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SUFFICIENT STATISTICS FOR WELFARE ANALYSIS:  
A BRIDGE BETWEEN STRUCTURAL AND REDUCED-FORM METHODS

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Sufficient Statistics for Welfare Analysis: A Bridge Between Structural and Reduced-Form  
Methods

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**ABSTRACT**

The debate between "structural" and "reduced-form" approaches has generated substantial controversy in applied economics. This article reviews a recent literature in public economics that combines the advantages of reduced-form strategies -- transparent and credible identification -- with an important advantage of structural models -- the ability to make predictions about counterfactual outcomes and welfare. This recent work has developed formulas for the welfare consequences of various policies that are functions of high-level elasticities rather than deep primitives. These formulas provide theoretical guidance for the measurement of treatment effects using program evaluation methods. I present a general framework that shows how many policy questions can be answered by identifying a small set of sufficient statistics. I use this framework to synthesize the modern literature on taxation, social insurance, and behavioral welfare economics. Finally, I discuss topics in labor economics, industrial organization, and macroeconomics that can be tackled using the sufficient statistic approach.

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There are two competing paradigms for policy evaluation and welfare analysis in economics: the “structural” approach and “reduced-form” approach (also known as the “program evaluation” or “treatment effect” approach). The division between structural and reduced-form approaches has split the economics profession into two camps whose research programs have evolved almost independently despite focusing on similar questions. The structural approach specifies complete models of economic behavior and estimates the primitives of such models. Armed with the fully estimated model, these studies then simulate the effects of counterfactual changes in policies and the economic environment on behavior and welfare. This powerful methodology has been applied to an array of topics, ranging from the optimal design of tax and transfer policies in public finance to the sources of inequality in labor economics and optimal antitrust policy in industrial organization.

Critics of the structural approach argue that it is difficult to identify all primitive parameters in an empirically compelling manner because of selection effects, simultaneity bias, and omitted variables. These researchers instead advocate “reduced-form” strategies that estimate statistical relationships, paying particular attention to identification concerns using research designs that exploit quasi-experimental exogenous variation. Reduced-form studies have identified a variety of important empirical regularities, especially in labor economics, public economics, and development. Advocates of the structural paradigm criticize the reduced-form approach for estimating statistics that are not policy invariant parameters of economic models, and therefore have limited relevance for welfare analysis (Rosenzweig and Wolpin 2000, Heckman and Vytlacil 2005).<sup>1</sup>

This paper argues that a set of papers in public economics written over the past decade (see Table 1) provide a middle ground between the two methods. These papers develop “sufficient statistic” formulas that combine the advantages of reduced-form empirics – transparent and credible identification – with an important advantage of structural models – the ability to make precise statements about welfare. The central concept of the sufficient statistic approach (illustrated in Figure 1) is to derive formulas for the welfare consequences of policies that are functions of high-level elasticities estimated in the program evaluation literature

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<sup>1</sup>See Section 1 of Rosenzweig and Wolpin (2000) and Table V of Heckman and Vytlacil (2005) for a more detailed comparison of the structural and treatment effect approaches.

rather than deep primitives. Even though there are multiple combinations of primitives that are consistent with the inputs to the formulas, all such combinations have the same welfare implications.<sup>2</sup> For example, Feldstein (1999) shows that the marginal welfare gain from raising the income tax rate can be expressed purely as a function of the elasticity of taxable income even though taxable income may be a complex function of choices such as hours, training, and effort. Saez (2001) shows that labor supply elasticity estimates can be used to make inferences about the optimal progressive income tax schedule in the Mirrlees (1971) model. Chetty (2008a) shows that the welfare gains from social insurance can be expressed purely in terms of the liquidity and moral hazard effects of the program in a broad class of dynamic, stochastic models. The goal of this survey is to elucidate the concepts of this new sufficient statistic methodology by codifying the steps needed to implement it, and thereby encourage its use as a bridge between structural and reduced-form methods in future work.

The idea that it is adequate to estimate sufficient statistics rather than primitive structure to answer certain questions is not new; it was well understood by Marschak (1954), Koopmans (1954), and other pioneers of structural estimation. Structural methods were preferred in early microeconomic work because the parameters of the simple models that were being studied could in principle be easily identified. There was relatively little value to searching for sufficient statistics in that class of models. In the 1980s, it became clear that identification of primitives was difficult once one introduced plausible dynamics, heterogeneity, and selection effects. Concerns about the identification of parameters in these richer models led a large group of empirical researchers to abandon structural methods in favor of more transparent program evaluation strategies (see e.g. Imbens and Wooldridge 2008 for a review of these methods). A large library of treatment effect estimates was developed in the 1980s and 1990s. The recent sufficient statistic literature essentially maps such treatment effect estimates into statements about welfare in modern structural models that incorporate realistic features such as dynamics and heterogeneity.

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<sup>2</sup>The term “sufficient statistic” is borrowed from the statistics literature: conditional on the statistics that appear in the formula, other statistics that can be calculated from the same sample provide no additional information about the welfare consequences of the policy.

The structural and sufficient statistic approaches to welfare analysis should be viewed as complements rather than substitutes because each approach has certain advantages. The sufficient statistic method has three benefits. First, it is simpler to implement empirically because less data and variation are needed to identify marginal treatment effects than to fully identify a structural model. This is especially relevant in models that allow heterogeneity and discrete choice, where the set of primitives is very large but the set of marginal treatment effects needed for welfare evaluation remains fairly small. By estimating the relevant marginal treatment effects as a function of the policy instrument, one can integrate the formula for the marginal welfare gain between any two observed values to evaluate policy changes. Second, identification of structural models often requires strong assumptions – such as no borrowing or no private insurance – given available data and variation. Since it is unnecessary to identify all primitives, sufficient statistic approaches typically do not require such stark assumptions and therefore are less model dependent. Third, the sufficient statistic approach can be applied even when one is uncertain about the positive model that generates observed behavior – as in recent studies in the behavioral economics literature which document deviations from perfect rationality. In such cases, welfare analysis based on a structural model may be impossible, whereas the more agnostic sufficient statistic approach permits some progress. For instance, Chetty, Looney, and Kroft (2008) derive formulas for the deadweight cost of taxation in terms of price and tax elasticities in a model where agents make arbitrary optimization errors with respect to taxes.

The parsimony of the sufficient statistic approach naturally comes with costs.<sup>3</sup> The first relates to out-of-sample predictions. Structural methods can in principle be used to simulate the effect of any policy change, since the primitives are by definition policy invariant. Because the inputs to the formulas are generally endogenous to the policy, one must estimate marginal treatment effects as a function of the policy instrument and extrapolate to make out-of-sample predictions using the sufficient statistic approach. Such extrapolations may be less reliable than those from a structural model because they are guided by a statistical

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<sup>3</sup>A practical cost of the sufficient statistic approach is the analytical work required to develop the formula. The costs of the structural approach are to some extent computational once identification problems are solved, making it a versatile tool in an age where computation is inexpensive.

model rather than an economic model.<sup>4</sup> A second and more important weakness of the sufficient statistic method is that it is a “black box.” Because one does not identify the primitives of the model, one cannot be sure whether the data are consistent with the model underlying the welfare analysis. Although sufficient statistic approaches do not require full specification of the model, they do require some modelling assumptions; it is impossible to make theory-free statements about welfare. For example, Chetty (2008b) points out that Feldstein’s (1999) influential formula for the excess burden of income taxation is based on a model that makes assumptions about the costs of evasion and avoidance that may not be fully consistent with the data. In contrast, because structural methods require full estimation of the model prior to welfare analysis, a rejection of the model by the data would be evident.

There are several ways to combine the structural and sufficient statistic methods to address the shortcomings of each strategy. For instance, a structural model can be evaluated by checking whether its predictions for local welfare changes match those obtained from a sufficient statistic formula. Conversely, when making out-of-sample predictions using a sufficient statistic formula, a structural model can be used to guide the choice of functional forms used to extrapolate the key elasticities. Structural estimates can also be used for “overidentification tests” of the general modelling framework. By combining the two methods in this manner, researchers can pick a point in the interior of the continuum between program evaluation and structural estimation, without being pinned to one endpoint or the other.

The paper is organized as follows. The next section discusses a precursor to the modern literature on sufficient statistics: Harberger’s (1964) “triangle” formula for the deadweight cost of taxation.<sup>5</sup> I show that Harberger’s formula can be easily extended to setting with heterogeneity and discrete choice – two of the hallmarks of modern structural models. Using

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<sup>4</sup>See Lumsdaine, Stock, and Wise (1992) and Keane and Wolpin (1997) for comparisons of reduced-form statistical extrapolations and model-based structural extrapolations. They find that structural predictions are more accurate, but statistical extrapolations that include the key variables suggested by the economic model come quite close.

<sup>5</sup>Another precursor is the asset price approach to incidence (e.g. Summers 1981, Roback 1982), which shows that changes in asset values are sufficient statistics for the distributional incidence of government policies and changes in other exogenous variables in dynamic equilibrium models. I focus on the Harberger result here because it is more closely related to the applications in the recent literature, which concentrate on efficiency and aggregate welfare rather than incidence.

these results as motivating examples, in Section II, I develop a general framework which provides a “recipe” for deriving sufficient statistic formulas. This framework explains why a small set of elasticities are sufficient for welfare analysis in many problems. Sections III to V present three applications of the sufficient statistic method: income taxation, social insurance, and behavior (non-rational) models. These three sections provide a synthesis of the modern public finance literature, showing how a dozen seemingly unrelated papers are essentially variants on the theme of finding sufficient statistics. The paper concludes in section VI with a discussion of potential applications of the sufficient statistic approach to other types of counterfactual analysis beyond changes in policy.

## I A Precedent: Measuring Deadweight Loss

Harberger (1964) popularized the measurement of the excess burden of a commodity tax using a simple elasticity-based formula. This result can be viewed as a precedent to the modern literature on sufficient statistics, and provides a starting point from which to build intuition about the more sophisticated applications discussed below.

Consider an economy in which an individual is endowed with  $Z$  units of the numeraire ( $y$ ), whose price is normalized to 1. Firms convert the numeraire good  $y$  (which can be interpreted as labor) into  $J$  other consumption goods,  $x = (x_1, \dots, x_J)$ . Producing  $x_j$  units of good  $j$  requires an input of  $c_j(x_j)$  units of  $y$ , where  $c_j$  is a weakly convex function. Let  $c(x) = \sum_{j=1}^J c_j(x_j)$  denote the total cost of producing a vector  $x$ . Production is perfectly competitive. The government levies a unit tax  $t$  on good 1. Let  $p = (p_1, \dots, p_J)$  the vector of pre tax prices for the produced goods.

To simplify the exposition, ignore income effects by assuming that utility is quasilinear in  $y$ . The consumer takes the price vector as given and solves:

$$\begin{aligned} \max_{x,y} u(x_1, \dots, x_J) + y & & (1) \\ \text{s.t. } px + tx_1 + y = Z & \end{aligned}$$

The representative firm takes prices as given and solves

$$\max_x px - c(x) \tag{2}$$

These two problems define maps from the price vector  $p$  to demand and supply of the  $J$  goods,  $x^D(p)$  and  $x^S(p)$ . The model is closed by the market clearing condition  $x^D(p) = x^S(p)$ .

Suppose the policy maker wants to measure the efficiency cost of the tax  $t$ . The efficiency (or “deadweight”) cost of a tax increase equals the loss in surplus from the transactions that fail to occur because of the tax. To calculate the efficiency cost, the conceptual experiment is to measure the net loss in welfare from raising the tax rate and returning the tax revenue to the taxpayer through a lump-sum rebate. With quasi-linear utility, the consumer will always choose to allocate the lump-sum rebate to consumption of the numeraire good  $y$ . Social welfare can therefore be written as the sum of the consumer’s utility (which is a money metric given quasilinearity), producer profits, and tax revenue:

$$\begin{aligned} W(t) &= \left\{ \max_x u(x) + Z - tx_1 - px \right\} + \left\{ \max_x px - c(x) \right\} + tx_1 \\ &= \left\{ \max_x u(x) + Z - tx_1 - c(x) \right\} + tx_1 \end{aligned} \tag{3}$$

where the second equation effectively recasts the decentralized equilibrium as a planner’s allocation problem. In this expression, the term in curly brackets measures private surplus, while the  $tx_1$  term measures tax revenue. The individual treats tax revenue as fixed when choosing  $x$ , failing to internalize the effects of his behavior on the lump-sum transfer he ultimately receives. This assumption, which is standard in efficiency cost calculations, captures the intuition that in an economy populated by a large number of individuals, any one individual has a negligible impact on the government revenue and therefore treats it as fixed.

