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'PRECAUTIONARY' SAVING REVISITED:
SOCIAL SECURITY, INDIVIDUAL WELFARE,
AND THE CAPITAL STOCK

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ABSTRACT

This paper focuses on precautionary saving against uncertain longevity and on the annuity insurance aspects of social security within the life-cycle framework. The principal findings are three. First, the evolution of social security is reviewed in response to missing markets for providing insurance for consumption in the face of lifetime uncertainty. A simple life-cycle model is used to show that even an actuarially fair, fully funded social security system can reduce national saving. Second, to the extent that the introduction of social security reduces the size of accidental bequests, the net effect on the consumption of subsequent generations is diminished. Finally, consideration of the welfare gains from compulsory social security requires an examination of the tradeoff between the benefits to early participants from access to the annuities and the costs to generations that follow of a lower capital stock. Across a range of parameter values, the partial equilibrium impact of social security on consumption is reversed. The introduction of an explicit bequest motive mitigates both the initial impact of social security on saving and the long-run welfare loss from the introduction of social security.

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1. INTRODUCTION

Discussions of the impact of social security on saving have extended analysis of and estimation of life-cycle models of consumption (cf., Modigliani and Ando, 1957; Modigliani and Brumberg, 1954).¹ Empirical work is usually conducted in the certainty version of the model, wherein social security affects wealth accumulation in two ways: (i) reducing disposable income by the amount of the tax, and (ii) increasing lifetime resources and raising consumption over the life cycle to the extent that the present value of benefits exceeds the present value of taxes paid.

The emphasis here is on precautionary saving against uncertain longevity and on the annuity insurance aspects of social security within the life-cycle framework. This is a departure from the literature beginning with Feldstein's (1974) seminal paper (and the corresponding arguments in Barro, 1974, 1978), which focuses on the impact on consumption of the way in which social security is financed, i.e., the degree to which an unfunded social security system decreases private saving.² The principal findings are three. First, the evolution of social security is reviewed in response to missing markets for providing insurance for consumption in the face of uncertain lifetimes. A simple life-cycle model is put forth in section II to show that even an actuarially fair, fully funded social security system can reduce individual saving by more than the tax paid.³

Second, the large partial equilibrium saving impacts found in Section II are mitigated when initial endowments are considered. Specifically, accidental bequests, which arise in the model because of lifetime uncertainty, provide an intergenerational link for saving

decisions. To the extent that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of subsequent generations is diminished.

Finally, while it is true that social security reduces individual saving to a lesser degree in the generations after its introduction, there is still a reduction in the aggregate capital stock in the long run.⁴ Ultimately, consideration of the welfare gains from compulsory social security requires an examination of the tradeoff between the benefits to early participants from access to the annuities and the costs to generations that follow of a lower capital stock.

That general equilibrium analysis is carried out in Section IV. Across a range of parameter values, the partial equilibrium impact of social security on per capita consumption is reversed. Consumption and welfare of the representative individual are reduced in the new steady state. The introduction of an explicit bequest valuation function in Section V mitigates the initial impact of social security on saving and reduces the long-run welfare loss from the introduction of social security.

Conclusions and directions for future research are presented in Section VI.

II. SOCIAL SECURITY AND SAVING IN A LIFE-CYCLE MODEL

A. Individual Saving Decisions

The type of precautionary saving considered here is that against uncertain longevity. Yaari's (1965) seminal paper showed that with an uncertain lifetime, intertemporal utility maximization can dictate

saving for the possibility of living longer than the expected lifetime to avoid depreciation in old age.⁵ That excess saving can be large. Kotlikoff and Spivak (1981, p. 379) found for a set of plausible underlying parameter values that the present expected value of unintended bequests represented almost 25 percent of initial wealth for a single male aged 55.

To emphasize this point, consider the following life-cycle model. Agents are assumed to be selfish, in that no bequests are desired; the implications of relaxing this assumption are discussed in Section V. The retirement age Q is taken as exogenous, and individuals live at least Q periods. The probability of having died in the interval $[0, t]$ is p_t for each t ; by assumption, p_t is equal to zero in the interval $[0, Q]$. Individuals have an expected lifetime of D years, with $D' > D$ being the maximum age to which one can survive.⁶ Individuals are assumed to supply labor inelastically and receive a gross wage w_t in each period t during their working life; individual wages are assumed to grow over the working period at a constant rate g .

Following Yaari (1965) and Barro and Friedman (1977), let utility be additively separable across periods, and let $U(C_t)$ be evaluated contingent on being alive at time t . That is, the consumer's intertemporal choice model is given by⁷

$$(1) \quad \max \sum_{t=0}^{D'} (1-p_t) U(C_t) (1+\delta)^{-t}$$

subject to⁸

$$\sum_{t=0}^{D'} C_t (1+r)^{-t} = A_0 + w_0 \sum_{t=0}^Q \left(\frac{1+g}{1+r}\right)^t,$$

where C , δ , and r represent consumption and the (constant) subjective discount rate and interest rate, respectively. A_0 represents initial resources from unplanned bequests from the previous generation.

Assuming $U(C) = \frac{1}{\gamma} C^\gamma$, we can rewrite (1) as⁹

$$(2) \quad \max \sum_{t=0}^{D'} (1-p_t) \frac{1}{\gamma} C_t^\gamma (1+\delta)^{-t}$$

subject to

$$\sum_{t=0}^{D'} C_t (1+r)^{-t} = A_0 + w_0 \sum_{t=0}^Q \left(\frac{1+g}{1+r}\right)^t.$$

Carrying out the maximization in (2) yields an optimal consumption stream of

$$(3) \quad C_t = C_0 \left(\frac{1+r}{1+\delta}\right)^{\frac{t}{1-\gamma}} (1-p_t)^{\frac{1}{1-\gamma}},$$

where

$$(4) \quad C_0 = \frac{A_0 + w_0 \sum_{i=0}^Q \left(\frac{1+g}{1+r}\right)^i}{\sum_{i=0}^{D'} EPV_i}.$$

and

$$(4a) \quad EPV_i = (1+r)^{-i} \left(\frac{1+r}{1+\delta}\right)^{\frac{i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}} = (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}.$$

The extent to which uncertainty over length of life affects the stream of consumption depends on agents' degree of relative risk aversion (a transformation of γ , the elasticity of marginal utility function). The higher is an individual's degree of relative risk aversion (or, equivalently, the lower is his intertemporal elasticity of substitution in consumption), the slower will consumption grow over time.

