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THE TRUE COST OF SOCIAL SECURITY

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The True Cost of Social Security

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**ABSTRACT**

Implicit government obligations represent the lion's share of government liabilities in the U.S. and many other countries. Yet these liabilities are rarely measured, let alone properly adjusted for their risk. This paper shows, by example, how modern asset pricing can be used to value implicit fiscal debts taking into account their risk properties. The example is the U.S. Social Security System's net liability to working-age Americans. Marking this debt to market makes a big difference; its market value is 23 percent larger than the Social Security trustees' valuation method suggests.

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## 1. Introduction

In most developed countries and in many developing ones, commitments to make transfer payments and collect receipts represent the lion's share of government obligations and resources. Often referred to as implicit liabilities and assets, they typically are either ignored in assessing fiscal sustainability or valued on a piecemeal basis using ad hoc techniques. The justification for this practice generally offered is twofold. First, implicit fiscal commitments do not represent legal liabilities. Second, implicit commitments are difficult to value given their uncertain and extended nature.

This rationale may assuage accountants, but it offers little comfort to economists or, indeed, to anyone concerned with economic policy. The immense gulf between countries' true indebtedness and what's being measured means countries are largely driving blind with respect to their fiscal affairs. Generational accounting, developed by Auerbach, Gokhale, and Kotlikoff (1991), attempts to remedy this situation. Its framework is the government's intertemporal budget constraint, and it treats all government commitments on a consistent basis regardless of their legal status.

These advantages notwithstanding, a major shortcoming of generational accounting as well as related measurements<sup>1</sup> is the failure to adjust future government flows properly for risk. Generational accountants usually value the government's future payments and receipts by adding a risk premium to the risk-free discount rate. But their choice of risk premiums has no clear theoretical or empirical basis.

This paper presents a method for properly valuing implicit government debt. It treats government benefit obligations and tax claims as non-traded financial assets and applies what are now standard asset-pricing techniques to their valuation. In particular, we use Ross' (1976a, 1976b) Arbitrage Pricing Theory (APT) and its associated risk-neutral, derivative-pricing and process-free pricing theories (see Cox and Ross, 1976 and Ross, 1978). Our method treats future government payments and receipts as securities whose returns comprise two components – a market component, which is spanned by traded securities, and an idiosyncratic component, which can be fully diversified.

We apply our pricing method (henceforth referenced as APT) to value Social Security's net retirement benefit liability to working-age Americans (those aged 26 to 60).<sup>2</sup> Our valuation determines how much the U.S. government would have to pay private parties or foreign governments to retire this liability.

Marking this implicit debt to market makes a big difference – a 23 percent difference to be precise. The 2005 (our benchmark year) liability equals \$10.4 trillion when marked to market, but \$8.5 trillion when valued using Social Security Administration (SSA) methodology.

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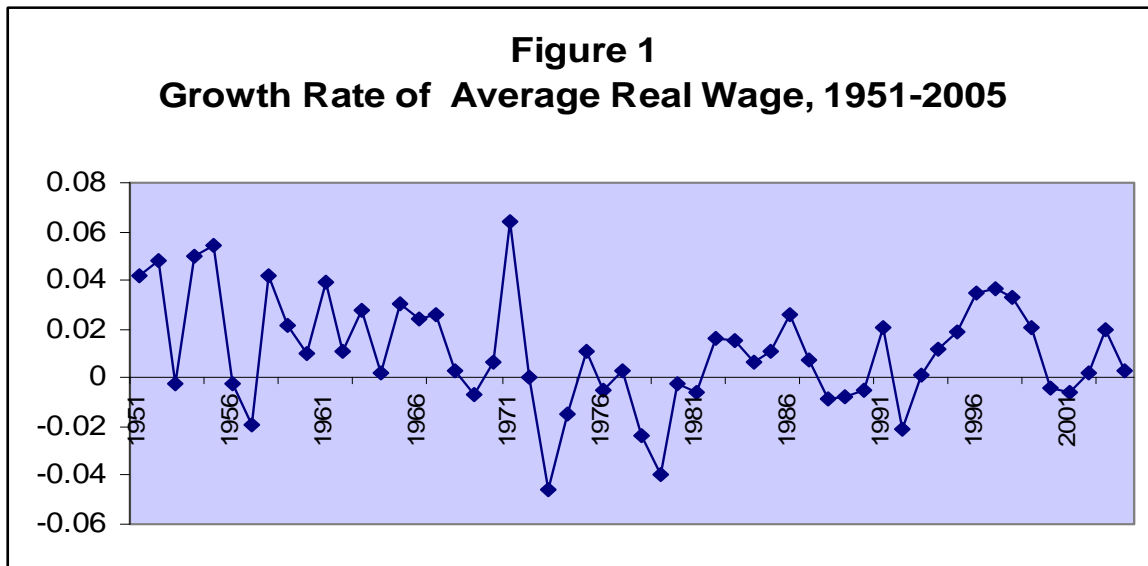
<sup>1</sup> See, for example, the 75-year and infinite horizon liability calculations reported in the annual OASDI Trustees Reports.

<sup>2</sup> Net retirement liability refers to Social Security's obligation to pay OAI retirement benefits net of OAI taxes.

Finding such a large discrepancy could be expected. The Social Security Trustees' unfunded liability calculation, reported in their annual *Trustees Report*, makes no adjustment for uncertain future economy-wide average wage growth, which is so determinative of workers' future benefits and taxes.<sup>3</sup> The Trustees also make no attempt to mark their risk-free benefit obligations to market notwithstanding the availability of risk-free securities to do such pricing. These obligations include retirement benefits being made to current retirees as well as retirement benefits that will be made to current workers once their initial benefit level is determined.

Social Security's failure to adjust formally for uncertainty in average real-wage growth is surprising given that a) this growth rate has been highly variable and b) Social Security's benefit obligations and tax receipts represent, in large part, wage-growth derivatives.

Figure 1 documents annual swings in real growth rates between 1951 and 2005.<sup>4</sup> Over this period, the average real wage has grown by as much as 6.4 percent in a single year and declined by as much as 4.6 percent.



The variability of real wage growth suggests there could be risk here to price, but it tells us nothing about the degree to which implicit claims to growth in the real wage are valued in the market. For it is the covariance of real wage growth and market returns,

<sup>3</sup> The trustees do examine the sensitivity of their liability measures to alternative economic and demographic assumptions. But this is no substitute for proper risk adjustment.

<sup>4</sup> <http://www.ssa.gov/OACT/COLA/AWI.html#Series> reports Social Security's nominal average wage series, which we adjust for inflation using the CPI.

together with the mean real-wage growth rate that determines the current price of an implicit wage-growth security.

As we show, a one-year, \$1 investment in the wage-growth security is worth \$0.988 according to APT. Social Security's valuation of such a claim is quite similar -- \$0.982.<sup>5</sup> But the APT and SSA valuations of multi-year wage growth securities (and the associated valuations of out-year benefits and taxes) diverge to an increasing degree the longer the duration of the relevant wage-growth security. For example, the APT value of a 10-year wage growth security is \$.862; the corresponding SSA valuation is \$.838. In the case of a 35-year wage growth security, the APT and SSA valuations are \$0.588 and \$.542, respectively.

Since the benefits to be paid to current workers postdate the taxes to be collected from them, the duration-dependent divergence of APT and SSA wage-growth valuations and the dependence of benefits and taxes on wage growth suggest that Social Security is systematically understating its net liability to current workers. But the trustees make a second valuation mistake that more than offsets the first.