There are two approaches to estimating the effect of an increase in the tax on social welfare ( $\frac{dW}{dt}$ ). The first is to estimate a  $J$  good demand and supply system to recover the utility function  $u(x)$  and cost function  $c(x)$ . Once  $u$  and  $c$  are known, one can directly compute  $W(t)$ . Preferences can be recovered using the parametric demand systems proposed, for



instance, by Stone (1954) or Deaton and Muellbauer (1980). Alternatively, one can fit a supply and demand system to the data and then integrate to obtain the expenditure function, as in Hausman (1981) or Hausman and Newey (1994). The econometric challenge in implementing any of these structural methods is simultaneity: identification of the slope of the supply and demand curves requires  $2J$  instruments.

Harberger (1964) suggested another solution.<sup>6</sup> Recognizing that the behavioral responses ( $\frac{dx}{dt}$ ) in the curly brackets of (3) can be ignored when calculating  $\frac{dW}{dt}$  because of the envelope conditions from maximization of utility and profits. Therefore, differentiating (3) yields

$$\frac{dW(t)}{dt} = -x_1 + x_1 + t \frac{dx_1}{dt} = t \frac{dx_1(t)}{dt} \quad (4)$$

This formula shows that  $\frac{dx_1(t)}{dt}$  is a “sufficient statistic” for welfare analysis. By estimating  $\frac{dx_1}{dt}(t)$  for different values of  $t$ , one can calculate the welfare consequences of any policy change that lies within the observed support of  $t$  by integrating (4):  $\Delta W = W(t_2) - W(t_1) = \int_{t_1}^{t_2} t \frac{dx_1}{dt}(t) dt$ . The full system of supply and demand curves does not have to be identified to compute the welfare change  $\Delta W$ .

The reason  $\frac{dW}{dt}$  depends only on  $\frac{dx_1}{dt}$  is that the government is optimizing a function that has already been optimized by individuals and firms (subject to constraints imposed by the government). Although the tax induces changes in behavior and equilibrium prices, these behavioral responses cannot have a first-order effect on private welfare; if they did, consumers or firms would not be optimizing. The loss in social surplus from the tax is therefore determined purely by the difference between the agent’s willingness to pay for good  $x_1$  and the cost of producing good  $x_1$ . The difference can be measured by the area between the supply and demand curves and the initial and post-tax quantities, which is proportional to  $\frac{dx_1}{dt}$ .

The tradeoffs between the sufficient statistic and structural approaches are apparent in the debate that followed Harberger’s work. One limitation of (4) is that it requires that there are no pre-existing distortions in the other markets; otherwise the spillover effects would have first-order effects on welfare. This limitation can be addressed by an extension of the

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<sup>6</sup>Hines (1999) colorfully recounts the intellectual history of the deadweight loss triangle.

formula that includes cross-price elasticities, as shown in Harberger’s original analysis. The more complex formula can be implemented by making plausible approximations about the structure of the distortions that allow the formula to be written purely in terms of own-price elasticities (Goulder and Williams 2003).<sup>7</sup>

A second limitation of (4) is that it cannot be used to evaluate counterfactual policy changes such as the imposition of a large new tax on good  $x_1$ . This limitation can be addressed by estimating  $\frac{dx_1}{dt}(t)$  for various values of  $t$  and integrating (4) between any two tax rates of interest, making functional-form assumptions to extrapolate out-of-sample if necessary. In practice, the Harberger formula is typically implemented under a linear or log-linear approximation to demand (e.g.  $\frac{dx_1}{dt}$  constant) because data limitations preclude estimation of higher-order properties of the demand curve. Structural simulations indicate that linear approximations are fairly accurate, presumably because the demand functions implied by standard models are not very curved (Shoven 1976, Ballard, Shoven, and Whalley 1985). Thus, despite its limitations, the simple Harberger “triangle” formula has become central to applied welfare analysis and has inspired a vast literature estimating tax elasticities.

The benefits of Harberger’s approach are especially evident in modern structural models that permit heterogeneity across individuals and discrete choice. I now extend Harberger’s analysis to incorporate these features.

*Extension 1: Heterogeneity.* Now suppose the economy has  $N$  individuals with heterogeneous preferences. Let  $x^i$  denote individual  $i$ ’s vector of demands and  $x = \sum_{i=1}^N x^i$  denote aggregate demand. Individual  $i$  is endowed with  $Z^i$  units of the numeraire and has utility

$$u^i(x^i) + y \tag{5}$$

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<sup>7</sup>The practical concern is that one may inadvertently ignore some pre-existing distortions and apply an inaccurate version of the Harberger formula. Indeed, Goulder and Williams argue that previous applications of the simple formula in (4) to assess the deadweight costs of commodity taxation are biased by an order-of-magnitude because they fail to account for interactions with the labor income tax. This mistake would not have been made in a properly specified structural model.

Under a utilitarian criterion, social welfare is given by:

$$W(t) = \left\{ \sum_{i=1}^N \max_{x^i} [u^i(x^i) + Z^i - tx_1^i] - c(x) \right\} + t \sum_{i=1}^N x_1^i \quad (6)$$

The structural approach requires identification of the demand functions and utilities for all  $i$  agents. The sufficient statistic approach simplifies the identification problem substantially here. Because there is an envelope condition for  $x^i$  for every agent, we can ignore all behavioral responses within the curly brackets when differentiating (6) to obtain

$$\frac{dW(t)}{dt} = - \sum_{i=1}^N x_1^i + \sum_{i=1}^N x_1^i + t \frac{d \sum_{i=1}^N x_1^i}{dt} = t \frac{dx_1(t)}{dt} \quad (7)$$

The slope of the *aggregate* demand curve ( $\frac{dx_1}{dt}$ ) is a sufficient statistic for the marginal excess burden of a tax; there is no need to characterize the underlying heterogeneity in the population to implement (7). Intuitively, even though each individual has a different demand elasticity, what matters for government revenue and aggregate welfare is the *total* change in behavior induced by the tax.<sup>8</sup>

An important caveat is that with heterogeneity,  $\frac{dx_1}{dt}$  may vary considerably with  $t$ , since the individuals at the margin will differ with the tax rate. Hence, it is especially important to distinguish average and marginal treatment effects for welfare analysis by estimating  $\frac{dx_1}{dt}(t)$  as a function of  $t$  in this case.

*Extension 2: Discrete Choice.* Now suppose individuals can only choose one of the  $J$  products  $\{1, \dots, J\}$ . These products might represent models of cars, modes of transportation, or neighborhoods. Each product is characterized by a vector of  $K$  attributes  $x_j = (x_{1j}, \dots, x_{Kj})$  observed by the econometrician and an unobservable attribute  $\zeta_j$ . If agent  $i$  chooses product

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<sup>8</sup>Of course, to analyze a policy that has heterogeneous impacts across groups, such as a progressive income tax, one needs group-specific elasticity estimates to calculate  $\frac{dW}{dt}$ . The key point, however, is that the only heterogeneity that matters is at the level of the policy impact; any additional heterogeneity within groups can be ignored. For instance, heterogeneous labor supply responses *within* an income group need not be characterized when analyzing optimal progressive income taxation.

$j$ , his utility is

$$\begin{aligned} u_{ij} &= v_{ij} + \varepsilon_{ij} \\ \text{with } v_{ij} &= Z^i - p_j + \zeta_j + \phi^i(x_j) \end{aligned}$$

where  $\varepsilon_{ij}$  is a random unobserved taste shock. Let  $P_{ij}$  denote the probability that individual  $i$  chooses option  $j$ ,  $P_j = \sum_i P_{ij}$  denote total (expected) demand for product  $j$ , and  $P = (P_1, \dots, P_J)$  the vector of aggregate product demands. Product  $j$  is produced by competitive firms using  $c_j(P_j)$  units of the numeraire good  $y$ . Let  $c(P) = \sum_j c_j(P_j)$ . This model differs from that above in two respects: (1) utility over the consumption goods is replaced by utility over the product attributes  $\phi^i(x_j) + \zeta_j + \varepsilon_{ij}$  and (2) the attributes can only be consumed in discrete bundles.

Assume that  $\varepsilon_{ij}$  has a type 1 extreme value distribution. Then it is well known from the multinomial logit literature (see e.g. Train 2003) that the probability that a utility-maximizing individual  $i$  chooses product  $j$  is

$$P_{ij} = \frac{\exp(v_{ij})}{\sum_j \exp(v_{ij})} \quad (8)$$

and that agent  $i$ 's expected utility from a vector of prices  $p = (p_1, \dots, p_J)$  is

$$S_i(p_1, \dots, p_J) = E \max(u_{i1}, \dots, u_{iJ}) = \log\left(\sum_j \exp v_{ij}\right).$$

Aggregating over the  $i = 1, \dots, N$  consumers, (expected) consumer surplus is

$$S = \sum_i \log\left(\sum_j \exp(v_{ij})\right)$$

Since utility is quasilinear, we can add producer profits to this expression to obtain social welfare:

$$W = \sum_i \log\left(\sum_j \exp(v_{ij})\right) + pP - c(P) \quad (9)$$

The classical approach to policy analysis in these models is to estimate the primitives  $\phi_i$  and

$\zeta_j$ , and simulate total surplus before and after a policy change (see e.g., Train 2003, p60). Identification of such models can be challenging, especially if the econometrician does not observe all product attributes, since  $\zeta_j$  will be correlated with  $p_j$  in equilibrium (Berry 1994; Berry, Levinsohn, and Pakes 1995).<sup>9</sup>

Sufficient statistic approaches offer a means of policy analysis that does not require identification of  $\phi_i$  and  $\zeta_j$ . For example, suppose the government levies a tax  $t$  on good 1, raising its price to  $p_1 + t$ . The government returns the proceeds to agents through a lump-sum transfer  $T$  so that  $y_i$  becomes  $y_i + T$ . As above, agents do not internalize the effects of their behavior on the size of the transfer  $T$ . Using the envelope condition for profit maximization,

$$\begin{aligned} \frac{dW(t)}{dt} &= \sum_i \left[ -\frac{\exp(v_{i1})}{\sum_j \exp(v_{ij})} - \sum_j \frac{dp_j}{dt} \frac{\exp(v_{ij})}{\sum_j \exp(v_{ij})} \right] + \sum_j \frac{dp_j}{dt} P_j + P_1 + t \frac{dP_1}{dt} \quad (10) \\ &= t \frac{dP_1(t)}{dt} \end{aligned}$$

where the second equality follows from (8). Identification of the welfare loss from taxation of good 1 requires estimation of only the effect of the tax on the aggregate market share ( $\frac{dP_1}{dt}$ ), as in the standard Harberger formula.

Now suppose that an ad-valorem tax  $\tau$  is levied on all the products except the numeraire good, raising the price of product  $j$  to  $(1 + \tau)p_j$ . Again, tax revenue is returned to agents through a lump sum grant. Following a similar derivation,

$$\frac{dW(\tau)}{d\tau} = \tau \sum_j p_j \frac{dP_j(\tau)}{d\tau} = \tau \frac{dE_P(\tau)}{d\tau}$$

where  $E_p = \sum_j p_j P_j$  denotes total pre tax expenditure in the market for the taxed good. The efficiency cost of a tax on all products depends on the aggregate expenditure elasticity for the taxed market; it does not require estimation of the substitution patterns within that market.

Intuitively, even though the microeconomic demand functions are not continuous in dis-

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<sup>9</sup>This model nests mixed logit specifications that permit preference heterogeneity; for instance, one could allow  $\phi_i(x_j) = (\beta + \nu_i)x_j$  where  $\nu_i$  is a random effect.

crete choice models, the social welfare function is smooth because the distribution of valuations for the goods is smooth. Since a small tax change induces a behavioral response only among those who are indifferent between products, behavioral responses have a second-order effect on social welfare. As a result, one obtains a formula for excess burden that requires estimation of only one reduced-form elasticity.<sup>10</sup>

The modern sufficient statistic literature builds on Harberger’s idea of only identifying the aspects of the model relevant for the question at hand. Before describing specific applications of this approach, I present a general framework that nests the papers in this literature and provides a “recipe” for developing such formulas.