B. The Introduction of Social Security

Access to fair annuity market could remove the influence of lifetime uncertainty on consumption. Individuals could exchange a portion of their labor income when young to smooth consumption in old age.¹⁰ If all individuals were identical in terms of their survival probabilities, then a competitive equilibrium would be possible. That is, as individual deaths are presumably independent, annuities would be actuarially fair in the competitive equilibrium.

The existence of a competitive equilibrium may be precluded by asymmetries of information between individuals and insurers. This is, of course, the familiar "adverse selection" phenomenon, discussed by Rothschild and Stiglitz (1976). They found that the competitive outcome may be inefficient, in the sense that the imposition of a common contract in addition to the competitively supplied contracts may be Pareto-improving. An interpretation of this compulsory additional contract as social security has been offered by Eckstein, Eichenbaum, and Peled (1983). There may be additional "moral hazard" or "free-rider" barriers to the existence of an annuities market. If individuals conjecture that the state will support them in deprivation, the need to purchase annuities is diminished.

Public provision of the annuities through public pensions is one possibility.¹¹ Moral hazard problems still make voluntary participation difficult. Consider, though, a social security system of the following form. Individuals are compelled to pay a payroll tax at rate t_s on gross wages, from which the social security system is funded. During retirement they receive annuity benefits S_t in each period t until death. The budget constraint in (2) becomes

$$(5) \quad \sum_{t=0}^{D'} C_t (1+r)^{-t} = A_0 + (1-t_s) \sum_{t=0}^Q w_0 \left(\frac{1+g}{1+r}\right)^t + \sum_{t=Q+1}^{D'} S_t (1+r)^{-t}.$$

If benefits are set according to a replacement rate of the terminal wage, then the economy-wide actuarially fair benefit S satisfies the condition that¹²

$$(6) \quad S \sum_{t=Q+1}^{D'} (1-p_t) (1+r)^{-t} = t_s \sum_{t=0}^Q w_0 \left(\frac{1+g}{1+r}\right)^t.$$

Substituting the actuarially fair social security benefit into the budget constraint in (4) yields

$$(7) \quad \sum_{t=0}^{D'} C_t (1+r)^{-t} = A_0 + (1-t_s) w_0 \sum_{t=0}^Q \left(\frac{1+g}{1+r}\right)^t +$$

$$t_s \left(\frac{w_0 \sum_{t=0}^Q \left(\frac{1+g}{1+r}\right)^t}{\sum_{t=Q+1}^{D'} (1-p_t)(1+r)^{-t}} \right) \sum_{t=Q+1}^{D'} (1+r)^t$$

$$= A_0 + (1 + t_s(\omega-1)) w_0 \sum_{t=0}^Q \left(\frac{1+g}{1+r}\right)^t,$$

where ω arises because of the difference in discount rates under certainty and uncertainty and is equal to

$$\left(\sum_{t=Q+1}^{D'} (1+r)^{-t} \right) / \left(\sum_{t=Q+1}^{D'} (1-p_t)(1+r)^{-t} \right).$$

Since ω is greater than unity, the system generates an increase in lifetime resources. Note that this increase in resources occurs even in a system which is actuarially fair and fully funded (i.e., in which

contributions are invested and earn the market rate of return r in each period).¹² In reality, the initial cohorts participating in social security received a rate of return greater than the actuarially fair return (see Hurd and Shoven, 1983). This analysis focuses only on an actuarially fair system to point out that the negative impact of social security on individual saving does not hinge on such initial transfers.¹³

Table 1 shows the percentage increase in lifetime resources generated by an actuarially fair social security system under various assumptions about the rate of interest and the social security payroll tax rate.¹⁴ For example, when $r = 0.06$ and $t_s = 0.10$, a 16 percent increase in the propensity to consume out of lifetime resources is afforded by an actuarially fair social security system. Because the system generates an increase in lifetime consumption, individual saving is reduced by more than the amount of the tax paid.

TABLE 1
PERCENTAGE INCREASE IN LIFETIME CONSUMPTION GENERATED BY
ACTUARIALLY FAIR SOCIAL SECURITY

r	t_s		
	0.10	0.12	0.14
0.02	29	35	41
0.04	21	26	32
0.06	16	19	23

The existence of social security annuities does not preclude the possibility of excess demand for old-age annuities. Suppose for example that effective participation in the system is higher for low-income individuals than for high-income individuals. Let \bar{w} represent the ceiling on taxable income; the growth rate of the taxable wage base and the determination of the replacement rate are as before. The budget constraint in (7) then becomes

$$(8) \quad \sum_{t=0}^{D'} C_t (1+r)^{-t} = A_0 + \sum_{t=0}^Q (1 + \tilde{t}_s (\omega - 1)) w_0 \left(\frac{1+g}{1+r} \right)^t,$$

where \tilde{t}_s is equal to $t_s \left(\frac{\bar{w}}{w_0} \right)$. The impact of social security on an individual's lifetime resources depends on his income. As an annuity, social security administered in this way generates a smaller reduction in saving for high-income people than for low-income people.

III. SOCIAL SECURITY AND DYNAMIC WEALTH ACCUMULATION

A. Individual Saving Behavior

We can use the derivation from the previous section of the impact of mandatory actuarially fair social security on saving to study individual wealth accumulation over time. For any time t , the present value (at time 0) of an individual's accumulated stock of wealth, A_{0t} (i.e., the present value of the "accidental bequest" of an individual who died in period t), can be expressed as

$$(9) \quad A_{0t} = \sum_{i=0}^t (1+r)^{-i} ((1 - t_s)w_i + S_i - C_i).$$

Wages and social security benefits are the sources of income to the

individual. Wages are zero in the interval $[Q+1, D']$, and social security benefits are zero in the interval $[0, Q]$. Using the expressions derived above for w_t , S_t and C_t , we can rewrite (9) as

$$(10a) \quad A_{0t} = A_0 + (1 - t_s)w_0 \sum_{i=0}^t \left(\frac{1+g}{1+r}\right)^i - (1 + t_s(\omega-1)) (w_0 \sum_{i=0}^Q \left(\frac{1+g}{1+r}\right)^i) \times$$

$$\left[\frac{\sum_{i=0}^t EPV_i}{\sum_{i=0}^{D'} EPV_i} \right], \quad t \in [0, Q],$$

