1983]) and found that the effect was negative. As this fact seems to be less appreciated in the literature, we devote a little time to extending it. We consider a sample of US cities, and ask whether the proportion of vacant homes for sale helps to forecast subsequent price changes in the cross-section. We find that higher vacancies predict lower rates of price appreciation rather than higher rates of price appreciation. This finding is consistent with the finding of positive autocorrelation in prices.

Our baseline model has mixed success. On the positive side, it generates a positive correlation between price changes and the volume of transactions. A large number of transactions leads to low inventory and high current prices. Calibrated versions of the model can also deliver sizable movements in prices as shifts in market tightness lead to shifts in surplus between buyers and sellers.

On the negative side, our baseline model generates negative autocorrelation in price changes and a negative correlation between the inventory of unsold homes and future price appreciation. The reason for this failure is instructive and likely to pose problems for any model that jointly models prices and inventories. When the inventory of unsold homes is zero prices are at their maximum and can only fall. Positive shocks therefore tend to raise prices today and lead to falling prices in the future. Conversely, when inventories of unsold homes are high, prices will tend to be low. The expected rate of price appreciation, however, must be high in order to compensate sellers who must wait to sell in the next period. A shock that raises inventory causes

3

price to fall today and rise tomorrow, thereby leading to negative short-run serial correlation in price changes. The model generates positive autocorrelation in price levels, but negative autocorrelation in price changes.

One possible explanation for these facts is that prices respond sluggishly to market conditions. To generate sluggish price adjustment, we incorporate an informational friction into our model. In particular, we assume that prices are posted before demand is revealed, as in the sequential service economies of Prescott (1975) and Bental and Eden (1993). Demand uncertainty leads to price dispersion in each market. Some sellers will charge low prices and sell with high probability, whereas other sellers will charge high prices and sell only if demand is sufficiently high. Prices do not incorporate all contemporaneous information, causing prices changes to lag inventory dynamics. When demand is unexpectedly high, prices will rise, but this rise will be tempered by the fact that some homes sold at low prices. During the next period when inventory is known to be low, it is now possible for prices to rise further.

This version of the model performs much better. A calibrated version of our sequential service economy does generate significant positive autocorrelation in price changes and the correlation between inventories and subsequent price changes, while not negative, is close to zero. A significant gap, however, remains between theory and data. First, the effects of sluggishness are short-lived. While the correlation between price changes in consecutive quarters is positive, the autocorrelation function quickly turns negative. Second, informational frictions make the price process much more sluggish, which reduces the volatility of house prices.

While our focus is on the time series properties of prices, the models that we consider shed light on other features of the trading process. The sequential service economy, captures the ambiguity in the relationship between price and time-on-the-market. In any given market in any given period, high prices are correlated with a longer time on the market, since high priced homes are less likely to sell. Across markets and time, however, high prices are correlated with low inventory and hence a greater probability of sale. The sequential service economy also makes

the prediction that price dispersion is correlated with sales.

The model is complementary to search models of housing such as Wheaton (1990). These models also predict, a positive relationship between price and market tightness. They generally imply that tight markets face expected future price declines. There are several differences between the two frameworks. In search models buyers and sellers match bilaterally. Brokers and multiple listing services, however, give agents the ability to sample more widely. In our framework, buyers and sellers see all of the houses that are for sale at any given period of time. Search models tend to keep bargaining power fixed so that all price fluctuations are generated by fluctuations in the outside option. Bargaining power shifts between buyers and sellers with market conditions in our framework. Search models cannot easily accommodate alternative market structures such as in the sequential service economy. Interestingly, search models do not seem to generate the ambiguity in the relationship between price and time on the market.[1]

Section 2 presents and analyzes the basic model. Section 3 discusses some of the comparative static properties of the model. We calibrate the model in section 3. Section 4 presents some evidence from US cities on the correlation between market tightness and subsequent price appreciation Section 5 considers a version of the model with prices that are posted before demand is realized. Section 6 concludes.

2. A model of a housing market

We present the basic model below. In subsequent sections we show how varying the timing of events alters the properties of the equilibrium.

Our focus is on the implications of trading frictions for price behavior. We keep other aspects of the model as simple as possible. Like in the Lucas' (1978) asset pricing model, we consider

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[1]In Wheaton's model tight markets lead to high prices and quick sales. In Albrecht et al. sellers who remain on the market become desparate and reduce prices; again price and time-to-sell are negatively related. See also Krainer (2001) and Novy-Marx (2008).

a economy in which equilibrium quantities are clear. What is not clear is how to price the asset, housing. To focus on fluctuations in price that are driven by shifts in bargaining power between buyers and sellers, we remove all other sources of price fluctuations. There are only two sources of heterogeneity: sometimes some agents cease deriving utility from their homes, and there is some randomness in the search process. Otherwise all houses and all agents are identical. We also remove all life-cycle and financial considerations. We assume that agents always have enough assets or borrowing ability to purchase their house.

