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DOMESTIC AND FOREIGN DISTURBANCES IN AN OPTIMIZING MODEL OF EXCHANGE RATE DETERMINATION

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ABSTRACT

This paper analyzes the effects of various disturbances of domestic and foreign origin in a small open economy under imperfect capital mobility in which the behavioral relationships are derived from optimization by the private sector. In this model the domestic economy jumps instantaneously to its new equilibrium following a change in either the domestic monetary growth rate or domestic fiscal policy. In response to a disturbance in either the foreign interest rate or inflation rate, the economy undergoes an initial partial jump towards its new equilibrium, which it thereafter approaches gradually. The implications of these results for exchange rate adjustment and the insulation properties of flexible exchange rates are discussed.

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1. INTRODUCTION

Much has been written recently on analyzing exchange rate determination in small open economies under flexible exchange rates. The models developed typically consider the effects of different disturbances on the behavior of the economy both in the short run and over time. The disturbances usually analyzed are domestic monetary and fiscal policy changes on the one hand, and foreign interest rate and price changes, on the other. Two issues which have received a lot of attention include: the adjustment of the exchange rate to monetary disturbances, and the degree to which flexible rates insulate the domestic economy against foreign disturbances. The overwhelming bulk of the existing literature employs ad hoc models of an essentially arbitrary nature. As a consequence, many of the conclusions with respect to these and other issues depend upon the chosen specification of the model. Indeed much of the thrust of the existing literature is to investigate the sensitivity of various propositions to specific assumptions.[1/]

Two exceptions to the above characteristics of the literature are papers by Obstfeld (1981) and Hodrick (1982), which analyze the impacts of various domestic policy disturbances in a model in which the behavioral functions are derived from explicit optimization on the part of agents in the economy.[2/] Both studies are based on the assumptions of purchasing power parity (PPP) and uncovered interest parity (UIP). In the Hodrick analysis, which is based on the assumption of a constant rate of consumer time preference, the disturbances lead to instantaneous jumps in the economy from one steady state to another. There is no transitional dynamics as is typically the case in ad hoc models based on similar assumptions. By contrast, the Obstfeld analysis endogenizes the rate of time preference in a manner suggested by Uzawa (1968) and finds the optimizing system to exhibit saddle-point behavior.

In this paper we analyze the effects of both domestic and foreign disturbances using an optimizing approach under the more plausible empirical assumption that capital is only imperfectly mobile internationally, in the sense that UIP does not hold.[3/] The framework we employ is an adaptation of the Brock and Turnovsky (1981) perfect foresight equilibrium. This is characterized by: (i) all relationships are derived from optimizing behavior; (ii) all expectations are realized; (iii) all markets clear. We retain the assumption of PPP, as the degree of substitutability between domestic and foreign goods is inessential to the results we obtain.

The remainder of the paper proceeds as follows. Section 2 outlines the framework, spelling out the optimization and deriving the perfect foresign equilibrium. The steady state equilibrium is discussed in Section 3, while the transitional dynamics is considered in the following section. Section 5 analyzes the short-run effects on the domestic economy of the various domestic and foreign disturbances. The main findings of the study are reviewed in the final section, where some generalizations of the analysis are also discussed.

2. THE FRAMEWORK

The model contains three sectors: (i) consumers, (ii) firms, and (iii) the government. To preserve analytical tractability we assume that all consumers and firms are identical, enabling us to focus on the representative individual in each group.

A. Structure of Economy

We assume that the domestic economy is small. It produces a single traded good, the foreign price of which is given on the world market. In percentage change terms, the PPP relationship implies

$$p = q + e \tag{1}$$

where

p = rate of inflation of the good in domestic currency,

q = rate of inflation of the good in foreign currency,

e = rate of exchange depreciation.

We assume that domestic residents may hold three assets. The first is domestic money, which is not held by foreigners. Secondly, they may hold a non-traded bond, issued by the domestic government, which is denominated in terms of domestic currency and pays a nominal interest rate r. Thirdly, they may hold (or issue) a traded bond denominated in foreign currency and paying an exogenous world interest rate r*.

The representative consumer's plans are obtained by solving the following intertemporal optimization problem[4/]

$$\text{Max} \int_0^\infty e^{-\beta t}[U(c,\ell) + V(m)]dt \qquad U_c > 0 \; , \; U_\ell < 0 \; , \; V' > 0 \; ;$$

$$U_{cc} < 0 \; , \; U_{\ell\ell} < 0 \; , \; U_{c\ell} < 0 \; , \; V'' < 0 \tag{1a}$$

subject to

$$c + \dot{m} + \dot{b} + \dot{a} = w\ell + \pi + (r^*-q)b + (r-q-e)a - (q+e)m$$

$$- qb - \frac{1}{2}\alpha b^2 - \tau \tag{1b}$$

and initial conditions

$$m(0) = \frac{M_0}{P(0)} \; ; \; a(0) = \frac{A_0}{P(0)} \; ; \; b(0) = \frac{B_0}{Q_0} \tag{1c}$$

where

c = real consumption,

m = real money balances; M = nominal money balances,

b = real stock of traded bonds; B = nominal stock of traded bonds,

a = real stock of non-traded bonds; A = nominal stock of non-traded bonds,

ℓ = labor,

w = real wage rate, taken as parameterically given,

π = real profit, taken as parameterically given,

r = domestic nominal interest rate,

r* = foreign nominal interest rate,

e = anticipated rate of exchange depreciation, which is equal to the actual rate of exchange depreciation,

τ = lump sum tax,

β = rate of time preference, taken to be constant,

P = domestic price level; Q = foreign price level.

The budget constraint is straightforward, although several features merit comment. First, since the analysis is based on perfect foresight, the anticipated rate of exchange depreciation is assumed equal to the actual rate of exchange depreciation. Secondly, since the domestic assets are denominated in domestic currency, they are subject to the unit rate of capital loss of $p = q + e$. By contrast, since the traded bond is denominated in foreign currency, the corresponding rate of capital loss is q. Thirdly, the budget constraint includes a cost item $1/2\ \alpha b^2$. This term is meant to capture, in a certainty equivalent framework, the imperfect substitutability between domestic and foreign bonds. In a stochastic model the cost parameter α would be a function of the degree of exchange risk and the degree of risk aversion of dmestic investors. While our representation of imperfect capital mobility is restrictive, it will be seen below that it is perfectly sensible in the sense that the demand function it yields for traded bonds is precisely equivalent to those derived from standard stochastic optimization models.