and

$$(10b) \quad A_{0t} = A_0 + (1 - t_s)w_0 \sum_{i=0}^Q \left(\frac{1+g}{1+r}\right)^i + t_s w_0 \sum_{i=0}^Q \left(\frac{1+g}{1+r}\right)^i \left(\frac{\sum_{i=Q+1}^t (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1-p_i)(1+r)^{-i}} \right)$$

$$- (1 + t_s(\omega-1))(w_0 \sum_{i=0}^Q \left(\frac{1+g}{1+r}\right)^i) \left[\frac{\sum_{i=0}^t EPV_i}{\sum_{i=0}^{D'} EPV_i} \right], \quad t \in [Q+1, D'].$$

$$t \in [Q+1, D'].$$

To provide an intuitive framework for considering an individual's wealth accumulation over the life cycle, note that if we denote the present values of lifetime labor income and social security taxes by V_L and V_S , respectively, we can rewrite (10a) and (10b) as:

$$(11a) \quad \frac{A_{0t}}{V_L} = \frac{A_0}{V_L} + \frac{(1 - t_s)w_0 \sum_{i=0}^t \left(\frac{1+g}{1+r}\right)^i}{V_L}$$

$$- \left(1 + \frac{V_S}{V_L} (\omega - 1)\right) \left[\frac{\sum_{i=0}^t EPV_i}{\sum_{i=0}^{D'} EPV_i} \right],$$

and

$$(11b) \quad \frac{A_{0t}}{V_L} = \frac{A_0}{V_L} + 1 - \frac{V_S}{V_L} + \frac{V_S}{V_L} \left(\frac{\sum_{i=Q+1}^t (1+r)^{-i}}{D' \sum_{i=Q+1} (1-p_i)(1+r)^{-i}} \right) - \left(1 + \frac{V_S}{V_L} (\omega-1) \right) \left[\frac{\sum_{i=0}^t EPV_i}{D' \sum_{i=0} EPV_i} \right].$$

The ratio A_{0t}/V_L tracks an individual's accumulated stock of assets relative to lifetime earnings. In a world of no uncertainty over longevity, A_{0t}/V_L is simply a function of age, and the results of the basic life-cycle model are reproduced, as long as the present values of social security contributions and benefits are equal. With lifetime uncertainty, an actuarially fair social security system generates an increase in individual lifetime resources, and lifetime consumption rises. Much of this increase in consumption comes during an individual's working life, as the need to save for retirement is reduced. Depending on risk aversion, while retirement consumption is higher in the presence of social security, dissaving in retirement is likely to be less than in the certainty case.¹⁵

The problem becomes more complicated when the insurance coverage provided by social security is not the same across individuals. Suppose again that there is a ceiling on the level of earnings against which payroll tax rates and replacement rates are calculated. If that ceiling is \bar{w} in period 0 and grows at the same rate as the wage base, then the effective tax rate is not t_s , but $\tilde{t}_s = t_s \left(\frac{\bar{w}}{w_0} \right)$. From equation (11), the ratio of wealth to lifetime earnings should rise with the level of lifetime earnings, though at a decreasing rate.¹⁶ This nonlinearity of

saving rates with respect to lifetime earnings occurs in the absence of any explicit bequest motive.

B. Long-Run Effects on Individual Saving

Given uncertainty over length of life, an actuarially fair social security system can reduce individual saving by more than the amount of the taxes paid. For plausible underlying assumptions about individual discount rates, survival probabilities, and the intertemporal elasticity of substitution in consumption, the magnitude of that reduction is substantial. The partial equilibrium conclusion is clear -- estimates of the reduction in individual saving brought about by social security that focus only on the extent to which the system delivers a present value of anticipated benefits greater than the present value of taxes paid underestimate the true impact. Before discussing general equilibrium interpretations of this finding (in the sense that the wage rate and interest rate are endogenous and respond to changes in the saving rate), it is important to address the issue raised in the simulation exercises of the links among generations provided by accidental bequests.

An initial bequest from an "early death" of one's parent raises the beneficiary's consumption relative to lifetime earnings. In the model, the size of that bequest depends on the testator's coverage by social security and his age at death. By facilitating greater consumption out of lifetime earnings, social security reduces the accidental bequest. On that account, the initial resources available to the heir (and consumption when young) are lower. Even within the partial equilibrium analysis, the impact of social security on the consumption and saving patterns of individuals in a given generation depends on the balance between the effective increase in lifetime resources made possible by

access to a fair annuity and the reduction in inheritances because of that impact on the saving of the previous generations.¹⁷

To see this more clearly, note that given an average age of death of D , an individual testator's expected bequest relative to lifetime resources a_D is

$$(12) \quad a_D = (1+r)^D \left(1 - \frac{\sum_{i=0}^D EPV_i}{\sum_{i=0}^{D'} EPV_i} \right).$$

The reduction in the accidental bequest because of the parent's participation in social security is¹⁸

$$(13) \quad \frac{da_D}{dt_s} = - (1+r)^D (\omega-1) \left(\frac{\sum_{i=0}^D EPV_i}{\sum_{i=0}^{D'} EPV_i} \right).$$

In general, the net increment to lifetime resources made possible by social security depends on the age at which the parent died (magnitude of the unplanned bequest). The role of family mortality history is important here, as individuals whose progenitors all died early will receive large bequests relative to those whose parent lived a long time.

Members of the first generation to participate in the social security system benefit in two respects, as their lifetime resources are augmented both by the bequests from the (uninsured) previous generation and the gains from participation in the social security annuity system. The reduced value of accidental bequests permits smaller consumption gains for subsequent generations. While it is true that social security reduces individual saving to a lesser degree in the

generations after its introduction, there is still a reduction in the long-run capital stock. Ultimately, to consider the potential welfare gains from compulsory pensions, the tradeoff between the benefits to early participants from access to the annuities and the costs to generations that follow of a lower capital stock must be examined.

