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# COUNTERFACTUALS, FORECASTS, AND CHOICE-THEORETIC MODELLING OF POLICY

Herschel I. Grossman

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# Counterfactuals, Forecasts, and Choice-Theoretic Modelling of Policy


## ABSTRACT

This paper focuses on the problem of formulating an analysis of economic policy that is consistent with rational expectations. Cooley, LeRoy, and Raymon show that the Lucas and Sargent strategy for econometric policy evaluation is itself vulnerable to the logic of the Lucas critique. The present discussion develops the distinction between counterfactuals and forecasts to clarify the nature of the inconsistencies in the Lucas and Sargent strategy. The paper goes on to propose and to illustrate a strategy for positive economic analysis that incorporates choice-theoretical modelling of policy. Such modelling can allow better forecasting, but it also shifts attention away from policy actions and their effects and towards the more fundamental relation between the policymaker's constraints and targets and economic outcomes. The forecasting problem in a choice-theoretic model of policy concerns the effects of hypothetical realizations of variables that determine the policymaker's constraints and targets. The analysis of counterfactuals in this context recognizes that the parameters of the policy process are not invariant with respect to the processes that generate these exogenous variables. A program of positive economics that includes choice-theoretic modelling of policy also preserves a distinct role for policy advice as part of the process being modelled.


Herschel I. Grossman
Department of Economics
Brown University
Providence, RI 02912

If economic agents in forming the expectations that influence their behavior take account of the processes that actually generate the realizations of these expectational variables, then the parameters of these processes enter into the decision rules that relate individual choice variables to individual information sets. The neglect of this proposition in traditional models of the effects of economic policy produced the famous Lucas (1976) critique of econometric policy evaluation. This proposition also provides the basis for the development of rational-expectations modelling strategies for analyzing and estimating relations between the economy's endogenous and exogenous variables.

Lucas and Sargent--henceforth L&S--describe one such strategy, which they claim provides a correct way to do econometric policy evaluation. See L&S (1981) for a concise description and Sargent (1981) for a fuller discussion. An essential element in the L&S approach is the explicit specification of the policy regime, implicitly defined as a persistent relation that proximately describes the generation of policy variables. This specification permits the derivation of private agents' rational expectations about policy, conditional on assumptions about relevant information. In the econometric implementation of such a model, L&S emphasize estimation of underlying behavioral parameters of private agents that are independent of the existing policy regime. Policy evaluation for L&S involves using these estimated behavioral parameters to simulate the consequences of alternative policy regimes.

In an important criticism of this procedure, Cooley, LeRoy, and Raymon (1982)--henceforth CL&R--show that the L&S strategy, which developed out of the Lucas critique, is itself vulnerable to the logic of the Lucas critique. CL&R point out that, although L&S are interested in the effects of changes in policy regimes, the L&S analysis implicitly assumes that private agents ignore the process that generates policy regimes. In other words, L&S take careful account of the effect of the parameters of the existing policy regime on the behavior of private agents, but L&S fail to allow for the relation between the parameters that govern regime changes and the behavior of private agents. To exemplify an analytical strategy that they claim to be fully consistent with rational expectations, CL&R present a model in which the expectations of private agents are based on knowledge of the possible policy regimes as well as the transition probabilities that govern regime changes.

Sargent (1984) acknowledges that the L&S strategy contains the "internal contradiction" identified by CL&R. Indeed, Sargent and Wallace (1976) already recognized this problem, and they pointed out that an analysis fully consistent with rational expectations would require an extended analytical framework along the lines subsequently suggested by CL&R. Sargent, however, defends the continued use of the L&S strategy on the grounds that such extended models "subvert normative economics", by which he means the activity of prescribing economic policy. As Sargent and Wallace earlier put it, "If rational agents live in a world in which rules can be and are changed, their behavior should take into account such possibilities and should depend on the process generating the rule changes. But invoking this kind of complete rationality seems to rule out normative economics completely by, in effect, ruling out freedom for the policymaker."

The present paper contributes to the ongoing search for correct and useful strategies for studying economic policy and its effects within a rational expectations context. Section 1 develops a simple framework that illustrates the distinction between two modes of positive economic analyses--counterfactuals and forecasts--and attributes the inadequacy of the L&S analysis to their neglect of this distinction. Section 2 extends this framework to show how to forecast correctly if the policy regime is subject to change. Section 3 motivates the formulation of an underlying choice-theoretic model of policy and uses this model to analyze the evolution of policy actions and policy regimes as responses to shifts in the policymaker's constraints. Section 4 explores the important implications, both for forecasting and for the analysis of counterfactuals, of knowing the underlying choice-theoretic model. Section 5 discusses the place of policy advice within a positive analysis that treats policy as the solution to a choice problem. Section 6 summarizes the main conclusions.

## 1. Counterfactuals versus Forecasts

A careful reading of the papers by L&S (1981) and Sargent (1981) indicates that the essential oversight, which leaves them vulnerable to the CL&R criticism, is that these papers do not distinguish between counterfactuals and forecasts. (Sargent (1984, esp. fn. 15) seems to recognize this critical distinction as an afterthought to acknowledging the contradiction identified by CL&R.) The essential point is that, within a rational expectations context, an analytical framework and strategy for determining what would be happening under a different fixed policy regime is in general not relevant for determining what will happen if the policy regime changes.