The utility function is assumed to be concave in its arguments c, ℓ, and m. The assumption $U_{c\ell} < 0$ that the marginal utility of consumption decreases with labor (increases with leisure) is a plausible one and renders the results more determinate. The separability of utility into a function of c and ℓ on the one hand and m on the other is made primarily for expositional simplicity. It leads to a separation of the real part of the system from the nominal. In the concluding section we note how relaxing this assumption changes our results in a minimal way.

In determining his optimal plans for c, ℓ, m, b, and a, the consumer is assumed to take e, q, π, τ, w, r, r*, the price level P and the exchange rate E, as parameterically given. The initial conditions in (1c) relate to the initial stocks of real bonds and real stock of money balances held by consumers. By definition these are the corresponding initial nominal stocks, divided by the initial price level.

Firms are assumed to produce output by hiring labor to maximize real profit

$$\pi = f(\ell) - w\ell \tag{2}$$

where $f(\ell)$ is the firm's production function, assumed to possess the usual neo-classical property of positive, but diminishing, marginal product of labor.

The domestic government's budget constraint, expressed in real terms is

$$\dot{m} + \dot{a} = g + ra - \tau - (q+e)(m+a) \tag{3}$$

where g denotes real government expenditure and all other variables are defined above. This equation asserts that total real expenditures, including interest payments, less the lump sum tax and 'inflation tax' revenues, must be financed either through printing money or by issuing bonds.

The rate of accumulation of traded bonds is by definition equal to the balance of payments on current account, which in turn equals the balance of trade plus the real interest payments on the net holdings of foreign debt. With a single traded commodity the balance of trade is simply the excess of domestic production over domestic absorption by the domestic private and government sector. In real terms the relationship is given by

$$\dot{b} = f(\ell) - c - g + (r^*-q)b \tag{4}$$

Ignoring the term $\frac{1}{2}\alpha b^2$, which is an attempt to express the consumer budget constraint in certainty equivalent terms, we can see that the constraints (1b), (2), (3) and (4) are linearly dependent. One can therefore be ignored.

B. Determination of Perfect Foresight Equilibrium

We consider a perfect foresight equilibrium to the model to be one in which the planned demand and supply functions that solve the optimization problems, consistent with the accumulation equations, clear all markets at all points of time. We now proceed to develop these conditions for the economy.

Beginning with consumers, we write the optimization problem (1a)-(1b) in the Lagrangian form

$$H \equiv e^{-\beta t}[U(c,\ell) + V(m)] + \mu e^{-\beta t}\{w\ell + \pi + (r^*-q)b + (r-q-e)a - (q+e)m - \dot{m} - \dot{b} - \dot{a} - c - \frac{1}{2}\alpha b^2 - qb - \tau\} \quad (5)$$

where $\mu e^{-\beta t}$ is the associated discounted Lagrange multiplier. The optimality conditions for consumers are

$$U_c(c,\ell) - \mu = 0 \quad (6a)$$

$$U_\ell(c,\ell) + w\mu = 0 \quad (6b)$$

$$V'(m) - \mu(q+e) = -\dot{\mu} + \mu\beta \quad (6c)$$

$$\mu[r^* - \alpha b - q] = -\dot{\mu} + \mu\beta \quad (6d)$$

$$\mu[r - (q+e)] = -\dot{\mu} + \mu\beta \quad (6e)$$

together with the budget constraint (1b) and initial conditions (1c). In addition there are transversality conditions at infinity

$$\lim_{t\to\infty} \mu m e^{-\beta t} = \lim_{t\to\infty} \mu b e^{-\beta t} = \lim_{t\to\infty} \mu a e^{-\beta t} = 0 \quad (6f)$$

Equations (6a)-(6e) are the Euler equations corresponding to the optimization with respect to c, ℓ, m, b, a, respectively. Combining equations (6d) and (6e), we derive

$$b = \frac{1}{\alpha}[r^* + e - r] \tag{7}$$

This equation expresses the net demand for foreign bonds by domestic residents as being proportional to the uncovered interest differential on foreign and domestic bonds. This form of demand function has been postulated by Driskill and McCafferty (1980) and Turnovsky and Bhandari (1982), among others. It is essentially analogous to that derived in a two-period mean-variance stochastic framework by Eaton and Turnovsky (1981) and Dornbusch (1980). In terms of that framework, $\alpha \cong A\sigma_e^2$, where A is a measure of absolute risk aversion and σ_e^2 is the one period variance of the exchange rate. Noting that $R = Az$, where R denotes the coefficient of relative risk aversion and z is real wealth, and assuming that R is constant, the demand function derived using the mean-variance approach can be expressed in terms of a portfolio fraction.[5/]

Taking α to be constant, as we shall throughout our analysis, our formulation is essentially a certainty-equivalent specification of constant absolute risk aversion. On the other hand, postulating α to be inversely proportional to wealth is analogous to assuming constant relative risk aversion. In either case, the coefficient α can be used to parameterize the degree of capital mobility. In the case of perfect capital mobility (no exchange risk or risk neutrality), $\alpha = 0$ and (7) reduces to the UIP condition

$$r = r^* + e \tag{7'}$$

In the case of zero capital mobility, $\alpha \to \infty$, so that $b = 0$; i.e., domestic residents hold no traded bonds. Thus the specification of the cost term in the budget constraint gives rise to a demand for foreign bonds which is entirely reasonable and accords with previous specifications, both derived theoretically or simply postulated.

The optimality condition for the representative firm usual marginal production condition

$$f'(\ell) = w \tag{8a}$$

together with the definition of profit

$$\pi = f(\ell) - w\ell \tag{8b}$$

To complete the specification, government policy needs to be spelled out. We assume that the government sets its real expenditure g exogenously. It also sets a constant rate of nominal monetary growth, θ, so that the rate of growth of the real money stock is

$$\dot{m} = (\theta-q-e)m \tag{9}$$

It then sets the lump sum tax τ so as to balance its budget, so that the real stock of government bonds outstanding remains fixed at $a = \bar{a}$, with $\dot{a} = 0$.[6/]

Combining the optimality conditions (6a)-(6e), (8a), (8b), together with the accumulation equations (3) and (4), and the policy specification (9), the perfect foresight equilibrium may be expressed by the following independent equations

$$U_c(c,\ell) = \mu \tag{10a}$$

$$U_\ell(c,\ell) = -f'(\ell)\mu \tag{10b}$$

$$V'(m) = r\mu \tag{10c}$$

$$\alpha b = r^* + e - r \tag{10d}$$

$$\dot{m} = (\theta-q-e)m \tag{10e}$$

$$\dot{b} = f(\ell) - c - g + (r^*-q)b \tag{10f}$$

$$\dot{\mu} = \mu[\beta - (r-q-e)] \tag{10g}$$

together with the balanced budget condition

$$g + r\bar{a} - \theta m - (q+e)\bar{a} - \tau = 0 \qquad (10h)$$

Taken in pairs, the first three equations describe the marginal rate of substitution conditions necessary for consumer optimality. The marginal rate of substitution between consumption and labor must equal the real wage the marginal rate of substitution between consumption and money balances must equal the nominal rate of interest. Equations (10d) to (10g) are unchanged from before. Of these, the first three have already been discussed. The last equation, which we may write as

$$r - (q+e) = \beta - \dot{U}_c/U_c$$

is the usual arbitrage condition for the rate of return on consumption.[7/]

