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Lindsay C. Page
Richard J. Murnane
John B. Willett

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Trends in the Black-White Achievement Gap:Clarifying the Meaning of Within- and Between-School Achievement Gaps
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ABSTRACT

We decompose black-white achievement gap trends between 1971 and 2004 into trends in within- and between-school differences. We show that the previous finding that narrowing within-school inequality explains most of the decline in the black-white achievement gap between 1971 and 1988 is sensitive to methodology. Employing a more detailed partition of achievement differences, we estimate that 40 percent of the narrowing of the gap through the 1970s and 1980s is attributable to the narrowing of within-school differences between black and white students. Further, the consequences for achievement of attending a high minority school became increasingly deleterious between 1971 and 1999.

Lindsay C. Page
Graduate School of Education
Harvard University
6 Appian Way
Cambridge, MA 02138
LPage@fas.harvard.edu

Richard J. Murnane
Graduate School of Education
Harvard University
6 Appian Way - Gutman 409
Cambridge, MA 02138
and NBER
richard_murnane@harvard.edu

John B. Willett
Graduate School of Education
Harvard University
6 Appian Way
Cambridge, MA 02138
John_Willett@Harvard.Edu

I. Introduction

From the early 1970s to the late 1980s, gaps between the average reading and mathematics achievement of black and white students in the United States closed markedly. Between 1970, when the *National Assessment of Education Progress, Long-Term Trend* (NAEP-LTT) was first administered, and 1988, the black-white gap in the average reading achievement of 13-year-olds fell by half. The change in the mathematics achievement gap over this period was similar (a 48 percent decline). These trends, a result of the average achievement of black students increasing while the average achievement of white students remained quite constant, were a source of optimism for the education policy community. Since cognitive skills were moderately strong predictors of adult wages in the mid-1980s (Murnane, Willett and Levy, 1995; Neal and Johnson, 1996; Neal, 2005), these trends portended real progress in closing racial gaps in economic outcomes that had plagued the United States for decades.

Unfortunately, progress made in the first two decades of the NAEP's administration came to a halt in the late 1980s. During the subsequent decade, black-white gaps in average achievement remained relatively constant for nine-year-olds and grew for 13- and 17-year-olds. Using data from the NAEP-LTT, we display these patterns in Figure 1. The left panel displays trends in the difference in average reading achievement between nationally-representative cohorts of white and black children of ages 9, 13 and 17. The right panel displays the analogous information for mathematics. Notice that the gaps in average achievement declined precipitously for all age-cohorts between 1970 and the late 1980s, then reversed in direction and grew through the late 1990s, only to flatten out and decline somewhat in the first decade of the 21st century. These abrupt turnarounds have led researchers and policymakers to ask: Why did average black-white achievement gaps converge during the 1970s and 1980s? Why did they diverge again during the 1990s? Why have they begun to narrow more recently?

Cook and Evans (2000) used the NAEP-LTT data to address the first of these questions. They reported that changes in the average relative performance of black and white students attending the same schools (usually referred to as *within-school differences*) accounted for most of the closing of the average black-white achievement gap during the 1970s and 1980s. Here, we revisit this question and extend the work of Cook and Evans by taking advantage of recent improvements in data and advances in methodology. Using a methodology that supports a more detailed partition of average black-white achievement differences, we show that less of the 1970s-1980s decline in the achievement gap can be unequivocally attributed to a decline in within-school achievement differences than Cook and Evans reported. We then address the second and third questions. We find that trends in the educational attainments of students' parents and in the payoffs to that attainment play important roles in explaining the narrowing of the racial gap in average achievement in the 1970s and 1980s and it subsequent widening in the 1990s. Finally, while the NAEP-LTT data do not show increases in racial segregation during the last three decades of the 20th century, we find that the deleterious consequences for black students of attending a racially segregated school increased steadily.

II. Prior Research

Until the latter part of the twentieth century, obvious differences in school resources provided to American children of different races explained substantial portions of black-white gaps in average achievement. In 1920, for example, more than one-quarter of the racial gap in children's literacy rates could be explained by differences in easy-to-measure variables such as the length of the school year and per pupil expenditures (Margo, 1986). Given the history of racial discrimination in the provision of school resources, it is understandable why, in the Civil

Rights Act of 1964, the U.S. Congress ordered a survey to document "the lack of availability of equal educational opportunities by reason of race, color, religion, or natural origin in public educational institutions at all levels."[1] In July 1966, the resulting volume entitled *Equality of Educational Opportunity* (better known as the Coleman Report, after its lead author, the eminent sociologist James Coleman) documented the substantial gaps between the average achievement of black and white children. To the surprise of many educators and civil rights activists, however, the report found no clear-cut pattern that white children attended schools with substantially more of the school resources measured in the survey than did black children. Moreover, school-to-school variation in these resources explained very little of the school-to-school variation in achievement. This led Harvard government professor Seymour Martin Lipset to summarize the results as: "schools make no difference; families make the difference."[2]

Subsequent research demonstrated that changes in the characteristics of black families, especially increases in the educational attainments of black parents, contributed to the closing of the black-white gap in average achievement during the 1970s and 1980s. However, combining the evidence on changes in the relative characteristics of black and white families with evidence on the role of family characteristics in predicting students' achievement led to the conclusion that changes in family characteristics explained less than half of the closing of the black-white gaps in achievement. Thus, researchers reconsidered the role of differences in school quality.

Controversy over the Coleman Report findings catalyzed the collection of new data to investigate the role of schooling in predicting student achievement. Many of the new data sets provided information on school resources and children's achievement over time. These data allowed researchers to demonstrate conclusively that students learned more in some classrooms

[1] Section 402, Civil Rights Act of 1964, quoted on page iii of Coleman *et al* (1966).
[2] Quotation reported by Hodgson (1973), p. 35.

and schools than in others (Rockoff, 2004; Kane, Rockoff and Staiger, 2005; Rivkin, Hanushek and Kain, 2005). This was important evidence that supported the view that schools -- and the resources they provide -- do indeed make a difference to children's academic achievement. However, with few exceptions,[3] conventional resources -- such as class size and teachers' educational attainments and years of experience -- were unable to explain a substantial proportion of the classroom-to-classroom and school-to-school variation in achievement. Neither were they successful in explaining a substantial proportion of race-related gaps in achievement. This led researchers to look for subtler differences among schools and classes than those listed in school budgets and included in standard surveys of school resources.

Two patterns from recent research stand out. First, on average, black students attended schools staffed by less-skilled teachers than did white students (Jencks and Phillips, 1998, Yinger, 2004; Clotfelter, Ladd and Vigdor, 2005). This is an example of an inequality that exists *between* the schools attended by black children and those attended by white children. Second, even when attending the same schools, black students were often treated differently than were white students. For example, they were less often enrolled in classes for the gifted and talented (Clotfelter *et al*, 2005). In addition, multiple studies have documented that teachers' expectations for black students are, on average, lower than for their white counterparts (Baron, Tom and Cooper, 1985; Ferguson, 1998; Figlio, 2005). These are examples of *within-school* inequalities.

This distinction between inequalities that exist *between schools* and those that exist *within schools* has led researchers back to the NAEP as a source of data for examining trends in

[3] Among the exceptions are the impact of class size in kindergarten and first grade, and the first two years of a teacher's professional experience. The Tennessee STAR experiment demonstrated that class size affected student achievement in both kindergarten and first grade, with smaller classes being especially important for economically disadvantaged children (Krueger, 1999). A number of recent studies have shown that elementary school students in classes taught by teachers in their first or second year of teaching learned less than those taught by more experienced teachers (Rockoff, 2004; Kane, Rockoff and Staiger, 2005; Rivkin, Hanushek and Kain, 2005).

the black-white gaps in average achievement. While the NAEP does not collect the panel data needed to estimate the contributions to individual learning over time, it does have compensating strengths. First, NAEP data are nationally representative. Second, the NAEP survey collects information on the family backgrounds of sampled students, including parental educational attainment. Third, the data sets contain information on multiple students within each sampled school. These strengths provide a basis for estimating the extent to which a black-white gap in average achievement can be explained by parental educational attainment, by differences among schools, and by differences between the average achievement of black students and white students from observationally similar family backgrounds who attend the same schools.

Using NAEP-LTT data and employing a variant of the *Oaxaca decomposition* (Oaxaca, 1973), Cook and Evans (2000) partitioned the closing of the black-white gap in average achievement between the early 1970s and the late 1980s into portions attributable to changes in parental educational attainment (a proxy for family characteristics more generally), changes in school quality, and changes in test scores of black and white students attending the same schools. While declining differences in school quality (due to convergence in inputs as a result of initiatives such as Title 1 funding) and in parental education are commonly hypothesized explanations for the observed narrowing of the average black-white achievement gap over time, Cook and Evans found that changes in these factors explained only modest portions of the observed trends. Rather, they report that approximately 83 percent of the black-white convergence in average reading achievement and 76 percent of the convergence in average mathematics achievement between 1971 and 1988 occurred *within*, rather than *between*, schools, for 13-year-old students with the same levels of parental education.

Cook and Evans do not discuss the substantive interpretation of their striking findings.

However, it is important to do so as alternative interpretations have very different implications for the mechanisms that schools, without outside help, might employ to close the achievement gap and the extent to which the gap, as a whole, would close as a result. While not exhaustive, we consider two possibilities. One is that, holding school resources constant, a decline in within-school inequality is the result of increases in the achievement of black students while that of white students decreases. That is, black student achievement improves at the expense of white students, while average achievement in the school remains constant. This interpretation is consistent with the notion that schools, acting on their own, could reduce the achievement gap.

A second possibility is that the achievement of white students in particular schools stays constant and the achievement of black students attending these same schools increases, thus increasing not only the achievement of black students but also average achievement in the school. While clearly the desired outcome, few schools are successful in carrying this out on their own. Rather, they would likely require additional resources, and the extent to which schools need additional resources may relate to school racial makeup. Nevertheless, as we explain, the decomposition methodology that Cook and Evans used implies the second interpretation of a decline in within-school differences.

III. Research Questions

The original intention of our research was to update the Cook and Evans (2000) findings in two ways. First, given the availability of additional years of NAEP-LTT data, we sought to extend the original work through 2004 in order to investigate the forces contributing to the increase in the black-white achievement gap during the 1990s and its more recent narrowing. Secondly, at the time of their research, the NAEP-LTT data files recorded the test score outcome only as raw item-level student performance and not as *Item Response Theory* (IRT) scaled scores. Thus, Cook and Evans (2000) employed the percentage of the total number of items that a student answered correctly as the metric in which to measure the student's achievement. A recent *Educational Testing Service* (ETS)-created NAEP-LTT data product not only includes the original item-level performance information but also contains the IRT-scaled achievement measures. This provides an opportunity to assess the sensitivity of the Cook and Evans (2000) findings to the choice of the metric in which student achievement is measured.

Additionally, as we began this work, we became aware of recent papers (Hanushek and Rivkin, 2006; Reardon, 2007) that questioned the technical details of the Cook and Evans decomposition methodology. This led us to examine the sensitivity of earlier findings and their interpretation to methodological choices. Consequently, we address two questions. First, we investigate how sensitive the results of Cook and Evans (2000) are to the choice of metric in which to measure student achievement and to the technical details of the decomposition method. Second, we explore what recent NAEP-LTT data reveal about the sources of the increase in the average black-white achievement gap during the 1990s and the more recent narrowing.

We have organized the remainder of the paper into several sections. In Section IV, we describe recent advances in methods for decomposing achievement gaps into their within- and between-school components. In Section V, we describe critical features of the NAEP-LTT data. In Section VI, we present results of our decompositions of achievement for the age-13 cohorts. Finally, in Section VII, we discuss the relevance of our finding for current educational policy.

IV. Three Approaches to Decomposing the Average Black-White Achievement Gap

Cook and Evans, 2000

In order to decompose trends in the average black-white achievement gap into their component parts, Cook and Evans (henceforth *"C&E"*) employed a variant of the *Oaxaca* *decomposition* (Oaxaca, 1973). While *C&E* control for several student-level covariates in their statistical models (such as student gender and the level of parental education), to simplify the exposition we focus here only on the division of a cross-sectional gap into *within*- and *between-school* components. *C&E* specified a model in which in a given year the achievement of student *i* attending school *s* is a function of school "quality" (as measured by school fixed effects[4]), a dichotomous indicator of the race of student *i*,[5] and a student-specific residual term:

$$Y_{is} = \mu_s + \gamma B_{is} + \varepsilon_{is} \qquad (1)$$

where:

Y_{is} is the test score for student i at school s,
μ_s is a school-level fixed effect measuring school-level average performance and representing the quality of school s,
B_{is} is a race indicator (1=black, 0=white) for student i at school s,
γ is a parameter representing the average within-school test score difference between blacks and whites, and
ε_{is} is a student-level error term.

Once this model has been fitted to data, we can estimate the achievement gap, $\hat{\delta}$ -- that is, the average difference in the achievement of black and white students -- in any given year, as:

$$\hat{\delta} = \Delta\overline{Y} = (\overline{\mu}^b - \overline{\mu}^w) + \hat{\gamma} \qquad (2)$$

where:

$\overline{\mu}^b$ is the average school-level performance of schools attended by black students,
$\overline{\mu}^w$ is the average school-level performance of schools attended by white students, and
$\hat{\gamma}$ is the estimated average difference in achievement between black and white students attending the same school.

[4] Cook and Evans note that while some research has defined school quality in terms of measurable school-level inputs, they define school quality "as the effect that attending a given school has on student performance after controlling for the student's observable characteristics" (p. 732). While the models presented here do not include student characteristics, the full models utilized by Cook and Evans and those used in our analyses do.
[5] All of the models presented in this section carry the simplifying assumption for the sake of clarity that all students are either black or white. The models used in our analyses allow for the inclusion of students of other races.

Equation (2) provides the *C&E* decomposition. It stipulates that, in any given year, the average difference in test scores can be decomposed into: (a) a *between-school* component, $(\bar{\mu}^b - \bar{\mu}^w)$, representing the portion of the average black-white achievement gap that is due to differences in the average performance of the schools attended by black children versus those attended by white children, and (b) a *within-school* component, $\hat{\gamma}$, representing the difference in average achievement between black and white students attending the *same* schools.