This second mistake involves the valuation of the benefits beyond those received in the first year of eligibility. These benefits are paid out as inflation-indexed annuities and should be actuarially priced using the prevailing Treasury Inflation Protected Securities (TIPS) term structure.<sup>6</sup> In 2005 the average annual real yields on TIPS was 1.50 percent, 1.63 percent, 1.81 percent, and 1.97 percent for 5, 7, 10, and 20 year maturities, respectively.<sup>7</sup> Each of these yields is considerably lower than the 2.9 percent real yield used by SSA in their 2005 unfunded liability calculations. Using the TIPS term structure, the value of a \$1 dollar single-life real annuity for a 62 year-old male in 2005 is \$15.43. This is 8.8 percent higher than SSA's \$14.18 valuation.<sup>8</sup>

Thus, Social Security's trustees appear, in part, to be undervaluing benefits relative to taxes and, in part, overvaluing benefits relative to taxes, with the latter mistake quantitatively exceeding the former.<sup>9</sup>

Our paper proceeds in section 2 with Baxter's (2001) and Baxter and King's (2002) observation that a household's future Social Security benefits and, by extension, its future Social Security taxes, can be viewed as financial assets, albeit ones with special market

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<sup>5</sup> Social Security assumes real wages will grow, on average, by 1.1 percent and discounts, as indicated, at a 2.9 percent rate.

<sup>6</sup> See [www.federalreserve.gov/release/h15.data.htm](http://www.federalreserve.gov/release/h15.data.htm)

<sup>7</sup> Unfortunately, issuance of 30-year TIPs was suspended between October 2001 and February 2006.

<sup>8</sup> Inflation-indexed annuities are sold on the market. Indeed, Vanguard's price for this annuity is \$19.44, which is 26.0 percent higher than our valuation (and 37.1 percent higher than Social Security's). Using these market prices for real annuities, while tempting, would, we think be inappropriate given that adverse selection surely explains much of the 26.0 percent differential and doesn't come into play in valuing annuities provided to all members of particular cohorts.

<sup>9</sup> The former mistake involves failing to account for risk with respect to initial benefit awards and tax payments, whereas the latter mistake involves failure to account for safety with respect to the stream of benefit payments once they commence.

and idiosyncratic return properties. We clarify these return properties and show how to value Social Security net retirement benefits using risk-neutral pricing and arbitrage-pricing theory.

Section 3 shows how one can measure the idiosyncratic components of Social Security benefits and taxes. It also presents our wage-growth valuation regressions and compares APT valuation of a \$1 wage-growth security with SSA valuation.

Section 4 describes our use of the Panel Study of Income Dynamics (PSID) data to estimate a random effects model of individuals' annual relative earnings – their annual earnings relative to Social Security's measure of economy-wide, average annual earnings. We use this model to determine the predictable, idiosyncratic component of future relative earnings and, thus, of future benefit claims and tax obligations. When it comes to taxes, we treat 7.9 percentage points of the 10.6 percentage point combined employer and employee OASI (Old Age Survivor Insurance) payroll tax rate as the tax used to finance OAI retirement benefits.<sup>10</sup> We then combine market-pricing and idiosyncratic-pricing elements to calculate the average value of benefit claims and tax obligations by age, sex, and education. Finally, we apply age-, sex-, and education-specific population weights to determine, using both APT and SSA methodologies, the aggregate values of future Social Security net benefits payable to working-age Americans.

Since our main focus is on market-pricing differences in valuing Social Security, we incorporate the same idiosyncratic component in both our SSA and APT valuations.<sup>11</sup> Thus, the aforementioned 23 percent difference in aggregate APT and SSA net liability measures is purely attributable to differences in market pricing, specifically how APT and SSA value wage growth and inflation-indexed annuities.

Section 5 illustrates section 4's analysis by comparing APT and SSA 2005 benefit, tax, and net retirement-liability valuations for selected demographic groups. We then present the aggregate valuations under the APT and SSA methodologies, decomposing the 23 percent net liability difference into benefit and tax components. Section 6 responds to potential criticisms of our approach. And section 7 concludes by pointing out that the valuation methods used here can be applied to other government implicit claims and obligations.

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<sup>10</sup> This is 10.6 percent times .745, which is the 2006 ratio of retiree benefits to total OASI benefits as reported in the 2007 OASDI Trustees Report.

<sup>11</sup> Social Security's treatment of the idiosyncratic components of future benefit and tax valuations appears to be similar to ours with the exception that we take into account workers' education in projecting their future earnings. The 2008 OASDI Trustees Report states "Future average awarded PIAs are calculated from projected earnings histories, which are developed using a combination of the actual earnings histories associated with a sample of awards based on 2004 entitlements, and more recent actual earnings levels by age and sex for covered workers."

## 2. Valuing Social Security Retirement Benefits and Taxes

Let  $b_i$  stand for the full retirement benefit or Primary Insurance Amount (PIA) available to worker  $i$ . This benefit is a concave function of the worker's Average Indexed Monthly Earnings (AIME). The AIME is, in turn, calculated by first accumulating worker  $i$ 's past covered earnings in each year starting from the year the worker was age 16 and continuing to the year the worker reaches age 60. The accumulation factor is based on the economy-wide growth in average total (uncovered as well as covered) monthly earnings. Next the 35 largest values of these indexed earnings plus worker  $i$ 's nominal earnings received after age 60 are averaged to form the AIME.

The rate of accumulation is determined by the real growth in economy-wide averaged earnings. For simplicity, we assume that workers' 35 years of highest earnings occur between ages 26 through 60 and that the worker retires at age 60.

The PIA is inflation-adjusted to ensure the same real PIA is used regardless of when the worker elects to start collecting benefits. Workers can begin collecting benefits as early as age 62 and as late as age 70. Deviations in workers' initial collection ages from their ages of full retirement trigger actuarial reductions or increases in the retirement benefit. We assume that all current workers begin collecting their benefits at age 62.<sup>12</sup>

In (1),  $w_{i,j}$  denotes the covered (up to the Social Security earnings ceiling) wage earned by worker  $i$  in year  $j$ ,  $\tau_i$  references worker  $i$ 's year of birth,  $g_k$  stands for the growth rate of average real earnings in year  $k$ , and 374 refers to 35 years times 12 months.

$$(1) \quad b_i = f_{\tau_i+60} \left( \frac{\sum_{j=\tau_i+25}^{\tau_i+60} w_{i,j} \prod_{k=j}^{\tau_i+60} (1+g_k)}{374} \right),$$

where  $f_{\tau_i+60}$  captures the PIA benefit formula and its argument is worker  $i$ 's PIA.

Let  $\bar{w}_j$  stand for the level of economy-wide, real average earnings in year  $j$ . Define the ratio of worker  $i$ 's covered earnings in year  $j$  to  $\bar{w}_j$  by  $z_{i,j}$ , i.e.,

$$(2) \quad w_{i,j} \equiv z_{i,j} \bar{w}_j.$$

Substituting (2) into (1) yields

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<sup>12</sup> Assuming that Social Security's actuarial adjustment is based on the same real interest rate as used in its unfunded liability valuation, SSA valuation of its net liability to current workers should be independent of when workers collect their benefits. On the other hand, the APT valuation of this net liability will be larger the later workers collect because Social Security will provide larger benefit increases in return for delaying benefit collection that the market indicates is actuarially fair. Thus, in assuming that current workers begin collecting retirement benefits at age 62, we are biasing down our estimate of Social Security's understatement of its net liability to current workers.

$$(3) \quad b_i = f_{\tau_i+60} \left( \frac{\sum_{j=\tau_i+25}^{\tau_i+60} z_{i,j} \bar{w}_j \prod_{k=j}^{\tau_i+60} (1+g_k)}{374} \right)$$

$$= f_{\tau_i+60} (\bar{z}_i \bar{w}_{\tau_i+60}),$$

where  $\bar{z}_i$  is one twelfth the average annual value of  $z_{i,j}$ .

Social Security indexes not just a worker's past earnings to economy-wide average covered earnings; it also indexes the brackets in its year- $t$  benefit function  $f_t(\cdot)$ . Thus, other things equal, if  $\bar{w}_{\tau_i+60}$  is twice as large, the value of  $b_i$  will be twice as large; i.e.,

$$(4) \quad f_{\tau_i+60} (\bar{z}_i \bar{w}_{\tau_i+60}) = f(\bar{z}_i \bar{w}_{\tau_i+60}, \bar{w}_{\tau_i+60}),$$

where  $f(\cdot, \cdot)$  is homogeneous of degree one in  $\bar{w}_{\tau_i+60}$ . Using this property, we can write

$$(5) \quad b_i = h(\bar{z}_i) \bar{w}_{\tau_i+60},$$

where

$$(6) \quad h(\bar{z}_i) \equiv f(\bar{z}_i, 1).$$

In what follows, we assume that  $h(\bar{z}_i)$  and  $\bar{w}_{\tau_i+60}$  are independently distributed.