2.1. Environment

Time is discrete and indexed by t. The economy is composed of a continuum of housing markets of measure one. Markets are indexed by $i \in [0, 1]$. Each market holds a finite number homes; we denote this number by H. There is a population of $\bar{N}$ agents per market. Agents are risk neutral.

Each home has a single owner. There are two types of home owners: those who are well matched with their current home and those who are mismatched. Well-matched owners derive γ units of utility from their homes in each period. Owners who are mismatched derive zero utility. Matches evolve over time. New owners are always well matched in the period in which they purchase their homes. At the very beginning of each period some well-matched owners become mismatched. Let z denote the probability that a well-matched owner becomes mismatched in a period. For simplicity, mismatched owners remain mismatched forever.

We make several assumptions on the structure of matches so that well-matched owners choose not search and so that agents who become mismatched choose to sell their homes and search for a new match in some other market. First, we assume that an agent can derive utility from only one home at a time, and that an agent cannot search proactively and stockpile well-matched homes in anticipation of the agent's current match dissolving.[2] Well-matched owners therefore

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[2]An assumption that would justify ruling out proactive search is that agents do not know where to search until

have no incentive to search for housing. Second, we assume that mismatched owners are also mismatched with other homes in their market, so that they gain nothing by swapping homes amongst themselves.[3] They therefore must search elsewhere. Third, we allow mismatched agents to search for new homes while they sell their old homes. In fact, we do not link in any way an agent's decision to sell their old home and purchase a new one.[4]

Search is a black box. We model the outcome of the search process, rather than search itself. Let N denote the average number of searchers per market. Note $N < \bar{N}$ since only mismatched owners search. The outcome of the search process is a probability distribution $\phi(n; N)$ which gives the probability that n buyers arrive in a market conditional on their being a mass N of searchers in the economy. Note that n is an integer, since the number of homes in a market H is an integer. This characterization includes several special cases. Shimer makes the assumption of "random search":

$$\phi(n; N) = e^{-N}\frac{N^n}{n!}.$$

This is the density that arises when a mass N agents each choose uniformly over a unit mass of markets. Another convenient functional form is "exponential search":

$$\phi(n; N) = (1 - \pi_N)\pi_N^{n-1}, \quad \pi_N \in (0, 1)$$

Here π_N is set so that the average number of buyers is equal to N. This formulation will prove

their current match disolves. Suppose that locations were grouped into different types and that when a match dissolved the agent learned his new type. By increasing the number of types, we could make it prohibitively expensive to the insurance that proactive search provides.

[3]Hanushek and Quigley (1979) argue that separations are mainly due to exogenous life events – loss of job, change in family composition.

Many people, however, move within a metropolitan area. Here one might think of the market more narrowly as both a location and a particular type of housing (efficiency, one-bedroom, two-bedroom, yard/no yard).

[4]Linking the purchase and sales decisions can lead to anomolous results. For example, if agents must sell their current home before purchasing a new one, then house prices may be negative for reasonable parameterizations. Agents are willing to pay buyers to take their homes so that they may enter the search process and purchase another home at a negative price. Decoupling the sales decision links house prices moer closely to the value of fundamentals, in this case the present value of γ.

convenient when we consider uncertain demand. In this formulation the arrival of n buyers says nothing about the arrival of the $(n+1)$st buyer. Note that search is not directed in any of these formulations. The simplicity of the model is due to the fact that prices do not affect search. This allows us to solve for allocations independent from prices.

Prices are determined by Bertrand competition among sellers. The total number of buyers and sellers in market i at date t is known when sellers set prices.[5] After prices are set, buyers queue up randomly and choose sequentially whether to purchase a home or to wait and search again in the next period. If a buyer chooses to purchase a home, the buyer becomes the new well-matched owner. Unsold homes remain in the pool of mismatched homes in the next period. We assume that owners of unsold homes incur a carrying cost l units of utility (which may be equal to zero) each period that their homes remain unsold. This carrying cost is in addition to the delay in receiving payment for the home.

The timing of events within a period is as follows: owners become mismatched, buyers arrive, sellers set prices, and trade takes place. Buyers and sellers make purchase and pricing decisions in order to maximize the present value of utility. Prices are quoted in units of utility. Utility in period t is the sum of utility from housing plus the price of any homes sold less the price of any home purchased.[6] Future utility is discounted by a factor $\beta \in (0, 1)$.

The model is set up so that we can solve for quantities first and then prices. Note that it will be efficient for all potential trades to take place each period. Trade simply replaces a mismatched owner with a well-matched owner. In the next section we use the assumption of efficient trade to solve for the evolution of sales and inventory in each market as a function of separation rate z and the arrival rate ϕ. The following section then solves the Bertrand pricing game among sellers given the evolution of sales and inventory and confirms that efficient trade is in fact an equilibrium.

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[5] An alternitive formulation would be to have sellers set prices before demand is realized. We will return to this issue below.

[6] The linearity of utiltiy in prices means that we do not need to track the distribution of wealth.

2.2. Solution

2.2.1. Quantities

We drop the subscripts i and t except where they are necessary to avoid confusion.