Because the focus of their work was on the decomposition of trends in the average black-white achievement gap, *C&E* then differenced versions of (2) that corresponded to different years of data collection. For example, they expressed the change in the average black-white achievement gap from 1971 to 1988 as:

$$\hat{\delta}_{88} - \hat{\delta}_{71} = [(\bar{\mu}^b_{88} - \bar{\mu}^w_{88}) + \hat{\gamma}_{88}] - [(\bar{\mu}^b_{71} - \bar{\mu}^w_{71}) + \hat{\gamma}_{71}], \tag{3}$$

or:

$$\hat{\delta}_{88} - \hat{\delta}_{71} = (\Delta\bar{\mu}_{88} + \hat{\gamma}_{88}) - (\Delta\bar{\mu}_{71} + \hat{\gamma}_{71}) \tag{4}$$

$$= (\Delta\bar{\mu}_{88} - \Delta\bar{\mu}_{71}) + (\hat{\gamma}_{88} - \hat{\gamma}_{71}). \tag{5}$$

In (5), the first term in parentheses represents the impact on the average black-white achievement gap of changes in the relative performance (or quality) of schools attended by black students and white students between 1971 and 1988. Thus, it represents changes in the *between-school* component of the gap. The second term expresses the impact of changes in the relative performance of black and white students attending the same schools and, thus, represents changes in the *within-school* component of the gap. Using this approach, *C&E* concluded that over three-quarters of the decline in the average black-white achievement gap for 13-year-olds between 1971 and 1988 was due to a reduction in within-school inequality.

Hanushek and Rivkin, 2006

Hanushek and Rivkin (2006) (henceforth *"H&R"*) offer a critique of research by Fryer and Levitt (2004, 2005), who utilized a decomposition approach virtually identical to that of *C&E* to examine trends in the average black-white achievement gap, using data from the *Early Childhood Longitudinal Survey - Kindergarten Cohort* (ECLS-K). Like *C&E*, Fryer and Levitt (2004, 2005) attribute the largest portion of the average black-white achievement gap to *within-* rather than *between-school* differences. *H&R*, however, point out that the *C&E* (and Fryer and Levitt) approach does not account for the uneven distribution of black and white students among schools. In particular, many white students attend schools that are all-white and many black students attend schools that are all-black. *H&R* highlight that, under the *C&E* decomposition approach, the achievement of students in racially homogenous schools do not contribute directly to the estimation of the within-school component of the average black-white achievement gap.

To support their argument, *H&R* provide an extreme example. Suppose that in a sample of 1000 schools only one serves both white and black students, and the student bodies of all other schools are either entirely white or entirely black. Under the *C&E* approach, the achievement levels of black and white students *within that single school* will then be completely responsible for the estimated within-school component of the black-white achievement gap for the complete sample. If the average black-white achievement gap in that single school is large relative to the achievement gap as a whole, the *C&E* approach will lead to the conclusion that most of the difference in average achievement is attributable to within-school differences. In fact, however, in this extreme case, almost all of the overall difference in black and white achievement is driven by the school-level differences. This issue is highly relevant to analyses of NAEP data, in which a large proportion of the sampled students attend schools that are mono-racial in relevant years.

H&R proposed the following alternative decomposition of the average black-white achievement gap, in a single year:

$$\hat{\delta} = \left[\sum_s \frac{n_{bs}}{n_b} \overline{Y}_s - \sum_s \frac{n_{ws}}{n_w} \overline{Y}_s \right] + \left[\left(\frac{1}{n_w} + \frac{1}{n_b} \right) \sum_s (\overline{Y}_{bs} - \overline{Y}_{ws}) p_s (1 - p_s) n_s \right] \quad (6)$$

where:

n_{bs} (n_{ws}) is the number of black (white) students in school s,
n_s is the total number of students in school s,
n_b (n_w) is the total number of black (white) students in the sample,
$\overline{Y}_s$ is the average test score for school s,
$\overline{Y}_{bs}$ ($\overline{Y}_{ws}$) is the average test score for black (white) students in school s, and
p_s is the proportion of students in school s who are black.

Here, the terms in the first set of brackets estimate the portion of the black-white achievement gap due to between-school differences, and the terms in the second set of brackets estimate the portion explained by within-school differences.[6] As was the case under the *C&E* decomposition, any racially homogenous schools ($p_s = 0$ or 1) do not contribute to the estimation of the within-school component of the average achievement gap. However, in the *H&R* decomposition, the within-school contribution of any single integrated school to the overall gap depends both on the level of integration in that school and on the number of sampled students in the school. Therefore, in the illustrative example above, the impact of the single integrated school on the overall estimation of the within-school component would be negligible. Using this approach, *H&R* conclude, contrary to Fryer and Levitt, that the majority of the black-white achievement gap in a given year can be explained by *between-school*, rather than *within-school*, differences.

Reardon, 2007

In recent work, Reardon (2007) has provided insight into the connection between the

C&E and *H&R* decomposition approaches. Because we draw heavily on Reardon's presentation, we describe his creative exposition in some detail. He first specifices a model for achievement, in a given year, as a function of individual and aggregate (within-school) student race:

$$Y_{is} = \beta_0 + \beta_1 B_{is} + \beta_2 p_s + \varepsilon_{is} \tag{7}$$

where:

B_{is} is a race indicator (1=black, 0=white) for student i at school s, and
p_s is the proportion of students in school s who are black.

Once this model has been fitted, the average black-white achievement gap can be expressed as:

$$\hat{\delta} = \hat{\beta}_1 + \hat{\beta}_2 (\overline{p}_s^b - \overline{p}_s^w), \tag{8}$$

which is equivalent to the *C&E* expression for the average black-white achievement gap with the first term, $\hat{\beta}_1$, representing the *C&E within-school* component of the gap and the second term, $\hat{\beta}_2 (\overline{p}_s^b - \overline{p}_s^w)$, representing the *C&E between-school* component of the gap.

Reardon points out that in this latter term, the difference in the average proportion of black students in schools attended by black and white children, $(\overline{p}_s^b - \overline{p}_s^w)$, is equivalent a standard measure of segregation known as the *variance ratio index of segregation*, $\hat{V}$. That is:

$$(\overline{p}_s^b - \overline{p}_s^w) = \frac{Var(p_s)}{Var(B_{is})} = \hat{V}, \tag{9}$$

where the numerator is the sample variance across schools of the proportion of students in a school who are black, and the denominator is the sample variance of the dichotomous race indicator.[7] Therefore, we can re-express the average black-white achievement gap in (8) as:

$$\hat{\delta} = \hat{\beta}_1 + \hat{\beta}_2 (\hat{V}). \tag{10}$$

[6] For a derivation of (6), see Hanushek and Rivkin (2006).
[7] See Appendix A for an algebraic proof of the equality expressed in (9).

Finally, because the *variance ratio index of segregation*, $\hat{V}$, ranges in value from 0 to 1 (where 0 indicates complete integration, and 1 complete segregation), we can rewrite (10) so that the average black-white achievement gap is a sum of three terms:

$$\hat{\delta} = \hat{\beta}_1(1-\hat{V}) + \hat{\beta}_1(\hat{V}) + \hat{\beta}_2(\hat{V}). \tag{11}$$

Reardon's critical insight is that, by reorganizing the three terms on the right-hand side of (11), he can describe the key differences between the *C&E* and *H&R* approaches. For instance, he can obtain the within-school component of the *C&E* decomposition by combining the first two terms in (11) and thus recovering equation (10). Alternatively, by combining the second and third terms in (11), he expresses the average black-white achievement gap as:

$$\hat{\delta} = \hat{\beta}_1(1-\hat{V}) + (\hat{\beta}_1 + \hat{\beta}_2)\hat{V}, \tag{12}$$

which is equivalent to the *H&R* decomposition.[8] Thus, Reardon's presentation shows that the difference between the *C&E* and *H&R* decomposition approaches rests on whether one regards the central $\hat{\beta}_1\hat{V}$ term in as part of the *within-school* component of the black-white gap in achievement, along with the $\hat{\beta}_1(1-\hat{V})$ term, or as part of the *between-school* component, along with the $\hat{\beta}_2\hat{V}$ term.[9] As he explains, the choice between these two allocations is not clear cut. Rather, Reardon argues that, while "the $\hat{\beta}_1(1-\hat{V})$ component (of the decomposition) is

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[8] See Appendix B for an algebraic proof of equivalence of the *H&R* and Reardon decomposition expressions.

[9] Following Reardon's analytic strategy, we estimate the proportion of students who are black in each school based on the reported race / ethnicity of the students in the sample. While this approach introduces bias into the $\hat{V}$ term, it is preferred for several reasons. First, while the 1988 and 1999 NAEP-LTT data files include school-reported continuous measures of school racial composition, the 1971 data includes only categorical measures. Second, several schools failed to report this information. In 1988, school-reported racial composition was missing for roughly 4 percent of the sample. In 1999, it was missing for nearly 12 percent of the sample. The consequence of estimating $\hat{V}$ from the sampled students is slight upward bias. Reardon reports an upward bias of 5-10 percent based on his analysis of ECLS-K data. Comparing our estimates of school racial composition to those reported by schools yields a very high correlation between the two measures but suggests upward bias on the order of 7percent. As Reardon notes, this bias will impact the within-school and ambiguous components of the decomposition. We account for this upward bias by adjusting our decomposition estimates accordingly.

unambiguously due to within-school differences, and the $\hat{\beta}_2\hat{V}$ component is unambiguously due to segregation and between-school differences, the remaining term, $\hat{\beta}_1\hat{V}$, is due to an interaction of between-school segregation and within-school gaps and so remains ambiguous" (p. 27).

To comprehend Reardon's three-term decomposition more fully and to support our own work described in this paper, we provide a graphical interpretation in Figure 2 that is similar to his.[10] Figure 2 displays the empirical and fitted relationships between student NAEP reading score and the proportion of students in a school who are black, separately for black and white students in the 1971 age-13 cohort.[11] White students are represented by gray points and black students by black points. The gray line represents the fitted relationship between reading achievement and the proportion of students in their schools who are black for white students, and the *solid* black line represents the same for black students; these lines both have slope $\hat{\beta}_2$.[12] Finally, the *dashed* black line represents the fitted trend line for a regression of average student reading achievement in a school on the proportion of students in a school who are black and has slope $\hat{\beta}_1 + \hat{\beta}_2$. At any point along its length, this dashed line can be thought of as a weighted average of the reading performance of black and white students in a school that has the proportion of black students indicated on the horizontal axis. Thus, it must intersect the gray line at the point that represents the fitted reading achievement of students in schools that have *no black students* (i.e., $p_s = 0$), and it must intersect the solid black line at the point that represents the fitted reading achievement of students in schools with *no white students* ($p_s = 1$).

[10] See Figures 5 through 9 and the accompanying discussion in Reardon (2007).
[11] Figure 2 is a stylized version of the relationship. To reduce visual clutter, we randomly sampled 25 percent of students into the plot. Also, the outcome for each student is the average value across the five NAEP plausible values. Regression models to estimate the fitted relationship utilized the full sample and the five plausible values.
[12] In practice, the fitted lines for black and white students need not be parallel. The model specification in (7) could include a two-way interaction between individual and aggregate race. The inclusion of such interaction terms

In Figure 2, notice the high degree of racial segregation that was evident in American schools in 1971. White students are generally clustered on the left side of the plot, in schools that are majority white. Although the tendency is less strong, black students are clustered on the right side of the plot, in schools that are majority black. Based on the full 1971 age-13 reading cohort sample, we estimate that the average white student attended a school that was seven percent black, whereas the average black student attended a school that was 59 percent black. The difference between these two proportions is the *variance ratio index of segregation*, $\hat{V}$, in this sample, and is equal to 52 percent. In Figure 2, the gray triangle depicts the predicted performance and average school racial makeup of the average white student (point A), and the black circle depicts the same for the average black student (point H). Then, the average black-white achievement gap, $\hat{\delta}$, is the *net vertical distance* and the variance ratio index of segregation, $\hat{V}$, is the *net horizontal distance* between these two symbols. As Reardon (2007) shows, interpretations of the several components of the average black-white achievement gap can be made directly from the geometry of the figure. To facilitate this, in Figure 3, we have re-plotted stylized versions of Figure 2, removing the individual data points, but retaining the important fitted trend lines, and sequentially adding several additional features to aid in explaning the three decomposition approaches discussed above.

Panel (a) of Figure 3 provides a geometric representation of the *C&E* decomposition. On the plot, horizontal distance AB equals the value of the variance ratio index of segregation, $\hat{V}$, and vertical distance BH equals the value of the average black-white achievement gap, $\hat{\delta}$. The *C&E* decomposition regards the average black-white achievement gap as the sum of two adjacent line segments, as follows:

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neither improved model fit nor substantially altered the results of our subsequent decompositions.

$$\hat{\delta} = BH = HE + BE \tag{13}$$

Based on the model specification in (7), the slopes of the solid fitted lines in Figure 3 are both $\hat{\beta}_2$, and their vertical separation is $\hat{\beta}_1$. We can map these estimates onto the geometry in panel (a). First, line segment HE represents the vertical separation of the parallel fitted trend lines for black and white children, and therefore equals $\hat{\beta}_1$. Second, BE is the vertical distance that the fitted trend line of (negative) slope $\hat{\beta}_2$ will drop over a horizontal distance of $\hat{V}$ and therefore must equal $\hat{\beta}_2\hat{V}$. Then, the average black-white achievement gap is:

$$\hat{\delta} = \hat{\beta}_1 + \hat{\beta}_2\hat{V}. \tag{14}$$

This is identical to (10) and also equal to our earlier expression for the *C&E* decomposition. Thus, the *C&E* decomposition strategy argues that the black-white achievement gap is the sum of line segments HE (the *within-school* component) and BE (the *between-school* component).

This plot is useful in highlighting what is problematic about the *C&E* decomposition. As noted, *C&E* regard the entire vertical distance $\hat{\beta}_1$ as the within-school component of the gap. Implicit in this designation is the notion that this entire portion of the gap may be eliminated through purely within-school efforts. However, it is likely the case that raising black achievement to the level of their white counterparts in low-minority schools would be a comparatively easier task than doing so in high-minority schools. Because eliminating the entire distance between the fitted lines for white and black students would require more intensive effort in higher minority schools, we should not designate the entire distance as a within-school effect.