IV. GENERAL EQUILIBRIUM EFFECTS OF SOCIAL SECURITY ON THE CAPITAL STOCK

The partial equilibrium effects of social security on individual saving will be dampened in a general equilibrium analysis of the impact on aggregate capital formation. The reduction in individual wealth accumulation brought about by social security will induce changes in factor returns, exhibiting both income and substitution effects on consumption. A higher interest rate decreases the present value of lifetime resources; in addition, a higher rate of interest reduces the price of consumption in old age.¹⁹

The following simple model of production and factor prices is used to examine the impact of savings against lifetime uncertainty on aggregate saving. Output Q is assumed to be produced according to a Cobb-Douglas production function in capital K and effective labor \hat{L} ; the capital share is equal to α . Again, labor L is inelastically supplied, and labor-augmenting technical change occurs at a constant rate g , so that

$$(14) \quad \hat{L} = (1+g) L.$$

Hence output is determined by

$$(15) \quad Q = K^\alpha \hat{L}^{1-\alpha},$$

or, using lower-case letters to denote variables in intensive form

$$(16) \quad q = k^\alpha.$$

Factor markets are assumed to be competitive, so that capital and labor are paid their marginal products, i.e.,

$$(17) \quad r = \alpha k^{\alpha-1},$$

and

$$(18) \quad w = (1-\alpha)k^\alpha.$$

The work force grows at a constant rate n , producing a total effective growth rate of $n + g + ng$. Aggregate saving SS is assumed to be equivalent to the increase in the capital stock; that is, the depreciation rate is set equal to zero. The capital stock is assumed to result from individual saving and assets of the funded social security system.²⁰

We can now solve for the steady-state values of aggregate consumption, the aggregate capital stock, and the interest rate. Aggregate consumption out of labor income in any period j in the absence of social security can be written as

$$(19) \quad \frac{c_j}{w_j L_j} = \frac{\left(\sum_{i=0}^Q \left(\frac{1+g}{1+r} \right)^i \right) \left(\sum_{i=0}^{D'} \left(\frac{1}{(1+g)(1+n)} \right)^i \left(\frac{1+r}{1+\delta} \right)^{\frac{i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}} \right)}{\left(\sum_{i=0}^{D'} EPV_i \right) \left(\sum_{i=0}^Q (1+n)^{-i} \right)}.$$

The definition of steady-state equilibrium requires that

$$(20) \quad wL + rK - C \equiv SS = (n + g + ng) K,$$

so that

$$(21) \quad k = [(1-\alpha) \left(\frac{\frac{C}{wL} - 1}{r-n-g-ng} \right)]^{\frac{1}{1-\alpha}}.$$

Hence the steady-state interest rate r solves

$$(22) \quad r = \frac{\left(\frac{\alpha}{1-\alpha}\right)(n + g + ng)}{\left(\frac{1}{1-\alpha} - \frac{C}{wL}\right)}.$$

The steady state can be solved for as follows. A guess is made for k . Solutions for r and w are then generated from the marginal productivity conditions to produce individual consumption and wealth profiles. The resulting aggregate consumption and capital stock in intensive form are compared with the initial guess, and iteration proceeds until convergence is reached.

To quantify the impact of social security on aggregate consumption, the capital stock, and the welfare of a representative individual, the general equilibrium system described above can be simulated for plausible parameter values. In the Cobb-Douglas production function, α is set equal to $\frac{1}{3}$. The following relationships among the parameters of the individual's optimization problem are assumed: $r > g$, $r > \delta$, and $\delta > 0$.²¹ In addition, the interest rate is assumed to exceed the growth rate of the economy; i.e., $r > n+g+ng$. g is assumed to equal 0.02, while $n = .01$ and $\delta = 0.03$.²²

There is some evidence on the value of γ in the literature. In their study of household portfolio allocation, Friend and Blume (1975) estimated the coefficient of relative risk aversion to be in excess of 2.0, implying a value of γ of at most - 1.0. Farber's (1978) estimation of preferences of United Mine Workers from collective bargaining agreements yielded estimates of the coefficient of relative risk aversion of 3.0 and 3.7. Hansen and Singleton (1983) found estimates of the coefficient of relative risk aversion between 0 and 2.0. Here, three alternative values of γ will be used: -1.0, -3.0, and -5.0.

Table 2 below describes the no-social-security steady states under the three values of γ , presenting the interest rate (r), output per worker (q), consumption per head (c), capital per worker (k), and the saving rate (s/q).

TABLE 2

STEADY STATE IN THE ABSENCE OF SOCIAL SECURITY

	<u>$\gamma = - 1.0$</u>	<u>$\gamma = - 3.0$</u>	<u>$\gamma = - 5.0$</u>
r	0.060	0.056	0.054
q	2.357	2.440	2.485
c	1.938	1.975	1.994
k	13.095	14.522	15.337
s/q	0.180	0.190	0.197

Because of the risk of living longer than expected and facing deprivation in retirement, substantial unplanned bequests are added on average to the capital stock. The capital stock is higher in the more risk-averse ($\gamma = -3$, $\gamma = -5$) economies because of the higher average accidental bequests. As result, the interest rate is lower and per capita consumption is higher in those cases.

As in the discussion in Section II, the introduction of an actuarially fair social security system permits an increase in individual lifetime consumption. The resulting increase in consumption lowers the capital stock, raising the interest rate and lowering the wage rate in the new steady state. For the three cases considered here, with an assumed tax rate t_s of 0.10, social security initially increases the rate of consumption by 16.7 percent when $\gamma = -1.0$, by 17.5 percent when $\gamma = -3.0$, and by 17.8 percent when $\gamma = -5.0$.²³ In Table 3 below, we consider the impact of introducing an actuarially fair, fully funded social security system (when $t_s = 0.10$) on the steady state values of the variables described in Table 2.²⁴

TABLE 3
STEADY STATE IN THE PRESENCE OF SOCIAL SECURITY

	<u>$\gamma = - 1.0$</u>	<u>$\gamma = - 3.0$</u>	<u>$\gamma = - 5.0$</u>
r	0.084	0.070	0.075
% Δ	40.0	39.3	38.8
q	1.992	2.067	2.108
% Δ	-16.2	-15.3	-15.2
c	1.739	1.784	1.808
% Δ	-10.3	-9.7	-9.3
k	7.905	8.834	9.370
% Δ	-39.6	-39.2	-38.9
s/q	0.127	0.136	0.142
% Δ	-28.6	-28.1	-28.0

NOTE: % Δ indicates the percentage change from the no-social security case.

Note that in all cases the introduction of the social security annuity system leads to a significant reduction in the saving rate and the aggregate capital stock. The reductions in the capital stock are approximately half the value of the partial equilibrium reductions. As pointed out earlier, while social security permits higher consumption out of lifetime resources, intergenerational transfers (bequests) are lowered, and the increase in the interest rate and decrease in the wage rate work to lower lifetime resources. Though the rate of consumption increases more initially as γ gets smaller, the greater initial increase in r is counterbalanced by a greater fall in ω , narrowing the gap among the cases. In the results presented here, per capita consumption declined because of the fall in output. These losses in consumption per head cannot, however, be interpreted as a measure of a change in steady-state welfare, for which the appropriate concept is the utility of a representative individual.