Formalizing this distinction requires a model that specifies both the effects of current and expected future policy as well as the process generating policy. Suppose that the true structural relation between the economic outcome and current and expected future policy variables has the following form:

$$y_{t+1} = a_o + a_1 E_t x_{t+1} - a_2 x_t + \varepsilon_{t+1}, \quad a_2 > a_1 > 0, \tag{1}$$

where $y_{t+1}$ describes a result of private action taken in period $t$,

$x_t$ is a policy action taken in period $t$,

$E_t x_{t+1}$ is the rational expectation of $x_{t+1}$ formed in period $t$,

$\varepsilon_{t+1}$ is a normally distributed exogenous random variable that has zero mean and is uncorrelated serially,

and $a_o$, $a_1$, and $a_2$ are constant positive parameters that are invariant with respect to the process generating $x_t$.

Equation (1) derives from a choice-theoretic problem that private agents behave as if they solve. This underlying choice model, which need not be specified explicitly here, is the basis for treating the parameters of equation (1) as policy invariant as well as for imposing functional relations among these parameters. Economists observe $\{y_{t+1}\}_{t=1}^{T}$ and $\{x_t\}_{t=1}^{T}$, but, as L&S stress, because they do not directly observe $E_t x_{t+1}$, calculating estimates of the parameters of equation (1) is not a routine econometric exercise. As we shall see, the need for such parameter estimates depends on the question being addressed.

To tell a specific story about equation (1), imagine that $y_{t+1}$ is the number of engineers who graduate in period $t+1$, that $x_t$ is the net cost of studying engineering in period $t$, and that $\varepsilon_{t+1}$ is the random deviation from the normal survival rate of engineering students. In this story, equation (1) says that the number who enroll in period $t$, adjusted for the normal survival rate, responds negatively to the current cost and, because the timing of this investment is flexible, responds positively, but less strongly, to the expected future cost. The number who graduate in period $t+1$ depends also on $\varepsilon_{t+1}$.

To complete the model, suppose that the true process generating $x_t$ is

(2) $$x_t = b + u_t, \quad b \neq 0,$$

where $b$ is a constant parameter

and $u_t$ is a normally distributed exogenous random variable that has zero mean and is uncorrelated both serially and with $\varepsilon_{t+1}$.

In terms of the story, $b$ is the normal cost of studying engineering. To make the idea of rational expectations operational, assume that in forming $E_t x_{t+1}$ private agents behave as if they know equation (2), the value of $b$, and the distribution of $u_t$. Economists can calculate an estimate of $b$

equal to the mean of $\{x_t\}_{t=1}^{T}$. Complicating this setup by assuming that $x_t$ also depends in a fixed way on current or past values of $y_t$ would not affect the present argument. The essential elements that the model given by equations (1) and (2) shares with the models considered by L&S are that the actions of private agents depend on their rational expectations of future policy and that the policy regime, described by the constant parameter b and the process governing $u_t$, is fixed.

(In earlier work, Neftci and Sargent (1978) look for regime breaks to use in testing the hypothesis that certain reduced-form coefficents depend on parameters of the policy regime. Their analysis, nevertheless, uses the framework of equations (1) and (2). Specifically, they assume that rational private agents regard the prevailing regime to be fixed and they treat each historical regime as an independent segment of history. This work, thus, is equally subject to the CL&R criticism.)

As stated by L&S, their objective, in terms of equations (1) and (2), is to determine how the behavior of $y_{t+1}$ over time depends on the parameter b of the process generating $x_t$. The difficulty, as suggested above, is that this loose statement encompasses two radically different problems. These problems are (i) simulating the behavior of $y_{t+1}$ under hypothetically different constant values of the parameter b and (ii) forecasting the effects on $y_{t+1}$ of hypothetical changes in the parameter b. The first problem involves the counterfact of an alternative fixed policy regime. The second problem, in contrast, involves a changing policy regime. The inconsistency in the L&S strategy for policy evaluation reflects their failure to recognize that analysis of a changing policy regime is not meaningful, because it asks about the effects of an event that cannot occur, if the true process generating policy actions has fixed parameters.

Given that equations (1) and (2) are the true model, the problem of forecasting the effects of policy actions consists only of predicting the future time path of $y_{t+1}$ conditional on hypothetical future realizations of $u_t$ or, equivalently, of $x_t$. (Because point realizations of continuously distributed random variables have zero probability, a careful description of the forecasting problem would specify the hypothetical realizations to be within intervals that have finite probability.) Observe that equation (2) implies

(3) $$E_t x_{t+1} = b,$$

and that substituting equation (3) into equation (1) gives

(4) $$y_{t+1} = A - a_2 x_t + \varepsilon_{t+1}, \quad \text{where} \quad A = a_o + a_1 b.$$

Using equation (4), a regression of $y_{t+1}$ on $x_t$ yields estimates of $A$ and $a_2$.