Indeed, *H&R* dispute *C&E*'s decomposition and offer an alternative. Panel (b) of Figure 3 illustrates *H&R*'s partition of the achievement gap. Here, we have added a second vertical dotted line, descending from the gray triangle that represents the predicted reading achievement of the average white child (point A) to intersect the overall fitted relationship for all children at point C. From this point, we have added a horizontal dotted line to intersect the original vertical dotted line BH at point D. Finally, we have labeled the point at which line BH intersects the overall fitted relationship (the dashed line) between outcome and predictor as point G. The *H&R* decomposition partitions the average black-white achievement gap into the following two-part sum of grouped line segments:

$$BH = [BD + GH] + [DG], \tag{15}$$

which, we can also write as:

$$BH = [AC + GH] + [DG]. \tag{16}$$

Recall that the horizontal axis of the plot records the proportion of students in a school who are black, and therefore has a total range of 1 (from a minimum of 0 to a maximum of 1). This means that the overall fitted relationship between average achievement in a school and the proportion of students in a school who are black -- the black dashed line -- must have a slope of $(\hat{\beta}_1 + \hat{\beta}_2)$. Thus, in right-triangle CDG, the vertical side DG must be of length $(\hat{\beta}_1 + \hat{\beta}_2)\hat{V}$. We can therefore write (16) as:

$$\hat{\delta} = [AC + GH] + [(\hat{\beta}_1 + \hat{\beta}_2)\hat{V}]. \tag{17}$$

H&R regard the second component, $(\hat{\beta}_1 + \hat{\beta}_2)\hat{V}$, as the *between-school* component of the achievement gap. Consequently, we can deduce -- by the required equivalence with the gap represented in (14) -- that *H&R* regard the sum of the AC and GH line segments as the *within-school* contribution to the average black-white achievement gap:

$$[AC + GH] = \hat{\beta}_1(1 - \hat{V}). \tag{18}$$

Notice that the summed length of these two line segments is not the same length as the HE line segment that *C&E* earlier regarded as the within-school contribution.

Finally, we provide Reardon's insightful decomposition in panel (c) of Figure 3. Here, we have added a fourth dotted line, which starts at point C, runs parallel to the fitted trend line for white children, and intersects the vertical dotted line at point F. Thus, we establish parallelogram $ACFE$, ensuring that segment EF has the same length as AC. Reardon decomposes the black-white achievement gap into the following three-part sum:

$$BH = [EF + GH] + [FG] + [BE], \tag{19}$$

which we can rewrite as:

$$BH = [AC + GH] + [FG] + [BE], \tag{20}$$

Substituting from (18) for $[AC + GH]$, and replacing BE by $\hat{\beta}_2\hat{V}$, as done earlier, we find that:

$$\hat{\delta} = [\hat{\beta}_1(1-\hat{V})] + [FG] + [\hat{\beta}_2\hat{V}], \tag{21}$$

And, finally, by the required equivalence with (14), we have:

$$\hat{\delta} = [\hat{\beta}_1(1-\hat{V})] + [\hat{\beta}_1\hat{V}] + [\hat{\beta}_2\hat{V}], \tag{22}$$

which is Reardon's decomposition.

Reardon agrees with *H&R* that the sum of line segments AC and GH -- with total length $\hat{\beta}_1(1-\hat{V})$ -- represents the uniquely *within-school* contribution to the average black-white achievement gap. In addition, Reardon agrees with *C&E* that line segment BE -- with length $\hat{\beta}_2\hat{V}$ -- represents the uniquely *between-school* contribution to the average black-white achievement gap. However, he argues that the interpretation of the remaining part of the gap -- with length $\hat{\beta}_1\hat{V}$ -- is ambiguous. The fact that it is regarded as a *within-school* contribution by *C&E* and as a *between-school* contribution by *H&R* is why these latter decompositions do not support the same empirical conclusions.

What do these geometric equivalences imply substantively? We frame our discussion in terms of the distribution of resources both within and between schools. This framework carries the assumption that all variance in test scores is the product of unequal access to educational resources. Of course, this is an implausible assumption. Nevertheless, the framework of resource redistribution is illustrative for the purpose of discussing Reardon's decomposition approach from a substantive perspective. Consider the first of the decomposition three terms in (22), $\hat{\beta}_1(1-\hat{V})$. Reardon regards this term as "unambiguously" a *within-school* effect. The *C&E* and *H&R* decompositions agree with this designation. As Reardon explains, eliminating this component of the black-white achievement gap might occur through the redistribution of resources (such as access to the highest quality teachers) within schools so that the achievement of black students increases and the achievement of white students decreases, *while average achievement in each school remains constant, and no redistribution of students or resources across schools occurs.*[13] Such redistribution would, of course, be exceedingly unpopular among white parents. This would be especially the case for the parents of white children in predominantly black schools, as the model implies that achievement of these white students would decline markedly under such a within-school redistribution of resources and achievement.

Notice that if we eliminate the unequivocally within-school inequality component --- $\hat{\beta}_1(1-\hat{V})$ -- from (22) while leaving segregation ($\hat{V}$) and the relationship between school-level average performance and school racial composition unchanged, the gap remaining would be $(\hat{\beta}_1+\hat{\beta}_2)\hat{V}$. Referring back to Figure 2, this implies that within any school the achievement of both black and white students would be scattered around the diagonal black dashed line rather than around their separate, parallel lines, and the triangle and circle representing white and black mean achievements would themselves fall on the black dashed line.

[13] We recognize this discussion regarding resource redistribution either within or between schools is speculative, as

We can next consider how to eliminate what Reardon argues is the "ambiguous" term, $\hat{\beta}_1\hat{V}$, represented by line segment GF in panel (c) of Figure 3. Having already eliminated $\hat{\beta}_1(1-\hat{V})$, removing the $\hat{\beta}_1\hat{V}$ component of the gap would leave only the $\hat{\beta}_2\hat{V}$ term remaining. Eliminating the "ambiguous" term, $\hat{\beta}_1\hat{V}$, without altering the extent of school racial segregation is equivalent to rendering the black dashed line parallel to the lines representing average achievement of white students and average achievement of black students. Note that this essentially involves a rotation that could occur about any point along the black dashed line. Consider the polar alternative of rotating it about its far left point, corresponding to a school in which all students were white. This would require progressively larger performance gains the greater the percentage of black students in a school. For example, schools with only white students would not need to improve their performances at all. Those that served predominantly black students would need extraordinary performance improvements. Bringing about such changes by providing additional resources (if possible at all) would necessitate more resources the greater the percentage of black students in a school. For this reason, Reardon argues that the $\hat{\beta}_1\hat{V}$ term should not be considered a part of within-school inequality.

Why, then, is the $\hat{\beta}_1\hat{V}$ term ambiguous? If we consider the possibility of eliminating segregation (such that $\hat{V}$ goes to zero), the entire gap in (22) would become equal to $\hat{\beta}_1$. That is, if black and white students attended schools of similar racial composition, on average, we could then eliminate the remaining gap by equalizing the academic performance of black and white students within the same schools. It is under this scenario that the entirety of the $\hat{\beta}_1$ term can be considered the within-school component of the gap. Continued residential segregation,

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we do not have strong causal evidence to indicate the impact on student achievement of such redistribution.

coupled with the recent Supreme Court rulings on school integration programs in Kentucky and Seattle, however, suggest that ending racial segregation in U.S. schools is unlikely in the near future.[14] Nevertheless, this potential mechanism lends credence to the notion that component $\hat{\beta}_1\hat{V}$ is a within- rather than a between-school effect. Taken together, the $\hat{\beta}_1\hat{V}$ term might be eliminated through a change in the allocation of resources across schools based on minority status or through the elimination of school racial segregation together with within-school efforts to reduce achievement differentials by race. Because either is at least theoretically possible, $\hat{\beta}_1\hat{V}$ is neither an unambiguously within- or between-school effect.

Once the $\hat{\beta}_1(1-\hat{V})$ and $\hat{\beta}_1\hat{V}$ portions of the average black-white achievement gap are eliminated, and again assuming that segregation has remained unchanged, the gap remaining is of size $\hat{\beta}_2\hat{V}$. This is the component that Reardon identifies as "unambiguously between-school." Similar to the discussion above, without a change to segregation, if all students were scattered about a line of slope $\hat{\beta}_2$, this final portion of the gap could be eliminated only by improving those schools attended by higher proportions of black students, by reducing the quality of those schools attended by higher proportions of white students, or by some combination of the two such that $\hat{\beta}_2$ went to zero. Alternatively, this portion of the gap could be eliminated through the elimination of segregation. This would require the redistribution of students among schools. Either case implies a between-school effort.

Given Reardon's insights, we favor his three-component decomposition of the average black-white achievement gap. As shown, the decomposition results of *C&E* and *H&R* can be recovered from the Reardon decomposition by grouping the components appropriately.

[14] Parents Involved in Community Schools v. Seattle School District No. 1 *et al* No. 05-908. Decided June 28,

__Including covariates in Reardon's three-part decomposition__

We now consider how to incorporate student background characteristics into the Reardon decomposition in order to explore the relationship between trends in these variables and trends in the black-white achievement gap. The full *C&E* regression model included an indicator for gender and measures of parental educational attainment consisting of a vector of three dichotomous predictors (representing “less than high school,” “more than high school,” and “missing”, with attainment of the high school diploma as the omitted category) for each parent. As was noted, *C&E* examined how changes in the levels of, and returns to, parental education explained trends in the average black-white achievement gap.

Consistent with Reardon's approach, we treat covariates at the individual and school levels. Regarding parental education, this makes substantive sense, given that parental education may act on a student's test performance through his household circumstances and the socioeconomic makeup of his school. We begin by specifying the following model:

$$Y_{is} = \beta_0 + \beta_1(B_{is}) + \beta_2(p_s^{black}) + \beta_3(male_{is}) + \beta_4(p_s^{male}) + \gamma(pared_{is}) + \theta(p_s^{pared}) + \varepsilon_{is} \quad (23)$$

where:

p_s^{male} represents the proportion of students who are male in school s, and

p_s^{pared} represents a vector of six values indicating the proportion of students whose mothers and fathers have given levels of education ($<$ high school, $>$ high school, missing) in school s.[15]

2007, Slip Opinion, http://www.supremecourtus.gov/opinions/06pdf/05-908.pdf. Retrieved November 26, 2007.

[15] We recognize the inherent weakness in including missingness of mother's and father's education. We had considered imputing level of parental education in the case where it was missing but ultimately opted against this for two reasons. First, because we seek to compare our results to those of Cook and Evans who included missingness, it seemed sensible to maintain it as a category. More importantly, we see the indicator of missingness as potentially measuring two different phenomena. Where coded as missing, the student may simply not have had information on a parent's level of education. On the other hand, this may be an indication that the parent is absent from the home or from the student's life such that the student has little information about the parent at all. Because we are not able to reliably distinguish between these two different possibilities, we opt to retain missingness as a category. Significant black-white differences in missingness persisted across the years examined, with missingness more prevalent for fathers than for mothers. This is consistent with the notion that missingness of information on parental education is confounded with issues of absent parents, particularly regarding the fathers of black students.

While including additional covariates destroys the predictor-predictor orthogonality that Reardon relies upon in illuminating the relationship between his and other decomposition approaches, the geometric interpretation of the three-part decomposition still holds. Therefore, the entire achievement gap in a given year can be expressed as:

$$\begin{aligned}\hat{\delta} &= \hat{\beta}_1 + \hat{\beta}_2(\overline{p}_b^{black} - \overline{p}_w^{black}) \\ &+ \hat{\beta}_3(\overline{male}^b - \overline{male}^w) + \hat{\beta}_4(\overline{p}_b^{male} - \overline{p}_w^{male}) \\ &+ \hat{\gamma}(\overline{pared}^b - \overline{pared}^w) + \hat{\theta}(\overline{p}_b^{pared} - \overline{p}_w^{pared}).\end{aligned} \tag{24}$$

In (24), the first line is the portion of the black-white gap that is due to racial differences, the second line is the portion due to gender differences, and the third line is the portion due to differences in parental education at the school and individual levels. Substituting the variance ratio, $\hat{V}$, for the difference in average segregation between schools typically attended by black and white students, note that the first line of this decomposition is identical to the expression in (10). Then, Reardon's three-term decomposition can be used to decompose the first line into its within-, ambiguous, and between-school components, after controlling for other covariates.

One additional complication is that NAEP cohorts include students who are neither black nor non-Hispanic white. For instance, the 1971 sample includes students classified as Puerto Rican, and the 1988 and 1999 samples include students classified as Hispanic, Asian, and American Indian. In our regression models, we include individual-level dichotomous predictors to distinguish these racial categories as well as school-level measures of the proportion of students in the school of each of the given racial categories, with non-Hispanic white as the omitted category. Thus, we explore how differential exposure to students of racial backgrounds, other than black and white, impacts average differences in black and white achievement.

Finally, it is important to note that the decomposition analysis presented here is entirely descriptive in nature. That is, we are not able to draw causal conclusions regarding how trends in parental education and school racial makeup have impacted trends in the average black-white achievement gap. Rather, this work allows us to examine whether selected characteristics of students, their schools, and their families have trended in similar or opposing ways to average student achievement over the past 30 years, and thus contributes to clarification of stylized facts related to trends in the black-white achievement gap over time.

V. Data

Following *C&E*, we employ data from the *National Assessment of Educational Progress, Long Term Trend* (NAEP-LTT), 1971 – 2004.[16] Often called "the Nation's Report Card", the NAEP is an ongoing assessment of nationally representative cross-sectional samples of students in a variety of subjects, including reading, mathematics, science, and U.S. history. The NAEP-LTT is administered every four years to a nationally-representative sample of students in three age cohorts, ages 9, 13 and 17, corresponding to the modal grades of 4, 8 and 11, respectively.[17] In Table 1, we present the sample sizes for the NAEP-LTT reading assessments of the age-13 cohort, which we have used in our analyses, overall and by race/ethnicity.

The NAEP sample is drawn in order to estimate population and sub-population (rather than individual) characteristics at recognized levels of precision. A three-stage sampling design is used to draw the sample. First, primary sampling units (PSUs), made up of single counties or groups of counties, are identified and selected with probability proportional to size. Second, within the PSU's, schools are identified and selected with probabilities proportional to the

[16] Modifications to the assessment design were made in 2004. In 2004, therefore, two different assessments were utilized. One assessment, the bridge assessment, employed the same test booklets and administration protocols as were used in previous years. The other, a modified assessment, utilized a revised test design. In our analyses, we utilize data from the bridge assessments only to ensure comparability across years.

number of age-eligible students they contain. Third, and finally, students within the schools are selected by simple random sampling (Sedlacek, 2006). To account for this complex survey design, our analyses incorporated appropriate sampling weights.

Beginning in 1984, NAEP assessment scores have been scaled so that they are equatable over time, and until recently, scores from administration years prior to 1984 were available only in their raw, item-level form. For this reason, Cook and Evans used the percentage of multiple choice test items that a student answered correctly as their outcome of interest. Our work benefits from the availability of a new NAEP-LTT data product that provides both student-level "plausible values" and item-level performance for all years of the NAEP-LTT administration.[18]

We focus primarily on reading assessments of the age-13 cohorts because modifications made to the NAEP-LTT mathematics assessment prevent the estimation of trends prior to 1978. Nevertheless, we examine trends in mathematics but only for the period 1978-2004. We restrict our analyses to the age-13 cohorts for several reasons. Data on the age-9 cohorts contain unreliable information on parental educational attainment; large proportions of students in each year reported not knowing their parents' levels of education. The age-17 cohorts do not include school dropouts, and because high school drop-out rates were higher for black students than for white students over the years examined, their achievement differences may be understated in the NAEP-LTT data. In addition, evidence suggests that older students do not give their best efforts

[17] See http://nces.ed.gov/nationsreportcard/ltt/.