In general, the evolution of the system proceeds as follows. Equations (10a)-(10d) determine the short-run solutions for c, ℓ, r, and e in terms of the marginal utility of consumption μ, the real stock of money m, and the real stock of foreign bonds b, as well as the exogenous parameters g, θ, q, r*. Substituting these solutions into (10e)-(10g) determines the evolution of the state variables m, b, and μ. In addition there is the government budget constraint (10h). Under our assumption that a is fixed and that the budget is balanced by the imposition of the lump sum tax, the required level of τ is determined residually, enabling this equation to be ignored.

One further requirement we shall impose is the intertemporal budget constraint on the economy, ruling out the possibility that the country can run up an indefinite credit or debt with the rest of the world. This is expressed by

$$\lim_{t\to\infty} \exp(-(r^*-q)t)b(t) = 0 \qquad (11)$$

Integrating (10f) we obtain

$$b(t) = \exp((r^*-q)t)[b_0 + \int_0^t \exp(-(r^*-q)t')[f(\ell) - c - g]dt']$$

so that the constraint reduces to

$$b_0 + \int_0^\infty \exp(-(r^*-q)g')[f(\ell) - c - g]dt' = 0 \qquad (11')$$

With imperfect capital mobility this condition is automatically met. But in the limiting case of perfect capital mobility it does impose an additional constraint which need to be taken into account.

The system of equations (10a)-(10g) provide the basis for the short-run and long-run analysis of the system and its response to the various disturbances undertaken in the following sections.

3. STEADY STATE EQUILIBRIUM

Since the analysis is based on the assumption of perfect foresight, the transitional dynamic adjustment path of the system is determined in part by the expectations of the long-run steady state. It is therefore convenient to begin with a consideration of the steady state and the long-run effects of the various disturbances.

The steady state equilibrium is attained when $\dot{m} = \dot{b} = \dot{\mu} = 0$, implying that

$$e = \theta - q \; ; \; p = \theta \qquad (12a)$$

$$f(\ell) + (r^*-q)b = c + g \qquad (12b)$$

$$b = \frac{1}{\alpha}[r^* - q - \beta] \qquad (12c)$$

That is, in steady state equilibrium, the rate of exchange depreciation equals the difference between the domestic monetary growth rate (equal to the long-run domestic rate of inflation) and the foreign rate of inflation. Total domestic absorption $c + g$ is constrained by output augmented by net interest earned on domestic residents' holdings of foreign bonds. The net demand for foreign bonds is proportional to the difference between the real after-tax rate of return on foreign bonds and the rate of time preference. Combining (12a), (12c) and (10d) yields

$$r = \beta + \theta \tag{12d}$$

the domestic after-tax nominal rate of interest is fixed exogenously by $\beta + \theta$.

In a small open economy, the rate of time preference is in general not independent of the real rate of interest abroad. Indeed, in the limiting case of perfect capital mobility (UIP), previous authors have demonstrated that for a well defined steady-state equilibrium to exist, thetwo must be equal, see, e.g., Hodrick (1982). More generally, the relationship will depend largely upon the degree of capital mobility. Given our parameterization of this in terms of α, it seems reasonable to postulate that β is given by the weighted average

$$\beta = \frac{1}{1+\alpha}(r^*-q) + \frac{\alpha}{1+\alpha}\bar{\beta} \tag{13}$$

where $\bar{\beta}$ is taken to be a constant, independent ofthe foreign real interest rate.[8/]
As $\alpha \to 0$ and capital becomes perfectly mobile, $\beta \to r^* - q$, in which case (12d) implies the long-run real interest parity relationship

$$r - \theta = r^* - q \tag{14}$$

As $\alpha \to \infty$, and capital becomes perfectly immobile, $\beta \to \bar{\beta}$, and the domestic interest rate becomes independent of the rate abroad.

Using equations (12) and (13) the steady state equilibrium reduces to

$$U_\ell(c,\ell) + f'(\ell)U_c(c,\ell) = 0 \tag{15a}$$

$$V'(m) - (\beta+\theta)U_c(c,\ell) = 0 \tag{15b}$$

$$f(\ell) = c + g - (r^*-q)b \tag{15c}$$

$$(1+\alpha)b = r^* - q - \bar{\beta} \tag{15d}$$

which determines the four variables c, ℓ, m, and b in terms of the exogenous policy parameters and disturbances. The equilibrium is obtained in the following recursive manner. First, the equilibrium stock of traded bonds is determined from the uncovered real interest differential (15d). This determines net real interest income to the economy. Given this, (15a) and (15c) together determine real activity, namely employment (and output) and consumption. The real money supply is then obtained residually from the marginal rate of substitution condition (15b).

Our objective is to analyze the effects of changes in:

(i) The domestic monetary growth rate θ;

(ii) domestic government expenditure g;

(iii) foreign nominal interest rate r*;

(iv) foreign rate of inflation q.

These are summarized in Table 1, which is divided into disturbances of domestic and foreign origin.

In the steady state an expansion in the monetary growth rate has the usual neutrality properties. All real activity, as described by output, employment, consumption, and the trade balance, as well as the marginal utility of consumption, remains unchanged. The real interest rate and the real stock of traded bonds also do not change, while the real stock of money falls. The domestic nominal after-tax interest rate, the domestic rate of inflation, and rate of exchange depreciation all rise proportionately.[9/]

An increase in government expenditure has no effect on the demand for traded bonds, or on the real or nominal interest rates. Employment increases, while consumption falls, implying that the output expenditure multiplier is less than unity. The reason for this result is simply that in order for the marginal rate of substitution condition (15a) to be met, c and ℓ must move in opposite directions. And the only way that the increase in government demand can be accommodated is if output rises while consumption falls. The balance of trade of the private sector falls, while the country's net trade balance, inclusive of government purchases, is zero.