It is useful to define two sets of sellers. Let $b \in \{0, H\}$ denote the number of sellers at the beginning of the period, after the match shock, but before trade. Let $e \in \{0, H\}$ denote the number of sellers at the end of the period after trade has taken place. $b_t - e_{t-1}$ will be equal to the number of newly mismatched owners in the market at date t. The number of homes sold in the market will be equal to $e_t - b_t$.

We solve for the steady state distribution of homes for sale at the beginning and end of the period. The dynamics of each market are described buy the probability that a match will dissolve z and the probability that buyers will arrive $\phi(n; N)$.

We create two transition matrices. The first P_z describes transitions from e_{t-1} to b_t. Recall e is the number of sellers at the end of a period and b is the number of sellers at the beginning of a period after the realization of the match parameters. P_z is an $H + 1 \times H + 1$ matrix. The (i, j)th element gives the probability of moving from $e = i - 1$ to $b = j - 1$. It is upper triangular since there is always a weakly positive flow of new sellers, $b_t \geq e_{t-1}$. Given that the realization of the probability z is independent across homes, the transition probabilities have a binomial distribution. Given e, there are $H - e$ well-matched homes that may become mismatched. The

probability of $b - e$ new sellers is therefore $\begin{pmatrix} H - e \\ b - e \end{pmatrix} z^{b-e}(1-z)^{H-(b-e)}$. We have:

$$P_z = \begin{bmatrix} (1-z)^H & \begin{pmatrix} H \\ 1 \end{pmatrix} z(1-z)^{H-1} & \cdots & \begin{pmatrix} H \\ H-1 \end{pmatrix} z^{H-1}(1-z) & z^H \\ 0 & (1-z)^{H-1} & \cdots & \begin{pmatrix} H-1 \\ H-2 \end{pmatrix} z^{H-2}(1-z_1) & z^{H-1} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1-z & z \\ 0 & 0 & \cdots & 0 & 1 \end{bmatrix}$$

The second matrix describes transitions from b_t to e_t. Given the assumption of efficient trade, the short side of the market determines the number of trades. The number of unsold homes at the end of the period is equal to $b - n$ if $n < b$ or 0 if $n \geq b$. This gives rise to a transition matrix $P_{n,N}$ which depends on $\phi(n, N)$. $P_{n,N}$ is an $H+1 \times H+1$. The (i,j)th element gives the probability of moving from $b = i - 1$ to $e = j - 1$. $P_{n,N}$ is a lower triangular matrix, since the number of buyers is weakly positive. We have

$$P_{n,N} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ \sum_{i>1} \phi(i, N) & \phi(0, N) & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \sum_{i>H-1} \phi(i, N) & \phi(H-2, N) & \cdots & \phi(0, N) & 0 \\ \sum_{i>H} \phi(i, N) & \phi(H-1, N) & \cdots & \phi(1, N) & \phi(0, N) \end{bmatrix}$$

The first column represents the probability that $n \geq b$.

From $P_{n,N}$ and P_z, we can derive the transition matrices form e_{t-1} to e_t and from b_{t-1} to b_t. Transitions from e_{t-1} to e_t are governed by $P_e = P_z P_n$. Transitions form b_{t-1} to b_t by

10

$P_b = P_{n,N} P_z$. Note that since P_z has strictly positive elements above the diagonal and P_n has strictly positive elements below the diagonal, P_b and P_e are everywhere strictly positive.

Let λ^e be a $(H+1) \times 1$ vector that gives the the steady-state, cross-sectional density of end of period sellers. The ith row gives the fraction of markets with $e - 1$ sellers (the first row is associated with $e = 0$). We have

$$\lambda^{e\prime} P_e = \lambda^{e\prime}$$

or

$$(I - P'_e)\lambda^e = 0$$

λ^e is the eigenvector associated with the unit eigenvalue of P'_e. Since all of the elements of P_e are strictly positive, there exists a unique λ^e (for any N). Similarly we may define λ^b as a density over b. We have

$$\lambda^{b\prime} P_e = \lambda^{b\prime}$$

and

$$\lambda^{e\prime} P_z = \lambda^{b\prime}$$
$$\lambda^{b\prime} P_{n,N} = \lambda^{e\prime}$$

To determine N, we calculate the implied population. Let Z be an $H \times 1$ vector whose elements are $\{0, 1, 2 \ldots H\}$. Given N there is a density λ^b of unmatched homes. This implies that the average number of matched homes per market is

$$H - \lambda^b \cdot Z$$

The total population is divided into matched homeowners and searchers. Hence the implied

11

population is

$$H - \lambda^b \cdot Z + N$$

Note that $\lambda^b \cdot Z$ is decreasing in N, since increasing N leads to more sales and hence fewer unmatched homes. There is therefore a unique N such that[7]

$$H - \lambda^b \cdot Z + N = \bar{N}$$

This completes the analysis of quantities. We now turn to prices.