## II General Framework

Abstractly, many government policies amount to levying a tax  $t$  to finance a transfer  $T(t)$ . In the context of redistributive taxation, the transfer is to another agent in the economy; in the context of social insurance, it is to another state of the economy; and in the context of excess burden calculations above, the transfer can be thought of as being used to finance a public good. I now present a six step rubric for calculating the welfare gain from raising the tax rate  $t$  (and the accompanying transfer  $T(t)$ ) using sufficient statistics.

To simplify exposition, the rubric is formally presented in a static model with a single agent. The same sequence of steps can be applied to obtain formulas for multi-agent problems with heterogeneous preferences and discrete choice if  $U(\cdot)$  is viewed as a (smooth) social welfare function aggregating the utilities of all the agents, as in (6) and (9). Similarly, dynamics can be incorporated by integrating the utility function over multiple periods.

**Step 1: Specify the general structure of the model.** Let  $x = (x_1, \dots, x_J)$  denote the vector of choices for the representative agent in the private sector. A unit tax  $t$  is levied on choice  $x_1$  and the transfer  $T(t)$  is paid in units of  $x_J$ . Let  $\{G_1(x, t, T), \dots, G_M(x, t, T)\}$  denote the  $M < J$  constraints faced by the agent, which include budget constraints, restrictions on insurance or borrowing, hours constraints, etc. The agent takes  $t$  and  $T$  as given and makes

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<sup>10</sup>This result does not rely on the assumption that the  $\varepsilon_{ij}$  errors have an extreme value distribution. The distributional assumption simplifies the algebra by yielding a closed-form solution for total surplus, but the envelope conditions used to derive (10) hold with any distribution.

his choices by solving:

$$\max U(x) \text{ s.t. } G_1(x, t, T) = 0, \dots, G_M(x, t, T) = 0 \quad (11)$$

The solution to (11) gives social welfare as a function of the policy instrument:

$$W(t) = \max_x U(x) + \sum_{m=1}^M \lambda_m G_m(x, t, T)$$

This specification nests competitive production because any equilibrium allocation can be viewed as the choice of a benevolent planner seeking to maximize total private surplus subject to technological constraints. For example, in the single agent Harberger model analyzed above,

$$\begin{aligned} U(x) &= u(x_1, \dots, x_{J-1}) + x_J \\ G_1(x, t, T) &= T + Z - t_1 x_1 - c(x_1, \dots, x_{J-1}) - x_J. \end{aligned} \quad (12)$$

The researcher has considerable choice in specifying the general model used to derive the sufficient statistic formula, and must tailor the model to the application of interest given the parameters he can identify empirically. A more general specification of preferences and constraints will yield a formula that is more robust but harder to implement empirically.

**Step 2: Express  $\frac{dW}{dt}$  in terms of multipliers.** Using the envelope conditions associated with optimization in the private sector, differentiate  $W$  to obtain

$$\frac{dW}{dt} = \sum_{m=1}^M \lambda_m \left\{ \frac{\partial G_m}{\partial T} \frac{dT}{dt} + \frac{\partial G_m}{\partial t} \right\} \quad (13)$$

where  $\lambda_m$  denotes the Lagrange multiplier associated with constraint  $m$  in the agent's problem in (11). In this equation,  $\frac{dT}{dt}$  is known through the government's budget constraint, and  $\frac{\partial G_m}{\partial T}$  and  $\frac{\partial G_m}{\partial t}$  can be calculated mechanically. For example, in the Harberger model,  $T(t) = tx_1$  and hence  $\frac{dT}{dt} = x_1 + t \frac{dx_1}{dt}$ . Differentiating (12) yields  $\frac{dG_1}{dT} = 1$  and  $\frac{dG_1}{dt} = -x_1$ . It follows that  $\frac{dW}{dt} = \lambda_1 t \frac{dx_1}{dt}$ .

The critical unknowns are the  $\lambda_m$  multipliers. In the excess burden application,  $\lambda_1$

measures the marginal value of relaxing the budget constraint. In a social insurance application,  $\lambda_1$  could represent the marginal value of relaxing the constraint that limits the extent to which agents can transfer consumption across states. If  $\lambda_1$  is small, there is little value to social insurance, whereas if it is large,  $\frac{dW}{dt}$  could be large.

**Step 3: Substitute multipliers by marginal utilities.** The  $\lambda_m$  multipliers are recovered by exploiting restrictions from the agent's first-order-conditions. Optimization leads agents to equate marginal utilities with linear combinations of the multipliers:

$$u'(x_j) = - \sum_{m=1}^M \lambda_m \frac{\partial G_m}{\partial x_j}$$

Inverting this system of equations generates a map from the multipliers into the marginal utilities. To simplify this mapping, it is helpful to impose the following assumption on the structure of the constraints.

**Assumption 1.** The tax  $t$  enters all the constraints in the same way as the good on which it is levied ( $x_1$ ) and the transfer  $T$  enters all the constraints in the same way as the good in which it is paid ( $x_J$ ). Formally, there exist functions  $k_t(x, t, T)$ ,  $k_T(x, t, T)$  such that

$$\begin{aligned} \frac{\partial G_m}{\partial t} &= k_t(x, t, T) \frac{\partial G_m}{\partial x_1} \forall m = 1, \dots, M \\ \frac{\partial G_m}{\partial T} &= -k_T(x, t, T) \frac{\partial G_m}{\partial x_J} \forall m = 1, \dots, M \end{aligned}$$

Assumption 1 requires that  $x_1$  and  $t$  enter every constraint interchangeably (up to a scale factor  $k_t$ ).<sup>11</sup> That is, increasing  $t$  by \$1 and reducing  $x_1$  by  $\$k_t$  would leave all constraints unaffected. A similar interchangeability condition is required for  $x_J$  and  $T$ . In models with only one constraint per agent, Assumption 1 is satisfied by definition. In the Harberger model, where the only constraint is the budget constraint,  $k_t$  corresponds to the mechanical increase in expenditure caused by a \$1 increase in  $t$  ( $\$x_1$ ) vs. a \$1 increase in  $x_1$  ( $\$p_1 + t$ ). Hence,  $k_t = \frac{x_1}{p_1 + t}$  in that model. Since increasing the transfer by \$1 affects the budget constraint in the same way as reducing consumption of  $x_J$  by \$1,  $k_T = 1$ .

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<sup>11</sup>If the tax  $t$  is levied on multiple goods ( $x_1, \dots, x_t$ ) as in Feldstein (1999), the requirement is that it enters the constraints in the same way as the combination of all the taxed goods, i.e.  $\frac{\partial G_i}{\partial t} = \sum_{i=1}^t k_t(x, t, T) \frac{\partial G_j}{\partial x_i}$ .



Models where the private sector choices are second-best efficient subject to the resource constraints in the economy typically satisfy the conditions in Assumption 1. This is because fungibility of resources ensures that the taxed good and tax rate enter all constraints in the same way (see Chetty (2006) for details). The sufficient statistic approach can be implemented in models that violate Assumption 1 (see section IV for an example), but the algebra is much simpler when this assumption holds. This is because the conditions in Assumption 1 permit direct substitution into (13) to obtain:

$$\begin{aligned}
\frac{dW}{dt} &= \sum_{m=1}^M \lambda_m \left\{ -k_T \frac{\partial G_m}{\partial x_J} \frac{dT}{dt} + k_t \frac{\partial G_m}{\partial x_1} \right\} \\
&= -k_T \frac{dT}{dt} \sum_{m=1}^M \lambda_m \frac{\partial G_m}{\partial x_J} + k_t \sum_{m=1}^M \lambda_m \frac{\partial G_m}{\partial x_1} \\
\frac{dW}{dt} &= k_T \frac{dT}{dt} u'(x_J(t)) - k_t u'(x_1(t)). \tag{14}
\end{aligned}$$

This expression captures a simple and general intuition: increasing the tax  $t$  is equivalent to reducing consumption of  $x_1$  by  $k_t$  units, which reduces the agent's utility by  $k_t u'(x_1(t))$ . The additional transfer that the agent gets from the tax increase is  $\frac{dT}{dt} k_T$  units of good  $x_J$ , which raises his utility by  $k_T \frac{dT}{dt} u'(x_J(t))$ . Since  $k_T$ ,  $k_t$ , and  $\frac{dT}{dt}$  are known based on the specification of the model, this expression distills local welfare analysis to recovering a pair of marginal utilities.<sup>12</sup>

In models with heterogeneity, the aggregate welfare gain is a function of a pair of average marginal utilities across agents. In dynamic models, the welfare gain is also a function of a pair of average marginal utilities, but with the mean taken over the lifecycle for a given agent. This result is obtained using envelope conditions when differentiating the value function.

**Step 4: Recover marginal utilities from observed choices.** The final step in obtaining an empirically implementable expression for  $\frac{dW}{dt}$  is to back out the two marginal utilities. There is no canned procedure for this step. Different formulas can be obtained by recovering the marginal utilities in different ways. The applications below provide several illustrations of this step. The trick that is typically exploited is that the marginal utilities

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<sup>12</sup>In many applications, steps 2 and 3 are consolidated into a single step because the constraints can be substituted directly into the objective function.

are elements in first-order conditions for various choices. As a result, they can be backed out from the comparative statics of behavior. For instance, in the single agent Harberger model above, the assumption of no income effects implies  $u'(x_j) = 1$ . To identify  $u'(x_1)$ , exploit the first-order condition for  $x_1$ , which is  $u'(x_1) = p_1 + t$ . Plugging in these expressions and the other parameters above into (14), we obtain (4):

$$\frac{dW(t)}{dt} = 1 \cdot (x_1 + t \frac{dx_1}{dt}) - \frac{x_1}{p_1 + t} \cdot (p_1 + t) = t \frac{dx_1(t)}{dt}.$$

**Step 5: Empirical Implementation.** Suppose the sufficient statistic formula one derives has the following form:

$$\frac{dW}{dt}(t) = f\left(\frac{dx_1}{dt}, \frac{dx_1}{dZ}, t\right). \quad (15)$$

The ideal way to implement (15) is to estimate the inputs as non-parametric functions of the policy instrument  $t$ . With estimates of  $\frac{dx_1}{dt}(t)$  and  $\frac{dx_1}{dZ}(t)$ , one can integrate (15) between any two tax rates  $t_1$  and  $t_2$  that lie within the support of observed policies to evaluate the welfare gain  $\Delta W$  for a policy change of interest. This procedure is similar in spirit to Heckman and Vytlacil's (2001, 2005) recommendation that researchers estimate a complete schedule of marginal treatment effects (MTE), and then integrate that distribution over the desired range to obtain policy relevant treatment effects. In the present case, the marginal welfare gain at  $t$  depends on the MTE at  $t$ ; analysis of non-marginal changes requires estimation of the MTE as a function of  $t$ .

In most applications, limitations in power make it difficult to estimate  $x_1(t)$  non-parametrically. Instead, typical reduced-form studies estimate the effect of a discrete change in the tax rate from  $t_1$  to  $t_2$  on demand:  $\frac{\Delta x_1}{\Delta t} = \frac{x_1(t_2) - x_1(t_1)}{t_2 - t_1}$ . The estimate of  $\frac{\Delta x_1}{\Delta t}$  permits inference about the mean change in welfare over the observed interval,  $\overline{dW/dt} = \frac{W(t_2) - W(t_1)}{t_2 - t_1}$ , or equivalently the effect of raising the tax rate from  $t_1$  to  $t_2$  on welfare. To see this, consider the Harberger model, where  $\frac{dW}{dt}(t) = t \frac{dx_1}{dt}(t)$ . A researcher who has estimated  $\frac{\Delta x_1}{\Delta t}$  has two options. This

first is to bound the average welfare gain over the observed range:

$$\begin{aligned}
 W(t_2) - W(t_1) &= \int_{t_1}^{t_2} \frac{dW}{dt} dt = \int_{t_1}^{t_2} t \frac{dx_1}{dt}(t) dt \\
 \Rightarrow t_1 \frac{\Delta x_1}{\Delta t} &> \overline{dW/dt} > t_2 \frac{\Delta x_1}{\Delta t}
 \end{aligned} \tag{16}$$

Intuitively, the excess burden of taxation depends on the slope of the demand curve between  $t_1$  and  $t_2$ , multiplied by the height of the “Harberger trapezoid” at each point. When one observes only the average slope between the two tax rates, bounds on excess burden can be obtained by setting the height to the lowest and highest points over the interval.