Note that for someone born in year  $\tau$ , the current (year  $t$ ) average covered wage,  $\bar{w}_t$ , is related to the average covered wage in year  $\tau+60$  according to

$$(7) \quad \bar{w}_{\tau+60} = \bar{w}_t \prod_{k=t}^{\tau+60} (1+g_k).$$

Equations (5) and (7) imply

$$(8) \quad b_i = h(\bar{z}_i) \bar{w}_t \prod_{k=t}^{\tau_i+60} (1+g_k).$$

According to (8), worker  $i$ 's full retirement benefit,  $b_i$ , is equivalent to what would be earned by investing the amount  $h(\bar{z}_i) \bar{w}_t$  at time  $t$  and holding it until time  $\tau_i+60$  in a wage-growth security, i.e., a security that compounds at the rate of growth of average real wages. Of course, as with any time- $t$  risky investment, the ultimate value of  $h(\bar{z}_i)$  at time  $\tau_i+60$  is unknown. Yet enough is known at time  $t$  to determine the value of  $b_i$ .



Note that the cross-sectional average of  $\bar{z}_i$  is 1. We assume that the terms  $\bar{z}_i - 1$  have no value much like the deviations of individual insurance claims from the industry average have no value. What matters for value, then, is simply the expected value of  $h(\bar{z}_i)$ ,  $E[h(\bar{z}_i)]$ .

Given that  $h(\bar{z}_i)$  is independent of  $\bar{w}_{\tau_i+60}$  and that the valuation operator is linear, we can write the value of the benefit claim as

$$(10) \quad V_t(b_i) = V(\bar{w}_t h(\bar{z}_i) [\prod_{k=t}^{\tau_i+60} (1+g_k)]) = \bar{w}_t E(h(\bar{z}_i)) V([\prod_{k=t}^{\tau_i+60} (1+g_k)]),$$

where  $V(\cdot)$  stands for the valuation function.

A straightforward argument, laid out in the appendix and motivated by financial pricing theory, lets us determine the current value of  $\prod_{k=t}^{\tau_i+60} (1+g_k)$ , which we refer to as a \$1 wage-growth security with maturity  $\tau_i+60-t$ . To form the valuation, all we need find is a portfolio of traded securities whose payoff mimics the final real wage up to some idiosyncratic terms. These terms are assumed not to matter for valuation because they are uncorrelated with the returns to marketed securities.

Assume that the annual growth rate,  $g_t$ , of the real wage has the following structure where  $f_{i,t}$  denotes the time- $t$  value of marketed asset  $i$ , and  $\varepsilon_t$  is an unpriced, idiosyncratic shock:

$$(11) \quad g_t = \alpha + \sum_i \beta_i \frac{\Delta f_{i,t}}{f_{i,t-1}} + \varepsilon_t$$

The appendix demonstrates that the final real-wage payment can be replicated by a portfolio of the (real) bond and the assets,  $f_{i,t}$ , that is rebalanced every year. The cost of doing so is the value of the terminal real wage, which the appendix shows is

$$(12) \quad V[\prod_{k=t}^{\tau_i+60} (1+g_k)] = \left[ \frac{1 + \alpha + r \sum_i \beta_i}{1+r} \right]^{\tau_i+60-t},$$

where  $r$  denotes the real rate of interest, which we assume is constant. If there is a term structure of real rates available from, say, inflation-protected bonds, then the formula becomes:

$$(13) \quad V\left[\prod_{k=t}^{\tau_i+60} (1+g_k)\right] = \Pi_t^{\tau_i+60} \left[ \frac{1 + \alpha + r_s \sum_i \beta_i}{1 + r_s} \right]$$

Combining (10) and (13) gives:

$$(14) \quad V_t(b_i) = \bar{w}_t E_t h(\bar{z}_i) \Pi_t^{\tau_i+60} \left[ \frac{1 + \alpha + r_s \sum_i \beta_i}{1 + r_s} \right].$$

These formulas assume that only contemporaneous asset returns price the wage growth security. As also shown in the Appendix, the formulas are more complex if lagged as well as contemporaneous asset returns predict current wage growth. The complexity involves the need to adjust for the fact that lagged returns predict current and future wage growth. While the case of multiple lags is quite complex, the formulas follow quite naturally in the case of a single lag:

$$(15) \quad V(b_i) = \bar{w}_t E_t h(\bar{z}_i) w_0 \left( \frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}}}{1 + r_{t-1}} \right) \prod_t^{T-1} \left( \frac{1 + \alpha + r_t \sum_j \beta_j}{1 + r_t} \right),$$

where the differences between equations (14) and (15) reflect the impact of lagged asset returns on expected future wage growth. To be more precise, the  $\beta_j$ s are the loadings on one-year lagged returns in the following modification of (10).

$$(16) \quad g_t = \alpha + \sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-2}} + \varepsilon_t$$

### ***Incorporating Survival to Age 62 and Annuity Valuation***

The above treats Social Security benefits as a one-year payoff of a wage-growth security derivative, with the payoff occurring in the year the worker reaches age 60. This is inappropriate for four reasons. First, under our assumption that workers take early retirement benefits, benefits commence at 62. Second, the worker may not survive from her current age to age 62. Third, receiving benefits at age 62, rather than full retirement age, triggers an actuarial reduction, the size of which depends on the worker's year of birth. Fourth, the benefit starting at age 62 is not a one-year payment, but continues each year in the future conditional on the worker's survival.

Equation (17) modifies (15) to arrive at  $V_t(B_i)$  -- the time- $t$  APT value of worker  $i$ 's lifetime benefits,  $B_i$ . In the formula, we multiply  $V_t(b_i)$  by a)  $c$  -- a two year real discount factor that discounts, at the market's safe real rate, for the fact that benefits don't begin at age 60, but rather at age 62, b)  $q_{i,t,\tau_i,62}$  -- the time- $t$  probability that worker  $i$ , who was born at time  $\tau_i$ , survives to age 62, c)  $\mu_{\tau_i}$  -- the early-retirement benefit reduction factor for workers born in year  $\tau_i$  who begin benefit receipt at age 62, and d)  $\delta_{i,\tau_i}$  -- the actuarially discounted present value of a \$1 real annuity beginning at age 62 payable to worker  $i$  who is born in year  $\tau_i$ , where the discounting is at the market's safe real term structure and goes back to age 62.

$$(17) \quad V_t(B_i) = cq_{i,t,\tau_i,62} \mu_{\tau_i} \delta_{i,\tau_i} \bar{w}_t E_t[h(\bar{z}_i)] \left( \frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}}}{1 + r_{t-1}} \right) \prod_t^{T-1} \left( \frac{1 + \alpha + r_t \sum_j \beta_j}{1 + r_t} \right)$$

### ***SSA's Benefit Valuation Formula***

The corresponding SSA valuation,  $\hat{V}_t(B_i)$ , is given by

$$(18) \quad \hat{V}_t(B_i) = \hat{c} q_{i,t,\tau_i,62} \mu_{\tau_i} \hat{\delta}_{i,\tau_i} \bar{w}_t E_t[h(\bar{z}_i)] \left( \frac{1 + \bar{g}}{1 + \bar{r}} \right)^{\tau_i + 60 - t},$$

where  $\bar{r}$  and  $\bar{g}$  reference, respectively, Social Security's assumed 2.9 percent real discount rate and 1.1 percent real-wage growth rate. Clearly the final terms in equations (17) and (18) differ, which reflects differences in APT and SSA valuations of real wage growth. But the two-year discount factor,  $\hat{c}$ , and the actuarial value of the annuity,  $\hat{\delta}_{i,\tau_i}$ , also differ from their equation (17) counterparts because they too incorporate SSA's assumed 2.9 percent discount rate rather than the prevailing TIPs term structure.

