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UNCERTAIN LIFETIMES, PENSIONS, AND INDIVIDUAL SAVING

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ABSTRACT

Attempts to measure the impacts of pensions on household saving have occupied much of the literature in empirical public finance over the past decade. The emphasis here is on the annuity insurance aspects of social security and pensions. A simple life-cycle model is put forth to show that even an actuarially fair, fully funded social security system can reduce individual saving by more than the tax paid. Hence, previous partial equilibrium estimates of the impact of social security on saving drawn solely from consideration of the intergenerational wealth transfer at the introduction of the system are, if anything too small.

The large partial equilibrium effects are mitigated when initial endowments are considered. To the extent that the introduction of social security reduces the size of unplanned bequests, its net effect on the consumption of subsequent generations is diminished.

The final sections of the paper extend the approach to private pensions and address empirical issues. Using a model specification for individual wealth accumulation from the literature, potential offsets are interpreted according to the presence or absence of a bequest motive and according to the ability of individuals to adjust their participation in private pensions to counteract involuntary changes in social security.

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I. INTRODUCTION

Attempts to measure the impacts of pensions on household saving have occupied much of the literature in empirical public finance over the past decade. From a theoretical perspective, identifying the channels through which pensions affect the intertemporal consumption decision can help to distinguish among motives for saving (e.g., for retirement consumption or for bequests) and to explain empirical findings of the relationship between wealth and lifetime earnings. Proper quantification of the effects of pensions on saving is important for analyses of intergenerational equity, bequests and income distribution, and tax policy and saving.

Most of the attention in the pension-saving controversy has focused on the social security system, beginning with the time-series studies of Feldstein (1974).[1] The theoretical argument of Feldstein (and of Barro, 1974, 1978) has centered around the funding status of social security, i.e., the degree to which an unfunded social security system reduces private saving. Empirical tests of the effects of social security on saving in this vein have been conducted in the perfect certainty version of the life-cycle model (Modigliani and Ando, 1957; Modigliani and Brumberg, 1954).[2] In that approach, social security affects wealth accumulation only through its impact on individual intertemporal budget constraints. Disposable income falls by the amount of the tax. To the extent that the present value of benefits exceeds the present value of taxes paid, an increae in lifetime resources is generated, raising consumption in all periods.

The emphasis here is on the annuity insurance aspects of social security and pensions; uncertainty over length of life figures prominently in the explanation of the impact of pension annuities on non-pension saving. The evolution of public and private pensions is reviewed in response to missing markets for providing insurance for consumption in the fact of uncertain lifetimes. A simple life-cycle model is put forth in Section II to show that even an actuarially fair, fully funded social security system can reduce individual saving by more than the tax paid. Hence, previous partial equilibrium estimates of the impact of social security on saving drawn solely from consideration of the intergenerational wealth transfer at the introduction of the system are, if anything, too small.[3]

A related finding stems from the fact that under current U. S. law, social security taxes and benefits are calculated only up to an earnings ceiling. High-income individuals have incomplete access to the social security annuity system. Hence, even in the absence of an explicit bequest motive, the ratio of wealth to lifetime earnings could rise with the level of lifetime earnings. Constrained access to publicly provided pension annuities may provide an impetus to the growth of private pension annuities.

Individual wealth-age profiles are constructed in Section III given uncertain lifetimes and social security. The large partial equilibrium saving impacts found in Section II are mitigated when initial endowments are considered. Specifically, accidental bequests, which arise in the model because of lifetime uncertainty, provide an intergenerational link for saving decisions. To the extent that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of subsequent generations is diminished.

Section IV extends the approach to private pensions. The fifth section addresses empirical issues arising from the models of Section III and IV, primarily with respect to how one should interpret econometric estimates of "offsets" to individual saving attributed to pensions. Using a model specification for individual wealth accumulation from the literature, potential offsets are interpreted according to the presence or absence of a bequest motive and according to the ability of individuals to adjust their participation in private pensions to counteract involuntary changes in social security. Some conclusions and directions for future reserch are given in Section VI.

II. SOCIAL SECURITY AND SAVINGS IN A LIFE-CYCLE MODEL

A. Consumer Saving Decisions

The solution to an economic agent's intertemporal consumption problem subject to a lifetime resource constraint requires the equalization of expected marginal utilities of consumption across time. Otherwise, an increase in consumption at one point in his life at the expense of consumption at another time would raise lifetime utility, indicating that the initial allocation was suboptimal. The introduction of uncertainty generates a demand for insurance to diversify risks. Where insurance markets are incomplete or missing, the first-best optimum may be unattainable.

The type of uncertainty considered here is that over longevity; agents do not know when they will die. Yaari's (1965) seminal paper showed that with an uncertain lifetime, intertemporal utility maximization can dictate saving for the possibility of living longer than the expected lifetime to avoid deprivation in old age (excessively high marginal utility of future consumption).[4] That excess saving can be large. Kotlikoff and Spivak (1981, p. 379) found that for plausible underlying parameter values, the present expected value of unintended bequests represented almost 25 percent of initial wealth for a single male aged 55.

To emphasize this point, consider the following simple model. Agents are assumed to be selfish, in the sense that no bequests are desired. The retirement age Q is taken as exogenous, and individuals live Q periods for certain. The probability of having died in the interval [0,t] is p_t for each t; by assumption, p_t is equal to zero in the interval [0,Q]. Individuals have an expected lifetime of D years, with D' > D being the maximum age to which one can survive. That is, D is just the weighted average of the years t in (Q+1, D'], with weights $(1-p_t)$ for each t. Individuals receive a gross wage w_t in each period t during their working period; wages are assumed to grow at rate g. Income taxes on wages are levied at rate θ.

Following Yaari (1965) and Barro and Friedman (1977), let utility be additively separable, and let $U(C_t)$ be evaluated contingent on being alive at time t. That is, the consumer's intertemporal choice model is given by

$$(1) \qquad \max \sum_{t=o}^{D'} (1-p_t)\, U(C_t)\, (1+\delta)^{-t}$$

subject to

$$\sum_{t=o}^{D'} C_t (1+r)^{-t} = (1-\theta)\, w_o \sum_{t=o}^{Q} \left(\frac{1+g}{1+r}\right)^t ,$$

where C, δ, and r represent consumption and the (constant) subjective discount rate and real interest rate, respectively.

Carrying out the optimization in (1) assuming $U(C) = \frac{1}{\gamma} C^{\gamma}$ yields an optimal consumption stream of

$$(2) \qquad C_t = C_o\left(\frac{1 + r}{1 + \delta}\right)^{t/(1-\gamma)} (1 - p_t)^{1/(1-\gamma)} ,$$

where

$$(3) \qquad C_o = \frac{(1-\theta)\, w_o \sum_{t=o}^{Q} \left(\frac{1 + g}{1 + r}\right)^t}{\sum_{i=o}^{D'} (1 + r)^{i\gamma/(1-\gamma)} (1 + \delta)^{-i/(1-\gamma)} (1 - p_i)^{1/(1-\gamma)}} .$$

The extent to which uncertainty over length of life affects the stream of consumption depends on agents' degree of relative risk aversion, a transformation of γ, the elasticity of the marginal utility function. The higher is an individual's degree of relative risk aversion (or, equivalently, the lower is his intertemporal elasticity of substitution in consumption), the slower will his consumption grow over time.

B. The Introduction of Social Security

Access to a fair annuity market could remove the influence of lifetime uncertainty on consumption. Individuals could exchange a portion of their labor income when young to smooth consumption in old age. This role of annuities as a mechanism for sharing uncertainty about longevity is an integral part of Diamond's (1977) evaluation of the social security system, in which he focuses on the absence of complete markets for such contracts. Merton (1983) considers Pareto-improving social security programs in an intertemporal model in which human capital is not tradeable. Eckstein, Eichenbaum, and Peled (1983) consider the Pareto-improving potential of mandatory social security in the context of market failure in competitive insurance markets in the presence of adverse selection in the paradigm of Rothschild and Stiglitz (1976) or Wilson (1977).

If all individuals were identical in terms of their probabilities of survival,[5] then (with risk-neutral insurers) a competitive equilibrium in the provision of fair annuities would be possible. The existence of a competitive equilibrium may be precluded by asymmetries of information between individuals and insurers. This is, of course, the familiar "adverse selection" phenomenon discussed by Rothschild and Stiglitz (1976).[6] There may be additional "moral hazard" or "free-rider" barriers to the existence of an annuities market. If individuals conjecture that the state will support them in deprivation, the need to purchase annuities is diminished. A rigorous development of optimal second-best provision of annuities is beyond the scope of this paper.

