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ABSTRACT

We present a pricing kernel that summarizes well the main features of the dynamics of interest rates and risk in postwar U.S. data and use it to uncover how the pricing kernel has moved with the short rate in this data. Our findings imply that standard monetary models miss an essential link between the central bank instrument and the economic activity that monetary policy is intended to affect and thus we call for a new approach to monetary policy analysis. We sketch a new approach using an economic model based on our pricing kernel. The model incorporates the key relationships between policy and risk movements in an unconventional way: the central bank's policy changes are viewed as primarily intended to compensate for exogenous business cycle fluctuations in risk which threaten to push inflation off target. This model, while an improvement on standard models, is considered just a starting point for their revision. It leads to critical questions that researchers need to answer as they continue to revise their approach to monetary policy analysis.

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Modern models of monetary policy start from the assumption that the central bank controls an asset price, namely the short rate, as its policy instrument. In these models this policy instrument is then linked to the economy through agents' Euler equation for nominal bonds. More abstractly, the Euler equation links the policy instrument to the economy through the model's pricing kernel. To be useful, a model of how monetary policy affects the economy should account for how the pricing kernel has moved with the short rate in postwar U.S. data[1].

In this paper we use data on the dynamics of interest rates and risk to uncover how the pricing kernel has moved with the short rate in postwar U.S. data. Our two main findings are that

- Most (over 90%) of the movements in the short rate correspond to random walk movements in the conditional mean of the pricing kernel. We refer to these movements as the *secular* movements in the short rate.
- The remaining movements in the short rate, which we refer to as the *business cycle* movements in the short rate correspond to movements in the conditional variance of the pricing kernel associated with changes in risk.

Standard models used for monetary policy analysis are inconsistent, by construction, with these regularities and, hence, do not capture how the pricing kernel moves with the short rate. We argue that this inconsistency is a serious problem if we want to use these models to understand monetary policy and the macroeconomy. We argue that a new approach to analyzing monetary policy is needed.

Here we sketch a new approach to analyzing monetary policy. To do so we build an economic model consistent with the comovements of interest rates and risk found in U.S. data. Using this model we interpret postwar monetary policy as follows.

- Secular movements of the short rate arise as a result of random walk movements in the Fed's inflation target.

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[1] Throughout this paper we consider models in which all variables are conditionally log-normal and we use the term *pricing kernel* as short-hand for the log of the pricing kernel.

- Business cycle movements of the short rate arise as a result of the Fed's endogenous policy response to exogenous business cycle fluctuations in risk. The Fed chooses this policy response to maintain inflation close to its target.

In our economic model, what the Fed is doing over the business cycle is simply responding to exogenous changes in the real risk. Specifically the Fed is responding to exogenous changes in the conditional variance of the real pricing kernel with the aim of maintaining inflation close to a target level. Clearly, this view differs substantially from the standard view of what the Fed does over the business cycle. In the standard view, risk plays no role. Instead the Fed's policy is a function of its forecasts of economic variables that enter the mean of the pricing kernel, such as expected real growth and expected inflation. This policy is often summarized by a Taylor rule. Our interpretation of the historical record is that over the business cycle what the Fed actually did has little to do with these forecasts about changes in conditional means of growth and inflation. Instead, policy mainly responded to exogenous changes in real risk.

While we find our model helpful in interpreting the data, it represents, at best, a start to a new approach. Going beyond this specific model, our empirical findings lead us to raise two broader questions to be answered in future research in monetary policy analysis.

The first question regards the secular movements in the Fed's policy instrument: Why did the Fed choose such large secular movements in its policy instrument, namely the short rate? In our economic model we mechanically describe the secular movements in Fed policy as arising from a random walk inflation target. Our approach here is similar to that followed in many recent monetary models. The main problem we see with this approach is that it attributes the vast bulk of the movements in the Fed's policy instrument to a purely mechanical factor. Thus while this approach may be adequate as a statistical description of Fed policy, it seems useless for answering fundamental questions at any more than a superficial level: Why did the great inflation of the 1970s occur? Why did it end? Is it likely to occur again? and How can we change institutions to reduce that likelihood?

We argue that to answer such questions, a deeper model of the forces driving the secular component of policy is needed. We briefly discuss some ambitious attempts by Orphanides (2002), Sargent, Williams, and Zha (2005), and Primiceri (2006) at modeling these forces but find them wanting. We are led to call for a new approach to modeling the economic forces underlying the secular movements in Fed policy.

The second question regards the business cycle comovements between the Fed's policy instrument and the macroeconomy as captured in the Euler equation: How do we fix our models so that they capture this link? The Euler equation in standard monetary models links the short rate to expectations of growth in the log of the marginal utility of consumption and inflation. Canzoneri, Cumby, and Diba (2007) document that this Euler equation in these models does a poor job of capturing this link between policy and the economy at business cycle frequencies.

We offer a potential explanation for the failure of the Euler equation. Existing research nearly universally imposes that the conditional variances of these variables that enter the Euler equation are constant. Thus, all the movements in the pricing kernel in these models arise from movements in conditional means. With our model of the pricing kernel we find precisely the opposite, at least for the business cycle. That is, over the business cycle nearly all of the movements in the Euler equation come from movements in conditional variances and not from conditional means.

Given this finding we argue that recent attempts to fix this Euler equation by making the conditional means of the pricing kernel more volatile while continuing to assume that the conditional variances are constant are misguided. We argue that instead researchers should be looking for a framework that delivers smooth conditional means and volatile conditional variances of the pricing kernel at business cycle frequencies. That is, researchers should come to terms with the fact that at business cycle frequencies interest rates move one for one with risk.

In terms of antecedents for this work, our pricing kernel builds on the work of Backus, Foresi, Mozumdar, and Wu (2001) and Backus, Foresi, and Telmer (2001). Our economic model is a pure exchange economy with exogenous time-varying real risk. Since the early contribution by McCallum (1994) a large literature has studied interest rates in such economies. Examples include Wachter (2006), Bansal and Shaliastovich (2007), Gallmeyer, Hollifield, Palomino, and Zin (2007), Piazzesi and Schneider (2007). Our model draws most heavily from the work of Gallmeyer, Hollifield, and Zin (2005).

Our paper proceeds as follows. Section 1 documents four key regularities regarding the dynamics of interest rates and risk that we use to guide our construction of the pricing kernel. Section 2 documents that standard monetary models are inconsistent with these regularities and lays out our pricing kernel. Section 3 presents our two main findings regarding the comovements of the short rate and the pricing kernel in postwar U.S. data. Section 4 presents the economic model we use the interpret these findings. Section 5 discusses the two broader questions for monetary policy research that follow from our findings. Section 6 concludes.

1. The Behavior of Interest Rates and Risk: Evidence

Empirical work in finance over the last several decades has established some regularities regarding the dynamics of interest rates and risk that any useful analysis of monetary policy must address.

In this paper we focus on the implications of four of these regularities for the analysis of monetary policy. We will argue that standard monetary models are not consistent with these regularities and that a new approach is needed if we are to build models for monetary policy analysis that are consistent with these regularities. We document these four regularities here.

Two of the regularities regard the dynamics of interest rates and two regard the comovements of interest rates and risk.

A. Dynamics of Interest Rates

To document the first two regularities we use a traditional principal components analysis to summarize the dynamics of the yield curve. This analysis reveals the following two regularities.

1. The first principal component accounts for a large majority of the movements in the yield curve. Because it is associated with similar movements in the yields on all maturities (essentially parallel shifts in the term structure), this component is commonly referred to as the *level factor* in interest rates. It also has the property that it is (nearly) permanent and is well modeled by a random walk. We here will refer to the first principal component as the *secular component* of interest rates in order to emphasize that permanence. In the data this secular component corresponds closely to the long rate.

2. The second principal components accounts for most of the remaining movements in the yield curve. Because it is associated with changes in the difference between the short rate and the long rate—with changes in the slope of the yield curve—it is commonly referred to as the *slope factor* in interest rates. This component also captures most of the movements in interest rates at business cycle frequencies. We will here refer to this component as the *business cycle component* of interest rates in order to emphasize that property. In the data, this business cycle component is essentially the yield spread between the long rate and the short rate.

We document these two regularities here. We use monthly data on the rates of U.S. Treasury bills of maturities of three months and imputed zero coupon yields for maturities of from one to thirteen years over the postwar period from 1946:12 to 2007:12. For 1946:12–1991:2, we use data from McCulloch and Kwon (1993) for these series; for 1991:3–2007:12 we use CRISP data for the three-month T-bill rate and data from Gurkaynak, Sack, and Wright (2006) for the other zero coupon rates. (In the rest of our analysis, we use the three-month T-bill rate as our measure of the short rate and the thirteen-year zero coupon rate as our measure of the long rate.)

Our principal components analysis of the yield curve uses the traditional procedure (closely following that of Piazzesi forthcoming, Section 7.2.) We focus on the first two principal components, which together account for over 99% of the variance of the short rate and over 99.8% of the total variance of all yields. In Figure 1 we plot the short rate and the first two principal components of the yield curve which result from our analysis.[2]

To document our first regularity, we note that the first principal component accounts for over 90% of the variance of the short rate. (It also accounts for over 97% of the total variance of all yields). This component has a monthly autocorrellation over .993. Figure 1 demonstrates visually that this component captures the long secular swings in the short rate. Figure 2 demonstrates that it also corresponds closely to the long rate.

To document our second regularity, we show in Figure 3 that the second principal component is very similar to the yield spread between the short and long rate. This component

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[2] We have scaled these principal components so that the short rate's loadings on each of these components are equal to one.

has a monthly correlation of .957. Figure 1 demonstrates that, barring one exception in the early 1980s, this component captures well the business cycle movements in the short rates.

B. Interest Rates and Risk

With regard to the dynamics of interest rates and risk, decades of empirical work has revealed that movements in the business cycle component of interest rates are associated with substantial movements in risk. Specifically, this work has found two regularities regarding the comovements of interest rates and expected excess returns.

3. Movements in the difference between the short rate and the long rate—that is, the *yield spread*—are associated with movements risks, defined as in the expected excess returns to holding long term bonds of a similar magnitude.
4. Movements in the short rate relative to foreign-currency short rates are associated with movements in risk, defined as the expected excess returns to holding foreign-currency bonds of a similar magnitude.

We follow much of the literature in interpreting movements in expected excess returns as movements in the compensation for risk.[3]

Before we cite some of the work documenting these regularities, let's describe them more precisely.

We begin with the regularity on the yield spread and the expected excess returns to holding long bonds. We use the following notation to describe these empirical results. Let P_t^k denote the price in time period t of a zero-coupon bond that pays off one dollar in period $t+k$ and let $p_t^k = \log P_t^k$. Then the (log) *holding period return*, that is the return to holding this k period bond for one period is $r_{t+1}^k = p_{t+1}^{k-1} - p_t^k$. The (log) *excess return* to holding this bond over the short rate i_t is $r_{xt+1}^k = r_{t+1}^k - i_t$. The *risk premium* on long bonds is the expected excess return $E_t r_{xt+1}^k$. Many researchers have run return forecasting regressions of excess returns against the yield spread similar to the regression

$$(1) \qquad r_{xt+1}^k = \alpha^k + \beta^k (y_t^k - i_t) + \varepsilon_{t+1}^k,$$

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[3] The bulk of the asset pricing literature interprets measured returns as capturing the total payoffs to owning an asset and accounts for differences in returns as arising from differences in risk. In doing so this literature assumes that measured returns do not leave out some portion of total returns, such as taxes transactions costs or liquidity services that both differ across assets and vary over the business cycle.

where $y_t^k \equiv -p_t^k/k$ is the yield to maturity on this bond. Regressions this form have been run for 20 years, starting with the work of Fama and Bliss (1987). (See also the work of Campbell and Shiller (1991) and Cochrane and Piazzesi (2005).)

Note that under the hypothesis that the risk premia on long bonds are constant over time, the slope coefficient β^k of this regression should be zero. In the data, however, these regressions yield estimates of β^k that are significantly different from zero with point estimates typically greater than one for moderate to large k.