To forecast, set $b$ in equation (2) equal to its estimated value and simulate $\{x_t\}_{t=T+1}^{\infty}$ for any hypothetical realization of $\{u_t\}_{t=T+1}^{\infty}$. Then, set $A$ and $a_2$ in equation (4) equal to their estimated values, set $\varepsilon_{t+1}$ equal to its zero mean, and simulate $\{y_{t+1}\}_{t=T+1}^{\infty}$ for the implied realization of $\{x_t\}_{t=T+1}^{\infty}$.

This forecasting exercise does not require use of the L&S strategy of estimating underlying private behavioral parameters, but, because it concerns realizations within a given policy regime, it is also not an example of the type of problem that L&S have in mind. As suggested above, however, within the context of equations (1) and (2) as the true model, the only admissible question about the effects of different policy regimes involves counterfact. Specifically, how would the historical pattern of $y_{t+1}$ be different if the constant parameter $b$ of the process generating $x_t$ were different? Answering this question requires estimating underlying behavioral parameters.

In the present example, the essential underlying parameter that remains to be estimated is $a_1$. If, however, $a_o$, $a_1$, and $a_2$ are independent parameters, $a_1$ is not identified. This lack of identification presented no problem for forecasting, but it inhibits analysis of counterfactuals. To facilitate such analysis, suppose that a known function relates $a_o$, $a_1$, and $a_2$. In that case the parameters are just identified, and the estimates of $b$, $A$, and $a_2$ imply an estimate of $a_1$. (If the underlying parameters were overidentified, efficient estimation would incorporate these restrictions.)

To determine the hypothetical differences in the past and future history of $y_{t+1}$ associated with hypothetically different policy regimes, rewrite equation (4) as

$$y_{t+1} = a_o - (a_2 - a_1)b - a_2 u_t + \varepsilon_{t+1}. \tag{5}$$

Then, set $a_1$ and $a_2$ equal to their estimated values, set $\{u_t\}_{t=1}^{T}$ and $\{\varepsilon_t\}_{t=1}^{T}$ equal to their estimated realizations, set $\{u_t\}_{t=T+1}^{\infty}$ and $\{\varepsilon_t\}_{t=T+1}^{\infty}$ equal either to their zero means or to any other hypothetical sequence of realizations, and simulate $\{y_{t+1}\}_{t=1}^{\infty}$ for counterfactuals involving constant values of $b$ other than the estimated value of $b$.

## 2. Regime Changes

The model given by equations (1) and (2) implies that the relation between the realization of $y_{t+1}$ and the parameter $b$ of the process generating $x_t$ involves both the parameter $a_1$ associated with the rational expectation of $x_{t+1}$ as well as the parameter $a_2$ associated with the realization of $x_t$. Specifically, as equation (5) indicates, if, as a counterfact, $b$ were one unit larger, then, for all $t = 1, \ldots, \infty$, $x_t$ and $E_t x_{t+1}$ each would be one unit larger, and $y_{t+1}$ would be $a_2 - a_1$ units smaller. The mistake made by L&S is to propose using the coefficient $a_2 - a_1$ to forecast the effects of changes in the parameter $b$, even though $b$ is a constant in the model that implies that this coefficient relates $y_{t+1}$ and $b$. If the parameter $b$ of the process generating $x_t$ is truly a variable rather than a constant, then equation (2) is not part of the true model. In this case, pretending, as L&S implicitly do, that equations (1) and (2) are the true model generally produces wrong forecasts. Specifically, if $b$ is truly variable, using the relation between $y_{t+1}$ and $b$ given by equation (5) to forecast the effects of changes in $b$ is not consistent with rational expectations.

To illustrate these points, assume that equation (2) is false, and that the true model includes instead

(6) $$x_t = b_t + u_t \quad \text{and}$$

(7) $$b_t = \beta + \rho b_{t-1} + \delta_t, \qquad b_o = \frac{\beta}{1-\rho} \qquad \text{and} \quad 0 < \rho < 1,$$

where $\beta$ and $\rho$ are constant parameters,
$b_t$ is now a variable parameter,
and $\delta_t$ is a normally distributed random variable that has zero mean and is uncorrelated serially and with both $\varepsilon_{t+1}$ and $u_t$.

To make the idea of rational expectations operational in this context, assume that private agents behave as if they know equations (6) and (7), the values of $\beta$ and $\rho$, and distributions of $\delta_t$ as well as $u_t$, and as if they observe $b_t$. Note for future reference that equations (6) and (7) imply an expression for current policy in terms of current and past disturbances in the policy process as

(8) $$x_t = \frac{\beta}{1-\rho} + \sum_{i=0}^{t-1} \rho^i \delta_{t-i} + u_t, \quad t \geq 1.$$

If economists observe $\{b_t\}_{t=1}^{T}$, using equation (7), a regression of $b_t$ on $b_{t-1}$ yields estimates of $\beta$ and $\rho$.