Public provision of the annuities through public pensions is one possibility.[7] Moral hazard problems still make voluntary participation difficult. Consider, though, a public pension system ("social security") of the following form. Individuals are compelled to pay a payroll tax at rate t_s on gross wages, from which the social security system is funded. During retirement they receive annuity benefits S_t in each period t until death. The budget constraint in (1) becomes

$$(4) \qquad \sum_{t=o}^{D'} C_t(1+r)^{-t} = (1-\theta-t_s) \sum_{t=o}^{Q} w_o\left(\frac{1+g}{1+r}\right)^t + \sum_{t=Q+1}^{D'} S_t(1+r)^{-t}.$$

If benefits are set according to a replacement rate of the terminal wage, (i.e., where $S = Rw_Q$, where R is the earnings replacement rate) then the economy-wide actuarially fair benefit S satisfies the condition that[8]

$$(5) \qquad S \sum_{t=Q+1}^{D'} (1-p_t)\,(1+r)^{-t} = t_s \sum_{t=o}^{Q} w_o \left(\frac{1+g}{1+r}\right)^t .$$

Substituting the actuarially fair social security benefit into the budget constraint in (4) yields

$$(6) \qquad \sum_{t=o}^{D'} C_t (1+r)^{-t} = (1-\theta-t_s) \sum_{t=o}^{Q} w_o \left(\frac{1+g}{1+r}\right)^t +$$

$$t_s \left(\frac{\sum_{t=0}^{Q} w_o \left(\frac{1+g}{1+r}\right)^t}{\sum_{t=Q+1}^{D'} (1-p_t)(1+r)^{-t}} \right) \sum_{t=Q+1}^{D'} (1+r)^t$$

$$= (1-\theta+t_s(\omega-1)) \sum_{t=o}^{Q} w_o \left(\frac{1+g}{1+r}\right)^t ,$$

where ω arises because of the difference in discount rates under certainty and uncertainty and is equal to

$$\left(\sum_{t=Q+1}^{D'} (1+r)^{-t} \right) / \left(\sum_{t=Q+1}^{D'} (1-p_t)(1+r)^{-t} \right) .$$

Since ω is greater than unity, the system generates an increase in lifetime resources. Note that this increase in resources occurs even in a system which is actuarially fair and fully funded (i.e., in which contributions are invested and earn the market rate of return r in each period).[9] In reality, the initial cohorts participating in social security received a rate of return greater than the actuarially fair return (see Hurd and Shoven, 1983). This analysis focuses only on an actuarially fair system to point out that the negative impact of social security on individual saving does not hinge on such initial transfers.[10]

Table 1 shows the percentage increase in lifetime resources generated by an actuarially fair social security system under various assumptions about the real rate of interest and the social security payroll tax rate.[11] For example, when r=0.04 and t_s = 0.14, a 32 percent increase in lifetime resources is afforded by an actuarially fair social security system. Because the system generates an increase in lifetime resources, saving is reduced by more than the amount of the tax paid.

TABLE 1

PERCENTAGE INCREASE IN LIFETIME RESOURCES GENERATED BY ACTUARIALLY FAIR SOCIAL SECURITY

			t_s	
		0.10	0.12	0.14
r	0.02	29	35	41
	0.04	21	26	32
	0.06	16	19	23

Suppose that not everyone has equal access to the retirement annuities provided by social security, and that effective participation is higher for low-income individuals than for high-income individuals. Let $\bar{w}$ represent the ceiling on taxable income; the growth rate of the taxable wage base and the determination of the replacement rate are as before. The budget constraint in (6) then becomes

$$(7) \qquad \sum_{t=o}^{D'} C_t(1+r)^{-t} = \sum_{t=o}^{Q} (1-\theta + \tilde{t}_s(\omega - 1))w_o(\frac{1+g}{1+r})^t \quad ,$$

where $\tilde{t}_s$ is equal to $t_s(\frac{\bar{w}}{w_o})$. The impact of social security on an individual's lifetime resources depends on his income. As an annuity, social security administered in this way generates a smaller reduction in saving for high-income people than for low-income people.

III. SOCIAL SECURITY AND DYNAMIC WEALTH ACCUMULATION

A. Individual Saving Behavior

We can use the derivation from the previous section of the impact of mandatory actuarially fair social security on saving to study individual wealth accumulation over time. For any time t, the present value (at time 0) of an individual's accumulated stock of wealth, K_t (i.e., the present value of the "accidental bequest" of an individual who died in period t), can be expressed as

$$(8) \qquad K_t = \sum_{i=0}^{t} (1+r)^{-i}((1 - \theta - t_s)w_i + S_i - C_i).$$

Wages and social security benefits are the sources of income to the individual. w_t is zero in the interval [Q + 1, D'], and S_t is zero in the interval [0,Q]. Using the expressions derived before for w_t, S_t, and C_t, we can rewrite (8) as

$$K_t = (1-\theta-t_s)w_o \sum_{i=0}^{t} \left(\frac{1+g}{1+r}\right)^i - [1-\theta+t_s(\omega-1)]\left(w_o \sum_{i=0}^{Q}\left(\frac{1+g}{1+r}\right)^i\right)\left(\frac{\sum_{i=0}^{t}(1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'}(1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}\right),\ t \in [0,Q], \text{ and} \tag{9a}$$

$$K_t = (1-\theta-t_s)w_o \sum_{i=0}^{Q} \left(\frac{1+g}{1+r}\right)^i + t_s\left(w_o\sum_{i=0}^{Q}\left(\frac{1+g}{1+r}\right)^i\right)\frac{\sum_{i=Q+1}^{t}(1+r)^{-i}}{\sum_{i=Q+1}^{D'}(1-p_i)(1+r)^{-i}} \tag{9b}$$

$$-\ [1-\theta+t_s(\omega-1)]\left(\sum_{i=0}^{Q} w_o\left(\frac{1+g}{1+r}\right)^i\right)\frac{\sum_{i=0}^{t}(1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'}(1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}},\ t \in [Q+1,D'].$$

To provide an intuitive framework for considering an individual's wealth accumulation over the life cycle, note that if we denote the present values of lifetime labor income and social security taxes by V_L and V_S, respectively, we can rewrite (9a) and (9b) as:

$$\frac{K_t}{V_L} = \frac{(1-\theta-t_s)w_o\sum_{i=0}^{t}\left(\frac{1+g}{1+r}\right)^i}{V_L} \tag{10a}$$

$$-\ \left(1-\theta+\frac{V_S}{V_L}(\omega-1)\right)\left(\frac{\sum_{i=0}^{t}(1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'}(1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}\right), \text{ and}$$

$$\text{(10b)} \qquad \frac{K_t}{V_L} = 1 - \theta - \frac{V_S}{V_L} + \frac{V_S}{V_L} \frac{\sum_{i=Q+1}^{t} (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1-p_i)(1+r)^{-i}}$$

$$- \left(1 - \theta + \frac{V_S}{V_L}(\omega - 1)\right) \left(\frac{\sum_{i=0}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}} \right) .$$

The ratio K_t/V_L tracks an individual's accumulated stock of assets relative to lifetime earnings. In a world of no uncertainty over longevity, K_t/V_L is simply a function of age, and the results of the basic life-cycle model are reproduced, as long as the present values of social security contributions and benefits are equal. With lifetime uncertainty, wealth is still built up relative to earnings during the working period, but the rate at which consumption draws down accumulated wealth depends on survival probabilities and relative risk aversion. Because an actuarially fair social security system generates an increase in individual lifetime resources, lifetime consumption rises. Much of this increase in consumption comes during an individual's working life, as the need to save for retirement is reduced. Depending on risk aversion, while retirement consumption is higher in the presence of social security, dissaving in retirement is likely to be less than in the certainty case.[12]

The problem becomes more complicated when the insurance coverage provided by social security is not the same across individuals. Suppose again that there is a ceiling on the level of earnings against which payroll tax rates and replacement rates are calculated. If that ceiling is $\bar{w}$ in period 0 and grows at the same rate as the wage base, then the effective tax rate is not t_s, but $\tilde{t}_s = t_s(\frac{\bar{w}}{w_0})$. In that situation, equation (10) reveals that the ratio of wealth to lifetime earnings rises with the level of lifetime earnings, though at a decreasing rate.[13] This nonlinearity of saving rates with respect to lifetime earnings occurs in the absence of any explicit bequest motive. The implications of this effect for studies of the relationship between bequests and lifetime resources will be discussed later.