2.2.2. Prices

Price setters play a Bertrand game. We look for an equilibrium in pure strategies. Since all sellers are alike, each sets the same price. The state variable that matters for pricing is e, the number of unsold homes at the end of the period. e determines the reservation value of buyers and sellers. Sellers who wait to sell in the future only care about the number of unsold homes that remain at the end of the period. Buyers care only about the price that they may receive in the future which depends only on e.

We define a number of useful expressions. Let v denote the present value of being a buyer searching this period. Let V be an $(N+1) \times 1$ vector that gives the value of owning a home when there are e unsold homes at the end of the period. The ith row gives the value of owning a home when there will be $i - 1$ unsold homes. Let p a $(N+1) \times 1$ vector of equilibrium prices. Again the ith row gives the price of a home conditional on there being $i - 1$ unsold homes at the end of the period. Let W be an $(N+1) \times 1$ vector that gives the value of the optimal selling strategy when there are e unsold homes at the end of the period. Note that the linearity of

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[7]Suppose that we start out of steady state, how would the economy behave? Would it converge? Monotonically? At what rate? Convergence is made difficult by the fact that N_t and hence P_{n,N_t} may change over time if the economy begins out of steady state. This is a time inhomogenous Markov chain and standard convergence theorems do not apply.

utility and the lack of borrowing constraints make the problem of selling a home separable.

There are two cases to consider: $e = 0$ and $e > 0$.[8] If $e = 0$, then $n > b$. There are (weakly) more buyers than sellers. Every seller makes a sale. Sellers in this case behave collectively as monopolists. They extract all of the surplus from buyers. Price is equal to the buyers maximum willingness to pay.

$$V_0 - p_0 = \beta v \tag{2.1}$$

where V_0 is the value of owning a home when $e = 0$, and p_0 is the price in this case.

If $e > 0$, there are more sellers than buyers. Some homes go unsold.

$$W_e = \max \{p_e, -l + \beta x'_e P_e W\}$$

where x_e is a column vector of zeros with a one in the $e + 1$ row. With Bertrand competition among identical sellers, sellers must be indifferent between selling today and holding on to their homes in order to sell in the future. Hence $p_e = -l + \beta P_e W$ and $W_e = p_e$. It follows that

$$p_e = -l + \beta x'_e P_e p \qquad e > 0 \tag{2.2}$$

Note that we can iterate indifference forward. Sellers are indifferent between selling today and waiting in every period that $e > 0$. It follows that sellers are indifferent between selling today and waiting until $e = 0$. Let T be a stopping time defined as the first time that $e = 0$. We have

$$p_e = E \left\{ \beta^T p_0 - \frac{(1 - \beta^T)}{1 - \beta} l \,\middle|\, e \right\} \tag{2.3}$$

This characterization immediately implies that p is monotonically decreasing in e; as e rises so does T.

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[8]This is similar to Shimer (2007). Shimer has two wages depending on whether or not there is excess supply of workers. Here we have several prices. The difference is that Shimer only has spot markets whereas we have a durable.

Note that equilibrium considerations affect prices only through p_0 according to equation (2.1), and, given p_0, all other all other prices are determined by equation (2.2). Hence given p_0, p_e depends only on P_e, β, and l.

2.2.3. The value of owning a home

We now consider the value of owning. Let P_z^+ be a $(H+1) \times (H+1)$ matrix such that

$$
P_z^+ =
\begin{bmatrix}
0 & (1-z)^{H-1} & \binom{H-1}{1} z(1-z)^{H-2} & \cdots & \binom{H-1}{H-2} z^{H-2}(1-z) & z^{H-1} \\
0 & 0 & (1-z)^{H-2} & \cdots & \binom{H-1}{H-3} z^{H-3}(1-z) & z^{H-2} \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 1 \\
0 & 0 & 0 & \cdots & 0 & 0
\end{bmatrix}
$$

Let P_z^- be a $(H+1) \times (H+1)$ matrix such that

$$
P_z =
\begin{bmatrix}
(1-z)^{H-1} & \binom{H-1}{1} z(1-z)^{H-2} & \binom{H-1}{2} z^2(1-z)^{H-2} & \cdots & z^{H-1} & 0 \\
0 & (1-z)^{H-2} & \binom{H-2}{1} z(1-z)^{H-2} & \cdots & z^{H-2} & 0 \\
0 & 0 & (1-z)^{H-3} & \cdots & z^{H-3} & 0 \\
\vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 1 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0
\end{bmatrix}
$$

14

P_z^+ gives e to b transition probabilities from the perspective of an owner who himself receives a z shock. P_z^- gives the transition probabilities from the perspective of an owner who remains matched. These present the probabilities of other owners becoming mismatched. We have[9]

$$
\begin{aligned}
V &= \gamma + \beta \left[z P_z^+ P_{n,N} (W + v) + (1 - z) P_z^- P_{n,N} V \right] \\
&= \frac{\gamma}{1 - \beta(1 - z)} + (I - (1 - z) P_z^- P_{n,N})^{-1} z P_z^+ P_{n,N} (W + v)
\end{aligned}
$$

The value of owning a home is equal to the flow value γ plus the present value of an optional policy in the future. In the future, with probability $1 - z$, the owner will remain matched and receive EV conditional on remaining matched. With probability z, the owner will become mismatched and receive the value of an optimal sales policy, as well as the value of search v. Note that $W = p$.