We emphasize the magnitude of this slope coefficient here because these regression results thus imply that the risk premium on long bonds moves more than one for one with the yield spread. More precisely, note that a finding that the slope coefficient $\beta^k \geq 1$ implies that

$$(2) \qquad cov(E_t r_{xt+1}^k, y_t^k - i_t) \geq var(y_t^k - i_t)$$

which, by the use of simple algebra, implies that the variance in the risk premium on long bonds is greater than that of the yield spread:

$$(3) \qquad var(E_t r_{xt+1}^k) \geq var(y_t^k - i_t).$$

The fourth regularity regarding movements in the spread between the short rate and foreign currency denominated short rates and the expected excess returns to holding foreign currency denominated bonds is simply a consequence of the empirical finding that exchange rates are well-approximated by random walks as documented by Meese and Rogoff (19??) and much subsequent work.

To see this, let

$$(4) \qquad r_{xt+1}^* = i_t^* + e_{t+1} - e_t - i_t$$

denote the (log) excess return on a foreign short bond with rate i_t^* where e_t is the log of the exchange rate. If exchange rates are a random walk, then $E_t e_{t+1} = e_t$, so that

$$(5) \qquad E_t r_{xt+1}^* = i_t^* - i_t.$$

That is, the expected excess return on a foreign bond is simply the interest differential across currencies.

2. Towards an Economic Model

In this section we present the result that standard models, by assumption, cannot match the dynamics of interest and risks that we have discussed and we present a simple model of the pricing kernel that is consistent with these dynamics.

A. The Standard Euler Equation

Consider first the link between the short rate and macroeconomic aggregates built into standard monetary models.

We begin with representative agent models. The short term nominal interest rate enters standard representative consumer models through an Euler equation of the form

$$(6) \qquad \frac{1}{1+\mathrm{i}_t} \equiv \exp(-i_t) = \beta E_t \left[\frac{U_{ct+1}}{U_{ct}} \frac{1}{\pi_{t+1}} \right],$$

where i_t is the logarithm of the short term nominal interest rate $1+\mathrm{i}_t$, β and U_{ct} are the discount factor and the marginal utility of the representative consumer, and π_{t+1} is the inflation rate. Analysts then commonly assume that the data are well-approximated by a conditionally lognormal model so that this Euler equation can be written as

$$(7) \qquad i_t = -E_t \left[\log \frac{U_{ct+1}}{U_{ct}} \frac{1}{\pi_{t+1}} \right] - \frac{1}{2} \mathrm{var}_t \left[\log \frac{U_{ct+1}}{U_{ct}} \frac{1}{\pi_{t+1}} \right].$$

A critical question in monetary policy analysis is what terms on the right hand side of (7) change when the monetary authority changes the interest rate i_t. The traditional assumption is that conditional variances are constant, so that the second term on the right side of (7) is constant. This leaves the familiar version of the Euler equation:

$$(8) \qquad i_t = -E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1} + \text{constant}.$$

Thus, by assumption, standard monetary models imply that movements in the short rate are associated one-for-one with the sum of movements in the expected growth of the log of the marginal utility of the representative consumer and expected inflation. The debate in the literature on the effects of monetary policy might thus be summarized roughly as a debate over how much of the movement in the short rate is reflected in the expected growth of the log of marginal utility of consumption (representing a *real* effect of monetary policy) and how much of the movement is reflected in expected log inflation (representing a *nominal* effect of monetary policy). A resolution of this debate in the context of a specific model depends on the specification of its other equations. However, virtually universally, the possibility that movements in the short rate might be associated with changes in the conditional variances of these variables is ruled out by assumption.

We have described the standard Euler equation in the context of a model with a representative consumer. Our discussion also applies to more general models which do not assume a representative consumer. To see this note that we can write equations (6)-(8) more abstractly in terms of a nominal *pricing kernel* (or stochastic discount factor) m_{t+1} as

$$\exp(-i_t) = E_t \exp m_{t+1}. \tag{9}$$

In a model with a representative agent this pricing kernel is given by $\exp(m_{t+1}) = \beta U_{ct+1}/(U_{ct}\pi_{t+1})$ and (9) is the representative agent's first-order condition for optimal bond holdings. In some segmented market models (9) is first-order condition for the subset of agents who actually participate in the bond market; in others, (9) is no single agent's first-order condition. In general, (9) is implied by lack of arbitrage possibilities in the financial market.

Using conditional lognormality, we see that (9) implies that

$$i_t = -E_t\left[m_{t+1}\right] - \frac{1}{2}var_t\left[m_{t+1}\right] \tag{10}$$

and with constant conditional variances, we have that

$$i_t = -E_t m_{t+1} + \text{constant}. \tag{11}$$

Thus the more general assumption made in the literature is that movements in the short term interest rate are associated with movements in the conditional mean of the log of the pricing kernel and not with movements in its conditional variance.

Standard monetary models with constant conditional variances are clearly inconsistent with the evidence on the comovements of interest rates and risk. We can see this by considering the following proposition:

Proposition 1. In any model with a pricing kernel in which variables are conditionally lognormal and conditional second movements are constant, risk is constant.

Proof. Let m_{t+1} be (the log of) the pricing kernel and let r_{t+1} be any log asset return. Lack of arbitrage implies the standard asset pricing formula:

$$1 = E_t \exp(m_{t+1} + r_{t+1}) \tag{12}$$

Taking logs of (12) and using conditional lognormality gives that

$$0 = E_t m_{t+1} + E_t r_{t+1} + \frac{1}{2} var_t(m_{t+1} + r_{t+1})$$

Using (10) implies that the expected excess return on this asset:

$$E_t r_{t+1} - i_t = -\frac{1}{2} var_t(r_{t+1}) - cov_t(m_{t+1}, r_{t+1}). \tag{13}$$

So if conditional second moments are constant, then expected excess returns are constant. Hence risk is constant. $Q.E.D.$

Proposition 1 implies that when we log-linearize our models and impose that the primitive shocks have constant conditional variances, risk is constant. Our reading of the literature on monetary policy is that these assumptions are nearly universal. Yet as we have seen, the evidence is clear that risk is not constant. This seems a serious problem if we want to use these models to understand what in the macroeconomy moves when the short rate moves.

B. A Simple Model of the Pricing Kernel

Here we present a simple model of the pricing kernel that is consistent with the evidence on interest rates and risk that we have discussed. This model serves as a statistical summary of the joint dynamics of interest rates and risk. In the next section we use this model to decompose movements in the short rate observed in postwar U.S. data into movements in the conditional mean of the pricing kernel and its conditional variance. This model is similar to the "negative" Cox-Ingersoll-Ross model analyzed by Backus, Foresi, Mozumdar, and Wu (2001) augmented with a random walk process and an independently and identically distributed (i.i.d.) shock to the pricing kernel. To analyze the expected excess returns on foreign bonds we extend the model to having two countries and two currencies in a manner similar to that in the work 2001 work of Backus, Foresi, and Telmer.

The Home Country Pricing Kernel

The model has two state variables z_{1t} and z_{2t} that govern the dynamics of the pricing kernel, interest rates, and risk. One state variable follows a random walk with

$$(14) \quad z_{1t+1} = z_{1t} + \sigma_1 \varepsilon_{1t+1}$$

and the other follows an AR1 process with heteroskedastic innovations given by

$$(15) \quad z_{2t+1} = (1-\varphi)\theta + \varphi z_{2t} + z_{2t}^{1/2}\sigma_2 \epsilon_{2t+1}.$$

The innovations $\varepsilon_{1t+1}, \varepsilon_{2t+1}$, are independent, standard, normal random variables. Because these state variables are independent and all yields will be linear combinations of these variables, they correspond to the principal components of the yield curve implied by this pricing kernel. We will show below that z_{1t} is a level factor and z_{2t} is a slope factor. To emphasize its persistence we refer to z_{1t} in the model as the *secular* component of interest rates. Because it is stationary we refer to z_{2t} in the model as the *business cycle* component of interest rates. (We calibrate our model so that the secular and business cycle components in the model correspond closely to the secular and business cycle components that we have identified in the data.)

We use these two state variables to parameterize the dynamics of the pricing kernel. The (log of the) pricing kernel m_{t+1} is given by

$$(16) \quad -m_{t+1} = \delta + z_{1t} + \sigma_1 \epsilon_{1t+1} - (1-\lambda^2/2) z_{2t} + z_{2t}^{1/2}\lambda \epsilon_{2t+1} + \sigma_3 \epsilon_{3t+1}$$

where ϵ_{3t+1} is a third independent standard normal random variable.

The Short Rate

Given this stochastic process for the pricing kernel, we use the standard asset pricing formula

$$i_t = -\log E_t \exp(m_{t+1})$$

to solve for the dynamics of the short rate. Because the pricing kernel is conditionally lognormal, we have that

$$(17) \quad i_t = -E_t m_{t+1} - \frac{1}{2} Var_t(m_{t+1})$$

so that movements in the short rate correspond to a combination of movements in the conditional mean of the log of the pricing kernel and movements in the conditional variance of the log of the pricing kernel. Observe that the conditional mean of the log of the pricing kernel is given by

$$E_t m_{t+1} = -\delta - z_{1t} + (1 - \lambda^2/2) z_{2t} \tag{18}$$

and that the conditional variance of the log of the pricing kernel is given by

$$\frac{1}{2} Var_t(m_{t+1}) = \frac{1}{2}(\sigma_1^2 + \sigma_3^2) + \frac{\lambda^2}{2} z_{2t}. \tag{19}$$

We thus have that

$$i_t = \delta - \frac{1}{2}(\sigma_1^2 + \sigma_3^2) + z_{1t} - z_{2t} \tag{20}$$

Note that the structure of this model implies that the state variable z_{1t} is the secular component of the short rate and the state variable z_{2t} is the business cycle component of the short rate.

In contrast to standard monetary models, this model allows for variation over time in the conditional variance of the pricing kernel. As (19) makes clear, that variation corresponds to business cycle movements in the short rate, with the extent of that variation governed by the parameter λ. In particular, λ governs how movements in the business cycle component of the short rate are divided up between movements in the conditional mean of the (log of the) pricing kernel and the conditional variance of the (log of the) pricing kernel. Specifically, the response of the conditional mean of the pricing kernel to z_{2t} is $1 - \lambda^2/2$, and the response of 1/2 the conditional variance is $\lambda^2/2$. Thus, if $\lambda = 0$, then here, as in the standard model, the conditional variance of the pricing kernel is constant and all movements in z_{2t} correspond to movements in the conditional mean of the log of the pricing kernel. In contrast, if $\lambda = \sqrt{2}$, then the conditional mean of the pricing kernel does not respond to movements in z_{2t} while one half the conditional variance of the pricing kernel responds one for one with z_{2t}. If $\lambda > \sqrt{2}$, then the conditional mean and the conditional variance of the pricing kernel move in opposite directions when the business cycle component of the short rate moves.

Longer-Term Interest Rates

To solve for longer term interest rates we use the standard asset pricing formula

$$(21) \quad p_t^k = \log E_t \exp(m_{t+1} + p_{t+1}^{k-1})$$

to set up a recursive formula for bond prices. These prices are linear functions of the states z_{1t} and z_{2t} of the form.

$$(22) \quad p_t^k = -A_k - B_k z_{1t} - C_k z_{2t},$$

where A_k, B_k, and C_k are constants. Then we can use standard undetermined coefficients to derive this proposition:

Proposition 2. The coefficients of the bond prices are given recursively by

$$A_k = \delta + A_{k-1} + C_{k-1}(1-\varphi)\theta - \frac{1}{2}(B_{k-1}+1)^2\sigma_1^2 - \sigma_3^2,$$

$$B_k = B_{k-1} + 1,$$

and

$$C_k = -(1-\lambda^2/2) + C_{k-1}\varphi - \frac{1}{2}(\lambda + C_{k-1}\sigma_2)^2,$$

with $A_1 = \delta - (\sigma_1^2 + \sigma_3^2)/2$, $B_1 = 1$, $C_1 = -1$.