A related problem surfaces in the consideration of received bequests which augment lifetime resources. If we let A_0 represent the initial bequest, then we can rewrite equation (10) as

$$\text{(11a)} \quad \frac{K_t}{V_L} = \frac{A_0}{V_L} + \frac{(1-\theta-t_s)w_o \sum_{i=0}^{t} (\frac{1+g}{1+r})^i}{V_L}$$

$$- (1-\theta+\frac{V_S}{V_L}(\omega-1)) (\frac{\sum_{i=0}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}), \text{ and}$$

$$\text{(11b)} \quad \frac{K_t}{V_L} = (1-\theta+\frac{A_0}{V_L}-\frac{V_S}{V_L}) + \frac{V_S}{V_L} \frac{\sum_{i=Q+1}^{t} (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1-p_i)(1+r)^{-i}}$$

$$- (1-\theta+\frac{V_S}{V_L}(\omega-1)) (\frac{\sum_{i=0}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}).$$

As in the case of labor income, the rate at which lifetime resources are consumed depends on survival probabilites and risk aversion. The initial capital endowment A_0, which comes here from an accidental bequest from the previous generation, raises the individual's lifetime resources, increasing the consumption out of the present value of labor income and reducing the ratio of accumulated wealth to lifetime earnings. In the case in which participation in social security annuities is higher for low-income individuals, initial wealth endowments may smooth the nonlinearity in earnings of saving rates brought about by such a social security system.

To quantify the impact of social security and bequests on individual consumption and wealth-age profiles, the model embodied in equation (11) can be simulated for plausible parameter values. Simulations were performed over a set of different values of r, g, δ, and γ. The following relationships among the parameters are assumed: $r > g$, $r > \delta$, and $\delta > 0$.[14] There is some evidence on the value of γ in the literature. In their study of household portfolio allocation, Friend and Blume (1975) estimated the coefficient of relative risk aversion to be in excess of 2.0, implying a value of γ of at most -1.0. Farber's (1978) estimation of preferences of United Mine Workers from collective bargaining agreements yielded estimates of the coefficient of relative risk aversion of 3.0 and 3.7. Here we use three alternative values of γ: 0.25, - 1.0, and -3.0. g is assumed to equal 0.02, while $r = 0.04$, and $\delta = 0.03$.[15]

Table 2 reports K_t/V_L for selected ages. The optimization begins at age 20; individuals are assumed to retire at age 65. Figures are expressed as differences from the no-social security case. Column 1 reports values in the absence of social security, but with an initial bequest equal to 25 percent of lifetime earnings. Column 2 reports the reduction in K_t/V_L when the individual participates in a social security system in which $\tilde{t}_s = t_s = 0.14$. The third column shows the reduction in K_t/V_L for an individual whose effective tax rate (participation) in the system is only half of the nominal rate. Finally, the fourth column shows the change in the wealth-age profile for an individual with an initial bequest equivalent to 25 percent of his lifetime earnings and for whom $\tilde{t}_s = t_s = 0.14$.

TABLE 2

SOCIAL SECURITY AND K_t/V_L

K_t/V_L
(DIFFERENCE FROM THE NO-SOCIAL-SECURITY CASE)

AGE	$t_S = 0, \frac{A_0}{V_L} = .25$	$\tilde{t}_s = t_s = .14$	$\tilde{t}_s = .07,\ t_s = .14$	$\tilde{t}_s = t_s = .14,\ \frac{A_0}{V_L} = .25$
		$\gamma = 0.25$		
40	.150	-.208	-.104	-.057
50	.104	-.296	-.148	-.192
65	.041	-.393	-.196	-.337
70	.025	-.336	-.168	-.311
75	.014	-.273	-.137	-.260
80	.006	-.222	-.111	-.215
		$\gamma = -1.00$		
40	.155	-.201	-.101	-.046
50	.114	-.282	-.141	-.168
65	.060	-.386	-.193	-.326
70	.044	-.330	-.175	-.306
75	.031	-.321	-.161	-.290
80	.020	-.298	-.150	-.279
		$\gamma = -3.00$		
40	.160	-.195	-.097	-.035
50	.122	-.272	-.136	-.150
65	.072	-.362	-.181	-.307
70	.057	-.333	-.177	-.276
75	.044	-.304	-.152	-.260
80	.033	-.282	-.141	-.250

Several interesting patterns emerge. As expected, higher values of relative risk aversion (lower values of γ) encompass higher wealth in all periods, particularly in old age. Given uncertainty over longevity with no social security, an initial bequest of 25 percent of lifetime earnings is almost completely consumed by age 75 when $\gamma = 0.25$. When $\gamma = -1.0$, however, about 13 percent remains; nearly 20 percent remains in the case in which $\gamma = -3.0$.

The second and third columns, which address the implied resource gains made possible by access to actuarially fair social security, display the reduction in K/V_L attributable to social security (when $t_s = 0.14$). When the effective tax rate is less than the nominal tax rate, the reduction in K/V_L is smaller. Hence, effective participation in social security which rises with income, *ceteris paribus*, leads to saving rates which rise with earnings (and, *a fortiori*, stocks of wealth which rise with earnings). As γ is decreased (higher relative risk aversion), the social security system permits greater wealth decumulation in old age. In other words, the more risk-averse the individual, the less of the "income effect" of social security participation consumed prior to retirement. Those findings are intuitive, since the value of annuity is highest for very risk-averse individuals.

The last column of Table 2 shows the combined impact on the wealth-age profile of the combination of effective participation in social security at the nominal rate (14 percent here) and the receipt of an initial bequest. From the information in the first column of Table 2 and from a comparison of the second and fourth columns, most of the impact of initial bequests on consumption occurs prior to retirement.

That is, the differences in K/V_L in old age (with respect to the no-social security case) are almost invariant to the initial bequest (at least in the range examined here).

We can now consider the issue of the consumption pattern of the elderly, addressed earlier by Mirer (1979) and by Davies (1981). Given uncertainty over length of life, the rapid reduction in consumption (relative to lifetime resources) in old age confirms the findings in Davies (1981) that positive net worth may continue indefinitely after retirement. The resulting slow decline (or possible increase) in net worth in retirement ignores, however, the decline in the value of the social security annuity. Since the model implies that individuals acknowledge the actuarial value of their social security holdings, that dissaving must take place.

For each year t in retirement, withdrawals to finance consumption relative to lifetime earnings can be expressed as

$$(12) \qquad \frac{C_t}{V_L} = \frac{(1 - \theta + t_s(\omega - 1))\,(\frac{1+r}{1+\delta})^{\frac{t}{1-\gamma}}(1-p_t)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{i\gamma/(1-\gamma)}(1+\delta)^{-i/(1-\gamma)}(1-p_i)^{1/(1-\gamma)}} \, .$$

Correspondingly, in each year t, the decline in the annuity value of social security relative to lifetime earnings is

$$(13) \qquad \frac{\Delta V_{St}}{V_L} = \frac{S(1 - p_t)}{V_L} = \frac{t_s\,(1 - p_t)}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}} \, .$$

The relationship between these two uses of total (pension plus non-pension wealth) depends on γ and the distribution of survival probabilities. To see the importance of considering the "dissaving" of annuity wealth, Table 3 contrasts consumption and annuity revaluations in retirement of the case of $\gamma = -1$, $t_S = 0.14$, $r = 0.04$, and $\delta = 0.03$.[16] Note that annuity dissaving (the reduction in the actuarial value of the social security annuity) is substantially greater than the reduction in non-pension wealth.

TABLE 3

ANNUITY AND NONANNUITY DISSAVING IN RETIREMENT

AGE	C_t/V_L	$\Delta V_{St}/V_L$	$\Delta V_{St}/C_t$
66	.033	.084	2.55
70	.032	.072	2.25
75	.029	.055	1.90
80	.026	.038	1.46
85	.020	.021	1.05

To estimate correctly the net effect of social security on individual consumption and wealth accumulation after the commencement of the system, we must also consider its impact on intergenerational transfers (here, accidental bequests). By affecting the accidental bequests of previous generations, social security further influences individual consumption patterns. It is to this issue which we now turn.

B. Long-Run Effects on Individual Saving

Given uncertainty over length of life, an actuarially fair social security system can reduce individual saving by more than the amount of the taxes paid. For plausible underlying assumptions about individual discount rates, survival probabilities, and the intertemporal elasticity of substitution in consumption, the magnitude of that reduction is substantial. The partial equilibrium conclusion is clear -- estimates of the reduction in individual saving brought about by social security which focus only the extent to which the system delivers a present value of anticipated benefits greater the present value of taxes paid are, if anything, an underestimate. Before discussing general equilibrium interpretations of this finding (in the sense that the wage rate and real interest rate are endogenous and respond to changes in the saving rate), it is important to address the issue raised in the simulation exercises of the links among generations provided by accidental bequests.