2.2.4. The value of search

We now turn to the value of search. A searcher is randomly allocated across markets. The value of search depends on the probability that the searcher purchases a home and the price at which that purchase is made. The probability that the market has b sellers is λ^b. The probability that the market has n buyers is $\phi(n; N)$. These two probabilities are independent. If $n < b$ then the searcher makes a purchase and receives a benefit $V_{b-n} - p_{b-n}$. If $b > n$ then the searcher receives βv regardless of whether or not the seller makes a purchase: in these states sellers are monopolists, so they drive the value of buying down to the value of search. Putting this all together:

$$
v = \frac{1}{N} \sum_b \lambda^b \left[\sum_{n<b} \phi(n; N) n (V_{b-n} - p_{b-n}) + \beta \sum_{n \geq b} \phi(n; N) n v \right]
$$

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[9]Note that V_H is not well defined in this expression, since a person cannot be well matched in a market in which all owners are mismatched. It may be set arbitrarily.

Rearranging

$$v = \frac{\sum_b \lambda^b \left[\sum_{n<b} \phi(n; N) n (V_{b-n} - p_{b-n}) \right]}{N \left[1 - \beta \frac{1}{N} \sum_{n \geq b} \phi(n; N) n v \right]}$$

3. Discussion

An immediate consequence of equation (2.3) is that price is negatively correlated with end of period inventory. The expected time to stock-out T is monotonically increasing in e. Price fluctuations result in shifts in surplus between buyers and sellers. In this model the fundamental value of the house is a constant flow γ. Given that z is likely to be quite low, buyers valuations V_e will not change much as e changes. Sellers get a better deal in tight markets and buyers get a better deal in loose markets.

The inventory dynamics lead naturally to mean reversion in the long run. Given that P_e has no non-zero elements, it follows immediately that the expected inventory level converges to the average of the elements of λ^e. Higher inventory levels are expected to fall. Lower inventory levels are expected to rise. This leads to long-run negative correlation in price changes. If prices rise today, inventories fall, leading to expectations of future increases in inventories and price reductions.

An immediate consequence is that price changes are predictable. In fact there are two cases, depending on whether or not there are end of period inventories. When end of period inventories are positive, equation (2.3) implies that the expected gross rate of price appreciation is:

$$E_t(p_{t+1}/p_t) = \frac{1}{\beta} + \frac{l}{\beta p_t}$$

When l is equal to zero price appreciation is bounded above by $1/\beta$, the risk free rate. As in the Hotelling (1931) model of natural resource extraction, if prices were to rise faster, each seller would have an incentive to wait. This would push prices up and reduce the rate of price

appreciation. If prices were to rise at a slower rate, each seller would have an incentive to sell today. Competition among sellers would bid prices down and increase the rate of appreciation.

When end of period inventories are zero, $p_0 = V_0 - \beta v$ and the expected price is $x_0' P_e p$. Recall x_e is a column vector of zeros with a one in the $e + 1$ row. Since $p_e < p_0$ for all $e > 0$, prices are expected to fall. Unlike when $e > 0$, all sellers are already making sales so there is no additional supply of unmatched owners to push down prices. One might ask whether matched owners have an incentive to enter the market and take advantage of the high prices. Matched owners, however, are identical to the buyers. Since the buyers are willing to buy at price p_0 in spite of the prospect of falling prices, matched owners are also willing to remain in their homes.

While expected price changes are bounded above by $1/\beta$ when $l = 0$, actual price changes are likely to be much greater. Consider a market with one unsold home. The upside is bounded by p_0. The downside is effectively unbounded. The price must therefore be less than βp_0 in order to generate an expected increase of $1/\beta$. The model therefore has the potential to generate large price changes.

Since equilibrium considerations affect prices only through p_0 according to equation (2.1), many of the properties of prices in this model are robust to change in the structure of the model. Note that when $l = 0$, the characterization of the price process becomes particularly simple. In this case, the ratio p/p_0 follows from equation (2.2) independently of equilibrium considerations. We will make use of this observation at several points below.

4. Calibration

To fit the model, we first need to determine the time period. The natural time period is the one over which prices are fixed. Unfortunately, there is not much evidence on the time between price adjustments. Knight (2002) reports that an average time of 3.5 months before a list price change for a sample of homes in Stockton California during 1998. We therefore take the time

period to be a quarter. Knight's number, however, may overstate the period over which prices are fixed as some sellers may alter the price without altering the list price.

