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FROM PRICE PLANS TO TAX TABLES

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ABSTRACT

Requiring agents with private information to select from a menu of incentive schedules can yield efficiency gains. It will do so if, and only if, agents will receive further private information after selecting the incentive schedule but before taking the action that determines where on the incentive schedule they end up. We argue that this information structure is relevant in many applications. We develop the theory underlying optimal menus of non-linear schedules and prove that there exists a menu of schedules that offers a strict first-order interim Pareto improvement over the optimal single non-linear schedule. We quantify the gains from schedule selection in two settings. The first is a stylized example of a monopolistic utility company increasing profits by offering a menu of price plans. The second is a simulation based on U.S. earnings data, which shows that moving to a tax system that allows individuals to choose their tax schedule increases social welfare by the same amount as would occur from a 4.0 percent windfall gain in the government budget (or about $600 per filer per year). The resulting reduction in distortions accounts for about two thirds of the increase in social welfare while the remainder comes from an increase in redistribution.

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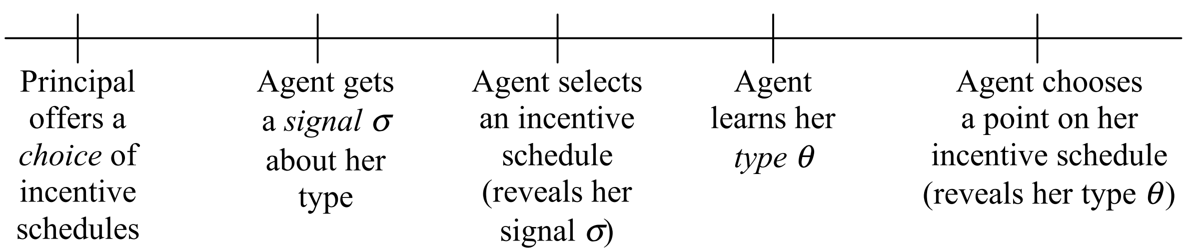
1. Introduction

Agents select schedules in many contexts. Individuals choosing a phone plan choose among a number of alternative schedules, each typically consisting of a subscription fee, a low price per minute for a certain number of minutes with a high price per minute thereafter. Similarly, consumers may select among price plans offered by a utility company. Employers may offer their sales force various compensation schedules – some with a relatively low base compensation and high commissions, and others with a relatively high base compensation and low commissions. Governments also offer a menu of incentive schedules in many contexts. When closely held firms decide whether to function as Schedule C or Schedule S corporations, they are essentially choosing between schedules. So too are spouses in the U.K., who for tax purposes must choose ex-ante how to allocate joint capital income between them. Individuals deciding how much to contribute to their medical flexible spending account in effect decide on the schedule of marginal costs they will face for medical spending in the coming year. American senior citizens face a schedule selection challenge when deciding whether to participate in the Medicare drug program and, if so, which plan to choose. More generally, anyone deciding whether or not to enroll in some money-saving plan with an up-front fee is choosing from a two-item menu of schedules.

This paper determines when offering a menu of schedules can generate an efficiency improvement over the optimal non-linear incentive schedule. The essential setup is an expanded screening model. As with the standard screening model, an agent's type is private information, and agents of different types have preferences that differ in the space of the incentive schedule. Thus, in the cell phone example, an agent with a higher type would value additional cell phone minutes more highly for each usage level. The model expands the standard screening model because agents secure private information at two stages, not merely one. Thus, each agent possesses private information about her future type before she selects one of the schedules the principal offers. Then after choosing her schedule, she receives additional private information that precisely defines her type. Given her type, she picks a single point on her previously selected incentive schedule. We will refer to the initial private information, i.e., the information at the point of schedule selection, as the agent's signal, and the subsequent private information as the agent's type. For example, the agent might know that she generally will be a heavy cell phone user (her signal), but before she chooses her phone plan, she is unlikely to know whether some lengthy calls will become desirable that month (her type). When she must choose among the menu of schedules, the agent will select the one that maximizes her expected utility, given her subjective distribution of her future types. We shall assume rational expectations, i.e., that these subjective distributions are accurate. Note, an agent's signal only affects the probability distribution of the agent's type and is therefore no longer pay-off relevant once she learns her type.

The principal offers a menu of non-linear schedules to maximize his objective function. The maximand would be social welfare were the principal a welfare-maximizing government offering a menu of tax schedules. The maximand would be profits were the principal a cell phone company. The timing of this setup is summarized in Figure 1. Because an agent learns her type with certainty only after she has chosen her incentive schedule, some agents will regret their choice of incentive schedule once they learn their type.

Figure 1: Timing of events



Providing the principal with additional instruments cannot hurt the principal (there are no commitment problems). Thus, it is immediate that offering a menu of non-linear schedules yields a weak gain to the principal over the constrained optimal outcome given a single non-linear schedule. The more surprising result is that, in a wide range of cases, this gain is strict and first order.[1] Part of this gain is due to the fact that the menu of schedules allows resources to be redistributed to the principal's advantage. If the principal were a profit-maximizing company, this redistribution would take the form of the principal extracting more rents from the agents. If

[1] The qualifier "a wide range of cases" is spelled out in detail in Proposition 2. In particular, our proof assumes separable utility and a standard solution to the optimal single incentive schedule (no corner solution, no bundling, binding incentive constraints). With this setup, an interim-by-type, first-order efficiency gain is achieved except in the following two cases: (i) the degenerate case where the signal is either perfectly informative or perfectly uninformative and (ii) one combination of parameters that has Lebesgue measure zero. We also prove that when there are two or fewer types, offering a menu of schedules can never produce a strict welfare gain. Of course, this does not rule out cases with two or fewer agents, because a single agent could have many potential types.

the principal were a government, part of the social welfare increase would come from more redistribution from richer individuals to poorer ones.

However, even if one has no interest in any gains stemming from a redistribution of resources, the schedule-selection approach still offers advantages. We prove that it generically produces a first-order interim Pareto efficiency gain. The Pareto gain is interim in the sense that average utility of each type increases (where the average is taken with respect to the signal received). The interim Pareto efficiency gain implies an ex-ante efficiency gain. Behind the veil of ignorance, i.e., before learning one's signal or type, all agents would prefer the menu of schedules to the optimal single non-linear schedule.

If functional form restrictions are imposed on a price schedule, allowing agents to select from a menu of price schedules can yield welfare gains simply because the menu effectively relaxes the functional form restrictions. For example, as Faulhaber and Panzar (1977) acknowledge, the menu of two-part tariffs in their model is equivalent to a single non-linear pricing schedule. Similarly, the menu of tax schedules in Alesina and Weil (1992) is effectively a single non-linear schedule. This equivalence between a menu of schedules and a single non-linear schedule breaks down when agents receive further private information about their type after selecting a schedule from the menu. Such an information structure is employed in Baron and Besanko (1984), Clay et al. (1992), Miravete (1996, 2002, 2005), Courty and Li (2000), and Grubb (2007). These papers differ in the types of screening problems modeled, restrictions imposed on the schedules, and restrictions imposed on type and signal distributions. Clay et al. (1992) compare a menu of two-part tariffs to a single three-part (or tapered) tariff rather than allowing arbitrary functional forms for the tariffs. Miravete (1996, 2002) also limits the menu of schedules to a menu of two-part tariffs. The remaining papers impose no functional form restrictions on the schedules. Their contributions, nicely synthesized in Rochet and Stole (2003), lie in characterizing the design of optimal menus of non-linear schedules.

The theoretical contribution of this paper consists of two new results concerning the welfare properties of a menu of schedules. First, we prove that the gain to the principal of offering a menu of schedules generically is strict and first-order. Second, we show that generically there exists a menu of schedules that yields an interim-by-type efficiency gain over the optimal single non-linear schedule.

The empirical contribution of this paper is to characterize the magnitudes of the gains to the principal and to decompose this gain into a component due to a reallocation of resources to the principal's advantage and a component due to an increase in efficiency. We provide two simulations. One highly-stylized simulation shows how a monopolistic utility company can increase its profits by offering a menu of rate schedules. We demonstrate to what extent this increase in profits comes from an efficiency gain, and to what extent it is driven by the principal's increased ability to extract rents from the agents. The second simulation is more ambitious. It starts from the optimal non-linear income tax schedule for the actual ability distribution in the Unites States. We estimate a signal distribution that people have about their future ability level, and examine the welfare gain from offering a choice of two non-linear tax schedules instead of one. We find that, for our baseline parameters, the choice of tax schedules increases social welfare by the same amount as would occur from a 4.0 percent windfall gain in the government budget (or about $600 per filer per year). This gain in social welfare is over two times as large as the gain in social welfare that comes from going from the optimal linear to the optimal non-linear tax schedule. About two thirds of the increase in social welfare derives from the efficiency gain of offering a menu of tax schedules (the average marginal tax rate falls) while the remainder comes from an increase in redistribution.

2. Formal Results

We start by considering the standard screening problem, which captures the majority of problems described in the literature, though only a fraction of the problems that arise in the real world. Consider a situation where, once an agent contracts with the principal, she receives no further private information, in other words, her signal is perfectly informative. This would occur, for example, if a firm had to hire a salesperson with private knowledge about her capabilities, but those capabilities remained fixed, and the firm could monitor conditions that changed sales prospects, e.g., whether a potential client was recalcitrant. In this case, i.e., where signals are perfectly informative or agents have perfect information about their type, offering a menu of schedules yields no benefit. As Clay et al. (1992) note, if an agent always knew her type with certainty when selecting her schedule, she could always predict with certainty which point she would choose on the incentive schedule. In that case, the principal could simply offer a single incentive schedule consisting of the set of bundles that agents end up choosing on the menu of schedules. The outcome with this single incentive schedule would be identical to the outcome with the menu of schedules.

In two additional cases, offering a menu of incentive schedules offers no benefit beyond an optimal single non-linear schedule. First, if there is no initial signal, or that signal is completely uninformative, a menu of incentive schedules yields no benefit because the agent has no information on which to base her choice of incentive schedule. Second, and far less obvious, there is no benefit to offering a menu of schedules if there are only two types. In a two-type screening model, the model most often considered in the literature, the principal makes a tradeoff between the cost of distorting the behavior of the low type and giving informational rents to the high type.[2] If the fraction of high types increases, the optimal plan raises the distortion to the low types so as to reduce the informational rent paid to the high types. It does so since this distortion is relative cheap (few low types) and informational rent to be paid to high types is relatively expensive (many high types). Thus, if an agent truthfully reports that she has a high signal (raising the probability she is of the high type), the optimal incentive scheme would reduce her informational rent should she prove to be a high type. Similarly, upon a truthful report of a low signal, the optimal incentive scheme would boost the informational rent. But this means that those with a high signal would have an incentive to declare falsely that they received a low signal. A similar argument shows that low types would also have an incentive to declare falsely. In short, it is not possible to get truthful declarations of the signals in a two-type model if the signals will be used to improve social welfare.

Proposition 1 captures these three negative cases for offering a menu of schedules:[3]

Proposition 1: *Offering a choice of schedules can not yield a strict welfare gain over the optimal single incentive schedule if any one of the following three conditions holds: (i) the signal is perfectly informative, (ii) the signal is completely uninformative, or (iii) there are only two types.*

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[2] It is never optimal to distort the behavior of the high type, because this would reduce the surplus that can be extracted from her.

Proof in the appendix.

The remaining results in the paper exclude the conditions in Proposition 1, and assume that agents receive further private information after their contracts with the principal are struck, i.e., that the initial signals are imperfect, and that there are N types, with $N \geq 3$. To provide motivation and intuition, we frame the setup in terms of optimal non-linear income taxation. However, the framework applies immediately to a wide range of screening problems.

2.1 Setup with the Optimal Single Non-Linear Schedule

Consider the standard screening problem, i.e., one where agents receive no signals and the optimal single non-linear schedule is offered. All agents derive utility from consumption, C, and disutility from effort, X, according to utility function $U(C, X) = V(C) - F(X)$. We assume that agents are expected-utility maximizers and that $V(C)$ is concave and increasing in C while $F(X)$ is convex and increasing in X. Let θ denote an agent's type ($\theta = 1, 2, \ldots N$), and let w_θ be the wage or return to effort for that type. Without loss of generality, we assume that w is increasing in θ. The population fraction of each type of agent is denoted by α_θ. The principal, logically thought of as the government in this context, observes income $Y = w_\theta X$, but neither X nor w_θ separately. The principal maximizes an objective function or social welfare function given by $\Sigma_\theta\, \alpha_\theta\, \Phi(U(C_\theta, X_\theta))$. We assume that the social welfare function $\Phi(.)$ is weakly increasing and weakly concave; i.e., the principal wishes to redistribute resources towards the lower types. The principal maximizes the objective function by offering a feasible, incentive-compatible incentive schedule consisting of N bundles $\{C_\theta, Y_\theta\}_{\theta=1..N}$. By the revelation principle, we can limit our analysis to direct mechanisms, those where the principal maximizes his objective function subject to a resource constraint (RC) and the agents' incentive constraints (IC).

(1) $$\underset{\{C_\theta, Y_\theta\}_{\theta=1..N}}{\text{Max}} \sum_{\theta=1}^{N} \alpha_\theta \Phi(U(C_\theta, Y_\theta / w_\theta)),$$

s.t.

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[3] Since the single incentive schedule is a special case of a menu of incentive schedules (namely a menu with two identical items), the menu of schedules can by definition attain at least the social welfare achieved by a single incentive schedule.

$$\sum_{\theta=1}^{N} \alpha_\theta (C_\theta - Y_\theta) \leq 0 \text{ , and} \qquad \text{(RC)}$$

$$U(C_{\theta'}, Y_{\theta'}/w_\theta) - U(C_\theta, Y_\theta/w_\theta) \leq 0 \quad \text{for any } \theta, \theta'. \qquad \text{(IC)}$$

While this sets up the screening problem as one where the principal maximizes his measure of social welfare, this is much less restrictive than it may at first seem. Screening problems in which the principal maximizes profits subject to a participation constraint by the agent are a special case of the dual of the screening problem of a welfare-maximizing principal. To see this, note that the resource constraint is simply the negative of the principal's profits, and that setting the derivative of $\Phi(.)$ to zero for all but the lowest type turns the objective function into the participation constraint for the lowest type (which is the binding participation constraint). Maximizing the utility of the lowest type subject to a zero-profit constraint is merely the dual of maximizing profits subject to a constraint on the utility of the lowest type.

2.2 Setup with Schedule Selection

Now, we extend the standard screening problem described above by letting the agents receive a signal about their type, and letting the principal offer a choice of schedules to the agents after the agents have received their signal but before they have learned their type. In this setup, the agents initially do not know their own type θ; instead they receive a signal σ, where σ is either equal to B (bad) or G (good). The principal cannot observe the signal, and conditional on type the signal does not affect pay-offs. Let the joint probability of being of type θ and receiving signal σ be denoted by $\alpha_{\theta\sigma}$. We assume that the signals are imperfectly informative.