### ***Measuring the Idiosyncratic Component of Benefit Valuation***

The value of  $\bar{w}_t$  for our base year,  $t=2005$ , is reported by the Social Security administration, so the remaining question is how to determine the value of the idiosyncratic component,  $E_t h(\bar{z}_i)$ . Our method is to use our aforementioned random-effects model of relative earnings to simulate the average value of  $h(\bar{z}_i)$  by individual age in 2005, sex, and education group. The education groups are less than high school, high school, and college or more.

Our random effects model, which we estimate separately for each of the six education and sex groups, is given by

$$(19) \quad z_{it} = \phi_i + \theta_0 + \theta_1 a_{it} + \theta_2 a_{it} b_i + \theta_3 a_{it}^2 + \theta_4 a_{it}^2 b_i + \theta_5 a_{it}^3 + \theta_5 a_{it}^3 b_i + \theta_6 b_i + \varepsilon_{it},$$

$$\phi_i \sim N(0, \sigma_\alpha^2), \quad \varepsilon_{it} \sim N(0, \sigma_\varepsilon^2),$$

where  $\phi_i$  is the random effect, and  $a_{it}$  and  $b_i$  reference, respectively, worker  $i$ 's age in year  $t$  and her year of birth (i.e.,  $t - a_{it}$ ). The term  $\varepsilon_{it}$  is a transitory error.

To determine the average value of  $h(\bar{z}_i)$  for agents in 2005 of a given sex and education group who were born in year  $b_i$  (who were a given age in 2005), we draw 100,000 values of  $\alpha_i$ ; i.e., we consider 100,000 agents with specific random effects. For each agent we draw 35 values of  $\varepsilon_{it}$  -- starting for the year the agent was age 26 and continuing through the year the agent will be age 60. For each year,  $m$ , of these 35 years, we evaluate the right-hand-side of (19) using the values of the agent's  $\alpha_i$  and  $b_i$  as well as the value of  $\varepsilon_{it}$  drawn for that year. In this evaluation,  $a_{im}$  is set to  $m - b_i$ . Next we form the average over the 35 simulated values of  $z_{it}$  to form the value the agent's  $\bar{z}_i$ . This value of  $\bar{z}_i$  is then run through Social Security's benefit formula to calculate the value of  $h(\bar{z}_i)$ .<sup>13</sup> The average of the  $h(\bar{z}_i)$  values across all 100 agents provides our sex-, education-, and cohort-specific estimates of  $E_i h(\bar{z}_i)$ .

### ***Calculating the Aggregate Value of Benefits***

The total value of benefits for Americans age 26 through 60 is calculated by forming

$$(20) \quad \sum_{i=1}^N \omega_i V_i(B_i),$$

where  $\omega_i$  is the CPS population weight for sex-, education-, and cohort-population cell  $i$ .

### ***Valuing Taxes***

Taxes,  $T_{i,l}$ , paid by worker  $i$  in year  $l$  equal the tax rate in year  $l$ ,  $\nu_l$ , multiplied by worker  $i$ 's covered wages in year  $l$ ,  $z_{i,l} \bar{w}_l$ .

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<sup>13</sup> This function includes the early retirement reduction factor since we are computing reduced benefits assumed to be taken at age 62.

$$(21) \quad T_{i,l} = v_l z_{i,l} \bar{w}_l$$

Following the above lines of argument, we can write the time- $t$  value of taxes paid in year  $l$  as

$$(22) \quad V_t(T_{i,l}) = q_{i,l,\tau_i,l-\tau_i} v_l E_t z_{i,l} \bar{w}_l \left( \frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}}}{1 + r_{t-1}} \right) \prod_t^l \left( \frac{1 + \alpha + r_t \sum_j \beta_j}{1 + r_t} \right).$$

To determine the value of  $E_t z_{it}$  in (22), we again resort to averaging draws we generate from our random-effects model within each sex-, education-, and cohort-specific cell.

Let  $T_i$  stand for the remaining lifetime OAI taxes of worker  $i$  and  $V_t(T_i)$  stand for the market value of these taxes. Then,

$$(23) \quad V_t(T_i) = \sum_t^{\tau_i+60} V_t(T_{i,l}).$$

### ***Calculating the Aggregate Value of Taxes***

The total value of taxes for Americans age 26 through 60 is calculated by forming

$$(24) \quad \sum_{i=1}^S \omega_i V_t(T_i),$$

where, again,  $\omega_i$  is the CPS weight for sex-, education-, and cohort-cell  $i$ .

### **3. Valuing the Wage-Growth Security**

The first step in valuing the real wage-growth security is estimating the parameters of (11) on data covering 1952 through 2005. We consider two sets of APT-factor regressions for estimating our intercept  $\alpha$  and the factor loadings (the  $\beta$  coefficients). The first set of regressions, reported in table 1, incorporate contemporaneous and lagged nominal equity returns of indexes of large and small cap stocks as well as of short- and

long-term government nominal bond returns.<sup>14</sup> The second set, reported in table 2, substitute contemporaneous and lagged Fama-French factors for the asset-index returns.

When lagged regressors are included, the adjusted  $R^2$ s are quite high, ranging from .289 to .452 across the two tables. Omitting lagged regressors lowers these values dramatically. The table 1 regressions with lagged returns provide better fits than the corresponding table 2 regressions based on Fama-French factors.

For each regression, we used the estimated parameters to calculate the implied present value of \$1 invested in the wage growth security for 1 and 35 year horizons based on the appendix formula (a13). These valuations are provided in the tables.

Consider the results in table 1, which include lagged regressors and provide the best fits to the data. For a 1-year, \$1 wage-growth security, our valuations range from 98.2 cents to 99.3 cents. SSA's valuation is 1.011 divided by 1.029, or 98.2 cents, based on SSA's assumed 1.1 percent average real-wage growth rate and 2.9 percent real discount rate. In the case of the 35-year, \$1 wage-growth security, our table 1 lagged regression valuations range from 58.0 cents to 60.4 cents. The corresponding SSA valuation is  $(1.011/1.029)^{35}$  or 53.9 cents.

Clearly the discrepancy between the SSA and APT valuations grows the longer out is the wage-growth security's duration. This is clear from the first model in table 1. Its 1-year wage-growth security valuation is identical to Social Security's. But its 35-year valuation of 59.4 cents is 10.2 percent higher than SSA's 53.9 cent figure. The reason the APT and SSA valuations diverge more for longer duration wage-growth securities is due to the fact that contemporaneous and past real market returns affect future expected real wage growth differently through time in the APT valuation. This is clear from considering how the security's duration enters into equations (17) and (18).

We base our APT valuations on the regression appearing in the last column in table 1 (Restricted 2). This model values the \$1, one-year wage-growth security at 98.8 cents and the \$1, 35-year wage-growth security at 58.8 cents. Our selection of this model was guided by the Bayesian information criterion (BIC, also known as Schwartz's Criterion) and Akaike's information criterion (AIC) as well as Akaike's criterion corrected for small-sample bias (AICc).<sup>15</sup>

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<sup>14</sup> Regressing a real growth rate against nominal returns may seem surprising, but the short-term nominal bond return is highly correlated with, and thus controls for, the inflation rate.

<sup>15</sup> See Andrews and Monahan (1992), Hurvich and Tsai (1989), and Schwarz (1978). These criteria represent ways to trade-off model complexity and goodness-of-fit in model selection. All three criteria are increasing in the number of parameters in the model and decreasing in the maximized log-likelihood. In the case of normal errors, the latter can be recast as increasing in the sum of squared residuals. Hence, minimization of these criteria is a logical guide for model selection. Furthermore, the use of model selection criteria avoids the problems of multiple testing and non-nested model comparison that are commonly seen with other approaches. The criteria have similar forms (see Schwartz, 1978 and Hurvich and Tsai, 1989), although each behaves somewhat differently. AIC and AICc are based on an information-theoretic derivation and are asymptotically equivalent to likelihood-based selection. BIC is based on Bayesian as well as minimum-description-length arguments and is not asymptotically equivalent to selection based purely upon maximized likelihood. This is because BIC penalizes the addition of

#### 4. Modeling Relative Earnings

Table 4 presents parameter estimates by demographic group for our random-effects model. The dependent variable is the natural logarithm of relative earnings ( $z$ ). The table's estimated coefficients were used to simulate lifetime paths for relative covered earnings as described above. Our data come from the Panel Survey of Income Dynamics (PSID) for the years 1968 through 1993. We include observations reporting educational attainment and positive labor income.<sup>16</sup>

Figures 1 and 2 shows the average age-relative earnings profiles for different cohorts holding sex and education constant as predicted by table 4's results. The profiles tell some interesting stories. First, all the profiles peak between ages 40 and 45 with the exception of males with high school educations; their profiles peak in their 30s. Second, the female (male) profiles are significantly higher (lower) for younger cohorts, indicating that successive cohorts have experienced a smaller gender gap in earnings. Third, females with college or more education experience relative limited declines in their relative earnings after age forty. Each of the other groups experience very sharp declines.