The second option is to use an approximation to the demand curve to calculate  $\overline{dW/dt}$ . For instance, if one can estimate only the first-order properties of demand precisely, making the approximation that  $\frac{dx_1}{dt}$  is constant over the observed range implies

$$\overline{dW/dt} \simeq \frac{t_1 + t_2}{2} \frac{\Delta x_1}{\Delta t}$$

If the demand curve is linear, the average height of the trapezoid and  $\frac{\Delta x_1}{\Delta t}$  exactly determine excess burden. If one has adequate data and variation to estimate higher-order terms of the demand curve, these estimates can be used to fit a higher order approximation to the demand curve to obtain a more accurate estimate of  $\overline{dW/dt}$ .

The same two options are available in models in which  $\frac{dW}{dt}$  is a function of more than one behavioral response, as in (15). Bounds may be obtained using the estimated treatment effects  $(\frac{\Delta x_1}{\Delta Z}, \frac{\Delta x_1}{\Delta t})$  by integrating  $\frac{dW}{dt}$  and setting the other parameters at their extrema as in (16). Under a linear approximation to demand ( $\frac{dx_1}{dt}, \frac{dx_1}{dZ}$  constant), treatment effects can be mapped directly into the marginal welfare gain:  $\frac{dW(t)}{dt} = f(\frac{\Delta x_1}{\Delta t}, \frac{\Delta x_1}{\Delta Z}, t)$ . If  $\frac{dW(t)}{dt}$  can only be estimated accurately at the current level of  $t$ , one can at least determine the direction in which the policy instrument should be shifted to improve welfare.

The bottom line is that the precision of a sufficient statistic formula is determined by the precision of the information available about the sufficient statistics as a function of the policy instrument. In all three applications discussed below, the data and variation available only permit estimation of first-order properties of the inputs, and the authors are therefore

constrained to calculating a first-order approximation of  $\overline{dW/dt}$ . The potential error in this linear approximation can be assessed using the bounds proposed above or a structural model.

**Step 6. Structural Evaluation and Extrapolation.** The final step is to evaluate the accuracy of the sufficient statistic formula as implemented in Step 5 using a structural model. Unfortunately, this step is frequently neglected in existing sufficient statistic studies. The structural evaluation begins by finding a vector of structural parameters  $\omega$  that is consistent with the sufficient statistics estimated in step 5. If the empirical estimates of the sufficient statistics are internally consistent with the model – which may not occur because the estimates are typically high-level elasticities (see Chetty (2006) for an example of inconsistency) – there must be at least one  $\omega$  that matches the estimated statistics. The validity of the model can be assessed by evaluating whether the set of  $\omega$ 's that matches the moments contains at least one plausible set of primitives, where plausibility is judged using information beyond the estimated sufficient statistics themselves.

The parameterized structural model is then used to run three types of simulations. First, one compares the exact welfare gain from the simulation to the welfare gain implied by a sufficient statistic formula as implemented using approximations. In many practical applications, one will likely find that the standard errors in the estimates of  $\frac{\Delta x_1}{\Delta t}$  dwarf the potential errors from ignoring the second-order properties of the inputs. A second simulation is to explore how the sufficient statistics vary with the policy instrument – e.g. evaluating the shape of  $\frac{dx_1}{dt}(t)$ . If certain behaviors are highly non-linear functions of  $t$ , the higher-order terms implied by either the structural model or empirical estimates can be included in the sufficient statistic formula. Finally, one can use structural simulations to guide the functional forms used to make extrapolations using the sufficient statistic formula outside the observed support of policies. Conversely, one can make out-of-sample predictions and solve for the globally optimal policy using the structural model, having the confidence that the model has been calibrated to match the moments relevant for local welfare analysis. Note that there will generally be more than one value of  $\omega$  will be consistent with the sufficient statistics. In such cases, the simulations should be repeated with multiple values of  $\omega$  to assess robustness, using additional data beyond the sufficient statistic estimates to narrow

the set of permissible  $\omega$  vectors.

The next three sections show how a variety of recent papers in public economics can be interpreted as applications of this framework. Each application illustrates different strengths and weaknesses of the sufficient statistic approach and demonstrates the techniques that are helpful in deriving such formulas.

### III Application 1: Income Taxation

Since the seminal work of Mirrlees (1971) and others, there has been a large structural literature investigating the optimal design of income tax and transfer systems. Several studies have simulated optimal tax rates in calibrated versions of the Mirrlees model (see Tuomala 1990 for a survey). A related literature uses microsimulation methods to calculate the effects of changes in transfer policies on behavior and welfare. The most recent structural work in this area has generalized the Mirrlees model to dynamic settings and simulated the optimal design of tax policies in such environments using calibrated models. Parallel to this literature, a large body of work in labor economics has investigated the effects of tax and transfer programs on behavior using program evaluation methods. See Table 1 for examples of structural and reduced-form studies.

Recent work in public economics has shown that the elasticities estimated by labor economists can be mapped into statements about optimal tax policy in the models that have been analyzed using structural methods. This sufficient statistic method has been widely applied in the context of income taxation in the past decade, with contributions by Feldstein (1995, 1999), Piketty (1997), Diamond (1998), Saez (2001), Gruber and Saez (2002), Goulder and Williams (2003), Chetty (2008b), and others. All of these papers can be embedded in the general framework proposed above. I focus on two papers here in the interest of space: Feldstein (1999) and Saez (2001).

*Feldstein (1999).* Traditional empirical work on labor supply did not incorporate the potential effects of taxes on choices other than hours of work. For instance, income taxes could affect an individual's choice of training, effort, or occupational choice. Moreover, individuals may be induced to shelter income from taxation by evading or avoiding tax

payments (e.g. taking fringe benefits, underreporting earnings). While some studies have attempted to directly examine the effects of taxes on each of these margins, it is difficult to account for all potential behavioral responses to taxation by measuring each channel separately.

Feldstein proposes an elegant solution to the problem of calculating the efficiency costs of taxation in a model with multi-dimensional labor supply choices. His insight is the elasticity of taxable income with respect to the tax rate is a sufficient statistic for calculating deadweight loss. Feldstein considers a model in which an individual makes  $J$  labor supply choices  $(x_1, \dots, x_J)$  that generate earnings. Let  $w_j$  denote the wage paid for choice  $j$  and  $\psi_j(x_j)$  denote the disutility of labor supply through margin  $x_j$ . In addition, suppose that the agent can shelter  $\$e$  of earnings from the tax authority (via sheltering or evasion) by paying a cost  $g(e)$ . Total taxable income is  $TI = \sum_{j=1}^J w_j x_j - e$ . Let  $c = (1 - t)TI + e$  denote consumption. For simplicity, assume that utility is linear in  $c$  to abstract from income effects. Feldstein shows that it is straightforward to allow for income effects. As in the Harberger model, we calculate the excess burden of the tax by assuming that the government returns the tax revenue to the agent as a lump sum transfer  $T(t)$ . Using the notation introduced in section II, we can write this model formally as:

$$\begin{aligned} u(c, x, e) &= c - g(e) - \sum_{j=1}^J \psi_j(x_j) \\ T(t) &= t \cdot TI \\ G_1(c, x, t) &= T + (1 - t)TI + e - c \end{aligned}$$

Social welfare is

$$W(t) = \left\{ (1 - t)TI + e - g(e) - \sum_{j=1}^J \psi_j(x_j) \right\} + t \cdot TI \quad (17)$$

To calculate the marginal excess burden  $\frac{dW}{dt}$ , totally differentiate (17) to obtain

$$\begin{aligned}\frac{dW}{dt} &= T I + t \frac{d T I}{dt} - T I + (1-t) \frac{d T I}{dt} + \frac{d e}{dt} (1-g'(e)) - \sum_{j=1}^J \psi'_j(x_j) \frac{d x_j}{dt} \\ &= \frac{d T I}{dt} + \frac{d e}{dt} (1-g'(e)) - \sum_{j=1}^J \psi'_j(x_j) \frac{d x_j}{dt}\end{aligned}\tag{18}$$

This equation is an example of the marginal utility representation in (14) given in step 3 of the rubric in section II. To recover the marginal utilities (step 4), Feldstein exploits the first order conditions

$$\begin{aligned}g'(e) &= t \\ \psi'_j(x_j) &= (1-t)w_j \\ \Rightarrow \sum_{j=1}^J \psi'_j(x_j) \frac{d x_j}{dt} &= \sum_{j=1}^J (1-t)w_j \frac{d x_j}{dt} = (1-t) \frac{d(T I + e)}{dt}\end{aligned}\tag{19}$$

where the last equality follows from the definition of  $T I$ . Plugging these expressions into (18) and collecting terms yields the following expression for the marginal welfare gain from raising the tax rate from an initial rate of  $t$ :

$$\frac{dW(t)}{dt} = t \frac{d T I(t)}{dt}.\tag{20}$$

A simpler, but less instructive, derivation of (20) is to differentiate (17), recognizing that behavioral responses have no first-order effect on private surplus (the term in curly brackets) because of the envelope conditions. This immediately yields  $\frac{dW}{dt} = -T I + T I + t \frac{d T I}{dt}$ .

Equation (20) shows that we simply need to measure how taxable income responds to changes in the tax rate to calculate the deadweight cost of income taxation. It does not matter whether  $T I$  changes because of hours responses, changes in occupation, or avoidance behaviors. Intuitively, the agent supplies labor on every margin  $(x_1, \dots, x_J)$  up to the point where his marginal disutility of earning another dollar through that margin equals  $1-t$ . The marginal social value of earning an extra dollar net of the disutility of labor is therefore  $t$  for all margins. Likewise, the agent optimally sets the marginal cost of reporting \$1 less

to the tax authority ( $g'(e)$ ) equal to the marginal private value of doing so ( $t$ ). Hence, the marginal social costs of reducing earnings (via any margin) and reporting less income via avoidance are the same at the individual's optimal allocation. This makes it irrelevant which mechanism underlies the change in  $TI$  for efficiency purposes.

The main advantage of identifying  $\frac{dTI(t)}{dt}$  as a sufficient statistic is that it permits inference about efficiency costs without requiring identification of the potentially complex effects of taxes on numerous labor supply, evasion, and avoidance behaviors. Moreover, data on taxable income are available on tax records, facilitating estimation of the key parameter  $\frac{dTI}{dt}$ . Feldstein implements (20) by estimating the changes in reported taxable income around the Tax Reform Act of 1986 (Feldstein 1995), implicitly using the linear approximation described in step 5 of the rubric. He concludes based on these estimates that the excess burden of taxing high income individuals is very large, possibly as large as \$2 per \$1 of revenue raised. This result has been influential in policy discussions by suggesting that top income tax rates should be lowered (see e.g., Joint Economic Committee 2001). Subsequent empirical work motivated by Feldstein's result has found smaller values of  $\frac{dTI}{dt}$ , and the academic debate about the value of this central parameter remains active.