The principal now offers a choice of two incentive schedules, with $\{C_{\theta G}, Y_{\theta G}\}_{\theta=1..N}$ intended for those with a good signal and $\{C_{\theta B}, Y_{\theta B}\}_{\theta=1..N}$ for those whose signal is bad. After receiving her signal but before learning her type, the agent chooses one of the two incentive schedules.[4] Because the principal does not observe the signal, these incentive schedules need to be ex-ante incentive compatible – agents with a good signal must weakly prefer the schedule

[4] We assume that agents are fully rational when they make this choice (i.e., they do not suffer from time-inconsistency nor are they subject to temptation). Miravete (2003) and Narayanan et al. (2007) provide evidence in the context of phone plans that consumers generally choose the best schedule for them. DellaVigna and Malmandier (2006), however, show that when choosing gym memberships, consumers systematically choose contracts that are more expensive ex-post than other available contracts.

intended for them over the schedule intended for those with a bad signal, and vice versa. After learning her type, the agent chooses a point on the incentive schedule she previously selected.

The principal's objective function is $\Sigma_\theta \Sigma_\sigma \alpha_{\theta\sigma} \Phi(U(C_{\theta\sigma}, X_{\theta\sigma}))$. We consider direct mechanisms in which the principal maximizes his objective function subject to a resource constraint (RC), two ex-ante incentive constraints that ensure signals are truthfully revealed, and $2N(N\text{-}1)$ ex-post incentive constraints that ensure truthful revelation of type conditional on the schedule selected.

(3) $$\underset{\{C_{\theta\sigma},Y_{\theta\sigma}\}_{\left(\substack{\theta=1..N\\ \sigma=B,G}\right)}}{\text{Max}} \sum_{\sigma=B,G} \sum_{\theta=1}^{N} \alpha_{\theta\sigma} \Phi(U(C_{\theta\sigma}, Y_{\theta\sigma}/w_\theta))$$

s.t.

$$\sum_{\sigma=B,G} \sum_{\theta=1}^{N} \alpha_{\theta\sigma}(C_{\theta\sigma} - Y_{\theta\sigma}) \leq 0 \qquad \text{(RC)}$$

$$\sum_{\theta=1}^{N} \alpha_{\theta\sigma}\left(U(C_{\theta\sigma'}, Y_{\theta\sigma'}/w_\theta) - U(C_{\theta\sigma}, Y_{\theta\sigma}/w_\theta)\right) \leq 0 \qquad \text{for any } \sigma, \sigma' \qquad \text{(Ex-Ante IC)}$$

$$U(C_{\theta'\sigma}, Y_{\theta'\sigma}/w_\theta) - U(C_{\theta\sigma}, Y_{\theta\sigma}/w_\theta) \leq 0 \qquad \text{for any } \theta, \theta', \sigma. \qquad \text{(Ex-Post IC)}$$

2.3 Results

We express the efficiency gain from offering a menu of schedules in terms of an interim-by-type Pareto gain, which implies a welfare or ex-ante Pareto gain. The interim-by-type Pareto gain, i.e., the expected utility of each type weakly increases, is most relevant in a context, such as optimal taxation, where the government is concerned about redistribution across types.

Proposition 2: *Posit that (i) there are at least three types, (ii) utility is separable, and (iii) the optimal single incentive schedule has the incentive constraints for type θ binding toward type θ-1, is not a corner solution, and does not involve bundling. Then, for generic distributions of signals and types, there exists a menu of schedules that yields a strict first-order, interim-by-type Pareto improvement over the optimal single non-linear incentive schedule.*

Corollary: *The principal will offer a menu of schedules that yields a strict first-order gain to the principal compared to the optimal single non-linear incentive schedule.*

Proofs in the appendix.

To understand the idea behind the proof of Proposition 2, consider the special case of three types, *L* (low), *M* (medium) and *H* (high). We start from the optimal single incentive schedule $\{\hat{C}_\theta, \hat{Y}_\theta\}_{\theta=L,M,H}$, which is the solution to the F.O.C.s of (1), and assume we have a standard interior solution (no bundling, no corner solutions, two binding incentive constraints). In this optimal solution, the effort levels of the low and medium types are distorted downwards.

The optimal single schedule is also a feasible, though generally not optimal, solution to the problem in which the principal can offer a menu of schedules. Namely, the principal could offer a (rather limited) menu consisting of the best single incentive schedule. Starting there, we examine whether we can perturb this degenerate menu of schedules to produce the desired interim-by-type Pareto improvement. We denote these perturbations by lower-case letters while denoting the resulting non-degenerate menu of schedules by upper-case letters: $\{C_{\theta B} = \hat{C}_\theta + c_{\theta B}, Y_{\theta B} = \hat{Y}_\theta + y_{\theta B}\}_{\theta=L,M,H}$ and $\{C_{\theta G} = \hat{C}_\theta + c_{\theta G}, Y_{\theta G} = \hat{Y}_\theta + y_{\theta G}\}_{\theta=L,M,H}$. We then examine perturbations $\{c_{\theta\sigma}, y_{\theta\sigma}\}_{\theta=L,M,H;\ \sigma=B,G}$ such that (i) all the incentive constraints identified in (3) remain satisfied, and (ii) the expected utility for each type remains constant. If such a perturbation relaxes the resource constraint, extra resources are available, and all types can be made better off. A strict interim Pareto improvement is available.

To find the required perturbation, we linearize all the binding incentive constraints of the menu of schedules problem (3) around the optimal single incentive schedule. This imposes six linear constraints on the perturbation (two ex-ante incentive constrains, and two ex-post incentive constraints for each of the two schedules). In addition, we linearize expected utility for each type around the optimal single incentive schedule and hold expected utility constant, which imposes three additional constraints (one for each type). Finally, we do not perturb income for the high type (y_{HB} and y_{HG}) because the first-order conditions of (3) tell us that these should be set at the first-best level, as is well known to be the case for the optimal single incentive schedule. In total, we impose 9 linear restrictions on a 10-dimensional perturbation. Let ε be the one free parameter of the deviation, i.e., the perturbation in each choice variable is a linear function of ε. By substituting the perturbations in the choice variables into the resource constraint, we determine how the perturbation affects the resource constraint, and find that the change in the resource constraint is proportional to ε and that the factor of proportionality is non-zero for generic distributions of $\alpha_{\theta\sigma}$. Because ε enters linearly, the efficiency gains are first order. Since the change in the resource constraint depends on a first-order approximation of the effects of a perturbation in the optimal single incentive schedule, this expression only holds for small values of ε. For larger values, second-order effects could become important, and could begin to offset the first-order gains.

2.4 Intuition for the Three-Type Case with Signals Satisfying the Monotone Likelihood Property

The intuition is most easily understood when the signals satisfy the monotone likelihood property, though schedule selection can also offer welfare gains in cases where signals fail to satisfy this property. When signals satisfy the monotone likelihood property, they change the relative likelihoods of the three types: a bad signal increases the likelihood of the low type and reduces the likelihood of the high type, whereas a good signal does the opposite. The effect of the signal of the likelihood of a medium type is ambiguous. We know from the screening literature that the optimal incentive schedule depends on the relative fractions of the types. The optimal distortion to any given type decreases as its fraction in the population increases, because a distortion's efficiency cost is proportional to the numbers of those it is imposed upon. The optimal distortion increases with the proportion of higher types, because increasing the distortion reduces the informational rents paid to higher types, and these informational rents are proportional to the higher types' fraction of the population. Thus, if the population fraction or the likelihood of a type increases, it is optimal to both reduce the distortion to and the redistribution to this type.

Ideally, the incentive schedule conditional on a signal would reduce both the distortion and the redistribution to the types that become more likely conditional on that signal. Thus, efficiency considerations push towards less redistribution and less distortion to the more likely types. However, when signals are private this is not incentive-compatible because agents with a signal that makes a given type more likely would not choose a schedule that reduces redistribution to that type. Thus, the incentive constraint that a signal be truthfully revealed pushes in the opposite direction, namely in the direction of more redistribution towards types that become more likely conditional on the signal. With just two types, high and low, these contradictory forces cannot be resolved because only one type (low) is distorted. As a result, offering a menu of schedules provides no benefit if there are only two types, as we proved in Proposition 1.

Once three or more types are involved, the picture changes. At least two types get distorted and these opposing efficiency and incentive forces no longer cancel out in general. With two or more types distorted, the perturbation conditional on the signal can decrease the distortion to one type, thereby raising efficiency and tightening the ex-ante incentive constraint, but increasing the distortion to the other type, which lowers efficiency but simultaneously relaxes the ex-ante incentive constraint. Only for highly specific parameter values, will the effect of a distortion on efficiency relative to its effect on the ex-ante incentive constraint be exactly the same for each type. Apart from that knife-edge case, one can increase efficiency by reducing the distortion that is relatively costly in terms of efficiency but has relatively little effect on the incentive constraint, and vice versa.

This intuition explains why, as the proof in the appendix shows, the perturbation always moves y_{LB} and y_{MG} in one direction but y_{LG} and y_{MB} in the other. For example, if, conditional on a bad signal, the distortion to the low type is increased, then the distortion to the medium type will be decreased. Moreover, one can show that the perturbation always moves the utility of the high and the low types in the opposite direction of the utility of the medium type. This non-monotonic effect results from reducing redistribution where the efficiency costs of redistribution are highest, and increasing redistribution where it most strongly relaxes the ex-ante incentive constraint.

3. Application I: Monopolistic Utility Company Offering a Menu of Price Plans

We now present a highly stylized example to illustrate how giving agents a choice of schedules can benefit the principal. In this example, the principal is a profit-maximizing firm, there are just three types of agents, and parameter values are chosen to enhance exposition. Section 4 presents a more empirically-based example in which the principal is a social-welfare maximizing government, the agents come from a continuum of types, and parameter values are calibrated to reality.

Consider a monopolistic utility company that produces an output X at a constant marginal cost of β. The company, which maximizes profits, sells to consumers, who differ in how much they value the good. Their valuation is determined by the taste parameter, w_θ, which can take on three values $0 < w_L < w_M < w_H$ depending on whether the agent's type θ is L (low), M (medium) or H (high). The population fraction of each type of agent is denoted by α_θ. Consumer surplus is given by $U_\theta(X,P) = w_\theta \sqrt{X} - P$. Since there are just three types of consumers, the price plan consists of three points in (X, P) space. The company finds the price plan that maximizes profits:

(4) $$\underset{\{X_\theta, P_\theta\}_{\theta=L,M,H}}{\text{Max}} \sum_{\theta=L,M,H} \alpha_\theta (P_\theta - \beta X_\theta) \text{ - (\textit{fixed cost})},$$

s.t.

$U_\theta(X_\theta, P_\theta) \geq 0$ for any θ, and (IR)

$U_\theta(X_\theta, P_\theta) - U_\theta(X_{\theta'}, P_{\theta'}) \geq 0$ for any θ, θ'. (IC)

Thus, in setting its price plan, the company needs to ensure that each type's individual rationality constraint is satisfied (consumer surplus cannot be negative) and that all incentive constraints are satisfied (each type of consumer chooses the point on the price schedule intended for her).

Next, we extend the standard screening problem described above by letting agents receive a signal about their type and letting the company offer a menu of schedules to the agents once the agents have received their signal, but before they have learned their type. That is, agents receive a private signal σ, where σ takes on one of two values B (bad) or G (good). Let the joint probability of receiving signal σ and being of type θ be denoted by $\alpha_{\theta\sigma}$. In this application, we assume that the signals are informative and, more specifically, that they satisfy the monotone likelihood property: the probability of receiving a good signal increases strictly with type: $\alpha_{HG} / \alpha_{HB} > \alpha_{MG} / \alpha_{MB} > \alpha_{LG} / \alpha_{LB}$. In the context of a utility company, one can think of the agent's type as being independently re-drawn every month from a given distribution. The distribution from which the type is drawn is determined by the signal. Thus, consumers with a bad signal – low users on average – draw their type each month from a distribution that is stochastically dominated by the distribution from which consumers with a good signal draw their types.

The utility company now offers a choice of two price plans, with $\{X_{\theta G}, P_{\theta G}\}_{\theta=L,M,H}$ intended for those with a good signal, and $\{X_{\theta B}, P_{\theta B}\}_{\theta=L,M,H}$ for those with a bad signal. After receiving her signal but before learning her type, the consumer chooses one of the two price plans. One can think of this as a one-time decision. Because the principal can never observe the signal, these price plans need to be ex-ante incentive compatible – consumers with a good signal must prefer the plan intended for them ex-ante over the plan intended for those with a bad signal, and vice versa. Each month, upon learning the draw of her type for that month, the consumer chooses how much output to consume – that is, she chooses a point on her previously selected price plan.

The utility company now maximizes profits by offering a menu with two price plans:

(5) $$\underset{\{X_{\theta\sigma}, P_{\theta\sigma}\}\left(\substack{\theta=L,M,H \\ \sigma=B,G}\right)}{\text{Max}} \sum_{\sigma=B,G} \sum_{\theta=L,M,H} \alpha_{\theta\sigma}(P_{\theta\sigma} - \beta X_{\theta\sigma}) \text{ - } (\textit{fixed cost}),$$

s.t.

$U_\theta(X_{\theta\sigma}, P_{\theta\sigma}) \geq 0$ for any θ, σ, (IR)

$\sum_{\theta=L,M,H} \alpha_{\theta\sigma}[U_\theta(X_{\theta\sigma}, P_{\theta\sigma}) - U_\theta(X_{\theta\sigma'}, P_{\theta\sigma'})] \geq 0$ for any σ, σ', and (Ex-Ante IC)

$U_\theta(X_{\theta\sigma}, P_{\theta\sigma}) - U_\theta(X_{\theta'\sigma}, P_{\theta'\sigma}) \geq 0$ for any θ, θ', σ. (Ex-Post IC)

We calculated the optimal single price plan and the optimal menu of price plans for a marginal cost (β) equal to 1 and taste parameters w_L =12, w_M =15, w_H = 20. Given this marginal cost and these taste parameters, the first-best consumption levels are 36, 56.25 and 100. The joint distribution of signals and types is chosen as:

(6) $$\begin{pmatrix} a_{LB} & a_{LG} \\ a_{MB} & a_{MG} \\ a_{HB} & a_{HG} \end{pmatrix} = \begin{pmatrix} 9/30 & 1/30 \\ 5/30 & 5/30 \\ 1/30 & 9/30 \end{pmatrix}.$$

In other words, the unconditional probability of each signal is one half and the unconditional probability of each type is one third. However, conditional on a good signal, the consumer is nine times more likely to be of the high type than of the low type, and vice versa for a bad signal.

The circles in Figure 2 show the choices for the three types under the optimal single non-linear price plan. That plan produces three quantity-price combinations: 9 units for a price of 36 (chosen by the low type), 25 units for a price of 66 (chosen by the medium type) and 100 units for a total price of 166 (chosen by the high type). The figure also plots the indifference curves for the three types. These curves show that the high type is indifferent between the bundle of the high type and the one for the medium type. In other words, the incentive constraint for the high type binds. Similarly, the incentive constraint binds for the medium type. Finally, the low type is indifferent between its bundle and not purchasing at all, showing that the individual rationality constraint binds for the low type.

Panel A of Table 1 shows quantity sold, revenue, consumer surplus, producer surplus and social surplus associated with the optimal single non-linear price plan. The utility company maximizes profits by distorting the consumption of low and medium types downwards, but setting the consumption level for the high type at its first-best level. This is the standard result in this type of screening model – the distortion to the lower types allows the principal to charge a higher price from the high type because the distortion makes it less attractive for the high type to select a quantity-price combination intended for a lower type. We find that the consumption level for the medium type is distorted strongly downwards from a first-best level of 56.25 to 25, which is the same distortion as would be caused by a 50% tax on the consumption good. The distortion to the low type is even more severe. Its consumption is distorted downwards by a factor of 4 (from 36 to 9), which is the same distortion as would be caused by a 100% tax on the consumption good.