#### 5. Benefit and Tax Valuations

Tables 5 through 13 show the results of our calculations of benefit and tax values for individuals aged 26, 40, and 55 in 2005. The APT and SSA benefit values are calculated based on (17) and (18), respectively. The APT tax value is calculated based on (23), and the SSA tax values are calculated using wage-growth security values analogous to those in (18). Table 14 presents aggregate values of tax and benefits based on (20) and (24).

The values in tables 5 through 13 make sense. The market values of benefit obligations are larger for those with more education, but so are the market values of tax obligations. The net liabilities are smaller for those with more education in the case of 26 year olds, but larger for the better educated in the case of 40 and 55 year olds. This simply reflects the fact that older workers have all their benefits coming, but only a portion of their lifetime taxes left to pay.

The differences in SSA and APT valuations of net liabilities to particular groups can be quite sizeable. Take 40 year-old females with a high school education. SSA's methodology places Social Security's average liability to these women at \$39,708, whereas the APT valuation is 34 percent higher at \$53,253.

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parameters more heavily than AIC (the coefficient on the number of parameters is 2 for AIC vs.  $\log(n)$  for BIC). Hence, BIC tends to select more parsimonious models than AIC.

<sup>16</sup> Selecting observations in this manner excluded between 11 and 18 percent of potential observations depending on the year in question. This selection results in an unbalanced panel and raises the issue of coefficient bias, although it's not clear how such bias, were it to exist, would cut with respect to our comparison of APT and SSA valuations.

A final point concerns the absolute size of the benefit obligations to older workers. As table 11 shows, whether one uses APT or SSA valuations, the amounts are sizeable when set against the relatively small values of financial wealth held by typical older workers.

Table 14 reports our main finding, namely that Social Security's valuation method appears to understate the market value of the net liability to working-age Americans by approximately \$2 trillion or 23.1 percent. The main source of this undervaluation involves the valuation of benefits. SSA-based valuation leads to a \$13.7 trillion figure, whereas APT valuation puts the figure at \$15.8 trillion. This is a \$2.1 trillion differential. In contrast, the difference in the SSA and APT values of taxes owed by working-age Americans is only \$.1 trillion.

Were we to value benefits by simply marketing annuities to market (using TIPS rates), but retaining SSA's wage-growth security valuation, we'd arrive at an SSA aggregate benefit valuation of \$14.9 trillion. Since doing so would leave the SSA valuation of taxes unchanged, it would reduce the SSA valuation of net liabilities to \$9.68 trillion. Hence, proper annuity valuation would, by itself, eliminate approximately 60 percent of the difference in APT and SSA valuations. Social Security Trustees could, therefore, significantly improve their net liability measure simply by using TIPS returns to value the system's promised annuities.

## **6. Critiquing the Approach**

There are at least five objections to the approach taken here. The first is that, given their size, any actual attempt to market Social Security's net liabilities would dwarf the financial markets. Our response is that valuation is a marginal exercise; we routinely establish values for total stocks of financial and real assets as well as financial liabilities based on the going price in the market. Take, for example, the Federal Reserve's *Flow of Funds* valuation of owner-occupied homes. All of these homes are all carried at marginal market price even though the immediate sale of all U.S. homes could greatly alter values. Like most homes, Social Security liabilities are currently being held, rather than actively traded. Moreover, although Social Security's net liabilities are large relative to U.S. net worth, they are a small component of total world net worth.

The second objection involves what we take to be the idiosyncratic component of real wage growth. Does the market value this component, which accounts for about half of wage growth variability? Arguably not. If this risk were significant to investors it would, presumably, be marketed and priced by the major financial securities we've included in our analysis. The opposite view must maintain that financial markets are profoundly incomplete and fail to span aggregate risks of major importance to investors. Were the opposite view correct, our results would be incomplete and potentially biased. But the direction of such bias cannot be determined a priori.



This second objection is reminiscent of the old debate in international trade about whether there are more factors of production or goods being produced; it is fundamentally unresolvable. Our position, though, should be clear. We have a practical problem, and we offer a consistent and robust practical solution that closely aligns with decades of research in modern asset pricing. The alternative of relying on economic projections of real wages many years in the future is similar to relying on analysts' forecasts of a company's earnings and the using the resulting discounted cash flow to determine the value of the company's stock rather than simply using its trading price. Such practice bets against the market and represents a highly questionable foundation on which to base generational and fiscal policy.

The third objection is our failure to take account of potential future policy changes; i.e., changes to the  $h()$  function. Here we plead guilty; but determining which policy changes are likely to arise and the impact on different parties of such changes is not our objective. Our objective is valuing Social Security's net claims taking current policy as given and determining whether that policy is sustainable. Were we instead to incorporate future policy changes, our valuation exercise would be trivial; we'd necessarily find the government to be intertemporally balanced. The reason is that along any path the economy travels government spending will necessarily be financed by the private sector. From this perspective, the government can never be intertemporally insolvent. That said, many of these paths, all of which entail ex-post satisfaction of the government's intertemporal budget constraint, will entail terrible economic and fiscal conditions, including policy changes described as explicit or implicit defaults.

A fourth objection is that the sum of workers' valuations of their net Social Security's benefit claims may differ dramatically from the valuation we measure. As demonstrated in Liu, Rettenmaier, and Saving (2007), individual valuations are based on wealth-equivalent changes in expected utility and take into account workers' idiosyncratic risks. Admittedly, how much today's workers would be willing to pay to keep Social Security is an interesting question. But that amount is potentially quite different from what the market would be willing to pay.<sup>17</sup>

Finally, there is the question of relevance. Does it matter if the market thinks a country is financially troubled, while its government proclaims its solvency? The answer is surely yes. Over the years, scores of countries have experienced abrupt runs on their currencies and financial instruments because the market made a decision that their policies were unsustainable. Argentina's 2002 fiscal/financial meltdown, following a decade of excellent economic growth, is a good example. This crisis came as a shock to its leaders who had forecasts aplenty for how the government was going to reverse its prolonged fiscal slide and pay its bills.

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<sup>17</sup> The consumption of grapefruit provides a useful analogy. What workers are willing to pay to have access to grapefruit, if the alternative is never eating another grapefruit, is the sum of their consumer surplus from grapefruit. This is not the same as the cost of buying, at market prices, the grapefruit the workers intend to consume.

## 7. Conclusion

No one would suggest that the prices of explicit financial securities are independent of their risk properties. Such a proposition would deny fact, let alone theory. But the same financial laws that determine the prices of marketed securities govern the pricing of non-marketed assets and liabilities; they cannot be priced by treating their variable returns as sure things and discounting at safe rates.<sup>18</sup> Nor can safe government payments and receipts be valued using discount factors that differ from the discounts associated with safe marketed securities. This, however, has been standard U.S. practice since our government began considering its implicit debts.

Were marking to market implicit government liabilities and assets of minor import, the government's ad-hoc valuation methods would be of little concern. But the example considered here – the valuation of Social Security's net retirement liability to working-age Americans – suggests the opposite. Proper asset pricing delivers a measure of this net liability that exceeds SSA's valuation by almost one quarter.

Of course, Social Security's net retirement liability to working-age Americans is only part of its overall implicit debt. And Social Security is only one part of a much broader set of future U.S. government receipts and payments, whose market values need to be assessed. The ultimate goal, in this regard, is valuing all components of the government's intertemporal budget to determine whether its overall current policy is sustainable, i.e., whether the government's entire fiscal enterprise breaks even as a matter of present valuation. Answering this broader question is a much bigger task, but one that can surely be approached using techniques similar to those considered here.