An initial bequest from an "early death" of one's parent raises the beneficiary's consumption relative to lifetime earnings. In the model, the size of that bequest depends on the testator's coverage by social security and his age at death. By facilitating greater consumption out of lifetime earnings, social security reduces the accidental bequest. On that account, the initial resources available to the heir (and, from Table 2, consumption when young) are lower. Even within the partial equilibrium analysis, the impact of social security on the consumption and saving patterns of individuals in a given generation depends on the balance between the effective increase in lifetime resources made possible by access to a fair annuity and the reduction in inheritances because of that impact on the saving of the previous generation.[17]

To see this more clearly, note that for an individual receiving an accidental bequest from a "parent" who died at age t in the interval [Q + 1, D'], the reduction in the bequest because of the parent's participation in social security is[18]

$$(14) \qquad \frac{dA_0}{dV_S} = (1 + r)^t \{-1 + \frac{\sum_{i=Q+1}^{t} (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}}$$

$$-(\omega-1) \frac{\sum_{i=0}^{t} (1 + r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1 - p_i)^{\frac{1}{1-\gamma}}} \}.$$

We know from the individual's optimization problem that social security generates an increase in lifetime resources of $V_S(\omega-1)$. If the "parent" and "child" have the same lifetime earning potential (i.e., the same w_0), then the net effect of social security is to increase lifetime resources by the amount E, where

$$(15) \qquad E = V_s(\omega-1) \{1 - (1+r)^t (\frac{\sum_{i=0}^{t} (1 +r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1 - p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1 + \delta)^{\frac{-i}{1-\gamma}} (1 - p_i)^{\frac{1}{1-\gamma}}})$$

$$+ (1+r)^t (\omega-1)^{-1} (\frac{\sum_{i=Q+1}^{D'} (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1 - p_i)(1 + r)^{-i}} - 1)\}.$$

Note that if the parent lived to the maximum age, then E = 0. In general, the net increment to lifetime resources E made possible by social security depends on the age at which the parent died (magnitude of the accidental bequest).[19] To consider the net effect of social security on saving n generations after its introduction, an n-generational analogue to equation (15) could be constructed given the ages of death of previous testators. The role of family mortality history is important here, as individuals whose "ancestors" all died impact of a pay-as-you-go social security system on the capital stock in a general equilibrium. For plausible parameter values, he found that the positive lifetime wealth increment traceable to social security (because of growth of the wage base) caused a twenty-percent steady-state reduction in the capital stock in the general equilibrium.[21] While this is certainly substantial, it is roughly half of his partial equilibrium effect, which is directly related to the extent to which benefits are unfair (i.e., to the extent that the present value of benefits exceeds the present value of social security taxes paid).

While general equilibrium calculations of the impact of social security on aggregate saving are not performed here, it is useful to differentiate the effects implied by the life-cycle model under uncertain lifetimes from those implied by the perfect certainty life-cycle model. First the partial equilibrium effect in the uncertain lifetime framework must be larger; the commencement of even an actuarially fair, fully funded social security system will substantially reduce individual saving. When the effect of social security on accidental bequests (which arose from the life-cycle model under uncertain lifetimes) is considered, however, the partial equilibrium impact is reduced over time. Smaller changes are induced in factor returns, so that the divergence between the general equilibrium impact and the partial equilibrium impact (including the intergenerational component) will be reduced.

early will receive large bequests relative to those whose parent lived a long time.

Members of the first generation to participate in the social security system benefit in two respects, as their lifetime resources are augmented both by the bequests from the (uninsured) previous generation and the gains from participation in the social security annuity system. The reduced value of accidental bequests permits smaller consumption gains for subsequent generations. While it is true that social security reduces individual saving to a lesser degree in the generations after its introduction, there is still a reduction in the long-run capital stock. Ultimately, to consider the potential welfare gains from compulsory pensions, the tradeoff between the benefits to early participants from access to the annuities and the costs to generations that follow of a lower capital stock must be examined.

C. General Equilibrium Effects of Social Security on the Capital Stock

The partial equilibrium effects of social security on individual saving will be dampened in a general equilibrium analysis of the impact of social security on aggregate capital formation.[20] The reduction in individual wealth accumulation brought about by social security will induce changes in factor returns, exhibiting both income and substitution effects on consumption. A higher real interest rate decreases lifetime resources; in addition, a higher rate of interest reduces the price of consumption in old age.

Kotlikoff (1979a), using a life-cycle model with no uncertainty over longevity and a Cobb-Douglas production technology, considered the

IV. APPLICATION TO PRIVATE PENSIONS

To the extent that high-income individuals (those for whom $w_o > \bar{w}$) are constrained to less than their desired participation in social security, there is excess demand for social security annuities. Adverse selection and the possibility of multiple insurance[22] still render unlikely the provision of such annuities by competitive insurance companies. Employer-sponsored private pension funds may act to fill this gap. Employers are likely to have better information on individual workers' life expectancies than would a disinterested insurance company. Second, by definition, such annuities can only be purchased at an individual's place of work; multiple insurance is not possible. Finally, the pension instrument may provide an added degree of freedom for the firm in influencing worker behavior.[23]

The tax treatment of pension plans is an important consideration. Social security taxes are levied on gross earnings, and prior to the 1983 amendments to the Social Security Act, benefits were not considered taxable income. For private pension plans, employer contributions are a deductible business expense and are not regarded as taxable income to employees until benefits are paid. Pension fund earnings accumulate tax-free until disbursement. Upon distribution, taxes paid on benefits are presumably less than corresponding wage tax payments, since earnings (and hence tax rates) are lower in retirement. Moreover, special retirement income credits further diminish effective tax rates on pension benefits.

At this point, we will assume that covered workers take their participation in plans as given; the implications of relaxing that assumption will be discussed later. For simplicity, let P be the actuarially fair pension benefit in retirement (determined by the product of a replacement rate and the terminal wage) corresponding to an implicit reduction in wages at rate t_p.[24]

In the context of this model, the worker bears only $(1-\theta)t_p$ of the wage reduction, where θ is the marginal income tax rate. Benefits are taxed at rate $\hat{\theta}$, where $\theta > \hat{\theta}$. We introduce a parameter β to measure the extent to which benefits received are actuarially fair. That is, an actuarially fair pension benefit P can be constructed just as in the case of social security annuity benefits in equation (5). Benefits received are equal to βP, where P solves

$$(16) \qquad P \sum_{t=Q+1}^{D'} (1 - p_t)(1 + r)^{-t} = t_p \sum_{t=0}^{Q} w_o\left(\frac{1+g}{1+r}\right)^t .$$

For received annuity payments to be actuarially fair, it must be the case that $\beta = 1$; less-than-fair benefits are associated with $\beta < 1$.

Given participation in social security, the budget constraint in (7) can be rewritten as

$$(17) \quad \sum_{t=o}^{D'} C_t(1+r)^{-t} = (1-\theta-\tilde{t}_s)(1-t_p) \sum_{t=o}^{Q} w_o\left(\frac{1+g}{1+r}\right)^t + (S+(1-\hat{\theta})\beta P) \sum_{t=Q+1}^{D'} (1+r)^{-t}$$

$$= (1-\theta-\tilde{t}_s)(1-t_p) \sum_{t=o}^{Q} w_o\left(\frac{1+g}{1+r}\right)^t + (\tilde{t}_s + (1-\hat{\theta})\beta t_p) \sum_{t=0}^{Q} w_o \left(\frac{1+g}{1+r}\right)^t \left(\frac{\sum_{t=Q+1}^{D'} (1+r)^{-t}}{\sum_{t=Q+1}^{D'} (1-p_t)(1+r)^{-t}}\right)$$

$$= [1-\theta + \tilde{t}_s(\omega-1) + t_p((1-\hat{\theta})\beta\omega-(1-\theta-\tilde{t}_s))] \sum_{t=o}^{Q} w_o\left(\frac{1+g}{1+r}\right)^t .$$

As shown before, $\omega > 1$. As long as β is close to unity, for any reasonable assessment of the relationship between θ and $\hat{\theta}$, $(1-\hat{\theta})\beta\omega > 1-\theta-\tilde{t}_s$. This is certainly true for the estimated tax rates used by the Treasury in calculating the tax expenditure associated with pension tax subsidies, namely $\theta = 0.23$ and $\hat{\theta} = 0.115$ (See Munnell, 1982, p. 44 for details). Because of the tax deductibility of pension contributions, even in a world of certainty over longevity (ω=1), a funded private pension can still generate an increase in lifetime resources for the individual.

The tax treatment of pension contributions reinforces the role of private pension annuities in alleviating the rationing of public annuities. The effective contribution rates (participation rates) in the public and private pension systems both depend on the income of the individual. Recall that $\tilde{t}_s = t_s(\bar{w}/w_o)$, where $\bar{w}$ is the ceiling on taxable earnings. Under a progressive tax system, the marginal tax rate also depends on income (i.e., $\theta'(w_o) > 0$). Hence for given (assigned) nominal participation rates in social security and private pensions, high-income individuals receive a greater effective increase in lifetime resources from private pensions of the sort described here. This effect may be desirable if one reason for the private pension system is to supplement the rationed access to social security annuities for high-income workers. Capital market imperfections and borrowing restrictions would still limit the demand for pension annuities.