The optimal menu of non-linear price plans consists of two plans: one plan is preferred by those who received a bad signal (σ=*B*), and the other is preferred by those who received a good signal (σ=*G*). The triangles in Figure 2 show the price-quantity combinations for the plan chosen by those with a good signal while the diamonds show the plan chosen by those with the bad signal. Panel B of Table 1 shows the outcomes associated with this menu of price plans. The consumption level for the high type in the menu of plans remains at the first-best level, as was also the case in the single price plan. Compared to the optimal single price plan, however, the price plan chosen by those with a good signal increases the distortion to the low type so as to extract more rent from and reduce the distortion to the medium type. Since relatively few individuals receiving a good signal are of the low type, the efficiency cost of the increased distortion to the low type is relatively low, while the efficiency gain that comes from reducing the distortion to the medium type is relatively large. In contrast, the plan chosen by those getting a bad signal curtails the distortion of the low type, but increases the distortion to the medium type. The increased distortion to the medium type allows the monopolist to reduce the surplus going to the high type, which ensures that this plan is unattractive to those with a good signal (among whom the fraction of high types will be high). Indeed, this consideration is important as those with a good signal are indifferent between the price plan for those with a good signal and the price plan for those with a bad signal (see row 5 of panel B).

In this example, the menu of plans allows the utility company to increase its profit as a fraction of sales by about one percentage point. Given that profits are typically around 5%-10% of sales, this boosts profits by roughly 10% to 20%. This increase in profits is made possible by two factors. First, the menu of price plans gives the utility company an additional mechanism for extracting surplus from consumers. Since the utility company has no concern for efficiency per se, it will still extract additional surplus even if the efficiency cost of doing so is extremely high. Second, for a given amount of surplus going to consumers, the menu of price plans allows the utility company to make the price plans more efficient. Table 2 disentangles these two effects for five different pricing mechanisms. These pricing mechanisms include two infeasible ones, namely first-best (requires that types be observable) and non-linear pricing conditional on signal (requires that signals be observable). These two infeasible pricing mechanisms provide a theoretical benchmark. The three feasible schedules presented are the menu of non-linear price plans, the single non-linear price plan, and the single linear price plan. The two non-linear price plans are the focus of this paper, while the linear price plan, which is a two-part tariff consisting of a base price plus a price per unit of consumption, serves as a comparison.

The first three columns show consumer surplus, profit and deadweight loss produced by these five pricing mechanisms when there is no constraint on consumer surplus. Not surprisingly, profits decrease monotonically as we move down these five pricing mechanisms, because, as we move down the list, successively more constraints are imposed on the pricing mechanisms (in order, we add: type is unobservable, signal is unobservable, only one plan can be offered, the plan must be linear).

As pricing mechanisms become more constrained, two tendencies emerge: First, consumer surplus tends to rise, which makes sense given that a more constrained schedule makes it harder for the utility company to extract surplus from the consumers. Second, the deadweight loss tends to rise. These tendencies are reflected in the table, with two exceptions. First, as we move from the menu of non-linear price plans to the single non-linear price plan, the deadweight loss falls. Apparently, constraining the pricing mechanism to a single plan eliminates some very distortionary ways of extracting surplus from consumers, leading to an overall drop in deadweight loss. Second, the linear price plan lowers consumer surplus compared to the single non-linear plan.

To disentangle the inherent efficiency effects of the pricing mechanisms from the effects caused by a change in the amount of surplus extracted from consumers, the last three columns show the effect of the pricing mechanism on profits and DWL when consumer surplus is constrained to remain at the level associated with the single non-linear plan. Such constraint might be put in place, for example, were a regulator considering letting the monopolist move to the choice-of-plans format. Here we find that the menu of non-linear prices indeed allows the company to raise its profits without reducing consumer surplus. In other words, the menu of non-linear price plans is also inherently more efficient, even though unimpeded the company will squander these gains in the service of higher profits.

4. Application II: Menu of Income Tax Schedules

Whether the metric be monies paid, people covered, or deadweight loss imposed, the tax-and-transfer system is probably the preeminent schedule for individuals in the United States and in most developed nations. It has also received considerable attention from economists. Thus we select it for our second application, and assess the welfare gains that could be reaped if the government offered people a menu of tax-and-transfer schedules before they earned income in a period. We define income taxes broadly, including means-tested transfers that in effect are negative income taxes.

Following Mirrlees (1971), we assess the design of the optimal income tax as a screening problem: the government maximizes social welfare by redistributing income across agents whose differing ability (or skill) levels are private information. There is a continuum of abilities or types. We follow Mirrlees with one major exception. His analysis was static: agents' ability levels were known to them at the outset and stayed constant. In contrast, we specifically assume that agents secure additional private information about their ability after they select a tax schedule. We employ the term “later learning” for such situations. Golosov, Kocherlakota, Tsyvinski (2003), Kocherlakota (2005), Albanesi and Sleet (2006), Battaglini and Coate (2006), and Golosov and Tsyvinski (2006) have expanded Mirrlees’ static setup in a related direction; they allow unobserved ability to vary over time. Thus, they too allow for later learning, but in their setup ability actually changes whereas in our setup ability does not change; individuals merely learn more about their ability over time.

In either later-learning situation, offering a menu of schedules can be beneficial if individuals have some, but imperfect, private information about their future ability. In the framework with time-varying abilities, the government could allow individuals to repeatedly choose a tax schedule (say for the coming year or the coming 5 years). In the static later-learning situation, where ability is constant but only learned over time, the choice of tax schedules would be offered once, namely at a point in time when the agent has some but not perfect information about her ability.

In this analysis, we follow Mirrlees’ static setup of the optimal income tax problem and augment it with a one-time choice of tax schedules for a group of individuals who are initially indistinguishable to the government, or amongst whom the government is not permitted to distinguish. Using U.S. data, we calculate the welfare gains that would accrue were a menu of schedules offered, as opposed to a single schedule for all. To compute the welfare gains reaped, we first need to find the optimal schedule for the base case, namely the optimal single non-linear income tax schedule for the U.S. To do this, we follow the general approach of Saez (2001). Thus, we use the actual distribution of wage earnings of joint-filers in the 1999 public-use tax file and assume, following Saez, that the upper right tail of the earnings distribution is a Pareto distribution with parameter 2.[5] The distribution of earnings of couples with labor earnings is depicted in Figure 3.

We use this earnings distribution to derive the distribution of unobserved ability levels. The unobserved ability level is the pre-tax wage when wages are measured as compensation per unit of effort (not necessarily per hour). We use a standard iso-elastic labor supply function with

[5] The tax file does not separate out the earnings of each individual of couples who file jointly. Rather than attributing earnings across individuals, we will treat couples as one unit with a single utility function. We exclude the 15% of couples without substantive (>$1000/yr) labor income. The vast majority of these are retired couples, and they are beyond the scope of the optimal income tax model.

a labor supply elasticity of ε: $X = w^{\varepsilon}$, where X denotes effort or labor supply and w denotes ability or the pre-tax wage. Following Saez, we approximate the current tax system as one with a constant marginal rate of 40%. We can now infer the pre-tax wage from observed earnings, since we know that total pre-tax earnings equal the pre-tax wage times the labor supply as a function of the after-tax wage: $Y = w\,X((1-\tau)w) = (1-\tau)^{\varepsilon}\, w^{\varepsilon+1}$, where Y denotes pre-tax earnings and τ denotes the marginal tax rate. This relationship between Y and w allows us to infer the distribution of unobserved ability from the distribution of earnings. The inferred cumulative distribution of ability levels is denoted by $G(w)$, while the corresponding density function is denoted by $g(w)$.

We use a quasi-linear utility function $U(C, X) = C - \frac{\varepsilon}{\varepsilon+1} X^{1+1/\varepsilon}$. We then solve the standard Mirrlees optimal tax problem, in which the government designs a non-linear tax schedule. That schedule is implicitly defined by relating consumption and income, $C(w)$ and $Y(w)$, as a function of ability. The government sets the tax schedule to maximize social welfare subject to the resource constraint (RC) and an incentive constraint (IC):

$$\underset{\{C(w),Y(w)\}}{\text{Max}} \int \Phi\big(U(C(w),Y(w)/w)\big)g(w)dw\,,$$

s.t.

$$\int \big(C(w)+E-Y(w)\big)g(w)dw \le 0\text{, and} \qquad \text{(RC)}$$

$$\frac{U_X(C(w),Y(w)/w)}{U_C(C(w),Y(w)/w)} + w\frac{C'(w)}{Y'(w)} = 0 \quad \forall w\,, \qquad \text{(IC)}$$

where $\Phi(.)$ is the social welfare function and E is government consumption. In the screening problem with continuous types, the incentive constraint becomes the condition that the after-tax tax wage rate ($w\,C'/Y'$) equals the marginal disutility of labor (U_X/U_C); that is, agents locally have no incentive to deviate from choosing the bundle intended for them. We follow Saez (2001) by setting government consumption, E, equal to 25% of pre-tax income. Since the government budget is balanced, net tax revenue is also equal to 25% of pre-tax income. We posit a logarithmic social welfare function.

We solve this problem using the dynamic programming approach developed by Mirrlees (1971), and extended by Saez (2001). This yields a system of two differential equations, which we solve numerically. We compute the optimal single non-linear income tax schedule to provide a basis of comparison for our menu-of-schedules approach. Hence, we will present it below, after we first calculate the menu of optimal non-linear tax schedules.

The cornerstone of our approach is that individuals get an imperfect signal on their ability. We take a person's decision whether or not to attend college as a proxy for the signal the person received about his or her ability. We can then calculate the fraction of individuals at each earnings level that went to college, and take that as the fraction of people at that earnings level that received a good signal. Because the optimal tax schedules are calculated for joint filers, we define a good signal as both individuals in the couple having at least some college education. Because people probably know more about their likely ability than is reflected in their choice of going to college, we consider this proxy to be a lower bound on the informativeness of the signal.

We use the 5% Public Use Micro Sample from the 2000 Census to estimate the probability of having received a good signal as a function of position in the cumulative earnings distribution. The circles on Figure 4 show these probabilities by percentile in the earnings distribution using data from 2.3 million couples in the 2000 Census. We fit a 3rd order polynomial (solid line) through these probabilities, and will use this polynomial as the "weakly informative" signal distribution in the simulations. Under this signal distribution, the lowest earner (lowest ability type) has an 18% chance of receiving a good signal, the highest earner has an 87% chance of a good signal, while the unconditional probability of a good signal is 45%. The precision of the signal distribution, defined as the probability that the signal correctly predicts the person to be in the top 45% of the earnings distribution, is 63%. This 63% should be compared to a precision of 50% for a completely random signal and a precision of 100% for a completely informative signal. In short, this signal distribution is only weakly informative about future earnings. Because we believe this "weakly informative" signal distribution may underestimate the precision of the true signal distribution, we also use two more informative signal distributions, also plotted in Figure 4. The "medium informative" signal distribution is formed by keeping the unconditional probability of a good signal at 45% but increasing the signal's average precision by 10% (from 63% to 69%) and by letting the signals at the extremes of the ability distribution have a precision of 95% (rather than about 85% in the weakly informative distribution). Finally, the "strongly informative" signal distribution is a symmetric distribution where the probability of a good signal is 50% and where the precision at the extremes is 99%.

Using Bayes' rule to combine the earnings distribution from administrative tax data (Figure 3) with the probability of a good signal conditional on position in the earnings distribution (Figure 4), we find the earnings distributions conditional on the signal received. Figure 5 plots these distributions for the medium informative signal distribution. As before, given a labor supply function, the underlying ability distribution can be inferred from the earnings distribution. This yields the underlying ability distributions conditional on the signal received: $g(w|\sigma)$, where σ equals G for a good signal and B otherwise.

We now adapt the standard Mirrlees problem to allow for a choice of tax schedules so as to assess the improvement. The government now offers two schedules: $\{C(w, G), Y(w, G)\}$ for those who announce a good signal and $\{C(w, B), Y(w, B)\}$ for those who announce a bad signal. Formally, the government solves the following maximization:

$$\underset{\substack{\{C(w,B),Y(w,B),\\ C(w,G),Y(w,G)\}}}{\text{Max}} \sum_{\sigma} \alpha_{\sigma} \int_{w} \Phi\big(U(C(w,\sigma),Y(w,\sigma)/w)\big) g(w|\sigma)dw$$

s.t.

$$\sum_{\sigma} \alpha_{\sigma} \int_{w} \big(C(w,\sigma)-Y(w,\sigma)\big) g(w|\sigma)dw + E \le 0 \qquad \text{(RC)}$$

$$\frac{U_X(C(w,\sigma),Y(w,\sigma)/w)}{U_C(C(w,\sigma),Y(w,\sigma)/w)} + w\frac{C'(w,\sigma)}{Y'(w,\sigma)} = 0 \quad \forall w,\sigma, \qquad \text{(Ex-Post IC)}$$

$$\int_{w} \big(U(C(w,\sigma'),C(w,\sigma')) - U(C(w,\sigma),C(w,\sigma))\big) g(w|\sigma)dw \le 0 \quad \forall \sigma,\sigma' \qquad \text{(Ex-Ante IC)}$$

The resource constraint (RC) and the ex-post incentive constraint are the same as in the standard Mirrlees problem, except that the RC is summed over signals, and the ex-post incentive constraints are conditional on the signal. In addition, there are now two ex-ante incentive constraints that ensure that the signal is truthfully revealed. Once again we invoke the dynamic programming techniques employed in the standard Mirrlees problem. This yields a system of two differential equations for each signal, but these systems of differential equations are interlinked and different from the one in the standard Mirrlees problem because of the ex-ante incentive constraints. The systems of differential equations are solved numerically.

We now turn to the comparison we sought, namely between the optimal single tax schedule and the pair of optimal tax schedules contingent on the announced signal. Figure 6 shows the total tax paid as a function of earnings for the three situations, where labor supply elasticity is assumed to equal 0.5 and the signal-dependent schedules are based on the medium informative signal distribution. We express the total tax as a fraction of the consumption level under the optimal single schedule.[6] The optimal single schedule has a transfer level of \$13,467 per year at no income, the optimal schedule conditional on a bad signal starts at \$15,395 and the optimal schedule conditional on a good signal starts at \$10,235. Thus, as the figure shows, the tax schedule for those with a bad signal starts off with a relatively high level of transfers and relatively low marginal rates, but around \$50,000 of earnings the marginal tax rate starts to rise relatively steeply, and beyond \$100,000 of earnings those with a low signal pay more tax than those with a high signal. This excess tax above \$100,000 is what prevents those receiving a good signal from pretending they received a bad signal.