The model given by equations (6) and (7) explicitly treats the policy regime as variable. Specifically, the parameter $b_t$ of the policy regime evolves from an autoregressive process. Because the autoregressive parameter $\rho$ is positive, disturbances to the policy regime, generated by the random variable $\delta_t$, are persistent. As the parameter $\rho$ approaches unity, the policy regime approaches a random walk. In contrast, for a given policy regime, disturbances to the policy variable, generated by the random variable $u_t$, are transitory. Complicating this setup by making the evolution of the policy regime depend also on current or past private actions, as in the model of policy analyzed by Flood and Garber (1983), would not change the present argument.

Given that equations (1), (6), and (7) are the true model, the problem of forecasting the effects of policy now involves consideration of regime changes. The future path of $y_{t+1}$ depends on the future path of $b_t$--or, equivalently, on future realizations of $\delta_t$--as well as future realizations of $u_t$. Specifically, observe that equations (6) and (7) imply

(9) $$E_t x_{t+1} = E_t b_{t+1} = \beta + \rho b_t$$

and that substituting equation (9) into equation (1) gives

(10) $$y_{t+1} = B_o + B_1 b_t - a_2 x_t + \varepsilon_{t+1}, \quad \text{where} \quad B_o = a_o + a_1 \beta$$

$$\text{and} \quad B_1 = a_1 \rho.$$

Using equation (10), a regression of $y_{t+1}$ on $b_t$ and $x_t$ yields estimates of $B_o$, $B_1$, and $a_2$.

(In contrast to the present setup, a relevant model for some practical applications would assume that private agents behave as if they do not know $b_t$ and, hence, cannot distinguish directly between the persistent disturbances to $x_t$ that stem from $\delta_t$ and the transitory disturbances to $x_t$ that stem from $u_t$. Under this restricted information structure, the rational expectation of $x_{t+1}$ would be a weighted average of $\beta$ and all current and past realizations of $x_t$. Rational expectations of future policy have this form in the model developed by CL&R, as well as in models of policy analyzed by Gertler (1982) and by Brunner, Cukierman, and Meltzer (1983). Similarly, when implementing equation (10) econometrically, an economist who did not have observations on $b_t$ would include instead observed past values of $x_t$. These complications would not change the essential conclusions of this section.)

Although equation (10), which relates $y_{t+1}$ to both $b_t$ and $x_t$, is the correct regression equation, it is possible, by substituting equation (6) into equation (10), to write the true relation

$$y_{t+1} = B_o - (a_2 - a_1\rho)x_t - a_1\rho u_t + \varepsilon_{t+1}.$$

This equation relating $y_{t+1}$ to $x_t$ looks superficially like equation (4). Except in the special case of $\rho = 0$, however, a regression of $y_{t+1}$ on $x_t$ alone would involve an error term containing $u_t$ and, hence, correlated with $x_t$. If $b_t$ is truly variable, as in equations (6) and (7), pretending that $b_t$ is constant as in equation (2) would produce a biased estimate of the relation between realizations of $y_{t+1}$ and $x_t$.

To forecast using equations (6), (7), and (10), set $\beta$ and $\rho$ in equation (7) equal to their estimated values and simulate $\{b_t\}_{t=T+1}^{\infty}$ for any hypothetical realization of $\{\delta_t\}_{t=T+1}^{\infty}$. Next, add this implied realization of $\{b_t\}_{t=T+1}^{\infty}$ to any hypothetical realization of $\{u_t\}_{t=T+1}^{\infty}$ to simulate $\{x_t\}_{t=T+1}^{\infty}$. Finally, set $B_o$, $B_1$, and $a_2$ in equation (10) equal to their estimated values, set $\varepsilon_{t+1}$ equal to its zero mean, and simulate $\{y_t\}_{t=T+1}^{\infty}$ for the implied realizations of $\{b_t\}_{t=T+1}^{\infty}$ and $\{x_t\}_{t=T+1}^{\infty}$. This model implies that a realization of $\delta_t$ equal to unity adds one unit to $b_t$ and to $x_t$, which in turn adds $\rho$ units to $E_t x_{t+1}$, and, as a result, reduces $y_{t+1}$ by

$a_2 - a_1\rho$ units. Thus, except in the special case of $\rho = 1$, the decrease in $y_{t+1}$ associated with a one unit increment to $b_t$ does not equal $a_2 - a_1$ units, which is the effect suggested by the pretense that $b_t$ is constant.

The main lesson from this analysis is that correct forecasts, whether of the effects of realizations within a given policy regime or of the effects of changes in the parameters of the policy regime, require specification, estimation, and simulation of a model that incorporates the true process generating the policy regime. In the present example, equation (2) is a good approximation to equations (6) and (7) for estimation only if $\rho$ is close to zero, but it is a good approximation for forecasting only if $\rho$ is close to unity. Another important lesson is that, given knowledge of the form of the true model, forecasts of the effects of hypothetical regime changes, like forecasts of the effects of hypothetical policy actions, do not involve the L&S strategy of estimating underlying private behavioral parameters. Specifically, in the present model, forecasts of the effects of realizations of either $\delta_t$ or $u_t$ do not require calculating an estimate of $a_1$.