The lifetime utility of a representative individual entering at time \bar{j} (denoted $V_{\bar{j}}$) is

$$(23) \quad V_{\bar{j}} = \sum_{t=0}^{D'} \frac{1}{\gamma} C_{\bar{j}t}^{\gamma} (1+\delta)^{-t} (1-p_t).$$

Substituting for C in (23) yields an expression for V that depends on lifetime resources, the interest rate, and the social security tax rate:

$$(24) \quad V_{\bar{j}} = \frac{1}{\gamma} \sum_{t=0}^{D'} EPV_t \times \left[\frac{w_0 (1+g)^{\bar{j}} (1+t_s (\omega-1)) \left(\sum_{i=0}^Q \left(\frac{1+g}{1+r} \right)^i \right) (1+\pi)^{\gamma}}{\sum_{i=0}^{D'} EPV_i} \right],$$

$$\text{where } \pi = \left(\frac{1+r}{(1+g)(1+n)} \right)^D \left[(1-(1+t_s (\omega-1))) \left(\frac{\sum_{i=0}^D EPV_i}{\sum_{i=0}^{D'} EPV_i} \right) \right].$$

Differences in welfare can be compared across steady states by computing the percentage change in lifetime earnings (or w_0 , given g and r) that would be necessary to yield the level of utility achieved in the new steady state. That is, equation (24) below must be solved for Δ , which serves as a measure of the difference in welfare between the two steady states.

$$(25) \quad V(w_0^1(1+\Delta), r^1, t_s^1) = V(w_0^2, r^2, t_s^2).$$

Following the earlier experiment, we consider the introduction of social security with $t_s = 0.10$ for the three cases of $\gamma = -1.0$, $\gamma = -3.0$, $\gamma = -5.0$. The value of Δ is summarized in Table 4 below. In all cases, the reduction in the capital stock from the imposition of the social security program leads to welfare losses in the new steady state on the order of 9-12 percent of lifetime income, or roughly four years' earnings.

TABLE 4

THE STEADY-STATE WELFARE COST OF SOCIAL SECURITY

(Expressed as a Percentage of Lifetime Income)

	<u>$\gamma = - 1.0$</u>	<u>$\gamma = - 3.0$</u>	<u>$\gamma = - 5.0$</u>
A	- 11.6	- 8.8	- 8.6

The results in Table 4 indicating that the introduction of a funded, actuarially fair social security system reduces steady-state welfare are subject to some important qualifications. First, the analysis in Sections II - IV assumed the absence of any explicit bequest motive. For any given set of parameter values, a positive bequest motive would raise individual saving rates and mitigate the impact of social security on the propensity to consume of the initial generation and of generations to come. Second, the model has assumed complete failure in the provision of private annuities. Finally, real-world limitations on borrowing against future earnings dampen the impact of social security on lifetime consumption. The first of these points is considered in detail in the next section. The remaining two are discussed briefly in Section VI.

V. BEQUESTS AND THE LIFE-CYCLE GROWTH MODEL

A. Individual Saving Behavior

The theoretical and simulation results presented heretofore have ignored the existence of explicit bequest motives. Some recent analyses have suggested, however, that bequests and other intergenerational transfers may account for a significant portion of the capital stock (see for example Kotlikoff and Summers, 1981). In the context of social security, the discussion of bequests has centered around the question of whether intergenerational transfers are adjusted in response to liabilities placed on future generations by an unfunded system (see Feldstein, 1974; Barro, 1974, 1978).

Again, to focus on the insurance features of social security, the emphasis here is on a funded, actuarially fair system. That individuals might gain utility from leaving a bequest (to provide capital to heirs or for prestige or security) mitigates the impact of social security annuities on life-cycle saving.

Consider the basic individual optimization problem from before with the addition of a bequest motive of the following form

$$(26) \quad \max \sum_{t=0}^{D'} (1 - p_t) U(C_t)(1+\delta)^{-t} + U(B_D)(1+\delta)^{-D}$$

subject to

$$\sum_{t=0}^{D'} C_t (1+r)^{-t} + B_D (1+r)^{-D} = A_0 + w_0 \sum_{t=0}^Q \left(\frac{1+g}{1+r}\right)^t,$$

where C , δ , and r are as before. A_0 and B_D represent initial resources and the bequest at the expected date of death D , respectively.

Assuming $U(C) = \frac{1}{\gamma} C^\gamma$ and $U(B_D) = \frac{1}{\gamma} B_D^\gamma$, we can rewrite (26) as

$$(27) \quad \max \frac{1}{\gamma} \sum_{t=0}^{D'} (1-p_t) C_t^\gamma (1+\delta)^{-t} + \frac{\beta}{\gamma} B_D^\gamma (1+\delta)^{-D}$$

subject to the same budget constraint. The parameter β indexes the desire to leave bequests; that is, a large value of β indicates a significant desire to leave a bequest to one's heirs.

This bequest framework is used by Seidman (1984) in his analysis of the transition from capital taxation to consumption taxation in a life-cycle growth model.²⁵ Seidman points out that a given steady state with a particular saving rate can be generated by different combinations of a subjective discount rate and bequest motive. The more powerful is the bequest motive, the greater is the discount rate parameterizing the initial steady state. If $r > \delta$, the gain in lifetime resources offered by access to the social security annuity system is smaller than in the no-bequest case.

Carrying out the optimization in (27) yields an optimal consumption stream of

$$(28) \quad C_t = C_0 \left(\frac{1+r}{1+\delta} \right)^{\frac{t}{1-\gamma}} (1 - p_t)^{\frac{1}{1-\gamma}},$$

where

$$(29) \quad C_0 = \frac{A_0 + w_0 \sum_{i=0}^Q \left(\frac{1+g}{1+r} \right)^i}{\sum_{i=0}^{D'} EPV'_i},$$

and

$$(29a) \quad EPV'_i = (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}} + \beta (1+r)^{\frac{D\gamma}{1-\gamma}} (1+\delta)^{\frac{-D}{1-\gamma}}$$

While the existence of a bequest motive does not change the shape of the age-consumption profile, it does lower the fraction of lifetime resources devoted to lifetime consumption.