Figure 7 shows more clearly that the marginal tax rates for those with a bad signal are lower than the marginal tax rates of the single schedule when earnings are low, but become very high at high levels of earnings. In contrast, the marginal tax rates for those with a good signal are lower than the marginal tax rates of the single schedule when earnings are high. Thus, effectively, those with a high signal purchase low marginal tax rates in their likely earnings range. The price to them is a low transfer level and high initial marginal tax rates. For either signal, the menu of tax schedules offers relatively low marginal rates for the earnings range where the agent is most likely to be given her signal. This feature drives the efficiency gains that come from offering a menu of schedules. These relationships between marginal and average rates over various ranges are the norm with schedule selection applied to optimal income taxation.

Figure 8 plots the marginal tax rate as a function of an agent's percentile in the unconditional ability distribution. It also plots the expected marginal tax rate conditional on the

[6] We need to divide the total tax by consumption or income in order to create a graph that is easily comprehensible. Dividing by income results in average tax rates going to minus infinity at low income levels. Because own consumption differs by schedule, dividing by own consumption is not desirable. Hence, we divide all three schedules by the consumption level that the agent would have received under the single schedule.

agent's type (computed by taking a weighted average of the two schedules)[7]. This shows that, for each level of unobserved ability, the expected marginal tax rate is lower for the menu of schedules than it is for the optimal single schedule. This reduction in the expected marginal tax rate generates the efficiency gain secured from offering a choice of tax schedules.

Figure 9 shows the redistributive effects of a menu of schedules. It plots the log difference in utility at each ability level between the single schedule and each of the signal-dependent schedules. Because the social welfare function is logarithmic, the average across signals of the log change in utility also represents the contribution to social welfare of offering a choice of tax schedules. The figure shows that those with low ability levels gain on average, as do those with the highest ability levels. However, the social welfare of the upper middle class is lower with menu of tax schedules than with the single schedule.

Table 3 shows the effect of offering a menu of tax schedules on behavior, characteristics of the tax schedule, welfare, and efficiency for our baseline parameter assumptions (medium informative signal distribution and a labor supply elasticity of 0.5). In panel A, the comparison is made holding net government revenue (= government expenditure) constant. This is the relevant comparison when evaluating the effects of introducing a menu of schedules. We find that offering a menu of schedules reduces the marginal tax by 3.8 percentage points on average, which produces increased average earnings and consumption. This reduction in average marginal tax rates reduces DWL by 12.1 percent, which is equivalent to $440 per tax filer or 2.8 percent of government revenue. The menu of schedules not only increases efficiency, it also allows the government to redistribute more towards lower ability types, as was evident in Figure 9. The combined effect of the efficiency gain and the increased redistribution is to increase social welfare by 2.4%. Panel B provides an alternative metric for this welfare gain. It asks how much more revenue can the government raise when it offers a menu of schedules compared to the optimal single schedule if it holds social welfare constant. The answer to this question is $636 per filer per year, or about 4.0 percent of tax revenue. In other words, offering a menu of schedules yields the same gain in social welfare as would a 4.0 percent windfall gain in government revenue.

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[7] The weighted average is calculated as the square root of the weighted mean of the squared tax rates associated with each type, where the weights are the fractions of each type at that level of ability. We do this because deadweight loss is proportional to the square of the tax rate.

In Table 4, we compare the optimal single non-linear schedule to three alternative schedules. Panel A shows that the welfare gain of offering a menu of taxes would rise to 4.5 percent if the government could observe the signal. Since the welfare gain is only 2.3 percent when the signal is unobservable, roughly half of the potential welfare gain of offering a menu of schedules is foregone because of the need to satisfy the ex-ante incentive constraints. The government can observe the signal when it consists of having attained some college education. However, by treating the signal as unobservable, we ensure that offering a menu of tax schedules will not affect the individuals' incentives to go to college. It thus makes sense to treat observable signals that are reflections of endogenous decisions as unobservable to the government. A number of recent papers have examined the benefits of making the tax schedule dependent on exogenous characteristics such as age, height, and gender (Kremer 2001, Alesina et al. 2007, Mankiw and Weinzierl 2007, and Weinzierl 2007). Mankiw and Wienzierl note that the idea of conditioning tax schedules based on exogenous characteristics such as height generates considerable opposition. These authors explore a number of reasons for this opposition, but much such opposition would vanish if the people of different heights *chose* to be on different tax schedules rather than were forced to be on a different schedule. Offering a choice of schedules would yield a smaller welfare gain than forcing people onto different schedules, but this reduction in the size of the welfare gain may be acceptable if it makes it politically feasible to use multiple schedules.

We saw in Table 3 that the expected utility of those with a good signal is lower under a menu of schedules than under the optimal single schedule. Thus, at the point when agents know their signal but not their type, all those with a good signal would oppose moving from the optimal single schedule to a choice of schedules. If the support of those with a good signal is needed for political feasibility, the government needs to offer a menu of schedules that offers an interim-by-signal Pareto improvement. That is, conditional on her signal (but before learning her type), each agent must expect to achieve a higher level of utility when given the menu of schedules. Panel B of Table 4 shows that this additional restriction on the choice of tax schedules would reduce the welfare gain of offering a choice of schedules from 2.3% to 1.9%.

To place the welfare gain from offering a choice in tax schedules in perspective, we also calculated the welfare gain of going from the optimal linear schedule to the optimal non-linear schedule. Panel C of Table 4 shows that this welfare gain to be 1.0 percent, implying that the welfare gain from offering a menu of tax schedules is about twice as large as the gain from offering a single non-linear rather than a single linear schedule.

Table 5 explores the sensitivity of the welfare and efficiency gains to assumptions about the labor supply elasticity and the informativeness of the signal distribution. Row 5 reproduces the results for our baseline assumptions. Offering a menu of schedules leads to a welfare gain of 2.4 percent, which is the same welfare gain that would be reaped were there a windfall gain to the government budget of 4.0 percent or $636 per filer. The efficiency gains from offering a menu of schedules are equivalent to 2.8 percent of the government budget or $440 per filer, and thus account for more than half of the welfare gain. Increased redistribution accounts for the remainder of the welfare gain. Rows 4 and 6 show that the welfare gains are quite sensitive to the informativeness of the signal distribution. The welfare and efficiency gains are about one fifth as great for the weakly informative signal distribution as for the medium informative signal distribution, while the strongly informative signal distribution yields welfare and efficiency gains that are about 30 percent larger than in the base case. In contrast, the results are relatively insensitive to assumptions about the labor supply elasticity. The welfare and efficiency gains in absolute dollars are about 30 percent lower when the labor supply elasticity is 0.2 and about 30 percent higher when the labor supply elasticity is 1.0. These effects are much less than proportional to the labor supply elasticity because the optimal tax rate also decreases as the labor supply elasticity increases. Overall, the table shows that even under relatively conservative assumptions about the labor supply elasticity and the informativeness of the signal distribution, offering a menu of schedules will still yield worthwhile welfare and efficiency gains.

5. Conclusion

In a wide variety of settings, principals let agents select their incentive schedule from a menu of schedules. The advantages are diverse. It may be an alternative way of presenting a single complicated non-linear schedule (Alesina and Weil, 1992). The principal may use a menu of schedules to take advantage of agents' time-consistency problems or other behavioral anomalies (Esteban and Miyagawa 2005, Eliaz and Spiegler 2006, 2007, or Grubb 2007). We explore a quite different rationale for offering a menu of schedules in a setting where agents are fully rational: Where agents acquire more information about their type over time, offering the agent a choice of schedules at the point where the agent's private information about his own type is imperfect will generically be beneficial to the principal.

We make two main theoretical contributions beyond the existing literature that compares menus of schedules to the optimal single non-linear schedule in settings with fully rational agents (Baron and Besanko 1984, Miravete 2005, and Courty and Li 2000). The first contribution is our result that, generically, the benefit to the principal of offering a menu of schedules is strict and first-order. Schedule selection benefits the principal because it allows the principal to screen the types more efficiently. Our second contribution is our result that this efficiency gain can, in principle, be used to achieve a first-order interim-by-type Pareto improvement. Efficiency gains emerge because the menu of schedules induces each type to select the schedule that contains relatively few distortions to the action that that type is likely to take. In addition, a menu of schedules benefits the principal because it gives him a more flexible tool to redistribute rents to suit his objectives, namely more profits if the principal is a profit-maximizing company, or more income redistribution if the principal is a government maximizing a redistributive social welfare function. Generally, this redistribution of rents means that the menu of schedules that the principal selects will not Pareto dominate the optimal single incentive schedule.

Our empirical contribution is to empirically assess the possible benefits of schedule selection in a tax setting. To do so, we compared the optimal non-linear income tax schedule for the U.S. to the optimal menu of two non-linear tax schedules. For our baseline parameter choices, we found that offering a choice of tax-schedules would boost social welfare by 2.4%. To put this welfare gain in perspective, it is equivalent to the welfare gains under the optimal single schedule of a windfall gain to the government budget of 4.0 percent or $636 per filer. To provide another metric, the welfare gain from offering a menu of schedules is more than twice as large as the welfare gain from going from the optimal linear to the optimal non-linear tax schedule, a topic that has received considerable attention in the economics literature. Our simulation of the welfare gain admittedly relies on many assumptions. Nevertheless, we believe that it shows that schedule selection has the potential to yield significant welfare gains.

Three decades ago, nonlinear schedules started to receive considerable research attention, and research findings and practical applications proliferated side by side. Schedule selection, an equivalently promising technology, has had limited but important real world applications. If it is to reach its potential, it now merits much greater attention in both research and practice.

References

Albanesi, Stefania and Christopher Sleet (2006), "Dynamic Optimal Taxation with Private Information," *Review of Economic Studies*, 73(1), pp. 1-30.

Alesina, Alberto, Andrea Ichino, and Loukas Karabarbounis (2007), "Gender Based Taxation and the Organization of the Family," NBER working paper no. 13638.

Alesina, Alberto and Philippe Weil (1992), "Menus of Linear Income Tax Schedules," NBER working paper no. 3968.

Baron, David P. and David Besanko (1984), "Regulation and Information in a Continuing Relationship," *Information Economics and Policy*, 1(3), pp. 267-302.

Battaglini, Marco and Stephen Coate (2006), "Pareto Efficient Income Taxation with Stochastic Abilities," *Journal of Public Economics*, forthcoming.

Clay, Karen B., David S. Sibley, and Padmanabhan Srinagesh (1992), "Ex Post vs. Ex Ante Pricing: Optional Calling Plans and Tapered Tariffs," *Journal of Regulatory Economics*, 4(2), pp. 115-138.

Courty, Pascal and Hao Li (2000), "Sequential Screening," *Review of Economic Studies*, 67(4), pp. 697-717.

DellaVigna, Stefano and Ulrike Malmendier (2006), "Paying Not to Go to the Gym," *American Economic Review*, 96(3), pp. 694-719.

Eliaz, Kfir and Ran Spiegler (2006), "Contracting with Diversely Naive Agents," *Review of Economic Studies*, 73(3), pp. 689-714.

Eliaz, Kfir and Ran Spiegler (2007), "Speculative Contracts," Unpublished Manuscript, New York University.

Faulhaber, Gerald R. and John C. Panzar. (1977), "Optimal Two-Part Tariffs with Self-Selection," Bell Laboratories Economics Discussion Paper No. 74.

Esteban, Susanna and Eiichi Miyagawa (2005), "Optimal Menu of Menus with Self-Control Preferences," Unpublished manuscript, Pennsylvania State University.

Golosov, Mikhail, Narayana Kocherlakota, and Aleh Tsyvinski (2003), "Optimal Indirect and Capital Taxation," *Review of Economic Studies*, 70, pp. 569-587.

Golosov, Mikhail and Aleh Tsyvinski (2006), "Designing Optimal Disability Insurance: A Case for Asset Testing," *Journal of Political Economy*, 114(2), pp. 257-279.

Grubb, Michael D. (2007), "Selling to Overconfident Consumers," Unpublished manuscript, MIT.

Kocherlakota, Narayana (2005), "Zero Expected Wealth Taxes: A Mirrlees Approach to Dynamic Optimal Taxation," *Econometrica*, 73(5), pp. 1587-1621.

Kremer, Michael (2001), "Should Taxes Be Independent of Age?" Unpublished manuscript, Harvard.

Mankiw, N. Gregory and Matthew Weinzierl (2007), "The Optimal Taxation Of Height: A Case Study Of Utilitarian Income Redistribution," Unpublished manuscript, Harvard.

Miravete, Eugenio J. (1996), "Screening Consumers through Alternative Pricing Mechanisms," *Journal of Regulatory Economics*, 9(2), pp. 111-132.

Miravete, Eugenio J. (2002), "Estimating Demand for Local Telephone Service with Asymmetric Information and Optional Calling Plans," *Review of Economic Studies*, 69(4), pp. 943-71.

Miravete, Eugenio J. (2003), "Choosing the Wrong Calling Plan? Ignorance and Learning," *American Economic Review*, 93(1), pp. 297-310.

Miravete, Eugenio J. (2005), "The Welfare Performance Of Sequential Pricing Mechanisms," *International Economic Review*, 46(4), pp. 1321-1360.

Mirrlees, James A. (1971), "An Exploration in the Theory of Optimum Income Taxation," *Review of Economic Studies*, 38(114), pp. 175-208.

Narayanan, Sridhar, Pradeep K. Chintagunta, and Eugenio J. Miravete. (2007), "The Role of Self Selection, Usage Uncertainty and Learning in the Demand for Local Telephone Service," *Quantitative Marketing and Economics*, 5, pp. 1-34.

Rochet, Jean-Charles and Lars Stole. (2003), "The Economics of Multidimensional Screening." *Advances in Economics and Econometrics: Theory and Applications, Eighth World Congress*, M. Dewatripont, L. P. Hansen and S. J. Turnovsky (eds.), Cambridge University Press.

Saez, Emmanuel (2001), "Using Elasticities to Derive Optimal Income Tax Rates," *Review of Economic Studies*, 68(1), pp. 205-229.

Weinzierl, Matthew (2007), "The Surprising Power of Age-Dependent Taxes," Unpublished manuscript, Harvard.

Appendix

Proof of Proposition 1

The proofs of parts (i) and (ii) are immediate (as explained in the body of the paper). Below follows the proof to part (iii).