The steady state ceteris paribus effects of changes in the foreign interest rate and foreign inflation rate are qualitatively equal and opposite to one another, so that only the former need be discussed. In both cases, the changes incorporate the induced changes in the domestic rate of time preference β, as indicated by (13). The effect of an increase in the foreign nominal interest rate r^* is to raise the after-tax domestic nominal and real interest rates by a factor $1/(1+\alpha)$ and the real holdings of foreign bonds by the same amount.[10/] In the case of perfect capital mobility, the effect on the interest rates is proportional. A one percentage point increase in r^* leads to a one percentage point increase in the domestic (real and nominal) interest rates. Otherwise, the degree of responsiveness declines with the degree of capital mobility and in the limiting case there is no effect.

A change in r^* impinges on the real part of the system through the real interest income term $-(r^*-q)b$. Provided b is not too negative, so that the domestic economy is not too great a debtor to the rest of the world, then the impact effect through the real interest income term is negative and operates qualitatively like a decrease in government expenditure. We consider this to be the more likely case, with the signs in parentheses therefore being "probable" signs. The higher interest earnings leads to increased consumption, while output and employment falls. Since government expenditure is assumed fixed, the balance of trade therefore falls. The domestic steady-state rate of inflation and exchange depreciation are obviously unaffected by the change.

The effects we have been discussing are reversed in the case of an increase in the rate of inflation abroad. Furthermore, the rate of exchange depreciation falls proportionately, while the domestic rate of inflation remains unchanged.

These results on foreign disturbances shed some light on the issue of the degree of insulation provided by flexible exchange rates against foreign inflationary disturbances. It is evident that flexible rates do not provide complete insulation againt foreign inflationary pressures alone. This is because a change in the foreign inflation rate alone constitutes a real disturbance. However, a change in the foreign inflation rate is likely to be accompanied by a change in the foreign nominal interest rate, as these two variables respond to the common influences abroad. A necessary and sufficient condition for a flexible exchange rate to insulate the domestic economy completely against disturbances in the foreign inflation is:

(i) The rest of the world be "Fisherian" in the sense that the after-tax nominal interest rate abroad fully adjusts to the foreign inflation rate, viz.,

$$\frac{dr^*}{dq} = 1$$

In essence, the Fisherian condition ensures that the foreign price disturbance is purely nominal.[11/] Thus, the condition for perfect insulation originally obtained by Turnovsky (1977) continues to apply in the present optimizing model.

Finally, we wish to make the following observation. A key difference between the domestic and foreign disturbances is that the equilibrium real stock of traded bonds is independent of the former, but is responsive to changes in the latter.[12/] This difference turns out to be crucial in contrasting the dynamic response of the economy to these two sets of disturbances.

4. TRANSITIONAL DYNAMICS: GENERAL CHARACTERISTICS

To analyze the transitional dynamics of the economy we must consider the short-run equilibrium conditions (10). These yield solutions for the short-run variables c, ℓ, e, and r of the form[13/]

$$\ell = \ell(\mu) \qquad \ell_\mu > 0 \tag{16a}$$

$$c = c(\mu) \qquad c_\mu < 0 \tag{16b}$$

$$r = r(\mu, m) \qquad r_\mu < 0 \;,\; r_m < 0 \tag{16c}$$

$$e = e(\mu, m, b) \qquad e_\mu < 0 \;,\; e_m < 0 \;,\; e_b > 0 \tag{16d}$$

The signs of the partial derivatives are as indicated and are derived in the Appendix.

Intuitively, a ceteris paribus increase in the marginal utility of consumption, μ, means that consumption is reduced and the supply of labor rises. With the real stock of money fixed, the marginal rate of substitution condition implies that the after-tax nominal interest rate must fall. Given the real stock of traded bonds, the uncovered interest differential remains unchanged. The rate of return on domestic bonds must fall, i.e., the rate of exchange depreciation must fall. An increase in the money stock leaves c and ℓ unchanged. The marginal utility of money falls, implying that the domestic interest rate declines and hence the rate of exchange depreciation falls as well. An increase in b leaves c, ℓ, and r unaffected. The rate of return on foreign bonds must rise; i.e., the rate of exchange depreciation must increase.

Linearizing the differential equations (10e), (10f), (10g) about the steady state equilibrium, the dynamics may be written in the form

$$\begin{bmatrix} \dot{m} \\ \dot{b} \\ \dot{\mu} \end{bmatrix} = \begin{bmatrix} -e_m\tilde{m} & -e_b\tilde{m} & -e_\mu\tilde{m} \\ 0 & r^*-q & f'\ell_\mu - c_\mu \\ 0 & \tilde{\mu}\alpha & 0 \end{bmatrix} \begin{bmatrix} m-\tilde{m} \\ b-\tilde{b} \\ \mu-\tilde{\mu} \end{bmatrix} \tag{17}$$

where $\tilde{m}$, $\tilde{b}$, $\tilde{\mu}$, denote the steady state equilibrium determined in Section 3 and the partial derivatives in (17) are evaluated at steady state. From the matrix in (17) we see that the three eigenvalues λ_1, λ_2, λ_3, have the properties

$$\lambda_1 + \lambda_2 + \lambda_3 = r^* - e_m\tilde{m} > 0 \tag{18a}$$

$$\lambda_1\lambda_2\lambda_3 = e_m\tilde{m}\tilde{\mu}\alpha[f'\ell_\mu - c_\mu] < 0 \tag{18b}$$

From this pair of inequalities we deduce that there is one negative (stable) root and two positive (unstable) roots, implying a unique convergent path. Focusing on this path, stable solutions for m, b, μ are of the form[14/]

$$m(t) = [m(0) - \tilde{m}]e^{\lambda_1 t} + \tilde{m} \tag{19a}$$

$$b(t) = [b_0 - \tilde{b}]e^{\lambda_1 t} + \tilde{b} \tag{19b}$$

$$\mu(t) = [\mu(0) - \tilde{\mu}]e^{\lambda_1 t} + \tilde{\mu} \tag{19c}$$

where $\lambda_1 < 0$ is the stable root.[15/] We assume that the real stock of foreign bonds evolves continuously from its initial level b_0, while the initial values of m(0) and $\mu(0)$ are determined endogenously. With the nominal stock of money being determined by the monetary growth rule, the initial nominal stock M_0 is predetermined, so that the initial real stock m(0) is determined by some appropriate initial jump in the domestic price level (exchange rate). The initial jump in $\mu(0)$ takes place through initial jumps in consumption and employment.