Proof. To find these prices, we start with $k = 1$ to find the price of the short-term bond, using the asset pricing formula (21) with $p_{t+1}^0 = 0$, so that

$$p_t^1 = \log E_t \exp(m_{t+1}) = E_t m_{t+1} + \frac{1}{2} var_t(m_{t+1})$$

so plugging into both sides gives that

$$-A_1 - B_1 z_{1t} - C_1 z_{2t} = -\delta - z_{1t} + \frac{1}{2}(\sigma_1^2 + \sigma_3^2) + z_{2t}.$$

For $k > 1$, we write the coefficients at k as functions of the coefficients at $k-1$ as follows. Given our form in (22), we know that

$$p_{t+1}^{k-1} = -A_{k-1} - B_{k-1} z_{1t+1} - C_{k-1} z_{2t+1}$$

Using the form of the dynamics of the state variables (14) and (15) we have that

$$p_{t+1}^{k-1} = -A_{k-1} - B_{k-1}z_{1t} - B_{k-1}\sigma_1\epsilon_{1t+1} - C_{k-1}(1-\varphi)\theta - C_{k-1}\varphi z_{2t} - C_{k-1}\sigma_2 z_{2t}^{1/2}\epsilon_{2t+1}.$$

Note then that this bond price is conditionally lognormal. Combining this bond price with our form for m_{t+1} gives that

$$\log E_t \exp(m_{t+1} + p_{t+1}^{k-1}) = E_t(m_{t+1} + p_{t+1}^{k-1}) + \frac{1}{2}var_t(m_{t+1} + p_{t+1}^{k-1})$$

$$= -\delta - A_{k-1} - C_{k-1}(1-\varphi)\theta + \frac{1}{2}(B_{k-1}+1)^2\sigma_1^2 - (B_{k-1}+1)z_{1t} -$$

$$[-(1-\lambda^2/2) + C_{k-1}\varphi]z_{2t} + \frac{1}{2}(\lambda + C_{k-1}\sigma_2)^2 z_{2t} + \sigma_3^2.$$

Using

$$p_t^k = -A_k - B_k z_{1t} - C_k z_{2t}$$

then gives recursive formulas for the coefficients of bond prices and yields. *Q.E.D.*

Level and Slope Factors ≈ Secular and Business Cycle Components

We now show that in our model the secular component of interest rates z_{1t} corresponds to a level factor which leads to parallel shifts in the yield curve and that the business cycle component z_{2t} corresponds to a slope factor which leads to changes in the spread between the long and short rates.

Since yields are related to prices by $y_t^k = -p_t^k/k$, (22) implies that yields can be written as

$$y_t^k = \frac{1}{k}\left(A_k + B_k z_{1t} + C_k z_{2t}\right).$$

Thus, the implications of this model for the yield curve and its movements depend on the behavior of the coefficients A_k/k, B_k/k, C_k/k. Note here that our recursion implies that $B_k = k$. Thus we can write yields as

$$y_t^k = z_{1t} + \frac{1}{k}\left(A_k + C_k z_{2t}\right).$$

Clearly, movements in the secular component z_{1t} correspond to parallel shifts in the yield curve because when this component moves all yields shift by the same amount. Hence, this component corresponds to a level factor in yields.[4] Note that this result follows from the fact that z_{1t} is a random walk.

We next show that z_{2t} corresponds to a slope factor. To see this, note that C_k converges to a negative constant $\bar{C}$ as k grows. Hence, for large k, movements in z_{2t} have no impact on long yields since $\bar{C}/k$ goes to zero as k gets large. In particular, since $C_1 = -1$, we have that any yield differential is given by

$$y_t^k - i_t = \mathit{constant} + (C_k/k + 1)\, z_{2t}$$

and the observation that C_k/k converges to zero as k gets large implies that at the same time the yield differential converges to

$$y_t^k - i_t = \mathit{constant} + z_{2t}.$$

Thus, z_{2t} is a slope factor in that movements in it correspond to movements in the spread between the long rate and the short rates for long enough maturity bonds.

Expected Excess Returns

We now turn to our model's implications for expected excess returns on both long term bonds and foreign currency denominated bonds.

Long Term Bonds We begin with the excess returns to holding a long term bond for one period. To computed these in our model we use the asset pricing formula (21). Since bond prices and the pricing kernel are conditionally lognormal, we can write this formula as

$$p_t^k = E_t m_{t+1} + E_t p_{t+1}^{k-1} + \frac{1}{2} Var_t(m_{t+1} + p_{t+1}^{k-1}).$$

Hence, the expected excess return on a k period bond is given by

$$E_t r_{xt+1}^k = E_t p_{t+1}^{k-1} - p_t^k - i_t = \frac{1}{2} Var_t(m_{t+1}) - \frac{1}{2} Var_t(m_{t+1} + p_{t+1}^{k-1}),$$

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[4] Note that theoretically, the inclusion of a random walk component of the short rate leads to counterfactual implications for the average value of very long yields. This is because A_k is has a component that grows linearly with k as k gets large and then a component that grows with k^2 coming from B_{k-1}^2. This impies that for large k, the constant A_k/k goes to negative infinity fast. We will not worry about this limiting implication. Instead, we imagine that the random walk component of interest rates is in fact stationary, but that it appears to be a random walk over a 30-year horizon.

or, equivalently

$$(23) \quad E_t r^k_{xt+1} = -\frac{1}{2} Var_t(p^{k-1}_{t+1}) - Cov_t(m_{t+1}, p^{k-1}_{t+1})$$

Thus, we see that expected excess returns, which we interpret as compensation for *risk,* are determined by a combination of movements in the conditional variance of the log of the pricing kernel, the conditional variance of bond prices, and the covariance between the log of the pricing kernel and the log of bond prices.

Using our solutions for bond prices in the formula for excess returns (23) gives this proposition:

Proposition 3. The expected excess returns on holding a long term bonds are given by

$$(24) \quad E_t r^k_{xt+1} = D_k + F_k z_{2t}$$

where $D_1 = F_1 = 0$ and

$$D_k = -B_{k-1}\left[\frac{1}{2}B_{k-1} + 1\right]\sigma_1^2 \text{ and } F_k = \sigma_2 C_{k-1}\left[\lambda - \frac{1}{2}C_{k-1}\sigma_2\right] \text{ for } k > 1.$$

Note from (24) that movements in expected excess returns on long bonds are a function only of movements in the business cycle component of interest rates z_{2t}. Hence, a regression of excess returns on the yield spread of the form (1) in our model has a slope coefficients of

$$(25) \quad \beta^k = \frac{F_k}{C_k/k + 1}.$$

We refer to these slope coefficients as the *Fama-Bliss* coefficients.

Foreign Currency–Denominated Bonds The expected excess return on a foreign currency denominated bond is given by

$$E_t r^*_{xt+1} = i^*_t + E_t e_{t+1} - e_t - i_t$$

where i^*_t denotes the log of the foreign short rate and e_t denotes the log of the exchange rate. To model these expected excess returns we also model the foreign pricing kernel m^*_{t+1}. This foreign kernel prices foreign currency denominated assets and thus can be used to derive foreign bond prices in a manner similar to what we have done above for domestic bond prices. In particular, for the foreign currency denominated bond

$$(26) \quad i_t^* = -E_t m_{t+1}^* - \frac{1}{2} var_t m_{t+1}^*.$$

The lack of arbitrage in complete financial markets implies that

$$(27) \quad e_{t+1} - e_t = m_{t+1}^* - m_{t+1},$$

so that taking conditional expectations gives

$$(28) \quad E_t e_{t+1} - e_t = E_t \left[m_{t+1}^* - m_{t+1} \right].$$

Using (10), (26) and (27) gives that

$$(29) \quad E_t r_{xt+1}^* = \frac{1}{2} \left[var_t m_{t+1} - var_t m_{t+1}^* \right].$$

We model the foreign pricing kernel in a symmetric fashion as the domestic pricing kernel as in (14), (15), and (16) and impose that the parameters in the two countries are identical. We also impose that secular component of interest rates is common to both countries, in that $z_{1t} = z_{1t}^*$. Under these assumptions

$$(30) \quad E_t e_{t+1} - e_t = \left(1 - \frac{\lambda^2}{2} \right) (z_{2t}^* - z_{2t})$$

and

$$(31) \quad E_t r_{xt+1}^* = \frac{\lambda^2}{2} (z_{2t}^* - z_{2t}) = \frac{\lambda^2}{2} (i_t^* - i_t)$$

Note that with $\lambda = \sqrt{2}$, the expected change in the exchange rate in our model is constant and hence exchange rates are a random walk. With this choice of λ, the expected excess return to a foreign currency bond is simply $E_t r_{xt+1}^* = z_{2t}^* - z_{2t} = i_t^* - i_t$.

C. Calibration and Consistency With the Evidence

We have derived our model's implications for the key features of the data on the dynamics of interest rates and risk that motivate our study. We will use this model to decompose the observed postwar U.S. history of interest rates into a secular and a business cycle component and to measure the comovements of these components of the short rate with the conditional mean and the conditional variance of the pricing kernel.

To do so, however, we must first choose parameter values for our model. We set the time period to be a month. We choose parameter values so that our model is quantitatively consistent with the four facts that motivate our analysis. Since we demean the data we need only choose parameters that affect our model's implications of how interest rates and risk move as the secular and business cycle components move. Thus we need only set the parameters that determine B_k and C_k and the expected excess returns on long term bonds and foreign bonds. These parameters are λ, which determines how the conditional variance of the pricing kernel moves with the business cycle component of interest rates, and φ and σ_2, which govern how persistent the business cycle component is and how the conditional variance of the business cycle component moves with its level. We set these parameters to be $\lambda = \sqrt{2}$, $\varphi = .99$, and $\sigma_2 = .017$ so that the model reproduces the four regularities on interest rates and risk we have discussed above. We now discuss our model's quantitative implications for each of these regularities.

1. That the secular component of interest rates z_{1t} in the model is a random walk that acts like a level factor on the yield curve is built in to the specification. This level factor in our model corresponds closely to the first principal component of interest rates we discussed. We demonstrate this result in Figure 4, where we plot the loadings on the first principal component from the data for bonds of maturities three months and from one to thirteen years, together with the coefficients B_k/k (the "loadings" on z_{1t}) for the same maturities from our model.
2. That the business cycle component of interest rates z_{2t} in the model acts like a slope factor is also built into the specification. With our chosen parameters this slope factor in our model corresponds closely to the second principal component of interest rates that we discussed above. We demonstrate this also in Figure 4, where we plot the loadings on the second principal component from the data for bonds of maturities three months and from one to thirteen years, together with the coefficients C_k/k (the "loadings" on z_{2t}) for the same maturities from our model.
3. That movements in the yield spread are associated with movements in the expected excess returns on long bonds of similar magnitude (risk) follows from our parameter

choices. Specifically, at these parameter values, (25) implies that the Fama-Bliss coefficient for a five-year bond is 1.

4. That movements in the short rate relative to foreign-currency short rates are associated with movements in the expected excess returns to holding foreign-currency bonds of a similar magnitude (risk) also follows from our parameter choices. Specifically, since $\lambda = \sqrt{2}$, (30) and (31) implies that exchange rates are a random walk and that expected excess returns on foreign bonds thus move exactly one-for-one with the interest differential.

As we have seen in Figure 4 the coefficients on z_{1t} and z_{2t} in the model correspond closely to the factor loadings on the first and second principal components. Hence, in our decomposition the constructed interest rates capture the dynamics of yield curve nearly as well as the first two principal components do in the data. Recall that these two components account for over 99% of the both the variance of the short rate and the overall variance of the yield curve. In this sense our decomposition captures the dynamics of interest rates extremely well.

We have purposefully chosen a very parsimonious parameterization of the pricing kernel and have chosen parameters so that the model closely matches the dynamics of interest rates and risk. Specifically, we chose parameters so that the responses of yields and excess returns to the state variables as summarized by the coefficients B_k and C_k match those found in the data. We have abstracted from the model's implications for means of yields and excess returns, as summarized by the coefficients A_k. Our model does not have enough parameters to simultaneously match all three sets of coefficients. (For some work on pricing kernels with a larger number of parameters that attempt to match both the dynamics and the means of interest rates and risk see Dai and Singleton 2003 and Cochrane and Piazzesi 2008.) We have adapted a simpler approach because we find it more useful in deriving lessons for monetary policy analysis.

In summary, we have a quantitative pricing kernel model that captures very well the dynamics of interest rates and is consistent with empirical evidence on how risk moves with interest rates.

3. The Decomposition of Interest Rates

We now use our pricing kernel to decompose the movements in the short rate observed in postwar U.S. data into movements in the conditional mean and the conditional variance of the pricing kernel. Our two main findings are the following. First, movements in the secular component of the short rate correspond to random walk movements in the conditional mean of the pricing kernel. Second, movements in the business cycle component of the short rate correspond to movements in the conditional variance of the pricing kernel.

To construct our decomposition, we set z_{1t} and z_{2t} equal to the observed history of the first and second principal components after scaling these components appropriately.[5] With this definition of z_{1t} and z_{2t} we obtain the same decomposition of the short rate into secular and business cycle components shown in Figure 1.

When we do so the secular and business cycle components in our model account for the same portion of movements in the short rate that is accounted for by the first two principal components of interest rates in the data, over 99%.