We can now reconstruct the wealth-age profiles given both social security and private pensions. Wages and public and private pension annuity payments are the sources of income to the individual. w_t is zero in the interval [Q+1, D'], and S_t and P_t are zero in the interval

[0,Q]. Using the expressions derived before for w_t, S_t, P_t, and C_t, and denoting the present values of lifetime labor income, social security taxes, and implicit wage reductions to finance private pensions by V_L, V_S, and V_P, respectively, we can construct wealth-age profiles relative to lifetime earnings. That is,

$$(18a)\qquad \frac{K_t}{V_L} = \frac{(1-\theta-t_s)(1-t_p)\, w_o \sum_{i=o}^{t} \left(\frac{1+g}{1+r}\right)^i}{V_L}$$

$$-\left\{ 1-\theta + \frac{V_S}{V_L}(\omega-1) + \frac{V_P}{V_L}\left[(1-\hat{\theta})\beta\omega - (1-\theta-\tilde{t}_s)\right] \right\} \times$$

$$\left[\frac{\sum_{i=o}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=o}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}\right],\quad t \in [0,Q],$$

and

$$(18b)\qquad \frac{K_t}{V_L} = \left(1-\theta-\frac{V_S}{V_L}\right)\left(1-\frac{V_P}{V_L}\right) + \left[\frac{V_S}{V_L} + (1-\hat{\theta})\,\beta\,\frac{V_P}{V_L}\right]\left(\frac{\sum_{i=Q+1}^{t} (1+r)^{-i}}{\sum_{i=Q+1}^{D'} (1-p_i)(1+r)^{-i}}\right)$$

$$-\left\{1-\theta + \frac{V_S}{V_L}(\omega-1) + \frac{V_P}{V_L}\left[(1-\hat{\theta})\beta\omega - (1-\theta-\tilde{t}_s)\right]\right\}$$

$$\left[\frac{\sum_{i=o}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=o}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}} (1+\delta)^{\frac{-i}{1-\gamma}} (1-p_i)^{\frac{1}{1-\gamma}}}\right],\quad t \in [Q+1, D'].$$

The addition of private pension annuities complicates the evaluation of the effect of a change in compulsory social security holdings on non-pension wealth. Suppose that individual participation in private pension annuities is not invariant to changes in social security annuities. Let ψ_{ps} represent the magnitude of that discretionary adjustment, i.e.,

$$(19) \qquad \psi_{ps} = dV_p/dV_S.$$

Then from equation (18a), the impact of a change in social security wealth on the non-pension wealth of a non-retired individual is

$$(20) \qquad \frac{dK_t}{dV_S} = - \; [\omega - 1 + \psi_{ps}((1 - \hat{\theta})\beta\omega - (1 - \theta - \tilde{t}_s)] \times$$

$$\left[\frac{\sum_{i=0}^{t} (1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}{\sum_{i=0}^{D'} (1+r)^{\frac{i\gamma}{1-\gamma}}(1+\delta)^{\frac{-i}{1-\gamma}}(1-p_i)^{\frac{1}{1-\gamma}}}\right].$$

If $\psi_{ps} = 0$, then the impact of a change in holdings of social security annuities has the same influence on lifetime resources as before. When $\psi_{ps} < 0$ (i.e., increases in involuntary social security annuitization can be at least partially undone through changes in private pension participation), the impact of social security on individual wealth accumulation will also depend on the extent to which private pension annuities are actuarially fair (i.e., on the value of β) and on the tax advantages of pensions as compensation (values θ and $\hat{\theta}$).

When coverage by social security is higher for low-wage earners than for high-wage earners, we can use equation (13) to examine the impact on non-pension wealth of change in the social security payroll tax rate (index of participation). First, since the effective tax rate $\tilde{t}_s = t_s(\bar{w}/w_0)$, a given increase in the nominal tax rate translates into a smaller increase in V_S (and, ceteris paribus, a smaller displacement of non-pension wealth) for high-income workers (for whom $w_o > \bar{w}$) than for low-income workers (for whom $\bar{w} > w_o$). When private pension participation is responsive to changes in social security annuity holdings (i.e., when $\psi_{ps} < 0$), then for a given offset factor ψ_{ps}, high-income individuals receive a smaller total offset than low-income individuals.[25]

In the next section, we take up issues associated with empirical treatment of forms of (18), emphasizing the role of assumptions about the structure of social security and private pensions, the presence or absence of a bequest motive, and the extent to which participation in private pension annuities is voluntary.

V. EMPIRICAL ISSUES

Gathering econometric evidence of the impact of social security and private pension annuities on household saving in the context of lifetime uncertainty entails estimation of the wealth profiles consistent with equation (18). Suppose one has a cross-section of household or individual data with information on earnings, assets and liabilities, pensions, and individual and labor-market characteristics. Most previous empirical examinations of the impact of social security on non-pension wealth have employed versions of the following specification:

$$(21) \qquad W_i = f(Y_i^*, A_i, Z_i) - \beta PW_i,$$

where i refers to the individual and W, Y^*, A, Z, and PW are non-pension wealth, lifetime earnings, a vector of socioeconomic variables and individual characteristics, and the actuarial present value of anticipated pension benefits, respectively.

Consider for example a wealth accumulation equation of the following form:

$$(22)\quad \left(\frac{W}{Y^*}\right)_{it} = g(Y_i^*) + j(A_{it}) - a_s\left(\frac{SSW}{Y^*}\right)_i - a_p\left(\frac{PPW}{Y^*}\right)_i + \gamma' Z_{it} + \varepsilon_i.$$

Anticipated pension benefits are divided into two components, social security (SSW) and private pensions (PPW), to allow for different effects on saving. a_s and a_p are coefficients to be estimated. j is a function of age. Finally, the function g can be specified to test the nonlinearity in income of the ratio of wealth to permanent income.[26]

Recalling the wealth-age profiles constructed from the theoretical model in the previous section, the specification of wealth accumulation in (22) illustrates the importance of the inclusion of the pension variables. With respect to social security, if individual earnings replacement rates are negatively correlated with earnings for high-income workers (as in the U.S. system), the measured effect of Y^* on W/Y^* would be biased upward if the social security variable were omitted. The correlation of PPW/Y^* with Y^* is less clear. Similarly, if one wanted to use (22) to interpret the impact of social security on saving, then omitting the private pension variable biases the estimate of a_s toward zero. The extent of the bias depends on the degree of "integration" of the benefits of the two systems and on the extent to which private pension participation is discretionary.

Many recent the empirical studies have tried to isolate the impact of pensions on the level of non-pension saving (using cross-section data) in models similar to (21) or (22). Estimating a version of (22) in level form, Feldstein and Pellechio (1979) found that an extra dollar of social security wealth reduced non-pension wealth by approximately a dollar, using data from the Federal Reserve Board's 1962 Survey of Consumer Finances; they had no data on private pensions. Some of their specifications also found a positive relationship between the ratio of net worth to permanent income and the level of permanent income. Using data from the Retirement History Survey, Diamond and Hausman (1982) found a social security offset of 30 to 50 cents (with a smaller non-pension wealth reduction for changes in private pension wealth). They also found evidence of a positive relationship between W/Y^* and Y^*.

Employing a logarithmic form of (22) for Canadian data, King and Dicks-Mireaux (1982) estimated the offset to non-pension wealth from a one-dollar increase in social security wealth to be 24 cents (10 cents for private pensions), with offsets of approximately dollar-for-dollar for individuals in the top decile of the wealth distribution. Hubbard (1983) estimated a similar model for the U.S. (using data from the President's Commission on Pension Policy), finding a mean offset for social security wealth of 33 cents (16 cents for private pensions), with social security offsets in excess of dollar-for-dollar for those in the top decile of the wealth distribution.

Whether the versions of (21) and (22) used in the empirical studies described above can be justified according to a consistent set of economic assumptions depends on the structure of annuity markets and on whether or not a bequest motive exists. The basic model presented earlier assumes complete market failure in the private provision of annuities and the absence of a bequest motive. Theoretical possibilities encompass assumptions along the dimensions of "perfectness" of private annuity markets and the presence or absence of a bequest motive.