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<sup>18</sup> Fortunately, government officials aren't asked to value the stock market. Were they to do so, they'd badly misprice the market. Indeed, were Social Security's valuation method applied to the S&P, its price-earnings ratio would equal 34.5 – more than twice the ratio observed at our writing. To see this, note that Social Security uses an assumed safe 2.9 percent discount rate for its liability valuations. Let  $e$  stand for the expected earnings on the S&P per dollar invested. Then Social Security would value the S&P by setting  $P$ , the price per dollar invested, equal to the  $e/.029$ ; i.e., the value of a perpetual safe stream  $e$  discounted at 2.9 percent. Since  $P = e/.029$ ,  $P/e = 1/.029 = 34.5$ .

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## Appendix -- Pricing the Wage-Growth Security

Suppose that the real wage follows the growth process:

$$(a1) \quad \frac{dw}{w} = \alpha dt + \sum_i \beta_i \frac{df_i}{f_i} + \sigma_\varepsilon d\varepsilon$$

where  $f$  is the vector of priced assets, e.g., the S&P 500, and  $\varepsilon$  is the process for the residual and unpriced component. The symbol  $df/f$  denotes the total returns (dividends included) on the priced assets.

Converting this into logs we have:

$$(a2) \quad \begin{aligned} d(\ln w) &= \frac{dw}{w} - \frac{1}{2} \sigma_w^2 dt \\ \text{and} \\ d(\ln f_i) &= \frac{df_i}{f_i} - \frac{1}{2} \sigma_i^2 dt \end{aligned}$$

where  $\sigma^2$  denotes the appropriately subscripted instantaneous variance.<sup>19</sup>

For reference, letting  $\Omega$  denote the instantaneous variance covariance matrix for the returns on the priced assets we have:

$$(a3) \quad \sigma_w^2 = \beta' \Omega \beta + \sigma_\varepsilon^2$$

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<sup>19</sup> Note, the second term in the log derivatives arises because the process instantaneously has infinite movement and its variance is of order  $dt$ . Since log is concave, we have the second order negative correction.

Now we integrate to obtain the stochastic wage at time T:

$$(a4) \quad w_T = \exp[\alpha T - \frac{1}{2}\sigma_w^2 T + \frac{1}{2}(\sum_i \beta_i \sigma_i^2)T + \sigma_\varepsilon \int_0^T d\varepsilon] \prod_i \left(\frac{f_{iT}}{f_{0T}}\right)^{\beta_i}$$

In other words, the terminal real wage is an exponential in some time terms multiplied by power functions of the total accumulated values of the priced assets.

To obtain the current value of the wage at time T, we take the expected discounted value under the martingale measure, i.e., we take the expectation of the discounted value assuming that all of the priced assets have an expected growth rate equal to the risk free rate.

$$(a5) \quad \begin{aligned} V &= E^*[e^{-rT} w_T] \\ &= w_0 \exp[-rT + \alpha T - \frac{1}{2}\sigma_w^2 T + (\frac{1}{2}\sum_i \beta_i \sigma_i^2)T + \frac{1}{2}\sigma_\varepsilon^2 T + (\sum_i r\beta_i)T - (\frac{1}{2}\sum_i \beta_i \sigma_i^2)T + \frac{1}{2}(\beta'\Omega\beta)T] \\ &= w_0 \exp[-rT + \alpha T + r(\sum_i \beta_i)T] \end{aligned}$$

In the valuation all of the variance terms have dropped out and only the betas and the alpha remain. The easiest way to understand this is to build it up. Suppose first that there were no betas or they were all zero. Then the wage will grow at the rate  $\alpha$  to T and be discounted back at the rate  $r$ . Now suppose there is only one marketed asset and its beta is 1. Then the wage is just the same as something that has the terminal value,  $w_0 e^{\alpha T}$ , invested in the asset. In that case the value is just the invested value,  $w_0 e^{\alpha T}$ , which is precisely what the formula gives. If beta isn't one, then the formula just corrects for the difference.

### ***A Discrete Time Approach***

While the above analysis is somewhat formal, the result indicates that a simpler intuitive approach applies. Furthermore, while the above may appear to depend on distributional assumptions, an elementary discrete time analysis will verify that it is in fact independent of any such assumptions.

In discrete time, we have

$$(a6) \quad \frac{w_{t+1}}{w_t} = 1 + g_t = 1 + \alpha + \sum_i \beta_i f_{i,t} + \varepsilon_t,$$

To value the future payment of  $w_{t+1}$  we can ask how much we would have to invest in marketed assets to replicate it. Consider investing  $A_t$  in the risky asset and  $B_t$  in the riskless asset. The return on that portfolio will be

$$(a7) \quad \sum_i A_{it}(1 + f_{i,t}) + B_t(1 + r) = 1 + g_t = 1 + \alpha + \sum_i \beta_i f_{i,t} + \varepsilon_t$$

This will replicate the priced portion for any realization of the returns,  $f_{i,t}$ , if

$$A_{i,t} = \beta_i$$

$$(a8) \quad \text{and}$$

$$B_t = \frac{1 + \alpha - \sum_i \beta_i}{1 + r}$$

for a total expenditure of

$$(a9) \quad V_t = \sum_i A_{i,t} + B_t = \left( \frac{1 + \alpha + r \sum_i \beta_i}{1 + r} \right)$$

The above expression tells us that a claim to one dollar invested in the wage-growth security for one year has an immediate and, therefore, sure value of  $V_1$ . At the end of one year the expected value of this claim is just  $V_1(1+r)$ . Having this amount for sure in a year also has this same expected value. The value after one period of investing one dollar for two years in the wage-growth security is  $V_1^2(1+r)$ . Discounted to the present, the value is just  $V_1^2$ . In general, the value,  $V^T$ , of a dollar invested in the wage-growth security for  $T$  years is  $V_1^T$ ; hence we can write

$$(a10) \quad V^T = w_0 \left( \frac{1 + \alpha + r \sum_i \beta_i}{1 + r} \right)^T \rightarrow w_0 e^{\frac{\alpha + r(\sum_i \beta_i - 1)T}{1 + r}},$$

which is the continuous time formula.

Some care should be exercised in interpreting formulas a9 and a10. They would appear to imply that a riskier wage, i.e., one with a higher beta would actually be more valuable than a less risky one. This apparently anomalous result comes about because the return on the asset in a6 has not been demeaned. As a consequence, the intercept,  $\alpha$ , is actually the intercept from a demeaned regression,  $\gamma$ , less  $\beta$  times the expected return on the asset,

$$a = \gamma - \beta E_f = \gamma - \beta(r + \pi),$$

where  $\pi$  is the risk premium on the asset. Substituting this result into a9 gives

$$V_1 = \sum_i A_{i,t} + B_t = \left( \frac{1 + \alpha + r \sum_i \beta_i}{1 + r} \right) = \left( \frac{1 + \gamma - r \sum_i \beta_i \pi_i}{1 + r} \right)$$

which clearly reflects the decline in value with increasing risk.

### *An Extension to Lagged Variable*

Suppose the regression is in discrete terms (e.g., yearly), and we have only lagged variables:

$$(a11) \quad \frac{\Delta w_t}{w_t} = \alpha + \sum \beta_j \frac{\Delta f_{j,t-1}}{f_{j,t-1}} + \varepsilon_t$$

A similar analysis produces the amended formula:

$$(a12) \quad V(w_T) = w_0 \left( \frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,-1}}{f_{j,-1}}}{1+r} \right) \left( \frac{1 + \alpha + r \sum_j \beta_j}{1+r} \right)^{T-1},$$

which is the same as when the returns are contemporaneous but with the addition of the multiplying term containing the past year's returns. Without concerning ourselves with the issues of a stochastic interest rate, using the term structure of interest rates this formula becomes:



$$(a13) \quad V(w_T) = w_0 \left( \frac{1 + \alpha + \sum_j \beta_j \frac{\Delta f_{j,-1}}{f_{j,-1}}}{1 + r_{-1}} \right) \prod_{t=0}^{T-1} \left( \frac{1 + \alpha + r_t \sum_j \beta_j}{1 + r_t} \right),$$

Extending this analysis to multiple lags is more difficult and explicitly involves the covariance structure of the returns. As an alternative we could change to a different formulation in terms of the unit root process:

$$(a14) \quad w_T = w_0 \left[ (1 + \alpha)^T + \sum_j \sum_{s=0}^k \beta_j f_{j,T-s} + \varepsilon_T \right]$$

While this is more difficult to estimate than the usual sort of regression, it allows for a simple valuation equation:

$$(a15) \quad V(W_T) = \left( \frac{1 + \alpha}{1 + r} \right)^T + \sum_j f_{j,0} \sum_{s=0}^k \left( \frac{\beta_j}{(1 + r)^s} \right)$$

We won't make use of this formulation since our estimations involve a single lag.