Suppose that schedule $S_M = \{C_{LB}, Y_{LB}, C_{LG}, Y_{LG}, C_{HB}, Y_{HB}, C_{HG}, Y_{HG}\}$ is an optimal menu of incentive schedules that satisfies the resource constraint, ex-ante incentive constraints and the ex-post incentive constraints. This incentive schedule yields social welfare of:

$$V(S_M) = \alpha_{LB}\, \Phi(U(C_{LB},Y_{LB}/w_L)) + \alpha_{LG}\, \Phi(U(C_{LG},Y_{LG}/w_L)) + \alpha_{HB}\, \Phi(U(C_{HB},Y_{HB}/w_H)) + \alpha_{HG}\, \Phi(U(C_{HG},Y_{HG}/w_H))$$

and satisfies the following resource and incentive constraints:

$$\alpha_{LB}(C_{LB}-Y_{LB}) + \alpha_{LG}(C_{LG}-Y_{LG}) + \alpha_{HB}(C_{HB}-Y_{HB}) + \alpha_{HG}(C_{HG}-Y_{HG}) \leq 0$$

$$\alpha_{LG}/\alpha_G\, (U(C_{LB},Y_{LB}/w_L) - U(C_{LG},Y_{LG}/w_L)) + \alpha_{HG}/\alpha_G\, (U(C_{HB},Y_{HB}/w_H) - U(C_{HG},Y_{HG}/w_H)) \leq 0$$
$$\alpha_{LB}/\alpha_B\, (U(C_{LG},Y_{LG}/w_L) - U(C_{LB},Y_{LB}/w_L)) + \alpha_{HB}/\alpha_B\, (U(C_{HG},Y_{HG}/w_H) - U(C_{HB},Y_{HB}/w_H)) \leq 0$$

$$U(C_{LB},Y_{LB}/w_H)- U(C_{HB},Y_{HG}/w_H) \leq 0$$
$$U(C_{LG},Y_{LG}/w_H)- U(C_{HG},Y_{HG}/w_H) \leq 0$$

Using the identities $\alpha_{HG}/\alpha_G = 1-\alpha_{LG}/\alpha_G$ and $\alpha_{HB}/\alpha_B = 1- \alpha_{LB}/\alpha_B$, adding the two ex-ante incentive constraints yields:

$$(1-\alpha_{LG}/\alpha_G)\, (U(C_{HB},Y_{HB}/w_H) - U(C_{HG},Y_{HG}/w_H)) + \alpha_{LG}/\alpha_G\, (U(C_{LB},Y_{LB}/w_L) - U(C_{LG},Y_{LG}/w_L)) +$$
$$(1- \alpha_{LB}/\alpha_B)\, (U(C_{HG},Y_{HG}/w_H) - U(C_{HB},Y_{HB}/w_H)) + \alpha_{LB}/\alpha_B\, (U(C_{LG},Y_{LG}/w_L) - U(C_{LB},Y_{LB}/w_L)) \leq 0$$

Rearranging:

$$(-\alpha_{LG}/\alpha_G)(U(C_{HB},Y_{HB}/w_H)-U(C_{HG},Y_{HG}/w_H)) + \alpha_{LG}/\alpha_G\, (U(C_{LB},Y_{LB}/w_L)-U(C_{LG},Y_{LG}/w_L)) +$$
$$(-\alpha_{LB}/\alpha_B)(U(C_{HG},Y_{HG}/w_H)-U(C_{HB},Y_{HB}/w_H)) + \alpha_{LB}/\alpha_B\, (U(C_{LG},Y_{LG}/w_L)-U(C_{LB},Y_{LB}/w_L))\leq 0$$

$$(\alpha_{LG}/\alpha_G-\alpha_{LB}/\alpha_B)[(U(C_{HG},Y_{HG}/w_H)-U(C_{HB},Y_{HB}/w_H))-(U(C_{LG},Y_{LG}/w_L)-U(C_{LB},Y_{LB}/w_L))] \leq 0$$

Since $(\alpha_{LG}/\alpha_G - \alpha_{LB}/\alpha_B) < 0$, this implies:

$$\text{(A.1)} \quad U(C_{HG},Y_{HG}/w_H) - U(C_{HB},Y_{HB}/w_H) \geq U(C_{LG},Y_{LG}/w_L) - U(C_{LB},Y_{LB}/w_L)$$

Next, we prove by contradiction that $U(C_{LG},Y_{LG}/w_L) - U(C_{LB},Y_{LB}/w_L) \leq 0$. Suppose that $U(C_{LG},Y_{LG}/w_L) - U(C_{LB},Y_{LB}/w_L) > 0$. Then, the following ex-ante incentive constraint

$$\alpha_{HB}/\alpha_B\, (U(C_{HG},Y_{HG}/w_H) - U(C_{HB},Y_{HB}/w_H)) + \alpha_{LB}/\alpha_B\, (U(C_{LG},Y_{LG}/w_L) - U(C_{LB},Y_{LB}/w_L)) \leq 0$$

implies that α_{HB}/α_B $(U(C_{HG},Y_{HG}/w_H)$ - $U(C_{HB},Y_{HB}/w_H)) < 0$. But this contradicts equation (A.1). Hence, it follows that

$$(A.2) \qquad U(C_{LG},Y_{LG}/w_L) - U(C_{LB},Y_{LB}/w_L) \leq 0$$

Now consider a single incentive schedule, i.e., a schedule that is independent of the signal. Hence, it consists of two consumption/income bundles:

$S_S = \{C_L, Y_L, C_H, Y_H\}$

Let this incentive schedule be defined by:

$Y_L= (\alpha_{LB}Y_{LB} + \alpha_{LG}Y_{LG})/ \alpha_L$
$Y_H= (\alpha_{HB}Y_{HB} + \alpha_{HG}Y_{HG})/ \alpha_H$
$C_L= (\alpha_{LB}C_{LB} + \alpha_{LG}C_{LG})/ \alpha_L - P$
$C_H= (\alpha_{HB}C_{HB} + \alpha_{HG}C_{HG})/ \alpha_H$

where $P \geq 0$ is the solution to:

$$(A.3) \qquad \alpha_{LB}U(C_{LB},Y_{LB}/w_L) + \alpha_{LG}U(C_{LG},Y_{LG}/w_L) = \alpha_L\, U(C_L,Y_L/w_L)$$

By concavity of $U()$, the agent prefers the mean consumption-exertion bundle over the lottery. Hence, she is willing to pay a positive premium ($P \geq 0$) to obtain the mean consumption-exertion bundle.

Next, we show that S_S, the single schedule, satisfies the resource and incentive constraints but weakly increases social welfare, and that therefore the optimal menu of schedules, S_M, offers no benefit over the single incentive schedule.

$$\begin{aligned}
V(S_M) \;=\; & \alpha_{LB}\, \Phi(U(C_{LB},Y_{LB}/w_L) + \alpha_{LG}\, \Phi(U(C_{LG},Y_{LG}/w_L) + \\
& \alpha_{HB}\, \Phi(U(C_{HB},Y_{HB}/w_H) + \alpha_{HG}\, \Phi(U(C_{HG},Y_{HG}/w_H) \\
\leq\; & \qquad \text{(by concavity of } \Phi) \\
& \alpha_L\, \Phi((\alpha_{LB}/\alpha_L)U(C_L,Y_L/w_L)+(\alpha_{LG}/\alpha_L)U(C_{LG},Y_{LG}/w_L)) + \\
& \alpha_{HB}\, \Phi(U(C_{HB},Y_{HB}/w_H) + \alpha_{HG}\, \Phi(U(C_{HG},Y_{HG}/w_H) \\
=\; & \qquad \text{(by definition of } P) \\
& \alpha_L\, \Phi(U(C_L,Y_L/w_L)) + \\
& \alpha_{HB}\, \Phi(U(C_{HB},Y_{HB}/w_H) + \alpha_{HG}\, \Phi(U(C_{HG},Y_{HG}/w_H) \\
\leq\; & \qquad \text{(by concavity of } U) \\
& \alpha_L\, \Phi(U(C_L,Y_L/w_L) + \alpha_H\, \Phi(U(C_H,Y_H/w_H) \;=\; V(S_S)
\end{aligned}$$

Hence, social welfare is weakly higher for S_S than for S_M. The resource constraint of S_S is satisfied:

$$\alpha_{LB}(C_L\text{-}Y_L) + \alpha_{LG}(C_L\text{-}Y_L) + \alpha_{HB}(C_H\text{-}Y_H) + \alpha_{HG}(C_H\text{-}Y_H)$$
$$= \alpha_{LB}(C_{LB}\text{-}P\text{-}Y_{LB}) + \alpha_{LG}(C_{LG}\text{-}P\text{-}Y_{LG}) + \alpha_{HB}(C_{HB}\text{-}Y_{HB}) + \alpha_{HG}(C_{HG}\text{-}Y_{HG})$$
$$\leq \alpha_{LB}(C_{LB}\text{-}Y_{LB}) + \alpha_{LG}(C_{LG}\text{-}Y_{LG}) + \alpha_{HB}(C_{HB}\text{-}Y_{HB}) + \alpha_{HG}(C_{HG}\text{-}Y_{HG}) \leq 0$$

where the equality sign follows from the definition of S_S, the first inequality sign follows from $P \geq 0$ and the final inequality follows from S_M satisfying the resource constraint. The schedule S_M satisfies the following two ex-post incentive constraints:

$$U(C_{LB},Y_{LB}/w_H)\text{-}\ U(C_{HB},Y_{HG}/w_H) \leq 0$$
$$U(C_{LG},Y_{LG}/w_H)\text{-}\ U(C_{HG},Y_{HG}/w_H) \leq 0$$

Hence, taking a linear combination we have:

$$(\alpha_{HB}/\alpha_H)[U(C_{LB},Y_{LB}/w_H)\text{-}\ U(C_{HB},Y_{HB}/w_H)] + (\alpha_{HG}/\alpha_H)\ [U(C_{LG},Y_{LG}/w_H)\text{-}\ U(C_{HG},Y_{HG}/w_H)] \quad \leq 0$$

by concavity of U, this implies:

$$(\alpha_{HB}/\alpha_H)U(C_{LB},Y_{LB}/w_H) + (\alpha_{HG}/\alpha_H)\ U(C_{LG},Y_{LG}/w_H) - U(C_H,Y_H/w_H) \leq 0$$

rearranging:

$$(\alpha_{LB}/\alpha_L)U(C_{LB},Y_{LB}/w_H) + (\alpha_{LG}/\alpha_L)\ U(C_{LG},Y_{LG}/w_H) + (\alpha_{HB}/\alpha_H - \alpha_{LB}/\alpha_L)U(C_{LB},Y_{LB}/w_H) + (\alpha_{HG}/\alpha_H - \alpha_{LG}/\alpha_L)U(C_{LG},Y_{LG}/w_H) - U(C_H,Y_H/w_H) \leq 0$$

using the identities $\alpha_{HB}/\alpha_H = 1 - \alpha_{HG}/\alpha_H$ and $\alpha_{LB}/\alpha_L = 1 - \alpha_{LG}/\alpha_L$, we have:

$$(\alpha_{LB} / \alpha_L)U(C_{LB},Y_{LB}/w_H) + (\alpha_{LG} / \alpha_L)\ U(C_{LG},Y_{LG}/w_H) + (\alpha_{HB}/\alpha_H - \alpha_{LB}/\alpha_L)(U(C_{LB},Y_{LB}/w_H) - U(C_{LG},Y_{LG}/w_H)) - U(C_H,Y_H/w_H) \leq 0$$

Because signals are informative, a high type is less likely to receive a bad signal than a low type, so $(\alpha_{HB}/\alpha_H - \alpha_{LB}/\alpha_L) \leq 0$. Moreover, from equation (A.2), we know that $(U(C_{LB},Y_{LB}/w_H) - U(C_{LG},Y_{LG}/w_H)) \leq 0$. Thus, the inequality is preserved if we drop the product of these to terms, which is positive:

$$\text{(A.4)} \quad (\alpha_{LB}/\alpha_L)U(C_{LB},Y_{LB}/w_H) + (\alpha_{LG}/\alpha_L)\ U(C_{LG},Y_{LG}/w_H) - U(C_H,Y_H/w_H) \leq 0$$

Now note that by construction (see eq. A.3), we have:

$$(\alpha_{LB}/\alpha_L)U(C_{LB},Y_{LB}/w_L) + (\alpha_{LG}/\alpha_L)\ U(C_{LG},Y_{LG}/w_L) = U(C_L,Y_L/w_L)$$

Substituting this equation into (A.4), we get:

$$U(C_L,Y_L/w_H) - U(C_H,Y_H/w_H) \leq 0$$

Thus, the IC is satisfied for this incentive schedule that does not rely on declarations of the signal. We showed earlier that this single incentive schedule satisfies the resource constraint and yields a weak welfare improvement over the optimal menu of schedules. Hence, a menu of schedules is never necessary to maximize social welfare if there are only two types.

Proof of Proposition 2 and its corollary

To prove proposition 2, we show that a menu of schedules can be formed by applying a specific perturbation to the optimal single incentive schedule and that this perturbation yields the desired interim Pareto improvement. It turns out that we can restrict our attention to a particular class of perturbations, namely those in which (i) the labor supply (or effort) of types 3 through N is held constant and (ii) the change in utility is the same for types 3 through N. Because we consider perturbations to labor supply of the lowest two types, we first need to derive the optimal levels of labor supply for these types under the single optimal incentive schedule. We use the assumption that the utility function is separable in order to rewrite $U(C, X) = V(C)-F(X)$. It is convenient to define $Z_\theta(Y) = F(Y_\theta/w_\theta)$, so that we can write $U(C, Y/w_\theta) = V(C) - Z_\theta(Y)$. Moreover, we assume that we have a standard interior solution with the resource constraint holding with equality and without bundling (i.e., each type has a distinct bundle at the optimum). Because the utility function satisfies the single crossing property, only the incentive constraints of type θ towards type θ-1 may bind. We assume that $\Phi(V(.))$ is sufficiently concave that these constraints indeed bind. (If this were not the case, the principal could redistribute without distorting labor supply, which is not a relevant case). We can thus rewrite the optimization problem as:

$$\text{(A.5)} \quad \underset{\{C_\theta, Y_\theta\}_{\theta=1..N}}{\text{Max}} \sum_{\theta=1}^{N} \alpha_\theta \Phi\left(V(C_\theta) - Z_\theta(Y_\theta)\right),$$

s.t.

$$\sum_{\theta=1}^{N} \alpha_\theta (C_\theta - Y_\theta) = 0, \text{ and}$$

$$\left(V(C_{\theta-1}) - Z_\theta(Y_{\theta-1})\right) - \left(V(C_\theta) - Z_\theta(Y_\theta)\right) = 0 \qquad \text{for } \theta = 2 .. N.$$

Let the Lagrange multiplier on the resource constraint be denoted by μ_0 and the Lagrange multiplier on the incentive constraint of type θ+1 toward the bundle of type θ by μ_θ. The first-order conditions associated with (C_1, C_2, … , C_N, Y_1 and Y_2) are :

(A.6a) $\alpha_1 \mu_0 + (\alpha_1 \varphi_1 + \mu_1) v_1 = 0,$

(A.6b) $\alpha_2 \mu_0 + (\alpha_2 \varphi_2 + \mu_2 + \mu_1) v_2 = 0,$

(A.6c) $\alpha_\theta \mu_0 + (\alpha_\theta \varphi_\theta + \mu_\theta - \mu_{\theta-1}) v_\theta = 0$ for $\theta = 3 .. N$ (and where $\mu_N = 0$),

(A.6d) $-\alpha_1 \mu_0 - (\alpha_1 \varphi_1 + \mu_1 r_2) z_{11} = 0,$

(A.6e) $-\alpha_2 \mu_0 - (\alpha_2 \varphi_2 - \mu_1 + \mu_2 r_3) z_{22} = 0,$

where v_θ denotes $dV(C_\theta)/dC_\theta$, or the marginal utility of consumption evaluated at the consumption level of type θ, and $z_{\theta'\theta}$ denotes $dZ_\theta(Y_{\theta'})/dY_{\theta'}$, or type's θ marginal disutility of earning income evaluated at the income level of type θ'. Let r_θ be the ratio $[dZ_\theta(Y_{\theta-1})/dY_{\theta-1}]$ / $[dZ_\theta(Y_\theta)/dY_\theta]$ or the disutility of earnings at the income level of the next lower type relative to the disutility of earnings at the own income level and let φ_θ denote $d\Phi(U)/dU$ evaluated at $U= V(C_\theta)- Z_\theta(Y_\theta)$.