It is also possible, in the context of the model given by equations (1), (6), and (7), to consider questions of counterfact about differences in the process generating policy regimes. Such analysis, involving different values of the constants $\beta$ and $\rho$, like the analysis of counterfactuals involving $b$ in the model given by equations (1) and (2), would require calculating an estimate of the underlying parameter $a_1$. The discussion below considers in greater depth the problem of analyzing counterfactuals concerned with different processes generating policy regimes.

## 3. A Choice-Theoretic Model of Policy

The model given by equations (1), (6), and (7) fully specifies both the economic structure that determines the effects of current and expected future policy actions as well as the process that generates policy actions. Given that this model is true, it provides a consistent framework for forecasting the effects of exogenous disturbances to the policy process. This model, however, does not include a deep analysis of policy. It treats the evolution of policy actions as an exogenous process, like the processes that generate natural phenomena, and it provides no criteria for modelling policy beyond historical experience.

In modelling the actions of private agents, knowing that they behave as if they solve a well-defined choice-theoretic problem is important. As noted above, such knowledge is the basis for treating the parameters of equation (1) as invariant with respect to the policy process as well as for imposing functional relations among these parameters. An analogous analytical strategy would hypothesize that the policymaker is also an agent that behaves as if it has consistent preferences over the outcomes that it can influence and as if it solves a well defined choice-theoretic problem. In such a model, policy actions, whether classified as occurring within a policy regime or as involving changes in the policy regime, evolve as the rational and, hence, predictable responses to shifts in the constraints that the policymaker faces.

A choice-theoretic model of policymaking has important implications both for forecasting and for the analysis of counterfact. Admittedly, such a model does not attempt to portray either the conscious behavior of policymakers or the political process that proximately generates policy actions. Indeed, according to Grossman (1980), "experience suggests that the political process has limited ability to specify consistent goals, establish priorities, and choose between competing objectives about economic matters, especially when these decisions require comprehension of complex technical issues and constant processing of complex information." The important issue in the present context, however, is whether the assumption that policymakers, like other economic agents, behave as if they solve choice-theoretic problems yields true probabilistic models that relate the behavior of private agents, the behavior of policymakers, and the economy's exogenous variables. Some recent literature--see, for example, Barro and Gordon (1983)--suggests that this research strategy can illuminate complex problems in positive economics.

To exemplify a choice-theoretic model of policy that can underlie equations (6) and (7), suppose that the policy action $x_t$ results from the policymaker's attempt to minimize the loss L, given by

$$L = E_t(y_{t+1} - z_t)^2, \tag{11}$$

subject to the structure of private behavior given by equation (1). In this formulation, $z_t$ is an exogenous stochastic variable that represents the policymaker's target value for $y_{t+1}$. In the terms of the story about equation (1), $z_t$ is the target number of engineering graduates in period t+1. Equation (11) represents a partial, indirect loss function, with $z_t$ deriving from the solution of the policymaker's complete choice problem involving fixed preferences and changing constraints.

In the present context, the natural extension of the idea of rational expectations is that the policymaker behaves as if it knows the parameters of equation (1), the distribution of $\varepsilon_{t+1}$, and the private agents' expectation of future policy, $E_t x_{t+1}$. For private agents, an operational specification of rational expectations in this context is that they behave as if they understand the choice problem that the policymaker solves, including the process, to be specified below, that generates realizations of $z_t$.

The solution to the problem given by equation (11) is for the policymaker to select $x_t$ such that $E_t y_{t+1} = z_t$, or, equivalently, such that

$$x_t = \frac{1}{a_2}(a_o + a_1 E_t x_{t+1} - z_t). \tag{12}$$

Equation (12) says that current policy is a linear function of the policymaker's target and of the private agents' expectation of policy next period, with coefficients that depend on the parameters that govern private behavior. This specification of policy implies, from substituting equation (12) into equation (1), that

$$(13) \qquad y_{t+1} = z_t + \varepsilon_{t+1}.$$

Equation (13) says that rational policy actions induce private actions such that the economic outcome, $y_{t+1}$, equals the sum of the realizations of the truly exogenous variables, $z_t$ and $\varepsilon_{t+1}$.

A complete analysis of the evolution of policy actions implied by equation (12) requires deriving a solution for $E_t x_{t+1}$ in terms of exogenous variables. The specification of rational expectations implies that in forming $E_t x_{t+1}$ private agents behave as if they know that equation (12) will hold in all future periods. Accordingly, leading equation (12) $i$ periods, $i > 1$, and applying the operator $E_t$ gives a partial difference equation in expected future policy,

$$(14) \qquad E_t x_{t+i} = \frac{a_1}{a_2} E_t(E_{t+i} x_{t+i+1}) - \frac{1}{a_2} E_t(z_{t+i} - a_o)$$

$$= \frac{a_1}{a_2} E_t x_{t+i+1} - \frac{1}{a_2} E_t(z_{t+i} - a_o).$$

To solve equation (14) for $E_t x_{t+i}$, fix $t$ and treat equation (14) as an ordinary difference equation in $i$. Given that $a_1/a_2$ is less than unity, as long as $E_t z_{t+i}$ grows with $i$ at a rate less than $a_2/a_1$, the forward-looking particular solution to this equation converges. Using the forward operator, $F$, this solution is, for $i = 1$,