[18] The NAEP assessment uses matrix sampling in which each student answers a planned subset of test items. *Item Response Theory* (IRT) is then used to estimate student performance on a common scale. Given the small number of items administered to each student, estimates of individual level performance are imprecise and so multiple imputations of student performance are provided. This leads to the generation of the five plausible values (or scores), based on both the student's background characteristics and individual test responses, for each student (Mislevy, Beaton *et al*, 1992). These plausible values are "random draws from the posterior distribution of potential scores for all students who have similar characteristics and identical patterns of item responses" (Kolstad, 2006, 12). We make use of the five plausible values in the estimation of all test statistics. For example, we fit each regression model five times, and reported parameter estimates are average results across the five fitted models. In addition, we incorporate student replicate weights in the estimation of all standard errors using NAEP-recommended methods

when answering questions on the NAEP examinations.[19]

Table 2 summarizes the racial/ethnic composition of the schools attended by black and white students in the 1971, 1988, 1999, and 2004 age-13 reading cohorts.[20] According to these data, the level of exposure that black and white students have to students of other races has reamined essentially unchanged over the period that we consider. On average, white students attended schools in which the student body was approximately seven percent black (eight percent in 1999), whereas black students attended schools that were nearly 60 percent black (52 percent in 2004). This stability implies that changes observed in average race-related achievement gaps are, for the most part, not due to changes in aggregate patterns of school segregation. The statistics reported in Table 2 also suggest that black and white students did not differ markedly in their exposure to students of other races/ethnicities. Exposure to Hispanic students grew somewhat more for black students than for white students between 1988 and 2004, but exposure to Asian and American Indian students was similar.[21]

Figure 4 summarizes parental educational attainments of black and white students in the 1971, 1988, 1999, and 2004 age-13 reading cohorts. Between 1971 and 1988, the parental educational attainments of black students became much more similar to those of white students. Examine, for instance, the change in the percentage of mothers with some education beyond high school. As indicated in Table 13 (Appendix C), in 1971, the percentage of white students whose mothers had more than a high school education (26 percent) was ten points higher than the

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(Johnson and Rust 1992; Campbell, Hombo *et al*, 2000). See Johnson (1989) for a complete discussion of utilizing plausible values and jackknife standard error estimation with NAEP data.

[19] National Commission on NAEP 12th Grade Assessment and Reporting. 2004. *12th grade student achievement in America: A new vision for NAEP*. Washington, DC: National Assessment Governing Board.

[20] In the data for each of the years examined, race/ethnicity information was missing for a handful of students. We eliminated these students from all analyses. See Appendix C, Table 10 for comparable table based on the mathematics data for the age-13 cohorts.

[21] The 1971 age-13 reading cohort contained students classified as black, white, or Puerto Rican. There were no Asian or American Indian students in the 1971 sample. We recoded Puerto Rican students as Hispanic.

percentage of black students whose mothers had a similar educational attainment. By 1988, these percentages had risen in both groups and were identical at 38 percent. Over the next decade, however, the gap between the educational attainments of white parents and black parents opened again. For example, by 1999, 53 percent of the mothers of white children possessed some education beyond high school as compared to 42 percent of the mothers of black children.

VI. Results

Impact of test score metric

We begin by examining the sensitivity of the average black-white achievement gap to the choice of *metric* for the reading outcome. To facilitate the comparison, we employ the basic *C&E* decomposition approach. To simplify our presentation, we do not include the covariates representing student gender or level of parental education in our models. Table 3 presents a decomposition of the average black-white reading achievement gap in both 1971 and 1988 into the components that *C&E* defined as *within*- and *between-school.* The decomposition is presented for both metrics in which it was possible to measure student achievement (*percent of items answered correctly* and *IRT-scaled plausible values*).

Notice the extent to which the narrowing of the gap from 1971 to 1988 is metric dependent. When measured using the total percent of items answered correctly, the average black-white achievement gap narrowed by approximately 40 percent from just over 16 percentage points to just less than 10. Using IRT-scaled plausible values, the corresponding narrowing is almost 53 percent, from a gap of more than 38 points to just over 18 points.[22]

[22] It is worth reiterating here the process by which the NAEP plausible values are generated. After parameters for item response functions are estimated, predictive scale score distributions are estimated for each respondent. Important is the fact that these predictive distributions are conditional not only on student assessment performance but also on background characteristics, such as race and gender. Once these distributions are generated, five random draws are taken from them. These five random draws are the plausible values used in all analyses (Allen, McClellan and Stoeckel (2005)). The process of scaling and conditioning is beneficial, in that it reduces measurement error

The decomposition of the average black-white achievement gap also depends somewhat on choice of metric. With the percent correct metric, between-school differences increased substantially from about 27 percent of the gap to more than 57 percent of the gap between 1971 and 1988, a pattern reported by *C&E*. In the plausible value metric, between-school differences increased somewhat less dramatically (from 21 percent to 41.2 percent).[23] It is also the case that within-school differences decline more precipitously between 1971 and 1988 when achievement is measured by the percentage of items answered correctly than when measured by IRT-scaled plausible values. However, C&E's substantive story about the critically important role of trends in within-school achievement differences is present with both measurement metrics.

For two reasons we employ the IRT-scaled plausible values in all subsequent analyses. First, this is the outcome metric recommended by NAEP. As explained by Rogers and Stoeckel (2006), plausible values are equatable over time, while the same is not necessarily true for scores in the total percentage correct metric. Second, the multiply imputed plausible values allow for the incorporation of measurement error in the estimation of standard errors (Johnson, 1989).

Impact of decomposition methodology

Next, we examine the impact of decomposition *methodology* on the partition of the average black-white achievement gap into its components. In Table 4, we compare the *C&E* and Reardon decompositions of the black-white reading difference for the age-13 cohort in 1971 and 1988. Recall that the component that *C&E* attribute to within-school inequality is essentially the

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(born from the fact that each student is administered relatively few test score items). In that error is reduced, differences between relevant groups are increased and parameter estimates from associated regression models are changed. Therefore, it is not surprising that movement in the gap overtime appears more substantial when using the IRT-scaled plausible values or that the decomposition yields slightly different results between metrics (Personal correspondence with Daniel Koretz, April 2, 2008). A separate analysis (not presented here) revealed that the differences in results were not due simply to the process of scaling.

[23] Because the data utilized by Cook and Evans did not include the student replicate weights now available, the authors used an alternate simulation technique for standard error estimation. Their approach generally yields smaller standard errors. Nevertheless, it does not lead to different conclusions regarding statistical significance.

sum of Reardon's unambiguously within-school term and his ambiguous interaction term.[24] Because the magnitude of the latter component is large compared to the others, conclusions about the relative roles of within- and between-school inequality in explaining achievement gaps is indeed sensitive to how this term is assigned. We find compelling Reardon's argument that the ambiguous term should not be considered part of within-school inequality, and so in all subsequent work, we track all three components of the decomposition separately.

Decomposition of trends in the reading achievement gap: 1971-2004

As noted previously, we incorporated student- and school-level covariates into the Reardon decomposition approach. Specifically, we controlled for student gender and race / ethnicity and examined the impact of parental education at both the individual- and school-levels. Table 5 presents decomposition results associated with this model for 1971, 1988, 1999, and 2004. The top portion of the table displays cross-sectional results, and the bottom portion displays the decomposition of trends in the black-white achievement gap from 1971 to 1988, 1988 to 1999, and 1999 to 2004. Figure 5 provides a graphical representation of key findings.

In Table 5, note that exposure to students of other races/ethnicities has no bearing on the cross-sectional black-white differences or trends in the average achievement gap. Student gender similarly had minimal impact. In contrast, parental education plays a substantial role. As Figure 5 illustrates, cross-sectionally, differences in parental education explain between 12 and 28 percent of the achievement gap. In addition, trends in the levels of and returns to parental education explain approximately 41 percent of the narrowing of the black-white achievement gap between 1971 and 1988 and 30 percent of the widening between 1988 and 1999.

Following *C&E*, we explored the relative importance of changes in the levels of parental

[24] The columns do not add perfectly, as the Cook and Evans and Reardon approaches treat race/ethnicity variables other than black and white differently.

education versus changes in the associated returns to parental education on the observed achievement gap trends. Algebraically, we decompose the impact of changes in parental education on the changes in the size of the achievement gap as:

$$(\Delta\overline{X}_{88} - \Delta\overline{X}_{71})\hat{\theta}_{71} + \Delta\overline{X}_{71}(\hat{\theta}_{88} - \hat{\theta}_{71}) + (\Delta\overline{X}_{88} - \Delta\overline{X}_{71})(\hat{\theta}_{88} - \hat{\theta}_{71}), \qquad (25)$$

or:

$$(\Delta\overline{X}_{88} - \Delta\overline{X}_{71})\hat{\theta}_{88} + \Delta\overline{X}_{88}(\hat{\theta}_{88} - \hat{\theta}_{71}) + (\Delta\overline{X}_{71} - \Delta\overline{X}_{88})(\hat{\theta}_{88} - \hat{\theta}_{71}). \qquad (26)$$

where:

$\Delta\overline{X}$ represents the differences in levels of average parental education between black and white students at the school- and individual-levels for a particular year, and
$\hat{\theta}$ represents the parameter estimates associated with returns to parental education at both school- and individual-levels for a particular year.

The difference between these two expressions is that the first utilizes 1971 as the base year, whereas the second utilizes 1988 as the base year. Because there remains no consensus on which approach is to be preferred (Johnson and Solon, 1986), we present results using both base years. In both expressions, the first term is the effect of the changes in levels of parental education, the second term is the effect of the changes in returns to parental education, and the third term represents an interaction between the two. Note that in comparing (25) and (26), the interaction terms based on these two expressions will have the same absolute values but opposite signs.

As shown in Table 6, the closing of the gap in the parental educational attainments of black and white students explains much of the overall effect of parental education during the period 1971-1988. Changes in the returns to education also contributed substantially. From 1988 to 1999, the widening of the gap is much more attributable to changes in the levels of parental education.[25] As shown in Figure 4, between 1988 and 1999, white parents became much

[25] We do not decompose trends in parental education from 1999 to 2004, as they hardly serve to account for

more likely than black parents to have education beyond high school completion. While the college-going rate of mothers of white children increased by 15 percentage points between 1988 and 1999, the college going rate for black mothers increased by only 4 percentage points. The trends for fathers' educational attainments are similar: an increase of 12 percentage points in the college-going rate for white fathers and a decrease of two percentage points for black fathers.

Next, by comparing the results presented in Tables 4 and 5, we examine the extent to which inclusion of parental education covariates influenced the attribution of the narrowing of the racial gap. The important lesson concerns the role of between-school differences. When estimated from models that do not include parental education (Table 4), our results suggest that changes in between-school inequality played almost no role in accounting for the closing of the gap between 1971 and 1988. When estimated from models that controlled for parental education (Table 5), between-school inequality increased. In other words, the increase in the educational attainments of black parents from 1971 to 1988 were just sufficient to overcome the effect on black students' achievement of the relative worsening in quality of the schools they attended.

Recall from Table 2 that the extent of segregation between black and white students changed little over this period of time. Therefore, the movement that we observe in the three decomposition components is a function of movement in the parameter estimates over time.[26] Between 1971 and 1988, the parameter estimate on the indicator for a black student became less negative (from -27.12 in 1971 to -11.31 in 1988) and then became more negative in 1999 (-17.18), thus explaining the narrowing of the within-school and ambiguous decomposition components from 1971 to 1988 and their subsequent widening from 1988 to 1999. In contrast, the impact of the proportion of black students in a school became increasingly negative (-2.43

movement in the average black-white achievement gap over this period.

[26] See Appendix C, Table 13 for parameter estimates used to calculate decompositions presented in Table 5.

in 1971, -9.93 in 1988, and -14.88 in 1999). In other words, the deleterious impact on reading achievement of attending an overwhelmingly black school increased from 1971 to 1999.

Interestingly, in 2004, between-school differences again explain little of the difference between black and white student achievement. Therefore, the modest narrowing of the black-white achievement gap between 1999 and 2004 is due to a decline in between-school inequality.

Decomposition of trends in the mathematics achievement gap: 1978-2004

Table 7 presents decomposition results for age-13 students using the NAEP-LTT mathematics assessment for years 1978, 1986, 1999, and 2004.[27] While *C&E* use 1990 as the middle year, we preferred 1986, as the 13-year-old mathematics achievement gap was at its nadir in this year (See Figure 1). As noted previously, 1978 is the earliest year for which mathematics plausible values are estimated and available.

Trends in the mathematics achievement gap are similar to the reading trends. The average mathematics gap declined by more than 50 percent between 1978 and 1986, and then lost half of this gain over the next 14 years. In many respects, the results of decomposing the trends are similar as well. As shown in Table 8, trends in the educational attainments of parents partially explain both the narrowing of the racial achievement gaps in mathematics and in reading between 1978 and 1999 and the subsequent widening over the next decade.

One respect in which the results of decomposing the mathematics trends differ from those in reading concerns the role of between-school inequality. While the reading achievement of students in majority-black schools fell relative to that of students in majority-white schools throughout the period from 1971-1999, this was not the case for mathematics achievement. During the period 1978-1986, the mathematics achievement of students in majority-black schools increased relative to that of students in majority-white schools. This improvement explained 8 percent of the closing of the mathematics gap during this period. However, this improvement was short-lived. From 1986 to 1999, between-school inequality increased, explaining 18 percent of the increase in the mathematics achievement gap during that period. From 1999 to 2004, a decline in between-school inequality contributed to the modest narrowing of the gap in mathematics, just as was the case for reading achievement.

Exploring the growth in between-school differences

Our analyses of reading achievement data show that between-school differences grew consistently from 1971 to 1999 despite the overall narrowing of the gap in the first part of this period. Similar to analyses conducted by *C&E*, we explore the extent to which the growth in between-school differences was driven by the worsening of high-minority, high-poverty urban schools during this period of time. As they note, between school differences could grow for two distinct reasons. First, migration and differentials in population growth could lead to black students being more concentrated in areas with relatively low-performing schools. Alternatively, the schools that black students attend could have deteriorated in relative quality.

Regarding the first possibility, Jargowsky (1997) highlights that between 1970 and 1990, urban black poor became more concentrated in high-poverty areas. This pattern is present in the NAEP data. In 1971, 17 percent of black students attended predominantly minority schools in disadvantaged urban areas. This rose to 29 percent in 1988, to 32 percent in 1999, and fell to 26 percent in 2004. In all years, less than one percent of white students attended such schools.