We now use our model of the pricing kernel to interpret this decomposition.

A. Expectations of Future Policy

Our model gives a simple interpretation of the decomposition in Figure 1. Movements in z_{1t} in the figure represent movements in expectations of where the short rate will be in the long run. Under this interpretation in the postwar period, over 90% of the variance in the Fed's policy instrument—the short rate—is associated with movements in agents' expectations of where the Fed will be setting its policy instrument in the distant future.

B. The Short Rate and the Pricing Kernel

Consider next what the decomposition implies for the comovements of the short rate with the conditional mean and variance of the pricing kernel. Recall that

$$(32) \quad i_t = -E_t\left[m_{t+1}\right] - \frac{1}{2}var_t\left[m_{t+1}\right].$$

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[5] Movements in the principal components are determined only up to a scale factor. Motivated by (20) we set the scale factor on these components so that the response rate of the short rate to the first principal component is 1 and the response of the short rate to the second principal component is -1.

As we have discussed above standard monetary analyses impose that that the conditional variances are constant, so that

$$i_t = -E_t m_{t+1} + \text{constant}. \tag{33}$$

In our model (18) and (19) imply that when $\lambda = \sqrt{2}$,

$$-E_t m_{t+1} = \text{constant} + z_{1t} \tag{34}$$

and

$$-\frac{1}{2} var_t(m_{t+1}) = \text{constant} - z_{2t}. \tag{35}$$

This result gives a very stark interpretation of the decomposition of the short rate shown in Figure 1: movements in the secular component of the short rate are movements in the conditional mean of the pricing kernel and movements in the business cycle component are movements in the conditional variance of the pricing kernel.

These results thus imply that, at least for business cycle analysis, existing monetary models miss the link between the short rate and the economy present in postwar U.S. data. In these models, movements in the short rate are associated solely with movements in the conditional mean of the pricing kernel. Our quantitative model implies that for business cycle analysis, in the data movements in the short rate are associated solely with movements in the conditional variance of the pricing kernel.

4. Towards a New View of Monetary Policy

Our pricing kernel is a statistical summary of the joint dynamics of interest rates and risk observed in postwar U.S. data. To give an economic interpretation of this pricing kernel we build an economic model in which equilibrium asset prices are described by this pricing kernel. In this sense our economic model is consistent with the dynamics of interest rates and risk observed in postwar U.S. data. We use this economic model to lay a foundation for a new view of monetary policy.

Using our pricing kernel model, we have made two points about the postwar history of the Fed's policy instrument: Most of the movements in this policy instrument are permanent, driven by the secular component. And the business cycle movements in this policy instrument are associated with movements in risk. In our economic model we give an interpretation of these findings with two assumptions: the secular movements in the Fed's policy instrument arise from permanent movements in the Fed's inflation target and the business cycle movements in the Fed's policy instrument arise from the Fed's endogenous policy response to exogenous changes in real risk in the economy. We then discuss how this interpretation leads to a new view of monetary policy.

The model economy we build here is a pure exchange economy with exogenous time-varying risk. Since the early contribution by McCallum (1994), a large literature has studied interest rates in such economies. Examples include the work of Wachter (2006), Bansal and Shaliastovich (2007), Gallmeyer, Hollifield, Palomino, and Zin (2007), Piazzesi and Schneider (2007).

A. An Economic Interpretation of the Model

Here we identity the various key parts of our pricing kernel model with their economic counterparts.

Again, we interpret the secular component of interest rates in our model as corresponding to the Fed's long run inflation target $\pi_t^* = z_{1t}$, which follows a random walk. We interpret the shock ε_{3t+1} in the pricing kernel as the deviation of realized inflation π_{t+1} from the inflation target π_{t+1}^*. Given this interpretation, realized inflation in our model is the sum of a random walk component and an i.i.d. component,

$$\pi_{t+1} = z_{1t+1} + \varepsilon_{3t+1},$$

as in the model of inflation studied by Stock and Watson (2007).

We interpret the business cycle component of nominal interest rates in our model (z_{2t}) as corresponding to the real pricing kernel derived from the growth of the marginal utility of the representative agent in our economy. Assume that the representative consumer has expected utility with external habit of the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(C_t - X_t\right)^{1-\gamma},$$

where X_t is an exogenous stochastic process for external habit.

Since habit is external, the representative consumer's marginal utility is given by

$$(C_t - X_t)^{-\gamma}$$

Following Campbell and Cochrane (1999), we define

$$S_t = \frac{C_t - X_t}{C_t}$$

Using lower case letters for logarithms of variables, we write the pricing kernel as

$$m_{t+1} = \log \beta - \gamma(c_{t+1} - c_t + s_{t+1} - s_t)$$

We assume that the logarithm of consumption growth is i.i.d. with

$$c_{t+1} - c_t = \delta_c + \sigma_c \epsilon_{2t+1}$$

Note that in this representative agent framework, c_t is also aggregate consumption. We assume that the external habit level X_t is a non-linear function of lagged values of consumption, habit, and a preference shock z_{2t} given implicitly by

$$s_{t+1} = s_t + \eta(z_{2t})\epsilon_{2t+1}$$

where z_{2t} evolves according to

$$z_{2t+1} = (1 - \varphi)\theta + \varphi z_{2t} + \sigma_2 z_{2t}^{1/2} \epsilon_{2t+1}$$

With

$$\eta(z_2) = \frac{\sqrt{2}}{\gamma} z_2^{1/2} - \sigma_c$$

and ε_{2t+1} independent of ε_{1t+1}, the pricing kernel in this economy is given by (14), (15), and (16) with $\lambda = \sqrt{2}$.

B. A New View of U.S. Monetary Policy

This economic interpretation of our model leads to a new interpretation of the history of U.S. monetary policy in the postwar period. Under this new interpretation, the business cycle movements in the Fed's policy instrument, the short rate, arise as a result of the Fed's need to compensate for exogenous business cycle fluctuations in risk as it aims for its inflation target.

Specifically, under this interpretation of our model, expected growth of consumption is always constant and the Fed is always hitting its inflation target, at least in expectation. In a standard model, with constant risk, the movements in the short rate would then correspond only to movements in the Fed's inflation target, that is

$$i_t = \text{constant } + \pi_t^*.$$

In this model, however, risk is time-varying because of exogenous shifts in habit, so that the short rate has a business cycle component that is driven by these business cycle fluctuations in risk:

$$i_t = \text{constant } + \pi_t^* - \frac{1}{2} var_t m_{t+1} = \text{constant } + \pi_t^* - z_{2t}.$$

These business cycle fluctuations in the Fed's policy instrument are required to ensure that inflation stays on target and correspond in the data to fluctuations in the slope of the yield curve.

A simple way to summarize our view about what the Fed does over the business cycle is that it simply responds to exogenous changes in real risk—specifically to exogenous changes in the conditional variance of the real pricing kernel—with the aim of maintaining inflation close to a target level. This seems to be not what standard monetary policy analysis focuses on. In our experience as Fed staff members, for example, we know that the typical policy meeting at the Fed involves of detailed discussions of forecasts of economic variables that enter the mean of the pricing kernel, such as expected real growth and expected inflation. These discussions are often summarized by a Taylor rule for policy that makes no reference to risk. Our interpretation of the historical record, however, is that over the business cycle, the Fed's response had little to do with these forecasts about changes in conditional means of growth and inflation. Instead, policy mainly responds to exogenous changes in real risk.

5. A Research Agenda

Our economic model is only one potential interpretation of the implications of the joint dynamics of interest rates and risk for monetary policy analysis. In looking forward more broadly to a new research agenda for monetary policy analysis, we take away two important questions to be confronted in future research.

1. One question regards the secular movements in the Fed's policy instrument. We interpret these as arising from random walk movements in the Fed's inflation target. We view this interpretation as a purely mechanical accounting of these secular movements. It avoids a central question: Why did the Fed choose the secular movements in its policy instrument?

2. The other question regards the business cycle comovements between the Fed's policy instrument and the macroeconomy as captured in the standard Euler equation. We have suggested here—and Canzoneri, Cumby, and Diba (2007) have documented—that, in practice, standard monetary models miss this link. Now we need to know, How do we fix our models so that they capture it?

A. Why Did the Fed Choose the Secular Movements in Policy?

The literature has offered two basic approaches to modeling the secular movements in the short rate in postwar U.S. data. One approach mechanically describes aspects of Fed policy over this period that led to these movements. The other approach explicitly models the Fed's objectives and information that led to its behavior. Neither approach has so far been successful.

In our economic model, we have followed the first approach that mechanically describes the secular movements in Fed policy as arising from a random walk inflation target. We have documented that the random walk policy component is large, accounting for over 90% of the variance in the short rate over the postwar period. This model seems adequate as a purely statistical description of Fed policy, but seems useless for answering fundamental questions at any more than a superficial level. Again, Why did the great inflation of the 1970s occur? Why did it end? Is it likely to occur again? How can we change institutions to reduce that likelihood?

Researchers have begun wrestling with these questions. For example, Orphanides (2002) argues that the Fed's difficulties in interpreting real time economic data in the 1970s played a key role in shaping the Fed's choice of the short rate during that time. It is unclear, however, what mechanism in this framework would lead to a large random walk component in policy. Thus, we do not see how an explanation of this sort would be able to account for secular component of Fed policy.

Primiceri (2006) and Sargent, Williams, and Zha (2006) have made the most ambitious attempts to reconcile the observed secular movements in Fed policy with optimizing behavior by the Fed. In their work, the Fed uses a misspecified model to choose policy and continually revises that model in light of the data. This approach is clearly aimed at fundamental questions in analysis of monetary policy in the postwar period. Unfortunately, data on the secular movements in Fed policy pose a formidable challenge to models of this type. The basic problem is that these models have a very difficult time generating a volatile random walk component of policy simply from learning dynamics.

To illustrate this point we graph in Figure 5 the time series for long run averages of expected inflation over horizons of 20 and 30 years from the model of Sargent, Williams, and Zha (2005) together with the secular component of Fed policy from our quantitative model.[6] Clearly, the expectations of long-run averages of inflation from the learning model are much less volatile than the secular component of postwar monetary policy.

In sum, existing approaches to the forces driving the secular component of policy have not been successful. Thus a new approach is needed.

In thinking about a new approach, we note that the secular component of interest rates has not always been so volatile. In fact, the postwar period stands out from the U.S. historical record as a period with exceptionally high volatility of the secular component of interest rates. To illustrate this point, in Figure 6A we graph a short rate and a long rate for the United States from 1836 through 2007. For the short rate we use the U.S. three-month commercial paper rate and for the long rate, we use the yield on a ten-year U.S. Treasury bond (available at www.globalfinancialdata.com.). Clearly, in the prewar period, fluctuations in the long rate (which we associate with the secular component of interest rates) are a much smaller fraction of overall fluctuations in the short rate than they are in the postwar period.

This difference in pre- and post-war behavior of long and short rates is also evident in the data for many other countries, including the United Kingdom (Figure 6B), France (Figure 6C), Germany (Figure 6D), and the Netherlands (Figure 6E).

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[6] Tao Zha kindly provided us with these long run expectations of inflation from the 2006 Sargent, Williams, and Zha model.

A central question in the analysis of monetary policy at the secular level then is What institutional changes led to this pattern? To answer this question at a mechanical level, we note that the Gold Standard was the main institution governing monetary policy in the prewar era and that after the war most countries switched to a fiat standard governed for part of the time by the Bretton Woods agreement. But this answer is, at best, superficial. In the prewar era, countries chose to be on the Gold Standard most of the time and chose to leave it when it suited their purposes. Thus, the relevant questions are, rather, What deeper forces led agents to have confidence that their governments would choose stable policy over the long term? And what forces led them to lose this confidence after World War II. Only if we can quantitatively account for this history can we give advice on how to avoid another great inflation.

B. How Do We Fix the Euler Equation in Our Models?

As we have discussed in modern monetary models, the policy instrument enters the economy through the Euler equation that links the short rate to expectations of growth in the marginal utility of consumption and inflation. Canzoneri, Cumby, and Diba (2007) document that this Euler equation in standard models does a miserable job of capturing this link between policy and the economy at business cycle frequencies. Here we offer some intuition for why this is so here. We then argue that existing attempts to fix this Euler equation are misguided and we propose a new direction.

Consider, first, what aspects of the comovements of the short rate and macroeconomic aggregates that are not captured by in the Euler equation of standard monetary models. The basic problem with the simplest of these models is that the terms

$$-E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1}$$

are too smooth relative to the short rate at business cycle frequencies so they account for virtually none of the fluctuations in the policy variable, the short rate, at these frequencies.