**Table 1**  
**Regression of Average Real Wage Growth on Nominal Returns**

<b>Regressor</b>	<b>Unrestricted Model</b>	<b>Contemporaneous Only</b>	<b>Lagged Only</b>	<b>Restricted 1</b>	<b>Restricted 2</b>
<b>(Intercept)</b>	0.003 (0.934)	0.013 (2.545)	0.002 (0.407)	0.003 ( 0.766)	0.002 (0.521)
<b>stGovBondReturn</b>	-0.435 (-5.515)	-0.025 (-0.251)		-0.336 (-2.566)	
<b>ltGovBondReturn</b>	-0.033 (-2.440)	0.036 (1.463)			
<b>largeCapReturn</b>	0.010 (0.681)	0.028 (1.262)			
<b>smCapReturn</b>	-0.006 (-0.654)	-0.040 (-3.403)			
<b>L.stGovBondReturn</b>	0.421 (3.816)		0.0587 (0.533)	0.283 (2.313)	
<b>L.ltGovBondReturn</b>	0.028 (2.261)		0.044 (1.933)	0.036 (1.945)	0.046 (1.820)
<b>L.largeCapReturn</b>	0.098 (4.795)		0.087 (3.508)	0.087 (7.149)	0.085 (7.215)
<b>L.smCapReturn</b>	-0.008 (-0.657)		0.015 (-0.230)		
<b>1 yr. Valuation</b>	-	\$0.993	\$0.986	-	\$0.988
<b>35 yr. Valuation</b>	-	\$0.782	\$0.602	-	\$0.588
<b>Residual SE</b>	0.019	0.025	0.019	0.018	0.019
<b>R<sup>2</sup></b>	0.509	0.105	0.454	0.494	0.450
<b>Adjusted R<sup>2</sup></b>	0.422	0.0315	0.409	0.452	0.429
<b>AIC</b>	-264.624	-240.198	-266.887	-270.962	-270.551
<b>AICc</b>	-259.508	-238.411	-265.100	-269.175	-269.735
<b>BIC</b>	-244.735	-228.264	-254.953	-259.028	-262.595

Note: t-scores based on HAC std. errors are in parentheses. Dependent variable for all regressions is real AWI growth. n = 54.

**Table 2 Regression of Average Real Wage Growth on Fama-French Factors**

<b>Regressor</b>	<b>Unrestricted Model</b>	<b>Contemporaneous Only</b>	<b>Lagged Only</b>	<b>Restricted 1</b>	<b>Restricted 2</b>
<b>(Intercept)</b>	0.007 (1.474)	0.014 (2.601)	0.005 (1.429)	0.005 (1.366)	0.004 (1.301)
<b>MktRF</b>	-0.007 (-0.8616963)	-0.006 (-0.405)			
<b>HML</b>	-0.020 (-1.032)	-0.023 (-0.946)			
<b>SMB</b>	-0.009 (-0.762)	-0.044 (-1.445)			
<b>L.MktRF</b>	0.081 (6.012)		0.085 (6.758)	0.086 (6.551)	0.081 (6.518)
<b>L.HML</b>	-0.006 (-0.685)		-0.002 (-0.218)		
<b>L.SMB</b>	-0.014 (-1.261)		-0.021 (-1.705)	-0.021 (-0.936)	
<b>1 yr. Valuation</b>	-	\$0.993	\$0.994	\$0.994	\$0.994
<b>35 yr. Valuation</b>	-	\$0.770	\$0.623	\$0.621	\$0.623
<b>Residual SE</b>	0.021	0.025	0.021	0.020	0.020
<b>R<sup>2</sup></b>	0.369	0.0769	0.356	0.355	0.343
<b>Adjusted R<sup>2</sup></b>	0.289	0.0215	0.317	0.330	0.330
<b>AIC</b>	-255.111	-240.553	-259.959	-261.948	-262.893
<b>AICc</b>	-251.911	-239.303	-258.709	-261.131	-262.413
<b>BIC</b>	-239.199	-230.608	-250.014	-253.992	-256.926

Note: t-scores based on HAC std. errors are in parentheses. Dependent variable for all regressions is real AWI growth. n= 54.

**Table 3**

**Social Security Valuation Elements**

	<b>Annuity Factor</b>	<b>Early Retirement Reduction Factor</b>	<b>Discount for Waiting from 60 to 62 to Start Collecting Benefits</b>	<b>2006 Average Wage</b>
<b>APT</b>	15.43	0.7	0.961	\$ 36,953
<b>SSA</b>	14.18	0.7	0.944	\$ 36,953

<b>Table 4</b>						
<b>Coefficient Estimates from Random Effects Model of Relative Earnings</b>						
<b>Regressor</b>	<b>Males</b>			<b>Females</b>		
	<b>Less than High School</b>	<b>High School</b>	<b>College or Greater</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or Greater</b>
<b>Intercept</b>	-4.790	-3.903	-8.982	-4.991	-8.380	-7.538
	(0.334)	(0.479)	(0.229)	(0.268)	(0.186)	(0.246)
<b>Age</b>	0.270	0.069	0.635	0.213	0.535	0.430
	(0.026)	(0.037)	(0.02)	(0.022)	(0.016)	(0.021)
<b>age*year of birth</b>	0.002	0.003	0.005	0.008	0.002	0.005
	(6.5e-04)	(7.4e-04)	(5.3e-04)	(6.1e-04)	(4.2e-04)	(5.1e-04)
<b>age2</b>	-0.005	8.946e-04	-0.014	-0.003	-0.010	-0.008
	(6.7e-04)	(9.4e-04)	(5.5e-04)	(6.0e-04)	(4.3e-04)	(5.8e-04)
<b>age2*year of birth</b>	-2.626e-05	-1.176e-05	-1.525e-04	-1.405e-04	-3.303e-05	-9.978e-05
	(1.2e-05)	(1.3e-05)	(1.1e-05)	(1.1e-05)	(8.0e-06)	(9.3e-06)
<b>age3</b>	2.572e-05	-2.463e-05	9.857e-05	1.031e-05	5.515e-05	5.246e-05
	(5.5e-06)	(7.6e-06)	(5.0e-06)	(5.3e-06)	(3.8e-06)	(5.2e-06)
<b>age3*year of birth</b>	1.740e-07	-2.560e-07	1.461e-06	7.314e-07	2.164e-07	7.167e-07
	(8.3e-08)	(9.4e-08)	(7.8e-08)	(7.3e-08)	(5.3e-08)	(6.1e-08)
<b>Year of birth</b>	-0.071	-0.059	-0.061	-0.113	-0.045	-0.059
	(0.012)	(0.014)	(0.008)	(0.01)	(0.007)	(0.008)
<b># of Observations</b>	11972	22514	29230	8529	23539	25570
<b># of Individuals</b>	2513	4133	4282	2055	4356	4465
<b>Std. Dev. of Random Effect</b>	0.831	0.738	0.664	1.0363	0.827	0.797
<b>Std. Dev. of Residual</b>	0.841	0.788	0.786	0.966	0.972	0.969

Dependent variable is  $\log(Z)$ , where Z is ratio of individual taxable earnings to AWI for given year. Standard errors are reported in parentheses.