Differentiating (19) with respect to t and substituting into (17) yields

$$\begin{bmatrix} -e_m\tilde{m}-\lambda_1 & -e_b\tilde{m} & -e_\mu\tilde{m} \\ 0 & r^*-q-\lambda_1 & f'\ell_\mu - c_\mu \\ 0 & \alpha\tilde{\mu} & -\lambda_1 \end{bmatrix} \begin{bmatrix} m-\tilde{m} \\ b-\tilde{b} \\ \mu-\tilde{\mu} \end{bmatrix} = 0 \tag{20}$$

confirming that λ_1 is an eigenvalue of (16) with $(m-\tilde{m},\ b-\tilde{b},\ \mu-\tilde{\mu})$ being the corresponding eigenvector. Equation (20), which holds for all t, involves three linearly dependent relationships between the three elements of the eigenvector. Assume that the system starts out in steady state equilibrium so that $b_0 = \bar{b}$ say. Then, the first and third equations of (20) considered at time 0, but immediately following a disturbance, imply

$$\mu(0) - \tilde{\mu} = \frac{\alpha\tilde{\mu}}{\lambda_1}[\bar{b}-\tilde{b}] \tag{21a}$$

$$m(0) - \tilde{m} = \frac{-\tilde{\alpha}\tilde{\mu}m[1 - \frac{\beta+\theta}{\lambda_1}](\bar{b}-\tilde{b})}{\tilde{m}V'' + \tilde{\mu}\lambda_1} \tag{21b}$$

where in deriving (21b) we substitute for the partial derivatives e_i, $i = m, b, \mu$, from the expressions given in the Appendix, and μ, m are evaluated at steady state.

Our objective is to analyze the effects of the various disturbances on the adjustment path of the economy. The most critical determinant of this is how the initial jumps in $\mu(0)$, $m(0)$ respond to these disturbances. This is because these initial jumps transmit the impacts of these shocks to the other short-run variables, via the short-run equilibrium relationships (16).

Differentiating equations (21a), (21b) with respect to some arbitrary variable, x say, yields to the first order of approximation

$$\frac{d\mu(0)}{dx} = \frac{d\tilde{\mu}}{dx} - \frac{\alpha\tilde{\mu}}{\lambda_1}\frac{d\tilde{b}}{dx} \tag{22a}$$

$$\frac{dm(0)}{dx} = \frac{d\tilde{m}}{dx} + \frac{\alpha\mu m\left(1 - \frac{\beta+\theta}{\lambda_1}\right)}{mV'' + \mu\lambda_1}\frac{d\tilde{b}}{dx} \tag{22b}$$

where henceforth the ˜ on the equilibrium values μ, m are omitted. Equations (22a) and (22b) can be used to determine the short-run impact effects of the various disturbances, to which we now turn.

5. ANALYSIS OF DISTURBANCES

In this section we discuss the dynamic adjustment following disturbances of domestic and foreign origin, in turn.

A. Domestic Monetary Expansion

Consider an increase in the monetary growth rate θ. From Table 1 we see that

$$\frac{d\tilde{b}}{d\theta} = \frac{d\tilde{\mu}}{d\theta} = 0$$

which, using (22a) and (22b), imply

$$\frac{d\mu(0)}{d\theta} = 0 \tag{23a}$$

$$\frac{dm(0)}{d\theta} = \frac{d\tilde{m}}{d\theta} = \frac{\mu}{V''} < 0 \tag{23b}$$

Equation (23a), together with the short-run solutions (16a), (16b) yield that

$$\frac{d\ell(0)}{d\theta} = \frac{dc(0)}{d\theta} = \frac{dT(0)}{d\theta} = 0 \tag{24}$$

The initial responses of the financial variables are obtained by taking the derivatives of (10c) and (10d) at time 0, to yield

$$\frac{V''dm(0)}{d\theta} = \mu\,\frac{dr(0)}{d\theta} = \mu\,\frac{de(0)}{d\theta} \tag{25}$$

Combining (23b) and (25) we obtain

$$\frac{de(0)}{d\theta} = \frac{dr(0)}{d\theta} = 1 \tag{26a}$$

The implied instantaneous jump in the level of the exchange rate E(0) is

$$\frac{dE(0)/d\theta}{E(0)} = \frac{-dm(0)/d\theta}{m(0)} = \frac{-d\tilde{m}/d\theta}{m} = \frac{-\mu}{mV''} > 0 \tag{26b}$$

A change in the monetary growth rate causes the economy to jump instantaneously to its new steady state. In this new equilibrium all real variables such as employment, output, consumption, and the trade balance remain unchanged. The jump is brought about by a discrete depreciation of the exchange rate, causing the real stock of money balances in the economy to fall, doing so by an amount which is precisely equal to its long-run decline. As a result of this, the rate of exchange depreciation and hence the domestic rate of inflation, increases instantaneously by the amount of the increase in the monetary growth rate. The exchange rate neither overshoots nor undershoots, in the sense that the real money balances immediately adjusts to its new equilibrium level.

B. Increase in Domestic Government Expenditure

Referring to Table 1, it is recalled that an increase in government expenditure leaves the long run position in traded bonds unchanged, while leading to an increase in the marginal utility of consumption and a decrease in the real stock of money. Consequently, equations (22) imply

$$\frac{d\mu(0)}{dg} = \frac{d\tilde{\mu}}{dg} = \frac{\Omega}{\Delta} > 0 \tag{27a}$$

$$\frac{dm(0)}{dg} = \frac{d\tilde{m}}{dg} = \frac{\beta+\theta}{V''}\frac{\Omega}{\Delta} < 0 \tag{27b}$$

which, together with the short-run solutions (16a), (16b), yield

$$\frac{d\ell(0)}{dg} = \frac{d\tilde{\ell}}{dg} > 0 \tag{28a}$$

$$\frac{dc(0)}{dg} = \frac{d\tilde{c}}{dg} < 0 \tag{28b}$$

$$\frac{dT(0)}{dg} = f'\frac{d\tilde{\ell}}{dg} - \frac{d\tilde{c}}{dg} - 1 = 0 \tag{28c}$$

Turning to the financial variables and differentiating (10c) and (10d) with respect to g and noting the above relationships we find

$$\frac{de(0)}{dg} = \frac{dr(0)}{dg} = 0 \tag{29a}$$

Thus as in the case of the domestic monetary expansion, there is no transitional dynamics. The expansion in government expenditure causes the exchange rate to depreciate instantaneously by an amount

$$\frac{dE(0)/E(0)}{dg} = \frac{-dm(0)/dg}{m(0)} > 0 \tag{29b}$$

Output expands and consumption falls. The private balance of trade increases by an amount equal to the increase in government expenditures, so that the country's net balance of trade is unchanged. The rate of exchange depreciation and the domestic rate of interest also both remain unchanged. Again the reason for the complete instantaneous adjustment is the fact that the long-run equilibrium stock of traded bonds, $\tilde{b}$, which in an important sense drives the short-run dynamics, is unaffected by the change in government expenditure.