To illustrate this point, we[7] have estimated a version of the Smets and Wouters (2007)

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[7] Actually, we asked Ellen McGrattan to reestimate the model using codes kindly provided by Frank Smets and Raf Wouters and she kindly obliged. This applies later to the computations underlying Figures 10 and 11 as well.

model, with their habit preferences replaced by standard CRRA preferences, and computed the errors in the consumption Euler equation, where the error is computed as

$$\text{error}_t = i_t - \left[-E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1} \right].$$

In Figure 7, we plot the HP-filtered short rate (the Fed Funds Rate) and the HP filtered error in the Euler equation. (We HP-filter both i_t and error_t so that we can focus on business cycle frequencies.) We find this figure striking. As we have explained, in theory the standard monetary models imply that movements in the short rate are associated one-for-one with the sum of the movements in the expected growth of the log of marginal utility for the representative consumer and expected inflation. Figure 7 shows that, in practice, in a standard monetary model, movements in the short rate are associated almost one-for-one with Euler equation error and the model captures essentially none of the link between the short rate and the macroeconomy. Since this Euler equation is the fundamental link between monetary policy and the macroeconomy, this type of model can hardly be said to be useful in accounting for analyzing monetary policy at business cycle frequencies if the observed movements in the monetary policy instrument at these frequencies correspond simply to the unexplained error in this equation.

How should we fix this problem? To address this question, consider the Euler equation allowing for movements in conditional variances:

$$\text{(36)} \quad i_t = -E_t \log \frac{U_{ct+1}}{U_{ct}} + E_t \log \pi_{t+1} - \frac{1}{2} \text{var}_t \left[\log \frac{U_{ct+1}}{U_{ct}} \frac{1}{\pi_{t+1}} \right].$$

Consider, first, a way that has been tried to fix this equation but doesn't work. The approach taken in most of the literature so far has been to use more exotic preferences, such as preferences with habit persistence, but to continue to log-linearize the model and assume constant conditional variances. Mechanically, this approach amounts to making the conditional means of marginal utility growth ($E_t \log U_{ct+1}/U_{ct}$) more volatile while assuming that the conditional variances are still constant.

That this approach is a failure is well-documented by Canzoneri, Cumby, and Diba (2007). For example, consider what happens when we repeat the experiment Figure 7 using the Smets-Wouters model as specified with habit persistence. In Figure 8, we plot the HP-filtered short rate and the HP-filtered Euler equation error from the model. Clearly adding habit is not improving matters.

Our decomposition suggests the approach being taken in the literature to fixing the Euler equation is misguided. Our decomposition indicates that we should not be trying to make the conditional mean more volatile at business cycle frequencies; at these frequencies, it is approximately constant. Instead, we should be looking for a framework that delivers smooth conditional means and volatile conditional variances of the pricing kernel at business cycle frequencies.

Note that the economic model we have described here, while useful in helping us interpret the data, is probably not the full answer to this problem. In that model, we have made special assumptions which guarantee that the conditional mean of the pricing kernel is constant. (We made consumption growth i.i.d. and engineered the habit process appropriately.) If Canzoneri, Cumby, and Diba (2007) are right that expected consumption growth varies over time, then our model is likely to have problems similar to those they document for other models. The reason is that when expected consumption growth varies over time, the conditional mean of the pricing kernel in our model would likely to become volatile.

6. Concluding Remarks

We have used a simple model of the pricing kernel to interpret the postwar U.S. data on the dynamics of interest rates and risk and to draw out implications from these data for new research directions for monetary policy analysis.

Our work here also points to new directions for empirical work on the dynamics of interest rates and risk. We have used a simple model of the pricing kernel and have shown that, given the data, it yields a very sharp characterization of the dynamics of the short rate, the conditional mean of the pricing kernel, and its conditional variance. The short rate has a random walk component that accounts for the vast bulk of its movements. The conditional mean of the pricing kernel closely tracks that random walk component. The short rate also has stationary component which accounts for almost all of the rest of its movements. The conditional variance of the pricing kernel closely tracks this stationary component.

We think that refining our simple characterization empirically might yield some useful results. Specifically, a huge literature uses a wide variety of affine models of the pricing kernel to model the dynamics of interest rates and risk. Prominent recent examples are Dai and Singleton (2002) and Cochrane and Piazzesi (2008). The most promising of these models might be used to develop new tools for using yield curve data in real time to help guide the Fed's choice of monetary policy.

In building our economic model, we made assumptions that gave one possible interpretation to the joint dynamics of interest rate and risk that we uncovered with our pricing kernel. Under this interpretation the Fed must continually adjust the short-term nominal interest rate in response to exogenous time variation in risk even if the Fed's sole objective is to maintain a constant level of expected inflation. We think of this view as the *exogenous risk* approach. An alternative approach, the *endogenous risk* approach, reverses the direction of causality. In it, the Fed is an active player in generating time-varying risk. Alvarez, Atkeson, and Kehoe (2002, 2007) propose such an approach. At this point we do not see any strong evidence favoring one approach over the other. Clearly, before progress can be made in modeling monetary policy, we must sort out which way the causality actually runs: from risk to the Fed or from the Fed to risk.

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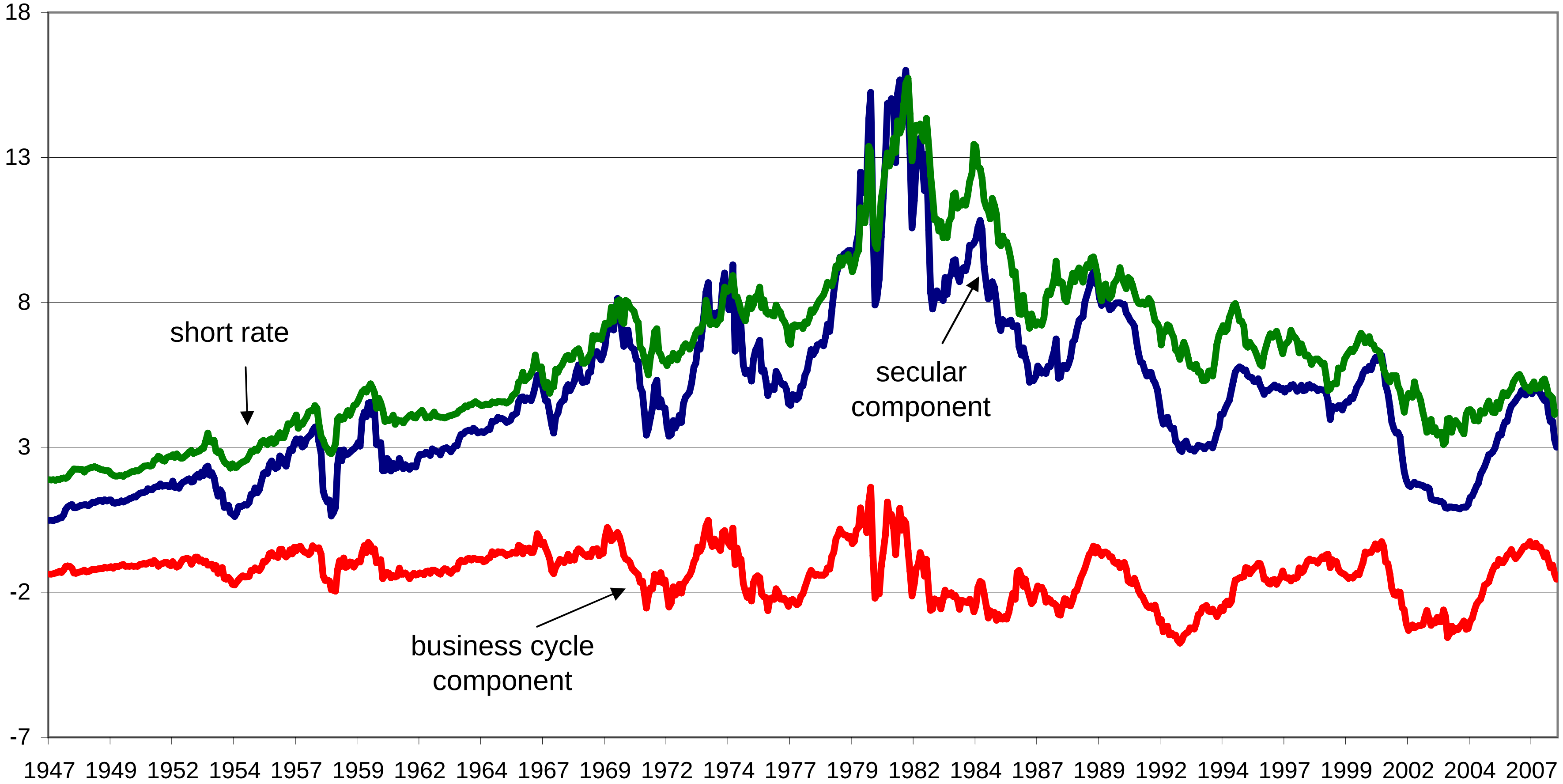
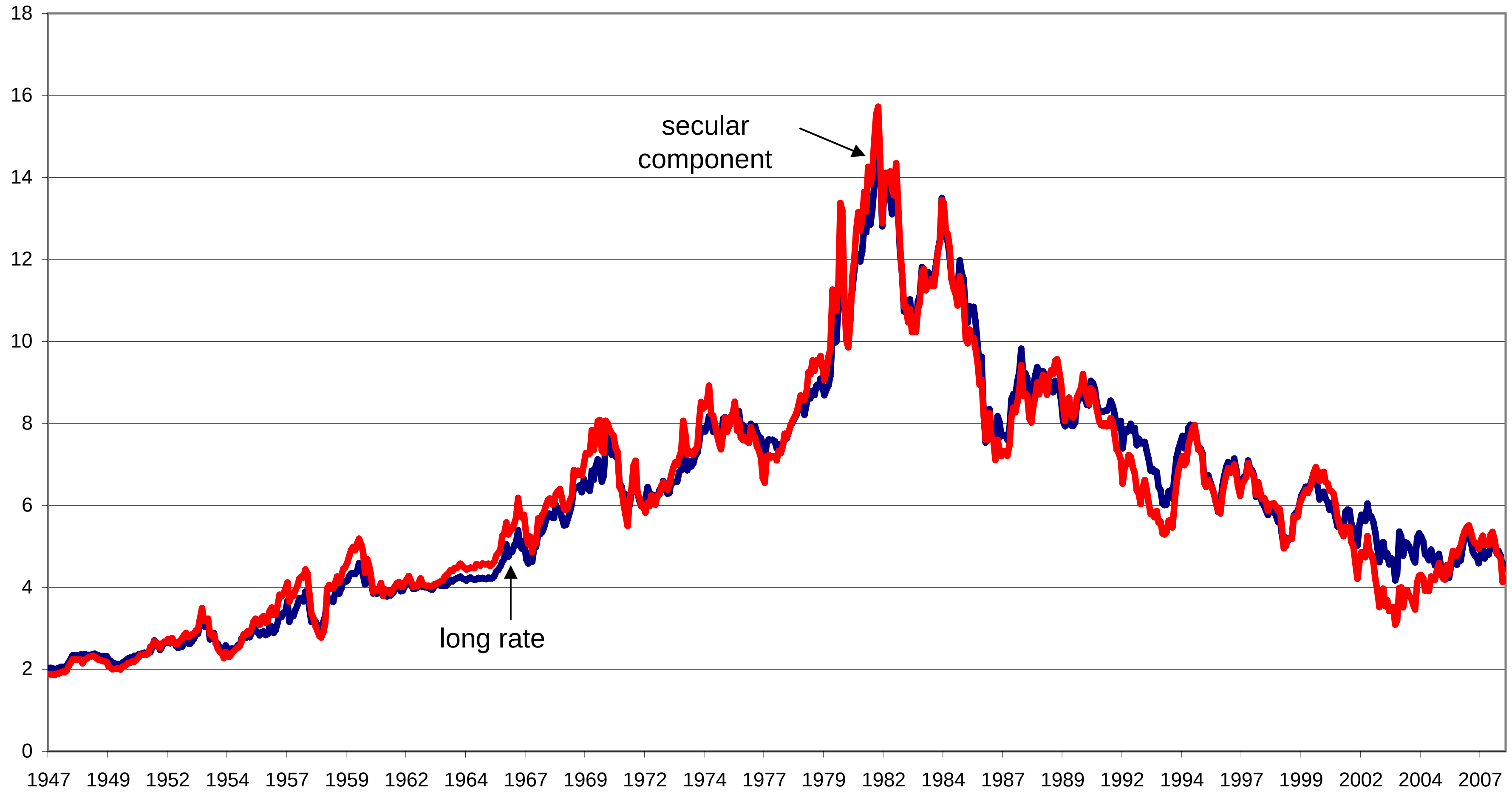
Figure 1: Short rate and the secular and business cycle components *