The sixth step of the rubric – structural evaluation – has only been partially implemented in the context of Feldstein's formula. Slemrod (1995) and several other authors have found that the large estimates of  $\frac{dTI}{dt}$  are driven primarily by evasion and avoidance behaviors ( $\frac{de}{dt}$ ). However, these structural parameters ( $g(e), \psi_j(x_j)$ ) of the model have not been directly evaluated. Chetty (2008b) gives an example of the danger in not investigating the structural parameters. Chetty argues that the marginal social cost of tax avoidance may not be equal to the tax rate at the optimum – violating the first-order-condition (19) that is critical to derive (20) – for two reasons. First, some of the costs of evasion and avoidance constitute transfers, such as the payment of fines for tax evasion, rather than resource costs. Second, there is considerable evidence that individuals overestimate the true penalties for evasion. Using a sufficient statistic approach analogous to that above, Chetty relaxes the  $g'(e) = t$  restriction and obtains the following generalization of Feldstein's formula:

$$\frac{dW(t)}{dt} = t\left\{\mu(t)\frac{dTI(t)}{dt} + (1 - \mu(t))\frac{dLI(t)}{dt}\right\} \quad (21)$$



where  $LI = \sum_{j=1}^J w_j x_j$  represents total earned income and  $\mu(t) = \frac{g'(e(t))}{t}$  measures the gap between social marginal costs of avoidance and the tax rate. Intuitively, deadweight loss is a weighted average of the taxable income elasticity ( $\frac{dT I}{dt}$ ) and the total earned income elasticity ( $\frac{dLI}{dt}$ ), with the weight determined by the resource cost of sheltering. If avoidance does not have a large resource cost, changes in  $e$  have little efficiency cost, and thus it is only  $\frac{dLI}{dt}$  – the “real” labor supply response – that matters for deadweight loss.

Not surprisingly, implementing Chetty’s more general formula requires identification of more parameters than Feldstein’s formula. The most difficult parameter to identify is  $g'(e)$ , which is a marginal utility. By leaving  $g'(e)$  in the formula, Chetty does not complete step 4 of the rubric above; as a result, further work is required to implement (21). Gorodnichenko et al. (2008) provide a method of recovering  $g'(e)$  from consumption behavior. Their insight is that real resource costs expended on evasion should be evident in consumption data; thus, the gap between income and consumption measures can be used to infer  $g'(e)$ . Implementing this method to analyze the efficiency costs of a large reduction in income tax rates in Russia, Gorodnichenko et al. find that  $g'(e)$  is quite small and that  $\frac{dT I}{dt}$  is substantial, whereas  $\frac{dLI}{dt}$  is not. They show that Feldstein’s formula substantially overestimates the efficiency costs of taxation relative to Chetty’s more general measure. Intuitively, reported taxable incomes are highly sensitive to tax rates, but the sensitivity is driven by avoidance behavior that has little social cost at the margin and hence does not reduce the total size of the pie significantly.

The general lesson from this work is that sufficient statistic approaches are not model free. It is critical to evaluate the structure of the model, even though the formula for  $\frac{dW}{dt}$  can be implemented without the last step of the rubric. In the taxable income application, estimating  $g'(e)$  has value instead of simply assuming that  $g'(e) = t$  given plausible concerns that this condition does not hold in practice. Successful application of the sufficient statistic approach requires judicious choice of which restrictions to exploit – that is, assessing how general a class of models to consider – based on an evaluation of the structural parameters.

*Saez (2001)*. Harberger and Feldstein study the efficiency effects and optimal design of a linear tax. Much of the literature on optimal income taxation has focused on non-linear income tax models and the optimal progressivity of such systems. Mirrlees (1971) formalizes this question as a mechanism design problem, and provides a solution in differential equations

that are functions of primitive parameters. The Mirrlees solution offers little intuition into the forces that determine optimal tax rates. Building on the work of Diamond (1998), Saez (2001) expresses the optimality conditions in the Mirrlees model in terms of empirically estimable sufficient statistics.

Saez analyzes a model in which individuals choose hours of work,  $l$ , and have heterogeneous wage rates  $w$  distributed according to a distribution  $F(w)$ . Wage rates (skills) are unobservable to the government. Let pre tax earnings be denoted by  $z = wl$ . For simplicity, I again restrict attention to the case without income effects, as in Diamond (1998).

Saez begins by analyzing the optimal tax rate on top incomes. He considers a model where the government levies a linear tax  $\tau$  on earnings above a threshold  $\bar{z}$  and characterizes the properties of the optimal tax rate  $\tau^*$  as  $\bar{z} \rightarrow \infty$ . For a given  $\bar{z}$ , individuals maximize utility

$$\begin{aligned} u(c, l) &= c - \psi(l) \\ \text{s.t. } G_1(c, l) &= (1 - \tau) \max(wl - \bar{z}, 0) + \bar{z} - c = 0 \end{aligned}$$

Let  $c(w, \tau)$  and  $l(w, \tau)$  denote an agent's optimal choices as a function of his wage and the tax rate and  $z(w, \tau) = wl(w, \tau)$  denote the optimized earnings function. Let  $z_m(\bar{z}) = E[wl(w, \tau) | z(w, \tau) > \bar{z}]$  denote the mean level of earnings for individuals in the top bracket. Let  $\bar{w}$  denote the wage threshold that corresponds to an earnings threshold of  $\bar{z}$  when the tax rate is  $\tau$ :  $\bar{w}l(\bar{w}, \tau) = \bar{z}$ . The tax revenue generated by the top bracket tax is  $R = \tau(z_m(\bar{z}) - \bar{z})$ . The planner uses this tax revenue to fund a project that has a (normalized) value of \$1 per dollar spent.

The social planner's objective is to maximize a weighted average of individual's utilities, where the weights  $\tilde{G}(u)$  are "social welfare weights" that reflect the redistributive preferences of the planner:

$$W = \left\{ \int_0^{\infty} \tilde{G}(u(c(w, \tau), wl(w, \tau))) dF(w) \right\} + \tau(z_m(\bar{z}) - \bar{z})$$

In this equation, the first term (in curly brackets) represents private surplus and the second

term reflects government revenue. To calculate  $\frac{dW}{d\tau}$ , observe that individuals with incomes below  $\bar{z}$  are unaffected by the tax increase. Normalize the measure of individuals in the top bracket to 1. Utility maximization implies that behavioral responses ( $\frac{\partial l}{\partial \tau}$ ) have no first-order effect on private surplus, as  $wu_c(w, \tau) = \psi'(l(w, \tau))$ . Using this envelope condition, we obtain

$$\begin{aligned} \frac{dW}{d\tau}(\tau) &= -\int_{\bar{w}}^{\infty} \tilde{G}_u(u)(z(w, \tau) - \bar{z})dF(w) + [(z_m - \bar{z}) + \tau \frac{dz_m}{d\tau}] \\ &= -(z_m(\bar{z}) - \bar{z})\bar{g} + [(z_m(\bar{z}) - \bar{z}) + \tau \frac{dz_m}{d\tau}] \end{aligned} \quad (22)$$

where  $\bar{g} = \int_{\bar{w}}^{\infty} \tilde{G}_u(u)(z - \bar{z})dF(w) / \int_{\bar{w}}^{\infty} (z - \bar{z})dF(w)$  denotes the mean marginal social welfare weight placed by the planner on individuals in the top tax bracket. The parameter  $\bar{g}$  measures the social value of giving \$1 more income to individuals in the top bracket relative to the value of public expenditure. If  $\bar{g} = 1$ , the government weighs the consumption of the individuals it taxes and public expenditure equally, and (22) collapses to the Harberger formula for excess burden in (4). When  $\bar{g} < 1$ , the first term in (22) captures the welfare loss to individuals in the top tax bracket from having to pay more taxes. The second term reflects the gain in revenue to the government, which consists of two familiar components: the mechanical gain in revenue and the offset due to the behavioral response.

Equation (22) shows that three parameters are together sufficient statistics for the welfare gain of increasing top income tax rates: (1) the effect of tax rates on earnings ( $\frac{dz_m}{d\tau}$ ), which quantifies the distortions created by the tax; (2) the shape of the earnings distribution ( $z_m(\bar{z})$ ), which measures the mass of individuals whose behavior is distorted by the marginal tax, and (3) the marginal social welfare weight ( $\bar{g}$ ), which measures the planner's redistributive preferences. Note that Saez does not implement step 4 of the rubric – recovery of marginal utilities from observed behavior – because he views the relevant marginal utility in this case ( $\bar{g}$ ) as a feature of the planner's social welfare function that is external to the choice environment. Thus,  $\bar{g}$  is determined by the shape of the earnings distribution and the (exogenous) specification of the social welfare function (e.g. utilitarian or Rawlsian).

The advantage of (22) relative to a structural approach is that one does not need to identify preferences ( $\psi$ ) or the shape of the skill distribution  $F(w)$  to calculate  $dW/d\tau$ .

Moreover, one can permit arbitrary heterogeneity across skill types in preferences without changing the formula. The disadvantage of (22) is that  $z_m, \bar{g}$ , and  $\frac{dz_m}{d\tau}$  are endogenous to  $\tau$ : the level of earnings and the weight the social planner places on top earners presumably decrease with  $\tau$ , while  $\frac{dz_m}{d\tau}$  may vary with  $\tau$  depending upon the shape of the  $\psi(l)$  function. Hence,  $\frac{dW}{d\tau}(\tau)$  measures only the marginal welfare gain at a given tax rate  $\tau$  and must be estimated at all values of  $\tau$  to calculate the tax rate  $\tau^*$  that maximizes  $W$ . To simplify empirical implementation and derive an explicit formula for the optimal tax rate, Saez observes that the ratio  $\frac{z_m(\bar{z})}{\bar{z}}$  is approximately constant in the upper tail of the empirical distribution of earnings in the U.S.: that is, the upper tail of the income distribution is well described by a Pareto distribution. A Pareto distribution with parameter  $a$  has  $\frac{z_m(\bar{z})}{\bar{z}} = \frac{a}{a-1}$  for all  $\bar{z}$ . Hence, (22) can be expressed as

$$\frac{dW}{d\tau} = \frac{(1 - \bar{g})}{a - 1} \bar{z} + \tau \frac{dz_m}{d\tau}$$

The optimal top-bracket tax rate  $\tau$  satisfies  $\frac{dW}{d\tau}(\tau) = 0$ , implying

$$\frac{\tau^*}{1 - \tau^*} = \frac{1 - \bar{g}}{a\varepsilon} \tag{23}$$

where  $\varepsilon = \frac{dz_m}{d(1-\tau)} \frac{1-\tau}{z_m}$  denotes the taxable income elasticity in the top bracket. In the Mirrlees model,  $a$  and  $\varepsilon$  converge to constants (invariant to  $\tau$ ) in the limit as  $\bar{z} \rightarrow \infty$ . Equation (23) is therefore an *explicit* formula for the optimal asymptotic top income tax rate if the social welfare weight  $\bar{g}$  is taken as exogenous. For example, one plausible assumption is that  $\bar{g} \rightarrow 0$  as  $\bar{z} \rightarrow \infty$ . Saez exploits this property of (23) to calculate optimal top income tax rates using reduced-form estimates of the taxable income elasticity for high incomes (Gruber and Saez 2002) and a Pareto parameter of  $a = 2$  consistent with the earnings distribution in the U.S. He finds that optimal top income tax rates are generally above 50% when the formula is calibrated using plausible elasticities.

Building on this sufficient statistic approach, Saez characterizes the optimal tax rate at any income level  $z$  in a non-linear tax system. Let  $E$  denote the fixed amount of government expenditure that must be financed through taxation. Let  $T(z)$  denote the total tax paid by an individual who earns income  $z$ , so that net of tax income is  $z - T(z)$ . Let

$\varepsilon(z) = \frac{dz}{d(1-\tau)} \frac{1-\tau}{z}$  denote the earnings elasticity at income level  $z$  and  $h(z)$  the density of the earnings distribution at  $z$ . Finally, let  $\tilde{G}(u(z))$  denote the weight that the planner places on an individual with earnings  $z$  and  $g(z) = \tilde{G}_u \cdot u_c(z)$  the corresponding marginal social welfare weight.

The government chooses the schedule  $T(Z)$  that maximizes social welfare

$$W(T(z)) = \int_0^{\infty} \tilde{G}(u(c(w, T), wl(w, T))) dF(w)$$

subject to resource and incentive-compatibility constraints:

$$\begin{aligned} G_1(c, z, T) &= \int_0^{\infty} z(w, T) dF(w) - \int_0^{\infty} c(w, T) dF(w) - E = 0 \\ G_2(c, z, T) &= (1 - T'(z))w - \psi'(l(w)) = 0 \end{aligned}$$

Exploiting envelope conditions and perturbation arguments as above, the first order conditions for the optimal tax rates can be expressed in terms of sufficient statistics. In the case without income effects, the optimal tax schedule satisfies the following condition at all  $z$ :

$$\frac{T(z)}{1 - T(z)} = \frac{1}{\varepsilon(z)zh(z)} \int_z^{\infty} (1 - g(z'))h(z')dz' \quad (24)$$

Equation (24) depends on the same three parameters as (22): the taxable income elasticity, the shape of the earnings distribution, and the social welfare weights. It is again important to recognize that all three of these parameters are endogenous to the tax regime itself, and hence (24) is not an explicit formula for optimal taxation. Unfortunately, the techniques used to obtain the explicit formula for the asymptotic top income tax rate in (23) cannot be applied at an arbitrary income level  $z$  because there are no analogous limit convergence results. Hence, interpreted literally, (24) can only be used to test whether a given tax system  $T(z)$  happens to be optimal.