These findings regarding the instantaneous adjustment of the system to changes in the domestic policy variables θ and g are sensitive to certain aspects of the specification of the model. Under alternative assumptions they cease to hold. For example, if one assumes that all forms of income, including interest income, are taxed at a fixed rate and that the budget is balanced through changing the tax rate, rather than through lump sum taxation, then the equilibrium stock of traded bonds will be responsive to changes in θ and g, and the adjustment to the new steady state will be a gradual one. Alternatively, if one assumes that the cost parameter α varies inversely with the stock of wealth (as it would with constant relative risk aversion), then the equilibrium stock of traded bonds is given by the expression

$$\tilde{b} = \frac{1}{\alpha'}(r^* - q - \beta)(\tilde{m} + \bar{a} + \tilde{b}) \qquad (30)$$

In this case $\tilde{b}$ is affected by θ and g through the wealth effect, thereby again rendering the adjustment to be a gradual one.

C. Foreign Interest Rate Disturbances

Consider now a foreign monetary disturbance, taking the form of an increase in the foreign nominal interest rate r^*. From Table I we see that this leads to an increase in the equilibrium real stock of traded bonds, namely,

$$\frac{d\tilde{b}}{dr^*} = \frac{1}{1+\alpha} > 0 \qquad (31)$$

Accordingly, the short-run responses $\mu(0)$, $m(0)$ are given by

$$\frac{d\mu(0)}{dr^*} = \frac{d\tilde{\mu}}{dr^*} - \frac{\alpha\mu}{\lambda_1(1+\alpha)} > \frac{d\tilde{\mu}}{dr^*} \qquad (32a)$$

$$\frac{dm(0)}{dr^*} = \frac{d\tilde{m}}{dr^*} + \left[\frac{\alpha\mu m\left(1 - \frac{\beta+\theta}{\lambda_1}\right)}{mV'' + \mu\lambda_1}\right]\frac{1}{1+\alpha} < \frac{d\tilde{m}}{dr^*} \qquad (32b)$$

where the corresponding steady-state responses are reported in Table I. These effects imply the following short-run responses of the real part of the economy

$$\frac{d\ell(0)}{dr^*} = \ell_\mu \frac{d\mu(0)}{dr^*}$$

$$= \frac{d\tilde{\ell}}{dr^*} - \frac{\ell_\mu \alpha \mu}{\lambda_1(1+\alpha)} > \frac{d\tilde{\ell}}{dr^*} \qquad (33a)$$

and similarly

$$\frac{dc(0)}{dr^*} = \frac{d\tilde{c}}{dr^*} - \frac{c_\mu \alpha \mu}{\lambda_1(1+\alpha)} < \frac{d\tilde{c}}{dr^*} \qquad (33b)$$

$$\frac{dT(0)}{dr^*} = \frac{d\tilde{T}}{dr^*} - \frac{(f'\ell_\mu - c_\mu)\alpha\mu}{\lambda_1(1+\alpha)} > \frac{d\tilde{T}}{dr^*} \qquad (33c)$$

An increase in the foreign nominal interest rate raises the equilibrium stock of traded bonds, $\tilde{b}$. Hence, the marginal utility of consumption falls less on impact than it does in the steady state; see (22a). As a consequence, the fall in domestic employment, the rise in consumption, and the deterioration in the balance of trade, are all less than they are in steady state. Indeed, it is possible for all these impact effects to reverse qualitatively the long-run effects. This occurs if and only if the positive effect of an increase in r* on steady-state bonds outweighs the negative effect on the steady-state marginal utility of consumption

The effects on the financial variables are obtained by first differentiating (10c), (10d) to yield

$$V'' \frac{dm(0)}{dr^*} = \left[\mu \frac{dr(0)}{dr^*} + r \frac{d\mu(0)}{dr^*}\right]$$

$$1 + \frac{de(0)}{dr^*} = \frac{dr(0)}{dr^*}$$

and then utilizing (32a), (32b). This result is

$$\frac{dr(0)}{dr^*} = \frac{1}{(1+\alpha)[V''m + \mu\lambda_1]}[\mu\lambda_1 + mV''(1+\alpha) + \alpha V'] \gtrless 0 \qquad (34a)$$

$$\frac{de(0)}{dr^*} = \frac{(V' - \mu\lambda_1)}{V''m + \mu\lambda_1} \left(\frac{\alpha}{1+\alpha}\right) < 0 \qquad (34b)$$

In addition, the effect on the nominal exchange rate E(0) is given by

$$\frac{dE(0)/dr^*}{E(0)} = - \frac{dm(0)/dr^*}{m(0)} \gtrless 0 \tag{34c}$$

where the initial jump in the real money stock, derived from (32b) is

$$\frac{dm(0)}{dr^*} = \frac{(\beta+\theta)\psi\Omega}{V''\Delta} + \frac{\mu}{1+\alpha}\left[\frac{1}{V''} + \frac{\alpha m(1 - (\beta+\theta)/\lambda_1)}{mV'' + \mu\lambda_1}\right] \tag{34d}$$

It is evident that as long as $\alpha \neq 0$, so that traded and domestic bonds are less than perfect substitutes, the impact effect of an increase in the foreign interest rate is to move the domestic economy to a point which does not coincide with the new steady state. Thereafter, the economy approaches its new equilibrium asymptotically; there is therefore a gradual dynamic adjustment. With perfect capital mobility, on the other hand, ($\alpha=0$), the economy jumps immediately to steady state.

Strictly speaking, the immediate response of the domestic interest rate is indeterminate, although intuitively one would expect it to rise. A sufficient condition for this to be so is

$$\eta < - \frac{\alpha}{1+\alpha} \tag{35}$$

where $\eta = mV''/V'$ is the elasticity of the marginal utility of money. Clearly, the higher the degree of capital mobility, the more likely this condition is to be met and the more likely the domestic rate of interest is to rise. In the other limiting case where there is zero substitutability ($\alpha \to \infty$), it can be shown that $dr(0)/dr^* \to 0$. In general, comparing (34a) with the steady state, we see

$$\frac{dr(0)}{dr^*} - \frac{d\tilde{r}}{dr^*} = \frac{\alpha[mV'' + V']}{(1+\alpha)(mV'' + \mu\lambda_1)} \tag{36}$$

implying that as long as $\eta < 1$ and capital is imperfectly mobile, the interest rate undershoots its steady state adjustment on impact.