Because we restrict our attention to this class of perturbations that only affect the labor supply of types 1 and 2, we can simplify the analysis by aggregating types 3 through N into a single aggregate type. This effectively turns the N-type problem into a 3-type problem: type 1 (denoted by the subscript L), type 2 (denoted by the subscript M) and a weighted average of types 3 though N (denoted by the subscript H). We first divide equation (A.6c) by v_θ and then sum this equation for θ=3 … N. This yields:

(A.7) $$\mu_0\sum_{\theta=3}^{N}\alpha_\theta/v_\theta+\sum_{\theta=3}^{N}\alpha_\theta\varphi_\theta-\mu_2=0.$$

Now define $\alpha_H=\sum_{\theta=3}^{N}\alpha_\theta$, $\varphi_H=\sum_{\theta=3}^{N}\alpha_\theta\varphi_\theta/\alpha_H$, and $v_H=\alpha_H\Big/\sum_{\theta=3}^{N}\alpha_\theta/v_\theta$. Multiplying (A.7) by v_H yields:

(A.8) $$\alpha_H\mu_0+(\alpha_H\varphi_H-\mu_2)\,v_H=0$$

In words, equation (A.8) effectively aggregates types 3 through N into a single type, the high type. Renaming the type 1 type L, and renaming type 2 type M, we can rewrite the system of N+2 equations (A.6a-e) into a system of 5 equations:

(A.9a) $\alpha_L\mu_0+(\alpha_L\varphi_L+\mu_L)\,v_L=0$

(A.9b) $\alpha_M\mu_0+(\alpha_M\varphi_M+\mu_M+\mu_L)\,v_M=0$

(A.9c) $\alpha_H\mu_0+(\alpha_H\varphi_H-\mu_M)\,v_H=0$

(A.9d) $-\alpha_L\mu_0-(\alpha_L\varphi_L+\mu_L r_M)\,z_{LL}=0$

(A.9e) $-\alpha_M\mu_0-(\alpha_M\varphi_M-\mu_L+\mu_M r_H)\,z_{MM}=0$

Before proceeding, we make two normalizations that will simplify the expressions that follow. First, we multiply the utility function $U(C,Y)$ with a constant such that:

(A.10) $$\alpha_L v_M v_H+\alpha_M v_L v_H+\alpha_H v_L v_M=1$$

Second, we multiply the social welfare function $\Phi(U)$ with a constant such that:

(A.11) $$\alpha_L\varphi_L+\alpha_M\varphi_M+\alpha_H\varphi_H=1$$

We solve the first-order conditions (A.9a-e) to find expressions for disutility of earnings (z_{LL} and z_{MM}) that must hold at the optimal single incentive schedule, $\{\hat{C}_\theta,\hat{Y}_\theta\}_{\theta=1..N}$. We find:

(A.12) $$\frac{z_{LL}}{v_L}=\frac{v_M v_H}{v_M v_H+\alpha_H(\varphi_L v_L-\varphi_H v_H)v_M+\alpha_M(\varphi_L v_L-\varphi_M v_M)v_H(1-r_M)}$$

(A.13) $$\frac{z_{MM}}{v_M} = \frac{\alpha_M v_L v_H}{\alpha_M v_L v_H + \alpha_L \alpha_H (\varphi_L v_L - \varphi_H v_H) v_M + \alpha_M \alpha_H (\varphi_L v_L - \varphi_M v_M) v_H (1 - r_H)}$$

Note that both ratios are less than one, implying that the marginal disutility of earnings is less than the marginal utility of consumption. In other words, at the optimum, the labor supply of the low and medium types is below the first-best level.

We now define a set of perturbations $\{c_{\theta\sigma}, y_{\theta\sigma}\}_{\sigma=G,B;\ \theta=1..N}$ such that the menu of schedules is equal to the optimal single incentive schedule plus these perturbations:

(A.14) $\{C_{\theta\sigma}, Y_{\theta\sigma}\}=\{\hat{C}_\theta + c_{\theta\sigma}, \hat{Y}_\theta + y_{\theta\sigma}\}$ for $\sigma=G,B$ and $\theta=1..N$.

Because of the assumption that the optimal single incentive schedule has a standard interior solution without bundling, there are no restrictions on the signs of the perturbations and, for sufficiently small perturbations, the perturbed solution is also interior and has no bundling. We require that the perturbations satisfy the ex-ante and ex-post incentive constraints of the schedule selection problem:

(A.15) $$\sum_{\theta=1}^{N} \alpha_{\theta\sigma} \left(V(\hat{C}_\theta + c_{\theta\sigma'}) - Z_\theta(\hat{Y}_\theta + y_{\theta\sigma'}) - V(\hat{C}_\theta + c_{\theta\sigma}) + Z_\theta(\hat{Y}_\theta + y_{\theta\sigma}) \right) = 0 \quad \text{for any } \sigma, \sigma',$$

(A.16) $$\left(V(\hat{C}_{\theta-1} + c_{\theta-1,\sigma}) - Z_\theta(\hat{Y}_{\theta-1} + y_{\theta-1,\sigma}) \right) - \left(V(\hat{C}_\theta + c_{\theta\sigma}) - Z_\theta(\hat{Y}_\theta + y_{\theta\sigma}) \right) = 0 \quad \text{for } \theta=2 .. N,$$

and $\sigma=B,G$. In addition, we require that the deviations hold the expected utility of each type constant:

(A.17) $$\frac{\alpha_{\theta B}}{\alpha_\theta} \left(V(\hat{C}_\theta + c_{\theta B}) - Z_\theta(\hat{Y}_\theta + y_{\theta B}) \right) + \frac{\alpha_{\theta G}}{\alpha_\theta} \left(V(\hat{C}_\theta + c_{\theta G}) - Z_\theta(\hat{Y}_\theta + y_{\theta G}) \right) = V(\hat{C}_\theta) - Z_\theta(\hat{Y}_\theta)$$

for $\theta = 1 .. N$. Note that when the perturbations are all equal to zero, equations (A.15) and (A.17) hold trivially and equation (A.16) holds because it coincides with the incentive constraints of the single incentive schedule. We now linearize these constraints:

(A.18) $$\sum_{\theta=1}^{N} \alpha_{\theta\sigma} \left(v_\theta (c_{\theta\sigma'} - c_{\theta\sigma}) - z_{\theta\theta}(y_{\theta\sigma'} - y_{\theta\sigma}) \right) = 0,$$

(A.19) $$(v_{\theta-1} c_{\theta-1,\sigma} - z_{\theta\theta} r_\theta y_{\theta-1,\sigma}) - (v_\theta c_{\theta\sigma} + z_{\theta\theta} y_{\theta\sigma}) = 0 \qquad \text{for } \theta=2 .. N;\ \sigma = B,G, \text{ and}$$

(A.20) $$\alpha_{\theta B} (v_\theta c_{\theta B} - z_{\theta\theta} y_{\theta B}) + \alpha_{\theta G} (v_\theta c_{\theta G} - z_{\theta\theta} y_{\theta G}) = 0 \qquad \text{for } \theta=1..N.$$

We do not perturb the labor supply of types 3 though N, thus $y_{\theta\sigma} = 0$ for $\sigma=G,B$ and $\theta=3 .. N$. We set $c_{\theta\sigma} = (v_3/v_\theta)\, c_{3,\sigma}$ for $\sigma=G,B$ and $\theta = 3.. N$. Substituting $c_{\theta\sigma} = (v_3/v_\theta)\, c_{3,\sigma}$ and $y_{\theta\sigma} = 0$ for $\sigma=G,B$ and $\theta = 3.. N$ into equation (A.19) shows that this equation holds for $\theta = 4 ... N$. Making the same substitutions into equation (A.20) shows that this equation holds for $\theta = 4 .. N$ if and only if this equation holds for $\theta = 3$. Next, we define average consumption among the high types as $c_{H\sigma} = \sum_{\theta=3}^{N} \alpha_\theta c_{\theta\sigma} / \alpha_H$. Using the definition $v_H = \alpha_H \Big/ \sum_{\theta=3}^{N} \alpha_\theta / v_\theta$ and $c_{\theta\sigma} = (v_3/v_\theta)\, c_{3,\sigma}$, it follows that:

$$\text{(A.21)} \qquad c_{H\sigma} = \sum_{\theta=3}^{N} \alpha_\theta c_{\theta\sigma} / \alpha_H = \sum_{\theta=3}^{N} \alpha_\theta (v_3 / v_\theta) c_{3\sigma} / \alpha_H = (v_3 / v_H) c_{3\sigma}\,.$$

Using equation (A.21), renaming the type 1 type L, and renaming type 2 type M, we can rewrite the system of 3 N equations (A.18-A.20) into a system of 9 equations:

$$\text{(A.22)} \qquad \begin{aligned}
&\sum_{\theta=L,M,H} \alpha_{\theta G} \left(v_\theta (c_{\theta B} - c_{\theta G}) - z_{\theta\theta} (y_{\theta B} - y_{\theta G}) \right) = 0 \\
&\sum_{\theta=L,M,H} \alpha_{\theta B} \left(v_\theta (c_{\theta G} - c_{\theta B}) - z_{\theta\theta} (y_{\theta G} - y_{\theta B}) \right) = 0 \\
&(v_L c_{L,\sigma} - z_{MM} r_M y_{L,\sigma}) - (v_M c_{M\sigma} + z_{MM} y_{M\sigma}) = 0 && \text{for } \sigma = B,G \\
&(v_M c_{M,\sigma} - z_{HH} r_H y_{M,\sigma}) - (v_H c_{H\sigma} + z_{HH} y_{H\sigma}) = 0 && \text{for } \sigma = B,G \\
&\alpha_{\theta B} \left(v_\theta c_{\theta B} - z_{\theta\theta} y_{\theta B} \right) + \alpha_{\theta G} \left(v_\theta c_{\theta G} - z_{\theta\theta} y_{\theta G} \right) = 0 && \text{for } \theta = L,M,H.
\end{aligned}$$

Since the permutation is 10-dimensional (c_{LB}, c_{LG}, c_{MB}, c_{MG}, c_{HB}, c_{HG}, y_{LB}, y_{LG}, y_{MB}, y_{MG}) and system of equations (A.22) consist of 9 constraints, the perturbation has one degree of freedom. Let this degree of freedom be taken up by the parameter ε. The expressions for the perturbations can be simplified considerably by defining "contrasts" γ_{ij} that measure how strongly the signal distinguishes between type i and type j:

$$\text{(A.23)} \qquad \begin{aligned}
\gamma_{ML} &= \frac{\alpha_{LB}}{\alpha_L} \frac{\alpha_{MB}}{\alpha_M} \left(\frac{\alpha_{MG}}{\alpha_{MB}} - \frac{\alpha_{LG}}{\alpha_{LB}} \right), \\
\gamma_{HL} &= \frac{\alpha_{LB}}{\alpha_L} \frac{\alpha_{HB}}{\alpha_H} \left(\frac{\alpha_{HG}}{\alpha_{HB}} - \frac{\alpha_{LG}}{\alpha_{LB}} \right), \text{ and} \\
\gamma_{HM} &= \frac{\alpha_{MB}}{\alpha_M} \frac{\alpha_{HB}}{\alpha_H} \left(\frac{\alpha_{HG}}{\alpha_{HB}} - \frac{\alpha_{MG}}{\alpha_{MB}} \right).
\end{aligned}$$

We assume that the signals are informative and, in that case, these three contrasts are generically different from zero. In the special case that they satisfy the monotone likelihood property (the probability of receiving a good signal is strictly increasing with type), $\alpha_{HG} / \alpha_{HB} > \alpha_{MG} / \alpha_{MB} > \alpha_{LG} / \alpha_{LB}$ and, as a result, the contrasts γ_{ij} are strictly positive. Solving (A.22) and making use of the definitions (A.23), we find the following expressions for perturbations that satisfy the ex-ante and ex-post incentive constraints and that keep the expected utility of each type constant:

(A.24a) $y_{LB} = \varepsilon\,(\alpha_{LG}\gamma_{HM} + \alpha_{MG}\gamma_{HL})(1 - r_H)/\,z_{LL}$

(A.24b) $y_{LG} = -\varepsilon\,(\alpha_{LB}\gamma_{HM} + \alpha_{MB}\gamma_{HL})(1 - r_H)/\,z_{LL}$

(A.24c) $y_{MB} = -\varepsilon\,(\alpha_{HG}\gamma_{ML} + \alpha_{MG}\gamma_{HL})(1 - r_M)/\,z_{MM}$

(A.24d) $y_{MG} = \varepsilon\,(\alpha_{HB}\gamma_{ML} + \alpha_{MB}\gamma_{HL})(1 - r_M)/\,z_{MM}$

(A.24e) $c_{LB} = \varepsilon\,(\alpha_{LG}\gamma_{HM} r_M + \alpha_{MG}\gamma_{HL})(1 - r_H)/\,v_L$

(A.24f) $c_{LG} = -\varepsilon\,(\alpha_{LB}\gamma_{HM} r_M + \alpha_{MB}\gamma_{HL})(1 - r_H)/\,v_L$

(A.24g) $c_{MB} = -\varepsilon\,(\alpha_{HG}\gamma_{ML} + \alpha_{MG}\gamma_{HL} r_H)(1 - r_M)/\,v_M$

(A.24h) $c_{MG} = \varepsilon\,(\alpha_{HB}\gamma_{ML} + \alpha_{MB}\gamma_{HL} r_H)(1 - r_M)/\,v_M$

(A.24i) $c_{HB} = -\varepsilon\,\alpha_{HG}\gamma_{ML}(1 - r_M)(1 - r_H)/\,v_H$

(A.24j) $c_{HG} = \varepsilon\,\alpha_{HB}\gamma_{ML}(1 - r_M)(1 - r_H)/\,v_H$

Substituting these perturbations into the resources constraint and using equations (A.12) and (A.13) to simplify the resulting expression yields the effect of the perturbation on the resource constraint (*RC*):

(A.25)

$$\Delta RC = \varepsilon\,\frac{\gamma_{ML}(1 - r_M)(1 - r_H)}{v_L v_M v_H \alpha_M^2 \alpha_L} \times \Big[\alpha_M\gamma_{HL}\big(\alpha_M(\varphi_L v_L - \varphi_M v_M)v_H + \alpha_H(\varphi_L v_L - \varphi_H v_H)v_M\big) - \alpha_L\gamma_{HM}\big(\alpha_L(\varphi_L v_L - \varphi_H v_H)v_M + \alpha_M(\varphi_M v_M - \varphi_H v_H)v_L\big)\Big]$$

All terms in parentheses in (A.25) are positive, and the sign of this expression thus depends on the term in square brackets. If we select a value of ε with the same sign as this expression, the resource constraint will be strictly relaxed, except for the combination of parameters where the term in square brackets is exactly zero. Since the set of parameter combinations for which the term in square brackets is zero has Lebesgue measure zero, the resource constraint will be relaxed for generic distributions of α. Moreover, since ε enters linearly, the efficiency gains are first order. Since the change in the resource constraint depends on a first-order approximation of the effects of perturbing the optimal single incentive schedule, this expression only holds for small values of ε. For larger values, second order effects could become important, and could generally offset the first-order gains. The first-order gains from the relaxation of the resource constraint can be divided between the principle and each type of agent, thus achieving an interim-by-type Pareto improvement.