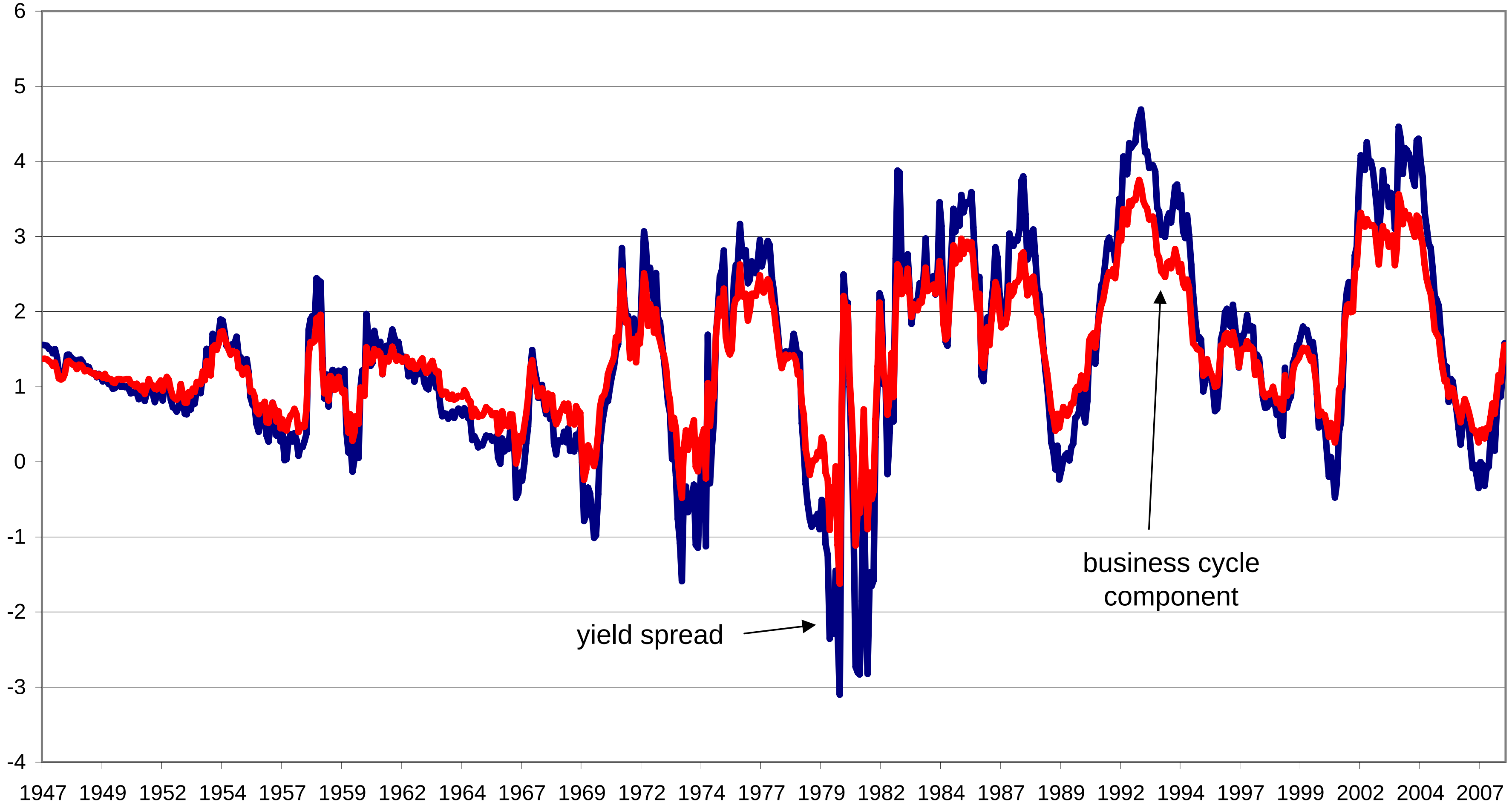
* The short rate is the 3-month T-bill rate. The secular and business cycle components are the first two principal components derived from a decomposition of the covariance matrix of a vector of 14 yields: the 3-month rate and the imputed zero coupon yields for maturities k=1,...,13 years over 1946:12–2007:12. For the period 1946:12–1991:2, we use data from McCulloch and Kwon (1993), and for the period 1991:3–2007:12, we use data from Gurkaynak, Sack, and Wright (2006).

Figure 2: Long rate and the secular component *



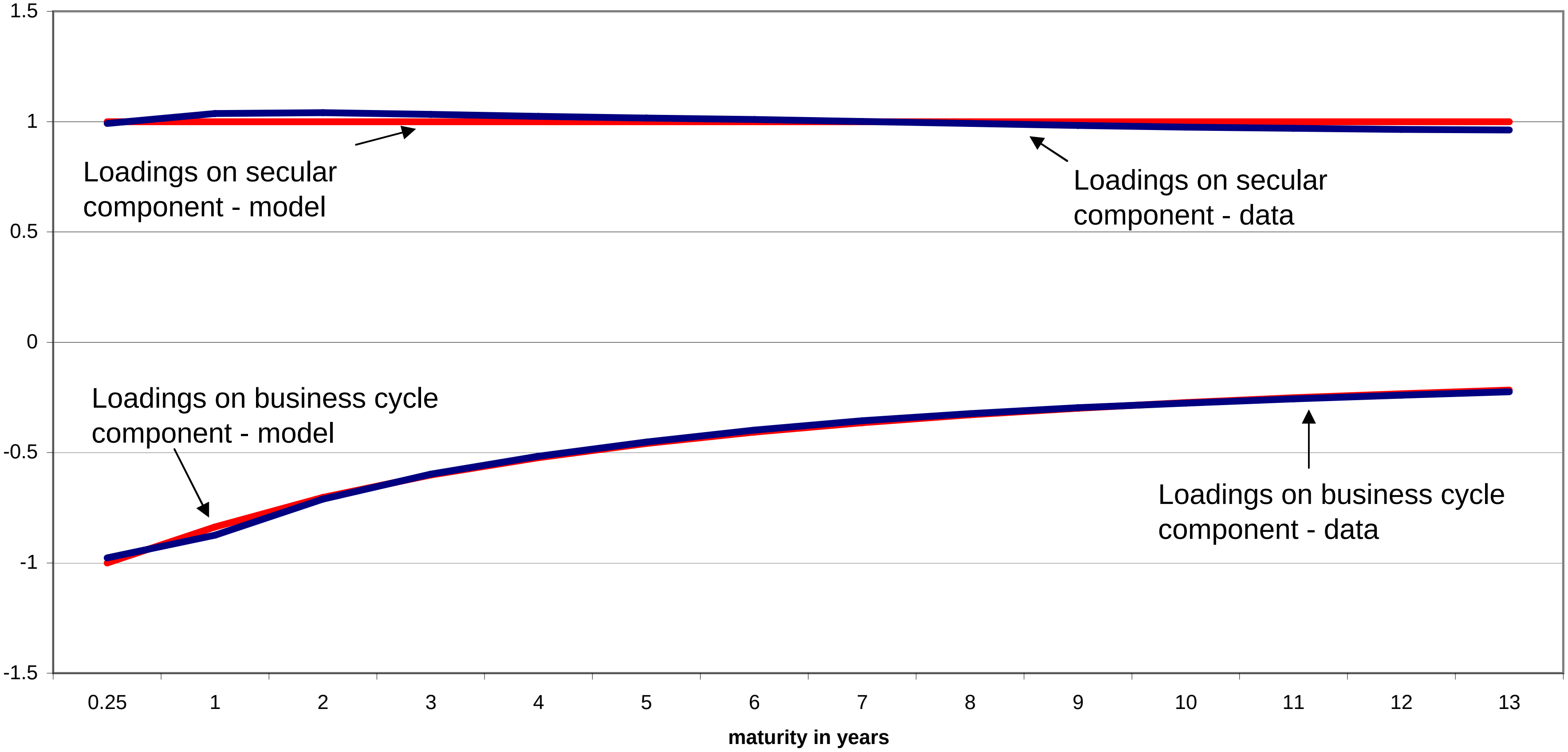
* The long rate is the imputed zero coupon yield for 13-year bonds over 1946:12–2007:12.

Figure 3: Yield spread and the business cycle component *



* The yield spread $y^{L}_{t\text{-}}i_t$ is defined as the difference between the imputed zero coupon yield for 13-year bonds and the 3-month T- bill rate. For the business cycle component, see note to Figure 1.

Figure 4: Loadings on the secular and business cycle components data and model *



* The loadings on the secular and business cycle components in the data are the factor loadings in the principal components decomposition. The loadings are the secular components in the model and the coefficients B_k/k and Ck/k, respectively.

Figure 5: Sargent-Williams-Zha (SWZ) expectations of 20- and 30-year average inflation and secular component of interest rates