Because of this limitation, Saez couples the sufficient statistic formula with a structural approach to calibrate the optimal income tax schedule. Based on (24), he infers that the key parameters that should be matched by a structural model for evaluation of optimal taxation are the distribution of taxable income elasticities and the earnings distribution. He specifies

primitives (skill distributions and utility functions) that match empirical estimates of these moments. He then simulates the optimal tax schedule in the calibrated model. The resulting optimal income tax schedule is inverse-U shaped, with a large lump sum grant to non-workers and marginal rates ranging from 50-80%. This technique – which recognizes the limitations of the sufficient statistic approach and uses extrapolation based on structural methods to address those limitations – is an exemplary illustration of step 6 in the rubric.<sup>13</sup>

## IV Application 2: Social Insurance

Programs such as unemployment insurance, health insurance, social security, workers compensation, and disability insurance account for the majority of government expenditure in many countries. Starting with the seminal work of Wolpin (1987) and others, a large literature has studied the optimal design of social insurance programs in dynamic structural models. Parallel to this literature, a large body of reduced-form empirical work has investigated the impacts of social insurance programs on health expenditures, unemployment durations, consumption, disability claims, etc. See Table 1 for examples of structural and reduced-form studies.

In the context of social insurance, an important harbinger to the sufficient statistic approach is the work of Baily (1978), who showed that the optimal level of unemployment benefits can be expressed as a function of a small set of parameters in a static model. Baily’s result was viewed as being of limited practical relevance because of the strong assumptions made in deriving the formula. However, recent work has shown that the parameters Baily identified are actually sufficient statistics for welfare analysis of social insurance in a rich class of dynamic models. Studies in this literature include Gruber (1997), Chetty (2006), Shimer and Werning (2007), Chetty (2008a), Einav, Finkelstein, and Cullen (2008), and Chetty and Saez (2008a). I now embed these papers in the general framework above, focusing primarily on the first four papers.<sup>14</sup>

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<sup>13</sup>The limitation of this approach is that there are many potential structures consistent with the sufficient statistics observed at current tax rates, and each structure may imply different optimal tax rates. A full structural estimation is needed to discriminate between the  $\omega$  vectors consistent with the sufficient statistics.

<sup>14</sup>The tax and social insurance problems are closely related because social insurance is effectively state-

*Baily (1978) and Chetty (2006)*. For simplicity, consider a static model with two states: high and low. Let  $w_h$  denote the individual's income in the high state and  $w_l < w_h$  income in the low state. Let  $A$  denote wealth. Let  $c_h$  denote consumption in the high state and  $c_l$  consumption in the low state. The low state can be thought of as corresponding to job loss, injury, disability, natural disaster, etc. The agent can control the probability of being in the bad state by exerting effort  $e$  at a cost  $\psi(e)$ . For instance, "effort" could reflect searching for a job, taking precautions to avoid injury, or locating a house away from areas prone to natural disasters. Choose units of  $e$  so that the probability of being in the high state is given by  $p(e) = e$ .

Individuals may have some ability to insure against shocks through informal private sector arrangements, such as transfers between relatives. To model such informal private-insurance arrangements, suppose that the agent can transfer  $b_p$  between states at a cost  $q(b_p)$ , so that increasing consumption by  $b_p$  in the low state requires payment of a premium  $\frac{1-e}{e}b_p + q(b_p)$  in the high state. The loading factor  $q(b_p)$  can be interpreted as the degree of incompleteness in private insurance. If  $q(b_p) = 0$ , private insurance markets are complete; if  $q(b_p) = \infty$ , there is no capacity for private insurance.

The government pays a benefit  $b$  in the low state that is financed by an actuarially fair tax  $t(b) = \frac{1-e}{e}b$  in the high state. The model can be formally specified using the notation in section II as follows.<sup>15</sup>

$$\begin{aligned}
 U(c_l, c_h, e) &= eu(c_h) + (1 - e)u(c_l) - \psi(e) & (25) \\
 t(b) &= \frac{1 - e}{e}b \\
 G_1(c_l, c_h, t) &= c_h + \frac{1 - e}{e}b_p + q(b_p) + t - w_h - A \\
 G_2(c_l, c_h, t) &= c_l - b_p - b - w_l - A
 \end{aligned}$$

Substituting the constraints into the utility function yields social welfare as a function of the

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contingent taxation. Rather than levying taxes on the basis of income, taxes and transfers are levied on the basis of a state (joblessness, sickness, injury, disability, etc.). Conversely, redistributive taxation is social insurance against uncertain skill realizations behind the veil of ignorance.

<sup>15</sup>I follow the convention in the social insurance literature of specifying the problem in terms of the transfer benefit  $b$  rather than the tax  $t$ .

government benefit level:

$$W(b) = eu(A + w_h - \frac{1-e}{e}b_p - q(b_p) - t(b)) + (1-e)u(A + w_l + b_p + b) - \psi(e)$$

Differentiating this expression and using the envelope conditions for  $b_p$  and  $e$  gives

$$\begin{aligned} \frac{dW(b)}{db} &= (1-e)u'(c_l) - \frac{dt}{db}eu'(c_h) \\ &= (1-e)\{u'(c_l) - (1 + \frac{\varepsilon_{1-e,b}}{e})u'(c_h)\} \end{aligned}$$

where  $\varepsilon_{1-e,b} = \frac{d(1-e)}{db} \frac{b}{1-e}$  denotes the elasticity of the probability of being in the bad state (which can be measured as the unemployment rate, rate of health insurance claims, etc.) with respect to the benefit level. In tax models with quasilinear utility, the welfare gain measure  $\frac{dW}{dt}$  is a money metric. Since curvature of utility is an essential feature of the social insurance problem, we need a method of converting  $\frac{dW}{db}$  to a money metric. An intuitive metric is to normalize the welfare gain from a \$1 (balanced budget) increase in the size of the government insurance program by the welfare gain from raising the wage bill in the high state by \$1:

$$\begin{aligned} M_W(b) &= \frac{\frac{dW}{db}(b)/(1-e)}{\frac{dw_h}{dw_h}(b)/e} \\ &= \frac{u'(c_l) - u'(c_h)}{u'(c_h)} - \frac{\varepsilon_{1-e,b}}{e} \end{aligned} \tag{26}$$

This expression, which is Baily's (1978) formula, corresponds to the marginal utility expression obtained after the third step of the rubric in section II. The first term in (26) measures the gap in marginal utilities between the high and low states, which quantifies the welfare gain from transferring an additional dollar from the high to low state. The second term measures the cost of transferring this \$1 due to behavioral responses.

Chetty (2006) establishes that the parameters in (26) are "sufficient statistics" in that they are adequate to calculate  $M_W(b)$  in a general class of dynamic models that nest existing structural models of insurance. Chetty analyzes a dynamic model where transitions from the good state to the bad state follow an arbitrary stochastic process. Agents make  $J$



choices and are subject to  $M$  constraints. The choices could include variables such as reservation wages, savings behavior, labor supply, or human capital investments. Subject to a regularity condition analogous to Assumption 1, Chetty shows that (26) holds in this general model, with the difference in marginal utilities replaced by the difference between the average marginal utilities in the high and low states over the agent’s life. This result distills the calculation of welfare gains in complex dynamic models to two parameters: the gap in average marginal utilities and the elasticity that enters the government’s budget constraint  $\varepsilon_{1-e,b}$ . Identification of parameters such as asset limits or the degree of private insurance ( $q(b_p)$ ) is not required. This permits calculation of  $\frac{dW}{db}$  without the assumptions made in the structural studies for tractability, such as no private insurance or no borrowing (Hansen and Imrohglu 1992, Hopenhayn and Nicolini 1997).<sup>16</sup>

Equation (26) is not directly implementable because the gap in marginal utilities must be recovered from choice data. The recent literature has proposed the use of three types of choice data to recover the marginal utility gap: consumption (Gruber 1997), liquidity and substitution effects in effort (Chetty 2008a), and reservation wages (Shimer and Werning 2007).

*Gruber (1997)*. Taking a quadratic approximation to the utility function, Gruber observes that

$$\frac{u'(c_l) - u'(c_h)}{u'(c_l)} = \gamma \frac{\Delta c}{c_h}(b) \tag{27}$$

where  $\gamma = -\frac{u''(c_h)}{u'(c_h)}c_h$  is the coefficient of relative risk aversion evaluated at  $c_h$  and  $\Delta c = c_h - c_l$ . Gruber posits that the effect of UI benefits on consumption is linear (an assumption that should ideally be evaluated using a structural simulation):

$$\frac{\Delta c}{c_h}(b) = \alpha + \beta b$$

In this specification,  $\alpha$  measures the consumption-drop that would occur absent government

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<sup>16</sup>In dynamic models, sufficient statistic formulas are generally functions of behavioral responses to *permanent* changes in policy, starting at the beginning of the agent’s life. In practice, program evaluation studies estimate the effect of changes in policies in the middle of individual’s life. These short-run treatment effects could potentially differ from the relevant permanent effects (Lucas 1976, Rosenzweig and Wolpin 2000). Estimating the long-run responses is likely to require some structural assumptions.

intervention while  $\beta$  measures the slope of the consumption function with respect to the benefit level. Putting this equation together with (27) and (26) yields the following expression for the marginal welfare gain from increasing the benefit level:

$$M_W(b) = (\alpha + \beta b)\gamma - \frac{\varepsilon_{1-e,b}}{e}. \quad (28)$$

Building on work by Hamermesh (1982), Gruber estimates the consumption-smoothing effect of unemployment insurance (UI) benefits by exploiting changes in UI benefit laws across states in the U.S. coupled with panel data on consumption. He estimates  $\alpha = 0.24$  and  $\beta = -0.28$ , and then calibrates the welfare gain from raising UI benefits using estimates of  $\varepsilon_{1-e,b}$  from Meyer (1990). He finds that at conventional levels of risk aversion ( $\gamma < 2$ ), increasing the UI benefit level above the levels observed in his data (roughly 50% of the wage) would lead to substantial welfare losses.

Gruber proceeds to solve for the  $b^*$  such that  $\frac{dW}{db}(b^*) = 0$  in (28), and finds that  $b^*$  is close to zero. These calculations of the optimal benefit level assume that  $\frac{\Delta c}{c}$  is linear in  $b$  and  $\gamma(b)$  and  $\varepsilon_{1-e,b}(b)$  do not vary with  $b$ . This application of the sufficient statistic formula – which is not guided by a structural model – could be very inaccurate, because it uses ad hoc assumptions to make predictions about counterfactuals that are well out-of-sample. Equation (28) should not be used to make statements about global optima unless one can estimate the sufficient statistics for a range of different benefit levels. Lacking such estimates, a more reliable method of inferring  $b^*$  would be to calibrate a structural model to match the sufficient statistics and simulate the optimal  $b^*$  in that model, as in Saez (2001).

A difficulty with (28) is that risk aversion ( $\gamma$ ) is known to vary substantially across contexts (Rabin 2000, Chetty and Szeidl 2007). Chetty (2003) uses a structural model to show that Gruber’s choice of  $\gamma = 2$  is inconsistent with data on the effects of UI benefits on search behavior. With his preferred estimate of  $\gamma = 4$ , (28) implies a small positive welfare gain from raising unemployment benefit levels. In light of the sensitivity of the results to the value of  $\gamma$ , more recent studies have sought alternative ways of recovering the gap in marginal utilities that do not require an estimate of  $\gamma$ .