(15) $$E_t x_{t+1} = -\frac{1}{a_2} E_t[(1 - \frac{a_1}{a_2} F)^{-1}(z_{t+1} - a_o)]$$

$$= -\frac{1}{a_2} E_t \sum_{j=0}^{\infty} (\frac{a_1}{a_2})^j (z_{t+1+j} - a_o).$$

Equation (15) expresses the private agents' expectation about policy next period as an average, with exponentially declining weights, of all expected future values of the policymaker's target. The weights depend on parameters that govern private behavior, which the policymaker takes into account in determining policy actions. Substituting equation (15) into equation (12) gives an expression for current policy as

(16) $$x_t = \frac{a_o}{a_2 - a_1} - \frac{1}{a_2} [z_t + E_t \sum_{j=0}^{\infty} (\frac{a_1}{a_2})^{j+1} z_{t+1+j}].$$

The general solutions for $E_t x_{t+1}$ and, hence, for $x_t$ can also contain either deterministic or stochastic homogeneous components. The literature often refers to such components as rational self-confirming bubbles--see, for example, Diba and Grossman (1983). For simplicity, the present discussion abstracts from policy bubbles.

To complete the model, suppose that the policymaker's target evolves according to

(17) $$z_t = \alpha c_t + v_t \quad \text{and}$$

(18) $$c_t = \gamma + \lambda c_{t-1} + \eta_t, \quad c_o = \frac{\gamma}{1-\lambda} \quad \text{and} \quad 0 < \lambda < 1,$$

where $\alpha$, $\gamma$, and $\lambda$ are constant parameters,
and $v_t$ and $\eta_t$ are normally distributed exogenous random variables that have zero mean and are uncorrelated serially and with both $\varepsilon_{t+1}$ and each other.

In terms of the above story, in which $z_t$ is the target number of engineering graduates in period t+1, $c_t$ might be a measure of planned future development and construction of weapons systems and other military hardware. The specification of rational expectations implies that private agents behave as if they know equations (17) and (18), the values of $\alpha$, $\gamma$, and $\lambda$, and the distributions of $v_t$ and $\eta_t$, and as if they observe $c_t$.

Equations (17) and (18) imply

$$(19) \qquad z_t = \frac{\alpha\gamma}{1-\lambda} + \alpha \sum_{i=0}^{t-1} \lambda^i \eta_{t-i} + v_t .$$

Leading equation (19) 1+j periods, $j \geq 0$, and applying the operator $E_t$ gives

$$(20) \qquad E_t z_{t+1+j} = \frac{\alpha\gamma}{1-\lambda} + \alpha \sum_{i=1+j}^{t+j} \lambda^i \eta_{t+i+j-i}$$

$$= \frac{\alpha\gamma}{1-\lambda} + \alpha\lambda^{1+j} \sum_{i=0}^{t-1} \lambda^i \eta_{t-i} .$$

Substituting equations (19) and (20) into equation (16) yields an expression for current policy in terms of current and past disturbances in the process that generates $z_t$,

$$(21) \qquad x_t = \frac{a_o - \alpha\gamma/(1-\lambda)}{a_2 - a_1} - \frac{\alpha}{a_2 - a_1\lambda} \sum_{i=0}^{t-1} \lambda^i \eta_{t-i} - \frac{1}{a_2} v_t .$$

Compare the solution for $x_t$ given by equation (21) with the solution for $x_t$ given by equation (8) in the preceding section. This comparison shown that the descriptive model of policy regimes and policy actions, given by equations (6) and (7), and the choice-theoretic model of policy, given by equations (11), (17), and (18), have fully consistent implications. Specifically, if this choice-theoretic model actually underlies the descriptive model, then the alternative expressions of the solutions for $x_t$ mean that the parameters and exogenous variables that appear in equations (6) and (7) are functions of the parameters and exogenous variables that appear in equations (1), (17), and (18). Consistency of the solutions implies that these functions are

$$(22) \quad \begin{cases} \beta = \dfrac{(1-\lambda)a_o - \alpha\gamma}{a_2 - a_1}, \\ \rho = \lambda, \\ \delta_t = \dfrac{-\alpha}{a_2 - a_1\lambda}\eta_t, \quad \text{and} \end{cases}$$

$$(23) \quad u_t = \frac{-1}{a_2} v_t.$$

Equations (22) and (23) imply that the persistent component of policy, denoted as the policy regime in the descriptive model, is a linear function of the persistent component of the policymaker's target, with slope and intercept that depend on the parameters of the process that generates the target and on the parameters that govern private behavior. Specifically, either combining equation (22), equations (6) and (8), and equations (17) and (19), or combining equation (23), equation (10), and equations (13) and (17) yields

$$(24) \quad \alpha c_t = B_o - (a_2 - B_1)b_t.$$

(Equation (24) implies that observing $c_t$ and knowing the model's structural parameters is equivalent to knowing $b_t$, an implication that clarifies the consistency of assuming that private agents behave as if they observe $b_t$.) Equation (23) also shows that the transitory component of policy is proportional to the transitory component of the policymaker's target, with the factor of proportionality being the response parameter of private behavior to current policy. In sum, the choice-theoretic model implies that the evolution of policy, including both changes in the policy regime and transitory policy actions as described in equations (6) and (7), reflects the policymaker's purposeful response to the evolution of his target. Moreover, although equation (10) correctly implies that $y_{t+1}$ depends on $b_t$ and on $x_t$, this dependence reflects the more basic phenomenon, reflected in equation (13), that policy is purposely inducing changes in $y_{t+1}$ in response to changes in $z_t$.