The corollary follows immediately. The menu of schedules defined by the perturbation is inside the principal's choice set and yields a strict first-order gain to the principal. Hence, the menu of schedules selected by the principal must offer the principal a gain that is at least as large, and this gain must therefore also be strict and first-order.

FIGURES AND TABLES

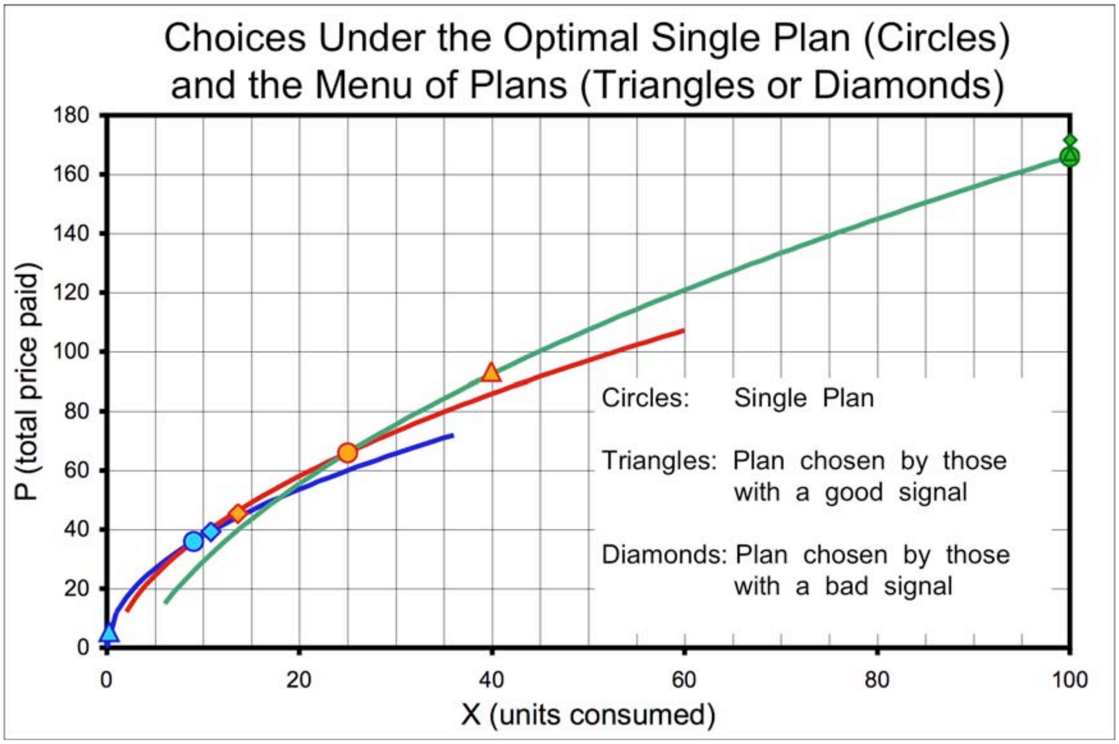

Figure 2: Utility Company's Optimal Single Price Plan and Optimal Menu of Price Plans

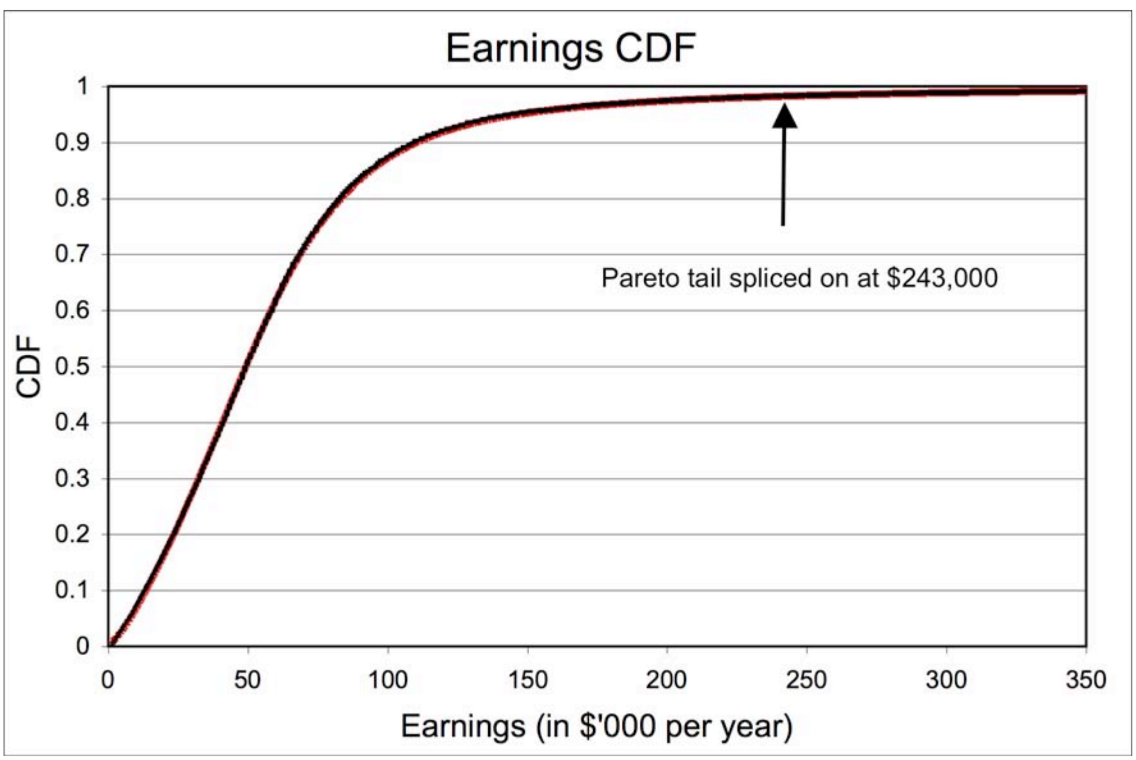

Figure 3: The Cumulative Distribution of Pre-tax Earnings of Couples in the U.S.