The addition of the bequest motive described here modifies equations (11a) and (11b) as follows:

$$(30) \quad \frac{A_{0t}}{V_L} = \frac{A_0}{V_L} + \frac{(1-t_s)w_0 \sum_{i=0}^t \left(\frac{1+g}{1+r}\right)^i}{V_L} - \left(1 + \frac{V_S}{V_L} (\omega-1)\right) \left[\frac{\sum_{i=0}^t EPV_i}{\sum_{i=0}^{D'} EPV'_i} \right].$$

and

$$(31) \quad \frac{A_{0t}}{V_L} = \frac{A_0}{V_L} + 1 - \frac{V_S}{V_L} + \frac{V_S}{V_L} \left(\frac{\sum_{i=0}^t (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1-p_i)(1+r)^{-i}} \right) - \left(1 + \frac{V_S}{V_L} (\omega-1)\right) \left[\frac{\sum_{i=0}^t EPV_i}{\sum_{i=0}^{D'} EPV'_i} \right].$$

Note that given an average age of death of D , an individual testator's expected bequest relative to lifetime resources a_D becomes

$$(32) \quad a_D = (1+r)^D \left(1 - \frac{\sum_{i=0}^D EPV_i}{\sum_{i=0}^{D'} EPV'_i} \right).$$

The reduction in the bequest because of the parent's participation in social security is

$$(33) \quad \frac{da_D}{dt_s} = - (1+r)^D (\omega-1) \left(\frac{\sum_{i=0}^D EPV_i}{\sum_{i=0}^{D'} EPV'_i} \right).$$

In addition to the role of family mortality history in determining bequests, as the desire to leave bequests (indexed by β) gets larger, the impact of the social security system on intergenerational transfers is mitigated.

B. Bequests and General Equilibrium Impacts of Social Security on the Capital Stock

As in the previous section, suppose that the aggregate production function is Cobb-Douglas, with a capital share of α . We can now solve for the steady-state values of aggregate consumption, the aggregate capital stock, and factor prices. Aggregate consumption in any period j in the absence of social security can be written as

$$(34) \quad \frac{C_j}{w_j L_j} = \frac{\left(\sum_{i=0}^Q \left(\frac{1+g}{1+r} \right)^i \right) \left(\sum_{i=0}^{D'} \left(\frac{1}{(1+g)(1+n)} \right)^i \left(\frac{1+r}{1+\delta} \right)^{\frac{i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}} \right)}{\left(\sum_{i=0}^{D'} EPV'_i \right) \left(\sum_{i=0}^Q (1+n)^{-i} \right)}.$$

Steady-state equilibrium is defined as in equation (20), where the steady-state interest rate r now solves

$$(35) \quad r = \frac{\left(\frac{\alpha}{1-\alpha} \right) (n + g + ng)}{\frac{1}{1-\alpha} - \left[\frac{\left(\sum_{i=0}^Q \left(\frac{1+g}{1+r} \right)^i \right) \left(\sum_{i=0}^{D'} \left(\frac{1}{(1+g)(1+n)} \right)^i \left(\frac{1+r}{1+\delta} \right)^{\frac{i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}} \right)}{\left(\sum_{i=0}^{D'} EPV'_i \right) \left(\sum_{i=0}^Q (1+n)^{-i} \right)} \right]}$$

Two countervailing effects surface in examining the impact of social security on steady-state consumption and welfare in the bequest case. In the presence of the bequest motive modeled here, the increase in consumption accompanying a social security system of a given size is not as great as in the no-bequest case. However, because the capital-labor ratio is higher in the bequest regime, the interest rate is lower, and the gain from participating in a social security system of a given size (i.e., a given t_s) is greater (because w is greater).

Table 5 reports the steady state values (for the case of $\gamma = -1.0$) for the interest rate, output per capita, consumption per capita, saving rate, and capital-output ratio for five bequest regimes-- $\beta = 0.5, 2.0, 10.0, 15.0,$ and 25.0 . Higher values of β are associated with higher capital-labor ratios and higher saving rates.

TABLE 5
STEADY STATE WITH BEQUEST VALUATION

$$(\gamma = -1.0, t_s = 0)$$

	<u>β</u>				
	0.5	2.0	10.0	15.0	25.0
r	0.057	0.051	0.035	0.031	0.025
q	2.418	2.557	3.086	3.279	3.652
c	1.966	2.022	2.145	2.151	2.093
k	14.142	16.709	29.391	32.259	48.686
s/q	0.187	0.209	0.305	0.344	0.427

The results from introducing an actuarially fair, funded social security system with $t_s = 0.10$ are displayed in Table 6; again only the case of $\gamma = -1.0$ is considered. The same five bequest regimes as in Table 5 are considered. For the first four cases, the capital-reducing (output-reducing) effect of social security still outweighs the consumption-increasing effect; per capita consumption falls in the new steady state in each case. As expected, larger values of β are associated with smaller declines in consumption per head, as the impact of the social security system on the long-run capital stock is smaller. In the fifth case, consumption increases as a result of the introduction of social security.

The associated welfare changes are calculated as in the no-bequest case and are reported in Table 7. While the costs are lower than in the no-bequest case, the bequest motive in most cases did not eliminate the long-run cost of the introduction of the system. The only gain came in the $\beta = 25$ case.

TABLE 6
STEADY STATE WITH BEQUEST VALUATION

($\gamma = -1.0$, $t_s = 0.10$)

	<u>β</u>				
	0.5	2.0	10.0	15.0	25.0
r	0.080	0.070	0.045	0.039	0.030
% Δ	40.4	37.3	28.6	25.8	20.0
q	2.041	2.182	2.722	2.924	3.333
% Δ	-15.6	-14.7	-11.8	-10.8	-8.7
c	1.769	1.849	2.077	2.124	2.148
% Δ	-10.0	-8.6	-3.2	-1.3	2.6
k	8.505	10.391	20.16	24.987	37.037
% Δ	-39.9	-37.8	-31.4	-29.1	-23.9
s/q	0.133	0.152	0.237	0.273	0.356
% Δ	-28.9	-27.3	-22.3	-20.6	-16.7

NOTE: % Δ refers to the percentage change from the corresponding value in the absence of social security.