<b>Table 5 Social Security's Benefit Obligations to 26 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$52,884.39	\$57,823.68	\$87,621.58
	<b>SSA</b>	\$43,811.28	\$47,903.16	\$72,588.78
<b>Females</b>	<b>APT</b>	\$62,439.88	\$91,625.04	\$100,734.87
	<b>SSA</b>	\$51,727.38	\$75,905.38	\$83,452.29

<b>Table 6 Social Security's Tax Claims on 26 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$38,692.79	\$45,867.30	\$84,817.32
	<b>SSA</b>	\$37,183.86	\$44,364.69	\$81,613.33
<b>Females</b>	<b>APT</b>	\$47,132.75	\$82,988.78	\$95,301.44
	<b>SSA</b>	\$45,222.06	\$79,694.66	\$91,402.69

<b>Table 7 Social Security's Net Liability to 26 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$14,191.61	\$11,956.39	\$2,804.26
	<b>SSA</b>	\$6,627.42	\$3,538.48	-\$9,024.55
<b>Females</b>	<b>APT</b>	\$15,307.13	\$8,636.25	\$5,433.43
	<b>SSA</b>	\$6,505.32	-\$3,789.28	-\$7,950.40

<b>Table 8 Social Security's Benefit Obligations to 40 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$78,541.11	\$87,817.08	\$119,659.09
	<b>SSA</b>	\$67,250.16	\$75,192.63	\$102,457.08
<b>Females</b>	<b>APT</b>	\$73,688.46	\$102,193.70	\$117,059.56
	<b>SSA</b>	\$63,095.12	\$87,502.49	\$100,231.26

<b>Table 9 Social Security's Tax Claims on 40 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$32,986.24	\$33,524.46	\$63,463.14
	<b>SSA</b>	\$32,239.18	\$32,791.23	\$62,056.14
<b>Females</b>	<b>APT</b>	\$31,548.85	\$48,940.34	\$61,163.53
	<b>SSA</b>	\$30,824.90	\$47,794.78	\$59,663.88

<b>Table 10 Social Security's Net Liability to 40 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$45,554.87	\$54,292.62	\$56,195.95
	<b>SSA</b>	\$35,010.98	\$42,401.40	\$40,400.94
<b>Females</b>	<b>APT</b>	\$42,139.60	\$53,253.36	\$55,896.04
	<b>SSA</b>	\$32,270.22	\$39,707.71	\$40,567.39

<b>Table 11 Social Security's Benefit Obligations to 55 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$133,480.47	\$151,434.14	\$186,439.44
	<b>SSA</b>	\$118,406.64	\$134,332.82	\$165,385.00
<b>Females</b>	<b>APT</b>	\$96,713.49	\$124,821.42	\$150,533.59
	<b>SSA</b>	\$85,791.72	\$110,725.45	\$133,533.97

<b>Table 12 Social Security's Tax Claims on 55 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$9,845.27	\$11,380.19	\$17,337.79
	<b>SSA</b>	\$9,765.11	\$11,285.27	\$17,197.26
<b>Females</b>	<b>APT</b>	\$7,211.85	\$11,028.50	\$16,192.97
	<b>SSA</b>	\$7,153.73	\$10,936.99	\$16,055.42

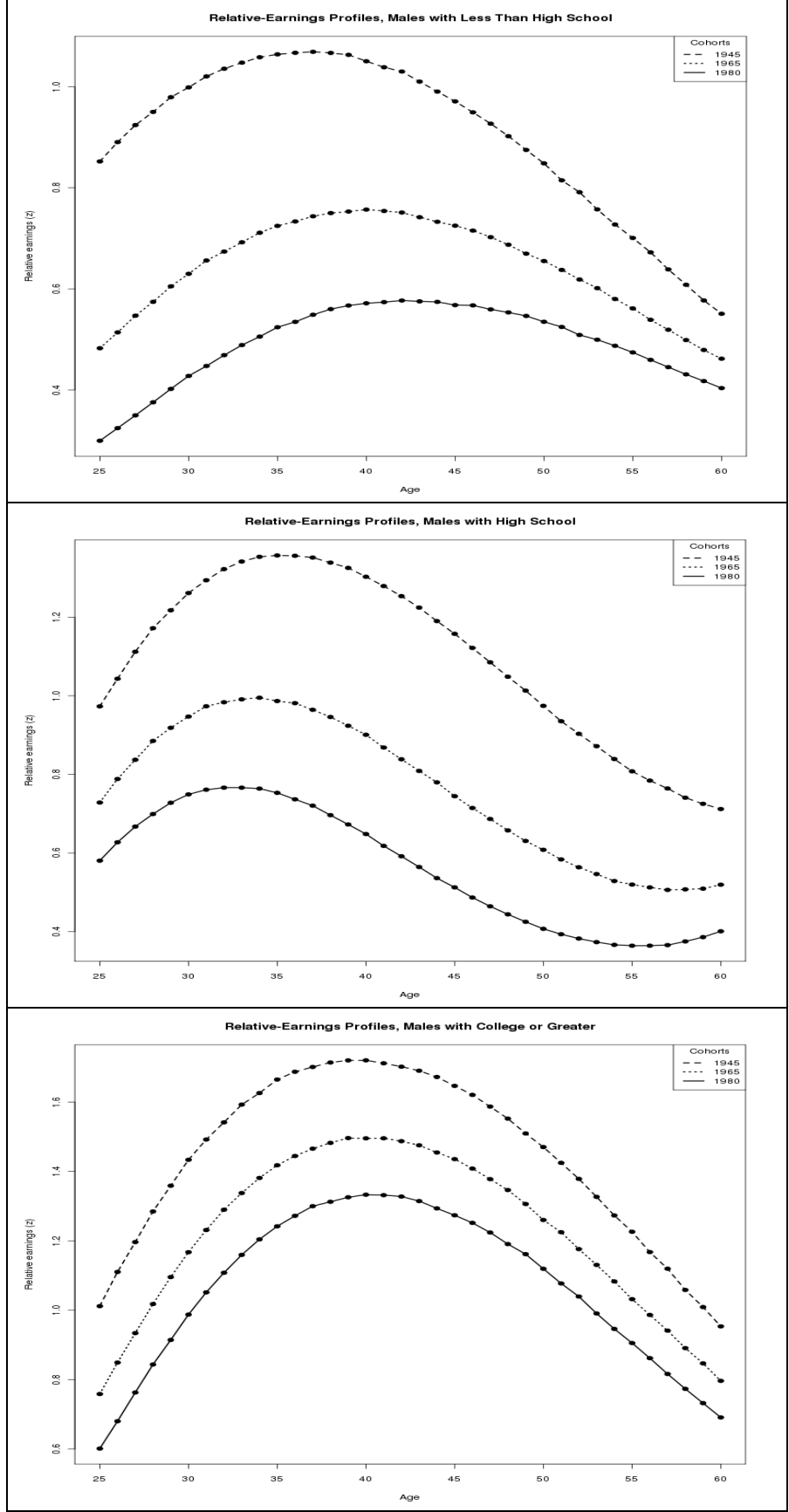
<b>Table 13 Social Security's Net Liability to 55 Year-Olds</b>				
<b>Gender</b>	<b>Valuation</b>	<b>Less than High School</b>	<b>High School</b>	<b>College or More</b>
<b>Males</b>	<b>APT</b>	\$123,635.20	\$140,053.95	\$169,101.65
	<b>SSA</b>	\$108,641.53	\$123,047.55	\$148,187.74
<b>Females</b>	<b>APT</b>	\$89,501.64	\$113,792.92	\$134,340.62
	<b>SSA</b>	\$78,637.99	\$99,788.46	\$117,478.55



**Table 14****Social Security's Aggregate Net Liability to Working-Age Americans**

	<b>APT</b>	<b>SSA</b>
<b>Aggregate Benefits Owed by Social Security</b>	\$ 15.765 trillion	\$ 13.676 trillion
<b>Aggregate Tax Obligations Owed to Social Security</b>	\$ 5.333 trillion	\$ 5.199 trillion
<b>Social Security's Aggregate Net Liability</b>	\$ 10.433 trillion	\$ 8.477 trillion

**Figure 1 Relative-Earnings Profiles of Males**



**Figure 2 Relative-Earnings Profiles of Females**