In addition, econometric estimates of the impact of private pension annuities on non-pension wealth accumulation as well as of the links between changes in social security annuities and private pension participation are necessary for an empirical consideration of the impact of the social security system on individual saving. The latter link is both important and not often noted. That annuity markets are extremely imperfect in the real world is not evidence per se of a severe market failure, as individuals have some control over their participation in private pensions either explicitly (for participants in defined-contribution plans) or implicitly (through choice of employer). To the extent that individuals adjust their pensions for variation in social security annuities, the effective annuity market may be quite large. The magnitude of that adjustment must be resolved empirically.

As an empirical proposition, it is important to ascertain the degree of discretion in individual private pension plan participation. We can consider an auxiliary model of the form

$$(23) \qquad \left(\frac{PPW}{Y^*}\right)_i = \lambda' Z_i - \psi_{ps} \left(\frac{SSW}{Y^*}\right)_i ,$$

where ψ_{ps}, as before, represents the adjustment of private pension annuities to involuntary changes in social security annuities. Again, apart from issues of substitutability (i.e., if $\beta=1$), a value of zero for ψ_{ps} indicates no discretion in pension participation; $\psi_{ps} = -1$ indicates complete discretion.

Given the assumption of market failure in the provision of non-pension annuities, four potential cases can be considered along the two dimensions of (i) bequest motives and (ii) discretion in private pension participation. As a first case, suppose that there is no bequest motive and that private pension participation is exogenous to individual decisions. The offset to non-pension wealth of a change in compulsory social security annuities corresponds to the level described earlier; that is, the present value of anticipated (actuarially fair) social security benefits should displace non-pension wealth by more than dollar for dollar (in the absence of capital market restrictions). If effective replacement rates are nonlinear in earnings, high-income individuals are rationed in their access to social security annuities, and saving rates will rise with the level of permanent income.

Second, suppose that while there is no bequest motive, private pension participation is completely under individual control. In the limit, if private pension annuities are also actuarially fair ($\beta=1$ in equation (17)), there would be no restricted access to fair annuities, and W/Y^* would be independent of the level of Y^*. Involuntary increases in compulsory annuities (social security) would be completely reflected in reduced holdings of private pension annuities and not in the level of non-pension wealth. For intermediate versions of this second case, both a smaller offset to non-pension wealth from a change in social security benefits and a smaller effect of Y^* on W/Y^* would be expected relative to the first case.

The existence of a bequest motive changes the predicted effect of changes in compulsory social security annuities on the level of non-pension wealth and complicates the distinction of "annuity rationing" effects from the data. The third and fourth cases embody the sort of "bequest motive" described above, evidenced by levels of non-pension wealth relative to permanent income that rise with permanent income.[27]

The third case is described by the existence of an operative bequest motive in conjunction with discretionary private pension participation. In this case, involuntary changes in social security participation will have no impact on non-pension wealth; the changes are counteracted by offsetting movements in private pension holdings. With discretion in pension participation, there is no restriction of "fair" annuity purchases, so that a nonlinear relationship between W/Y^* and Y^* is traceable to the desire to leave bequests.

The fourth case combines a bequest motive with exogenous participation in private pensions. Again, the reduction in non-pension wealth attendant to an increase in holdings of social security annuities will be less than in the first case. An observation that saving rates out of permanent income increase with permanent income could reflect a combination of a bequest motive and rationed access to pension annuities.

The cases are summarized with respect to interpretations of the offset parameter a_s and nonlinearity of the ratio of non-pension wealth to permanent income with respect to permanent income in Figures 1 and 2 below. Note that the predicted effects of changes in social security wealth and of changes in permanent income on individual wealth accumulation depend greatly on assumptions about bequest motives and on the size of the effective private annuity market afforded by access to private pensions. In reality, of course, the degree of discretion in private pension annuity holdings can vary anywhere between "none" and "complete." Estimation of the impact of changes in compulsory social security annuities on holdings of private pension annuities (e.g., equation (23) above) can help to allocate observed nonlinearities of saving rates with respect to the level of earnings between annuity rationing and bequest motives.[28]

FIGURE 1

OFFSET TO NON-PENSION WEALTH FROM INVOLUNTARY INCREASE IN SOCIAL SECURITY ANNUITIES

	Complete Discretion in Pension	No Discretion in Pension
Bequest Motive	$a_s = 0$	$a_s > 0$ but less than value below
No Bequest Motive	$a_s = 0$	$a_s > 1$

observed nonlinearity in the wealth-income relationship reflects both a bequest motive and incomplete access to retirement annuities outside social security.

VI. CONCLUSIONS AND EXTENSIONS

Assessing the impact of social security and private pensions on individual wealth accumulation is important for many analyses of welfare, capital formation, and equity in the distributions of income and wealth. Previous research efforts along the lines of Feldstein (1974) have addressed the funding status of social security and pensions. The focus here is on insurance features of pension annuities with respect to the problem of uncertainty over length of life.

The first part of the paper considers the introduction of social security into an economy with market failure in the provision of private annuities. The principal findings are three. First, in such a world, even an actuarially fair, fully funded social security system can substantially reduce individual saving, though individual welfare is initially improved. Hence, partial equilibrium estimates of the impact of social security on saving which rely solely on the extent to which individuals earn a more than fair return on social security are underestimates of the true effect.

Second, under current U. S. law, social security taxes and benefits are calculated only up to an earnings ceiling. High-income individuals have incomplete access to the social security annuity system. Hence, even in the absence of an explicit bequest motive, the ratio of wealth to lifetime earnings would rise with the level of lifetime earnings.

FIGURE 2
INTERPRETATION OF NONLINEARITY OF W/Y^* WITH RESPECT TO Y^*

	Complete Discretion in Pension	No Discretion in Pension
Bequest Motive	Any nonlinearity due to bequest motive	Combination of annuity rationing and bequest motive
No Bequest Motive	W/Y^* independent of Y^*	Any nonlinearity due to annuity rationing

The theoretical results in sections II - IV and the summary of implications in Figures 1 and 2 facilitate interpretation of the coefficients of (22). We can infer information about bequest motives and the impact of involuntary changes in social security annuities on non-pension wealth. First, consider the case in which the wealth-earnings relationship exhibits little nonlinearity in earnings. As ψ_{ps} approaches minus one, the model implies no bequest motive (of the sort outlined here) and no substantial impact of changes in social security on the level of non-pension wealth. As ψ_{ps} approaches zero, the implication of no bequest motive is joined by the prediction of a significant impact of a change in social security on non-pension wealth.

Second, suppose that the ratio of wealth to permanent income increases with permanent income. As ψ_{ps} approaches unity in absolute value, a bequest motive is ratified (since discretionary pensions provide an effective annuity market); the impact of involuntary changes in social security will fall almost entirely on holdings of private pension annuities. The closer is ψ_{ps} to zero, the greater will be the impact of changes in social security on non-pension wealth, so that the

Constrained access to publicly provided pension annuities may provide an impetus to the growth of private pension annuities.

Third, the partial equilibrium impact of social security and private pension annuities on non-pension saving is reduced when initial endowments are considered. For example, to the extent that the introduction of social security reduces the size of accidental bequests, the net effect of social security on the consumption of suceeding generations is mitigated. In addition, general equilibrium considerations, primarily the endogeneity of factor returns, can be expected to reverse part of the partial equilibrium impact. Because of these two considerations, the impact of social security on the steady-state capital stock is likely to be smaller than the partial equilibrium impact.

To provide an interpretation of econometric measures of the impact of pensions on non-pension saving, two additional considerations are important. Theoretical possibilities encompass assumptions along the dimensions of "perfectness" of private annuity markets (in this case, the ability to adjust private pension participation in response to involuntary changes in social security annuities) and the presence or absence of a bequest motive. Four cases are generated, as shown in Figures 1 and 2 in the text. The predicted effects of changes in social security wealth and of changes in permanent income on individual wealth accumulation depend on assumptions about bequest motives and on the size of the effective private annuity market afforded by access to private pensions.

Two immediate extensions to the models presented here are left as tasks for future research. First, additional research is needed on private annuity markets to determined the actual extent of market failure. Second, given the current political environment, introducing uncertainty over future social security benefits may be appropriate. That uncertainty would modify the wealth impacts derived here.

The debate over the influence of pensions on individual saving brings together questions of consumer choice under uncertainty and the effectiveness of fiscal policy. Researching the relationships among social security, private pensions, annuity markets, and bequests facilitates close empirical scrutiny of models of individual and aggregate saving, permitting consideration of the welfare effects of compulsory pensions. In addition, while this paper has concentrated on annuity insurance, similar approaches could be used to study the impacts of other social insurance programs on national saving.

NOTES

[1]Earlier studies for private pensions include those of Cagan (1965), Katona (1964), and Munnell (1974). Feldstein's results have by no means gone unchallenged; see for example Leimer and Lesnoy (1982) and the reply in Feldstein (1982). Microeconomic (cross-section) evidence has generally been supportive of the proposition that social security has reduced individual saving. See Feldstein and Pellechio (1979), Kotlikoff (1979b), Blinder, Gordon, and Wise (1981), Diamond and Hausman (1982), King and Dicks-Mireaux (1982), and Hubbard (1983).