*Chetty (2008a)*. Chetty (2008a) shows that the gap in marginal utilities in (26) can be

backed out from the comparative statics of effort choice. To see this, observe that the first order condition for effort is

$$\psi'(e) = u(c_h) - u(c_l). \quad (29)$$

Now consider the effect of an exogenous cash grant (such as a severance payment to job losers) on effort, holding fixed the private insurance level  $b_p$ :

$$\partial e / \partial A = \{u'(c_h) - u'(c_l)\} / \psi''(e) \leq 0 \quad (30)$$

The effect of increasing the benefit level on effort (again holding  $b_p$  fixed) is:

$$\partial e / \partial b = -u'(c_l) / \psi''(e) \quad (31)$$

Combining (30) and (31), we see that the ratio of the “liquidity” effect ( $\partial e / \partial A$ ) to the “substitution” effect ( $\partial e / \partial w_h = \partial e / \partial A - \partial e / \partial b$ ) recovers the gap in marginal utilities:

$$\frac{u'(c_l) - u'(c_h)}{u'(c_h)} = \frac{-\partial e / \partial A}{\partial e / \partial A - \partial e / \partial b}$$

Plugging this into (26) yields the following expression for the welfare gain from increasing the benefit level:

$$M_W(b) = \frac{-\partial e / \partial A}{\partial e / \partial A - \partial e / \partial b} - \frac{\varepsilon_{1-e,b}}{e} \quad (32)$$

The intuition for this formula is that the gap between marginal utilities in the good and bad states can be inferred from the extent to which effort is affected by liquidity vs. moral hazard. In a model with perfect consumption smoothing ( $c_h = c_l$ ), the liquidity effect  $\partial e / \partial A = 0$ , because a cash grant raises  $u(c_h)$  and  $u(c_l)$  by the same amount. Chetty implements (32) by estimating the effects of unemployment benefits and severance payments on search intensity using hazard models for unemployment durations. He finds that the welfare gains from raising the unemployment benefit level are small but positive, suggesting that the current benefit level is below the optimum given concavity of  $W(b)$ .

Chetty evaluates the ability to extrapolate from the sufficient statistic formula using a structural job search model calibrated to match his moral hazard and liquidity elasticity

estimates. He finds that  $\frac{dW}{db}(b)$  declines sharply with  $b$  in the structural simulation because the liquidity effect  $\frac{\partial e}{\partial A}(b)$  falls substantially as  $b$  rises (see Figures 1 and 2 in Chetty (2008a)). Based on this structural simulation, he concludes that the optimal benefit level is likely to be close to the current wage replacement rate of approximately 50% since  $M_W(b = 0.5w)$  is small but positive. Finally, Chetty uses the calibrated structural model to study the welfare gains from new policy instruments – such as the direct provision of liquidity through loans. These results illustrate how combining sufficient statistic formulas with structural methods expands the domain of policy questions one can answer beyond local welfare analysis.

*Shimer and Werning (2007)*. Shimer and Werning (2007) infer the gap in marginal utilities from the comparative statics of reservation wages instead of effort in a model of job search. They consider a model where the probability of finding a job,  $e$ , is determined by the agent’s decision to accept or reject a wage offer rather than by search intensity. Wage offers are drawn from a distribution  $F(w)$ . If the agent rejects the job offer, he receives income of  $w_l + b$  as in the model above. For simplicity, assume that the agent has no private insurance ( $q = \infty$ ); allowing  $q < \infty$  complicates the algebra but does not affect the final formula. The remainder of the model is specified as in (25).

The agent rejects any net-of-tax wage offer  $w - t$  below his outside option  $w_l + b$ . Therefore,  $e = 1 - F(w_l + b + t)$  and the agent’s expected value upon job loss is

$$W(b) = eE[u(w - t)|w - t > w_l + b] + (1 - e)u(w_l + b)$$

Note that even though the microeconomic choices of accepting or rejecting wage offers are discrete, the welfare function is smooth because of aggregation, as in (9).

Shimer and Werning’s insight is that  $\frac{dW}{db}$  can be calculated using information on the agent’s reservation wage. Suppose we ask the agent what wage he would be willing to accept with certainty *prior* to the start of job search.<sup>17</sup> Define the agent’s reservation wage prior to job search as the wage  $\bar{w}_0$  that would make the agent indifferent about accepting a job immediately to avoid having to take a random draw from the wage offer distribution.

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<sup>17</sup>Shimer and Werning study a stationary dynamic model with CARA utility where the reservation wage is fixed over time, in which case it does not matter at what point of the spell the reservation wage is elicited.

The reservation wage  $\bar{w}_0$  satisfies

$$u(\bar{w}_0 - t) = W(b)$$

The government's problem is to

$$\begin{aligned} \max W(b) &= \max u(\bar{w}_0 - t) \\ &\Rightarrow \max \bar{w}_0 - t \end{aligned} \tag{33}$$

Differentiating (33) gives a sufficient-statistic formula.<sup>18</sup>

$$M_W(b) = \frac{d\bar{w}_0}{db} - \frac{dt}{db} = \frac{d\bar{w}_0}{db} - \frac{1-e}{e} \left(1 + \frac{1}{e} \varepsilon_{1-e,b}\right)$$

Intuitively,  $\frac{d\bar{w}_0}{db}$  encodes the marginal value of insurance because the agent's reservation wage directly measures his expected value when unemployed. Shimer and Werning implement (33) using an estimate of  $\frac{d\bar{w}_0}{db}$  from Feldstein and Poterba (1984) and find a large, positive value for  $M_W(b)$  at current benefit levels. However, they caution that the credibility of existing reservation wage elasticity estimates is questionable, particularly in view of evidence that UI benefit levels have little impact on subsequent wage rates (e.g. Card, Chetty, Weber 2007, van Ours and Vodopivec 2008).

The multiplicity of formulas for  $M_W(b)$  illustrates a general feature of the sufficient-statistic approach: since the model is not fully identified by the inputs to the formula, there are generally several representations of the formula for welfare gains.<sup>19</sup> This flexibility allows the researcher to apply the representation most suitable for his application given the available variation and data. For example, in analyzing disability insurance, it may be easiest

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<sup>18</sup>This corresponds to equation (12) in Shimer and Werning (2007), where the unemployment rate is  $u = 1 - e$ . The slight difference between the formulas (the  $\frac{1}{1-u}$  factor in the denominator) arises because Shimer and Werning write the formula in terms of a partial-derivative-based elasticity. Here,  $\varepsilon_{1-e,b}$  is the elasticity including the UI tax response needed to balance the budget; in Shimer and Werning's notation, it is holding the tax fixed.

<sup>19</sup>All three formulas hold in models that allow both reservation wage and search intensity choices. Chetty's (2006) generalization of Baily's formula nests the model with stochastic wages. If agents control the arrival rate of offers via search effort, the first order condition for search effort remains the same as in (29), with  $Eu'(c_h)$  replacing  $u'(c_h)$ . It follows that Chetty's (2008a) formula also holds with stochastic wages.

to implement Chetty's (2008a) formula since the available variation permits identification of liquidity and moral hazard effects (Autor and Duggan 2007).

*Inefficiencies in Private Insurance.* An important assumption made in all three formulas above is that the choices within the private sector are constrained Pareto efficient – that is, total surplus is maximized in the private sector subject to the constraints. In practice, private insurance contracts are likely to be second-best inefficient as well because of adverse selection and moral hazard in private markets. In this case, the envelope condition invoked in deriving (26) is violated because of externalities on the private insurer's budget constraint that are not taken into account by the individual.

Recent work by Einav, Finkelstein, and Cullen (2008) and Chetty and Saez (2008a) identifies sufficient statistics for the welfare gains from social insurance in environments with adverse selection and moral hazard in private insurance markets. Einav et al. develop a method of characterizing the welfare gain from government intervention that uses information about insurance purchase decisions. They show that the demand curve for private insurance and the cost of providing insurance as a function of the price are together sufficient statistics for welfare. The first parameter captures consumers' willingness to pay for insurance while the latter captures how the marginal costs of providing insurance varies with the pool of the insured via selection effects. Einav et al. implement their method using quasi-experimental price variation in health insurance policies and find that the welfare gains from government intervention in health insurance markets is small.

Chetty and Saez focus on ex-post behaviors, namely how marginal utilities vary across the high and low states, as in the Baily formula. They develop a simple extension to Gruber's (1997) implementation of the formula that includes two more parameters – the size of the private insurance market and the crowdout of private insurance by public insurance. Intuitively, the government exacerbates the moral hazard distortion created by private insurance, and must therefore take into account the amount of private insurance and degree of crowdout to calculate the welfare gains from intervention. Chetty and Saez apply their formula to analyze health insurance, and show that naively applying (26) dramatically overstates the welfare gains from government intervention in this case.

These examples illustrate that the sufficient statistic approach can be extended to en-

vironments where private sector choices are not second-best efficient. One has to deviate slightly from the general rubric to handle such second-best inefficiencies because the incentive compatibility constraints for private insurance violate Assumption 1. These constraints lead to additional terms in the sufficient statistic formula, increasing the number of moments that need to be estimated for welfare analysis.

## V Application 3: Behavioral Models

There is now considerable reduced-form evidence that individuals' behavior deviates systematically from the predictions of neoclassical perfect optimization models; see Table 1 for a few examples and DellaVigna's (2008) review for many more. In light of this evidence, an important new challenge is normative analysis in models where agent's choices deviate from perfect optimization. The budding literature on this topic has proposed some structural approaches to this issue, primarily in the context of time discounting. An early example is Feldstein's (1985) model of social security with myopic agents, in which individuals have a higher discount rate than the social planner. Feldstein numerically calculates the welfare gains from social security policies under various assumptions about the primitives. More recently, a series of papers have used calibrations of Laibson's (1997)  $\beta$ - $\delta$  model of hyperbolic discounting to make numerical predictions about optimal policy for agents who are impatient (see Table 1). Another set of studies has modelled the behavioral patterns identified in earlier work – such as “ironing” and “spotlighting” effects in responses to non-linear price schedules – and simulated optimal tax policy in such models (Liebman and Zeckhauser 2004, Feldman and Katuscak 2006).

The difficulty with the structural approach in behavioral applications is that there are often multiple positive models which can explain deviations from rationality, and each of these models can lead to different welfare predictions. The sufficient statistic approach can be very useful in such situations, because welfare analysis does not require full specification of the positive model underlying observed choices (Bernheim and Rangel 2007). In applications where agents optimize, the main benefit of the sufficient statistic approach is that it simplifies identification. If one had unlimited power to identify primitives, there would be no advantage

to using the sufficient statistic approach in such models. In non-optimizing models, however, the sufficient statistic strategy has value even if identification is not a problem because there is no consensus alternative to the neoclassical model.

Given the infancy of this area, there is currently very little work applying sufficient statistic approaches to behavioral models. However, this is a fertile area for further research, as illustrated by Chetty, Looney, and Kroft’s (2008) recent analysis of the welfare consequences of taxation when agents optimize imperfectly with respect to taxes. Chetty et al. present evidence that the effect of commodity taxes on demand depend on whether the tax is included in posted prices or not. Taxes that are not included in posted prices – and are hence less salient to consumers – induce smaller demand reductions. Chetty et al. note that there are various psychological and economic theories which could explain why salience affects behavioral responses to taxation, including bounded rationality, forgetfulness, and cue theories of attention. They therefore develop an sufficient-statistic approach to welfare analysis that is robust to specifications of the positive theory of tax salience.

Chetty et al. characterize the efficiency costs and incidence of taxation in a two-good model analogous to the Harberger model presented in section I of this paper. Let  $x$  denote the taxed good and  $y$  the numeraire. Let demand for  $x$  as a function of its pretax price and tax rate be denoted by  $x(p, t)$ . I assume here that utility is quasilinear in  $y$  and production is constant-returns-to-scale. These assumptions simplify the exposition by eliminating income effects and changes in producer prices; Chetty et al. shows that similar results are obtained when these assumptions are dropped.