## 4. Importance of Knowing the Choice-Theoretic Model

The implications derived from a true underlying choice-theoretic model of policy are important both for forecasting and for the analysis of counterfact. As described in the preceding section, forecasting the effects of hypothetical disturbances to the policy process, as described by equations (6) and (7), requires estimates of the three coefficients, $B_o$, $B_1$, and $a_2$, in equation (10), which relates the economic outcome to the policy regime and the policy action. If economists either observe the policy target or observe variables that influence the policy target, knowing the underlying model makes possible more efficient estimation of these coefficients.

As the most dramatic example of this advantage, suppose that economists observe $\{z_t\}_{t=1}^{T}$. This knowledge permits combining equations (10) and (13) into a relation among observed variables,

$$z_t = B_o + B_1 b_t - a_2 x_t,$$

which allows an exact calculation of true values of $B_o$, $B_1$, and $a_2$, leaving nothing to be estimated. Alternatively, suppose that economists do not observe $z_t$, but that they observe $\{c_t\}_{t=1}^{T}$ and know the value of $\alpha$. In this case, equation (24) permits an exact calculation of $B_0$ and the difference $a_2 - B_1$, leaving only one coefficient to be estimated. Finally, suppose that economists observe $\{c_t\}_{t=1}^{T}$, but do not know $\alpha$. In this case, equation (24) permits an exact calculation of the ratio of $B_0$ to $a_2 - B_1$, leaving only two coefficients to be estimated.

Thus far, forecasting has concerned the effects of hypothetical realizations of the variables $\delta_t$ and $u_t$, which are components of the descriptive model of policy. A basic insight associated with formulation of the underlying choice-theoretic model is that a more fundamental forecasting problem involves the effects of hypothetical disturbances to the policymaker's constraints and derived targets, which, in this framework, are the phenomena that underlie the policy process. Such forecasts concern hypothetical realizations of the variables, $\eta_t$ and $v_t$, and require estimates of parameters $\alpha$, $\gamma$, and $\lambda$. This problem focuses on the policymaker's goals and achievements, rather than on how it manipulates policy instruments, and probably corresponds more closely to the actual forecasting strategy of practical people.

Using equation (18), a regression of $c_t$ on $c_{t-1}$ yields estimates of $\gamma$ and $\lambda$. If $z_t$ is observed, using equation (17), a regression of $z_t$ on $c_t$ yields an estimate of $\alpha$. (If $z_t$ is not observed, using equations (13) and (17), a regression of $y_{t+1}$ on $c_t$ yields an alternative, less efficient, estimate of $\alpha$.) To forecast, set $\gamma$ and $\lambda$ equal to their estimated values and simulate $\{c_t\}_{t=T+1}^{\infty}$ for any hypothetical realization of $\{\eta_t\}_{t=T+1}^{\infty}$. Then, substitute equation (17) into equation (13), set $\alpha$ equal to its estimated value and set $\varepsilon_{t+1}$ equal to its zero mean, and simulate $\{y_{t+1}\}_{t=T+1}^{\infty}$ for the implied realization of $\{c_t\}_{t=T+1}^{\infty}$ and any hypothetical realization of $\{v_t\}_{t=T+1}^{\infty}$.

This forecasting exercise, importantly, does not involve estimation of the coefficients of equations (6), (7), and (10). More basically, because the underlying choice-theoretic model directly relates the economic outcome to the evolution of the policymaker's target, it sidesteps entirely the problem of modelling the evolution of the policy variable. In other words, given knowledge of the underlying model represented by equations (13), (17), and (18), forecasting economic outcomes does not require either observing or inferring the policy regime, or even observing policy actions. In this context, forecasting requires only observing the variables that determine the evolution of the policymaker's target.