A more intuitive explanation of the initial phases of the dynamic adjustment is the following. Suppose that there is an exogenous increase in the foreign interest rate. Generally this gives rise to an instantaneous fall in the trade balance which is offset by an increase in the flow of interest income if the country is a net creditor. The net effect on the current account balance is therefore ambiguous and the domestic economy may initially either accumulate or decumulate traded bonds. At the same time, the fall in the rate of exchange depreciation, e, implies that the real money stock starts to rise, while the rise in the foreign interest rate means that through the consumer arbitrage condition, the marginal utility of consumption starts to fall. This latter effect implies a decrease in output, accompanied by an increase in consumption, leading to a decrease in the trade balance. The fall in μ, together with the increase in m, and the change in b, has offsetting effects on the rate of exchange depreciation, although on balance e will rise. The decrease in the trade balance means that the rate of accumulation of foreign bonds will fall. The adjustment so described is stable and eventually the economy converges to the steady state described in Section 3.

D. Foreign Inflationary Disturbance

Given the symmetry with which the foreign variables impinge on the domestic economy, the effects of a unit increase in the foreign inflation rate alone, q, are identical to those of an equivalent decrease in the foreign interest rate, r*. The effects on the real and financial variables therefore follow immediately from (32)-(34) in Section 5.C. In addition we may establish

$$\frac{dp(0)}{dq} = \frac{de(0)}{dq} + 1 = \frac{\alpha[\mu\lambda_1 - V']}{(1+\alpha)(mV'' + \mu\lambda_1)} > 0$$

so that the domestic inflation rate rises. As long as domestic and foreign bonds are less than perfect substitutes, an increase in the foreign inflation rate imposes an initial jump on the domestic economy, followed by a gradual adjustment towards the new steady state equilibrium. The economic explanation for this behavior is identical to that which would stem from a decrease in the foreign interest rate.

It is evident from these results that the flexible exchange rate does not in general insulate the domestic economy in the short run against once and for all increases in the foreign inflation rate. However, it will provide perfect short-run insulation against disturbances in the foreign inflation rate if and only if condition (i) noted in Section 3 applies. That is, the flexible exchange rate will provide perfect insulation against disturbances in the foreign inflation rate in the short run as well as the long run if and only if: the foreign nominal interest rate fully adjusts to theforeign inflation rate.

6. CONCLUSIONS

In this paper we have analyzed the effects of various disturbances of domestic and foreign origin in a model of a small open economy in which the behavioral relationships are derived from optimization by the private sector. The main conclusion of the analysis is a simple one. In this model, the domestic economy jumps instantaneously to its new equilibrium steady state, following a change in either the domestic monetary growth rate or domestic fiscal policy. In response to a disturbance in either the foreign interest rate or inflation rate, on the other hand, it undergoes an initial jump taking it part way towards its new equilibrium, which it thereafter approaches gradually. The critical driving force in determining the nature of the adjustment to a specific disturbance is the response of the steady state equilibrium stock of traded bonds, to this disturbance. When one derives the demand for these assets from intertemporal utility maximization, as we have done, this equilibrium stock turns out to be independent of the domestic policy variables we consider, but it is dependent upon both the foreign interest and inflation rates. The magnitude of the initial jump and the speed of adjustment following disturbances in these latter variables depends critically upon the degree of capital mobility. In the limiting case where domestic and foreign bonds are perfect substitutes, the economy adjusts instantaneously to foreign, as well as domestic, disturbances.

The results we have obtained are relevant to the issues of exchange rate adjustment and the insulation properties of flexible exchange rates, noted previously. The fact that the economy adjusts fully on impact to a change in the monetary growth rate means that the exchange rate neither overshoots nor undershoots its long-run response. Moreover, ad hoc equilibrium models of asset accumulation and imperfect substitutability which involve gradual adjustments to domestic policy changes may in fact be somewhat misleading, since the adjustments may occur instantaneously. Secondly, the Fisherian condition obtained previously as being necessary and sufficient for perfect insulation, holds in the present optimizing framework.

Of course, the analysis is based on a number of specific assumptions and in concluding the effects of relaxing a number of these should be noted. First, the separability of the consumer utility function into consumption and labor on the one hand, and real money balances, on the other, is made for expositional convenience. If instead, one assumes a nonseparable function $U(c,\ell,m)$ say, then the only difference is that the recursive nature of the solution breaks down. As a consequence, the monetary growth rate will have effects on the real variables such as employment, consumption, and output. In all other respects the nature of the solution remains unchanges.

Secondly, it is straightforward to relax the assumption of PPP and to introduce a domestic good which is an imperfect substitute for the traded commodity. The relative price is introduced as an additional variable, but again, this does not alter the character of our analysis in any essential way.

Thirdly, as Turnovsky and Brock (1980) argued, it is reasonable to introduce government expenditure as a direct argument of the utility function to relect the fact that consumers derive direct utility from public expenditure. Again this does not affect the character of our analysis, although it does change the specific responses to change in g, r*, and q. In the limiting case where consumers view private and public goods as being perfect substitutes it can be easily established that fiscal policy has no real effects; there is "direct" crowding out.

Finally, and most importantly, the instantaneous adjustment of the economy in response to the domestic policy shocks does depend upon our specification of the cost parameter α as being constant. If instead, one postulates this to vary inversely with wealth, then the domestic policy variables will influence the equilibrium stock of traded bonds through the wealth effect. As a consequence of this dependence, a change in domestic monetary or fiscal policy will generate a true dynamic adjustment in the domestic economy.

Thus while models based on ad hoc relationships are often criticized as being arbitrary, models based on optimizing behavior are not entirely immune from this criticism either. As our results here indicate, their implications are also dependent upon specification of the optimization which to some degree is also arbitrary.

TABLE I

STEADY STATE EFFECTS OF CHANGES IN

On	Domestic Monetary Growth Rate θ	Domestic Government Expenditure g
Traded bonds b	0	0
Employment ℓ	0	$-\frac{[U_{\ell c} + f'U_{cc}]}{\Delta} > 0$
Consumption c	0	$\frac{U_{\ell\ell} + [f''U_c + f'U_{c\ell}]}{\Delta} < 0$
Trade balance T	0	0
Marginal utility of consumption	0	$\frac{\Omega}{\Delta} > 0$
Real money stock m	$\frac{U_c}{V''} < 0$	$\frac{\beta+\theta}{V''}\frac{\Omega}{\Delta} < 0$
After-tax nominal interest rate (after tax)	1	0
After-tax real interest rate	0	0

$$\Omega \equiv U_{cc}[U_{\ell\ell} + f''U_c] - U_{\ell c}^2 > 0$$

$$\Delta \equiv -[U_{\ell\ell} + [f''U_c + f'U_{c\ell}]] - f'[U_{c\ell} + f'U_{cc}] > 0$$