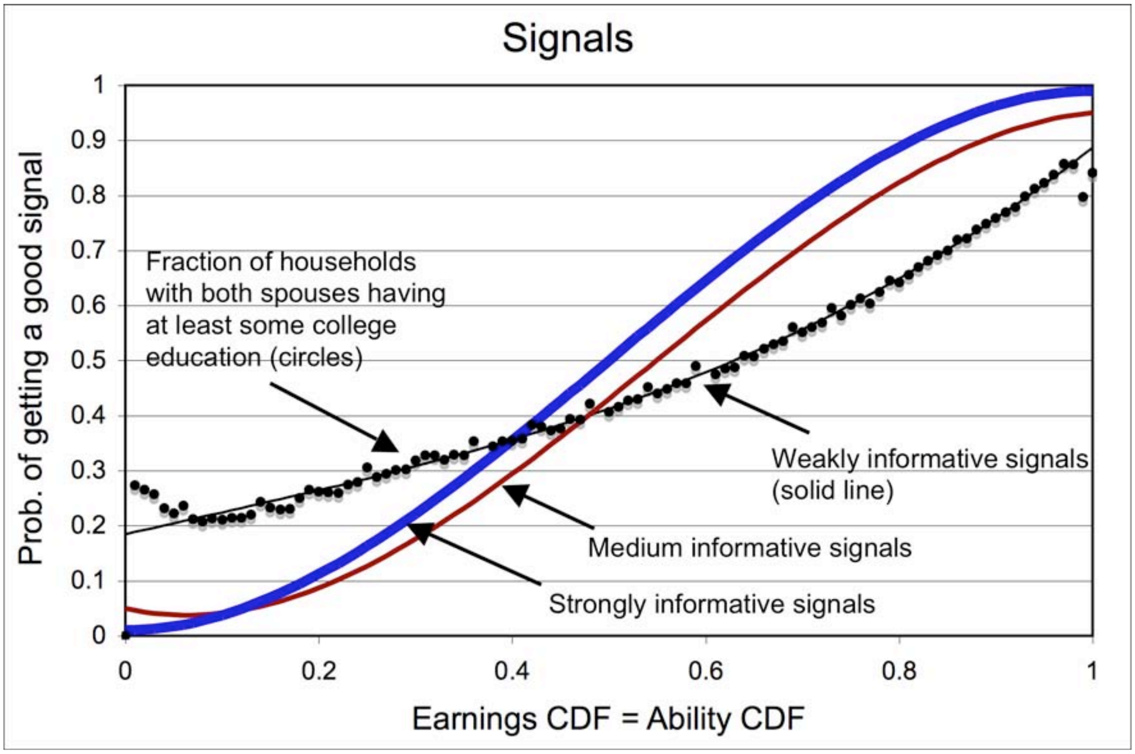

Figure 4: Informativeness of Signals

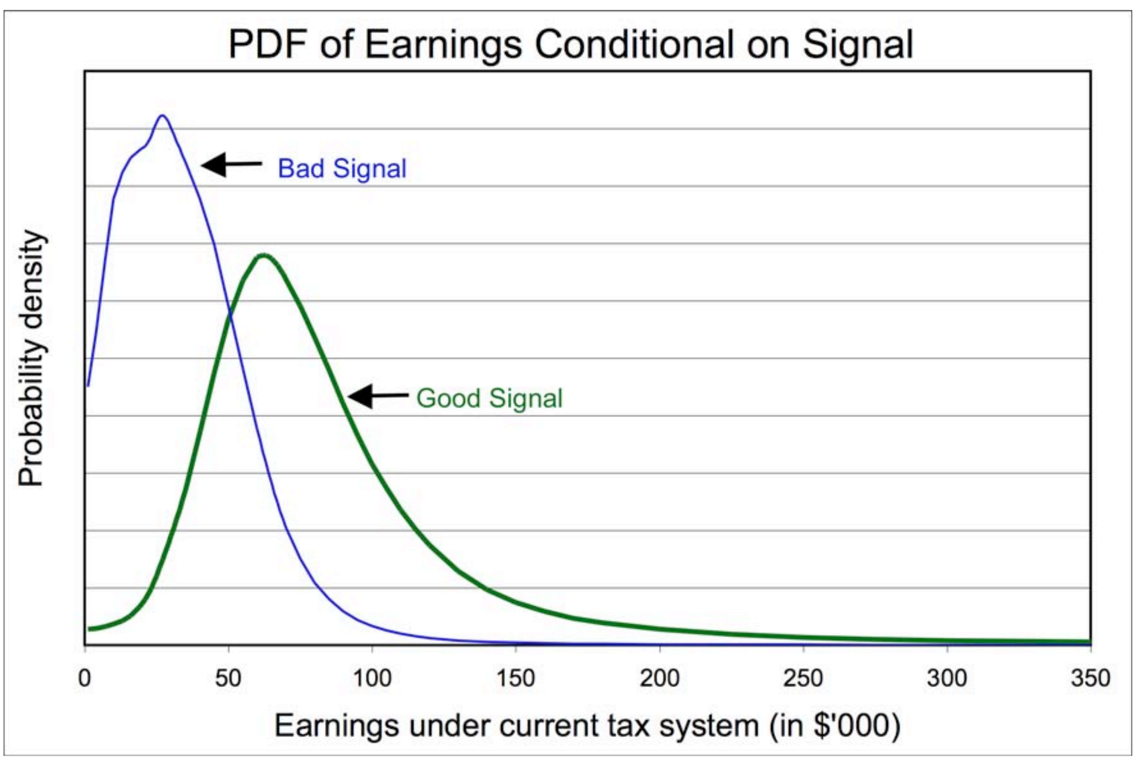

Figure 5: The Density Function of Ability Conditional on Signal

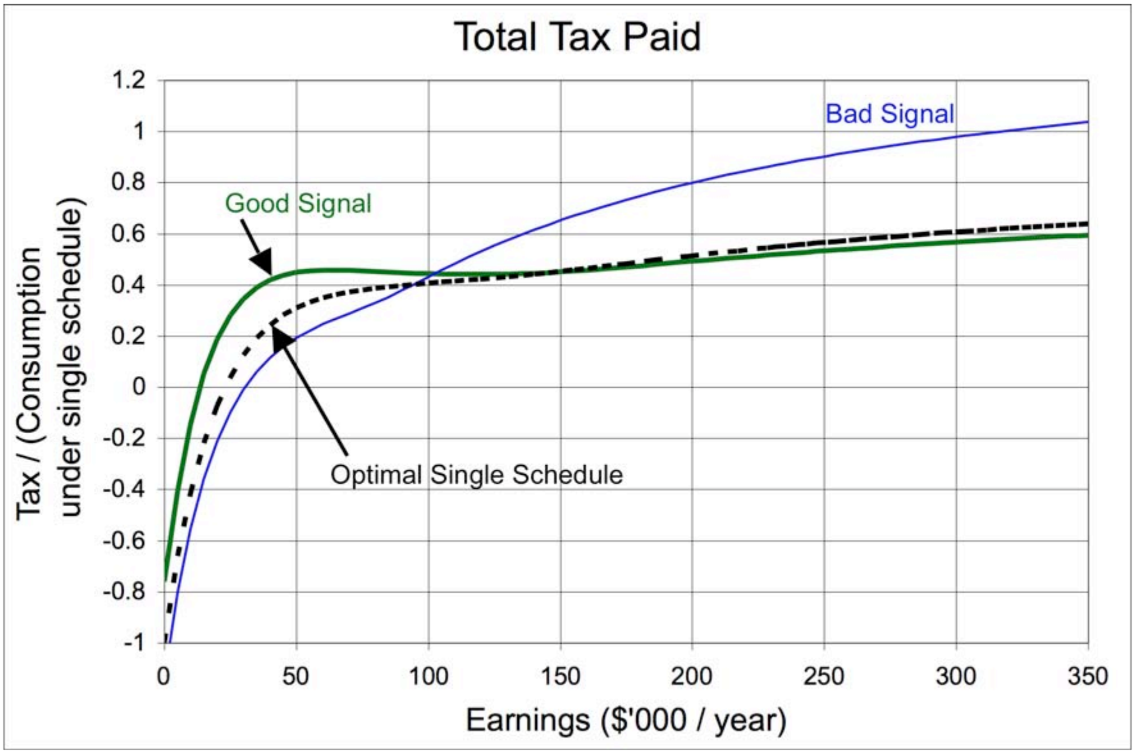

Figure 6: Total Tax as a Function of Earnings

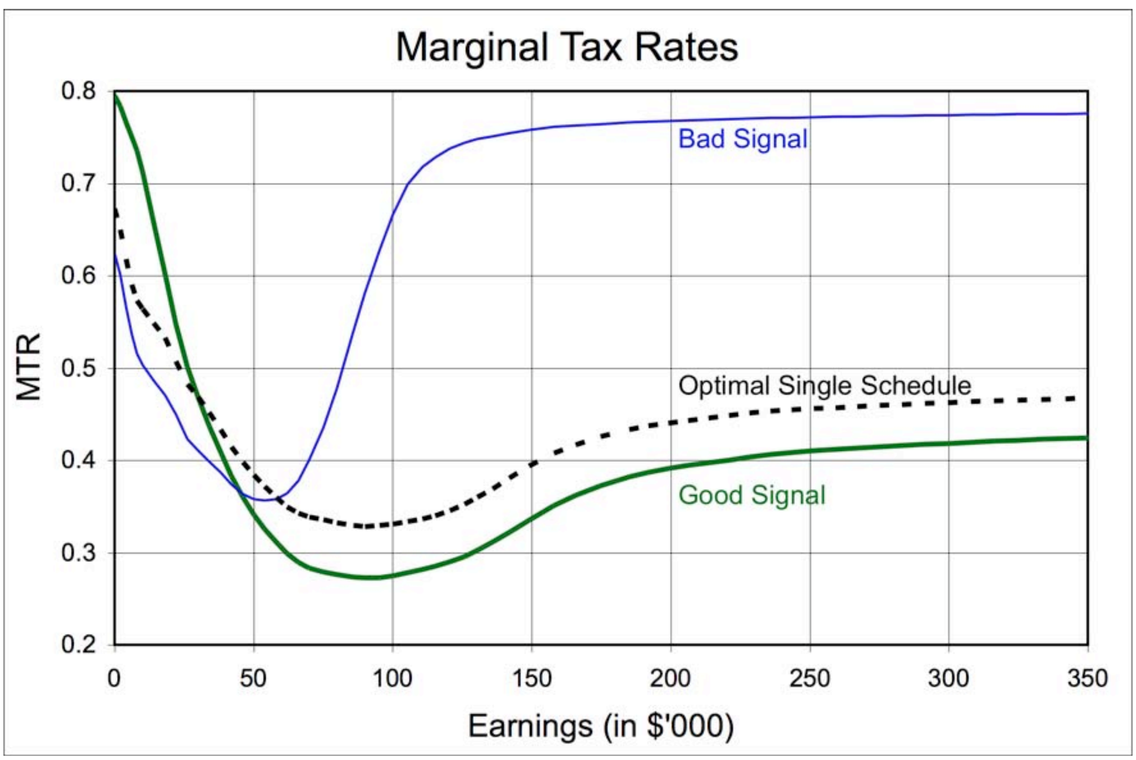

Figure 7: Optimal Marginal Tax Rates as a Function of Earnings

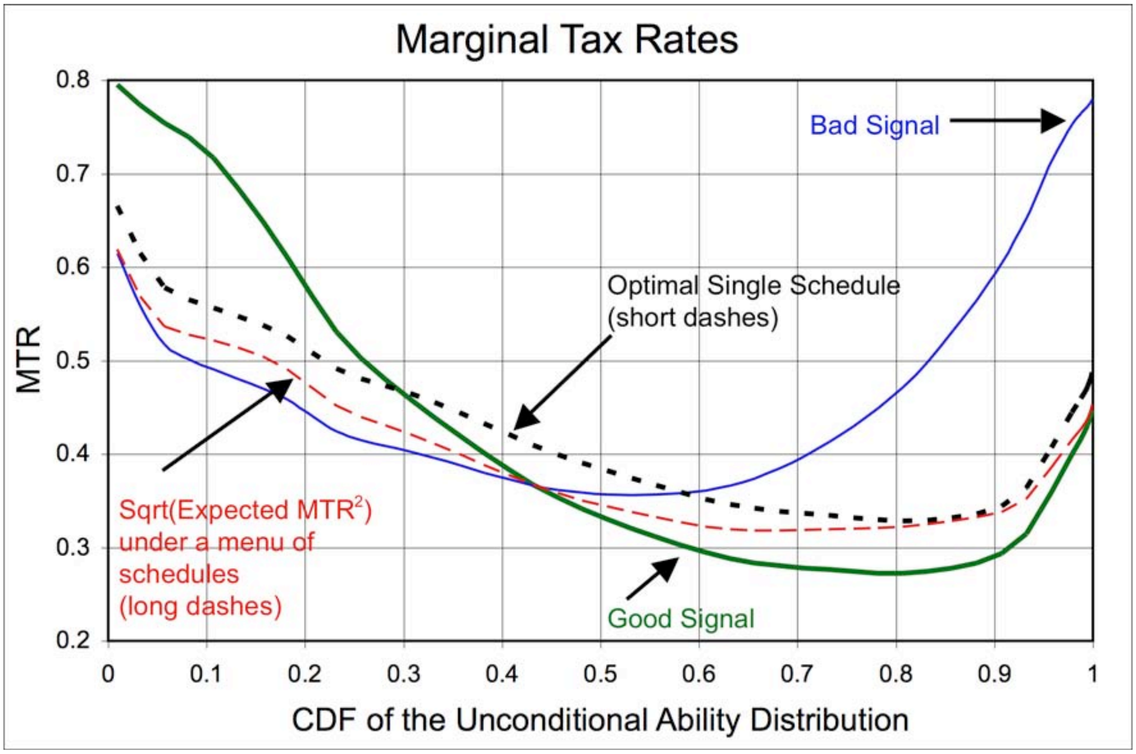

Figure 8: Optimal Marginal Tax Rates as a Function of Ability

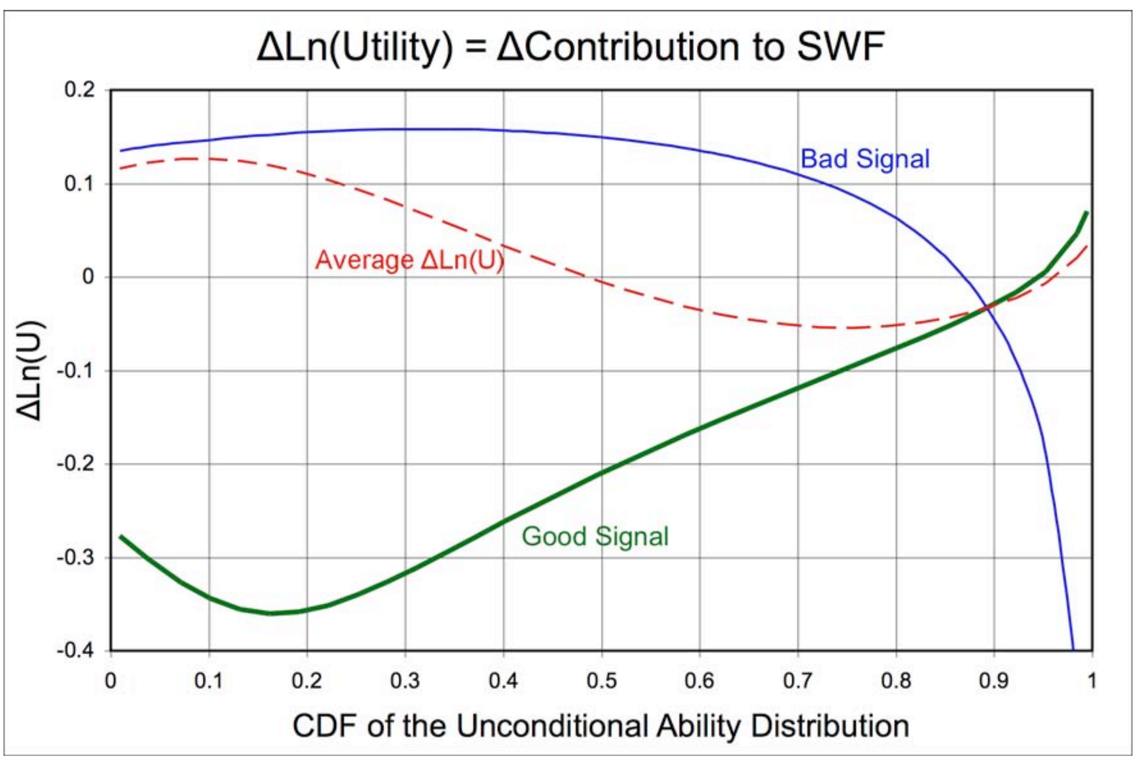

Figure 9: Effect of a Menu of Schedules on Social Welfare and Utility

Table 1. The Optimal Single Non-Linear Price Plan and the Optimal Menu of Non-Linear Price Plans

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Low type		Medium type		High type				
	σ=*B*	σ=*G*	σ=*B*	σ=*G*	σ=*B*	σ=*G*	E[•| σ=*B*]	E[•| σ=*G*]	E[•]
Panel A: Single non-linear price plan									
1. Quantity sold (*X*)	9.0		25.0		100.0		20.4	68.9	44.7
2. Revenue (*P*)	36.0		66.0		166.0		54.7	124.0	89.3
3. Producer Surplus (*P-X*)	27.0		41.0		66.0		34.3	55.1	44.7
4. Consumer Surplus (*U*)	0		9.0		34.0		5.3	23.4	14.3
5. Social Surplus	27.0		50.0		100.0		39.5	78.5	59.0
Panel B: Menu of non-linear price plans									
1. Quantity sold (*X*)	10.8	0.2	13.6	39.9	100.0	100.0	17.7	73.3	45.5
2. Revenue (*P*)	39.4	5.6	45.4	93.3	171.7	167.0	50.2	131.7	91.0
3. Producer Surplus (*X-P*)	28.6	5.4	31.9	53.4	71.7	67.0	32.6	58.4	45.5
4. Consumer Surplus (*U*) from own plan	0	0	9.8	1.4	28.3	33.0	5.2	20.2	12.7
5. Consumer Surplus from *other* plan							2.7	20.2	
6. Social Surplus	28.6	5.4	41.7	54.8	100.0	100.0	37.7	78.6	58.2

Table 2. Distributional and Efficiency Effects of a Menu of Price Plans

				Consumer Surplus is constrained to remain constant		
	U	Profit	DWL	*U*	Profit	DWL
1. First-Best (type is observable)	0	23.9	0	14.3	9.6	0
2. Non-Linear Pricing Conditional on Signal	8.7	10.3	4.9	14.3	8.2	1.4
3. Menu of Non-Linear Price Plans	12.7	5.3	5.9	14.3	5.2	4.4
4. Single Non-Linear Price Plan	14.3	4.5	5.1	14.3	4.5	5.1
5. Single Linear Price Plan (two-part tariff)	14.0	-6.2	16.0	14.3	-6.5	16.0

Fixed costs are calibrated such that profit is equal to 5% of revenue in the case of the optimal single non-linear price plan.

Table 3. Menu of Schedules Versus Optimal Single Non-linear Schedule

	Effects on Behavior			Tax Schedules		Welfare Effects		Distortions	
	Average Earnings ($/filer)	Average Cons. ($/filer)	Average Net Tax Revenue ($/filer)	Intercept ($)	Average Marginal Tax Rate	Average Utility ($)	Average Social Welfare	Average DWL ($)	Average DWL / Tax Revenue
Panel A: Menu of non-linear schedules vs. single non-linear schedule									
1. Optimal menu of schedules	64,608	48,816	15,792		38.7%	34,953	10.314	3,423	21.7%
Average | bad signal	35,200	33,771	1,429	15,395	43.6%	26,944	10.150	2,809	196.6%
Average | good signal	100,044	66,944	33,100	10,235	32.7%	44,602	10.511	4,163	12.6%
2. Optimal single nonlinear schedule	63,170	47,377	15,792	13,467	42.5%	34,492	10.290	3,863	24.5%
Average | bad signal	35,481	30,790	4,690	13,467	46.9%	23,826	10.011	2,517	53.7%
Average | good signal	96,533	67,363	29,170	13,467	37.2%	47,343	10.626	5,485	18.8%
Percentage change	*2.3%*	*3.0%*	*0.0%*		*-9.4%*	*1.3%*	*2.4%*	*-12.1%*	*-12.1%*
Absolute change	*$1,438*	*$1,438*	*$0*		*-3.8%*	*$461*		*-$440*	*-2.8%*
Panel B: Menu of non-linear schedules vs. the single non-linear schedule that yields the same social welfare									
1. Optimal menu of schedules	64,608	48,816	15,792		38.7%	34,953	10.314	3,423	21.7%
3. Optimal single nonlinear schedule	63,290	48,133	15,156	13,964	42.2%	35,176	10.314	3,815	25.2%
Change			*$636*				*0.0%*		

Note: These calculations are based on a labor supply elasticity of 0.5 and the medium informative signal distribution.

Table 4. Alternative Tax Schedules

	Effects on Behavior			Tax Schedules		Welfare Effects		Distortions	
	Average Earnings ($/filer)	Average Cons. ($/filer)	Average Net Tax Revenue ($/filer)	Intercept ($)	Average Marginal Tax Rate	Average Utility ($)	Average Social Welfare	Average DWL ($)	Average DWL / Tax Revenue
Panel A: Menu if signals are publicly observable									
4. Optimal menu, observable signals	65,495	49,702	15,792		35.1%	35,306	10.335	3,048	19.3%
2. Optimal single nonlinear schedule	63,170	47,377	15,792	13,467	42.5%	34,492	10.290	3,863	24.5%
Change			*$0*				*4.5%*	*-$815*	*-5.2%*
Panel B: Menu that yields Pareto-by-signal improvement									
5. Conditional	66,573	50,780	15,792		35.6%	35,676	10.309	2,690	17.0%
Average | bad signal	36,713	33,686	3,027	13,669	40.6%	25,993	10.097	2,150	71.0%
Average | good signal	102,551	71,377	31,174	10,225	29.6%	**47,343**	10.563	3,340	10.7%
2. Optimal single nonlinear schedule	63,170	47,377	15,792	13,467	42.5%	34,492	10.290	3,863	24.5%
Average | bad signal	35,481	30,790	4,690	13,467	46.9%	23,826	10.011	2,517	53.7%
Average | good signal	96,533	67,363	29,170	13,467	37.2%	**47,343**	10.626	5,485	18.8%
Change			*$0*				*1.9%*	*-$2,145*	*-7.4%*
Panel C: Non-linear vs. linear schedule									
2. Optimal single nonlinear schedule	63,170	47,377	15,792	13,467	42.5%	34,492	10.290	3,863	24.5%
6. Optimal single linear schedule	61,892	46,100	15,792	10,811	42.1%	34,066	10.280	4,271	27.0%
Change			*$0*				*1.0%*	*-$408*	*-2.6%*

Note: These calculations are based on a labor supply elasticity of 0.5 and the medium informative signal distribution.

Table 5: Sensitivity of the Welfare and Efficiency Gains to Parameter Assumptions

Effect of a menu of tax schedules relative to the optimal single nonlinear tax schedule

	Informativeness of signal	Elasticity of labor supply	Welfare gains ($)	Welfare gains (percent)	Welfare gains (percent of tax revenue)	Reduction in DWL ($)	Reduction in DWL (percent)	Reduction in DWL (percent of tax revenue)
1.	Weak	0.2	$72	0.21%	0.49%	$59	1.73%	0.40%
2.	Medium	0.2	$406	1.17%	2.75%	$318	9.64%	2.16%
3.	Strong	0.2	$535	1.54%	3.63%	$424	13.07%	2.88%
4.	Weak	0.5	$123	0.47%	0.78%	$83	2.16%	0.52%
5.	Medium	0.5	$636	2.39%	4.03%	$440	12.09%	2.79%
6.	Strong	0.5	$833	3.11%	5.27%	$542	15.11%	3.43%
7.	Weak	1.0	$173	0.91%	0.92%	$104	2.44%	0.55%
8.	Medium	1.0	$844	4.35%	4.47%	$568	14.07%	3.01%
9.	Strong	1.0	$1,066	5.46%	5.64%	$460	11.26%	2.44%