To explore whether declines in the quality of high-minority, urban schools account for the trends in the black-white achievement gap in reading, we recalculated the decomposition results presented in Table 5, but did so excluding from the analysis students in predominantly minority schools located in disadvantaged urban areas. Comparison of the results with those

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[27] See Appendix C, Table 14 for parameter estimates used to calculate decompositions presented in Table 7.

presented in Table 5 reveal three patterns. First, the trends in the overall gap are similar. With the students from disadvantaged urban, high-minority schools removed, the achievement gap narrows somewhat more through the 1970s and 1980s, widens less through the 1990s, and remains essentially unchanged from 1999 to 2004. Nevertheless, the pattern is similar, showing that the black-white achievement gap is not solely a result of low achievement among black students attending high-minority, urban schools. Second, within-school differences are more important for explaining cross-sectional and trend differences in the sample that does not include students in high-minority, urban schools. Third, once these students are removed from the sample, there is no longer consistent growth in between-school differences. Taken together, these findings show that the growth of between-school differences from 1971 to 1999 is driven in large part by a combination of the worsening of student performance in high-minority schools in disadvantaged urban areas and the increased concentration of black students in these schools.

VII. Conclusion

Summary of results

Our analyses provide five critical results. First, trends in the magnitude of the black-white achievement gap in reading are somewhat sensitive to the choice of test metric, as is the partitioning of the gap into within- and between-school components. Also, using NAEP plausible values, rather than percent of items answered correctly, to represent achievement generally results in larger standard errors, because the plausible values provide a mechanism for considering measurement error in individual test scores. Despite these many modest differences, the substantive results reported by *C&E* are not sensitive to the choice of test score metric.

Second, the choice of decomposition methodology affects the findings a great deal. In particular, Cook and Evans' finding that a decline in within-school inequality is a dominant factor in explaining the closing of black-white achievement gaps from 1971 to 1988 is very sensitive to methodology**.** The C&E methodology rests on two strong assumptions. First, it implies that, holding resources constant, schools, acting alone, have the capacity to raise the level of achievement of black student to the level of their white counterparts, without sacrificing at all white student achievement. However, we must consider what steps a school might take to raise the achievement of black students. Without the infusion of additional resources, a school might reallocate resources, such as its most highly skilled teachers. It is quite possible, then, that such a reallocation would lead to a reduction in the achievement of white students in that school.

In addition, their approach implies that reducing the gap between black and white students within a school is equally possible for all schools, regardless of school racial makeup. For example, acting alone, a school that is 75 percent white and 25 percent black can eliminate its racial gap just as readily as a school that is 75 percent black and 25 percent white. Not only would relatively few schools be able to accomplish such gains on their own, but also the higher minority school is charged with raising achievement for a larger proportion of its student body. Therefore, it is likely that both schools would require additional resources and that the lower performing, higher minority school would require more intensive investments. The presence of such variation across schools is contrary to the *C&E* defnition of within-school inequality.

Third, trends in parental education are important in explaining trends in the black-white achievement gap through the 1970s, 1980s and 1990s. Moreover, shifts in the levels of parental education are more important for explaining trends than are shifts in their returns.

Fourth, while within-school inequality remains substantial across the years analyzed, it becomes less important over time relative to between-school inequality in explaining changes in the size of the black-white achievement gap.

Finally, between-school differences became increasingly important over time in explaining trends in the black-white achievement gap. From 1971 to 1999, this is driven by the increasingly detrimental impact on achievement of attending a high minority school.

Discussion

It is important to reiterate the limitation that this work is entirely descriptive in nature. Therefore, we are not able to draw causal conclusions regarding potential mechanisms for alleviating the persistent gap in achievement between black and white students. A second limitation, as noted by *C&E*, is the lack of detailed family information in the NAEP datasets. These authors provide a useful discussion of the extent of likely omitted variables bias present in their estimates based on not including other measures of family income and structure (i.e., living in a two-parent household). This slight bias is likely present in our estimates of the effect of parental education on test scores in our cross-sectional regressions. Given this caveat, our estimates nevertheless suggest that the relationship between trends in parental education and student academic achievement is substantial. Therefore, the widening of the gap in parental educational attainment by race in the 1990s is troubling.

Our results point to the important roles of both within- and between-school differences in explaining trends in the black-white achievement gap over the past several decades. While our results suggest a declining relative importance of within-school differences in explaining achievement gap trends over time, they remain substantial in an absolute sense. From a policy perspective, this signals the need for continued efforts to combat within-school inequalities, such as differential access to the best teachers and to the most challenging curriculum. At the same time, it is important to keep in mind that only a portion of the black-white achievement gap consists of pure within-school differences. Consequently, reducing discrepancies within schools can play only a partial role in closing the entire black-white achievement gap.

The most important implication of our work is to show that between-school differences are more important than was previously thought and become relatively more important over the last 30 years of the 20th century in explaining trends in the black-white achievement gap. From a policy perspective, we might consider two approaches to combatting the negative consequences of attending a high minority school, especially one in a high poverty urban setting.

The first is a substantial reduction in the level of school segregation in the U.S. At least for now, however, the U.S. Supreme Court has deemed unconstitutional policies aimed at reducing segregation by race, including voluntary policies such as those introduced in Seattle and St. Louis (Linn *et al*, 2007). The second approach is improving those schools serving largely minority populations, particularly in high-poverty urban areas. The modest narrowing of the black-white achievement gap during the period 1999-2004 suggests that local, state, and federal policies aimed at improving urban schools may be on the right track. Still, research has yet to show whether it is possible to close the black-white achievement gap without a concerted and sustained effort to integrate America's schools.

Figure 1: Difference in average white and black achievement in reading and mathematics for 9-, 13-, and 17-year-olds on the NAEP-LTT assessment, by year.

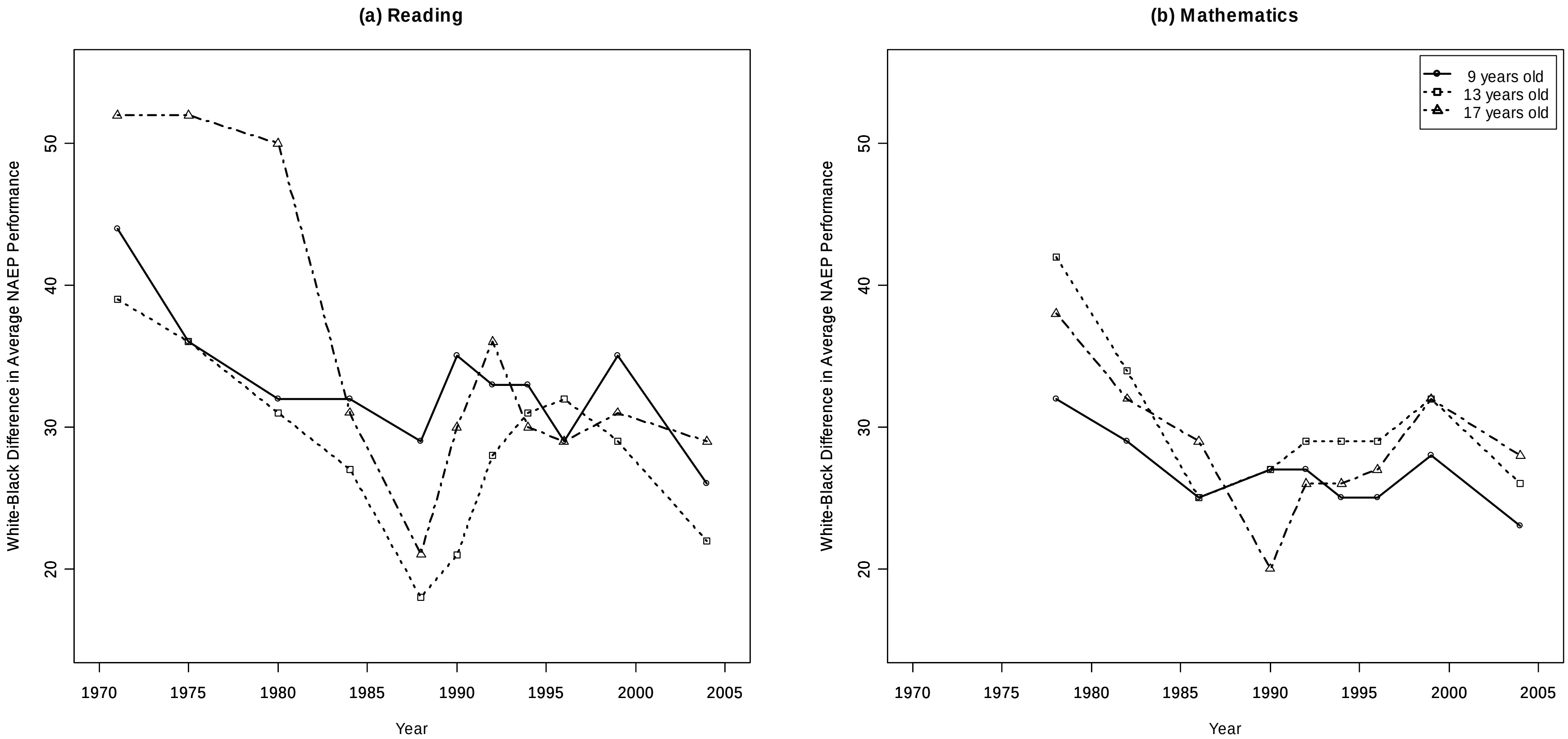

Figure 2: Fitted relationship between NAEP scaled reading score and the proportion of students in school who are black, for black and white students in the age-13 cohort, 1971.