TABLE 7

THE STEADY-STATE WELFARE COST OF SOCIAL SECURITY
(Expressed as a Percentage of Lifetime Income)

$$\gamma = - 1.0$$

β

	0.5	2.0	10.0	15.0	25.0
A	- 7.7	- 7.5	- 3.0	- 0.6	+ 3.2

VI. CONCLUSIONS AND EXTENSIONS

Assessing the impact of social security and private pensions on individual wealth accumulation is important for many analyses of welfare, capital formation, and equity in the distributions of income and wealth. Previous research efforts along the lines of Feldstein (1974) have addressed the funding status of social security and pensions. The focus here is on insurance features of pension annuities with respect to the problem of uncertainty over length of life.

The first part of the paper considers the introduction of social security into an economy with market failure in the provision of private annuities. The principal findings are three. First, in such a world, even an actuarially fair, fully funded social security system can substantially reduce individual saving, though individual welfare is

initially improved. Hence, partial equilibrium estimates of the impact of social security on saving relying solely on the extent to which individuals earn a more than fair return on social security are underestimates of the true effect.

Second, the partial equilibrium impact of social security annuities on non-pension saving is reduced when initial endowments are considered. For example, to the extent that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of succeeding generations is mitigated.

Third, general equilibrium considerations, namely the endogeneity of factor returns, reverse the partial equilibrium impact in the new steady state. In the simulation exercises presented in Section IV, the accompanying reduction in the capital stock and output are sufficient to reduce per capita consumption and individual welfare. For plausible parameter values, the reduction in welfare is on the order of nine percent of lifetime resources.

The introduction in Section V of a bequest valuation into the life-cycle model produced smaller initial impacts on consumption than in the no-bequest case. While steady-state welfare is still reduced by the introduction in social security, the loss is smaller than in the no-bequest case. For very high weights on bequests, a small welfare gain occurs as the result of introducing social security.

Three extensions of the models presented here are left as tasks for future research. First, additional research on private annuity markets is needed to determine the actual extent of market failure. As discussed elsewhere (see Hubbard, 1984), links between private pensions and social security are important in this context.²⁶ That annuity

markets are extremely imperfect in the real world need not be evidence of a severe market failure, as individuals have some control over their participation in private pensions either explicitly (for participants in defined-contribution plans) or implicitly (through choice of employer). To the extent that individuals adjust their pensions for variation in social security annuities, the effective annuity market may be quite large. The magnitude of that adjustment must be resolved empirically.²⁷ A second and related point is that, given the current political environment, introducing uncertainty over future social security benefits may be appropriate. That uncertainty would modify the wealth impacts derived here.

A third extension is to consider the market imperfection introduced by restrictions on borrowing against future earnings.²⁸ Young workers are hardest hit by this restriction, as their current earnings are below their lifetime average earnings. If the liquidity constraint is then binding in their life-cycle optimization, they will consume all of their after-tax income; that is, social security taxes reduce consumption one for one. Hence, reducing payroll taxes during youth would raise lifetime utility, even if the reduction were compensated (in the sense of raising required contributions later in the life cycle).

The debate over the influence of pensions on individual saving brings together questions of consumer choice under uncertainty and the effectiveness of fiscal policy. Researching the relationships among social security, private pensions, annuity markets, and bequests facilitates close empirical scrutiny of models of individual and aggregate saving, permitting consideration of the welfare effects of compulsory pensions. In addition, while this paper has concentrated on annuity insurance, similar approaches could be used to study the impacts of other social insurance programs on national saving.

FOOTNOTES

¹Empirical tests of the life-cycle model under certainty have tested the hypothesis of a hump-shaped wealth-age profile, but results have by no means unambiguously validated the model. See for example White (1978), Mirer (1979), and Kurz (1981). Even after controlling for the effects of permanent income, Blinder, Gordon, and Wise (1981), Diamond and Hausman (1982), King and Dicks-Mireaux (1982), and Hubbard (1983) found results only mildly supportive of the basic theory. Other studies have addressed the possibility of other motives for saving. Kotlikoff and Summers (1981) rejected the ability of the life-cycle model to explain wealth accumulation in the U. S., putting forth a major role for bequests.

²Earlier studies for private pensions include those of Cagan (1965), Katona (1964), and Munnell (1974). Feldstein's results have by no means gone unchallenged; see for example Leimer and Lesnoy (1982) and the reply in Feldstein (1982). Microeconomic (cross-section) evidence has generally been supportive of the proposition that social security has reduced individual saving. See Feldstein and Pellechio (1979), Kotlikoff (1979b), Blinder, Gordon, and Wise (1981), Diamond and Hausman (1982), King and Dicks-Mireaux (1982), and Hubbard (1983).

³Hence, previous partial equilibrium estimates of the impact of social security on saving drawn solely from consideration of the intergenerational wealth transfer at the introduction of the system are, if anything, too small. Abel (1983) takes up the intergenerational consequences of this point in a two-period overlapping-generations model, with the implication that the insurance features of social security may reduce inequality in the distribution of wealth.

⁴Another discussion of this point can be found in Kotlikoff, Shoven, and Spivak (1983).

⁵The precise direction of the influence of this uncertainty for saving is unclear. Heightened uncertainty over the length of life may lead to more saving (because of a longer than expected lifetime) or to less saving (to maintain present consumption). In the argument of Yaari (1965), two individuals with identical tastes, income, and investment opportunities are compared. The difference between them is that one lives T periods for certain while the other faces an uncertain lifetime of t periods, up to a maximum of T periods. Given a shorter expected life, uncertainty over length of life unambiguously leads to increased initial consumption. Champernowne (1969) and Levhari and Mirman (1977), on the hand, consider two agents with identical expected lives, but differing in the distribution of length of life. In either case, the impact of uncertainty over length of life on wealth accumulation of a risk-averse individual is ambiguous and depends on the parameters of the model.