[2]Empirical tests of the life-cycle model under certainty have tested the hypothesis of a hump-shaped wealth-age profile, but results have by no means unambiguously validated the model. See for example White (1978), Mirer (1979), and Kurz (1981). Even after controlling for the effects of permanent income, Blinder, Gordon, and Wise (1981), Diamond and Hausman (1982), King and Dicks-Mireaux (1982), and Hubbard (1983) found results only mildly supportive of the basic theory. Other studies have addressed the possibility of other motives for saving. Kotlikoff and Summers (1981) reject the ability of the life-cycle model to explain wealth accumulation in the U. S., putting forth a major role for bequests.

[3]Abel (1983) takes up the intergenerational consequences of this point in a two-period overlapping-generations model, with the implication that the insurance features of social security may reduce inequality in the distribution of wealth.

[4]The precise direction of the influence of this uncertainty for saving is unclear. Heightened uncertainty over the length of life may lead to more saving (because of a longer than expected lifetime) or to less saving (to maintain present consumption). In the argument of Yaari (1965), two individuals with identical tastes, income, and investment opportunities are compared. The difference between them is that one lives T periods for certain while the other faces an uncertain lifetime of t periods, up to a maximum of T periods. Given a shorter expected life, uncertainty over length of life unambiguously leads to increased initial consumption. Champernowne (1969) and Levhari and Mirman (1977), on the other hand, consider two agents with identical expected lives, but differing the distribution of length of life. In either case, the impact of uncertainty over the length of life on wealth accumulation of a risk-averse individual is ambiguous and depends on the parameters of the model.

[5]Note that this does not require that they <u>actually</u> die at the same time.

[6]Rothschild and Stiglitz (1976) show that there will be no "pooling equilibrium," where all buy the same contract. They illustrate conditions under which a "separating equilibrium" occurs, in which different contracts are purchased by the risk groups. Following their argument and that of Riley (1979), if there is a fairly continous distribution of survival probabilities, there is little hope for an equilibrium. Eckstein, Eichenbaum, and Peled (1983) consider the Pareto-improving potential of mandatory social security in the context of market failure in competitive insurance markets in the presence of adverse selection.

[7]Previous work in this area in the context of pensions includes the contributions of Davies (1981) and Sheshinski and Weiss (1981). Davies used a life-cycle model under uncertain lifetime to address the phenomenon of slow dissaving in retirement. The presence of pensions in his simulation model (using Canadian data) reduced, but by no means eliminated, the effect of uncertainty on retirement consumption. In the model of Sheshinski and Weiss, the ultimate impact of social security on saving depends on the availability of a private annuity market. (The problem will arise here in Section IV in the context of discretion in private pension participation.) They found that, at the optimum, Yaari's (1965) result holds, namely that private savings are reserved for bequests, while social security benefits are used to finance retirement consumption.

[8]The actuarially fair benefit is constructed with respect to economy-wide survival probabilities. It is true that individuals who believe they will die "young" will want to purchase less than the "average optimal" amount of social security annuities, while those who expect to live a long time will want more. Both groups are better off, however, with the mandatory social security than without it, since in its absence, adverse selection is assumed to foreclose the possibility of a market of private annuities. A discussion of the potential separating equilibria in the private provision of annuities which may arise after the imposition of mandatory social security is given Eckstein, Eichenbaum, and Peled (1983).

[9]While the imposition of the social security system increases lifetime resources, nothing has been said about the optimal tax rate. Current law prohibits the explicit leverage of anticipated social security benefits. The ability to implicitly borrow against future benefits will depend on differences in w_0 (differences in ability to procure "unsecured" loans). Under the assumption of complete (explicit and implicit) nonmarketability of benefits, we can demonstrate that there is an interior solution ($0 < t_s < 1$) for the individual's optimal tax rate (a sufficient statistic of participation as long as benefits are actuarially fair). The intuition is that while the purchase of "social security retirement annuities" increases resources available in old age, it decreases the resources available for current consumption. The optimal tax rate $\hat{t}_s$ in such a world is just

$$\hat{t}_s = \frac{1}{\omega+1}\left[\frac{\sum_{i=Q+1}^{D'} (1+r)^{i\gamma/(1-\gamma)}(1+\delta)^{-i/(1-\gamma)}(1-p_i)^{1/(1-\gamma)}}{\sum_{i=0}^{D'} (1+r)^{i\gamma/(1-\gamma)}(1+\delta)^{-i/(1-\gamma)}(1-p_i)^{1/(1-\gamma)}}\right],$$

which is zero for individuals who "know" that they will die prior to retirement.

[10]Uncertainty over future social security benefits would mitigate the effect shown here. Watson (1982) discusses the influence of uncertainty over benefits in assessing the impact of social security on saving. Merton, Bodie, and Marcus (1984) show that many private pension integration arrangements remove much of this uncertainty.

[11]A retirement age of 65 was assumed. Probabilities for survival were taken from Faber (1982).

[12]This effect is most pronounced in the absence of explicit capital market restrictions. With no initial endowment (and, hence, binding restrictions on the nomarketability of social security when young), relative impacts on "working-period" and "retirement-period" consumption will depend on the relationship of the individual's actual and optimal tax rate (participation). The importance of (accidental) bequests as intergenerational links will be discussed later.

[13]This nonlinearity has surfaced in some recent studies of the impact of social security on saving. See for example Diamond and Hausman (1982) and Hubbard (1983).

[14]For a more complete discussion of the implications of the choice of parameter values, see Levhari and Mirman (1977) or Davies (1981).

[15]As in Table 1, survival probabilities are taken from Faber (1982).

[16]Note that if participation in social security is rationed by income, low-income individuals have more of their retirement dissaving in the form of reduction in the value of their social security annuity than do high-income individuals. This analysis assumes that annuity and non-annuity holdings are perfect substitutes in dissaving. The studies cited in the beginning of the paper have found good but not perfect substitutability of social security for non-pension wealth in accumulation. Empirical evidence in Hubbard (1983) suggests that the substitutability is greatest for high-income individuals.

[17]In a world with capital market restrictions, then, a social security system of this type may increase saving, since received initial bequests are more liquid than anticipated social security benefits. The impact of social security on intergenerational transfers is an important component of the system's net effect on individual saving.

[18]The implicit assumption, of course, is that the parent dies at the beginning of the child's (optimizing) life, age twenty here. This assumption is made to highlight the point that the existence of social security for the previous generation mitigates the impact of the present generation's participation in social security on its own wealth accumulation. More general assumptions about the timing of a testator's death would complicate expressions like (14) in the text, but the qualitative point would remain.

[19]This damping through intergenerational transfers of the impact of social security on wealth accumulation is mitigated if "children" earn more on average than their "parents" (because of productivity growth).

[20]The consumption of individuals of each age can be calculated from equations (17) and (18), given the initial wage. The growth rate of the population will determine the relative number of persons at each age. Aggregate consumption can be calculated by summing consumption over ages, weighted by the relative population size.

[21]Kotlikoff's (1979a) analysis also incorporates the influence of social security on retirement age, which is taken as exogenous here. To the extent that social security lowers the desired retirement age, the partial equilibrium wealth replacement effect of social security on saving is dampened.

[22]The idea here is that an individual who thinks he will live a long time would buy several small annuities rather than one large one in order to misrepresent his assessment of his longevity. Companies know his participation in social security, but not the extent to which he has obtained insurance from other private sources. Pauly (1974) and Wilson (1977) discuss certain situations in which market equilibria might occur after a compulsory insurance program is imposed.

[23]Lazear (1983) has focused particularly on this point, emphasizing the role of pensions in influencing turnover, retirement, and investment in human capital. Many arguments for the existence of private pensions have emphasized their favorable federal tax treatment. Tax treatment cannot be the complete explanation, since "defined contribution" plans would dominate. "Defined benefit" plans are instead prevalent. Munnell (1982) emphasizes both the tax benefits (to employers and to employees) and the inadequacy of social security in explaining the growth of private pension plans.

[24]This ignores the possibility that firms may be willing to offer "more-than-fair" plans to achieve some other impact on worker behavior. See Lazear (1983).

[25]This is just the characteristic of "integration" of the benefits of social security and private pension annuities. Since the passage of the Revenue Act of 1942, Congress has allowed public (social security) and private benefits to be considered together in determining whether a private plan discriminates in favor of low-income workers. For descriptions of typical integration provisions and discussions of their prevalence in the U.S. pension system, see Munnell (1982) and Kotlikoff and Smith (1983).