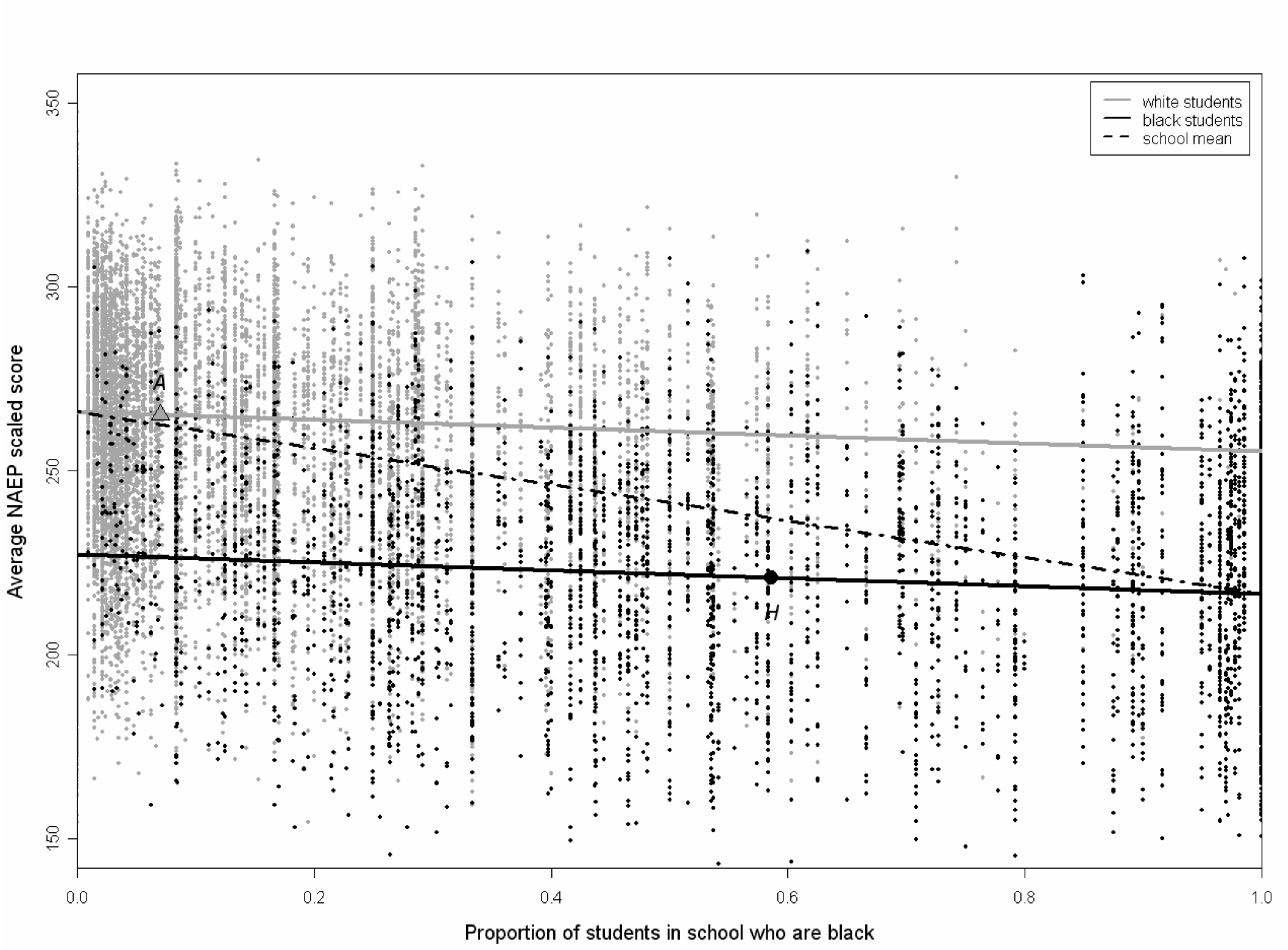

Figure 3: Graphical comparison of decomposition approaches

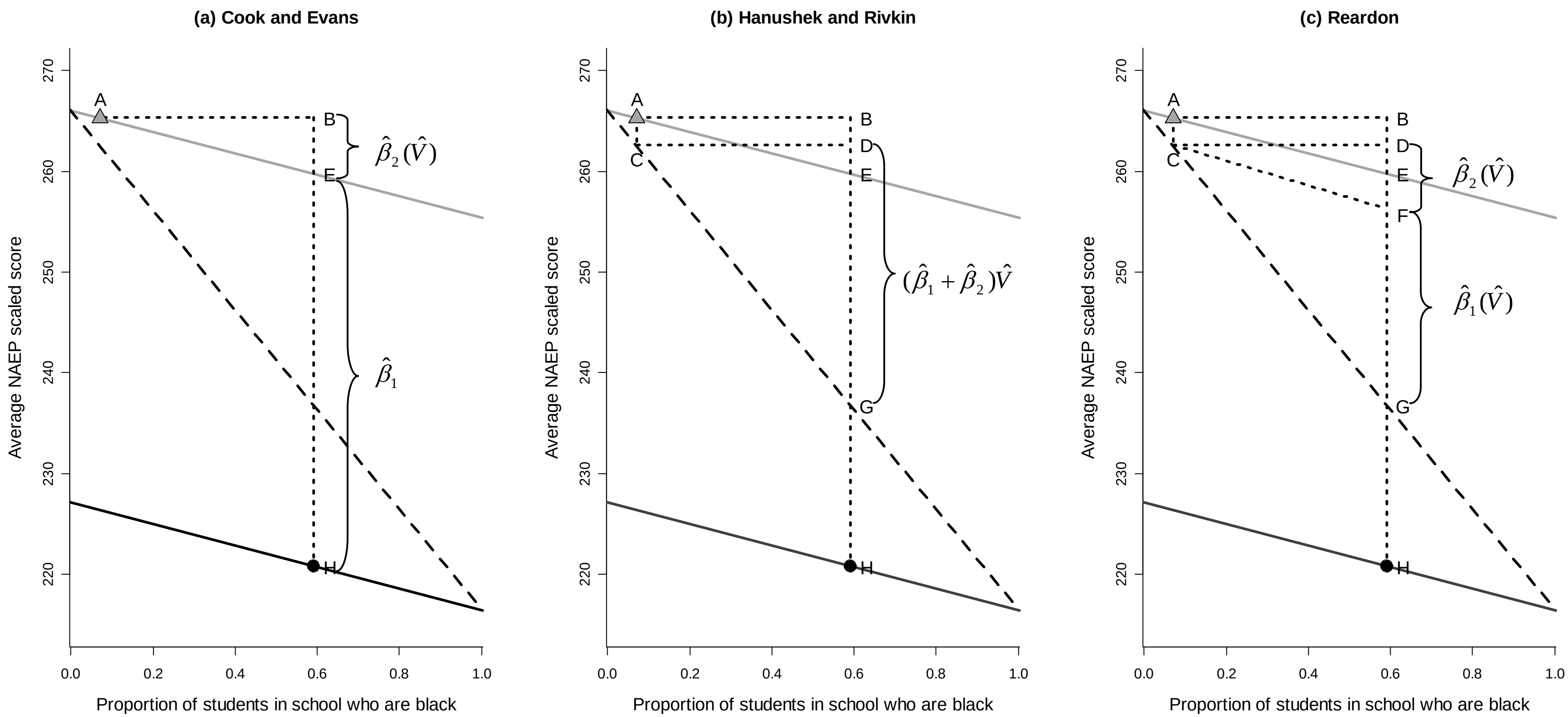

Figure 4: Average parental education of black and white students in the age-13 NAEP reading cohorts, 1971, 1988, 1999, 2004.

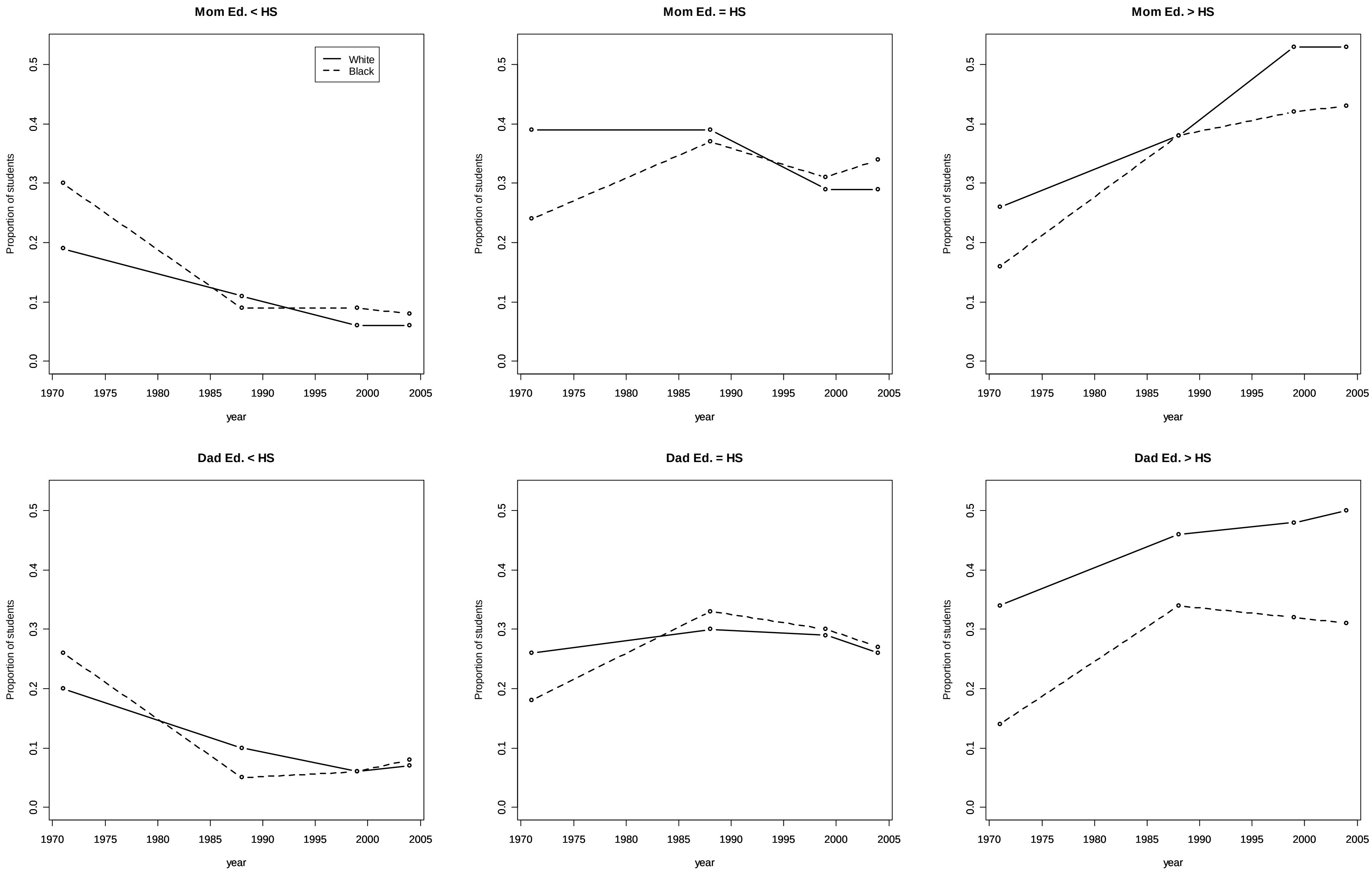

Figure 5: Decomposition results, age-13 NAEP-LTT reading cohorts, 1971, 1988, 1999, 2004.

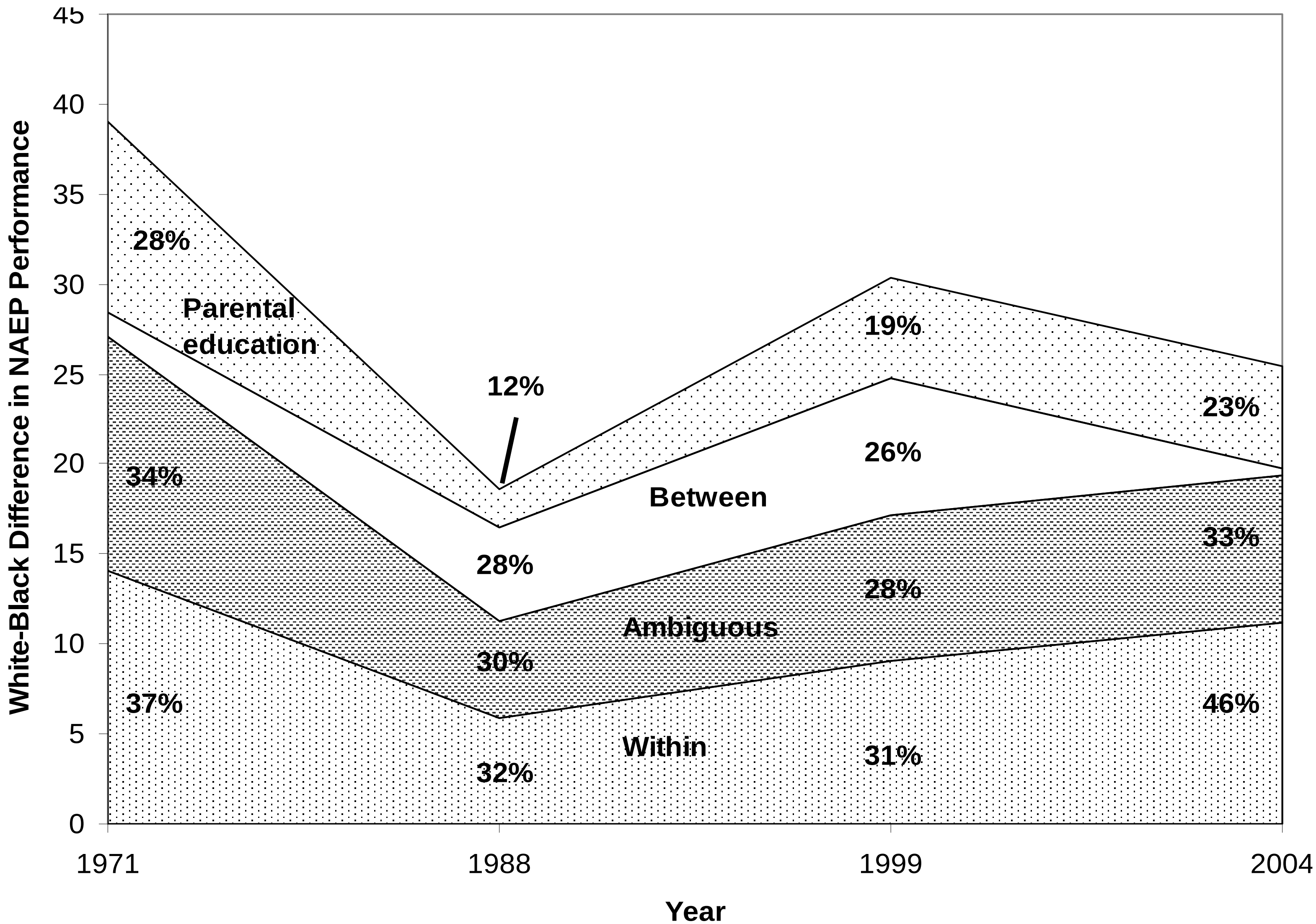

Table 1: Sample sizes for NAEP-LTT reading assessments of the age-13 cohorts, overall and by student race/ethnicity.

	1971	1988	1999	2004
Overall	25,545	4,001	4,090	4720
White	20,907	2,890	2,547	2,795
Black	4,254	576	704	682
Hispanic	384	358	633	921
Asian		100	136	241
American Indian		77	70	81

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1971-2004, NAEP Long-Term Trend Reading Assessments.

Table 2: Aggregate racial characteristics of schools attended by black and white students in the age-13 NAEP reading cohorts, by year.

	1971			1988			1999			2004		
	White	Black	Diff	White	Black	Diff	White	Black	Diff	White	Black	Diff
School % Black	0.07	0.59	−0.52	0.07	0.58	−0.52	0.08	0.59	−0.51	0.07	0.52	−0.45
	(0.01)	(0.03)		(0.01)	(0.05)		(0.01)	(0.03)		(0.01)	(0.04)	
School % Hispanic	0.01	0.01	0.00	0.07	0.07	−0.00	0.09	0.11	−0.01	0.12	0.15	−0.03
	(0.00)	(0.00)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.02)	
School % Asian				0.02	0.02	0.00	0.02	0.02	0.00	0.03	0.04	−0.01
				(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	0.01)	
School % Am. Indian				0.00	0.01	−0.01	0.00	0.01	−0.01	0.00	0.01	−0.01
				(0.12)	(0.00)		(0.13)	(0.00)		(0.14)	(0.00)	

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1971-2004 NAEP Long-Term Trend Reading Assessments.

Table 3: Impact of test score metric on the decomposition of the average black-white achievement gap in reading for the age-13 NAEP-LTT cohorts, 1971 and 1988.

	Percent of items answered correctly			IRT-scaled plausible values		
	Gap	Within school	Between school	Gap	Within school	Between school
1971	−16.21	−11.91	−4.31	−38.53	−30.45	−8.08
	(0.70)	(0.75)	(0.78)	(1.31)	(1.33)	(1.34)
		73.4%	26.6%		79.0%	21.0%
1988	−9.77	−4.17	−5.61	−18.18	−10.69	−7.49
	(1.39)	(1.45)	(1.73)	(2.48)	(3.69)	(3.00)
		42.6%	57.4%		58.8%	41.2%
1971-1988	−6.44	−7.74	1.30	−20.35	−19.76	−0.59
	(1.56)	(1.63)	(1.90)	(2.80)	(3.92)	(3.29)
		120.2%	−20.2%		97.1%	2.9%

Notes: All results calculated using sample weights. Standard errors, in parentheses, calculated using NAEP-recommended jackknife variance calculation procedures. In the case of the plausible values, results presented are based on average results across all five plausible values.

Table 4: Impact of decomposition methodology on the decomposition of the average black-white achievement gap in reading for the age-13 NAEP-LTT cohort, from 1971 to 1988.

	Cook and Evans Approach			Reardon Approach		
	Gap	Within school	Between school	Within school	Ambiguous	Between school
1971	−38.53	−30.45	−8.08	−14.77	−15.73	−8.06
	(1.31)	(1.33)	(1.34)	(1.22)	(1.05)	(1.33)
		79.0%	21.0%	38.3%	40.8%	20.9%
1988	−18.18	−10.69	−7.49	−5.16	−5.53	−7.40
	(2.48)	(3.69)	(3.00)	(1.91)	(1.95)	(3.01)
		58.8%	41.2%	28.4%	30.38%	40.71%
1971-1988	−20.35	−19.76	−0.59	−9.61	−10.21	−0.66
	(2.80)	(3.92)	(3.29)	(2.26)	(2.22)	(3.30)
		97.1%	2.9%	47.2%	50.2%	3.3%

Notes: All results presented are based on average results across five plausible values and are calculated using sample weights. Standard errors, in parentheses, calculated using complete NAEP-recommended variance calculation procedures. Percentages calculated using the Reardon approach do not sum to 100%, as this model also attributes a portion of the gap to the proportion of students in a school who are Hispanic, Asian, or American Indian.

Table 5: Decomposition results, all covariates, age-13 NAEP-LTT reading cohorts, 1971, 1988, 1999, 2004.