- ⁶That is, D is just the weighted average of the years t in $(Q+1, D')$, with weights $(1-p_t)$ for each t .
- ⁷Discussions of the lifetime utility function can be found in Strotz (1956) and Atkinson (1971).
- ⁸Individuals will die on average prior to reaching age D' , but the lifetime budget constraint reflects the possibility that the individual will live through D' . In no case can the present value of consumption exceed lifetime resources. The problem is simplified here by making lifespan (and earnings) in the interval $(0, Q)$ nonstochastic.
- ⁹This formulation of the utility function is used by Summers (1981), Kotlikoff and Spivak (1981), Kotlikoff and Summer (1981), Evans (1983), and Seidman (1984). As noted by Evans (1983), it is the only additive utility function consistent with the original homogeneity hypothesis of Modigliani and Brumberg (1954).
- ¹⁰This role of annuities as a mechanism for sharing uncertainty about longevity is an integral part of Diamond's (1977) evaluation of the social security system, in which he focuses on the absence of complete markets for such contracts. Merton (1983) considers Pareto-improving social security programs in an intertemporal model in which human capital is not tradeable.
- ¹¹Previous work in this area in the context of pensions includes the contributions of Davies (1981) and Sheshinski and Weiss (1981). Davies used a life-cycle model under uncertain lifetime to address the phenomenon of slow dissaving in retirement. The presence of pensions in his simulation model (using Canadian data) reduced, but by no means eliminated, the effect of uncertainty on retirement consumption. In the model of Sheshinski and Weiss, the ultimate impact of social security on saving depends on the availability of a private annuity market. They found that, at the optimum, Yaari's (1965) result holds, namely that private savings are reserved for bequests, while social security benefits are used to finance retirement consumption.
- ¹²The actuarially fair benefit is constructed with respect to economy-wide survival probabilities. It is true that individuals who believe they will die "young" will want to purchase less than the "average optimal" amount of social security annuities, while those who expect to live a long time will want more. Both groups are better off, however, with the mandatory social security than without it, since in its absence, adverse selection is assumed to foreclose the possibility of a market of private annuities. A discussion of the potential separating equilibria in the private provision of annuities which may arise after the imposition of mandatory social security is given Eckstein, Eichenbaum, and Peled (1983).

- ¹³Uncertainty over future social security benefits would mitigate the effect shown here. Watson (1982) discusses the influence of uncertainty over benefits in assessing the impact of social security on saving. Merton, Bodie, and Marcus (1984) show that many private pension integration arrangements remove much of this uncertainty.
- ¹⁴A retirement age of 65 was assumed. Probabilities for survival were taken from Faber (1982).
- ¹⁵This effect is most pronounced in the absence of explicit capital market restrictions. With no initial endowment (and, hence, binding restrictions on the nonmarketability of social security when young), relative impacts on "working-period" and "retirement-period" consumption will depend on the relationship of the individual's actual and optimal tax rate (participation). The importance of (accidental) bequests as intergenerational links will be discussed later.
- ¹⁶This nonlinearity has surfaced in some recent studies of the impact of social security on saving. See for example Diamond and Hausman (1982) and Hubbard (1983).
- ¹⁷In a world with capital market restrictions, then, a social security system of this type may increase saving, since received initial bequests are more liquid than anticipated social security benefits. The impact of social security on intergenerational transfers is an important component of the system's net effect on individual saving.
- ¹⁸The implicit assumption, of course, is that the parent dies at the beginning of the child's (optimizing) life, age twenty here. This assumption is made to highlight the point that the existence of social security for the previous generation mitigates the impact of the present generation's participation in social security on its own wealth accumulation. More general assumptions about the timing of a testator's death would complicate expressions like (13) in the text, but the qualitative point would remain.
- ¹⁹Kotlikoff (1979a) used a life-cycle model with certain longevity and a Cobb-Douglas production technology to consider the general equilibrium impact of a pay-as-you-go social security system. For plausible parameter values, he found that the positive lifetime wealth increment traceable to social security (because of the growth of the wage base) caused a 20-percent reduction in the steady-state capital stock. While this effect is certainly substantial, it is roughly half of his calculated partial equilibrium effect, which is directly related to the excess of the present value of benefits over the present value of payroll contributions. Kotlikoff's analysis also incorporates the influence of social security on retirement age, which is taken as exogenous here. To the extent that social security lowers the desired retirement age, the partial equilibrium wealth replacement effect of social security on saving is dampened.

- ²⁰Hence non-social-security government saving is zero, and no role for debt policy exists. Auerbach and Kotlikoff (1983) discuss the importance of the stance of debt policy in assessing transitions between fiscal regimes.
- ²¹As in Table 1, survival probabilities are taken from Faber. For a more complete discussion of the implications of the choice of parameter values, see Levhari and Mirman (1977) or Davies (1981).
- ²²Kotlikoff and Summers (1981) report that annual growth rates of productivity (real GNP per man-hour) and population in the U. S. during the twentieth century have been 2.2 percent and 1.4 percent, respectively.
- ²³Note that the consumption increase is higher in the case of risk aversion. The calculation proceed as follows. When $\gamma = -1.0$, $\omega = 2.677$; when $\gamma = -3.0$, $\omega = 2.747$; when $\gamma = -5.0$, $\omega = 2.784$. The gains are evaluated as $t_s(\omega-1)$ as described in Section II.
- ²⁴Bequests are incorporated as follows. Using information on $p_t(t=Q+1, \dots, D')$, a distribution of bequests is generated. All individuals receive the weighted average bequest regardless of their particular family mortality history. The implicit assumption is that individual bequests are taxed away by the government and distributed lump sum to individuals.
- ²⁵Clearly other bequest formulations outside the infinite horizon optimization in Barro (1974) are available. Evans (1983), for example, considers two alternatives: (i) individuals pass on a bequest related by the steady-state growth rate to initial inheritances; or (ii) both inheritances and bequests are invariant to interest rate movements. In Seidman's certain lifetime model, β is equivalent to the ratio of the bequest to consumption in the last period of life.
- ²⁶To the extent that high-income individuals are constrained to less than their desired participation in social security, there is excess demand for social security annuities. Adverse selection and the possibility of multiple insurance still render unlikely the provision of such annuities by competitive insurance companies. Employer-sponsored private pensions may act to fill this gap. Employers are likely to have better information on individual workers' life expectancies than would a disinterested insurance company. Second, by definition, such annuities can only be purchased at an individual's place of work; multiple insurance is not possible. Pauly (1974) and Wilson (1977) discuss certain situations in which private market equilibria might occur after a compulsory insurance program is imposed.
- ²⁷In addition, imperfect annuity arrangements may be present at the level of the family (see for example Kotlikoff, Shoven, and Spivak, 1983). Given any of these modifications, the partial equilibrium impact of the capital stock of introducing actuarially fair social

security annuities is mitigated. The point remains, however, that the impact of social security on individual consumption in the new steady state will differ from that on initial participants.

²⁸Hayashi (1982) found that approximately 20 percent of all consumption is accounted for by such liquidity constrained individuals. Flavin (1984) found that the estimate of the marginal propensity to consume is affected dramatically by the inclusion of proxies for liquidity constraints, suggesting that liquidity constraints are an important part of the observed excess sensitivity of consumption to current income.

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