To simulate the model we need to calibrate N, z, β, H, γ, and l. We set β to 4% per year. We set z to match data on turnover. Dieleman, Clark and Deurloo (2000) find that the turnover rate is about 8% per year for urban home owners. We therefore set z equal to .02 for quarterly data. We choose N and H to match data on the time to buy and the time to sell. Anglin and Arnott (1999) report that time to buy and time to sell are both in the neighborhood of a quarter.[10] This results in $H = 72$ and $N = 2.41$. The small value of H is not troubling since there is nothing in the model that says the market is large. We can think of markets in the model as being quite specific: two bedroom homes located in a certain neighborhood. We normalize γ to one and set l to zero.

Figure 1 presents price as a function of vacancies. As expected price declines monotonically with vacancies. Figure 2 presents the steady-state density of vacancies across markets. Most markets have no vacancies. Given that time to sell is equal to one period, there must be lots of markets that stock out each period.

On the positive side, the model can generate reasonably large price changes. The mean absolute percentage change in prices in the calibrated model is 1.88% per quarter. The median is zero, reflecting the large number of markets that stock out in consecutive periods. The standard deviation is 3.10%. The model also generates a positive correlation between sales and the change in house prices. The correlation is .29.

The model misses badly on two other facts. The autocorrelation in price changes is -.18. This is far from positive. The correlation between price changes and future vacancies is -.29. Which is also far from positive. These correlations reflect the mean reversion in inventories discussed above.

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[10]Given that we have chosen the unit of time to be a quarter, we would need all homes to sell every period to match a time to sell of one quarter. We assume that sales are spread evenly throughout the period, so that immediate sales could as .5 quarters and sales after n periods count as n+.5 quarters.

5. Some Evidence on Market Tightness and Price Appreciation

A strong prediction of the baseline model is that inventories are negatively correlated with future price appreciation. Price can only fall if end of period inventory falls to zero, and (if $l = 0$) the expected rate of price appreciation is $1/\beta$. Peach (1983) finds the opposite on aggregate data. Given this divergence between model and data, we update his evidence.

We consider an unbalanced panel of cities. The data is annual. The time period is 1988 to 2005. We measure market tightness as the proportion of homes that are vacant and for sale. The data is from the Bureau of Economic Analysis. The frequency is annual. This measure obviously ignores homes that are occupied and for sale. Our maintained assumption is that these are highly correlated. We take the S&P Case-Shiller index as our measure of house prices. This is a repeat sales index. There has been much discussion of the potential biases in this index. Unfortunately, there is no other widely available index that is obviously better. The data begin in 1987 with 14 cities. Coverage increases to 20 cities over the same period.

We estimate the following relationship

$$\Delta p_{it} = \alpha_i + \beta_t + \gamma \Delta p_{it-1} + \delta v_{t-1} + \varepsilon_{it} \tag{5.1}$$

Here $\Delta p_{it} = p_{it} - p_{it-1}$ is the price change in city i at date t. α is a city fixed effect meant to capture differences in the natural vacancy rate. β is a time effect meant to capture common influences such as the state of the national business cycle or the level of interest rates. v represents vacancies. These are lagged since we are interested in the predictive power of vacancies for future price changes. ε is the error term.

Table 1 presents the results from estimating equation (5.1). We see the serial correlation in price changes. We also see that vacancies have a significantly negative correlation with subsequent price changes.

Table 1

Panel estimates of price appreciation

on lagged appreciation and lagged vacancies

	coef	std. error
Δp_{it-1}	0.833	0.063
v_{t-1}	-1.265	0.518
β_t	0.466	0.078

There results are robust to changes in specification. Dropping the last year (2005) increases the coefficient on v_{t-1} to -1.96 with a standard error of .42, indicating that the effect is stronger before the recent run up in prices. Lagging vacancies by two periods also increases the coefficient on vacancies; it rises to -1.6 with a standard error of 0.6. Adding an additional lag price change also does not affect the results. Prices appear to decline following high vacancies, the opposite of what the model predicts.[11]

6. Posting prices before demand is realized

One possible reason for the positive autocorrelation in prices and for the positive correlation between inventories and future price changes is that prices respond sluggishly to market conditions. To generate sluggish price adjustment, we incorporate an informational friction into our model. We alter the above analysis by assuming that sellers post prices before demand is realized. We continue to assume that sales are efficient, so that the distributional dynamics do not change. Prices, however, do not completely incorporate market conditions.

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[11]In his discussion of this paper, Francois Ortalo-Magne presented evidence from a single neighborhood in Madison Wisconsin that appeared to support the implications of the baseline model. In his data, a high inventory of unsold homes appeared to be correlated with low prices and future price appreciation. This raises the possibility that the relationship between prices and vacancies differs at different degrees of aggregation. The baseline model's predictions may be more applicable to very local markets.

It turns out to be simpler to work with a search process that is governed by an exponential distribution

$$\phi(n) = (1 - \pi_N)\pi_N^{n-1}, \quad \pi_N \in (0, 1)$$

This simplifies the analysis in several ways. First, there is always one buyer. Second, the probability of additional buyers within the period is independent of how many buyers have already shown up. This will imply that the optimal policy of the seller with the b highest price is the same regardless of how many sellers have lower prices. This reduces the number of state variables and leads to a simple recursive representation of price within a period.