	Gap	Within	Ambiguous	Between	Hispanic	Asian	American Indian	Gender	Parental Education
1971	−38.53	−14.11	−13.01	−1.26	0.02			0.42	−10.60
	(1.31)	(1.11)	(0.92)	(1.18)	(0.02)			(0.17)	(0.91)
		36.62%	33.77%	3.26%	−0.06%			−1.09%	27.50%
1988	−18.18	−5.87	−5.44	−5.14	−0.01	−0.03	−0.06	0.55	−2.19
	(2.48)	(1.81)	(1.86)	(3.48)	(0.05)	(0.08)	(0.12)	(0.25)	(1.48)
		32.29%	29.92%	28.24%	0.05%	0.18%	0.32%	−3.04%	12.04%
1999	−29.52	−9.01	−8.17	−7.61	0.13	−0.01	0.15	0.55	−5.58
	(2.53)	(1.92)	(1.54)	(3.75)	(0.17)	(0.05)	(0.18)	(0.18)	(1.34)
		30.52%	27.68%	25.79%	−0.45%	0.04%	−0.52%	−1.88%	18.91%
2004	−24.55	−11.22	−8.17	−0.41	0.01	0.02	0.30	0.53	−5.60
	(2.45)	(1.77)	(1.45)	(1.92)	(0.13)	(0.04)	(0.20)	(0.55)	(1.20)
		45.70%	33.28%	1.66%	−0.05%	−0.07%	−1.23%	−2.18%	22.83%
1971-1988	−20.35	−8.24	−7.57	3.88	0.03	0.03	0.06	−0.13	−8.41
	(2.80)	(2.13)	(2.07)	(3.68)	(0.05)	(0.08)	(0.12)	(0.30)	(1.74)
		40.49%	37.20%	−19.07%	−0.16%	−0.16%	−0.29%	0.65%	41.32%
1988-1999	11.34	3.14	2.73	2.48	−0.14	−0.04	−0.21	−0.00	3.39
	(3.54)	(2.64)	(2.41)	(5.12)	(0.18)	(0.10)	(0.22)	(0.31)	(2.00)
		27.69%	24.07%	21.85%	−1.25%	−0.40%	−1.88%	−0.02%	29.93%
1999-2004	−4.97	2.21	0.00	−7.20	0.12	−0.00	−0.15	0.02	0.02
	(3.52)	(2.61)	(2.12)	(4.21)	(0.21)	(0.06)	(0.27)	(0.58)	(1.80)
		−44.47%	−0.00%	144.85%	−2.41%	0.07%	2.98%	−0.41%	−0.44%

Notes: All results presented are based on average results across five plausible values and are calculated using sample weights. Standard errors, in parentheses, calculated using complete NAEP-recommended variance calculation procedures.

Table 6: Decomposition of the parental education effect into portions due to change in level of parental education and change in returns to parental education, 1971-1988 and 1988-1999.

	Change	**% of Total Change**
Difference in gap 1971-1988	−20.35	
Change due to parental education	−8.41	41.32%
1970 as base year:		
Level	−6.45	31.71%
Return	−4.94	24.30%
Interaction	2.99	−14.69%
1988 as base year:		
Level	−3.46	17.02%
Return	−1.96	9.61%
Interaction	−2.99	14.69%
Difference in gap 1988-1999	11.34	
Change due to parental education	3.33	29.93%
1988 as base year:		
Level	1.67	14.74%
Return	−0.23	−2.00%
Interaction	1.95	17.19%
1999 as base year:		
Level	3.62	31.93%
Return	1.72	15.19%
Interaction	−1.95	−17.19%

Table 7: Decomposition results, all covariates, age-13 NAEP-LTT mathematics cohorts, 1978, 1986, 1999, 2004.

	Gap	Within	Ambiguous	Between	Hispanic	Asian	American Indian	Gender	Parental Education
1978	−41.98	−14.14	−13.33	−4.49	−0.01	0.04	0.00	0.13	−10.18
	(1.94)	(1.17)	(1.14)	(1.98)	(0.05)	(0.06)	(0.02)	(0.11)	(1.31)
		33.68%	31.75%	10.70%	0.02%	−0.10%	−0.00%	−0.31%	24.25%
1986	−20.30	−10.02	−4.17	−2.67	−0.16	−0.02	0.32	−0.09	−3.49
	(1.95)	(1.24)	(0.60)	(2.48)	(0.43)	(0.10)	(0.17)	(0.27)	(2.57)
		49.36%	20.54%	13.17%	0.81%	0.07%	−1.59%	0.43%	17.18%
1999	−31.24	−11.84	−9.45	−4.65	0.74	−0.06	0.02	−0.10	−5.90
	(2.32)	(1.38)	(1.03)	(2.08)	(0.44)	(0.08)	(0.05)	(0.11)	(1.66)
		37.90%	30.25%	14.87%	−2.37%	0.21%	−0.05%	0.31%	18.88%
2004	−26.87	−11.98	−7.20	−2.79	0.23	0.07	0.01	−0.03	−5.18
	(1.69)	(1.51)	(0.94)	(1.63)	(0.25)	(0.12)	(0.05)	(0.07)	(1.25)
		44.59%	26.80%	10.37%	−0.86%	−0.26%	−0.03%	0.12%	19.28%
1978-1986	−21.69	−4.12	−9.16	−1.82	0.16	0.06	−0.32	0.22	−6.69
	(2.75)	(1.71)	(1.29)	(3.18)	(0.43)	(0.12)	(0.17)	(0.29)	(2.89)
		18.99%	42.23%	8.40%	−0.72%	−0.26%	1.48%	−1.00%	30.86%
1986-1999	10.94	1.82	5.28	1.97	−0.90	0.05	0.31	0.01	2.41
	(3.03)	(1.86)	(1.19)	(3.24)	(0.61)	(0.13)	(0.17)	(0.29)	(3.07)
		16.64%	48.26%	18.03%	−8.27%	0.45%	2.79%	0.09%	22.04%
1999-2004	−4.37	0.14	−2.25	−1.86	0.51	−0.13	0.01	−0.06	−0.72
	(2.86)	(2.04)	(1.40)	(2.65)	(0.51)	(0.15)	(0.07)	(0.13)	(2.08)
		−3.20%	51.49%	42.58%	−11.68%	3.04%	−0.22%	1.44%	16.45%

Notes: All results presented are based on average results across five plausible values and are calculated using sample weights. Standard errors, in parentheses, calculated using complete NAEP-recommended variance calculation procedures.

Table 8: Decomposition of the parental education effect into portions due to change in level of parental education and change in returns to parental education, age-13 NAEP mathematics cohorts, 1978-1986 and 1986-1999.

	Change	% of Total Change
Difference in gap 1978-1986	−21.69	
Change due to parental education	−6.69	30.86%
1978 as base year:		
Level	−5.47	25.21%
Return	−3.46	15.97%
Interaction	2.24	−10.33%
1986 as base year:		
Level	−3.23	14.88%
Return	−1.22	5.65%
Interaction	−2.24	10.33%
Difference in gap 1986-1999	10.94	
Change due to parental education	2.41	22.04%
1986 as base year:		
Level	2.96	27.07%
Return	−1.05	−9.57%
Interaction	0.5	4.54%
1999 as base year:		
Level	3.46	31.61%
Return	−0.55	−5.03%
Interaction	−0.5	−4.54%

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Appendix A: Derivation of $\bar{p}_s^b - \bar{p}_s^w = \frac{Var(p_s)}{Var(B_{is})}$

First, it is useful to reexpress $Var(p_s)$ and $Var(B_{is})$:

$$Var(p_s) = E[p_s^2] - (E[p_s])^2 = \frac{1}{N}\left[\sum_s n_s \left(\frac{n_{bs}}{n_s}\right)^2\right] - \left(\frac{n_b}{N}\right)^2$$

$$Var(B_{is}) = E[B_{is}^2] - (E[B_{is}])^2 = E[B_{is}] - (E[B_{is}])^2$$

$$= \frac{n_b}{N} - \left(\frac{n_b}{N}\right)^2 = \frac{n_b}{N}\left(1 - \frac{n_b}{N}\right) = \frac{n_b}{N}\frac{n_w}{N} = \frac{n_b n_w}{N^2}$$

Then, we can complete the proof of equivalence as follows:

$$\bar{p}_s^b - \bar{p}_s^w = \frac{Var(p_s)}{Var(B_{is})}$$

$$\frac{1}{n_b}\sum_i B_{is} p_s - \frac{1}{n_w}\sum_i (1 - B_{is}) p_s = \frac{\frac{1}{N}\left[\sum_s n_s \left(\frac{n_{bs}}{n_s}\right)^2\right] - \left(\frac{n_b}{N}\right)^2}{\frac{n_b n_w}{N^2}}$$

$$\frac{n_w \sum_i B_{is} p_s - n_b \sum_i (1 - B_{is}) p_s}{n_b n_w} = \frac{N\left[\sum_s n_s \left(\frac{n_{bs}}{n_s}\right)^2\right] - n_b^2}{n_b n_w}$$

$$n_w \sum_i B_{is} p_s - n_b \sum_i (p_s - B_{is} p_s) = N \sum_s \frac{n_{bs}^2}{n_s} - n_b^2$$

$$n_w \sum_i B_{is} p_s - n_b \sum_i p_s + n_b \sum_i B_{is} p_s = N \sum_s \frac{n_{bs}^2}{n_s} - n_b^2$$

$$N \sum_i B_{is} p_s - n_b \sum_i p_s = N \sum_s \frac{n_{bs}^2}{n_s} - n_b^2$$

$$N \sum_i B_{is} p_s - n_b \sum_s \frac{n_{bs}}{n_s} n_s = N \sum_s \frac{n_{bs}^2}{n_s} - n_b^2$$

$$N \sum_i B_{is} p_s - n_b \sum_s \frac{n_{bs}}{n_s} n_s = N \sum_s \frac{n_{bs}^2}{n_s} - n_b^2$$

$$N \sum_i B_{is} p_s - n_b^2 = N \sum_s \frac{n_{bs}^2}{n_s} - n_b^2$$

$$\sum_i B_{is} p_s = \sum_i B_{is} \frac{n_{bs}}{n_s} = \sum_s n_{bs} \frac{n_{bs}}{n_s} = \sum_s \frac{n_{bs}^2}{n_s}.$$

Appendix B: Equivalence of Hanushek and Rivkin and Reardon Decomposition Expressions

Reardon notes that because of the orthogonality of $B_{is} - p_s$ and p_s, the $\hat{\beta}_1$ and $\hat{\beta}_1 + \hat{\beta}_2$ terms resulting from fitting equation (7) can be expressed as:

$$\hat{\beta}_1 = \frac{Cov(Y_{is}, B_{is} - p_s)}{Var(B_{is} - p_s)}$$

$$\hat{\beta}_1 + \hat{\beta}_2 = \frac{Cov(Y_{is}, p_s)}{Var(p_s)}$$

Then, because

$$\hat{V} = \frac{Var(p_s)}{Var(B_{is})}, \text{and}$$

$$(1 - \hat{V}) = \frac{Var(B_{is}) - Var(p_s)}{Var(B_{is})},$$

Reardon is able to reexpress the average black-white achievement gap as:

$$\hat{\beta}_1(1 - \hat{V}) + (\hat{\beta}_1 + \hat{\beta}_2)\hat{V} = \frac{Cov(Y_{is}, B_{is} - p_s)}{Var(B_{is} - p_s)} \times \frac{Var(B_{is}) - Var(p_s)}{Var(B_{is})} + \frac{Cov(Y_{is}, p_s)}{Var(p_s)} \times \frac{Var(p_s)}{Var(B_{is})}$$

$$= \frac{Cov(Y_{is}, B_{is} - p_s)}{Var(B_{is})} + \frac{Cov(Y_{is}, p_s)}{Var(B_{is})}$$

Where the first term to the right of the equals sign is equivalent to the Hanushek and Rivkin within-school term, and the second is equivalent to their between-school term. The following provides a proof of these equalities.

First, note that:

$$Var(B_{is}) = \frac{n_b n_w}{N^2},$$

as was shown in Appendix A.

Equivalence of within-school terms:

$$\left(\frac{1}{n_w}+\frac{1}{n_b}\right)\sum_s(\bar{Y}_{bs}-\bar{Y}_{ws})p_s(1-p_s)n_s = \frac{Cov(Y_{is},B_{is}-p_s)}{Var(B_{is})}$$

$$\left(\frac{n_b+n_w}{n_b n_w}\right)\sum_s(\bar{Y}_{bs}-\bar{Y}_{ws})p_s(1-p_s)n_s = \frac{Cov(Y_{is},B_{is}-p_s)}{\frac{n_b n_w}{N^2}}$$

$$\left(\frac{N}{n_b n_w}\right)\sum_s(\bar{Y}_{bs}-\bar{Y}_{ws})p_s(1-p_s)n_s = \frac{N^2}{n_b n_w}Cov(Y_{is},B_{is}-p_s)$$

$$\sum_s(\bar{Y}_{bs}-\bar{Y}_{ws})p_s(1-p_s)n_s = N\left(E[Y_{is}(B_{is}-p_s)]-E[Y_{is}]E[B_{is}-p_s]\right)$$

$$= N\left(\frac{1}{N}\sum_i(Y_{is}(B_{is}-p_s))-\bar{Y}\frac{1}{N}\sum_i(B_{is}-p_s)\right)$$

$$= \sum_i Y_{is}(B_{is}-p_s)-\bar{Y}\sum_i(B_{is}-p_s)$$

$$= \sum_i Y_{is}B_{is}-\sum_i Y_{is}p_s-\bar{Y}\sum_i B_{is}+\bar{Y}\sum_i p_s$$

$$= \sum_s \bar{Y}_{bs}n_{bs}-\sum_s \bar{Y}_s n_{bs}-\bar{Y}\sum_s n_{bs}+\bar{Y}\sum_s n_{bs} = \sum_s(\bar{Y}_{bs}-\bar{Y}_s)n_{bs}$$

$$\sum_s(\bar{Y}_{bs}-\bar{Y}_{ws})p_s(1-p_s)n_s = \sum_s(\bar{Y}_{bs}-\bar{Y}_s)p_s n_s$$

Now, we can reexpress $\bar{Y}_s = \frac{1}{n_s}\left(n_{ws}\bar{Y}_{ws}+n_{bs}\bar{Y}_{bs}\right)$. Then,

$$\sum_s(\bar{Y}_{bs}-\bar{Y}_{ws})(1-p_s) = \sum_s\left(\bar{Y}_{bs}-\frac{n_{ws}}{n_s}\bar{Y}_{ws}-\frac{n_{bs}}{n_s}\bar{Y}_{bs}\right)$$

$$= \sum_s\left(\bar{Y}_{bs}\left(1-\frac{n_{bs}}{n_s}\right)-\frac{n_{ws}}{n_s}\bar{Y}_{ws}\right) = \sum_s\left(\frac{n_{ws}}{n_s}\bar{Y}_{bs}-\frac{n_{ws}}{n_s}\bar{Y}_{ws}\right)$$

$$= \sum_s\left(\bar{Y}_{bs}-\bar{Y}_{ws}\right)\frac{n_{ws}}{n_s} = \sum_s\left(\bar{Y}_{bs}-\bar{Y}_{ws}\right)(1-p_s)$$