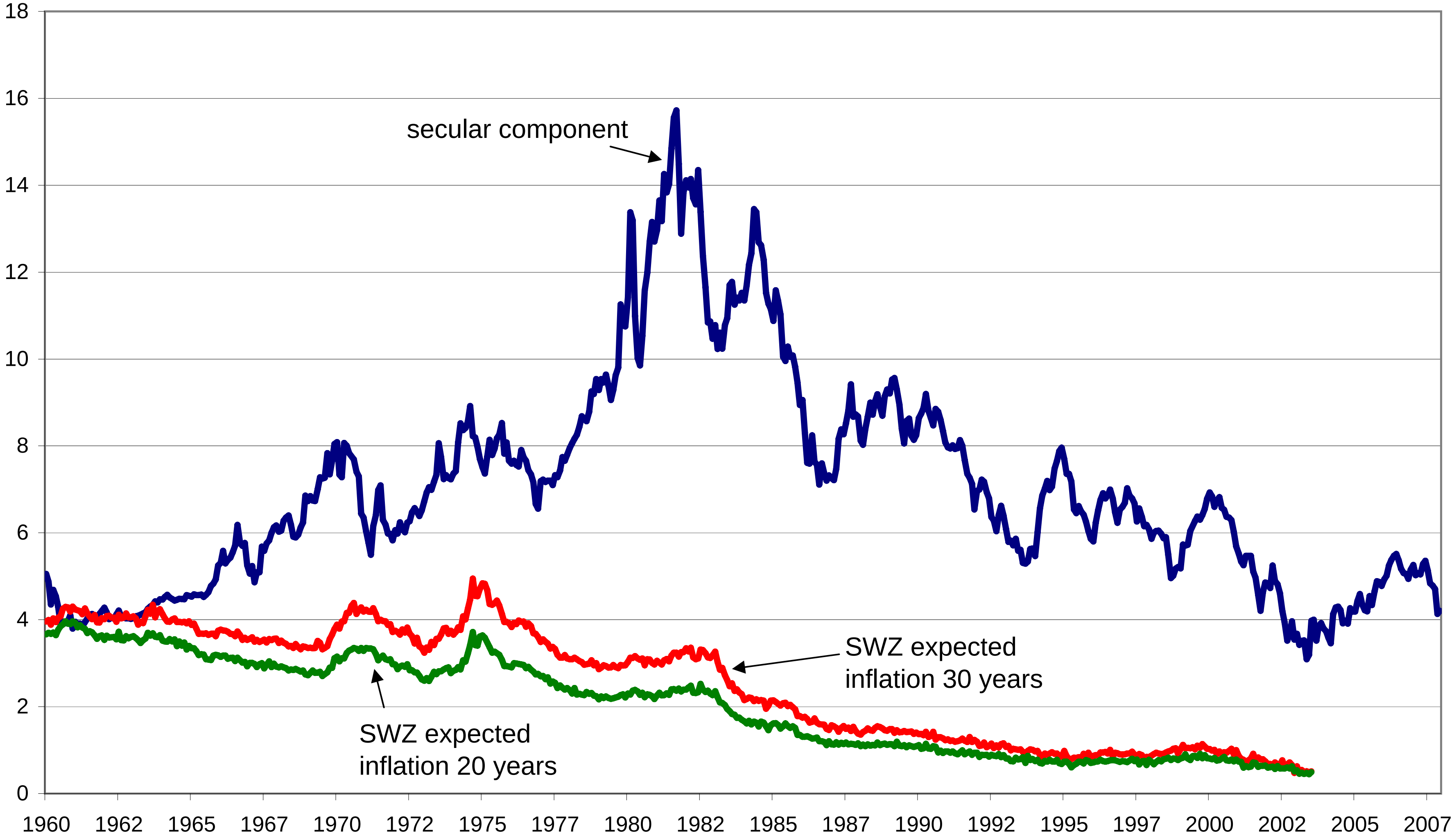
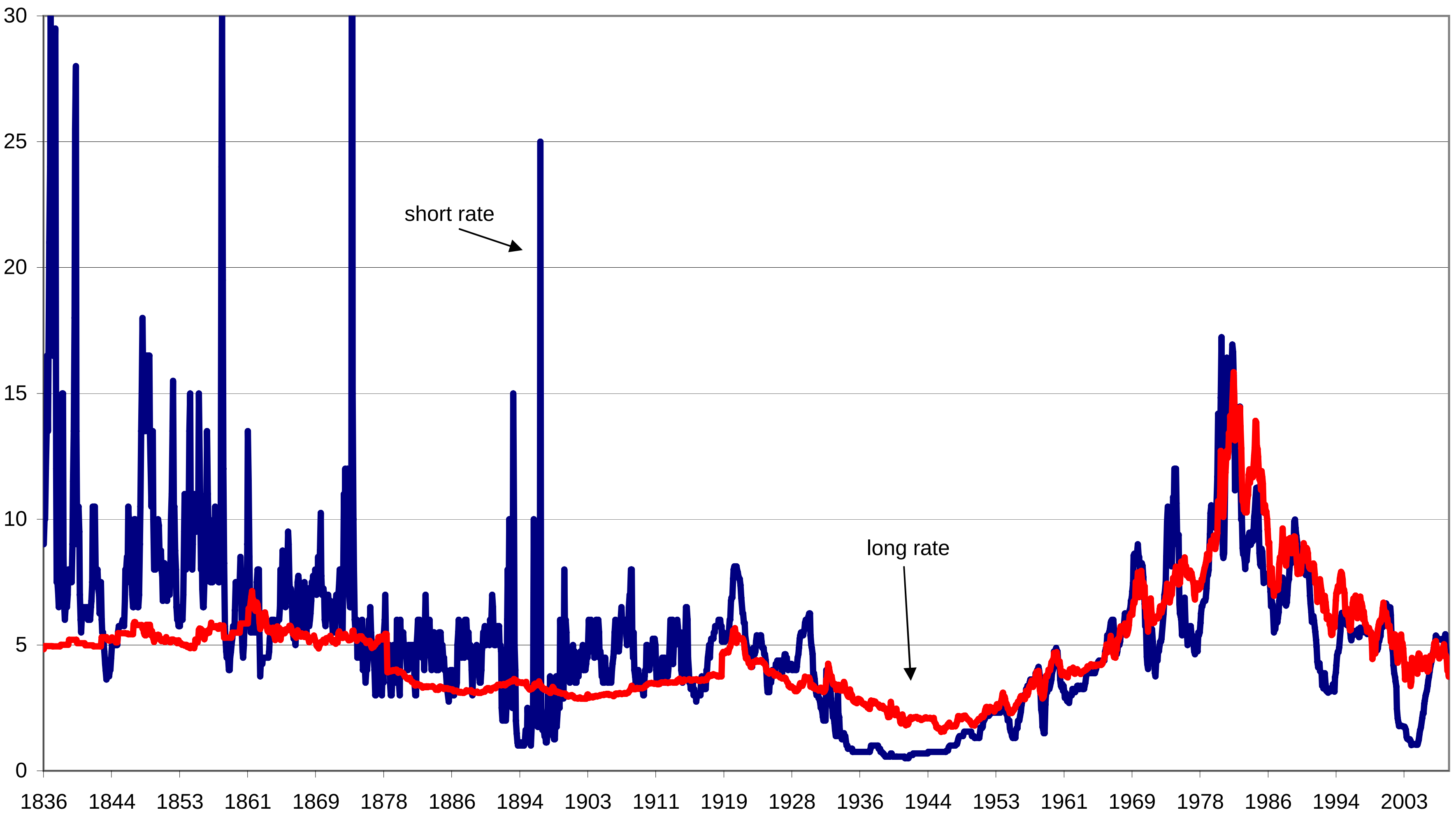
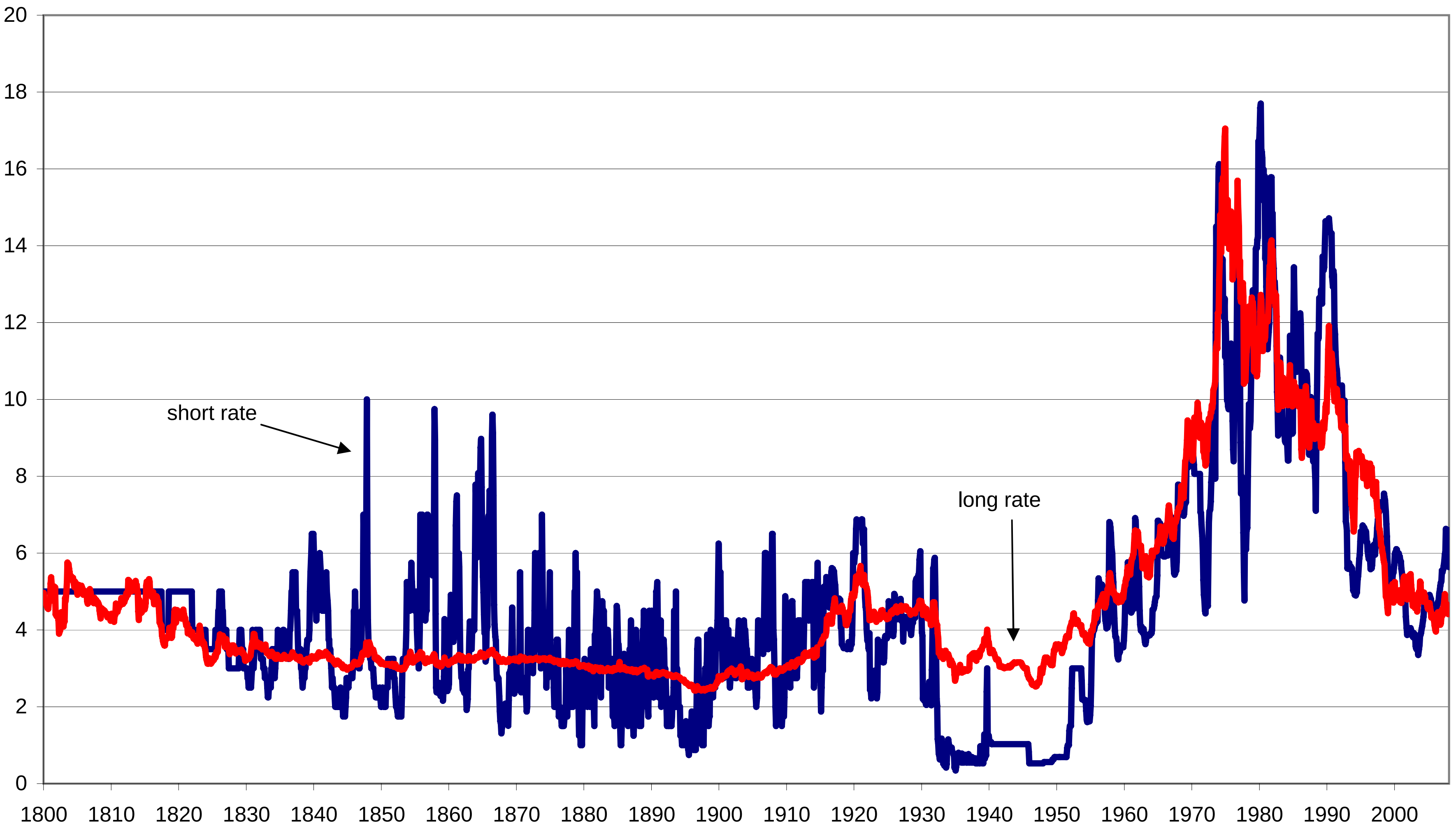


Figure 6A: Long and short rates in the United States*



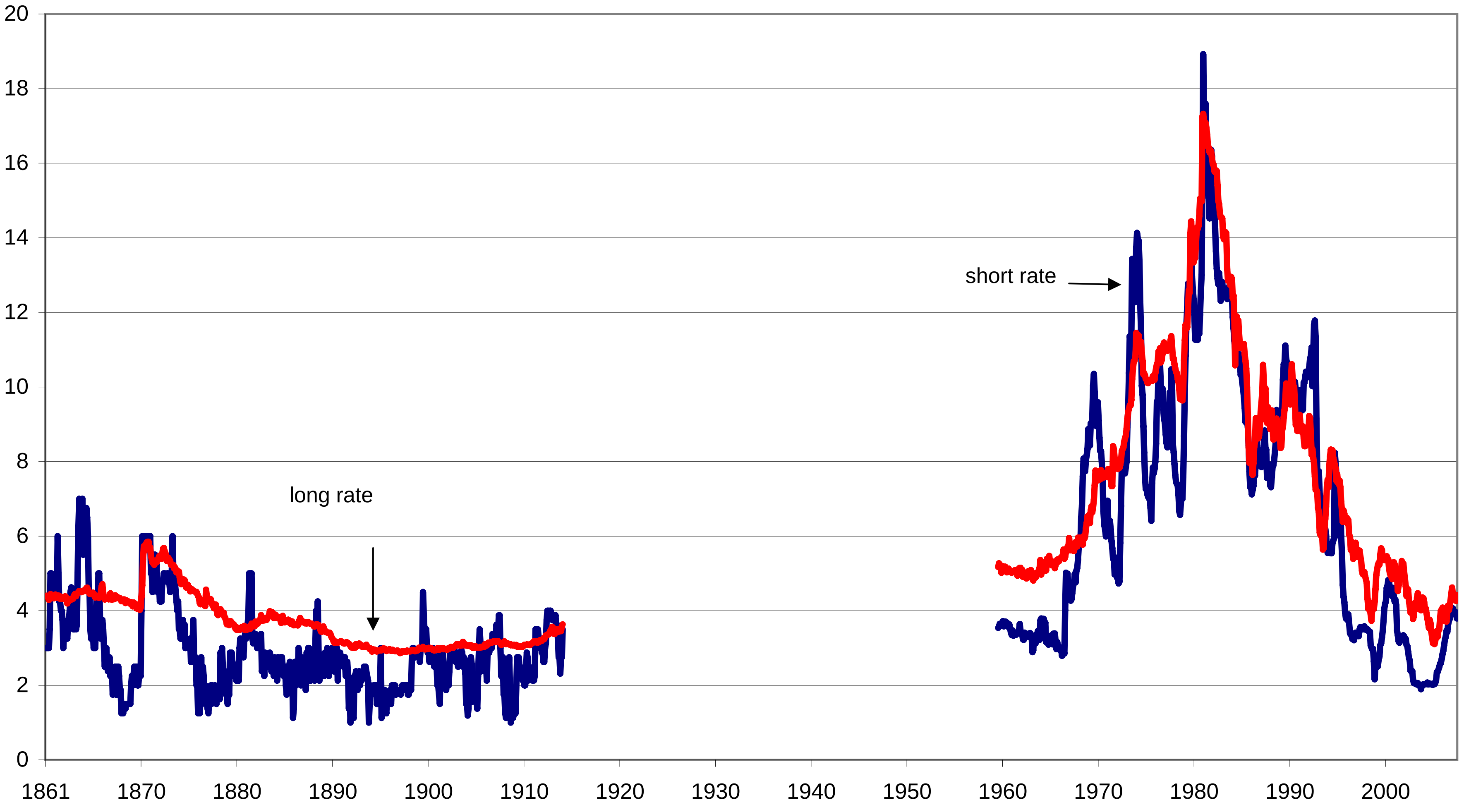
* The short rate is the 3-month commercial paper rate, and the long rate is the yield of a long-term bond. For detailed information see the data appendix.