The agent’s true ranking of the consumption bundles  $(x, y)$  is described by a smooth, quasiconcave utility function

$$\begin{aligned} U(x, y) &= u(x) + y \\ &= u(x) + Z - (p + t)x \end{aligned}$$

where the second line imposes that the allocation  $(x, y)$  the agent chooses must satisfy the true budget constraint  $(p + t)x + y = Z$ . Chetty et al. depart from the traditional Harberger analysis by dropping the assumption that the consumption bundle  $(x, y)$  is chosen



to maximize  $U(x, y)$ . Instead, they take the demand function  $x(p, t)$  as an empirically estimated object generated by a model unknown to the policy maker, permitting  $\frac{\partial x}{\partial p} \neq \frac{\partial x}{\partial t}$ .

To calculate excess burden, assume that tax revenue is returned to the agent lump-sum. Then, under the assumption that individual's utility is a function purely of their ultimate consumption bundle, social welfare is given by

$$W(p, t) = \{u(x) + Z - (p + t)x\} + T(t)$$

where  $T(t) = tx(t)$ . In non-optimizing models, one must deviate from step 2 of the rubric in section II at this point because the envelope condition used to derive (13) does not hold. Instead, totally differentiate the social welfare function to obtain

$$\frac{dW}{dt} = [u'(x) - p] \frac{dx}{dt} \tag{34}$$

This marginal-utility based expression captures a simple intuition. An infinitesimal tax increase reduces consumption of  $x$  by  $\frac{dx}{dt}$ . The loss in surplus from this reduction in consumption of  $x$  is given by the different between willingness to pay for  $x$  ( $u'(x)$ ) and the cost of producing  $x$ , which equals the price  $p$  under the constant-returns-to-scale assumption.

Equation (34) applies in any model, irrespective of how  $x(p, t)$  is chosen. Given that  $\frac{dx}{dt}$  can be estimated empirically, the challenge in calculating  $\frac{dW}{dt}$  – which reflects the main challenge in behavioral public economics more generally – is the recovery of the true preferences ( $u'(x)$ ). In neoclassical models, we use the optimality condition  $u'(x) = p + t$  to recover marginal utility and immediately obtain the Harberger formula in (4). Since we do not know how  $x$  is chosen, we cannot use this condition here. Chetty et al. tackle this problem by making the following assumption.

**Assumption 2** When tax inclusive prices are fully salient, the agent chooses the same allocation as a fully-optimizing agent:

$$x(p, 0) = x(p, 0) = \arg \max u(x(p, 0)) + Z - px(p, 0)$$

This assumption requires that the agent only makes mistakes with respect to taxes, and

not fully salient prices. To see why this assumption suffices to calculate welfare, let  $P(x) = x^{-1}(p, 0)$  denote the agent's inverse-price-demand curve. Assumption 2 implies that  $P(x) = u'(x)$  via the first order condition for  $x(p, 0)$ . Plugging this into (34) yields

$$\frac{dW}{dt} = [P(x) - p] \frac{dx}{dt}$$

This formula for  $\frac{dW}{dt}$  can be implemented using an estimate of the inverse-price-demand curve  $P(x)$ . To simplify implementation, Chetty et al. make the approximation that demand  $x(p, t)$  is linear in both arguments to obtain

$$\begin{aligned} \frac{dW}{dt} &= \left[ \frac{dp}{dx} \cdot (x(p, t) - x(p, 0)) \right] \frac{dx}{dt} \\ &= \left[ \frac{dp}{dx} \cdot \frac{dx}{dt} t \right] \frac{dx}{dt} = t\theta \frac{dx}{dt} \end{aligned} \quad (35)$$

where  $\theta = \frac{dx}{dt} / \frac{dx}{dp}$  measures the degree of underreaction to the tax relative to the price. This expression, which nests the Harberger formula as the case where  $\theta = 1$ , shows that the price and tax elasticities of demand are together sufficient statistics to calculate excess burden in behavioral economics models. Intuitively, the tax-demand curve ( $\frac{dx}{dt}$ ) is used to determine the actual effect of the tax on behavior. Then, the price-demand curve ( $\frac{dx}{dp}$ ) is used to calculate the effect of that change in behavior on welfare. The price-demand curve can be used to recover the agent's preferences and calculate welfare changes because it is (by assumption) generated by optimizing behavior.

Because it does not rely on a specific structural model, (35) accommodates *all* errors in optimization with respect to taxes, and is hence easily adaptable to other applications. For example, confusion between average and marginal income tax rates (Liebman and Zeckhauser (2004), Feldman and Katuščák (2006)) or overestimation of estate tax rates (Slemrod 2006) can be handled using the same formula, without requiring knowledge of individuals' tax perceptions or rules of thumb. Any such behaviors are reflected in the empirically observed tax and price elasticities.

In sum, one can make progress in behavioral welfare economics by making assumptions that narrow the class of models under consideration without full specifying one particular

model. One is effectively forced to make stronger assumptions about the class of models in exchange for relaxing the full optimization assumption. These stronger assumptions make it especially valuable to pair the sufficient statistic approach with structural methods in behavioral models. For instance, identifying the structural reasons for why tax salience matters would cast light on the plausibility of Assumption 2.

## VI Conclusion: Other Applications

In an effort to unite theoretical models of optimal policy with evidence from program evaluation, the modern public economics literature has uncovered a potential “middle ground” between pure reduced-form and structural methods. Sufficient statistic formulas provide theoretical guidance for reduced-form empirical work, highlighting the parameters of greatest interest for a given question. Although sufficient statistic formulas are sometimes applied as a substitute for structural analysis, their results are most compelling when accompanied by a structural evaluation of the formula’s accuracy and the plausibility of the model and primitives underlying the sufficient statistics. Hence, the sufficient statistic approach should be viewed as a complement to rather than a substitute for structural estimation.

This review has focused on the welfare consequences of policies that change prices via taxes or transfer benefits. A natural next step for the application of sufficient statistic methods is to other types of counterfactual analysis, such as changes in regulations or exogenous fluctuations in state variables. I conclude by briefly discussing such applications in macroeconomics, labor economics, and industrial organization.

*Macroeconomics.* A central debate in the macroeconomics literature is whether households optimize intertemporally as predicted by standard permanent income hypothesis models. The structural approach to answering this question, taken for example by Scholz et al. (2006), is to specify a dynamic model of optimization and determine whether observed consumption and savings patterns and consistent with those predicted by the model. The sufficient statistic counterpart to this approach is to isolate one moment – such as the drop in consumption at retirement or the sensitivity of behavior to cash-on-hand – that is adequate to test between models (see e.g., Aguiar and Hurst 2005; Card, Chetty, and Weber 2007).

A second application is to models of business cycles and growth, where different models can again be compared by identifying a single moment, as in Shimer's (2008) analysis of the "labor wedge." Trade is another natural area of application, where the reduced-form impacts of changes in quotas and tariffs on prices and quantities can presumably be used to inform policy decisions as in the literature on taxation.

*Labor Economics.* Labor economists have long been interested in the effects of minimum wages on the economy. The sufficient statistic approach offers a method of translating evidence on the impacts of minimum wages on employment and wages into statements about optimal policy (Lee and Saez 2008). Another potential application is to the analysis of returns to schooling. Many studies have investigated the effects of schooling on behaviors such as job mobility, occupation choice, and health. A sufficient statistic approach would intuitively suggest that examining effects on total earnings is adequate in a model where agents optimize.

*Industrial Organization.* The discrete choice analysis in Section I indicates that the effects of price intervention policies on welfare in the class of models commonly studied in the industrial organization (IO) literature can be characterized using sufficient statistic formulas. There are two challenges in applying sufficient statistic methods to other common questions in IO. First, many policies of interest in IO tend to be non-marginal changes such as antitrust policy or the introduction of a new product. Second, much of the IO literature focuses on models of strategic interaction rather than price theory models. In games, small changes in exogenous parameters can lead to jumps in behavior, violating the smoothness properties used to derive the formulas. Discontinuous changes in behavior could perhaps be handled using sufficient statistic formulas that provide bounds for welfare instead of exact values. These are important topics for further methodological research at the interface of structural and reduced-form methods.

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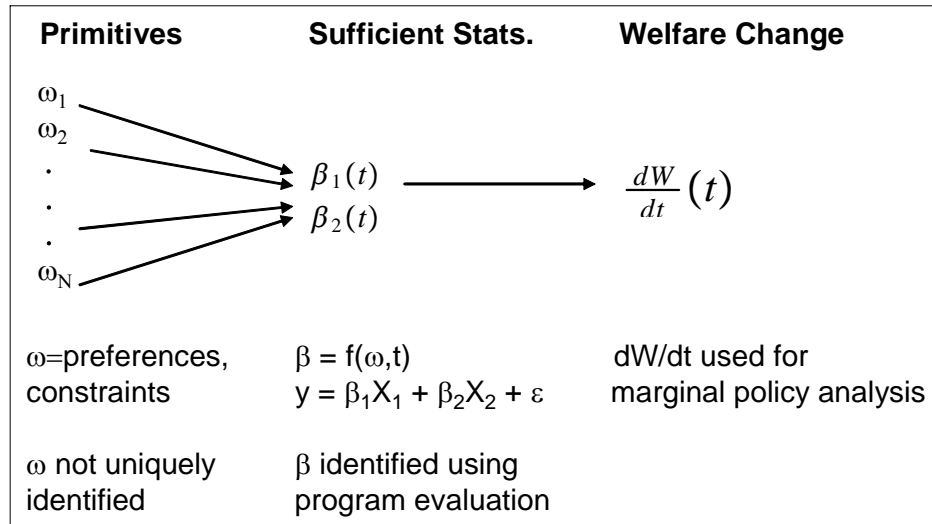
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**TABLE 1**  
Recent Examples of Structural, Reduced-Form, and Sufficient Statistic Studies

	<b>Structural</b>	<b>Reduced Form</b>	<b>Sufficient Statistic</b>
<b>Taxation</b>	Hoynes (1996) Keane and Moffitt (1998) Blundell et al. (2000) Golosov and Tsyvinski (2007) Weinzierl (2008)	Eissa and Liebman (1996) Blundell, Duncan, and Meghir (1998) Goolsbee (2000) Meyer and Rosenbaum (2001) Blau and Khan (2007)	Feldstein (1999) Diamond (1998) Saez (2001) Goulder and Williams (2003) Chetty (2008b)
<b>Social Insurance</b>	Rust and Phelan (1997) Golosov and Tsyvinski (2006) Lentz (2008) Blundell, Pistaferri, and Preston (2008) Einav, Finkelstein, and Schrimpf (2008)	Anderson and Meyer (1997) Gruber and Wise (1999) Autor and Duggan (2003) Lalive, van Ours, and Zweimuller (2006) Finkelstein (2007)	Gruber (1997) Chetty (2006) Shimer and Werning (2007) Chetty (2008a) Einav, Finkelstein, and Cullen (2008)
<b>Behavioral Models</b>	Angeletos et al. (2001) İmrohoroğlu, İmrohoroğlu, and Joines (2003) Liebman and Zeckhauser (2004) DellaVigna and Paserman (2005) Amador, Werning, and Angeletos (2006)	Genesove and Mayer (2001) Madrian and Shea (2001) Shapiro (2006) Ashraf, Karlan, and Yin (2006) Chetty and Saez (2008b)	Bernheim and Rangel (2008) Chetty, Looney, and Kroft (2008)

Notes: Categories used to classify papers are defined as follows. Structural: estimate or specify primitives to make predictions about welfare. Reduced form: estimate high-level behavioral elasticities qualitatively relevant for policy analysis, but do not provide quantitative welfare results. Sufficient statistic: make predictions about welfare without estimating or specifying primitives. This list includes only selected examples that relate to the topics discussed in the text, and omits many important contributions in each category.

FIGURE 1  
THE SUFFICIENT STATISTIC APPROACH



NOTE—The structural approach maps the primitives ( $\omega$ ) directly to statements about welfare ( $\frac{dW}{dt}$ ). The sufficient statistic approach leaves  $\omega$  identified and instead identifies a smaller set of parameters ( $\beta$ ) using program evaluation methods, e.g. via a regression of an outcome  $y$  on exogenous variables  $X$ . The  $\beta$  vector is “sufficient” for marginal welfare analysis in that any vector  $\omega$  consistent with  $\beta$  implies the same value of  $\frac{dW}{dt}$ . Identifying  $\beta$  does not identify  $\omega$  because there are multiple  $\omega$  vectors consistent with a single  $\beta$  vector.