Equivalence of between-school terms:

$$\sum_s \frac{n_{bs}}{n_b}\overline{Y}_s - \sum_s \frac{n_{ws}}{n_w}\overline{Y}_s = \frac{Cov(Y_{is}, p_s)}{Var(B_{is})}$$

$$\frac{1}{n_b}\sum_s n_{bs}\overline{Y}_s - \frac{1}{n_w}\sum_s n_{ws}\overline{Y}_s = \frac{Cov(Y_{is}, p_s)}{\frac{n_b n_w}{N^2}}$$

$$\frac{n_w \sum_s n_{bs}\overline{Y}_s - n_b \sum_s n_{ws}\overline{Y}_s}{n_b n_w} = \frac{N^2 Cov(Y_{is}, p_s)}{n_b n_w}$$

$$n_w \sum_s n_{bs}\overline{Y}_s - n_b \sum_s n_{ws}\overline{Y}_s = N^2\left(E[Y_{is} p_s] - E[Y_{is}]E[p_s]\right)$$

$$n_w \sum_s n_s p_s \overline{Y}_s - n_b \sum_s n_s (1 - p_s)\overline{Y}_s = N^2\left(\frac{1}{N}\sum_i Y_{is} p_s - \overline{Y}\frac{1}{N}\sum_i p_s\right)$$

$$\sum_s (n_w n_s p_s \overline{Y}_s - n_b n_s \overline{Y}_s + n_b n_s p_s \overline{Y}_s) = N\sum_i Y_{is} p_s - N\overline{Y}\sum_i p_s$$

$$\sum_s N n_s p_s \overline{Y}_s - \sum_s n_b n_s \overline{Y}_s = N\sum_i Y_{is} p_s - N\overline{Y}\sum_i p_s$$

$$N\sum_s n_s p_s \overline{Y}_s - \sum_s n_b n_s \overline{Y}_s = N\sum_s \overline{Y}_s n_s p_s - N\overline{Y}\sum_s p_s n_s$$

$$N\sum_s n_s p_s \overline{Y}_s - \sum_s n_b n_s \overline{Y}_s = N\sum_s \overline{Y}_s n_s p_s - N\overline{Y}\sum_s p_s n_s$$

$$n_b \sum_s n_s \overline{Y}_s = N\overline{Y}\sum_s p_s n_s$$

$$n_b \sum_s n_s \overline{Y}_s = N\overline{Y} n_b$$

$$\sum_s n_s \overline{Y}_s = N\overline{Y} = \sum_s n_s \overline{Y}_s$$

Appendix C: Descriptive Statistics and Regression Model Parameter Estimates

Table 9: Sample sizes for NAEP-LTT mathematics assessments of the age-13 cohorts, overall and by race/ethnicity.

	1978	1986	1999	2004
Overall	24,180	6,200	5,934	5,712
White	18,559	3,667	3,699	3451
Black	3,983	1,462	1,064	800
Hispanic	1,447	868	859	1,102
Asian	153	112	218	267
American Indian	29	91	94	92

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1978-2004, NAEP Long-Term Trend Mathematics Assessments.

Table 10: Aggregate racial characteristics of schools attended by black and white students in the age-13 NAEP-LTT mathematics cohorts, by year.

	1978			1986			1999			2004		
	White	**Black**	**Diff**	**White**	**Black**	**Diff**	**White**	**Black**	**Diff**	**White**	**Black**	**Diff**
School % Black	0.06	0.58	−0.52	0.13	0.45	−0.32	0.07	0.55	−0.48	0.07	0.47	−0.40
	0.01)	(0.03)		(0.03)	(0.04)		0.01)	(0.03)		(0.01)	(0.04)	
School % Hispanic	0.04	0.04	−0.00	0.08	0.12	−0.04	0.07	0.12	−0.05	0.12	0.17	−0.05
	0.00)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.02)	
School % Asian	0.01	0.00	0.00	0.01	0.01	0.00	0.03	0.02	0.01	0.03	0.03	−0.00
	(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.00)		(0.00)	(0.01)	
School % Amer. Indian	0.00	0.00	0.00	0.00	0.01	−0.01	0.00	0.01	−0.01	0.00	0.01	−0.01
	(0.01)	(0.00)		(0.09)	(0.00)		(0.09)	(0.00)		(0.13)	(0.00)	

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1978-2004, NAEP Long-Term Trend Mathematics Assessments.

Table 11: Average parental education and gender characteristics for black and white students in the age-13 NAEP reading cohorts in 1971, 1988, 1999, 2004.

	1971			1988			1999			2004		
	White	**Black**	**Diff**	**White**	**Black**	**Diff**	**White**	**Black**	**Diff**	**White**	**Black**	**Diff**
Mom Ed. < HS	0.19	0.30	−0.11	0.11	0.09	0.02	0.06	0.09	−0.03	0.06	0.08	−0.02
	(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Mom Ed. > HS	0.26	0.16	0.10	0.38	0.38	−0.00	0.53	0.42	0.11	0.53	0.43	0.10
	(0.01)	(0.01)		(0.02)	(0.03)		(0.02)	(0.02)		(0.01)	(0.02)	
Mom Ed. Miss	0.16	0.30	−0.14	0.12	0.16	−0.04	0.12	0.18	−0.06	0.12	0.15	−0.03
	(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.02)	
Dad Ed. < HS	0.20	0.26	−0.05	0.10	0.05	0.04	0.06	0.06	−0.00	0.07	0.08	−0.01
	(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Dad Ed. > HS	0.34	0.14	0.20	0.46	0.34	0.12	0.48	0.32	0.16	0.50	0.31	0.18
	(0.01)	(0.01)		(0.02)	(0.03)		(0.02)	(0.02)		(0.02)	(0.02)	
Dad Ed. Miss	0.20	0.42	−0.22	0.14	0.28	−0.14	0.17	0.32	−0.15	0.17	0.34	−0.17
	(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.02)		(0.01)	(0.02)	
Sch. % Mom Ed. < HS	0.20	0.27	−0.07	0.11	0.13	−0.01	0.07	0.09	−0.02	0.07	0.10	−0.03
	(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.01)	
Sch. % Mom Ed. > HS	0.26	0.19	0.07	0.37	0.37	−0.00	0.50	0.42	0.08	0.50	0.43	0.08
	(0.01)	(0.01)		(0.01)	(0.03)		(0.02)	(0.02)		(0.01)	(0.02)	
Sch. % Mom Ed. Miss	0.17	0.26	−0.10	0.12	0.17	−0.05	0.13	0.19	−0.05	0.14	0.17	−0.03
	(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.02)		(0.01)	(0.01)	
Sch. % Dad Ed. < HS	0.21	0.25	−0.04	0.10	0.07	0.03	0.07	0.09	−0.02	0.08	0.10	−0.02
	(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Sch. % Dad Ed. > HS	0.33	0.19	0.14	0.45	0.36	0.08	0.46	0.33	0.13	0.47	0.33	0.14
	(0.01)	(0.01)		(0.02)	(0.02)		(0.02)	(0.02)		(0.02)	(0.02)	
Sch. % Dad Ed. Miss	0.21	0.36	−0.15	0.16	0.27	−0.12	0.19	0.31	−0.11	0.20	0.32	−0.12
	(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.02)	
Male	0.50	0.47	0.03	0.50	0.44	0.05	0.50	0.45	0.05	0.49	0.45	0.04
	(0.00)	(0.01)		(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.02)	
Sch. % Male	0.50	0.49	0.01	0.49	0.47	0.01	0.49	0.49	0.00	0.49	0.48	0.01
	(0.00)	(0.01)		(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.02)	

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1971-2004 NAEP Long-Term Trend Reading Assessments.

Table 12: Average parental education and gender characteristics for black and white students in the age-13 NAEP mathematics cohorts in 1978, 1986, 1999, 2004.

	1978			1986			1999			2004		
	White	Black	Diff	White	Black	Diff	White	Black	Diff	White	Black	Diff
Mom Ed. < HS	0.15	0.21	−0.06	0.12	0.12	0.00	0.06	0.09	−0.03	0.06	0.07	−0.01
	(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Mom Ed. > HS	0.29	0.20	0.09	0.42	0.40	0.02	0.59	0.50	0.09	0.61	0.51	0.09
	(0.01)	(0.01)		(0.02)	(0.03)		(0.01)	(0.03)		(0.01)	(0.02)	
Mom Ed. Miss	0.15	0.29	−0.13	0.10	0.18	−0.08	0.10	0.12	−0.02	0.11	0.14	−0.03
	(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.02)		(0.01)	(0.01)	
Dad Ed. < HS	0.15	0.18	−0.03	0.11	0.10	0.01	0.07	0.06	0.01	0.07	0.08	−0.01
	(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Dad Ed. > HS	0.36	0.17	0.19	0.45	0.35	0.10	0.55	0.36	0.19	0.54	0.39	0.14
	(0.01)	(0.01)		(0.03)	(0.02)		(0.02)	(0.02)		(0.01)	(0.02)	
Dad Ed. Miss	0.20	0.40	−0.21	0.15	0.28	−0.13	0.16	0.29	−0.13	0.17	0.29	−0.13
	(0.01)	(0.02)		(0.02)	(0.02)		(0.01)	(0.02)		(0.01)	(0.02)	
Sch. % Mom Ed. < HS	0.16	0.20	−0.04	0.12	0.13	−0.01	0.07	0.10	−0.03	0.07	0.10	−0.03
	(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Sch. % Mom Ed. > HS	0.28	0.21	0.07	0.40	0.40	0.00	0.56	0.49	0.08	0.57	0.49	0.09
	(0.01)	(0.01)		(0.02)	(0.02)		(0.01)	(0.03)		(0.01)	(0.02)	
Sch. % Mom Ed. Miss	0.17	0.27	−0.11	0.12	0.15	−0.03	0.11	0.14	−0.03	0.13	0.16	−0.03
	(0.01)	(0.02)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Sch. % Dad Ed. < HS	0.16	0.18	−0.02	0.11	0.11	0.00	0.07	0.08	−0.01	0.08	0.09	−0.01
	(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.01)	
Sch. % Dad Ed. > HS	0.34	0.21	0.13	0.43	0.37	0.06	0.53	0.38	0.16	0.51	0.40	0.11
	(0.01)	(0.01)		(0.03)	(0.02)		(0.02)	(0.02)		(0.01)	(0.02)	
Sch. % Dad Ed. Miss	0.21	0.37	−0.16	0.18	0.23	−0.06	0.18	0.28	−0.10	0.19	0.27	−0.07
	(0.01)	(0.02)		(0.02)	(0.02)		(0.01)	(0.02)		(0.01)	(0.01)	
Male	0.50	0.47	0.03	0.49	0.49	0.01	0.51	0.49	0.02	0.48	0.46	0.01
	(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.02)		(0.01)	(0.02)	
Sch. % Male	0.50	0.49	0.01	0.50	0.50	−0.00	0.50	0.51	−0.01	0.48	0.47	0.00
	(0.01)	(0.01)		(0.01)	(0.01)		(0.01)	(0.02)		(0.01)	(0.01)	

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1978-2004, NAEP Long-Term Trend Mathematics Assessments.

Table 13: Parameter estimates, all covariates, age-13 NAEP-LTT reading cohorts, 1971, 1988, 1999, 2004.

	1971	1988	1999	2004
Intercept	285.52	263.70	267.22	268.00
Black	−27.12	−11.31	−17.18	−19.40
School % Black	−2.43	−9.93	−14.88	−0.90
Hispanic	−0.90	−13.71	−14.64	−13.06
School % Hispanic	−14.45	−2.94	9.83	0.43
Asian		14.19	−2.51	3.81
School % Asian		12.86	−11.19	4.39
Amer. Indian		−6.71	−12.13	−9.69
School % Amer. Indian		13.42	−28.61	−36.56
Male	−11.13	−11.57	−12.60	−10.45
School % Male	−12.52	4.79	4.35	−11.58
Mom Ed. < HS	−6.94	−2.64	−7.89	−6.15
Mom Ed. > HS	4.50	5.51	6.24	4.77
Mom Ed. Missing	−11.09	−6.74	−8.39	−6.98
Dad Ed. < HS	−3.64	−3.50	−6.37	−1.70
Dad Ed. > HS	7.78	7.22	7.04	8.08
Dad Ed. Missing	−7.13	−2.19	−5.50	−3.79
School % Mom Ed. < HS	−22.66	−0.75	−24.35	5.15
School % Mom Ed. > HS	−6.43	−3.74	17.33	11.65
School % Mom Ed. Missing	−10.23	2.28	−18.86	−3.95
School % Dad Ed. < HS	−10.11	−10.86	−5.40	−14.86
School % Dad Ed. > HS	3.43	4.18	−5.61	5.64
School % Dad Ed. Missing	−10.41	−7.99	1.10	−5.91

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1971-2004, NAEP Long-Term Trend Reading Assessments.

Table 14: Parameter estimates, all covariates, age-13 NAEP-LTT mathematics cohorts, 1978, 1986, 1999, 2004.

	1978	1986	1999	2004
Intercept	283.88	281.24	267.09	269.22
Black	−27.48	−14.19	−21.29	−19.18
School % Black	−8.62	−8.47	−9.73	−6.90
Hispanic	−15.98	−15.99	−18.98	−15.38
School % Hispanic	−6.62	−4.46	14.78	4.56
Asian	6.05	18.08	1.74	9.24
School % Asian	−16.71	37.93	11.93	18.55
Amer. Indian	−4.71	−7.03	−10.01	−12.47
School % Amer. Indian	−23.08	−39.91	11.90	−1.91
Male	−1.46	1.86	2.27	4.23
School % Male	−7.18	−19.56	−3.99	−8.12
Mom Ed. < HS	−7.61	−3.29	−0.66	−4.27
Mom Ed. > HS	5.88	5.76	6.16	6.51
Mom Ed. Missing	−10.73	−8.04	−3.11	−7.45
Dad Ed. < HS	−3.81	−3.54	−2.38	−3.42
Dad Ed. > HS	8.88	7.15	8.28	5.14
Dad Ed. Missing	−7.56	−5.56	−3.13	−2.33
School % Mom Ed. < HS	−38.86	−31.46	−2.82	8.88
School % Mom Ed. > HS	−10.48	−10.11	24.87	14.83
School % Mom Ed. Missing	−3.36	21.83	−13.20	−14.34
School % Dad Ed. < HS	2.81	−8.73	−13.71	4.23
School % Dad Ed. > HS	12.45	18.79	−0.77	17.59
School % Dad Ed. Missing	−10.53	−9.67	−8.06	0.92

Source: U.S. Department of Education, Institute of Education Sciences, National Center for Education Statistics, National Assessment of Educational Progress (NAEP), selected years, 1971-2004, NAEP Long-Term Trend Reading Assessments.