STEADY STATE EFFECTS OF
UNIT INCREASE IN FOREIGN NOMINAL INTEREST RATE
OR UNIT DECREASE IN FOREIGN INFLATION RATE q

On	
Bonds b	$\frac{1}{1+\alpha} > 0$
Employment ℓ	$\psi \frac{d\ell}{dg}$ (< 0)
Consumption c	$\psi \frac{dc}{dg}$ (> 0)
Trade balance T	$\psi[f' \frac{d\ell}{dg} - \frac{dc}{dg}]$ (< 0)
Marginal utility of consumption U_c	$\psi \frac{\Omega}{\Delta}$ (< 0)
Real money stock m	$\frac{1}{V''}\left[\frac{\mu}{1+\alpha} + (\beta+\theta)\psi \frac{\Omega}{\Delta}\right] \gtrless 0$
After tax nominal interest rate	$\frac{1}{1+\alpha} > 0$
After tax real interest rate	$\frac{1}{1+\alpha} > 0$

$$\psi \equiv -\left[b + \frac{(r^*-q)}{\alpha}\right] \quad (< 0)$$

APPENDIX

1. Derivation of Partial Derivatives of Short-Run Solutions (16)

Consider equations (10a) and(10b). The differentials of this pair of equations is given by

$$\begin{bmatrix} U_{cc} & U_{c\ell} \\ U_{\ell c} & U_{\ell\ell} + \mu f'' \end{bmatrix} \begin{bmatrix} dc \\ d\ell \end{bmatrix} = \begin{bmatrix} d\mu \\ -f'd\mu \end{bmatrix} \tag{A.1}$$

the solution to which is

$$\frac{\partial \ell}{\partial \mu} = - \frac{[f'U_{cc} + U_{\ell c}]}{\Omega} < 0 \tag{A.2a}$$

$$\frac{\partial}{\partial \mu} = \frac{U_{\ell\ell} + f''\mu + U_{c\ell}f'}{\Omega} < 0 \tag{A.2b}$$

where

$$\Omega \equiv U_{cc}[U_{\ell\ell} + \mu f''] - U^2_{\ell c} > 0$$

Differentiating (10c) yields

$$V''dm = [rd\mu + \mu dr]$$

so that

$$\frac{\partial r}{\partial m} = \frac{V''}{\mu} < 0 \tag{A.3a}$$

$$\frac{\partial r}{\partial} = \frac{-r}{\mu} < 0 \tag{A.3b}$$

Finally, taking the differential of (10d)

$$de = \alpha db + dr$$

implying

$$\frac{\partial e}{\partial m} = \frac{V''}{\mu} < 0 \qquad \text{(A.4a)}$$

$$\frac{\partial e}{\partial \mu} = -\frac{r}{\mu} < 0 \qquad \text{(A.4b)}$$

$$\frac{\partial e}{\partial b} = \alpha > 0 \qquad \text{(A.4c)}$$

FOOTNOTES

1/ The exchange rate overshooting question in particular has generated a voluminous literature, too extensive to document here. The robustness of the phenomenon to changes in assumptions about: (i) the degree of capital mobility; (ii) output flexibility; (iii) price flexibility, has been considered by a number of authors.

2/ More recently, Obstfeld has applied the intertemporal optimizing model to a number of issues in international macroeconomics. Some of these are summarized in the survey paper by Obstfeld and Stockman (1984).

3/ The term "perfect capital mobility" is often used quite loosely. Even if UIP does not hold, capital may still be perfectly mobile in the weaker sense of covered interest parity (CIP); see, e.g., Eaton and Turnovsky (1983). The empirical evidence supporting the stronger definition of UIP is not particularly strong. Levich (1978), Bilson (1978) and Hansen and Hodrick (1980) report systematic deviations from UIP for several exchange rates over long periods, reflecting risk averse behavior by speculators. Obstfeld (1982) also briefly considers the case of imperfect capital mobility. However, he does so by arbitrarily postulating some relationship between the stock of bonds and the interest rate, rather than deriving such a relationship as an equilibrium condition from the optimizing framework.

4/ We adopt the following notational convention. Except where noted, partial derivatives are denoted by corresponding lower case letter, while total derivatives of a function of a single argument are denoted by primes.

5/ More specifically, the two period mean-variance optimization yields a demand function for foreign bonds of the form

$$b = \frac{1}{A\sigma_e^2}(r^* + e - r) = \frac{z}{R\sigma_e^2}(r^* + e - r)$$

where $z = m + a + b$.

6/ The same results as those derived in this paper are obtained if one assumes instead, that the government: (i) finances its budget by issuing bonds; (ii) engages in an initial open market operation to ensure that the stock of bonds generated by the government budget constraint is consistent with the consumers' transversality conditions.

7/ For example, this relationship is equivalent to the optimality condition in Yaari (1964).

8/ This scheme is admittedly a simple one. It is meant to capture the notion that the more closely the domestic economy is tied financially to the rest of the world, the less independent the domestic rate of time preference can be. Alternatively, one could endogenize β along the lines of Obstfeld (1981).

9/ The neutrality breaks down if one postulates the cost parameter α to be inversely dependent upon wealth, so that the alternative bond demand function of footnote 5 applies. By means of the wealth effect, m, b, ℓ, and c are all dependent upon the monetary growth rate and domestic government expenditure.

10/ These propositions are established by considering (12d) and (13), and (15d), respectively.

11/ If one assumes that interest income is taxed by the domestic government, then certain mild asymmetries between changes in the foreign inflation and interest rate fully adjusts to the foreign inflation rate, the effects of a unit increase in the latter are non-neutral. This is because the taxes provide a real transfer from the foreign to the domestic economy. Neutrality is restored if the taxes on foreign bonds are collected by the foreign government. These issues are discussed in an earlier version of this paper.

12/ The comment regarding the independence of the equilibrium stock of traded bonds with respect to domestic disturbances applies to θ and g. If interest income is taxed, the equilibrium stock of traded bonds <u>does</u> depend upon the income tax rate.

13/ We prefer to depart from our notational convention and denote $d\ell/d\mu$ by ℓ_μ rather than by ℓ'.

14/ The two unstable roots may be either real or complex, while the single stable root is necessarily real. Thus the stable adjustment path to equilibrium is necessarily a monotonic one.

15/ Given the recursivity of the model, the roots are

$$\lambda_2 = -e_m/m \qquad \text{(unstable)}$$

while λ_1, λ_2 ($\lambda_1 < 0$, $\lambda_2 > 0$) are the roots to the quadratic equation

$$\lambda^2 - (r^*-q)\lambda - [f'\ell_\mu - c_\mu]\mu\alpha = 0$$

while we assume that $r^* - q > 0$.

16/ As $\alpha \to \infty$, it can be seen from the solution to the characteristic equation given in footnote 9 that $\lambda_1 \to 0(\alpha^{1/2})$. Thus letting $\alpha \to \infty$ in (34a) and noting this fact, it is easily seen that $dr(0)/dr^* \to 0$.

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