Sellers set prices before demand is realized. Uncertain demand leads naturally to price dispersion as in the analysis of Prescott (1975). Some sellers set low prices and sell with a high probability. Others set high prices and sell only if demand is sufficiently high. We assume that sellers know how many other sellers there are as well as the prices set by these other sellers. Equilibrium is characterized by indifference among sellers.[12]

Given the exponential distribution of buyers, we can think of buyers arriving sequentially. We solve for the optimal pricing policy by induction. Suppose that there is one seller remaining in the market and one buyer. This seller will set the monopoly price:

$$p_0 = V_0 - \beta v$$

Note we have indexed price p to the number of sellers left after the current sale is made and V to the number of homes left on the market. This will facilitate comparison with the last section.

Now suppose that their are two sellers and one buyer. The low price seller will sell with probability one. The other seller will set the monopoly price and sell only if another buyer

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[12]This imposes a minimum amound of coordination among sellers. Things would be slightly different if sellers randomized independently. In this case, it would be possible for inefficiencies to arise. If, for example, all sellers charged the monopoly price and there were only a few buyers, then the buyers might prefer search as $V_e - p_0 < V_0 - p_0 = v$.

arrives. The low price seller must be indifferent between selling immediately with probability one or switching places with the "monopoly" seller.

$$p_1 = \pi p_0 + (1 - \pi) \left[-l + \beta x_1' P_z p \right]$$

In general

$$p_e = \pi p_{e-1} + (1 - \pi) \left[-l + \beta x_e' P_z p \right]$$

The following proposition shows the sense in which the lowest price when demand is uncertain is related to the expected price when demand is known.

Proposition Consider two economies with identical values of π and $V_0 - v$, but differing in when demand is realized. Let p denote the equilibrium price in the economy with uncertain demand and $\hat{p}$ denote equilibrium price when demand is known before setting the price. Then,

$$p = P_n \hat{p}$$

Proof: Let $\hat{p}^0 = [V_0 - v, 0 \dots, 0]'$, and let $\hat{p}^i = -l + \beta P_e \hat{p}^{i-1}$. Standard dynamic programming arguments imply that $\hat{p}^i \to \hat{p}$.

Now let $p^0 = P_n \hat{p}^0$ and suppose that for all $j \leq i - 1$, $p^j = P_n \hat{p}^j$. We show that $p^i = P_n \hat{p}^i$. The first row of $P_n \hat{p}^i$ is is $V_0 - v$. This is also the first row of p^i. Suppose that the $k - 1$st rows of $P_n \hat{p}^i$ and p^i are equal. The kth row of $P_n \hat{p}^i$ is

$$
\begin{aligned}
\left(P_n \hat{p}^i \right)(k) &= \pi \left(P_n \hat{p}^i \right)(k - 1) + (1 - \pi) \left(-l + \beta x_k' P_e \hat{p}^{i-1} \right) \\
&= \pi \hat{p}^i(k) + (1 - \pi) \left(-l + \beta x_k' P_z P_n \hat{p}^{i-1} \right) \\
&= \pi \hat{p}^i(k) + (1 - \pi) \left(-l + \beta x_k' P_z p^{i-1} \right) \\
&= p(k)
\end{aligned}
$$

The equality of $P_n \hat{p}^i$ and p^i follows by induction. The convergence of $P_n \hat{p}^i$, establishes the proposition.∎

While the prices in the two markets are related, the stochastic properties of the price process are not the same. Price does not fully reflect current demand when demand is uncertain. Suppose that beginning of period inventory is high. Then some sales will be made at low prices even if demand turns out to be high. The full effects of high demand will only be incorporated in a few higher priced sales. The average price during the period will be an average of both the high and low prices. This averaging mutes the price increase during high demand periods. Since demand was high, end of period inventories will be low, and next period prices will tend to be high on average. In this way, the model can possibly generate positive serial correlation in price changes and a positive correlation between end of period inventories and future price changes.

Calibrating the model to the parameters above, implies a value of π_N of about .75. On the positive side, the resulting correlation between price changes in adjacent periods is fairly large, approximately .3. The correlation between inventories and future price changes, while not positive, is at least very small, on the order of -.03. Incorporating demand uncertainty greatly improves the models performance on these dimensions.

On the negative side, the smoothing of price reduces the correlation between volume and the change in price to about .1, and the average size of price changes is now about .5% per quarter. Moreover, the persistence of price changes is fairly short-lived. The correlation between $p_t - p_{t-1}$ and $p_{t-2} - p_{t-3}$ is small and negative. It remains an open question if combining trading frictions with serial correlation in the fundamentals can produce values that match the data.

There are also implications for the relationship between price and time-on-the-market. At any given time, higher prices will be associated with lower probability of sale and hence greater time on the market. Across periods however, prices will tend to be higher in tighter markets in which time to sale is low.

There are also implications for price dispersion. Price dispersion is positively correlated with

volume. When volume is high, sales work there way further up the pricing ladder. Initial sales are at low prices, and final sales are at relatively high prices. We know of no evidence on this implication of the model.