Figure 6B: Long and short rates in the United Kingdom *



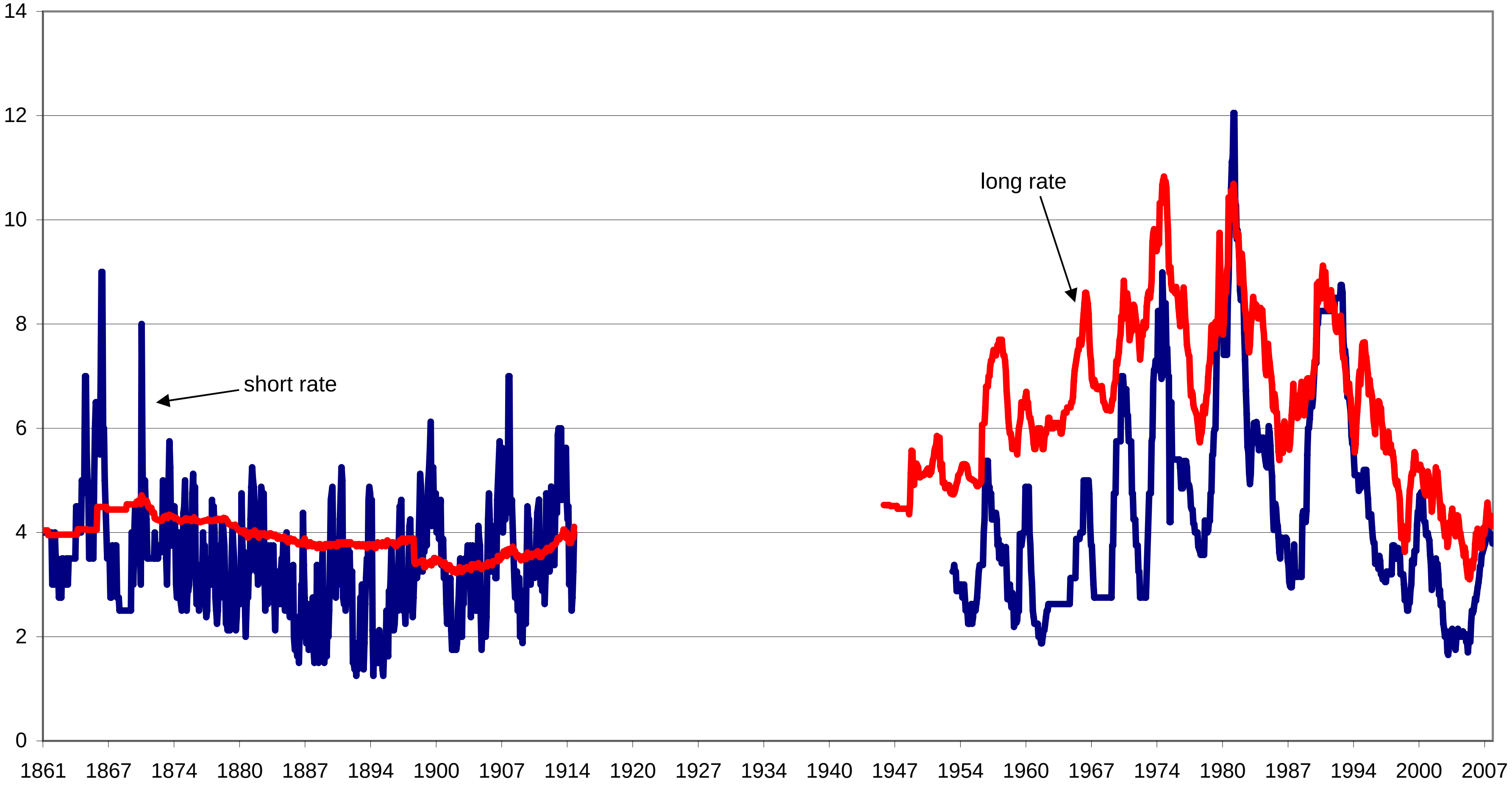
* The short rate is the private discount rate, and the long rate is the 2.5% consol yield. For detailed information, see the data appendix.

Figure 6C: Long and short rates in France *



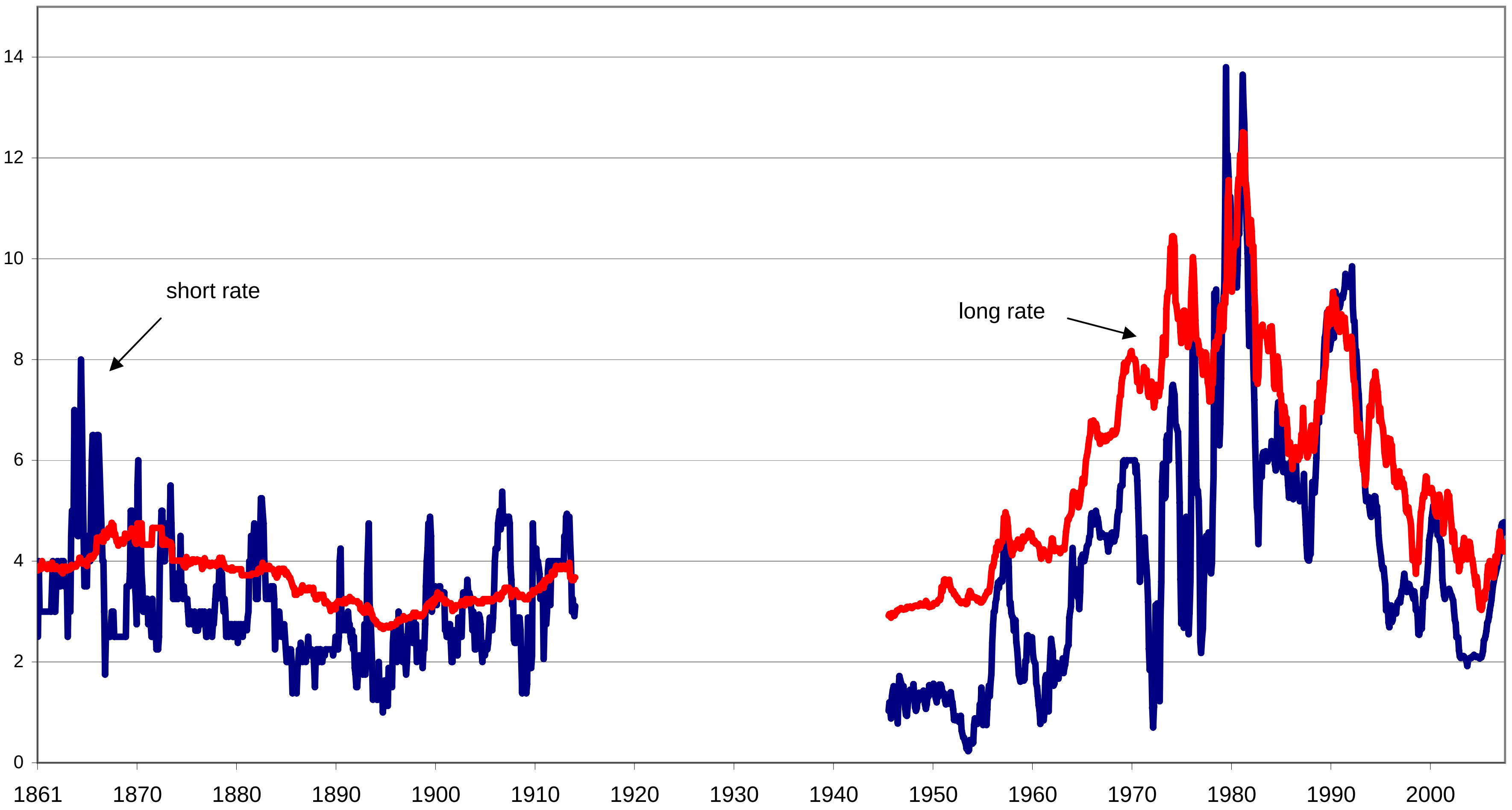
* The short rate is the private discount rate for the period 1860–1914 and the 3-month T-bill for 1960–2007. The long rate is the 10-year government bond yield. For detailed information, see the data appendix.

Figure 6D: Long and short rates in Germany *



* The short rate is the Berlin discount rate for the period 1860–1914 and the 3-month T-bill for 1953–2007. The long rate is the 10-year government bond yield. For detailed information, see the data appendix.

Figure 6E: Long and short rates in the Netherlands *



* The short rate is the private discount rate for the period 1860–1914 and the 3-month T-bill for 1946–2007. The long rate is the 10-year government bond yield. For detailed information, see the data appendix.

Figure 7: HP-filtered federal funds rate and HP-filtered Euler equation error CRRA utility

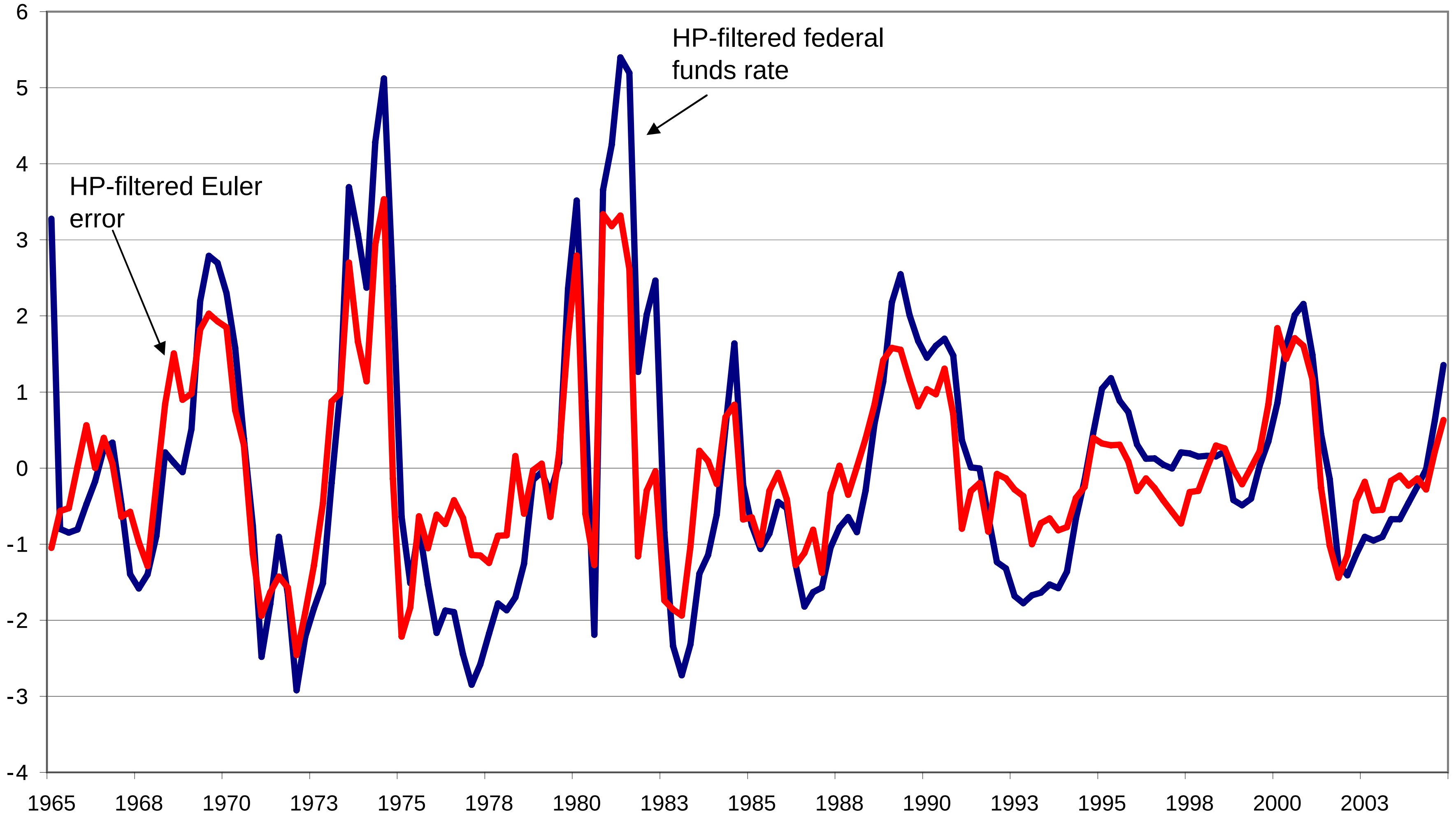


Figure 8: HP-filtered federal funds rate and HP-filtered Euler equation error with habit