The underlying choice-theoretic model of policy also has fundamental implications for the analysis of counterfactuals involving the evolution of policy. Equations (22) show that the parameters and exogenous variables that appear in equations (6) and (7) and describe the evolution of the policy regime are not mutually independent. For example, the parameter $\rho$, which determines the persistence of the effect of the disturbance $\delta_t$ to the policy regime, is equivalent to the parameter $\lambda$, which determines the persistence of the effect of the disturbance $\eta_t$ to the policy target, while the parameter $\beta$ of the process generating the policy regime and the realizations of $\delta_t$ also depend on $\lambda$. Consequently, if the choice-theoretic model actually underlies the descriptive model, in order to determine how the historical pattern of the economic outcome $y_{t+1}$ would be different if $\rho$ were different, it is necessary to take into account that, if $\rho$ were different, $\beta$ and $\delta_t$ would also be different. In general, this analysis shows that, if policy corresponds to the solution of a choice problem, analyses of counterfact that treat the parameters of the descriptive policy process as independently fixed are vulnerable to the logic of the Lucas critique. Note that $\beta$, $\delta_t$, and $u_t$ also depend on the parameters that govern private behavior. Another, and possibly more important observation, is that, analogously to the reformulation of the problem of forecasting, the underlying choice-theoretic model directs analysis of counterfactuals to differences in the underlying parameters that generate the policymaker's constraints and derived targets.

## 5. The Place of Policy Advice

As mentioned above, Sargent (1984), following the earlier argument of Sargent and Wallace (1976), contends that models that include a specification of the evolution of policy for all time leave no place for prescriptive policy analysis. Sargent and Wallace made this point in the context of a framework that treats the evolution of policy regimes as an exogenous process. In reiterating this position, however, Sargent explicitly refers to a framework that views policy as resulting from a rational choice process.

This section discusses a view of the distinction between descriptive and prescriptive analysis that derives from choice-theoretic modelling of policy and that, in contrast to Sargent's view, preserves a distinct and meaningful role for policy advice in a rational-expectations context. Note, however, that even if we were to accept Sargent's interpretation of policy advice, his desire to prescribe policy would not justify his employing the logically inconsistent L&S analytical framework.

CL&R respond to the argument of Sargent and Wallace by denoting policy analysis to be the exercise of forecasting endogenous variables conditional on realizations of the exogenous stochastic variables that influence the policy process. The problem with this response is that, as CL&R recognize, the assumption that policy evolves according to the fixed process specified in their model leaves no independent role for these forecasts in influencing policy. In other words, given the assumptions of the CL&R model, policy advice based on what CL&R denote as policy analysis seems pointless.

Viewing policy as the solution to a choice problem allows for a quite different conception of the scope for policy advice, one that neither Sargent nor CL&R consider. Given a choice-theoretic model of policy, we can view policy analysis as concerned with formulating and solving the actual choice problems faced by policymakers, while viewing positive analysis as concerned with modelling the consequences of these choices in probabilistic terms. From this perspective, the policy advisor is part of the action, whereas the positive economist is part of the audience. In other words, economists who advise government officials and the electorate, like other economists who play a prescriptive role--for example, business economist advising firms or home economists advising households--are a component, at least implicitly, of the interaction between policy and private behavior that the positive analysis attempts to model. In this context, if a fully specified positive model is true, even in a probabilistic sense, the actual activities of all economists who advise economic agents are helping to confirm the predictions that the positive economist derives from the model.

Nevertheless, from the standpoint of the decision-making economic agent or its advisor, the choice problem remains meaningful, even if positive analysis is successful in predicting its outcome on average. The essential point is that the formulation of positive models of the choice processes of private agents and policymakers that fit the data, and thus seem to be true at least in an as-if sense, do not vitiate these choice processes. In particular, the development of a positive model that includes a choice-theoretic analysis of policymaking does not supercede policy advice. Rather, this development clarifies the distinction between the positive economist as a spectator and the policy advisor as a player, and it points to the importance of keeping straight these different activities in which an economist can engage.

A program of positive economic analysis that yields probabilistic relations between economic outcomes and exogenous variables does not directly address either the problem of how to give good policy advice or the related positive problem, which traditionally lies in the domains of behavioral psychology and political science, of literally describing the conscious behavioral processes of private agents and policymakers. Both of these problems involve further considerations such as understanding the technologies available for decision making and analyzing how decision makers use and should use the results of positive economic analysis.

## 6. Conclusions

To be consistent with the logic of rational expectations, a model for forecasting economic outcomes that are influenced by policy must include a complete specification of the process that generates policy actions. In particular, forecasts of the effects of changes in the policy regime require knowledge of the true model of the evolution of policy regimes. The CL&R criticism of the L&S strategy for econometric policy evaluation reflects the failure of L&S to specify an adequate forecasting model. The L&S strategy, emphasizing estimation of policy invariant behavioral parameters, actually addresses the distinct problem of analyzing counterfactuals, rather than the problem of forecasting.

An especially attractive strategy for modelling the process that generates policy is to hypothesize that the policymaker, like private agents, behaves as if it solves a well-defined choice-theoretic problem. Knowledge of a true choice-theoretic model of policy permits more efficient estimation of the effects of policy on economic outcomes and, thereby, allows better forecasting of the effects of disturbances to the policy process. The more radical result of choice-theoretic modelling of policy, however, is to shift attention towards a more basic forecasting problem that concerns the effects of hypothetical realizations of the exogenous variables that determine the policymaker's constraints and targets. A choice-theoretic model of policy also implies that the parameters of the policy process are not invariant with respect to the processes that generate these exogenous variables. The basic problem for analysis of counterfact in this context concerns differences in the parameters of these underlying processes.

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