7. Conclusion

We have constructed a model of trading frictions and used it to price houses. The model leads to a simple characterization of equilibrium prices. A robust feature of this model is the positive correlation between vacancies and future price increases. This correlation is likely to be a feature of any model with vacancies. Owners of unsold homes must be compensated for waiting.

The correlation between vacancies and future price appreciation makes it difficult to generate serial correlation in price changes. Positive shocks tend to reduce vacancies generating expectations of price declines. We have shown that the introduction of demand uncertainty and posted prices, both realistic features of real estate markets, can overcome some of these difficulties in the short run.

The model abstracts from several features that may hinder its ability to match these facts. The only source of uncertainty is random shocks to supply and demand. Adding serial correlation or even non-stationarity to the driving process could improve its ability to generate serial correlation in price changes. The model abstracts from credit market frictions and life-cycle saving. Ortalo-Magne and Rady (2006) argue that these feature may generate volatility and covariance between price and volume. It remains an open question whether our mechanism in combination with these or other mechanisms can explain can generate realistic housing cycles.

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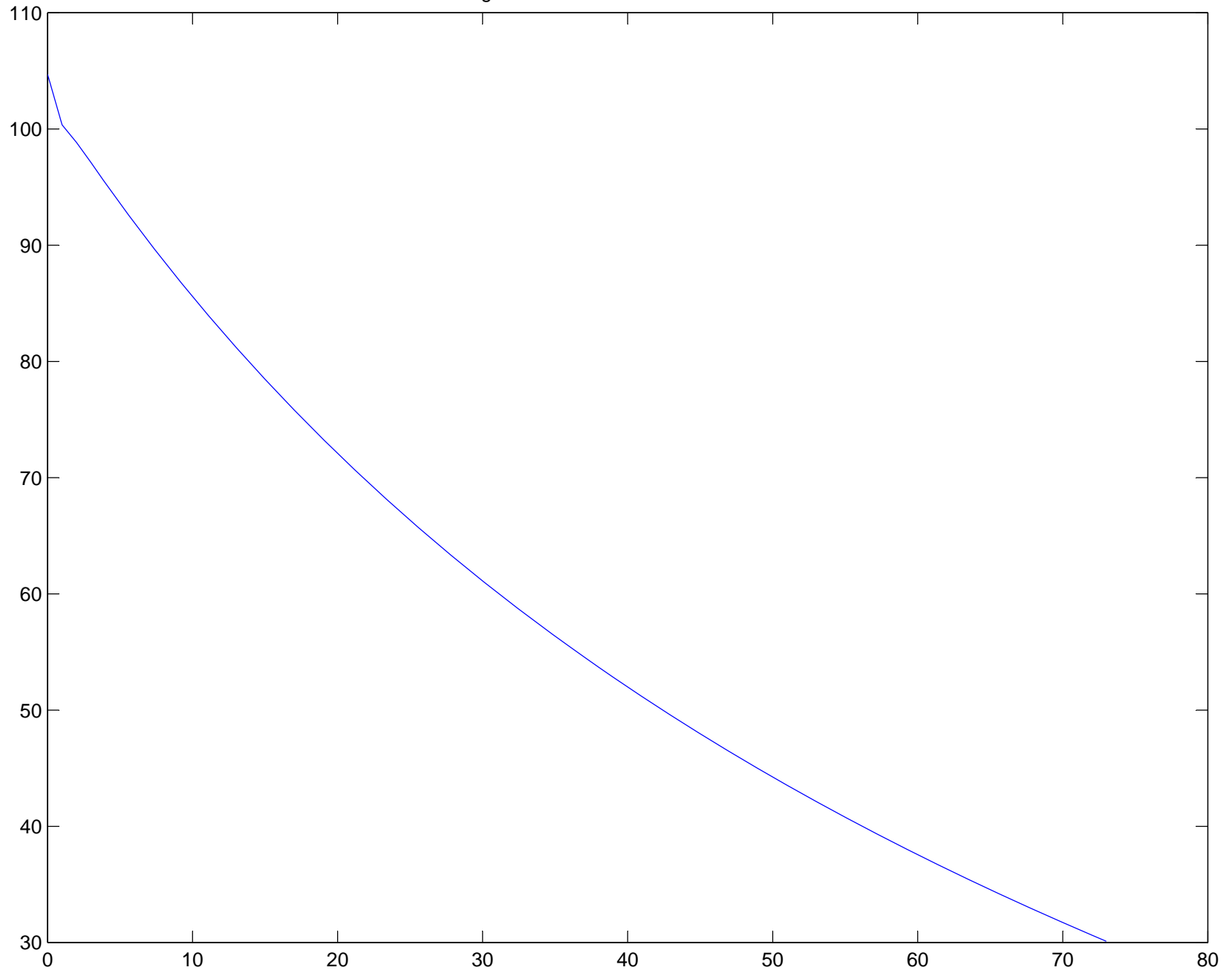
Fig1: Price as a function of vacancies

Fig2: The density of vacancies