[26]Note that empirical evidence of saving rates increasing with income does not validate the hypothesis the bequests are a luxury good (even if data on bequests are known), because of, among other things, rationing of the purchase of pension annuities by income.

[27] Such a bequest motive is usually grounded in work in the human capital literature (see for example Becker and Tomes, 1976, 1979). That is, if human capital investment initially yield a higher rate of return than that on financial assets, parents who "care" about their children invest first in human capital up to the level at which the returns to additional investment just equal the market return. Further transfers are exclusively financial. Hence observed (financial) bequests will be higher for children whose parents had significant resources than for children with access to low parental resources. Despite serious data limitations, there have been some recent efforts to estimate the relationship between bequests and lifetime resources. The finding that the ratio of bequests to earnings rises with the level of earnings is corroborated in the careful empirical study of Menchik and David (1983).

[28] The problem of isolating a relationship between wealth (or bequests) and lifetime resources if further complicated by the fact that price effects may be present as well (e.g., a correlation between earnings and after-tax financial returns). Government retirement saving policy can bring about those price effects--tax-favored treatment of IRAs and Keogh plans, for example (see Hubbard, 1984). To the extent that changes in government pension policy involve tradeoffs among policy options (e.g., liberalized ceilings on tax-deductible IRA or Keogh contributions in exchange for a reduction in social secuity benefits), the stability of any observed relationship between wealth and earnings is all the more tenuous.

REFERENCES

Abel, A. B. 1983. Bequests with Uncertain Lifetimes. Mimeograph, Harvard University.

Barro, R. J. 1974. Are Government Bonds Net Wealth? Journal of Political Economy 82: 1095-1117.

Barro, R. J. 1978. The Impact of Social Security on Private Savings: Evidence from the U.S. Time Series. Washington, D.C.: American Enterprise Institute for Public Policy Research.

Barro, R. J. and Friedman, J. W. 1977. On Uncertain Lifetimes. Journal of Political Economy 85: 843-849.

Becker, G. S. and Tomes, N. 1976. Child Endowments and the Quantity and Quality of Children. Journal of Poltical Economy 84, part 2: S143-S162.

____________________. 1979. An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility. Journal of Political Economy 87: 1153-1189.

Blinder, A. S., Gordon, R. H., and Wise, D. E. 1981. Social Security, Bequests, and the Life Cycle Theory of Saving: Cross-Sectional Tests. Working Paper No. 619, National Bureau of Economic Research, Inc. January.

Cagan, P. 1965. The Effect of Pension Plans on Aggregate Saving: Evidence from a Sample Survey. Occasional Paper 95, Columbia University Press (for the National Bureau of Economic Research).

Champernowne, D.G. 1969. Uncertainty and Estimation in Economics. San Francisco: Holden Day.

Davies, J. B. 1981. Uncertain Lifetime, Consumption and Dissaving in Retirement. Journal of Political Economy 89: 561-578.

Diamond, P. A. 1977. A Framework for Social Security Analysis. Journal of Public Economics 8: 275-298.

Diamond, P. A. and Hausman, J. A. 1982. Individual Retirement and Savings Behavior. Mimeograph, Massachusetts Institute of Technology, August 1982.

Eckstein, Z., Eichenbaum, M. and Peled, D. 1983. Uncertain Lifetimes and Welfare Enhancing Properties of Annuity Markets and Social Security. Mimeograph, Carnegie-Mellon University, March.

Faber, J. F. 1982. Life Tables for the United States: 1900-2050. U.S. Department of Health and Human Services, Social Security Administration, Actuarial Study No. 87 September.

Farber, H. S. 1978. Individual Preferences and Union Wage Determination: The Case of the United Mine Workers. Journal of Political Economy 86: 923-42.

Feldstein, M. S. 1974. Social Security, Induced Retirement and Aggregate Capital Accumulation. Journal of Political Economy 82: 905-926.

____________. 1982. Social Security and Private Saving: Reply. Journal of Political Economy 90: 630-641.

Feldstein, M. S. and Pellechio, A. 1979. Social Security and Household Wealth Accumulation: New Microeconomic Evidence. Review of Economics and Statistics 61: 361-368.

Friend, I. and Blume, M. E. 1975. The Demand for Risky Assets. American Economic Review 65: 900-922.

Hubbard, R. G. 1984. Do IRAs and Keoghs Increase Saving? National Tax Journal 37: 43-54.

____________ 1983. The Financial Impacts of Social Security: A Study of Effects on Household Wealth Accumulation and Allocation. Monograph 1983-3, Monograph Series in Finance and Economics, Salomon Brothers Center for the Study of Financial Institutions, New York University, 1983.

Hurd, M. and Shoven, J. B. 1983. The Distributional Impact of Social Security. Working Paper No. 1155, National Bureau of Economic Research, June.

King, M. A. and Dicks-Mireaux, L. 1982. Asset Holdings and the Life Cycle. Economic Journal 92: 247-267.

Kotlikoff, L. J. 1979a. Social Security and Equilibrium Capital Intensity. Quarterly Journal of Economics 94: 233-253.

Kotlikoff, L. J. 1979b. Testing the Theory of Social Security and Life Cycle Accumulation. American Economic Review 69: 396-410.

Kotlikoff, L. J. and Smith, D. E. 1983. Pensions in the American Economy. Chicago: University of Chicago Press.

Kotlikoff, L. J. and Spivak, A. 1981. The Family as an Incomplete Annuities Market. Journal of Political Economy 89: 371-391.

Kotlikoff, L. J. and Summers, L. H. 1981. The Role of Intergenerational Transfers in Aggregate Capital Accumulation. Journal of Political Economy 90: 706-732.

Kurz, M. 1981. The Life-Cycle Hypothesis and the Effects of Social Security and Private Pensions on Family Savings. Technical Report No. 335 (Revised), Institute for Mathematical Studies in the Social Sciences, Stanford University, December.

Lazear, E. P. 1983. Incentive Effects of Pensions. Working Paper No. 1126, National Bureau of Economic Research, Inc., May.

Leimer, D. R. and Lesnoy, S. D. 1982. Social Security and Private Saving: New Time-Series Evidence. Journal of Political Economy 90: 606-629.

Levhari, D. and Mirman, L. 1977. Savings and Consumption with an Uncertain Horizon. Journal of Political Economy 85: 265-281.

Menchik, P. L. and David, M. 1983. Income Distribution, Lifetime Savings, and Bequests. American Economic Review 73: 672-690.

Merton, R. C. 1983. On the Role of Social Security as a Means for Efficient Risk-Bearing in an Economy Where Human Capital Is Not Tradeable. In Z. Bodie and J. B. Shoven (eds.), Financial Aspects of the U. S. Pension System. Chicago: University of Chicago Press, 1983

Merton, R. C., Bodie, Z., and Marcus, A. 1984. "Pension Plan Integration as Insurance Against Social Security Risk. Paper presented to the NBER Conference on Pensions, San Diego, April 1984.

Mirer, T. W. 1979. The Wealth-Age Relationship Among the Aged. American Economic Review 69: 435-443.

Modigliani, F. and Ando, A. 1957. Tests of the Life-Cycle Hypothesis of Savings. Bulletin of the Oxford Institute of Economics and Statistics 19: 99-124.

Modigliani, F. and Brumberg, R. 1954. Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data. In K. Kurihara (ed.), Post-Keynesian Economics. New Brunswick: Rutgers University Press.

Munnell, A. 1982. The Economics of Private Pensions. Washington, D. C.: Brookings Institution.

_________________. 1974. The Effects of Social Security on Personal Saving. Cambridge: Ballinger Press, 1974.

Pauly, M. V. 1974. Overinsurance and Public Provision of Insurance: The Role of Moral Hazard and Adverse Selection. Quarterly Journal of Economics 88: 44-54.

Riley, J. G. 1979. Informational Equilibrium. Econometrica 47: 331-359.

Rothschild, M. and Stiglitz, J. E. 1976. Equilibrium in Competitive Insurance Markets: An Essay on the Economics of Imperfect Information. Quarterly Journal of Economics 90: 629-650.

Sheshinski, E. and Weiss, Y. 1981. Uncertainty and Optimal Social Security Systems. Quarterly Journal of Economics 96: (May 1981): 189-206.

Watson, H. 1982. Saving, Social Security, and Uncertainty. Southern Economic Journal 49: 330-341.

White, B. B. 1978. Empirical Tests of the Life-Cycle Hypothesis. American Economic Review 68: 547-560.

Wilson, C. 1977. A Model of Insurance Markets with Incomplete Information. Journal of Economic Theory 16: 167-207.

Yaari, M. E. 1965. Uncertain Lifetime, Life Insurance, and the Theory of the Consumer. Review of Economic Studies 32: 137-158.